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Drilling bottom-hole assembly dynamics

Payne, Michael Lyle, Ph.D.

Rice University, 1992
Rice University

Drilling Bottom-Hole Assembly Dynamics

by

Michael Lyle Payne

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

Doctor of Philosophy

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May, 1992
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Abstract

A mathematical model is proposed for studying the dynamic behavior of drillstrings, focusing on the lower section of the drillstring known as the bottom-hole assembly or "BHA". In parallel, a comprehensive assessment of existing concepts and techniques for addressing drillstring dynamics is undertaken. Accounting for stiffness, damping, and inertial properties of the BHA and by incorporating appropriate excitations and boundary conditions, the dynamic characteristics of the BHA can be analyzed. The representation of the BHA stiffness is influenced by the underlying beam formulation assumptions, the stress-stiffening effect of axial loads, the elastic properties of the various materials used in the BHA, and the effective stiffnesses of special BHA components with complex geometries. Damping in the BHA involves both internal structural damping and damping resulting from its interaction with the surrounding viscous drilling fluid. Damping for the structure is accounted for using recent damping data leading to a smooth damping function which involves the vibration frequency and the drilling fluid density. Inertial properties of the BHA include its mass and the added mass effects of the fluid, both inside and outside the BHA, which is displaced through its motion. Frequency response characteristics for the structure are developed assuming a monochromatic exciting force. Both damped and undamped responses are simulated using a transfer function representation developed by modal
superposition techniques. Sensitivity studies are performed to determine appropriate grid spacing for specific BHA problems of interest. Parameter studies reflect the influence of fluid added mass, weight-on-bit, boundary conditions, and the location of the excitation force. Excitation mechanisms for actual drilling assemblies are studied leading to a response superposition procedure which fully accounts for the behavior of BHA in drilling operations. Topics for further research are recommended.
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Grandmother Alford and Mammaw Payne

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Chapter 1- Introduction

"Progress begins with getting a clear view of the obstacles."

Interest in drilling dynamics began many years ago when explanations for drill string "bounce" were sought. By the late 1970's, work had progressed towards full three-dimensional (3-D) consideration of the bottom-hole assembly (BHA) using specialized finite-element programs. Current modelling work focuses on increased accuracy, computational issues, correlation with field observations, and nonlinear problem aspects. Many advances have also been made in data collection on drillstring dynamics. Unfortunately, complex phenomena and unknown parameters remain as barriers to the development of a comprehensive model for drillstring dynamics. For example, the exact diameter of the wellbore is unknown during drilling and can vary substantially from expected dimensions. Because of limitations regarding parameters which affect drillstring dynamics, engineering efforts should be focused on bounding the problem through sensitivity studies on key parameters, and on quantifying only the most critical features of the problem which can be reliably described. Towards this objective, an overview of drilling practices and problems is attempted.

Although many other methods have been tried historically, rotary drilling techniques as shown in Figure 1.1 dominate the industry. In specific applications, motor/turbine drilling is also used. The fundamental components of rotary drilling include the bit which abrasively destroys the formation as it rotates and the drillstring which is used to control the bit, circulate the drilling fluid or "mud", and generate the weight loads required to push the bit into the formation with sufficient force. The drillstring is made up, primarily, of drillpipe and drillcollars. Drillcollars are very thick,
Figure 1.1
The Rotary Drilling Process

Derrick

Diesel Engines or Generators

Solids Control Equipment

Drillpipe Stands

Blowout Prevention Equipment

Conductor Casing

Drillpipe Rotated by Rotary Table in Rig floor

Drillpipe

Drilling Mud Circulating down Drillpipe and back up annulus

Surface Casing

Bottom-Hole Assembly or "BHA"

BHA provides weight for forcing bit into formation with "Weight-on-Bit" (WOB)
for example 8" outside diameter (OD) x 2.5" inside diameter (ID), steel pipes which are used to construct the BHA. The drillcollars are used to provide the necessary gravitational weight to produce the desired "Weight-on-Bit" (WOB), to sustain the loads generated while drilling without damage or fatigue, and to provide a stiff structural constraint to ensure straight drilling. The BHA is positioned directly above the bit to perform these functions. Drillpipe has more conventional wall thickness, for example 5" OD x 4.276" ID, and is run from the BHA back to the surface to control and rotate the entire drillstring.

Modern drilling assemblies, such as shown in Figure 1.2, involve more complicated components including:

1. Stabilizers, both near the bit and between drill collars, to centralize the assembly and induce directional drilling tendencies.

2. Non-magnetic drill collars to accommodate the use of directional survey instruments which rely on measurements of the earth’s magnetic field.

3. Drilling "jars" to shock load the drill string in the event it sticks downhole.

4. Measurement While Drilling (MWD) tools which contain sophisticated instruments to measure formation properties and transmit relevant data to the surface by using pressure pulses in the drilling fluid.

5. Heavy-weight drillpipe (HWDP) which provides for a gradual crossover between drillcollars and drillpipe and can allow additional weight-on-bit (WOB).

6. Shock-absorbing tools which use axial springs (Bellville, foam, or gas cushion) to dampen axial vibrations and shift resonances away from operating speeds.
Figure 1.2
Modern Bottom-Hole Assembly (BHA)

Heavy-Weight Drillpipe
and Drilling Jars
9" O.D. Drillcollar to 5-1/2" O.D. Drillpipe Crossover
9" O.D. Drillcollar

9" O.D. Drillcollar
16-1/2" O.D. Stabilizer

9" O.D. Drillcollar
11" O.D. to 9" O.D. Crossover
11" O.D. Drillcollar

11" O.D. Drillcollar
16-1/2" O.D. Stabilizer

11" O.D. Drillcollar
11" O.D. Shock Sub

16-1/2" O.D. Stabilizer
16-1/2" Gage Diameter Bit
Rotary drilling involves a "rotary table" mounted in the rig floor to rotate a hexagonal joint, the "kelly" which then turns the rest of the drillstring. In the past several years, "top drives" or "power swivels" have become popular where the motor rotating the drillstring is mounted in the derrick and attached on the top of the drill string. Another variation involves the use of downhole turbines or motors which rely on the flow of the drilling fluid to produce torque and rotary motion. With such motors, intentionally bent components, from 1/2° up to 3° or more, can be added to the string to force directional drilling towards desired targets.

Bits come in many forms. Tri-cone bits, such as in Figure 1.3, have three conical heads which destroy rock with their teeth as the bit and its cones rotate. Fixed cutter bits, such as shown in Figure 1.4, have stationary cutters which indent into the formation and remove material during rotation. Each bit type exhibits specific displacement, force, and frequency characteristics as it drills. Rock properties influence the drillstring dynamic behavior and the formations can change rapidly during drilling.

The drillstring components are typically 4-1/2" to 8" in outer diameter and are used to drill wellbores with internal diameters of 6" to 24". Depthwise, however, the assemblies can extend up to 5 miles or more downhole. A drilling research effort in the Soviet Union is targeted for 50,000+’ downhole or nearly 10 miles, and similar efforts are under discussion in the U.S.A. With small radial clearances and extremely long depths, accurate modeling of contact between the drillstring and the wellbore is quite difficult. Directional drilling and its now popular variation, horizontal drilling, also complicate the problem as wells are drilled with substantial curvatures and three-dimensional characteristics.
Figure 1.3
Tri-Cone Rotary Rock Bit
Figure 1.4
Bladed Fixed-Cutter Bit
Drillstring failures are continually ranked as one of the most frequent and costly drilling problems in the industry. Vibration is destructive to the BHA and its elimination will allow these failures to be minimized. Early work provided solutions to axial and torsional vibrations treated as single degree-of-freedom problems. However, drillstring failures are continually experienced which cannot be explained by these models. Typically, these events are attributed to poor inspection for cracks in the components or simply accepted as a part of drilling. In the early 1980's, certain BHA failures occurred so rapidly and with well-inspected equipment that it was clear that dynamic effects had to be accelerating the failures. Current technical thinking considers lateral bending motions as an important factor for the fatigue of drilling assemblies.

Costs of BHA failures are high. At a minimum, a BHA failure forces suspension of drilling, a "trip" out of the hole for "fishing" equipment, a "fishing" trip, a new BHA component and a "trip" back in the hole to resume drilling. On the low side, this cost would involve 12-24 hours of rig time which would vary from $10,000 for small onshore operations, to $30,000 for fixed offshore operations, and to $100,000 or more for floating offshore operations. On the high side, the failed BHA may not be retrievable, forcing a sidetrack or resulting in the loss of the well. The economic impact of such an incident would run from several hundred thousands of dollars onshore to several millions offshore. Clearly, the economic incentives are substantial for reducing BHA failures, and the technology for analyzing the problem is of great value to the industry.

The present study has two prime objectives. First, it aims to consolidate and organize the existing knowledge on drillstring dynamics. Second, it focuses on the
development of a method which enables one to predict efficiently the response of a
given mathematical model of the drillstring under monochromatic or monofrequency
excitation. Interest in this kind of excitation is strong as it can provide a convenient
scheme for detecting dynamic amplification effects involving the frequency content of
more arbitrary excitations of the drillstring.
Chapter 2 - Critical Assessment of Current Drillstring Mechanics Technology

"To know the road ahead ask those coming back" - Chinese

Several classes of dynamic drillstring models exist. The ensuing technology review is organized according to topics. Specifically, torque/drag models, static directional models, dynamic directional models, experimental studies, bit mechanics and excitations, ROP models, stick/slip models, and response models are considered. In some instances, works overlap one or more areas and judgement has been used in placing the work in context with related concepts.

2.1 Torque and Drag Models

Models for torsional and axial drag prediction are included in this review of BHA dynamics because they address the loads on moving drilling assemblies. These models involve static approximations to estimate torsional and axial friction loads while rotating or reciprocating the strings. The merit of the models is in planning and executing directional drilling operations. There are large economic stakes in being able to drill extreme directional wells to provide full drainage of reservoirs from limited surface locations. This capability minimizes expensive surface facilities such as offshore platforms, gravel islands along Arctic coastlands, onshore Arctic drilling pads, and subsea completion templates and floating production systems for deepwater developments. Torque/drag models can identify directional limitations in terms of either the drillstring or rig capacities.

In 1982, Johancsik, Friesen and Dawson of Exxon [1] provided the first torque/drag model. As shown in Figure 2.1.1, the model is based on satisfying equilibrium between weight, axial, and friction forces. The drillstring is assumed to be
Figure 2.1.1
Idealized Basis for Torque/Drag Estimation
(from Johansik, et.al. 1983 [1])

Free-body Diagram of a Single Element

\[ N = \left[ \left( T \Delta \phi \sin(\bar{\theta}) \right)^2 + (T \Delta \theta + W \sin(\bar{\theta}))^2 \right]^{1/2} \]

where

- \( f \) = Drag or Friction Coefficient
- \( F \) = Axial Friction Force
- \( M \) = Torque
- \( N \) = Normal Force
- \( T \) = Tension
- \( R \) = Effective Radius of Element
- \( \theta \) = Inclination Angle
- \( \bar{\theta} \) = Mean Inclination Angle
- \( \phi \) = Azimuth Angle
- \( \Delta \) = Incremental Values
in continuous contact with the wellbore and shear forces and bending moments are neglected. The drillstring is divided into discrete elements. Despite the simplicity of the model, the associated approach is adequate if an appropriate friction factor is selected. Thus, the friction factor is a calibration parameter for the model and should really be viewed as a drag factor. This distinction is made since friction factor implies a precise relation between the normal load and the friction force that does not really apply in these models, since normal loads are only approximated.

In 1984, Corbett and Dawson of Exxon [2] refined the application of torque/drag models to drillstring design with several new concepts. Specifically, they presented a method for generating an artificial planned wellbore with a roughened trajectory including doglegs (abrupt changes in the wellbore path), a design criterion for drillstring stresses in high-angle drilling, and an optimization method for designing tapered drillstrings. In 1985, Soeininah of Mobil [3] patented a special measuring device to acquire torque/drag data while drilling and introduced the concept of monitoring these loads to detect hole problems.

In 1986, Tolle and Dellinger of Mobil [4] published results from a field evaluation of friction coefficients in the Gulf of Mexico and proposed eccentric tool joints and aluminum drillpipe to minimize torque/drag loads. Further, they reviewed Mobil’s Stafjord field and concluded that it could have been developed with two (2) platforms instead of the three (3) which were used, if torque/drag technology had been available in 1974 when the field was developed.

Later in 1986, Sheppard, Wick and Burgess of Schlumberger [5] studied the impact of the well shape on torque and drag and demonstrated that torque and drag
reduction could be achieved with catenary trajectories. In 1987, Whitten of Teleco [6] presented an enhanced torque/drag model that more accurately placed buoyancy loads at cross-section changes. Whitten also focused on quantifying normal force severity with the "Borehole Severity Log" to estimate the risk of the drillpipe becoming stuck.

In 1987, Lesage, Falconer and Wick of Anadrill/Schlumberger [7] introduced the concept of integrating measurement-while-drilling (MWD) and surface data to refine torque/drag measurements and promoted foot-by-foot analysis of the friction factor to detect hole problems early. Their work included several field cases where the methods were used successfully to circumvent problems and reduce costs. Maidla and Wojtanowicz of LSU [8] presented a comprehensive study of torque/drag data and calculations that refined the theory to include hydrodynamic viscous drag, a calculation of contact surface based on pipe and wellbore geometry, an assessment of 2-D versus 3-D modeling results, and a correlation for friction factor as a function of inclination and walk rate. Figure 2.1.2 shows guidelines for friction factor versus depth. Brett, Beckett, Holt, and Smith of Amoco [9] presented application examples for Amoco's "Tension-Torque" model which focused on pre-well planning, real-time monitoring during drilling and post-mortem analysis. The analyzed wells included two wells located offshore in Trinidad, one well offshore in the Beaufort Sea, and an onshore well in England.

In 1988, Child and Ward of British Petroleum [10] presented BP's "Drillstring Simulator" which linked torque/drag models to stress analysis calculations and graphics as shown in Figure 2.1.3. BP's model was based on continuous beam theory and approximated the bending stiffness of the assembly. The model also allows for
Figure 2.1.2
Guidelines for Torque/Drag Friction Coefficient
(from Maidla and Wojtanowicz 1987 [8])

Friction Factor for Torque/Drag Prediction

Measured Depth of Drillstring (feet)
Figure 2.1.3
Graphical Display of Torque/Drill Loads and Stresses on Wellbore Trajectory
(from Child and Ward 1988 [10])
multi-fluid analysis such as when mud, cement, and spacer are in the hole simultaneously. Through model validation on six (6) wells, BP staff concluded that friction factors were functions of mud system and the open-hole, cased-hole configuration. BP also determined that ideal trajectories required roughening with $0.3^\circ$ doglegs in cased hole and $0.6^\circ$ doglegs in open hole to match field observations. Applications of the model for rig selection, trajectory optimization, BHA design, and liner rotation were presented.

Also in 1988, Ho of NL Petroleum Services [11] demonstrated several shortcomings of the original Johanssik model and proposed a new approach. Ho's model linked a rigorous structural model of the BHA with the "soft model", which neglects bending, for the drillstring. This approach provides more accurate modeling of BHA bending, clearance, and contact, while allowing for the more efficient torque/drag calculations in the drillpipe section.

In 1989, McKown of Smith [12] demonstrated that torque/drag analysis should be integrated with drillpipe, rig, and hydraulic limitations to optimize drillstring designs for directional drilling. Also in 1989, Lesso, Mullens, and Daudley [13] presented results from a joint Anadrill/Schlumberger-Gulf Canada project to optimize platform sizing and site selection by using torque/drag models to study the development scenarios. Their work optimized the development of the Amauligak Field in the Beaufort Sea as shown in areal and perspective views in Figure 2.1.4. A key element of the 50-well analysis was the randomized dogleg severities along well paths since friction estimates had to be conservative. Friction loads in some of the roughened wells were 70% higher than those predicted by using smooth planned trajectories.
Figure 2.1.4
Application of Torque/Drag Analysis for Platform Optimization
(from Lesso, et.al. 1989 [13])

Amanigak 50-Well Platform

Target Production Sands

Scale

2000 meters
In 1990, Belaskie, Dunn, and Choo [14] presented results from a joint Ana-
drill/Schlumberger-ARCO Alaska project to use weight-on-bit (WOB) and torque-on-bit
(TOB) measurements from MWD tools to refine drilling operations relative to surface
only data. Specific areas of focus included bit performance, motor/directional
performance, and drillstring friction interpretation. In one case, although surface loads
remained essentially constant, substantial variations of WOB occurred as a result of
drillstring design changes. These observations highlight the limitations of surface
measurements and simplified torque/drag models. Also in 1990, Mueller, Quintana and
Bunyak of Unocal [15] published casing running results on two wells with severe drag
problems induced by reaches of 2.41 and 2.15 miles, respectively. Unocal reported that
it was necessary to rotate the casing and use special casing components which allowed
running portions of the casing "dry", without fluid inside. The first action results in
lower kinetic friction (versus static) on the string, while the latter action increases
buoyancy on the pipe decreasing the effective pipe weight and resultant contact loads.
A recent refinement in torque/drag prediction is the detailed simulation of the wellbore.
Navarro and Yalcin [16] presented results from a joint ARCO-Anadrill/Schlumberger
effort to characterize the severity of doglegs in planned offsets as a function of
formation, build/drop rates, and doglegs in previous wells. This allows planned wells
to be described by realistic "roughened" trajectories as opposed to smooth theoretical
curves. This work demonstrated that drillstring friction factors were between 0.24
and 0.28 when the wellbore was accurately modeled. When using theoretical smooth
wellbores, "drag factors" were on the order of 0.40 to 0.60. Thus, accurate modeling
of the wellbore is a critical part of the torque/drag problem.
Torque/drag models are valuable for drill string design, evaluation of friction in specific wells, and the study of directional drilling limitations. However, they provide limited insight into the drillstring behavior during drilling. At best, torque/drag models provide an average of torsional and axial loads about which fluctuations occur.
2.2 Static Directional Models

Interest in how the BHA influences directional drilling dates to the earliest days in the industry. Early interest focused on drilling straight holes by overcoming factors which tend to deflect the BHA into "crooked" paths. This work was later applied to the problem of intentionally deflecting the BHA to produce an inclined path through a desired target. Modeling of directional drilling involves the coupling of a BHA model (static, quasi-dynamic, or fully dynamic) to a bit reaction model that accounts for bit and formation properties to predict the drilling direction.

In 1978, Millheim of Amoco [17] published a series of articles on directional drilling including a survey of the literature from 1912 to 1970. Because of the many papers on the topic, this review focuses only on recent publications. In Millheim’s series, he reviewed methods of directional drilling (whipstocks, steerable motor BHAs, and jetting), force equilibrium concepts as shown in Figure 2.2.1 (stiffness, tangency length, buoyancy, resultant bit forces), standard BHA configurations (for building, holding and dropping), and the application on finite-element methods to analyze the problem. In 1980, Callas of Colorado School of Mines [18] presented work on the solution of the static BHA problem using boundary-value ordinary differential equations. The BHA was treated as an elastic, slender beam. Assumptions included a centralized bit without a bending moment, constant elastic BHA properties, lateral displacement much smaller than length, a rigid borehole, and negligible dynamic and fluid effects.

In 1981, Toutain of Total [19] presented a two-dimensional model developed to analyze stabilized BHA in deviated wells. Assumptions included an arbitrary well axis, a
Figure 2.2.1
Basic Force Considerations for Static Directional BHA Analysis

- Tension or Compression at Drillpipe Crossover
- Gravitational Force
- Reaction Force at Stabilizer
- Reaction Force at Stabilizer
- Resultant Bit Side Load
- Weight-on-Bit
rigid borehole, variable stabilizer clearance, a rigid formation, no bit embedding, an
elastic BHA body, negligible dynamics effects, and BHA forces from gravity and WOB
only. Toutain reviewed the significance of bit anisotropy, formation anisotropy,
formation heterogeneity, WOB, RPM, inclination, and azimuth changes on directional
performance. One important finding was that walk is influenced strongly by bit
clearances, frictional loads, and wellbore curvature.

In 1985, Amara of Total [20] attempted an integration of analytical BHA
models to field performance databases. Amara covered the concepts of relative BHA
rigidity, BHA effectiveness relative to ROP, and equilibrium curvature. A computerized
prediction of BHA behavior from Total’s model is shown in Figure 2.2.2. Relying on a
comparison with four (4) field cases, Amara concluded that rigid wellbore assumptions
must be removed and elasticity of the wellbore must be accounted for. Amara also
recognized the need to model bent components in steerable assemblies.

In 1986, Jogi, Burgess, and Bowling of Anadrill/Schlumberger [21] presented
results from correlating a three-dimensional (3-D) static BHA model with 17 bit runs
on 3 Shell wells in the Gulf of Mexico. The study showed half of the predictions were
accurate using nominal problem descriptions while other cases required manipulation of
stabilizer clearances to achieve correlation. Due to this effect, rules were developed to
use nominal geometry if ROP exceeded 40 feet/hour in 12-1/4” holes, but to add an
1/8” enlargement for lower ROP. A second important observation was the BHA had
to be drilled for half of its length before the theoretical equilibrium curvature would be
approached. Williamson of Smith [22] published a two-dimensional (2-D) BHA model
linked to a bit/formation model to predict build/drop behavior. The bit/formation
Figure 2.2.2
Graphical Display of Static BHA Forces and Deflections
(from Amara 1985 [20])

Position Along BHA
(meters from bit)
model was based solely on resultant bit force and neglected bit anisotropy which requires consideration of bit tilt. The model accounts for formation dip and provides "formation class", a measure of hardness, as a correlation coefficient. This model is based on the theoretical equilibrium curvature of the BHA and is thus insensitive to the actual wellbore geometry being considered. In a companion publication, McKown and Williamson [23] overviewed BHA stabilization considering influences of formation type, stabilizer gage protection, directional objectives, and drilling parameters. Figure 2.2.3 shows stabilizer blade configurations and a stabilizer design chart. Later in 1986, Chandra of NL Industries [24] presented 2-D and 3-D theory for BHA modeling including boundary conditions at the bit and stabilizers, treatment of contact, initial curvature, loading mechanisms, and buckling considerations. Chandra also discussed the need to understand stiffness properties of special BHA components such as motors, turbines, MWD, and shock tools. Walker of Terra Tek [25] reviewed formation, bit, BHA, wellbore, and drilling parameters which affect hole inclination and azimuth and provided analysis results for BHA designs summarized in Figure 2.2.4. Ho of NL Industries [26] provided the first large deformation 3-D formulation for BHA analysis. Ho concluded the large deformation theory could be neglected for curvatures under 10-15°/100'.

In 1987, Kadjar, Delafon, and Leveque of Total [27] presented an aggressive application of BHA analysis to field operations. Analytical models, combined with performance databases for vertical and directional wells, addressed operating objectives of maximizing WOB and ROP while minimizing the number of BHAs run and the severity of doglegs. The result was a substantial improvement in drilling performance.
Figure 2.2.3
Stabilizer Blades Types and Stabilization Design Chart
(from McKown and Williamson 1986 [23])

BHA Stabilization Chart for Straight Hole Drilling

- Formation Hardness?
  - Soft
  - Medium or Hard

- Crooked Hole Tendency?
  - Medium or Severe
  - Over
  - Under

- Hole Gage?
  - Gage
  - Stiff-Long Blade or Integral Blade
  - Reamer or Combo

- Welded Blade
- Open Design

- Integral Blade
- Square Drillcollars
Figure 2.2.4
Summary of Commonly Used Directional BHAs
(after Walker 1986 [25])

Strong Build

Moderate Build

Slight Build

Strong Hold

Weak Hold

Moderate Drop

Strong Drop
Comparing 1983 performance to 1986, daily footage increased from 238' to 416' and the number of BHAs run decreased from 1.8 to 0.5 per 1000' of drilled hole. Later in 1987, Ho of NL Petroleum Services [28] presented a new 3-D rock-bit interaction model accounting for rock anisotropy and bit anisotropy. The rock-bit interaction model, coupled with a BHA model, predicted build/drop and walk rates and could be used for drill-ahead predictions, correlation to field data, or generation of new information such as a drilling log based on field results. In late 1987, Amara of Total and Strand of Norsk Hydro [29] present optimization results from the Oseberg platform including saving 78 days of a planned 538 days (15%), reducing BHA trips per well from 7 to 3, increasing bit runs from 160 meters to 367 meters, and "perfectly controlling" inclination and azimuth.

In 1988, Rafie of Driltek [30] presented results from a 2-D BHA and formation-bit model accounting for stabilizer position, WOB, stabilizer wear, inclination, and curvature. Gibson of Texaco [31] demonstrated how on-site use of BHA models could improve drilling performance by determining lead angles and maximizing ROP within directional constraints. Gibson reinforced Jogi, et.al.'s conclusions on the impact of stabilizer clearances on BHA behavior. Improvements were characterized by drilling wells with only 5 different BHAs where previously 12 had been "required".

In 1989, Delafon of Total [32] addressed the impact of Total’s BHA model/database technology on the Alwyn North platform as shown in Figure 2.2.5. Delafon cites BHA runs as long as 4,500 to 5000' in 12-1/4" and 17-1/2" holes. For the project, 11 wells were drilled from Platform A, 14 from Platform B, and 13 from Platform C (Alwyn North). For the Alwyn North platform, the number of BHAs was
Figure 2.2.5
Optimization of BHA Performance in Directional Drilling
(from Delafon 1989 [32])

The Average Number of BHAs required to Drill 1,000 ft.

- **Platform A**
  - Average: 1.11 BHAs per 1,000 ft.
  - (900 ft. per BHA)

- **Platform B**
  - Average: 0.97 BHAs per 1,000 ft.
  - (1,030 ft. per BHA)

- **Platform C** (Alwyn North)
  - Average: 0.56 BHAs per 1,000 ft.
  - (1,785 ft. per BHA)
reduced to 0.56 per 1000' for 17-1/2" hole versus 1.11 and 0.97 on the first two platforms, and to 0.58 per 1000' for 12-1/4" hole versus 0.92 and 1.15 previously. These benefits were obtained by optimizing rotary BHAs for long bit runs with high WOB, flow rate, and torque capacity to maximize ROP as opposed to using steerable systems which can restrict these operating parameters and cause premature bit wear. Later in 1989, Williams, Apostal and Haduch of Jordan, Apostal, Ritter and its affiliate Drilling Resources Development [33] presented results from a 3-D static BHA model accounting for bent components, eccentric contact stabilizers and wear pads, and orientation of steerable systems. The model enhanced previous finite-element analysis (FEA) formulations with penalty functions and numerical solutions of the associated equations. In late 1990, Amara of Total [34] presented the integration of Total’s existing BHA analysis and database systems with an expert system shell for rule-based guidance of the drilling engineer. The advanced graphical expert system interface allows all engineers to benefit from Total’s technologies as opposed to only the few BHA experts, such as Amara.
2.3 Dynamic Directional Models

Several efforts have focused on using dynamic models to explain trends in directional drilling not predicted by static models. These efforts focused initially on qualitative and not on quantitative aspects of dynamic effects. This distinction is made since the ability to characterize dynamic phenomena was a secondary priority to their ability to correlate with directional drilling trends. The first work to be examined used a steady-state formulation while the latter two used transient time-domain integration.

In 1976, Millheim of Amoco, Jordan and Ritter of Marc Analysis Research Corp. [35] reported the application of FEA to BHA analysis using 3-D beam elements to model the drillstring. Nodes had six degrees of freedom: three displacements and three rotations. Nonlinear gapping elements, with bilinear stress-strain relationships, were used to enforce the geometric wellbore constraint and were applied at the nodes as shown in Figure 2.3.1. The gap constraints were accounted for using a predictor-corrector algorithm which modified the stiffness matrix when the gaps closed. The BHA weight and WOB loads were applied in 15-20 increments until full loading and convergence was achieved. Different stabilizer configurations representing distinct directional BHAs such as holding, building, and dropping were analyzed. Of twelve cases analyzed, six correlated in mode (build, drop, or hold) and magnitude of response, two correlated in mode but varied in magnitude, and four did not even correlate in the response mode. A refinement to nodal gapping was presented using a continuous nonlinear elastic foundation whose reaction was calculated by five-point Gaussian integration along the beam element. Conclusions were that FEA could be successfully
Figure 2.3.1
BHA Analysis through Finite-Element Idealization and Nodal Gapping Elements
(after Millheim, et.al. 1978 [35])

Stress-Strain Relationship for Gap Elements

Fixed Boundary Constraint

Stabilizer

Structural Beam Elements for BHA Components

Gap Elements

Stress

E=10 psi

8

E=10 psi

8

E=10 psi

Strain
applied to the problem and that FEA could account for variations in drillstring geometry, material properties, wellbore trajectory, and stabilizer placement.

In 1980, Millheim of Amoco and Apostol of Jordan, Apostol, Ritter [36] presented further progress on FEA techniques including the effect of dynamics on directional BHA trends. The dynamic formulation was based on a force vector including static terms (gravitational load, nodal reaction forces, and curvature forces to accommodate large displacement effects, if any), and dynamic terms (inertial forces and frictional forces). The total force vector involved a constant portion and sine and cosine dynamic portions. Three sets of equations are solved iteratively for constant, sine, and cosine components of the total displacement. Iteration is required since forcing terms can be dependent on displacements. Friction factors were 0.2 along drillcollars and 0.5 at stabilizers. An important observation from the dynamic calculations was that frictional forces deflect the BHA out of the 2-D plane by generating forces which cause walking in the azimuthal directions, such as shown in Figure 2.3.2. Orbital paths were predicted for specific cases and classified as low, moderate, or high energy based on whether the BHA remained on the low side of the hole or whether it whipped around the wellbore. Five (5) shallow directional wells were drilled near Catoosa, Oklahoma to calibrate the model and comparisons were also made to full-scale wells in the Gulf of Mexico and Holland. These studies called attention to the importance of the stabilizer placement, friction, trajectory, geology, and RPM in controlling directional tendencies.

In 1985, Birades of the L’Ecole Centrale des Arts et Manufactures de Paris [37] presented his doctoral thesis on time-domain finite-element modeling of directional
Figure 2.3.2
BHA Deflection into Azimuthal Plane Caused by Frictional Contact During Rotation
(from Millheim and Apostal 1980 [36])

View of Drillstring Looking Down the Wellbore

High Friction Causes Increase in Azimuthal Deflection

Circle of Radial Nodal Constraint
\( Rc = Rw - Rs \)

Nodal Vibration Path for \( \mu \) of 0.75
Nodal Vibration Path for \( \mu \) of 0.25

\( \mu = \) Friction Coefficient
\( Rc = \) Radius of Nodal Constraint
\( Rw = \) Wellbore Radius
\( Rs = \) Local Drillstring Radius
drilling. Birades developed a 2-D static model, a 3-D static model, and finally a 3-D dynamic model. The static models were frictionless, assumed a rigid wellbore imposed at nodes, and used a pinned boundary condition at the bit as shown in Figure 2.3.3. Birades studied the penalty and projected gradient methods in addressing the wellbore constraint, and proposed the Uzawa algorithm as a more efficient solution technique. For further efficiency, Birades noted a particular advantage with the Uzawa technique in describing the wellbore as a polygon as opposed to a circle. Even with these refinements, Birades noted that it required more iterations to study a stabilized BHA than a slick BHA. For the dynamic formulation, Coulomb friction was chosen based on unpublished Elf Aquitaine research, and friction factors up to 0.6 were used. A special shock formulation was used to handle contact. Specifically, a force to reverse the normal velocity into the wellbore was calculated for the node violating the wellbore constraint, and this normal velocity was internally modified at the time of impact. Then, using equations based on a perfectly elastic impact and the beam properties, updated values were calculated for the tangential velocity and torsional velocity as shown in Figure 2.3.4. These events were assumed to occur with zero duration so these changes were imposed without modification of the timestep. The time of impact is interpolated between the current and next time step to determine when the boundary constraint would be violated. Implementation of this feature is made easier through lumped mass descriptions for the BHA. Looking at the dynamic robustness of Birades work, Figure 2.3.5 shows the only results presented in the frequency domain by using the Fourier transform of time-domain results. The upper figure in Figure 2.3.5 on bit inclination load shows a peak near 3.2 Hz which is attributed to the rotary speed (1X),
Figure 2.3.3
Finite-Element Idealization of Directional BHA
(from Birades 1985 [37])
Figure 2.3.4
Modeling Treatment of BHA Contact as Shock Phenomenon
(from Birades 1985 [37])

$\omega$ = Rotational Velocity
$V_N$ = Velocity Normal to Wellbore
$V_T$ = Velocity Tangential to Wellbore
- Designates Timestep Prior to Impact
+ Designates Timestep After Impact
Figure 2.3.5
Spectral Results from Time-Domain Model Missing Key Bit Excitations
(from Birades 1985 [37])

*No units are provided by the original author
but no other distinct peaks. The lower figure on bit azimuth load shows no dominant peak. The lack of a bit model clearly prohibits properly accounting for bit excitations. Birades noted that further work should be focused on improved contact modeling, inclusion of mud and damping effects, and the determination of critical speeds for the BHA. In 1986, Birades and Fenoul of Elf Aquitaine [38] presented numerical results of a 2-D static program, known as "Orphee-2D", which compared favorably with data for directional wells by introducing a correction for hole enlargement as shown in Figure 2.3.6. In 1986, Birades [39] presented an overview on the development of the 3-D static and dynamic models and compared the 3-D model, known as "Orphee-3D", favorably to directional wells in Elf's Handil field. In 1989, Birades and Gazaniol of Elf [40] presented an enhancement to Orphee-3D for static modeling of steerable BHA with bent components as shown in Figure 2.3.7.

