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Control of Smart Base Isolated Buildings With New Semiaactive Devices And Novel H₂/LQG, H∞ And Time-Frequency Controllers

by

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ABSTRACT

Control of Smart Base Isolated Buildings With New Semiaactive Devices And Novel $H_2/LQG$, $H_\infty$ And Time-Frequency Controllers

by

Sriram Narasimhan

The large base displacement demands imposed by near field earthquakes on base isolated buildings and the development and implementation of novel semiaactive control devices and new algorithms to overcome this problem, is a key challenge in the design of smart isolation systems, which is the main goal of this study. In this dissertation a comprehensive class of smart base isolated buildings, new control algorithms based on $H_2/LQG$, $H_\infty$ and time-frequency methods are developed. The frequency content of the earthquakes is incorporated into the $H_2/LQG$ and $H_\infty$ controllers using newly developed weighting filters. The time-frequency content of the earthquakes is also estimated by new control algorithm developed based on time-frequency methods, for variable stiffness isolation systems. Novel semiaactive variable friction, variable damping and variable stiffness control devices are developed and their behavior studied analytically and experimentally. The new control algorithms and semiaactive
devices developed in this study are inherently smooth and do not cause sharp increases in floor accelerations or interstory drifts, while achieving reductions in base displacements. The newly developed semiactive devices and control algorithms are implemented in linear and nonlinear multi degree of freedom two and three dimensional smart base isolated buildings, subjected to a suite of near field earthquakes, and shown to be effective in reducing the response. A new smart base isolated benchmark building is developed to demonstrate the feasibility of implementation of the newly developed semiactive devices and controllers in full scale three dimensional structures subjected to strong near field earthquakes.
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Chapter 1
Introduction

In recent years base isolation has been increasingly adopted as a structural design
technique for buildings and bridges in areas of high seismicity. There are currently
over forty base isolated buildings in the United States (Kelly, 1997). The concept of
base isolation involves a flexible isolation system with significant energy dissipation
capability which shifts the fundamental period away from the predominant periods
of ground excitation. The isolation system can be in the form of linear elastomeric
bearings, lead rubber bearings or frictional bearings. The flexibility of the isolation
system results in the base isolated structure with a low fundamental period, usually
between 2.0 sec. to 4 sec. The base isolated structure responds predominantly in its
fundamental mode when subjected to earthquake excitation. This response primarily
involves the deformation of the isolation system resulting in reduction of forces in the
superstructure. The demands imposed by strong earthquakes, especially near-fault
excitation, result in large base displacements. These displacements may lead to buck-
ling of the bearings or in some cases pounding with adjacent structures. In order
to limit these large base displacements, passive (e.g., viscous dampers), active (e.g.,
actuators), or semiactive control devices (e.g., variable dampers) are installed at the
isolation level. The structure with control devices in the isolation system is, hence-
forth, termed as smart base isolated structure. The central objective of this research is to develop novel control algorithms and semiactive control devices for response reduction in smart base isolated buildings subjected to near-fault earthquakes. The control algorithms are based on robust optimal control methods ($H_2/LQG$ and $H_\infty$) and time-frequency methods (Short-Term Fourier Transform (STFT)). New variable friction, variable damping and variable stiffness devices are developed. Experimental data is presented to validate the analytical behavior of the proposed control device analytical models. The control algorithms and devices developed in this study are inherently smooth in nature, thereby eliminating the disadvantages associated with rapidly switching devices and on-off type control algorithms.

Active and semiactive control for structural applications has been studied extensively for the past two decades (Housner et al., 1997, Spencer and Nagarajaiah, 2003). Recent studies on analytical and experimental behavior of two-dimensional base isolated buildings (e.g., Nagarajaiah et al., 2000, Sahasrabudhe et al., 2000, Ramallo et al., 2002, He et al., 2003) has been reported. However, there have been very few or no comprehensive analytical or experimental studies performed on control applications for full-scale three-dimensional smart base isolated buildings taking into account different active and semiactive strategies, isolation systems and performance criteria. A majority of the aforementioned control studies of smart base isolated buildings involve addition of variable damping or smart damping (Magneto-rheological (MR) dampers)
at the isolation level. There have been very few studies focusing on variable friction systems (He et al., 2003) or variable stiffness systems. Almost all the algorithms employ some kind of switching criteria. In the case of variable MR dampers, the level of damping is controlled by the voltage supplied and in the case of variable friction devices, the level of force is controlled by the normal force applied to the friction interface. Recognizing the lack of a realistic test-bed for the application of control algorithms to smart base isolated buildings, the ASCE task committee on structural control voted to develop a benchmark for base isolated buildings. The smart base isolated benchmark problem along with a set of nonlinear analysis programs for studying various types of control systems was developed as a part of a broader objective to study control applications to base isolated buildings in this dissertation.

Semiactive systems require nominal energy for their performance and have been shown to reduce the structural responses considerably. Unlike active systems, stability is not an issue in semiactive systems as they are reactive systems. A number of semiactive devices have been studied extensively - Magnetorheological damper (Dyke, et al., 1996), electrorheological damper (Gavin et al., ), resettable damper (Yang et al., 2000) and variable stiffness device (Kobori et al., 1993, Nagarajaiah et al., 1998). Several control algorithms that have been proposed for these devices involve high frequency switching; for example, the resetting controller proposed by Yang et al. (2000) has a peak resetting frequency of 8 Hz. In base isolated buildings such abrupt
switching may give rise to increases in accelerations and drifts. Control algorithms that result in smooth switching are introduced in this thesis, which reduce the structural responses considerably without resulting in increased drifts and accelerations. Robust control and time-frequency based control algorithms are proposed. The new devices developed in this study are capable of varying the level of stiffness, damping, or friction smoothly to achieve response reductions in base isolated buildings subjected to near-fault earthquakes. Experimental verification is also presented to verify the analytical models developed for the novel semiactive devices.

Earthquakes are highly non-stationary in nature. Typically, earthquakes contain few strong pulses - such as velocity pulses in near fault earthquakes - that contribute to most of the damage sustained by the structure. Hence, it is very important that the frequency characteristics of the excitation be explicitly accounted for in the control techniques. In base isolated structures, the dominant frequencies in an earthquake assume even greater significance as the structure responds predominantly in its fundamental mode. Very few researchers have included earthquake time-frequency characteristics directly in control (Varadarajan and Nagarajaiah, 2003). However, it is a common practice to include the earthquake excitation as a filtered white noise for control design such as $H_2/LQG$ (e.g., Yoshioka et al., 2002, Ramallo et al., 2002). The filter that is most commonly used in the literature is the Kanai-Tajimi (Soong and Grigoriu 1993) filter; however, it is shown in this thesis that it is possible to achieve
better response reductions through the use of new filters, that model a set of near fault earthquakes, developed especially for use in new controllers for base isolated buildings. Control algorithms that directly use the time-frequency characteristics of the earthquakes in control of base isolated buildings are also developed in this study.

1.1 Overview of the Dissertation

The dissertation is divided into nine chapters. The second chapter reviews the literature relevant to the current study. This includes studies related to modeling of base isolated buildings, active and semiactive control of base isolated structures, application of optimal control techniques, such as $H_2/LQG$ and $H_\infty$, semiactive control using conventional techniques such as clipped optimal control and application of time-frequency methods in structural control.

The third chapter presents the mathematical modeling of base isolated buildings including modeling of various isolation systems. Two types of base isolated buildings that are used throughout this dissertation are introduced. The framework for control implementation is introduced in the form of a benchmark problem. A set of newly developed performance indices are also introduced. The framework along with the performance indices developed in this chapter are used extensively in the thesis for testing new control algorithms.

The theoretical basis for robust optimal control methods, namely, $H_2/LQG$ and $H_\infty$ are presented in the fourth chapter. All the necessary formulations together with
techniques for augmenting state equations with weighting filters are presented in this chapter. Carefully chosen numerical examples are presented to highlight the ideas of frequency domain control that are necessary for development of new controllers in the ensuing chapters.

A new smooth-clipped optimal control algorithm is developed for use with the MR Damper in the fifth chapter. New weighting filters that enhance the performance of the controller are presented for the linear isolation case. For the nonlinear friction isolation system, a skyhook controller is presented. Methods to improve the performance by smoothing filters are also introduced.

Newly developed active and semiactive control methods based on $H_\infty$ for variable friction isolation systems are introduced in the sixth chapter. A newly developed semiactive variable friction device, SAIVF is introduced. Details of the device, algorithm and discussion of the results are also presented. Analytical model and experimental verification of the new variable friction device developed is also presented in this chapter.

Newly developed semiactive control method based on $H_\infty$ for variable damping isolation systems is introduced in the seventh chapter. A newly developed semiactive variable damping device, SAIVD is also introduced. Details of the algorithm and a discussion of the results is also presented. Analytical model and experimental verification of the new variable damping device developed is also presented in this
chapter.

The concept of time-frequency, its relevance to structural control and the necessary formulation for use with variable stiffness isolation systems is developed in the eighth chapter. The newly developed time-frequency controller is presented along with the simulation results.

The main conclusions of this study are summarized in the ninth chapter along with recommendations for future research that could be natural extensions of this work.

The work accomplished in the dissertation can be briefly summarized as follows (a more detailed description of the objectives met in this dissertation is given in the ninth chapter):

1. A general framework for the control of base isolated structures in the form of a benchmark problem has been introduced with a broad set of carefully chosen performance indices. Three types of base isolated buildings with varying levels of complexity have been analytically formulated for algorithm implementation.

2. New control algorithms that result in smooth switching laws are developed in this study to reduce the structural responses considerably without resulting in increased drifts and accelerations. Robust control methods, namely $H_2$ and $H_\infty$, which incorporate newly designed weighting filters, form the basis for the proposed control algorithms.
3. New semiactive devices are developed in this study; these devices are capable of varying the level of stiffness, damping, or friction smoothly to achieve response reductions in base isolated buildings subjected to near-fault earthquakes. Experimental results are presented to validate the proposed analytical models for these semiactive devices.

4. The application of time-frequency based control algorithms for base isolated buildings through the use of Short-Term Fourier Transform has been developed for variable stiffness systems. The novel algorithm and the variable stiffness device are capable of smooth stiffness variation thereby eliminating some of the disadvantages associated with rapidly switching devices.
Chapter 2
Review of Previous Work

The idea of seismic isolation dates back to two decades (Kelly 1986; Buckle and Mayes 1990) and since then base isolated structures have been studied extensively. Three dimensional analytical models have been proposed by various researchers (Nagarajaiah et al., 1991). For generic structures, recognizing the limits of performance by passive devices, control methods have been investigated extensively by various researchers for the past two decades (Spencer and Nagarajaiah, 2003). Several researchers have studied active and semiactive control of base isolated structures (Reinhorn et al. 1987, Nagarajaiah et al. 1993, Nagarajaiah 1994, Reinhorn et al. 1994, Yoshida et al. 1994, Yang et al. 1996, Symans et al. 1999, Yoshida et al. 1999, Spencer et al. 2000, Sahasrabudhe, 2002, Ramallo et al. 2002, Madden et al. 2003).

This chapter presents an extensive review of relevant literature.

2.1 Seismic Base Isolation

In seismic base isolation, linear or nonlinear bearings are introduced between the superstructure and the foundation to provide lateral flexibility. This flexibility lengthens the natural period of the structure and the response due to earthquakes is reduced due to this period shift and dominant fundamental mode response. A variety of isolation systems have been developed and implemented in bridges and buildings. Some of
the commonly used isolation systems are: lead rubber bearings (Buckle 1990, Soong 1992, Kelly 1993) and sliding isolation bearings with restoring springs (Mayes 1994, Soong and Constantinou, 1992). The first fully three dimensional analytical model of base isolated structures was presented by Nagarajaiah et al., (1991) which lead to the development of the 3DBASIS suite of computer programs. Verification for the proposed analytical modeling techniques for base isolated buildings with a combination of elastomeric and lead-rubber bearings was provided by the response of the USC hospital building in Los Angeles during the 1994 Northridge earthquake (Nagarajaiah et al., 2000). Recent near fault earthquakes and their potential impact on response of base isolated buildings has been widely reported (Kelley, 1999). Recent revisions in the Uniform Building Code (ICBO, 2001) impose stringent performance requirements on the design of base isolated buildings. The more stringent code requirements and the potential impact of near fault earthquakes has paved the way for the need for additional passive, active or semiactive control devices.

2.2 Active Control of Base Isolated Structures

Active control methods employ actuators to impart external force (energy) to the system. The force required is obtained using optimal control design techniques such as $H_2/LQG$ (Stengel, 1986) or $H_\infty$ (Doyle et al., 1989, Green and Limebeer 1995). The control methods used for active control may be classified as time domain and frequency domain methods. The LQR method (Stengel, 1986) is an example of a
time domain method. In this method, a controller is designed to minimize a quadratic performance index based on structural responses and control forces with appropriate weighting matrices with the constraint imposed by the dynamic equations of motion. This is a classic optimization problem which can be solved using a variety of methods such as linear programming (Bryson and Ho, 1975). The counterpart of LQR is LQG method (Bryson and Ho, 1975). In this method, the LQR method is combined with the optimal estimation using Kalman filter. This method enables partial state measurements where the remaining states are estimated. The estimated states are then used for controller design using LQR. A number of studies have focused on the use of active devices such as actuators in base isolated buildings to reduce the base drifts (Yoshida et al., 1994; Spencer and Sain 1997). Experimental studies on hybrid control of bridges using sliding bearings, with recentering springs in parallel with hydraulic actuators were conducted by Nagarajaiah et al. (1994). A scaled model single degree of freedom system and a control algorithm based on instantaneous optimal control (Yang et al., 1992) was used in conjunction with absolute acceleration measurements to achieve response reductions. The studies show that active control integrated into the base isolation system is effective in reducing the base displacements, isolation and superstructure forces. Suhardjo (1990) investigated the performance of frequency domain control methods, namely, $H_2$ and $H_{\infty}$ methods in seismically excited buildings and showed that frequency domain design methods can be effectively used for optimal
control of a structure using output feedback of acceleration measurements and to selectively shape the output structural response in a desirable manner. The effects of bidirectional ground motions and their impact on control designs has not been investigated. Yoshida et al. (1994) presented active control of a base isolated structure using LQG and $H_{\infty}$ methods. The actuator was assumed to be at the isolation level of the building having 4 degrees of freedom. The LQG controller was based on a reduced order structure model and Kalman estimator. The controller was designed to have both feedforward and feedback links. The feedforward link introduced the frequency characteristics of earthquake excitation into the analytical model. They showed that LQG and $H_{\infty}$ controllers are effective in reducing the response. However, the drawback of active control is large power requirements and concerns regarding reliability.

2.3 Semiaactive Control of Base Isolated Structures

The performance of the isolation system is enhanced by introducing supplemental passive dissipation mechanisms at the isolation level. Passive devices rely on energy dissipation mechanisms for achieving response reductions. Viscous dampers and friction elements are examples of devices that fall under this category. For the case of base isolation systems with sliding or elastomeric bearings, the addition of supplemental passive damping at the isolation level to reduce the base displacements in near fault earthquakes may lead to increased inter-story drifts and floor accelera-
tions (Makris, 1997 and Kelly, 1999). A semi-active control system is one which can adjust stiffness, damping or friction in real time but cannot input energy into the system being controlled. Minimum power requirement in the semi-actively controlled systems is the main attractive feature of these systems over the actively controlled systems, which require large amount of power. Reliability of the semi-actively controlled systems is an added advantage, as they can operate as a passive systems in extreme cases. Semiaactive control of linear and nonlinear structures using novel devices such as variable stiffness systems, Magneto-Rheological (MR) dampers, and Electro-Rheological (ER) dampers has gained significant attention in the recent years (Spencer and Nagarajaiah, 2003).

2.3.1 Variable Damping Systems

Semiaactive control using MR dampers (Dyke et al., 1996) has been studied extensively. Dyke et.al. (1997) evaluated a number of semi-active control algorithms for use with MR damper; the relative performance of these algorithms were compared through simulations and advantages of each algorithm were discussed. These results indicate that the performance of the control system is highly dependent on the choice of the algorithm employed. All the algorithms were implemented on a 3DOF fixed base linear structure. Ramallo et al. (2002) implemented the Magneto-rheological (MR) dampers on two and six degrees of freedom base-isolated structures. The semi-active dampers were added at the isolation level and consisted of hysteretic-type
lead-rubber bearings. $H_2/LQG$ clipped-optimal control algorithm was used to control the voltage commanded to the MR dampers. The control system for the damper was designed to be optimal over a suite of ground motions achieving reductions in base drifts without an accompanying increase in accelerations over optimal passive damping cases. Their results showed that the control strategies provided superior base-isolation system than a conventional base-isolation system or a base-isolation system coupled with supplemental passive dampers. A semiactive control strategy utilizing variable slip-force level dampers based on maintaining constant ductility level was examined and found to be effective both analytically and experimentally (Nishitani et al., 2003).

2.3.2 Variable Stiffness Systems

Variable stiffness systems have also gained attention in the recent years (Spencer and Nagarajaiah, 2003). However, there is very little published literature on application of variable stiffness systems for base isolated buildings. The primary advantage of variable stiffness systems is their ability to avoid resonance (Kobori et al., 1993 and Nagarajaiah et. al., 1998). The active variable stiffness system developed by Kobori et. al., (1993) has performed successfully in several earthquakes; however, the on-off device can switch stiffness between on-off states and may in some cases lead to increased accelerations. Similar on-off control strategy was developed using the concept of active members by Onada and Watanabe (1991). In this method, the
axial truss members are built with a mechanism to vary their stiffness between two
stiffness states and the optimum stiffness state is chosen to drive the system to the
origin in the phase plane.

To overcome the limitation of abrupt switching, a new semiactive independently
variable stiffness (SAIVS) device has been developed by Nagarajaiah (2000). SAIVS
device is capable of switching the stiffness smoothly. Nagarajaiah et al. (1998) have
shown the effectiveness of SAIVS system for non-resonant control. Nagarajaiah et
al. (2000) and Varadarajan and Nagarajaiah (2004) have developed smart tuned
mass damper based on SAIVS and developed time-frequency controllers. The control
algorithm proposed by Kobori et al. (1993) is based on estimation of response in
each uncontrolled stiffness state and selection of the state which results in the least
response. The controller developed by Yang et. al. (1996) is a sliding mode controller.
The resetting algorithms developed by Jabbari and Bobrow (2002) and Yang et. al.
(2000) are effective primarily due to energy dissipation with constant stiffness. The
tuned interaction damper developed by Zhang et. al. (2002) is based on Lyapunov
theory. The aforementioned studies do not estimate the energy spectrum using the
time-frequency distribution.

2.3.3 Variable Friction Systems

Semiactive control strategies that control the normal force at the sliding interface
have also been proposed (Inaudi 1997). The normal force at the damper interface is
adjusted as a function of the prior peak drift experienced across the friction damper. A modified version of this algorithm by introducing smoothing conditions that limit sticking have also been developed (He et al., 2003). Lyapunov based control criteria that adjust the normal force at the damper interface have been proposed by researchers (Dupont et al., 1997; Nitsche and Gaul, 1999). This strategy maximizes the energy dissipation in an instantaneous sense by modulating the normal force at the friction interface. A semiactive dry friction damper for automotive applications was developed using a feedback of relative velocity by Ferri and Heck (1992). The control laws that were introduced used the relative velocity and a feedback gain to simulate a linear viscous damper. The accelerations were reduced by using actuator saturation and output filters. The use of optimal control laws based on LQ have been used with friction devices (Hirai et al., 1996). In this method piezoelectric actuators were used to vary the level of friction force, which in turn was calculated using the optimal control laws. The weighting matrices were cleverly chosen such that the required force can always be provided by the friction device. Algorithms that apply balance logic where the friction force is adjusted to cancel the spring force in a simple system was implemented analytically by Stammers and Sireteanu (1998). Both cases of instantaneous switching and finite switching times were studied. Major limitations of all the above algorithms is that they are all of bang-bang type and were implemented on simple idealized systems. Additionally, the rapid switching due to
bang-bang control of semiactive devices increases the contribution from higher modes, increased inter-story drifts, and floor accelerations. Hence, there is a need for developing new controllers and devices with smooth switching. Also, controllers which reflect the time-frequency content of earthquake excitation need to be developed.

