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The electrical environment of thunderstorm models and measurements

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THE ELECTRICAL ENVIRONMENT OF THUNDERSTORM MODELS AND MEASUREMENTS
by
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ABSTRACT

The Electrical Environment of
Thunderstorm Models and Measurements

by

Paul Geis

A model describing a thunderstorm's interaction with the global electric circuit is presented. The model includes a thunderstorm, the surrounding atmospheric and ionospheric region, and the magnetically conjugate atmospheric and ionospheric region. The model is time-dependent, and includes lightning and thundercloud evolution. A method of using experimental data to more accurately simulate observed thunderstorms has been developed.

Of the upward current generated by a thunderstorm, about 50% flows through the Earth's magnetosphere to the conjugate hemisphere. This percentage is fairly constant over the storm's active life, and varies little with storm size or structure.

Infrequent lightning activity (less than one flash/minute) within a thunderstorm does not appear to greatly affect the thunderstorm's efficiency in transferring separated charge to the global circuit. Lightning does limit the magnitude of the electric fields and resulting currents within and below the storm.
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Once again, thank you Arthur Few.

To lose a friend is the greatest of all evils, but endeavour rather to rejoice that you possessed him than to mourn his loss.

-Seneca (4 B.C.-A.D. 65) Epistulae ad Lucilium

This work is dedicated to the memory and spirit of Eric Spriestersbach.
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PREFACE

The research described herein on the global electric circuit has two separate components; theoretical and experimental. All the results described in Chapter 4 come from computer modeling of thunderstorms and their electrical interaction with the atmosphere on a global scale. Because the conclusions are drawn exclusively from the model generated data, most of this thesis deals with the details of the computer modeling work. Confidence in model performance is increased by closely matching the model output to aircraft measurements. Chapter 3 has the relevant information on the aircraft field program, the data analysis and application of the data to the computer model.
CHAPTER I
BACKGROUND: THE GLOBAL ELECTRIC CIRCUIT

This chapter provides a quick review of the global electric circuit; the basic electrodynamics, the circuit's components, and its properties of interest. Some of the more specific terminology used can be found in Appendix A at the back of this thesis. No original work is presented in this chapter; it is for those unfamiliar with the field and terminology.

1.1 Basic Electrodynamics of the Atmosphere

Many particles and molecules in the atmosphere carry an electrical charge, through either ionization or by attachment of free ions or electrons. Unattached ions can vary in size from $10^{-10}$ to $10^{-6}$ meters - larger when attached to an aerosol particle. Aside from their random thermal motion, charged particles in the atmosphere - space charge - may be motivated by some external force to move in a uniform direction, creating electric currents. An ambient electric field, for example, will cause space charge to move in the direction of the field. This produces an electric current proportional to the field and atmospheric conductivity. Space charge itself modifies the local electric field, and in the absence of other constraints will move so as to decrease the local field. Small ions are the primary source of atmospheric conductivity, but both small and large ions contribute to the effects of space charge.
1.1.1 Atmospheric conductivity

The electrical conductivity of the atmosphere determines how much current will flow when an electric field is present. Atmospheric conductivity is a function of several factors. The intensity of ionizing radiation, which generally increases with latitude and altitude, determines the number of ions created in the region. Higher ion production rates mean a higher conductivity. At low altitudes cosmic rays are the primary source of ionizing radiation, but at higher altitudes solar ultraviolet and x-ray radiation also contribute. Although the release of radioactive gases (i.e. radon) from the ground can enhance the local conductivity, the conductivity in general increases with altitude. At ground level, the conductivity is only $10^{-14} \text{ mho/m}$ [Volland, 1982], lower than that of many common insulators. At altitudes above 60 km, the conductivity increases by nearly 10 orders of magnitude (figure 1). The rapid increase in conductivity is due to the increased presence of free electrons at these altitudes, and this region is known as the ionosphere. In this work, the lower boundary of the ionosphere is taken to be 60 km [Ahrens, 1988], although altitudes from 50-100 km are sometimes quoted.

Two conductivity profiles are shown in figure 1.1; the specific and Pederson conductivities. Below 60 km altitude the conductivity is isotropic, and the magnitude of the current generated by an electric field is independent of the field's orientation. Above 60 km, the conductivity of the atmosphere is higher along the Earth's magnetic field lines, and an electric field will induce more current to flow when the electric field is aligned with the magnetic field. The conductivity parallel to the magnetic and electric fields is called the specific
conductivity, while the conductivity parallel to the electric field and perpendicular to the magnetic field is the Pederson conductivity.

![Graph showing electrical conductivity as a function of altitude and conductance](image)

FIGURE 1.1 The electrical conductivity of the atmosphere (specific and Pederson) as a function of altitude.

At lower altitudes several phenomena can affect the atmospheric conductivity. Conductivity depends on the number of charge carriers, their mass (lighter charged particles move and respond more quickly to an electric field), and the particles' mean free path. For example, the atmosphere's decrease in neutral particle density with altitude increases the mean free path for charged particles and causes the observed exponential increase in conductivity. Thunderclouds, on the other hand, have a large concentration of very heavy (slow) particles. The charge carriers tend to attach themselves to cloud droplets and precipitation particles by diffusion, reducing their mobility and causing a decrease in conductivity within the cloud.
Corona currents (discussed below) occur near the ground and inject charge into the atmosphere, increasing the conductivity. Lightning also causes a conductivity increase, although it is localized within the lightning channel and exists for a relatively short time.

1.1.2 Current density and atmospheric currents

In atmospheric electricity research it is often useful to work with current density rather than total current. Current density is the flow of current through a unit area (Coulombs/second\textperthousand\textperthousand meter\textsuperscript{2}). For example, the total current flowing into the Earth could be found by integrating the vertical current density over the entire surface of the globe.

Although conduction current accounts for most charge transfer in the atmosphere, several other forms of current are important, particularly near thunderstorms.

Lightning: The most well-known manifestation of atmospheric electricity, lightning is caused by concentrations of charge in thunderstorms. When the electric fields produced by thunderstorm charge concentrations become large enough, a series of events produces an ionized highly conductive channel through the air. The current flowing in this channel typically reaches 40,000 Amps, but lasts less than 80 milliseconds [Krider, 1986]. Lightning between concentrations of charge in thunderclouds (intracloud lightning) is more common than cloud to ground lightning. Among the differences between the two types of lightning, one in particular (relevant to this work) is that cloud-to-ground lightning helps charge the global electric circuit, while intracloud lightning reduces the charge available.
Source current: Source current is caused by charge separation taking place in thunderstorms. The exact mechanism is not well understood, but the source current results in two or more regions of charge being built up within the thunderstorm. Because the current depends on the storm's updraft, the source current is large in the developing and mature stages of the storm and diminishes during the dissipating stage.

Displacement current: The displacement current is proportional to $\frac{dE}{dt}$, the time rate of change of the electric field, and arises from Maxwell's equations. Displacement currents are important where charge distributions are changing rapidly, most notably near lightning activity.

Corona: Objects on the ground enhance the electric field around them - the more 'pointed' the object, the higher the enhancement. If the field becomes strong enough, ionization of the air can occur and result in corona; St. Elmo's fire is the most well known manifestation of this effect. Corona can start to occur when the electric field near the ground reaches 5000 Volts/meter [Simmer, 1980].

Precipitation current: Precipitation carries charge from the charge centers of a thunderstorm. Precipitation can be positively or negatively charged, depending upon the polarity of the last cloud charge concentration it came in contact with.

Convection current: Convection under and around a storm can move charge. This type of current is most prevalent in regions where concentrations of one sign of charge carrier exists, such as near the ground or cloud boundary.

The sum of all currents above is the total current. It is called the Maxwell current, and appears in the basic equation solved by the computer model (discussed later in Chapter 2).
1.1.3 Electric fields and the atmosphere

Electric field measurements are probably the most common type of data used in the study of atmospheric electricity. Electric field, current density, and atmospheric conductivity measurements are (relatively) easy in situ quantities to measure from the ground or in the air using balloons and aircraft. The majority of work on the global circuit comes from the study of these quantities.

In good weather, the electric field near the ground is typically 100-150 V/m [Israel, 1973], reaching several thousands of Volts per meter under a thunderstorm (fortunately for us, the conductivity of the atmosphere is too low to sustain dangerous currents, even when the atmospheric voltage from a person's head to toe reaches 5000 Volts...). In areas away from electrified clouds, the electric field decreases with altitude.

When the local electric field increases or decreases, the local space charge distribution requires a finite amount of time to adjust; this is the electrical relaxation time of the atmosphere. The relaxation time decreases with altitude. At ground level it varies from 5-40 minutes, decreasing to less than a millisecond in the ionosphere [Roble and Tzur, 1986]. Longer relaxation times mean it takes longer for the atmosphere to neutralize an electrified object, such as a cloud.

1.2 Components of the Global Electric Circuit

Figure 1.2 shows the important elements of the global electric circuit. Thunderstorms around the world separate electrical charge and drive current into the upper atmosphere. This current spreads out uniformly across the globe in the equalization layer, flowing back to Earth in regions away from thunderstorms. Once in the ground, it is conducted back to areas of
thunderstorm activity. Upward electric fields under thunderstorms draw conduction current from the ground into the clouds, with cloud-to-ground lightning also transferring charge between storms and the Earth.

![Diagram of Equalization layer and current flow]

**FIGURE 1.2** Simplified picture of the global electric circuit. Arrows indicate the direction current flows.

For modeling purposes, the components of the global electric circuit can be represented as shown in figure 1.3. Thunderstorms which separate charge are treated as current sources. The atmosphere above and below a storm has some resistance. The ground and equalization layer have a very high conductivity relative to the lower atmosphere, and are treated as constant potential surfaces. The fair weather atmosphere provides the bulk of the load resistance in the circuit, giving the current a complete path from top to bottom of the thunderstorm current generators. There is also a resistance across the current generators that acts to decrease the current flowing in the rest of the global circuit. This resistance represents current lost to intracloud lightning and conduction current between a storm’s charge centers.

Any model of the global electric circuit needs to include each of the components shown below. Each is discussed in more detail.
1.2.1 Thunderstorms

Thunderstorms are believed to be the primary generators in the global electric circuit, separating electrical charge which drives a current to both the upper atmosphere and Earth.