In 1986, Brakel of the University of Tulsa [41] presented his doctoral thesis on time-domain finite-element modeling of directional drilling behavior. Brakel's program built upon previous works at Tulsa. In 1972, Nicholson [42] analyzed BHA components using finite-element formulations with straight wellbore constraints using the penalty function method. In 1974, Wolfson [43] generalized the model for curved wellbores. To enhance predictions for drilling trajectory, Brakel extended the work to a transient dynamic formulation. Rock-bit interaction was added to a dynamic BHA model to generate the "Bottom-Hole Assembly and Drillbit Analysis Program" (BHADAP). For the initial static analysis, the previous penalty function approach was replaced with gap elements acting as nonlinear radial springs. Wilson-Theta numerical integration and Rayleigh damping were used. An iterative procedure, using the
Figure 2.3.6
Calibration of "ORPHEE" Static BHA Model to Field Data
using Hole Enlargement Correction
(from Birades and Fenoul 1986 [38])

Predicted Build/Drop Rate (degrees/10 m)

Actual Build/Drop Rate (degrees/10 m)

Calculated Hole Enlargement (%)

Rate of Penetration (m/hour)
Figure 2.3.7
Extension of "ORPHEE" Static BHA Model for Bent Sub BHAs
(from Birades and Gazanol 1989 [40])

Tool face definition

Finite Element Model of a Bent Sub BHA.
Deformed Shape.
Newton-Raphson method, was used to ensure that the mathematical model corresponds to the assembly remaining within the wellbore. Dynamic contact is modeled by calculating a radial reaction force which restores the node violating the constraint back into the wellbore and applying that force during a recalculation of the time-step in question. This contact response approach introduces a restitution coefficient for the degree of non-elastic behavior during impact. Contact forces are then used for calculating sliding Coulomb friction. To model bit boundary conditions, a rock-bit interaction model was developed for roller cone bits and PDC bits. Time-domain behavior was calculated until auto-correlations were adequate to deem the behavior "stationary". Then, averages are determined for forces, displacements, etc. Parametric studies were performed for several factors including restitution coefficient, damping, friction coefficients, and stabilizer clearance. An example result is shown in Figure 2.3.8 where orbital paths for several nodes in the BHA are plotted. BHAs for building, holding, and dropping hole angles, each using both roller cone bits and PDC bits, were studied. The BHA was assumed to drill in the direction of the resultant bit force; thus, bit tilt and formation effects were ignored. The transition between the current trajectory and the direction of the resultant bit force was assumed to occur in 100' of drilling, which was arbitrary. The effects of bit weight, rotary speed, hole inclination, hole curvature, bit type, and rock strength on inclination and azimuth response were studied. The model was found to overestimate building and dropping tendencies relative to field data. Power spectra of bit forces and torque showed peaks associated with the rotary speed (1X) and the roller cone speeds. As shown in Figure 2.3.9, no peaks are present which correlate with the 3X excitation; Brakel attributed
Figure 2.3.8
Time-Domain Plots of BHA Orbital Paths
(from Brakel 1986 [41])

Bit Node 1
Height = 0'
Radial Clearance = 0.100"
Friction Coefficient = 0.25

Node Number 2
Height = 2.5'
Radial Clearance = 2.225"
Friction Coefficient = 0.25

First Nearbit Stabilizer Node 3
Height = 5.0'
Radial Clearance = 0.100"
Friction Coefficient = 0.40

Second Nearbit Stabilizer Node 4
Height = 8.0'
Radial Clearance = 0.100"
Friction Coefficient = 0.40

Node Number 7
Height = 29.0'
Radial Clearance = 2.225"
Friction Coefficient = 0.25

Second Stabilizer Node 8
Height = 38'
Radial Clearance = 0.350"
Friction Coefficient = 0.40
Figure 2.3.9
Spectral Results from Time Domain Model
(from Brakel 1986 [41])
this to the assumption of the tricone bit drilling upon a flat surface. Although Brakel’s work is outstanding, the model for tricone bit drilling could be improved to generate meaningful dynamic excitations. For PDC bits, limited knowledge of PDC spectral characteristics prevents meaningful comparison with Brakel’s results. In 1987, Brakel and Azar [44] provided a summary of the work. In 1989, Maidla of the University of Campinas and Sampaio of Petrobras [45] presented an application study where Brakel’s model was calibrated with 15 directional wells in the Campos Basin of Brazil. Brakel’s model was used to define the resultant bit forces for the field BHAs. Coefficients were then sought to account for formation effects, bit effects, and all other neglected effects using statistical regression. Results were successful in modeling build/drop and walking behavior, thereby allowing lead angle predictions on future wells in the field.
2.4 Experimental Studies and Observations

Experimental observations of BHA dynamics are useful for providing insight into the physical behavior of drilling systems. Unlike modeling efforts which have the potential of introducing artificial trends, experimental data collection provides direct access into the complex dynamics of drilling systems. Unfortunately, the task is challenging and as it will be shown, many factors still remain undefined.

In 1960, Finnie and Bailey of Shell [46] presented results involving a simple single degree-of-freedom (SDOF) model of the drillstring and test data. The model provided natural axial and torsional frequencies of the drill string through a trial-and-error approach. A graphical approach was used when the top boundary condition was not fixed to account for stiffness and mass of surface equipment. Damping was not included. The test data were obtained from strain gages placed below the kelly and velocity sensors above the kelly. Torque, rotational displacement, axial load, and axial displacement were measured. The authors observed vibration frequencies that corresponded to the natural frequencies of the string as well as others which could not be accounted for. Data were collected for several cases including an air-drilled well at 3,962' with a 8-3/4" button bit and a well drilling at 3,496' with a 12-1/4" rock bit. Unfortunately, their calibrated measurement tools burned in a fire during the air drilling operation resulting in the remainder of their data being collected with uncalibrated backup tools. Data from two wells revealed well-defined torsional vibration modes, but not well defined longitudinal modes. Coupling between axial and torsional vibrations was also noted. A remarkable observation was the need to "understand the
nature of the damping in the system and the boundary conditions at the ends of the string"; this need remains, even today.

In 1965, Angona of then Socony Mobil [47] studied the attenuation of drillstring vibrations with the objective of using surface oscillators to accelerate drilling. Methods used to initiate a stress wave included dropping a 134 lb. weight 15' onto the drillstring and catching the drillstring in the slips after dropping it. Following their initiation, wave reflections were measured and studied. Although Angona wanted to study attenuation as a function of frequency, means of producing controlled frequency signals were not available. Thus, different frequency components of the above signals were used as shown in Figure 2.4.1. Angona concluded that the damping was low enough to the extent that the use of surface oscillators, such as unbalanced flywheels or hydraulic modulators, for accelerating drilling, would be limited only by drillstring fatigue and bit life constraints.

In 1968, Deily, Daireing, Paff, Ortloff, and Lynn of Esso (Exxon) Production Research [48] reported on using a downhole recording tool to measure dynamic BHA behavior. The tool, shown in Figure 2.4.2, stored a maximum of nine (9) minutes of data and measured eight parameters: axial, torsional, and bending loads, axial, angular, and lateral accelerations, internal and external pressures. The system was used in 15 wells in the Gulf Coast and Canada and allowed many insights. WOB and torque interactions were identified. Exciting mechanisms such as a 30 Hz signal were identified which was attributed to rows of teeth on tri-cone bits. Normal variations in WOB of 30-50% were seen. In extreme cases, WOB variations resulted in bits bouncing off bottom and re-impacting inducing peak loads of 3.5 times the mean WOB. Normal
Figure 2.4.1
Socony Mobil Drillstring Damping Data
4-1/2" Drillpipe in 8-3/4" Hole with 9.5 ppg Mud
(from Angona 1965 [47])

Attenuation of Axial Drillstring Waves (dB/1000')
Figure 2.4.2
Esso (Exxon) Dynamic Downhole Recording Tool
(from Deily, et.al. 1968 [48])
torsional variations were 20-40%, and in extreme cases the drillstring and bit were found to be moving backwards. Double integration of accelerations allowed bit displacements to be quantified with 1/16" being normal, and peak displacements of 1" in hard rock drilling. Resonances and the 3X excitation associated with the tri-cone bit interface were seen. During resonances, 3X frequencies were evident in fluid pressure readings, indicating a link between BHA motion and downhole pressures. In 1968, Cunningham of Hughes Tool [49] analyzed additional data from the Esso downhole tool. Bit displacements had an average of 0.7" with a maximum displacement of 1.7". The 3X RPM excitation for tri-cone bits was reinforced as was the 4X stroke frequency for the double-acting duplex pump. For the first time, "beats" shown in Figure 2.4.3 were noticed in torsional data with periods longer than the ones calculated using the elastic drillstring properties. Cunningham's observation of "the bit rotating rapidly ... part of the time ... and practically still during part of the time" was likely the first observation of the now well-recognized slip/stick phenomenon. Cunningham noted beats occurred under 90 RPM, a prelude to current concepts of critical rotary speeds to avoid stick-slip. Also in 1968, Miller and Rollins of Drilco Oil Tools [50] use the Esso recorder to study the effectiveness of a vibration damping tool, commonly referred to as a "shock-absorber" or "shock-tool". The shock-tool was designed to transmit axial and torsional loads through rubber. Tests were run in a variety of wells in Texas, Oklahoma, and Louisiana and the location of the shock-tool in the BHA was varied. The shock-tool successfully reduced dynamic load severity as shown in Figure 2.4.4 and became an industry standard.
Figure 2.4.3
Esso (Exxon) Data Showing Torsional Beats
(from Cunningham 1968 [49])

Test 8 Run 24
40 RPM 70 kips WOB
2025 psi Pump Pressure

Torque

Angular Acceleration

Elapsed Time
Figure 2.4.4
Validation of the Shock Sub using the Dynamic Downhole Recording Sub
(from Miller and Rollins 1968 [50])
In 1971, Lutz, et. al. of Societe Nationale des Petroles d’Aquitaine [51] presented a dynamic drilling theory which combined an impedance model for longitudinal vibrations with surface data for instantaneous evaluation of the force generated by the bit. Measured data included longitudinal force and acceleration and rotational velocity from a tool above the kelly and torque and torsional acceleration from a tool below the kelly. Signals from the rotating tools were transferred using turning contacts and brushes. A theoretical model for expected vibrations was developed using lab-measured bit characteristics and drillstring properties. Comparison of theoretical response and measured data provided real-time estimates of the formation hardness, which was presented as an acceleration log as shown in Figure 2.4.5. The acceleration log correlated with formation characteristics well including neutron, sonic, and gamma ray logs. As a result of its correlation to the formation being drilled, the acceleration log provided a means for drilling optimization and an indicator of pore pressure increases requiring weighting of the mud.

In 1985, Besaisow, Jan and Schuh of ARCO [52] presented the use of high-frequency surface measurements to study BHA dynamics. Axial and torsional loads, axial, torsional, and radial accelerations, RPM, pressure and temperature were measured. The focus of the work was the detection and avoidance of casing wear caused by drillstring impact. The system was first used in a shallow research well and then in the drilling of a 2,300’ test well in Rockwall, Texas. Measured signals were transmitted via FM radio to a data analysis system for interpretation. Stick-slip motion was detected including backward rotation of the bit. Drillstring/casing interaction was generated using an
Figure 2.4.5
Correlation of Surface Dynamic Accelerations to Formation Properties
(from Lutz, et.al. 1971 [51])
eccentric weight and was successfully detected by the system. A diagram of the system is shown in Figure 2.4.6.

Also in 1985, Wolf, Zacksenhouse, and Arian of NL Industries [53] reported on a downhole dynamic measuring system based on wire telemetry of the signals to the surface using special drillpipe. Downhole measurements included WOB, torque-on-bit (TOB), bending, accelerations in three dimensions, external pressure, pressure drop across the bit, temperature, formation resistivity, gamma ray, inclination, and azimuth. Surface measurements included ROP, hook load, swivel acceleration, RPM and internal pressure. The system recorded 650 data samples per second and was used for 60 hours on a 8-3/4" straight hole outside Quitman, Texas. The 3X tri-cone excitation was observed to build up to a maximum then rapidly vanish, indicating a positive feedback mechanism where the bit eventually destroys the current tri-lobed interface. Figure 2.4.7 shows the variation in WOB fluctuations as a function of RPM. Figure 2.4.8 shows a phenomenon where WOB fluctuations abruptly decrease simultaneously with an increase in bending fluctuations. Bending loads were observed as high as 25,000 ft-lbs, 50% of the pipe's endurance limit. Lateral motions were observed at 2.4X RPM. They were attributed to the mechanism known as "rotational walk" where the drillstring walks around the larger diameter wellbore once per surface revolution.

In 1986, Aarrestad, Tønnesen, and Kyllingstad of Rogaland Research [54] reported on an effort to match axial vibration predictions with measurements from a 1000 meter test well. Axial pulses were imparted to the drillstring using a special anchor bit rotating on a tri-lobed hardened steel cam. Two instrumented drillcollars were used for downhole data collection. The "Televigile" tool of the Institute Francais
Figure 2.4.6
ARCO ADAMS High-Frequency Dynamic Surface Measurement System (from Besaisow, et.al. 1985 [52])

Transmitter and Battery
Typical 4 Places

Strain Gages and
Temperature Sensor

O-Ring Seat

View B-B
Sensor Mounting Plate

Servo Accelerometers
3 places

Typical 2 places
Piezoelectric
Accelerometers
Charge Amplifiers

Battery

View C-C
Transmitter Mounting Collar

Transmitter and Battery
Typical 4 Places
Figure 2.4.7
Measured Variations in Dynamic WOB versus Rotary RPM
(from Wolf, et.al. 1985 [53])

AVERAGE WOB = 30 KLB
BIT RUN #6

PEAK-TO-PEAK FLUCTUATIONS KLB

ROTOR SPEED, RPM

0.1 0.05 0.02 0.001

1 SEC

0 20 40 60 80 100

10 20 30 40 50 60 70 80 90 100
Figure 2.4.8
Interaction Between WOB Variations and BHA Dynamic Bending
(from Wolf, et.al. 1985 [53])
de Petrole (IFP) measured WOB, TOB, internal and external pressure, and internal and external temperature. A second tool built by Rogaland Research measured 3-D and torsional accelerations. The authors concluded that the data collected could not be explained by Dareing's axial resonance model, and they proposed a more detailed study of the surface boundary conditions, frequency dependencies, and damping.

Also in 1986, Besaisow and Payne of ARCO [55] reported on a study to identify BHA dynamic excitations and apply them to prevent BHA failures in a deep well offshore Texas. As shown in Figure 2.4.9, excitations can result from mass imbalance, misalignment of the drillstring components, asymmetric boundary conditions, tri-cone bit cutting actions, rotational walk, precession, and whirl. The primary and secondary excitations from these sources are related to multiples of rotary speed. In cases such as the walk shown in Figure 2.4.10, variations in the pipe and wellbore dimensions and shapes will create variations in the value and consistency of these multiples. As shown in Figure 2.4.11, many drillstring excitations were observed using the high-frequency ADAMS surface sensors. Also in 1986, Skauzen and Kyllingstad of Rogaland Research [56] presented tests to evaluate shock-tool performance. As above, a test rig with a special tri-lobed exciter was used. In addition, test frames were constructed to directly measure static and dynamic properties of the shock tools, which included rubber and Belleville springs. The results showed that all three shock tools have nonlinear static and dynamic stiffnesses as shown in Figure 2.4.12. Furthermore, their behavior cannot be described by simple damping models. Later, Skauzen, Kyllingstad, Aarrestad, and Tønnesen of Rogaland Research [57] expanded on these conclusions with test results for seven (7) shock tools. Shock tool effectiveness was shown to increase with
Figure 2.4.9
Summary of Potential BHA Excitations
(from Besaisow and Payne 1986 [55])

<table>
<thead>
<tr>
<th>Physical Mechanism</th>
<th>Primary Excitation(s)</th>
<th>Secondary Excitation(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass Imbalance or Bent Pipe</td>
<td>1 x RPM Lateral</td>
<td>2 x RPM Axial</td>
</tr>
<tr>
<td>Misalignment</td>
<td>1 x RPM Lateral</td>
<td>2 x RPM Axial</td>
</tr>
<tr>
<td></td>
<td>2 x RPM Lateral</td>
<td>2 x RPM Torsional</td>
</tr>
<tr>
<td>Tricone Bit</td>
<td>3 x RPM Axial</td>
<td>3 x RPM Torsional</td>
</tr>
<tr>
<td>Very Soft Formation, Low WOB, Causing a Loose</td>
<td>1, 2, 3, 4, 5, x RPM</td>
<td>3/2 x RPM Lateral</td>
</tr>
<tr>
<td>Drillstring</td>
<td>Axial, Torsional,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Lateral</td>
<td></td>
</tr>
<tr>
<td>Rotational Walk</td>
<td>( \frac{D}{d} ) x RPM Lateral ( \frac{D}{d} )</td>
<td>( \frac{D}{d} ) x RPM Axial ( \frac{D}{d} )</td>
</tr>
<tr>
<td></td>
<td>2 ( \frac{D}{d} ) x RPM Lateral? ( \frac{D}{d} )</td>
<td>2 ( \frac{D}{d} ) x RPM Torsional ( \frac{D}{d} )</td>
</tr>
<tr>
<td>Drillstring Precession</td>
<td>( \frac{d}{d} ) x RPM Lateral? ( \frac{d}{d} )</td>
<td>2 ( \frac{d}{d} ) x RPM Lateral RPM ( \frac{d}{d} )</td>
</tr>
<tr>
<td>Nonsynchronous Walk or Whirl</td>
<td>((0.8 - 1.2)) ( \frac{D}{d} ) x RPM Lateral ( \frac{D}{d} )</td>
<td>((1.6 - 2.4)) ( \frac{D}{d} ) x RPM Axial ( \frac{D}{d} )</td>
</tr>
<tr>
<td></td>
<td>((1.6 - 2.4))( ( \frac{D}{d} ) x RPM) ( \frac{D}{d} )</td>
<td>((1.6 - 2.4)) ( \frac{D}{d} ) x RPM Torsional ( \frac{D}{d} )</td>
</tr>
<tr>
<td>Drillstring Whip</td>
<td>RPM Harmonics (1X,</td>
<td>RPM Harmonics</td>
</tr>
<tr>
<td></td>
<td>2X, 3X) Lateral</td>
<td>Axial, Torsional</td>
</tr>
</tbody>
</table>
Figure 2.4.10
Examples of Rotational and Nonsynchronous Walk
(from Besaisow and Payne 1986 [55])

ROTATIONAL WALK (SYNCHRONOUS)

FORWARD WHIRL

BACKWARD WHIRL (RUB)

NONSYNCHRONOUS WALK
Figure 2.4.11
Axial Load Spectrum During Drilling Test with ARCO ADAMS System
(from Besalsow and Payne 1986 [55])
Figure 2.4.12
Static and Dynamic Properties of Various BHA Shock Tools
(from Skaugen and Kyllinstad 1986 [56])

Static (Slow Force Application)

Axial Compression Force (kN)

A is a Steel Spring Type Shock Tool
B & C are Rubber Type Shock Tools

Dynamic (Free damped oscillation)
decreasing spring rate and decreasing tool damping. As expected, the shock tools were effective in reducing drillstring dynamic response. The authors concluded that the shock tools reduce both the dynamic bit loads and the reaction amplitudes in the drillstring.

Also in 1986, Aarrestad and Kyllingstad of Rogaland Research [58] studied the bit boundary condition for axial vibrations using the full-scale test rig. The occurrence of the 3X excitation was reinforced with additional refinements. First, sidebands adjacent to the primary axial resonance but offset by the primary torsional frequency were observed and explained using frequency modulation theory. Also, the occurrence of higher order modes, i.e. multiples of the fundamental frequency, was observed and attributed to nonlinear axial/torsional coupling. Finally in 1986, Halsey, Kyllinstad, Aarrestad, and Lysne of Rogaland Research [59] compared measured torsional vibrations with existing theory. It was concluded that torsional resonances are easily observed and calculated and that close agreement between theory and data can be achieved, particularly if a correction is made to account for the tool joints. For resonance prediction, the authors concluded that the drillstring should be modeled as fixed at surface and free at the bit. Further, torsional frequencies appeared independent of RPM, WOB, and damping.

In 1988, Close, Owens, and MacPherson of Exploration Logging [60] reported on BHA dynamic measurements using a special MWD recorder, designated as the "Downhole Vibration Monitor" (DHVM). Four cases were presented, the first was stick-slip occurrence during reaming which lead to axial accelerations of 2.5 g's (g being the acceleration of the earth's gravity) and lateral accelerations of 13 g's. During
normal drilling lateral accelerations were below 0.5 g's. The second case described high
frequency drilling vibrations using a positive displacement motor including a 200 Hz,
1.5 g axial motion and a 20 Hz, 2.0 g lateral motion. The third case was the drilling
of a casing shoe and lateral accelerations up to 25 g's were induced as a result of
drillcollars impacting casing. The last case involved detection of "destructive vibra-
tions" in excess of 100 g's. Representative measurements from the DHVM are shown
in Figure 2.4.13 for a span of one second and a bandwidth of 1000 Hz. Figure 2.4.14
shows a 0.3 second span of lateral acceleration and its frequency spectrum during
casing impact.

In 1989, Cook, Nicholson, Sheppard, and Westlake [61] reported on a joint
effort by Shell and Anadrill/Schlumberger which used the first real-time data measure-
ment of downhole dynamics to monitor directional drilling. The Anadrill "Drilling
Mechanics Sub" (DMS) measured WOB, TOB, bending moment, transverse shear,
differential pressure, and axial, transverse, and torsional accelerations. Figure 2.4.15
shows bending response decreasing with higher WOB, possibly indicating a stiffer bit
boundary condition reduced the lateral motion. High lateral forces were measured as
the BHA passed doglegs, indicating significant dynamic loads are associated with
wellbore contact and not just the BHA harmonics. Bending curvatures in the DMS
exceeded twice the wellbore curvature. In addition, significant transverse motions were
detected on and off bottom. Transverse motions in cased hole exceeded those in open
hole, perhaps due to the differences in impact restitution.

In 1989, Vandiver, Nicholson, and Shyu reported on a joint study by Shell and
M.I.T [62] on BHA bending vibration and whirl. Forward synchronous whirl is
Figure 2.4.13
Exlog Downhole Dynamic Lateral Acceleration Spectrum (from Close, et al., 1988 [60])

Lateral Acceleration

Elapsed Time

Lateral Acceleration
1 Second Duration
2,000 Samples per Sec

Frequency (Hz)

0 200 400 600 800 1000
Figure 2.4.14
Exlog Downhole Measurement of BHA/Casing Impact
(from Close, et.al. 1988 [60])

Elapsed Time (seconds)

Lateral Acceleration (m/s²)

272 136 0 -136 -272

0 0.15 0.30
Figure 2.4.15
Shell/Anadrill Downhole Data Showing Decreased Bending for Increased WOB
(from Cook, et.al. 1989 [61])
associated with rapid drillstring wear. Backward whirl generates an extremely high number of stress cycles. Whirling is detected by examining relationships between bending frequencies in separate planes. As shown in Figure 2.4.16, a generalized whirl equation was proposed which accounts for the above limiting cases. Five (5) cases were discussed including no whirl, forward synchronous whirl, backward whirl with little slip, backward whirl with large slip, and linear coupling of axial and transverse motions.

Also in 1989, Aarrestad and Kyllingstad of Rogaland Research [63] reported on efforts to quantify surface boundary conditions for axial vibrations. Discrepancies between data and models lead the authors to consider more sophisticated boundary condition approaches. The first involved quantifying the dynamic modes of the derrick and allowing it to react to the drillstring response. In this approach, only the lowest modes of the derrick were used which can be estimated using Rayleigh energy methods. The second approach was to quantify non-linear coupling between axial drillstring motion and the lateral motion of the drill-lines. This approach lead to dynamic equations for which areas of instability could be found. The authors concluded that a simple linear spring/mass boundary condition at the surface was inadequate and that the nonlinear coupling of axial drillstring motion to transverse drill-line motion was the most promising boundary condition for correlating theoretical results with data.

In 1990, Besaisow, Ng, and Close of ARCO and Exlog, respectively, [64] reported on the field application of the ARCO Advanced Drillstring Analysis and Measurement System, known as "ADAMS". The authors overviewed ADAMS development which was initiated in 1984 and highlighted aspects of the instrumented
**Figure 2.4.16**

*Generalized Whirl Diagram showing Tangential Slip Velocity vs. Drillcollar Whirl Rate*

(from Vandiver, et.al. 1989 [62])

- $R_B = $ Borehole Radius
- $R_C = $ Drillcollar Radius
- $\omega = $ Drillcollar Rotation Rate
- $\Omega = $ Whirl Rate
- $\Omega_b = $ Backward Whirl Rate Without Slip
- $V = $ Tangential Slip Velocity

\[ \Omega_b = \frac{-R_C \omega}{(R_B - R_C)} \]
tool, signal conditioning and telemetry, and data acquisition and analysis. Two interpretation algorithms were described: Standing wave and Propagating wave. Standing wave algorithms were designed to detect BHA harmonics and to measure the energy exerted at the bit. A means of optimizing ROP was proposed based on maximizing the energy associated with the bit excitation. Propagating wave algorithms search for correlations between axial and shear waves to detect drillpipe interaction with casing which could cause casing wear. Four field applications were reviewed where ROP and bit life were increased while BHA stresses were reduced. Figure 2.4.17 shows qualitatively how bit energy is optimized using the energy associated with the 3X axial signal. Savings of 10-30% of the drilling costs in the subject intervals were attributed to the use of the ADAMS system.

Also in 1990, Clayer, Vandiver, and Lee reported on a joint effort by Elf Aquitaine and M.I.T. [65] to study the effect of surface and downhole boundary conditions on drillstring vibrations. A surface measurement tool developed by Elf was used below a power swivel to measure axial and torsional forces and accelerations, hook position, RPM, and internal pressure. The authors reported that torsional natural frequencies could be adequately predicted using boundary conditions of fixed at the surface and free at the bit, but that damping was critical to magnitude prediction. Figure 2.4.18 shows a measured force spectrum modulating at the primary torsional frequency indicating the significance presence of that dynamic mode. The mobility method was applied to drillstring components to derive transfer functions for downhole to surface parameters such as shown in Figure 2.4.19. A simple spring and dashpot was used to account for the boundary condition at the bit. However, studies
Figure 2.4.17
Field Drilling Optimization by Increased Bit Energy with ARCO ADAMS System
(from Besalsow, et.al. 1990 [64])

- **HOLE MAKING ENERGY AT 60 RPM**
- **HOLE MAKING ENERGY AT 45 RPM**
- **1x 2x 3x**

<table>
<thead>
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<th>Parameter</th>
<th>Value 1</th>
<th>Value 2</th>
</tr>
</thead>
<tbody>
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<td>9210'</td>
</tr>
<tr>
<td>RPM</td>
<td>55</td>
<td>45</td>
</tr>
<tr>
<td>WOB</td>
<td>60 k</td>
<td>43 k</td>
</tr>
<tr>
<td>ROP</td>
<td>7.8 FT/HR</td>
<td>6.7 FT/HR</td>
</tr>
</tbody>
</table>
Figure 2.4.18
Modulation of Surface Axial Force by Primary Torsional Mode
(from Claye, et al. 1990 [65])

Main Frequency
at 3X RPM from Tri-Cone Bit

Modulation at 8 second Primary Torsional Period

Elapsed Time (seconds)

Frequency (Hz)
Figure 2.4.19
Measured Surface Torque Spectrum vs. Transfer Function Prediction
(from Clayer, et.al. 1990 [65])

Measured RMS Surface Torque Spectrum
17-1/2" Tri-Cone Bit, 20 tons WOB,
140 RPM (N-m/Hz)

Predicted Surface Torque per Unit Torque at Bit

Surface Torque (N-m)

Frequency (Hz)
involving behavior at high WOB indicated that the formation boundary condition changed rapidly with time. The authors concluded that further study was required with regard to the axial boundary condition but that a fixed/free arrangement was adequate for torsional analysis.

The conclusions from this review of experimental studies of drillstring dynamics are many. Some data for drillstring damping are available, but their accuracy and completeness warrant additional work. The 3X excitation has been repeatedly observed, but is complicated by modulation, particularly by low order torsional frequencies. Shock tools, incorporating axial springs, shift the frequency response as expected and can beneficially reduce dynamic loads. Drillstring dynamic behavior can be correlated to the formation being drilled. Thus, the bit is established as a critical excitation source. High frequency measurements, both surface and downhole, disclose complex effects including drillstring/casing interaction, transient boundary conditions, a multitude of excitation mechanisms, and unpredictable walking and whirling patterns. These observations suggest that many dynamic phenomena extend beyond available drillstring modeling technology. Certain problem complexities should probably be avoided to allow focusing on the most critical aspects. For example, exact modeling of the bit/ROP process and drillstring/wellbore contact are probably not feasible within current computational technology due to their complex and random nature. Eliminating these from consideration and focusing on fundamental BHA dynamic aspects will lead to a more practical BHA design model.
2.5 Bit Mechanics and Excitations

All modeling and experimental studies indicate that the bit plays a crucial role in BHA dynamics. Energy enters the drillstring through surface rotation and after frictional dissipation along the hole, it is further dissipated as the bit interacts with and destroys the formation. Understanding the nature of this interaction is fundamental to building a rational approach to the BHA dynamics problem.

In 1987, Skaugen of Rogaland Research Institute [66] analyzed the effects of quasi-random bit excitations on drillstring dynamics. Skaugen argued that the tri-lobed excitation model is rather simplistic. It should account for variation of the lobed surface with time, the variation in bit rotation frequency due to torsional vibrations, and the randomness of the bit loads due to variations in rock strength and failure mechanisms. Skaugen studied these effects in the time domain using a linear axial vibration model with quasi-random bit loading. Bits loads were introduced by imposing a quasi-random displacement with a regular oscillation component and random variations in amplitude and phase. The primary oscillation frequency was a multiple of the rotation frequency, but two effects disturbed this regular frequency. The first effect was variation of the lobes over time for which Skaugen proposed two models. The first model was the abrupt formation and destruction of lobes. The second model was the continuous "processing" of lobes, described by a lobe precession frequency. This lobe precession frequency was treated as a function of lobe amplitude and ROP. For a field case, it was calculated to be 0.06 Hz, much smaller than the surface rotation frequency of 1-3 Hz. The second effect which disturbed the primary frequency of the bit displacement was a torsional frequency shift caused by increased torque as the bit passes lobe peaks and by torsional harmonics. Bit rotation frequency was
treated with three components: primary rotation frequency, forcing frequency from lobe effects, and net frequency from torsional harmonics. The axial bit displacement was defined in terms of lobe amplitude, sinusoidal variations, and amplitude of surface irregularities. Both amplitudes were multiplied by random functions to generate the bit displacement in time. Previous Rogaland experiments using the steel tri-lobed excitor were modeled with torsional harmonics disturbing the 3X excitations (i.e. no lobe effects). Figure 2.5.1 shows a comparison of experimental data with a simulation study which assumed constant bit displacement frequency and a comparison to the quasi-random simulation which shows closer agreement. The predicted harmonics at 70 RPM and 100-120 RPM are less pronounced, although the quasi-random simulation still overestimates peaks relative to the data. In summary, the quasi-random treatment of bit excitations yields analytical results which compare more favorably with experimental observations and reduces estimates of resonance levels relative to deterministic models.