2.4 Concluding Remarks

The results of the relevant literature survey provides the state of current research in the area of active and semiactive structural control and the motivation for new contributions in this area. It is seen that a comprehensive class of smart base isolated buildings and new control algorithms are needed. These algorithms should incorporate better characterization of near-fault earthquakes in a stationary sense for use with traditional optimal control designs, and time-frequency representation of the earthquakes in new non-traditional algorithms. Novel semiactive variable friction, variable damping and variable stiffness control devices need to be developed and their behavior studied analytically and experimentally. The new control algorithms and semiactive devices developed should be inherently smooth without sharp increases in floor accelerations or interstory drifts, while achieving reductions in base displacements.
Chapter 3
Analytical Formulation and Framework for Numerical Implementation

3.1 Introduction

In base isolated structures, lateral flexibility and energy dissipation capacity are provided by a specially designed isolation system that is placed between the foundation and the ground. The lateral flexibility provided by the isolation system results in a structure that has a fundamental period in the range of 2.0 to 4.0 sec. Such an isolated structure responds to earthquakes predominantly in its fundamental mode where the energy of the earthquake is small. Well designed base isolated structures typically result in lower structural forces and consequently lower inter-story drifts and floor accelerations than their fixed-base counterparts. On the other hand, in near fault earthquakes due to the lateral flexibility provided by the isolation system, the base displacements increase considerably. This imposes a large demand on the isolators, which result in problems such as buckling of isolators and pounding. Moreover, recent revisions to the Uniform Building Code (ICBO 1997) have made the requirements for base-isolated systems more demanding compared to the previous versions (Kelly, 1999), thus making the design of base isolated buildings complex and economically unfeasible. In order to overcome the restrictions imposed by strict code requirements on the design of isolators, designers are increasingly turning toward sup-
plemental passive dissipation devices that aid in reducing these base displacements. Passive systems rely on energy dissipation to achieve response reductions and the dissipation is achieved in a variety of ways depending on the nature of the device. For example, passive energy dissipation capacity is provided by the lead-plug within the rubber unit, as in lead-rubber bearings or by inherent damping capacity of the rubber, as in high-damping elastomeric bearings or by steel dampers. Sliding bearings dissipate energy by means of frictional behavior. Restoring force or re-centering capability in the frictional systems is provided by helical springs or springs in the form of rubber cylinders. Efficient analytical modeling techniques have been developed to model these passive dissipation systems in base isolated buildings (Nagarajaiah et al., 1991). Several full scale base isolated buildings have been built in recent years with and without supplemental passive devices. The USC hospital building is a base-isolated seven-story (seven stories above the ground and a basement), steel framed building with 68 lead-rubber isolators and 81 elastomeric isolators. The San Bernardino county medical center which has five base-isolated buildings, totalling 86,400 $m^2$, has 392 high-damping rubber isolators and 184 viscous dampers. The Hayward City Hall has been built with 53 FPS (Friction Pendulum System) isolators and 15 viscous dampers. Other types of control devices, such as active and semiactive devices have gained considerable attention in the last two decades (Spencer and Nagarajaiah, 2003). The discussion on these systems is reserved for later chapters when
various control algorithms are introduced. In this chapter analytical models for base isolated buildings is developed together with a framework for control implementation and a set of performance indices.

Two types of base isolated buildings are developed and used in this thesis. The first building is a five-story, steel framed, shear-beam type building with moment-resistant connections. The isolators are connected between the base and the ground. A single control device is connected between the base and the ground. The second structure considered is a fully three dimensional eight story steel framed building with lateral-torsional coupling. This building is similar to the USC hospital building in Los Angeles, California. This building was selected to be the test bed for the first generation smart base isolated benchmark study (Narasimhan et al., 2004). The control devices are placed at the isolation level between the foundation and the base.

3.2 Structure A: Two Dimensional Five Story Base Isolated Building

The analytical model for five story base isolated building is shown in Fig. 3.1.

The structural stiffness is provided by the columns at each story level that are represented by spring elements with spring stiffness $k_1$ through $k_5$. The isolator stiffness is represented by a linear spring of stiffness $k_b$. The structural damping in the members are shown as dashpots with values $c_1$ through $c_5$ for the superstructure. The damping in the isolators is $c_b$. The control devices are assumed to be connected to the
base as shown in Fig. 3.1. These control devices may be active, passive or semiactive, though an example semiactive device is shown in Fig. 3.1. The displacements are denoted by $x_b$ through $x_5$ with respect to a fixed inertial frame of reference shown. The equations of motion are formulated with respect to this frame of reference. The five story's along with the base make this structure a six degrees of freedom (DOF) model. This structure has an isolated period of 2.5 sec. and the parameters are shown
Table 3.1 Structural Properties of Five Story Model

<table>
<thead>
<tr>
<th>Mass (kg)</th>
<th>Stiffness (kN/m)</th>
<th>Damp. (kN.s/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_b=6800$</td>
<td>$k_b=232$</td>
<td>$c_b=7.48$</td>
</tr>
<tr>
<td>$m_1=5897$</td>
<td>$k_1=33732$</td>
<td>$c_1=67$</td>
</tr>
<tr>
<td>$m_2=5897$</td>
<td>$k_2=29093$</td>
<td>$c_2=58$</td>
</tr>
<tr>
<td>$m_3=5897$</td>
<td>$k_3=28621$</td>
<td>$c_3=57$</td>
</tr>
<tr>
<td>$m_4=5897$</td>
<td>$k_4=24954$</td>
<td>$c_4=50$</td>
</tr>
<tr>
<td>$m_4=5897$</td>
<td>$k_4=19059$</td>
<td>$c_4=38$</td>
</tr>
</tbody>
</table>

in Table 3.1. In the unisolated case, the structure has a natural period of 0.3 sec. This base isolated structure has been used frequently by many researchers in the past (e.g., Kelly et al., 1987, Ramallo et al., 2002, He et al., 2003) to study their control algorithms. The only variation in this study from the original structure properties is the change of damping coefficient of the isolators, $c_b$, to obtain 4% critical instead of 2% critical.

In the analytical formulation, we assume that the superstructure remains elastic. This is a valid assumption as base isolated structures in practice are designed in such a way that the superstructure is linear and all the nonlinearities are limited to the isolation level. For the first test structure, i.e., the five story building model, the isolation system is linear; hence, standard numerical techniques for solving differential
equations are used to compute the structure responses.

Denoting the displacements relative to the ground as

\[
x = \begin{bmatrix} x_b & x_1 & x_2 & x_3 & x_4 & x_5 \end{bmatrix},
\]

the equations of motion for the base six DOF structure can be written as:

\[
M\ddot{x} + C\dot{x} + Kx = \Gamma f - MR\ddot{u}_g \tag{3.1}
\]

where the mass, damping and stiffness matrices are,

\[
M = \begin{bmatrix}
m_b & 0 & 0 & 0 & 0 & 0 \\
0 & m_1 & 0 & 0 & 0 & 0 \\
0 & 0 & m_2 & 0 & 0 & 0 \\
0 & 0 & 0 & m_3 & 0 & 0 \\
0 & 0 & 0 & 0 & m_4 & 0 \\
0 & 0 & 0 & 0 & 0 & m_5 \\
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
c_b + c_1 & -c_1 & 0 & 0 & 0 & 0 \\
-c_1 & c_1 + c_2 & -c_2 & -c_2 & 0 & 0 \\
0 & -c_2 & c_2 + c_3 & -c_3 & 0 & 0 \\
0 & 0 & -c_3 & c_3 + c_4 & -c_4 & 0 \\
0 & 0 & 0 & -c_4 & c_4 + c_5 & -c_5 \\
0 & 0 & 0 & 0 & -c_5 & c_5 \\
\end{bmatrix}
\]
and
\[
K = \begin{bmatrix}
k_b + k_1 & -k_1 & 0 & 0 & 0 & 0 \\
-k_1 & k_1 + k_2 & -k_2 & 0 & 0 & 0 \\
0 & -k_2 & k_2 + k_3 & -k_3 & 0 & 0 \\
0 & 0 & -k_3 & k_3 + k_4 & -k_4 & 0 \\
0 & 0 & 0 & -k_4 & k_4 + k_5 & -k_5 \\
0 & 0 & 0 & 0 & -k_5 & k_5 
\end{bmatrix}.
\]

The control forces location vector and earthquake influence vectors are given by,
\[
\Gamma = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T; R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T.
\]

Re-writing the above equations in the state-space form by defining the states as,
\[
\dot{x} = \begin{bmatrix} x & \dot{x} \end{bmatrix},
\]

we get,
\[
\dot{x} = Ax + Bf + E\ddot{u}_g \tag{3.2}
\]

where,
\[
A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}_{12 \times 12}; \quad B = \begin{bmatrix} 0 \\ M^{-1}\Gamma \end{bmatrix}_{12 \times 1}; \quad E = \begin{bmatrix} 0 \\ -I \end{bmatrix}_{12 \times 1}
\]

Eq. 3.2 is solved using explicit Runge-kutta method for numerical integration of the initial value problem.
3.3 Structure B: Three Dimensional Eight Story Benchmark Building

The second structure used in this work is a eight-story three dimensional structure similar to existing buildings in Los Angeles, California. This structure is also the test-bed for the ongoing smart base isolated benchmark study (Narasimhan et al., 2004).

3.3.1 Background on Benchmark Studies

Recently well-defined analytical benchmark problems (Caughey, 1998, Spencer et al., 1998, Ohtori et al., 2003, Yang et al., 2003, Dyke et al., 2003) have been developed for studying response control strategies for building and bridge structures subjected to seismic and wind excitation, by broad consensus effort of the ASCE structural control committee. The goal of this effort was to develop benchmark models to provide systematic and standardized means by which competing control strategies, including devices, algorithms, sensors, etc. can be evaluated. Carefully defined analytical benchmark problems are an excellent alternative to expensive experimental benchmark test structures. Due to effectiveness of the fixed base building benchmark effort (Spencer et al., 1998(a,b), Ohtori et al., 2003, Yang et al., 2003), the ASCE structural control committee voted to develop a new smart base isolated benchmark problem. Narasimhan, et al. (2002, 2003, 2004) have developed the smart base isolated benchmark problem, with capability to model three different kinds of base isolation systems: linear elastomeric systems with low damping or supplemental high damping, frictional
systems, bilinear or nonlinear elastomeric systems or any combination thereof. The superstructure is assumed to remain linear at all times. A host of control devices can be considered at the isolation level.

3.3.2 Structural Model

The benchmark structure is a base-isolated eight-story, steel-braced framed building, 82.4-m long and 54.3-m wide, similar to existing buildings in Los Angeles, California. The floor plan is L-shaped as shown in Fig. 3.2. The superstructure bracing is located at the building perimeter. Metal decking and a grid of steel beams support all concrete floor slabs. The steel superstructure is supported on a reinforced concrete base slab, which is integral with concrete beams below, and drop panels below each column location. The isolators are connected between these drop panels and the footings below as shown in Fig. 3.2. The superstructure is modeled as a three dimensional linear elastic system. The superstructure members, such as beam, column, bracing, and floor slab are modeled in detail. Floor slabs and the base are assumed to be rigid in plane. The superstructure and the base are modeled using three master degrees of freedom (DOF) per floor at the center of mass. The combined model of the superstructure (24 DOF) and isolation system (3 DOF) consists of 27 degrees of freedom. All twenty four modes in the fixed base case are used in modeling the superstructure. The superstructure damping ratio is assumed to be 5% in all fixed base modes. The first nine computed natural periods for the structure
Table 3.2  Periods

<table>
<thead>
<tr>
<th></th>
<th>North-South</th>
<th></th>
<th>East-West</th>
<th></th>
<th>Torsion</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Period</td>
<td>0.78</td>
<td>0.27</td>
<td>0.15</td>
<td>0.89</td>
<td>0.28</td>
<td>0.15</td>
</tr>
</tbody>
</table>

are shown in Table 3.2. The eigen values and the eigen vectors for the structure for all the twenty four modes is is used in the analytical formulation. Fig. 3.2 shows an example isolation system which is a combination of linear elastomeric and frictional bearings (can also be viewed as dampers).

3.3.3 Isolation Model

Several isolation elements are included so that any combination of these can be used to model the isolation system completely. The isolation elements are elastic, viscous, hysteretic elements for bilinear elastomeric bearings and hysteretic elements for sliding bearings. The force-displacement characteristics for friction pendulum, lead rubber bearing and linear isolation bearings is shown in Fig. 3.3. The hysteretic elements can be uni-axial or biaxial. The linear elastic and viscous elements are for modeling linear elastomeric bearings and fluid dampers. They can also be used for modeling bilinear elastomeric isolation systems with corresponding equivalent linear properties, obtained using appropriate linearization techniques.

The biaxial hysteretic behavior of bilinear elastomeric bearings and/or frictional bearings is modeled using the biaxial interaction equations of Bouc-Wen model pro-
Figure 3.2  (a) Isolation Plan, (b) FEM Model of Superstructure and (c) Elevation View with Devices
Figure 3.3  Force-Displacement Characteristics of Bearings
posed by Park and Wen (1986) as follows:

\[
U^y \begin{cases}
\dot{z}_x \\
\dot{z}_y
\end{cases} = \alpha \begin{cases}
\dot{U}_x \\
\dot{U}_y
\end{cases} - Z_w \begin{cases}
\dot{U}_x \\
\dot{U}_y
\end{cases}
\]

\[Z_w = \begin{bmatrix}
2(\gamma \text{sgn}(\dot{U}_x z_x) + \beta) & z_x z_y (\gamma \text{sgn}(\dot{U}_x z_y) + \beta) \\
z_x z_y (\gamma \text{sgn}(\dot{U}_x z_x) + \beta) & z_y^2 (\gamma \text{sgn}(\dot{U}_y z_y) + \beta)
\end{bmatrix}
\]  

(3.3)

where \( z_x \) and \( z_y \) are dimensionless hysteretic variables that are bounded by values \( \pm 1 \), \( \alpha \), \( \beta \) and \( \gamma \) are dimensionless quantities, \( U_x, U_y \) and \( \dot{U}_x, \dot{U}_y \) represent the displacements and velocities in the \( x \) and \( y \) directions, respectively, at the isolation bearing or device and \( U^y \) is the yield displacement. Eq. 3.3 accounts for biaxial interaction of both sliding and bilinear hysteretic bearings. When yielding commences Eq. 3.3 leads to \( z_x = \cos \theta \) and \( z_y = \sin \theta \) provided \( \alpha/(\beta + \gamma) = 1 \) with \( \theta = \tan^{-1}(\dot{U}_x/\dot{U}_y) \) and resultant velocity \( \dot{U} = \sqrt{\dot{U}_x^2 + \dot{U}_y^2} \). The biaxial interaction can be neglected when the off-diagonal terms of the matrix in Eq. 3.3 are replaced by zeros. This results in an uniaxial model with two independent elements in two orthogonal directions.

The forces, \( f \), mobilized in the elastomeric isolation bearings or devices can be modeled by a elasticviscoelastic model with strain hardening

\[
f_x = k_p \dot{U}_x + c_v \dot{\dot{U}}_x + (k_e - k_p) U^y z_x
\]

(3.4)

\[
f_y = k_p \dot{U}_y + c_v \dot{\dot{U}}_y + (k_e - k_p) U^y z_y
\]

(3.5)
where \( k_c = \) pre-yield stiffness, \( k_p = \) post-yield stiffness, \( c_v = \) viscous damping coefficient of the elastomeric bearing or device, \( U^y \) is the yield displacement.

Eq. 3.3 can also be used to model sliding bearings with flat or spherical sliding surface, by means of a small yield displacement \( U^y \) (because of rigid plastic behavior and large pre-yield stiffness) setting \( c_v = 0 \) and \( (k_c - k_p) U^y = \mu N \)

\[
f_z = k_p U_z + \mu N z_x \tag{3.6}
\]

\[
f_y = k_p U_y + \mu N z_y \tag{3.7}
\]

where \( \mu \) is the coefficient of friction and \( N \) is the average normal force at the bearing (normal force variation is neglected). In a similar manner other devices such as nonlinear fluid dampers can also be modeled using Eq. 3.3.

### 3.3.4 Nonlinear Dynamic Analysis

Base isolated buildings are designed such that the superstructure remains elastic. Hence, in this study the superstructure is modeled by a condensed linear elastic system. Also, the localized nonlinearities at the isolation level allow condensation of the linear superstructure. In addition, the equations of motion are developed in such a way that the fixed-base properties are used for modeling the linear superstructure. The base and the floors are assumed to be infinitely rigid in plane. The superstructure and the base are modeled using three master degrees of freedom (DOF) per floor at
the center of mass. Each nonlinear isolation bearing or device is modeled explicitly using discrete biaxial Bouc-Wen model, and the forces in the bearings or devices are transformed to the center of mass of the base using a rigid base slab assumption. All the linear isolation bearings or devices can be modeled individually or globally by equivalent lumped elements at the center of mass of the base. The displacement coordinates are shown in Fig. 3.4 and the asymmetric model is shown in Fig. 3.5.

