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Novel Devices and Systems for Terahertz Spectroscopy and Imaging

by

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Novel Devices and Systems for Terahertz
Spectroscopy and Imaging

Kanglin Wang

Abstract

This doctoral thesis documents my research on novel devices and systems for terahertz (THz) spectroscopy and imaging. The research is particularly focused on the manipulation of THz radiation, including subwavelength concentration and low-loss wave guiding.

One of the major obstacles for THz imaging is the poor spatial resolution due to the diffraction of the long-wavelength light source. To break this restriction, we build a THz near-field microscopy system by combining apertureless near-field scanning optical microscopy (ANSOM) with terahertz time-domain spectroscopy (THz-TDS). The experimental result indicates a sub-wavelength spatial resolution of about 10 micron. Abnormal frequency response of the ANSOM probe tip is observed, and a dipole antenna model is developed to explain the bandwidth reduction of the detected THz pulses. We also observe and characterize the THz wave propagation on the near-field probe in ANSOM. These studies not only demonstrate the feasibility of ANSOM in the THz frequency range, but also provide fundamental insights into the near-field microscopy in
general, such as the broadband compatibility, the propagation effects and the antenna effects.

Motivated by our study of the propagation effects in THz ANOSM, we characterize the guided mode of THz pulses on a bare metal wire by directly measuring the spatial profile of electric field of the mode, and find that the wire structure can be used to guide THz waves with outstanding performance. This new broadband THz waveguide exhibits very small dispersion, extremely low attenuation and remarkable structural simplicity. These features make it especially suitable for use in THz sensing and imaging systems. The first THz endoscope is demonstrated based on metal wire waveguides. To improve the input coupling efficiency of such waveguides, we develop a photoconductive antenna with radial symmetry which can generate radially polarized THz radiation matching the waveguide mode. Through THz-TDS measurements and theoretical calculation, we study the dispersion relation of the surface waves on metal wires, which indicates the increasing importance of skin effects for surface waves in the THz frequency range.
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## Contents

Abstract

Acknowledgements

Contents

Illustrations

1 Introduction

1.1 Terahertz Spectroscopy and Imaging ................................................. 1

1.2 Scope of this Thesis ........................................................................... 4

2 Background

2.1 Challenges in Terahertz Spectroscopy and Imaging .......................... 7

2.1.1 Spatial Resolution .......................................................................... 7

2.1.2 Wave Guiding .................................................................................. 11

2.2 Terahertz Time-Domain Spectroscopy .............................................. 13

2.3 Sommerfeld Waves ........................................................................... 17

3 Terahertz Apertureless Near-field Microscopy ................................. 23

3.1 Introduction ...................................................................................... 23

3.2 Sub-wavelength Resolution .............................................................. 24

3.3 Antenna Effects ................................................................................ 30

3.4 Propagation Effects .......................................................................... 37

4 Terahertz Waveguides ........................................................................ 45

4.1 Introduction ...................................................................................... 45

4.2 Conventional Metal Tubes ................................................................. 47

4.3 Metal Wire Waveguides ................................................................... 51

4.4 Application of Terahertz Metal Wire Waveguides ............................. 61

4.5 Enhanced Coupling by Using Radial Photoconductive Antenna ........ 69

5 Dispersion of Surface Plasmon Polaritons on Metal Wires ............... 74

5.1 Introduction ...................................................................................... 74

5.2 Experimental Study .......................................................................... 75

5.3 Theoretical Analysis ......................................................................... 80

6 Discussion and Future Work ............................................................... 86
6.1 Summary of Results ................................................................. 86
6.2 Future Research Directions ............................................... 88

Bibliography ................................................................. 90

APPENDIX ................................................................. 99
Illustrations

2.1 The photoconductive antenna (left) in a typical THz transmitter (right). ... 14
2.2 A diagram of the optical setup of our THz-TDS system. ......................... 15
2.3 (a) A typical time-domain THz waveform obtained with our THz-TDS system. (b) The spectral amplitude (blue) and the spectral phase (red) of the THz waveform obtained through Fourier transform. ......................... 16
2.4 (a) Electric fields of Sommerfeld wave outside a cylindrical conductor. (b) The coordinate system for solving the Sommerfeld wave. ................. 18
3.1 A schematic of the experimental setup of our THz ANSOM. .................... 25
3.2 Amplitude of the scattered THz pulses as a function of the distance between the probe tip and the sample surface in the ANSOM measurement. The insets show the THz pulse incident on the metal tip, along with the scattered waveforms measured at three different tip-sample separations as indicated in the plot. ........................................... 26
3.3 Scattering cross-section of the tip-surface system as a function of the tip-surface separation, calculated with Mie theory and a quasi-electrostatic model proposed by Knoll et al. .................................................. 27
3.4 Comparison of the calculated approaching curve (red line) and the measured approaching curve (squares). Upper part of the inset shows the calculated $C_{sca}$ which is modulated by a sinusoid dithering of the probe (the minimum tip-sample distance is 0). ........................................... 28
3.5 (a) Experimental configuration for the one dimensional imaging test. (b) The amplitude of the scattered THz pulses as a function of the lateral probe position, obtained at four different tip-sample separations: 100 nm, 200 nm, 400 nm and 800 nm. .................................................. 30
3.6 A schematic of the electro-optic sampling setup used to detect the near-field component of THz radiation in the vicinity of a copper tip (courtesy of P. Planken, Delft University of Technology, The Netherlands). .................................................. 31
3.7 THz waveforms detected in a THz ANSOM using photoconductive (PC) antenna with the setup shown in Figure 3-1 and the waveforms detected using electro-optic (EO) sampling with the setup shown in Figure 3-6. The plots (a) and (b) correspond to measurements of the THz pulses incident on the metal tip; (c) and (d) correspond to measurements of the near-field; (e) and (f) correspond to the calculated time-integral of the incident fields.

3.8 Comparison of the measurement and the calculation for the EO sampling setup. (a) Power spectrum of the measured incident THz pulses. (b) Power spectrum of the measured THz near-field pulse. (c) Power spectrum of the calculated THz near-field, with R=150 $\Omega$, C=$10^{-13}$ F, and L=$2\times10^{-11}$ H (purple), and the power spectrum of the near-field calculated using resistive coupling only (green).

3.9 A schematic of our experimental setup for studying the propagation effects in THz ANSOM.

3.10 A series of time-domain THz pulses, for different incident positions of the THz beam focused on the probe shaft. Each successive waveform corresponds to a translation of the beam spot by 1.5 mm.

3.11 (a) A close-up view of the upper and lower waveforms shown in Figure 3-10. (b) Arrival time as a function of incident position for a series of time-domain THz pulses propagating on a bare metal probe (squares) and that on a probe wrapped with a 0.5 mm PVC insulation layer (triangles). The solid lines show the least-squares fit to these data, from which the group velocities can be obtained.

3.12 THz pulses measured with a circular aluminum barrier situated between the incident spot and the probe tip, for several different barrier sizes.

3.13 Left: Experimental setup for measuring THz pulses with direct illumination of the probe tip. Right: Result of the measurement. The upper waveform is obtained with no barrier on the probe. The lower waveform is obtained with an aluminum barrier ($w=3$ mm) on the probe. The bottom curve is the transfer function obtained by deconvolving the
upper waveform from the lower one. .............................................. 43

4.1  (a) Experimental setup for the characterization of a copper-coated silica
glass tube used as a THz waveguide. The tube is 4.33 cm long and 1.74
mm inner diameter. The thickness of the copper layer is about 0.6 μm. (b)
Measured waveform of the input THz pulse. (c) Measured waveform of
the output THz pulse. ............................................................ 48

4.2  (a) Amplitude spectrum of the output waveform. The red arrow indicates a
cutoff frequency below which no waveguide modes exist. (b) Wigner-
Ville distribution of the output waveform. The dashed line indicates the
cutoff frequency of the major mode in the waveguide. ...................... 49

4.3  Dispersion relation for THz radiation propagating in a metallic tube with
1.74 mm inner diameter. Blue dots show the experimental result extracted
from Figure 4-2. The red solid line, the black solid line and the black
dashed line show the theoretical dispersion curves for mode TE_{11}, TM_{01}
and TE_{21} respectively. .......................................................... 50

4.4  Experimental setup for characterizing the propagating THz wave on a
stainless steel wire waveguide. ...................................................... 52

4.5  Time-domain electric field waveforms detected with the receiver 3 mm
above and 3 mm below the axis of the wire waveguide. ....................... 54

4.6  (a) Amplitude of the detected THz pulses as a function of the vertical
displacement of the receiver, which is measured at a propagation distance
of 24 cm. The solid curve shows the prediction of the Sommerfeld wave
model. (b) Spatial profile of the electric field around the wire obtained by
moving the THz receiver in a plane perpendicular to the waveguide axis.
................................................................................. 55

4.7  (a) Broadband group velocity of the propagating THz pulses on a metal
wire waveguide. (a) Arrival time as a function of incident position. The
insets show the THz waveforms (blue) and the corresponding spectra (red)
detected after 4 cm and 24 cm of propagation. (b) Group velocity of the
guided mode as a function of frequency, extracted from the spectra of the
THz waveforms. ................................................................. 56
4.8 Amplitude of the THz pulses as a function of the vertical displacement of the receiver, measured for different propagation distances.

4.9 (a) Blue dots show the maximum peak-to-peak amplitude of the THz pulses detected at different propagation distance. Green dots show the measurements with the receiver moved away from the end of the waveguide. (b) Red squares show the amplitude attenuation coefficient of the guided mode as a function of frequency. The solid line shows the conductivity loss of the stainless steel wire, computed using Sommerfeld's surface wave model.

4.10 Propagation of THz pulses on a 21 cm long stainless steel wire with different bend radii. The hollow squares show the amplitude of the detected THz pulses as a function of the radius of curvature. The solid squares show the amplitude attenuation coefficient as a function of the radius of curvature. The red curve shows the fit with an exponential loss model.

4.11 A Y-splitter constructed with a straight waveguide and a curved waveguide in contact with each other, as shown on the left. THz waveforms detected at A, B and C are shown on the right. The separation between A and C is 2 cm, and the THz receiver is located 3 mm below the plane of the splitter in the measurement.

4.12 The first THz endoscope. (a) Photograph of a THz endoscope which is inserted into a flask. (b) A schematic of the THz endoscopic measurement of the bottom of the flask. (c) THz waveforms obtained at the end of the output waveguide with the receiver 3 mm above and 3 mm below waveguide.

4.13 A THz endoscopic measurement with the endoscope inserted into a metal tube. A schematic of the experimental configuration is displayed in the upper figure. Waveforms obtained at the end of the output waveguide with the receiver 3 mm above and 3 mm below the waveguide are displayed in the lower figure.

4.14 Photograph of the metal wires tested as THz waveguides and the
waveforms detected in the characterization experiment for each wire. In all these measurements, the propagation distance is 10.1 cm, and a 0.9 mm stainless steel wire oriented perpendicular to the waveguide is used as an input coupler.

4.15 (a) Microscopic photograph of a photoconductive antenna with radial symmetry fabricated on a GaAs substrate. (b) The structure used to couple the THz radiation generated by the radial photoconductive antenna to the wire waveguide. A silicon lens is mounted on the GaAs substrate to collimate the THz beam, and the wire waveguide is end-coupled to the tip of the lens.

4.16 A schematic of the setup for testing radially symmetric THz transmitter. The waveguide is a 0.9 mm diameter, 27 cm long stainless steel wire which is end-coupled to the silicon lens of the transmitter. The fiber-coupled receiver is the same as that in Figure 4-4, which is oriented so that only horizontally (x) polarized component of the electric field can be detected.

4.17 Time-domain THz waveforms detected 27 cm away from the radial transmitter. The red curves show the measurements with the wire waveguide present between the transmitter and the receiver; the blue curves show the measurements with the waveguide removed. The waveforms are detected at two positions, with 3 mm and -3 mm horizontal offset of the receiver with respect to the axis of the waveguide.

5.1 Experimental setup for the measurement of the dispersion relation of surface plasmon polaritons on metal wires. The inset shows how the metal wires are supported at the distal end.

5.2 (a) Time-domain waveforms of the THz SPPs after a propagation of 20 cm on aluminum wires with different diameters. (b) Time-domain waveforms of the THz SPPs after a propagation of 20 cm on stainless wires with different diameters.

5.3 Time delay of the peak of the detected THz waveforms with respect to that of the waveform measured on the largest wire, for both aluminum and
stainless steel wires. ................................................................. 78

5.4 Discrete symbols (dots, triangles and crosses) show the experimentally determined phase velocity of SPPs propagating on aluminum wires of three different diameters, as indicated by the legend. The wire of 2388 µm diameter is used as the reference for determining the phase velocity. The calculated phase velocities for these wires are shown by the thick solid line, the thin solid line, and the dashed line respectively. ......................... 80

5.5 Dispersion relation of SPPs on a planar aluminum surface and on an aluminum wire of 25 µm diameter, together with the dispersion relation of plane waves in air. Inset A shows a zoomed-in view of the low frequency portion of the dispersion diagram. Inset B shows a further zoomed-in view of the low frequency portion indicated by the blue frame in inset A. In both insets, we plot the frequency against $k - k_0$, where $k_0$ denotes the propagation constant of plane waves in air. .................................................. 81

5.6 Calculated phase velocity over a broader bandwidth, for four wire diameters as indicated. The arrow shows the shift of the turnover point of the phase velocity curves with decreasing wire diameters. ......................... 83

5.7 An illustration of the skin effects for SPPs on cylindrical surfaces. $\delta$ denotes the skin depth. Cylindrical Surface 1 has a larger radius of curvature than Cylindrical Surface 2. .................................................. 85
Chapter 1

Introduction

1.1 Terahertz Spectroscopy and Imaging

As the name indicates, terahertz (THz) radiation means the electromagnetic radiation with frequencies around 1 THz, or $10^{12}$ Hz. Although the boundaries are somewhat arbitrary depending on different communities and different techniques involved, the THz region usually includes the portion between 100 GHz and 10 THz in the electromagnetic spectrum, which lies between microwave and infrared. This has been the most difficult region in which to operate, because these frequencies are too high for conventional electronics, and too low for standard optical techniques. For this reason, there have been few good sources and detectors for THz radiation for a long time. This has greatly hindered the development of technologies in this spectral region, and therefore this region is often referred to as “the gap in the electromagnetic spectrum”.

A milestone for the development of the techniques of generating and detecting THz radiation is the usage of short optical pulses. In the 1970s, Y. R. Shen and coworkers first demonstrated the far-infrared generation by optical rectification of picosecond laser pulses in a nonlinear dielectric crystal [1]. Subsequently, terahertz optoelectronic techniques were developed by a number of groups in the 1980s [2, 3]. One of the most important among these is terahertz time-domain spectroscopy (THz-TDS). In this technique, single-cycle terahertz electromagnetic pulses are generated either through optical rectification, or with an optoelectronic switch (photoconductive antenna) gated by
ultrafast laser pulses, usually with time duration less than 100 femtosecond. The generated pulses have an extremely broad bandwidth which covers most part of the THz frequency range. The use of time-gating in the detectors permits direct measurement of the electric field of the THz pulse as a function of time, and therefore both the spectral amplitude and the spectral phase of the pulse can be obtained simultaneously through Fourier analysis [4].

With the rapid progress in intense optical sources, compact, portable, and inexpensive femtosecond lasers became available in 1990s, which dramatically advanced the development of the THz technologies. The radiation intensity, detection sensitivity, and the bandwidth of the THz systems were all greatly improved. More importantly, as a result of the decrease in the size, cost, and the operation complexity of the THz instruments, more and more researchers entered this field, and the focus of the efforts switched from the generation and detection of THz radiation to the applications of THz radiation. With this development, THz technology became a new separate research field, and the term “THz” gained popularity over the previous “submillimeter waves” widely used in the electronics community and the “far infrared” widely used in the optics community.