The implication of Skaugen's findings to the current study is that monochromatic forced frequency response (MFFR) analysis will provide conservative estimates of the severity of drillstring vibrations, since in practice these frequencies are not likely to occur in a pure form without some randomizing effects present. Thus, as a planning tool, MFFR will allow analysis of frequency responses from a conservative perspective.

In 1987, Winters, Warren, and Onyia of Amoco [67] presented a new ROP model for roller-cone bits that included rock ductility, rock compressive strength, and cone offset. Using the model, a rock strength log can be generated from drilling data for better offset planning. Relative to the dynamics problem, this work provides a means of identifying intervals of high rock strength during drilling so that BHA dynamics can be better accounted for and watched during those intervals. BHA dynamic
Figure 2.5.1
Improved Correlation to Test Data by Quasi-Random Bit Excitation
(from Skaugen 1987 [66])

Predicted with Quasi-Random Bit Speed
Measured

Predicted with Constant Bit Speed
Measured

Peak Axial Force at Bit (kN)

Rate of Rotation (RPM)
behavior is influenced by these rock parameters. The authors reviewed the impact of rock mechanics on drilling efficiency. Factors studied included the increase in compressive strength and ductility with confining pressure, which can be controlled through mud weight, and the decrease in drilling ability as ductility increases. The mechanics of the cutting structure was reviewed including the control of rolling and sliding tooth motions through the manipulation of cone offset as shown in Figure 2.5.2. A two-step theory was proposed for the cutting process that involves rock indentation followed by displacement of the cutting material. The new bit model with ductility and cone offset terms was successfully calibrated to lab data on 30 bits from 15 different bit types and field data from an Oklahoma test well.

In 1987, Peltier, Cooper, and Curry of Schlumberger [68] studied the use of dynamic bit torque data to detect bit bearing failure. The work focused on separation of formation effects on drilling torque from bit effects. Data on torque, ROP, and WOB was taken at either 4 Hz or 10 Hz to analyze these effects. A linear relationship between bit torque (TOB) and WOB was proposed which can be analyzed to detect unusual torque increases. Using thermocouples mounted at the bit bearings, the authors verified that the bit torque can be related to the state of bearing wear. The work concluded that packaging of sensors for WOB, TOB, RPM, and ROP near the bit may provide for prediction and early detection of bearing failures. The implication of this work is that formation effects can be isolated from bit effects and that the bit itself exhibits a unique dynamic spectral signature. In 1988, Falconer, Burgess, and Sheppard of Anadroll/Schlumberger [69] proposed a model for milled tooth and PDC bits for rigsite use to separate lithology effects from bit effects and estimate bit wear. The authors reviewed the basis and limitations of the classic D-exponent used in drilling
Figure 2.5.2
Description of Roller Cone Bit Rolling/Scraping Characteristics
(from Winters, et.al. 1987 [67])

TRUE-ROLLING BIT

![Diagram of True-Rolling Bit]

ALIGNED

![Diagram of Aligned Cone]

CONIC CONE

ACTUAL BIT

![Diagram of Actual Bit]

SKEWED

![Diagram of Skewed Cone]

OFFSET

CONIC CONE

DOUBLE CONE

APEX-INNERS CONE

APEX-OUTER CONE
analysis [133] and proposed two new parameters, a dimensionless bit torque and an apparent formation strength, for this purpose. The bit models were linked with real-time MWD information to analyze and display these parameters at surface. The authors suggested that it is also possible to detect cone locking in roller cone bits and provided three case studies on wells in the Gulf Coast and in the North Sea with milled tooth and PDC bits. A potential implication here is the use of MWD to provide lithology and bit information for the tuning of BHA dynamic analysis. This and other Schlumberger work on drilling mechanics was conducted at Schlumberger Cambridge Research using the Drilling Test Machine (DTM) shown in Figure 2.5.3.

In 1988, Cooper of Sedco-Forex/Schlumberger [70] presented a procedure for evaluating bit force spectra for determining bit wear states. As shown in Figure 2.5.4, dynamic force characteristics can be traced to individual rows of teeth on roller cone bits. In this figure, an axial load of 674 lbs. occurs as a 19X excitation corresponding to the outer row of teeth, while the inner row generates 340 lbs. at a 9X frequency. As shown in spectral form in Figure 2.5.5, an upward shift in tooth-specific frequencies can be traced to wear as the cone rotates faster with smaller teeth. Cooper also reviewed ongoing work with high-resolution data to detect cone locking and to analyze drill-off behavior. Later in 1988, Sheppard and Lesage of Anadrill/Schlumberger [71] presented a detailed study on this topic including the strain-gauge instrumentation of the bit cones as shown in Figure 2.5.6. The transverse force behaviors measured are shown in Figure 2.5.7. The transverse force behavior shows a more complex 19X frequency behavior from the outer row of 120 lbs. in magnitude, while the inner row shows a clear 9X frequency behavior of the same magnitude, but opposite sign, indicating the inner teeth gouge backwards during drilling. Also shown is a 9X
Figure 2.5.3
Schlumberger Cambridge Research Drilling Test Machine
(from Peltier, et.al. 1987 [68])
Figure 2.5.4
Measured Axial Loads for Tri-Cone Bit Teeth Drilling Portland Limestone
(from Sheppard and Lesage 1988 [71])

Averaged Vertical Force (kN)

Outer Row Teeth

Rotating Position (degrees)

20

10

0

0  72  144  216  288  360

Averaged Vertical Force (kN)

Inner Row Teeth

Rotating Position (degrees)

5.0

2.5

0

0  72  144  216  288  360
Figure 2.5.5
Increase in Bit Teeth Frequency as Bit Wear Increases
(from Jardine, et. al. 1990 [73])

Vertical Acceleration (m/s²)

Frequency (Hz)
Figure 2.5.6
Instrumentation of Tri-Cone Bit for Dynamic Load Measurement
(from Sheppard and Lesage 1988 [71])
Figure 2.5.7
Dynamic Measurement of Loads and Couples in Tri-Cone Bit
(from Sheppard and Lesage 1988 [71])

Outer Row Transverse Force (kN)

Inner Row Transverse Force (kN)

Inter-Row Couple (N-m)

Rotating Position (Degrees)
"inter-row couple" torque of about 20 ft-lbs. Sheppard and Lesage also discussed the significance of cone speed which varied from 1.11X to 1.31X for the bits examined.

In 1989, Brett, Warren, and Behr of Amoco [72] presented a new theory for PDC bit failure based on whirling as shown in Figure 2.5.8. In this mode, the instantaneous center of rotation varies erratically and the bit undergoes severe whirling resulting in rapid and uneven wear. The authors presented data that suggested that during whirling, the bit may exhibit high excitation frequencies such as twelve times the rotary speed (12X). In addition, a relevant frequency spectrum was provided as shown in Figure 2.5.9. A theoretical whirl frequency was introduced based on the whirl radius and the bit-hole clearance. However, it is not clear how stable the whirl state will be. This work is extremely important in identifying a complex destructive motion of PDC bits which can cause rapid bit wear and induce unique BHA harmonics.

In 1990, Jardine, Lesage, and McCann of Schlumberger [73] disclosed further work on the identification of bit wear through processing of bit spectral data. Single row and complete bit signatures were examined using data from Schlumberger Cambridge Research Center’s Drilling Test Machine (DTM). Informative plots comparing new and worn bits are shown in Figure 2.5.10. A quite revealing result is shown in Figure 2.5.11, which shows an effectively constant bit signature, or power spectra, while drilling through various kinds of rock including slate, marble, limestone, and sandstone.

In 1991, Rector and Hardage of Western Atlas [74] discussed the use of radiation patterns from roller-cone bits as a seismic source to update geophysical and geopressure estimations as drilling proceeds. The authors concluded that longitudinal waves are stronger than shear waves for this purpose due to variations in the direction
Figure 2.5.8
PDC Bit Whirl Description by Amoco
(from Brett, et.al. 1989 [72])

\[ C_b = \] Center of the Bit
\[ C_h = \] Center of the Hole
\[ \delta = \] Difference Between Hole Diameter and Bit Diameter (inches)
\[ \phi_1 = \] Angular Position of a Cutter from a Reference Plane on the Bit (radians)
\[ \theta_1 = \] Angular Position of the Center of the Bit with respect to the Center of the Hole (radians)
\[ \theta_1 = \] Angular Position of a Reference Cutter with respect to a Reference Plane (radians)
\[ r_W = \] Whirl Radius of the Bit (inches)
\[ R_W = \] Whirl Radius (inches)
Figure 2.5.9
Spectral and Transient Behavior of PDC Bit Whirl
(from Brett, et.al. 1989 [72])

Lateral Test
Vessel Acceleration
(Relative Magnitude)
Figure 2.5.10
Change in Bit Spectral Signature as Bit Wear Increases
(from Jardine, et.al. 1990 [73])

![Graph showing Weight-on-Bit Power Spectra (kN²) with ratios of frequency to rotary frequency for New Bit (T0) and Worn Bit (T7) for different cones.](image-url)
Figure 2.5.11
Bit Power Spectra for a Variety of Rock Types Drilled
(from Jardine, et.al. 1990 [73])
and randomness of transverse bit forces that affect shear wave radiation. A seismic interpretation system was built by measuring longitudinal waves acquired with a surface drillstring sensor and geophone sensors around the rig as depicted in Figure 2.5.12. In field use, waves which correspond to BHA and drillstring axial harmonics, depicted in Figure 2.5.13, are observed, as shown in Figure 2.5.14. Although not closely related to the current study, the work underscores the importance of the bits forcing spectra and its ability to excite the drillstring into harmonic states. Future research may identify means of coupling seismic data analysis with drillstring data acquisition for improved interpretation of both drilling and formation characteristics.

To recapitulate, random effects not included in monochromatic analysis act as a conservative margin for BHA dynamic analysis, but should be considered when attempting to calibrate field data. Tri-cone bit mechanics will exhibit cone and row/teeth characteristic signatures during drilling, although the signals can be somewhat randomized by formation effects and the gouging action of the bit. Laboratory drilling simulations have quantified bit teeth excitation magnitudes for certain kinds of bits. New and worn bits exhibit different dynamic signatures. PDC bit whirl has been identified as a periodic phenomenon which cannot currently be predicted nor characterized. Bit excitations are strong enough to provide the basis for seismic analysis while drilling, providing another avenue for future research.
Figure 2.5.12
Use of Bit Excitation Mechanisms for Seismic Analysis while Drilling
(from Rector and Hardage 1991 [74])

Radiation of Drill Bit Energy from Drilling Rig into Earth

Head Wave Emanation Point in Drill String

Longitudinal Wave in Drillstring

$\theta_c$

Head Wave Raypath

Head Wave Wavefront

SV-Amplitude

Drill Bit

P-Amplitude
Figure 2.5.13
Schematic of BHA and Drillstring Multiples observed with Seismic Sensors while Drilling
(from Rector and Hardage 1991 [74])
Figure 2.5.14
Example Application of Seismic Analysis while Drilling
(from Rector and Hardage 1991 [74])

Pipe Length = 3,290 ft
2.6 Rate of Penetration (ROP) Models

The mechanics of the drilling process and factors controlling rate of penetration (ROP) have been studied since the earliest days in the petroleum industry. Thus, a comprehensive literature review of this technology is a challenging task. The following review covers work on drillbit rock mechanics, but focuses on implications for BHA dynamics. Two works will be reviewed in detail where researchers linked dynamic BHA models to dynamic ROP models.

In 1957, Gatlin of the University of Tulsa [75] reported on studies to quantify the influence of WOB and RPM on ROP. Gatlin reported a linear increase in ROP with increasing WOB, but a nonlinear relationship between RPM and ROP where ROP was proportional to a power of RPM with the exponent ranging from 0.4 to 0.5. Due to bit, rig or drillstring limitations, it is not always possible to simultaneously use high WOB and high RPM. To balance these effects, Gatlin presented "drilling practices" curves based on a relationship where the product of WOB and RPM is constant. In 1958, Somerton of the University of California at Berkeley [76] reported on a lab study of rotary drilling using dual cone, 1.25" bits drilling upward into shale, sandstone, and concrete. Power into the drill was measured and drill cuttings were collected for study. Dimensionless variables were developed to describe drilling behavior. Good correlations were developed between these dimensionless variables and ROP. Rock strength and bit wear were found to be important but not well defined considerations. Ultimate compressive strength was deemed as inadequate to describe rock drillability. In multi-constituent rocks, the weaker component failed in more volume than the stronger component.
In 1959, Outmans of Unocal [77] presented calculations from rock penetration and failure theory. Outmans calculated the volume of fractured rock around bit teeth and the tooth penetration past retained cuttings to derive expressions for ROP. ROP was then studied as a function of hydraulic horsepower, fluid pressure, viscosity, flow regime, cuttings cleaning behavior, RPM and WOB. Many conclusions were derived. Fluid pressure increased the fracture strain and reduced the tooth penetration, both acting to reduce ROP. Rock stresses under the action of a tooth were dissimilar to triaxial compressive stresses, thereby shedding doubt about the appropriateness of that measure to characterize drillability. At low WOB or low RPM, ROP was found proportional to WOB squared, while at high WOB or high RPM, ROP and WOB were linearly related. Outmans examined Gatlin’s ROP model and determined that it was valid for a small range of operating parameters. Under ideal hole cleaning conditions, Outmans predicted ROP would be proportional to the product of RPM and WOB squared.

In 1963, Simon of Battelle Memorial Institute [78] examined drilling using energy concepts. Simon concluded that most of the energy from bit forces was dissipated into elastic loads below the cutting surface and only a small fraction of the energy was expended in the plastic deformation and fracture of the rock. This observation led to a comparison of the energy efficiency in other cutting mechanisms such as the use of wedges and chisels. The comparison showed drilling efficiency was comparable to the underlying fundamental action of the drilling process. Simon hypothesized, however, that it may be possible using coring technology to better utilize energy by cutting smaller volumes of rock and removing the rest using low
energy means, such as breaking the core. In 1965, Teale of the Great Britain Mining Research Establishment [79] proposed that the highest drilling efficiency, measured by the ratio of the energy consumed to the rock volume removed, correlated to the crushing strength of the rock as shown in Figure 2.6.1. The implication of this was that rock properties may control drilling efficiency and thus, to improve ROP, the most direct means was to increase energy delivered to the bit. Teale also noted that the most efficient energy utilization for drilling may not always correspond to maximum ROP.

Two key observations from the above works are the relationship between energy delivered to the bit and drilling performance and the complexity of factors that control ROP. Relative to BHA dynamics, the first consideration makes it desirable to avoid dissipating energy into unnecessary BHA motions. The second consideration puts into perspective the difficulty of modeling the entire drilling process without adequate data and simulation tools. Changes in lithology, formation characteristics, borehole geometry, bit mechanics and other factors make this a formidable task. Despite these barriers, two major works have focused on comprehensive dynamic drilling system models.

In 1978, Eronini of the University of California at Berkeley [80] presented a comprehensive drilling model combining drillstring, fluid, bit, and ROP models. The approach used a transmission line model based on transfer matrices from the formation to the rig as shown in Figure 2.6.2. As an example transfer element, the last drillstring element delivered force and RPM data to the drillstring/bit element which produces a prediction of torque and axial velocity. With this output as input, a bit/rock penetra-
Figure 2.6.1
Concept of Minimum Drilling Energy Correlating to Rock Properties
(from Teale 1965 [79])

Rotary Specific Energy
( in-lb/cu. in x 1000)

Penetration per Revolution (in)
Figure 2.6.2
Transmission Line Model of Drillstring and Bit ROP
(from Eronini 1978 [80])
tion element predicted tooth force and rotation for the cutting action of an idealized bit as shown in Figure 2.6.3. The first element at surface, characterizing the block, was connected via spring/dashpot elements to a fixed boundary. This approach lead to a set of ordinary differential equations which were solved using an implicit trapezoidal scheme. The drillstring model was a lumped form of Dareing/Livesay’s axial and torsional vibration model [99]. Lateral vibrations were not included, although bending due to initial hole curvature was superimposed on axial stresses. The position of the drillstring was assumed fixed on the low side of the hole so axial and torsional vibration responses occurred about this position. The bit was modeled as an end impedance and provided for coupling of axial and torsional motions. The drilling fluid was modeled as a incompressible fluid without any elastodynamic wave propagation capability. In evaluating the robustness of the Eronini model, a fundamental issue is whether it generates forcing mechanisms observed in the field, such as the 3X excitation for tri-cone bits. Eronini did observe a bit force oscillation with a period of 0.3 seconds in one case, but traced it to travel time of a longitudinal wave in the drillstring. Since drillstring length was arbitrary, this was coincidental, since this model is not capable of producing the 3X excitation attributed to cones acting against a tri-lobed surface. Another concern involves the "flat response" of axial vibrations across rotary speeds from 30 to 150 RPM as shown in Figure 2.6.4. This conflicts with other response models and field observations. The lack of these characteristics suggest that the model is missing critical components of the bit/rock interaction. Eronini’s work is well focused for studying ROP dynamic trends using a parametric model. However, it appears unable to effectively model dynamic BHA behavior as
Figure 2.6.3
Idealization of Rotary Drill Bit
(from Eronini 1978 [80])

Actual Rotary Bit  Equivalent Bit per Eronini Model  Detailed Treatment of Bit Teeth
Figure 2.6.4
Rotary Drilling Response Prediction from Transmission Line Model
(from Eronini 1978 [80])

Penetration Rate (cm/sec)

Maximum Axial Drillstring Velocity (m/sec)

Rotary Speed (RPM)
fundamental aspects of the bit dynamics are missing. In 1982, Eronini, Somerton, and Auslander [81] provided a summary publication on the work.

In 1984, Baird and Caskey [82,83] reported on a joint effort by Jordan, Apostal, and Ritter Associates (J.A.R.) and the Geothermal Technology Development Division of Sandia National Laboratories (S.N.L.) to develop a dynamic drilling simulator. Towards an objective of reducing geothermal drilling cost, the quantification of dynamic behavior was a first step in the ultimate development of a instrumented, "smart" BHA. The program, called "Geodyne", modeled the BHA linked to a PDC bit drilling in nonhomogeneous formations. Specifically, 8-node isoparametric 3-D brick elements modeled the PDC bit as shown in Figure 2.6.5. This 3-D model was tied using special constraints to linear beam elements which modeled the drillcollar as shown in Figure 2.6.6. PDC cutters were not modeled directly. However, their effects were accounted for using "ghost cutters" through which cutter forces were transformed into nodal loads as shown in Figure 2.6.7. The formation was modeled as three surfaces which surround the bit. These surfaces were the bottom and the incline of the cutting surface and the side wall as shown in Figure 2.6.8. Due to contact and formation changes, the problem was nonlinear. Thus, Newton-Raphson iteration and a Newmark-Beta scheme were used for integration in time. To improve efficiency, a modal reduction of the system was used. In this initial publication, 19 modes were used with a time step of 0.001 seconds, thus requiring 625 time step solutions to model 1.25 turns at a speed of 120 RPM. The results showed large fluctuations in bit side force and tilt, particularly when the bit is not well constrained radially. Later in 1984, Baird, Caskey, Tinianow, and Stone [84] elaborated on this effort with an
Figure 2.6.5
Geodyr Idealization of PDC Bit as Solid FEA Structure
(from Baird, et.al. 1984 [84])

Smith 8-3/4" PDC Bit

Finite-Element Mesh for Smith 8-3/4" PDC Bit
Figure 2.6.6
Geodyn Tying of 3-D PDC Bit to 3-D Beam Element
(from Tinianow, et.al. 1984 [82])
Figure 2.6.7
Geodyn Transformation of Ghost Cutters into 3-D Bit Nodal Locations
(from Tinianow, et.al. 1984 [82])

Dotted Circles Represent Actual Cutter Locations
Cutter Forces are Transformed into Model Nodes
Figure 2.6.8
Geodyn Formation Surface Model Surrounding the Bit
(from Baird, et.al. 1985 [87])

- DISCRETE POINTS
- BOTTOM, SIDEWALL & INCLINED SURFACES
- USER DEFINED TOPOLOGY & PROPERTIES
- PENETRATION ALLOWED
- FORCES APPLIED TO CUTTERS
example application using revised drilling parameters; the erratic bit force and motion was again predicted as shown in Figure 2.6.9.

In 1985, Stone, Carne, and Caskey of Sandia [85] discussed their qualification efforts on the now complete Phase-I Geodyne system. A four-part qualification plan was developed. The first step was to measure modal behavior of a PDC bit and a pony (6' long) drillcollar and to compare mode shapes and frequencies with model predictions. The second step was to measure the response of the bit/collar structure to an impact on a PDC cutter to evaluate the effectiveness of the ghost cutter transformations. The third step was to verify the response to a single cutter impact with the structure rotating at a speed of 120 RPM. The last verification step was to drill a well characterized rock and compare the measured dynamic behavior during drilling with predictions. To collect the data, the bit and pony drillcollar were fitted with 3-D accelerometers at 24 different locations. Data were collected through an 8,000 Hz anti-aliasing filter at 20,000 samples per second. For the first test, the assembly was vertically suspended and an impact hammer was used to excite the system. Using the acquired data, modal properties were found and the frequency response for the system was determined from 100 Hz to 1500 Hz. The test was conducted several times to ensure repeatability. Figure 2.6.10 shows the instrumented collar/bit assembly and an overlay plot of several acceleration responses showing the repeatability of the experiment. The modal predictions and data were found to be well correlated as shown in Figure 2.6.11. The largest frequency error of 7.4% in the 4th bending mode was attributed to inaccuracies in modeling the threaded connection between the bit and the drillcollar. For the second test, the force time history of the hammer impacting the
Figure 2.6.9
Example Geodyne Numerical Results for Bit Motion
(from Baird, et al. 1984 [84])
Figure 2.6.11
Validation of Geodyn Predictions Against Measured Properties
(from Stone, et.al. 1985 [85])

Bit/Drillstring Natural Frequencies (Hz)

<table>
<thead>
<tr>
<th>Source</th>
<th>First Bending</th>
<th>Second Bending</th>
<th>Third Bending</th>
<th>Fourth Bending</th>
<th>First Torsional</th>
<th>First Axial</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>155</td>
<td>442</td>
<td>870</td>
<td>1392</td>
<td>679</td>
<td>1291</td>
</tr>
<tr>
<td>Geodyne</td>
<td>149</td>
<td>435</td>
<td>889</td>
<td>1495</td>
<td>656</td>
<td>1265</td>
</tr>
<tr>
<td>Percent Difference</td>
<td>3.9</td>
<td>1.6</td>
<td>2.2</td>
<td>7.4</td>
<td>3.4</td>
<td>2.0</td>
</tr>
</tbody>
</table>

![First Bending Mode](image)

![Second Bending Mode](image)

![Third Bending Mode](image)

![Fourth Bending Mode](image)
cutter was measured and used as input to Geodyn. Again, the model and response data correlated well as shown in Figure 2.6.12. The third test posed serious challenges including impacting a rotating cutter and acquiring the accelerometer data off a rotating assembly. The first issue was addressed by mounting an artificial cutter 1-1/2" by 0.75" on the side of the bit. The latter issue was addressed through slip rings with battery powered amplifiers on the drillcollar to generate a high signal-to-noise ratio (SNR) signal prior to passing over the slip rings. At the time of the publication, this third test had been performed but not evaluated, while preparations continued on the fourth test involving actual drilling.

Also in 1985, Baird, Apostal of J.A.R. and Wormley of M.I.T. [86] reported on theoretical predictions for the startup of drilling using Geodyn-2. Geodyn-2 included a more complete BHA model involving stabilizers, 3-D curved wellbore description, and more descriptive formation properties. Stabilizers were modeled similar to PDC cutters with a 3-D surface mesh around each stabilizer as shown in Figure 2.6.13. The stabilizer body was modeled as a beam while blades were treated as massless, rigid bodies allowing specific widths and shapes to be accounted for. The drillstring formulation was enhanced to include large displacement geometric effects in the load vector. The Geodyn-2 treatment of ghost cutters was unchanged, while the formation was modeled as a 3-D surface, not only at the bit, but also at stabilizers, thus allowing modeling of eccentric, oval or irregularly shaped wellbores. The solution approach was enhanced by using substructures for the bit and collars linked using component mode synthesis techniques. In the simulations, four BHA configurations were modeled in terms of placement, rotary startup and WOB application. The BHA was allowed 0.4
Figure 2.6.12
Time-Domain and Frequency Response Qualification of Geodyn
(from Stone, et.al. 1985 [85])

- Tangential Acceleration at top of Drillstring (g's)
- Response Magnitude
Figure 2.6.13
Geodyn-2 Stabilizer Treatment Including Wellbore Surface Modeling
(from Tinianow, et.al. 1984 [82])

- 3-D CURVED WELLBORE
- SURVEY TECHNIQUE - RADIUS OF CURVATURE
- DISCRETE POINTS DEFINE WELLBORE FORMATION
- CONTINUOUS SURFACE DEFINITION
seconds to come to rest inside the wellbore with a millisecond timestep used. The string was subjected to rotary acceleration from rest to 60 RPM in 1 second. Finally, the bit was loaded to 5,000 lbs. WOB in an additional second. Various transients were noted during the simulations including bit force and motion. Figure 2.6.14 shows bit displacement prediction with time highlighting the program's ability to estimate ROP. The mean bit position was correlated successfully to the drilling direction for building, dropping, and holding assemblies. Later in 1985, Baird (J.A.R.), Caskey (S.N.L.), Wormley (M.I.T.) and Stone (S.N.L.) [87], provided a summary publication on Geodyn-2.

The thoroughness of the Geodyn approach is significant. However, it does not appear that the industrial community possesses the necessary data nor the computational facilities to fully evaluate and use the program. The model's complexity, while enhancing its generality, significantly increases its cost. Although its computational performance is not addressed in any of the relevant publications, experience has shown that even short durations of simulated behavior require substantial computational resources. This extreme computational cost has limited the use of the program. In this context, the practicality of modeling the entire drilling system needs to be considered.
Figure 2.6.14
Geodyn-2 Bit Position Results Showing ROP Prediction
(from Baird, et.al. 1985 [87])

Parameters for Analysis
Rotary Speed 120 RPM
Time Step 1 millisecond
Mud Weight 10 lb/gallon
Bit Depth 1000'
Wellbore Inclination 30 degrees
A Holding BHA was Used:

10' 30'
2.7 Stick/Slip Models

"Stick/Slip" is a phenomenon where nonlinear wellbore friction induces a torsional pendulum motion in the drillstring. The frequency of this oscillation is lower than the fundamental torsional frequency of the drillstring and without dynamic data collection, it may never by noticed. However, the impact of stick/slip is significant, and it is an important feature of the drillstring dynamic behavior. Stick/slip is summarized in this study separately from harmonic models as it represents a special torsional behavior for the system distinct from the harmonics of the BHA.

In 1981, Belokobylskii and Prokopov of the Leningrad Polytechnic Institute [88] presented one of the first analytical treatments of friction-induced torsional drillstring vibrations. A simple model was proposed using a weightless drillstring of known rigidity linked to a rigid BHA of known weight. Five friction models were investigated. Although no correlations to data were provided, the work introduced the subject matter and provided a theoretical basis for the phenomenon.

In January 1987, Narasimhan and Spanos of Rice University [89] provided a comprehensive investigation of stick/slip in drillstrings including comparison to field data in a project with Exxon. It is generally accepted that wellbore friction, whether viscous or Coulomb, is nonlinear. However, previous works have not accounted for this effect. Narasimhan and Spanos demonstrated nonlinear wellbore friction was required to explain observed dynamic behaviors. Specifically, while rotating drillstrings off bottom, Exxon observed strong torsional resonances. This cannot be explained with linear vibration theory. As shown in Figure 2.7.1, wellbore friction is best described by a combination of dry and viscous friction resulting in nonlinear friction.
Figure 2.7.1
Nonlinear Dry-Viscous Behavior for Wellbore Friction
(from Narasimham 1987 [89])
Friction models studied are shown in Figure 2.7.2 including piecewise linear and cubic. After studying these models, the continuous nonlinear dry-viscous friction model was promoted, having a sound rational basis and having been applied to other problems. Torsional behavior with nonlinear friction was calculated and correlated well with field data from three rigs. This work represents the first concise treatment of nonlinear drillstring friction, and its results suggest that nonlinear friction should be considered, particularly in torsional models. In June 1987, Dawson of Exxon and Lin and Spanos of Rice University [90] presented a follow-up study using a piecewise linear friction model.

In 1987, Kyllingstad and Halsey of Rogaland Research Institute [91] presented some theoretical and experiment work on stick/slip. In their model, they assumed that stick/slip motion occurred and focused on predicting the stick/slip period, the time the bit is at rest, and the increase in torque due to stick/slip. The friction model used in this analysis was a constant Coulomb friction model with an increment torque required to break the BHA from rest. The solution technique involved numerical integration in time from assumed initial conditions. The modeling work was correlated to data from a test well and several conclusions were drawn. Specifically, if damping exceeds a critical value stick/slip will not occur; since linear viscous damping works against stick/slip motion, and stick/slip may be avoided by higher RPM. Further, if stick/slip does occur, its frequency will be lower than the primary torsional frequency, and the bit will be stationary for a large fraction of the time and its peak speed will exceed twice the nominal speed. Figure 2.7.3 shows an example of the Rogaland results. Also in 1987, Zamudio, Tlusty, and Dareing of the University of Florida [92] presented
Figure 2.7.2
Alternative Friction Models for Stick/Slip Analysis
(after Narasimham 1987 [89])

**Signum Model**

**Piecewise Linear Model**

**Cubic Model**

**Continuous Model**
Rogaland Stick/Slip Analysis
(from Kyllinstad and Halsey 1987 [91])

Dynamic Torque at Top of Drillstring

![Dynamic Torque Graph]

Bit Rotational Speed

![Rotational Speed Graph]

Friction Torque at Bit

![Friction Torque Graph]
theoretical work on self-excited vibrations related to PDC bits. Their work was based on the assumption that a PDC bit is cutting at constant RPM but it is subject to longitudinal vibrations. From this starting point, the authors developed conditions where the PDC bit exhibits "regeneration of waviness", analogous to machine tool chatter. Finally, the impact of a shock absorber to reduce these theoretical vibrations was quantified.

In 1988, Halsey, Kyllingstad, and Kylling of Rogaland Research Institute [93] developed the concept of rotary torque feedback to eliminate stick/slip under a study funded by Esso Norge, an Exxon affiliate. As described in their 1987 work, Rogaland contends stick/slip motion results in accelerated fatigue of drillstring components, inefficient ROP, and rapid bit wear. Thus, means to eliminate or reduce stick/slip are critical to drilling efficiency. Standard rotary systems are aimed at maintaining constant RPM. As a result, the rotary acts as a fixed boundary condition since torsional waves impacting the surface are reflected along with the majority of their energy. The authors proposed modifying the constant RPM control based on torque feedback to reduce RPM as the torsional waves impact the rotary, thus changing the surface boundary from a reflective, fixed condition to a damping mechanism which removes energy from the waves. This torque feedback system was implemented on a test rig and was effective in eliminating stick/slip as shown in Figure 2.7.4. Without torque feedback, dynamic torques 16 times the input torque were observed indicating small damping and potentially damaging dynamic torques during stick/slip motion.

In 1990, Dareing, Tlusty and Zamudio of the University of Florida [94] republished their 1987 work and again offered only theoretical results with no field
Figure 2.7.4
Elimination of Stick/Slip through Torque Feedback in Rotary Speed Control
(from Halsey, et.al. 1988 [93])

Torque at Top of Drillstring (kN-m)

Rotary Speed (RPM)

Downhole Acceleration "X"

Downhole Acceleration "Y"

Time (Seconds)
Feedback Activated at 37 seconds
verification of their proposed mechanisms. Also in 1990, Lin and Wang of Texas A&M University - Galveston [95] presented theoretical results on stick/slip. The authors attributed the phenomenon to the nonlinearity of the wellbore "resistance" and observed that as drillstring length increases, it will be more difficult to avoid stick/slip motion since frictional resistance increases. The authors suggested that stick/slip motion may initiate the tri-lobed surface with tri-cone bits due to variation in ROP with increased RPM. Specifically, it is suggested that stick/slip motion causes unequal RPM through a given revolution and hence unequal penetration. This point is not well substantiated and has several deficiencies. An important observation, however, is that stick/slip distorts the 3X tri-cone frequency, since the bit continually slows and accelerates during the stick/slip cycle.