![Figure 3.4](image)

**Figure 3.4** Displacement Coordinates of the Base Isolated Structure

The equations of motion for the elastic superstructure are expressed in the following form:

\[
M_{n \times n} \ddot{U}_{n \times 1} + C_{n \times n} \dot{U}_{n \times 1} + K_{n \times n} U_{n \times 1} = -M_{n \times n} R_{n \times 3}(\ddot{U}_g + \ddot{U}_b)_{3 \times 1} \tag{3.8}
\]
Here, \( n \) is three times the number of floors (excluding base), \( M \) is the superstructure mass matrix, \( C \) is the superstructure damping matrix in the fixed base case, \( K \) is the superstructure stiffness matrix in the fixed base case and \( R \) is the matrix of earthquake influence coefficients, i.e. the matrix of displacements and rotation at the center of mass of the floors resulting from a unit translation in the \( X \) and \( Y \) directions and unit rotation at the center of mass of the base. Furthermore, \( \ddot{U}, \dot{U}, U \) and \( \ddot{U}_b \) represent the floor acceleration, velocity and displacement vectors relative to the base, \( \ddot{U}_g \) is the vector of base acceleration relative to the ground and \( \ddot{U}_g \) is the vector of ground acceleration. The control devices are located at the isolation level only as shown in Fig. 3.1. The equations of motion for the base are as follows:

\[
R^T_{3 \times n} M_{n \times n} \left[ (\ddot{U})_{n \times 1} + R_{n \times 3}(\ddot{U}_g + \ddot{U}_b)_{3 \times 1} \right]_{n \times 1} + M_{b_{3 \times 3}} (\ddot{U}_g + \ddot{U}_b)_{3 \times 1} + C_{b_{3 \times 3}} \ddot{U}_b_{3 \times 1}
\]

\[= K_{b_{3 \times 3}} U_{b_{3 \times 1}} + f_{B_{3 \times 1}} + f_{c_{3 \times 1}} = 0 \tag{3.9} \]

in which, \( M_b \) is the diagonal mass matrix of the rigid base, \( C_b \) is the resultant damping matrix of viscous isolation elements, \( K_b \) is the resultant stiffness matrix of elastic isolation elements, \( f_B \) is the vector containing the nonlinear bearing forces and \( f_c \) is the vector containing the control forces. Eqn. 3.8-3.9 can be reformulated in the modal domain and the fixed base frequencies, damping ratios, and modes can be used for modeling the superstructure (Nagarajaiah et al. 1991). Using \( X = \{U^T \ U_b^T \ \dot{U}^T \ \dot{U}_b^T \}^T \), the state space equations can be formulated as
Figure 3.5  Asymmetric Base Isolated Structure Excited by Bidirectional Ground Motion
\[
\dot{X}(t) = AX(t) + Bu(t) + B^*F_B(t) + E\ddot{U}_g(t) = g(X, u, \ddot{U}_g) \quad (3.10)
\]

\[
A = \begin{bmatrix}
0 & I \\
-\overline{M}^{-1}K & -\overline{M}^{-1}\overline{C}
\end{bmatrix}, \\
E = \begin{bmatrix}
0 \\
-\overline{M}^{-1}
\begin{bmatrix}
MR \\
R^TMR + M_b
\end{bmatrix}
\end{bmatrix}, \\
(3.11)
\]

\[
B = B^* = \begin{bmatrix}
0 \\
-\overline{M}^{-1}
\begin{bmatrix}
0 \\
I
\end{bmatrix}
\end{bmatrix}, \\
(3.12)
\]

\[
\overline{M} = \begin{bmatrix}
M & MR \\
R^T M & R^T MR + M_b
\end{bmatrix}, \\
\overline{C} = \begin{bmatrix}
C & 0 \\
0 & C_b
\end{bmatrix}, \\
(3.13)
\]

\[
K = \begin{bmatrix}
K & 0 \\
0 & K_b
\end{bmatrix}, u = \begin{bmatrix}
0 \\
f_c
\end{bmatrix}, F_B = \begin{bmatrix}
0 \\
f_B
\end{bmatrix}. \\
(3.14)
\]

In the above equations, \(A, B, B^*\) and \(E\) are condensed system matrices having 54 states derived from the full three dimensional finite element model. The Eq. 3.10 is solved using unconditionally stable Newmark’s constant-average acceleration method, which can also be derived from trapezoidal rule given by

\[
X_{k+1} = X_k + \frac{\Delta t}{2}(s_k + s_{k+1}) \\
(3.15)
\]

where \(s_{k+1} = g(X_{k+1}, u_{k+1}, \dot{U}_{g(k+1)})\). This method is implicit, needing iteration.

The nonlinear forces in the isolation bearings, devices and control forces are updated
by solving Eqns. 3.3 to 3.7 using the unconditionally stable semi-implicit Runge-Kutta method (Rosenbrook 1964) suitable for solutions of stiff differential equations. Then Eqn. 3.10 is solved using an iterative predictor-corrector solution procedure (Nagarajaiah et al. 1991) until equilibrium of nonlinear forces is reached within specified tolerance and convergence is achieved.

3.4 Numerical Implementation

The first objective in this work is to develop an unified and realistic framework for base isolated buildings to implement control strategies. This framework mimics real-life implementations, where all the disturbances and device dynamics are incorporated. By developing such an unified framework, flexibility in choosing sensors, control device properties becomes less complicated and easy to compare. The analytical model for both the buildings is implemented using MATLAB and SIMULINK as shown in Fig. 3.6. The analysis program comprises of input data files, a file to read and assemble the required matrices for input into the nonlinear dynamic analysis block, which is a SIMULINK based S function program. The full implementation procedure is also shown in Fig. 3.6. Additional inputs to the nonlinear analysis block are the seismic excitation and the control forces provided by the control devices. The nonlinear response is calculated using a predictor-corrector algorithm as explained in the earlier sections. All the sensor and control devices can be modeled in this program as SIMULINK blocks and the output of these models fed into the analysis
S-function block. This framework is used throughout the thesis for all the numerical simulations. The results of any implementation is available in terms of performance indices, to be defined in the following sections, or in terms of time history plots.

Figure 3.6  Schematics of MATLAB/SIMULINK Implementation

3.5  Control Design

The control devices, sensor devices and the control algorithms are interfaced to the structural evaluation model though measurement and device connection outputs, designated \( y_m \) and \( y_{cd} \) respectively. The evaluation outputs \( y_e \) are used for the calculation of performance indices.

3.5.1  Sensor Models

The sensors take the following form

\[
\dot{X}^s = g_1(X^s, y_m, u_m, t)
\]  

(3.16)
\[ y^s = g_2(X^s, y_m, u_m, v, t) \]  \hspace{1cm} (3.17)

where \( X^s \) are the states of the sensor, \( v \) is the measurement noise vector, \( u_m \) is a vector of control device continuous time responses and \( y^s \) is the output of sensor in units of volts (Fig. 3.8). \( u_m \) consists of device forces, and/or stroke that may be needed for feedback into the controller.

### 3.5.2 Control Algorithm

Control algorithms are designed to work with active or semi-active systems. The discrete form of the control algorithms may be written as

\[ X^{c}_{k+1} = g_3(X^{c}_k, y^{s}_k, k) \]  \hspace{1cm} (3.18)

\[ u_k = g_4(X^{c}_k, y^{s}_k, k) \]  \hspace{1cm} (3.19)

where \( X^{c}_k \) is the discrete state vector at time \( t = k\Delta t \), \( y^{s}_k \) is the discretized sensor model output and \( u_k \) is the discrete control command from the control algorithm (Fig. 3.9).

### 3.5.3 Control Devices

The device models are interfaced with the building model by including the dynamics of the device (Fig. 3.10) as follows

\[ \dot{X}^{cd} = g_5(X^{cd}, y_{cd}, u_k, t) \]  \hspace{1cm} (3.20)

\[ u = g_6(X^{cd}, y_{cd}, u_k, t) \]  \hspace{1cm} (3.21)
\[ y_f = g_7(X_{cd}, y_{cd}, u_k, t) \] (3.22)

where the continuous states of the devices are represented by \( X_{cd} \). If the dynamics are neglected, then the model is

\[ u = g_8(y_{cd}, u_k, t) \] (3.23)

\[ y_f = g_9(y_{cd}, u_k, t) \] (3.24)

3.6 Evaluation Criteria

The performance of any control algorithm cannot be measured by a single quantity. For example, it has been observed that the addition of significant amount of passive damping at the isolation level reduces the base displacements, but at the cost of increased structural forces, inter-story drifts and accelerations. This effect can be attributed to the excitation of higher modes due to the presence of high levels of damping (Kelly, 1999). A similar situation (increased accelerations) occurs when semiactive algorithms with rapid switching are implemented. In order to understand and quantify all aspects of the performance of control algorithms in a base isolated building, a set of normalized performance indices are developed called the evaluation criteria. The evaluation criteria are based on both peak and root mean square responses (RMS) responses. In addition, they also measure the amount of control force input or developed in the device (for active and semiactive devices) and dissipated
(for semiactive devices) during the control implementation. There are a total of nine evaluation criteria; represented $J_1$ through $J_9$. The first six evaluation criteria are based on peak quantities ($J_1$ through $J_6$), the next two on RMS responses ($J_7$ and $J_8$) and the last on energy; both input and dissipated ($J_9$). In the definition of the evaluation criteria, the term uncontrolled refers to the structure without any control devices and the term controlled refers to the presence of control devices. The details of the evaluation criteria are as follows:

1. Peak base shear (isolation-level) in the controlled structure normalized by the corresponding shear in the uncontrolled structure,

$$J_1(q) = \frac{\max_t \| V_0(t,q) \|}{\max_t \| \dot{V}_0(t,q) \|}$$

2. Peak structure shear (at first story level) in the controlled structure normalized by the corresponding shear in the uncontrolled structure,

$$J_2(q) = \frac{\max_t \| V_1(t,q) \|}{\max_t \| \dot{V}_1(t,q) \|}$$

3. Peak base displacement or isolator deformation in the controlled structure normalized by the corresponding displacement in the uncontrolled structure,

$$J_3(q) = \frac{\max_{t,s} \| d_i(t,q) \|}{\max_{t,s} \| \dot{d}_i(t,q) \|}$$
4. Peak inter-story drift in the controlled structure normalized by the corresponding inter-story drift in the uncontrolled structure,

\[ J_4(q) = \frac{\max_{t_i} \| d_f(t, q) \|}{\max_{t_f} \| d_f(t, q) \|} \]

5. Peak absolute floor acceleration in the controlled structure normalized by the corresponding acceleration in the uncontrolled structure,

\[ J_5(q) = \frac{\max_{t_f} \| a_f(t, q) \|}{\max_{t_f} \| a_f(t, q) \|} \]

6. Peak force generated by all control devices normalized by the peak base shear in the controlled structure,

\[ J_6(q) = \frac{\max_{t} \| \sum_k F_k(t, q) \|}{\max_{t} \| V_0(t, q) \|} \]

7. RMS base displacement in the controlled structure normalized by the corresponding RMS base displacement in the uncontrolled structure,

\[ J_7(q) = \frac{\max_{t} \| \sigma_d(t, q) \|}{\max_{t} \| \sigma_d(t, q) \|} \]

8. RMS absolute floor acceleration in the controlled structure normalized by the corresponding RMS acceleration in the uncontrolled structure,

\[ J_8(q) = \frac{\max_{t} \| \sigma_a(t, q) \|}{\max_{t} \| \sigma_a(t, q) \|} \]
9. Total energy absorbed by all control devices normalized by energy input into the controlled structure,

\[ J_g(q) = \sum_k \left[ \frac{\int_0^{T_q} F_k(t,q) v_k(t,q) \, dt}{\int_0^{T_q} \langle V_o(t,q) U_g(t,q) \rangle \, dt} \right] \]

where, \( i = \) isolator number, \( 1, \ldots, N_i (N_i = 8); k = \) device number, \( 1, \ldots, N_d; f = \) floor number, \( 1, \ldots, N_f; q = \) earthquake number; \( t = \) time, \( 0 \leq t \leq T_q; \langle \cdot \rangle = \) inner product; \( \| \cdot \| = \) vector magnitude incorporating components of the earthquakes.

3.7 Earthquakes

The earthquakes used in this study are both the fault-normal (FN) and fault-parallel (FP) components of Newhall, Sylmar, El Centro, Rinaldi, Kobe, Ji-ji and Erzinkan as shown in Fig. 3.11. All the excitations are near fault earthquakes and are used at the full intensity for the performance evaluation.

3.8 Concluding Remarks

In this chapter analytical models for base isolated buildings is developed together with a framework for control implementation and a set of carefully chosen performance indices. Two types of base isolated buildings are developed and used in this thesis. The first building is a five-story, steel framed, shear-beam type building with moment-resistant connections. The isolators are connected between the base and the ground. A single control device is connected between the base and the ground. The second
structure considered is a fully three dimensional eight story steel framed building with lateral-torsional coupling. This building is similar to the USC hospital building in Los Angeles, California. This building was selected to be the test bed for the first generation smart base isolated benchmark study.
Benchmark Problem for Control of Base Isolated Buildings

by
Sriram Narasimhan, Satish Nagarajaiah, Erik Johnson and Henri Gavin

Figure 3.7  SIMULINK Block Diagram for Simulations
Replace the contents of this block with a model of your sensor(s)

**Figure 3.8** Sensor Model Implementation

Replace the contents of this block with a model of your devices

**Figure 3.9** Control Algorithm Implementation

Replace the contents of this block with a model of your devices

**Figure 3.10** Control Devices Implementation
Figure 3.11  Time Histories of Near Fault Earthquake Records. FP - Fault Parallel, FN - Fault Normal, EW - East West and NS - North South.
Chapter 4
Optimal Control Methods: $H_2/LQG$ and $H_\infty$ Methods

4.1 Introduction

The field of linear optimal control has been a subject of active research for the last two decades (Spencer and Nagarajaiah, 2003). Time domain methods such as LQR (Linear Quadratic Regulator) and its counterpart, LQG (Linear Quadratic Gaussian) have been employed in various structural control applications. The main idea behind the LQR method is a minimization of a performance index under the constraint imposed by the dynamic equations of motion (Stengel, 1986). This minimization is in essence driving the structural responses of the system close to zero. Typically, two types of weighting matrices are used in the LQR procedure. They correspond to weighting structural responses and control forces. By choosing the appropriate matrices through a trial and error procedure, the desired performance of the system can be achieved. The major limitations in the LQR method are (i) trial and error involving the selection of the weighting matrices, (ii) assuming the earthquake as a zero-mean white noise to reduce the time varying riccati equation to an algebraic one and, (iii) ability to measure all the states of the system for full state feedback.

Some of the limitations in the LQR method are addressed in the LQG method. In this method the LQR method is combined with a state estimator (Kalman Buca
Filter); the estimated states along with the partially measured states are used in place of unobserved states for full state feedback in LQR using the separation principle (Stengel, 1986). Such a procedure is very useful in structural control where, only partial state measurements or a state combination (e.g., accelerations) are available or for output feedback. The ground excitation can be modeled by augmenting state equations (with an appropriate filter excited by a white noise) More discussion on the augmentation procedures are given in the following sections.

Though the LQG procedure addresses some of the limitations of the LQR method, there is no assurance of robustness, which can only be ensured by the $H_2$ and $H_\infty$ frequency domain methods. The LQG procedure is essentially equivalent to $H_2$ control as minimizing the LQG cost function is equivalent to minimizing the closed-loop system 2-norm. The control designs in frequency domain provide more physical insights into the problem, specially because, earthquakes can be described satisfactorily using their power spectral densities (PSD); and the base isolated structural system can be adequately modeled by transfer function (dominated by fundamental mode). $H_2$ and $H_\infty$ frequency domain methods also incorporate the system uncertainty directly in the problem formulation to provide a robust design. Both the methods studied in this dissertation, $H_2$ and $H_\infty$ -minimize a prescribed norm of the transfer function from the excitation to the output. $H_2$ and $H_\infty$ derive their name from the norm that is minimized, 2-norm or $\infty$ norm.
In this chapter, the basics of dynamic systems, time domain methods and frequency domain methods are introduced. A single degree of freedom (SDOF) system with partial state measurements is formulated based on $H_2$ and $H_\infty$ frequency domain methods. Results obtained from this formulation form the basis for various control strategies to be developed for smart base isolated buildings in the following chapters.

4.2 State Space Representation and Transfer Functions

For the application of classical control, the equations of motion are formulated in state space. A dynamical system can have multiple realizations, or equivalent forms of system representation. One such realization is the state space representation where a $n^{th}$ order differential equation is converted into $n$ simultaneous first order differential equations. These equations are in the time domain and simple to solve using standard methods available in matrix algebra. In the state space representation, a general linear, time varying structural system excited by an earthquake can be represented as

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) + E(t)\bar{u}_g(t)$$

$$y(t) = C(t)x(t) + D(t)u(t) + v(t)$$

(4.1)

where, $x$ are the states of the system, $A$, $B$, $C$, $D$ and $E$ are time-varying system matrices, $\bar{u}_g$ is the earthquake excitation, $u$ is the vector of control forces and $v$ is a measurement noise. For a linear time-invariant system (LTI), where the systems
matrices do not change with time, the above equations can be re-written as,

\[\dot{x}(t) = Ax(t) + Bu(t) + E\dot{u}_s(t)\]
\[y(t) = Cx(t) + Du(t) + v(t)\]  

(4.2)

Details of the state space formulation for an example single degree of freedom (SDOF) system is shown in the numerical example. Two important properties of a system are controllability and observability. A system is said to be controllable if a state can be driven to any specified value from its initial state. A system is said to be observable if a state vector can be determined or estimated from the measured output. The controllability matrix can be formed using the A and B matrices as

\[CO = \begin{pmatrix} B & AB & A^2B & \cdots & A^{n-1}B \end{pmatrix}\]  

(4.3)

and the observability matrix with C and A matrices as

\[OB = \begin{pmatrix} C^T & A^T C^T & (A^T)^2 C^T & \cdots & (A^T)^{n-1} C^T \end{pmatrix}\]  

(4.4)

If the dimension of the state vector is n, then \((A, B)\) is said to be controllable if \(\text{rank}(CO)=n\) and \((A, C)\) is said to be observable if \(\text{rank}(OB)=n\).

Let us consider the simplified form of the state space equations in eq. 4.2 without the earthquake excitation and the measurement noise.

\[\dot{x}(t) = Ax(t) + Bu(t)\]
\[y(t) = Cx(t) + Du(t)\]  

(4.5)
Figure 4.1  Frequency response function for a linear system

Taking the laplace transform on both sides of eq. 4.5, we get

\[ sX(s) = AX(s) + BU(s) \Rightarrow X(s) = (sI - A)^{-1} BU(s) \]  

(4.6)

\[ Y(s) = CX(s) + DU(s) = [C(sI - A)^{-1} B + D] U(s) \Rightarrow \Phi(s) U(s) \]  

(4.7)

where, \( \Phi(s) \) is the transfer function from the control input, \( u \) to the measurement vector, \( y \). The transfer function is related to the frequency response function if the variable \( s \) is replaced by the complex variable \( j\omega \). As shown in Fig 4.1, the output is equal to the harmonic excitation input, \( e^{j\omega t} \), multiplied by the frequency response.

4.3 Time Domain Methods: LQR and LQG

4.3.1 LQR Method

The Linear Quadratic Regulator (LQR) method, also known as the quadratic optimal regulator method, provides a systematic way of computing the state feedback control gain matrix. The LQR method involves computing the feedback gain matrix \( K \) of the optimal control vector, \( u = -Kx(t) \), given the state equation,

\[ \dot{x} = Ax + Bu \]  

(4.8)

so as to minimize the quadratic performance index,

\[ J = \int_0^\infty (x^T Q x + u^T R u) \, dt \]  

(4.9)
where, $Q$ is a positive-definite (or positive-semidefinite) Hermitian or real symmetric matrix and $R$ is a positive-definite Hermitian or real symmetric matrix. These weighting matrices determine the relative importance of the responses and the expenditure of control energy. The block diagram representation is shown in Fig. 4.2. The control gain matrix is obtained by solving the optimization problem (Stengel, 1986) and is given as

$$ K = R^{-1}B^TP $$

(4.10)

and the matrix $P$ is the steady state solution of the following simplified algebraic Riccati equation,

$$ A^TP + PA - PBR^{-1}B^TP + Q = 0 $$

(4.11)

The basic design steps for the LQR method can be summarized as follows:

• Compute the solution of the algebraic Riccati equation, eq. 4.11 for $P$. If a positive definite matrix $P$ exists, the system is stable, or the matrix $A - BK$ is stable.
• The matrix $P$ obtained is substituted in eq. 4.10 to obtain the optimal feedback gain matrix.

As can be readily seen from the above procedure, in order to calculate the optimal gain matrix, all the states need to be known. Another limitation of the method for the structural control purposes is the lack of excitation information in computing the feedback gain matrix. The above limitations are addressed in the Linear Quadratic Gaussian (LQG) method.

