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Nonlinear Seismic Response of Dams Using a Coupled Boundary Element - Finite Element Formulation

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE
Doctor of Philosophy

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Houston, Texas
April, 1996
NONLINEAR SEISMIC RESPONSE OF DAMS USING A COUPLED
BOUNDARY ELEMENT - FINITE ELEMENT FORMULATION

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Abstract

A study of the effects of dam-foundation interaction on the response of earth, rockfill and concrete-faced rockfill dams to obliquely incident P, SV and Rayleigh waves is presented. Emphasis is placed on the effects of the foundation flexibility, the spatial variability of the ground motion and the material nonlinearity. The study is based on a rigorous hybrid numerical formulation that combines the efficiency and versatility of the Finite Element Method (FEM) and the ability of Boundary Element Method (BEM) to account for the radiation conditions at the far field. The developed hybrid method is very powerful, and can be used efficiently to obtain accurate solutions of problems of complex geometry, material heterogeneity and, for time-domain analysis, material nonlinearity. The 2-D frequency-domain formulation is used at first to investigate the linear response of
earth and rockfill dams to incident P, SV and Rayleigh waves and the response of concrete-faced rockfill dams to incident Rayleigh waves. Furthermore, the nonlinear time-domain coupled BE-FE formulation is used to investigate the response of earth and rockfill dams to vertically incident SV waves. By accounting rigorously for the energy radiated back into the halfspace, the study demonstrates the dramatic effect of the flexibility of the foundation rock in reducing the overall response of the dam. The effects of the spatial variability of the ground motion across the width of the dam are also shown to be important, but less dramatic than those of the foundation flexibility. Finally, the results from the nonlinear analysis of two different dams, each experiencing various degrees of nonlinearity, have demonstrated the great importance of the material nonlinear behavior on the response of dams subjected to strong ground motion.
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To my Mother

(1943-1994)
Chapter 1

Introduction

1.1 Motivation

Critical civil infrastructure systems, such as earth dams, nuclear and toxic waste containment systems, highway/railway bridges and tunnels, etc., built in seismic regions, must be designed safely and economically to withstand potentially strong earthquakes. Extensive earthquake damage or failure of such critical systems may result into substantial loss of life and property or cause serious environmental problems. A realistic assessment of the behavior of such systems to strong ground shaking depends significantly on the proper consideration of a variety of factors, including the nature of expected seismic waves, site effects, the dynamic soil-structure interaction, accurate measurement of the material properties, and the nonlinear behavior of the materials and soil-structure interfaces.

Refined seismic evaluation of such critical structures is carried out currently in practice by using rigorous methods of analysis. Due to inherent limitations, such methods focus on expressing the effects of selected important factors only, while they may either ignore or consider inadequately other essential factors controlling the response. More
specifically, theoretical methods may lead to powerful rigorous solutions for rather simple problems of linear elastodynamics, but they are impractical or impossible for problems of complex geometry, material inhomogeneity and nonlinear behavior. The standard numerical method for treating the latter problems is the Finite Element Method (FEM), which allows significant flexibility for problems of finite domain but it is not perfectly suited for dealing accurately and efficiently with infinite domains. On the other hand, the Boundary Element Method (BEM) is undoubtedly superior in handling infinite domain problems, but it can only treat efficiently linear elastic problems. Hence, although each one of the above methods can treat accurately and efficiently certain aspects of the dynamic soil-structure interaction problem, their inability to capture adequately the effects of other important factors also controlling the response may lead to significant uncertainty regarding the validity of the results and the overall assessment of safety.

Earth dams are exceptionally massive structures that consist of various types of materials, and may be placed on sites of poor soil conditions. Under strong ground shaking, the dam and foundation soils may behave in a nonlinear inelastic way. A refined analysis of the seismic behavior of earth dams is currently performed by using a finite-element effective-stress analysis in which the geometry, the inhomogeneity and the nonlinear behavior of soil are considered adequately, the energy radiated back into the canyon may be approximated by simple absorbing boundaries, whereas the nature of the associated wave propagation phenomena, the true soil-structure interaction, and any nonlinearity of the material near the dam body are ignored. A reliable dynamic model for earth dams should account for the presence of the flexible halfspace accurately and
efficiently, as well as for the nonlinear hysteretic behavior of soils. It should also be capable to account for the spatial variability of ground motion along the dam base.

In the current study, a general formulation for the soil-structure interaction problem is presented, making use of two well established methods, the Finite Element Method, and the Boundary Element Method. The formulation gives the flexibility of using finite elements along with the rigorous treatment of the radiation of waves into the halfspace provided by the boundary elements. The formulation is applied to the important problem of dam-foundation interaction, solving for a more realistic dynamic response in both frequency and time domain. It is used to study the important factors affecting the dynamic behavior of earth/rockfill and concrete faced dams. A parametric study is performed to elucidate the relative importance of such factors.

1.2 Background

1.2.1 Dynamic Response of Dams

The seismic response of earth and rockfill dams has been the subject of considerable research in the last twenty years with primary focus on the effects of such factors as the dam and canyon geometries, the inhomogeneity of the dam material, the nonlinear and inelastic behavior of the material, the relative flexibility of the dam and the canyon or foundation materials, and, to only a limited extent, the spatial variability of the excitation. In the following a brief summary of representative past contributions is presented. Detailed accounts of past contributions are available in two state-of-the-art reports by Gazetas (1987) and Gazetas and Dakoulas (1992).
1. Canyon Geometry

Canyon geometry effect on dams dynamic response was first addressed by Hatanaka and Ambraseys in the 50's, extending the shear wedge concept to account for the rigidity of a rectangular canyon. Since then, a lot of work has been done to investigate further the importance of this factor. Martinez and Bielak (1980) developed a numerical procedure for the 3-Dimensional analysis of earth dams. By assuming symmetry, neglecting the longitudinal deformation, discretizing only the dam midsection into finite elements, and expanding the longitudinal displacements and inertial forces in Fourier series, they reduced the problem to solving a finite number of uncoupled 2-Dimensional finite element problems. Ohmachi (1981) and Ohmachi and Tokimatsu (1982) presented an approximate solution in which they neglected both horizontal and vertical displacements. The formulation, however, does not assume model symmetry. The dam model is divided into trapezoidal shaped super-elements, and the displacement distribution is taken to be on the form of the shear-beam modal shapes. This implies that the displacement distribution along the depth of the dam is not affected by the existence on the canyon. Although this simplification gives reasonable results for low frequencies (first 2 or 3 modes), it fails in the higher frequency ranges of excitation.

Abdel-Ghaffar and Koh (1981) introduced a semi-analytical solution to deal with dams in arbitrary canyon geometry, provided that they are symmetric. The solution is based on a Rayleigh-Ritz method and using simple sinusoids, or even shear-beam mode shapes as basis functions. This work was extended by Elgamal et al. (1984) to evaluate the nonlinear inelastic response of dams in non rectangular canyons.
By changing the 2-Dimensional isoparametric elements with 3-Dimensional prismatic elements in the finite element program LUSH, Makdisi et al. (1982) introduced a 3-Dimensional formulation, in which he neglected the longitudinal displacements and assumed only shear waves can travel across the dam body and into the canyon. The method was used to obtain the response of homogeneous dams in triangular canyons. Subsequent advancements (Mejia et al. 1982; Mejia and Seed 1983) eliminated the restricting assumption of neglecting the longitudinal deformations.


The results of the aforementioned studies demonstrated that the dynamic response characteristics and response of dams with length to height ratios, \( L/H \) of the order of 10 or more are practically identical to those obtained under plane-strain conditions (\( L/H = \infty \)). Even for \( L/H \) ratios as low as 6, the plane-strain assumption represents a convenient and
reasonable approximation (Makdisi et al. 1982). However, for dams in narrow canyons, the stiffening effect of the canyon abutments increases the natural frequencies of the dam and leads to modal displacement patterns that attenuate more sharply with depth than those obtained under plane-strain conditions. Furthermore, the acceleration amplification at first resonance, which is typically of the order of 8 for plane-strain conditions and 10% material damping, becomes sensitive to canyon geometry. This is particularly true for the amplifications associated with the higher modes of vibration.

2. Material Inhomogeneity

One of the most common assumptions made to simplify the dynamic dam response problem is material homogeneity. Laboratory measurements show the dependence of soil properties, shear modulus in particular, on the confining pressure $\sigma_0$ (Ghaboussi and Wilson 1973). The confining pressure increases with distance from the dam crest and side slopes. Based on published distributions of static confining stress distributions in an idealized dam section, Gazetas (1981, 1982) recommended that the average shear modulus $G$ across the dam width be taken proportional to $z^{2/3}$ where $z$ is the depth measured from the crest. Accordingly, Gazetas (1982) extended the shear-beam model to take into effect such material variation with depth. By considering the stiffness variation with depth one can explain the sharp attenuation with depth of the displacements in the fundamental mode recorded in Sannokai and Bouquet dams. Although, this factor has a significant effect on the displacement mode shape of the dam, it has no effect on its fundamental frequency (Gazetas 1982).

A more complete study was presented by Dakoulas and Gazetas (1985, 1986) further
explaining the material stiffness changes with depth, and its effects on the seismic response of earth dams. Closed-form expressions were derived for the natural frequencies, mode shapes, steady-state transfer functions and participation factors for a dam modeled as a triangular inhomogeneous shear-beam with an arbitrary value of homogeneity factor m (Dakoulas 1985; Dakoulas and Gazetas 1985). A parametric study showed that the increase of the inhomogeneity factor m leads to sharper attenuation of mode displacements with depth, almost identical fundamental frequencies, closer-spaced higher natural frequencies, higher displacement and acceleration amplification factors, greater relative importance for higher modes of vibration, and the occurrence of maximum values of shear-strain mode shapes closer to the dam crest.

3. Material Nonlinearity

Material nonlinearity is a significant and, in many cases, a crucial factor that must be taken in consideration in the process of analyzing and designing earth dams to sustain strong ground motion. Soil is a material that, under large strains, exhibits a very complex and highly nonlinear behavior, which may affect substantially the response and behavior of the entire structure. The first attempt to study the effects of material nonlinearity on the dynamic response of dams was conducted by Idriss et al. (1973). They suggested an equivalent linear scheme in which the soil stiffness is corrected according to the level of strains the soil element experiences in an iterative manner. This scheme represents a crude approximation of the nonlinear hysteretic behavior of soils, and was incorporated into established methods of linear seismic response analysis. The scheme has serious disadvantages, such as the tendency for spurious response to develop, hence exaggerating
the overall response (Gazetas 1987). Another limitation for such scheme is its inability to provide information on permanent deformations, because of the linear nature of the analysis.

Dakoulas and Gazetas (1988) introduced a simplified nonlinear method, which allows for the updating of the shear modulus $G$ and the damping factor at various time intervals, depending on the root mean square of the shear strains during the same interval. In this method, the updating of the soil parameters occurs during every time step during the analysis in comparison to the one time post-updating in the equivalent linear analysis. This allows for better handling of the nonlinear material properties, with a significant computational cost savings. It also decreases the high frequency filtration which occurs in the case of equivalent linear analysis.

Prevost et al. (1985 a,b) suggested a more rigorous, but still simplified method to take into account the nonlinear inelastic behavior of soils. The hysteretic Galerkin formulation can handle 1,2 and 3-Dimensional structures, expanding the solution by using basis functions defined over the whole dam. The eigenmodes of the linearized homogeneous dam are thought to be a convenient and efficient basis function. The nonlinear material behavior is handled by using elastic-plastic constitutive stress-strain relationship based on multi-surface kinematic plasticity theory. The resulting system is solved through a step-by-step integration using Newmark's algorithm and Newton Raphson iterations.

Recently, Elgamal (1991) and Yangos and Provost (1991) extended the method further to take into account soil degradation and porewater pressure buildup. The soil is considered to be a two-phase poro-elastoplastic medium with fully coupled soil skeleton
and porewater pressure developing in the saturated portion of the dam.

The layered inelastic shear beam is another attempt to combine the simplicity and efficiency of the shear beam model and the versatility of plain-strain finite elements in handling zones and elements of different material properties (Stara-Gazetas 1986). The method suggests performing a nonlinear incremental finite element analysis to derive a monotonic constitutive relations for a layer of horizontal elements joined in a super-element configuration. The dam is then modeled as a 1-Dimensional shear beam, with each layer having a cyclic stress-strain relationship estimated from the monotonic constitutive relations obtained in the static analysis and extended for cyclic loading using the masing criterion. A similar method was reported by Papadakis and Wylie (1975) using the method of characteristics.

The method has proven to be accurate and cost-effective and lead to reasonable results for dams subjected to strong earth shaking. However, it does not give detailed information about the stress and strain distribution across the body of the dam.

Finn et al. (1986, 1988) proposed an approximate 2-Dimensional nonlinear effective-stress analysis. The author extended the 1-Dimensional hyperbolic model of cyclic behavior in simple shear to approximately account for the nonlinear inelastic soil behavior in two dimensions. The model can handle transient and residual porewater pressure generated and diffused during the shaking, as well as volumetric compaction due to shear. The method has been tested against published data for dams and embankments, as well as centrifuge data, and proved to be an efficient method with good engineering accuracy and cost-effective computational wise.
Finally, a number of rigorous plasticity based finite element and finite difference methods have been investigated (see Kawai 1985, Prevost et al. 1985, Yiagos and Provost 1991 and Zienkiewicz et al. 1980). They can incorporate multi-directional seismic excitations, multi-surface kinematic plasticity models with massing-style criteria. The computational cost needed to achieve such results, however, is very high, and in a lot of cases prohibitive.

4. Spatial Variability of Ground Motion

Previous numerical and theoretical studies on the effects of the spatial variability of the ground motion were limited by restricting simplifying assumptions. Early finite element studies (Chopra et al. 1969 and Dibaj and Penzien 1969) investigated the lateral response of long dams subjected to waves traveling across the dam width, but with no account of the effects of the dam-foundation interaction. These studies revealed that the response of a dam to a spatially variable motion may be substantially higher than that obtained for a space-invariant motion. Some results on the combined effects of wave passage and dam-canyon interaction have been presented using the Boundary Integral Equation Method (Nahhas 1987), but they are based on substantial idealization/simplification of the dam-canyon geometry. Recently, Dakoulas and his co-workers (Dakoulas and Hsu 1995, Dakoulas 1993a, 1993b and Dakoulas and Hashmi 1992) presented a series of closed-form solutions using generalized shear beam models for dams built in canyons of semicircular, semielliptical and rectangular shapes and excited by obliquely incident SH waves propagating along the longitudinal direction of the dam. These solutions account either rigorously (Dakoulas and Hsu 1995, Dakoulas 1993a and
effects, as well as for the wave scattering and diffraction phenomena associated with the presence of the dam-filled canyon. The results demonstrate that obliquely incident waves impinging on the dam-canyon interface induce new vibrational modes in addition to those excited by uniform synchronous base motion. (For the symmetric dams and the waves examined, the additional modes are anti-symmetric). Even at small incidence angles, the excitation of these modes may lead to substantially higher response in the dam than that caused by vertically propagating waves.

5. Flexibility of the Canyon or Foundation Material

For dams built in flexible canyons, dam-canyon interaction phenomena may affect significantly the response. The importance of such interaction depends on the impedance ratio, \( IR = \frac{V_c \rho_c}{V_d \rho_d} \), where \( V_c, V_d \) are the S-wave velocities and \( \rho_c, \rho_d \) are the mass densities of the canyon rock and dam soil, respectively. For an infinitely long dam on a flexible foundation subjected to vertically propagating SH waves, the impedance ratio determines the amount of energy radiated back into the canyon (radiation damping). The latter may reduce the amplification of ground acceleration by a factor of 2 or more, compared to that for rigid base, depending on the exact value of IR and the frequency (Dakoulas 1985). The effect of the impedance ratio on the amplification is found to be more pronounced for dams in narrow canyons than for plane-strain dams. Results for dams in canyons with rectangular, semi-circular and semi-elliptical shapes indicate consistently a dramatic effect of the impedance ratio on the response for the entire frequency range, and even more intensely for the high-frequency excitation, with reduction of the amplification by a factor of 2 or 3, compared to that for rigid base
(Dakoulas 1993a,b; Dakoulas et al. 1991, 1992 and Dakoulas and Hsu 1995).

Given the importance of the base flexibility and the spatial variability of the ground motion, as clearly demonstrated in the aforementioned studies, it is of interest to use a rigorous numerical formulation to conduct a comprehensive study of the effects of SV, P and Rayleigh waves travelling across the width and along the length of the dam.

1.2.2 Numerical Methods

Numerical methods are much more versatile than theoretical methods of analysis and offer significant advantages. Although they appear in many types, fundamentally, they consist of two main categories, namely the domain-type methods and the boundary type methods. The use of any of the above methods has to be based on the particular nature of the physical problem.

While the domain-type methods are capable of dealing with complex geometries as well as inhomogeneous and nonlinear material behavior, they fail to deal efficiently and accurately with problems having infinite domains, because of the wave reflection at the artificial boundaries of the domain. Different formulations and new element types were devised to make the method capable of handling such problems, such as the use of transmitting boundaries by Liao and Wong (1984), Kausel (1988), and others. A comparison of different types of transmitting boundaries can be found in Kausel and Tassoulas (1981). The use of infinite elements is an alternative approach to handle infinite domains. The work by Zhao and Valliappan (1993) and Zhao et al. (1992) is a good example of such techniques. Despite the trials to overcome the wave reflection problem from the boundaries, this is only achievable for special cases of wave incidences, and
requires considering large domains and in some cases artificial damping to get acceptable results.

On the contrary, the boundary-type methods are undoubtedly superior in handling problems involving infinite domains in both frequency and time domain. A comprehensive review of applications of the boundary element method (BEM) in dynamic analysis can be found in Beskos (1987). Despite the apparent advantages of the BEM, a significant limitation is that it can only treat linear elastic problems, without enlarging the computational effort significantly by using the sub-domain techniques (Ahmad and Banerjee 1988, Henry and Banerjee 1988).

A combination of both domain and boundary type numerical methods can eliminate the limitations of both, while exploiting there main advantages. Formulation and applications of the coupling technique were initially considered in frequency domain. Among others, Kobayashi and Mori (1986) and Wang (1992) have employed a combination of the FEM and the BEM to solve some generic three dimensional soil-structure interaction problems. Auersch and Schmid (1990) solved a two dimensional wave field excited soil-structure interaction problem through coupling the FEM with the indirect BEM. Bielak et al. (1991) applied this technique to the problem of soil amplification in inhomogeneous alluvial valleys due to incident SH waves. Zhang et al. (1992) added infinite boundary elements to the coupled FE-BE and solved for the dynamic interaction between alluvial soil and rock canyons.

Transient soil-structure interaction problems were also treated using the same coupling technique in the time domain.Spyrakos and Beskos (1986) investigated flexible strip
foundations subjected to external dynamic force as well as seismic wave input, while Karabalis and Beskos (1985) examined three dimensional flexible foundations in time domain. Von Estorff and Prabucki (1990) presented complete rigorous formulation of the coupling procedure, and applied it on a force excited layered halfspace, and a two dimensional trench subjected to wave excitations. Von Estorff and Antes (1991) presented similar formulation for the fluid-structure interaction problem. Von Estorff and Kausel (1989) demonstrated the applicability of the coupling technique in various soil-structure interaction problems such as flexible foundations, open and filled trenches and tunnels.

Although all the previous work was performed with the assumption of linear elastic material behavior, the extension of the coupling method to handle nonlinear material behavior is possible (Von Estorff and Antes 1991, Abouseeda and Dakoulas 1996).

1.3 Objectives

The goal of this study is to develop a rigorous, versatile and efficient method for linear and nonlinear dynamic analysis of soil-structure systems, and use it to study selected important factors affecting the seismic response of earth/rockfill dams. The factors to be studied are:

a- The effect of spacial variability of the ground motion.

b- The effect of relative stiffness of the dam soil and the underlying material.

c- The effect of soil nonlinear inelastic behavior.

For this purpose, powerful numerical tools are developed in both frequency and time domain for dynamic response of generic two dimensional soil-structure interaction problems. These tools are capable of treating the halfspace accurately and efficiently,
considering spatially variable wave field excitation, and taking into account the nonlinear hysteretic behavior of soil in the near field.

1.4 Scope of Work

The proposed work is focused on developing numerical tools that can furnish better understanding of the dynamic behavior of earth dams. The dam in this study is approximated by considering it as a two-dimensional structure. This assumption is acceptable for dams located in wide valleys, with length to height ratio larger or equal to 5. Although extension of the model for three dimensional problems is straightforward, this will be left for future research.

A frequency domain formulation of the hybrid BE-FE methods is introduced. The boundary element formulation makes use of the full-space steady-state Green's functions to simulate the halfspace part of the model. These functions inherently satisfy the radiation conditions at infinity, thus eliminating the problems of wave reflections from the boundaries of the model. A standard steady-state finite element formulation is also used to simulate the dam body. The two parts are then added together to obtain the final solution. The model is verified against existing solutions for the dynamic stiffness of strip footings (Gazetas 1983).

A parametric study is performed to establish a better understanding of the effects of spatially variable excitation on the dynamic response of earth, rockfill and concrete-faced rockfill dams. The response is evaluated at different positions within the dam body for P and SV waves incident of different angles and for Rayleigh waves. The effect of the relative stiffness of the halfspace and the dam material is also investigated.
For nonlinear dynamic soil-structure interaction problems, a time domain formulation is presented. The formulation in this case is used to compute the transient dynamic response of earth dams. The full-space transient Green’s functions is used in a time domain boundary integral formulation to simulate the halfspace region of the model. The dam body is simulated by using a Galerkin finite element formulation. The Newmark beta scheme is used to handle nonlinear inelastic soil model, which initially assumes constant elastic parameters for the soil within each time step, and change it by the end of each time step, using the strains computed during that step. Both FE and BE regions are combined afterwards, resulting in a system of linear equations, which is solved for the total response of the structure.

The time domain model can be considered as a versatile tool in computing realistically the seismic effects on earth dams. A real dam subjected to an actual earthquake record is investigated using the model. The different aspects of dynamic response are discussed, showing the effects of considering the non-linear hysteretic behavior of soil.

1.5 Outline of the Thesis

Chapter 2 of this dissertation addresses the general formulation of the combined FE-BE methods. A brief description of the problem, the governing equations and the dynamic reciprocity theorem are presented, followed by the derivation of the basic Finite Element and Boundary Element equations for both steady-state and transient cases, along with the full-space fundamental solutions for the two-dimensional elastic problem in both frequency and time domain. The combined FE-BE formulation is derived, and the nonlinear model and the wave input method are also discussed.
A study of the effects of dam-foundation interaction on the response of earth dams to obliquely incident P and SV waves is presented in Chapter 3. Emphasis is placed on the effects of the foundation flexibility and the spatial variability of the ground motion. The study is based on the rigorous hybrid numerical formulation presented in Chapter 2. The 2-D frequency-domain formulation is used here to investigate the response of infinitely long earth dams to obliquely incident P and SV waves. By accounting rigorously for the energy radiated back into the halfspace, the study demonstrates the dramatic effect of the flexibility of the foundation rock in reducing the overall response of the dam. The effects of the spatial variability of the ground motion for P and SV waves travelling across the width of the dam are also important, but somewhat less dramatic than those of the foundation flexibility.