In 1991, Dufeyte and Henneuse of Elf Aquitaine [96] presented torsional surface measurements during 3,500 hours of drilling (4.8 drilling-months). The work was the result of Elf Aquitaine’s "Dynafor" project initiated in 1986 to promote "improved drilling performance through a better knowledge of dynamic phenomena". The data included four wells, 3 onshore and 1 offshore, 55 tri-cone bits and 15 PDC bits, and both rotary table and power swivel drives. The Elf surface tool is shown in Figure 2.7.5. Stick/slip motion was detected using analysis of surface data and addressed by changing WOB, RPM or mud properties. The algorithm to detect stick/slip searched for dynamic torque variations in excess of 15% of average torque and periods of 2 seconds per 1000 m of 5" drillpipe. Many observations were provided by the authors. If RPM is above a critical value, no stick/slip will occur. This critical RPM value, however, could not be defined by Elf with available data. As WOB increases thereby
Figure 2.7.5
Elf Aquitaine Dynamic Surface Sub "le Dynametre"
(from Dufeyte and Henneuse 1991 [96])
increasing stabilizer and contact loads, stick/slip is more likely and more severe. Studies of mud types and their influence on stick/slip were inconclusive. Studies of bit behavior lead to a hypothesis that PDC bit wear from stick/slip may be as severe as wear induced by extremely hard formations. The local interaction of the BHA with the formation plays a role in stick/slip as indicated by Elf’s monitoring of near-bit stabilizers (NBS) passing through clay formations. Directional trajectories are also a consideration, with stick/slip more prevalent in directional wells due to increased friction. In one case cited by Elf, stick/slip in an s-shaped well temporarily vanished after running casing but subsequently reappeared after enough open hole had been drilled. Elf verified the elimination of stick/slip using the torque feedback system developed by Rogaland. Elf also conducted a number of tests spotting different lubricants across the BHA. As shown in Figure 2.7.6, one lubricant was successful in eliminating stick/slip as it passed the BHA and for a short time thereafter, indicating it modified the friction characteristics of the system. Stick/slip was found to be prevalent with motors due to the practice of slowly rotating the drillstring during these operations. This puts the drillstring below the critical RPM level to avoid stick/slip and as a result, drilling occurs with torsional waves traversing the pipe, BHA, motor and bit. Elf concluded that stick/slip has appreciable impact on drilling efficiency. Peak speeds of 3 to 10 times the rotary speed result which cause premature bit wear. Drillpipe torques of 30-40% of the torsional yield limits, 2 to 3 times the make-up torques, are experienced leading to fatigue problems. Finally, lower ROP is achieved as shown in Figure 2.7.7. With a working system to monitor and avoid stick/slip, Elf alluded to focusing on other dynamic concerns including axial bounce and
Figure 2.7.6
Temporary Elimination of Stick/Slip as Lubricant Passed by BHA
(from Dufeyte and Henneuse 1991 [96])

Events During Lubricant Test

a) Stick/slip behavior active

b) Lubricant Pill being Pumped

c) Stick/slip behavior minimized, even as RPM decreased.

d) Stick/slip reactivates at low RPM, but then is removed by increasing RPM.

e) Stick/slip still avoided by residual effects of lubricant pill.

f) Stick/slip reactivates after short RPM decrease.
Figure 2.7.7
Detrimental Impact of Stick/Slip on Drilling Efficiency
(from Dufeyte and Henneuse 1991 [96])
lateral whirl.

Stick/slip is a special phenomenon of the drilling system controlled by structural and frictional system characteristics and operating practices. Stick/slip motion is a torsional pendulum motion with a lower frequency than the primary torsional mode for the drillstring without friction and damping considered. Although detrimental to ROP and bit life, it remains unclear how destructive stick/slip is to BHA components. Torque feedback to adjust surface RPM is one means to minimize stick/slip and is being implemented by several operators. Field operating guidelines are also needed since they do not require additional rig equipment and would be more easily implemented and on a broader scale.
2.8 Response Models

Due to the complexity and computational inefficiency of addressing drillstring dynamics using time-domain integration, significant work has focused on the system response in terms of harmonic behavior. This is also the focus of the current work. In particular, if natural frequencies of the drilling system can be avoided, unnecessary drillstring stresses will be reduced and the energy reaching the bit will also be optimized. This approach will, thus, simultaneously maximize the reliability of the drillstring relative to fatigue and optimize the drilling performance.

In 1961, Bogdanoff and Goldberg of Purdue University [97] presented an analysis of the drillstring response subject to stochastic torsional and axial forces. In response to the deterministic approach of Finnie and Bailey [46], the authors argued that the loads at the bit and along the drillstring are random processes characterized by mean levels with stochastic fluctuations. Theoretical derivations were presented leading to a series of complex integrals requiring the power spectral density of the forcing mechanisms. As this information was not available, no practical results were provided. Even today, power spectra of excitations are only beginning to be measured and remain largely undefined. Thus, while this work represents a significant contribution in drillstring dynamics, it is of limited use as the necessary data are not available.

In 1963, Paslay of Rice University and Bogy of Shell [98] analyzed the longitudinal behavior of a drillstring subject to intermittent contact of bit teeth. The work focuses on whether it is possible to sense large tooth loadings from surface vibrations. For this purpose, a low bound on damping and an upper bound on the magnitude of tooth loadings were used with a mobility method description of the drillstring. Two
extreme boundary conditions, fixed and free, are considered for the top of the drillstring. Results indicated the ratio of surface forces to bit input forces will range from 0.1 to 1.0 for a 10,000’ drillstring. For a bit displacement of 0.020”, a bit force and potential surface force of 100,000 lbs. was predicted.

In 1968, Dareing and Livesay of the Universities of Arkansas and Tulsa, respectively, [99] presented a model for damped longitudinal and torsional vibrations of drillstrings using a bit-to-rig model as shown in Figure 2.8.1. Each section represented a uniform pipe section, while springs represented shock tools. Rig components were included such as a large mass for the traveling block, hook and elevators, and a spring for the drilling line. Downhole recorder data was used to substantiate the 3X tri-cone bit excitation and to characterize the bit displacement (by double integration of acceleration) to be 0.25” peak to peak. Figure 2.8.2 shows a core taken from a well where the lobed surface and its dimensions were observed by Dareing. The model validity was supported by the consistency between measured dynamic WOB and that predicted by the model. The model was excited across a range of RPM to produce frequency response diagrams and to study the effects of shock-tools as shown in Figure 2.8.3. Constant viscous damping factors were estimated for drillpipe and drillcollars. Damping was shown to significantly reduce vibration magnitudes at resonances.

Later in 1968, Huang of the University of Texas Arlington and Dareing [100] predicted buckling loads and natural frequencies for lateral vibrations of drillpipes suspended in fluid. Boundary conditions were assumed to be pinned at the surface and guided at the bit. Both buckling loads and natural frequencies were presented in
Figure 2.8.1
Dareing and Livesay Axial/Torsional Drillstring Response Model
(from Dareing and Livesay 1968 [99])

\[ k = \text{Spring Stiffness of Drill-lines and Derrick} \]
\[ M = \text{Mass of Kelly, Swivel and Block} \]
\[ L_2 = \text{Total Drillstring Length} \]
\[ L_1 = \text{Length of the BHA} \]
\[ x = \text{Distance from bit to point in drillstring} \]

\[ \text{Bit Excitation } u(0,t) = U_0 \sin(\omega t) \]

**Force Equilibrium at Surface:**

\[ ku_2 \]
\[ (A_2 E_2 \frac{\partial u_2}{\partial x}) \]

**Force Equilibrium within Drillstring:**

\[ (A E \frac{\partial u}{\partial x}) + \frac{\partial}{\partial x} (A E \frac{\partial u}{\partial x}) \ dx \]
\[ \gamma \frac{\partial u}{\partial t} \ dx \]
\[ \rho g \ dx \]
\[ (A E \frac{\partial u}{\partial x}) \]

\( A = \text{Cross-Sectional Area} \)
\( E = \text{Modulus of Elasticity} \)
\( \gamma = \text{Damping factor} \)
\( \rho = \text{mass per unit length of drillstring} \)
\( g = \text{Gravitational acceleration} \)
\( \omega = \text{Circular frequency} \)
Figure 2.8.2
Cored Drilling Surface Showing Tri-Lobed Phenomenon
(from Dareing 1982 [102])
Figure 2.8.3
Predicted Influence of Shock Sub on Dynamic Response
(from Dareing and Livesay 1968 [99])

Without Shock-Sub

With Shock-Sub

Maximum Axial Displacement Ratio

Rotary Speed (RPM)
dimensionless variables. The results can be used as a guide to determine the length of drillcollars required for a desired WOB without buckling. Primary lateral modes for long drillstrings were shown at extremely low frequencies. This study did not deal with the wellbore constraint; thus, its applicability is rather limited.

In 1978, Ohanehi and Mitchell of Virginia Polytechnical Institute [101] presented a study of rotary-vibratory drilling including a detailed analysis of a 1957 prototype system developed by Drilling Research Incorporated (DRI). Rotary-vibratory drilling combines standard rotary techniques with vibratory systems to increase power delivered to the bit and ROP. Works by Russians, Gulf Oil, Pan American Petroleum, and Borg-Warner were reviewed. All of the efforts surveyed were terminated due to financial limitations. The authors concluded that the most carefully executed and documented rotary-vibratory effort was the 1957 DRI work and a detailed review of that effort was made. The DRI exciter operated downhole above the bit but was isolated from the drillstring with a vibration isolator. The exciter operated at a frequency of 300 Hz, which according to the authors should have increased the bit energy by a factor of 1.4 to 1.9. The measured field ROP was about twice the normal ROP lending credence to the analysis. However, the critical modeling conclusion was that 300 Hz was not near any of the primary system resonance and if the exciter could have been tuned to one of the resonances, in this case 90 Hz, bit energy and presumably ROP could have increase by a factor of ten (10). Placement of the exciter could also be optimized. The authors concluded that these exciters should be controllable, preferably automatically, with regard to vibratory frequency and dynamic load magnitude. The industry has yet to produce such an ideal mechanism.
In 1982, Dareing of Maurer Engineering [102] reexamined longitudinal and torsional vibrations to demonstrate that the primary harmonics can be predicted directly from the BHA length of a drillstring as shown in Figure 2.8.4. Dareing claimed that by estimating the primary torsional and axial modes of the BHA and ignoring drillpipe vibration modes, it is possible to readily identify the most harmful resonance frequency. This work involved a simple calculation for primary axial and torsional harmonics and focused, for the first time, on the harmonic behavior of the drillcollars without regard for the drillpipe. The argument for this is that the limber drillpipe section with balanced pipe and connection strengths is of much less concern than the thick heavy and connection-weak drillcollar section. If harmonics build in the collars, the drillpipe offers negligible resistance and the result will be a washout or twistoff in the BHA. In comparison, damping and wellbore interaction work against drillpipe harmonics and even if they occur, they are not likely to cause failure of the drillpipe connections nor excite the BHA. In 1983, Dareing reiterated these points in additional publications [103,104]. Also in 1984, Dareing [105,106] suggested that drillstring vibrations may increase beneficial power at the bit for drilling, and that MWD systems may be developed which could control such processes. Also in 1984, he suggested that drillstring vibrations contribute to crooked holes [107].

In 1983, Rey of M.I.T. [108] studied the dynamics of an unbalanced drillcollar rotating in an inclined wellbore. A model of the BHA from the bit to the first stabilizer was used. The wellbore was considered rigid, contact was not allowed except at the bit and stabilizer, and the solution was steady-state. Sensitivity studies were
Figure 2.8.4
Control of Drillstring Response Peaks by BHA Properties
(from Dareing 1982 [102])

Frequency Response of 800' of Drillcollars

Frequency Response of an 8,000' Drillstring
containing 800' of Drillcollars
performed with regard to boundary conditions, WOB, TOB, mass eccentricity distributions, damping, and bit restoring moment.

In 1984, Dunayevsky, Judzis and Mills of Sohio [109] presented a new approach to the drillstring harmonics problem by formulating an eigenvalue description based on the 2-to-1 parametric relationship between lateral and axial motions. Dunayevski combined governing equations for an elastic beam with displacement constraints of the borehole and loads, including gravity, buoyancy, inertia, and friction, to generate the governing partial differential equations. Finite element discretization was used to reduce these equations to a set of Mathew-Hill ordinary differential equations. Parametric resonances for these equations were found which determined the onset of precession. Combinations of weight-on-bit (WOB) and rotary speed were determined which lead to precession as shown in Figure 2.8.5. The analysis allowed evaluation of conditions where axial resonances coincide with unstable transverse vibrations, leading to precessional motion. Parameters shown to control precession were frequency and the magnitude of WOB variations. The model is based, however, on the assumption that the drillstring is in continuous contact with the wellbore as shown in Figure 2.8.6, which must be seriously questioned.

In 1985, Mitchell and Allen of Enertech [110] presented results for 3-D harmonic analysis for axial, torsional, and lateral drillstring motions using the MARC general purpose finite-element program. They showed failures in three (3) Mesa Petroleum wells could not be explained by axial and torsional responses. The forcing mechanism for their model was the 3X tri-cone excitation using an amplitude of 0.12" based on Dareing’s work. Critical axial modes were shown to be below the operating
Figure 2.8.5
Resonance Prediction from Dynamic Parametric Model
(from Dunayevsky, et.al. 1984 [109])

Hatched Areas are Zones of Parametric Resonance
Figure 2.8.6
Drillstring Precession Model with Continuous Contact Assumption
(from Dunayevsky, et.al. 1984 [109])

\( \gamma \) = Angle of precession
\( d \) = Drillstring diameter
\( D \) = Wellbore diameter
\( O \) = Center of the wellbore
\( O' \) = Center of the drillstring
\( s \) = Distance along the drillstring
\( t \) = Time
\( X, Y, Z \) - Major physical axes
\( u, v, w \) - Displacements along major physical axes
RPM of 110-120 RPM, due to shock tools, while critical torsional modes were shown to be higher. However, lateral modes were predicted near operating frequencies and high stresses were predicted near the failure locations.

Later in 1985, Dunayevsky, Judzis, Mills, and Griffin [111] presented an enhancement to their model to include the effects of wellbore curvature and performed sensitivity studies on WOB fluctuation, RPM, inclination, heavy-weight drillpipe (HWDP) length, and total well depth as shown in Figure 2.8.7. The authors claimed good correlation with a limited number of field failures, but did not disclose the supporting information.

In 1986, Khan of M.I.T. [112] studied longitudinal and torsional transfer functions between downhole parameters such as bit displacement and force and surface indications such as force, torque and acceleration. Finite-difference equations described the governing wave equations and a standard eigensolver was employed for mode shape and natural frequency determination. The transfer functions were developed using the mobility method employed by Paslay and Bogy for varying RPM. Constant 1% ratio of critical damping was assumed and maximum surface forces were predicted to be 2.5 times the bit forces while maximum displacements were predicted to be 1.5 times those at the bit. Lateral motion was not addressed at all.

In 1986, Gonzalez of Pool Company-Resotek [113] presented theoretical models and field results for an eccentric-weight oscillator to free stuck drillpipe and liners as shown in Figure 2.8.8. The system proved successful in 51 of 88 attempts (58%) in the California area, but in only 3 of 13 attempts (23%) in the Texas area. The lower success rate in Texas was attributed to deeper wells and different lithology. The
Figure 2.8.7
Sensitivity Study from Dynamic Parametric Model
(from Dunayevsky, et.al. 1985 [111])
Constructive Use of Eccentric Weight Oscillator to Free Stuck Pipe
(from Gonzalez 1986 [113])
deepest drillpipe freed was 7,800' while the deepest tubing freed was 9,000'.
Resonant extraction, if it occurred, required an average of 3 hours, while typical jarring operations last many times longer. This performance would improve with surface or downhole instrumentation which was available on only a fraction of the jobs. The current status of this system is unknown. Also in 1986, Chuan-Xiao of the Chinese Jianghan Rock Bit Plant [114] presented a parametric study of axial and torsional vibrations with a focus on rock bit tooth contributions.

In 1987, Burgess, McDaniel and Das of Anadrill/Schlumberger [115] presented a two-dimensional finite-element model for lateral BHA vibrations. The static configuration of the BHA was determined using an iterative solution with nonlinear gap elements for contact points and loads. The BHA was truncated at the wellbore contact point above the last stabilizer, which prevents analysis of vertical holes so a minimum inclination of 0.5° is imposed. Following the static calculation, an undamped system matrix was constructed and inverted at each frequency to determine response. The operating range was swept at 2 RPM increments and 0.5 RPM increments near resonances as shown in Figure 2.8.9. The displacement experienced by the "critical component" (i.e. MWD) was normalized by the maximum displacement anywhere in the BHA. The authors indicated that without further information on the damping and the excitation of the system, use of the model should be limited to estimating the location of resonances.

In 1987, Joglekar of M.I.T. [116] studied the motion of rotors subject to partial or full radial rubbing. Theoretical work involved a single degree of freedom model. Experimental work involved tests where behavior was measured visually using
Figure 2.8.9
Two-Dimensional Undamped Response Prediction by Anadrill
(from Burgess, et.al. 1987 [115])

Weight-on-Bit 25,000 lbs.
Hole Size 12-1/2"
Inclination 1 degree

Normalized Flexural Amplitude (Normalized against MWD displacement)
strobe lights, as shown in Figure 2.8.10, with and without fluid between the rotor and casing. Conditions were defined for the onset of partial and full rub and an added mass coefficient was obtained for one set of conditions.

Also in 1987, Allen of Enertech [117] presented results from case studies on thirteen (13) BHA failures showing incidents had occurred over a wide range of depths and operating speeds as shown in Figure 2.8.11. Allen modified the mass term in the standard drillstring dynamics equations to account for the added mass effect of the mud, lowering frequencies 10% for an added mass coefficient of 1.0. Even with this enhancement, Allen concluded that these equations were inadequate for predicting lateral resonances. Later in 1987, Mitchell and Allen of Enertech [118] correlated their finite-element frequency response model to eight (8) field cases. For packed BHA, they concluded that the model showed good correlation with harmonic frequencies and the location of the failure or damage in the BHA. For slick BHA, the correlation was less successful.

In 1988, Chin [119,120] presented results from applying wave propagation theory of strings to the drillstring. Using the concept of group velocity, Chin showed that the group velocity for bending waves generated in the BHA under axial compression vanishes when these waves reach the neutral point where axial loads are zero as shown in Figure 2.8.12. The analogy is drawn to the string problem where the wave speed is related to the square root of tension divided by density. Chin suggested that with lateral waves "trapped" at the neutral point, energy builds there and is responsible for many failures. Chin also proposed this explanation for why lateral vibrations cannot be detected at the surface. As a followup, Chin introduced a vibration model
M.I.T. Experimental Apparatus to Study Rotor Dynamics
(from Jogekar 1987[116])

Figure 2.8.10
Figure 2.8.11
Enertech Review of BHA Failures versus Rotary Speed and Depth
(from Allen 1987 [117])

Bands Denote Operating Regions for Failed BHAs

 Rotary Speed (RPM)

 Operating Depth (ft)

 BHA Number
Figure 2.8.12
Application of Wave Speed Theory to Drillstring Vibrations
(from Chin 1988 [119])

Nondimensional Group Velocity

\[ C_g^* = 0.5 \sqrt{\frac{\rho A}{4 \left( EI \omega_o^2 \right)}} \frac{\partial \omega}{\partial k} \]

Nondimensional Axial Force

\[ N^* = \frac{N}{\sqrt{2 \omega_o (EI A) \rho}} \]

\( \rho = \) Mass density
\( \omega_o = \) Vibration frequency
\( \frac{\partial \omega}{\partial k} = \) Group Velocity
\( A = \) Cross-sectional Area
\( E = \) Elastic Modulus
\( I = \) Moment of Inertia
\( k = \) Wave number
\( N = \) Axial Force
where the bit boundary conditions are modified. At the centroid of the bit, alternating tension and compression are applied to simulate bit loads, while a ROP condition is specified on the bit displacement as shown in Figure 2.8.13. Results were shown, but model details were not provided.

In 1989, Dunayevski, Judzis, and Abbassian of British Petroleum [121] presented further work on their parametric resonance model. The authors indicated that drillstring failures are the second largest drilling problem for BP and account for 3% of the drilling budget. Their model, based on full drillstring to wellbore contact and focused on axial-lateral parametric resonance, was used for a sensitivity study on BHA length, shock-tools, and stabilizer placement. A controversial result of their analysis was that stabilizers were predicted to have little effect on conditions which lead to parametric resonance as shown in Figure 2.8.14. Although the authors admitted that the assumption of full wellbore contact is violated at the stabilizers, they still argued that it adequately models drillstring behavior. The poor modeling of the BHA portion of the drillstring seems to be in conflict with the fact, discussed in their introduction, that 75% of all BHA failures occur in the drillcollar threads. Finally, the authors noted that drillstring loads are not monoharmonic, but polyharmonic and should be addressed in future work.

In 1989, Shyu of M.I.T. [122] studied bending vibrations of rotating drillstrings. The work focused on three bending mechanisms: linear coupling between axial force and bending, parametric excitation of bending by axial force (2-to-1 relation), and whirling. General theoretical relationships were developed for whirling rotating shafts for which backward whirl and forward synchronous whirl are special cases. Shyu
Figure 2.8.13
Proposed Enhanced Axial Drillstring Response Model
(from Chin 1988 [119])

Existing Drillstring Model

Proposed New Model

Same Surface Mass-Spring-Damper Model

Same Differential Equations

Bit Motion Modeled by Periodic "Push-Pull" of External Loading

Rock/Bit Interaction Model Prescribed at "x=0"

Drillbit Stress, Speed, and Position Calculated
Figure 2.8.14
Prediction of No Stabilization Effect on Parametric Resonance
(from Dunayevsky, et.al. 1989 [121])
applied an experimentally derived frequency-dependent added mass term to the structural
definition of the BHA. A finite-difference model and eigenvalue solver were provided
for the BHA up to the second stabilizer to predict natural modes and frequencies.
Despite the frequency dependent added mass coefficient, a non-iterative approach was
taken with constant added mass coefficients. Thus, the presented eigensolution is not
fully compatible with the equation of motion. Figure 2.8.15 shows eigenvalue
predictions versus WOB. Shyu presented a governing equation for lateral vibration of
a rotating beam with initial curvature and eigenvalues for it. Relations were derived to
convert bending strains from fixed to rotating coordinates since downhole instrumen-
tation systems experience behaviors in the rotating frame of reference. Figure 2.8.16
shows an experimental set-up to verify these relations, while Figure 2.8.17 shows the
bifurcation of natural frequencies in a rotating beam relative to those in a non-rotat-
ing beam. From Shyu's experiments, Figure 2.8.18 shows the axial force spectrum
generated on a non-rotating specimen and the accompanying bending spectrum. The 1X
lateral to 2X axial parametric relationship is apparent. Figure 2.8.19 shows the force
spectrum for rotation at 2.5 Hz, while the bending spectrum is shown in Figure
2.8.20.

In 1990, Lie Zhang of the Chinese Lanzhou Petroleum Machinery Research
Institute [123] reported on experimental measurements of tri-cone bits acting against
tri-lobed formation surfaces. The author concluded that dynamic displacements can
cause severe enough bit bounce to result in the bit not contacting the troughs in the
lobes. Thus, he promoted the use of shock tools to ensure continuous bit contact and
higher drilling efficiency.
Figure 2.8.15
Theoretical Sensitivity of Lateral Eigenvalues to WOB
(from Shyu 1989 [122])

First Mode, 0.95 Hz

Second Mode, 3.33 Hz

Stabilizer

Stabilizer

Bit

First Two Bending Modes

The Effect of WOB on Natural Frequency
Figure 2.8.16
M.I.T. Experimental Apparatus to Study Parametric Relations and Rotating Observation Frame
(from Shyu 1989 [122])
Figure 2.8.17
Bifurcation of Observed Lateral Frequencies in Rotating Observation Frame
(from Shyu [122])

\[ \omega_n - \omega \]

\[ \omega_n , \omega_n^+ \] natural frequency

\[ \omega_n + \omega \]
Figure 2.8.18
Non-Rotating Spectra of Axial Forcing Mechanism and Lateral Response
(from Shyu 1989 [122])
Figure 2.8.19
Rotating Spectra of Axial Forcing Mechanism
(from Shyu 1989 [122])

Axial Force Spectra with 2.5 Hz Rotation (Hz)
Figure 2.8.20
Rotating Spectra of Lateral Response Verifying Bifurcation
(from Shyu 1989 [122])

Bending Spectra with 2.5 Hz Rotation (Hz)
Also in 1990, Apostol, Haduch, and Williams of Drilling Resources Development [124] presented a forced frequency response (FFR) model allowing for 3-D wellbores, damping, and a software link between a directional drilling BHA model and the dynamic FFR analysis. The model used a lumped mass matrix including an experimentally derived added mass coefficient for the mud. A general damping form was presented allowing for Rayleigh, structural, and experimentally derived viscous damping values to be implemented for a specific application. Figures 2.8.21 and 2.8.22 show representative results. Three cases studies were evaluated in which drillstring failures occurred. Two failures correlated reasonably with the predictions while a third could not be explained.

Finally, in 1990 Jansen of Koninklijke Shell E&P Laboratorium [125] presented a study of drillcollar whirl based on a two-dimensional rotor model driven by eccentric loads. Jansen’s approach relies on the equation of motion of an eccentrically loaded rotor. It was concluded that the largest deflection occurs when the whirl frequency coincides with the primary bending mode, and that several variables are important including fluid added mass, stabilizer clearance and friction, and drillcollar contact with the wellbore.

Although additional work is needed to develop harmonic models which can fully explain dynamic drilling phenomena, frequency domain approaches appear well suited for focusing on critical BHA behaviors and providing modeling tools for their study. In particular, harmonic models provide the most effective means of predicting resonances in the drilling system. By avoiding resonant conditions, lower stress levels will be ensured leading to longer BHA life, fewer failures, and increased ROP.
Figure 2.8.21
Frequency Response Influence of Added Mass
(from Apostal, et.al. 1990 [124])

![Graph showing frequency response influence of added mass. The graph plots lateral displacement (inches) against rotary speed (RPM). The lines represent Chen Model and Infinite Medium.]
Figure 2.8.22
Frequency Response Influence of Damping
(from Apostal, et.al. 1990 [124])

[Graph showing lateral displacement (inches) vs. rotary speed (RPM) for different damping conditions.]
2.9 Technology Summary and Implications for BHA Analysis

Despite their simplicity, torque/drag models can be correlated to provide estimates of mean axial and torsional loads during drillstring motion. Torque/drag models are useful for evaluating drillstring and rig capacities in directional and horizontal drilling. However, torque drag models do not address the dynamic behavior of the drillstring and BHA.

Static directional models address the equilibrium of the BHA subject to gravitational and applied loads and the placement of stabilizers. From these models, bit resultant forces can be used to predict directional drilling trends. Again however, these models do not provide key information for the BHA dynamics problem.

Dynamic directional models are based on refining static calculations using temporal averaging techniques. Rotational friction loads cause deflections in the azimuthal plane and are required to generate BHA walking predictions. Millheim identified these effects and solved for steady-state dynamic behavior. Birades used time-domain simulation with shock loading treatment of wall contact to model this problem. Since Birades did not address bit excitations, the resulting model cannot be used to determine critical operating conditions. Although Brakel included tri-cone and PDC models with his time-domain BHA model, review of the resultant spectral predictions indicates fundamental effects like the 3X tri-cone bit excitation are lacking.

Many experimental efforts have been undertaken by the industrial community. Stick/slip behavior, 3X tri-cone bit excitations, and axial/torsional harmonics were identified in the earliest studies. Dynamic characteristics correlate well with the formation being drilled indicating that the bit plays a critical role in generating loads
which dominate BHA and drillstring dynamic behavior. High-frequency data acquisition identified additional effects including walking, precession, and whirling. Surface and bit boundary conditions are complex, frequency-dependent, and transient. Shock-tools are nonlinear in terms of static and dynamic characteristics. Destructive conditions are observed with downhole recorders which are generated both by BHA harmonics and by impact between the BHA and wellbore. Attempts to measure damping indicate it is also complex with frequency dependencies.

Bit excitations are believed to be quasi-random, making monochromatic frequency response a conservative engineering tool which will bound actual responses. In lab tests, bit properties are exhibited which allow use of dynamic WOB, TOB, and ROP to evaluate bit wear. Bit force spectra shift with wear due to smaller teeth and faster rotation of cones. This step forward in understanding tri-cone bit dynamics is accompanied by the finding that PDC bits are subject to chaotic whirling which leads to poor ROP, rapid wear, and significant impact on BHA dynamics.

Although ROP models have evolved, they are based on "steady-state" quantities such as mean WOB and RPM. All ROP works indicate that increased bit energy will result in increased ROP. Eronini’s work linking BHA, bit, and formation models into a comprehensive drilling simulator is laudable, but, like Birades’ and Brakel’s, falls short of capturing key dynamic characteristics. The Sandia/J.A.R. Geodyne project led to a massive numerical simulator which appears beyond the industry’s ability to validate due to a lack of data and computational resources.

Stick/slip models simulate an important phenomenon where nonlinear wellbore friction induces a fundamental torsional mode. The impact of stick/slip is primarily a
reduction in drilling efficiency in terms of reduced ROP and increased bit wear, but may also damage drillstring components.

Harmonic models initially developed for axial and torsional modes, have evolved into 3-D formulations and are proposed as the most effective tool for quantifying operating conditions which put the BHA at risk due to resonance. This harmonic problem requires knowledge of structural stiffness, damping, and inertial properties, insight into BHA forcing excitations, and rigorous mathematical treatment of the problem. This is the focus of the second part of this study.
Chapter 3 - Modeling Considerations for Dynamic BHA Characterization

"Follow the river and you will find the sea" - French

An important objective of the present study is to construct a practical analysis tool for quantifying dynamic effects in planning proposed BHAs and evaluating the field behavior of existing BHAs. The BHA design considerations involve component selection, drillcollar sizing, stabilization, and operating parameters (WOB,RPM) for minimizing drillstring failures and improving drilling efficiency.

To appreciate the need to focus solely on the BHA, it is critical to reinforce the dominant role of the BHA in drillstring dynamics and operational reliability. In one of the few published studies of its kind, Sweet of Conoco [126] presented a case history of drillstring failures from Conoco’s operations offshore West Africa. Sixty-six (66) drillstring failures occurred on five (5) wells at a cost of millions of dollars. The critical statistic from the study, however, was that 77% of the failures occurred in the BHA while only 23% occurred in the drillpipe. Thus, nearly 80% of drillstring failures may be expected to occur in the BHA. Even studies of drillpipe failures highlight the dominant role of the BHA. Figure 3.1 shows a histogram of 1,785 drillpipe failures collected as part of a joint study by API and IADC [127]. As shown in the figure, the drillpipe failures overwhelmingly occur adjacent to the BHA on a statistical basis. Thus, although the BHA comprises only a few hundred feet of a drillstring which is several thousands of feet long, the BHA sustains the critical stresses downhole during drilling and is most susceptible to failure. Likewise, the substantial forces and motions of the BHA dynamics also largely dominate drillpipe
failures. These trends justify focusing, for a first approximation, only on the BHA portion of the drillstring.

Upon focusing on the BHA dynamic behavior, judgement must be exercised as to the optimal means of quantifying the dynamic sensitivity of the BHA in operation. Previous attempts to model the bit/formation interaction and ROP behavior have shown very limited success in their ability to capture key dynamic characteristics. Further, new observations, such as the Schlumberger spectral data on tri-cone bit excitations and the Amoco detection of PDC whirl, make it clear that the physics of the bit drilling are more complex than previous thought. However, current information does indicate that it is feasible to characterize the frequency spectra of both tri-cone and fixed-cutter bits, although this information is now available on only a few bits and remains proprietary.

The approach of dynamic analysis of BHA behavior which will be adopted here is based on the solving of the governing equation of motion for the system. That is,

\[
[M] \{\ddot{x}\} + [C] \{\dot{x}\} + [K] \{x\} = \{F\} \cos(\omega t) \tag{3.1}
\]

where \{F\} is the excitation, \{x\} is the displacement, \{\dot{x}\} is the velocity, and \{\ddot{x}\} is the acceleration of an appropriate nodal vector of a discrete model of the BHA. The stiffness, damping, and mass of the BHA system are represented by \{K\}, \{C\}, and \{M\}, respectively. The cosine term involves time, t, and the forcing frequency, \omega, associated with conditions of steady-state vibration.

This chapter discusses the development of the stiffness, mass, and damping matrices of a finite element model of the BHA and its excitation. Application of the model will be made versatile through special numerical techniques described in the next
chapter. The following sections present the fundamental considerations involved in the
dynamic BHA characterization.