### 4.3.2 Optimal Estimation

Estimation of unknown states based on available measurements is accomplished with the aid of the Kalman Bucy filter. The linear optimal estimator minimizes the mean-square estimation error with respect to the choice of a filter gain matrix. The estimate of states is through a linear ordinary differential equation based on a system model with the actual residual measurement errors driving the state propagation through the optimal gain matrix. The covariance estimate is derived from nonlinear, ordinary differential equation driven by the statistics of the assumed measurement errors and disturbance inputs (Stengel, 1986). The dynamic system considered in eq. 4.2 is repeated here where the excitation is a white, zero-mean gaussian random
process, and the matrix, D = 0.

\[
\dot{x}(t) = Ax(t) + Bu(t) + Ew(t)
\]
\[
y(t) = Cx(t) + v(t)
\]  
(4.12)

The known expected values of the initial state and covariance assuming uncorrelated disturbance input and measurement are as follows:

\[
E(x(0)) = \hat{x}_0
\]
\[
E([x(0) - \hat{x}_0][x(0) - \hat{x}_0]^T) = P_0
\]  
(4.13)

The disturbance input and measurement error are white, zero-mean gaussian processes with spectral density matrices \(Q_e\) and \(R_e\) defined as follows:

\[
E(w(t)) = E(v(t)) = 0
\]
\[
E(w(t)w^T(t)) = Q_e(t)\delta(t - \tau)
\]
\[
E(v(t)v^T(t)) = R_e(t)\delta(t - \tau)
\]  
(4.14)

The linear estimator is optimal in the sense that the variance of the state estimate error is minimized on the average (expected value). Based on the optimization (Stengel, 1986), the covariance estimate is found by integrating the following equation:

\[
P_e = AP_e + P_eA^T + EQ_eE^T - P_eC^TR_e^{-1}CP_e
\]
\[
P_e(0) = P_0
\]  
(4.15)

The optimal filter gain equation is

\[
K_e = P_eC^TR_e^{-1}
\]  
(4.16)
The state estimate is found by integrating the following equation:

\[
\dot{x} = A\dot{x} + Bu + K_c[x - C\dot{x}]
\]

\[
\hat{x}(0) = \hat{x}_0 \tag{4.17}
\]

For stable filter estimates, \(R_c^{-1}\) should be positive definite. If \((A, C)\) is detectable and \((A, Q_c)\) is stabilizable (system's unstable subspace is contained in the controllable subspace), then the filter known as Kalman Bucy Filter (KBF) is stable and \(K_c\) approaches a steady state value, which is an unique positive semi-definite solution of the algebraic Riccati equation

\[
AP_e + P_eA^T + EQ_cE^T - P_eC^TR_c^{-1}CP_e = 0 \tag{4.18}
\]

If \((A, Q_c)\) is controllable, then the solution is positive definite.

### 4.3.3 LQG Method

From the LQR method, it is seen that the optimal control for a linear system with a quadratic performance index is a linear feedback of the state variables. From the KBF, the estimates of the state variables can be obtained from noisy measurements of linear combinations of the state variables, using a filter that is a model of the system and a feedback signal proportional to the difference between the actual and estimated measurements. The LQG method involves the combination of the KBF optimal estimation filter and the optimal deterministic controller. This optimal feedback controller is combined in the ensemble average sense, for linear-quadratic problems
with additive gaussian white noise. Consider the following state equations:

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + Ev(t) \\
y(t) &= Cx(t) + v(t)
\end{align*}
\]  
(4.19)

For this system, using the certainty-equivalence principle, the controller is designed using the LQR method with the states computed using the KBF. The estimated states are used for the controller design as though they are the exact states of the system. In other words, the LQG method involves the minimization of the quadratic performance index,

\[
J = E\left[\int_0^\infty \left(x^TQx + u^TRu\right) dt\right]
\]  
(4.20)

under the constraint imposed by the equations of motion. The solution to this problem is:

\[
\begin{align*}
u &= -K\dot{x}(t) \\
\dot{x} &= Ax + Bu + K_c[x - C\hat{x}]
\end{align*}
\]  
(4.21)

If \((A, B)\) is stabilizable, \((A, Q)\) is detectable, \((A, C)\) is detectable, and \((A, Q_c)\) is stabilizable, then the closed loop system using the LQG control is stable. The order of the resulting controller is the same as the order of the plant.
4.4 Control in the Frequency Domain

4.4.1 $H_2$ and $H_\infty$ norm

The power spectral density, $S_z$, of the output, $z$, from a transfer function, $H$ subjected to an input, $d$, of power spectral density, $S_d(\omega)$, is given by

$$S_z(\omega) = H(j\omega)S_d(\omega)H^*(j\omega) \quad \text{(4.22)}$$

where, the $*$ indicates the conjugate transpose of $H$. The root mean square (RMS) value of the output, $z$ is,

$$\|z\|_{\text{rms}} = \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{trace}[H(j\omega)S_d(\omega)H^*(j\omega) d\omega] \right)^{1/2} \quad \text{(4.23)}$$

For the case when the input, $d$ is a unit intensity white noise signal, the $H_2$ norm of the transfer function is defined as,

$$\|z\|_{\text{rms}} = \left( \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{trace}[H(j\omega)H^*(j\omega) d\omega] \right)^{1/2} \quad \text{(4.24)}$$

Thus, the $H_2$ norm of the transfer function is the RMS value of the output, when the input is a unit intensity white noise.

The singular values of any matrix, $A$, denoted $\sigma_i[A]$, are the non-negative square-roots of the eigen values of $A^*A$, where, $A^*$ is the transpose of the complex conjugate of $A$, given by

$$\sigma_i[A] = \sqrt{\lambda_i(A^*A)} \quad \text{(4.25)}$$
The smallest and the largest singular values are denoted by $\sigma[A]$ and $\overline{\sigma}[A]$ respectively. In terms of the singular values, the $H_2$ norm can be written as

$$\|H\|_2 = \left(\frac{1}{2\pi} \int_{-\infty}^{\infty} \sum_{i}^{n} \sigma_i[H(j\omega)]^2 d\omega\right)^{1/2}$$  \hspace{1cm} (4.26)

where, $n$ is the smallest dimension of the matrix $H$.

The $\infty$ norm of a transfer function matrix, $H$ is defined in terms of its singular values as,

$$\|H\|_\infty = \sup_{\omega} (\overline{\sigma}[H(j\omega)])$$  \hspace{1cm} (4.27)

This means the $\infty$ norm is the supremum of the maximum singular value over all frequencies.

### 4.4.2 Frequency Domain Representation

The basic block diagram used for the purposes of this study is shown in Fig. 4.3.

The generalized plant is represented by $G$ and the controller by $K$. The measurement
outputs are represented by \( y \), the outputs to be regulated by \( z \), external disturbance by \( w \), which includes the earthquake excitation and sensor noise and the control input is represented by \( u \). Frequency domain representation enables the frequency information of the excitation and/or the regulated variables to be included in the system representation. In order to do that within the framework of the standard block diagram representation, the frequency weighting functions are augmented in the generalized plant. A more detailed description of the procedure is introduced in the following sections. For all the discussions to follow, unless otherwise noted, the system \( G \) is assumed to be Linear Time Invariant (LTI). In order to explain the main idea of frequency control using \( H_2 \) or \( H_\infty \) methods, the partitioned form of the transfer function of the plant shown in Fig. 4.3 is,

\[
\begin{pmatrix}
G_{zw} & G_{zu} \\
G_{yw} & G_{yu}
\end{pmatrix}
\]  

(4.28)

By a simple rearrangement of the input-output equations, we obtain the transfer function for the disturbance input, \( w \) to the regulated outputs, \( z \) as:

\[
H_{zw} = G_{zw} + G_{zu}K(I - G_{yu}K)^{-1}G_{yw}
\]  

(4.29)

The central idea behind the \( H_2 \) and \( H_\infty \) control methods is to minimize the norm of \( H_{zw} \). Depending upon whether the 2-norm or the \( \infty \) norm that is minimized, the method is named accordingly.
For the purposes of structural control, frequency dependent weighting functions are introduced in order to characterize both the excitation and the responses. In order to accomplish this within the framework of the standard block diagram shown in Fig. 4.3, the weighting functions are appended to the plant system. The resulting plant is typically of a higher size than its original. However, the larger size is usually not a serious limitation as structural systems may be reduced using model reduction techniques (one such method called state order reduction method is used in chapter 5). A schematic representation of the augmentation is shown in Fig. 4.4. The weighting functions are represented by $W_1$ and $W_2$. $W_1$ is a filter whose output represents the excitation of interest. This filter is designed to simulate the frequency characteristics of the excitation (earthquake for our case) and $W_2$ weights the structural responses at the frequencies of interest to be regulated. The resulting augmented system is
represented by $G_a(s)$ and contains both the weighting functions and is used instead of the plant system, $G(s)$ in Fig. 3. The weighting procedures are described in detail in the numerical example for a SDOF system.

4.5 $H_2$ and $H_\infty$ solution procedures

In order to compute the controller, the system in Fig. 4.3 is cast in the state space form (Doyle et al., 1989). The state space equations written in the standard form as follows:

$$
\dot{x} = Ax + B_1w + B_2u \\
z = C_1x + D_{11}w + D_{12}u \\
y = C_2x + D_{21}w + D_{22}u
$$

(4.30)

4.5.1 Riccatti Domain

Let $A$, $Q$ and $R$ be real, $(n,n)$ matrices, with $Q$ and $R$ being symmetric. The Hamiltonian is of size $(2n, 2n)$ and is defined as

$$
H_m = \begin{pmatrix} A & R \\ Q & -A^T \end{pmatrix}
$$

(4.31)

The riccatti domain is denoted by $dom(Ric)$ and consists of hamiltonian matrices, $H_m$ that do not have any eigen values on the imaginary axis. In other words, there exists an $X$ denoted by $X=ric(H_m)$, with the following properties:

- $X$ is symmetric
• $X$ satisfies the algebraic riccati equation $A^TX + XA + XRX - Q = 0$

• $A + RX$ is stable.

4.5.2 Computation of $H_2$ and $H_\infty$ norms

The computation of $H_2$ norm (Doyle et al., 1989) of an arbitrary stable system, $G$ is based on state space approach. If $L_c$ denotes the controllability Gramian of $(A, B)$ and $L_o$, the observability Gramian of $(C, A)$, then,

$$AL_c + L_cA^T + BB^T = 0$$

$$A^TL_o + L_oA + C^TC = 0$$ (4.32)

The $H_2$ norm can then be calculated in a finite number of steps using either of the two Grammians as follows:

$$\|G\|_2^2 = \text{trace} \left( C L_c C^T \right) = \text{trace} \left( B^T L_o B \right)$$ (4.33)

The $H_\infty$ norm is more difficult to compute and can only be approximated. In order to compute the norm, for the transfer function matrix $G(s)$, with $A$ stable and $\gamma > 0$, define the Hamiltonian matrix:

$$H_m := \begin{bmatrix} A & \gamma^{-2}BB^T \\ -C^TC & -A^T \end{bmatrix}$$ (4.34)

In this case, the following conditions are equivalent (Doyle et al., 1989):

1. $\|G\|_\infty < \gamma$
2. $H_m$ has no eigen values on the imaginary axis

3. $H_m \in \text{dom}(Ric)$

4. $Ric(H_m) \geq 0$

The preceding suggests the following algorithm:

1. Choose $\gamma$.

2. Find the eigenvalues of the Hamiltonian.

3. If the Hamiltonian has no imaginary eigenvalues, lower $\gamma$ and go to step 2; otherwise, increase $\gamma$.

4. Stop when $\gamma = \gamma_{\text{min}}$. Then, $\|G\|_\infty = \gamma_{\text{min}}$.

In step 3, various search rules can be used; the bisection rule is adequate in most cases.

4.5.3 Determination of $H_2$ Optimal and $H_\infty$ Sub-optimal Controllers

The state-space formulae for the determination of $H_2$ optimal and $H_\infty$ sub-optimal controllers can be found in Doyle et al., (1989) and are listed in Appendix A. The following assumptions are made regarding the standard state-space equations in eq. 4.30 for the purposes of controller determination:

1. $D_{11} = 0$ (a necessary condition for existence of norm)
2. $D_{22} = 0$ (simplifying the formulas)

3. the pair $(A, B_2)$ is stabilizable and the pair $(C_2, A)$ is detectable (a necessary condition of existence of a stabilizing controller);

4. the pair $(A, B_1)$ is stabilizable and the pair $(C_1, A)$ is detectable;

5. 

$$D_{12}^T \begin{bmatrix} C_1 & D_{12} \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix}$$

6. 

$$\begin{bmatrix} B_1^T & D_{21}^T \end{bmatrix}^T D_{21}^T = \begin{bmatrix} 0 & I \end{bmatrix}^T$$

For the purposes of a general plant controller design, the above six assumptions may be very restrictive. In order to address this issue, it is possible to relax assumptions 2, 5 and 6 through loop-shifting and loop-scaling procedures (Green and Limebeer, 1995). This results in a modified set of riccati equations which include additional terms and contain the same valuable properties as the solutions of the simplified riccati equations listed in Appendix A. These equations will be introduced in the later chapters (Chapter 6, eqns. 6.14-6.19) as general multi-degree of freedom (MDOF) base isolated structures are considered.

4.6 Numerical Example

A SDOF structure consisting of a spring, mass and dashpot system is considered. Since base isolated buildings behave predominantly in their fundamental mode, SDOF
systems give a good insight on the overall behavior. In this section, the SDOF structure is subject to ground excitation. Frequency dependent weighting matrices are chosen for the control design incorporating the outputs and input characterizations. Four types of control designs using both $H_2$ and $H_\infty$ methods are considered; (i) No weighting filters, (ii) Output weighting filter only, (iii) Input excitation filter only and, (iv) both output and input excitation filters. Comparison of the responses for all cases in the frequency domain and some general observations regarding the choice of weighting functions will be made. The method of augmenting the system equations with the weighting functions is also presented for each case. The SDOF system chosen for this example has the following system properties (Eq. 1):

$$A = \begin{bmatrix} 0 & 1 \\ -6.317 & -0.0503 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 1 \end{bmatrix}^T = -E$$

(4.35)

The control objective is to minimize the displacements, which is one of the states of the system and the control energy input. The plant with the weighting filters $W1$ and $W2$ is shown in Fig. 4.5. The vector $w$ consists of the earthquake excitation vector and the measurement noise,

$$w = \begin{bmatrix} w & v \end{bmatrix}^T$$
and the regulated quantities are

\[ z = \begin{bmatrix} z_1 & z_2 \end{bmatrix}^T \]

which are the base displacement and the control input respectively.

4.6.1 Augmentation with \( W1 \)

In order to better inform the controller about the frequency content of the ground motion, a Kanai-Tajimi (Soong and Grigoriu, 1993) shaping filter is incorporated into the system. Fig. 4.6 shows the magnitude of the filter as well as the frequency content of magnitude-scaled versions of Kobe (NS component shown in solid line) and Sylmar (Fault-parallel component shown in dashed line) earthquakes. The equations for the shaping filter, whose output is the ground accelerations can be written as:

\[
\begin{align*}
\dot{x}_f &= A_f x_f + B_f w \\
\ddot{y}_g &= C_f x_f
\end{align*}
\] (4.36)
Figure 4.6  Frequency content of Kanai-Tajimi filter and earthquakes

where,

\[
A_r = \begin{bmatrix} 0 & 1 \\ -\omega_g^2 & -2\zeta_\omega \omega_g \end{bmatrix}
\]

\[
B_r = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\]

\[
C_r = \begin{bmatrix} -\omega_g^2 & -2\zeta_\omega \omega_g \end{bmatrix}
\]

(4.37)
Here, $\omega_g=17 \text{ rad/s}$ and $c_g=0.3$. Augmenting the state space equations with the filter, we get

$$
\begin{align*}
\begin{bmatrix}
\dot{x} \\
\dot{x}_f
\end{bmatrix} &= 
\begin{bmatrix}
A & EC_f \\
0 & A_f
\end{bmatrix}
\begin{bmatrix}
x \\
x_f
\end{bmatrix} + 
\begin{bmatrix}
B \\
0
\end{bmatrix} u + 
\begin{bmatrix}
0 \\
B_f
\end{bmatrix} w \\
\dot{x}_a &= A_a x_a + B_a u + E_a w
\end{align*}
$$
(4.38)

The matrices in the state and output equations can be written as

$$
A = A_a;
$$

$$
B_1 = \begin{bmatrix} E_a & 0 \end{bmatrix}; B_2 = B_a; D_{11} = 0; D_{12} = B;
$$

$$
C_1 = \begin{bmatrix} 1 & 0 \\
0 & 0
\end{bmatrix}; C_2 = \begin{bmatrix} 0 & 1 & 0 \\
0 & 1
\end{bmatrix}; D_{21} = \begin{bmatrix} 0 \\
1
\end{bmatrix}; D_{22} = 0.
$$
(4.39)

4.6.2 Augmentation with $W_2$

The SDOF system is sensitive to disturbance around its natural period. At higher frequencies where the structure is often not sensitive to disturbance, we want to lower the control. In order to accomplish this, a first order weighting function shown in Fig. 7 of the form,

$$
W_2 = \frac{a}{s + a}
$$

is chosen. Here, the parameter $a=3.3 \text{ rad/sec}$ determines the roll-off frequency. As
with the input excitation filter, the plant is augmented with the filter as follows:

\[
\begin{bmatrix}
x \\
x_o
\end{bmatrix} =
\begin{bmatrix}
A & 0 \\
B_o & A_o
\end{bmatrix}
\begin{bmatrix}
x \\
x_o
\end{bmatrix} +
\begin{bmatrix}
B \\
0
\end{bmatrix} u +
\begin{bmatrix}
E \\
0
\end{bmatrix} w
\]
\[\hat{x}_a = \hat{A}_a \hat{x}_a + \hat{B}_a u + \hat{E}_a w\]  \hspace{1cm} (4.40)

The matrices in the state and output equations can be written as

\[
A = \hat{A}_a;
\]
\[
B_1 = \begin{bmatrix}
\hat{E}_a & 0
\end{bmatrix}; B_2 = \hat{B}_a; D_{11} = 0; D_{12} = B;
\]
\[
C_1 = \begin{bmatrix}
0 & C_0 & 0 \\
0 & 0 & 0
\end{bmatrix}; C_2 = \begin{bmatrix}
0 & 1 & 0
\end{bmatrix}; D_{21} = \begin{bmatrix}
0 & 1
\end{bmatrix}; D_{22} = 0.
\]  \hspace{1cm} (4.41)
4.6.3 Augmentation with both $W1$ and $W2$

The augmented state equations when both the weighting filters, $W1$ and $W2$ are introduced, can be written as follows:

$$
\begin{bmatrix}
\dot{x}_a \\
\dot{x}_o
\end{bmatrix} = \begin{bmatrix}
A_a & 0 \\
B_o & A_o
\end{bmatrix} \begin{bmatrix}
x_a \\
x_o
\end{bmatrix} + \begin{bmatrix}
B_a \\
0
\end{bmatrix} u + \begin{bmatrix}
E_a \\
0
\end{bmatrix} w
$$

(4.42)

$$\dot{x}_{aa} = A_{aa} x_{aa} + B_{aa} u + E_{aa} w$$

The matrices in the state and output equations can be written as

$$A = \tilde{A}_{aa};$$

$$B_1 = \begin{bmatrix}
\tilde{E}_{aa} & 0 \\
0 & C_0 & 0
\end{bmatrix}; B_2 = B_{aa}; D_{11} = 0; D_{12} = B;$$

(4.43)

$$C_1 = \begin{bmatrix}
0 \\
0 & 0 & 0
\end{bmatrix}; C_2 = \begin{bmatrix}
0 & 1 & 0
\end{bmatrix}; D_{21} = \begin{bmatrix}
0 & 1
\end{bmatrix}; D_{22} = 0.$$

4.6.4 Frequency Domain Response of the SDOF

The SDOF system is excited by a white noise filtered through the Kanai-Tajimi filter. The non-stationarity is simulated by using a simple half-sine function varying between 0 and $\pi$. In this example, we assume that one of the states, namely, the velocity is measured. The displacement of the system is to be controlled. The singular value plots of the transfer functions between the input excitation, $w$ and the regulated output, namely, the displacement, $z1$ is shown in Fig. 4.8 and Fig. 4.9 for both $H_2$ and $H_{\infty}$ controls. Fig. 4.8 shows the singular value plots for the transfer functions
Figure 4.8  Magnitude of the transfer function, $H_{z1w}$ for the case of $H_2$ control for the case of $H_2$ control in all the four cases; namely, without weighting filters, with output weighting filter, $W_2$ only, with input excitation filter, $W_1$ only and both $W_1$ and $W_2$. The transfer function from the base displacement, $x$ to the input excitation, $w$ for the general case of both input and output filters can be written as

$$H_{z1w} = W_1C_1\tilde{G}W_2 + W_1C_1GK(I - C_2GK)^{-1}\tilde{G}W_2$$

(4.44)
where,

\[ G = (sI - A)^{-1}B \]
\[ \hat{G} = (sI - A)^{-1}E \]  \hspace{1cm} (4.45)

![Graph showing magnitude vs. frequency with different weighting schemes](image)

**Figure 4.9** Magnitude of the transfer function, \( H_{s1w} \), for the case of \( H_\infty \) control. The cases with one filter only or no filters can be derived as special cases of eq. 4.42. From Fig. 4.8, we can see that the case with both input and output filters minimizes the response at higher frequencies. However, the responses corresponding to the peak are not minimized. In comparison, from Fig. 4.9 we can see that the \( H_\infty \) control minimizes the peaks for all cases. As with the \( H_2 \) case, the presence of
both $W_1$ and $W_2$ leads to better response reductions; however, in the case of $H_\infty$, the response reductions occur at all frequencies. It is clear from these figures that the $H_\infty$ control is more effective in suppressing the peaks of the systems compared to the $H_2$ control. In other words, $H_2$ control minimizes the responses in an average sense and $H_\infty$ control minimizes the worst case responses. This behavior is very important for the case of base isolated structures where the primary objective is minimizing peak responses, especially the peak isolator deformations and peak floor accelerations.