The effects of Rayleigh waves on the response of earth dams are examined in Chapter 4, with emphasis on the importance of the foundation flexibility and the spatial variability of the ground motion. The 2-D formulation is used in the frequency domain to investigate the response of infinitely long earth and rockfill dams subjected to Rayleigh waves travelling across the dam width. The study demonstrates the dramatic effect of the flexibility of the foundation rock in reducing the overall response of the dam. It also shows that the spatial variability of the vertical component of the ground motion caused by the Rayleigh waves induces additional rocking motion that contributes significantly to increasing the horizontal response especially in the upper part of the dam body, while reducing the vertical vibration.

In Chapter 5, the 2-D formulation explained in Chapter 2 is used in the frequency
domain to investigate the response of infinitely long Concrete-Faced/Rockfill (CFR) dams subjected to Rayleigh waves travelling across the dam width. The study demonstrates the dramatic effect of the flexibility of the foundation rock in reducing the overall response of the dam. It also confirms the conclusions of Chapter 4 regarding the spatial variability of the ground motion caused by the Rayleigh waves. It also demonstrates the significant effects of the concrete slab on the dynamic response characteristics of the dam.

In Chapter 6, the general transient hybrid boundary-finite element formulation presented in Chapter 2 is applied to study the effects of the foundation flexibility and the material nonlinearity on the response of earth dams. The 2-D formulation is used in the time domain to investigate the response of infinitely long earth dams subjected to a vertically propagating SV wave front. The simple Ramberg-Osgood model is used to represent the material nonlinearity occurring in the dam body. The study shows the significant effect of the flexibility of the foundation rock on dissipating the earthquake energy through radiation, hence lessening the nonlinearity effects all over the dam, and resulting in higher values of amplification. It also shows the substantial increase in the hysteretic damping with increasing levels of strain, and it’s impact on decreasing the overall response of the dam.

1.6 Notations

\[ u_i \] Displacement vector
\[ \dot{u} \] First derivative of displacement (velocity)
\[ \ddot{u} \] Second derivative of displacement (acceleration)
\[ c_1, C_p \] Dilatational, irrotational, or P wave velocity
$c_2, C_{sv}$  Shear, equivoluminal, distortional, or S wave velocity

$b_i$  Body forces vector

$x_i$  Directional vector

$\lambda, \mu$  Lamé elastic constants

$E$  Modulus of elasticity

$\nu$  Poisson’s ratio

$\rho$  Mass density

$\omega$  Circular frequency

$t$  Time

$i$  Imaginary unit ($\sqrt{-1}$)

$\beta$  Linear hysteretic damping ratio

$\sigma_{ij}$  Stress tensor

$e_{ij}, \{\varepsilon\}$  Strain tensor

$[M]$  Global mass matrix

$[K]$  Global stiffness matrix

$[C]$  Rayleigh damping matrix

$\{f\}$  Force vector

$[A]$  Equivalent stiffness matrix

$G_{ij}$  Displacement Green’s function

$F_{ij}$  Traction Green’s function

$c_{ij}$  Boundary element constant

$K_0, K_1, K_1$  Modified Bessel functions of the second kind

$[F]$  Tractions coefficient matrix

$[G]$  Displacements coefficient matrix

$t_i$  Tractions vector

$[T]$  Finite element-boundary element transformation matrix

$[A]$  Finite element-boundary element combined stiffness matrix

$\gamma_p$  P wave number
\( \gamma_{sv} \)  
SV wave number

\( k_r \)  
Rayleigh wave number

\( \{\sigma\}^e \)  
Generalized stress vector acting on the wave input boundary \( \Gamma \)

\( \{P\Gamma\}^e \)  
Generalized force vector acting on the wave input boundary \( \Gamma \)

\( [D] \)  
Elastic properties matrix (finite element)

\( B \)  
Element strain shape function (finite element)

\( e^{int} \)  
Time variation for steady-state equations

\( C_r \)  
Rayleigh wave velocity

\( \delta \)  
Dirac delta function

\( e \)  
Constant unit vector

\( H \)  
Heaviside function

\( \delta_{ij} \)  
Kronecker delta function

\( N_k(\zeta, \eta) \)  
Finite element shape function defined on a standard element

\( |\Omega| \)  
Jacobian of transformation from any general element into the standard element

\( \{R^N\} \)  
Constant value vector (transient boundary element formulation)

\( \alpha_m \)  
Mass proportional damping factor

\( \alpha_k \)  
Stiffness proportional damping factor

\( \theta, \beta \)  
Newmark scheme factors (finite element)

\( \Delta t \)  
Transient formulation time step

\( \Omega^B \)  
Far field subregion

\( \Omega^F \)  
Near field subregion

\( \Gamma_i^B \)  
Finite element-boundary element inner interface

\( \Gamma_o^B \)  
Finite element-boundary element outer interface

\( G_{max}, G_0 \)  
Maximum shear modulus

\( G_t \)  
Tangent shear modulus

\( G \)  
Secant shear modulus

\( \tau_{max} \)  
Maximum shear strength
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_t$</td>
<td>Tangent bulk modulus</td>
</tr>
<tr>
<td>$K_b$</td>
<td>Bulk modulus constant</td>
</tr>
<tr>
<td>$P_a$</td>
<td>Atmospheric pressure</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Mean normal stress</td>
</tr>
<tr>
<td>$\alpha, r$</td>
<td>Ramberg-Osgood model parameters</td>
</tr>
<tr>
<td>$\varepsilon_y$</td>
<td>Yielding normal strain</td>
</tr>
<tr>
<td>$\gamma_y$</td>
<td>Yielding shear strain</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>Yielding normal stress</td>
</tr>
<tr>
<td>$\tau_y$</td>
<td>Yielding shear stress</td>
</tr>
</tbody>
</table>
Chapter 2

Theoretical Background

2.1 Governing Equations

Under the assumption of small displacements, the motion of a particle in an homogeneous isotropic linear elastic medium can be expressed in terms of the derivatives of the displacement components \( u_i(x, t) \) as (Eringen and Suhubi 1975, Fung 1965)

\[
(c_1^2 - c_2^2)u_{j,j,i} + c_2^2u_{i,j,j} - \ddot{u}_i + b_i = 0
\]  

(2.1)

where a rectangular coordinate system \( (i = 1,2,3 \text{ in 3D cases, } i = 1,2 \text{ in 2D cases}) \) is used, and commas and dots indicate respectively partial space \( \frac{\partial u_i}{\partial x_j} \) and time \( \frac{\partial u}{\partial t} \) derivatives. \( b_i \) is the body force component in the \( x_i \) direction, \( c_1 \) is the dilatational wave velocity (compression or irrotational velocity) and \( c_2 \) the distortional wave velocity (shear or equivoluminal velocity). These velocities are defined as:

\[
c_1^2 = \frac{(\lambda + 2\mu)}{\rho}, \quad c_2^2 = \frac{\mu}{\rho}
\]

(2.2)

where the material moduli \( \lambda \) and \( \mu \) are the Lamé constants. Also, \( \rho \) represents the mass density of the elastic medium, and \( b_i \) for the body forces.

Furthermore, the displacements and tractions need to satisfy the boundary as well as the
initial conditions, which can be summarized as follows:

- boundary conditions (specified along the boundary $\Gamma = \Gamma_1 \cup \Gamma_2$)

$$ u_i(x, t) = \bar{u}_i(x, t) \quad \text{for} \quad t > t_o \quad \text{on} \quad \Gamma_1 $$

and

$$ t_i(x, t) = \sigma_{ij} n_j = \bar{t}_i(x, t) \quad \text{for} \quad t > t_o \quad \text{on} \quad \Gamma_2 $$

- initial conditions

$$ u_i(x, t) = \bar{u}_{i0}(x) \quad \text{and} \quad \dot{u}_i(x, t) = \bar{v}_{i0}(x) \quad \text{for} \quad t = t_o \quad \text{on} \quad \Gamma \quad \text{and} \quad \Omega $$

(2.4)

where overbars indicate prescribed values and $n_j$ are the direction cosines of the outward normal to the boundary $\Gamma$. The traction components $\sigma_{ij}(x, t)$ can be expressed as functions of displacement derivatives according to:

$$ \sigma_{ij} = \rho [\delta_{ij}(c_1^2 - 2c_2^2)u_{k,k} + c_2^2(u_{i,j} + u_{j,i})] $$

(2.5)

2.2 Dynamic Reciprocal Theorem

The reciprocal theorem in dynamics specifies a relationship between a pair of elastodynamic states. It is essentially the dynamic extension of the classical reciprocal theorem of Betti-Rayleigh in elastostatics (Eringen and Suhubi 1975, Manolis and Beskos 1988). The reciprocal theorem first specifies a regular region $\Omega$ with boundary $\Gamma$ and material properties $\rho$, $c_1$, and $c_2$. Consider two distinct elastodynamic states

$$ A = [u_p, t_p, b_i] \quad \text{and} \quad B = [\bar{u}_p, \bar{t}_p, \bar{b}_i] $$

(2.6)

defined in that region and with initial conditions
\[ u_i(x, 0) = \ddot{u}_{i0}(x), \quad \dot{u}_i(x, 0) = \ddot{v}_{i0}(x) \]
\[ \dot{u}_i(x, 0) = \ddot{u}_{i0}(x), \quad \ddot{u}_i(x, 0) = \dddot{v}_{i0}(x) \]

Then, for \( t \geq t_o \)
\begin{align*}
\int_{\Gamma} \int_{\Omega} & \hat{p}(b_{i*} \hat{u}_i + \dddot{v}_{i0} * \hat{u}_i + \dddot{u}_{i0} * \dot{\hat{u}}_i) d\Omega d\Gamma \\
= & \int_{\Gamma} \int_{\Omega} \hat{p}(b_{i*} u_i + \dddot{v}_{i0} * u_i + \dddot{u}_{i0} * \dot{u}_i) d\Omega d\Gamma 
\end{align*}

where the operation \(*\) above denotes time convolution, i.e.,
\[ f \ast g = \int_{t_o}^{t} f(x, t-\tau) g(x, \tau) d\tau = \int_{t_o}^{t} g(x, t-\tau) f(x, \tau) d\tau \]
for \( t \geq t_o \) and for two functions \( f \) and \( g \).

### 2.3 Steady-State Response Formulation

#### 2.3.1 Finite Element Formulation

By using the finite element method to discretize the whole domain of the near field into a finite number of elements (Figure 2.5), a linear system of equations is obtained
\[ [M] \ddot{u} + [K] u + f = 0 \]

where \([M], [K]\) are the global mass and stiffness matrices and \( f \) is the vector of nodal loads. For steady-state harmonic response, the nodal displacements can be expressed as
\[ u = u(x) e^{i\omega t} \]
which, if substituted in Equation (2.10), leads to
\[ [A] u = f \]
where

\[ [A] = - \omega^2 [M] + (1 + 2i\beta)[K] \]  \hspace{1cm} (2.13)

and \( \beta \) is the viscoelastic damping coefficient.

### 2.3.2 Boundary Element Formulation

Making use of the dynamic reciprocity theorem with one state being the real state of the problem, and the other state being the fundamental singular solution pair expressing the displacements, \( G_{ij} \), and the forces, \( F_{ij} \), the frequency-domain governing equation can be expressed in terms of a boundary integral equation as (Eringen and Suhubi 1975)

\[ c_{ij} u_j + \int_{\Gamma} F_{ij} u_j \, d\Gamma = \int_{\Gamma} G_{ij} t_j \, d\Gamma \]  \hspace{1cm} (2.14)

where \( u \) and \( t \) denote the displacement and traction vectors; \( \Gamma = \Gamma_u + \Gamma_t \), in which \( \Gamma_u \) and \( \Gamma_t \) are the boundaries with prescribed displacements and tractions, respectively; and \( c_{ij} \) is a constant related to the geometric location and smoothness of the boundary at the source points, defined later in this Chapter in (2.63). The fundamental solutions \( G_{ij} \) and \( F_{ij} \) for plane strain steady-state elastodynamics have the form (Cruse and Rizzo 1968)

\[ G_{ij} = \frac{1}{2\pi\rho c_2^2} [\Phi \delta_{ij} - \Psi r_i r_j] \]  \hspace{1cm} (2.15)
\[ F_{ij} = \frac{1}{2\pi} \left[ \frac{d\Phi}{dr} - \frac{\Psi}{r} \right] \left( \delta_{ij} \frac{\partial}{\partial n} r_{i,j} + r_{j,i} n_{i} \right) \]

\[ -2 \frac{\Psi}{r} \left( r_{i,j} - 2 \frac{\partial r}{\partial n} r_{i,j} - 2 \frac{d\Psi dr}{\partial n} r_{i,j} \right) + \left( \frac{c_1^2}{c_2^2} - 2 \right) \left( \frac{d\Phi}{dr} - \frac{d\Psi}{dr} - \frac{\Psi}{r} \right) r_{i,j} \]

where

\[ \Phi = K_0 \left( \frac{i\omega r}{c_2} \right) + \frac{c_2}{i\omega r} \left[ K_1 \left( \frac{i\omega r}{c_2} \right) - \frac{c_2}{c_1} K_1 \left( \frac{i\omega r}{c_1} \right) \right] \]

\[ \Psi = K_2 \left( \frac{i\omega r}{c_2} \right) - \frac{c_2^2}{c_1^2} K_2 \left( \frac{i\omega r}{c_1} \right) \]

\( r \) is the distance between the source and field points, \( K_0, K_1, K_2 \) are modified Bessel functions of the second kind, and \( c_1, c_2 \) are the P and S complex valued wave velocities in the elastic body. By discretizing the boundary of the problem into a finite number of elements and using the same concept of shape functions as in the FEM to describe the nodal displacements, Equation (2.14) can be expressed as a system of linear equations

\[ [F]u = [G]t \]

where \([F]\) is the constant coefficient matrix derived by integrating the left hand side of Equation (2.14) and adding \( c_{ij} \); \([G]\) is the coefficient matrix derived by integrating the right hand side of Equation (2.14). A sufficient portion of the halfspace surface is discretized, so that the error resulting from the approximation is negligible.
2.3.3 Coupled Finite Element - Boundary Element Formulation

In order to incorporate the wave excitation into the formulation of the coupling procedure, the wave field is divided in two components: (a) the free-field component (superscript $f$) representing the wave field in an equivalent halfspace having the same material properties and (b) the scattered wave field component (superscript $s$) due to the existence of geometrical and material irregularities. The displacements can then be expressed as

$$ u = u^f + u^s $$  \hspace{1cm} (2.20)

The free-field component can be calculated analytically at all nodes of the near-far field interface, considering the type and angle of the incident wave. Equation (2.19) can be rewritten as

$$
\begin{bmatrix}
F_{ii} & F_{ir} \\
F_{ri} & F_{rr}
\end{bmatrix}
\begin{bmatrix}
u^s_i \\
u^s_r
\end{bmatrix} =
\begin{bmatrix}
G_{ii} & G_{ir} \\
G_{ri} & G_{rr}
\end{bmatrix}
\begin{bmatrix}
t^s_i \\
t^s_r
\end{bmatrix}
$$  \hspace{1cm} (2.21)

where the subscript $i$ indicates the interaction nodes lying on the FE-BE interface, and the subscript $r$ indicates the rest of the boundary element nodes; $t^s_i$ and $t^s_r$ are stresses due to the scattered wave field, and $t^s_i$ is the stress at the interaction nodes resulting only from the near field region. The unknowns in Equation (2.21) are the displacement vectors $u^s_i$ and $u^s_r$, and the stress vector $t^s_i$.

By using the condensation technique, one can eliminate the extra degrees of freedom, keeping only those associated with the interaction nodes. Equation (2.21) can be expressed as
\[ \hat{F} u^g_i = \hat{G} t_i + \hat{C} \]  
(2.22)

where

\[ \hat{F} = \left[ F_{ii} \right] - \left[ F_{ir} \right] \left[ F_{rr} \right]^{-1} \left[ F_{ri} \right] \]  
(2.23)

\[ \hat{G} = \left[ G_{ii} \right] - \left[ F_{ir} \right] \left[ F_{rr} \right]^{-1} \left[ G_{ri} \right] \]  
(2.24)

\[ \hat{C} = - \left[ G_{ir} \right] t_r^f + \left[ G_{ii} \right] t_i^f + \left[ F_{ir} \right] \left[ F_{rr} \right]^{-1} \left[ G_{rr} \right] t_r^f \]  
+ \left[ F_{ir} \right] \left[ F_{rr} \right]^{-1} \left[ G_{ri} \right] t_i^f \]  
(2.25)

To ensure compatibility between the far field discretized with boundary elements and the near field discretized with finite elements, a transformation from BE tractions to FE nodal forces is introduced

\[ f_i = -T t_i \]  
(2.26)

where \( T \) is a transformation matrix constructed using the FE shape functions \( N \) and the BE shape functions \( L \) as

\[ T = \int_{\Gamma_i} N^T L \, d\Gamma_i \]  
(2.27)

By making use of Equation (2.27), Equation (2.22) yields

\[ \hat{A} u^g_i = -f_i + \hat{B} \]  
(2.28)

in which
\[
\hat{A} = \begin{bmatrix} T \end{bmatrix} \begin{bmatrix} G \end{bmatrix}^{-1} \begin{bmatrix} \hat{F} \end{bmatrix} \\
\hat{B} = \begin{bmatrix} T \hat{C} \end{bmatrix}^{-1} \begin{bmatrix} \hat{C} \end{bmatrix}
\] (2.29)

The FE system of linear Equations (2.29) can also be written as

\[
\begin{bmatrix}
A_{dd} & A_{di} \\
A_{id} & A_{ii}
\end{bmatrix}
\begin{bmatrix}
u_d \\
u_i
\end{bmatrix} =
\begin{bmatrix}
f_d \\
f_i
\end{bmatrix} 
\] (2.30)

where the subscript \(i\) indicates the interaction on the BE and FE interface and the subscript \(d\) indicates the rest of the FE nodes in the near field.

By combining Equations (2.28) and (2.30), the complete coupled system takes the form

\[
\begin{bmatrix}
A_{dd} & A_{di} \\
A_{id} & A_{ii} + \hat{A}
\end{bmatrix}
\begin{bmatrix}
u_d \\
u_i
\end{bmatrix} =
\begin{bmatrix}
f_d - A_{di}u_i^f \\
\hat{B} - A_{ii}u_i^f
\end{bmatrix} 
\] (2.31)

The solution of Equation (2.31) yields the displacements vector \(u_d\) in the FE region and the scattering wave field \(u_i^s\) at the interaction boundary. The interaction tractions, \(t_i\), the interaction forces, \(f_i\), the scattered wave field, \(u_i^s\) and the total displacement field, \(u\), can be calculated using Equations (2.21, 2.22, 2.26 and 2.20), respectively.

### 2.3.4 Wave Input Method

Since natural earthquake waves can be decomposed into a large number of harmonic waves by the Fast Fourier Transformation technique, a harmonic wave propagating in infinite media can be used to develop the wave input method. As shown in Figure 2.1(a),
Figure 2.1  Wave input model for an infinite medium
the structure and the near field of the continuum, \( \Omega_n \), is modeled by finite elements, and the far field of the continuum, \( \Omega_f \), is modeled by boundary elements. For a harmonic wave propagating from the far field into the near field, the horizontal boundary in the underlying rock is defined as wave input boundary \( \Gamma \). It is clear that Figure 2.1 represents a typical soil infinite medium dynamic interaction problem. From wave motion theory, this is a wave scattering problem due to the geometrical irregularities and variabilities of material properties in the near field.

In order to obtain the response of the near field due to the incident wave from the far field, the model shown in Figure 2.1(a) can be divided into two parts as shown in Figure 2.1(b) and (c). In Figure 2.1(b), an artificial fixed boundary is added onto the wave input boundary \( \Gamma \), so that the incident wave reflects and reaction stresses \( \sigma_\Gamma \) will appear on this fixed boundary. Adding the opposite of \( \sigma_\Gamma \) on the input boundary in Figure 2.1(c), the effect of the fixed boundary can be eliminated. Because there is no response in the near field from Figure 2.1(b), the model shown in Figure 2.1(c) can be used to calculate the response of the near field in replacement of the model shown in Figure 2.1(a). However, the response of the far field under the wave input boundary can be obtained only by superposition of the results from Figure 2.1(b) and (c). This is the basic idea of the wave input method for an infinite medium.

2.3.5 Incident P and SV Wave Reflection Characteristics on a Fixed Boundary

As discussed above, the reaction forces on the wave input boundary can be obtained by studying the incident P and SV wave reflection characteristics on a fixed boundary. For plane P or SV waves propagating into the space and impinging onto a fixed boundary. The
Figure 2.2  P wave and SV wave reflections on a fixed boundary
ratios of related potential amplitudes can be expressed as follows (Zhao et al. 1989)

For a P wave incidence,

\[
\frac{A_R^P}{A_I^P} = \frac{\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2}{\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2}
\]

\[
\frac{B_{SV}^R}{A_I^P} = \frac{-2 \gamma_p \sin \theta_1 \cos \theta_1}{\gamma_{sv} \left( \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \right)} \tag{2.32}
\]

For an SV wave incidence, in which the incidence angle \( \theta_2 \) is less than the critical incident angle,

\[
\frac{A_R^P}{B_I^{SV}} = \frac{\gamma_{sv} \sin \theta_2 \cos \theta_2}{\gamma_p \left( \cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2 \right)}
\]

\[
\frac{B_{SV}^R}{B_I^{SV}} = \frac{\cos \theta_1 \cos \theta_2 - \sin \theta_1 \sin \theta_2}{\cos \theta_1 \cos \theta_2 + \sin \theta_1 \sin \theta_2} \tag{2.33}
\]

where \( A_I^P \) and \( A_R^P \) are the potential amplitudes of the incident and reflected P waves, respectively; \( B_I^{SV} \) and \( B_R^{SV} \) are the potential amplitudes of the incident and reflected SV waves, respectively; \( \gamma_p = \omega / C_p \) and \( \gamma_{sv} = \omega / C_{sv} \) are the wave numbers of P and SV waves, respectively; \( \theta_1 \) is a P wave incident or reflection angle and \( \theta_2 \) is an SV wave incidence or reflection angle.

In order to compute the reaction stresses on the fixed boundary, only the normal stress \( \sigma_y \) and the tangential stress \( \tau_{xy} \) are to be considered. From elastic wave motion theory, the ratios of total stresses to the stresses induced by incident waves can be derived as follows.