3.1 Stiffness Properties and Axial Load Influence

Euler-Bernoulli beam-column theory is used to develop the finite-element model
of the stiffness of the BHA components. The Euler-Bernoulli beam theory assumes
linear, small-displacements where plane sections remain plane during deformation and
neglects the shear contributions included in the Timoshenko beam theory. Using
standard finite-element methods, derivations of the stiffness matrix can be found by
Przemieniecki [128], Rao [129], and others. Linear shape functions are used for
torsional and axial strain leading to constant axial load and torque within each element.
For lateral deformations, cubic shape functions are used which provide exact solution
to the Euler-Bernoulli beam under the four boundary conditions (two at each end).
The element stiffness matrix is thus 12 x 12 in size resulting from the two nodes per
element and 6 degrees of freedom per node. Figure 3.1.1 shows the Euler-Bernoulli
linear beam element.

Shear deformations can be included in the formulation and lead to the following
correction term on all lateral displacement or rotation terms in the elemental stiffness
matrix

\[
1 / (1 + \Phi )
\]

(3.2)

where \( \Phi = (12 EI) / (G A_s L^2) = 24 (1 + v) (A / A_s) (r / L)^2 \), and

(3.3)
Figure 3.1.1
Three-Dimensional Euler-Bernoulli Linear Beam Element
(from Przemieniecki 1968 [128])
E is the elastic modulus (psi), I is the bending rigidity (in$^4$), G is the shear modulus, or $E / 2(1 + \nu)(\text{psi})$, $A_{\text{b}}$ is the effective cross-sectional area for shear loads (in$^2$), L is the length of the beam (in), and r the radius of gyration, $\frac{1}{\sqrt{2}} \left[ \text{OD}^2 + \text{ID}^2 \right]^{1/2}$.

For a 8" OD by 3" ID drillcollar, Eq. 3.3 reduces to the approximate result:

$$\phi \approx \frac{1}{L^2}, \text{ where } L \text{ is the beam length expressed in feet.}$$  \hspace{1cm} (3.4)

Verifying negligible shear contribution thus requires a guideline for spacing the beam elements in the BHA model. This will be addressed later in the sensitivity studies.

Having been used by Birades [37], J.A.R. [82], and Enertech [110], this element formulation has been verified repeatedly. Recalling the qualification efforts of Sandia National Laboratories which compared experimentally measured mode shapes against Geodyne predictions [85], it is clear that the formulation can accurately model BHA behavior subject to certain conditions. Ho [26] established that linear beam theory was accurate for wellbore curvatures from 10° to 15° per 100'. Many problems of interest conform with these limits.

The structural stiffness matrix is modified to account for the influence of a constant axial force on lateral beam vibrations as derived by Przemieniecki [128]. As shown in Figure 3.1.2, tensile axial loads increase the lateral bending frequency, while compressive axial loads decrease the lateral bending frequency. In addition, when compression reaches a critical value, solution of the eigenproblem will lead to zero or negative eigenvalues. In this case, the eigenvalues and eigenvectors are meaningless as this indicates that the system is buckled in the static configuration. This relation will
Figure 3.1.2
Influence of Axial Load on Lateral Vibrations
(from Przemieniecki 1968 [128])
be used as a check on the integrity of the posed BHA problem through the use of the eigensolver early in the computational process.

Figure 3.1.3 shows the final form of the elemental stiffness matrix with the axial load effect included. Inclusion of the shear contribution term can be added to the formulation, but first the need for it will be studied. The terms in the stiffness formulation may use any consistent sets of units and are defined as follows: Elastic Modulus (E), cross-sectional area (A), length (L), axial load (P), bending moments of inertia about two planes (I₁ and I₂), external loads (Sᵢ), and respective displacements and rotations (Uᵢ), as shown in Figure 3.1.1.

The mechanical properties of BHA materials must be considered. Table 3.1 shows material property guidelines for BHA including density, elastic modulus, Poisson’s ratio, minimum yield strength, minimum tensile strength, and endurance limit. Drillpipe is predominantly used in grades with yield strengths from 75,000 to 135,000 psi, with endurance limits ranging from 17,400 psi to 28,000 psi. Aluminum drillpipe is sometimes used in severe directional wells to reduce weight and torque/drag loads. Aluminum has dramatically different mechanical properties than carbon steels, but it is never used in BHAs. In most cases of directional drilling, one or more drillcollars are of non-magnetic material to allow the use of magnetic surveying instruments without interference from the steel BHA. For this purpose, beryllium-copper alloys, referred to as "Monel", are used. With an elastic modulus of 18,900,000 psi and a density of 0.302 lbs/in³, its properties vary significantly from steel and must be accounted for. More recent metallurgical work has led to non-magnetic drillcollars using magnesium and chrome alloying techniques with normal steels. Their properties are much closer to
**Figure 3.1.3**

Element Stiffness Matrix with Axial Load Effect

(after Przemieniecki 1968 [128])

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<th>0</th>
<th>0</th>
<th>0</th>
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<td>0</td>
<td>-12EI/L + P/10</td>
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<td>0</td>
<td>0</td>
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<td>0.3</td>
<td>75,000</td>
<td>100,000</td>
<td>17,400</td>
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<td>X-95</td>
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<td>0.3</td>
<td>95,000</td>
<td>105,000</td>
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</tr>
<tr>
<td></td>
<td>G-105</td>
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<td>30,000</td>
<td>0.3</td>
<td>105,000</td>
<td>115,000</td>
<td>22,000</td>
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<td>S-135</td>
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<td>135,000</td>
<td>145,000</td>
<td>28,000</td>
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<td>10,600</td>
<td>0.3</td>
<td>58,000</td>
<td>63,800</td>
<td>12,200</td>
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<tr>
<td>DC (&lt;7&quot;)</td>
<td>4140</td>
<td>0.283</td>
<td>30,000</td>
<td>0.3</td>
<td>110,000</td>
<td>120,000</td>
<td>22,900</td>
</tr>
<tr>
<td>DC (7-11&quot;)</td>
<td>4140</td>
<td>0.283</td>
<td>30,000</td>
<td>0.3</td>
<td>100,000</td>
<td>110,000</td>
<td>20,900</td>
</tr>
<tr>
<td>NMDC (&lt;7&quot;)</td>
<td>BeCu</td>
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<td>18,900</td>
<td>0.3</td>
<td>110,000</td>
<td>140,000</td>
<td>25,000</td>
</tr>
<tr>
<td>NMDC (7-11&quot;)</td>
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<td>18,900</td>
<td>0.3</td>
<td>100,000</td>
<td>135,000</td>
<td>23,500</td>
</tr>
<tr>
<td>NMDC (&gt;11&quot;)</td>
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<td>18,900</td>
<td>0.3</td>
<td>90,000</td>
<td>120,000</td>
<td>23,000</td>
</tr>
<tr>
<td>NMDC (&lt;7&quot;)</td>
<td>MgCr</td>
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<td>0.3</td>
<td>110,000</td>
<td>120,000</td>
<td>22,900</td>
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<tr>
<td>NMDC (7-11&quot;)</td>
<td>MgCr</td>
<td>0.282</td>
<td>28,700</td>
<td>0.3</td>
<td>100,000</td>
<td>110,000</td>
<td>20,900</td>
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<tr>
<td>NMDC (&gt;11&quot;)</td>
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<td>Various</td>
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<tr>
<td>Motors</td>
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<td>0.3</td>
<td>Various</td>
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</tr>
</tbody>
</table>
those of nominal steel, but still vary slightly. Yield strengths and endurance limits vary for drillcollars depending on size with smaller drillcollars requiring higher strength properties for reliable performance. As shown in Table 3.1, endurance limits for drillcollars vary from 19,000 psi to 25,000 psi.

Additional issues involve proper modeling of special BHA components such as shock tools, MWD tools, mud motors, turbines, and drilling jars. Each of these components involves complex geometry with multiple structural housings, internal threaded connections, and other special mechanisms. Detailed finite-element modeling and/or physical testing is required to quantify the effective behavior of these components under tension, compression, torsion, and bending, in several planes. Rules of thumb are used in the industry of assigning 50-60% of the rigidity of a solid drillcollar to these special components. It is not clear that the basis for these guidelines is reliable. With shock-tools, whose purpose is to provide axial vibration absorption, an axial spring constant is generally available from the manufacturer. However, in view of the work by Rogaland Research showing nonlinear shock-tool behavior [56], even these properties must be used with caution. Additional work regarding the properties of special BHA components is needed.

3.2 Mass Properties and Fluid Added Mass Effects

A consistent mass matrix is used as the basis for the inertial beam description. Derivations can be found in references such as Przemieniecki [128] and Rao [129]. Modifications are made to the matrix terms involving lateral displacements and rotations to compensate for the added mass of the fluid inside and outside the drillstring component.
The added mass relations developed by Chen, Wambsganss, and Jendrzejczyk [130] are used. Chen, et. al. analytically and experimentally studied the behavior of a cylindrical rod vibrating in a viscous fluid within an outer cylinder. Theoretical expressions for added mass and damping coefficients were derived which compared well with test results. The experiments involved a 1/2" OD aluminum rod fixed at the end to behave as a cantilever inside a 2-1/2" ID cylindrical shell. Annular clearance was modified by inserting brass liners of varying thicknesses inside the outer shell. The test fixture was originally 28" long, but was cut to 14" to perform the test with the fluids. Fluids tested were water, mineral oil, and silicone oil. The rod was excited by an electromagnetic exciter. The rod displacement was measured using an electro-optical tracker adjusted to track a painted white/black interface on the end of the rod. For the theoretical work, Chen, et. al. assumed an infinitely long cylinder subject to small displacement harmonic motion. The linearized equations of motion for the fluid were solved and the following function, $H$, was used to define resulting added mass and damping terms:

$$ H = \left( \frac{A}{B} \right) - 1, \text{ where} $$

$$ A = 2 \alpha^2 \left[ I_0(\alpha) K_0(\beta) - I_0(\beta) K_0(\alpha) \right] - 4\alpha \left[ I_1(\alpha) K_0(\beta) + I_0(\beta) K_1(\alpha) \right] + 4\alpha\gamma \left[ I_0(\alpha) K_1(\beta) + I_1(\beta) K_0(\alpha) \right] - 8\gamma \left[ I_1(\alpha) K_1(\beta) - I_1(\beta) K_1(\alpha) \right], \text{ and} $$

$$ B = \alpha^2 (1 - \gamma^2) \left[ I_0(\alpha) K_0(\beta) - I_0(\beta) K_0(\alpha) \right] + 2\alpha\gamma \left[ I_0(\alpha) K_1(\beta) - I_1(\beta) K_0(\alpha) + I_1(\beta) K_0(\alpha) - I_0(\alpha) K_1(\beta) \right] + 2\alpha^2 \left[ I_0(\beta) K_1(\alpha) - I_0(\alpha) K_1(\beta) \right] + 2\alpha\gamma^2 \left[ I_0(\beta) K_1(\alpha) - I_0(\alpha) K_1(\beta) \right] \left[ I_1(\alpha) K_0(\beta) - I_1(\beta) K_0(\alpha) \right] $$
where \( I_0, I_1, K_0, \) and \( K_1 \) are the subject Bessel functions, and \( \alpha = kd, \beta = kD, \) and \( \gamma = d/D \)
where \( d=OD \) of the vibrating rod, \( D=ID \) of the constraining cylinder, and \( k = \sqrt{\frac{i \alpha \sqrt{\nu}}{v}} \),
with \( \omega \) the frequency of vibration and \( \nu \) the kinematic viscosity.

Special cases of the function \( H \) can be derived for \( \nu = 0 \) and for \( D/d = \infty \), but
these are of no interest here. One special form of \( H \) can be found for \( \alpha \) and \( \beta \) large (\( \geq 10 \) according to Shyu [122]),

\[
H = \frac{[\alpha^2(1+\gamma^2)-8\gamma] \sinh(\beta-\alpha)+2\alpha(2-\gamma+\gamma^2) \cosh(\beta-\alpha)-2\gamma^2[\alpha\beta]^{1/2}-2\alpha[\alpha/\beta]^{1/2}}{\alpha^2(1-\gamma^2) \sinh(\beta-\alpha)-2\alpha \gamma(1+\gamma) \cosh(\beta-\alpha)+2\gamma^2[\alpha\beta]^{1/2}+2\alpha[\alpha/\beta]^{1/2}}
\]  
(3.6)

Through the function \( H \), Chen, et.al. then provide the following added mass
coefficient and damping coefficient

\[
C_M = \text{Re}(H) \quad \text{and} \quad C_V = -M \omega \text{Im}(H), \quad \text{where}
\]  
(3.7), (3.8)

\( M \) is the mass per length of fluid displaced (\( M=\rho \pi d^2 \)).  
(3.9)

The behavior of the function \( H \) is shown in Figure 3.2.1 as a function of \( D/d \)
and the parameter \( S = \omega d^2 / \nu \). It is seen from these figures that both coefficients
increase as the hole clearance and the frequency become small. For increasing frequency,
\( \omega \), (increasing \( S \)), both coefficients decrease. In the limiting case of \( D/d = \infty \) and \( \nu = 0 \),
\( H \) becomes one (1.0), the value used for riser analyses and similar studies of vibrations
in infinite fluid reservoirs.

Although questions can be raised concerning the applicability of Chen, et. al.'s
results to BHA dynamics in view of the fluid tested, small displacement assumptions,
and flow regimes, it does provide an approach for estimating the influence of added
Proposed Added Mass and Damping Functions
(from Chen, et al. 1976 [130])

Figure 3.2.1
mass and allows the problem to bound between their value of $C_M$ as an upper bound and a lower bound such as $C_M$ of 1.

With regards to modification of the mass matrix to account for added mass, all terms associated with lateral displacements and rotations are modified. Axial and torsional inertial terms are not modified. To account for the fluid added mass, the mass of the drill collar, $M$, is replaced with the following composite mass for involved lateral terms,

$$M_i = M + M_{Mi} + C_M M_{Mo}, \tag{3.10}$$

where $M_{Mi}$ is the mass of mud inside the drillcollar, $M_{Mo}$ is the mass of mud displaced by the OD of the drillcollar, and $C_M$ is the added mass coefficient being considered.

Figure 3.2.2 shows the resultant elemental mass matrix with fluid added mass terms added into lateral degrees of freedom. Like the element stiffness matrix, terms used in the mass matrix may use any consistent sets of units and are defined as follows: mass of the drillcollar ($M$), total lateral mass of the drillcollar per Eq. 3.10 ($M_i$), mass density of drillcollar material ($d$), bending, length ($L$), bending moments of inertia about two planes ($I_1$ and $I_2$), external loads ($S_i$), and respective displacement and rotatory accelerations ($U_i$), as shown in Figure 3.1.1.

In calculations by Allen [117], added mass, without $C_M$ included, affected natural frequencies by about 10%, a significant amount. To appreciate the relative magnitude of the added mass effect, one can consider an 8" OD by 3" ID drillcollar operating inside a 12-1/4" hole with 8.9 lb/gallon mud, a light drilling fluid case. For a 30’ long drillcollar, the drillcollar weighs about 4,400 lbs. The mud inside the collar
Figure 3.2.2
Element Mass Matrix Modified for Lateral Fluid Added Mass

<table>
<thead>
<tr>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>S9</th>
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<tr>
<td>M3</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>M6</td>
<td>0</td>
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<td>0</td>
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<tr>
<td>0</td>
<td>13M₁/35 + 6d₁ / SL</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>11LM₁/210 + d₁ / 10</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>-13M₁,L/420 + d₁ / 10</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-13M₁/35 + 6d₁ / SL</td>
<td>0</td>
<td>-11LM₁/210 + d₁ / 10</td>
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<td>0</td>
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<td>-11LM₁/210 + d₁ / 10</td>
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<td>-M₈,L²/140 + d₁ / 10</td>
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<td>0</td>
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<td>M₈</td>
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adds only about 100 lbs., 2.3% of the drillcollar weight. The mud displaced by the 8" drillcollar, however, adds about 700 lbs. to the mass, 16% of the weight of the drillcollar. Increased $C_M$ above 1.0 due to the confinement effects identified by Chen, et.al., cause this ratio to increase proportionately. With the density of steel being 65.5 lbs/gallon (in fluid units), higher mud weights in the range of 15 - 20 lbs/gallon also amplify the importance of this effect. Clearly, added mass must be taken seriously for accurate modeling of BHA dynamics and additional research appears justified.

3.3 Damping Properties of an Operating BHA

Frictional losses and energy dissipation through damping mechanisms are a critical aspect of dynamic modeling. Although damping is not always thoroughly understood, damping must be included in order to predict responses near resonances. This key role of damping will be reinforced later in the sensitivity studies.

Apostal, Haduch, and Williams [124] discuss a very generalized approach to BHA damping allowing for the following damping forms

Rayleigh Damping,

$$[C_R] = \alpha_R [M] + \beta_R [K]$$  \hspace{1cm} (3.11)

where $\alpha_R$ and $\beta_R$ satisfy $\zeta_R = \alpha_R / 2\omega + \beta_R \omega / 2$

Structural Damping,

$$[C_S] = 2\zeta_S / \omega [K], \text{ and}$$  \hspace{1cm} (3.12)

Viscous Damping,

$$[C_V] \text{ as defined by Chen.}$$

It is not clear, however, how these damping forms were implemented nor that adequate information exists to do so.
Rayleigh damping suffers from the requirement that critical damping be known. For the multi-degree-of-freedom BHA problem, this requires modal analysis. Although structural damping magnitudes of 1-2% typically represent the "internal damping" of steel structures, this is the negligible portion of the overall damping imposed on a vibrating BHA. Viscous damping models such as Chen's may be used for BHA models, but the variation between the basis for Chen's derivation and actual BHA dynamics may cause problems. For example, Chen's experiments involved small oscillations without cylinder (pipe) rotation or fluid flow. In pursuit of this work, additional alternatives to Chen's damping model were sought.

In May 1990, ARCO Oil and Gas Company conducted experimental research on drillstring damping. As shown in Figure 3.3.1, 32 joints of 3-1/2" drillpipe were run into a test well and suspended in a workover rig. The well was alternately filled with varying weights of drilling fluids. At the end of the drillstring, an accelerometer package was attached involving one axial accelerometer and two torsional accelerometers. A wire cable ran from the accelerometer package back to a surface instrumentation system. An impact hammer with a force transducer was used to initiate a wave signal at the top of the drillstring. The transducer on the hammer allowed synchronization of data collection with the generation of the signal. Time domain data were converted to the frequency domain and expressed in terms of attenuation (db/1000") versus frequency as shown in Figure 3.3.2 for 9.9 lb/gallon mud. For a comparison, Figure 3.3.3 shows the ARCO data for 8.6 and 9.9 lb/gallon fluids with the 1965 data from Angona of Socony Mobil [47] for a 9.5 lb/gallon data. Other parameters relevant to the comparison include 3-1/2" drillpipe inside a 7.8" hole with the ARCO
Figure 3.3.1
ARCO Drillstring Damping Measurement

32 joints (1280’) of 3-1/2” Drillpipe
Data cable strapped to pipe.

Accelerometer Package
One Axial Accelerometer
Two Torsional Accelerometers

Wellbore Fluids:
8.3# Water
8.6# Gel
9.9# Mud
11.8# Mud
12.5# Mud
Figure 3.3.2
Application of Attenuation Model to Drillstring Damping Data

Axial Vibration Signal Attenuation

Decibels / 1000'

Q = Resonance Amplification or "Quality" Factor
*= Raw Damping Data

Q = 100

Wave Frequency (Hz)
Figure 3.3.3
Comparison of 1990 ARCO Data to 1965 Mobil Data

- 1990 ARCO Data; 9.9# Mud
- 1990 ARCO Data; 8.6# Mud
- 1965 Mobil Data; 9.5# Mud
data, while the 1965 data involved 4-1/2" drillpipe inside a 8-3/4" hole. Although these data clearly do not agree exactly, qualitative trends with frequency are similar.

For interpretation of the ARCO data, Paslay [131] argues that Kolsky's attenuation model [132] for sinusoidal waves traveling in elastic rods subject to linear viscous damping should be used

\[
\text{Attenuation} = e^{-\kappa x}, \text{ where}
\]

\[
\kappa = \pi f / (cQ), \text{ with}
\]

\(x\) the distance traveled by the wave, \(f\) the wave frequency (Hz), \(c\) the wave speed (ft/sec), and \(Q\) the resonance amplification factor. Using this approach, the functions for constant \(Q\) were plotted by Paslay on the previous figures.

In the current work, the damping measurements were studied further. Note that \(Q\) is related to the damping ratio, \(\zeta\), through the relation

\[
Q = 1 / (2\zeta)
\]

Values for \(Q\) versus frequency were gathered for all fluid weights tested (8.3, 8.6, 9.9, 11.8, and 12.5 lb./gallon) and \(\zeta\) were calculated. Each data set of frequency and \(\zeta\) was then statistically fit with a smooth function. Several equation forms were evaluated, and a power fit was found to provide the closest fit

\[
\zeta = af^b
\]

As shown in Figures 3.3.4 through 3.3.8, regression coefficients varied from 0.92 to 0.96 for the five data sets. Next, the relationship between the five sets of the coefficients, \(a\) and \(b\), and fluid weight was examined. As shown in Figure 3.3.9, a power fit was again found most accurate (\(R^2=0.971\)) for the "a" coefficient:

\[
a = (5.23 \times 10^{-9}) \rho_{\text{mud}}^{8.75}
\]
Figure 3.3.4
Statistical Fit of Damping Ratio versus Frequency for 8.3 lb/gallon Drilling Mud

\[ y = 0.42831 \times x^{-0.84308} \quad R^2 = 0.924 \]
Figure 3.3.5
Statistical Fit of Damping Ratio versus Frequency for 8.6 lb/gallon Drilling Mud

\[ y = 1.1948 \times x^{-0.95438} \quad R^2 = 0.953 \]
Figure 3.3.6
Statistical Fit of Damping Ratio versus Frequency for 9.9 lb/gallon Drilling Mud

\[
y = 2.2198 \times x^{-1.0361} \quad R^2 = 0.961
\]
Figure 3.3.7

Statistical Fit of Damping Ratio versus Frequency for 11.8 lb/gallon Drilling Mud

\[ y = 13.765 \times x^{-1.3254} \quad R^2 = 0.956 \]

Damping Ratio, $\zeta$

Frequency (Hz)
Figure 3.3.8
Statistical Fit of Damping Ratio versus Frequency for 12.5 lb/gallon Drilling Mud

\[
y = 19.806 \times x^{-1.3757} \quad R^2 = 0.958
\]
Figure 3.3.9
Statistical Fit of Regression Coefficient as a Function of Drilling Mud Weight

\[ y = 5.2278e^{-9} \times x^{8.7484} \quad R^2 = 0.971 \]

Coefficient "a"
while the b coefficient was found to be linear ($R^2=0.979$), as shown in Figure 3.3.10:

$$b = (0.15) - (0.123) \rho_{\text{mud}},$$  \hspace{1cm} (3.18)

with $\rho_{\text{mud}}$, the mud weight in lbs./gallon.

With these statistical fits, the damping ratio $\zeta$ is defined as a function of frequency and mud weight. Figure 3.3.11 shows the damping data used in the regression, while Figure 3.3.12 shows the smooth function, $\zeta(\rho_{\text{mud}},f)$ described above.

To highlight the uncertainty present in the damping models, comparisons between the proposed $\zeta(\rho_{\text{mud}},f)$ function and the Chen model can be made. As part of their analysis, the authors studied response magnification near resonances by slowing varying excitation frequency in either an increasing or decreasing fashion across the location of natural frequency. Damping ratios can then estimated according to the "bandwidth method" by the following equations:

$$\zeta_n = \left( \frac{1}{(2 [N^2 - 1]^{1/2})} \right) \Delta f_N / f_n$$  \hspace{1cm} (3.19)

where $\Delta f_N = f_N^1 - f_N^2$  \hspace{1cm} (3.20)

The meaning of these terms relate to the magnification factor at resonance as shown on Figure 3.3.13. Using this technique, the damping ratio curves shown in Figure 3.3.14 were obtained. One motivation for comparing the $\zeta(\rho_{\text{mud}},f)$ function to the Chen functions is the dependence of Chen's damping on the geometric parameter, $D/d$, which is lacking in the $\zeta(\rho_{\text{mud}},f)$ function. Two comparisons are attempted focusing on this effect using the $D/d$ ratio present in the ARCO experiments of 2.23 (7.8" casing ID divided by 3-1/2" drillpipe). The first involves the generation of
Figure 3.3.10
Statistical Fit of Regression Coefficient as a Function of Drilling Mud Weight

Coefficient "b"

\[ y = 0.14948 - 0.12293x \]

\[ R^2 = 0.979 \]
Figure 3.3.11
Damping Ratio Data as Function of Drilling Fluid Weight and Frequency
Figure 3.3.12
Smooth Function for Damping Ratio as a Function of Drilling Fluid Weight and Frequency

Damping Ratio $\zeta$

Mud Weight (lb/gal)

Frequency (Hz)
Figure 3.3.13
Acquisition of Damping Ratio by the Bandwidth Method

System Response
(Displacement, etc.)

\[ f_N^1 \quad f_n \quad f_N^2 \]

Frequency

Response Ratio of \( N \)

\[ \Delta f_N \]
Figure 3.3.14
Damping Ratio as a Function of Geometry for Several Fluids
(from Chen, et.al. 1976 [130])
\( \zeta(\rho_{\text{mud}} f) \) values for Chen's conditions. This comparison is shown in Figure 3.3.15.

The correlation between Chen's damping and the \( \zeta(\rho_{\text{mud}} f) \) function is obviously inadequate. The primary cause for this is that the \( \zeta(\rho_{\text{mud}} f) \) function accounts for viscosity statistically in the density term while Chen's data includes fluids that have extreme variations in viscosity. The silicone oil used by Chen had a viscosity of 0.145 Pa-seconds, while the mineral oil was 0.041 and the water 0.001. Although the mineral oil comes reasonably close to the \( \zeta(\rho_{\text{mud}} f) \) function, the silicone oil damping is severely underestimated by \( \zeta(\rho_{\text{mud}} f) \), while the water damping is overestimated. A further comparison is made using Chen's Test 1 and Test 4 which were both conducted with water, but at two different frequencies, at 16.04 Hz and 58.38 Hz. As shown in Figure 3.3.16, the Chen damping decreases by a factor of 2 due to the frequency increase while the \( \zeta(\rho_{\text{mud}} f) \) function decreases by a factor of 3. Thus, there is at least qualitative agreement between the two models in terms of frequency dependence, although there are obvious discrepancies between the models.

For oilfield use, it is desirable that the damping function directly relate to fluid weight. This is because water-based drilling fluids treated with clay additives prevail in the industry and reliable fluid viscosity information is frequently not available. Another complication is the variation that drilling fluids exhibit away from Newtonian behavior. As shown on Figure 3.3.17, drilling fluids are best modeled by either the Bingham plastic fluid model or the power-law model [133]. Developing relations between these fluid models and Newtonian fluids for relevant operating parameters, resolving the variations of the Chen results against the ARCO or 1965 Mobil results,
Figure 3.3.15
Comparison of Chen Damping versus Damping Function

Damping Ratio %

Fluid Weight (lb/gallon)

- Mineral Oil
- Silicone Oil
- Water

Chen

ζ Function
Figure 3.3.16
Comparison of Chen Damping versus $\zeta$ Damping Function

The fluid is water for both cases.

Damping Ratio %

Reduced by 3

$\zeta$ Function

Reduced by 2

Chen

Frequency (Hz)
Figure 3.3.17

Variation of Drilling Fluid Properties from Newtonian Fluids
(after Bourgoyne, et.al. 1986 [133])

Newtonian Fluid  Bingham Plastic Fluid

Pseudoplastic Power-Law Fluid  Dilatant Power-Law Fluid
and addressing the practical requirement of having damping based on drilling mud density are beyond the scope of this study and not justified for its purpose. For illustrative calculation of the effect of damping, the generated $\zeta(\rho_{mud}, f)$ function or constant damping constants will be used.

These comparisons and limitations highlight, however, the need for a better understanding of drillstring damping in operating conditions. Although damping does not substantially affect the location of natural frequencies, it is of major concern when calibration attempts between models and field data are made, since damping controls response magnitude at resonances.

3.4 Forcing Excitations and System Boundary Conditions

Many excitation mechanisms must be considered in analyzing dynamic BHA behavior. Besaisow and Payne [55] provide a summary of these mechanisms as shown earlier in Figure 2.4.9. In that study, the mechanisms reviewed included mass imbalance or bent pipe, misalignment, tri-cone bit 3X excitation, rotation walk, drillstring precession, and whirl mechanisms. Primary and secondary excitation frequencies and modes were provided for these mechanisms.

Current information has identified additional excitation sources. The Ana-drill/Schlumberger data by Cooper [70] indicates bit tooth forces can be significant and occur at frequencies associated with the number of teeth in that row on the bit cone such as the 670 lb. 19X outer row spectrum and the 340 lb. 9X inner row spectrum. The cone of the bit has its own frequency relating to its diameter and its contact point with the formation. Stabilizers are believed to generate excitations related to the number of blades, such as 4X RPM, as the blades impact or drag during rotation.
Similarly, it has been proposed that PDC bits generate excitations according to the number of blades or vanes in their design.

The parametric relation between 1X lateral excitations and 2X axial excitations is now well recognized after being identified by Besaisow and Payne and studied by Dunyevsky [109], Shyu [122], and others. PDC bits are now known to exhibit the whirl phenomenon while drilling which currently cannot be predicted, but appears to exhibit a polyharmonic power spectrum when it does occur according to Brett, Warren, and Behr [72].

Mud motors and turbines are popular drilling tools in many areas for either ROP or directional considerations. Mud motors, or "PDM"s for Positive Displacement Motors, convert mud flow to rotational motion using helicoidal, lobed cavities and operate in the range of 120 to 500 RPM. Turbines convert mud flow to rotational motion using blades oriented in opposite directions and mounted respectively on rotors and stators and operate in the range of 900 to 1300 RPM. The basis of the PDM and the turbine is shown in Figure 3.4.1. A schematic of a complete PDM is shown in Figure 3.4.2, along with alternative lobe configurations. PDM lobe configurations are identified by the number of lobes, followed by the number of cavities. Figure 3.4.3 shows a schematic for a drilling turbine. With the complex cross-sections shown, the difficulty in defining bending rigidities as discussed earlier for these and other special BHA components becomes apparent.

Although a properly designed PDM or turbine drilling assembly will provide the necessary torque and RPM to drive the bit, it is common to rotate the drillpipe at the surface also to avoid stuck-pipe or to temporarily remove the directional tendency of a
Figure 3.4.1
Mechanical Basis of Turbine and Positive Displacement Motors
Figure 3.4.2
Positive Displacement Motor (PDM) Schematic and Lobe Configurations

Power Section

Intermediate Transmission Section

Bearing Section

Top Connection
Bypass valve
Couplings
Intermediate shaft
Rotor/stator

Bearing flow control valve
Radial bearings
Thrust bearings
Output shaft
Bit connection

1:2  3:4  5:6  7:8  9:10
Figure 3.4.3
Drilling Turbine Schematic

A - Bit Connection
B - Lower Radial Bearing
C - Thrust Bearing
D - Upper Radial Bearing
E - Bearing Package Housing
F - Cross-over Sub Drive Shaft Turbine Section
G - Drive Shaft Coupling
H - Housing Sub
I - Lower Radial Bearing
J - Working Turbine Stages
K - Upper Radial Bearing
L - Turbine Housing
steerable BHA. Thus, many PDM or turbine drilling assemblies involve several rotary speeds, the surface rotary, the PDM or turbine rotary, and the combined speed. This is one impact of PDM and turbine drilling on BHA dynamics analysis. In addition, it is believed that both PDMs and turbines will exhibit their own excitation signatures in operation. Little data or analysis are available in this regard. One manufacturer believes that the dominant PDM frequency can be associated directly to the number of lobes, although the nature of the load is not described. Spectral behavior on turbines is not known to exist. As further perspective on the impact of BHA dynamics and their effects, Table 3.2 shows a summary by Beswick of the Camborne School of Mines and Forrest of Drilex (PDM Manufacturer) [134] detailing axial and transverse vibrations as the primary or secondary cause of failure in most PDMs subject to hard drilling.