The mechanism which separates positive and negative space charge is not well understood. Thunderclouds develop with upward and downward convection vertically building the cloud. There are several theories, the details of which are not important here, which explain how positively charged particles are convected to the upper parts of the cloud. This produces two primary regions of space charge, with negative charge in the lower cloud and positive charge in the upper cloud. As the cloud electrically develops, additional smaller pockets of charge may appear, and space charge is trapped on the boundary between the cloud and clear air.
Charge buildup within the cloud alters the local electrical environment, modifying electric fields and inducing several types of currents. Figure 1.4 shows the currents present in a thunderstorm.

![Diagram of thunderstorm currents]

**FIGURE 1.4** Schematic of the important currents flowing in and around a thunderstorm.

The concentration of negative charge in the lower cloud causes an upward vertical field between the cloud and the ground. Conduction current flows upward into the base of the cloud. As charge separation continues, the fields below the cloud can become quite intense, reaching several hundred thousand Volts/meter (at the edges of water droplets the field strength is even higher, amplified by the droplets' geometry). When the electric field reaches the dielectric breakdown voltage of the atmosphere, lightning occurs. Cloud-to-
ground lightning usually lowers negative charge from the cloud to the ground, an upward electrical current.

The positive charge in the upper cloud creates an upward field above the storm, driving positive charge to the upper atmosphere. As altitude increases, some charge spreads horizontally, and only a fraction of the upward current reaches the altitude of the equalization layer.

Between the charge centers, in the middle of the cloud, are very strong downward-directed electric fields. Despite the strength of these fields, the conduction current in this area is very small owing to the poor conductivity inside the cloud. When the field reaches the breakdown strength, lightning is initiated between the charge centers - intracloud lightning. Intracloud lightning works against the source current by reducing the amount of space charge in each charge center.

Precipitation also transports charge. Because precipitation can transport positive or negative charge to the ground, its overall effect on the electrical development of the storm is difficult to understand.

Strong electric fields near the ground cause corona currents to flow from elevated or pointed objects. Corona increases the upward current and enhances the conductivity by providing additional charge carriers.

When a thunderstorm enters the dissipating stage and charge separation ceases, the remaining currents reduce the charge centers. Lightning activity and corona will cease, and eventually only conduction, convection, and precipitation currents will remain. In the absence of other electrified clouds, the local electric field will reorient from upward to downward, and the region will become part of the fair weather component of the global electric circuit.
It is estimated that at any given moment there are 1500 - 2000 active thunderstorms across the world [Rable and Tzur, 1986]. Worldwide thunderstorm activity is not constant over the 24 hour day, but has maxima and minima corresponding to variations in land/ocean area as a function of longitude; continental thunderstorm activity in daytime is higher than at night. The effect of an individual continent's thunderstorm activity on the global electric circuit has been detected [Torreson et al., 1946]. In theory, large individual thunderstorms should also cause a variation in the fair weather current density worldwide. The small size of a thunderstorm's contribution, however, makes it difficult to detect.

1.2.2 Fair weather atmosphere

The fair weather atmosphere has a downward directed field of about 150 V/m at ground level. The average current density driven by the field is $\sim 2 \times 10^{-12}$ A/m$^2$. Both of these quantities vary with location and atmospheric conditions. Although the electric field strength decreases with height, the current density remains fairly constant. This is to be expected; a variation in current density at some altitude would mean charge was piling up or being depleted. Because conductivity increases with altitude, and current density is the product of conductivity and the electric field, the electric field must decrease with height.

The total global current, the current density integrated over the world's surface, is estimated to be about 1000 Amps [Muhleisen, 1977]. If this current is due predominantly to thunderstorm activity, then the average thunderstorm is supplying 0.5 - 1 Amp of current to the global circuit.
1.2.3 Equalization layer

The electric field structure around a thunderstorm is very complicated, with field lines flowing to both the upper atmosphere and back down to ground. Of the conduction current that leaves the top of the storm, some fraction following the electric field will return to the Earth close to the storm. The rest will continue upward into the highly conductive ionosphere.

As the conductivity increases, current begins to spread horizontally. Space charge is distributed across the globe, spreading to fair weather regions where it flows through the atmosphere and back to ground. The region of the ionosphere in which the current spreads is called the equalization layer, and starts at about 60 km.

Because the ionosphere's conductivity is so high, electric fields there are small compared to fields found in the lower atmosphere. The ionosphere can be approximated as a constant potential surface, at a uniform voltage above ground. The ionospheric potential is the voltage of the ionosphere relative to the ground, maintained by the worldwide thunderstorm activity. The potential is experimentally measurable, and ranges from 150 to 400 kiloVolts [Markson, 1976].

The equalization layer includes the ionosphere and the magnetosphere. At ionospheric and magnetospheric altitudes, the nonisotropy of the conductivity means current will preferentially flow along magnetic field lines. The Earth's magnetic field is approximately dipolar, so that field lines leaving a given latitude in one hemisphere will return to that latitude in the opposite hemisphere. This means, for example, that current entering the magnetosphere at 45° N magnetic latitude would be 'mapped' to 45° S magnetic latitude. When a
thunderstorm injects upward flowing current into the ionosphere, a fraction of it does not spread uniformly in the equalization layer but flows through the magnetosphere to the conjugate magnetic location in the opposite hemisphere, where it may enhance the fair weather electric field and current density of the atmosphere.

Magnetospheric and solar events can cause small but detectable perturbations on the global electric circuit, particularly in the polar regions. Such influences, however, are beyond the scope of this work.

1.2.4 Earth

The conductivity of the Earth's surface is much higher than that of the lower atmosphere, and is usually considered a constant potential surface in the global circuit. Charge flowing to the ground in fair weather regions is transported to regions of thunderstorm activity. Strong upward fields cause charge to enter the atmosphere or be neutralized via the previously discussed currents.

At the Earth's surface an electrode layer can form. In air an electric field will cause positive space charge to flow in the direction of the field and negative space charge in the opposite direction. At the boundary of the Earth and atmosphere, however, it is difficult for space charge to leave the Earth and enter the air. Ions of one sign (depending on the direction of the field) will be depleted near the boundary, and the imbalance between positive and negative space charge will alter the local electric field. This produces the electrode layer.
CHAPTER II
THE COMPUTER MODEL

The results and conclusions presented in this thesis are based on original research using a computer model. The model contains a thunderstorm within the Earth's atmosphere, and includes the most important components and mechanisms of this system needed to examine the electrodynamics involved.

Following is a basic description of precisely what is modeled, how it is done, and some of the approximations made. More detailed information on the model and parameters used in the simulations is located in Appendix B.

2.1 Numerical Modeling of the Atmosphere

The computer model used in this research approximates the most important aspects of the atmosphere's electrical state. It is a mathematical model based on Maxwell's equations, solving for the electric potential of the atmosphere.

At the beginning of a simulation, the electric potential of the atmosphere is uniformly zero; there is no charge anywhere and therefore no voltages. The model predicts the state of the atmosphere in the future by numerically determining the atmospheric electric potential distribution for each timestep (typically on the order of seconds) in the future. The results of this computation are then fed back into the model as it steps further into the future, and the process is continued. As the simulation progresses, source currents are mathematically initiated and the electric potential of the atmosphere responds to the developing storm. Typical simulations track the global circuit's electrical state through the thunderstorm's entire lifetime.
By the nature of numeric models, the electric potential can only be found at a limited number of locations within the atmosphere, called grid points (information between grid points is found by interpolation). The larger the number of grid points, the greater the resolution and accuracy of the model's results. Unfortunately, computer memory and speed restrictions limit the number of available grid points, and thus the resolution and accuracy of the simulations.

The actual model is a computer program and a set of input parameters. To solve for the electric potential distribution, the computer program needs specific information about the thunderstorm and the atmosphere. An input parameter file contains information on cloud size and height, the cloud's effect on local conductivity, etc... There are about 70 parameters determining the specifics of a simulation. As many parameters as possible are determined empirically (i.e. from measurements, such as the height of a typical thundercloud base), but some are poorly known. The uncertainty in choosing some of the model's parameters provides another inherent limitation on the accuracy of a simulation's results.

2.2 Model Geometry

Two regions of the atmosphere are modeled; the storm hemisphere and the conjugate hemisphere (figure 2.1). Each hemispherical shell is bounded by the Earth's surface below, and extends upward to 150 km altitude. In the following discussion, vertical is defined as perpendicular to the Earth's surface and horizontal parallel to the Earth's surface. Vertical distances are usually measured from the ground (and are thus equivalent to altitude), while horizontal distances are measured from the axis of symmetry at the center of the hemisphere. Note
that because of the Earth's curvature, vertical lines are not parallel, and the horizontal extent of a hemisphere increases with altitude. Unless otherwise noted, horizontal distances are measured at the ground.

FIGURE 2.1 Geometry of the global model. Shown is a cross-sectional slice through the Earth and atmosphere. Although the entire Earth's atmosphere can be modeled, only partial hemispheres are generally modeled, as shown here.

If the hemispheres have a horizontal extent of 10,000 km from their center, they include the Earth's entire atmosphere (the Earth has a circumference of about 40,000 km). In general, only a fraction of the atmosphere is modeled to increase the density of grid points. Far from the storm and its magnetically conjugate region, changes in the atmosphere's electrical structure are minimal, and these distant regions do not affect what happens near the storm and the conjugate region (an exception to this is the increase in the Earth's total fair weather resistance since part of the atmosphere's horizontal extent is missing. Ionospheric potentials and current densities must be multiplied by a scaling factor to account for the missing atmosphere [Stansbery, 1989]).
Model geometry is simplified by assuming the storm is symmetric about its central vertical axis. Figure 2.2 is a view of the storm hemisphere, looking down along the axis of symmetry. By removing any azimuthal variation in the thunderstorm's structure, all cross-sectional planes passing through the axis of symmetry become identical, reducing the problem to one of modeling only a two dimensional slice of the storm. Reduction from three to two spatial degrees of freedom greatly simplifies the necessary computations. Further, the cross sectional plane is itself symmetric about the central axis. The electric potential distribution of the entire storm hemisphere, therefore, can be determined by modeling half a cross-sectional plane.

![Diagram of storm hemisphere with axis of symmetry and azimuthal symmetry indicated.](image)

**FIGURE 2.2** Top view of storm hemisphere. All cross-sectional planes passing through the axis of symmetry are identical.