4.7 A Brief note on Robustness of $H_2$ and $H_\infty$ Methods

No discussion on the $H_2$ and $H_\infty$ methods is complete without reference on the robustness of these methods to model uncertainties. A controller that functions adequately for all admissible perturbations is termed robust. Robustness can be defined in terms of stability or performance. A control system is said to be robustly stable if it is stable for all admissible perturbations. A control system is said to perform robustly if it satisfies the performance specifications for all admissible perturbations. The stability of feedback systems is determined in terms of gain and phase margins for gain and phase perturbations. In the field of optimal control, two types of uncertainties are considered in the control design; (i) Structured uncertainty where there is information available about the uncertainty which will restrict the uncertainty to a section of a model process and (ii) unstructured uncertainty where no information about the uncertainty is known except the upper bound of its magnitude. There has
been significant study in the areas of structured and unstructured uncertainties (Burl, 1999). Unstructured uncertainty is modeled by connecting an unknown but bounded perturbation to the plant. The unstructured uncertainty is analyzed by placing them within a common framework discussed in the earlier sections. The system so formed now will have three inputs and three outputs. Combining the nominal plant, $G(s)$ with the feedback, $K(s)$, results in a system consisting of a nominal closed loop system, $N(s)$, with the perturbation, $\Delta(s)$, in a feedback loop as shown in Fig. 4.10.

![Diagram](image)

**Figure 4.10** Unstructured uncertainty model for robustness

The above feedback system, for the bounded unstructured uncertainty, $\|\Delta\| \leq 1$, is internally stable for all possible perturbations provided the nominal closed loop system is stable and

$$\|N_{zdwd}\|_\infty = \sup_{\omega} \{\tilde{\sigma} [N_{zdwd}(j\omega)]\} < 1$$

(4.46)

This is called the small-gain theorem and it used to test for robust stability with respect to bounded perturbations. Eq. 4.46 is a necessary and sufficient condition
for internal stability with respect to unstructured uncertainty.

Structured uncertainty arises when a plant is subjected to multiple uncertainties such as a number of uncertain parameters or multiple unstructured uncertainties. For this case, the structured uncertainty can be written in the block diagonal transfer function form:

\[
\Delta(s) = \begin{bmatrix}
\Delta_1(s) & 0 & 0 & 0 \\
0 & \Delta_2(s) & . & . \\
0 & . & \Delta_3(s) & . \\
0 & . & . & \Delta_n(s)
\end{bmatrix}
\]  

(4.47)

where, \( n \) is the number of uncertainties and \( \Delta(s) \) represents the individual uncertainties applied to the plant. In the standard block diagram notation, the structured uncertainty, \( \Delta(s) \) is represented in the same way as in Fig. 4.10. The uncertainty is scaled so that their infinity norms

\[
\|\Delta_1\|_\infty \leq 1; \|\Delta_2\|_\infty \leq 1; \ldots \ldots; \|\Delta_n\|_\infty \leq 1 \Rightarrow \|\Delta\|_\infty \leq 1.
\]  

(4.48)

The general feedback system given in Fig. 4.10 is stable for all possible perturbations

\[
\Delta(j\omega) \in \Delta
\]

and

\[
\|\Delta(j\omega)\|_\infty \leq 1,
\]

if and only if the nominal closed loop system is internally stable and

\[
\sup_{\omega} \{\mu_{\Delta} [N_{sdw_d}(j\omega)]\} < 1
\]  

(4.49)
where, $\mu_\Delta$ is called the structured singular value and given by

$$
\mu_\Delta (N) = \min_{\Delta \in \tilde{\Delta}} \left\{ \frac{1}{|\bar{\sigma}(\Delta)| \det(I + N\Delta) = 0} \right\}
$$

(4.50)

$$
\mu_\Delta (N) = 0 \quad \text{if} \quad \det(I + N\Delta) \neq 0 \quad \forall \quad \Delta \in \tilde{\Delta}
$$

The determination of robust stability is dependent on the computation of the structured singular value and can be impractical for a large number of cases. Hence, bounds on the structured singular values are generated and they provide good estimates of the structured singular value. In the case of performance robustness, the robust performance problem can be converted into an equivalent robust stability problem by appending an uncertainty block to the system in Fig. 4.10. The system meets the performance robustness objectives if and only if the new augmented system is robustly stable. The computational details have been presented by Burl (1999).

In the following chapters, robustness is investigated through small perturbations in the stiffness of the plant. Small variations of the order of $\pm 10\%$ are introduced in the stiffness of the plant, and the controller performance is investigated through time domain simulations. Fig. 4.11 shows a specific case of multiplicative uncertainty at the plant output. The modeling error, $p$, is assumed to enter the system at the same point as the measurement noise. The weighting function $W_3$ is assumed to be a constant for the current study.
4.8 Concluding Remarks

The main idea of this dissertation is to develop new control algorithms based on the optimal control methods such as LQG, $H_2$ and $H_\infty$. The emphasis of these algorithms will be on control of novel semiactive systems which are inherently stable. Hence, stability robustness of semiactive control is not an issue. However, the performance of the controller to stiffness perturbations in the plant are investigated in the following chapters.
Chapter 5
Three Dimensional Smart Base Isolated Building
With Variable Damping Systems: Active and
Semiactive Controllers Based on LQG

5.1 Introduction

Optimal control methods based on $H_2/LQG$ have been used extensively for both active and semiactive structural control by various researchers. A majority of work performed on the control base isolated buildings involve either actuators for active control or variable damping. These control devices are installed at the isolation level. Almost all of the structures used for the studies are two-dimensional planar models.

In the current chapter, two types of isolation systems are investigated, namely, linear and frictional isolation systems on a fully three dimensional base isolated structure described in chapter 3. $H_2/LQG$ method is presented for active control in the linear isolation case and the skyhook method is presented for semiactive control in the friction isolation case. New filters that characterize the frequency content of the ground excitations better than the filters used in the past are developed. These filters are included both for active and semiactive control formulation in the linear isolation case. A new semiactive control algorithm called the smooth-clipped-optimal control is developed and implemented using MR dampers on the three-dimensional benchmark structure described in Chapter 3. It is seen that the performance of the
traditional on-off switching controllers is improved by the introduction of smoothing techniques presented in the new algorithm.

In structures, accelerometers provide a cost effective tool to measure responses. Measurement of states, i.e., displacement and velocity, may not be practical in many cases. Hence, in the $H_2/LQG$ design, Kalman filter is used to estimate the states based on absolute acceleration measurements. Input filters are used to better inform the controller of the spectral content of the near-fault earthquake excitations and are directly incorporated into the control design for the linear base isolated benchmark building. Ramallo et al., 2002 used the Kanai-Tajimi shaping filter (Soong and Grigoriu, 1993) for modeling the ground excitation. The overall system was augmented using the Kanai-Tajimi filter and the controller is designed for this augmented system. The above procedure resulted in improved controller performance. However, the Kanai-Tajimi filter overestimates the energy in the low frequency range which effects the performance of long period structures such as base isolated buildings. Hence, new filters, which represent the earthquakes better in the low frequency range are required for improved controller performance in base isolated structures. Such filters are developed by fitting the PSD (power spectral density) of the filter with the PSD of a set of near-fault ground motions selected for the study. The augmented $H_2/LQG$ controller for the linear base isolated benchmark building is presented.

Semiaactive control of nonlinear base isolated structure with frictional isolation
system using MR dampers is presented. The skyhook control used for the nonlinear friction case takes into account both the relative and absolute velocities at the measured locations, which are computed from acceleration measurements through the use of a higher order filter that approximates an integrator. Computed results of the passive, semiactive, and active cases are presented in terms of the performance indices developed in Chapter 3.

5.2 Active Control Based on LQG

The state equation is as follows

$$\dot{X}(t) = AX(t) + Bu(t) + E\ddot{U}_g(t) = g(X, u, \ddot{U}_g)$$  (5.1)

where $X = (U^T \quad U_b^T \quad \dot{U}^T \quad \dot{U}_b^T)^T$

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = B^* = \begin{bmatrix} 0 \\ -M^{-1} \begin{bmatrix} 0 \\ I \end{bmatrix} \end{bmatrix},$$  (5.2)

$$E = \begin{bmatrix} 0 \\ -M^{-1} \begin{bmatrix} MR \\ R^TMR + M_p \end{bmatrix} \end{bmatrix}.$$  

$$M = \begin{bmatrix} M_{n\times n} & M_{n\times n}R_{n\times 3} \\ R_{3\times n}^T M_{n\times n} & R_{3\times n}^T M_{n\times n}R_{n\times 3} + M_{3\times 3} \end{bmatrix}, \quad \overline{C} = \begin{bmatrix} C_{n\times n} & 0 \\ 0 & C_{b_{3\times 3}} \end{bmatrix}.$$
\[ \mathbf{K} = \begin{bmatrix} \mathbf{K}_{n \times n} & 0 \\ 0 & \mathbf{K}_{b_3 \times 3} \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} 0 \\ f_{c_3 \times 3} \end{bmatrix}. \] (5.3)

In the above equations, \( \mathbf{A}, \mathbf{B}, \mathbf{B}^* \) and \( \mathbf{E} \) are condensed system matrices having 54 states derived from the full three dimensional finite element model. \( \mathbf{M} \) is the superstructure mass matrix, \( \mathbf{C} \) is the superstructure damping matrix in the fixed base case, \( \mathbf{K} \) is the superstructure stiffness matrix in the fixed base case, \( \mathbf{M}_b \) is the mass matrix of the rigid base, \( \mathbf{C}_b \) is the resultant damping matrix of viscous isolation elements, \( \mathbf{K}_b \) is the resultant stiffness matrix of elastic isolation elements and \( f_c \) is the vector containing control forces. \( \mathbf{R} \) is the matrix of earthquake influence coefficients, i.e. the matrix of displacements and rotation at the center of mass of the floors resulting from a unit translation in the \( X \) and \( Y \) directions and unit rotation at the center of mass of the base. Furthermore, \( \mathbf{U}, \mathbf{U} \) and \( \mathbf{U} \) represent the floor acceleration, velocity and displacement vectors relative to the base, \( \mathbf{U}_b \) is the vector of base acceleration relative to the ground and \( \mathbf{U}_g \) is the vector of ground acceleration.

For the controller implementation, this model is further reduced to 24 states using model reduction techniques (Davison 1966) as compared to 54 states in a full order model. The retained states correspond to the displacement and velocity on the eighth floor, fifth floor, first floor and base. The reduction is accomplished by constructing a matrix of lower order that has the same dominant eigenvalues and eigenvectors as
the original system. The state space equations can be formulated as

\[ \dot{x}_r = A_r x_r + B_r u + E_r \ddot{U}_g \]  
(5.4)

\[ z_r = C_{rz} x_r + D_{rz} u + F_{rz} \ddot{U}_g \]  
(5.5)

\[ y_{mr} = C_{mr} x_r + D_{mr} u + F_{mr} \ddot{U}_g + v_r \]  
(5.6)

where \( A_r, B_r \) and \( E_r \) are the system matrices, \( z_r \) is the regulated output vector which is obtained by choosing the appropriate mapping matrices, \( C_{rz}, D_{rz} \) and \( F_{rz} \). \( y_{mr} \) is the measurement vector obtained by choosing matrices \( C_{mr}, D_{mr} \) and \( F_{mr} \) appropriately. \( v_r \) is the measurement noise vector. The measured outputs are the responses of the eighth floor, base and ground denoted by \( y_{mr} = [\ddot{x}_{8ax} \; \ddot{x}_{8ay} \; \ddot{x}_{8a\theta} \; \ddot{x}_{8ax} \; \ddot{x}_{8ay} \; \ddot{x}_{8a\theta} \; \ddot{u}_{gx} \; \ddot{u}_{gy}]^T \) and the outputs to be regulated include the inter-story drifts and base displacements at the farthest corner of the building and absolute accelerations for all degrees of freedom given by \( z_r = [x_b \; x_i - x_{i-1} \; \ddot{x}_b \; \ddot{x}_i]^T \); where, \( i \) denotes the floor under consideration.

To better inform the controller of the frequency characteristics of earthquakes, the input excitation is modeled as a filtered white noise. The shaping filter is given by

\[ \dot{x}_f = A_f x_f + B_f w \]  
(5.7)

\[ \ddot{U}_g = C_f x_f \]

where \( w \) is the white noise excitation and \( x_f \) are the states of the shaping filter.
Combining Eq. 5.7 with Eq. 5.4, the augmented system becomes

\[\begin{bmatrix}
\dot{x}_r \\
\dot{x}_f
\end{bmatrix} = \begin{bmatrix}
A_r & E_r C_r \\
0 & A_f
\end{bmatrix} \begin{bmatrix}
x_r \\
x_f
\end{bmatrix} + \begin{bmatrix}
B_r \\
0
\end{bmatrix} u + \begin{bmatrix}
0 \\
B_f
\end{bmatrix} w \]

(5.8)

from which the augmented equations can be written as

\[\dot{x}_a = A_a x_a + B_a u + E_a w \]

(5.9)

\[y_a = C_{ya} x_a + D_{ya} u + F_{ya} \ddot{U}_g + v_a \]

(5.10)

\[z_a = C_{za} x_a + D_{za} u + F_{za} \ddot{U}_g \]

(5.11)

where the matrices \(A_a, B_a, \) and \(E_a\) are augmented system matrices. \(C_{ya}, D_{ya}, F_{ya},\)
\(C_{za}, D_{za}, \) and \(F_{za}\) are mapping matrices of appropriate dimensions. \(v_a\) is the measurement noise vector. As shown in Fig. 5.1, the new shaping filter (eq. 5.12), obtained by least squares fit of earthquake data, closely models the PSD of the ground excitation of the set of near-fault earthquakes chosen for this study. The new shaping filter is as follows:

\[F(s) = \frac{4\zeta_g \omega_g s}{s^2 + 2\zeta_g \omega_g s + \omega_g^2}, \text{ where, } \omega_g = 2\pi \text{ rad/sec, } \zeta_g = 0.3. \]

(5.12)

The measured responses contain identically distributed RMS noise of 0.14 Volts and they are modeled as Gaussian rectangular pulse processes with a pulse width of 0.005 seconds. The sensor gains are given by \((10/9.81) [I] V/(m/sec^2), \) where \(I\) is of
Figure 5.1  Excitation filter and power spectral density of earthquakes

cost function that weights the regulated outputs and the control forces is given as

\[ J = \lim_{\tau \to \infty} \frac{1}{\tau} E \left[ \int_0^\tau \left( z_n^T Q z_n + u^T R u \right) dt \right] \]  

(5.13)

where \( R \) is an identity matrix that weights the control forces and \( Q \) is a diagonal matrix of the form \( 10^3 * I_{54 \times 54} \) that weights the regulated outputs. The separation principle allows the control and estimation problems to be treated independently (for linear systems only). The control law takes the form

\[ u = -K_a \dot{x}_a \]  

(5.14)

where, \( K_a \) is the full state feedback gain matrix and \( \dot{x}_a \) is the Kalman filter estimate of
the state vector based on the augmented model. The block diagram for the augmented controller is shown in Fig. 5.2.

![Block Diagram of H2/LQG Controller](image)

**Figure 5.2** Design of H2/LQG Controller

Calculations of $K_a$ and the Kalman estimator gains are performed. Calculations to determine the discrete time compensator are performed. The state space and output matrices, $A_{cd}$, $B_{cd}$, $C_{cd}$ and $D_{cd}$ generated by discrete computations are used in the controller block.

### 5.3 Semiactive Clipped Optimal Control Based on LQG

In order to illustrate the application of semi active control system using MR dampers, a new control algorithm called smooth-clipped-optimal control strategy
based on $H_2$/LQG method is developed. This control approach involves the design of a controller for an active system with the desired optimal control force being generated by an MR damper (Dyke et al., 1996) according to the voltage control law,

$$u = V_{\text{max}} H \{f_c - f\} f$$

(5.15)

where $V_{\text{max}}$ is the maximum voltage, $H$ is the heaviside function, $f_c$ is the optimal force required as per Eq. 5.14 and $f_{MR}$ is the force that is generated by the MR damper. It is found that the high frequency switching (also chattering at velocities close to zero) due to the clipping at each time step leads to increased accelerations and inter-story drifts. In order to address this issue, the control signal generated by the digital controller is passed through a low pass filter before it is commanded to the device. The filter is given by,

$$\dot{\nu} = -\zeta \nu + \zeta u$$

(5.16)

where, the filter parameter, $\zeta = 10$ rad/sec. Such a filter leads to improved control performance. The force generated by the damper is a function of the voltage supplied. The MR damper is modeled using a spring, a dash pot, and hysteresis element in parallel as shown in Fig. 5.3. The force generated by the damper is given by

$$f_{MR} = (\alpha z) f(\nu) + C \dot{U}_b + kU_b$$

(5.17)

where, $\alpha = \alpha_a + \alpha_b$, $C = C_a + C_b$, $f(\nu)$ is a function of voltage $\nu$, supplied to the MR damper. The hysteresis variable $z$ (Wen, 1976) is obtained by solving the differential
Figure 5.3 MR Damper Model

equation:

\[ Y_i \ddot{z}_i + \gamma |\dot{U}_b| z_i |\dot{z}_i| + \beta \dot{U}_b z_i^2 - \dot{U}_b = 0 \]  \hspace{1cm} (5.18)

where \( U_b \) is the displacement of the MR damper. The parameter \( Y_i \) is the yield displacement of the hysteretic element, \( \gamma, \beta, \alpha_a, \alpha_b, C_a \) and \( C_b \) are constants. The force displacement characteristics of one of the eight MR Dampers subjected to harmonic excitation is shown in Fig. 5.4 for both passive-on and passive-off cases.