The excitations covered in the technology review and those introduced here are pictorially summarized in Figures 3.4.4 through 3.4.7 for various drilling scenarios. Figure 3.4.4 shows Case 1 for rotary drilling with a tri-cone bit. One or more drillpipe motion characteristics (walk, precession, or whirl) can be imparted into the drillcollars of the BHA at the drillpipe/drillcollar interface. These drillpipe motions are largely unpredictable, but can be characterized by the walk equations if they occur. Mass imbalance or misalignment can occur at any of the BHA components. Naturally, there is no reason to believe these imbalances are aligned in any way and these are typically not well quantified. Table 3.3 provides some insight into the potential magnitude of mass imbalance forces in an operating BHA. The table shows variations in wall thickness and OD measured with ultrasonic gages and calipers, respectively, on four 11" drillcollars used on a Gulf of Mexico well. Average wall thickness variations
Table 3.2
Destructive Influence of Dynamic Vibrations on PDM Components
(from Beswick, Forrest 1982 [134])

<table>
<thead>
<tr>
<th>Component*</th>
<th>High Axial Vibrations</th>
<th>High Transverse Vibrations</th>
<th>High Sand Content in Mud</th>
<th>Excessive Weight-on-Bit</th>
</tr>
</thead>
<tbody>
<tr>
<td>By-Pass Valve Internals</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotor</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stator Assembly</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Universal Joints</td>
<td>Primary</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thrust Bearing Assembly</td>
<td>Secondary</td>
<td></td>
<td>Tertiary</td>
<td>Primary</td>
</tr>
<tr>
<td>Bleed Valve Assembly</td>
<td>Secondary</td>
<td></td>
<td>Primary</td>
<td></td>
</tr>
<tr>
<td>Output Radial Bearing Assembly</td>
<td></td>
<td></td>
<td>Primary</td>
<td>Secondary</td>
</tr>
<tr>
<td>Center Coupling</td>
<td></td>
<td></td>
<td>Primary</td>
<td></td>
</tr>
<tr>
<td>Bypass Spring</td>
<td></td>
<td></td>
<td>Primary</td>
<td></td>
</tr>
<tr>
<td>By-Pass Valve-Stator Crossover</td>
<td></td>
<td></td>
<td>Primary</td>
<td></td>
</tr>
<tr>
<td>Main shaft-Bearing Crossover</td>
<td></td>
<td></td>
<td>Primary</td>
<td></td>
</tr>
<tr>
<td>Lower Shaft Coupling</td>
<td></td>
<td></td>
<td>Primary</td>
<td></td>
</tr>
<tr>
<td>Upper Shaft Coupling</td>
<td></td>
<td></td>
<td>Primary</td>
<td></td>
</tr>
<tr>
<td>Bearing Casing</td>
<td></td>
<td></td>
<td>Primary</td>
<td>Secondary</td>
</tr>
<tr>
<td>Inlet- Bypass Crossover</td>
<td></td>
<td></td>
<td>Primary</td>
<td></td>
</tr>
</tbody>
</table>

*Refer to Figure 3.4.2 for general PDM configuration
Figure 3.4.4
Dynamic BHA Excitation Summary
Case 1 - Surface Rotary Drilling with Tri-Cone Bit

Drillpipe Rotational Walk
D / (D-d) RPM
Drillstring Precession
d / (D-d) RPM
Drillstring Whip/Whirl
WX (W-Whirl Velocity)

Parametric Relation for all Axial and Lateral Modes
(1X Lateral = 2X Axial)

Surface Rotary Speed, RPM

1X Mass Imbalance
Misalignment in any Joint (Lateral)

Stabilizer Blade Impact (Lateral)
and Hanging (Torsional)
BX (B - No. Blades)

Bit Forces: Axial, Torsional, Lateral
3X
N1X (N1 Teeth on Row 1)
N2X
N3X
CX (C - Cone Rotary Speed)

Axial Harmonic
at Mud Pump Frequency
Figure 3.4.5
Dynamic BHA Excitation Summary
Case 2 - Surface Rotary Drilling with PDC or Fixed Cutter Bit

- Drillpipe Rotational Walk
  \( D / (D-d) \) RPM
- Drillstring Precession
  \( d / (D-d) \) RPM
- Drillstring Whip/Whirl
  \( WX \) (\( W \)-Whirl Velocity)

- Parametric Relation for all Axial and Lateral Modes
  (1X Lateral = 2X Axial)

- Surface Rotary Speed, RPM

- 1X Mass Imbalance Misalignment in any Joint (Lateral)

- Stabilizer Blade Impact (Lateral) and Hanging (Torsional)
  \( BX \) (B - No. Blades)

- Bit Forces: Axial, Torsional, Lateral
  1X
  \( VX \) (V-No. Vanes or Blades)
  \( WbX \) (Polyharmonic Bit Whirl)

- Axial Harmonic at Mud Pump Frequency
Figure 3.4.6
Dynamic BHA Excitation Summary
Case 3 - PDM Drilling with Tri-Cone Bit

Drillpipe Rotational Walk
D / (D-d) RPM
Drillstring Precession
d / (D-d) RPM
Drillstring Whip/Whirl
WX (W-Whirl Velocity)

Parametric Relation for all
Axial and Lateral Modes
(1X Lateral = 2X Axial)

Surface Rotary Speed, RPM1

Stabilizer Blade Impact (Lateral)
and Hanging (Torsional)
BX (B - No. Blades)

PDM or Turbine Rotary Speed, RPM2

Total Rotary Speed Below PDM or Turbine
RPM1 + RPM2

Bit Excitations: Axial, Torsional, Lateral
3X
N1X (N1 Teeth on Row 1)
N2X
N3X
CX (C - Cone Rotary Speed)
Figure 3.4.7
Dynamic BHA Excitation Summary
Case 4 - PDM/Turbine Drilling with PDC or Fixed Cutter Bit

Drillpipe Rotational Walk
\frac{D}{(D-d)} \text{ RPM}

Drillstring Precession
\frac{d}{(D-d)} \text{ RPM}

Drillstring Whip/Whirl
WX (W-Whirl Velocity)

Parametric Relation for all Axial and Lateral Modes
(1X Lateral = 2X Axial)

Surface Rotary Speed, RPM1

Stabilizer Blade Impact (Lateral) and Hanging (Torsional)
BX (B - No. Blades)

PDM or Turbine Rotary Speed, RPM2

Total Rotary Speed Below PDM or Turbine RPM1 + RPM2

Bit Excitations: Axial, Torsional, Lateral
1X VX (V-No. Vanes or Blades)
WbX (Polyharmonic Bit Whirl)

1X Mass Imbalance Misalignment in any Joint (Lateral)
PDM Harmonic LX (L - Lobes)

Axial Harmonic at Mud Pump Frequency
### Table 3.3

**Example Dimensional Variances in 11" O.D. Drillcollars**

<table>
<thead>
<tr>
<th>Serial No.</th>
<th>Dimensional Variances*</th>
<th>Position Along Drill Collar</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th>Average</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0'</td>
<td>3'</td>
<td>9'</td>
<td>12'</td>
<td>15'</td>
<td>18'</td>
<td>21'</td>
<td>24'</td>
<td>27'</td>
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<tr>
<td>520028</td>
<td>Wall</td>
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<td>0.064</td>
<td>0.126</td>
<td>0.057</td>
<td>0.043</td>
<td>0.029</td>
<td>0.029</td>
<td>0.054</td>
<td>0.041</td>
<td></td>
<td>0.058</td>
<td>0.126</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>OD</td>
<td>0.016</td>
<td>0.031</td>
<td>0.016</td>
<td>0.000</td>
<td>0.016</td>
<td>0.016</td>
<td>0.000</td>
<td>0.000</td>
<td>0.047</td>
<td></td>
<td>0.016</td>
<td>0.047</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>126606</td>
<td>Wall</td>
<td>0.010</td>
<td>0.054</td>
<td>0.018</td>
<td>0.078</td>
<td>0.029</td>
<td>0.095</td>
<td>0.095</td>
<td>0.095</td>
<td>0.036</td>
<td></td>
<td>0.057</td>
<td>0.095</td>
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<td></td>
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<tr>
<td></td>
<td>OD</td>
<td>0.047</td>
<td>0.031</td>
<td>0.047</td>
<td>0.062</td>
<td>0.078</td>
<td>0.078</td>
<td>0.078</td>
<td>0.047</td>
<td>0.078</td>
<td></td>
<td>0.061</td>
<td>0.078</td>
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<td></td>
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<tr>
<td>226350</td>
<td>Wall</td>
<td>0.068</td>
<td>0.144</td>
<td>0.212</td>
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<td>0.141</td>
<td>0.083</td>
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<tr>
<td></td>
<td>OD</td>
<td>0.016</td>
<td>0.031</td>
<td>0.062</td>
<td>0.047</td>
<td>0.078</td>
<td>0.031</td>
<td>0.031</td>
<td>0.047</td>
<td>0.062</td>
<td></td>
<td>0.045</td>
<td>0.078</td>
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<tr>
<td>229410</td>
<td>Wall</td>
<td>0.063</td>
<td>0.120</td>
<td>0.169</td>
<td>0.191</td>
<td>0.291</td>
<td>0.339</td>
<td>0.384</td>
<td>0.435</td>
<td>0.362</td>
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<td>0.262</td>
<td>0.435</td>
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<tr>
<td></td>
<td>OD</td>
<td>0.047</td>
<td>0.156</td>
<td>0.156</td>
<td>0.188</td>
<td>0.203</td>
<td>0.125</td>
<td>0.203</td>
<td>0.281</td>
<td>0.203</td>
<td></td>
<td>0.174</td>
<td>0.281</td>
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<td></td>
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</table>

*all dimensions in inches

<table>
<thead>
<tr>
<th>Overall Variances</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall</td>
<td>0.136</td>
<td>0.435</td>
</tr>
<tr>
<td>OD</td>
<td>0.074</td>
<td>0.281</td>
</tr>
</tbody>
</table>
are 0.136" with a maximum variation of 0.435". Average OD variations are 0.074" with a maximum variation of 0.281". These measurements indicate that drillcollars may be ovalized in OD shape and further, that the inner bore is not aligned with the center of the oval. These geometric or mass irregularities transform into 1X loadings according to the eccentric rotor equation:

\[ F = e_0 \cdot M \cdot \Omega^2 \]  

(3.21)

where \( e_0 \) is the off-center distance, \( M \) is the mass being rotated, and \( \Omega^2 \) is the square of the rotation velocity (in rad/sec). Thus, this 1X loading becomes more prevalent at higher rotating speeds and requires a weighting which increases with the square of the frequency.

Stabilizer impact and rotational hanging generate lateral and torsional forces with a frequency related to the number of blades. Pressure pulsations at the mud pump frequency transform due to piston effects to axial harmonics at that same frequency. The motions mentioned for the drillpipe (walk, precession or whirl) can also occur in the drillcollar section and at stabilizer locations. Drillcollar walk and precession, however, should be avoidable provided the BHA is adequately stabilized and buckling is not occurring. The two most quantifiable variables are the surface rotary speed and the bit excitations. The bit excitations are relatively well defined for the tri-cone bit since they are a function of the bit geometry. The magnitude of the bit loads needs further study although the Anadrill/Schlumberger data are available. The tri-cone bit excitations are distorted by torsional drillstring resonances causing variations in rotary speed at the bit. These torsional resonances may be at the fundamental pendulum frequency of the drillstring itself, or at the even lower stick/slip frequency if operating
under those conditions. For all excitations listed, parametric relations exist. Due to imperfect straitness, a one-to-one relation exists, between axial and lateral vibrations where axial motions will induce lateral ones. In addition, the 1X lateral to 2X axial nonlinear parametric coupling exists and can cause harmonics to be transformed from either mode to the other. Transient effects caused by drillstring to wellbore contact are intentionally omitted from Figure 3.4.4 since this study will purposely focus on packed BHA where contact is limited to the bit and stabilizer locations. Even with this simplification, Figure 3.4.4 poses a formidable engineering problem.

Figure 3.4.5 provides the same summary for surface rotary drilling with a PDC or other fixed cutter bit. Relative to the tri-cone bit case, the certainty in the loading mechanisms worsens as the geometry controlled excitation characteristics of tri-cone bits are replaced by a 1X excitation, a hypothetical VX excitation related to the number of bit vanes, and a \( W_b X \) bit whirl excitation that can neither be predicted nor quantified at this time.

The surface rotary cases become more complex in Figures 3.4.6 and 3.4.7, where PDM drilling is considered for tri-cone bits and PDM or turbine drilling is considered for PDC bits. In these cases, three rotary speeds must be dealt with. The surface rotary speed ("slow speed") used to keep the drillpipe moving to avoid sticking acts along the entire length of the string. Unfortunately, this slow speed may induce stick/slip harmonics which prevail at lower rotary speeds. At the location of the PDM or turbine, fluid energy is being converted into substantial additional rotary speed. The total rotary speed below this component is the surface slow speed plus the PDM or
turbine speed. Finally, the PDM may exhibit its own harmonics related to the number of lobes in the PDM and its own rotary speed.

Accounting for the impact of all these excitations poses a challenging engineering problem. Analytical solutions are not currently available. Time-domain modeling would be extremely costly computationally and still may not be accurate due to contact modeling and other problems. In view of the complexity of the physical problem and the inability to properly account for key aspects such as ROP modeling and drillpipe contact, this work focuses on using frequency domain analysis of the BHA, which is the portion of the drillstring controlling conditions leading to destructive stresses and failures. This approach allows acquisition of eigensolutions, estimation of response to forcing mechanisms, inclusion of damping, and good computational performance. Overall excitation response is addressed by transforming the frequency response (in terms of Hz) into an operational response (in terms of RPM) using the characteristics of the excitations. Conditions leading to resonant stresses can then be filtered by thorough consideration of all load sources. The next chapter will describe the implementation of this analysis method.
Chapter 4 - Methods of Analysis for Dynamic BHA Behavior

"One pound of learning requires ten pounds of common sense to apply it" - Persian

Upon determining the dynamic model for the BHA, analytical and numerical techniques can be used to solve the governing equations to identify locations of severe vibration. Several numerical approaches exist for this purpose such as eigenvalue analysis, forced-frequency response, and time-domain integration of the equations of motion. Solving the eigenproblem provides useful information regarding the BHA, but this alone is not adequate to guide drilling operations. Time-domain integration is computationally intense and not an effective means of characterizing BHA response over a broad frequency range. For these reasons, monochromatic forced frequency response (MFFR) is proposed as a general analysis technique. Previous efforts with MFFR analysis [110], [115], [117], [124] have not addressed critical factors affecting the BHA dynamics. Several studies involve undamped response models which by their very nature are incapable of predicting the system response at resonances, although this is their fundamental purpose. Previous models have also taken a simplistic view of the boundary conditions of the problem including those at the bit and the wellbore constraint. Further, numerical considerations have not been addressed with inefficient matrix inversion techniques being applied. The following sections discuss some of the complexities encountered in properly addressing the BHA dynamics problem leading to a description of the model developed here.
4.1 Rigorous Solution to the BHA Eigenproblem

Solving an appropriate characteristic eigenproblem provides the natural frequencies and mode shapes of a system. Eigensolutions are the basis of some BHA dynamic models currently in use. More importantly, solution of the eigenproblem provides the foundation of modal superposition techniques, which are a powerful numerical approach, previously unexplored in BHA dynamics. Understanding the special requirements for solving the eigenproblem for BHA dynamics is, thus, essential.

An eigensolver for BHA dynamics can be constructed using established algorithms [134] as shown in Figure 4.1.1. The procedure is initiated by forming the global stiffness and mass matrices. The mass matrix is inverted using an "LU" decomposition algorithm. The system matrix is converted to its Hessenberg form and eigenvalues are solved using the "QR" technique. Eigenvectors are found through the power method.

The problem with such a standard eigensolver for the posed BHA dynamics problem is that the mass matrix, M, is a function of the forcing frequency, \( \omega \), due to the added mass terms. As a result, in order for the eigensolution to represent the natural modes of free vibration, the mass matrix must be reformulated at the predicted natural frequency, the system matrix regenerated, the eigenproblem resolved, and this process must be repeated until convergence. An iterative approach such as shown in Figure 4.1.2 is required. Although the frequency-dependent added mass terms have been noted in previous work by Shyu [122], his eigensolver did not account for this required solution technique and it can be erroneous. Apostol, et. al. [124], incorporated the added mass terms but did not solve the eigenproblem and used matrix inversion for the response calculation.
Figure 4.1.1
Normal Eigenvalue Solution Routine

Assemble Global K

Assemble Global M

Invert M

Form System Matrix A

Balance A

Convert A to Hessenberg Form

Find Eigenvalues by QR Algorithm

Find Eigenvectors by Power Method

End
Figure 4.1.2
Iterative Eigensolver for Frequency Dependent Mass Problem

Assemble Global K

Assemble Global M(\omega)

Invert M

Form System Matrix A

Balance A

Convert A to Hessenberg Form

Find Eigenvalues by QR Algorithm

Find Eigenvectors by Power Method

End

Begin Loop of i+1 Mode

Update old \omega with new \omega for ith mode

Convergence with ith Eigenvalue?

No

Have All Modes Been Found?

Yes

No
An iterative eigensolver was coded as part of this study. However, it was found to be extremely slow computationally. Further, since each eigenvalue and eigenvector represent single mode solutions to distinct dynamic system matrices developed at different frequencies, the resulting eigenvectors do not exhibit orthogonality properties with respect to the mass and stiffness matrices which decouple the governing equations. A single mass matrix is also not available to characterize the inertial system properties since it is formulated as a function of frequency. Thus, alternative means of solving this problem will be studied.

4.2 Direct Matrix Inversion Method

The approach to monochromatic forced frequency response (MFFR) used in most previous studies [115], [117], [118] involves inversion of an undamped system matrix, via

\[ \{ x \} = \left[ [K] - \omega^2 [M] \right]^{-1} \{ F_0 \}, \tag{4.1} \]

where the \( M \) represents the mass matrix including drillcollar and added mass effects.

These formulations did not account for the added mass terms described here which require enhancement of the formulation as follows,

\[ \{ x \} = \left[ [K] - \omega^2 \left[ M_0 + M_1(\omega) \right] \right]^{-1} \{ F_0 \}, \tag{4.2} \]

where \( M_0 \) represents the nominal mass matrix, not frequency dependent, and \( M_1(\omega) \) represents the contributions from the frequency-dependent added mass terms. Still, damping is excluded and the BHA response cannot be calculated at the natural frequencies since the system matrix becomes singular and cannot be inverted.
Including damping in the formulation leads to the following system of equations which must be solved using complex variables

\[
\{ x \} = \left( [K] + i\omega \ [C(\omega)] - \omega^2 \ [M_0 + M_1(\omega)] \right)^{-1} \{ F_0 \}
\] (4.3)

where the damping matrix was formulated using damping functions or other forms such as Rayleigh damping. Although this formulation has been used in a previous study [124] and was studied as part of the current effort, it has several disadvantages. Due to an inability to relate the damping magnitudes to critical damping levels without modal information, difficulties can be encountered with developing practical damping magnitudes. Further, numerical performance of the complex matrix inversion is poor compared to alternative methods. In view of these considerations and the additional benefits that are obtained from an eigensolution, the matrix inversion method is no longer used nor recommended, and a new method has been generated which builds upon the modal data made available from the eigensolution.

4.3 Mode Superposition Method

The solution method proposed for the MFFR BHA problem is the transfer function representation developed by modal superposition techniques. The basis of the mode superposition method is the decoupling of the dynamic equations. In general, since off-diagonal terms in the above formulations such as \( K_{ij} \) or \( M_{ij} \), where \( i \neq j \), are nonzero, the formulations require the solution of \( N \times N \) simultaneous equations via some matrix computation. However due to the orthogonality properties of the eigenvectors with respect to the stiffness and mass matrices,

\[
[\phi_i]^T [K] [\phi_j] = 0 \text{ for } i \neq j, \text{ and}
\] (4.4)
\[ (\phi_i)^T [M] \{ \phi_j \} = 0 \text{ for } i \neq j. \quad (4.5) \]

These relations allow decoupling of the equations of motion whereby each mode can be solved for as a single degree of freedom. The complete system response is then found by summing the responses from each mode across an appropriate number of modes.

Derivations of the mode superposition method can be found by Craig [135] and others.

One assumption of this method is that damping, whether viscous or of other form, is such that the modal equations of motion are uncoupled. In view of the uncertainties associated with the present problem, there is no reason to reject this approach because of this assumption.

MFFR of BHA dynamics by modal superposition has several advantages. First, since the solution requires modal information, a proper eigensolution must be obtained as the first step. This provides information on mode shapes and frequencies as well as verifying that the posed BHA problem is not buckled due to static loads. In optimizing this approach, it is often found that an adequate response can be developed using a subset of the modes present depending on the degree of accuracy required and the frequency range of interest. This will be demonstrated later. Further, the technique can be modified to account for nonlinear and other special effects. This versatility will also be demonstrated.

Once the eigenproblem has been solved, the eigenvectors are used to convert the system stiffness and mass properties into modal quantities through the relations

\[ (\phi)^T [K] (\phi) = K_D \text{ for modal stiffness, and} \quad (4.6) \]

\[ (\phi)^T [M] (\phi) = M_D \text{ for modal mass.} \quad (4.7) \]
Both $K_D$ and $M_D$ are diagonal matrices because of the orthogonality properties of the eigenvectors so that scalar magnitudes for modal stiffness and modal mass are now defined for each vibration mode as well as the eigenvalues, $\lambda$, equal to $\omega_n^2$, where $\omega_n$ is the natural frequency for the mode.

The steady-state response for the system can then be found as follows:

$$ (x(t)) = \sum_{r=1}^{N} \left( \frac{\phi_r \phi_r^T P}{K_r} \right) \left[ \frac{1}{\sqrt{(1 - r_r^2)^2 + (2 \zeta_r r_r)^2}} \right] \cos(\Omega t - \alpha_r) \tag{4.8} $$

where the summation $\sum$ corresponds to the eigenvectors $\phi_r$,

$K_r$ is the modal stiffness,

$r_r$ is the ratio of forcing frequency $\Omega$ to natural frequency of the mode, $\omega_r$,

$\zeta_r$ is the modal damping ratio,

$P$ is the forcing vector, and

$\alpha_r$ is the phase shift.

The mode superposition method provides an elegant solution to the MFFR BHA problem and this equation provides important insight into the dynamic response. For example, the $\phi_r^T P$ term is a measure of the alignment between the mode shape and the excitation force, essentially a modal force. For mode shapes that do not coincide with the force vector in any degree of freedom, this term is zero and the mode does not contribute to the response. For force vectors which do coincide with particular modes, this term provides a weighted force magnitude. A substantial alignment of the
forcing vector with the mode shape generates a dominant response for that particular mode. This is an important concept when analyzing possible BHA excitations.

The $K_r$ term in the denominator converts the modal force into a static deflection for each mode. This static modal deflection is amplified by the dynamic amplification factor within the square brackets. The behavior of the dynamic amplification factor is important. At the natural frequency of a particularly mode, note the term "$1 - r_r^2$" goes to zero and the bracketed term would approach $\infty$, if it were not for the modal damping also appearing in the denominator. For this reason, damping should be incorporated in the MFFR and it is easily done. The cosine term on the right provides the time variance of the response including the effect of phase shift. Here, only the response magnitude is critical so this term is neglected. With the modal force to modal stiffness ratio representing a static modal deflection and the bracketed term representing the dynamic amplification, the last consideration is to multiply the resulting dynamic severity for each mode by the mode shape and sum across a desired number of modes to predict physical displacements. Note that the modal force term, $\phi_r^T P$, represents a vector multiplication yielding a scalar, and the modal stiffness and all terms in the dynamic amplification factor are scalar quantities. The equation thus produces a scalar deflection magnitude for each mode which is used to multiply the mode shape vector yielding a response vector. Because of the simplicity of these operations, MFFR using modal superposition technique is very efficient. The technique can be further accelerated by optimizing the number of modes considered for the response based on the frequency range of interest. Figure 4.3.1 shows the overall program
Figure 4.3.1
Harmonic Response by Modal Representation
of the Transfer Function

Eigenvalue Solver

Form Modal K

Form Modal M

Form Modal Forces

Enter Frequency Loop

Calculate Frequency Ratio and Damping $\zeta$

Find Response using Modes

End after Last Frequency
flow for the application of the modal superposition problem to a BHA dynamics problem.

4.4 Implementation Method for BHA MFFR Using Modal Superposition

Even with the advantages of modal superposition and value of the eigensolution defined, challenges exist in developing an effective implementation method. One option is to use an iterative eigensolution which would yield non-orthogonal eigenvectors and proceed with modal superposition using this unique set of mode information. This option is not selected due to the poor efficiency of iteratively solving the eigenproblem. A second option is to repose the eigenproblem at each frequency and use new modal information at each frequency for the response. This method yields accurate results but requires excessive computation since the repeated eigensolutions to cover a range of frequencies are costly. A unique method has been developed here which properly accounts for the frequency dependent mass terms while optimizing the computational performance.

In order to avoid the computational expense of the above options, a solution technique has been developed where the frequency-dependent added mass effects are accounted for in the forcing term. In this special procedure, nominal modal properties are formed from the stiffness, $K$, and the non-frequency-dependent mass matrix, $M$. The eigenproblem is solved and nominal modal properties, stiffness, mass, and force, are formed. As shown in Figure 4.4.1, an iterative approach is then taken at the modal superposition stage, where the $M(\omega)$ terms are accounted for in terms of an unbalanced force term

$$\mathbf{P}^* = \omega^2 [M(\omega)] \mathbf{x} \quad (4.9)$$
Figure 4.4.1
Harmonic Response with $M(\omega)$ as Force

- Eigenvalue Solver
- Form Modal K
- Form Modal M
- Form Modal Forces
- Enter Frequency Loop
- Calculate Frequency Ratio and Damping $\zeta$
- Find Response using Modes
- Form $M(\omega)$
- Enter Convergence Loop
- Find $M(\omega)$ Added Force
- Find Response using Modes
- Is Response Converged?
  - Yes
  - No
The iterative process involves generating the total equivalent force in the system, summing modal contributions for a response vector, \( \{x'\} \), and then reforming \( P^* \) as described until system displacements converges as follows:

\[
P' = P + P^*
\]

\[
\{x'(t)\} = \sum_{r=1}^{N} \left( \frac{\phi_r \phi_r^T P'}{K_r} \right) \left[ \frac{1}{\sqrt{(1 - \frac{1}{r^2})^2 + (2 \frac{\zeta_r}{r} \frac{\Omega}{r})^2}} \right] \cos(\Omega t - \alpha_r)
\]

\[
P^* = \omega^2 [M(\omega)] \{x'\}
\]

It was found that the procedure would not always converge across the full frequency range of interest. This problem was traced to the significant impact that the added mass terms for fluid outside the BHA had on the system mass matrix and the resulting inaccuracies in the nominal modal information.

To address this issue, the procedure was enhanced to update the modal information during the frequency scan whenever convergence failed. This feature was successfully implemented and led to the observation that once the modal information had been updated to account for the added mass terms, convergence was achieved for the remainder of the frequency band. For example, scanning from 0 to 50 Hz would require a single update at around 4 Hz, but convergence was then obtained for frequencies up to 50 Hz. The procedure thus required an eigensolution at the initiation of the frequency scan, with nominal mass, and a second eigensolution at a later frequency, with a full mass matrix including added mass terms at that frequency.
To avoid solving the eigenproblem twice, the procedure was further refined to initiate the modal information at the median frequency of interest and then solve for response across the entire frequency range using the iterative scheme with the following out of balance force

\[
P^* = \omega^2 \left[ M(\omega) - M(\omega_m) \right] \{x\}
\]  \hspace{1cm} (4.13)

where \(\omega\) is the frequency at which the response is being calculated and \(\omega_m\) is the median frequency of interest at which modal information was developed to initiate the solution. This technique was found to converge well and no further refinements were necessary.

4.5 Approach for Nonlinear Restoring Forces and Special Effects

Nonlinearities including friction and contact exist in BHA dynamics. The modal superposition technique can be extended to handle nonlinear problems efficiently by dealing with these effects as out of balance forces and iterating within the response calculation similar to the above procedure such as shown in Figure 4.5.1.

Nonlinear effects are expressed as forcing terms of the nature

\[
P^* = f(U, U', U'')
\]  \hspace{1cm} (4.14)

Depending on the specific nature of this function, a force vector equivalent to the nonlinear effect would be determined and the iteration process used until convergence was achieved. Although information is available on the nonlinear axial behavior of shock-tools [56] and nonlinear torsional wellbore friction [89,90], quantitative descriptions of nonlinear lateral effects are not available. However, one problem area where the use of the proposed technique can be demonstrated involves wellbore contact.
Figure 4.5.1
Harmonic Response for General Nonlinearity

Eigenvalue Solver

Form Modal K

Form Modal M

Form Modal Forces

Enter Frequency Loop

Calculate Frequency Ratio and Damping

Find Response using Modes

Enter Convergence Loop

Form Nonlinear Forcing Term and Add to Nominal Force

Find Response using Modes

Is Response Converged?

No

Yes
It was determined that previous MFFR solutions do not account for the wellbore constraint and thus allow response predictions which violate fundamental geometric boundary conditions. To address this problem, it is proposed that the BHA excitation force remains constant when contact occurs, but that a restoring force acting on the BHA by the wellbore is generated which restrains the BHA inside the wellbore.

To calculate the restoring force necessary to restore the BHA response back into the wellbore, it is possible to manipulate available modal information to determine this force directly. To simplify terminology, let the following modal weighting factor be defined

\[
\frac{1}{w_r} = \frac{1}{\sqrt{(1 - r_r^2)^2 + (2 \zeta_r r_r)^2}}
\]  

(4.15)

It is then easily shown that the restoring force necessary to correct the response at the jth node to remain within the wellbore is the following

\[
F^* = \left( U_{\phi}^j - R_c \right) / \sum_r \left( w_r (\phi_r^j)^2 / K_r \right)
\]  

(4.16)

where \( U_{\phi}^j \) is the displacement at the jth node violating the wellbore constraint, \( R_c \) is the maximum radial clearance at the jth node, \( \phi_r^j \) is the jth nodal displacement for the rth mode, and the summation occurs with regard to the r modes being considered.

With the restoring force calculated, the corrected system response is then found with a modified form of equation (4.8)

\[
\{x(t)\} = \sum_{r=1}^{N} \left( \frac{\phi_r^j \phi_r^T (P-F^*)}{K_r} \right) \frac{1}{\sqrt{(1 - r_r^2)^2 + (2 \zeta_r r_r)^2}} \cos(\Omega t - \alpha_r)
\]  

(4.17)
The direct manner in which this restoring force can be found demonstrates another advantage of the modal superposition solution approach which allows manipulation of modal information to characterize the stiffness of the system response to reduced displacements at any node. For the general case where the wellbore constraint is violated at a node distinct from the location of the excitation, this procedure constitutes a nonlinear restoring force relative to the original system excitation, and the various nodal responses are affected nonlinearly depending on the character of the modal information and the location of the excitation and node violating the constraint. For the special case where the excitation location and the node violating the wellbore constraint coincide, equation (4.17) acts on the original response calculation, equation (4.8), as a linear reduction, scaling down the excitation force to bring the node being excited back into the wellbore. In this case, the displacements at all nodes in the system are linearly reduced.

Figure 4.5.2 shows a summary of the current model calculation procedures combining the ability to iteratively converge for the solution of the frequency-dependent added mass terms with the contact correction for nodal displacements violating the wellbore constraint. In addition to calculating the allowable radial clearances from the wellbore diameter and the outer dimensions of the various BHA components, the algorithm allows a separate definition of radial clearance at the bit. The importance of this capability will be demonstrated in the following chapter on numerical results. Optimization techniques are also implemented such as initiating the response estimate at a new frequency of interest with the final converged displacements found at the
Figure 4.5.2
BHA MFFR Model with $M(\omega)$ Force and Contact Adjustment
previous frequency analyzed. Computational performance of the various solution
techniques will also be addressed in the next chapter.

4.6 Modal Refinement Procedure

As an alternative to iteratively solving for the influence of the added mass
terms, an approximate solution technique has also been generated in which modal
information is updated to account for the impact of those terms. The basis of the
approach is that the nominal stiffness and mass formulations are used to develop
nominal modal information. Then, at the nominal natural frequencies within the
frequency range of interest, the mass matrix is reformulated including the added mass
terms for the purpose of updating the modal masses using the following relation

$$\{\phi\}^T [ M + M_1(\omega) ] \{\phi\} = M_D^{*} \text{ for modal mass.}$$  \hspace{1cm} (4.18)

Additionally, the natural mode frequencies are also updated using the nominal modal
stiffness and the updated modal mass

$$\lambda^* = \omega^*^2 = K_D / M_D^{*} \text{ for modal mass.}$$  \hspace{1cm} (4.19)

Note that the eigenvectors are not updated in any way.