The model uses spherical coordinates ($\theta, r$), with the origin taken at the Earth's center. Coordinates are sometimes given in ($x, z$), where $x$ is the horizontal distance from the central axis and $z$ is altitude above ground level.
FIGURE 2.3 Cross sectional view of the modeled regions. Lateral boundaries can extend out to 10,000 km. Although the horizontal boundaries should follow the curvature of the Earth, all figures in this work are adjusted to appear rectangular. Since the left and right halves of each hemisphere shown above are symmetric, only one-half of the region displayed is modeled.
For ease of presentation, all subsequent plots showing a cross section of either hemisphere are presented as shown in figure 2.3.

The ground, not surprisingly, is taken to be the reference from which all electric potentials are measured. The ground of both hemispheres is, of course, at the same potential. The tops of each hemisphere, at \( z = 120 \) km, are at nearly the same potential above ground, being connected by the magnetosphere. This connection maps current from the top of the storm hemisphere directly to the top of the conjugate hemisphere as it flows along the Earth’s magnetic field. The electric potential of the magnetosphere relative to ground depends on the activity of the thunderstorm.

2.3 Model Features

Many properties of the thunderstorm and global circuit are user specified in the parameter file. Some of the more important are discussed below.

2.3.1 Conductivity

As seen in figure 1.1, the atmospheric conductivity varies greatly with altitude and is non isotropic in the upper atmosphere. The conductivity profile used in the model is a function approximating measured profiles [Volland, 1984] from the ground to 65 km. Above 65 km the specific conductivity exceeds the Pederson conductivity. Both conductivities are then approximated as basic exponential functions, with differing scale heights. The upper atmosphere is divided into two regions, so that the maximum in the Pederson conductivity at 110 km is present in the model profile. The conductivity of the ionosphere and
magnetosphere has a stronger dependence on latitude and time of day compared to the lower atmospheric conductivity. The conductivity profile used in this work is for midlatitude conditions during daytime.

In the simulations discussed in Chapter 4, the Earth's magnetic field lines are vertical (high-latitude geometry), so the specific conductivity is vertical at all altitudes and the Pederson conductivity horizontal. The model is capable of depicting other latitudes and other times of day, but these parameters were kept fixed in this study and latitudinal or daytime/nighttime variations in thunderstorm-global circuit coupling were not examined. The latitudinal variation in thunderstorm-global circuit coupling has been studied by Tzur and Roble [1985] in their model of the global electric circuit.

The thundercloud decreases the conductivity within it. The decrease is believed to be a factor of 10 to 500, depending on the cloud's water content and drop size [Pruppacher and Klett, 1978]. The model therefore includes a cloud which causes the atmospheric conductivity to fall off quickly in the cloud's interior. The cloud is given an initial height and vertical and horizontal extent before cloud electrification begins. As the storm develops, the cloud grows, particularly in the vertical due to upward convection.

Lightning, as will be discussed below, also affects the atmospheric conductivity, greatly increasing it within the lightning channel.

2.3.2 Source current

Charge separation within the thundercloud creates two regions of charge; a positive region over a negative one. This structure is not as complex as those observed in actual thunderstorms, where additional smaller pockets of charge
may form. The model’s charge separation produces equal amounts of positive and negative charge - a requirement of charge conservation - at altitudes and extents specified by the parameter file. The source charge regions have the greatest amount of charge separation at their center (a graphical representation is included in Appendix B).

Charge separation is also time dependent. Initially, there is no cloud electrification. The beginning of charge separation is the start of electric potential changes in the cloud. As the storm develops, the rate of charge separation increases, leveling off at its maximum value. This is the peak of storm activity. Charge separation then decreases, eventually going to zero. It should be noted that even if charge separation was turned off instantly when it had reached its peak value, the thunderstorm’s electrical activity would continue until the existing charge in the cloud had been dissipated.

Corona current at the Earth’s surface is also generated by the model. Unlike charge separation in the cloud, which is predetermined by parameters chosen by the model user, corona activity is determined by the electric fields that develop. As fields at the Earth’s surface increase, positive charge is created near the Earth’s surface. Higher fields create more charge. Increasing positive charge concentrations at ground level limit the size of the electric fields that develop, which without corona could be an order of magnitude stronger.

2.3.5 Lightning

Lightning comes in two forms; intracloud and cloud-to-ground. As the storm develops, the strongest electric fields tend to form between the positive and negative charge centers, and near the lower cloud boundary. When the
breakdown voltage of the atmosphere is exceeded in one of these regions, lighting is triggered.

The model's normal timestep is on the order of tens of seconds. When lightning occurs, the model goes to a timestep less than a second (in reality, a typical lightning flash lasts only 200 milliseconds or so - Ulman [1984]). The atmospheric conductivity is increased several orders of magnitude at the breakdown location, simulating the ionization of the lightning channel. The channel is then extended wherever the electric field is high enough for channel propagation. The electric field strength necessary to propagate the channel is significantly less than the breakdown voltage, so that breakdown at one grid point can create lightning over a large distance. A typical channel will connect the two charge centers or the lower charge center and the ground. During the lightning timestep, large currents will flow along the lightning channel and one (cloud-to-ground) or both (intracloud) charge centers will be partially depleted of charge. Because the charge centers have a large horizontal extent, lightning does not deplete charge uniformly. The first flash will be along the storm's central axis, where the greatest charge concentration is, while subsequent flashes may occur near the edge of the storm. Lightning activity is most intense when the source current is at its maximum.

2.3.4 Grid spacing

The distribution of the model's grid points is not uniform. Although it adds complexity to the model, variable grid spacing allows the user to make the grid densest where there is the greatest amount of electrical structure, and where the electric potential changes most rapidly (i.e. in and around the thunderstorm).
Far from the storm the potential is fairly uniform (see results in Chapter 4) and a relatively sparse grid can be used. Vertical grid spacing must be small when passing through the ionosphere due to the rapid change in conductivity.

2.1 Physics and Mathematics of the Model

The heart of the model is a program capable of solving a partial differential equation through successive approximations. The equation solved by the program specifies the electric potential distribution of the atmosphere.

2.4.1 Equation derivation

The equation solved by the computer model comes from Maxwell's equation

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \mu_0 \varepsilon \frac{\partial \mathbf{E}}{\partial t},$$

(2.1)

where \(\mathbf{E}\) is the electric field, \(\mathbf{B}\) the magnetic field, \(\mathbf{J}\) the current density, \(\mu\) the permeability, and \(\varepsilon\) the permittivity of the atmosphere. All vectors are functions of both time and position.

Taking the divergence gives

$$\nabla \cdot (\nabla \times \mathbf{B}) = \nabla \cdot \left( \mu_0 \mathbf{J} + \mu_0 \varepsilon \frac{\partial \mathbf{E}}{\partial t} \right),$$

(2.2)

which is equal to 0, since the divergence of a curl is 0.

\(\mathbf{J}\) includes both the conduction current, \(\mathbf{J}_c\), and the source current, \(\mathbf{J}_s\).

Therefore

$$\nabla \cdot \left( \mathbf{J}_c + \mathbf{J}_s + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \right) = 0,$$

(2.3)
The conduction current can be written as \( \bar{\sigma}\mathbf{E} \), where \( \bar{\sigma} \) is the conductivity tensor. A function \( S \) can be defined that is the divergence of the source current, \( S \equiv \nabla \cdot \mathbf{J} \), giving

\[
\nabla \cdot \left( \bar{\sigma}\mathbf{E} + \mathbf{J} + \varepsilon \frac{\partial \mathbf{E}}{\partial t} \right) = \nabla \cdot (\bar{\sigma}\mathbf{E}) + S + \nabla \cdot \left( \varepsilon \frac{\partial \mathbf{E}}{\partial t} \right) = 0.
\]

(2.4)

If the approximation \( \mathbf{E} = -\nabla \phi \) is made (see Section 2.4.3), the above equation can be rewritten as

\[
\nabla \cdot (\bar{\sigma}\nabla \phi) + \varepsilon \frac{\partial (\nabla \phi)}{\partial t} = S.
\]

(2.5)

Equation (2.5) can be transformed into an equation solvable by the computer by approximating

\[
\frac{\partial (\nabla \phi)}{\partial t} = \frac{\nabla \phi(t + \Delta t) - \nabla \phi(t)}{\Delta t}.
\]

(2.6)

and substituting, giving

\[
\nabla \cdot \left( (\bar{\sigma} + \varepsilon) \nabla \phi(t + \Delta t) \right) = S(t + \Delta t) + \frac{\varepsilon \cdot \nabla \phi(t)}{\Delta t}.
\]

(2.7)

The expanded form of this equation is derived in Appendix C.

2.4.2 Additional approximations

Earlier the approximation

\[ \mathbf{E} = -\nabla \phi \]
was made, which is true only when there is no movement of electric charge, since \( \phi \) is the electrostatic potential. In general

\[
E = -\nabla \phi \cdot \frac{\partial A}{\partial t}
\]

where \( A \) is the vector potential.

In landmark work [Geis, 1990] it was shown that \( \partial A/\partial t \) varies directly with \( \partial J/\partial t \), the time rate of change of current density. The approximation made, therefore, is most likely to be invalid in the presence of lightning, where \( \partial J/\partial t \) is very high. In the model's present application, it was shown that the effect of the approximation made in determining \( E \) can be safely ignored.

Additional boundary conditions are required by the computer program. Along the storm and conjugate hemispheres' central axes, and at their lateral boundaries, there can be no horizontal potential gradients. This means no horizontal fields or currents can develop at the very center of the storm, or at the edges of the modeled regions.

This approximation is not a problem. Because the storm is symmetric about the central axis, there should be no horizontal fields or currents at the storm center (there is no preferred direction for them to be in). At the edges of both hemispheres, the vertical fair weather field dominates at most altitudes, and the horizontal potential gradients are insignificant.
CHAPTER III
MODEL VERIFICATION AND TESTING

'Garbage in, garbage out' is a well known saying among those relying upon computers. Applied to this work, the phrase is a warning that predictions of how the global circuit will behave are only as valid as the assumptions and data that went into constructing the model. Appendix B contains a list of model parameters and references justifying their values when possible. The numerical methods used to find the atmospheric electric potential distribution are tried and true. In any work of this nature, however, inaccuracy cannot be eliminated but only minimized.

This chapter details efforts to reduce sources of error and improve the accuracy of our model's results. Aircraft measurements are compared to the data generated by the model, and model parameters are adjusted to increase the correlation between the two sets of data. If the model produces accurate values for quantities that can be measured, properties of interest that cannot be experimentally measured should also be accurate (or at least more so). A numeric test of the conduction current distribution generated by the model is also used. This 'continuity of current test' is sensitive to sources of error in both the model's computer program and some of the more critical model parameters.