One of the most distinguished features of THz radiation is the strong interaction with the rotational transitions in gas molecules. So the earlier study of THz applications focused on the atmospheric and astronomical spectroscopy [5]. With the rapid development of THz technology, numerous new techniques for THz spectroscopy have been established, such as time resolved pump-probe spectroscopy, emission spectroscopy, and correlation spectroscopy. With this trend, the THz spectroscopic study has been
extended to all types of materials, including gases [6-8], liquids [9-11], solids [12], and novel bio- and nano- materials [13-15].

Besides spectroscopy, another major application of THz radiation is imaging. Since first reported in 1995 [16], THz imaging has attracted increasing attention due to its numerous outstanding features. Since THz imaging is based on the technique of THz-TDS which permits coherent detection of the electric field of THz pulses, the detected signal contains far more information (both amplitude and phase) than the traditional optical imaging which relies on the intensity measurement. Therefore, THz images can be constructed with different methods to reveal different properties of samples [4]. These methods include 2-D translating imaging in transmission mode [16, 17], 2-D translating imaging in reflection mode [18, 19], 3-D computed tomography in transmission mode [20, 21], and 3-D computed tomography in reflection mode [22]. Since THz waves can penetrate most dielectric materials, and they are very sensitive to water content and small defects in these materials, THz imaging is well suited for a range of applications including package inspection [16, 17], moisture monitoring [16, 17], and defect analysis in complex materials such as Space Shuttle tiles [21]. Furthermore, many materials have unique spectral responses in the THz range, so THz imaging has the signature or fingerprint capacity to identify materials and provide much information that is generally absent in optical, X-ray and NMR images. In addition, THz imaging is a nondestructive technique and is perfectly safe to people. These unique features greatly extends the range of the applications of THz imaging and make it a very promising technique for medical diagnosis [23, 24] and security screening [25].
1.2 Scope of this Thesis

The goal of my doctoral research is to explore new devices and systems for THz spectroscopy and imaging. In particular, I am interested in new techniques for facile manipulation of THz radiation, including subwavelength concentration and low-loss wave guiding, with the goal of extending the application capacity of terahertz spectroscopy and imaging. The thesis which follows describes the investigation of these techniques, both the basic concepts and the potential applications.

For a better understanding of the motivation of this research, relevant background information is given in Chapter 2. Two major challenges in THz spectroscopy and imaging are presented. One of them is the poor spatial resolution of the THz optical systems due to the long wavelength of the radiation; the other one is the difficulty of efficiently guiding THz radiation due to the material properties and the special application requirements in this spectral range. The previous efforts searching for the solutions in these areas are also reviewed. This chapter also includes an introduction of terahertz time-domain spectroscopy, a technique which is extensively used in the experimental work of this thesis, as well as a brief description of Sommerfeld waves, the theoretical model used in the research work described in Chapter 4 and Chapter 5.

Chapter 3 describes our investigation of subwavelength concentration of THz beams with a sharp metallic probe tip. By combining this technique with THz-TDS, we build a THz apertureless near-field scanning optical microscopy (ANSOM) and achieve sub-wavelength spatial resolution. To explore the compatibility of near-field microscopy with broadband light sources, we study the scattering characteristics of THz pulses at the ANSOM probe tip. We observe a large bandwidth reduction of the scattering THz pulses,
and explain this result using a dipole antenna model. We also observe that free-space broadband THz pulse can be coupled into a propagating mode along the shaft of the ANSOM probe. The spatial extent of this guided mode and its velocity are determined. In addition, we consider the possibility of multiply reflected modes propagating along the probe, an effect which will be significant in near-field spectroscopic measurements. Most importantly, we develop a novel broadband THz waveguide motivated by the study of the propagation effects. Details of this study are described in the following chapter.

Chapter 4 describes our exploration of novel THz waveguides. First we present our THz-TDS study of the performance of metal tubes, the conventional microwave waveguides. Then we demonstrate a novel waveguiding structure for broadband THz pulses, in which THz waves are confined and guided along a bare metal wire in the form of surface waves or surface plasmon polaritons (SPPs). This waveguide exhibits extremely low attenuation, because of the minimal exposed metallic surface area. It also supports propagation of broadband radiation with negligible group velocity dispersion, making it especially suitable for use in terahertz spectroscopy and imaging systems. In addition, the structural simplicity lends itself naturally to the facile manipulation of the guided pulses, including coupling, directing and beam splitting. As an example of the applications of the wire waveguide, we demonstrate the first THz endoscope. We also develop a radially symmetric photoconductive antenna which greatly improves the input coupling efficiency of the wire waveguide.

Chapter 5 describes our in-depth study of THz surface waves on metal wires, and the comparison of this form of surface wave with SPPs at higher frequencies. We present the THz-TDS measurements and the theoretical analysis of the dispersive behavior of
SPPs on cylindrical metal surfaces in the THz frequency range. The measurements reveal a unique dispersion structure that is inconsistent with a simple extrapolation of the high frequency portion of the dispersion diagram for SPPs on a planar metal surface, and also distinct from that of SPPs on metal nanowires observed at visible and near-infrared frequencies. Numerical solution of Maxwell’s equations gives similar results, showing that the dispersive behavior of SPPs on a cylindrical metal surface at terahertz frequencies is quite different from that of SPPs on a flat surface. These results indicate the increasing importance of skin effects for SPPs in the THz frequency range, as well as the enhancement of such effects on curved surfaces.

Finally, Chapter 6 concludes this thesis and suggests future directions of this research.
Chapter 2

Background

2.1 Challenges in Terahertz Spectroscopy and Imaging

2.1.1 Spatial Resolution

As shown in Chapter 1, THz spectroscopy and imaging have attracted considerable attention for their various applications. It soon became clear that many such studies need to be conducted in a microscopic scale. For example, THz-TDS has proven to be a powerful tool to study carrier mobility and distribution in semiconductors [26-28], therefore it is a promising technique for characterization of semiconductor devices, especially for devices planned to operate at higher gigahertz frequencies. In addition, more and more interesting biomedical applications of THz spectroscopy and imaging have been demonstrated. THz-TDS can be used to distinguish different organic compounds, biomolecules, and diseased tissues, so it can be potentially used for DNA mapping, living cell inspection, and medical diagnosis [13, 24, 29]. For all the above applications, microscopic resolution of the THz system is required.

However, since THz-TDS measurements are based on far-field optics, the spatial resolution is restricted by the diffraction limit of the light source, i.e. the THz beams. According to Rayleigh’s criterion, the limit of resolution for any optical imaging system is [30]:

\[ (\Delta l)_{\text{min}} = 1.22 f\lambda / D \] (2-1)
Here \( f \) is the focal length; \( D \) is the aperture diameter of the optical instrument; \( \lambda \) is the wavelength. \((\Delta l)_{\text{min}}\) has the same value as the radius of Airy Disk, the central high-irradiance circular area in the focal plane of the optical lens which defines the size of the focal spot. A typical THz image is constructed pixel by pixel by translating the object at the focus of the THz beam, so it is straightforward to understand the spatial resolution of a standard THz imaging system using equation (2-2). Usually the values of \( f \) and \( D \) are of the same order, therefore the Rayleigh's criterion restricts the spatial resolution of a standard THz-TDS system to the scale of the wavelength of THz radiation, which is around 1 millimeter. Obviously, this restriction greatly limits THz spectroscopy and imaging in the microscopic scale applications.

To overcome the diffraction limit of conventional optical systems, near-field microscopy [31, 32] is required. In 1998, Hunsche et al. demonstrate the first THz near-field imaging system [33]. In their setup, the free space THz beam is coupled to a tapered metal tube with a subwavelength aperture in the end, analogous to the fiber probes used in a near-field scanning optical microscopy (NSOM). Images are constructed by placing the sample close to the aperture and scanning the sample in a plane perpendicular to the axis of the metal tube. In this way, a spatial resolution about 50 \( \mu m \) is obtained, which corresponds to 1/4 of the averaged wavelength. Two years later, Mitrofanov and coworkers demonstrated that a subwavelength aperture together with a protruding high refractive index tip can be integrated to THz photoconductive antennas, both for transmitters [34] and for receivers [35]. A spatial resolution of 60 \( \mu m \) was achieved with the aperture integrated THz transmitter (the illumination mode), and a resolution of 7 \( \mu m \) was achieved with the aperture integrated receiver (the collection mode) in 2002 [36].

such aperture based near-field THz imaging techniques, the spatial resolution is determined by the size of the aperture, but not by the wavelength, so the diffraction limit is overcome. However, the high requirement of precise alignment in the imaging setup makes these techniques difficult to implement in practice. In addition, waveguide effect in the small apertures strongly attenuates the signal, which leads to the decrease of the signal to noise ratio in the THz images.

In 2000, Chen et al. reported a dynamic aperture approach to achieve subwavelength resolution in THz imaging [37]. They applied an optical gating beam on a semiconductor wafer as the near-field aperture. The optical beam generates photocarriers at the beam spot that changes the local conductivity of the semiconductor wafer, therefore the THz transmission through this area is modulated. In this method, the size of the dynamic aperture is determined by the focus of the optical beam, which is much smaller than that of a THz beam. With this method, a spatial resolution less than 50 µm was obtained in the THz imaging of a metal circuit deposited on a GaAs wafer. However, the application of this technique is limited to semiconductor surfaces, in which the concentration of carriers can be modulated by an optical beam.

Besides aperture based near-field optical microscopy, another promising subwavelength imaging technique is the apertureless near-field scanning optical microscopy (ANSOM). It offers a simple way to reach ultrahigh spatial resolution and can be applicable to variable sample surfaces [38, 39]. The basic idea of this technique is analogous to the phenomenon that electric field lines are concentrated around sharp geometries of a conductor in a static electric field. Similarly, a sharp metallic structure can be used as an optical antenna to concentrate electromagnetic waves in a
subwavelength region, and the size of this region is determined by the dimension of the sharp geometry not by the wavelength of the light source [40]. Van der Valk et al. first introduced this technique to the THz imaging in 2002 [41]. In their method, the THz electric field is concentrated by a sharp copper tip, and the concentrated field is electrooptically measured in a GaP crystal placed very close to the tip. By raster scanning the tip along the surface of the crystal, the profile of the THz spot can be measured, and microscopic structures on the crystal surface can be imaged. In 2003, a similar method with different experimental configuration was demonstrated by Chen and coworkers [42]. In their experiment, a tungsten tip is held close to the sample where the focused THz beam is incident. By scanning the sample with respect to the tip and the focus of the THz beam, the reflected THz pulses can be modulated by the structures on the surface of the sample, and an image of the surface can be constructed. Based on the result with this method, a spatial resolution of 150 nm was claimed. However, signal detected in this experiment is the reflected radiation. Not like the radiation scattered from the probe tip, the radiation reflected from the sample surface contains little information of the near-field component in the vicinity of the tip. Furthermore, the ac coupling model used to explain the experimental data does not involve any near-field features of the tip-surface system, and the values of L can C in that model are less realistic [43]. So it is still under debate whether or not this experiment is a valid demonstration of sub-micron resolution of near-field THz imaging. In addition, since the line scan measurement is conducted over the edge of a 2 μm thick gold layer on a silicon substrate, the 150 nm transition length in the detected signal is more like a height artifact than a confirmative evidence of spatial resolution [44].
2.1.2 Wave Guiding

Besides poor spatial resolution, another major obstacle for THz spectroscopy and imaging is the guiding devices for THz radiation. Most of the THz applications demonstrated so far overwhelmingly rely on free-space transport of the terahertz beam, using bulk optical components. But in many real-world situations, the sample or region to be studied may not be readily accessible to a line-of-sight beam. Common devices such as optical fiber-based sensors or medical endoscopes rely on the guided wave delivery of light to the remote sensing location. In order to extend this paradigm to THz applications, the development of optimized guided wave devices is required. The development of practical THz waveguides will dramatically expand the application of THz-TDS, especially in many new areas such as nanometer thin-film measurements [45].

The development of THz waveguides has been hindered by the material properties and the application requirements in this spectral range. On the one hand, the characteristics of materials at THz frequencies make it extremely difficult to build a fiber to guide THz beams over a long distance. The most transparent materials for this range are crystalline (e.g., high resistivity silicon) [46], and thus are costly, fragile, and challenging to form into specific geometries for waveguide configurations. Other materials, such as low-loss polymers or glasses, are more malleable but exhibit prohibitively high absorption losses for propagation distances of more than a few centimeters [46]. For this reason, THz waveguides generally must rely on propagation in air, rather than via dielectric confinement as in an optical fiber. On the other hand, many THz applications rely on the use of broadband pulses for time-domain analysis and spectroscopic applications. To avoid pulse reshaping during propagation, the significant
additional constraint of low dispersion is also required. But for many conventional metal waveguides (e.g., metal tubes), pulse reshaping in propagation is difficult to avoid, due to the extreme dispersion near the waveguide cutoff frequencies \[47, 48\]. Furthermore, finite conductivity of metals can lead to considerable losses in the wave propagation \[47, 49\].

Early work on THz waveguides focused on the coplanar or microstrip transmission lines integrated onto dielectric substrates \[50\]. The major disadvantage of this type of waveguide is the high propagation loss. The power absorption coefficient of a typical THz coplanar transmission lines is much larger than \(1 \text{ cm}^{-1}\). This value is unacceptably high for applications in THz spectroscopy and imaging, in which the guiding length is at least several centimeters. Furthermore, the combination with planar dielectric substrates restricts the application of this type of waveguide in many realistic situations. Great efforts have been devoted to finding waveguiding structure for THz pulses within the last few years. Various guides with quasi-optical coupling have been demonstrated. Most of these THz waveguides have been based on conventional guiding structures used for microwaves, infrared, or visible light, such as metal tubes \[48, 51, 52\], plastic ribbons \[53\], or dielectric fibers \[54\]. There have also been reports on the application of the latest technology of photonic crystal fibers to THz radiation \[55, 56\]. In all of these cases, the utility for transport of THz pulses is limited by group velocity dispersion of the guided waves. The most promising studies have reported dispersionless propagation in parallel metal plate waveguides \[49, 57, 58\]. But in this case the propagation loss is still very high (amplitude absorption constant \(-0.1 \text{ cm}^{-1}\)), due in large part to the finite conductivity of the metal plates. In addition, the cross-sectional area of
the waveguide is too large for many of the proposed THz applications, including in particular medical diagnosis.

2.2 Terahertz Time-Domain Spectroscopy

Terahertz time-domain spectroscopy is not only a powerful tool to study various phenomena in the THz frequency range, it also provides the platform for many THz applications. This technique is extensively used is the experimental work in this thesis. As mentioned in Chapter 1, THz-TDS can generate broadband THz pulses and can directly measure the electric field of the pulses in the time-domain. There are generally two methods used for THz pulse generation in this technique: optical rectification and photoconductive switch. There are also two different ways to detect the THz pulses: electro-optic detection and photoconductive switch. In our system, we use photoconductive switches (photoconductive antennas) for both the generation and the detection.

Figure 2-1 shows the structure of a typical photoconductive antenna in a THz transmitter. The gold antenna pattern is deposited on a GaAs substrate by optical lithography. A DC bias is applied between the two electrodes. In the normal state, there is no current flowing through the gap because GaAs is not conductive. During operation, a beam of ultrafast laser pulses is focused on the area between the electrodes (usually near the anode). The laser used is at a wavelength (~ 800 nm) corresponding to the bandgap of GaAs so that photocarriers can be generated under illumination. When a laser pulse impinges the substrate, the generated photocarriers will make the gap conductive and produce a transient photocurrent through the antenna. Although the photocurrent can last as long as one nanosecond (for intrinsic GaAs), the onset of the photocurrent is much
faster. Since the electric field generated by the acceleration of carriers is proportional to the time derivative of the current, the onset of the photocurrent in a photoconductive antenna will produce a short burst of electromagnetic radiation at THz frequencies. For such a structure, most of the generated radiation power is transmitted into the dielectric [59, 60], so a substrate lens made of materials with similar refractive index with GaAs can be used to efficiently collect the THz radiation and couple it into free space [61].