These updated modal frequencies are then used along with the nominal modal
stiffnesses and eigenvectors to develop the response prediction

$$\{x(t)\} = \sum_{r=1}^{N} \left( \frac{\phi_r \phi_r^T P}{K_r} \right) \left[ \frac{1}{\sqrt{(1 - r^*^2 \xi_r^2)^2 + (2 \xi_r r^* \omega_r^*)^2}} \right] \cos( \Omega t - \alpha_r )$$  \hspace{1cm} (4.20)

where $r^*$ is the ratio of forcing frequency, $\Omega$, to the updated natural modal frequency

$\omega_r^*$. 
Although an approximation, results in the next chapter show that this technique is sufficiently accurate and offers clear computational advantages. The capability of modal updating to significantly capture relevant problem information again demonstrates the versatility of modal solution techniques. Figure 4.6.1 summarizes this modal refinement approach.
Figure 4.6.1  
Modal Refinement Procedure

- Nominal K, M Matrices
- Eigensolver
- Update Modal Mass
- Update Natural Frequency of Mode
- Find Response using Updated Modal Information
- Find Max Wellbore Contact
- Calc. Contact Restoring Force
- Adjust for Wellbore Contact

Loop for each Natural Frequency within Frequency Range of Interest

Frequency Loop

End after Last Frequency
Chapter 5 - Numerical Results and Parametric Sensitivity Analysis

"You must scale the mountains if you would view the plain" - Chinese

This chapter discusses numerical considerations of the various solution approaches and analyzes the sensitivity of the dynamic BHA response to problem variables. Observations from these studies lead to conclusions regarding optimal approaches to BHA problems and identify areas for further research.

5.1 Evaluation of Alternative Eigensolvers

The somewhat specialized eigensolver shown in Figure 4.1.1 was successfully coded. However, it was also desirable to quantify the performance of readily accessible eigensolvers in solving eigenproblems for BHA systems. For this purpose and all subsequent sensitivity and numerical studies, the BHA model shown in Figure 5.1.1 was analyzed. The BHA is made up of 43' of 8" OD by 2-13/16" ID drillcollars and 100' of 6-3/4" OD by 2-13/16" ID drillcollars. Five (5) stabilizers, each 5' in length, are located as shown. The total length of the BHA is 168'. Axial and torsional springs were attached to the top of the BHA representing the stiffness of the drillpipe to surface. However, only lateral modes are analyzed in this study. Boundary conditions are applied at each stabilizer to reflect the restriction of lateral displacements. No lateral constraints are placed at the bit nor at the top of the BHA allowing the model to freely displace and rotate laterally at those locations. To ensure accurate results, element spacing of about 2.5' is used resulting in 69 nodes in the model and a total of 131 degrees of freedom for lateral displacements and rotations. Grid refinement is studied later to provide general guidelines for BHA element spacing. This model is excited by a single 100 lb. force acting in the lateral direction applied at the bit.
Figure 5.1.1
BHA System for Numerical Studies

Total BHA Length - 168'

6-3/4" OD x 2-13/16" ID Drillcollars

Pinned Stabilizer Boundary Conditions

8" OD x 2-13/16" ID Drillcollars

100 lb. Lateral Bit Excitation

Weight-on-Bit
Four (4) EISPACK [137] eigensolvers shown in Figure 5.1.2 were analyzed to quantify their relative performance in providing eigensolutions for BHA problems. The first two algorithms are similar to that shown in Figure 4.1.1 with a balancing routine, conversion to Hessenberg form [137], and solution by the HQR2 method [137], which provides both eigenvalues and eigenvectors. The first routine uses normal similarity transformations to convert the system matrix into Hessenberg form, while the second uses orthogonal similarity transformations. The third and fourth routine also use a balancing routine and solve the eigenvalues by the HQR method. However, eigenvectors are solved using inverse iteration. Again, normal similarity transformations and orthogonal similarity transformation distinguish these latter two approaches.

Results showed that the first two algorithms using HQR2 solution for eigenvalues and eigenvectors were robust. Accuracy was measured by evaluating the percent error between \([A]\{\phi\}\) and \(\lambda(\phi)\), where \([A]\) is the system matrix, \(\{\phi\}\) is the eigenvector, and \(\lambda\) is the eigenvalue. Both HQR2 based algorithms solved for 119 of 131 vibrations modes with no percent error, to the second decimal place, between \([A]\{\phi\}\) and \(\lambda(\phi)\). The highest twelve vibration modes exhibited some errors in this measure, with the orthogonal similarity transformations yielding better results. Orthogonality of the resulting eigenvectors with respect to the mass and stiffness matrices was also evaluated by calculating the largest ratio of nonzero off-diagonal terms in the modal matrices to their corresponding diagonal term. This term was found to be very small of the order of \(10^{-10}\) indicating that the modal matrices were effectively diagonal. A final verification calculation was made comparing \(\lambda\) to the ratio of \(K_D/M_D\). As with the above check, these values were in agreement with the zero error for all but the
Figure 5.1.2
Eigen solvers Investigated
(from EISPACK Numerical Software [137])
highest modes. Due to slightly better accuracy with the algorithm based on orthogonal similarity transformations, it was selected as the eigensolver for this study and is recommended for this class of problem. Both algorithms based on inverse iteration solution for the eigenvectors were found to have convergence problems and are not recommended.

In order to verify the validity of the modal analysis results, the above mentioned quality control checks were implemented following the eigensolution. These are

\[
\begin{align*}
\{ A \} \{ \phi \} &= \lambda \{ \phi \} \text{ for all modes,} \\
M_{ij} / M_{ii} &= 0, (i \neq j) \text{ for the modal mass matrix} \\
K_{ij} / K_{ii} &= 0, (i \neq j) \text{ for the modal stiffness matrix, and} \\
\lambda &= K_D / M_D \text{ for all modes,}
\end{align*}
\]  

Since eigensolvers can be sensitive to particular problems, the analysis algorithm allows the selection of either the EISPACK eigensolver using normal similarity transforms or orthogonal similarity transforms. This capability addresses cases where the above measures are not fully satisfied and allows comparative analysis to be run.

Figures 5.1.3 through 5.1.7 show the first fifteen (15) mode shapes predicted for the BHA being studied which represent vibration frequencies to about 47 Hz. Each mode is shown in terms of lateral displacement versus axial position in inches from the bit. As expected, low frequency modes reflect the simplest possible deformations but the modes quickly become complex with increasing frequency. The importance of understanding modal force concepts becomes apparent during inspection of the higher order modes. For example, 1X eccentricity loadings on the unsupported spans between stabilizers cannot excite the higher order modes due to the complexity of those mode
Figure 5.1.3

Natural Vibration Modes 1-3 for BHA Case Study

$\omega_1 = 1.7$ Hz  $\omega_2 = 4.2$ Hz  $\omega_3 = 5.2$ Hz

Position in BHA from Bit (in)

Lateral Deflection (in)
Figure 5.1.4
Natural Vibration Modes 4-6 for BHA Case Study

- $\omega_4 = 6.8$ Hz
- $\omega_5 = 10.2$ Hz
- $\omega_6 = 15.1$ Hz

Position in BHA from Bit (in)

Lateral Deflection (in)
Figure 5.1.5
Natural Vibration Modes 7-9 for BHA Case Study

Modes
7
8
9

\( \omega_7 = 17.0 \text{ Hz} \)
\( \omega_8 = 19.0 \text{ Hz} \)
\( \omega_9 = 22.4 \text{ Hz} \)
Figure 5.1.6
Natural Vibration Modes 10-12 for BHA Case Study

$\omega_{10} = 25.8$ Hz
$\omega_{11} = 33.3$ Hz
$\omega_{12} = 36.5$ Hz

Position in BHA from Bit (in)

Lateral Deflection (in)
shapes and the improbability that the mode shape and load vector would align in an appreciable way. Thus, the mode shape information is useful in evaluating likely excitation sources as well as identifying the deformation shape and location of largest displacement where contact and wear would be expected.

5.2 Comparison of Solution Procedures and Computational Efficiency

Due to the large amount of data generated by the solution algorithms, it is essential to develop an effective means of representing the predicted response behavior. For example, Figures 5.2.1 and 5.2.2 show the predicted BHA response for lateral rotation and lateral displacement versus excitation frequency as a function of position along the BHA. Direct modal superposition without iterative procedures was used to obtain the results in these two plots. Figure 5.2.3 shows a plot of equivalent von Mises stress due to the dynamic lateral deformations. Equivalent von Mises stress, \( \sigma_{\text{vme}} \) (psi), is defined according to standard elasticity conventions [138]:

\[
\sigma_{\text{vme}}^2 = (\sigma_{\text{axial}} + \sigma_{\text{bndg}})^2 + 3(\tau_{\text{tors}})^2
\]

(5.5)

where \( \sigma_{\text{axial}} \) is the nominal axial stress (psi), \( \sigma_{\text{bndg}} \) is axial stress from bending (psi), and \( \tau_{\text{tors}} \) is the torsional shear stress (psi). Stress recovery calculations are based on standard routines acquired from previous models [118].

By using equivalent triaxial stress as the measure of response severity, the results can be presented in terms of a single parameter although it is still challenging to deduce critical information from the 3-D plot of stress versus frequency versus position in the BHA. As a result, it is useful to scan the BHA at each frequency and present the maximum stress in the BHA as a function of frequency such as shown in
Figure 5.2.1
Lateral BHA Displacement vs. Position vs. Frequency
Figure 5.2.2
Lateral BHA Rotation vs. Position vs. Frequency

Lateral Rotation (radians)

Position in BHA from Bit (in)

Frequency (Hz)
Figure 5.2.3
BHA Mises Stress vs. Position vs. Frequency
Figure 5.2.4. This approach is logical in that it is not desirable to incur high stresses anywhere in the BHA as a failure at any location is unacceptable from a drilling perspective. For purposes of comparing solution techniques and executing sensitivity studies, this 2-D presentation of maximum BHA stress versus frequency will be used heavily.

Figure 5.2.5 shows the results from the first modal superposition technique described above where modal data were updated as required during the frequency span. Three curves are shown in this figure. The solid curve corresponds to the base case where added mass is not accounted for. The dotted line corresponds to the iterative procedure which accounts for added mass as an out-of-balance force. The dashed line corresponds to a procedure where the eigenproblem is reposed and solved at every frequency thereby providing a response calculation based on modal information that directly accounts for the added mass terms. This dashed curve validates the accuracy of the iterative solution technique. Figure 5.2.6 shows the convergence behavior of the iterative response procedure as a plot of the number of iterations for convergence versus frequency. Convergence is initially achieved with one difficult point near 1.7 Hz corresponding to the primary vibration mode, but convergence fails near 3.7 Hz within the allowed 30 iterations. At this frequency, the algorithm automatically reformulates the system mass matrix using added mass, regenerates the system matrix, solves the eigenproblem, and updates the modal data. As shown, convergence from this point forward is achieved easily with 3-8 iterations required for convergence. Overlaying Figures 5.2.5 and 5.2.6 reveals that locations near newly predicted or old resonances
Figure 5.2.4
Maximum BHA von Mises Stress vs. Frequency

Mises Stress (psi)

Excitation Frequency (Hz)
Figure 5.2.5
Nominal BHA Response vs. Fluid Added Mass Solution using
Iterative Modal Superposition

Mises Stress (psi)

Excitation Frequency (Hz)
Figure 5.2.6
Convergence Behavior of Iterative Modal Superposition

Number of Iterations for Convergence

Excitation Frequency (Hz)
require the most iterations as the procedure works to either generate or remove that resonance relative to the base response.

As described previously, this iterative procedure was further enhanced to solve the eigenproblem at the median frequency of interest, and to use this modal information to sweep the entire frequency range. Results from these calculations are shown in Figures 5.2.7 and 5.2.8 for maximum lateral rotation and maximum lateral displacement. As above, three curves are shown corresponding to the base case without added mass, the iterative solution, and a comparative solution where the eigenproblem is reformulated and solved at each frequency. Again, the accuracy of the solution is affirmed by these comparisons. Figure 5.2.9 shows the iterative performance of this technique which used modal data at 25 Hz as the basis for determining response for the frequency range from 0-50 Hz. As shown in this figure, convergence is achieved throughout the frequency range with only 2-7 iterations required depending on the proximity of the response frequency to either old or new resonances. The advantage of this approach is that a single eigensolution is used to generate the entire response spectrum.

Figure 5.2.10 shows a comparison of the response predicted by the iterative modal superposition technique to a matrix inversion technique. The key difference between these models is the inclusion of damping in the modal superposition technique, while the matrix inversion methods of previous works [115], [117], [118] omit damping. As shown in this figure, while the location of resonances coincide, the response magnitude at resonances cannot be found by the undamped model and the predicted stress levels are off the scale.
Figure 5.2.7
Rotational Response using Enhanced Iterative Modal Superposition Procedure

Lateral Rotation (rad)

Excitation Frequency (Hz)
Figure 5.2.8  
Lateral Response using Enhanced Iterative Modal Superposition Procedure

---

- Base Case without Added Mass
- Iterative Modal Superposition Technique
- Modal Superposition with Eigensolution at Each Frequency

Lateral Deflection (in)

Excitation Frequency (Hz)
Figure 5.2.9
Convergence Behavior of Enhanced Iterative Modal Superposition

Number of Iterations for Convergence

Excitation Frequency (Hz)
Figure 5.2.10
Damped Modal Response versus Undamped Matrix Inversion

Mises Stress (psi)

Excitation Frequency (Hz)
For an undamped response model, magnitude at resonances cannot be calculated. Figures 5.2.11 and 5.2.12 demonstrate, indeed, the response magnitude predicted by undamped MFFR BHA models is determined purely by numerical considerations unrelated to the physical problem. Figure 5.2.11 shows a comparison of response predictions from damped modal superposition against undamped matrix inversion. The damped response is plotted against the left-hand stress scale from 0-3,000 psi. The undamped response is plotted against the right-hand stress scale from 0-100,000 psi, which is necessary to show the predicted resonance magnitudes. The dramatic variation in response magnitude is the first indication that the undamped model is inadequate. Note that the most severe resonances for the damped response are near 22 Hz, 47 Hz, 19 Hz, 39 Hz, and so forth. For the undamped response, the most severe resonances are at 47 Hz, 45 Hz, 22 Hz, 40 Hz, and so forth. The relative magnitudes of resonances predicted by the two models are different. A general tendency exists for higher order modes to be predicted as most severe by the undamped model due to the high stresses incurred by those deformations, although the damped model changes these predictions due to the effect of damping in lessening those magnitudes. Figure 5.2.11 actually shows three damped responses and three undamped responses corresponding to different frequency increments being used to sweep the frequency range from 0-50 Hz. Frequency increments of 0.5, 0.1, and 0.05 Hz were used. Some smoothing of the damped curves can be seen in this figure, although the magnitude at resonance is essentially constant. The undamped responses, however, increase in resonance magnitude with decreasing frequency increments. This is explicitly shown in Figure 5.2.12 which shows undamped response magnitudes near 6.85, 22.35, and 45.55 Hz as
Figure 5.2.11
Damped versus Undamped Response with Varying Increments

Mises Stress (psi)

Excitation Frequency (Hz)

Damped Torsional Modulation
Undamped Matta Invarnica
Figure 5.2.12
Increase in Undamped Response due to Frequency Increment

Harmonic Peaks at
- 6.85 Hz
- 22.35 Hz
- 45.55 Hz

Mises Stress (psi)

Frequency Increment for Scan (Hz)
a function of the frequency increment used to sweep the frequency range. As shown in this figure, resonance magnitudes increase significantly, by a factor of five (5) from 5,000 psi to over 25,000 psi as a result the finer frequency increment. In this case, the relative severity of the peaks near 6.85 Hz and 22.35 Hz also changes although they remain close in magnitude. Obviously, a BHA MFFR model should not be sensitive to parameters like the frequency increment used to sweep the range of interest. These results underscore the inadequacy of undamped BHA MFFR models. Although these observations are self evident from theoretical considerations, they are provided due to the current industrial use of undamped BHA dynamic models.

Results from the modal refinement procedure are shown in Figure 5.2.13. As described earlier, this procedure uses nominal mass and stiffness properties to develop modal data, then updates the modal mass and modal frequencies using a full mass matrix including added mass at each resonance within the frequency range of interest. The solid curve in Figure 5.2.13 is developed using the above iterative solution for the problem, while the dotted curve shows results from the modal refinement procedure. Although an approximation, results show that the procedure works well and begins losing accuracy only at the higher frequencies between 30 and 50 Hz. Even those errors, however, are acceptable in terms of engineering requirements for the BHA dynamic problem.

The computational performance of the various procedures is an important consideration in the effective implementation of these models in drilling operations. Table 5.2.1 shows benchmark results for the various solution techniques described here. All computations were performed on a Sun SPARCstation 2 with 32 Megabytes
Figure 5.2.13

Response based on Modal Refinement Procedure

Damped Iterative Modal Superposition
Modal Refinement Procedure

Mises Stress (psi)

Excitation Frequency (Hz)
Table 5.2.1
Computational Performance of Alternate Solution Methods

<table>
<thead>
<tr>
<th>Solution Technique</th>
<th>Total CPU Time*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modal Superposition Techniques</td>
<td></td>
</tr>
<tr>
<td>without Iterative Effects - $\Sigma \phi$</td>
<td>156.76 seconds</td>
</tr>
<tr>
<td>with Iterative Solution for Added Mass - $\Sigma \phi \ w/ \ M(\omega)$</td>
<td>313.85 seconds</td>
</tr>
<tr>
<td>with Iterative Solutions for Added Mass and Wellbore Contact - $\Sigma \phi \ w/ \ M(\omega) &amp; F^*$</td>
<td>315.79 seconds</td>
</tr>
<tr>
<td>Matrix Inversion using standard LUDCMP/LUBKSB Algorithms</td>
<td></td>
</tr>
<tr>
<td>without Added Mass - $[K-\omega^2M]^{-1}$</td>
<td>926.24 seconds</td>
</tr>
<tr>
<td>with Added Mass - $[K-\omega^2M(\omega)]^{-1}$</td>
<td>1038.09 seconds</td>
</tr>
<tr>
<td>Matrix Inversion using Active Column Solver [118]</td>
<td></td>
</tr>
<tr>
<td>without Added Mass - $[K-\omega^2M]^{-1}$</td>
<td>123.89 seconds</td>
</tr>
<tr>
<td>with Added Mass - $[K-\omega^2M(\omega)]^{-1}$</td>
<td>241.42 seconds</td>
</tr>
<tr>
<td>Modal Refinement Procedure - $\Sigma \phi$ with $M^* = \phi^T \left[ M + M(\omega) \right] \phi$</td>
<td>142.32 seconds</td>
</tr>
</tbody>
</table>

* CPU for computations was a Sun SPARCstation 2 with 32 Megabytes RAM Memory.
of RAM memory. Modal superposition using nominal mass with no iterative procedures to account for the added mass term required 156.76 CPU seconds. Activating the iterative solution technique for the added mass where modal data was developed once at the median frequency increases the execution time to 313.85 CPU seconds. Further, activating the nonlinear restoring force adjustment for wellbore contact increases this time very slightly to 315.79 CPU seconds. The additional computational time for contact should remain small, though more significant time increases could be seen depending on how many violations of the wellbore constraint occur in a particular case. Matrix inversion using standard routines such as LUDCMP and LUBKSB [134] results in poor numerical performance with 926.24 CPU seconds required for the nominal case without the added mass term. For the case with added mass which requires reformulation of the mass and system matrices at each frequency, time increases to 1038.09 CPU seconds. Due to the symmetric, banded matrix being inverted, more efficient inversion techniques are possible such as an active column solver which uses an index scheme to track matrix terms and operates only on nonzero terms in vector fashion. This active column solution procedure was acquired from a previous BHA MFFR model [118]. The active column inversion technique required 123.89 CPU seconds without the added mass and 241.42 CPU seconds with added mass requiring reformulation of mass and system matrices at each frequency. By avoiding operations on nonzero terms, the active column solution technique greatly improves computational efficiency relative to standard matrix inversion. These latter are procedures are over 20% faster than the modal superposition techniques.
The disparity, however, is that the inversion techniques are for undamped system models so their results are not of the same quality as those provided by modal superposition. If damping was added to the matrix inversion formulations, all matrix computations would require complex variables due to the damped system matrices. These complex variables effectively double the computational cost relative to real variables. Thus, these execution times would approximately double for inversion of a damped system matrix, resulting in the active column solver taking about 50% longer than the iterative modal superposition technique to solve the problem. Since the wellbore contact correction cannot be as effectively addressed using the inversion technique due to lack of modal information, inclusion of this consideration would make inversion even less attractive, requiring some iterative scheme, to modify the stiffness matrix or the load vector, in order to satisfy the wellbore constraint. In addition, engineering issues in properly developing a damping matrix would have to be overcome.

If computational speed is of primary importance, Table 5.2.1 shows the modal refinement procedure to be very efficient, requiring only 142.32 CPU seconds to develop a very good damped response prediction and is over twice as fast as the iterative modal superposition technique. All the iterative modal superposition and modal refinement techniques also offer the advantage of providing modal data including natural frequencies and mode shapes in addition to the response prediction. As the modal data allows verification of the assumption that the posed BHA problem is not buckled nor has any rigid body modes, the modal based solution techniques provide substantially more useful information than the inversion techniques, and are thus recommended.
5.3 Grid Refinement

The quantification of numerical accuracy as a function of discretization is a fundamental task for all numerical approaches. Here, the accuracy of modal shapes and frequencies relative to the length of the beam elements used to model the BHA components is of concern. This issue has been studied through a parametric sensitivity to element length.

Grid refinement results are shown in Figure 5.3.1; they show frequency response for various element lengths. Due to the difficulty in discerning variation in response due to the element length, this issue is quantified more rigorously in Figure 5.3.2 which shows a subset of the lateral mode frequencies, f (Hz), versus the inverse of element length (1/ft.). A key parameter in this regard is the maximum frequency for which accurate results are required. Setting this limit requires consideration of two factors. These are the highest rate excitation mechanism to be included in the model, and the highest RPM to be evaluated operationally. Allowing for an outer row tooth excitation mechanism of 20X for a tri-cone bit and a rotary speed of 250-300 RPM, yields a required accuracy up to 85-100 Hz in the response calculation. As shown in the figure, the lateral modes near this cutoff frequency do not stabilize well until the element length has been reduced to 5', where 1/l is 0.2, while very small changes are evident as the model is further discretized down to 3' sections. In light of this information, 5' maximum section lengths will be considered as the nominal model spacing for solutions requiring the above frequency accuracy. Thus, the 2.5' spacing used in these results is conservative with regard to frequency accuracy. The 5' spacing guideline represents a significant change to some BHA models which are being applied
Figure 5.3.1
Effect of Element Spacing on Frequency Response

BHA Mises Stress (psi)

Element Length (ft)

Frequency (Hz)

272
Figure 5.3.2
Change in Eigenvalues due to Element Spacing

Eigenvalue (Hz)

Inverse Element Length (feet⁻¹)
with spacings as long as 15-30'. It also somewhat with the Geodyn spacing of 15''
which was used in those qualification efforts.

Returning to the assumption of negligible shear contribution in the beam
stiffness formulation, it can now be determined that with the 5' spacing guideline
\( \Phi = 1/(5')^2 \) or 0.04. Relative to 1.0 it is reasonable to allow this \( \Phi \) term to be
neglected, although for smaller spacings the ability to include the shear deformation
contribution should be considered.

**5.4 Added Mass Effect**

Several industry response and eigenvalue models for BHA behavior do not include
added mass accurately. It is clear from previous discussions that added mass should be
treated properly. However, this sensitivity is repeated here in the form of eigenvalues
versus the added mass coefficient. As shown in Figure 5.4.1, the inclusion of
significant added mass coefficients, like those of Eq. 3.7, has an appreciable impact on
the natural frequencies of the system, shifting higher-frequency modes more than
20%, for example from 90 Hz to below 70 Hz by changing \( C_M \) from 0 to 4. Clearly,
fluid added mass is an important aspect of the problem and further research into the
validity of current models [130] is justified.

**5.5 Damping Effects**

Damping influences on the frequency response are shown in Figure 5.5.1 for
damping levels of 1%, 5%, and 10% of critical damping, and for the frequency-depen-
dent damping function of equations (3.16) to (3.18). Damping of 1% has been used in
the above solution comparisons and induces clear peaks near resonances at 12 frequen-
cies between 0-50 Hz. At 5% damping, the sharpness and magnitude of these peaks is
Figure 5.4.1
Change in Eigenvalues due to Fluid Added Mass Coefficient

Eigenvalue (Hz)

Added Mass Coefficient for External Drilling Fluid ($C_M$)
Figure 5.5.1
Response Variation due to Damping Magnitude

Mises Stress (psi)

Excitation Frequency (Hz)
substantially reduced. However, peaks are discernible at 19, 22, 39, and 46 Hz. At 10% damping, only the most severe lateral vibration mode at around 22 Hz is apparent. The frequency-dependent damping function described earlier, $\zeta(\rho_{\text{mud},f})$, shows variable damping as expected. Below about 17 Hz, $\zeta(\rho_{\text{mud},f})$ yields in damping in excess of 10%. At about 35 Hz $\zeta(\rho_{\text{mud},f})$ yields damping below 5%. The behavior of $\zeta(\rho_{\text{mud},f})$ is an important consideration since it predicts very heavy damping at low frequencies where modes are more likely to be excited and lower damping for higher frequencies where modes are less feasible. Obviously, damping plays an critical role in determining response magnitudes near resonances and further research on BHA damping is needed.

5.6 Weight-on-Bit (WOB)

Previous researchers [122] have concluded that WOB does not appreciably influence the BHA response based on inspection of the eigenvalue sensitivity to WOB such as shown earlier in Figure 2.8.15. This conclusion is verified in terms of the BHA response as shown in Figure 5.6.1 and 5.6.2. The first figure plots the BHA displacement response against WOB and forcing frequency in Hz. Weights-on-bit of 1000, 2000, 5000, and 10,000 lbs. were analyzed. The location and severity of the various harmonics are essentially constant. This is seen more clearly in Figure 5.6.2 which overlays these responses. Review of the stress stiffening formulation [128] results in the following four lateral stiffness terms where the right term represents the modification due to axial load

$$12EI / (l)^3 + 6P/l$$

(5.6)
Figure 5.6.1
Effect of Weight-on-Bit (WOB) on Frequency Response
Figure 5.6.2
Response Variation due to Weight-on-Bit

Mises Stress (psi)

Weight-on-Bit (WOB) (lbs)

Excitation Frequency (Hz)
\[ 6EI/(l)^2 + P/10 \]  
\[ 4EI/l + 2Pl/15 \]  
\[ 2EI/l - Pl/30 \]  

where \( E \) is the elastic modulus, \( I \) the bending moment of inertia, \( l \) the element length, and \( P \) the axial load. Analysis shows the adjustment from the terms involving the axial load, \( P \), are small, below 1\%, for the range of weights-on-bits and element spacing used here. Higher weight-on-bit and longer element spacing would increase this effect so it is recommended that it be retained in the formulation although it is of limited importance.

5.7 Stabilizer Boundary Conditions

The boundary conditions at the top of the BHA can be relatively well defined by the torsional and axial stiffness of the drillpipe that spans to surface. However, the nature of the lateral boundary conditions within the BHA is less clear. The issue is the behavior of stabilizers in contact with the wellbore as most BHAs are run in a stabilized or "packed" fashion. In surveying different models, it is clear that some researchers treat stabilizers as points at which lateral displacements are constrained but lateral rotations allowed. Others treat the stabilizer blades as finite lengths which constrain both lateral displacements and rotations. To quantify the impact of this effect without assuming either one a priori, a sensitivity study has been performed. As shown in Figure 5.7.1, stabilizer blades were treated as point boundary conditions and as 12" and 24" of finite length. As shown, the modeling of the stabilizer blades as increasing finite lengths does not appreciably affect the predicted BHA response. Slight shifts in resonance locations and severity can be seen for some of the resonance
Figure 5.7.1
Effect of Stabilizer Treatment on Frequency Response

Mises Stress (psi)

Excitation Frequency (Hz)
locations, but the overall BHA response is largely unchanged. Although there are arguments that stabilizers cannot restrain lateral rotation, BHA dynamics are frequently critical in hard rock areas where tight wellbore clearances may result in close constraint of the stabilizers. This issue will not be resolved without significant field data. However, this result indicates that this is not a critical issue for the model studied and does not affect the system stiffness in an appreciable way.

5.8 Loading Placement

Some drillstring analyst have assumed that if a load of certain type, axial, torsional, or lateral, is applied at some fixed location, usually the bit, the resultant response prediction can then be repeatedly superimposed for whatever forcing mechanism is of interest. Based on the discussion of modal force and the concept of "alignment" between the forcing vector and the mode shape, this approach should be critiqued. Shown in Figure 5.8.1 are the results of placing a lateral force of the same magnitude at the bit and at the center of two drillcollar sections higher in the BHA, to model eccentric weight. As shown in the figure, significant differences appear in the response prediction. For example, substantial harmonics exist at 1.7 Hz and 10 Hz when exciting the upper drillcollar section that are not excited from placing the load at the bit. Similarly, several peaks which appear significant based on the bit excitation become less critical when the excitation is applied in the drillcollar sections (15 Hz, 17 Hz, 19 Hz, 22 Hz). When loading the middle drillcollar section, the most severe harmonics appear near 4, 5, and 6 Hz.

These results suggest that the specific source of each excitation mechanism be considered and that the force term be properly placed on the system. Bit excitation
Figure 5.8.1

Effect of Forcing Location on Frequency Response

Mises Stress (psi)

Excitation Frequency (Hz)

DC Load Point (Node 51)

DC Load Point (Node 18)

Bit Load Point

Bit N 18

N 51

3000

2500

2000

1500

1000

500

0
sources will clearly be applied at the bit node. However, excitation mechanisms related to the drillstring walking mechanisms should be applied at the top of the BHA where those forces would be physically transferred from the drillpipe into the BHA. Eccentricity and misalignment loadings of the 1X nature should be placed at the center of specific drillcollar sections, particularly those with the longest unsupported length. The need to individually examine the BHA response from specific excitation forces highlights yet another advantage of the modal superposition method due to the ability to efficiently modify the force vector in Eq. 4.17 to evaluate each excitation source.

5.9 Effective Stiffness of Special Components

With drilling jars, shock-tools, MWD and other components commonly used in BHA, issues remain concerning their true rigidities. Guidelines provided by manufacturers for "50% of a drillcollar" are so qualitative that the basis for the rating must be questioned. Figures 5.9.1 and 5.9.2 examine the impact of this issue for the case where a special BHA component is run fairly close to the bit, and imposes a modified stiffness of unknown magnitude varying from 40% to 200% of a standard drillcollar. As shown in the response plot, the stiffness change has an appreciable effect on the response of the system. Figure 5.9.2 shows substantial variations in the response magnitudes due to these stiffness variations and also the location of the resonances for several of the harmonics. Note that these stiffness variations can result from both material properties or running special BHA components such as a shock-tool or MWD tool which would exhibit reduced stiffnesses. For a beryllium-copper drillcollars, the stiffness would be reduced to 63% of a standard drillcollar. Shock-tools, MWD tools and other components will have reduced rigidity due to the internal geometries, but
Figure 5.9.1
Effect of BHA Component Stiffness on Frequency Response

BHA Mises Stress (psi)

Stiffness Ratio of Component to Normal 8" OD Drillcollar

Frequency (Hz)
Figure 5.9.2
Effect of Component Stiffness on Frequency Response

Stiffness Ratio to Solid Drillcollar
8" OD x 2-13/16" ID

Excitation Frequency (Hz)

Misses Stress (psi)

3000  2500  2000  1500  1000  500  0

BHA Component of Unknown Rigidity
reliable values for their stiffnesses are not available. For square drillcollars, the stiffness can be as much as 170% that of a standard round drillcollar. The impact of these stiffness changes will also depend on the placement of the special component relative to the mode shape for the frequency of interest. Due to the placement of MWD tools and shock-tools close to the bit which is a critical portion of the BHA, further research is required to better define the stiffnesses of these special tools which have an appreciable effect on BHA responses.