3.1 Aircraft Measurements

Remote sensing programs have been used to study thunderstorms for many decades. The traditional sensor platform has been the balloon or low flying aircraft, both capable of penetrating the heart of a thundercloud and measuring
the electric fields or other quantities in situ. More recently, high altitude aircraft and satellites have operated electric field and lightning detectors.

Previous work using this model [Stansbery, 1989] has compared balloon data to the internal fields of a simulated thunderstorm; this is useful in studying the internal structure of a storm. In this work, electric field data collected by an ER-2 aircraft flown during the CaPE field program is used. Because this data is collected in the region where the storm is most directly coupling to the global circuit, the data is useful for examining a thunderstorm's interaction with the atmospheric environment. Use of electric field data in this manner has not been done before.

3.1.1 CaPE

Storm activity near Kennedy Space Center poses a potential hazard to rockets and the space shuttle on the launch pad and during the initial stages of takeoff (in the moon program, Apollo 12 was struck by lightning twice, causing their onboard inertial navigational system to fail). NASA's launch criteria do not permit a launch if electrified clouds are too close to the Kennedy facility. Because it is difficult to determine how likely a lightning strike would really be, a large safety factor is built into the criteria; launches are scrubbed when there is probably little real danger.

The Cloud Electrification and Precipitation Experiment took place in the summer of 1991. CaPE's operational goal was to improve the understanding of electrified cloud activity and allow more reasonable and less stringent restrictions on space vehicle launch criteria.
Cloud activity was studied at all times of the day. Kennedy, located on Florida's eastern shore, sees frequent storm activity in the late morning and early afternoon. Observations were made of both small air-mass and severe thunderstorms.

A program goal was to make coordinated observations of individual storms. Electric field, lightning location, radar, and other types of measurements were carried out by airborne and ground based sensors trained on the same storms. As a result, scientists studying these storms have a broad range of observations available. In practice, it has been more difficult than expected to use this data, but we have relied upon radar observations to get a rough idea of cloud height for the storms we modeled.

3.1.2 The ER-2

Electric field measurements were made from NASA's ER-2. The ER-2 is basically the more familiar U-2 spy plane flown by the U.S. Air Force. It is a jet, flying at about MACH 0.7, and at nearly 20 km altitude (roughly twice as high as commercial airliners).

The ER-2 aircraft was deployed at Wallops Island, Virginia, for about a month, during which time it made roughly a dozen science flights in the vicinity of Florida's Kennedy Space Center. Although lack of good thunderstorm activity, sensor failure, and other factors resulted in only two or three flights of useful data over good thunderstorms, this was enough for fairly selective criteria to be applied in choosing the thunderstorms to be modelled.

Flying through clouds or precipitation causes erratic electrical charging of the aircraft which degrades the sensor measurements. At its operational height, the
ER-2 was capable of flying well above all storms it encountered during CaPE. The ER-2's high speed allows it to traverse a storm fairly quickly, so the changes seen in the electric field are primarily a function of position and not time (an obvious exception to this are the rapid changes caused by lightning). These are the primary benefits of using the ER-2 in measuring the atmospheric electric field.

The ER-2 has several important limitations. It is a fairly fragile aircraft, not capable of steep and quick turns. The pilot's visibility is limited, and even with help from ground personnel it was difficult to accurately direct the ER-2. Ideally, the aircraft was to be flown over the center of a storm simultaneously with lower altitude aircraft; this proved difficult. For this work, multiple passes over the same storm would provide information about the time evolution of the storm. Because the ER-2 makes very long and wide turns, this was not practical. One storm was found where two overpasses of a single storm were made within 10 minutes of each other, and this storm was modeled. Another factor limiting the quality of the data was the harsh environment in which the ER-2 operates. In particular, temperature variations can affect the calibration and operation of the sensors in a manner that is difficult to recreate on the ground.

3.1.3 Electric Field Mills

Two electric field mills were installed on the ER-2 to measure the atmospheric electric field. These sensors work by alternately shielding and exposing a pair of insulated metal plates to the ambient field, and measuring the electric charge driven on/off the plates connected to the shield through a charge detection
circuit. Rapid variations in the electric field (fractions of a second) cannot be measured, but this is not important in the application discussed below.

The mills were mounted on the top and bottom of the aircraft, aft of the cockpit. With both mills mounted vertically, only the vertical component of the electric field can be measured. Two mills are necessary, however, to find the vertical field. As the aircraft flies through the atmosphere, it collides with charged particles, some of which stay on the plane's metal skin and charge the airframe. Aircraft charging is highly variable and influenced by several factors, including the location and intensity of charged clouds below the aircraft. Charge on the ER-2 is responsible for a component of the electric field measured by the mills.

Positive charge on the ER-2 will produce an upward field above the aircraft and a downward field below. Because the mills are oriented in opposite directions (the top one up and the bottom one down) both see the charging field increasing in the same direction. The atmospheric field, however, will be 'positive' to one mill and 'negative' to the other, as shown in figure 3.1. By taking the sum or the difference of the field mill signals, the vertical component of E due to aircraft charging or the atmospheric field can be found.

Of the several sources of error in making the electric field measurements, offset error and form factor error are the two most important. Offset error is caused by a mill's calibration drifting over time, so that the signal from the mill changes even in a constant field. Cooling of the sensitive preamps at the ER-2's high altitude exacerbates the problem. The calibration of the mills could be checked between flights, but the offset error that occurs during a flight (up to 50 V/m) is large enough to make fair weather field measurements unreliable.
The stronger the field being measured, the less important the offset error, so that over thunderstorms offset error is relatively small.

\[
\begin{align*}
\text{Upper mill signal} & = E_{\text{charging}} + E_{\text{atmospheric}} \\
\text{Lower mill signal} & = E_{\text{charging}} - E_{\text{atmospheric}} \\
\frac{\text{Upper mill signal} + \text{Lower mill signal}}{2} & = E_{\text{charging}} \\
\frac{\text{Upper mill signal} - \text{Lower mill signal}}{2} & = E_{\text{atmospheric}}
\end{align*}
\]

FIGURE 3.1 The relation between mill signals and the components of the atmospheric and charging fields.

Form factor error is caused by the perturbation the metal aircraft has on the local electric fields. The aircraft has a tendency to enhance the electric field, with the magnitude of the enhancement being primarily a function of aircraft shape (the form factor). Unlike offset error, form factor error is proportional to the strength of the electric field being measured. The form factor for the ER-2 has not been precisely determined. The field enhancement of a long cylinder is 2, very close to the form factor of a NASA Lear jet also used for electric field measurements. A form factor of 2.0 was used in the data analysis; this form factor may be slightly high, causing the actual field strength to be underestimated.
3.2 Electric Field Profiles

Figure 3.2 is an example of an ER-2 electric field profile. The fields rise from near zero away from the storm to several thousand Volts per meter as the aircraft passes over the electrified clouds. In this example two passes are made over the same storm. The difference in profile shape between the two passes has two causes: 1) the storm’s structure is asymmetric, and 2) the ER-2 does not fly over the storm from the same direction in the second pass. More important, lightning has a strong influence on the profile. Rapid changes in the electric field indicate a lightning flash has taken place, showing up as an instantaneous jump in the field. These jumps are followed by slower decays as charge is replenished in the charge centers and screening layers. In a very active storm, such as this one, the basic shape of the profile may be difficult to discern due to lightning activity. In modeling this storm, these two storm overpasses have been averaged together (Chapter 4) to reduce the variation caused by lightning activity and storm asymmetry.

With the exception of lightning effects, most of the variation in the profile is a function of aircraft position relative to the storm. At 12 km per minute, the ER-2 is over the storm only a short time, but upward electric fields are seen over 20 km from storm center (roughly the maximum distance of the ER-2 from storm center while taking this profile).

The computer model can reproduce a vertical electric field profile at the aircraft’s altitude. Changes in model parameters, such as the rate of charge separation, change the shape of the model generated electric field profile. By varying model parameters, agreement between the model generated profile and the ER-2 profile can be improved.
FIGURE 3.2 The vertical electric field over storm 2 as measured by the ER-2. Two passes are made over the storm.

Although visual comparison of the profiles works very well, a numeric 'goodness of fit' parameter is useful when the profiles match fairly closely. A simple measure of agreement is the average difference between the two profiles,

\[ P = \left( \frac{\left| E_{1}(x) - E_{2}(x) \right|}{E_{2}(x)} \right). \]

where P is a measure of relative difference. Through successive adjustments, P is decreased by varying model parameters until satisfactory agreement between the model and ER-2 profiles is reached.

3.3 Continuity of Current Test

Even if a list of parameters could perfectly describe a thunderstorm's size, charge distribution, and other important characteristics, error in the numerical
computations can render the model results meaningless. In solving for the
electric potential, some error is inevitable.

Grid density is the largest source of numeric error. The farther apart grid
points are, the less resolution there is of the atmospheric electric potential and
conductivity. Inaccuracy in these quantities shows up in almost all of the results.
The total number of grid points is limited by the computer's capability, and a
uniformly dense grid with sufficient resolution is not possible. Instead, a non-
uniform grid is used, increasing the grid point density (and thus the resolution)
in critical areas. In and around the storm there are rapid changes in both the
conductivity and electric potential, and grid density is greatest there. Increasing
the grid density in this region unfortunately decreases the density in the
ionosphere, where the conductivity rapidly increases with altitude. Grid point
distribution must be balanced so that no region of the atmosphere is too poorly
resolved.

The conduction current is sensitive to error caused by insufficient grid
resolution. The conduction current is a function of the conductivity and the
electric field, which in turn is derived from the electric potential. While there is
no easy way to see error in the potential distribution, the conduction current can
be checked.

Maxwell current is a conserved quantity. The model's boundary conditions
prevent current from flowing out of the modeled region at the hemispheres'
vertical boundaries. Current can only leave or enter the atmosphere at the
ground or areas of charge generation; everywhere else the net vertical Maxwell
current is constant with altitude. Above the thundercloud the Maxwell current is
almost exclusively conduction current, although displacement current may also
be significant. The net conduction current, which will be up in the storm hemisphere and down in the conjugate hemisphere, should be constant with altitude above the storm.

The continuity of current test determines how well a simulation conserves current as a function of altitude. Above the storm, ideal behavior would be no variation in current. Since the error is an artifact of grid point distribution, the continuity of current test is done in a steady state mode; the simulation is not time dependent, eliminating the effect of displacement currents.