5.4 Semiactive Skyhook Control

A skyhook controller (Karnopp, 1974) is used for the semiactive control strategy for linear case with nonlinear friction bearings/dampers. The skyhook control algo-
Figure 5.4  Force Displacement Relationship of MR Damper

The algorithm is given by

\[ C(t) = \begin{cases} 
C_{\text{max}} & \dot{u}_a \dot{u}_b > 0 \\
0 \text{ or } C_{\text{min}} & \dot{u}_a \dot{u}_b < 0 
\end{cases} \] (5.19)

Here, \( C_{\text{min}} \) is the minimum damping coefficient, \( C_{\text{max}} \) is the maximum damping coefficient of the damper, \( \dot{u}_a \) is the absolute velocity and \( \dot{u}_b \) is the relative velocity. The velocities are computed from acceleration measurements at the eight device locations through the use of a second order filter which approximates an integra-
tor. The measured outputs at the eight MR damper locations are represented as

\[ y_{mf} = [\ddot{x}_{dev1} \ \ddot{x}_{dev2} \ \ddot{x}_{dev3} \ \ddot{x}_{dev4} \ \ddot{x}_{dev5} \ \ddot{x}_{dev6} \ \ddot{x}_{dev7} \ \ddot{x}_{dev8}]^T. \]

### 5.5 Evaluation Of Control Designs

The results of the evaluations for three different control designs are presented in Tables 5.1 to 5.11. The results presented in Tables 5.1 to 5.6 are for the fault normal (FN) component and the fault parallel (FP) components acting in two perpendicular directions; the evaluation is reported in terms of the performance indices. The uncontrolled response quantities are presented in Tables 5.8 to 5.11 for the fault normal (FN) component and the fault parallel (FP) component acting in two perpendicular directions. Time history responses in the NS direction for Newhall earthquake FN and FP components acting on the benchmark building are shown in Fig. 5.5. The force displacement loops for the MR damper and the isolation bearings (linear and frictional) for both smooth clipped optimal and skyhook control is shown in Fig. 5.6. The maximum corner drifts normalized by their corresponding uncontrolled values are shown in Table 5.7 for the three different control designs.

The results of active control of the benchmark problem with linear elastomeric isolation system are summarized in Tables 5.1 and 5.2. Actuators are used to apply the active control forces to the base of the structure. In this control strategy most of the response quantities are reduced substantially from the uncontrolled cases. The benefit of active control strategy is the reduction of base displacements and shears of
upto 25% without increase in drift or accelerations.

The results of smooth clipped optimal control strategy for the benchmark problem with linear elastomeric isolation system are presented in Tables 5.3 and 5.4. The semi-active force is applied to the base of the structure by sixteen MR Dampers, eight in the X and eight in the Y direction. Fourteen of the MR Dampers are located in the periphery of the base slab and two near the center of mass of the base slab. Performance indices are presented in Tables 5.3 and 5.4 for both passive and semiactive control cases. The advantage of semiactive smooth clipped optimal control is evident in significant reductions in base displacements as compared to the active control case. The reduction in base displacements is about 25 to 50% compared to 10 to 25% for active case. The reductions are achieved at the cost of increased floor accelerations and inter-story drifts; however, the increases in the semiactive control case are less than that of the passive damping case. This increase is observed mostly at higher floors. The performance of controlled case is better than the passive case in terms of achieving the good reduction in base drifts with a correspondingly lower increase in floor accelerations and story drifts.

The results of skyhook control algorithm for the benchmark problem with nonlinear friction isolation system are presented in Tables 5.5 and 5.6. Performance indices are presented in Tables 5.5 and 5.6 for both passive and skyhook control cases. The Skyhook controller performs better than the passive case in all earthquakes.
The base and structural shear remains at the level of uncontrolled structure for all earthquakes except El Centro and Kobe for the controlled case. There is an increase in the base and structure shear for El Centro and Kobe earthquakes. The reduction in maximum base displacements is between 15 to 50%. The results of the passive case are better than the controlled case for peak base displacements in most cases; however, the inter-story drifts in the controlled case are significantly better than the passive case. For both the passive and controlled cases, the inter-story drifts are higher than the uncontrolled case. The peak accelerations increased for both controlled and passive cases with the magnitude of increase much higher for the passive case in all excitations.

5.6 Conclusions

A new semiactive control algorithm based on LQG, called the smooth clipped optimal controller was developed in this chapter. It was found that this algorithm is effective in reducing the switching frequency of the semiactive device and reducing the higher order responses (such as accelerations). A simple semiactive controller called skyhook was used to reduce the structure responses for the case of nonlinear isolation system. It was shown that the Skyhook controller performs better than the passive case in all earthquakes.
### Table 5.1  Results for Active Control (FP - X and FN - Y)

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### Table 5.3 Results for Smooth Clipped Optimal Control (FP - X and FN - Y)

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Table 5.5  Results for Skyhook Control (FP - X and FN - Y)
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Figure 5.5  Time history responses, both controlled (red, thick) and uncontrolled (blue, thin), at the center of mass of the base in the NS direction for Newhall earthquake FN-X and FP-Y components acting on the benchmark building: (a-1,a-2) Base displacement and top floor acceleration responses for active control, linear elastomeric system; (b-1,b-2) Base displacement and top floor acceleration responses for smooth clipped optimal control, linear elastomeric system; (c-1,c-2) Base displacement and top floor acceleration responses for skyhook control, friction isolation system.
Figure 5.6  (a) Smooth clipped-optimal - MR Damper Force-displacement for Newhall - Y direction, (b) Smooth clipped-optimal - Linear Bearing Force-displacement for Newhall - Y Direction, (c) Skyhook - MR Damper Force-displacement for Kobe - Y Direction and (d) Skyhook - Frictional Force-displacement for Kobe - Y Direction. Both passive on (blue, dashed) and controlled (red, solid) are shown; earthquake FP - X and FN - Y and the bearing and device location corresponding to 45.
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W: Weight of the structure (202,000 kN); h: Average story height (4.04 m); g: Acc. due to gravity (9.81 m/sec²)
Table 5.9  Linear Isolation System - Uncontrolled Response Quantities (FP-Y, FN-X)

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W: Weight of the structure (202,000 kN); h: Average story height (4.04 m); g: Acc. due to gravity (9.81 m/sec²)
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<td>Peak Acc. (g)</td>
<td>0.572</td>
<td>0.355</td>
<td>0.402</td>
<td>0.338</td>
<td>0.360</td>
<td>0.478</td>
<td>0.270</td>
</tr>
<tr>
<td>RMS Disp (m)</td>
<td>0.071</td>
<td>0.124</td>
<td>0.023</td>
<td>0.084</td>
<td>0.050</td>
<td>0.166</td>
<td>0.151</td>
</tr>
<tr>
<td>RMS Acc. (g)</td>
<td>0.080</td>
<td>0.076</td>
<td>0.079</td>
<td>0.071</td>
<td>0.083</td>
<td>0.071</td>
<td>0.074</td>
</tr>
</tbody>
</table>

W: Weight of the structure (202,000 kN); h: Average story height (4.04 m); g: Acc. due to gravity (9.81 m/sec$^2$)
### Table 5.11  Friction Isolation System - Uncontrolled Response Quantities (FP-Y, FN-X)

<table>
<thead>
<tr>
<th></th>
<th>Newhall</th>
<th>Sylmar</th>
<th>ElCentro</th>
<th>Rinaldi</th>
<th>Kobe</th>
<th>Jiji</th>
<th>Erzinkan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Base Shear (Norm. by W)</td>
<td>0.177</td>
<td>0.251</td>
<td>0.080</td>
<td>0.232</td>
<td>0.140</td>
<td>0.428</td>
<td>0.225</td>
</tr>
<tr>
<td>Peak Str. Shear (Norm. by W)</td>
<td>0.162</td>
<td>0.205</td>
<td>0.087</td>
<td>0.201</td>
<td>0.119</td>
<td>0.363</td>
<td>0.195</td>
</tr>
<tr>
<td>Peak Isolator Def. (m)</td>
<td>0.292</td>
<td>0.495</td>
<td>0.085</td>
<td>0.428</td>
<td>0.281</td>
<td>1.188</td>
<td>0.563</td>
</tr>
<tr>
<td>Peak Iso. Drift (Norm by h)</td>
<td>0.003</td>
<td>0.003</td>
<td>0.002</td>
<td>0.003</td>
<td>0.002</td>
<td>0.005</td>
<td>0.003</td>
</tr>
<tr>
<td>Peak Acc. (g)</td>
<td>0.464</td>
<td>0.434</td>
<td>0.375</td>
<td>0.377</td>
<td>0.368</td>
<td>0.493</td>
<td>0.319</td>
</tr>
<tr>
<td>RMS Disp (m)</td>
<td>0.068</td>
<td>0.133</td>
<td>0.020</td>
<td>0.084</td>
<td>0.051</td>
<td>0.182</td>
<td>0.168</td>
</tr>
<tr>
<td>RMS Acc. (g)</td>
<td>0.081</td>
<td>0.079</td>
<td>0.079</td>
<td>0.068</td>
<td>0.082</td>
<td>0.076</td>
<td>0.082</td>
</tr>
</tbody>
</table>

W: Weight of the structure (202,000 kN); h: Average story height (4.04 m); g: Acc. due to gravity (9.81 m/sec²)
Chapter 6
Smart Base Isolated Buildings With Variable Friction Systems: $H_\infty$ Controller And Novel SAIVF Device

Control algorithms based on $H_2/LQG$, both active and semiactive, have been studied in the earlier chapters. It was shown that the new filter for characterizing the ground excitation is effective in improving the performance of the controller as seen in chapter 4. $H_\infty$ control compared to $H_2$ control which minimizes structural responses on an average-minimizes worst case structural responses. For base isolated structures, most of the response is at the structure's fundamental mode. Hence, $H_\infty$ control is a better choice for studying control of base isolated structures, where the objective is to minimize peak responses. A majority of semiactive control algorithms studied in the literature with reference to the MR damper or other semiactive devices, result in high frequencies of switching. As seen in chapter 5, this problem is alleviated to an extent in MR dampers by introducing low pass filters at the controller output.

The semiactive friction devices studied so far by various researchers vary the normal force across friction interfaces using either physical or electromagnetic means (eg., Fujita, et al., 1994 and He et al., 2003). The level of normal force is prescribed by using structural responses directly such as prior peak displacement (Inaudi, 1997), or through other criteria derived based on Lyapunov methods (Dupont et al., 1997).
Though these algorithms have shown to reduce base displacements, accelerations and inter-story drifts increase as a result of abrupt switching. In this chapter a new algorithm based on $H_\infty$, and a new variable friction device is developed. The new variable friction device developed consists of four friction elements and four spring elements, each friction-stiffness pair in parallel, arranged in a rhombus configuration. The stiffness elements act only to restore the position of the device and their contribution to the overall system stiffness is small. The level of friction force can be adjusted by varying the angle of the arms of the device leading to smooth variation of friction. This type of frictional force variation eliminates the disadvantages associated with rapid switching devices. Experimental results are presented to verify the proposed analytical model of the device. The central idea of the new control algorithm is to determine the position of the semiactive variable friction device at each time instant based on the optimal force generated by the $H_\infty$ controller. The $H_\infty$ controller is designed using appropriate weighting filters that have been chosen for optimal performance in reducing earthquake responses.

6.1 Structure with Variable Friction System: Formulation

The equations of motion for the six degrees of freedom base isolated structure are developed with one degree of freedom at the center of mass of each floor and the base. The state space equations for the superstructure and the base were developed and described in detail in Chapter 3 for this structure A. The summary of state space
equations repeated here for clarity are as follows:

\[
\dot{x}(t) = Ax(t) + Bu(t) + E\ddot{u}_g(t)
\]  

(6.1)

where \(x\) consists of the states, \(u(t)\) is the control force, \(\ddot{u}_g(t)\) is the ground acceleration and \(A, B\) and \(E\) are system matrices that are defined as follows:

\[
A = \begin{bmatrix}
0 & I \\
-M^{-1}K & -M^{-1}C
\end{bmatrix} ; \quad B = \begin{bmatrix}
0 \\
M^{-1}A
\end{bmatrix} ; \quad E = \begin{bmatrix}
0 \\
\Gamma
\end{bmatrix}
\]

In the above equations, \(M\) is a diagonal mass matrix formed using 6800 kgs for the mass of the base and 5897 kgs for the remaining five floors. Similarly, \(C\) is a diagonal damping matrix of the form \(C = \text{diag}([7480\ 67000\ 58000\ 57000\ 50000\ 38000]^T)\) N-s/m.

The stiffness of the base is 231.5 kN/m and the stiffness of the five stories are 33,732, 29,093, 28,621, 24,954 and 19,059 kN/m respectively. The earthquake influence vector and device location vectors are

\[
\Lambda = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}^T \quad \text{and} \quad \Gamma = \begin{bmatrix}
1 & 1 & 1 & 1 & 1
\end{bmatrix}^T.
\]

The natural frequencies of the structure are 0.40, 5.46, 10.27, 14.67, 18.3 and 21.3 Hz respectively. The damping coefficient in the isolation system is so chosen to provide a damping coefficient, \(\zeta = 4\%\) in the isolated case for the fundamental mode.

### 6.2 Semiaactive Variable Friction Device (SAIVF)

The SAIVF device is based on the semiaactive variable stiffness device (SAIVS) developed by Nagarajaiah et al., (1998) and Nagarajaiah (2000). The SAIVF device
consists of four friction elements and four spring elements, each friction-stiffness pair in parallel, arranged in a rhombus configuration as shown in Fig. 6.1. The spring-friction elements are connected to joints 1 to 4 as shown in Fig. 6.1. Joint 1 is fixed in the $x$ direction and can be positioned at any desired position in the $y$ direction by a linear electromechanical actuator and controller. Joint 2 is free to move in both the $x$ and $y$ directions. Joints 3 and 4 are free to move only in the $x$ direction. The ends of the guide rail, on which joint 2 moves, are attached to the base slab. The ends of the guide rail, on which joints 3 and 4 move, are attached to the ground. The electromechanical actuator is fixed to the ground and can actuate in the $y$ direction thus moving joint 1 to the required position. Each of the pair of elements are located at an angle, $\theta$ to the horizontal. The stiffness elements act as restoring devices and their contribution to the overall system stiffness is minimum. The SAIVF device is capable of providing smooth variation of the level of friction force by varying the angle ($\theta$) of the arms of the device with a linear electromechanical actuator. The magnitude of force developed in the device is a function of the angle, $\theta$, given by

$$f_d(t) = k_e f(\theta, t)^2 y_d(t) + c_e z f(\theta, t)$$  \hspace{1cm} (6.2)$$

where, $y_d(t)$ is the displacement in $x$ direction at joint 2, $k_e$ is the stiffness of single spring, $\theta(t)$ is the time varying angle of the spring with the horizontal for a given device position, $c_e$ is a constant and $z$ is the hysteretic parameter (Wen, 1976) obtained
by solving the following differential equation

\[ Y \ddot{z} + \gamma |\dot{y}_d| z |\dot{z}| + \beta \dot{y}_d z^2 - \dot{y}_d = 0 \]  

(6.3)

where \( \dot{y}_d \) is the velocity at the joint 2, \( Y \) is the yield displacement of the hysteretic element, \( \gamma \) and \( \beta \) are constants obtained using least-squares fit of experimental data.

The angle, \( \theta \) can only vary between angles 0 and \( \pi/2 \). The time-varying function
\( f(\theta) \) is the cosine of angle \( \theta \) and given as,

\[
f(\theta, t) = \cos \theta \begin{array}{c|cc}
\pi/2 \\
0
\end{array}
\] (6.4)

6.3 Experimental Study and Verification

In order to verify the analytical model of the semiactive device proposed in eq. 6.2, experimental studies were conducted on a scaled model of the device. The experiments consisted of recording the force developed in the device while excited by a 0.25 Hz sinusoid. The magnitude of the force in the device for two positions, \( \theta = 10^0 \) (closed), and \( \theta = 77^0 \) (open), are shown in Fig. 6.2. The response of the device to real-time switching is also shown at \( t = 8 \text{sec}. \), when the device is switched from open to close positions while excited by the sinusoid. Fig. 6.3 shows the comparison of experimental and simulated force-displacement relationship for the variable friction device. Eq. 6.2 is used to simulate the force corresponding to the device positions measured in the experiment as shown in Fig. 6.2. The comparison study shows that the analytical model for the device provides a good representation of the SAIVF device. The analytical model also captures the behavior of the device at intermediate values of the angle \( \theta \).

The analytical model of the SAIVF device that is used for the simulations for
Figure 6.2  Experimental Results - Force developed in the SAIVF device excited by a 0.25 Hz. sinusoid with an amplitude of 0.25 in
Figure 6.3  Comparison of experimental and analytical results - SAIVF device force-displacement characteristics
Figure 6.4  Force generated in the SAIVF device due to sine excitation of $\omega = 2\pi$ rad/s
various positions of the device, i.e., $\theta$ is shown in Fig. 6.4. The force generated by the device is due to a sine excitation of frequency, $\omega = 2\pi$ rad/s.

### 6.4 $H_\infty$ Formulation

Frequency domain control methods such as $H_\infty$ provide the necessary framework to introduce frequency-shaping filters in the control design. Such filters enable the designer to better inform the controller about input and desired output frequency characteristics. In this thesis, the developed input excitation filter (obtained using least squares fit of experimental data) has the transfer functions given by

$$F_A(s) = \frac{4\zeta_g \omega_g s}{s^2 + 2\zeta_g \omega_g s + \omega_g^2} \quad (6.5)$$

and,

$$F_B(s) = \frac{8\zeta_g^2 \omega_g^2 s^2}{\left[s^2 + 2\zeta_g \omega_g s + \omega_g^2\right]^2} \quad (6.6)$$

where, $\omega_g = 2\pi$ rad/sec and $\zeta_g = 0.3$. These filters are chosen for this study instead of the well-known Kanai-Tajimi filter, as they accentuate the energy of the input excitation in the low frequency region and roll off rapidly at higher frequencies.

The transfer functions for the two filters are represented by subscripts A and B. Filter A is a second order filter obtained by adjusting the parameters corresponding to the dominant period and decay of the ground motion, namely $\omega_g$ and $\zeta_g$, to achieve a good fit to the power spectral densities of recorded near-fault earthquakes. Filter
Figure 6.5  Transfer functions for ground excitation filters

B is obtained by cascading another second order filter (Fig. 6.5) having a transfer function, 

\[
F_A = \frac{4\zeta \omega_n s}{s^2 + 2\zeta \omega_n s + \omega_n^2}
\]

with \( F_A(s) \).

The outputs are weighted using a first order filter of the form, \( W = \frac{a}{s+a} \), where, \( a=3.3 \) rad/sec, which determines the roll-off frequency. The characteristics of the best-fit input and output weighting filters and the power spectral densities of Sylmar (FN) and Jiji (FN) earthquakes are shown in Fig. 6.6.

The equations augmented with both the weighting filters can be written as

\[
\dot{x}_a = A_a x_a + B_a u + E_a w
\]

(6.7)

\[
y_a = C_{ya} x_a + D_{ya} u + F_{ya} a_g + \nu_a
\]

(6.8)

\[
z_a = C_{za} x_a + D_{za} u + F_{za} a_g
\]

(6.9)

Using the standard notation presented in chapter 4, the above equations can be re-cast in the form

\[
\dot{x}_a = A x_a + B_1 w + B_2 u
\]

(6.10)
Figure 6.6  Weighting Filters for Ground Excitation and Output
\[ z_a = C_1 x_a + D_{11} w + D_{12} u \]  
(6.11)

\[ y_a = C_2 x_a + D_{21} w + D_{22} u \]  
(6.12)

where, \( w \) contains both the external disturbance and measurement sensor noise.