![Image of photoconductive antenna](image)

**Figure 2-1:** The photoconductive antenna (left) in a typical THz transmitter (right).

The THz receiver has a similar structure with the transmitter, except that the DC voltage supply on the photoconductive antenna is replaced with a sensitive current detector. When an ultrashort pulse impinges the receiver, the gap becomes conductive, and the electric field of the THz pulse will induce a current through the antenna which is detected by the current detector. The detected current is proportional to the convolution of the electric field and the conduction time window provided by the laser pulse. The measurable bandwidth of the THz pulse is limited by the width of the conduction time window. For this reason, THz detectors have a higher requirement of the short carrier lifetime in the substrate compared with THz detectors. Usually low-temperature-grown
GaAs need to be used in THz detectors to provide a fast photoconductive switch [62]. The laser beam to the transmitter and the beam to the receiver are from the same source to make sure that the detection laser pulses are synchronized with the THz waveform. Since the time duration of the laser pulse is much smaller than the variation of the THz electric field, the detection process mentioned above only records an instant value of the THz electric field. In order to record a complete THz waveform, the relative time delay between the laser pulse reaching the receiver and the THz pulse is scanned. The measurement with a successive pulse train provides a series of sample points which are used to construct the waveform.

![Diagram](image)

**Figure 2-2: A diagram of the optical setup of our THz-TDS system**

Figure 2-2 shows a diagram of the basic optical setup of our THz-TDS system used in most experiments in this thesis. It consists of an ultrafast laser source, a group velocity dispersion compensator, and a fiber-coupled THz-TDS unit developed by Picometrix, Inc. [63] The laser source generates 100 femtosecond pulses with a central
wavelength around 800 nm at a repetition rate of 80 MHz. The laser pulses first go through a group velocity compensator before entering the THz-TDS, in order to precompensate for the dispersion of the fibers in the THz-TDS unit which will stretch the laser pulses and reduce the THz bandwidth. This is done by making the laser go through a pair of gratings which provide a negative dispersion with the same amount of the dispersion as in the fibers. In the control box of the THz-TDS, the laser is split into two beams, one to the THz transmitter and the other one to the THz receiver. The beam to the receiver goes through a scanning optical delay line to provide the relative time delay between the laser pulses and the THz pulses generated by the transmitter. Both the THz transmitter and THz receivers are fiber coupled so that they can be easily moved to measure a THz wave front at different spatial locations. As shown in the following chapters, this is a very important feature to our research.

Figure 2-3: (a) A typical time-domain THz waveform obtained with our THz-TDS system. (b) The spectral amplitude (blue) and the spectral phase (red) of the THz waveform obtained through Fourier transform.
Figure 2-3 (a) shows a typical time-domain THz waveform obtained with our THz-TDS, with two polyethylene lenses standing between the transmitter and the receiver to focus the THz beam. The detected THz waveform is a single cycle electromagnetic pulse with time duration on the order of one picosecond. The two small oscillations after the main pulse are due to optical and electrical impedance mismatches in the system, such as the silicon-air interface in the transmitter module. Through Fourier transform of the time-domain waveform, we can get both the spectral amplitude and the spectral phase of the THz pulse. As shown in Figure 2-3 (b), the THz pulse has a nearly linear phase and a broad bandwidth covering a large part of the THz frequency range.

2.3 Sommerfeld Waves

The propagation of THz pulses along a bare metal wire is extensively studied in this thesis. Electromagnetic wave propagation on the cylindrical surface of a conductor with finite conductivity can be described by a theoretical model proposed by Sommerfeld in 1899 [64]. He demonstrated an azimuthally symmetric guided mode as a valid solution of Maxwell’s equations for such a system. Therefore this mode is often referred to as a Sommerfeld wave. Sommerfeld wave is the zeroth-order transverse magnetic (TM) mode for a system consisting of an infinitely long cylindrical conductor embedded in a homogeneous dielectric, with an electric field profile as depicted in Figure 2-4 (a). Other modes, although existing, present extremely high loss and vanish almost immediately upon excitation. The solution of Sommerfeld waves and their characteristics are described by Goubau in 1950 using a series of approximations which are valid for microwave frequencies [65]. In 1962, King et al. described the solution of Sommerfeld waves in the millimeter range, by using fewer or different approximations [66]. In this
section, we will review the previously described approaches used to obtain the solution of Sommerfeld waves, as well as the method we use for the solution of cylindrical surface waves in a much broader frequency range.

![Diagram of electric fields of Sommerfeld wave outside a cylindrical conductor and the coordinate system for solving the Sommerfeld wave.](image)

Figure 2-4: (a) Electric fields of Sommerfeld wave outside a cylindrical conductor. (b) The coordinate system for solving the Sommerfeld wave.

Now let us consider an infinitely long conductor with circular cross section embedded in an infinite, homogeneous and lossless dielectric (usually air), as shown in Figure 2-4. The radius of the conductor cylinder is \( a \). The permeability and dielectric constant of the conductor are \( \mu_c \) and \( \epsilon_c \) respectively, while those of the dielectric are \( \mu_d \) and \( \epsilon_d \) respectively. Using the cylindrical coordinate system shown in Figure 2-4 (b), the field components for a Sommerfeld wave propagating in the \( z \) direction are expressed as:

\[
E_r = jA \frac{\hbar}{\gamma} Z_1(\gamma r) e^{j(\omega t - kz)}
\]

(2-2)

\[
E_z = A Z_0(\gamma r) e^{j(\omega t - kz)}
\]

(2-3)

\[
H_\phi = jA \frac{k^2}{\omega \mu \gamma} Z_1(\gamma r) e^{j(\omega t - kz)}
\]

(2-4)
where $h$ is the propagation constant of the guided wave, which has the same value in the conductor and in the dielectric. In the conductor, the cylinder functions $Z_0$ and $Z_I$ are the Bessel functions $J_0$ and $J_I$, while in the dielectric they are the Hankel function of the first kind, $H^{(1)}_0$ and $H^{(1)}_I$ respectively. The parameter $\gamma$ is defined by:

$$\gamma_d^2 = k_d^2 - h^2$$  \hspace{1cm} (2-5)

$$\gamma_c^2 = k_c^2 - h^2$$  \hspace{1cm} (2-6)

where $k$ is the propagation constant in a single medium $k=\omega(\epsilon\mu)^{1/2}$. The subscript $d$ and $c$ refer to the dielectric and the conductor respectively. Using the boundary conditions that $E_z$ and $H_\phi$ must be continuous at the surface of the conductor ($r = a$) and the relation given by (2-5) and (2-6), we can obtain the equation

$$\frac{k_c^2 J_1(\gamma_c a)}{\mu_c \gamma_c J_0(\gamma_c a)} = \frac{k_d^2 H^{(1)}_1(\gamma_d a)}{\mu_d \gamma_d H^{(1)}_0(\gamma_d a)}$$  \hspace{1cm} (2-7)

from which $\gamma_d$, $\gamma_c$ and $h$ can be determined and also the propagation characteristics of the guided wave.

To solve the transcendental equation (2-7), Goubau used the following approximations:

$$k_c = (\omega \mu_c \sigma_c)^{1/2} e^{-j\pi/4}$$  \hspace{1cm} (2-8)

$$|\gamma_c a| >> 1$$  \hspace{1cm} (2-9)

$$|\gamma_d a| << 1$$  \hspace{1cm} (2-10)

Approximation (2-8) assumes that $\sigma_c$ is a frequency-independent value, and the DC conductivity of the conductor was used in the calculation. Approximation (2-9) is equivalent to assuming that the conductor radius is large compared with the skin depth,
while approximations (2-10) embodies the assumption that the conductor radius is not too large compared with the wavelength of the radiation. With these approximations, (2-7) can be simplified to

\[ \xi \ln \xi = \eta \]  \hspace{1cm} (2-11)

with

\[ \xi = (-j0.89\gamma_d a)^2 \]

\[ \eta = 2(0.89)^2 \frac{k_d^2 a \mu_e}{|k_e| \mu_d} e^{j3\pi/4} = |e^{j\pi/4} \]

Then equation (2-11) is then solved by splitting it into two real equations [65].

The above method has been proved valid in the microwave frequency range. At higher frequencies, however, the approximation (2-10) is not valid any more. King et al. found that \(|\gamma_d a|\) is not small enough to use the zero representations of the Hankel functions in the millimeter range. Instead, they used approximation (2-8) and (2-9), as well as

\[ \gamma_c \equiv k_c = |k_c| e^{-j\pi/4} \]  \hspace{1cm} (2-12)

which takes the assumption \(|\gamma_c| > |\gamma_d|\) and \(|k_c| \gg |k_d|\). Under these approximations, equation (2-7) is simplified to

\[ \frac{H_0^{(1)}(\gamma_d a)}{H_1^{(1)}(\gamma_d a)} = \frac{k_d^2}{\gamma_d k_c} \left( j \frac{\omega^{3/2}}{\gamma_d \sigma_c^{1/2}} \mu_d \right)^{1/2} e^{j3\pi/4} \]  \hspace{1cm} (2-13)

Then by equating the real and imaginary parts of each side of (2-13), they solved the equation by using graphical methods and the Bessel function tables. After \(\gamma_d\) is obtained, they used the approximation

\[ h \equiv k_d - (\gamma_d a)^2 / 2k_d a^2 \]  \hspace{1cm} (2-14)
which assumes $\gamma_d$ is much smaller than $k_d$ and wrote the propagation constant as follows:

$$h \equiv (k_d - \frac{|\gamma_d a|^2}{2k_d a^2} \cos 2\theta) - (\frac{|\gamma_d a|^2}{2k_d a^2} \sin 2\theta) j$$  \hspace{1cm} (2-15)\]

The phase velocity of the guided wave can be obtained from the real part of $h$, and the attenuation of the wave can be obtained from the imaginary part of $h$.

From ref. [66] we can see that the above method provides fairly accurate solutions for Sommerfeld waves between 100 GHz and 1 THz, a frequency range matching our THz-TDS system very well. However, in this thesis we need to consider the properties of cylindrical surface waves in a much broader frequency range, from millimeter waves up to the visible light range. Some of the approximations used in the above method are not valid any more in such a broad frequency range and need to be replaced with more accurate expressions. On the other hand, today’s advance in computing technologies enables us to do more rigorous calculations than half a century ago. So in our calculation, we only keep the approximation (2-9) and replace all others with rigorous calculations. The validity of (2-9) for our calculation is discussed in Chapter 5. One important replacement in our method is that equation (2-8) is replaced by the original definition of the propagation constant:

$$k_c = \omega (\varepsilon_c \mu_c)^{1/2}$$  \hspace{1cm} (2-16)\]

where the frequency dependent complex value of $\varepsilon_c$ is obtained using Drude model:

$$\varepsilon_c = 1 - \frac{\omega_p^2}{\omega^2 + j\omega \omega_r}$$  \hspace{1cm} (2-17)\]

In (2-17), $\omega_r$ is the electron collision rate and $\omega_p$ is the plasma frequency defined by:

$$\omega_p = \sqrt{\frac{Ne^2}{\varepsilon_0 m_0}}$$  \hspace{1cm} (2-18)\]
where $N$ is the electron density in the conductor and $m_0$ is the electron rest mass. The parameters used in the calculation are taken from refs. [67] and [68].

To get the solution of equation (2-7), we take $\gamma_0 a$ as the only complex argument, and replace all other terms with $\gamma_0 a$ and constants by using the relation given by (2-5), (2-6) and (2-16). The equation is then transposed and numerically solved in MATLAB. Details of this process are provided in an example in the APPENDIX.
Chapter 3

Terahertz Apertureless Near-field Microscopy

Some of the results described in this chapter have been previously published in reference [69] and [70].

3.1 Introduction

One of the most active areas of research in the field of terahertz spectroscopy and imaging involves improving the resolution below the diffraction limit. Before the study described in this thesis, several near-field techniques have been used in conjunction with THz-TDS to obtain enhanced resolution [33-35, 37, 41]. As discussed in Chapter 2, the applicability of these techniques is limited by the aperture induced attenuation [33], the restrictions on samples [37, 41], or the difficult in practical operations [34, 35]. In this chapter we describe the implementation of apertureless near-field microscopy technique on terahertz imaging. This technique is readily applicable to any surfaces, and provides enhanced signal-to-noise ratio in comparison with techniques involving small apertures.

Apertureless near-field scanning optical microscopy, or ANSOM, has been pioneered by several groups in the past decade [38, 39, 71]. In this technique, a metallic probe is used to concentrate the electromagnetic field near a sample surface and couple the evanescent field into a propagating wave which can be easily detected. By scanning the tip near the sample surface, features much smaller than the wavelength of the incident beam can be obtained by analyzing the scattered far field. The resolution is determined not by the wavelength, but by the size of the metal tip and its proximity to the surface.
Extra attention needs to be paid to the use of THz radiation as the light source for ANSOM. Since the THz pulses have a very large bandwidth compared with the continuous-wave lasers used in a standard ANSOM system, the frequency response of the near-field probe has a significant influence on the measurements. In addition, it has been found that not only the probe apex but also the probe shaft and the shape of the whole tip structure play important roles in ANSOM experiments [72, 73]. This effect will be especially important in the THz frequency range because it is very difficult to focus the long wavelength THz radiation only on the probe apex and not on the shaft. A concern which immediately arises is the influence of propagation effects of the incident radiation along the probe tip. These effects can influence the phase of the electric field scattered from the tip, and therefore have an impact on the interferometric apertureless microscopy [39, 72, 74] and also on the growing field of near-field optical spectroscopy [73, 75, 76]. So it is important to characterize the propagation effects in ANSOM measurements. In this chapter, antenna effects and propagation effects are also investigated in addition to the exploration of subwavelength resolution in THz ANSOM.

3.2 Sub-wavelength Resolution

We construct a THz near-field microscopy by combining a typical ANSOM configuration with our THz-TDS system. Figure 3-1 shows a schematic of the experimental setup. THz pulses from the transmitter are first collimated by a polyethylene lens and then focused onto the near-field probe tip by a second lens. The probe is a beryllium-copper needle commercially available from Micromanipulator. The shaft is 0.5 mm diameter and coated with a thin layer of tin to prevent oxidation. The tip is carefully polished into about 5 μm radius against a fiber polishing film. The sample is a
featureless gold-coated silicon wafer, placed in close vicinity of the tip. The distance between the tip and the gold surface is precisely controlled by a piezoelectric transducer. In such a configuration, the tip strongly interacts with its image in the metal surface, and converts the localized evanescent field around the tip to propagating radiation through a scattering process [71]. The scattered radiation is modulated at 750 Hz by vibrating the probe tip normal to the surface with an amplitude of about 750 nm, and the detected signal is demodulated using a lock-in amplifier to ensure that only radiation coming from the near-field of the tip apex are measured [38, 74].

![Diagram](image)

**Figure 3-1: A schematic of the experimental setup of our THz ANSOM.**

The high resolution capacity of ANSOM comes from the fact that the near-field components of the radiation are rapidly attenuated outside a region much smaller than the wavelength. This means for the experimental configuration in Figure 3-1, the scattered THz pulses can only be detected at a very small tip-sample distance, and the detected signal is very sensitive to this distance. So as the first step to demonstrate the feasibility
of the ANSOM for THz radiation, we measure the amplitude of the scattered THz pulse as a function of the distance between the probe tip and the sample surface, as shown in Figure 3-2. The zero distance is determined by monitoring the DC resistance between the sample and the probe. The insets show the measured waveforms for three different tip-sample separations, along with the input waveform. The approaching curve shows high sensitivity of the amplitude of the scattered pulse to the tip-sample distance, especially when this distance is less than 1 μm. So this result clearly demonstrates the potential sub-wavelength resolution in THz imaging by using an ANSOM configuration.