For P wave incidence,
\[
\frac{\sigma^P_y}{\sigma^P_{yo}} = 1 + \frac{A^P_R}{A^P_I} + \frac{B^S_R G y^2_{SV} \sin 2\theta_2}{A^P_I \gamma^2_{P}(2G \sin^2 \theta_1 + (\lambda + 2\mu))} = 1 + \beta^P_\sigma
\]

\[
\frac{\tau^P_{xy}}{\tau^P_{x\gamma\phi}} = 1 - \frac{A^P_R}{A^P_I} - \frac{B^S_R \gamma^2_{SV} \cos 2\theta_2}{A^P_I \gamma^2_{P} \sin 2\theta_1} = 1 + \beta^P_\tau
\]

(2.34)

and for SV wave incidence at incidence angle \(\theta_2\) less than the critical angle,

\[
\frac{\sigma^S_V}{\sigma^S_V_{yo}} = 1 - \frac{B^S_R}{B^S_I} \frac{A^P_R \gamma^2_{SV} (D^2 - 2\sin^2 \theta_1)}{B^S_I \gamma^2_{SV} \sin 2\theta_2} = 1 + \beta^S_V
\]

\[
\frac{\tau^S_V}{\tau^S_V_{x\gamma\phi}} = 1 + \frac{B^S_R}{B^S_I} \frac{A^P_R \gamma^2_{SV} \sin 2\theta_1}{B^S_I \gamma^2_{SV} \cos 2\theta_2} = 1 + \beta^S_V
\]

(2.35)

where the subscripts \(I\) and \(R\) of related variables imply the incident or reflected wave, respectively; \(D = C_p / C_{SV}\); \(\beta^P_\sigma\) and \(\beta^P_\tau\) are the stress increase factors of stresses \(\sigma^P_y\) and \(\tau^P_{xy}\) on the fixed boundary due to P wave incidence, respectively; \(\beta^S_V\) and \(\beta^S_V\) are the stress increase factors of stresses \(\sigma^S_V\) and \(\tau^S_V\) on the fixed boundary due to SV wave incidence; \(\sigma^P_{yo}\), \(\tau^P_{x\gamma\phi}\) and \(\sigma^S_V\), \(\tau^S_V\) are the stresses induced by the incident P and SV waves respectively, on the fixed boundary before wave reflections occur. These stresses can be determined by elastic wave motion theory.

If the Equations (2.32, 2.33, 2.34 and 2.35) are combined together, the stress increase factors can be expressed as

\[
\beta^P_\sigma = \frac{\cos(\theta_1 + \theta_2)}{\cos(\theta_1 - \theta_2)} - \frac{D \sin 2\theta_1 \sin 2\theta_2}{\cos(\theta_1 - \theta_2)(2\sin^2 \theta_1 - D^2)}
\]

\[
\beta^P_\tau = \frac{-\cos(\theta_1 + \theta_2)}{\cos(\theta_1 - \theta_2)} - \frac{D \cos 2\theta_2}{\cos(\theta_1 - \theta_2)}
\]

(2.36)
\[ \beta_{SV}^{y} = \frac{-\cos(\theta_1 + \theta_2)}{\cos(\theta_1 - \theta_2)} + \frac{D^2 - 2\sin^2\theta_1}{D\cos(\theta_1 - \theta_2)} \]
\[ \beta_{SV}^{t} = \frac{\cos(\theta_1 + \theta_2)}{\cos(\theta_1 - \theta_2)} + \frac{\sin2\theta_1\sin2\theta_2}{D\cos(\theta_1 - \theta_2)\cos2\theta_2} \]  

(2.37)

2.3.6 Formulation of Generalized Stresses on Wave Input Boundary

Once the normal and tangential stresses resulting from an incident wave reflection on a fixed boundary are determined, the equivalent nodal load vector on the wave input boundary can be obtained from

\[ \{P_{r}\}^e = \int_{\Gamma} [N]^T \{\sigma\}^e d\Gamma \]  

(2.38)

where \([N]\) is the shape function matrix, \(\{\sigma\}^e\) is a generalized stress vector acting on the wave input boundary \(\Gamma\). In order to obtain the total stresses, \(\sigma_y\) and \(\tau_{xy}\), induced by a fixed boundary, the stresses \(\sigma_{yo}\) and \(\tau_{xyo}\) should be first determined by elastic wave theory.

2.3.7 SV Wave Incidence

Figure 2.3(a) shows a plane SV wave propagating from the far field to the near field and arriving at the fixed boundary. Before the incident wave reflects on the boundary, the particle displacement of the incident SV wave can be written as

\[ u_{ix} = u_i \cos\theta \]
\[ u_{iy} = u_i \sin\theta \]  

(2.39)

where \(u_i\) is the particle displacement of the incident plane SV wave while arriving at the fixed boundary before the reflection of the wave; \(u_{ix}\) and \(u_{iy}\) are the components of \(u_i\) in the horizontal and vertical directions, respectively; \(\theta\) is the incidence angle of the plane
Figure 2.3  Particle displacement on the fixed boundary due to (a) SV and (b) P incident wave.
SV wave.

Assuming the incident plane SV wave is a unit harmonic wave, the particle displacement $u_i$ can be written as

$$ u_i = \exp \left[ i\omega \left( t - \frac{x}{C_{svx}} + \frac{y}{C_{svy}} \right) \right] $$

(2.40)

where $C_{svx}$ and $C_{svy}$ are the velocity projection of the incident plane SV wave in the horizontal and vertical directions, respectively expressed as

$$ C_{svx} = \frac{C_s}{\sin \theta} $$

(2.41)

$$ C_{svy} = \frac{C_s}{\cos \theta} $$

From the elastic wave theory, the strain vector $\{ \varepsilon \}$ of the particle induced only by $u_i$ on the boundary ($y = 0$) can be derived as follows:

$$ \{ \varepsilon \} = \begin{bmatrix} \frac{\partial u_{ix}}{\partial x} \\ \frac{\partial u_{iy}}{\partial y} \\ \frac{\partial u_{ix}}{\partial y} + \frac{\partial u_{iy}}{\partial x} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \sin 2\theta \\ \frac{1}{2} \sin 2\theta \frac{C_s}{C_{svx}} \\ \cos 2\theta \end{bmatrix} i\omega \exp \left[ i\omega \left( t - \frac{x}{C_{svx}} \right) \right] $$

(2.42)

and the relationship between stresses and strains can be expressed as

$$ \{ \sigma \} = [D] \{ \varepsilon \} $$

(2.43)

where $\{ \sigma \}$ is a stress vector induced only by $u_i$ on the boundary ($y = 0$). The matrix $[D]$ expresses the elastic properties of the medium and has the following form for the plane strain case.
\[
[D] = \frac{E}{1 + \nu} \begin{bmatrix}
\frac{1 - \nu}{1 - 2\nu} & \frac{\nu}{1 - 2\nu} & 0 \\
\frac{1 - \nu}{1 - 2\nu} & \frac{\nu}{1 - 2\nu} & 0 \\
\frac{1}{2} & 0 & 0
\end{bmatrix}
\] (2.44)

where $E$ and $\nu$ are the modulus of elasticity and Poisson’s ratio, respectively.

From Equations (2.42, 2.43 and 2.44), the stress vector $\{\sigma\}$ can be obtained

\[
\{\sigma\} = \begin{bmatrix}
\frac{1}{2} \sin 2\theta \\
\frac{1}{2} \sin 2\theta \\
\cos 2\theta
\end{bmatrix} \frac{E\omega}{2(1 + \nu)C_{sv}} i\exp \left[ i\omega \left( t - \frac{x}{C_{svx}} \right) \right]
\] (2.45)

In the frequency domain analysis, the term $e^{i\omega t}$ in Equation (2.45) can be omitted, $\sigma_{yo}^{SV}$ and $\tau_{xyo}^{SV}$ can be expressed as

\[
\sigma_{yo}^{SV} = \frac{E\omega \sin 2\theta}{2(1 + \nu)C_{sv}} i\exp \left( -i\frac{\omega}{C_{svx}} x \right)
\] (2.46)

\[
\tau_{xyo}^{SV} = \frac{E\omega \cos 2\theta}{2(1 + \nu)C_{sv}} i\exp \left( -i\frac{\omega}{C_{svx}} x \right)
\]

From Equations (2.35) and (2.46), the generalized stress vector $\{\sigma\}^e$ acting on the wave input boundary of an element due to the SV wave incidence can be represented as

\[
\{\sigma\}^e = \begin{bmatrix}
\sigma_y^{SV} \\
\tau_{xy}^{SV}
\end{bmatrix} = \left[ \begin{bmatrix} f_{\sigma}^{SV} \\ f_{\tau}^{SV} \end{bmatrix} C_{sv} \right] \frac{E\omega}{C_{sv}} i\exp \left( -i\frac{\omega}{C_{svx}} x \right)
\] (2.47)

where
\[ f^S_{\sigma} = \frac{-\sin \theta}{2(1 + v)}(1 + \beta^S_{\sigma}) \]
\[ f^S_{t} = \frac{\cos \theta}{2(1 + v)}(1 + \beta^S_{t}) \]  

(2.48)

2.3.8 P Wave Incidence

The same procedure as described previously can be used to derive the generalized stresses in the case of P wave incidence. As shown in Figure 2.3(b), a plane P wave propagates from the far field to the near field and arrives at the fixed boundary. Before the reflection of the wave, the particle displacement pattern of this incident P wave can be expressed as

\[ u_{i_x} = u_i \sin \theta \]
\[ u_{i_y} = -u_i \cos \theta \]  

(2.49)

Similarly, the generalized stresses vector \( \{ \sigma \}^e \) acting on the wave input boundary of an element due to the P wave incidence can be represented as

\[ \{ \sigma \}^e = \begin{bmatrix} \sigma_{yx}^P \\ \tau_{xy}^P \end{bmatrix} = -\begin{bmatrix} f^P_{\sigma} \\ f^P_{t} \end{bmatrix} \frac{E \omega_i}{C_p} \exp \left(-i \frac{\omega}{C_p} x \right) \]  

(2.50)

where

\[ f^P_{\sigma} = \frac{-(v - 1)}{(1 + v)(1 - 2v)} \left( \cos^2 \theta + \frac{v}{1 - v} \sin^2 \theta \right)(1 + \beta^P_{\sigma}) \]
\[ f^P_{t} = \frac{-\sin \theta}{2(1 + v)}(1 + \beta^P_{t}) \]  

(2.51)

2.3.9 Rayleigh Wave Incidence

Rayleigh waves are surface waves, which attenuate their amplitudes exponentially
with distance from the surface, whereas the particle path forms a retrograde ellipse. The horizontal and vertical displacements $u_x$ and $u_y$ are given by the following equations:

$$u_x = iA k_r \left[ \exp(-\xi_p y) - 2 \xi_p \xi_x r / \left( 2 k_r^2 - \frac{\omega^2}{C_s^2} \right) \exp(-\xi_s y) \right] \exp(\omega t - k_r x)$$

$$u_y = A k_r \left[ \exp(-\xi_p y) - 2 k_r^2 / \left( 2 k_r^2 - \frac{\omega^2}{C_s^2} \right) \exp(-\xi_s y) \right] \exp(\omega t - k_r x)$$

(2.52, 2.53)

where $A$ is the horizontal amplitude of the Rayleigh wave, $k_r = \omega / C_R$ is the wave number, $C_R$ is the Rayleigh wave velocity and $\omega$ is the circular frequency. The terms $\xi_s$, $\xi_p$ are defined as follows,

$$\xi_s = \sqrt{\left( k_r^2 - \frac{\omega^2}{C_s^2} \right)}, \quad \xi_p = \sqrt{\left( k_r^2 - \frac{\omega^2}{C_p^2} \right)}$$

(2.54)

To obtain the Rayleigh wave velocity, a third degree polynomial (2.55) has to be solved, giving a root $0.9 C_s \leq C_R \leq 0.96 C_s$ for Poisson’s ratio $\nu$ between 0.15 and 0.5

$$\left( \frac{C_r^2}{C_s^2} \right)^3 - 8 \left( \frac{C_r^2}{C_s^2} \right)^2 + \left( 24 - 16 \frac{C_r^2}{C_p^2} \right) \left( \frac{C_r^2}{C_s^2} \right) + 16 \left( \frac{C_r^2}{C_p^2} - 1 \right) = 0$$

(2.55)

Following the same methodology used for SV and P waves, the input excitation can be represented as the reaction forces resulting from fixing the free surface. The normal stress $\sigma_y$ and the tangential stress $\tau_{xy}$ are given by the following equations

$$\{ \sigma \}^e = \begin{bmatrix} \sigma_y^R \\ \tau_{xy}^R \end{bmatrix} = A \exp(i \omega t - ik_r x - \xi_{s,y}) \left\{ \begin{array}{c} \lambda (\xi_p^2 - \xi_x \xi_s) + 4 \mu \xi_p^2 \\ -i \mu k_r (\xi_p^2 / \xi_s + 3 \xi_p) \end{array} \right\}$$

(2.56)
where $\lambda$ and $\mu$ are the Lamè constants.

### 2.4 Transient Response Formulation

#### 2.4.1 Fundamental Solution

The fundamental singular solution for displacement component $i$ at point $x$ (receiver) due to a concentrated pulse in direction $j$ at point $\xi$ (source) in a three-dimensional elastic solid of infinite extent is the solution of Equation (2.1) with a body force

$$b_j = \delta(x-\xi)\delta(t-\tau)e_j$$

(2.57)

where $\delta$ is the Dirac delta function and $e$ is a constant unit vector. This solution has the explicit form (see Eringen and Suhubi 1975, Butkovskiy 1982)

$$G_{ij} = \frac{1}{2\pi\rho} \left[ \frac{1}{c_1} H(c_1 \tau' - r) \left( \frac{2c_1^2 \tau'^2 - r^2}{\sqrt{c_1^2 \tau'^2 - r^2}} \right) \left( \frac{y_i y_j}{r^4} \right) - \frac{\delta_{ij}}{r^2} \sqrt{c_1^2 \tau'^2 - r^2} \right]$$

(2.58)

$$+ \frac{1}{c_2} H(c_2 \tau' - r) \left( \frac{2c_2^2 \tau'^2 - r^2}{\sqrt{c_2^2 \tau'^2 - r^2}} \right) \left( \frac{y_i y_j}{r^4} \right) - \frac{\delta_{ij}}{r^2} \sqrt{c_2^2 \tau'^2 - r^2} + \frac{\delta_{ij}}{r^2} \sqrt{c_2^2 \tau'^2 - r^2} \right]$$

where $r_i = x_i - \xi_i$, $r = |x_k - \xi_k|$, $\delta_{ij}$ is the Kronecker delta and $H$ is the Heaviside function. The fundamental singular solution for the tractions $F_{ij}$ can be obtained by using Equations (2.5) and (2.58).
\[ F_{ij} = \frac{\mu}{2\pi r} \left\{ \frac{1}{c_1} \left[ \frac{1}{r} - 1 \right] \left[ \frac{1}{\left( \frac{c_1 t'}{r} \right)^2 - 1} \right]^{3/2} \left( \frac{A_1}{r} \right) + \frac{2\left( \frac{c_1 t'}{r} \right) - 1}{\sqrt{\left( \frac{c_1 t'}{r} \right)^2 - 1}} \left( \frac{2A_2}{r} \right) \right\} \]

\[-\frac{1}{c_2} \left[ \frac{1}{r} - 1 \right] \left[ \frac{1}{\left( \frac{c_2 t'}{r} \right)^2 - 1} \right]^{3/2} \left( \frac{A_3}{r} \right) + \frac{2\left( \frac{c_2 t'}{r} \right) - 1}{\sqrt{\left( \frac{c_2 t'}{r} \right)^2 - 1}} \left( \frac{2A_2}{r} \right) \right\} \]

where

\[ A_1 = (\lambda/\mu) n_{i,r,j} + 2 r_{i,j} \frac{\partial r}{\partial n} \]

\[ A_2 = n_{i,r,j} + n_{j,r,i} + \frac{\partial r}{\partial n} (\delta_{ij} - 4 r_{i,j}) \]

\[ A_3 = \frac{\partial r}{\partial n} (2 r_{i,j} - \delta_{ij}) - n_{j,r,i} \]

The above fundamental singular solutions obey the causality condition

\[ G_{ij}(x, t, \xi, \tau) = 0 \text{ if } c_1(t - \tau) < r \quad (2.60) \]

and have the time translation property

\[ G_{ij}(x, t + t_o, \xi, \tau + t_o) = G_{ij}(x, t, \xi, \tau) \quad (2.61) \]

2.4.2 Boundary Integral Formulation

Love's integral identity can be derived by making use of the dynamic reciprocity theorem with one state being the real state of the problem, and the other being the fundamental singular solution pair \( G_{ij}(x, t, \xi, \tau=0) \) and \( F_{ij}(x, t, \xi, \tau=0) \) (see Eringen and Suhubi 1975, Manolis and Beskos 1988). It takes the form
\[
c_{ij}u_i(\xi, t) = \left\{ \int_{\Gamma} \{G_{ij} \times t_i(x, t) - F_{ij} \times u_i(x, t)\} d\Gamma \right. \\
\left. + \rho \int_{\Omega} G_{ij} \times b_i(x, t) d\Omega + \rho \int_{\Omega} \{G_{ij} \times \vec{v}_{io}(x) + \vec{G}_{ij} \times \vec{u}_{io}(x)\} d\Omega \right\}
\]  \quad (2.62)

where \(\xi\) and \(x\) are the receiver and source points respectively, \(\vec{u}_{io}\) and \(\vec{v}_{io}\) are the initial displacement and velocity, \(t_i\) is the traction vector, and \(c_{ij}\) is a constant which, for smooth boundaries, takes the value

\[
c_{ij} = \begin{cases} 
1 & \text{if } \xi \in \Omega \\
\frac{1}{2} & \text{if } \xi \in \Gamma \\
0 & \text{if } \xi \notin \Omega \cup \Gamma
\end{cases}
\]  \quad (2.63)

De Hoop (1958) proved that Equation (2.62) is valid for both bounded and unbounded domains. This equation can be viewed as a system of differential equations that solve for the boundary displacements and tractions knowing the initial conditions and the body forces in the domain of interest. Furthermore, it can be used to obtain the displacements and tractions on any of the interior points of the domain once the boundary problem has been solved.

However, it should be noticed that in the case of interest, the initial condition and the body forces can safely be assumed to vanish, so Equation (2.62) reduces to the form

\[
c_{ij}u_i(\xi, t) = \int_{\Gamma} \{G_{ij} \times t_i(x, t) - F_{ij} \times u_i(x, t)\} d\Gamma
\]  \quad (2.64)

To solve the above system of differential equations, a numerical treatment is inevitable. The numerical implementation is described in the following section.
### 2.4.3 Numerical Implementation

In Equation (2.64), the contribution of the initial conditions and body forces is neglected. This expression represents the exact formulation for transient boundary integral problems involving time integration. However, implementing such technique needs approximation in both temporal and spatial variation of the displacements and tractions.

To deal with the temporal integration, the time axis is divided into \( N \) equal steps, the time at any step \( n \) is \( n\Delta t \). The response at time \( T=n\Delta t \) can be obtained by computing the history of the boundary values in all previous and current steps. A constant temporal variation for the field variables is employed (Von Estorff and Prabucki 1990). The displacements and tractions are assumed to be constant throughout the time step and, thus, are not included in the time integration, which includes only the fundamental solution terms \( G_{ij} \) and \( F_{ij} \). The integral of \( G_{ij} \) during the time step \( n \) is expressed as

\[
G^n_{ij}(x, t, \xi, \tau) = \int_{(n-1)\Delta t}^{n\Delta t} G_{ij}(x, t, \xi, \tau) d\tau 
\]  
(2.65)

and the integral of \( F_{ij} \) during the time step \( n \) is expressed as

\[
F^n_{ij}(x, t, \xi, \tau) = \int_{(n-1)\Delta t}^{n\Delta t} F_{ij}(x, t, \xi, \tau) d\tau 
\]  
(2.66)

Substituting the two expressions (2.65) and (2.66) in Equation (2.64) results to

\[
c_{ij}u^n_i(\xi, t) = \sum_{n=1}^{N} \int_{\Gamma} \{ G^n_{ij}(x, t, \xi, \tau) t^{N-n+1}(x) - F^n_{ij}(x, t, \xi, \tau) u^{N-n+1}(x) \} d\Gamma 
\]  
(2.67)

A similar expression as (2.67) is obtained if one uses a different function basis to approximate the temporal variation of the field variables. However, the expression will be
more elaborate and the numerical effort to solve the system will increase.

The geometry representing the boundary of the problem can be modeled using isoparametric linear elements (Brebbia et al. 1984). In this case, co-ordinates or any other function defined on this element can be computed as

\[ x_i = \sum_{k=1}^{n} N_k(\zeta, \eta)X_{ik} \]  

(2.68)

where \( x_i \) can represent, for example, coordinates, displacements, or tractions at any point within the element, and \( X_{ik} \) are the corresponding nodal values. \( N_k(\zeta, \eta) \) is the shape function defined on a standard element with rectangular coordinates \( \zeta, \eta \) (see Figure 2.4).

For a two dimensional domain problem, in which the boundary is modeled by a linear isoparametric elements, the number of shape functions \( n \) in this case is 2. Higher isoparametric elements can be used to accurately resemble the boundary geometry, displacements and tractions distribution. If the above discretization scheme is used, Equation (2.67) transforms into

\[
c_{ij}u_j^k(\xi, \tau) = \sum_{n=1}^{N} \sum_{m=1}^{M} \left[ \sum_{l=1}^{L} T_{lk}^{N-n+1} \int_{-1}^{1} G_{ij}^{N}(x, t, \xi, \tau) N_k(\zeta, \eta) |\mathcal{J}| d\zeta d\eta \right. \\
+ \left. U_{ik}^{N-n+1} \int_{-1}^{1} F_{ij}^{N}(x, t, \xi, \tau) N_k(\zeta, \eta) |\mathcal{J}| d\zeta d\eta \right]
\]  

(2.69)

where \( M \) is the total number of elements on the boundary, and \( |\mathcal{J}| \) is the Jacobian of transformation from any general element into the standard element. It should also be noted that using the concept of standard elements and geometric transformations help in automating the numerical integration and making the numerical implementation easier.
Figure 2.4 General element versus standard element
After achieving the temporal and spatial discretization of the original boundary integral Equation (2.64) and performing the integration involved, Equation (2.69) can be expressed in a matrix form as

\[
\sum_{n=1}^{N} ((G^n\{t^{N-n+1}\} - [F^n]\{u^{N-n+1}\}) = \{0\} \tag{2.70}
\]

where \{u^{N-n+1}\} and \{t^{N-n+1}\} are the displacement and traction vectors for all boundary nodes at time \((N-n+1)\Delta t\), \([G^n]\) and \([F^n]\) are the coefficient matrices of the system resulting from Equation (2.69) at time \(n\Delta t\). Both matrices have to be evaluated up to \(n_t\) time step, where the total time

\[
T_t = n_t\Delta t. \tag{2.71}
\]

Depending on the nature of the problem, displacement vectors, traction vectors, or parts of each is going to be unknown at time step \(n\). In the problem of waves propagating towards a structure, the excitation can be represented in terms of the traction vector \{t^{N-n+1}\}. This will be discussed in more detail later. If the current time step is \(N\), all the traction vectors for \(n=1\) to \(N\), and the displacement vectors for \(n=1\) to \(N-1\) are known. Equation (2.70) then can be rewritten as

\[
[F^1]\{u^N\} = [G^1]\{t^N\} - \sum_{n=2}^{N} ((G^n\{t^{N-n+1}\} - [F^n]\{u^{N-n+1}\}) \tag{2.72}
\]

or simply

\[
[F^1]\{u^N\} = [G^1]\{t^N\} + \{R^N\} \tag{2.73}
\]

where \{\(R^N\)\} is a known vector with constant values. It should be pointed out that in order to achieve a realistic amount of calculations, a truncation of the boundary at a certain
distance from the structure may be desirable. An alternative approach is to make use of infinite boundary elements. The first approach achieves reasonable results in the frequency domain hybrid BE-FE analysis.