The purpose of the \( H_\infty \) control method is to minimize the \( \infty \) -norm of the transfer function from input \( w \) to regulated output \( z \), \( H_{zw} \) and is written as

\[ \| H_{zw}(s) \|_\infty = \sup_{\omega} \left[ \sigma \left( H_{zw}(s) \right) \right] \leq \gamma \]  
(6.13)

\( \sigma \) is the largest singular value of the transfer function, \( \sup \) denotes the supremum and \( \gamma \) is a positive bound for the norm. The solution of the above standard problem, the procedure developed by Doyle et al. (1989) was resented in chapter 4. It was also noted that some of the assumptions in forming the controller equations can be relaxed by various techniques (Green and Limebeer, 1995). In order for the standard problem with weaker conditions to have a solution, the following assumptions must be satisfied:

1. \( D_{11} = 0 \).

2. the pair \( (A, B_2) \) is stabilizable and the pair \( (C_2, A) \) is detectable (a necessary condition of existence of a stabilizing controller).

3. \( \text{rank}(D_{12}) = m_2 \) and \( \text{rank}(D_{21}) = p_2 \) where \( m_2 \) and \( p_2 \) are the dimensions of \( u \) and \( y \) respectively. These conditions are needed to ensure that the controllers are strictly proper.
4.

\[ \text{rank} \begin{pmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{pmatrix} = n + p_2 \]

and

\[ \text{rank} \begin{pmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{pmatrix} = n + m_2 \]

for all frequencies. Here \( n \) is the dimension of \( x \).

For the following equations, we also assume \( D_{22} = 0 \) which is a simplifying assumption. The solution for the controller for the generalized regulator problem (Fig.4.4 in chapter 4) is given by

\[ u = -F_\infty \hat{x} \quad \text{(6.14)} \]

and the state estimator is given by

\[ \dot{x} = A\hat{x} + B_2 u + B_1 \hat{w} + J_\infty L_\infty (y - \hat{y}) \quad \text{(6.15)} \]

where,

\[ \dot{w} = \gamma^{-2} B'_1 K_\infty \hat{x} \]

and

\[ \hat{y} = \gamma^{-2} D_{21} B_1' K_\infty \hat{x} + C_2 \hat{x} \]

The term, \( \hat{w} \) and \( \hat{y} \) are the estimates of the worst case disturbance and output of the estimator. There exists a stabilizing controller iff there exists positive semi-definite
solutions to the two Riccati equations for $K_\infty$ and $N_\infty$ and the condition

$$\rho(K_\infty N_\infty) < \gamma^2$$

(6.16)

where $\rho(A)$ is the spectral radius of $A$ which is defined as the largest singular value of $A$. The controller written in the packed matrix notation is

$$K_{\text{sub}}(s) = \begin{bmatrix} \hat{A}_\infty & J_\infty L_\infty \\ -F_\infty & 0 \end{bmatrix}$$

(6.17)

where,

$$F_\infty = (D_{12}'D_{12})^{-1}(B_2'X_\infty + D_{12}'C_1),$$

$$L_\infty = (Y_\infty C_2' + B_1D_1')(D_{21}D_{12}')^{-1},$$

and

$$J_\infty = (I - \gamma^{-2}Y_\infty X_\infty)^{-1}.$$ 

The terms, $K_\infty$ and $N_\infty$ are the solutions to the controller and estimator Riccati equations given by

$$K_\infty = \text{Ric} \begin{pmatrix} A - B_2\tilde{D}_{12}D_{12}'C_1 & \gamma^{-2}B_1B_1' - B_2\tilde{D}_{12}B_2' \\ -\tilde{C}_1'C_1 & -(A - B_2\tilde{D}_{12}D_{12}'C_1)' \end{pmatrix}$$

(6.18)

$$N_\infty = \text{Ric} \begin{pmatrix} (A - B_1D_{21}\tilde{D}_{21}C_2)' & \gamma^{-2}C_1'B_1' - C_2D_{21}C_2 \\ -\tilde{B}_1\tilde{B}_1' & -(A - B_1\tilde{D}_{21}'\tilde{D}_{21}C_2) \end{pmatrix}$$

(6.19)

where

$$\tilde{C}_1 = (I - D_{12}\tilde{D}_{12}D_{12}')C_1.$$
\[\tilde{B}_1 = B_1 \left( I - D'_{21} \tilde{D}_{21} D_{21} \right),\]
\[\tilde{D}_{12} = (D'_{12} D_{12})^{-1}; \tilde{D}_{21} = (D_{21} D'_{21})^{-1}\]

The measured outputs are the absolute acceleration responses of all floors and earthquake acceleration denoted by \(y_a = [\ddot{x}_{5a} \ \ddot{x}_{4a} \ \ddot{x}_{3a} \ \ddot{x}_{2a} \ \ddot{x}_{1a} \ \ddot{x}_{0a}]^T\), and the outputs to be regulated include the inter-story drifts, base displacements, and absolute accelerations for all floors given by \(z_a = [x_b \ x_i - x_{i-1} \ \ddot{x}_b \ \ddot{x}_i]^T\); where, \(i\) denotes the floor under consideration.

### 6.5 Control Algorithm

The device position can be determined from Eq. 6.2 by equating the force in the SAIVF device to the optimal force generated by the \(H_\infty\) controller. The resulting equations for the unknown variable \(\theta\) can be written as,

\[f_d(t) = k_c f(\theta, t)^2 y_d(t) + c_1 z f(\theta, t) = -K_\infty \ddot{x}_{az} = F_\infty\]  \hspace{1cm} (6.20)

Solving the above equation for the device position, \(f(\theta, t)\), results in the following,

\[f(\theta, t) = \frac{-c_1 z \pm \sqrt{c_1^2 z^2 + 4y_d F_\infty k_c}}{2k_c y_d}\]  \hspace{1cm} (6.21)

Due to the physical characteristics of the SAIVF device, the value of \(f(\theta, t)\) is positive and bounded between 0 and +1. Hence, Eq. 6.21 can be re-written as,

\[f(\theta, t) = |f(\theta, t)| = \frac{|-c_1 z \pm \sqrt{c_1^2 z^2 + 4y_d F_\infty k_c}|}{2k_c |y_d|}\]  \hspace{1cm} (6.22)
The sign $\pm$ in the numerator is determined based on the absolute magnitude of the term in the numerator (the maximum value is chosen). Control action is not taken when the absolute magnitude of $y_d$, $|y_d| \leq 0.01$. The values obtained by Eq. 6.22 is passed through a low pass filter to alleviate the high frequency switching and the control law can be written as,

$$
\hat{f}(\theta, t) = \begin{cases} 
1 & (u \geq 1) \\
u & \\
0 & (u \leq 0)
\end{cases}
$$

(6.23)

where, $u$ is the output of the following first order filter,

$$
\dot{u} = -a_1 u + a_2 \hat{f}(\theta, t)
$$

(6.24)

The values of $a_1$ and $a_2$ in the above equation are assumed to be 10.0 rad/sec. All the simulations are performed in Matlab/Simulink as shown in Fig. 6.7.

### 6.6 Results and Discussion

#### 6.6.1 No Stiffness Uncertainty

The $H_\infty$ active and semiactive control algorithms are implemented analytically on the base isolated building. The results for the active control for the case of no stiffness uncertainty are presented in Tables 6.1 and 6.2. The nine evaluation criteria developed in Chapter 3, i.e., $J_1$ through $J_9$ are used to assess the performance of the newly developed control algorithm for both active and semiactive cases. Table 6.1
Figure 6.7 Matlab/Simulink Control Algorithm Implementation

contains the performance indices for the case of active control with input excitation filter A. Table 6.2 contains the results for active control with the input excitation filter B. From the performance indices, it is clear that control with filter B outperforms control with filter A. This is because filter B accentuates the energy more than filter A corresponding to dominant frequencies in the PSD of most fault-normal components of the earthquakes used in the simulations. The base and structural shears in both cases are reduced between 10 to 60% compared to the uncontrolled case. The reduction in base displacement is between 20 to 65% for the case of filter B compared to 10 to 60% for filter A. Reductions of the order of 20 to 50% are achieved in inter-story drifts and accelerations for the controlled case with filter B and is better than that
achieved by using filter A.

For the case of semiactive control, the SAIVF device located at the isolation level adjusts its position based on the output of the control algorithm (angle, $\theta$) at each time step. The results of the control implementation is presented in Table 6.4 along with the % reductions compared to the passive case ($\theta = 0^\circ$) shown inside the brackets for the peak absolute performance indices, $J_1$ through $J_5$. The $H_{\infty}$ algorithm is implemented using the fourth order filter B for all cases of semiactive control. The response of the building when the device is in its closed position ($\theta = 0^\circ$) is given in Table 6.3; the device generates maximum frictional force at all time instants in this position. A total of twelve records from six near-fault earthquakes are used in this analytical study. The results in Tables 6.3 and 6.4 are given in terms of the performance indices defined earlier. The performance indices are normalized to yield a value of unity corresponding to the uncontrolled case when there is no control force applied; passive or semiactive. Values less than one correspond to reductions with respect to the uncontrolled case and vice versa. The uncontrolled response quantities are listed in Tables 6.11 and 6.12. The shears are normalized by the total weight of the structure, $W$ (3,56,000 N), and the accelerations are normalized by acceleration due to gravity, $g$. Figure 6.9 shows the comparison of uncontrolled and semiactive cases for peak values of base shear, base displacement, inter-story drifts and floor accelerations as a function of maximum ground acceleration ($g$). Figure 6.11 shows the time history
responses for base displacement, base shear and control force for a sample earthquake (Fault normal component of Jiji). Figure 6.12 shows the relationship between the total force at the isolation level, which is the sum of the SAIVF device force and the force in the elastomeric bearings, and the base displacement for the control case for Rinaldi (FN), Jiji (FN), Erzinkan (FP) and Kobe (FP) earthquakes. Figure 6.10 shows the normed \(||||\) interstory drifts for all floors in the case of Newhall (FP), Rinaldi (FP), Jiji (FN) and Erzinkan (FN) earthquakes.

The results of simulation for semiactive control with no stiffness uncertainty (Table 6.4) show that both the base shear and structural shear (measured by performance indices J1 and J2 respectively) are reduced by 10 to 50% in all earthquakes compared to the uncontrolled case except in the case of fault-normal component of Newhall, where it is at the uncontrolled level. The magnitude of reduction is between 50 to 75% for all the fault-parallel (weaker) components. The reductions are between 5% to 20% for J1, J2 and J4 compared to the passive case of maximum friction.

The effectiveness of the new control algorithm is evident from the results of base displacement (performance index J3 in Table 6.4) which are reduced by 22 to 65%, and the interstory drifts (J4 in Table 6.3), which are reduced by 20 to 40% in a majority of earthquakes (Fig. 6.10). These reductions are achieved without a corresponding increase in either the base or the structural shear. The magnitude of base displacements in the control case are similar to those achieved in the maximum
friction case (performance index J3 in Table 6.4). The interstory drifts are reduced further (10 to 20%) in the control case compared to the passive friction case.

The results of the semiactive control (Table 6.2) show that the new $H_{\infty}$ semiactive control out-performs the active case. This is evident in the case of base displacements, especially for Jiji earthquake, where the active control results in increased base displacements of the order of 50% compared to the passive case.

On-off type semiactive switching algorithms may result in increased base and superstructure accelerations in base isolated buildings. Results of the presented algorithm (performance index J5) show that contrary to bang-bang type controllers, the peak base and floor accelerations are reduced in a majority of earthquakes. The reductions in peak floor accelerations for the control case are between 3 to 40%. The magnitude of peak floor accelerations are lower when compared to the passive maximum friction case. The reduction in peak accelerations is as much as 50% in some cases with respect to the passive case.

The RMS values of base displacement and floor accelerations (J7 and J8 respectively) are also reduced in the controlled case and these values are lower in magnitude compared to the passive case. The control action is evident in the force displacement loops presented in Fig. 6.12. The smooth switching occurs during periods of maximum response resulting in a reduction of the total isolation level force and accelerations.
Figure 6.8  Time history response for Active control with filter B; (a) Base displacement, (b) Total base shear (normalized by weight of the structure) and, (c) 5th floor acceleration.
Figure 6.9  Peak Response Quantities vs. Peak Ground Acceleration; (a) Base Shear, (b) Base Displacement, (c) Inter-story Drift and (d) Floor Acceleration. (Legend: +: Controlled and o: Uncontrolled)
Figure 6.10  Plot of Interstory Drifts Normalized by Uncontrolled Value for a Set of Near-fault Earthquakes. (–x–: Controlled and –o–: Uncontrolled)
Figure 6.11  Time History Responses for Base Displacement, Base Shear and SAIVF Force Subjected to Jiji-FN Earthquake for Controlled and Uncontrolled Cases
6.6.2 **Stiffness Uncertainty of ±10%**

The results for the active control for the case of ±10% stiffness uncertainty are presented in Tables 6.5 and 6.6. Tables 6.5 and 6.6 contain the performance indices for the case of active control with input excitation filter B only. The performance of the controller with stiffness uncertainty is comparable to the case with no stiffness uncertainty.

The results of the control implementation is presented in Tables 6.9 and 6.10 along with the % reductions compared to the passive case ($\theta = 0^\circ$) shown inside the brackets for the peak absolute performance indices, $J_1$ through $J_5$. The $H_\infty$ algorithm is implemented using the fourth order filter B for all cases of semiactive control. The response of the building when the device is in its closed position ($\theta = 0^\circ$) is given in Tables 6.7 and 6.8; the device generates maximum frictional force at all time instants in this position. The results of simulation for semiactive control with ±10% stiffness uncertainty show that the performance of the controller is comparable to the case with no stiffness uncertainty.

### 6.7 Conclusions

Active control based on $H_\infty$ design was implemented on the base isolated structure. Two new filters to simulate ground excitation frequency characteristics were developed and their performance compared. A new semiactive control algorithm
Figure 6.12  Total Force at the Isolation Level vs. Base Displacement for (a) Rinaldi - FN; (b) Erzinkan - FP; (c) Jiji - FN and (d) Kobe - FP.
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<tr>
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<td>0.89 (16.3)</td>
<td>0.72 (-28.2)</td>
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(%) Reductions compared to passive case are given inside the brackets)
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<th>J7</th>
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(\% Reductions compared to passive case are given inside the brackets)
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(\% Reductions compared to passive case are given inside the brackets)
Table 6.6  Results for the $H_{\infty}$ Active Control Exc. Filter B w/ Stiffness Uncertainty of +10%  

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<td>0.59 (-23.3)</td>
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(% Reductions compared to passive case are given inside the brackets)
based on \( H_\infty \) is developed and shown to be effective in reducing the response of base isolated buildings in near fault earthquakes. A new variable friction device called SAIVF device that is capable of varying the level of friction force smoothly is introduced in this chapter. Experimental results are presented to verify the analytical model of the semiactive device. SAIVF device and the control algorithm are shown to be effective in reducing the response of the sample base isolated building in a set of near-fault earthquakes. Passive friction generally leads to increased floor accelerations and interstory drifts in base isolated buildings. The new control strategy is shown to be effective in reducing structural responses such as base displacements and interstory drifts without increasing either base shear or structural shear. Results of the simulation study indicate that the semiactive control strategy is effective for earthquakes of varying intensities. The nature of the new semiactive device eliminates the disadvantages associated with some abrupt switching devices. The results of the semiactive control also show that the new \( H_\infty \) semiactive control out-performs the active case, especially for the case of base displacements. The results of the simulation study in the presence of stiffness uncertainty show that both the active and semiactive controllers are effective in achieving response reductions during near-fault earthquakes.
<table>
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<th>J2</th>
<th>J3</th>
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<td>0.91 (-3.7)</td>
<td>1.21 (1.1)</td>
<td>0.21</td>
<td>0.55</td>
<td>0.79</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.61 (-11.5)</td>
<td>0.63 (-14.7)</td>
<td>0.46 (-3.8)</td>
<td>0.57 (-14.7)</td>
<td>0.74 (-13.8)</td>
<td>0.25</td>
<td>0.31</td>
<td>0.43</td>
<td>0.71</td>
</tr>
<tr>
<td>Kobe</td>
<td>FN</td>
<td>0.83 (-13.4)</td>
<td>0.86 (-12.3)</td>
<td>0.62 (2.0)</td>
<td>0.77 (-13.2)</td>
<td>0.99 (-42.4)</td>
<td>0.32</td>
<td>0.60</td>
<td>1.04</td>
<td>0.74</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.68 (-23.5)</td>
<td>0.73 (-25.3)</td>
<td>0.51 (2.1)</td>
<td>0.66 (-25.3)</td>
<td>0.98 (-48.1)</td>
<td>0.45</td>
<td>0.39</td>
<td>0.76</td>
<td>0.77</td>
</tr>
<tr>
<td>Jiji</td>
<td>FN</td>
<td>0.66 (-5.5)</td>
<td>0.68 (-3.7)</td>
<td>0.52 (-2.5)</td>
<td>0.61 (-3.7)</td>
<td>0.72 (-4.5)</td>
<td>0.15</td>
<td>0.43</td>
<td>0.55</td>
<td>0.57</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.76 (2.3)</td>
<td>0.80 (3.2)</td>
<td>0.53 (3.4)</td>
<td>0.72 (3.2)</td>
<td>0.90 (-0.3)</td>
<td>0.23</td>
<td>0.45</td>
<td>0.69</td>
<td>0.52</td>
</tr>
<tr>
<td>Erzinkan</td>
<td>FN</td>
<td>0.90 (-5.6)</td>
<td>0.93 (-6.2)</td>
<td>0.70 (-1.0)</td>
<td>0.84 (-6.2)</td>
<td>0.99 (-8.9)</td>
<td>0.18</td>
<td>0.49</td>
<td>0.64</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.78 (-8.3)</td>
<td>0.83 (-4.5)</td>
<td>0.58 (5.5)</td>
<td>0.75 (-4.6)</td>
<td>0.97 (-22.7)</td>
<td>0.29</td>
<td>0.26</td>
<td>0.39</td>
<td>0.77</td>
</tr>
</tbody>
</table>

(\% Reductions compared to passive case are given inside the brackets)
**Table 6.10** Results for the Semiactive Controlled Case - +10% Stiffness Uncertainty