Figure 3-2: Amplitude of the scattered THz pulses as a function of the distance between the probe tip and the sample surface in the ANSOM measurement. The insets show the THz pulse incident on the metal tip, along with the scattered waveforms measured at three different tip-sample separations as indicated in the plot.
The measured approaching curve can be understood with a quasi-electrostatic model proposed by Knoll et al. [77] In this model, the probe tip is represented by a small sphere with radius \( a \) at a small distance \( z \) from a plane sample. The complex dielectric constant of the probe material and the sample is \( \varepsilon_p \) and \( \varepsilon \) respectively. When the electric field is polarized perpendicularly to the sample surface, the incident field at the probe dipole is enhanced by its image field in the sample, and the near-field interaction between them is described by an effective polarizability \( \alpha_{\text{eff}} \) of the probe-sample system, as depicted by the inset in Figure 3-3, where \( \alpha \) is the polarizability of the sphere: \( \alpha = 4\pi a^3 (\varepsilon_p - 1)/(\varepsilon_p + 2) \). The scattered radiation from the tip can be described by Mie theory with the scattering cross-section \( C_{\text{sca}} = \frac{k^4}{6\pi} |\alpha_{\text{eff}}|^2 \). The calculated \( C_{\text{sca}} \) of the probe-sample system in our ANSOM measurement is shown by the blue curve in Figure 3-3.

![Figure 3-3: Scattering cross-section of the tip-surface system as a function of the tip-surface separation, calculated with Mie theory and a quasi-electrostatic model proposed by Knoll et al.](image-url)
To compare the calculation with the experimental data, the modulation and demodulation process in the data acquisition in our measurement need to be considered. Due to the non-linear characteristic of the cross-section approaching curve, $C_{sca}$ shows an anharmonic behavior when it is modulated by a sinusoid variation of the tip-sample distance, as shown by the upper part of the inset in Figure 3-4. Such a signal contains many harmonics of the fundamental modulation frequency, as shown by the lower part of the inset. Since we use the same frequency for modulation and demodulation, only the first harmonic component in the modulated signal is recorded, as depicted by the red bar in the plot (the first bar denoting the DC component). To incorporate this process into our calculation, a sinusoid modulation of the tip-sample distance is applied with the same parameters as in the experiment, and the first harmonic component is extracted as the

![Graph](image)

**Figure 3-4:** Comparison of the calculated approaching curve (red line) and the measured approaching curve (squares). Upper part of the inset shows the calculated $C_{sca}$ which is modulated by a sinusoid dithering of the probe (the minimum tip-sample distance is 0).
calculated $C_{sca}$ for each tip-sample distance. The red curve in Figure 3-4 shows the square root of the calculated $C_{sca}$ (normalized) which is proportional to the amplitude of the scattered radiation. For comparison, the normalized amplitude of the scattered THz pulses obtained in our measurement is also displayed, which shows a good agreement with the calculation.

To test the spatial resolution of our THz ANOSM system, we perform a simple one dimensional imaging with this setup. The sample in this experiment is a patterned gold surface, with a step about 400 nm in height, and a transition region of less than 10 μm in lateral extent. By scanning the sample relative to the probe tip, we can form a line scan image of the surface topography. As indicated by the approaching measurement shown in Figure 3-2, a characteristic feature of near-field imaging is the strong dependence of the spatial resolution on the tip-sample distance. Taking this into consideration, we perform the line scan measurement at four different separations between the tip and the sample surface (the level closer to the tip): 100 nm, 200 nm, 400 nm and 800 nm. Figure 3-5 shows the result of these line scans, displaying the amplitude of the scattered THz pulses as a function of the lateral position of the sample. As expected, the spatial resolution increases as the tip-sample separation decreases. For the closest approach (100 nm), the transition essentially follows the profile of the surface as measured with atomic force microscopy (AFM), as shown by the red line in the plot. This result is consistent with a 10 μm spatial resolution, closely matching the common estimate for an ANSOM system in which the spatial resolution is determined by the size of the probe tip. Since the scattered radiation is rather low frequency (~58 GHz), this 10 μm resolution corresponds to $\lambda/520$. 
3.3 Antenna Effects

From Figure 3-2 it is obvious that the scattered THz pulses measured in our ANSOM experiment has a very different shape compared with the incident THz pulses. Associated with this shape change, the bandwidth is greatly reduced in the detected signal. Very interestingly, the same phenomenon has also been observed by Planken’s group at Delft University of Technology, although they use a different setup in which the near-field of THz radiation in the vicinity of a copper tip is directly measured using electro-optic sampling. For practical applications of THz ANSOM, the bandwidth reduction in the near-field signal could severely limit the utility of this technique, so it is essential that the origin of this phenomenon is understood. Cooperated with the Planken group, we
developed a dipole antenna model to explain the near-field bandwidth reduction in THz ANSOM. We demonstrate that, regardless of whether the near-field signal is observed directly under the tip or in the far-field, it is approximately proportional to the time-integral of the incident THz signal. For the simplest case, when the THz electric field is measured in the near-field of the metal tip, a dipole antenna model is used to calculate the THz near-field of the tip. A comparison between these calculations and the measurements shows that the metal tip is predominantly resistively coupled to the THz pulses, with an increasing inductive component for increasing frequency. Our model qualitatively reproduces all the essential features observed in the measurements and provides insight into ANSOM experiments in general.

![Diagram](image)

**Figure 3-6:** A schematic of the electro-optic sampling setup used to detect the near-field component of THz radiation in the vicinity of a copper tip (courtesy of P. Planken, Delft University of Technology, The Netherlands).

Figure 3-6 shows a schematic of the electro-optic sampling system used by van der Valk et al. [41] to measure the near-field component of the THz radiation in the vicinity of a copper tip. THz pulses from an ultrafast photoconductive switch are incident
on a copper tip of 20 μm diameter, which is held close to the surface of a (100) oriented GaP crystal. Synchronized laser pulses are focused to a 10 μm spot size directly underneath the tip as a probe beam, where the polarization of the laser beam is changed by the electric field near the tip (which is perpendicular to the crystal surface). By measuring the polarization change of the reflected probe beam with a differential detection setup, the THz near field in the vicinity of the tip can be detected. The orientation of the GaP crystal ensures that only the electric fields perpendicular to the crystal surface (which develop near the tip apex) is detected, while the parallel polarized electric field of the incident THz pulse is not detected.

Figure 3-7: THz waveforms detected in a THz ANSOM using photoconductive (PC) antenna with the setup shown in Figure 3-1 and the waveforms detected using electro-optic (EO) sampling with the setup shown in Figure 3-6. The plots (a) and (b) correspond to measurements of the THz pulses incident on the metal tip; (c) and (d) correspond to measurements of the near-field; (e) and (f) correspond to the calculated time-integral of the incident fields.
To obtain the maximum signal to noise ratio in the detected near-field THz pulses, we measure the THz waveforms at the smallest tip-sample separation (contact mode) with a long time delay range using the photoconductive (PC) setup as shown in Figure 3-1. The data are compared with those measured by van der Valk et al. at Delft using the electro-optic (EO) setup as shown Figure 3-6. Figure 3-7 (a) and (b) show the measured THz waveforms incident on the metal tips in the PC setup and the EO setup respectively. The measured near-field THz signals obtained with these two setups are shown in (c) and (d). The results are strikingly different from the incident THz electric-field transients shown in (a) and (b). Instead of a signal consisting of approximately a single cycle of the electric field, we now measure a THz transient which resembles a half-cycle pulse, and the frequency spectra that correspond with the measured near-field data resemble low-pass filtered versions of the incident spectra. The results obtained in the two different laboratories are very similar, indicating that the observed effects are quite general and not specific to the detection method. From these measurements, an immediate and important conclusion can be drawn: In THz ANSOM experiments, the near-field spectrum cannot be assumed to be the same as the spectrum of the input radiation. A crucial clue regarding the explanation for the difference between the incident and the near-field signals is obtained when we calculate the time-integral of the incident THz pulse. The results are plotted in (e) and (f). These curves reproduce the essential features of the measured near-field transients. Since integrating in the time-domain is identical to a multiplication by $1/\omega$ in the frequency domain, the measured near-field spectra lose much of their high frequency content.
Obviously, this result can not be described simply by Mie theory and the quasi-electrostatic model used in 3.2, so we need to find a new method to efficiently explain the observed effects. To do this, we develop a model which involve the antenna effects of the probe-sample system in ANSOM. In this model, the tip is treated as an oscillating dipole, and the electric field emitted by this dipole can be written as [78]:

$$\vec{E} = \frac{\mu_0 c^2}{4\pi} \left[ \hat{\theta} \sin \theta \left( \frac{p}{r^3} + \frac{p'}{cr^2} + \frac{p''}{c^2 r} \right) + \hat{r} \cos \theta \left( \frac{2p}{r^3} + \frac{2p'}{cr^2} \right) \right]$$  \hspace{1cm} (3-1)

where $p$ is the oscillating dipole moment taken to be oriented along the tip ($p/\|z\)$, $p'$ and $p''$ are the first and second time-derivatives of $p$, and $r$ is the distance from the dipole. $\theta$ is the angle between $p$ and vector $r$. The $p$ and $p'$ terms decrease rapidly with distance from the dipole and are the near-field terms, whereas the $p''$ term corresponds to the far-field radiation.

We now concentrate on the EO sampling setup in which the THz near-field component perpendicular to the crystal surface is directly measured underneath the tip. At short distances, the $1/r^3$ terms dominate and the component of the electric field perpendicular to the crystal can be written as:

$$E_{\perp, nf} \propto \frac{\mu_0 c^2}{4\pi} \frac{p}{r^3}$$  \hspace{1cm} (3-2)

This means the measured near-field must be proportional to the dipole moment $p$. We now need a relation between the dipole moment and the incident THz electric field. This relation can be obtained if we treat our metal tip as a short antenna. The continuity equation gives us a relation between the dipole moment $p$ and the current $I_{THz}$ in an antenna:
\[ p(t) \propto \int_{-\infty}^{t} I_{THz}(t')dt' \] (3-3)

where the current is assumed to be the same everywhere along the antenna for simplicity. The expression (3-2), which is valid for a point dipole, can also be used in the case of a real antenna, assuming that the field is calculated for a distance away from the antenna, larger than the relevant antenna dimensions [79]. A relation between the current and the incident THz electric field can be obtained by treating the antenna as a simple electronic network consisting of a radiation resistance \( R_r \), a capacitor \( C \) and an inductance \( L \), all in series. The driving “voltage” \( V_{THz} \) of this network is the incident THz electric field \( E_{THz} \).

The relation between this electric field and the induced current \( I_{THz} \) for this network in the frequency domain is:

\[ I_{THz}(\omega) \propto \frac{E_{THz}(\omega)}{R_r + j\omega L - j(\omega C)^{-1}} \] (3-4)

If we assume, for simplicity that \( L=0 \), and \( R_r \) and \( C \) are large, then equation (3-4) can be approximated by \( I_{THz}(\omega) \propto E_{THz}(\omega)/R_r \), which is equivalent to \( I_{THz}(t) \propto E_{THz}(t)/R_r \).

This is the regime of resistive coupling. Using equation (3-3), this gives us \( E_{\perp,nf} \propto p(t) \propto \int_{-\infty}^{t} E_{THz}(t')dt' \), showing that the near-field is proportional to the time-integral of the incident field, as we have seen in Figure 3-7. In fact, with certain reasonable values of \( R_r \), \( C \) and \( L \), we can get an even better agreements between the calculation and the measurement. In Figure 3-8, we plot the spectrum of the measured incident THz pulse (blue), the spectrum of the measured near-field (red), and the spectrum of the calculated near-field (purple), obtained by “filtering” the incident THz field using a LCR network, with the values of \( R_r \), \( C \) and \( L \) indicated in the figure caption. The agreement between the
calculated and the measured near-field spectra is excellent. For comparison, we also plot the calculated THz near-field spectrum assuming resistive coupling only. In this case, there is less agreement between the calculation and the measurement. The difference in spectral power between the two calculations grows with increasing frequency, which is caused by the increasing relative importance of inductive coupling at higher frequencies. We note that by using either much smaller antennas ($R$, $C$ and $L$ small) or by decreasing the frequency $\omega$, we obtain $I_{THz}(\omega) \propto j\omega E_{THz}(\omega)$ and therefore $p(t) \propto E_{THz}(t)$. This corresponds to the regime of capacitive coupling employed by Knoll et al. in their model for ANSOM in the mid-infrared region [76, 77].

Figure 3-8: Comparison of the measurement and the calculation for the EO sampling setup. (a) Power spectrum of the measured incident THz pulses. (b) Power spectrum of the measured THz near-field pulse. (c) Power spectrum of the calculated THz near-field, with $R=150 \Omega$, $C=10^{-13} F$, and $L=2\times10^{-11} H$ (purple), and the power spectrum of the near-field calculated using resistive coupling only (green).
The above discussion applies particularly to the EO detection setup used in the Delft measurements. To apply the above model to the PC detection setup used at Rice University, in which the tip is held close to a metal surface, we need to take into consideration the image dipole induced in the metal. This image dipole generates an additional electric field near the real antenna dipole according to equation (3-2). This field only "sees" a short antenna of fixed length $l$ for which the radiation resistance is proportional to $R_r \propto \omega l^2$ [79]. Using equation (3-4), and assuming that $R_r$ dominates over $L$ and $C$, we get $I_{th}(\omega) \propto \frac{E_{th}(\omega)}{\omega^2}$. Considering that the factor $1/\omega$ in the frequency domain is identical to integrating in the time-domain, we get $I_{th}(t') \propto \int_{-\infty}^{t'} \int_{-\infty}^{t''} E_{th}(t'')dt''dt'$. By putting the expression of $I_{th}(t')$ into equation (3-3), we deduce that $p(t) \propto \int_{-\infty}^{t} \int_{t-\Delta t}^{t+\Delta t} E_{th}(t'')dt''dt$. So the far-field radiated by this dipole is proportional to the second time derivative of $p$, giving $p'' \propto \int_{-\infty}^{t} E_{th}(t')dt'$, which is consistent to what we observe in Figure 3-7. Note that for the THz beam originally incident on the antenna, we assume that $R_r$ is constant. This is justified for the focused THz beam for which the product $\omega l$ is constant because the focal spot size is proportional to the wavelength as shown by equation (2-1).

3.4 Propagation Effects

The propagation effects in THz ANSOM are first observed in our study of the pulse reshaping of the scattered THz signal. In an experiment to study the relation between pulse reshaping and the effective length of the dipole antenna, different parts of
the probe are blocked from the incident THz beam. Quite surprisingly, THz pulses scattered from the probe tip can still be detected even if the tip is blocked from the direct illumination by the incident THz beam. This phenomenon can only be explained by the propagation effect: THz pulses can be coupled to a surface mode on the probe and can move along the shaft down to the tip.

Figure 3-9: A schematic of our experimental setup for studying the propagation effects in THz ANSOM.

To study this effect with experiments, we make some modifications in the THz ANSOM, as shown by Figure 3-9. The THz transmitter and the focusing lenses are now mounted on a movable stage so that the incident spot can be moved along the shaft of the probe. A piece of metal perpendicular to the probe is placed close to the shaft at the incident spot to provide a sharp start point of the propagation. Scattering of the THz radiation at the edge of this metal helps to couple the incident wave into a propagating mode on the shaft. The THz transmitter, the focusing lenses, and this metal scatterer are all mounted on a movable stage so that the incident position along the shaft can be
precisely controlled. In addition, the probe we use in this experiment has a larger tip size (25 μm) for a stronger scattering of THz pulses.