2.4.4 Finite Element Formulation

The FEM discretization is to be used to discretize the near field (see Figure 2.5) of the problem of interest (Zienkiewicz et al. 1977, Bathe 1982). In this case the whole domain \( \Omega \) has to be divided into a finite number of elements. The displacement in each element can be obtained as \( u(x, t) = N \hat{u}(t) \) where \( N \) is the shape functions vector and \( \hat{u}(t) \) represent the nodal displacements. In order to minimize the errors resulting from the discretization, the Galerkin weighted residual formulation is used. In this approach the weighting function is to be taken the same as the shape function. This discretization scheme will render the following matrix differential equation

\[
[M] \ddot{u} + [C] \dot{u} + [K] u + f = 0
\]

(2.74)

where \([M]\), \([C]\) and \([K]\) are the global mass, damping and stiffness matrices, and \( f \) is the vector of nodal loads. These global matrices are assembled from the individual element stiffness and mass matrices, which can be obtained as

\[
M^e = \int_{\Omega^e} N^t \rho N \, d\Omega \quad , \quad K^e = \int_{\Omega^e} B^t DB \, d\Omega \quad , \quad f^e = - \int_{\Omega^e} N^t b \, d\Omega + \int_{\Omega^e} B^t \sigma \, d\Omega
\]

(2.75)

where \( B \) is the element strain shape function and \( D \) is the elasticity matrix. (Zienkiewicz et al. 1977); \( b \) are the body forces and \( \sigma \) are the applied stresses on the loaded region of the model. The damping matrix \([C]\) and can be expressed in a Rayleigh form as
Figure 2.5  Structure, near field and far field regions
\[ [C] = \alpha_m [M] + \alpha_k [K] \]  

(2.76)

where \( \alpha_m \) is a mass proportional damping factor, and \( \alpha_k \) is a stiffness proportional damping factor. The above integrations can be carried numerically using the Gauss quadrature and isoparametric elements, as in the case of the boundary elements (see Figure 2.4).

The time integration can be carried out by making use of the Newmark \( \beta \) scheme. Based on Taylor expansion, this scheme approximates the displacement derivatives at time \((m + 1)\Delta t\) to the known values of the displacement and its derivatives at time \(m\Delta t\). The velocities and accelerations can be represented as follows:

\[
\dot{u}^{m+1} = \frac{\theta}{\beta \Delta t} [u^{m+1} - u^m] + \left(1 - \frac{\theta}{\beta}\right) [\dot{u}^m] + \left(1 - \frac{\theta}{2\beta}\right) \Delta t [\ddot{u}^m] 
\]  

(2.77)

\[
\ddot{u}^{m+1} = \frac{1}{\Delta t^2 \beta} [u^{m+1} - u^m] + \frac{\beta}{\Delta t} [\dot{u}^m] + \left(1 - \frac{\theta}{2\beta}\right) [\ddot{u}^m] 
\]  

(2.78)

where \( \beta \geq \frac{1}{4} \) and \( \theta \geq \frac{1}{2} \).

Using Equations (2.77), (2.78), Equation (2.74) can be expressed as

\[ [K_{eff}] u^{m+1} + f_{eff} = 0 \]  

(2.79)

where

\[ [K_{eff}] = \frac{1}{\beta \Delta t^2} \left\{ (1 + \alpha_m \theta \Delta t) [M] + (\alpha_k \theta \Delta t + \beta \Delta t^2) [K] \right\} \]  

(2.80)

and

\[ f_{eff} = f + \frac{1}{\beta \Delta t^2} \left\{ (V + \alpha_m V_c) [M] + (\alpha_k V_c) [K] \right\} \]  

(2.81)
in which

\[ V = \left( \frac{1}{2} - \beta \right) \Delta t^2 + \Delta t \dot{u}^m + u^m \]

\[ V_c = \left( \frac{\theta}{2} - \beta \right) \Delta t^3 \ddot{u}^m + (\theta - \beta) \Delta t^2 \dot{u}^m + \theta \Delta t u^m \] (2.82)

This results into a linear system of equations that has to be solved N times where N is the total number of time steps.

In order to be able to solve the previous problem in the case of nonlinear hysteretic material behavior, a time lagging procedure will be used. In this procedure, the material is assumed to be elastic during the time step, and the material properties at time \( m \Delta t \) will be a function of the strains at time \( (m - 1) \Delta t \). The system matrices have to be computed for each time step.

2.4.5 Coupling of Finite Elements and Boundary Elements

In order to take advantage of the boundary and finite element methods, the problem geometry has to be subdivided into two subregions (Figure 2.6). The Boundary Elements Method is used to discretize the far field subregion \( \Omega^B \) where the basic advantage of the BEM is to be exploited. In this case, the halfspace is simulated accurately using the full space fundamental solution. The Finite Element Method is used to discretize the near field subregion \( \Omega^F \). Since it is more capable in dealing with material nonlinearity as well as complex geometries (see, e.g. Von Estorff and Antes 1991).

The boundary element nodes are divided into outer nodes \( \Gamma_o^B \) and interface nodes \( \Gamma_i^B \), which, when combined, give the whole BE subregion \( \Omega^B \). Equation (2.72) can now be
Figure 2.6  Problem geometry discretization for coupled FE-BE analysis
written as
\[
\{t^N\} = [V^1]\{u^N\} + \sum_{n=2}^{N} ([W^N]\{t^{N-n+1}\} - [V^N]\{u^{N-n+1}\})
\]  \hspace{1cm} (2.83)

where
\[
[V^N] = [G^1]^{-1} [F^n]
\]  \hspace{1cm} (2.84)

and
\[
[W^N] = [G^1]^{-1} [G^n]
\]  \hspace{1cm} (2.85)

The above equation can be further written as follows:
\[
\begin{bmatrix}
    t_i^N \\
    t_o^N
\end{bmatrix} =
\begin{bmatrix}
    V_{ii}^1 & V_{io}^1 \\
    V_{oi}^1 & V_{oo}^1
\end{bmatrix}
\begin{bmatrix}
    u_i^N \\
    u_o^N
\end{bmatrix} + \sum_{n=2}^{N} \left( \begin{bmatrix}
    W_{ii}^n & W_{io}^n \\
    W_{oi}^n & W_{oo}^n
\end{bmatrix}
\begin{bmatrix}
    t_i^{N-n+1} \\
    t_o^{N-n+1}
\end{bmatrix} - \begin{bmatrix}
    V_{ii}^n & V_{io}^n \\
    V_{oi}^n & V_{oo}^n
\end{bmatrix}
\begin{bmatrix}
    u_i^{N-n+1} \\
    u_o^{N-n+1}
\end{bmatrix} \right)
\]  \hspace{1cm} (2.86)

where the subscript \(i\) denotes the interface nodes and the subscript \(o\) denotes the outer nodes.

Equation (2.86) represents two linear systems of equations formed to evaluate \(t_i^N\) and \(t_o^N\). By eliminating \(u_o^N\), one can write the following expression:
\[
\{t_i^N\} = [K_{ii}^R]\{u_i^N\} + \{R\}
\]  \hspace{1cm} (2.87)

where
\[
[K_{ii}^R] = [V_{ii}^1] - [Y][V_{oi}^1]^{-1}
\]  \hspace{1cm} (2.88)

\[
[Y] = [V_{io}^1][V_{oi}^1]^{-1}
\]  \hspace{1cm} (2.89)
\[ \{R\} = [\bar{Y}] [t^N]_i + \sum_{n=2}^{N} \left( [V^I_n] \{u^I_{n-n+1}\} + [V^O_n] \{u^O_{n-n+1}\} \right) - [W^I_n] \{t^I_{n-n+1}\} - [W^O_n] \{t^O_{n-n+1}\} \] (2.90)

\[ [V^I_n] = [V^I_{iO}] - [Y] [V^I_{oO}] \] (2.91)

\[ [V^O_n] = [V^O_{iO}] - [Y] [V^O_{oO}] \] (2.92)

\[ [W^I_n] = [W^I_{iO}] - [Y] [W^I_{oO}] \] (2.93)

\[ [W^O_n] = [W^O_{iO}] - [Y] [W^O_{oO}] \] (2.94)

So, assuming that the traction on the outer surface is known, the vector \( \{R\} \) will be known, and Equation (2.87) represents a formulation where the only unknown values will be the displacements and tractions \( \{t^N_i\} \) and \( \{u^N_i\} \) at the interface nodes \( \Gamma^B_i \). The matrix \( [K^B_{ii}] \) physically represent the stiffness developed by the BE subregion \( \Omega^B \) at the BE-FE interface.

In the same way, the FE subregion nodes \( \Omega^F \) are divided into two groups, the interface nodes \( \Gamma^F_i \) with the subscript \( i \) and the remaining nodes \( \Gamma^F_r \) with the subscript \( r \). The FE Equation (2.79) can be written as

\[ \begin{bmatrix} K^F_{rr} & K^F_{ri} \\ K^F_{ir} & K^F_{ii} \end{bmatrix} \begin{bmatrix} \ddot{u}^N_r \\ \ddot{u}^N_i \end{bmatrix} + \begin{bmatrix} f_r \\ f_i \end{bmatrix} = \{0\} \] (2.95)

Before the final stage of the BE-FE coupling, it should be noted that the traction vector \( \{t\} \) is a stress distribution along the boundary elements, where the \( \{f\} \) vector is the forces at each node. In order to achieve consistency between the two methods at the interface, the
BE tractions have to be represented as a function of nodal values. Defining a shape function \( L \) such that

\[
\{ t_i^N \} = [L] \{ \tilde{t}_i^N \}
\]  
\hspace{1cm} (2.96)

and then performing an additional integration along the BE-FE interface to obtain the concentrated forces at the nodes, leads to a new transformation matrix \([A]\) such that

\[
A = \int_{\Gamma_i} N^T L d\Gamma_i
\]  
\hspace{1cm} (2.97)

The assembly of (2.87), (2.95) and using (2.96) yields a combined system of equations

\[
\begin{bmatrix}
K_{rr}^F & K_{ri}^F & 0 \\
K_{ir}^F & K_{ii}^F & A \\
0 & K_{ii}^B & I
\end{bmatrix}
\begin{bmatrix}
\tilde{u}_r^N \\
\tilde{u}_i^N \\
\tilde{t}_i^N
\end{bmatrix}
= \begin{bmatrix}
-f_r \\
-f_i \\
R
\end{bmatrix}
\]  
\hspace{1cm} (2.98)

By solving this system of equations, the displacements \( \tilde{u}_r^N, \tilde{u}_i^N \) and the nodal forces along the BE-FE interface \( \tilde{t}_i^N \) at time step \( N \) are known. Once knowing these values, the values of displacements at the outer nodes of the BE subregion \( \Gamma_o^B \) can be calculated without any difficulty. However, if the interest is only for values in the near field \( \Omega^F \), Equation (2.98) can be rewritten to reduce the number of equations in the linear system. The new system should be

\[
\begin{bmatrix}
K_{rr}^F & K_{ri}^F \\
K_{ir}^F & K_{ii}^F
\end{bmatrix}
\begin{bmatrix}
\tilde{u}_r^N \\
\tilde{u}_i^N
\end{bmatrix}
= \begin{bmatrix}
-f_r \\
Rf
\end{bmatrix}
\]  
\hspace{1cm} (2.99)

where

\[
K_{ii} = K_{ii}^F + A K_{ii}^B
\]  
\hspace{1cm} (2.100)
and

\[ Rf = -f_i + AR \quad (2.101) \]

In this case, the only unknowns are the displacements \( \tilde{u}^N_i \) and \( \tilde{u}^N_j \). It should be noted that the coupled stiffness matrix \( K_{ii} \) in Equation (2.99) is nonsymmetric and fully populated, and, therefore, the coefficient matrix loses its banded structure in the lower part. However, the number of interface nodes is generally small compared to the total number of nodes, so special nonsymmetric banded solvers can efficiently be employed in this case.

2.4.6 Non-Linear Stress Strain Models

A simple nonlinear model (Yogendrakumar, Bathurst and Finn 1992) is to be used in conjunction with the Finite Element model to simulate the material nonlinearity of the soil as well as to introduce the hysteretic damping to the model. The latter is automatically introduced as the hysteretic stress-strain loops take place during the analysis.

Note that the stiffness matrix \( [K] \) in Equation (2.75) is dependent upon the tangent moduli during loading, unloading and reloading. This dependency arises from the dependency of the elasticity matrix \( [D] \) on the same factors. The elasticity matrix has to be computed for each finite element at each time step. This matrix is primarily dependent on the tangent shear modulus \( G \), and the tangent bulk modulus \( K \), which are changing according to the strain level in each soil element.

6. The Hyperbolic Model

The "Hyperbolic Model" assumes that the relationship between shear stress \( \tau \) and shear strain \( \gamma \) during the initial monotonic loading is given by the expression
Figure 2.7  The hyperbolic model for loading and unloading
\[ \tau = \frac{\gamma G_{\text{max}}}{1 + \left(\frac{G_{\text{max}}}{\tau_{\max}}\right)|\gamma|} \]  

where \( G_{\text{max}} \) is the maximum shear modulus and \( \tau_{\max} \) is the shear strength of soil (Yogendrakumar, Bathurst and Finn 1992). Figure 2.7 illustrates the hyperbolic monotonic stress-strain relationship referred to as the "backbone" curve. In the case of unloading from a point \( A(\gamma_{u}, \tau_{u}) \) on the backbone curve, the stress-strain relation becomes

\[ \tau = \tau_{u} + \frac{(\gamma - \gamma_{r})G_{\text{max}}}{1 + \left(\frac{G_{\text{max}}}{2\tau_{\max}}\right)|\gamma - \gamma_{r}|} \]  

Equation (2.103) expresses also the stress-strain relationship in the case of reloading from a point \( B(\gamma_{r}, \tau_{r}) \) on the unloading curve.

The tangent shear modulus, \( G_{t} \), for a point on the backbone curve is given by

\[ G_{t} = \frac{G_{\text{max}}}{\left[1 + \left(\frac{G_{\text{max}}}{\tau_{\max}}\right)|\gamma|\right]^2} \]  

and at a point on an unloading or reloading curve, is given by

\[ G_{t} = \frac{G_{\text{max}}}{\left[1 + \left(\frac{G_{\text{max}}}{2\tau_{\max}}\right)|\gamma - \gamma_{r}|\right]^2} \]  

For this simple model, the horizontal or maximum shear strains may be used to represent the strain state of each element in the evaluation of the shear modulus.

In agreement with extensive experimental evidence (Yogendrakumar, Bathurst and Finn 1992), the tangent bulk modulus \( B_{t} \) of a soil element subjected to a mean normal
stress $\sigma_m$ is given by the expression

$$B_i = K_b P_a \left( \frac{\sigma_m}{P_a} \right)^n$$  

(2.106)

where $K_b$ is a bulk modulus constant, $P_a$ is the atmospheric pressure with the same units of the mean normal stress $\sigma_m$, and $n$ a material constant.

The initial properties of each element are evaluated by taking into account the strain level within the element in the initial static state.

Although, this model is simple and practical to use in conjunction with a numerical scheme, it can only function well in small strains cases ($\gamma \leq 0.05\%$). For higher strains, it tends to give unrealistic ally high damping values which lead to erroneous results.

7. The Ramberg-Osgood Model

A better simple model than the hyperbolic model is the Ramberg-Osgood model. Figure 2.8 illustrates the shape of the stress strain curve, which is defined by.

$$\frac{\varepsilon}{\varepsilon_y} = \frac{\sigma}{\sigma_y} + \alpha \left| \frac{\sigma}{\sigma_y} \right|^r$$  

for virgin loading  

(2.107)

$$\frac{\varepsilon - \varepsilon_0}{2\varepsilon_y} = \frac{\sigma - \sigma_y}{2\sigma_y} + \alpha \left| \frac{\sigma - \sigma_y}{2\sigma_y} \right|^r$$  

for unloading/reloading  

(2.108)

where $\varepsilon_y$ and $\sigma_y$ are the "yielding" strain and stress respectively. The equation can be used for axial or deviatoric stresses and strains, depending on the nature of the problem. If the horizontal shear strain $\gamma$ is the strain of choice, $\sigma_y$ would be the yielding shear strength $\tau_y$, and $\varepsilon_y$ would be the corresponding shear strain $\gamma_y$. The parameters $\alpha$ and $r$ must be found experimentally. Constantopoulos et al. (1973), found that the best
Figure 2.8  The Ramberg-Osgood model for loading and unloading
Figure 2.9 The nonlinear-hysteretic cyclic stress-strain behavior for the Ramberg-Osgood model: (a) shear modulus decrease and (b) damping ratio increase with shear strain amplitude
agreement with the published experimental data on behavior of soils under repeated loadings (Hardin and Drnevich 1972, and Seed and Idriss 1970) was obtained with $\alpha = 0.15$ and $r$ between 2.0 and 2.5. An extensive parametric study performed as part of this thesis showed that the values of $\alpha$ and $r$ that fit best the experimental data for rockfill are 0.12 and 2.5, respectively. The great advantage of the Ramberg-Osgood model, in addition to its simplicity, is the fact that it carries within the equation for the reloading curve the parameters indicating where it will meet the virgin curve. Thus, it is quite easy to ensure that the reloading curve will match the virgin curve on the other side of the strain axis if the reloading is large enough to cause yielding in the other direction.

Figure 2.9 shows the nonlinear-hysteretic cyclic stress-strain behavior for the Ramberg-Osgood model represented in terms of shear modulus decrease and damping ratio increase with cyclic shear strain amplitude. The model gives reasonable values of damping for higher values of strain (Foundation Engineering Handbook, 1991).

2.4.7 Wave Input Method

The same concept used in the case of steady state response in section 2.3.4 is used here to obtain the reaction stresses on the rigid interface in the case of vertically incident SV wave. By making use of Equation (2.5), the stresses induced by the incident SV wave $\sigma_{yo}^{SV}$ and $\tau_{xy}^{SV}$ can be derived as follows

$$\sigma_{yo}^{SV} = 0$$

(2.109)

$$\tau_{xy}^{SV} = \rho C_s^2 \left( \frac{\partial u}{\partial t} / C_s \right)$$

(2.110)

Now, combining Equation (2.35) with the above equations, the reaction stresses developed
on the rigid interface $\sigma_{\gamma}^{\lambda}$ and $\tau_{\delta \lambda}^{V}$ can be calculated. The respective values for the stress increase values $\beta_{\delta}^{SV}$ and $\beta_{\xi}^{SV}$ are equal to 1.

2.5 Verification

2.5.1 Frequency Domain FE-BE Formulation

A frequency domain finite element program was written, applying the formulations described before. The program has two types of elements: an eight node isoparametric plane strain element and a four node isoparametric plain strain element. The program is tested against some of the commercially available finite element codes with perfect agreement. The finite element program is then combined with a boundary element subroutine, as described earlier. In its final form, the program can be used for steady-state dynamic response analysis of two dimensional structures laying on a halfspace.

The coupled FE-BE formulation is used to solve the problem of a rigid strip footing on a homogeneous halfspace. The impedance functions for the footing are obtained, and plotted against the results presented by Gazetas, 1983 (Figure 2.10). $K_{\nu 1}/G$ and $K_{\nu 2}/G$ are the real and imaginary parts, respectively, of the impedance function for a vertically excited strip footing, normalized with respect to the shear modulus $G$. Similarly, $K_{h 1}/G$ and $K_{h 2}/G$ are the real and imaginary parts, respectively, of the impedance function for a horizontally excited strip footing, again normalized with respect to the shear modulus $G$. The results obtained using the combined FE-BE formulation are in nearly perfect agreement to those by Gazetas (1983).

Moreover, to assess the combined FE-BE formulation efficiency and accuracy, the steady state response of a rigid strip footing vibrating vertically is and compared to that
Figure 2.10 Impedance functions for a rigid strip footing on a homogeneous halfspace.
Figure 2.10 (continued) Impedance functions for a rigid strip footing on a homogeneous halfspace.
Figure 2.11 Steady-state response of a rigid strip footing on a homogeneous halfspace.
obtained by using finite elements combined with absorbing boundaries. Two different configurations are considered for the finite element analysis. The first configuration consists of mesh zones 1 and 2 surrounded by absorbing/infinite boundary. The second configuration consists of zones 1 and 2, having the desired damping characteristics for the half space. Zone 3 is considered as a damping zone, where the damping ratio increases gradually from its original value to a value of 0.4 at the external boundary. The later is considered as rigid. A representative sample of the results is shown in Figure 2.11. The results obtained from the FE-BE method and the first finite elements configurations seem to be in reasonable agreement. The second finite elements configuration gives results that are somewhat far from the two previous methods. However, the computational effort needed to obtain these results varies significantly. First, pure finite element meshes are much larger than that of the combined FE-BE and require more time in preparation and in interpreting the output results. Second, computational time is substantially lower for the FE-BE method, compared to the other two methods. This is due to the significantly fewer degrees of freedom in the FE-BE case. Table 2.1 shows the required run time to compute the response at 50 frequencies for each method.

<table>
<thead>
<tr>
<th>Required Computer time</th>
<th>FE-BE</th>
<th>FE Absorbing Boundary</th>
<th>FE Damping Zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 sec.</td>
<td>120 sec.</td>
<td>280 sec.</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2.1** Required execution time for the FE-BE, FE with absorbing boundaries and FE with damping zone to obtain the steady-state response of a strip footing on a homogeneous halfspace
2.5.2 Time Domain FE-BE Formulation

The formulation described earlier was used to develop a time domain nonlinear FE-BE algorithm. The FE component of the program has one type of element, a four node isoparametric plane strain element. In its final form, the program can be used for transient dynamic response analysis of two dimensional structures, with nonlinear material behavior, laying on a halfspace.

The FE-BE program is used to obtain the response of a linear elastic halfspace subjected to an instantaneous surface stress increment, as shown in Figure 2.12a. This same example has been also used by Antes (1985), Cruse and Rizzo (1968a,b), and Mansur (1982). A vertical impulse force of intensity $q=10^4$ psi acts on a strip having a width of $2b=6000$ inches (Figure 2.12a). The supporting homogeneous, linear elastic soil medium is characterized by a modulus of elasticity $E = 2.5 \times 10^6$ psi, mass density $\rho = 196.7$ pcf and Poisson's ratio $\nu = 0.25$. Although admittedly some of the problem parameters seem to be quite unrealistic, they were kept as originally adopted by the aforementioned researchers for the purpose of comparison. In Figure 2.12c, the vertical impulse response is plotted for the first second. It clearly shows that the results obtained by the new time domain FE-BE formulation are in good agreement with the previous rigorous BE solutions for all three points considered.

As a demonstration of the potential of the developed formulation to study wave propagation phenomena in the "near field" area of interest, and gain more insight on the response of soil-structure systems, the results of a dynamic analysis of a dam-foundation system are presented. The dam (Figure 2.13a) is subjected to a vertically propagating SV
Figure 2.12 (a) Spatial discretization of the halfspace under discontinuous boundary stress distribution. (b) Time history of applied stress. (c) Vertical displacements at the boundary points A, B and C.
Figure 2.13 Nonlinear response of an Earth Dam to vertically propagating SV pulse. (a) Geometry of the dam. (b) Incident pulse shape. (c) Horizontal displacement at different points within the dam body.
Figure 2.13 (continued) Nonlinear response of an Earth Dam to vertically propagating SV pulse. (d) Shear stress distribution across the dam body.
wave pulse, shown in Figure 2.13b. Figure 2.13c illustrates the horizontal displacement at
the midheight and base of the dam. It is of interest to examine a series of snapshots at
selected times of the dam body deformation as well as shear stress distribution within the
dam body, as illustrated in Figure 2.13d.