5.10 Solution for Wellbore Contact

As described earlier, previous MFFR models do not address the issue of the predicted BHA response violating the fundamental geometric constraint of the wellbore. To demonstrate the effectiveness of the algorithm described earlier to address this problem, Figure 5.10.1 shows a response prediction for the BHA where the 8" OD drillcollar between the second and third stabilizers has displaced beyond the allowable radial clearance of 2.125" in the 12.250" wellbore. In this case, the original displacement predicted was over 3.10". To correct the BHA response for this violation of the wellbore constraint, a restoring force is calculated according to equations (4.15) and (4.16) and applied in the response calculation of (4.17). The resulting response is shown in Figure 5.10.1 as the dotted curve. Using this adjustment, the point of maximum wellbore "penetration" is corrected back to where it just contacts the wellbore. Figure 5.10.2 shows the ratio of the original response prediction to the adjusted response predicted demonstrating the nonlinear nature of the adjustment relative to the original response prediction. Due to the direct method by which this restoring force is found using known modal sensitivity, the restoring force
Figure 5.10.1
BHA Displacement before and after Contact Restoring Force

Position in BHA from Bit (in)

- Unconstrained Solution without Contact Adjustment
- Final Solution after Contact Correction

2-3/4" Radial Clearance

2-1/8" Radial Clearance

Maximum Wellbore Constraint Violation

Lateral Deflection (in)
Figure 5.10.2
Nonlinear Nature of Restoring Force relative to Primary Excitation

Position in BHA from Bit (in)

Ratio of Unconstrained Response to Corrected Response
(Ratio assigned zero value at all stabilizers)
adjusts the node in question exactly to the allowable radial clearance and no iterations are required.

5.11 Behavior and Sensitivity of Lateral Bit Displacements

The bit is not constrained laterally so that excitations can be placed at that location in the BHA. However, it is not reasonable to believe that the bit can displace laterally to a large extent. Thus, caution must be exercised in evaluating vibration modes which involve large bit displacements. Figure 5.11.1 shows three response curves that illustrate the importance of considering allowable bit displacement. The solid curve represents the base case where no constraint is placed on the lateral bit displacement. The dotted curve shows the predicted response when the lateral bit displacement is restricted to 0.25" radially. Since the location of the excitation and nodal displacement being adjusted coincide, the restoring force procedure of equations (4.15) to (4.17) results in a linear reduction of the excitation force in this case. The 0.25" restriction on lateral bit motion affects only the resonance at 22 Hz, the most severe resonance in this case study. Further restricting the bit to 0.125" lateral motion results in resonances being truncated at 17, 19, and 22 Hz as shown by the dashed line in Figure 5.11.1.

The proper allowable radial motion and/or lateral stiffness constraint for the bit is currently not known in the drilling community and will require high-quality data to be defined. As an alternative to allowing free bit motion, Figure 5.11.2 shows the variation in response prediction that occurs if the bit is fixed radially and the model is excited laterally at the next node above the bit. In this figure, the solid curve corresponds to the base case of a free bit boundary condition, while the dotted line
Figure 5.11.1
Resonance Filtering due to Excessive Lateral Bit Motion

Excitation Frequency (Hz)

Mises Stress (psi)

Radial Bit Constraint (in)

Base
0.25
0.125

8000 6000 4000 2000
0 10 20 30 40 50
Figure 5.11.2
Response using Alternative Bit Boundary Conditions

Base Model with Excitation at Bit
Modified Model with Constrained Bit

Mises Stress (psi)

Excitation Frequency (Hz)
represents this new set of boundary conditions. Obviously, a dramatic change in the
dynamic response occurs as a result of changing the model boundary conditions. For
the low frequency modes below 15 Hz, the resonance locations are not changed
substantially. Above 15 Hz however, resonances shift by as much as 50%, for
example, with the resonance at 22 Hz now appearing near 32 Hz. Moreover, note the
dramatically lower stress values by a factor of twenty-five (25) relative to the free
boundary condition at the bit. Clearly, the nature of the bit boundary condition is a
very important dynamic modeling consideration on which additional information must
be obtained.

As an means of normalizing the dynamic BHA response for the essentially
unknown bit boundary condition, Figure 5.11.3 shows a response plot in which the
maximum BHA stress has been normalized against lateral bit motion. This response
presentation allows identification of resonance locations where high BHA stresses are
incurred which are not dependent on large lateral bit motions. Note that the previously
severe peaks at 17, 19 and 22 Hz are not as severe in this normalized measure. Lower
frequency resonances near 4, 5.5, and 7 Hz now appear more severe than previously
indicated by the simple stress based response presentation. At higher frequencies above
30 Hz, a virtually linear increase in this normalized BHA stress response is seen. Until
better definition of dynamic BHA boundary conditions is achieved, it is recommended
that normalized measures such as maximum BHA stress versus lateral bit displacement
be considered to improve the quality of dynamic BHA evaluations.
Figure 5.11.3
Normalized Response of BHA Stress to Bit Motion Ratio

Ratio of Mises Stress to Bit Displacement (psi/in)

Excitation Frequency (Hz)
5.12 Superposition Technique for Multi-Excitation Conditions

Due to the great number of possible BHA excitations, the application of MFFR to actual BHA analysis involves the superposition of the responses expected from the distinct load sources. As discussed earlier, each drilling assembly should be evaluated for the excitations to be expected from the various components and bit. In addition, parametric relations, including 1 axial-to-lateral and 2 axial-to-lateral should be included. In this figure, excitations are considered at 1X (mass and alignment), 2X (parametric), 3X (tri-cone), 4X (parametric), 9X (inner row bit teeth), and 19X (outer row bit teeth). For each particular excitation, the generic frequency response in terms of Hz is scaled appropriately into the operational measure of RPM as shown in Figure 5.12.1.

For this illustrative example, the 9X source is scaled by 3.4 and the 19X source by 6.7, both based on the spectral bit load data presented earlier [71]. The 2X source and the 3X source are scaled by 2.5 and 4.9 based on observations from field data, although other factors may be used. Finally, a scaling function is applied to the 1X excitation which increases with the square of the rotational speed to properly model the behavior of eccentric weight loading.

As described earlier, the generic frequency scales are shifted into their corresponding RPM scales to generate the composite response prediction. Figure 5.12.2 shows the effect of this scaling in terms of BHA stress versus rotary speed for the different types of excitation sources. Figures 5.12.3 show the maximum BHA stress responses overlayed on a plot of rotary speed up to 1000 RPM to allow the identification of hazardous operating regions for PDM or turbine drilling. Figure 5.12.4 shows
Figure 5.12.1
Transformation of Generic Response into Operating Responses

Frequency (Hz)

0  5  10  15  20  25

RPM Scale for 1X

0  100  200  300  400  500

RPM Scale for 3X

0  80  160  240  320  400

RPM Scale for 4X

0  15  30  45  60  75

RPM Scale for 20X
Figure 5.12.2
Scaling of Frequency Response into Operating Response

BIIA Mises Stress (psi)

Excitation Source (Excitations per Revolution)

Rotary Speed (RPM)
Figure 5.12.3
Composite BHA Response for 0-1000 RPM Range

Mises Stress (psi)

Rotary Speed (RPM)
Figure 5.12.4
Composite BHA Response for 0-200 RPM Range

Excitation Source

Rotary Speed (RPM)

Mises Stress (psi)
the same information rescaled to highlight the normal rotary operating speeds up to 200 RPM. Since the analysis of where harmful resonances occur must ultimately include a differentiation between high stress levels and acceptable stress levels, it is imperative that more field and lab data be gathered to better quantify the importance of these various excitation sources. The importance of high-order excitations such as the 19X outer bit tooth source becomes apparent when considering the proximity of resonances in the operating region and the lower damping at increased frequency.
Chapter 6 - Concluding Remarks and Directions for Future Work

The man who is afraid of asking is ashamed of learning - Danish

A comprehensive technology review has been made to provide a perspective of different aspects of BHA dynamics and mechanics. Torque/drag systems estimate mean loads during drillstring motion resulting from weight and friction. Static directional models use force equilibrium results to predict drilling trajectory direction. Time-domain dynamic directional models refine static results and allow study of azimuthal walk tendencies. Most recently, quasi-dynamic models have addressed the need for friction loads to be accounted for while providing computational efficiency comparable to static models. Laboratory and field data have revealed many important behaviors over the years. Random-like effects, such as modulation of axial excitations by torsional modes, will act to make monochromatic forced frequency response (MFFR) a conservative evaluation tool for planning and screening alternative BHA designs. Other observations from laboratory and field data, indicate that complex, transient behaviors occur downhole which are not currently understood. Additional studies are clearly warranted.

Bit mechanics studies indicate that bits produce polyharmonic excitation spectra. The data also indicate a variation in this spectrum as a function of bit wear. These spectra have been measured on a select number of bits. Rate-of-penetration models have allowed studies of the dynamics of drilling including a link between the BHA, the bit, and the formation. However, they fall short in their accurate modeling of key dynamic phenomena. ROP studies indicate that drilling efficiency is improved by increasing the energy delivery to the bit, an additional benefit of BHA MFFR analysis. Stick/slip motion, a special harmonic case, is a fundamental torsional mode controlled
by nonlinear wellbore friction under specific operating conditions. This phenomenon has a serious impact on drilling performance and should be addressed operationally, through torque-feedback in the rotary drive system. In the future, operating guidelines may be developed to allow for the avoidance of this detrimental condition without requiring rotary feedback systems.

After developing a broad perspective of issues involved in dynamic BHA behavior, monochromatic forced frequency response (MFFR) has been selected as the most effective approach to this complex problem. A model has been developed for MFFR in the form of numerical procedures based on finite-element methods. This approach not only provides for solution of the stated objective, but allows future extension to consider effects not presently modeled. The basic components of the MFFR approach for BHA dynamics are stiffness, mass, and damping characteristics of the system, and the nature of external excitations. These areas have been reviewed in detail, including development of a new frequency-dependent damping function. However, current technology presents many areas which require additional research.

Analysis methods for implementation of MFFR have evolved to the use of an elegant technique which develops a transfer function representation for the problem using modal superposition methods. This approach provides for a solution to the eigenproblem as an initial step, followed by an efficient computational scheme to produce the system response. The eigensolution provides useful information on mode shapes and frequencies, as well as a system check for static buckling. The mode superposition calculation is efficient, provides intuition into the physics of the problem, and can be used within iterative schemes to address nonlinear and other special
effects through the formulation of an unbalanced force vector with appropriate 
iteration control. For optimal computational efficiency, a modal refinement procedure 
has also been developed and verified, which although an approximate yields sufficiently 
accurate results.

Parametric sensitivity analyses have been performed on several important effects 
including grid refinement, added mass, weight-on-bit (WOB), boundary conditions at 
stabilizers, and loading description. The grid refinement approach allow spacing to be 
defined in terms of desired frequency accuracy. Superposition of multi-excitation load 
sources is proposed as the means of generating an operational response in terms of 
rotary speed.

Several conclusions can be drawn from this study. Clearly, drilling dynamics are 
complex. Engineering judgement must be carefully exercised to ensure that the system 
components being modeled are adequately described. Modeling of bit interaction, ROP, 
or full "rig-to-bit" drilling system models should be carefully scrutinized.

Monochromatic forced-frequency response analysis is proposed as an effective 
tool for planning and evaluating the dynamics of rotary drilling assemblies. An 
effective MFFR model has been developed and important guidelines and considerations 
have been identified for its application. MFFR provides an efficient tool for analyzing 
BHA dynamics and allows flexibility in both the formulation and future enhancements.

Important considerations in developing MFFR algorithms include: axial load 
influence on lateral stiffness of the beam element, depending on load magnitude and 
element length; shear contribution to the deflection if short beam spacings are used; 
fluid added mass and added mass coefficients to lateral inertias to account for fluid and
geometric confinement effects; damping formulation to account for fluid properties and frequency dependency; and the nature, magnitude, and location of excitation forces.

In several of the above areas, neither the technology nor the data necessary to develop it exist at the present time. For this reason, many areas for future work have been identified.

With the posed dynamic problem, unique computational schemes have been constructed to address several advanced issues. The frequency dependent mass matrix poses an eigenproblem which must be solved iteratively. Although matrix inversion was studied, it was determined that modal superposition offers many relative advantages and is preferred. Evaluation of BHA dynamics through multi-degree-of-freedom modal superposition techniques is done here for the first time. Iterative schemes to properly and efficiently account for frequency-dependent added mass and wellbore contact have also been developed for the first time. In addition, a new modal refinement procedure has been proposed to optimize computational efficiency. As nonlinear aspects of BHA behavior are better defined, an iterative approach has been demonstrated which can be applied to address these new problems.

Parametric sensitivity analyses have provided several important considerations which distinguish the capability of this MFFR model and the method by which it will be applied from earlier works. Grid refinement requirements have been defined in terms of the nature of the excitations considered, the frequency (rotary speed range) to be analyzed, and the desired accuracy of the solution. This technique will be repeated with the ongoing application of MFFR to drilling analyses and may lead to further model enhancements such as the inclusion of shear terms. Added mass effects are shown to
be quite significant and are included in this MFFR model. Weight-on-bit was shown not to be a critical parameter in the case study examined but could gain importance depending on the magnitude of compression in the BHA and element length of the model being used. The dynamic system response was shown to relatively insensitive to the nature of the stabilizer boundary conditions based on the types of conditions studied here. Field data and further study will be required to better define the conditions at stabilizers to allow refinement in this area. Load description, also addressed simplistically in previous works, has been demonstrated to be critical as the alignment between the load vector and the mode shape strongly affects the predicted response. The bit boundary condition for lateral motion has been shown to be a critical factor in determining the dynamic BHA response. This boundary condition is not well understood by the industrial community and additional research in this area is needed. To improve the validity of the current model in view of this missing data, a severity measure has been proposed which normalizes BHA stresses against lateral bit displacements. Complete system behavior accounting for all loads is addressed through the superposition of MFFR responses transformed into appropriate operational responses based on RPM. Nonlinear parametric relations, namely the 1X lateral to 2X axial relation, are treated analogously.

The MFFR technique developed herein is proposed as an effective and appropriate engineering tool for its purpose of planning and screening BHAs in operational use, but will require ongoing use for validation with currently lacking field data. MFFR should be applied broadly to BHA design and screening. This application over a period of time will be required to develop a statistical measure of the technique's success.
The present study has also revealed many areas where further research is warranted. Excitation spectra for bits in formations of interest and in a variety of wear states should be sought to provide more definitive description of the loadings experienced by the BHA. Models for fluid added mass and wellbore damping should be studied further, preferably through the acquisition of relevant full-scale data for drilling operation environments. As technical and commercial considerations allow, downhole field data should be gathered which will allow further insight and calibration of existing theories and models. Of the four downhole data systems cited in this work, three are no longer available for use. Efforts are now underway to address this critical barrier to the industry’s improved understanding of BHA and drilling dynamics. As field data becomes available, boundary conditions which influence BHA behavior including those at the bit, stabilizers, and at the drillpipe interface should be refined. Wellbore irregularities such as eccentric holes, local doglegs, and hole enlargements should be studied both in terms of their impact on BHA dynamics and the ability to quantify these effects with models which can be applied during the planning process. Alternatives to MFFR, particularly approaches which consider the polyharmonic nature of the excitations, should be considered. Time-domain approaches should be addressed with caution, as computational requirements may be excessive. Wellbore frictional behavior and its impact on BHA and stabilizer motions and forces should be studied when appropriate data becomes available. Linking MFFR to fatigue measures may provide additional refinement to BHA design capabilities. Similarly, work in the area of material endurance and stress concentration effects in BHA threaded connections is likely warranted.
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Torque and Drag Models


**Static Directional Models**


Dynamic Directional Models


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**Methods of Analysis for Dynamic BHA Behavior**


**Numerical Results and Parametric Sensitivity Analysis**


8. Glossary of Terms

Air-drilled - Pertaining to wells which are drilled without drilling fluids using air as the fluid medium for the removal of drill cuttings from the bit to the surface.

Azimuthal - The angle which describes the directional of the wellbore in terms of North or South and East or West orientation. The azimuth angle is used in conjunction with the inclination angle to define the wellbore path.

Bent Housing - Pertains to mud motors which have intentionally bent structural housings to create the desired directional drilling tendencies.

Bent Subs - Short BHA components which are intentionally bent to provide the desired directional tendency for the BHA. Bent subs are used in conjunction with straight drillcollars and motors to create a steerable directional BHA.

Bit - One of several types of mechanisms with either rolling or fixed cutters used to fracture or abrade the formation to destruction resulting in the removal of the earth from the path of the drilling assembly.

Bit Anisotropy - Refers to the directional tendency of the bit to drill preferentially forward with less tendency to drill sideways. Bit anisotropy is one correlation parameter used in conjunction with directional prediction models to relate drilling tendencies to predicted resultant forces.

Bit Teeth - The cutters located on the cones of roller cone bits or on the surface of fixed cutter bits.

Bit Wear - The abrasive removal of the bit material as drilling proceeds. Bit wear results in smaller and duller teeth and can also result in a loss of gauge where the effective bit diameter becomes smaller. Bit teeth wear is measured on a linear scale from 0 to 8 with T0 designating a new bit and T8 designating a completely worn bit tooth.

Bit Whirl - A chaotic behavior where the bit does not rotate about its centerline axis but momentarily rotates about the location of an effective center of rotation such as a dominant cutter location.

Blades - Refers to the structures on the sides of stabilizers which protrude from the body to the specified stabilizer diameter.

Bottom-Hole Assembly (BHA) - Refers to the lower part of the drillstring usually including the bit up to the last drillcollar. Inclusion or exclusion of heavy-weight drillpipe (HWDP) in the BHA varies by operators. The BHA does not include drillpipe under normal circumstances.
**Build/Drop** - Refers to increasing inclination angle (building) or decreasing inclination angle (dropping).

**Button Bit** - Alternate name for tungsten carbide insert roller cone bits for hard formations where the inserts are short and have rounded surfaces.

**Cased-hole** - Pertains to the part of the wellbore in which casing has been installed and cemented into place.

**Casing** - Steel or alloy pipe which is run into the well at required depths to provide structural integrity for the well or to isolate weak or high-pressure formations so that continued drilling can continue.

**Casing Running** - The process of installing casing into a well which involves the assembly and lowering of the casing by the rig on a joint by joint basis.

**Casing Shoe** - Refers to both a special casing sub which is run on the end of casing to guide the casing into the well and also to the location of the end of the casing string.

**Casing Wear** - The abrasive removal of casing wall material generally due to drillstring rotation and tripping.

**Catenary Trajectory** - A particular type of trajectory to minimize torque and drag loads based on the natural shape of a suspended string under gravitational load.

**Cement** - Concrete slurry pumped into the well around casing following running to lock the casing into place in the well and provide additional structural support following curing.

**Cone Locking** - Refers to a failure in the bit where roller cones cannot rotate due to excessive wear and seizing of the bearing materials.

**Connection-weak** - Refers to BHA components in which the strength of the threaded end connection is less than that of the main body in bending, fatigue, tension, or other loading mode.

**Core** - A solid cylinder of the earth removed using special drilling tools which destroy an annular volume surrounding the core but leave the core itself unharmed and captured for retrieval. Cores are removed from the coring equipment at the surface and studied to characterize formation properties.

**Crooked Hole** - Refers to geographical areas or operations where use of normal drilling BHAs results in undesired deviation of the wellbore from vertical. Highly dipping hard formation are a particular problem area and cause measures to be taken in attempts to increase stabilization and BHA rigidity to overcome the problem.
Cross-over - A special BHA sub used to connect one pipe size and end connection to another. For example, cross-overs are required between drillcollars and drillpipe.

Curvature - Refers to the theoretical circular curvature of the wellbore based on survey data and interpretative models such as the Radius of Curvature Method. Curvature is usually expressed in degrees of angle change per 100’ of measured depth or in degrees of angle change per 10 m of measure depth.

Cuttings Cleaning - Refers to the removal of drilled rock particles from the wellbore from the circulation of viscous drilling fluid down the drillstring and back up the annulus.

d-Exponent - A normalized measure of rate of penetration proposed by Jorden and Shirley in 1966 which compensates for WOB, rotary speed, and bit diameter and can be used to analyze drilling performance for predicting overpressured formations and other conditions.

Daily Footage - Refers to the amount of new hole drilled in a 24 hour period by a particular operation.

Deepwater - Refers to the geography, operations, or technology associated with offshore drilling on the outer edges of the continental shelves. The limits for deepwater classification vary by operator and change with time but would generally refer to water depths in excess of 1,000’ to 1,500’ at this time.

Degrees/100’ - Units for measuring wellbore curvature or doglegs defined by change in inclination angle, azimuth angle or a combination of the two over 100’ of wellbore length.

Directional - Refers to wells drilled intentionally non-vertical at slanted or even horizontal profiles to optimally intersect as many producing formations as possible from limited surface locations.

Doglegs - Abrupt changes in the wellbore caused by the irregular path of one or more drilling assemblies. Doglegs are characterized by equivalent degrees/100’ of curvature.

Double-acting Duplex Pump - A particular type of drilling mud pump in which two cylinders act in both reciprocating directions per each revolution of the pump motor.

Double-Tilt Unit (DTU) - A special directional drilling motor which includes two bent points in its housing. DTU’s provide for increased build/drop rates relative to motor housings or BHAs with a single bent sub.

Drill-line - Wire cable which is typically run in groups of 6-12 lines between the traveling block and the crown block on a drilling rig. The drill-line is manipulated by the drawworks to raise or lower the traveling block and drillstring or casing.
Drill-off - Drilling test performed when preparing to drill with a new bit or through a new formation which involves recording ROP for a variety of WOB and RPM so that drilling operating parameters can be optimized.

Drillcollars - Very thick-walled pipes used directly on top of the bit to constitute the bottom-hole assembly (BHA). Drillcollars provide weight for the desired WOB and the proper structural characteristics to promote drilling of the desired wellbore curvature.

Drillcuttings - The small fragments of rock destroyed by the bit which must be removed from the wellbore by the drilling mud and filtered from the mud using surface solids control equipment.

Drilling Fluids - General name for drilling muds which include a wide variety of water-based and oil-based fluids with proper weight and chemical properties designed for each specific drilling operation.

Drilling Line - Another name for Drill-line, as defined above.

Drilling Pads - A specially constructed surface location for drilling one or more wells. Particularly used when referring to Arctic drilling locations as well as shallow offshore islands and marshlands.

Drilling Rig - General name for a variety of land and offshore rigs which includes all necessary machinery and equipment for drilling a well. Offshore rigs encompass many types such as barge, jackups, platforms, submersibles, semisubmersibles, moored drillships, and dynamically positioned vessels.

Drillpipe - Primary component of the drillstring typically 4-1/2" through 5-1/2" in outside diameter and normal wall thicknesses. Drillpipe is not as thick as drillcollars since it does not have to withstand compressive loads or the most severe drilling loads generated near the bit.

Drillstring - General name given to the overall structure comprised by the kelly, drillpipe, heavy-weight drillpipe, drillcollars, stabilizers, any special BHA components and the bit.

Elevators - Clamping mechanism installed on bails beneath the traveling block to grasp and lift the drillstring or casing.

Floating Production Systems - Any offshore vessel designed to gather, process and send oil and gas production onshore or to tanker facilities.

Formation Dip - The angle of a particular formation with respect to a horizontal plane.
**Gamma Ray** - An electric logging device which measures the formation’s natural gamma ray radiation and is used to characterize the formation type particularly in terms of shale content.

**Geothermal** - Refers to wells drilled into geothermally active areas whereby the completed well produces steam from natural downhole fractures particularly in volcanic areas. The term is also applied (erroneously) to characterize injection or production wells in enhanced recovery fields using steam injection techniques.

**Heavy-Weight Drillpipe (HWDP)** - Special drillpipe with longer tool joints, a center upset and thicker walls. HWDP is generally used to transition from drillcollars into drillpipe and can withstand some compressive loading like drillcollars. Example weight for 4-1/2" HWDP would be 41 lb/ft, whereas standard 4-1/2" drillpipe would be about 16.6#.

**High-angle Drilling** - Refers to drilling at large inclination angles particularly for long reaches. This would generally include wells drilled at inclination angles of 60° or more.

**Holding** - Drilling while maintaining constant inclination angle, generally achieved with a stabilized BHA.

**Hole Cleaning** - The process of removing drillcuttings from the wellbore through adequate circulation of drilling mud with appropriate viscous properties.

**Hole Enlargement** - The increase in the wellbore diameter over the nominal bit diameter caused by erosion of the formation by the circulating fluid, failure of unconsolidated formation into the wellbore, or chemical reactions which cause failure of the wellbore rock.

**Hook** - Literally a large hook located beneath the traveling block which holds the swivel and subsequently the kelly and drillstring.

**Hook Load** - Total drillstring weight or casing weight experienced by the hook or traveling block.

**IADC** - International Association of Drilling Contractors, an organization which represents companies which own and contract drilling rigs worldwide.

**Inclination** - The angle of the wellbore with respect to vertical. Inclination angle and azimuth angle and measured depth are used in combination to define the location of the wellbore.

**Jetting** - A directional drilling technique which uses an open bit nozzle to preferentially wash out one side of the wellbore thereby allowing the bit to drill more in that direction.
**Kelly** - Special pipe with either a square or hexagonal cross-section used between the swivel and the drillpipe to rotate the drillstring by rotation of the kelly bushings in the rig floor.

**Lead Angle** - The offset azimuthal angle which is used to start a well to correct for the known or predicted azimuthal walk which will occur during the drilling of the well so that the final wellbore location is correct.

**Liner** - A special casing which is run to isolate formations downhole and is cemented into place but does not extend back to surface but only back inside the previous casing.

**Lobe** - Irregular surface shape which the bit/formation interface assumes due to the mechanics of the bit drilling and dynamic effects.

**Make-up Torque** - The assembly torque, usually measured in foot-pounds, which is used to tighten together the various components of the drillstring.

**Monel** - Although actually referring to a specific Copper-Beryllium alloy, Monel is used to refer to any non-magnetic material. Non-magnetic materials are used in the drillstring to avoid distorting the magnetically-based directional surveying instruments.

**Motor** - Refers to the Positive Displacement Motor (PDM) or "Mud Motor" which uses helicoidal cavities to convert hydraulic flow into rotational speed and torque. Motors are run in the BHA and allow minimizing or elimination of surface rotation.

**Mud** - Common name for the drilling fluid, defined above.

**Near-bit Stabilizers (NBS)** - BHA stabilizers which are located very close, usually adjacent to the drillbit.

**Neutron** - An electric logging device used primarily to identify porous formations and estimate their porosity.

**NMDC** - A non-magnetic drillcollar manufactured from Monel or other special alloy.

**Off bottom** - Refers to the condition when the drillstring is elevated such that the bit is not contacting the formation.

**On bottom** - Refers to the condition when the drillstring is lowered such that the bit is in contact with the formation.

**Open-hole** - Refers to the portion of the wellbore which has not been cased off and the formation is directly exposed to the drilling fluid and drillstring.
**Packed BHA** - Refers to a BHA in which many stabilizers are used to heavily centralize the BHA in the wellbore to promote straight drilling and minimize drillcollar/wellbore contact.

**Platforms** - Fixed offshore structures set in varying water depths which support drilling and production equipment for the purpose of developing offshore energy resources.

**Polycrystalline Diamond Compact (PDC) bits** - Drilling bits which use commercial grade diamonds as the primary cutting material. The diamonds are permanently fixed into the bit surface and their are no moving parts with a PDC bit.

**Pony Collar** - A short drill collar usually around 10' long used to configure particular BHAs for specific directional tendencies.

**Pore Pressure** - The fluid pressure contained in the pore space of downhole formations. For proper well control, the hydrostatic pressure from the drilling fluid should be equivalent to the pore pressure.

**Reciprocating** - The action of picking up and lowering a drillstring or casing. This is used while cementing casings to enhance the cement’s displacement of the drilling mud and can be performed with a drillstring to try and smooth out rough parts of the wellbore or enhance cuttings removal.

**Reservoir** - The downhole porous formation which contains the oil or gas to be produced.

**Retained Cuttings** - The drill cuttings which are not properly removed by the hydraulic circulation and become trapped between the bit and the formation analogous to a boundary layer.

**Rig** - Short name for drilling rig, defined above.

**Roller-Cone Bits** - Drilling bits with bearing mounted cones which rotate and use several rows of teeth to destroy the formation. Roller-cone bits usually have three cones, but this can vary from two cones to many.

**ROP** - Rate of Penetration or the speed that drilling proceeds into the formation usually expressed in feet per hour.

**Rotary Speed** - The nominal rotating speed of the drillstring expressed in RPM.

**Rotary-Vibratory** - Drilling processes which combines rotary drilling techniques with resonance vibration procedures in an attempt to accelerate ROP.
S-Shape - A particular directional well type in which a vertical wellbore is kicked-off and slanted followed by a hold section and ultimately a drop section back to vertical. The name derives from the shape of the well profile.

Shock Absorber or Shock Sub - A special BHA tool which uses axial springs to absorb dynamic drilling loads and also modifies the axial resonance frequency of the drillstring.

Slant - A particular directional well type in which a vertical wellbore is kicked-off and slanted at a constant inclination angle.

Slick BHA - A bottom-hole assembly without any stabilizers.

Slips - Special mechanisms with hardened sharpened surfaces which indent into and grip drillpipe or casing so that it can be suspended at the surface in the slip bowl.

Sonic - An electronic logging tool which measures the travel time of sound in the formation to provide information to characterize formation type, porosity, fluid content and pressure.

Spacer - A special fluid pumped ahead of the cement to assist in displacing the drilling fluid and enhancing the bonding strength of the cement to the casing.

Spotting Fluids - Special lubricating fluids which are pumped downhole in order to assist in releasing drillpipe which has become stuck.

Stabilizer Clearance - Radial clearance between the outside diameter of the stabilizer blades and the wellbore diameter.

Stabilizer Gage - The nominal outside diameter of the stabilizer blades, usually the same as the bit diameter but sometimes intentionally smaller.

Steerable Assemblies - Any BHA containing a bent sub or bent housing motor which can be controlled to drill in a particular direction because of its configuration.

Stick/slip - Phenomena whereby drillstring motion, particularly torsional, involves friction-induced oscillations, even including stoppage or backward rotation.

Stuck Pipe - Any condition such as differential pressure or keyseating which prevents moving the drillstring or casing.

Subs - Any special BHA component, cross-over, etc.

Subsea Completion Templates - Structural guides set on the ocean floor to properly space wellbores and allow connection of wellhead and flowline equipment.
Surface Location - The absolute coordinates of the wellbore at surface. Also the physical location itself whether onshore or offshore.

Swivel - Drilling equipment held by the hook and used to connection the mud line or "gooseneck" to the Kelly so that drilling fluid can be pumped through a rotating drillstring.

Tapered Drillstring - A drillstring in which different diameter drillpipes are used to operate within the wellbore geometric constraints. Typically, smaller diameter drillpipe may be required to drill inside liners in the lower part of the well.

Tool Joint - The threaded connection used to connect joints of drillpipe together.

TOB - Torque on Bit usually expressed in foot-pounds.

Torque/Drag - Technology associated with estimating frictional loads on moving drillstrings or casings.

Trajectory - The path of a planned or drilled well usually characterized by inclination and azimuth angles as well as measured depths and true vertical depths along the wellbore.

Traveling Block - The drilling equipment hung off the drilling lines using pulleys which holds the hook and is moved using the drilling lines to lift or lower the drillstring or casing.

Tricone - Roller cone bit with three cones.

Turbines - Although similar in purpose to mud motors, turbines use multistage turbine blades to convert hydraulic flow to rotational speed and torque and generally operate at substantially higher speeds than motors.

Turn - The intentional change in the azimuthal directional of the wellbore while drilling.

Vanes - Designed profiles along the surface of fixed-cutter bits to promote proper hydraulic flow during drilling.

Walk - A change, usually non-intentional, in the azimuthal directional of the wellbore while drilling as a result of specific drilling conditions and formations.

Wear Pads - Replaceable hardened pads on directional BHA components, particularly bent motors or stabilizers which can be easily and economically replaced without having to replace the entire component due to drilling wear.
**WOB** - Weight on Bit, the physical force between the bit and the formation. Downhole measured WOB and surface calculated WOB are not necessarily the same due to uncertainty with respect to overall drillstring torque/drag conditions.

**Wellbore** - The actual hole being drilled into the earth.

**Whipstocks** - Special tapered tools used for sidetracking which are set in the well in order to force the drilling assembly to drill away from the current wellbore.

**Workover** - Any remedial procedure associated with a well such as replacing tubing, acidizing perforations, or removing scale or paraffins.