![Graph](image)

**Figure 3-10:** A series of time-domain THz pulses, for different incident positions of the THz beam focused on the probe shaft. Each successive waveform corresponds to a translation of the beam spot by 1.5 mm.

Figure 3-10 shows a series of time-domain THz pulses, obtained by moving the transmitter stage along the shaft of the needle in steps of 1.5 mm. The propagation effect is evident from the relative delay of these waveforms. As the point of incidence moves away from the tip, the pulse takes longer to propagate along the shaft, and its amplitude decreases. The upper and lower waveforms in Figure 3-10, which correspond to the propagation distance of 1.5 mm and 27 mm respectively, are put together in Figure 3-11(a). Here, the lower waveform has been shifted to smaller delays and scaled by a constant factor, to facilitate comparisons with the upper waveform. It is clear that this propagation is largely non-dispersive, since the shape of the time-domain waveform does
not depend strongly on propagation distance. The shape of the detected THz pulse is different from that of the incident THz pulse due to the near-field response of the probe tip, as discussed in 3.3.

![Waveform and Delay Graph](image)

**Figure 3-11:** (a) A close-up view of the upper and lower waveforms shown in Figure 3-10. (b) Arrival time as a function of incident position for a series of time-domain THz pulses propagating on a bare metal probe (squares) and that on a probe wrapped with a 0.5 mm PVC insulation layer (triangles). The solid lines show the least-squares fit to these data, from which the group velocities can be obtained.

In order to characterize the phase distortions introduced by the propagation along the probe shaft, we first determine the group velocity of the propagating mode. As shown in Figure 3-11 (b), the propagation delay is a linear function of the propagation distance. A least-squares fit to these data yields the group velocity of the propagating mode, $v_g = 0.300$ mm/ps, equivalent to the free-space velocity $c$. This result, together with absence of pulse broadening and low-frequency cutoff, suggests that the propagating mode is similar to the TEM mode of a coaxial waveguide. However, the attenuation of the TEM-mode in a coaxial waveguide is predicted to vanish in the limit that the inner radius of the outer conductor diverges, even if the conductors are not ideal metals [47].
While in our data, an attenuation of $\sim 0.25 \text{ cm}^{-1}$ is observed. This discrepancy shows that a more rigorous analysis is required to accurately determine the modal pattern and the conductivity losses, which we will discuss in Chapter 4.

A similar measurement is performed in which the needle is wrapped with a 0.5 mm PVC layer. The existence of the insulator layer distorts the detected waveforms, and also reduces the group velocity of the propagation to 0.8c, as depicted by the triangles in Figure 3-11 (b). This result indicates that the propagating wave is confined and guided along the surface of the probe. To study the spatial distribution of this guided mode, we perform a more detailed measurement by adding aluminum barriers of different radii onto the probe, in order to impede the propagation of the mode. The result is shown in Figure 3-12. The barrier is located 11.7 mm away from the tip, and the THz beam is focused on the probe shaft at a spot 27 mm away from the tip. As the size of the barrier increases, the
amplitude of the detected THz pulse decreases, as roughly 1/r. Therefore the propagation mode is mainly localized within ~1 mm around the shaft. We also observe a slight decrease in the bandwidth of the detected THz pulses with the increasing barrier size, showing that higher frequency components have smaller spatial extent in this mode.

In order to emphasize the significance of these results in the context of broadband apertureless NSOM, we consider the possibility of multiple reflections of the propagation mode. It is easy to imagine that any discontinuity in the probe shaft, such as the sort shown in Figure 3-12, could generate a reflected wave, leading to the possibility of multiple time-delayed scattered pulses reaching the detector. In this case, the spectrum of the received radiation would be modulated by the presence of these multiple reflections, in analogy to the Fabry-Perot effect. This modulation could complicate the interpretation of spectroscopic measurements. Figure 3-13 shows the experimental setup to study this effect. The incident pulse is focused on the probe tip, and the receiver detects the scattered radiation. The incident radiation is also coupled to a propagating mode and move away from the sample along the probe shaft. Upon reaching the metal barrier, this propagating wave is partially reflected and moves back towards the tip. This process can repeat until the propagating wave dissipates. Additional scattered pulses are generated each time the propagating wave reaches the probe tip. This effect is immediately noticeable by comparing the THz pulses measured with and without a barrier, as shown by the plot in the right part of Figure 3-13. The blue curve is the THz waveform measured without barrier, and the purple one is that measured with a barrier on the probe. The reflected transients in the second waveform are evident as shown by the two vertical arrows. For a better observation of the multiple reflections, we deconvolve the upper
waveform from the lower one, and obtain a transfer function (a filter function) between them. This transfer function contains two peaks with the same time delays as the two peaks in the waveform measured with a barrier, therefore they correspond to the first and the second reflections at the barrier. The relative delays between the two peaks precisely coincide with the round-trip propagation time between the barrier and the sample at velocity $c$.

\[ \Delta t = \frac{2L}{c} \]

**Figure 3-13:** Left: Experimental setup for measuring THz pulses with direct illumination of the probe tip. Right: Result of the measurement. The upper waveform is obtained with no barrier on the probe. The lower waveform is obtained with an aluminum barrier ($w = 3$ mm) on the probe. The bottom curve is the transfer function obtained by deconvolving the upper waveform from the lower one.

The results described in Figure 3-10 through Figure 3-13 confirm the effects of guided wave propagation on a near-field probe in ANSOM. Understanding these effects
will be important for near-field microscopy studies which involve the phase of the scattered optical radiation. In addition, the effect of multiple reflections can substantially perturb the spectrum of the measured signal, which would be significant for near-field spectroscopy. This is likely to be important in experiments where structures in the optical setup can act as a reflector, such as a probe holder or an AFM cantilever.
Chapter 4
Terahertz Waveguides

Some of the results described in this chapter have been previously published in reference [80] and [81].

4.1 Introduction

From the previous chapters we have seen that numerous new techniques for generation and detection of THz radiation made rapid advance in the past twenty years. As a result, THz technology has received extensive attention, with applications in imaging, sensing and spectroscopy. However, waveguiding in this spectral range still remains a challenge. Progress in THz techniques is severely limited by the overwhelming reliance on free-space transport of the THz beam, using bulk optical components. In many real-world situations, the sample or region to be studied may not be readily accessible to a line-of-sight beam, so the development of optimized guided wave devices for THz radiation is required. Just as glass fibers have revolutionized optical applications in the visible and infrared, high performance waveguides will greatly expand the capabilities of THz techniques.

The characteristics of materials at THz frequencies have placed special demands on waveguiding structures, which have seriously hindered their development. The most transparent (i.e., lowest loss) materials for this range are crystalline (e.g., high resistivity silicon), and thus are costly, fragile, and challenging to form into specific geometries for waveguide configurations. Other materials, such as low-loss polymers or glasses, are more malleable but exhibit prohibitively high absorption losses for propagation distances
of more than a few centimeters. For this reason, THz waveguides generally must rely on propagation in air, rather than via dielectric confinement as in an optical fiber. Because many metals are nearly perfect conductors at THz frequencies, waves guided near metal surfaces offer great promise. Since many THz applications rely on the use of time-domain signals to obtain the most broadband response, the significant additional constraint of low dispersion is also required to avoid pulse reshaping during propagation.

In an attempt to meet the compelling need for useful THz waveguides, various guides with quasi-optical coupling have been demonstrated within the last few years. Most of these have been based on conventional guiding structures, such as metal tubes [48, 51, 52], plastic ribbons [53], or dielectric fibers [54]. There have also been reports on the application of the latest technology of photonic crystal fibers to THz radiation [55, 56]. In all of these cases, the utility for transport of THz pulses is limited by group velocity dispersion of the guided waves. The most promising studies have reported dispersionless propagation in parallel metal plate waveguides [49, 57, 58]. But in this case the attenuation is unacceptably high for propagation distances over a few tens of cm, due in large part to the finite conductivity of the metal. In addition, the cross-sectional area of the waveguide is too large for many of the proposed applications.

In this chapter, we explore novel waveguiding structures for broadband THz pulses. We show how a metal waveguide with very simple geometry, namely a bare wire, can be used to guide broadband THz pulses with outstanding performance, including low loss and negligible group velocity dispersion. The guided propagation of THz pulses on a metal wire can be described by the cylindrical surface waves also known as Sommerfeld waves [64, 65]. Since the exposed surface area of a wire is much smaller
than that of any previously reported metal waveguide, the attenuation due to conductivity losses is extremely low for this configuration. The behavior of the wave propagation on a metal wire is also qualitatively different from that of many other metal guiding structures. In addition, the structural simplicity of the wire waveguide presents great advantages in the manipulation of guided THz radiation. As an example of this waveguiding structure, we demonstrate the first endoscope for THz pulses. To further extend the potential applications of this waveguide, we develop a radially symmetric photoconductive antenna which greatly improves the input coupling efficiency of the waveguide.

4.2 Conventional Metal Tubes

Since most dielectric materials have very strong absorption for THz waves, a practical THz waveguides must rely on metallic structures in which THz radiation are confined and propagate in air rather than via dielectric confinement as in an optical fiber. In the microwave frequency range, the most widely used metallic structures for this purpose are metal tubes. To test the performance of metal tubes for THz wave guiding, we measure the propagation of THz pulses in a copper-coated silica glass tube which is fabricated using liquid-phase chemical deposition at Rutgers University [52]. Such a structure provides minimized propagation loss, because copper has the highest reflectivity of any metals at THz frequencies, and the inner surface of the tube fabricated with this method is very smooth (the roughness is just tens of nanometers).

Figure 4-1 (a) shows the experimental setup for characterization of the copper-coated silica glass tube used as a THz waveguide. Similar to the experimental setup described in Chapter 3, THz beam from the transmitter is collimated and focused by two polyethylene lenses. For convenient alignment and efficient coupling, an additional
silicon lens is placed at the beam waist of the converged THz beam, and the focus of this lens overlaps the waveguide entrance face. The output wave is detected by the THz receiver right at this end of the waveguide. Figure 4-1 (b) and (c) show the waveforms of the input THz pulse and the output pulse, respectively. After a propagation of 4.33 cm in the tube, the nearly single-cycle THz pulse is greatly distorted and stretched.

Figure 4-2 (a) shows the amplitude spectrum of the output THz pulse, obtained by Fourier transform of the time-domain waveform. There is an obvious cutoff frequency in the spectrum, indicated by the red arrow in the plot. This is identified as the cutoff frequency of the TE$_{11}$ mode in a metal tube waveguide of 1.74 mm inner diameter: $f_{c,11}$.
Figure 4-2: (a) Amplitude spectrum of the output waveform. The red arrow indicates a cutoff frequency below which no waveguide modes exist. (b) Wigner-Ville distribution of the output waveform. The dashed line indicates the cutoff frequency of the major mode in the waveguide.

\[ \frac{cx''_\text{eff}}{2\pi a} \approx 0.101 \text{ (THz)} \] where \( c \) is the speed of light, \( a \) is the inner radius of the waveguide, and \( x''_\text{eff} = 1.841 \) [47]. More detailed information about the distortion of the THz pulses in the waveguide can be obtained through time-frequency analysis of the output waveform. There are different methods for this type of analysis. Here we use the Wigner-Ville distribution because it provides very high time- and frequency-resolution [82]. Figure 4-2 (b) shows the Wigner-Ville distribution of the output waveform in our experiment, which illustrates the distribution of the frequency components (absolute value) at each time instant. The time-frequency structure is complicated, showing that more than one mode is excited in the waveguide. The major mode, however, is evident which can be seen at the front of the pulse. For this mode, it is clear that the higher frequency components arrives
earlier in time than the lower frequency components, corresponding to negative chirp, which is also observed in Figure 4-1 (c). This distortion indicates a severe group velocity dispersion of THz pulses propagating in the tube. By analyzing the relation between the frequency and the time delay, the dispersion relation of the major mode can be extracted.

Figure 4-3: Dispersion relation for THz radiation propagating in a metallic tube with 1.74 mm inner diameter. Blue dots show the experimental result extracted from Figure 4-2. The red solid line, the black solid line and the black dashed line show the theoretical dispersion curves for mode TE_{11}, TM_{01} and TE_{21} respectively.

Blue dots in Figure 4-3 show the group velocity as a function of frequency of the major mode in the tube, extracted from the experimental data shown in Figure 4-2. The large group velocity dispersion is evident, which causes the severe waveform distortion for THz pulses propagating in the waveguide. By comparison with the calculated dispersion relation of the modes supported by this waveguide [47], the major mode is identified as the TE_{11} mode, which is consistent with our conclusion based on the cutoff
frequency in the spectrum of the output waveform. In the plot we also put the dispersion relation of two other modes next to the TE$_{11}$ mode, calculated using $\frac{v_g}{c} = \sqrt{1 - \frac{f_c^2}{f^2}}$ where $v_g$ is the group velocity, $f_c$ is the cutoff frequency, and $f$ is the frequency. The calculation shows that all modes supported by this waveguide present large group velocity dispersion, indicating that the conventional metal tubes can not be used as practical waveguides for broadband THz pulses.

4.3 Metal Wire Waveguides

Our study of the propagation effects in ANSOM shows that THz pulses can be confined and guided along the surface of a metallic probe with nearly zero dispersion. These results open the possibility of a new method for broadband THz waveguiding, and motivate us to explore novel THz waveguides based on metal wires. Since the waveforms we detect in ANSOM experiments are not the electric field of the propagating THz pulses (but the scattered radiation from the probe tip), we need a new experimental configuration for the waveguide study in order to eliminate the spectral filtering effects introduced by the probe tip (as described in 3.3).

For a better observation and characterization of the guided THz propagation on metal wires, we change the experimental setup from the ANSOM configuration to a new configuration in which the electric field of the guided mode is directly detected at the end of the waveguide. With the fiber-coupled transmitter and receiver, we can change the incident position (the start point of the propagation) and the detection position of the THz pulses, which allows us to measure the spatial profile of the guided mode. We use a long
stainless steel wire with a smooth surface, rather than the tiny tapered probe used in the ANSOM experiments, as the waveguide for our new measurements.

![Diagram of THz experiment setup]

**Figure 4-4:** Experimental setup for characterizing the propagating THz wave on a stainless steel wire waveguide.

The schematic of the new experimental setup is shown in Figure 4-4. Free-space THz pulses are focused onto the stainless steel waveguide. Another stainless steel wire is placed at the focal spot, oriented perpendicular to the waveguide (along the y direction). This second wire serves as an input coupler. Scattering of the input THz radiation at the intersection structure helps to excite the radially polarized mode which can propagate along the waveguide. Both the waveguide and the coupler are 0.9 mm in diameter, and the separation between them is 0.5 mm. The incident THz beam is modulated by a chopper in front of the transmitter and a lock-in amplifier is used for demodulating the induced photocurrent in the receiver. The THz transmitter, the focusing lenses, and the coupler are all mounted on a movable stage so that the incident position along the
waveguide can be controlled. The receiver is placed at the end of the waveguide and is
mounted on a three-axis stage for detection at various spatial positions.