![Diagram](image)

**FIGURE 3.3** Continuity of current test: the integrated conduction current as a function of altitude. The dashed line represents ideal behavior.
Figure 3.3 shows the continuity of current test for storm 1. The dashed line is the ideal conduction current, constant with altitude. Most of the variation in the conduction current is in the upper atmosphere, where large conductivity gradients exist. The small 'spikes' in the current are caused by the conductivity profile being generated by three different functions over its 120 km altitude range. Accuracy in the upper atmosphere has been sacrificed to increase grid point density below and in the thunderstorm.

This test is valid for altitudes above the storm cloud. A more complicated test can be applied to regions in and below the thundercloud that includes the source current. This test shows that even with the dense grid in these regions, there is still significant current loss within the storm. This is not a serious problem, since this study is looking at the effect of the storm on the global electric circuit rather than the storm itself, and model accuracy is most important in atmospheric regions above and around the thunderstorm. Because of this, however, no quantitative conclusions are inferred regarding phenomena inside the thunderstorm. The amount of charge transferred by lightning or a quantitative relationship between lightning activity and thunderstorm efficiency are examples of subjects worth studying if higher grid resolution is ever available.
CHAPTER IV
RESULTS

The results for two simulations are presented below. A brief description of each storm is given, with a complete list of the input parameters used in each simulation located in Appendix B.

4.1 Storm 1

Storm 1 is a small air mass thunderstorm. The storm was modeled over one hour, with no charge separation at \( t = 0 \) min., maximum activity at \( t = 30 \) min., and a return to no charge separation at \( t = 1 \) hour. The lifetime of the actual storm (from which the ER-2 profile comes) is unknown and the choice of a one hour duration is somewhat arbitrary, but it provides an indication of how the storm differs during the building and dissipating stages, and how the upward current to the global electric circuit varies with thunderstorm activity. Because the storm is fairly small, there is no lightning activity; electric fields within or below the cloud never exceed the breakdown voltage of the atmosphere. This simulation therefore closely approximates the storms simulated in previous work [Stansbery, 1989], allowing a comparison of previous steady state results and this thesis's time dependent ones without the complicating effects of lightning.

The ER-2 profile was matched to the model profile at \( t = 45 \) min., during the dissipating stage of the thunderstorm (figure 4.1). The agreement is excellent, with the average difference between the two profiles being less than 0.5 V/m, and \( P \), the goodness of fit parameter defined in section 3.2, being 0.04. This storm is an ideal case to model because there is no lightning. Lightning perturbations
to the profile not only make a close fit physically impossible (see section 4.2), but increase the difficulty in successively adjusting the parameters.

![Graph showing comparison of ER-2 and model generated profiles for storm 1.](image)

FIGURE 4.1 Comparison of the ER-2 and model generated profiles for storm 1. The solid line is from the ER-2, and the dashed line from the model.

The integrated upward current over storm 1 at t = 30 minutes is shown in figure 4.2. The section of the curve where there is no upward current is between the charge centers, as you would expect. Above the storm, the upward current is fairly constant until ionospheric heights are reached, where horizontally spreading current begins to turn downward. The upward current continues to decay with altitude until the magnetosphere is reached at 120 km altitude. In the conjugate hemisphere, the upward current (now downward current flowing from the ionosphere to ground) is fairly constant, as it should be.
FIGURE 4.2 The integrated upward current over storm 1. In the conjugate hemisphere the current is actually downward, from the ionosphere to the ground.

The upward flowing current above the storm contributes to charging of the global electric circuit. From figure 4.2 the fraction of upward current flowing to the conjugate hemisphere can be found. Upward current above the storm is 0.14 amps, while current in the conjugate hemisphere (at low altitudes) is 0.068 amps. The fraction of upward current to the conjugate hemisphere is therefore about 50%.

The upward current produced by storm 1 over time is shown in figure 4.3. At $t = 0$, the upward current is zero since charge separation is just beginning. As the storm develops, upward current increases, reaching a maximum value just over
0.14 amps at $t = 26.5$ minutes. Notice that although the rate of maximum charge separation is still several minutes away, the storm's contribution to the global circuit is at its peak. The source charge generation is symmetric about $t = 30$ minutes (Appendix B), and the asymmetry in figure 4.3 is probably due to screening layers. As the storm develops, a negative layer of charge will build on the upper surface of the cloud, reducing the electric field that would be present above the cloud if the screening layer was absent. A reduced electric field causes a reduced upward current.

![Graph showing upward current over time](image)

**FIGURE 4.3** The integrated upward current over storm 1.

The average upward current of storm 1 is 0.07 amps, half of the peak output. In previous work [Stansbery, 1989] all simulations were steady state and the upward currents found corresponded to peak currents. In classifying a storm's activity and its contribution to the global electric circuit, the average current over
the storm’s life should be used. For steady state results, the upward current generated by the model should be reduced by 50% to find the average current from the storm to the ionosphere.

Although storm 1 is a relatively weak electrified storm without any lightning activity, its peak upward current falls within the range found above measured thunderstorms of 0.1 to 6 amps [Blakeslee, Christian, and Vonnegut, 1989]. Thunderstorms are thought to be the primary generators in the global electric circuit, but any storm producing charge separation, even one without lightning activity, may contribute to maintaining the Earth’s ionospheric potential.

4.2 Storm 2

Storm 2 is an active thunderstorm. Like the previous example, storm 2 was modeled with a lifetime of one hour and maximum charge separation at t = 30 min. This simulation is more involved than that for storm 1. The storm is large, and both intracloud and cloud-to-ground lightning occur. Corona currents at the ground are significant in this simulation. Although the temporal component of the charge separation function in storm 2 is the same as that of storm 1, the spatial extent of the charge separation region evolves, increasing in height as the thundercloud builds upward.

The particular storm modeled was chosen for several reasons. The model assumes cylindrical symmetry about the storm’s central axis, whereas real thunderstorms are seldom very symmetric. Two consecutive overflights of this storm were made, each from a different direction. By averaging the two measured profiles, the effect of thunderstorm asymmetry on the ER-2 profile can be reduced. There is a tradeoff in this procedure; averaging also reduces the
effect of each individual lightning flash on the overall profile (it’s magnitude will usually be cut in half), but doubles the number of flashes in the profile.

Some radar data was available for storms 1 and 2. Cloud tops were measured at approximately 10 to 12 km for the storms overflown on this day. Data on the individual storms was not yet available.

Storm 2 was also chosen because lightning location data was taken. When using an electric field profile, it is assumed that the ER-2 has passed over the center of the storm. Although it would not be difficult to calculate model generated profiles for any overflight path, it is preferable to use ER-2 profiles from overflights that are known to have passed near the storm center. In performing the overflights, the ER-2 pilot relied mainly on visual cues when lining up his overflights, and a quantitative measure of how close the aircraft came to the center of lightning activity is difficult to determine. For storm 2, however, ground based lightning location data was taken and indicates the aircraft’s flight track came within 5 km of the cloud’s active (lightning) region on both passes.

The ER-2 profile was matched to the model at t = 30 min., during the storm’s most intense charge separation (figure 4.4). Compared to storm 1, the fit between the two profiles is not as good. In this simulation, P = 0.415 with an average difference between the two profiles of 287 V/m. Increasing the correlation between the model results and real data is more difficult in this case. Lightning activity greatly increases the time required to complete a simulation. Lightning activity also adds additional complications and restraints when trying to determine which model parameters should be adjusted to increase correlation.
For example, this storm has both intracloud and cloud-to-ground lightning, and therefore both should also be present in the model.

![Graph showing horizontal distance from storm center (km) vs. current intensity (A/m)](image)

**FIGURE 4.4** Comparison of the ER-2 and model generated profiles for storm 2. The solid line is from the ER-2, and the dashed line from the model.

At $t = 30$ min., the upward current above the storm is 1.29 amps, with a current to the conjugate hemisphere of 0.62 amps. The fraction of upward current to the conjugate hemisphere is about 50%, the same percentage found in storm 1. This fraction of current to the conjugate hemisphere remains fairly constant during the active portion of the storm's lifetime. These results disagree with those found by Tzur and Rable [1985], who found that 90% of the upward
current reaches the conjugate hemisphere for the high latitude geometry we are modeling.

The average upward current of storm 2 is 0.61 amps. A comparison to real data can be made to see if this is a reasonable value. If the upward current is multiplied by the estimated number of active thunderstorms worldwide (about 1500), the total current to the global circuit would be 915 amps. Dividing by \(4\pi R_c^2\) gives a fair weather current density of \(1.8 \times 10^{-12} \text{ A/m}^2\), about 75% of the observed fair weather value. This seems reasonable for a moderately active thunderstorm.

The electric fields in the storm region follow behavior similar to the currents, rising from 0 to some peak near 30 minutes, and then decaying. Figure 4.5 shows the vertical component of the electric field at 2 locations along the storm's central axis. The vertical field at the ground and 6 meters above the ground is plotted; notice that the field at the ground is suppressed by corona. From figure 4.5 it is also seen that the electric field at 6 m altitude is nonzero at \(t = 1\) hour, even though charge separation within the cloud has ceased. There are still fields and currents in and around the storm due to screening layers of charge that were built up (and are now dissipating) on the cloud boundaries and near the ground.

The spikes seen in figure 4.5 are due to lightning in the model. Electric field strength increases everywhere as the storm begins to develop. At 24.5 minutes, the electric field between the charge centers reaches the breakdown field of the atmosphere, and an intracloud lightning flash is triggered. Charge is transferred between the positive and negative charge centers, reducing the electric fields everywhere. Although reduction of the electric fields in the storm reduces the conduction currents carrying charge away from the cloud's charge centers, the
source current is unaffected. Charge quickly rebuilds, and the storm's electric fields and currents return to nearly their preflash values rapidly, as seen in figure 4.5. Field recovery is initially very fast, followed by a slower period of recovery up until the next flash occurs; the quick initial reaction after the flash is due to the redistribution of charge in the cloud's screening layers. Charge continues to build within the cloud until the breakdown voltage is again reached, and another lightning flash is triggered. Because a lightning flash will deplete charge locally.

\[
\text{FIGURE 4.5 The vertical electric field at storm center. The solid line is the field}\n\text{at the ground, while the dashed line is the field 6 meters above the ground.}\n\]

(near its point of origin in a charge center), subsequent lightning flashes will tend to occur at locations within the cloud where charge has not yet been depleted; the old adage 'lightning doesn't strike the same place twice' has some validity...