The wave pulse propagating and the reflection of the SV waves at the two slopes of the
dam are clearly shown, as well as the interference of such waves originating from the two
slopes.
Chapter 3

Response of Earth Dams to SV and P Waves

3.1 Introduction

The general formulation presented in the previous chapter is now applied to study the effects of the foundation flexibility and the spatial variability of the ground motion on the seismic response of infinitely long earth dams. The dam body consists of a linearly-hysteretic elastic material and is founded on elastic halfspace (Figure 3.1). The excitation consists of harmonic, obliquely incident SV and P waves travelling across the width of the dam cross-section.

3.2 Model Description and Parametric Study

Two dams having the same height of $H = 100$ m and symmetric slopes of 2:1 and 3:1, respectively, are considered. The dam material has an elastic Young’s modulus $E_d = 8.19 \times 10^5$ kN/m$^2$, mass density $\rho_d = 1920$ kg/m$^3$, Poisson’s ratio $\nu_d = 1/3$, shear wave velocity $c_d = 400$ m/s and damping ratio $\beta_d = 10\%$. The foundation material has a mass density $\rho_f = 2400$ kg/m$^3$, Poisson’s ratio $\nu_f = 1/3$ and no material damping. The flexibility of the foundation is considered by examining a range of impedance ratio values, defined as
Figure 3.1  Earth dam cross-section on elastic halfspace subjected to oblique SV or P waves
\[ IR = \frac{\rho_f c_f}{\rho_d c_d} \]  

where \( c_f \) is the shear wave velocity of the foundation material. The dam body is discretized using four-node plane strain isoparametric elements, whereas the halfspace is discretized using two-node boundary elements. The discretized length of the halfspace surface is taken equal to five times the dam base width.

For harmonic SV and P wave excitation, the results of the parametric study are presented in the following in terms of steady-state amplifications, \( AF \), of the motion within the dam body with respect to the free-field surface motion, i.e.

\[ AF = \left| \frac{u_d(\omega)}{u_g} \right| \]  

where \( u_d(\omega) \) = total displacement in the dam; \( u_g \) = horizontal free-field motion for the case of incident SV waves or vertical free-field motion for the case of incident P waves; and \( \omega \) is the frequency of the incident waves. In the following, \( AF \) is plotted versus a dimensionless frequency, \( a_0 \), equal to

\[ a_0 = \frac{\omega H}{c_d} \]  

### 3.3 Effect of Impedance Ratio

It is of interest to examine first the effect of the flexibility of the elastic rock halfspace on the dam response. Figure 3.2 shows the first ten vibrational modes and the dimensionless natural frequencies of the dam having slopes 2:1, assuming rigid base (\( IR = \infty \)). For this dam, Figure 3.3 shows the steady-state amplification, \( AF \), of the
Figure 3.2  The first ten vibrational modes of the dam on rigid foundation.
(Dam slope 2:1)
horizontal motion at the points A, B, B1, C, C1, C2 and D, due to vertically incident SV waves. The five curves in each plot correspond to impedance ratios IR = 2, 3, 5, 10 and ∞. The results demonstrate that the flexibility of the base rock has a substantial effect on the dam response near the first resonance in shear vibration, occurring at $a_0 \approx 2.2$ for $IR = \infty$. More specifically, AF at first resonance is equal to 8.5 for $IR = \infty$, 7.3 for $IR = 10$, 5.6 for $IR = 5$, 3.6 for $IR = 3$ and 2.3 for $IR = 2$. For higher frequencies, however, the effect of the foundation flexibility becomes significantly smaller. Furthermore, the response near the crest and within the dam body becomes also smaller.

Higher resonances in shear vibration include also rocking motion, more evident near the upper part of the dam. For example, at point A, the small peak at $a_0 \approx 3.8$ corresponds to resonance mainly in rocking vibration, whereas the peak at $a_0 \approx 5$ corresponds to resonance mainly in shear vibration. The peak at $a_0 \approx 8$ is the combined effect of three adjacent shear-rocking vibrational modes.

At the lower half of the dam body and away from the slopes, deamplification of the motion is observed. At the dam base (point D), this deamplification becomes more significant as IR decreases, especially at first resonance. Near the slopes (point C1), the second resonance peak is significant.

Figure 3.4 shows the amplification, AF, of the horizontal motion due to vertically incident SV waves for the dam with slopes equal to 3:1. The results indicate more or less the same trends shown in Figure 3.3 regarding the effect of flexibility on the dam response. However, as the slopes become flatter, the response of the dam tends to be more in shear rather than in rocking vibration. Indeed, the rocking resonance peak observed at
a_0 = 3.8 in Figure 3.3 does not appear in Figure 3.4. Some difference is also observed in
the response near the slopes, where it seems to be more pronounced at the second natural
frequency in shear vibration for the flatter slope dam (point C1).

It is interesting to compare the amplification results in Figure 3.3 and Figure 3.4 with
those from the plane strain shear beam (SB) model on elastic foundation. The latter
assumes that the response of a dam to vertically incident SV waves is only in horizontal
lateral shear deformation with the upstream-downstream displacements uniformly
distributed across the dam width. The steady-state amplification AF at a depth z from the
crest is given by

\[
AF = \left| \frac{J_0\left( a_0^* \frac{z}{H} \right)}{J_0(a_0^*) + i\frac{\sqrt{1 + 2i\beta_d}}{iR}j_1(a_0^*)} \right| \tag{3.4}
\]

where

\[
a_0^* = \frac{a_0}{\sqrt{1 + 2i\beta_d}} \tag{3.5}
\]

\(J_n = \text{Bessel function of first kind and order } n, \text{ and } i = \sqrt{-1}.\) Figure 3.5 shows the AF
evaluated from Equation 3.4 at the points A, B, C, and D. Notice that, at the first
resonance, there is very good agreement between the results from Equation 3.4 and those
of Figures 3.3 and 3.4 from the more rigorous FE-BE analysis. For higher frequencies,
however, the shear beam predicts sharper resonance peaks at the crest (point A) compared
to those from the FE-BE method. At points lower than the crest area, the agreement
between the FE-BE and SB results for the first and higher natural frequencies is very good.
(e.g. compare the AF at points B and C2 from Figures 3.3 and 3.4, with AF at points B and C in Figure 3.5). The differences in the response obtained from the two methods are explained mainly by the fact that the SB assumes response only in shear and ignores the SV wave reflections and generation of P waves along the dam slopes.

Figure 3.6 shows the steady-state amplification AF of the vertical motion at the same points within a dam having slopes 2:1 and subjected to vertically incident P waves. As in the case of SV waves, the flexibility of the base rock has a substantial effect on the dam response near the first resonance in vertical vibration. More specifically, AF is equal to 6.2 for IR = ∞, 6 for IR = 10, 4.9 for IR = 5, 3.3 for IR = 3 and 2.2 for IR = 2. For higher frequencies, the response near the crest and within the dam body is smaller compared to that at first resonance. The effect of the flexible base is also smaller at higher frequencies, but greater than that observed in the case of SV waves in Figure 3.3. At the lower half of the dam body and away from the slopes, deamplification of the motion at high frequencies takes place. At the dam base (point D), this deamplification of the motion is substantial at first resonance. Near the slopes (points B1, C1 and C2), AF at the second resonance is significant. For the dam having slopes equal to 3:1, the results regarding the effect of the base flexibility indicate similar trends as those shown in Figure 3.5, with a slight reduction in the AF at first resonance. More striking is the difference observed in the response near the slopes, which becomes maximum at the second natural frequency in vertical vibration for the flatter slope dam.

Figure 3.8 plots the amplitude of the crest amplification at first resonance versus the inverse of the impedance ratio (1/IR) for the two dams. The upper shaded area represents
the crest amplification for vertically propagating SV waves, whereas the lower shaded area represents the crest amplification for vertically propagating P waves. For each case, the upper bound of the shaded area corresponds to the steeper dam and the lower bound to the flatter dam. The dashed curve represents the crest amplification obtained from the solution of the shear beam on flexible halfspace, given by equation 3.4. Figure 3.8 demonstrates that IR has a significant effect on AF and that this effect is generally larger for the shear-rocking vibration than for the vertical (compression-extension) vibration. However, for a typical practical range of IR values between 2 to 8, the amplification results from all analyses using the rigorous FE-BE method and the simplified shear beam model are in fairly close agreement.

3.4 Effect of the Angle of Incidence

The effect of the spatial variability of the ground motion is examined in Figures 3.9 to 3.14, which present the horizontal and vertical amplification AF for obliquely incident SV and P waves at various angles $\alpha$. The figures show the response at points A, B1, B2, C1, C2, C3, and C4 of a dam having impedance ratio $IR = 5$ and slopes 2:1.

Figures 3.9 and 3.11 show the amplification AF of the horizontal and vertical motion, respectively, for SV waves incident at angles $\alpha = 0^\circ$, $15^\circ$ and $30^\circ$. The horizontal response corresponding to different angles of incidence seems to be in good agreement for all points within the dam body. Such agreement suggests that the spatial variability of the ground motion across the dam base induced by oblique SV waves has relatively small effect on the horizontal motion within the dam body. Similar trends are also shown for the dam with slopes 3:1 (see Figures 3.10 and 3.12). The amplification AF induced in the
vertical direction is several times smaller than the AF in the horizontal direction. For vertically incident SV waves \((\alpha = 0^\circ)\), due to the anti-symmetry of the loading, the vertical response along the central axis of the dam cross-section is equal to zero. For the rest of the dam body, the vertical response is only a fraction of the horizontal one, because the dam deforms mainly in shear. For obliquely incident waves, the differences in the vertical motion induced at the left and right sides of the dam cross-section are shown at the pair points B1 - B2, C1 - C4, and C2 - C3 in Figure 3.9. However, again, the overall vertical response is only a fraction of the horizontal response. Indeed, for \(\alpha = 30^\circ\), the maximum horizontal amplification at crest is about 5.5, whereas the maximum vertical amplification is equal to 1.4. The small amplitude of the vertical response may be explained if one considers that when an SV wave of amplitude \(A_1\) impinges at an angle \(\alpha = 30^\circ\) at the dam-foundation interface, the transmitted SV wave has an amplitude of about 2 \(A_1\) and forms an angle of 7° to the vertical, whereas the transmitted P wave has an amplitude of only 0.14 \(A_1\). Although the total motion at the dam-foundation interface includes additional waves returning from complex reflection paths within the dam body, the presence of almost vertically transmitted high-amplitude SV waves and small-amplitude P waves can explain the significant lateral and small vertical response.

Figures 3.13 and 3.14 plot the amplification AF of the vertical and horizontal motion, respectively, for the same dam subjected to P waves incident at angles \(\alpha = 0^\circ, 15^\circ, 30^\circ\) and 45°. As in the case of SV waves, the results for the P waves suggest that the spatial variability of the ground motion across the dam base has relatively small effect on the vertical motion. In contrast, the amplification AF of the horizontal motion shown in Figure
3.14 demonstrates that the amplification in the horizontal direction increases significantly as the angle of incidence of the P waves increases from \( \alpha = 0^\circ \) to \( 45^\circ \). Indeed, as \( \alpha \) increases there is a significant resonance peak of the horizontal amplification consistently at \( a_0 = 2.1 \), which corresponds to the first resonance in shear vibration for \( IR = 5 \). Notice that for \( \alpha = 45^\circ \) the horizontal crest amplification is about 4.65, i.e. almost as high as the maximum vertical amplification, which is equal to 4.85. The presence of such high horizontal response can be explained if one considers that when a P wave of amplitude \( A_i \) impinges at an angle \( \alpha = 45^\circ \) at the dam-foundation interface, the resulting refracted P wave has an amplitude of about \( A_i \) and forms an angle of \( 10^\circ \) to the vertical, whereas the refracted SV wave has an amplitude of \( 0.89 A_i \) and forms an angle of \( 5^\circ \) to the vertical. The presence of a high amplitude obliquely transmitted SV wave can explain the significant horizontal shear response and its dependence on the angle of incidence.

The above results demonstrate that P waves, incident at angles \( \alpha \geq 30^\circ \) and travelling across the dam width, may induce horizontal response that is particularly strong for frequencies near the first resonance in shear vibration. This additional horizontal response is caused by the spatial variability of the ground motion, which may induce antisymmetric (or, in general, asymmetric) vibrational modes that do not appear in the case of vertically incident waves (\( \alpha = 0^\circ \)). The conclusion that the response of a dam to a spatially variable motion may be substantially higher than that obtained for a space-invariant motion, drawn from this study, is in agreement with the conclusion drawn from the earlier studies (see Chopra et al., 1969 and Dibaj and Penzien 1969) that did not consider the dam-foundation interaction. Furthermore, it is in agreement with the conclusions from
similar studies considering SH waves propagating along the longitudinal direction of symmetric dams in semicircular, semielliptical and rectangular canyons. The latter studies have also shown that, even at small incidence angles, oblique waves induce additional "antisymmetric" vibrational modes, which may lead to substantially higher response in the dam than the response caused by vertically propagating waves (Dakoulas and Hsu, 1995; Dakoulas 1993a,b).

In reality, a seismic shaking experienced by a dam body on a homogeneous halfspace may consist of P, S and Rayleigh waves. To different degrees, each of these waves may significantly increase the dam response due the spatial variability of the ground motion, depending on the magnitude of the earthquake, the distance and depth of the source, etc. Among them, Rayleigh waves are expected to have a particularly significant effect in increasing the dam response, because they transfer a substantial amount of energy and travel in the horizontal direction along the surface of the halfspace, thereby, increasing the spatial variability of the ground motion.

3.5 Conclusions

A hybrid numerical formulation combining the Finite Element and the Boundary Element methods has been used to study the effects of dam-foundation interaction on the response of earth dams subjected to obliquely incident P and SV waves. The developed hybrid method was proven to be very powerful and could be used to solve accurately and efficiently soil-structure interaction problems of complex geometry and material heterogeneity. The two-dimensional frequency-domain formulation has been used here to investigate the response of long dams to SV and P waves travelling across the width of the
dam, assuming linear hysteretic soil behavior.

The results showed the significant effect of the flexibility of the foundation rock in reducing the overall response of the dam, by accounting rigorously for the energy radiated back into the halfspace. These effects are particularly significant near the first resonance. For a typical range of impedance ratio values between 2 to 8, the amplification results from all analyses using the rigorous FE-BE method and the simplified shear beam model are in fairly close agreement.

The effects of the spatial variability of the ground motion for SV waves travelling across the width of the dam seem to be relatively small for angles of incidence $\alpha \leq 30^\circ$. In contrast, the effects of the spatial variability of the ground motion for P waves increase significantly as the angle of incidence increases from $\alpha = 0^\circ$ to $45^\circ$. The response increases mainly in the horizontal direction due to the excitation of additional vibrational modes that do not appear in the case of vertically incident waves. The conclusion of this study, that the response of a dam to a spatially variable motion may be substantially higher than that caused by vertically propagating waves, is in agreement with findings from the earlier studies by Chopra et al. (1969) and Dibaj and Penzien (1969) that did not consider the dam - foundation interaction and from more recent studies considering SH waves propagating along the longitudinal direction of symmetric dams in semicircular, semielliptical and rectangular canyons (Dakoulas and Hsu, 1995; Dakoulas 1993a,b).
Figure 3.3 Steady-state horizontal amplification at various points within the dam body for vertically incident SV waves and impedance ratios $\text{IR} = 2, 3, 5, 10, \infty$. (Dam slope 2:1).
Figure 3.3 (continued) Steady-state horizontal amplification at various points within the dam body for vertically incident SV waves and impedance ratios IR = 2, 3, 5, 10, ∞. (Dam slope 2:1).
Figure 3.4  Steady-state horizontal amplification at various points within the dam body for vertically incident SV waves and impedance ratios IR = 2, 3, 5, 10, ∞. (Dam slope 3:1).
Figure 3.4 (continued) Steady-state horizontal amplification at various points within the dam body for vertically incident SV waves and impedance ratios IR = 2, 3, 5, 10, ∞. (Dam slope 3:1).
Figure 3.5  Steady-state horizontal amplification at various points within the dam body obtained from the Shear Beam model for vertically incident SV waves and impedance ratios IR = 2, 3, 5, 10, ∞.
Figure 3.6  Steady-state vertical amplification at various points within the dam body for vertically incident P waves and impedance ratios IR = 2, 3, 5, 10, ∞. (Dam slope 2:1).
Figure 3.6 (continued) Steady-state vertical amplification at various points within the dam body for vertically incident P waves and impedance ratios IR = 2, 3, 5, 10, $\infty$. (Dam slope 2:1).
Figure 3.7  Steady-state vertical amplification at various points within the dam body for vertically incident P waves and impedance ratios $IR = 2, 3, 5, 10, \infty$. (Dam slope 3:1).
Figure 3.7 (continued) Steady-state vertical amplification at various points within the dam body for vertically incident P waves and impedance ratios IR = 2, 3, 5, 10, ∞. (Dam slope 3:1).
Figure 3.8  Maximum crest amplification at first resonance versus the inverse of the impedance ratio. (Material damping $\beta_d=10\%$)
Figure 3.9 Steady-state horizontal amplification at various points within a dam subjected to obliquely incident SV waves at angles $\alpha = 0^\circ$, $15^\circ$ and $30^\circ$. (Impedance Ratios IR = 5; Dam slope 2:1).
Figure 3.9 (continued) Steady-state horizontal amplification at various points within a dam subjected to obliquely incident SV waves at angles $\alpha = 0^\circ$, $15^\circ$ and $30^\circ$. (Impedance Ratios IR = 5; Dam slope 2:1).
Figure 3.10 Steady-state horizontal amplification at various points within a dam subjected to obliquely incident SV waves at angles $\alpha = 0^\circ$, $15^\circ$ and $30^\circ$. (Impedance Ratios IR = 5; Dam slope 3:1).
Figure 3.10 (continued) Steady-state horizontal amplification at various points within a dam subjected to obliquely incident SV waves at angles $\alpha = 0^\circ, 15^\circ$ and $30^\circ$. (Impedance Ratios $IR = 5$; Dam slope 3:1).
Figure 3.11 Steady-state vertical amplification at various points within a dam subjected to obliquely incident SV waves at angles $\alpha = 0^\circ$, $15^\circ$ and $30^\circ$. (Impedance Ratios IR = 5; Dam slope 2:1).
Figure 3.11 (continued) Steady-state vertical amplification at various points within a dam subjected to obliquely incident SV waves at angles $\alpha = 0^\circ$,  $15^\circ$ and $30^\circ$. (Impedance Ratios IR = 5; Dam slope 2:1).
Figure 3.12 Steady-state vertical amplification at various points within a dam subjected to obliquely incident SV waves at angles $\alpha = 0^\circ$, $15^\circ$ and $30^\circ$. (Impedance Ratios IR = 5; Dam slope 3:1).
Figure 3.12 (continued) Steady-state vertical amplification at various points within a dam subjected to obliquely incident SV waves at angles $\alpha = 0^\circ$, $15^\circ$ and $30^\circ$. (Impedance Ratios IR = 5; Dam slope 3:1).
Figure 3.13 Steady-state horizontal amplification at various points within a dam subjected to obliquely incident P waves at angles $\alpha = 0^\circ, 15^\circ, 30^\circ$ and $45^\circ$. (Impedance Ratios IR = 5; Dam slope 2:1).
Figure 3.13 (continued) Steady-state horizontal amplification at various points within a dam subjected to obliquely incident P waves at angles $\alpha = 0^\circ, 15^\circ, 30^\circ$ and $45^\circ$. (Impedance Ratios IR = 5; Dam slope 2:1).
Figure 3.14 Steady-state vertical amplification at various points within a dam subjected to obliquely incident P waves at angles $\alpha = 0^\circ, 15^\circ, 30^\circ$ and $45^\circ$. (Impedance Ratios IR = 5; Dam slope 2:1).
Figure 3.14 (continued) Steady-state vertical amplification at various points within a dam subjected to obliquely incident P waves at angles $\alpha = 0^\circ$, $15^\circ$, $30^\circ$ and $45^\circ$. (Impedance Ratios IR = 5; Dam slope 2:1).
Chapter 4

Response of Earth Dams to Rayleigh Waves

4.1 Introduction

Earthquake induced Rayleigh waves travel along the ground surface like rolling ocean waves, carrying in most cases a substantial amount of the total wave energy released by the seismic source. Rayleigh waves attenuate slowly with the distance, \( r \), from the source, their amplitude decreasing in proportion to \( 1/r^{0.5} \). By comparison, body waves attenuate in proportion to \( 1/r \), except near the surface where they attenuate in proportion to \( 1/r^2 \). Therefore for sites at large distances from the source, Rayleigh waves may constitute the major seismic disturbance on the ground surface. Figure 4.1 illustrates a Rayleigh wave of wavelength \( \lambda \) travelling to the right with a velocity \( c_R \), as well as the motion path of a particle, forming a retrograde ellipse. Because they propagate horizontally and carry a significant amount of energy, Rayleigh waves may have a significant effect on the seismic behavior of long structures such as earth dams and embankments.

The combined action of strong lateral and vertical oscillation induced by Rayleigh waves may have a substantial effect on the seismic stability of earth dams. The residual displacement of a sliding mass, the generation of excess water pressure that may
potentially lead to liquefaction of contractive cohesionless soils, and the loss of strength of clayey soils depend on both the shear and vertical cyclic stresses induced by actual seismic waves, a major part of which may consist of Rayleigh waves. Note that a Rayleigh wave having a wavelength $\lambda = 500$ m, which is comparable with the base width of an 100 m high earth dam, may induce substantial differential motion at the dam base. For typical average values of shear wave velocities $c_d = 400$ m/s for the dam and $c_f = 1000$ m/s for the foundation rock, the excitation frequency corresponding to $\lambda = 500$ m is close to first natural frequencies of the dam in lateral shear and vertical vibration. Therefore, resonance phenomena and the spatial variability of the base motion would tend to increase significantly the response, as will be shown below. In addition, the retrograde nature of the Rayleigh wave particle motion (Figure 4.1) has been proven to increase even further the response of the structure (Wolf 1985).

4.2 Model Description and Parametric Study

The general formulation presented in Chapter 2 is now applied to study the effects of the foundation flexibility and the spatial variability of the ground motion on the response of infinitely long earth and rockfill dams.

The dam body consists of a linearly-hysteretic elastic material and is founded on an elastic halfspace (Figure 4.3). Two dams having the same height of $H = 100$ m and symmetric slopes of 2:1 and 3:1, respectively, are considered. The dam material has an average shear wave velocity $c_d = 400$ m/s, mass density $\rho_d = 2000$ kg/m$^3$, Poisson’s ratio $\nu_d = 1/3$, and damping ratio $\beta_d = 10\%$. The foundation material has a mass density $\rho_f = 2400$ kg/m$^3$, Poisson’s ratio $\nu_f = 1/3$ and no material damping. The effect of the
Figure 4.1  Rayleigh waves propagating along the halfspace surface
flexibility of the foundation is considered by examining a range of impedance ratio values, defined as

\[ IR = \frac{\rho_f c_f}{\rho_d c_d} \quad (4.1) \]

where \( c_f \) is the shear wave velocity of the foundation material. The dam body is discretized using four-node plane strain isoparametric elements, whereas the halfspace is discretized using two-node boundary elements. The discretized length of the halfspace surface is taken equal to five times the dam base width.