In a measurement of this type, the signal of the guide wave is so weak that it is
comparable to the "stray THz radiation" reaching the detector, such as the radiation
scattered from the input coupler, the focusing lens holders, and the edges of metal sheets
used to block such radiation. To distinguish the guided wave from the scattered radiation,
we orient the THz receiver so that it is only sensitive to the vertically (y) polarized
electric field. Since the incident THz pulses are horizontally polarized, such a
configuration can efficiently minimize the detection of directly scattered radiation which
would interfere with the guided mode. In addition, we compare the polarization of the
THz waveforms detected with small variation of receiver positions around the end of the
waveguide, because only the signal from the guided wave is sensitive to these positions.
Figure 4-5 shows typical time-domain electric field waveforms detected in this way, for
two different receiver positions located 3 mm above and 3 mm below the axis of the wire
waveguide. These waves are vertically polarized, perpendicular to the horizontally
polarized input beam, and the polarization completely reverses as the detector scans
across the wire. These results clearly show that the detected signal is from a guided wave
on the wire, not just scattered radiation. The polarity reversal also shows the radial nature
of the guided mode.

To measure the spatial profile of guided mode, we scan the THz receiver in a
plane perpendicular to the waveguide and record the waveforms at different points in this
plane. The peak-to-peak amplitude of the waveform as a function of the vertical
displacement of the receiver is depicted by the squares in Figure 4-6 (a). The amplitude
Figure 4-5: Time-domain electric field waveforms detected with the receiver 3 mm above and 3 mm below the axis of the wire waveguide.

decreases with the transverse displacement approximately as $1/r$. Since the polarization response of the photoconductive receiver antenna is not perfectly symmetric, the measured electric field is not precisely zero at the center point in the experiment. This can also explain the slight asymmetry in the amplitude profile of the detected waveforms. A two-dimensional spatial profile of the mode is obtained by scanning the receiver and recording with a time delay fixed at the peak of the THz pulses (as indicated by the dashed line in Figure 4-5). The result is shown in Figure 4-6 (b). Because the receiver is sensitive only to one polarization, it is not possible to directly detect the spatial distribution of a radially polarized mode. Instead, the measured mode resembles the projection of a cylindrically symmetric radial mode onto the vertical direction.

The observed spatial profile can be understood in terms of Sommerfeld wave with a rough calculation using Goubau’s method. As Figure 2-4 (a) shows, the dominant
Figure 4-6: (a) Amplitude of the detected THz pulses as a function of the vertical displacement of the receiver, which is measured at a propagation distance of 24 cm. The solid curve shows the prediction of the Sommerfeld wave model. (b) Spatial profile of the electric field around the wire obtained by moving the THz receiver in a plane perpendicular to the waveguide axis.

electric field components of Sommerfeld wave is radially polarized. The variation of these components is described by a Hankel function, $H_1^{(1)}(\gamma r)$, where $\gamma$ is defined in terms of the propagation constant $h$ of the field outside the wire according to $\gamma^2 = \omega^2 / c^2 - h^2$. For a perfectly conducting wire, $\gamma = 0$ and the field propagates with a velocity determined solely by the external medium (in our case, air) [64]. For large but finite conductivity, $\gamma$ is small and the approximate form for the Hankel function can be used, appropriate for small argument:

$$H_1^{(1)}(x) \approx -2i/\pi x \quad (4-1)$$

Therefore a Sommerfeld wire wave also exhibits $1/r$ decay, within a distance $r_0 << |1/\gamma|$ of the wire surface.
We also use the Sommerfeld model to estimate the distance that the wave extends from the metal surface, for a metal of finite conductivity. \( \gamma \) can be determined by solving the transcendental equation which results from the boundary conditions at the wire surface. Following the method in ref. [65], we compute the amplitude of the wave as a function of radial distance, for the case of a 0.9 mm-diameter stainless steel (type 304) cylinder, with a conductivity of \( 1.39 \times 10^6 \) mho/m, about 2.4% of the conductivity of copper. To account for the finite aperture of our detector, we convolve this Hankel function with a Gaussian of 6 mm full-width at half-maximum. The resulting profile is shown as a red solid curve in Figure 4-6 (a). We can also calculate the radius inside of which 50% of the power is guided. At a frequency of 0.3 THz, half of the power is transmitted through an area extending roughly 1.2 millimeters from the surface of the wire.

Figure 4-7: (a) Broadband group velocity of the propagating THz pulses on a metal wire waveguide. (a) Arrival time as a function of incident position. The insets show the THz waveforms (blue) and the corresponding spectra (red) detected after 4 cm and 24 cm of propagation. (b) Group velocity of the guided mode as a function of frequency, extracted from the spectra of the THz waveforms.
In addition to the profile measurements at a fixed propagation length, we also study the propagation characteristics of the guided mode by moving the incident position of the THz beam along the waveguide. In this experiment, time-domain waveforms are obtained as a function of propagation distance. There is no evident change in the temporal shape of the waveforms for propagation up to 24 cm, the limit of our optical delay line. This shows that the propagation is largely dispersionless. As in the ANSOM experiment, we determine the broadband group velocity of the propagation mode by analyzing the dependence of the relative time delay of the waveforms on the propagation distance. A least-squares linear fit to these data yields the group velocity \( v_g = (2.995 \pm 0.001) \times 10^8 \text{ m/s} \), as shown in Figure 4-7 (a). To study the group velocity dispersion, we extract the group velocity for different frequency components by analyzing the spectra of these waveforms, using

\[
 v_g = \frac{c}{n_{\text{eff}}(\omega) + \omega \frac{dn_{\text{eff}}}{d\omega}} \tag{4-2}
\]

where \( n_{\text{eff}} \) is defined as

\[
 n_{\text{eff}}(\omega) = \Delta \phi(\omega) \frac{c}{\omega d} \tag{4-3}
\]

\( \Delta \phi(\omega) \) is the phase change for propagation distance \( d \) at angular frequency \( \omega \). Figure 4-7 (b) shows the extracted data, confirming that there is no measurable group velocity dispersion throughout the accessible spectral range. This is to be expected, given that the Sommerfeld surface wave model predicts a group velocity deviating from \( c \) by less than one part in \( 10^4 \), for our experimental situation.
Figure 4-8: Amplitude of the THz pulses as a function of the vertical displacement of the receiver, measured for different propagation distances.

By comparing the spatial profile of the guided mode detected at different propagation distances, we can study how the guided mode evolves in propagation. These profiles are depicted by the curves in Figure 4-8. Each of them is obtained in the same manner as that in Figure 4-6 (a). The curves are shifted for clarity. It is immediately clear that the electric field is more closely confined to the surface of the wire for the shortest propagation distances. Subsequently, the guided mode spreads laterally, especially during the first several centimeters of propagation, and approaches a spatial profile described roughly by $1/r$.

To study attenuation characteristics of the guided mode, we extract the waveform with the maximum peak-to-peak amplitude at each propagation distance. Except for the
Figure 4-9: (a) Blue dots show the maximum peak-to-peak amplitude of the THz pulses detected at different propagation distance. Green dots show the measurements with the receiver moved away from the end of the waveguide. (b) Red squares show the amplitude attenuation coefficient of the guided mode as a function of frequency. The solid line shows the conductivity loss of the stainless steel wire, computed using Sommerfeld’s surface wave model.

few shortest propagation distances, these are obtained at a fixed receiver offset of roughly 3 mm (see Figure 4-8). These amplitudes are plotted as a function of propagation distance in Figure 4-9 (a). The amplitude attenuation coefficient $\alpha$ of the wire waveguide can be extracted from these data, simply by fitting the dependence of the pulse amplitude $E$ on the propagation distance $x$ to:

$$E(x) = E_o e^{-\alpha x}$$

(4-4)

The value we obtain, $\alpha = 0.03$ cm$^{-1}$, is much lower than that of any previous demonstrated broadband THz waveguide [58]. This method can give us the spectrum-weighted amplitude attenuation coefficient, but a more detailed characterization is required to obtain the frequency dependence of the loss. We extract the attenuation
coefficient of each frequency component from the amplitude spectra of the THz waveforms detected at different propagation distances. The spectrum of the attenuation coefficient is shown by the red squares in Figure 4-9 (b). The attenuation decreases with increasing frequency.

The extremely low loss observed here originates from the unique structure of the wire waveguide. Compared to other waveguide geometries, a metal wire has much smaller surface area interacting with the electromagnetic field, so the propagation loss due to finite conductivity of the metal is negligible [47]. This is consistent with Sommerfeld’s wire wave model, which predicts a very small propagation loss due to the finite conductivity of the metal wire [65]. However, the spectrum of the attenuation obtained in our experiment can not be completely described by the Sommerfeld model, as shown by the solid line in Figure 4-9 (b). The predicted losses increase with increasing frequency, similar to other THz waveguides where the attenuation is dominated by ohmic effects [57]. But in our experiment, a somewhat opposite trend is observed. This indicates that much of the measured losses might arise from other sources, such as diffractive spreading of the propagating mode in the lateral dimensions, as seen in Figure 4-8. The significance of this loss mechanism for Sommerfeld waves has been discussed previously [83]. By moving the receiver away from the end of the waveguide, we observe a sharper drop in the amplitude of the detected pulses, as depicted by the green dots in Figure 4-9 (a), showing that the mode strongly diffracts when propagating off the end of the waveguide.
4.4 Application of Terahertz Metal Wire Waveguides

The low propagation loss and the negligible group velocity dispersion demonstrated by a metal wire make it a promising waveguide for THz applications. To explore the application potential of this new broadband waveguide, we study the manipulation of the guided mode. The ability to direct radiation along curves is one of the most important features for a practical waveguide. So as the first step, we compare the amplitude of THz pulses after propagating on a waveguide bent with different radii. The results are shown by the hollow triangles in Figure 4-10. The propagation distance is 21 cm, and the radius of curvature $R$ is varied from 90 cm down to 20 cm in steps of 10 cm. The amplitude of the electric field $E'$ as a function of the propagation distance $x$ along the bent waveguide is described by

$$E'(x) = E_0 e^{-\alpha' x}$$  \hspace{1cm} (4-5)

where $\alpha'$ is the amplitude attenuation coefficient for a bent waveguide. By comparing equation (4-5) to equation (4-4) we find

$$\alpha' = \alpha + \frac{\ln(E/E_0)}{x}$$ \hspace{1cm} (4-6)

So the amplitude attenuation coefficient for each bend radius can be extracted by comparing the amplitude of the detected THz pulse to that of a straight waveguide with the same propagation distance $x$. The extracted data are depicted by blue solid squares in Figure 4-10. As we can see, even a slight bend on the waveguide can lead to a considerable increase in the loss, from 0.03 cm$^{-1}$ for a straight waveguide to nearly 0.05 cm$^{-1}$ for a bend radius of 90 cm.
Figure 4-10: Propagation of THz pulses on a 21 cm long stainless steel wire with different bend radii. The hollow squares show the amplitude of the detected THz pulses as a function of the radius of curvature. The solid squares show the amplitude attenuation coefficient as a function of the radius of curvature. The red curve shows the fit with an exponential loss model.

Unlike a metal tube in which the guided mode is tightly bounded by the metallic wall, the guided mode on a metal wire is continuously converted into radiating modes as the wave travels around a curve. This is easy to understand by considering the wavefront of the transverse field, which must rotate around the center of the curvature during propagation. Consequently, at some distance from the center of curvature the phase velocity would exceed $c$, the propagation speed of the guided mode. So the portion of the field outside this point must be radiated, causing the power loss in the guided mode. This loss mechanism resembles that of a bent dielectric optical waveguide, in which the attenuation coefficient $\alpha$ can be described by a semi-empirical form [84]:
\[ \alpha = c_1 \exp(-c_2R) \]  

where \( R \) is the radius of curvature and \( c_1 \) and \( c_2 \) are constants independent of \( R \). A fit using equation (4-7) shows a good agreement with the experimental data, as seen in Figure 4-10, suggesting that radiation is the dominant mechanism for the propagation loss of a bent metal wire waveguide.

![Guided THz wave](image)

**Figure 4-11:** A Y-splitter constructed with a straight waveguide and a curved waveguide in contact with each other, as shown on the left. THz waveforms detected at A, B and C are shown on the right. The separation between A and C is 2 cm, and the THz receiver is located 3 mm below the plane of the splitter in the measurement.

As shown by Figure 4-6 and Figure 4-8, the guided mode has a large spatial extent compared to the cross section of the waveguide. So it is easy to imagine that the guided mode would be easily coupled between two curved waveguides in contact with each other (or between a curved waveguide and a straight one). Based on these features, a very simple structure of a Y-splitter can be constructed, as illustrated in Figure 4-11. The validity of this scheme is verified by electric field measurements with such a structure.
The waveforms in Figure 4-11 are detected at the end of the straight waveguide (A), at the end of the curved waveguide (C), and at a position between them (B), respectively. The separation between A and C is 2 cm, and the waveforms are detected with the THz receiver 3 mm below the plane of the splitter structure. The plot clearly shows that the Y-splitter efficiently directs part of the THz wave propagating on the straight waveguide to the branch waveguide.

![Figure 4-12: The first THz endoscope. (a) Photograph of a THz endoscope which is inserted into a flask. (b) A schematic of the THz endoscopic measurement of the bottom of the flask. (c) THz waveforms obtained at the end of the output waveguide with the receiver 3 mm above and 3 mm below waveguide.](image)

From Figure 4-9 (a) we can see that the guided mode can propagate off the end of the metal wire waveguide. Although the propagation loss increases due to diffraction, the
mode maintains its radial nature off the end of the waveguide for at least several centimeters. As a result of this remarkable feature, we can build a 90-degree output director by attaching a small mirror at the end of the waveguide, tilted at a 45 degree angle with respect to the axis of the waveguide. This structure, together with the Y-splitter, enables us to construct a THz endoscope based on wire waveguides. Figure 4-12 shows the first demonstration of a THz endoscope. A photograph of the setup with the endoscope inserted into a flask is shown in (a), and the experimental configuration of the endoscopic measurement is displayed in (b). The straight wire is used to guide the input THz pulses, and the curved wire is used to collect the pulses reflected by the bottom of the flask and sent them to the receiver which is located at the end of this wire. THz waveforms obtained with the receiver 3 mm above and 3 mm below the waveguide are shown in Figure 4-12 (c). The two reflections in each waveform arise from the two dielectric interfaces at the bottom of the glass flask. By attaching a piece of metal to the flask bottom, the strength of the second reflection is enhanced.

With the 90-degree output director attached to the distal end, more endoscopic measurements can be performed. Figure 4-13 shows a measurement of the reflection from the side wall of a metal tube into which the endoscope has been inserted, as shown in the upper part of the figure. In this case, the signal is not as strong as in the previous measurement due to the additional propagation and coupling process. Improvements of such measurement could be made by combining an endoscope with an imaging system. This can be accomplished by scanning the endoscope along the surface of the detected region, or alternatively, scanning or rotating the sample to obtain an internal THz image. One challenge for this goal is the low power transmitted by the endoscope which strongly
limits the data acquisition rate as well as the dynamic range. With optimization of the mode of the input beam and the coupling geometry, the power launched into the endoscope probe can be greatly increased. This work will be described in the following section.