Table 4.1 summarizes the lightning activity of the simulation. The third flash is the only one to occur in the same location as the previous flash. In these simulations, there is often one grid point or collection of grid points where the
breakdown voltage is likely to be seen. Breakdown is most frequent along the storm's central axis, where charge separation is most intense and there are no horizontal conduction currents depleting charge. In storm 2 only the first flash is off the central axis, but other simulations have multiple flashes at several distances from the storm’s center. Storm 2 has only one cloud-to-ground lightning flash, #4.

<table>
<thead>
<tr>
<th>Flash</th>
<th>time (min)</th>
<th>x (km)</th>
<th>height (km)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>24.5</td>
<td>1.5</td>
<td>5.1</td>
</tr>
<tr>
<td>2</td>
<td>28.3</td>
<td>0</td>
<td>5.1</td>
</tr>
<tr>
<td>3</td>
<td>31.8</td>
<td>0</td>
<td>5.1</td>
</tr>
<tr>
<td>4</td>
<td>34.8</td>
<td>0</td>
<td>0.3</td>
</tr>
<tr>
<td>5</td>
<td>37.3</td>
<td>0</td>
<td>5.3</td>
</tr>
</tbody>
</table>

**TABLE 4.1 Lightning activity and location within storm 2.**

The relative altitudes of the positive and negative centers and the conductivity perturbation of the cloud determine whether breakdown voltages occur most frequently above or below the negative charge center. This means, for example, that raising the height of the negative charge center will increase intracloud lightning and decrease cloud-to-ground activity. Again, other simulations have shown a more even ratio of cloud-to-ground and intracloud lightning, although lightning between charge centers is more common. Intracloud and cloud-to-ground lighting can be distinguished in the aircraft electric field profiles by the direction of the field change. Intracloud lighting neutralizes charge and will decrease the field everywhere (a downward jump in
the profile. Cloud-to-ground lightning reduces the lower charge center (and possibly some of the screening charge below the cloud) but not the upper charge center; this means that fields will increase above the cloud. In figure 4.5, cloud-to-ground lightning also causes a positive increase in the field near the ground, but only because the point of observation is inside the lightning channel; on the ground away from the storm's central axis the electric field decreases after the cloud-to-ground lightning flash.

Of the total charge separation current in a thunderstorm, only some fraction will contribute to charging of the global electric circuit. The storm's 'absolute efficiency' can be defined as the storm's upward conduction current contribution to the global circuit divided by its total source current. Figure 4.2 indicates that the upward conduction current can be obtained at any altitude between the thundercloud top and the bottom of the equalization layer. The total source current is also calculated by the model at every timestep. Grid point density within the storm is not accurate enough to determine the storm's absolute efficiency with any precision; the absolute efficiency of a simulation can vary significantly when only the grid spacing is altered. This nonphysical effect is a result of numerical error within the storm region, and as long as absolute efficiency varies with grid spacing, the efficiency values generated are suspect. It is possible, however, to look at relative variations in efficiency due to changes in lightning activity. By changing one parameter - the amplitude of the source current charge separation rate - lightning activity will increase or decrease while the storm's structure remains unchanged. Any change in efficiency should be due solely to the variation in lightning activity.
FIGURE 4.6 Efficiency vs. relative intensity. Relative efficiency shows only a slight increase when lightning is present.

Figure 4.6 shows the efficiency of a storm as a function of storm intensity. Variation in storm intensity is caused by varying the charge separation rate, with an intensity of 1.0 for a 'normal' storm 2. Simulations with intensities of 1.0+ have lightning activity. Although the graph is a plot of absolute storm efficiency, it is the trend in efficiency that is of interest. In figure 4.6, efficiency is constant up to a relative intensity of 0.5 - the smaller storms that have no lightning activity show no variation in efficiency. There is a slight increase in efficiency at 1.0; this
is caused by the beginning of lightning activity. Efficiency continues to increase in the storms of 1.5 and 2.0 relative intensity. Even at a relative intensity of 2.0, however, lightning is still relatively infrequent (less than one flash per minute).

Most of the lightning in these simulations is intracloud. If cloud-to-ground lightning was dominant, efficiency would likely be increased. Previous aircraft measurements also suggest that lightning activity may be proportional to a thunderstorm's total upward Maxwell current [Blakeslee et al., 1989]. Although closely related to the work presented here, these relationships will be addressed in future work.

FIGURE 4.7 Vertical electric field at 5 km. The solid line is for the 'standard' storm 2, while the dashed line is for a simulation where the charge separation rate has been doubled.

Within the storm electric fields do not increase linearly. For low intensity storms, the fields are proportional to the rate of charge separation. Increasing
storm intensity causes stronger fields to develop inside the thunderstorm. When a storm is intense enough to create lightning, the lightning activity limits the maximum fields found anywhere to the breakdown field strength; more intense storms do not have higher electric fields within the cloud. Instead, the fields reach their maximum values more quickly during the developing stage, and maintain the fields longer in their final stage. Because charge is being separated at a higher rate, electric field recovery after a flash is more rapid, and lightning activity is more frequent. These results are illustrated in figure 4.7.

![Graph showing electric field over time](image)

**FIGURE 4.8** Vertical electric field at 20 km. The solid line is for storm 2, while the dashed line is for a simulation where the charge separation rate has been cut in half.

The small kink in figure 4.6 is due to lightning activity at relative intensities of 1 and greater. For storm 2, there is actually a small increase in storm efficiency when lightning is present. Figure 4.8 is a plot of the vertical electric field at 20 km over the storm's lifetime. Simulations of storm 2 at normal intensity and at half intensity are plotted, with the vertical scale being doubled for the smaller
storm. Before lightning activity begins the two curves overlay each other; the development of the storms is identical, other than the factor of 2 difference. When lightning occurs, the fields in (the normal intensity) storm 2 drop as you would expect, but recover to a higher value than they would have had if there had been no lightning flash. From this point on, the vertical electric field of storm 2 is more than twice that of the half intensity storm, even after lightning activity ceases. Comparisons of the net upward current to the global circuit for each storm show the same behavior.

![Graph](image)

**FIGURE 4.9** Total conduction current at 5 km. The solid line is for storm 2 at half intensity, and the dashed line for the normal intensity storm 2.

One possible explanation for this behavior is that lightning preferentially depletes charge in the middle of the cloud. Most of the charge neutralized by an intracloud lightning flash will be in the lower portion of the positive charge center and in the upper portion of the negative charge center. Depletion of charge in this region may reduce the intra-electric center conduction current and
make it easier for charge to flow from the top and bottom of the cloud. In a sense, the 'centers' of the positive and negative charge regions are being moved upward and downward respectively. Figure 4.9 is a comparison of the conduction current between the charge centers for the normal intensity storm that includes lightning and the half intensity storm without lightning. As in figure 4.8, the intracloud conduction current of the two storms is proportional until lightning activity begins. After the first lightning flash, relatively less conduction current is present in storm 2. Most (but not all) of the difference in the amount of charge neutralized by conduction current is neutralized by lightning activity in storm 2.
CONCLUSIONS

Our global electricity model has been developed to run in a time-dependent mode, with the effects of lightning and cloud evolution included. It is now possible to look at the time evolution of a thunderstorm and its interaction with the global electric circuit. Previous work with the model in steady state simulations found peak values for several storms' conduction current contribution to the global circuit. This thesis's results show that the average value over a storm's lifetime may be about half its peak value.

A technique for comparing model output to real data has been developed, improving the reliability of our results. In this work, two simulations were matched to vertical electric field profiles taken by the NASA ER-2, but little other data on the storms modeled were available. In future work it would be preferable to use data from multiple sensors describing several aspects of the storm's structure; electric fields above, in, and below the storm, charge center heights and extents, and so on. If most of the storm input parameters are predetermined from actual data, and the model output reproduces the measured electric fields, confidence in the validity of the modeling results will be increased.

Results from the time-dependent simulations agree with previous steady state work in that about 50% of the upward generated conduction current flows to the conjugate hemisphere via the Earth's magnetic field. This figure seems to be constant regardless of storm size or structure.

The efficiency of a storm in supplying current to the global electric circuit is not strongly influenced by lightning. In varying the intensity of charge separation in storm 2, limited lightning activity caused only a small change in
thunderstorm efficiency. While intracloud lightning might decrease the amount of total separated charge, the spatial pattern of charge depletion may increase the percentage of available charge that does flow into the global electric circuit.

Future work on the global model should address two shortcomings of the model. A more detailed analysis of where numerical inaccuracy occurs inside the thundercloud and how it can be minimized should be done. If the model can be transported to a more capable computer, it might be possible to provide sufficient grid resolution everywhere in the model, with no significant current loss anywhere. Lightning activity can also be improved. In the model's present implementation, a lightning channel is created wherever the electric fields are strong enough. A more realistic approach would be to build the lightning channel over several timesteps, allowing the stepped leader to develop wherever the electric fields become intense enough.
REFERENCES


APPENDIX A:  
GLOSSARY OF TERMINOLOGY

Aerosol  Small solid or liquid particles suspended in the air.

Charge separation current  The current inside a thunderstorm caused by the separation of positive and negative charge through some convective mechanism. Also called source current.

Conduction current  The current caused by charge carriers moving under the influence of an ambient electric field. The current is proportional to the strength of the electric field and the conductivity of the atmosphere.

Conjugate hemisphere  Modeled region of the atmosphere magnetically connected to the thunderstorm region via the magnetosphere. Also called the fair weather hemisphere.

Current density  The electric current flowing through a unit area; in MKS, Coulombs per second-square meter.

Electrical relaxation time  The time the electric charge distribution in the atmosphere takes to come to 1/e of its equilibrium value when an electric field is applied.
**Electrode layer**  A layer of space charge built up at the boundary between the air and a cloud or the ground. The charge will influence the local electric field.

**Equalization layer**  The lower portion of the ionosphere (in these simulations from 60 to 120 km) where variations in potential are relatively small; current generated by thunderstorms spreads globally within the equalization layer.

**ER-2**  NASA's high altitude aircraft used to collect the electric field data above the modeled thunderstorms.

**Fair weather**  Any region of the atmosphere far from electrified clouds. In fair weather the vertical current flows from the equalization layer to the ground.