The excitation consists of harmonic Rayleigh waves of wavelength \( \lambda \) propagating from the left to the right direction across the width of the dam cross-section (Figure 4.3). The attenuation with depth of the horizontal and vertical amplitude of the Rayleigh wave motion is shown in Figure 4.2, for Poisson's ratios \( \nu_f = 1/4, 1/3 \) and 1/2. For \( \nu_f = 1/3 \) considered here, the Rayleigh wave velocity is \( c_R = 0.933 c_f \) while the ratio of the vertical motion over the horizontal motion is 1.56Si, where \( i = \sqrt{-1} \).

The results of the parametric study are presented in the following in terms of steady-state amplifications, \( AF \), of the (horizontal or vertical) motion within the dam body with respect to the horizontal component of the free-field surface motion, i.e.

\[ AF = \left| \frac{u_d(\omega)}{u_g} \right| \quad (4.2) \]

where \( u_d(\omega) = \) total displacement in the dam (in either the horizontal or vertical direction); \( u_g = \) horizontal free-field motion due to Rayleigh waves; and \( \omega \) is the frequency of the incident waves. In the following, \( AF \) is plotted versus a dimensionless frequency, \( a_0 \), equal to
Figure 4.2 Attenuation of the horizontal and vertical components versus depth for Poisson's ratios $\nu = 1/4$, $1/3$ and $1/2$
Figure 4.3  Earth dam cross-section on elastic halfspace subjected to Rayleigh waves
\[ a_0 = \frac{\omega H}{c_d} \] (4.3)

4.3 Effect of Impedance Ratio

It is of interest to examine first the effect of the flexibility of the elastic rock halfspace on the dam response. For the dam having slopes 2:1, Figure 4.4 shows the steady-state amplification, \( \text{AF} \), of the horizontal motion at four points A, B, C and D along the middle axis of the dam cross-section, as well as at symmetric points B1 and B2, and C1 and C4 on the slopes, and C2 and C3 in the middle of the upstream and downstream shells of the dam. The five curves in each plot correspond to impedance ratios \( \text{IR} = 2, 3, 5, 10 \) and \( \infty \) (rigid base). The results demonstrate that the flexibility of the base rock has a substantial effect on the lateral response of the dam near the first resonance in shear vibration, occurring at \( a_0 \approx 2.2 \) for \( \text{IR} = \infty \). More specifically, the mid-crest amplification \( \text{AF} \) at first resonance is equal to 8.5 for \( \text{IR} = 10 \) to \( \infty \), 7.33 for \( \text{IR} = 5 \), 4.2 for \( \text{IR} = 3 \) and 3.5 for \( \text{IR} = 2 \). Notice that the effect of IR is more significant in the range \( \text{IR} = 3 \) to 5. For frequencies \( a_0 > 4 \), the effect of the foundation flexibility is also important but at certain points within the dam it may appear more erratic. This can be seen, for example, at points A and B1 in Figure 4.4, where the lateral response increase as IR decreases.

Figure 4.5 shows the steady-state amplification, \( \text{AF} \), of the vertical motion of the same dam, evaluated at the same points as in Figure 4.4. Since the vertical response is normalized with respect to the horizontal free-field component, \( \text{AF} \) becomes equal to 1.565 as \( a_0 \) goes to zero. For \( \text{IR} = \infty \) the first resonance of the dam in vertical motion occurs at \( a_0 \approx 3.1 \). Notice that the effect of the flexibility of the base rock on the vertical
response of the dam is even greater than that for the horizontal motion, with the vertical amplification decreasing significantly as the impedance ratio decreases. This is partly due to the transferring of energy from vertical to rocking oscillation induced by the spatial variability of the ground motion, as discussed in more detail below. Moreover, for higher frequencies, the decrease of the vertical response with decreasing IR is more consistent than that of the horizontal response. (For example, compare the effect of IR on the horizontal and vertical response at points A and B1 in Figures 4.4 and 4.5).

Notice that Figures 4.4 and 4.5 show that the response in the middle zone of the dam is controlled mainly by the first resonance, whereas near the slopes it may also be significantly affected by higher vibrational modes. At the lower half of the dam body and away from the slopes, deamplification of the motion is observed. At the dam base (point D), this deamplification becomes more significant as IR decreases, especially at first resonance.

The different response values at the symmetric pair points B1 and B2, C1 and C4, and C2 and C3 in Figure 4.4 and Figure 4.5 demonstrate the effect of the horizontal propagation of the Rayleigh wave from the left to the right direction across the dam base. However, the comparison between the response of symmetric points at the left and right sides of the dam cross-section does not indicate any clear, consistent trend with respect to which of the two sides experiences higher amplification.

Figures 4.6 and 4.7 show, respectively, the horizontal and vertical amplification, AF, of a dam with similar properties with that examined in Figures 4.4 and 4.5, but having flatter slopes equal to 3:1. The results indicate more or less the same trends shown in Figures 4.4
and 4.5 regarding the effect of flexibility on the response at various points within the dam body. However, the vertical response in the flatter dam seems to be lower than in the steeper one. For example, the crest amplification reduces from \( \text{AF} = 10 \) for the 2:1 dam to \( \text{AF} = 8 \) for the 3:1 dam. Moreover, the contribution of the first vertical mode becomes much less important for almost all points within the body of the flatter dam, except near the middle axis (compare Figures 4.5 and 4.7). Similar trends, but to a much lesser degree, regarding the effect of dam slope, apply also for the horizontal response of the dam (e.g. compare Figures 4.4 and 4.6).

In conclusion, for the practically interesting range of IR values between 2 and 5, the response in the horizontal and especially in the vertical directions is substantially lower than that computed assuming a rigid base motion and depends mainly on the first few fundamental modes. The general conclusions regarding the effect of the impedance are in good agreement with those derived for the case of vertically and obliquely incident P and SV waves, although quantitative differences do of course exit due to the different nature of the seismic waves (Abouseeda and Dakoulas 1995).

### 4.4 Effect of the Spatial Variability of Ground Motion

To investigate the effect of the spatial variability of the ground motion, it is of interest to compare the horizontal and vertical dam response induced by the spatially variant Rayleigh wave to the response obtained by a superposition of vertically incident (i.e. spatially invariant) P and SV waves. This superposition is performed in such a way, so that the resulting free-field response has horizontal and vertical motions equal to \( u_g \) and 1.565 \( u_g \), respectively, i.e. identical to those of the Rayleigh waves for \( v_f = 1/3 \). For the
unrealistic, but theoretically interesting, limiting case of \( \text{IR} \to \infty \) (rigid base), the Rayleigh wave propagates with infinite speed and, therefore, the motion across the dam base is synchronous. In this case the input motion at the dam base and, consequently, the response within the entire dam body, are identical for the Rayleigh waves and the combined P and SV waves. This is shown in Figures 4.8 and 4.9 which plots the horizontal and vertical amplification at points A and B1 for the dam with slopes 3:1.

For identical dams and finite values of impedance ratio \( \text{IR} = 5 \), the horizontal amplification due to Rayleigh waves is larger than that obtained for the combined vertically incident P and SV waves. This is hardly surprising, as in this case the Rayleigh wave propagates at a finite speed and, therefore, the input ground motion varies across the dam base. Due to this variation, the significant vertical component of the Rayleigh wave at the base is inducing an additional rocking motion that contributes significantly to increasing the horizontal response of the upper dam body. Indeed, for \( \text{IR} = 5 \) and \( \nu_f = 1/3 \) the Rayleigh wave velocity is \( c_R = 0.933 \ c_f = 1.555 \text{ m/s} \) and the wavelength at the first resonance in shear rocking motion (in this case \( a_0 = 2.05 \)) is equal to \( \lambda = 1190 \text{ m} \), i.e. about double the width of the dam base (2b = 600 m, Figure 4.3). Such variation of the vertical component may rotate the dam base by an amplitude \( \theta_0 = 1.565 \ u_g/b \) and, thus, excite rocking vibration modes, while reducing the contribution of the vertical vibration modes. Consequently, for the whole dam body, and especially its critical upper part, the horizontal response due to the Rayleigh wave is larger than that of the combined P and SV waves. This is clearly demonstrated in Figures 4.10 and 4.11, which plot the horizontal and vertical amplification \( \text{AF} \) at points A and B1, respectively, for a dam with impedance...
ratio $IR = 5$ and slopes 3:1. The solid curve corresponds to the superposition of the vertically incident P and SV waves whereas the dashed curve corresponds to the Rayleigh waves. Notice that the horizontal crest amplification $AF$ is 30% higher for the Rayleigh wave at first resonance and 2 to 4 times higher at frequencies $a_0$ between 4 and 7, corresponding to higher shear/rocking modes. Similarly higher amplification values for the Rayleigh wave are observed for this frequency range at point B1 (Figure 4.11).

By contrast, the vertical crest amplification at first resonance for the Rayleigh wave ($AF = 3.5$) is about half the amplification for the combined P and SV waves ($AF = 6.9$). As the spatial variability of the vertical component of the base motion is exciting more rocking vibration modes, the contribution of the vertical vibration modes that would be normally excited by vertically-incident synchronous P waves is reduced. Consequently, the vertical response especially the upper part of the dam due to the Rayleigh wave appears to be smaller than that due to the combined P and SV waves.

In the above comparison of the effects of Rayleigh waves to those of combined P and SV waves, the latter were assumed to be vertically incident. For obliquely incident P waves, the induced spatial variability of the ground motion across the width of the dam base has relatively small effect on the vertical motion, but increases significantly the horizontal motion as the angle of incidence increases from $\alpha = 0^\circ$ to $45^\circ$ (Abouseeda and Dakoulas 1995). For example, for the 100 m high dam, having slopes 2:1 and $IR = 5$, subjected to P waves incident at $\alpha = 45^\circ$, the horizontal crest amplification is about 4.65, almost as high as the vertical amplification, which is equal to 4.85. In general, P waves incident at angles $\alpha \geq 30^\circ$ and travelling across the dam width, may induce horizontal
response that is particularly strong near the first few natural frequencies in shear/rocking vibration, due to excitation of rocking vibrational modes that do not appear in the case of vertically incident P waves. This is in agreement with the conclusion on the effects of the spatial variability due to Rayleigh waves. It is also in agreement with the conclusions of earlier studies that did not consider the dam - foundation interaction (Dibaj and Penzien 1969) and with those from studies considering SH waves propagating along the longitudinal direction of symmetric dams in semicircular, semielliptical and rectangular canyons (Dakoulas et al. 1992, 1995; Dakoulas 1993a,b). These studies have also shown that oblique waves induce additional vibrational modes, which may lead to substantially higher response in the dam than the response caused by vertically propagating waves.

In reality, a seismic shaking experienced by a dam body on a homogeneous halfspace may consist of P, S and Rayleigh waves. To different degrees, each of these waves may control the dam response, depending on the magnitude of the earthquake, the distance and depth of the source, etc. Among them, Rayleigh waves are expected to have a particularly significant effect in increasing the dam response, because they transfer a substantial amount of energy and travel in the horizontal direction along the surface of the halfspace, thereby, increasing the spatial variability of the ground motion.

4.5 Conclusions

A frequency-domain hybrid numerical formulation combining the Finite Element and the Boundary Element methods has been used to study the effects of dam-foundation interaction on the response of earth dams subjected to Rayleigh waves travelling across the width of the dam. The dam body is assumed to be infinitely long, consisting of a
linearly-hysteretic elastic material and resting on a linearly-hysteretic elastic halfspace.

The results showed the significant effect of the flexibility of the foundation rock in reducing the overall response of the dam, by accounting rigorously for the radiation of energy. Specifically, the flexibility of the base rock has a substantial effect on the lateral response, which decreases with decreasing impedance ratio near the first resonance in shear vibration. For frequencies \( a_0 > 4 \), the horizontal response may decrease or increase with decreasing IR. The effect of the flexibility of the base rock on the vertical response of the dam is even more profound than that for the horizontal motion, with the vertical amplification decreasing significantly as the impedance ratio decreases. Comparison between the response of symmetric points at the left and right sides of the dam cross-section does not seem to indicate any clear, consistent trend with respect to which of the two sides experiences higher amplification. Results for two dams with slopes 2:1 and 3:1, respectively, showed more or less similar trends regarding the effect of flexibility and yielded lower vertical response for the flatter dam.

The effect of the spatial variability of the ground motion is shown by comparison of the response of dam to (a) Rayleigh waves and (b) vertically incident combined P and SV waves of the same free-field surface motion amplitude. It is shown that the horizontal amplification due to Rayleigh waves is larger than that obtained for the combined vertically incident P and SV waves. The spatial variability of the vertical component of the ground motion caused by the Rayleigh waves is inducing additional rocking motion that contributes significantly to increasing the horizontal response, especially in the upper part of the dam body. Such variation of the vertical component may rotate the dam base and,
thus, excite rocking modes, while reducing the contribution of the vertical vibration modes. In the example considered, the horizontal crest amplification is 30% higher for the Rayleigh wave at first resonance and 2 to 4 times higher at frequencies $a_3$ between 4 and 7 compared to the response of the combined P and SV waves. The conclusion of this study, that the response of a dam to a spatially variable motion may be substantially higher than that caused by a synchronous excitation, is in agreement with findings from earlier studies that did not consider the dam - foundation interaction and from more recent studies considering SH waves propagating along the longitudinal direction of dams in semi-circular, semielliptical and rectangular canyons.

Finally, the developed hybrid FE-BE method was proven to be very powerful and can be used to solve accurately and efficiently soil-structure interaction problems of complex geometry and material heterogeneity.
Figure 4.4 Horizontal steady-state response to Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, \( \infty \), (Dam slope 2:1).
Figure 4.4 (continued) Horizontal steady-state response to Rayleigh waves at various points within the dam body for impedance ratios $IR = 2, 3, 5, 10, \infty$. (Dam slope 2:1).
Figure 4.5  Vertical steady-state response to Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, \( \infty \). (Dam slope 2:1).
Figure 4.5 (continued) Vertical steady-state response to Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞. (Dam slope 2:1).
Figure 4.6  Horizontal steady-state response to Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞. (Dam slope 3:1).
Figure 4.6 (continued) Horizontal steady-state response to Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞. (Dam slope 3:1).
Figure 4.7  Vertical steady-state response to Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Dam slope 3:1).
Figure 4.7 (continued) Vertical steady-state response to Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞.
(Dam slope 3:1).
Figure 4.8  Horizontal and vertical steady-state response at Point A to vertically incident combined P and SV waves and Rayleigh waves for impedance ratio IR = ∞.
Figure 4.9 Horizontal and vertical steady-state response at Point B1 to vertically incident combined P and SV waves and Rayleigh wave for impedance ratio $IR = \infty$. 
Figure 4.10 Horizontal and vertical steady-state response at Point A to vertically incident combined P and SV waves and Rayleigh waves for impedance ratio IR = 5.
Figure 4.11 Horizontal and vertical steady-state response at Point B1 to vertically incident combined P and SV waves and Rayleigh wave for impedance ratio \( IR = 5 \).
Chapter 5

Response of Concrete-Faced Rockfill Dams to Rayleigh Waves

5.1 Introduction

The concrete-faced rockfill (CFR) dam has been used very successfully in many parts of the world over the last 25 years, offering very frequently the most economical design (Cooke and Sherard 1987, Sherard and Cooke 1987). A number of CFR dams have been built in areas of low to moderate seismicity using very steep slopes, such as 1.3:1 to 1.6:1. The few existing actual records of seismic response of CFR dams and results from analytical studies suggest that despite the very steep slopes, the seismic behavior of CFR dams in regions of low to moderate seismicity is satisfactory. However, there is little recorded evidence about the response of such structures to very strong ground motion.

Due to the substantial success and great potential of the CFR dam, it is of interest to investigate the response of such dams to strong ground shaking. Such response differs from the response of Earth-Core Rockfill (ECR) dams, due to the presence of the concrete facing that acts like an impervious membrane. As most of the body of the CFR dam is unsaturated, the weight of the reservoir water acts directly on the dam rockfill increasing...
substantially its stiffness and strength. As a result, the dam displays a stiffer, more “elastic” response than the ECR dam. During an earthquake, although in general smaller displacements are expected, significant accelerations may be experienced in the near crest area that may pose problems to the concrete parapet wall, the upper part of the slab and the rockfill material. These near-crest accelerations can be further intensified by the high stiffness of the generally good-quality foundation rock used for CFR dams, resulting in smaller radiation damping, as well as by the three-dimensional effects of a narrow canyon.

The main motivation of this study is the crucial dependence of the CFR dam on the good performance of the concrete slab. If, during a ground shaking, the concrete slab cracks and allows water within the dam rockfill, the overall stiffness of the upstream side of the dam will be reduced significantly. In this case, the dam will behave as a rockfill dam with very steep slopes, which would be more vulnerable to seismic motion.

Due to their propagation along the surface and their retrograde elliptic motion, Rayleigh waves may induce substantial spatial variability of the ground motion along the dam base. Such spatial variability, combined with the high contribution of rocking vibrational modes due to the higher steepness of the slopes of such dams, could lead to significant tensile stresses and cracking of the concrete slab.

The current study examines the response of CFR dams to a moderate shaking, with emphasis placed on the response of the near crest area, and more critical upstream side.

Before proceeding with the seismic analysis, it is of interest to illustrate through an example the salient features of the CFR dam and highlight its important advantages. Figure 5.1a presents a typical cross-section of a CFR dam, consisting of a number of zones
TYPICAL MATERIAL COMPOSITION

<table>
<thead>
<tr>
<th>Zone</th>
<th>Maximum Rock Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A</td>
<td>5 cm</td>
</tr>
<tr>
<td>2B</td>
<td>20 cm</td>
</tr>
<tr>
<td>2C</td>
<td>40 cm</td>
</tr>
<tr>
<td>3A</td>
<td>90 cm</td>
</tr>
<tr>
<td>3B</td>
<td>150 cm</td>
</tr>
</tbody>
</table>

Figure 5.1 Concrete-Faced Rockfill Dam: (a) Typical Cross-Section and Material Composition; (b) Crest Detail; (c) Toe Slab Detail; (d) Parametric Joint Detail (Modified after Gazetas and Dakoulas, 1992).
Figure 5.1 (continued) Concrete-Faced Rockfill Dam: (a) Typical Cross-Section and Material Composition; (b) Crest Detail; (c) Toe Slab Detail; (d) Parametric Joint Detail (Modified after Gazetas and Dakoulas, 1992).
of different rock sizes, the concrete face slab, the toe slab, the overlying impervious fill and the grout curtain (Gazetas and Dakoulas 1992). A list of possible maximum rock sizes for each of the five indicated zones is also provided. Figure 5.1b is a detail of the crest area of the dam, illustrating the concrete face slab, the crest wall and the zones of the rockfill material. Figure 5.1c illustrates the toe slab, the concrete slab, the rockfill zones, the grout curtains and the grouted dowels, whereas Figure 5.1d shows a detail of the perimetric joint with the various materials used to prevent leakage.

From a technical point of view, the modern CFR dam seems indeed to be an excellent choice. Compared with the earth-core rockfill (ECR) dam, the CRF dam offers improved stability, very small settlements, smaller (and less dangerous) leakage, easier foundation treatment, more flexible work schedule, and less construction difficulties in rainy weather (Cooke and Sherard 1987). As worldwide experience is accumulating on the long-term performance of CFR dams and dam designers become more confident, their popularity will most likely increase in coming years. As a result of the good performance and gained confidence, there is a trend for higher dams, thinner slabs, less reinforcement, improved joint design, and more frequent use (Cooke 1984). Detailed accounts of current design trends and performance records of many recent dams are given by Cooke and Sherard (1985, 1987) and Sherard and Cooke (1987). Initial evaluations of the seismic performance of CFR dams have been given by Seed et al. (1985), Burea et al. (1985), Gazetas and Dakoulas (1992), Uddin and Gazetas (1995), and Uddin (1992).

5.2 Model Description and Parametric Study

Two dams are considered in this study, with a height H = 100 m and symmetric slopes of 1.3:1 and 1.6:1, respectively, a 10 m wide flat crest, and a 0.4 m concrete slab in the
upstream side of the dam. The dam material is nonhomogeneous, and the maximum shear modulus $G_0$ can be computed, using the empirical formula (see Figure 5.3)

$$G_0(\text{KPa}) = 1000 \times K \sqrt{\sigma_0(\text{KPa})}$$ (5.1)

where $\sigma_0$ is the overburden pressure, and the typical range of $K$ for very dense sand and gravel is between 30 and 40. The average Young's modulus $E_d = 8.53 \times 10^5 \text{kN/m}^2$, mass density $\rho_d = 2000 \text{ Kg/m}^3$, Poisson's ratio $\nu_d = 1/3$, average shear wave velocity $c_d = 400 \text{ m/s}$ and Rayleigh damping ratio $\beta_d = 10\%$. The foundation material has a mass density $\rho_f = 2400 \text{ Kg/m}^3$, Poisson's ratio $\nu_f = 1/3$ and no material damping. The effect of the foundation flexibility is accounted for by considering five values of the impedance ratio $IR = 2, 3, 5, 10 \text{ and } \infty$ defined as

$$IR = \frac{\rho_f c_f}{\rho_d c_d}$$ (5.2)

where $c_f$ is the shear wave velocity of the foundation material. The dam body is discretized using four-node plane strain isoparametric elements, whereas the halfspace is discretized using two-node boundary elements. The discretized length of the halfspace surface is taken equal to five times the dam base width.

The dam is subjected to Rayleigh waves propagating in both upstream-downstream (UD) and downstream-upstream (DU) directions. For $\nu_f = 1/3$, the Rayleigh wave velocity is $c_R = 0.933 c_f$, while the ratio of the vertical motion over the horizontal motion is $1.565i$, where $i = \sqrt{-1}$, indicating a phase difference of 90° between the motion in the two directions. The steady state response to such wave excitations are obtained for a range of the normalized frequency $a_0$, defined as
\[ a_0 = \frac{\omega H}{c_d} \]  

(5.3)

The results of the parametric study are presented in the following in terms of steady-state amplifications, \( AF \), of the (horizontal or vertical) motion at different points within the dam body (see Figure 5.2) with respect to the horizontal component of the free-field surface motion, i.e.

\[ AF = \left| \frac{u_d(\omega)}{u_g} \right| \]  

(5.4)

where \( u_d(\omega) \) = total displacement in the dam (in either the horizontal or vertical direction);

\( u_g \) = horizontal free-field motion due to Rayleigh waves.

5.3 Effect of Impedance Ratio

The effect of the flexibility of the elastic rock halfspace on the CFR dam response is somewhat similar to that of the rockfill or earth dam. For the CFR dam having slopes 1.3:1 and subjected to UD incident Rayleigh waves, Figure 5.4 shows the steady-state horizontal amplification, \( AF \), of the horizontal motion at four points A, B, C and D along the middle axis of the dam cross-section, as well as at symmetric points B1 and B2, and C1 and C4 on the slopes, and C2 and C3 in the middle of the upstream and downstream shells of the dam. The five curves in each plot correspond to impedance ratios \( IR = 2, 3, 5, 10 \) and \( \infty \) (rigid base). The results demonstrate that the flexibility of the base rock has a substantial effect on the lateral response of the dam near the first resonance in shear vibration, occurring at \( a_0 \approx 2.3 \) for \( IR = \infty \). More specifically, the mid-crest amplification \( AF \) at first resonance is equal to 10 for \( IR = 5 \) to \( \infty \), 7.8 for \( IR = 3 \), and 5.2 for \( IR = 2 \).
Figure 5.2  Earth dam cross-section on elastic halfspace subjected to Rayleigh waves
Notice that the effect of IR is more significant in the range IR = 2 to 5. For frequencies $a_0 > 4$, the effect of the foundation flexibility is also important but at certain points within the dam it may appear more erratic. This can be seen, for example, at point A in Figure 5.4, where the lateral response increase as IR decreases.