To further explore the practicability of wire waveguides for THz applications, we try many other wires besides the 0.9 mm diameter stainless steel wire described above. The materials of these wires include steel, aluminum, copper, zinc and nichrome. The wire diameter ranges from 0.46 mm to 6.38 mm. Figure 4-14 shows a picture of these wires and the waveforms obtained in the measurements with these different wires serving as waveguides. The experimental configuration of the measurements is similar to the
waveguide characterization experiment described in 4.3. For comparison, all waveforms are measured with the same input coupler (the same as in Figure 4-4) and the same propagation distance (10.1 cm). And all the plots in Figure 4-14 have the same scale for both time delay and amplitude. The two waveforms in each plot are detected with the receiver 3.5 mm above and 3.5 mm below the axis of the waveguide, except for the last two plots in which 5 mm offset is used due to the larger wire diameter. For clarity, the upper waveforms and the lower waveforms are shifted by +1.5 and -1.5 respectively. From the plots we can see that there is no strong difference in the performance of these waveguides, showing that THz pulses can be launched along any thin metal rod structures. With such waveguides available, it is convenient to direct the THz pulse inside of containers or around corners, where line-of-sight optics are not practical. In situations where the guided mode could be perturbed by other structures close to the waveguide, we could add a section of outer metallic shield to form a coaxial waveguide, as long as the additional ohmic losses can be tolerated. Many new applications of THz radiation will become possible based on the use of wire waveguides.
Figure 4-14: Photograph of the metal wires tested as THz waveguides and the waveforms detected in the characterization experiment for each wire. In all these measurements, the propagation distance is 10.1 cm, and a 0.9 mm stainless steel wire oriented perpendicular to the waveguide is used as an input coupler.
4.5 Enhanced Coupling by Using Radial Photoconductive Antenna

In our experimental work described in 4.3 and 4.4, the radial mode on the metal wire waveguide is excited by the scattering of the incident THz wave at the gap between the waveguide and another perpendicularly oriented wire close to the waveguide. Since the linearly polarized incident wave and the radially polarized guided wave are so poorly matched, the input coupling efficiency is very low. A rough estimate from our experimental data indicates that less than 1% of the power of the incident THz beam is coupled to the wire waveguide by this scattering process. The calculated coupling efficiency for this configuration is 0.42% at 0.1 THz, obtained by simulation with the finite element method (FEM) [85]. The very low coupling efficiency greatly limits the application of THz wire waveguides. To improve this efficiency, the polarization mismatch between the THz source and the waveguide mode need to be minimized. A straightforward way to solve this problem is to make a THz source which can generate radially polarized THz radiation. However, all traditional THz sources, either a photoconductive antenna or a NLO crystal for optical rectification, generate linearly polarized THz pulses. To directly generate radially polarized THz pulses, we design a new photoconductive THz transmitter antenna with radial geometry.

Figure 4-15 (a) shows a microscopic image of one of the new antennas we have designed and fabricated. The new antenna works with the same principle as the traditional photoconductive antenna as shown in Figure 2-1, except that the geometry is totally different. Two circular gold electrodes are photolithographically defined on a 500 μm thick semi-insulating GaAs substrate. When a DC bias is applied on the two electrodes and a femtosecond laser pulse is focused on the area around the central
electrode, the generated photocarriers and the radially polarized electric field between the electrodes produce a "radial array" of dipoles which can generate THz radiation with radial polarization. Although the break in the outer electrode and the presence of the feed electrode create asymmetry in the lower half of the antenna structure, the generated THz beam is still largely radially polarized. Our FEM simulation at 0.1 THz indicates that approximately 60% of the power generated by the antenna emerges in the form of a radial mode [85]. As in a standard photoconductive THz transmitter, a substrate-matched aplanatic hyperhemispherical silicon lens is placed on the other side of the GaAs substrate in order to couple the beam into free space and collimate it. Since the collimated THz beam is largely radially polarized, it can be easily coupled to a metal wire waveguide by placing the waveguide in contact with the silicon lens at its apex, as depicted in Figure 4-15 (b). For this configuration, our FEM simulation at 0.1 THz yields
Figure 4-16: A schematic of the setup for testing the radially symmetric THz transmitter. The waveguide is a 0.9 mm diameter, 27 cm long stainless steel wire which is end-coupled to the silicon lens of the transmitter. The fiber-coupled receiver is the same as that in Figure 4-4, which is oriented so that only horizontally (x) polarized component of the electric field can be detected.

a coupling efficiency of approximately 56%, an improvement of more than 2 orders of magnitude over that obtained via the scattering mechanism.

Figure 4-16 shows the setup for experimental test of the radially symmetric THz transmitter. It is modified from our standard THz-TDS system as shown in Figure 2-2. Here, the femtosecond laser beam is split before it enters the group velocity dispersion compensator. One of the beams is sent to the radial antenna transmitter. Extra path length is added to this beam by directing it around the optical table (not shown in the figure) to synchronize the laser pulses reaching the transmitter with those reaching the receiver through a long path in optical fibers. The transmitter antenna used in this experiment consists of an 8 μm diameter inner electrode separated from the outer electrode by 75 μm. A 20 V DC bias was applied to the antenna with the inner electrode serving as the anode. The laser beam focused on the transmitter antenna has an average power of 25mV. A 0.9
mm diameter, 27 cm long, stainless steel wire is end-coupled to the transmitter with a
collection depicted in Figure 4-15 (b). As in the waveguide characterization
experiments, the THz pulses are detected at the end of the wire by the fiber coupled
receiver which is sensitive to only vertical or horizontal polarization components
depending on the orientation of the receiver.

![Graph showing time-domain THz waveforms](image)

**Figure 4-17:** Time-domain THz waveforms detected 27 cm away from the radial
transmitter. The red curves show the measurements with the wire waveguide present
between the transmitter and the receiver; the blue curves show the measurements with
the waveguide removed. The waveforms are detected at two positions, with 3 mm and -3
mm horizontal offset of the receiver with respect to the axis of the waveguide.

Figure 4-17 shows the results from the experimental test of the coupling
capability of the radial antenna. The waveforms are the horizontal components of the
electric field detected at two positions: one with a 3 mm horizontal (x) offset of the
receiver relative the axis of the waveguide, the other one with a -3 mm horizontal offset
of the receiver. The red curves show the measurements with a 27 cm long wire
waveguide present between the transmitter and the receiver. Strong terahertz signal is present at both detection positions, with a magnitude much larger than that with the dual-wire coupling scheme. While these signals are virtually equal in magnitude, there is a clear polarity reversal between the two positions. Only a very small signal is detected when the receiver is placed at the center axis at the end of the wire (not shown in the figure). The equivalent magnitude of both signals and the reversal of polarity between them demonstrate the radial polarization of the guided mode on a metal wire, as we have seen in the waveguide characterization experiments described in 4.3. When the wire waveguide is removed and measurements are made with the receiver at the same offset positions described above, the detected terahertz pulse is considerably smaller, as shown by the blue curves in Figure 4-17. At the ±3 mm horizontal offset, the peak-to-peak amplitude of the terahertz signal measured at the end of the waveguide is approximately 20 times larger than that for the pulse measured at the same position when the waveguide is not present. While that number does not represent the actual power coupling efficiency, it provides insight into the considerable advantages of using a radially symmetric photoconductive THz antenna end-coupled to a metallic wire waveguide for convenient and efficient THz wave delivery.
Chapter 5

Dispersion of Surface Plasmon Polaritons on Metal Wires

Some of the results described in this chapter have been previously published in reference [86].

5.1 Introduction

In Chapter 4, we demonstrate that the propagation of a surface wave on exterior surface of a metal wire can be used to efficiently guide THz radiation with extremely low loss and low dispersion. This surface wave is described as guided mode propagating along the surface of a cylindrical conductor, a so-called Sommerfeld wave. This study connects THz-TDS with another actively researched field—surface plasmon polaritons (SPPs). Although less evident at low frequencies, surface waves are coupled with the collective oscillation of electrons in the conductor, so they have a plasma-like character. SPPs at metal-dielectric interfaces have been studied for several decades as a reliable technique for surface analysis and investigation of thin films [87]. Recently, SPPs have attracted special attention for their relevance in sub-wavelength optics and nanophotonics [88-90]. In particular, SPPs that are confined on metal nanowires are considered a promising alternative to dielectric waveguides in highly miniaturized integrated optical devices [90-94]. While most studies of SPPs focus on the visible and infrared frequency range, our study of THz metal wire waveguides and the recent study of THz waves confined on planar metal surfaces by several other groups have attracted increasing attention from the SPP community [70, 80, 81, 95-104].
Various applications of metal wire waveguides have been discussed in Chapter 4. For the development of plasmonic devices based on metal wire waveguides, more in-depth knowledge of the SPP dynamics on cylindrical metal surfaces is essential. In this Chapter, we will describe our experimental and theoretical study of the dispersion behavior of SPPs on cylindrical metal surfaces at THz frequencies, as well as the comparison of such behavior with SPPs on planar metal surfaces or at visible and near-infrared frequencies. Our experimental data reveal the unusual dispersive properties of SPPs on cylindrical metal surfaces in the THz frequency range. The observed results are different from the intuitive expectation of a simple extrapolation of the high frequency portion of the dispersion diagram of SPPs on a planar metal surface. For SPPs on a planar metal surface, the dispersion curve asymptotically approaches the light line $k = \omega/c$ as the frequency decreases, and accordingly, the phase velocity and group velocity gradually increase and approach $c$, the speed of light in air [87]. Similar behavior has also been observed for SPPs propagating on Au and Ag nanowires at visible and near-infrared frequencies [92, 94, 105]. Our results indicate that SPPs on cylindrical metal surfaces show the opposite dispersive trend: the value of the phase velocity drops as the frequency decreases. This trend becomes increasingly evident as the diameter of the metal wire decreases. Numerical solutions of Maxwell's equations give similar results and show that this phenomenon can only be observed in the low frequency (i.e., THz) range, at frequencies far below that of the bulk plasma frequency.

5.2 Experimental Study

To extend our study from the performance of metal wire waveguides to the effect of surface curvature on SPPs in general, we measure the guided propagation of
broadband THz pulses on many different wires, made of stainless steel and aluminum, with diameters ranging from 2.4 mm down to 18 μm. We still use THz-TDS for this study. The experimental setup is shown in Figure 5-1. It is similar to that used in the waveguide characterization experiment. The metal wire is supported and stretched by a tightly fitting Teflon slab close to the distal end, as shown in the inset. The broadband free space THz pulses are coupled to surface waves on the metal wire by scattering at a small (~ 400 μm) gap defined by the surface of the wire and a copper blade oriented perpendicular to the wire. After a propagation distance of 20 cm, the electric field of the guided wave is detected by a fiber-coupled THz receiver located 3 mm off the axis of the metal wire. By measuring the polarity reversal of the single-cycle THz pulse on opposite sides of the wire, it is confirmed that that the propagating wave is a radially polarized Sommerfeld wave for all the wires tested. We compare a series of measurements using wires of different diameters, while all other components stay fixed in the setup.
Figure 5-2: (a) Time-domain waveforms of the THz SPPs after a propagation of 20 cm on aluminum wires with different diameters. (b) Time-domain waveforms of the THz SPPs after a propagation of 20 cm on stainless wires with different diameters.

Figure 5-2 (a) shows the time-domain electric field waveforms detected on different Al wires. For clarity, only four typical waveforms are shown here. As expected, the detected THz pulses maintain the single-cycle shape, showing that the SPP propagation is largely nondispersive. That means the dispersion relation is largely linear within the bandwidth of the detected radiation (from 30 GHz up to about 500 GHz). However, we observe an increase in the transit time as the diameter of the wire decreases, which is immediately noticeable by comparing the position of the waveform peaks with the dashed line in the plot. Increased pulse reshaping becomes evident when the diameter is below 200 µm. This indicates that the dispersion relation is no longer linear when the diameter of the metal wire waveguide becomes sufficiently small. Figure 5-2 (b) shows the results for stainless steel wires. The behavior is qualitatively similar with that of the aluminum wires, except that the increase in transit time is slightly larger for a given wire diameter. Compared with plot (a), the overall amplitude of the pulses in plot (b) is
smaller, and the loss associated with the decreasing wire diameter is more evident, which results from the lower conductivity of stainless steel. Figure 5-3 shows the shift of the transit time of the detected waveforms as a function of wire diameter, for both stainless steel wires and aluminum wires.

![Graph showing relative delay vs. diameter for stainless steel and aluminum wires.](image)

**Figure 5-3:** Time delay of the peak of the detected THz waveforms with respect to that of the waveform measured on the largest wire, for both aluminum and stainless steel wires.

To get the detailed information about the propagation of the SPPs, Fourier analysis is made for all measured time-domain waveforms to extract both the spectral amplitude and spectral phase. From Figure 5-2, it is clear that the amplitude of the SPP decreases with decreasing wire size. This is due both to a decrease in the input coupling efficiency as well as to increasing propagation losses on smaller wires. Of more interest is the spectral phase, from which we obtain the phase velocity \( v_p = L/\tau_p \), where \( L \) is the
propagation distance and $\tau_p$ is the phase time delay. This delay is related to the measured spectral phase by

$$\tau_p(\omega) = \tau_0 + \frac{\Delta\phi(\omega)}{\omega}$$  \hspace{1cm} (5-1)$$

where $\tau_0$ is a reference phase time delay and $\Delta\phi(\omega)$ is the phase difference between the detected waveform and the reference waveform. As shown below, for large wire diameters (e.g., larger than 1 mm), the SPP dispersion in the THz range approaches that of a flat metal surface. The propagation is essentially dispersionless, and the velocity approaches the speed of light in air. Therefore, we use the THz waveform measured on the largest diameter wire (2.388 mm) as the reference pulse with $\tau_0 = L/c$. Here, $c$ is the speed of light in air ($c = c_0 / \sqrt{\varepsilon_{air}}$, $c_0 = 2.9979 \times 10^8$ m/s, $\varepsilon_{air} = 1.0005364$ [67]).

The experimentally determined phase velocity $v_p(\omega)$ for three different wire diameters are depicted by the discrete symbols in Figure 5-4. At all frequencies, the phase velocity decreases as the wire size decreases. This accounts for the increasing delay observed in Figure 5-2 and Figure 5-3. These data also show that, rather than approaching the speed of light $c$, the phase velocity deviates increasingly from $c$ as the frequency decreases. This means that the low-frequency components arrive later than the peak of the pulse rather than earlier. This is distinct from the behavior of SPPs on a planar metal surface, which show the opposite trend albeit with a much smaller magnitude [87].
Figure 5-4: Discrete symbols (dots, triangles and crosses) show the experimentally determined phase velocity of SPPs propagating on aluminum wires of three different diameters, as indicated by the legend. The wire of 2388 μm diameter is used as the reference for determining the phase velocity. The calculated phase velocities for these wires are shown by the thick solid line, the thin solid line, and the dashed line respectively.

5.3 Theoretical Analysis

The history of the theoretical study of electromagnetic surface modes on metal wires began as early as 1899 when Sommerfeld demonstrated the solution of Maxwell’s equations for wave propagation along a cylindrical metal surface [64]. This type of surface wave was studied ranging from radio frequencies to millimeter waves [65, 66, 106]. However, there has been no systematic experimental characterization of the dispersion of this propagation mode. On the other hand, the study of SPPs on metal wires at optical frequencies first appeared in the 1970s [107-109], and has recently received
considerable attention [90-94, 110]. The latest theoretical and experimental studies have shown that the dispersive behavior of SPPs on Au and Ag nanowires is similar to that on a planar metal surface at visible and near-infrared frequencies [92, 94, 110]. This is in contrast to what we observe in our THz-TDS measurements as shown in Figure 5-4. At THz frequencies, the distinct behavior of the cylindrical geometry is more clearly revealed.