**Field mill**  A sensor for measuring the atmospheric electric field. Field mills were flown on an ER-2 aircraft in gathering the electric field data used in this thesis.

**Form factor error**  The error introduced by not knowing how much the metal aircraft enhances the measured local atmospheric field. The enhancement is primarily a function of aircraft shape.

**Global electric circuit**  The components of the atmosphere and Earth important in the generation and maintenance of the electric fields and currents observed in the atmosphere.
**Grid point**  One of the locations at which the electric potential of the atmosphere is calculated in the computer model. Grid points are distributed on a plane that is a cross section of the storm or conjugate hemisphere.

**Ionosphere**  Region of the upper atmosphere (above about 60 km) where large concentrations of charged particles exist. Because of the density of free electrons here, the electrical conductivity is very high. The lower ionosphere corresponds to the beginning of the equalization layer.

**Maxwell current density**  The total current density; in our model the sum of conduction, displacement, lightning, source, and corona current densities.

**Offset error**  A drift in the calibration of a field mill, limiting the accurate measurement of small electric fields.

**Parameters**  When talking about the model, the input parameters are the specific information given the computer program. The parameters define the size and shape of the thunderstorm, the atmosphere’s conductivity, the location of the grid points, etc. Appendix B has specific information on all input parameters.

**Pederson conductivity**  The conductivity of the atmosphere perpendicular to the ambient magnetic field and parallel to the electric field.
**Source current** The current inside a thunderstorm caused by the separation of positive and negative charge through some convective mechanism. Also called charge separation current.

**Specific conductivity** The conductivity of the atmosphere parallel to both the magnetic and electric fields.

**Space charge** Electrically charged particles in the atmosphere. The density of the lighter, more mobile particles determines the local conductivity. Conduction current is caused by the uniform motion of space charge in an ambient electric field.

**Storm hemisphere** Modeled region of the atmosphere including the thunderstorm and extending out to some large horizontal distance.
APPENDIX B:  
MODEL PARAMETERS

The following is a list of the parameters used in both thunderstorm simulations. The parameter name and value is given, and a brief description of the parameter's function is included. If a parameter was the same in both simulations, only one value is given, otherwise storm 1 and storm 2 values are listed consecutively.

When possible, justification for the choice of parameter values is provided.

\[ RE = 6380 \text{ (km)} \]

Radius of the Earth [Gimbel, 1984].

\[ ZEQ = 120 \text{ (km)} \]

The top of the equalization layer. This is the upper limit of the modeled atmosphere, and all upward flowing current reaching this altitude is mapped to the conjugate hemisphere.

The higher the upper boundary, the better. As ZEQ increases, the space between vertical grid points also increases, leading to numerical errors. 120 km is well into the equalization layer, and the continuity of current test still indicates the grid density is adequate.

\[ THETAM = 0.2 \text{ (radians)} \]

The angular extent of the region modeled. THETAM \times RE = 1300 km, the vertical boundary of each hemisphere. Increasing THETAM allows more of the
atmosphere to be modeled, but decreases grid point density. In practice, several runs are made of each simulated thunderstorm varying this parameter to see if the grid density is sufficient and if including more of the atmosphere changes the results. If the angular extent is large enough, the electric potential distribution should uniformly decrease (because there is more fair weather atmosphere through which fair weather current flows, decreasing the load resistance of the global electric circuit) without a significant change in the structure of the potential distribution.

**ZM = 240 (km)**

Maximum height of the modeled region. The computer program is constructed so that the top of the conjugate hemisphere is above 120 km altitude. Increasing altitude from 120 to 240 km in the program is actually going down through the conjugate hemisphere to the ground. The program is designed this way for generality's sake (so it can be used to model only part of the storm hemisphere, for example). When modeling the complete global electric circuit, ZM must be twice ZEQ.

**NX = 45**

**NY = 400**

The number of horizontal and vertical grid points. NX*NY = the total number of grid points in the simulation, and is limited by memory availability in the computer. NY can be chosen by using the continuity of current test to see if the vertical grid is dense enough. NX is then made as large as computer memory allows.
IFLAGT = 1

Time dependence flag, determining the temporal variation in the source function. 0 is steady state (time independent), 1 the tangential function used in this work, and 2 an exponential growth function.

The tangential growth function varies between 0 (at storm beginning and end) to 1 (during which charge separation is most intense)

FIGURE A.1  The time dependence function of the global model.

ITM = 3600 (sec)

Ending time of the simulation. ITM should be at least twice TTAU.

TTAU = 1800 (sec)

This is an approximate measure of the thunderstorm's half-life. The storm's charge separation will peak at t = TTAU. An air mass thunderstorm's lifetime is highly variable, but development from the cumulus stage to the dissipating stage generally takes an hour or less [Ahrens, 1988].
TA = 4

Time source steepness, affecting how rapidly charge separation varies from 0 to 1 and back to 0.

IDELT = 10 (sec)

Time step. The smaller the time step, the greater the model's accuracy. Decreasing the timestep also increases the amount of real time the computer requires to run a simulation. In practice, accurate computation of the corona current limited the size of IDELT; too large a timestep causes the corona current to 'oscillate.'

The following parameters determine the growth of the cloud and charge centers. All input parameters describing cloud and charge center extent are the initial values at t = 0. If VZT = 12.0 km and GROWZ = 1.5, the cloud top will begin at 12 km altitude and grow to 18 km altitude by the simulation's end.

GROWCZ = 1.0  1.5

Vertical (upward) growth of the positive charge center over time. If GROWCZ = 2, the charge center will approach twice its original size by the end of the simulation.

GROWCX = 1.0

Horizontal growth factor of both charge centers. A value of 1 means no growth.
**GROWZ = 1.0 1.5**

Vertical growth factor of the thundercloud.

**GROWX = 1.0**

Horizontal growth factor of the thundercloud.

**VZT = 12.0 (km)**

Height of cloud top, determined through profile matching and observations of thunderstorm activity on the day the data was taken.

**VZB = 2.0 1.0 (km)**

Cloud base, determined through profile matching and observations on the day data were taken.

**HWC = 5.0 8.0 (km)**

Cloud width, determined through profile matching.

**SA = 2.857E-12 8.0E-12 (C/m^3s)**

Source function amplitude, determined through profile matching. This is the intensity of source charge generation, and although storm 2 has a higher value than storm 1, the size of the charge generation region also plays an important role in thunderstorm intensity.
FLAGM = F

The model is capable of simulating multi-cell thunderstorm complexes, in which case this flag is set true.

VZN = 4.0  3.7 (km)

Height of the negative charge center. This parameter was initially set from balloon electric field data taken in other thunderstorms [Byrne et al., 1987], and refined through profile matching.

The effect of the cloud on atmospheric conductivity is determined by a cloud conductivity perturbation function. The conductivity perturbation is largest at the cloud center and decreases as you approach the cloud boundaries. The source charge generation function is similar; charge separation is maximum in the middle of each charge center and drops off as you move horizontally or vertically away.

Several of the parameters described below are 'half-maximum' values. Similar to e-folding widths, they determine how far from the middle of the cloud or charge center the appropriate function drops to half its maximum value. For example, if VWX = 1.0 and VZN = 4.0 km, then maximum charge generation in the negative charge center will be at x = 0 km (along the storm's central axis) and y = 4.0 km. At y = 5.0 km or 3.0 km, charge generation will be half what it is at y = 4.0 km (less off the central axis, HW determines how quickly charge generation falls off horizontally).
\[ VWN = 1.0 \text{ (km)} \]

Vertical half-maximum width of negative charge generation intensity, determining the vertical decrease of negative charge generation. A large half-maximum width means more negative charge will be created within the charge center. The negative charge center is limited to altitudes between \( VZN - 2\times VWN \) and \( VZN + 2\times VWN \).

\[ VPN = 4.0 \]

Exponent in negative charge generation function, affecting the 'steepness' of charge generation. A smaller exponent will create charge more uniformly (vertically) within the charge generation region.

\[ VZP = 9.0 \quad 10.0 \text{ (km)} \]

Height of the positive charge center, determined in the same manner as \( VZN \).

\[ VWP = 1.0 \quad 2.0 \text{ (km)} \]

Vertical half-maximum width of positive charge generation intensity, determining the extent of positive charge generation. A large half-maximum width means more positive charge will be created within the charge center. The positive charge center is limited to altitudes between \( VZP - 2\times VWP \) and \( VZP + 2\times VWP \).

\[ VPP = 6.0 \quad 4.0 \]

Exponent in positive charge generation function, affecting the 'steepness' of charge generation. A smaller exponent will create charge more uniformly (vertically) within the charge generation region.
**HW = 3.0   7.5 (km)**

Horizontal half-max width of both charge centers. A larger value means the charge centers will have a larger horizontal extent.

**HWP = 4.0   3.0**

Exponent in charge generation function, affecting the 'steepness' of both positive and negative charge generation in the horizontal direction (see VPN and VPP).

![Diagram](image.png)

**FIGURE A.2** The source charge generation function for storm 2.
PHIL = 0

Electric potential of the model's lower boundary, in this case the ground in the storm hemisphere. In this application of the model, the ground is obviously at 0 Volts.

PHIU = 0

Electric potential of the model's upper boundary, in this case the ground in the conjugate hemisphere.

FLAGC = T

Corona is included in the simulation if FLAGC is 'true' (T).

CW = 0 (V/m)

The minimum electric field present near the ground sufficient to induce corona. This parameter should be about 5000 V/m [Sturrock, 1980]. If this is done, oscillation will occur as corona 'switches' on. Allowing small corona currents at low voltages has little effect on the lower atmosphere dynamics, and keeps the electric field at the Earth's surface from unrealistically bouncing around.

CZ = 0.5 (km)

Altitude below which charge is created to simulate corona.
SFF = 1.2E-14 (mho/m)

Conductivity of the atmosphere at ground level; near the ground, the conductivity is enhanced by the leakage of radioactive gas from the Earth [Hays and Roble, 1979].

SFH = 7.0 (km)

Atmospheric scale height [Wallace and Hobbs, 1977].

RHO1 = 4.69E+13 (ohm-m)
RHO2 = 2.22E+13
RHO3 = 5.9E+12

Volland specific resistance coefficients (see below).

ALPHA1 = 4.527E-3 (m⁻¹)
ALPHA2 = 3.75E-4
ALPHA3 = 1.21E-4

Volland scale heights. The above coefficients determine the average atmospheric conductivity up to 65 km in the equation

σ(z) = (RHO1 \cdot e^{-\alpha_{1}z} + RHO2 \cdot e^{-\alpha_{2}z} + RHO3 \cdot e^{-\alpha_{3}z})^{-1},

where z is altitude in meters [Volland, 1984].