Figure 5.6 shows the steady-state amplification, $AF$, of the horizontal motion at the same points for the CFR dam having slopes 1.6:1, subjected to UD incident Rayleigh waves. The mid-crest amplification $AF$ at first resonance is equal to 9.8 for $IR = 5$ to $\infty$, 6.3 for $IR = 3$, 4 and 4.2 for $IR = 2$. The same trends observed in the steeper dam can also be seen for this dam.

It is worth noticing that, despite of the difference in slope, the horizontal amplification for both dams is almost identical at points along the axis of the dam (e.g. A, B, C and D) near the fundamental frequency ($a_0 = 2.3$). However, there is some difference in the horizontal amplification at points away from the axis of the dam (e.g. B1, B2, C1..... C4) between the two dams, which is expected because the contribution of the rocking modes of vibration depends on the dam slope. Another observation is that the amplification values are higher for the steeper dam, which is due to lower overall stiffness in the near crest area of the steeper dam.

To examine the effects of the stiffness inhomogeneity within the dam body due to the substantial water pressure acting on the upstream concrete face (see Figure 5.3), additional analyses were performed with Rayleigh waves propagating in the opposite DU direction.

For the two CFR dams considered having slopes of 1.3:1 and 1.6:1, Figures 5.8 and 5.10 show the steady-state horizontal amplification of motion. Similar trends with respect
to the effect of IR are observed as those discussed above. The effect of the spatial variability of the ground motion and the direction of propagation of the Rayleigh waves are discussed in the following section.

Figure 5.5 shows the steady-state amplification, $A_F$, of the vertical motion of the dam having slopes 1.3:1 and subjected to UD incident Rayleigh waves, evaluated at the same points mentioned before. Since the vertical response is normalized with respect to the horizontal free-field component, $A_F$ becomes equal to 1.565 as $a_0$ goes to zero. For $IR = \infty$ the first resonance of the dam in vertical motion occurs at $a_0 \approx 3.6$. Notice that the effect of the flexibility of the base rock on the vertical response of the dam is even greater than that for the horizontal motion, with the vertical amplification decreasing significantly as the impedance ratio decreases. This is partly due to the transferring of energy from vertical to rocking oscillation induced by the spatial variability of the ground motion, as discussed in more detail in Chapter 4. It is also due to the fact that the radiation damping corresponding to vertical motion is higher than that for swaying motion (Gazetas 1991). Moreover, for higher frequencies, the decrease of the vertical response with decreasing IR is more consistent than that of the horizontal response. (For example, compare the effect of IR on the horizontal and vertical response at point A and in Figures 5.4 and 5.5).

Notice that Figures 5.4 and 5.5 show that the response in the middle zone of the dam is controlled mainly by the first resonance, whereas near the slopes it may also be significantly affected by higher vibrational modes. At the lower half of the dam body and away from the slopes, deamplification of the motion is observed. At the dam base (point D), this deamplification becomes more significant as IR decreases, especially at first
resonance.

Figure 5.7 shows the vertical amplification, $AF$, of the dam with slopes equal to 1.6:1 subjected to UD incident Rayleigh waves (evaluated at the same points). The results indicate more or less the same trends shown in Figure 5.5 regarding the effect of flexibility on the response at various points within the dam body. However, the vertical response in the flatter dam seems to be lower than in the steeper one. For example, the crest amplification reduces from $AF = 13.5$ for the 1.3:1 dam to $AF = 12$ for the 1.6:1 dam. Moreover, the contribution of the first vertical mode becomes much less important for almost all points within the body of the flatter dam, except near the middle axis (compare Figures 5.5 and 5.7).

For the CFR dam having slopes 1.3:1 and 1.6:1, Figures 5.9 and 5.11 show the steady-state vertical amplification, $AF$, of the horizontal motion at various points in the dam body, subjected to DU incident Rayleigh waves, respectively. The same trends mentioned above can be traced in these figures.

In conclusion, for the practically interesting range of IR values between 2 and 5, the response in the horizontal and especially in the vertical directions is substantially lower than that computed assuming a rigid base motion and depends mainly on the first few fundamental modes. The general conclusions regarding the effect of the impedance are in good agreement with those derived for the case of P, SV and Rayleigh waves on earth/rockfill dams, although some quantitative differences do of course exit (Abouseeda and Dakoulas 1995, 1996).
5.4 Effect of Concrete Face Slab

The existence of a concrete face slab, subjected to the overlying water pressure results in higher stiffness in the upstream side of the dam body affecting its dynamic response characteristics. Figure 5.12 shows the horizontal and vertical amplification contours of the 1.3:1 dam for UD Rayleigh waves at a frequency $\omega_0 = 2.3$, and impedance ratio IR = 3. Figure 5.13 shows the same contours but for a DU Rayleigh wave incidence. Similarly, Figures 5.14 and 5.15 show the horizontal and vertical amplification contours of the 1.6:1 dam for UD and DU Rayleigh waves, respectively, at a frequency $\omega_0 = 2.3$ (IR = 3).

For this frequency, the dam response is almost totally in the first vibrational shear mode. It can be noticed that, in both the UD and DU incident waves, the horizontal amplification is slightly larger on the side of the dam closer to the source of Rayleigh waves. This is partly due to the spatial variability of the ground motion, whereas the existence of a concrete face does not seem to have a significant effect on it.

The same type of behavior can be noticed on the vertical response as well. The side of the dam closer to the source of the Rayleigh waves experiences larger amplification of the ground motion, but in this case, the effect of having the concrete face is more evident on the response. The vertical amplification in the case of the UD Rayleigh waves near the upstream side is about 2.1 for the 1.3:1 dam and 1.9 for the 1.6:1 dam, whereas the vertical amplification in the case of DU Rayleigh waves near the downstream side is about 2.7 for the 1.3:1 dam and 2.4 for the 1.6:1 dam. Having the concrete slab on the upstream face of the dam resulted in a decrease of the vertical amplification on that side, due to the confinement provided to the outer rock shell of the dam by the gravitational weight of the
overlying water and the concrete slab.

Figures 5.16 and 5.17 show the stress contours for the major principal stress $\sigma_{\text{max}}$, the minor principal stress $\sigma_{\text{min}}$ and the maximum shear stress $\tau_{\text{max}}$ normalized by a factor $p_0$ for the CFR dam having 1.3:1 slopes, subjected to UD and DU Rayleigh waves, respectively, at a frequency $a_0 = 2.3$. The factor $p_0$ is defined as

$$p_0 = 100 \rho_d g u_b$$  \hspace{1cm} (5.5)$$

where $g$ is the gravitational acceleration, and $u_b$ is the horizontal base displacement. These figures show that the maximum values for $\sigma_{\text{max}}$ in the case of UD wave incidence, are at the base of the dam, very close to the concrete slab, where in the case of DU wave incidence, they are at the core of the dam, and near the crest close to the concrete slab. It is interesting to notice in these figures that for the UD wave incidence, the values of $\sigma_{\text{max}}$ at the downstream slope region of the dam are only a small fraction of those near the concrete slab region (in the upstream slope). The same conclusion applies also for the DU wave incidence. This indicates that the existence of the concrete slab and applied water pressure have a significant impact on the dynamic response of the rockfill layers in the part of the dam close to the slab. Maximum shear stresses are somewhat smaller in value for the UD wave incidence, compared with those for DU wave incidence. The maximum shear stresses are the highest at the base region near the concrete slab for the UD wave incidence, whereas they are almost symmetric for the DU wave incidence.

Figures 5.18 and 5.19 show the normalized principal stress and maximum shear stress contours for a CFR dam having a slope of 1.6:1, and subjected to UD and DU Rayleigh waves, respectively, at a frequency $a_0 = 2.3$. Both figures show similar response
characteristics to those observed in the 1.3:1 dam. Even though the dam sizes are significantly different, the dynamic stress values and their distributions are very similar. This indicates that, increasing the side slopes of a CFR dam does not have significant effect on the dynamic response characteristics of the dam near the fundamental resonance in shear vibration. However, the flatter dam would have better stability than the steeper dam, being able to resist larger seismic stresses against a slope failure.

Figure 5.20 shows the horizontal and vertical amplification contours of the 1.3:1 dam for UD Rayleigh waves at a frequency \( a_0 = 3.6 \), and impedance ratio \( IR = 3 \). Note the amplification values in that figure corresponding to the first resonance in vertical (compression - extension) vibration are as low (AF \( \approx 3 \)) due to the low value of the impedance ratio \( IR = 3 \). Figure 5.21 shows the same contours but for DU Rayleigh waves. Also, Figures 5.22 and 5.23 show the horizontal and vertical amplification contours of the 1.6:1 dam for UD and DU Rayleigh waves, respectively, at a frequency \( a_0 \approx 2.3 \) and impedance ratio \( IR = 3 \).

At the first resonance in the vertical oscillation, the dam response is almost totally in the first compression-extension vibrational mode. It can be noticed that, in both the UD and the DU incident cases, the vertical amplification is somewhat smaller on the concrete slab side of the dam. The substantial confinement provided by the concrete slab tends to restrain the motion in the region of the dam close to the concrete slab, resulting in lower values for the vertical amplification. The horizontal amplification follows similar trends (see Figures 5.20 to 5.23).

Figures 5.24 and 5.25 show the normalized stress contours of \( \sigma_{max} \), \( \sigma_{min} \) and \( \tau_{max} \) for a
the CFR dam having a slope of 1.3:1, and subjected to UD and DU Rayleigh waves, respectively, at a frequency \( a_0 = 3.6 \). These figures show that the maximum values for \( \sigma_{max} \) for both UD and DU wave incidence, are at the core of the dam, slightly towards the concrete slab. The maximum values for \( \sigma_{max} \) are significantly higher for the DU wave incidence case, compared to those for UD wave incidence. Indeed, for the UD wave incidence, the maximum value for \( \sigma_{max} \approx 87 \rho_0 \), whereas the same value for the DU wave incidence \( \sigma_{max} \approx 145 \rho_0 \), i.e. about 60\% increase. This demonstrates clearly the significant effect of the concrete slab in substantially increasing the seismic stresses within the dam. Similar behavior can be seen for the 1.6:1 dam (see Figures 5.26 and 5.27). Contrary to the results shown for \( a_0 = 2.3 \), principal stress values are significantly higher for the wider dam. This suggests that a flatter CFR dam would experience higher stress levels within its body, if it is excited at a frequency close to its fundamental resonance in vertical oscillation.

5.5 Conclusions

A frequency-domain hybrid numerical formulation combining the Finite Element and the Boundary Element methods has been used to study the effects of dam-foundation interaction on the response of CFR dams subjected to Rayleigh waves travelling across the width of the dam. The dam body is assumed to be infinitely long, consisting of a linearly-hysteretic elastic material and resting on a linearly-hysteretic elastic halfspace.

The results showed the significant effect of the flexibility of the foundation rock in reducing the overall response of the dam, by accounting rigorously for the radiation of energy. Specifically, the flexibility of the base rock has a substantial effect on the lateral
response, which decreases with decreasing impedance ratio near the first resonance in shear vibration. The effect of the flexibility of the base rock on the vertical response of the dam is even more profound than that for the horizontal motion, with the vertical amplification decreasing significantly as the impedance ratio decreases. These conclusions are in agreement with the results of the Earth/Rockfill dams examined in Chapter 4.

Despite of their difference in slope, the horizontal amplification for both dams considered earlier are almost identical at points along the axis of the dam near the fundamental frequency \( (a_0 = 2.3) \). However, there is some difference in the horizontal amplification between the two dams at points away from the axis of the dam, which is expected because the contribution of the rocking modes of vibration in the case of the steeper dam is more significant. Another observation is that the amplification values are higher for the steeper dam, which is due to its lower overall stiffness in the near crest area.

The horizontal amplification is slightly larger on the side of the dam towards the source of Rayleigh waves for both the UD and the DU wave incidence at the first vibrational shear mode frequency. This is partly due to the spatial variability of the ground motion, whereas the existence of a concrete face slab does not seem to have significant effect on it. The vertical amplification factors are also higher, but in this case, the stiffening effect of the concrete face slab is more evident on the response. Having the concrete slab on the upstream face of the dam resulted in a decrease of the vertical amplification on that side, due to the confinement provided to the outer rock shell of the dam by the gravitational weight of the overlying water and the concrete slab.

For both UD and DU Rayleigh wave incidence, the values of \( \sigma_{max} \) near the
downstream slope at the first vibrational shear mode frequency are significantly less than that of the region near the concrete slab. Maximum shear stresses are somewhat less for UD Rayleigh wave incidence, compared to those for DU wave incidence.

The vertical amplification of a CFR dam near the first compression-extension natural frequency is somewhat smaller near the upstream slope of the dam due to the substantial confinement provided by the concrete slab pressure. Finally, the developed hybrid FE-BE method was proven to be very powerful and can be used to solve accurately and efficiently soil-structure interaction problems of complex geometry and material heterogeneity.
Figure 5.3  Distribution of shear modulus at zero strains, \((G_0, \text{kPa})\) within the dam body
Figure 5.4  Horizontal steady-state response to Upstream-Downstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Concrete-Faced Rockfill Dam, slope 1.3:1).
Figure 5.4 (continued) Horizontal steady-state response to Upstream-Downstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Concrete-Faced Rockfill Dam, slope 1.3:1).
Figure 5.5  Vertical steady-state response to Upstream-Downstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Concrete-Faced Rockfill Dam, slope 1.3:1).
Figure 5.5 (continued) Vertical steady-state response to Upstream-Downstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, \( \infty \) (Concrete-Faced Rockfill Dam, slope 1.3:1).
Figure 5.6  Horizontal steady-state response to Upstream-Downstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Concrete-Faced Rockfill Dam, slope 1.6:1).
Figure 5.6 (continued) Horizontal steady-state response to Upstream-Downstream Rayleigh waves at various points within the dam body for impedance ratios $IR = 2, 3, 5, 10, \infty$ (Concrete-Faced Rockfill Dam, slope 1.6:1).
Figure 5.7  Vertical steady-state response to Upstream-Downstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ¥ (Concrete-Faced Rockfill Dam, slope 1.6:1).
Figure 5.7 (continued) Vertical steady-state response to Upstream-Downstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, \infty (Concrete-Faced Rockfill Dam, slope 1.6:1).
Figure 5.8 Horizontal steady-state response to Downstream-Upstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Concrete-Faced Rockfill Dam, slope 1.3:1).
Figure 5.8 (continued) Horizontal steady-state response to Downstream-Upstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Concrete-Faced Rockfill Dam, slope 1.3:1).
Figure 5.9 Vertical steady-state response to Downstream-Upstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Concrete-Faced Rockfill Dam, slope 1.3:1).
Figure 5.9 (continued) Vertical steady-state response to Downstream-Upstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Concrete-Faced Rockfill Dam, slope 1.3:1).
Figure 5.10 Horizontal steady-state response to Downstream-Upstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Concrete-Faced Rockfill Dam, slope 1.6:1).
Figure 5.10 (continued)  Horizontal steady-state response to Downstream-Upstream Rayleigh waves at various points within the dam body for impedance ratios $\text{IR} = 2, 3, 5, 10, \infty$ (Concrete-Faced Rockfill Dam, slope 1.6:1).
Figure 5.11 Vertical steady-state response to Downstream-Upstream Rayleigh waves at various points within the dam body for impedance ratios $IR = 2, 3, 5, 10, \infty$ (Concrete-Faced Rockfill Dam, slope 1.6:1).
Figure 5.11 (continued) vertical steady-state response to Downstream-Upstream Rayleigh waves at various points within the dam body for impedance ratios IR = 2, 3, 5, 10, ∞ (Concrete-Faced Rockfill Dam, slope 1.6:1).
Figure 5.12 Steady-state amplification contours for Upstream-Downstream Rayleigh waves at frequency $a_0 = 2.3$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.3:1)
Figure 5.13 Steady-state amplification contours for Downstream-Upstream Rayleigh waves at frequency $a_0 = 2.3$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.3:1)
Figure 5.14 Steady-state amplification contours for Upstream-Downstream Rayleigh waves at frequency $a_0 = 2.3$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.6:1)
Figure 5.15 Steady-state amplification contours for Downstream-Upstream Rayleigh waves at frequency $a_0 = 2.3$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.6:1)
Figure 5.16 Steady-state principle stresses and maximum shear stress contours for Upstream-Downstream Rayleigh waves at frequency $a_0 = 2.3$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.3:1)
Figure 5.17 Steady-state principle stresses and maximum shear stress contours for Downstream-Upstream Rayleigh waves at frequency $a_0 = 2.3$ and impedance ratios IR = 3. (Concrete-Faced Rockfill Dam, slope 1.3:1)
Figure 5.18 Steady-state principle stresses and maximum shear stress contours for Upstream-Downstream Rayleigh waves at frequency $a_0 = 2.3$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.6:1)
Figure 5.19 Steady-state principle stresses and maximum shear stress contours for Downstream-Upstream Rayleigh waves at frequency $a_0 = 2.3$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.6:1)
Figure 5.20 Steady-state amplification contours for Upstream-Downstream Rayleigh waves at frequency $a_0 = 3.6$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.3:1)
Figure 5.21 Steady-state amplification contours for Downstream-Upstream Rayleigh waves at frequency $\omega_0 = 3.6$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.3:1)
Figure 5.22 Steady-state amplification contours for Upstream-Downstream Rayleigh waves at frequency $a_0 = 3.6$ and impedance ratios IR = 3. (Concrete-Faced Rockfill Dam, slope 1.6:1)
Figure 5.23 Steady-state amplification contours for Downstream-Upstream Rayleigh waves at frequency $a_0 = 3.6$ and impedance ratios IR = 3. (Concrete-Faced Rockfill Dam, slope 1:6:1)
Figure 5.24 Steady-state principle stresses and maximum shear stress contours for Upstream-Downstream Rayleigh waves at frequency $a_0 = 3.6$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.3:1)
Figure 5.25 Steady-state principle stresses and maximum shear stress contours for Downstream-Upstream Rayleigh waves at frequency $\omega_0 = 3.6$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.3:1)
Figure 5.26 Steady-state principle stresses and maximum shear stress contours for Upstream-Downstream Rayleigh waves at frequency $a_0 = 3.6$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.6:1)
Figure 5.27 Steady-state principle stresses and maximum shear stress contours for Downstream-Upstream Rayleigh waves at frequency $a_0 = 3.6$ and impedance ratios $IR = 3$. (Concrete-Faced Rockfill Dam, slope 1.6:1)
Chapter 6

Nonlinear Transient Response of Earth Dams Subjected to Vertical SV Waves

6.1 Introduction

Material nonlinearity is a significant and, in many cases, a crucial factor that must be taken in consideration in the process of analyzing and designing earth dams to sustain strong ground motion. Soil is a material that, under large strains, exhibits a very complex and highly nonlinear behavior, which may affect substantially the response and behavior of the entire structure.

The general transient hybrid boundary-finite element formulation presented in Chapter 2 is now applied to study the effects of the foundation flexibility and the material nonlinearity on the response of earth dams. The dam is considered to be infinitely long, hence, it is modeled as a two-dimensional plane strain structure. The dam body consists of nonlinear material and is founded over an elastic halfspace (Figure 6.1). The earthquake excitation considered consists of vertically incident SV waves.
Figure 6.1  Geometry of the earth dam founded over a halfspace
6.2 Model Description

The dam considered in this study has a height $H = 100$ m, a 10 m wide crest and symmetric slopes. The dam consists of a heterogeneous material having stress-dependent Young's modulus $E_d$ with an average value of $8.53 \times 10^5$ kN/m$^2$, mass density $\rho_d = 2000$ Kg/m$^3$, Poisson's ratio $\nu_d = 1/3$, average shear wave velocity $c_d = 400$ m/s and Rayleigh damping ratio $\beta_d = 1\%$. The foundation material has a mass density $\rho_f = 2400$ Kg/m$^3$, Poisson's ratio $\nu_f = 1/3$ and no material damping. Two values for flexibility of the foundation are considered corresponding to impedance ratios $IR = 3$ and $\infty$ (rigid base), where $IR$ is defined as

$$ IR = \frac{\rho_f c_f}{\rho_d c_d} $$  

(6.1)

in which $c_f$ is the shear wave velocity of the foundation material. The dam body is discretized using four-node plane strain isoparametric elements, whereas the halfspace is discretized using two-node boundary elements. The discretized length of the halfspace surface is taken equal to five times the dam base width.

6.3 The Nonlinear Model

The simple Ramberg-Osgood model is used to represent the material nonlinearity occurring in the dam body. The stress-strain relationships for the model can be represented as

$$ \frac{\gamma_y}{\gamma_y'} = \frac{\tau}{\tau_y} + \alpha \left| \frac{\tau}{\tau_y} \right|^r $$

for virgin loading

(6.2)

and
\[ \tau_{xy, max} = \sqrt{\left(\frac{\sigma_y + \sigma_x}{2}\right)^2 \sin^2 \phi - \left(\frac{\sigma_y - \sigma_x}{2}\right)^2} - \tau_{xy} \]

**Figure 6.2** Computation of the maximum shear strength \( \tau_{xy, max} \) used in the Ramberg-Osgood model.
\[
\frac{\gamma - \gamma_0}{2\gamma_y} = \frac{\tau - \tau_y}{2\tau_y} + \alpha \left| \frac{\tau - \tau_y}{2\tau_y} \right|^r \quad \text{for unloading/reloading} \quad (6.3)
\]

where \( \gamma \) is the shear strain and \( \tau \) is the shear stress acting on a horizontal plane. It is important to choose the appropriate values for the different parameters in the model, \( \alpha, r, \gamma_y \) and \( \tau_y \) in order to achieve realistic results. There are certain criteria that need to be satisfied in choosing these parameters. The first criterion is to enable the model to behave elastically within the range of strains up to \( \gamma_y \). This can be achieved, if the reduction of the shear modulus of elasticity \( G/G_0 \) is approximately about 0.8 at a strain level \( \gamma = \gamma_y \). This corresponds to a secant shear modulus reduction \( G/G_0 = 0.9 \) (Dobry 1970). Indeed, experimental evidence suggests that for such strain levels the soil behaves elastically.

The second criterion is to reach the maximum shear strength, \( \tau_{\text{max}} \), at a sufficiently high strain. The third criterion is to match the experimentally derived curves for shear modulus reduction and damping ratio increase with increasing cyclic shear strain in gravel and sand with those obtained for the suggested model.