![Figure 5-5: Dispersion relation of SPPs on a planar aluminum surface and on an aluminum wire of 25 μm diameter, together with the dispersion relation of plane waves in air. Inset A shows a zoomed-in view of the low frequency portion of the dispersion diagram. Inset B shows a further zoomed-in view of the low frequency portion indicated by the blue frame in inset A. In both insets, we plot the frequency against k - k₀, where k₀ denotes the propagation constant of plane waves in air.](image)

To understand the difference between the dispersive behavior of SPPs on metal wires in the THz frequency range and that on planar metal surfaces or at higher frequencies, we calculate the dispersion relation of SPPs over the whole frequency range from THz to visible light, for both a planar Al surface and Al wires with various
diameters. The dispersion relation for a planar Al surface is given by:

$$k(\omega) = \frac{\omega}{c} \sqrt{\varepsilon_{\text{Al}}(\omega)\varepsilon_{\text{air}}(\omega) \over \varepsilon_{\text{Al}}(\omega) + \varepsilon_{\text{air}}(\omega)}$$

(5-2)

where $\varepsilon_{\text{Al}}$ denotes the complex permittivity of Al and $\varepsilon_{\text{air}}$ denotes the permittivity of air. As discussed in 4.3, our direct measurement of the spatial profile of the guided wave on metal wires has shown that the surface mode is the azimuthally symmetric zeroth-order transverse magnetic (TM) mode. The dispersion relation of this mode can be obtained by numerically solving the transcendental equation (2-7), or more specifically:

$$\frac{k_{\text{Al}}^2}{\mu_{\text{Al}} \gamma_{\text{Al}}} \frac{J_1(\gamma_{\text{Al}} a)}{J_0(\gamma_{\text{Al}} a)} = \frac{k_{\text{air}}^2}{\mu_{\text{air}} \gamma_{\text{air}}} \frac{H_{1}^{(1)}(\gamma_{\text{air}} a)}{H_{0}^{(1)}(\gamma_{\text{air}} a)}$$

(5-3)

The parameters in (5-3) are the same as in (2-7), except that the subscript $c$ and $d$ are replaced with $\text{Al}$ and $\text{air}$ here. We model the properties of aluminum using a Drude model, with the parameters taken from refs. [67] and [68]. To make the calculation valid over a very broad frequency range, we do not use the low frequency approximation $k_{\text{Al}} = (\omega \mu_{\text{Al}} \sigma_{\text{Al}})^{1/2} \exp(-i\pi/4)$ or the high frequency approximation $\varepsilon(\omega) = 1 - \omega_p^2 / \omega^2$ as in previous work [65, 66, 95, 109], nor other approximations in Goubau’s method we use in 4.3. The only approximation used in our calculation is $J_1/J_0 = -i$, which is valid because the radius of the wires used here are large compared to the skin depth ($|\gamma_{\text{Al}} a| > 1$) [65, 66]. In our calculation, the minimum value of $|\gamma_{\text{Al}} a|$ is 22.6 for $f > 20$ GHz and for all wire diameters used. As an example of our method, the MATLAB code for the calculation of a 51 $\mu$m diameter wire is attached in the APPENDIX of this thesis.

The calculated dispersion relation is shown in Figure 5-5. At visible and infrared frequencies, the dispersive behavior of an SPP on an Al wire of 25 $\mu$m diameter is almost
indistinguishable from that of a planar Al surface. But as shown in the insets, the low frequency portion of the dispersion curve approaches the light line in a different way from that of a planar surface, which leads to the unique dispersive behavior observed in the THz frequency range. For comparison with the experimental data, we calculate the phase velocity of the SPPs on Al wires of several different diameters. The results are shown as solid curves and dashed curves in Figure 5-4. The calculated phase velocity agrees qualitatively with the experimental results, reproducing in particular the notable feature of the decreasing $v_p$ with decreasing frequency. We obtain similar results for our measurements on stainless steel wires (not shown), except that wire diameter decrease has a somewhat larger effect on the wave propagation behavior, as shown in Figure 5-3.

![Graph showing phase velocity over frequency for different wire diameters](image)

**Figure 5-6:** Calculated phase velocity over a broader bandwidth, for four wire diameters as indicated. The arrow shows the shift of the turnover point of the phase velocity curves with decreasing wire diameters.

Figure 5-6 shows the calculated phase velocity for wires with different diameters in a broader frequency range. It is clear that this unusual dispersive behavior only appears at THz and lower frequencies. This result indicates that for frequencies much lower than
the surface plasmon frequency $\omega_{sp}$ ($\omega_{sp} = \omega_p / \sqrt{\varepsilon_{air}} + 1$ where $\omega_p$ is the bulk plasma frequency of Al), the resonant interaction between the electromagnetic wave and the plasma oscillation is no longer the dominant mechanism for determining the properties of surface waves. The electromagnetic properties of the metal play an increasingly important role due to the larger skin depth $\delta$ at lower frequencies [111]. The electric field and current that penetrate into the metal change the surface impedance and the internal inductance, and therefore affect the dispersive behavior of the surface waves [112-114]. This effect is enhanced for SPPs propagating on wires, due to the geometry of the metal surface. Since Sommerfeld waves are single-mode azimuthally symmetric TM waves [64, 65, 113], the electric field components inside the metal at a given point along the length of the wire are in phase. So, due to the curved nature of the surface, these evanescent field components can constructively interfere inside the metal, as illustrated in Figure 5-7. As a result, more power is transmitted inside the metal for SPPs on metal wires as compared to SPPs on planar metal surfaces. This enhanced skin effect is more significant for smaller wire diameters, since increased surface curvature leads to a larger overlap of the evanescent waves penetrating into the metal. It is also more significant at lower frequencies, due to the larger skin depth. This model is consistent with the calculation shown in the Figure 5-6, particularly the shift of the turning point of the $\nu_p$ curves to higher frequencies as the wire diameter decreases.

Our study is the first systematic analysis of the dispersion of SPPs on cylindrical metal surfaces in the THz frequency range. The unusual dispersive behavior described above not only shows the decreased plasmonic nature of SPPs at THz frequencies, but
also indicates the increasing importance of skin effects for SPPs at low frequencies, as well as the enhancement of such effects on curved surfaces.

Figure 5-7: An illustration of the skin effects for SPPs on cylindrical surfaces. δ denotes the skin depth. Cylindrical Surface 1 has a larger radius of curvature than Cylindrical Surface 2.
Chapter 6

Discussion and Future Work

6.1 Summary of Results

By introducing a vibrating metallic probe tip and combining a typical ANSOM configuration with THz-TDS system, we realize apertureless THz near-field imaging. The amplitude of the scattered THz pulses in this system is very sensitive to the tip-sample distance, and a sub-wavelength spatial resolution is achieved. Our study of the broadband frequency response of the ANSOM probe tip shows that the temporal shape of the observed near-field signals is approximately proportional to the time-integral of the incident waveform. Associated with this signal change is a bandwidth reduction by approximately a factor of three which is observed using both a near-field detection technique and a far-field detection technique. Using a dipole antenna model, we show that the observed effects can be explained by the signal filtering properties of the metal tips used in the experiments. We also study the propagation effects in the THz ANSOM and find the input THz pulses are coupled to a propagating mode along the shaft of the near-field probe. The spatial extent of this guided mode and its velocity are determined. Possibility of multiply reflected modes propagating along the near-field probe, an important effect in near-field spectroscopic measurements, is confirmed by our experiments.

In the exploration of efficient THz waveguides, we conduct THz-TDS measurements of the conventional metal tubes, and conclude they can not be used as
practical broadband THz waveguides due to the large group velocity dispersion. Inspired by our study of the propagation effects in THz ANSOM, we develop a new broadband THz waveguide in which THz pulses are confined and guided along a bare metal wire. The propagation of THz pulses on such a waveguide is experimentally characterized. The waveguide exhibits extremely low attenuation and negligible group velocity dispersion, making it especially suitable for use in THz sensing and diagnostic systems. In addition, the structural simplicity makes it possible to manipulate the guided THz wave in different ways, such as coupling, directing and beam splitting. We also develop a photoconductive transmitter antenna with radial symmetry to improve the input coupling efficiency of the metal wire waveguide.

The guided surface wave on metal wires is theoretically analyzed with Sommerfeld wave model, and the dispersive behavior of this mode is compared with SPPs on planar metal surfaces or at higher frequencies. Unique dispersion structures are revealed which are inconsistent with a simple extrapolation of the high frequency portion of the dispersion diagram for SPPs on a planar metal surface, and also distinct from that of SPPs on metal nanowires observed at visible and near-infrared frequencies. The results are consistent with a numerical solution of Maxwell's equations, showing that the dispersive behavior of surface waves on a cylindrical metal surface at terahertz frequencies is quite different from that of SPPs on a flat surface. These findings show the decreased plasmonic nature of SPPs at low frequencies, and indicate an important mechanism which must be considered in low frequency SPPs: the combined action of skin effects and surface geometry.
6.2 Future Research Directions

This thesis starts with the study of THz microscopy, and continues with the exploration of novel THz waveguides. It is interesting to note that the metal wire waveguide described in this thesis can be used to generate a radially polarized beam. Such a beam can be used to improve the spatial resolution in an imaging system, due to the sub-diffraction-limited focusing of radially polarized electromagnetic wave [115, 116]. This connection shows that THz microscopy can be combined with THz waveguides. With a focusing lens mounted at the end of a wire waveguide, a focal spot with reduced size can be obtained, and therefore higher spatial resolution can be realized than in the normal THz imaging system. Furthermore, since the radially polarized mode is an ideal input field for an apertureless near-field optical antenna or a coaxial near-field probe [117-119], nanometer-resolved endoscopic THz imaging may be possible through this combination. This will pave the way for a wide range of new applications for terahertz sensing and imaging.

Metal wire waveguides studied in this thesis open the possibility of many new applications of THz radiation. For the development of new functional structures and devices based on wire waveguides, complete theoretical models will be required. Basic characteristics of the waveguide, e.g. the mode structure for a single straight wire, can be obtained by solving Maxwell’s equations using Sommerfeld model. But it is hard to apply this method to a more complex system such as the wave propagation off the end of the waveguide or at a Y-splitter structure. With the rapid advance of computer technologies and the increasingly availability of various scientific softwares, more and more electromagnetics problems are solved by using finite element method (FEM) and
finite difference time-domain (FDTD) simulations. We have made great success in using FEM simulations in our design of the radially symmetric photoconductive antennas. More simulations can be done in the future, combined with THz-TDS measurements, in our development of waveguide based devices and systems, such as radial photoconductive antennas with improved symmetry, THz waveguide microscopes, and THz waveguide spectrometers. The ultimate goal of this research is to make a highly integrated and highly adaptive system combining THz-TDS with metal wire waveguides for THz spectroscopy and imaging, which will dramatically extend the application of THz radiation in many different fields.
Bibliography


87. H. Raether, Plasmons on Smooth and Rough Surfaces and on Gratings (Springer-Verlag, Berlin, 1988).


APPENDIX

MATLAB code for the calculation of surface waves on a 51 μm dia. Al wire

% This program is to calculate the dispersion and the loss of surface waves on a
% aluminum wire with the method discussed in 2-3.

clear; close all;

% Constants:
e = 1.60219e-19; %(coulomb)
m = 9.1095e-31; %(kg)
c = 2.997925e8; %(m/s)
eps0 = 8.8542e-12; %(F/m)
mu0 = 12.56637e-7; %(H/m)
epsd = 1.0005364; % permittivity of air (from CRC handbook)
NA1 = 1.81e29; %(m^-3)
torr = 0.8e-14; %(s)

wtorr = 1/torr;
wp = sqrt(NA1*e^2/(eps0*m));

a = 50.8/2*1e-6; % (m), radius of the wire
K = 30; % number of data points
f = 10.^linspace(-2.7,-0.2218,K); % (THz), the frequency range 0.002 - 0.6 THz

k0 = zeros(1,K);
kf = zeros(1,K); % the flat surface case

abGammaA = zeros(1,K); % |gamma*a|
angleGammaA = zeros(1,K); % angle(gamma*a)

for kk = 1:K
    w = f(kk)*2*pi*1e12;

    k0(kk) = w*sqrt(eps0*mu0);
    epsm = 1 - wp^2/(w^2 + i*w*wtorr); % Drude model (the "m" in epsm means metal)

    kf(kk) = real( (2*pi*f(kk)*1e12 / c * sqrt( epsm*epsd/(epsm+epsd) ) ));
    % the flat surface case

    kc = conj((w^2*mu0*epsm*eps0).^(1/2));
\[ M = 400; \]
\[ N = 200; \]
\[ eq27 = \text{ones}(M,N); \text{\% equation (2-7) in chapter 2} \]
\[ ab = 10.\times\text{linspace}(-4.2,-2,M); \text{\% the trial variable for} |\gamma a| \]
\[ \text{theta} = \text{linspace}(65,66.5,N)/180*\pi; \]
\[ \text{\% the trial variable for angle(} \gamma a). (\gamma \text{ is taken in the first quadrant}) \]

\[
\text{for } mm = 1:M \\
\text{for } nn = 1:N \\
\text{gammac} = (kk^2 + (ab(mm).\exp(i.*\text{theta}(nn))/a)^2 - k0(kk)^2*\text{epsd} )^{(1/2)}; \]
\[ \text{\% epsd is considered here.} \]
\[ \text{if } ab(mm) < 600 \\
\text{eq27(mm,nn) = abs(besselh(0,1,ab(mm).\exp(i.*\text{theta}(nn)))./} \]
\[ \text{besselh(1,1,ab(mm).\exp(i.*\text{theta}(nn)))} \] 
\[ \text{\% Unlike in King's paper, epsd is considered (see eq(2.4) in PRB 10, 3038).} \]
\[ \text{else} \]
\[ \text{eq27(mm,nn) = abs(} i \] 
\[ \text{- k0(kk)^2 } \] 
\[ \text{\% MATLAB cannot handle } |\gamma a| \geq 700 \text{ with an angle near } \pi/2, \text{ so} \]
\[ \text{\% } H(0,1,x)/H(1,1,x) \text{ is approximated by } i. \]
\[ \text{end} \]
\[ \text{end} \]

\[
[y0 \ \text{im0}] = \text{min(eq27);} \]
\[
[y \ \text{in}] = \text{min(y0);} \]
\[ \text{im = im0(in);} \]
\[
\text{absmin} = y; \] 
\[ \text{abGammaA(kk) = ab(im);} \]
\[ \text{angleGammaA(kk) = theta(in);} \]

\[ \text{end} \]

\[ \text{figure(1); plot(f,abGammaA,'d'); xlabel('Frequency (THz)'); ylabel('|a\gamma a|');} \]
\[ \text{title('Calculation for the } \phi 51\mu \text{m Al wire');} \]
\[ \text{figure(2); plot(f,angleGammaA*180/pi,'d'); xlabel('Frequency (THz)'); ylabel('theta');} \]
\[ \text{title('Calculation for the } \phi 51\mu \text{m Al wire');} \]
\[ h = (k0^2 \times \text{epsd} - (abGammaA.*\exp(i.*angleGammaA)/a).^2).^{(1/2)}; \]
\[ \text{\% Unlike in the paper, epsd is considered when extracting } h. \]
\[ \text{hreal = real(h);} \]
\[ \text{vpr = k0./hreal;} \]
\[ \text{figure(3); plot(f, vpr,'d'); xlabel('Frequency (THz)'); ylabel('v_p/c');} \]
\[ \text{title('Calculation of } v_p \text{ for the } \phi 51\mu \text{m Al wire');} \]
\[ L = -1 \times 1.324 \times 100 \times 2 \times \text{imag}(h); \text{ } \% \text{ (dB/100ft)} \]

figure(4); plot(f,L/3048/8.686,'d'); xlabel('Frequency (THz)'); ylabel('\alpha (cm^{-1})');

\textbf{title('Calculation of loss }\alpha\text{ for the }\phi 51\mu\text{m Al wire');}

figure(5); plot(hreal,2*pi*f*1e12,k0,2*pi*f*1e12, kf,2*pi*f*1e12,'--');

xlabel('propagation constant (m^{-1})'); ylabel('\omega');

\textbf{title('Calculation of the dispersion for the }\phi 51\mu\text{m Al wire');}

legend('surface wave', '\omega = k_{e}', 'flat surface wave');

figure(6); plot(hreal-k0,2*pi*f*1e12,-'o', k0,2*pi*f*1e12,'-*'); xlabel('h-k (m^{-1})');

ylabel('\omega'); \textbf{title('Calculation of the dispersion for the }\phi 51\mu\text{m Al wire');}

legend('cylindrical surface wave', 'flat surface wave');