SFRH1 = 4.5 (km)

The specific conductivity scale height from 65 to 110 km.
**SPH1 = 6.5 (km)**

The Pederson conductivity scale height from 65 to 110 km.

**SFZ1 = 65 (km)**

The height at which the specific conductivity profile becomes a simple exponential with scale height SFH1.

**SPZ1 = 65 (km)**

The height at which the Pederson conductivity profile becomes a simple exponential with scale height SPH1.

**SFH2 = 30 (km)**

The specific conductivity scale height from 110 to 120 km.

**SPH2 = 30 (km)**

The Pederson conductivity scale height from 110 to 120 km.

**SFZ2 = 110 (km)**

The height at which the specific conductivity profile becomes an exponential with scale height SFH2.

**SPZ2 = 110 (km)**

The height at which the Pederson conductivity profile becomes an exponential with scale height SPH2.
FIGURE A.3 The Pederson and specific conductivities as a function of altitude.

$SCA = 0.1$

Amplitude of the cloud's perturbation on the local conductivity. The perturbation is maximum at cloud center. SCA can vary from 1.0 (no perturbation) to 0.005 [Pruppacher and Klett, 1978]. 0.1 is a small perturbation; values are limited by the numerical error caused by large conductivity gradients.

$SCVZ = 7.0$ (km)

Height of the cloud center, determined from the cloud base and top. Approximate - this is the point of maximal conductivity perturbation inside the cloud (not necessarily the lowest conductivity within the cloud).
SCVWN = 5.0 (km)

The vertical half-max width of the conductivity perturbation in the lower cloud.

SCVWP = 5.0 (km)

The vertical half-max width of the conductivity perturbation in the upper cloud.

SCVN = 4.0

Exponent in the cloud conductivity perturbation function. A larger value means the conductivity drops off more quickly near the lower cloud boundary.

SCVP = 4.0

Exponent in the cloud conductivity perturbation function. A larger value means the conductivity drops off more quickly near the upper cloud boundary.

SCHW = 6.0  4.0

The horizontal half-max width of the conductivity perturbation in the cloud.

SCH = 6.0  4.0

Exponent in the cloud conductivity perturbation function. A larger value means the conductivity drops off more quickly near the vertical cloud boundaries (the sides of the cloud).
FLAGB = F

If true (T), this parameter increases the conductivity below the cloud to simulate the effect of lightning, useful in steady state simulations. It is false (F) in this work, and the following related parameters are all set to 0: SCBZ, SCBV, SCBA, SCBW, SCBH.

FLAGL = F  T

This parameter includes lightning in the simulation when true (T). For storm 1, this parameter was immaterial; electric fields never reached levels where lightning would occur.

FDELT = 0.1 (s)

The duration of a lightning flash [Krider, 1986]. The timestep is changed from IDELT to FDELT when lightning occurs.

EBREAK = 2.0E+5 (V/m)

The electric field needed to ionize the atmosphere and trigger a lightning flash. The breakdown electric field of clean air at standard temperature and pressure is actually about 3 megaVolts/m. Decreased atmospheric pressure within the cloud lowers the breakdown electric field by at least a factor of two, and water droplets within the air decrease it by another factor of 3.
\textbf{SIGMAL} = 1.0E-10 (mhos/m)

The enhanced conductivity within a lightning channel. The actual conductivity should be larger, but large conductivity gradients between adjacent grid points (at the edge of a lightning channel) introduce numerical errors.

\textbf{EPROP} = 4.0E+4 (V/m)

The electric field needed to propagate a lightning channel. Once the channel has been created by exceeding the breakdown voltage, it can be extended if the propagation voltage is present at an adjacent grid point.

\textbf{XF} = 0.05

Linearity factor for horizontal grid spacing. The grid produced by these grid parameters is shown below.

\textbf{XP} = 15

Exponent in horizontal grid spacing function.

\textbf{ZF} = 0.2

Linearity factor for vertical grid spacing.

\textbf{ZP} = 3.0

Exponent in vertical grid spacing function.
FIGURE A.4 The grid distribution near the thunderstorm.

A portion of the grid for the storm hemisphere is shown above. Vertical grid density continues to decrease with altitude. Although it is not obvious in the above figure, horizontal grid density also decreases with distance from the storm.

There are additional input parameters, but they do not affect the model results.
APPENDIX C:
EQUATION EXPANSION

The equation found in Chapter 2,

$$\nabla \cdot \left( \left( \frac{\sigma}{\Delta t} + \frac{E}{\Delta t} \right) \nabla \Phi(t + \Delta t) \right) = S(t + \Delta t) + \frac{E}{\Delta t} \nabla \cdot \Phi(t)$$  \hspace{1cm} (C.1)

must be further modified for use in the computer model. The model assumes the specific conductivity is along the magnetic field lines and is therefore always vertical, or along \(\hat{r}\), while the Pederson conductivity is in the direction \(\hat{\theta}\). The tensor conductivity can be written

$$\tilde{\sigma} = \begin{bmatrix} \sigma & 0 \\ 0 & \sigma \end{bmatrix}$$ \hspace{1cm} (C.2)

where \(\sigma_t\) and \(\sigma_p\) are the specific and Pederson conductivities respectively. (C.2) can be substituted into (C.1) and the \(\nabla\) operators applied on the left side of the equation, giving

$$(\sigma_t + \frac{E}{\Delta t}) \frac{\partial \Phi(t + \Delta t)}{\partial t} + \left( \frac{\partial \sigma_t}{\partial r} + \frac{2(\sigma_t + \sigma_p \Delta t)}{r} \right) \frac{\partial \Phi(t + \Delta t)}{\partial r} + \frac{\partial \sigma_p}{\partial r} \frac{\partial \Phi(t + \Delta t)}{\partial \theta}$$

$$+ \frac{1}{r^2} \left( \frac{\partial \sigma_t}{\partial \theta} + \frac{\sigma_t + \sigma_p \Delta t}{\tan \theta} \right) \frac{\partial \Phi(t + \Delta t)}{\partial \theta} = S(t + \Delta t) + \frac{E}{\Delta t} \nabla \cdot \Phi(t).$$ \hspace{1cm} (C.3)

The computer program requires the equation to be in rectangular coordinates \((x, y)\). Transforming the equation gives
\[(\sigma_i + \varepsilon/\Delta t) \left( \frac{\partial \phi}{\partial t} \right) + \frac{\partial^2 \phi}{\partial y^2} + \left[ (\sigma_i + \varepsilon/\Delta t) \frac{\partial^2 \phi}{\partial r^2} + \left( \frac{\partial \sigma_i}{\partial r} + \frac{2(\sigma_i + \varepsilon/\Delta t)}{r} \right) \frac{\partial \phi}{\partial r} \right] \frac{\partial \phi(t + \Delta t)}{\partial y} + \]
\[+ (\sigma_i + \varepsilon/\Delta t) \left( \frac{1}{r} \frac{\partial \phi}{\partial \theta} \right) \cdot \frac{\partial^2 \phi(t + \Delta t)}{\partial x^2} + \]
\[= S_i(t + \Delta t) + (\varepsilon/\Delta t) \nabla^2 \phi(t) . \]

(C.4)

The inputs to the equation are \( \phi(t) \), \( \frac{\partial \phi}{\partial t} \), \( \frac{\partial \phi}{\partial r} \), \( \frac{\partial \phi}{\partial \theta} \), and \( \frac{\partial^2 \phi}{\partial x^2} \) (determined from the grid spacing), \( \sigma_i \) and \( \sigma_i \), \( \frac{\partial \sigma_i}{\partial r} \) and \( \frac{\partial \sigma_i}{\partial \theta} \), \( \varepsilon \), \( \Delta t \), \( S_i(t + \Delta t) \), and \( (r, \theta) \) for each grid point.

Output is \( \phi(t + \Delta t) \).
APPENDIX D:
ER-2 FIELD MILL PROFILES

The data used in generating ER-2 profiles were taken during the Cloud Electrification and Precipitation field program during the summer of 1991. Both storms were overflown on flight 91-131, 7/28/91. The storms were traversed at:
storm 1: 22:37 UTC  storm 2: 21:45 UTC (approximate times).

The field mills have a low, medium, and high gain channel, with a gain increase of 46 each. In reducing the data, it is desirable to use the highest gain channel that does not saturate during the storm overpass; in both of our profiles the mid-gain channel was used.

Mill output was recorded on a scale from 0 to 4096, where a value of 2047 would indicate zero electric field on a perfectly calibrated mill. Although offsets were found from ground calibrations, it was more convenient to find the zero offset by assuming a zero electric field away from the storms and subtracting off the observed offset directly. The vertical electric field is several V/m even away from the storms, and this calibration technique can only used because the variability in drift during the flight can be over 20 V/m (this is the reason fair weather measurements can not be made with this sensor system). The offsets used were:
Upper mill (MSFC mill serial #2): +17 bits
Lower mill (MSFC mill serial #3): +10 bits.

$S_u$ and $S_d$ are the recorded signals from the upper and lower mills respectively. The scaled signals are then:

$S_u' = S_u - 2047 + 17$

$S_d' = S_d - 2047 + 10$ (in 'bits').
The scaled signals, in bits, are then converted to measured electric fields through the laboratory calibration factors. The difference in calibration factors between the two mills was small enough so that the same calibration factor could be used for both mills. \( E_i \) is the raw field measured by either the upper or lower mill, and is given by

\[
E_i = S_{\text{i}} \left( \frac{20 \text{ V signal}}{4096 \text{ bits}} \right) \left( \frac{1 \text{ kV} \ E_x}{1.59 \text{ V signal}} \right),
\]

where another factor has been included, the conversion from recorded bits (0 to 4096) to mill output voltage (-10 to +10 V).

Each mill measures the sum of the charging and atmospheric vertical electric field. The aircraft form factor magnifies the measured field so that

\[
E_u = \text{form factor} \times (E_{\text{charging}} + E_{\text{atmosphere}}),
\]

\[
E_l = \text{form factor} \times (E_{\text{charging}} - E_{\text{atmosphere}}).
\]

If a form factor of 2.0 is used, the above equations can be subtracted to give

\[
E_{\text{atmosphere}} = (E_u - E_l)/4.0.
\]