The first step for determining the model parameters is to obtain the static horizontal stress \( \sigma_x \), vertical stress \( \sigma_y \) and shear stress \( \tau_{xy} \) for all the elements within the dam body subjected to the load of its own weight. The maximum shear modulus \( G_0 \) can be computed, using the empirical formula

\[
G_0(\text{KPa}) = 1000 \ K \sqrt{\sigma_0(\text{KPa})} \quad (6.4)
\]

where \( \sigma_0 \) is the overburden pressure, and the typical range of \( K \) for very dense sand and gravel is from 30 to 40. A value of \( K = 35 \) is used, corresponding to an average shear wave velocity \( c_s = 400 \text{ m/s}^2 \).
Figure 6.3  Different material zones within the dam body

<table>
<thead>
<tr>
<th>Material</th>
<th>$E$ (MPa)</th>
<th>$G_0$ (MPa)</th>
<th>$\tau_{\text{max}}$ (MPa)</th>
<th>$\gamma$ (%)</th>
<th>$\tau_y$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.15 \times 10^3$</td>
<td>$4.30 \times 10^2$</td>
<td>70</td>
<td>$7.80 \times 10^{-3}$</td>
<td>3.01</td>
</tr>
<tr>
<td>2</td>
<td>$1.00 \times 10^3$</td>
<td>$3.75 \times 10^2$</td>
<td>45</td>
<td>$4.64 \times 10^{-3}$</td>
<td>1.57</td>
</tr>
<tr>
<td>3</td>
<td>$0.85 \times 10^3$</td>
<td>$3.19 \times 10^2$</td>
<td>27</td>
<td>$2.57 \times 10^{-3}$</td>
<td>0.74</td>
</tr>
<tr>
<td>4</td>
<td>$0.70 \times 10^3$</td>
<td>$2.63 \times 10^2$</td>
<td>17</td>
<td>$1.63 \times 10^{-3}$</td>
<td>0.39</td>
</tr>
<tr>
<td>5</td>
<td>$0.55 \times 10^3$</td>
<td>$2.06 \times 10^2$</td>
<td>10</td>
<td>$1.00 \times 10^{-3}$</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 6.1: Material properties for different zones within the dam body.
The maximum shear stress \( \tau_{xy,\text{max}} \) is also calculated (see Figure 6.2) using the following formula

\[
\tau_{xy,\text{max}} = \sqrt{\left(\frac{\sigma_y + \sigma_z}{2}\right)^2 \sin^2 \phi - \left(\frac{\sigma_y - \sigma_x}{2}\right)^2} - \tau_{xy}
\]  

(6.5)

where \( \phi \) is the angle of friction for the dam material. A typical range of \( \phi \) for very dense sand and gravel is between 38° and 42°. A value of 40° is used in the current study.

The dam is divided into five zones, each having a constant value of the \( G_0 \) and \( \tau_{xy,\text{max}} \) (Figure 6.3). This zoning of the dam helps decrease the numerical size of the problem and allows for easy modification of the properties for efficient parametric studies. The material properties in each zone are shown in Table 6.1 Accordingly, a different set of the model parameters \( \alpha, r, \gamma_y \) and \( \tau_y \) is computed for each zone.

By changing the parameters \( \alpha \) and \( r \), the shape of shear modulus reduction curve and the damping curve can be controlled. For \( \alpha = 0.12 \), and \( r = 2.5 \), the \( G/G_0 \) value is indeed 0.9, and the general shape of both the shape of shear modulus reduction curve and the damping ratio curve versus cyclic shear strain match very well that of experimental data for sand and gravel for a wide range of confining stress values. These values are used in all five material zones of the dam.

To satisfy the second criteria in choosing the model parameters, it is assumed that the model will reach the maximum shear stress \( \tau_{xy,\text{max}} \) at a shear strain amplitude of 2.0%. Using the aforementioned assumption, the values of the yielding shear strain \( \gamma_y \) and the yielding shear stress \( \tau_y \) are computed and summarized for the different material zones of the dam in Table 6.1.
Figure 6.4 Nonlinear-hysteretic cyclic stress-strain behavior of soil: Secant shear modulus reduction with shear strain amplitude
Figure 6.5  Nonlinear-hysteretic cyclic stress-strain behavior of soil: Damping ratio (β) increase with shear strain amplitude
Figure 6.4 shows the secant shear modulus reduction versus cyclic shear strain for the five materials comprising the dam body and the corresponding experimental values of a typical range of sand and gravel. Figure 6.5 shows the damping ratio increase with increasing cyclic shear strain for the five dam materials and the corresponding experimental values for sand and gravel.

6.4 Input Motion

The dam is subjected to a vertically propagating SV wave front. The time history acceleration used is the Imperial Valley earthquake record at the El Centro site (see Figure 6.6). The record was scaled to three different levels of outcrop rock peak ground acceleration (PGA) equal to 0.2g, 0.4g and 0.6g, to represent different levels of nonlinearity in the dam response, namely, mildly, moderately and strongly nonlinear response. The first fifteen seconds of the response were considered in the analysis.

6.5 Parametric Study

The results are presented in terms of time histories, and peak values plotted along the dam axis, and across the dam width at two elevations equal to 0.5H and 0.75H, respectively. Figure 6.7 to 6.9 represent the horizontal acceleration time history at various points within the dam body for a rigid base dam and an outcrop rock PGA = 0.2g, 0.4g and 0.6g respectively. Figure 6.10 plots the peak values for the horizontal acceleration along the central axis of the dam, and Figure 6.11 plots the peak values for the horizontal acceleration across the dam width.

Figures 6.12 to 6.14 represent the horizontal acceleration time history at the same
Figure 6.6 Acceleration time history and Fourier amplitude of the Imperial Valley earthquake, El Centro record.
points within the dam body for a flexible base dam with IR = 3 and an outcrop rock PGA = 0.2g, 0.4g and 0.6g, respectively. Figure 6.15 plots the peak values for the horizontal acceleration along the central axis of the dam, and Figure 6.16 plots the peak values for the horizontal acceleration across the dam width.

Figures 6.17 to 6.19 plot the horizontal displacement time history at various points within the dam body for a rigid base dam. Figure 6.20 plots the peak values for the horizontal displacement along the central axis of the dam, and Figure 6.21 plots the peak values for the horizontal displacement across the dam width.

Figures 6.22 to 6.24 plot the horizontal displacement time history at the same points within the dam body for a flexible base dam with IR = 3. Figure 6.25 plots the peak values for the horizontal displacement along the central axis of the dam, and Figure 6.26 plots the peak values for the horizontal displacement across the dam width.

Figures 6.27 to 6.29 plot the horizontal shear strain time history at various points within the dam body for a rigid base dam. Figure 6.30 plots the peak values for the horizontal shear strain along the central axis of the dam, and Figure 6.31 plots the peak values for the horizontal shear strain across the dam width.

Figures 6.32 to 6.34 plot the horizontal shear strain time history at the same points within the dam body for a flexible base dam with IR = 3. Figure 6.35 plots the peak values for the horizontal shear strain along the central axis of the dam and Figure 6.36 plots the peak values for the horizontal shear strain across the dam width.

Figures 6.37 to 6.39 plot the horizontal shear stress time history at various points within the dam body for a rigid base dam. Figure 6.40 plots the peak values for the
horizontal shear stress along the central axis of the dam and Figure 6.41 plots the peak values for the horizontal shear stress across the dam width.

Figures 6.42 to 6.44 plot the horizontal shear stress time history at the same points within the dam body for a flexible base dam with IR = 3. Figure 6.45 plots the peak values for the horizontal shear stress along the central axis of the dam, and Figure 6.46 plots the peak values for the horizontal shear stress across the dam width.

Stress-strain time histories at elevation $z = 0.75H$ for outcrop rock PGA = 0.2g, 0.4g and 0.6g are shown for the rigid base dam in Figure 6.47 and for the flexible base dam in Figure 6.48.

6.6 Effect of Impedance Ratio

One of the important factors affecting the response of earth dams is the impedance ratio. The significant importance of such factor was demonstrated in earlier chapters for the frequency domain analyses of earth dams subjected to different types of waves. To examine the effect of the impedance ratio in the time domain analysis, two configurations of the same dam discussed earlier are considered, founded first on rigid base and second on a flexible base with impedance ratio IR = 3.

The first anticipated observation when comparing the rigid base versus the flexible base acceleration results is that the amplitude of the acceleration near the base for the rigid case is higher than that for the flexible base. This is due to the fact that the rigid base does not allow any radiation of wave energy back to the halfspace. In this case, the amplitude of the base acceleration is twice that of the incident wave, i.e. equal to the outcrop-rock acceleration amplitude. In the case of a flexible base, with IR = 3, the amplitude for the
transmitted wave into the dam body is one and a half that of the incident wave. The amplitude for the transmitted wave is given by the equation

$$A_{transmitted} = \frac{2A_{incident}}{1 + \frac{1}{IR}}$$ (6.6)

It can also be noted that the frequency content of the acceleration response at the lower region of the dam has more high frequency component in the rigid base dam, differing from the acceleration response in the flexible base dam, which has less high frequency component in the same region. In the rigid base case, all the earthquake energy is being transmitted to the base of the dam, whereas in the flexible base case, the existence of the half space tends to dissipate the earthquake energy and filter portions of the high frequency content of the input excitation.

The frequency content for point C1 seems to be higher than that for points C2 and B1. This can be explained by looking at the frequency domain response of the 2:1 dam to vertical SV waves, presented in Chapter 2. While material properties are different for both dams, it is evident that point C1 has significantly higher values of amplification for high frequencies ($a_0 > 3$), compared to those for point C2 and B1, which have very small amplification values for high frequency.

In order to evaluate the effect of the material nonlinearity on the dynamic response of earth/rockfill dams, it is worthwhile comparing the ratio between the accelerations at any point within the dam, divided by the acceleration at the base of the dam. This ratio reflects only the material nonlinearity effects on the dam response, and does not depend on the site effects and the flexibility of the dam foundation. The term base-amplification ratio $A_{Fb}$
used to distinguish between this quantity and the regular amplification, defined in earlier chapters.

In Figure 6.10, the crest amplification $A_{F_b}$ for the horizontal acceleration for the rigid base dam equal to 2.5, 2.2 and 2.0 for the mild, moderate and strong nonlinear response, respectively. In Figure 6.15, the same values for the flexible base dam equal to 2.6, 2.45 and 2.14 for the mild, moderate and strong nonlinear response, respectively. By rigorously considering the flexibility of the dam foundation in the analysis, the earthquake energy is allowed to dissipate back into the halfspace, introducing radiation damping, contrary to what happens to a rigid base dam, where all the earthquake energy is trapped in the dam body. This explains the higher values of base-amplification ratios $A_{F_b}$ in the case of flexible base dam.

While this should result in higher values of base-amplification ratio for the rigid base linear elastic dam, the energy trapping leads to higher levels of strain in the dam body, in the nonlinear hysteretic case. This results in a substantial increase in the hysteretic damping of the dam material during an earthquake and, hence, a significant decrease in the base amplification ratio for the rigid base case. It seems that the existence of radiation damping, on one hand, decreases the dynamic response through energy dissipation into the half space, and on the other hand, reduces the amount of hysteretic damping resulting in higher dynamic response.

Having considered both types of damping into the analysis of the dam, the acceleration response values are still higher for the rigid base dam than that for the flexible base dam. Notice that the peak crest accelerations are 1.2g, 0.87g and 0.5g for the rigid base dam,
0.89g, 0.72g and 0.38g for the flexible base dam, and for mild, moderate and strong nonlinear response, respectively. This indicates that the introduction of the halfspace and proper consideration of the soil structure interaction is still important, even in the cases where strong nonlinear material behavior is anticipated.

Displacement response shows similar trends. For the strongly nonlinear response, the rigid base dam peak crest displacement amplification $A_F$ is 1.83, where the same quantity equals 2.13 for the flexible base dam (from Figure 6.20 and Figure 6.25, respectively). Still, the overall peak crest displacement for the rigid base dam is higher than for the flexible base dam. (For strongly nonlinear response, peak crest displacement is 0.36 m in the rigid case and 0.34 m in the flexible case).

By comparing the peak shear stresses developed along the dam axis during the strongly nonlinear excitation (PGA = 0.6g) for the rigid base dam (Figure 6.40) and the flexible base dam (Figure 6.45), the average shear stress is about 20% higher for the rigid base dam. The corresponding average peak shear strain developed along the dam axis during the strongly nonlinear excitation for both dams is about 50% higher for the rigid base dam (compare Figure 6.30 for the rigid base dam and in Figure 6.35 for the flexible base dam). This seems to suggest that the effect of flexibility tends to lessen the nonlinear material behavior of the dam body, decreasing strain levels significantly with little change in stress levels.

The response across the width of the dam cross section in both rigid and flexible bases shows similar trends. More or less uniform response is observed along the width of the dam due to the wave field considered in the study, i.e. vertically propagating SV waves,
which eliminates any wave passage effects. Although the current formulation is capable of handling obliquely incident waves and Rayleigh waves, the SV wave excitation is chosen to stress the effects of foundation flexibility on the response without interference of other factors.

6.7 Effect of Material Nonlinearity

Despite the simplicity of the Ramberg-Osgood model used in this study to represent the nonlinear inelastic behavior of soil, the model illustrates clearly the effects of material nonlinearity on the dynamic response of dams.

It is of interest to examine the change of the peak accelerations within the dam body shown in Figure 6.10 for the rigid base dam. It is evident from the figures that the crest acceleration amplification decreases as the level of nonlinearity increases from 0.2g to 0.6g peak ground acceleration. This is expected as the higher level of nonlinearity corresponds to higher amplitudes of cyclic strain and therefore higher damping ratio. The same trend is observed also in Figure 6.15 for the flexible base dam, although in this case the acceleration amplification is lower due to the presence of the radiation damping in addition to the material hysteretic damping.

Both Figures 6.10 and 6.15 show that the acceleration amplification is larger than one only in the upper quarter height of the dam. For the rest of the dam body, the lowest acceleration amplification is 0.8, 0.68 and 0.61 for mild, moderate and strong nonlinear response, respectively, for the rigid base dam, and 0.65, 0.5 and 0.49 for mild, moderate and strong nonlinear response, respectively, for the flexible base dam. The lowest value $AF = 0.49$ occurs for the flexible base dam and PGA of 0.6g. By increasing the level of
nonlinearity, the hysteretic damping increases, and the longer the wave path is, the more energy dissipation it experiences. This may explain the de-amplification in the middle part of the dam and is in agreement with other finite element studies as well as field observations where reduced amplification or de-amplification has been reported. (Gazetas 1987, Prevost et al. 1985, Griffiths and Prevost 1988, and Dakoulas 1990).

The effect of material nonlinearity on the dynamic response of dams is significant and crucial. The mechanism in which this factor affects the dam response is twofold. First, increasing material nonlinearity results to a substantial increase in the hysteretic damping experienced by the dam material. As mentioned before, this mechanism explains the amplification drop associated with higher levels of strain. It also explains the deamplification observed at the core of the dam for higher levels of nonlinearity. The second mechanism is that increasing material nonlinearity, although increases the hysteretic damping, it decreases the material stiffness significantly and may result in higher values for certain response parameters. It should be noted that the two mechanisms affect in different ways the various response parameters, i.e. accelerations, displacements, strains etc., which depend on different ranges of the input excitation frequency content. It appears that the first mechanism is more important, as indicated by several theoretical studies, as well as experimental studies and seismic recordings suggest this fact. The current study shows the same type of behavior for the core region of the dam, and near the crest. The maximum acceleration response distribution across the dam body (Figure 6.11 for IR = ∞ and Figure 6.16 for IR = 3) is more difficult to explain. It can be noticed that, for the mid-height distribution, the maximum accelerations near the slopes of the dam get
higher with increasing material nonlinearity. This seems to suggest that, even though hysteretic damping increases, a substantial part of the high amplifications normally developing in the region using linear elastic analyses (as shown in Chapter 3) is retained.

Finally, Figure 6.47 and Figure 6.48 show the stress strain relationship near point B for the three levels of excitation, and for IR = ∞ and 3, respectively. The figures demonstrate the ability of the simple nonlinear model to capture the nonlinear response characteristics of the dam subjected to strong ground motion. The minimal computational effort needed to account for material nonlinearity makes it possible to apply such models efficiently in solving real life problems.

### 6.8 Conclusions

A hybrid numerical formulation combining the Finite Element and the Boundary Element methods has been used to study the effects of dam-foundation interaction and material nonlinear behavior on the response of earth dams subjected to vertically incident SV waves. The developed hybrid method was proven to be very powerful and could be used to solve accurately and efficiently soil-structure interaction problems of complex geometry and material heterogeneity. The two-dimensional time-domain formulation has been used here to investigate the response of long dams to transient SV, assuming nonlinear hysteretic material behavior for the dam soil.

The results showed the significant effect of the flexibility of the foundation rock on dissipating wave energy and thus reducing the dam response considered in terms of outcrop-rock amplifications. They also showed that the smaller nonlinearity in the case of a flexible base leads to higher crest-to-base amplification values than those computed for
the rigid foundation. Therefore, the beneficial effect caused by the reduction of the base motion due to the flexibility is substantially reduced as a result of the corresponding decrease of the material hysteretic damping associated with smaller nonlinearity in this case.

The substantial amplification of the ground acceleration occurs in the upper quarter of the dam height with maximum values near the crest. For moderate to strong seismic input motion, the expected outcrop-rock amplification of the effective peak ground acceleration in this area is in the range of 1.5 to 3. However, for the lower three-quarters of the dam body, response de-amplification is reported for both rigid and flexible foundation dam indicating the substantial increase in the hysteretic damping with increasing levels of strain and its impact on decreasing the overall response of the dam. This is in agreement with other finite element studies as well as field observations where reduced amplification or de-amplification has been reported (Gazetas 1987, Prevost et. al. 1985, Griffiths and Prevost 1988 and Dakoulas 1990).

On using the simple Ramberg-Osgood model for realistic evaluation of nonlinear behavior of structures, a great care should be employed in choosing the appropriate values for the model parameters to represent actual soil behavior.

The proposed FE-BE method provides an efficient rigorous tool for making the, so far, formidable task of nonlinear seismic soil-structure interaction possible. This study demonstrated that the presented model can be a valuable tool for efficient parametric investigation of soil nonlinear seismic soil-structure interaction problems. By replacing the simple nonlinear soil model with more advanced model and by considering the
generation and dissipation of pore water pressure during the earthquake, a complete tool for rigorous seismic nonlinear soil-structure interaction analysis will be obtained. Such a model is a straightforward extension of the presented model, and could be extremely valuable in the study of the very complex seismic nonlinear soil-structure interaction problem, including the seismic behavior of earth/rockfill dams.
Figure 6.7 Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = \infty. (Peak Input Acceleration = 0.2g).
Figure 6.7 (continued) Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.2g).
Figure 6.8  Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = \infty.  
(Peak Input Acceleration = 0.4g).
**Figure 6.8 (continued)** Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = \( \infty \).
(Peak Input Acceleration = 0.4g).
Figure 6.9 Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio $IR = \infty$. (Peak Input Acceleration = 0.6g).
Figure 6.9 (continued) Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.6g).
Figure 6.10 Peak horizontal acceleration along the dam axis for different levels of excitation. Impedance ratio $IR = \infty$. 
Figure 6.11 Peak horizontal acceleration across the dam width for three levels of excitation. (a) height = 0.75H; (b) height = 0.5H. Impedance ratio IR = ∞.
Figure 6.12 Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.2g).
Figure 6.12 (continued) Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.2g).
Figure 6.13 Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3.
(Peak Input Acceleration = 0.4g).
Figure 6.13 (continued) Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.4g).
Figure 6.14 Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.6g).
Figure 6.14 (continued) Horizontal acceleration at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.6g).
Figure 6.15 Peak horizontal acceleration along the dam axis for different levels of excitation. Impedance ratio IR = 3.
**Figure 6.16** Peak horizontal acceleration across the dam width for three levels of excitation. (a) height = 0.75H; (b) height = 0.5H. Impedance ratio IR = 3.
Figure 6.17 Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = $\infty$. (Peak Input Acceleration = 0.2g).
Figure 6.17 (continued) Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.2g).
Figure 6.18 Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = \infty.
(Peak Input Acceleration = 0.4g).
Figure 6.18 (continued) Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.4g).
Figure 6.19 Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = \infty. (Peak Input Acceleration = 0.6g).
Figure 6.19 (continued) Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.6g).
Figure 6.20 Peak horizontal displacement along the dam axis for different levels of excitation. Impedance ratio $IR = \infty$. 
Figure 6.21 Peak horizontal displacement across the dam width for three levels of excitation. (a) height = 0.75H; (b) height = 0.5H. Impedance ratio IR = ∞.
Figure 6.22 Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.2g).
Figure 6.22 (continued) Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.2g).
Figure 6.23 Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.4g).
Figure 6.23 (continued) Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.4g).
Figure 6.24  Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.6g).
Figure 6.24 (continued) Horizontal Displacement at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.6g).
Figure 6.25 Peak horizontal displacement along the dam axis for different levels of excitation. Impedance ratio IR = 3.
Figure 6.26 Peak horizontal displacement across the dam width for three levels of excitation. (a) height = 0.75H; (b) height = 0.5H. Impedance ratio IR = 3.
Figure 6.27  Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio $IR = \infty$.
(Peak Input Acceleration = 0.2g).
Figure 6.27 (continued)  Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = \infty. (Peak Input Acceleration = 0.2g).
Figure 6.28 Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = \infty.
(Peak Input Acceleration = 0.4g).
**Figure 6.28 (continued)** Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.4g).
Figure 6.29 Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.6g).
Figure 6.29 (continued) Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.6g).
**Figure 6.30** Peak horizontal shear strain along the dam axis for different levels of excitation. Impedance ratio IR = ∞.
Figure 6.31 Peak horizontal shear strain across the dam width for three levels of excitation. (a) height = 0.75H; (b) height = 0.5H. Impedance ratio IR = ∞.
Figure 6.32 Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.2g).
Figure 6.32 (continued) Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.2g).
Figure 6.33 Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.4g).
Figure 6.33 (continued) Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.4g).
Figure 6.34 Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.6g).
Figure 6.34 (continued) Horizontal shear strain at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.6g).
**Figure 6.35** Peak horizontal shear strain along the dam axis for different levels of excitation. Impedance ratio IR = 3.
Figure 6.36 Peak horizontal shear strain across the dam width for three levels of excitation. (a) height = 0.75H; (b) height = 0.5H. Impedance ratio IR = 3.
Figure 6.37 Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.2g).
Figure 6.37 (continued) Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio $IR = \infty$. (Peak Input Acceleration = 0.2g).
Figure 6.38 Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.4g).
Figure 6.38 (continued) Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio $IR = \infty$. (Peak Input Acceleration = 0.4g).
Figure 6.39  Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio $IR = \infty$.
(Peak Input Acceleration = 0.6g).
**Figure 6.39 (continued)**  Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = ∞. (Peak Input Acceleration = 0.6g).
Figure 6.40 Peak horizontal shear stress along the dam axis for different levels of excitation. Impedance ratio $\text{IR} = \infty$. 
**Figure 6.41** Peak horizontal shear stress across the dam width for three levels of excitation. (a) height = 0.75H; (b) height = 0.5H. Impedance ratio IR = ∞.
Figure 6.42 Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3.
(Peak Input Acceleration = 0.2g).
**Figure 6.42 (continued)** Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.2g).
Figure 6.43 Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.4g).
Figure 6.43 (continued) Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.4g).
Figure 6.44 Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.6g).
Figure 6.44 (continued) Horizontal shear stress at various points within the dam body subjected to vertically incident SV waves. Impedance ratio IR = 3. (Peak Input Acceleration = 0.6g).
Figure 6.45 Peak horizontal shear stress along the dam axis for different levels of excitation. Impedance ratio IR = 3.
Figure 6.46 Peak horizontal shear stress across the dam width for three levels of excitation. (a) height = 0.75H; (b) height = 0.5H. Impedance ratio IR = 3.
Figure 6.47 Stress strain relationship for a rigid dam near point B ($z = 0.75 \, H$), subjected to different levels of excitation. Impedance ratio IR = $\infty$. 
Figure 6.48 Stress strain relationship for a rigid dam near point B (z = 0.75 H), subjected to different levels of excitation. Impedance ratio IR = 3.
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