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Case</th>
<th>J1</th>
<th>J2</th>
<th>J3</th>
<th>J4</th>
<th>J5</th>
<th>J6</th>
<th>J7</th>
<th>J8</th>
<th>J9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newhall</td>
<td>FN</td>
<td>0.89 (-5.9)</td>
<td>0.85 (-9.6)</td>
<td>0.72 (-5.2)</td>
<td>0.94 (-9.6)</td>
<td>1.02 (-7.3)</td>
<td>0.27</td>
<td>0.59</td>
<td>0.75</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.50 (-23.0)</td>
<td>0.50 (-20.5)</td>
<td>0.39 (1.3)</td>
<td>0.65 (-19.7)</td>
<td>1.36 (-17.9)</td>
<td>0.50</td>
<td>0.27</td>
<td>0.42</td>
<td>0.91</td>
</tr>
<tr>
<td>Sylmar</td>
<td>FN</td>
<td>0.65 (-6.3)</td>
<td>0.65 (-7.7)</td>
<td>0.63 (0.2)</td>
<td>0.71 (-7.7)</td>
<td>0.74 (-6.9)</td>
<td>0.19</td>
<td>0.40</td>
<td>0.43</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.56 (-5.1)</td>
<td>0.56 (-3.5)</td>
<td>0.50 (1.8)</td>
<td>0.62 (-3.5)</td>
<td>0.65 (-11.3)</td>
<td>0.25</td>
<td>0.30</td>
<td>0.36</td>
<td>0.88</td>
</tr>
<tr>
<td>Rinaldi</td>
<td>FN</td>
<td>0.91 (-2.0)</td>
<td>0.92 (-2.4)</td>
<td>0.81 (-1.4)</td>
<td>1.01 (-2.4)</td>
<td>1.04 (-3.4)</td>
<td>0.20</td>
<td>0.46</td>
<td>0.51</td>
<td>0.77</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.55 (0.5)</td>
<td>0.55 (3.2)</td>
<td>0.51 (9.4)</td>
<td>0.61 (3.2)</td>
<td>0.67 (10.9)</td>
<td>0.23</td>
<td>0.34</td>
<td>0.40</td>
<td>0.87</td>
</tr>
<tr>
<td>Kobe</td>
<td>FN</td>
<td>0.79 (-12.6)</td>
<td>0.79 (-15.8)</td>
<td>0.67 (-5.7)</td>
<td>0.86 (-15.9)</td>
<td>1.02 (-10.8)</td>
<td>0.30</td>
<td>0.58</td>
<td>0.75</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.63 (-19.0)</td>
<td>0.64 (-23.3)</td>
<td>0.57 (12.2)</td>
<td>0.72 (-23.5)</td>
<td>0.93 (-49.8)</td>
<td>0.47</td>
<td>0.43</td>
<td>0.61</td>
<td>0.85</td>
</tr>
<tr>
<td>Jiji</td>
<td>FN</td>
<td>0.59 (-3.6)</td>
<td>0.59 (-3.1)</td>
<td>0.56 (-0.6)</td>
<td>0.64 (-3.1)</td>
<td>0.61 (-8.5)</td>
<td>0.16</td>
<td>0.43</td>
<td>0.46</td>
<td>0.75</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.76 (2.0)</td>
<td>0.76 (1.8)</td>
<td>0.65 (2.4)</td>
<td>0.84 (1.8)</td>
<td>0.90 (0.9)</td>
<td>0.22</td>
<td>0.48</td>
<td>0.62</td>
<td>0.68</td>
</tr>
<tr>
<td>Erzinkan</td>
<td>FN</td>
<td>0.73 (-2.9)</td>
<td>0.72 (-4.2)</td>
<td>0.70 (2.8)</td>
<td>0.79 (-4.2)</td>
<td>0.80 (-2.5)</td>
<td>0.18</td>
<td>0.50</td>
<td>0.54</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.64 (-5.2)</td>
<td>0.61 (-7.0)</td>
<td>0.57 (9.0)</td>
<td>0.67 (-7.0)</td>
<td>0.74 (-1.6)</td>
<td>0.29</td>
<td>0.43</td>
<td>0.55</td>
<td>0.92</td>
</tr>
</tbody>
</table>

( % Reductions compared to passive case are given inside the brackets)
### Table 6.11  Uncontrolled Response Quantities (Fault-Normal Components)

<table>
<thead>
<tr>
<th></th>
<th>Newhall</th>
<th>Sylmar</th>
<th>Rinaldi</th>
<th>Kobe</th>
<th>Jiji</th>
<th>Erzinkan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Base Shear</td>
<td>0.224</td>
<td>0.537</td>
<td>0.349</td>
<td>0.241</td>
<td>0.801</td>
<td>0.485</td>
</tr>
<tr>
<td>(Norm. by W)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Str. Shear</td>
<td>0.226</td>
<td>0.539</td>
<td>0.352</td>
<td>0.243</td>
<td>0.803</td>
<td>0.487</td>
</tr>
<tr>
<td>(Norm. by W)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Isolator</td>
<td>0.340</td>
<td>0.820</td>
<td>0.530</td>
<td>0.366</td>
<td>1.225</td>
<td>0.742</td>
</tr>
<tr>
<td>Def. (m)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Interstory</td>
<td>0.194</td>
<td>0.462</td>
<td>0.302</td>
<td>0.208</td>
<td>0.688</td>
<td>0.417</td>
</tr>
<tr>
<td>Drift (cm)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Acc. (g)</td>
<td>0.235</td>
<td>0.543</td>
<td>0.362</td>
<td>0.247</td>
<td>0.807</td>
<td>0.490</td>
</tr>
<tr>
<td>RMS Disp (m)</td>
<td>0.085</td>
<td>0.309</td>
<td>0.140</td>
<td>0.075</td>
<td>0.285</td>
<td>0.278</td>
</tr>
<tr>
<td>RMS Acc. (g)</td>
<td>0.057</td>
<td>0.204</td>
<td>0.093</td>
<td>0.050</td>
<td>0.188</td>
<td>0.183</td>
</tr>
</tbody>
</table>

W: Weight of the structure; g: Acc. due to gravity; m: meters; cm: Centimeters
<table>
<thead>
<tr>
<th></th>
<th>Newhall</th>
<th>Sylmar</th>
<th>Rinaldi</th>
<th>Kobe</th>
<th>Jiji</th>
<th>Erzinkan</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Base Shear</td>
<td>0.213</td>
<td>0.408</td>
<td>0.471</td>
<td>0.183</td>
<td>0.393</td>
<td>0.294</td>
</tr>
<tr>
<td>(Norm. by W)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Str. Shear</td>
<td>0.214</td>
<td>0.410</td>
<td>0.472</td>
<td>0.186</td>
<td>0.394</td>
<td>0.294</td>
</tr>
<tr>
<td>(Norm. by W)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Isolator Def. (m)</td>
<td>0.326</td>
<td>0.623</td>
<td>0.719</td>
<td>0.279</td>
<td>0.601</td>
<td>0.449</td>
</tr>
<tr>
<td>Peak Interst. Drift (cm)</td>
<td>0.184</td>
<td>0.351</td>
<td>0.405</td>
<td>0.159</td>
<td>0.338</td>
<td>0.252</td>
</tr>
<tr>
<td>Peak Acc. (g)</td>
<td>0.217</td>
<td>0.416</td>
<td>0.477</td>
<td>0.193</td>
<td>0.396</td>
<td>0.298</td>
</tr>
<tr>
<td>RMS Disp (m)</td>
<td>0.122</td>
<td>0.233</td>
<td>0.255</td>
<td>0.075</td>
<td>0.134</td>
<td>0.188</td>
</tr>
<tr>
<td>RMS Acc. (g)</td>
<td>0.080</td>
<td>0.154</td>
<td>0.168</td>
<td>0.050</td>
<td>0.088</td>
<td>0.124</td>
</tr>
</tbody>
</table>

W: Weight of the structure; g: Acc. due to gravity; m: meters; cm: Centimeters
Chapter 7
Smart Base Isolated Buildings With Variable Damping Systems: $H_\infty$ Controller and Novel SAIVD Device

The most popular semiactive variable damping device is the Magneto-rheological (MR) damper. This device has been studied in Chapter 5 and shown to be effective in reducing the response of base isolated structures. The magnitude of damping force in a MR damper is a nonlinear function of voltage and the variation in the damping is achieved by changing the voltage supplied to the damper. A new semiactive device where the friction elements in the SAIVF device are replaced by linear viscous elements is developed and shown to be effective in reducing the responses of base isolated structures. As with the SAIVF, there are four stiffness elements and four linear dashpot elements, each stiffness-viscous element pair in parallel, which are arranged in a rhombus configuration. The stiffness elements act only to restore the position of the device and their contribution to the overall system stiffness is small. The level of damping force can be adjusted by varying the angle of the arms of the device leading to smooth variation of damping. This semiactive device, called SAIVD, is shown to be effective in reducing the response of base isolated buildings when implemented together with the $H_\infty$ controller developed in Chapter 6. The SAIVD device is essentially linear in velocity (viscous) compared to the MR damper which is nonlinear.
Experimental results are presented to verify the proposed analytical model for the SAIVD device.

7.1 Structure with Variable Damping System: Formulation

The equations of motion developed for the base isolated structure with variable friction device is essentially the same for the case of variable damping case too. The equations of motion for the six degrees of freedom base isolated structure are developed with one degree of freedom at the center of mass of each floor and the base. The state space equations for the superstructure and the base were developed and described in detail in Chapter 3 for this structure A. The state space equations repeated here for clarity are as follows:

$$\dot{x}(t) = Ax(t) + Bu(t) + E\ddot{u}_g(t)$$  \hspace{1cm} (7.1)

where $x$ consists of the states, $u(t)$ is the control force, $\ddot{u}_g(t)$ is the ground acceleration and $A$, $B$ and $E$ are system matrices that are defined as follows:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \hspace{1cm} B = \begin{bmatrix} 0 \\ M^{-1}\Lambda \end{bmatrix} \hspace{1cm} E = -\begin{bmatrix} 0 \\ \Gamma \end{bmatrix}.$$  

In the above equations, $M$ is a diagonal mass matrix formed using 6800 kgs for the mass of the base and 5897 kgs for the remaining five floors. Similarly, $C$ is a diagonal damping matrix of the form $C=\text{diag}([7480 \ 67000 \ 58000 \ 57000 \ 50000 \ 38000]^T)$ N·s/m. The stiffness of the base is 231.5 kN/m and the stiffness of the five stories are 33,732,
29,093, 28,621, 24,954 and 19,059 kN/m respectively. The device location vector and earthquake influence vector are

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad \text{and} \quad \Gamma = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix}^T.$$  

The natural frequencies of the structure are 0.40, 5.46, 10.27, 14.67, 18.3 and 21.3 Hz respectively. The damping coefficient in the isolation system is so chosen to provide a damping coefficient, $\zeta = 4\%$ in the isolated case for the fundamental mode.

### 7.2 Semiactive Variable Damping Device (SAIVD)

The SAIVD device is based on the semiactive variable stiffness device (SAIVS) developed by Nagarajaiah et al. (1998) and Nagarajaiah (2000). The SAIVD device consists of four viscous damping elements (dashpots) and four spring elements, each damping-stiffness pair in parallel, arranged in a rhombus configuration as shown in Fig. 7.1. The spring-dashpot elements are connected to joints 1 to 4 as shown in Fig. 7.1. Joint 1 is fixed in the $x$ direction and can be positioned at any desired position in the $y$ direction by a linear electromechanical actuator and controller. Joint 2 is free to move in both the $x$ and $y$ directions. Joints 3 and 4 are free to move only in the $x$ direction only. The ends of the guide rail, on which joint 2 moves, are attached to the base slab. The ends of the guide rail, on which joints 3 and 4 move, are attached to the ground. The electromechanical actuator is fixed to the ground and can actuate in the $y$ direction thus moving joint 1 to the required position. Each of the pair of elements are located at an angle, $\theta$ to the horizontal. The stiffness elements act as
restoring devices and their contribution to the overall system stiffness is minimum. The SAIVD device is capable of providing smooth variation of the level of damping force by varying the angle ($\theta$) of the arms of the device with a linear electromechanical actuator. The magnitude of force developed in the device is a function of the angle, $\theta$, given by

$$f_d(t) = k_e f(\theta, t)^2 y_d(t) + c_e v(t) f(\theta, t)$$  \hspace{1cm} (7.2)
where, $y_d(t)$ is the displacement in the $x$ direction at joint 2, $v(t)$ is the velocity at joint 2, $k_e$ is the stiffness of single spring, $\theta(t)$ is the time varying angle of the spring elements with the horizontal for any given device position, and $c_e$ is a constant. The angle, $\theta$ can only vary between angles 0 and $\pi/2$. The time-varying function $f(\theta)$ is the cosine of angle $\theta$ and given as,

$$f(\theta, t) = \cos \theta$$

$$\begin{array}{c|c}
\pi/2 & 0 \\
\end{array}$$

(7.3)

7.3 Experimental Study and Verification

In order to verify the analytical model of the semiactive device proposed in eq. 7.2, experimental studies were conducted on a scaled model of the device. The experiments consisted of recording the force developed in the device while excited by a 2.0 Hz harmonic excitation. The magnitude of the force in the device for two positions, $\theta = 10^0$ (closed), and $\theta = 77^0$ (open), are shown in Fig. 7.2. The response of the device to real-time switching is also shown at $t = 5sec.$, when the device is switched from open to close positions while excited by the sinusoid. Fig. 7.3 shows the comparison of experimental and simulated force-displacement relationship for the variable damping device at two positions of the angle $\theta$. Eq. 7.2 is used to simulate the force corresponding to the device positions measured in the experiment. The comparison study shows that the analytical model for the device provides a good representation of the force generated by the SAIVD device. The force-displacement
Figure 7.2  Experimental Results - Force developed in the SAVID device excited by a 2.0 Hz. sinusoid with an amplitude of 0.40 in
Figure 7.3  Comparison of Experimental and Analytical Results for SAIVD Device
relationship for the SAIVD device used in the simulation study is shown in Fig. 7.4 for various positions of $\theta$.

### 7.4 H$_\infty$ Formulation

For the $H_\infty$ control design, the same procedure is adopted as described in chapter 6. The excitation filter used for the designing the controller has a transfer function given by,

$$F_B(s) = \frac{8\zeta_g^2\omega_g^2s^2}{[s^2 + 2\zeta_g\omega_g s + \omega_g^2]^2} \quad (7.4)$$

and the outputs are weighted using a first order filter of the form, $W = \frac{a}{s+a}$. Where, $a=3.3 \text{ rad/sec}$, which determines the roll-off frequency.

The state equations augmented with both the weighting filters can be written as

$$\dot{x}_a = A_ax_a + B_au + E_aw$$  \quad (7.5)

$$y_a = C_yax_a + D_yau + F_ya_g + \nu_a$$  \quad (7.6)

$$z_a = C_za_x + D_za_u + F_za_g$$  \quad (7.7)

Using the standard notation given in chapter 4, the above equations can be re-cast in the form

$$\dot{x}_a = Ax_a + B_1w + B_2u$$  \quad (7.8)

$$z_a = C_1x_a + D_{11}w + D_{12}u$$  \quad (7.9)

$$y_a = C_2x_a + D_{21}w + D_{22}u$$  \quad (7.10)
Figure 7.4  Force generated in the SAIVD device used in the simulation study due a harmonic excitation of $\omega = 1 \text{ rad/s}$
where, \( \mathbf{w} \) contains both the external disturbance and measurement sensor noise. The equations for the \( H_\infty \) formulation (the control and measurement riccati equations) are given in chapter 6 and remain the same.

The measured outputs are the absolute acceleration responses of all floors and earthquake acceleration denoted by \( \mathbf{y}_a = [\ddot{x}_{3a} \ \ddot{x}_{4a} \ \ddot{x}_{3a} \ \ddot{x}_{2a} \ \ddot{x}_{1a} \ \ddot{x}_{6a}]^T \), and the outputs to be regulated include the inter-story drifts, base displacements and absolute accelerations for all floors given by \( \mathbf{z}_a = [x_b \ x_i - x_{i-1} \ x_b \ \ddot{x}_b]^T \); where, \( i \) denotes the floor under consideration.

### 7.5 Control Algorithm

The device position can be determined from Eq. 7.2 by equating the force in the SAIVD device to the optimal force generated by the \( H_\infty \) controller. The resulting equations for the unknown variable \( \theta \) can be written as,

\[
f_d(t) = k_e \phi(\theta, t)^2 y_d(t) + c_1 \nu f(\theta, t) = -K_\infty \dot{x}_{az} = F_\infty \tag{7.11}
\]

Solving the above equation for the device position, \( f(\theta, t) \), results in the following

\[
f(\theta, t) = \frac{-c_1 \nu \pm \sqrt{c_1^2 \nu^2 + 4 y_d F_\infty k_e}}{2 k_e y_d} \tag{7.12}
\]

Due to the physical characteristics of the SAIVD device, the value of \( f(\theta, t) \) is positive and bounded between 0 and +1. Hence, Eq. 7.12 can be re-written as,

\[
f(\theta, t) = |f(\theta, t)| = \frac{|-c_1 \nu \pm \sqrt{c_1^2 \nu^2 + 4 y_d F_\infty k_e}|}{2 k_e |y_d|} \tag{7.13}
\]
The sign \pm in the numerator is determined based on the absolute magnitude of the term in the numerator (the maximum value is chosen). The control action is not taken (retain the value of the previous step) when the magnitude of the base displacement, \( y_d \leq 0.01 \).

The values obtained by Eq. 7.13 is passed through a low pass filter to alleviate the high frequency switching and the control law can be written as,

\[
\hat{f}(\theta, t) = \begin{cases} 
1 & (u \geq 1) \\
u & \\
0 & (u \leq 0)
\end{cases}
\quad (7.14)
\]

where, \( u \) is the output of the following first order filter,

\[
\dot{u} = -a_1 u + a_2 \hat{f}(\theta, t) 
\quad (7.15)
\]

The values of \( a_1 \) and \( a_2 \) in the above equation are assumed to be 8.0 rad/sec, \( c_1 = 30,000Ns/m \) and \( k_c = 15,000N/m \).

7.6 Results and Discussion

7.6.1 Results for No Stiffness Uncertainty

The \( H_\infty \) semiactive control algorithm is implemented analytically on the base isolated building in terms of the evaluation criteria, \( J_1 \) through \( J_9 \), and presented in Tables 7.1 through 7.6. Tables 7.1 and 7.2 show the results of the simulation for the case of no stiffness uncertainty. The response of the building when the device is
in its closed position (θ = 25°) is given in Table 7.1. The results of the semiactive control is given in table 7.2. The value of θ for the passive case is chosen based on the average RMS value of the device angle for the set of near-fault earthquakes chosen for this study. A total of twelve records from six near-fault earthquakes are used in this analytical study. The performance indices are normalized to yield a value of unity corresponding to the uncontrolled case when there is no control force applied; passive or semiactive. Values less than one correspond to reductions with respect to the uncontrolled case and vice versa. The uncontrolled values for the base isolated building are the same as given in Chapter 6. Figure 7.4 shows the time history responses for base displacement, base shear and control force for a sample earthquake (Fault normal component of Newhall). Fig. 7.6 shows the relationship between the total force at the isolation level, which is the sum of the SAIVD device force and the force in the elastomeric bearings, and the base displacement for the control case for Rinaldi (FN), Jiji (FN), Erzinkan (FP) and Kobe (FP) earthquakes.

The results of simulation for semiactive control (Table 7.2) show that both the base shear and structural shear (measured by performance indices J1 and J2 respectively) are reduced by 10 to 60% in all earthquakes. The percentage reduction in the peak response quantities, i.e., J1 to J5 are shown in brackets. We can see that the performance of the controlled case is better than the passive case (Table 7.2) in all earthquakes considering both the base shear and story shear and the reductions are
in the order of 5 to 10% compared to the passive case in most of the earthquakes.

The effectiveness of the new control algorithm is evident from the results of base
displacement (performance index J3 in Table 7.2) which are reduced by 20 to 60%,
and the interstory drifts (J4 in Table 7.2), which are reduced by 5 to 55% in all
earthquakes. These reductions are achieved without a corresponding increase in either
the base or the structural shear. The magnitude of base displacements and the
interstory drifts in the control case are approximately 5% to 10% lower than the
corresponding passive damping case.

Previous research has shown that on-off type semiactive switching algorithms may
result in increased base and superstructure accelerations in base isolated buildings.
Results of this algorithm (performance index J5) show that contrary to bang-bang
type controllers, the peak base and floor accelerations are reduced in all earthquakes.

The RMS values of base displacement and floor accelerations (J7 and J8 respec-
tively) are also reduced in the controlled case and these values are lower in magnitude
compared to the passive case. The control action is evident in the force displacement
loops presented in Fig. 7.6. The smooth switching occurs during periods of maximum
response resulting in a reduction of the total isolation level force and accelerations.

7.6.2 Results for Stiffness Uncertainty

Results are presented in tables 7.3 to 7.6 when there is an uncertainty associated
with estimating the system stiffness. Two cases, namely uncertainties of ±10% are
Figure 7.5  Time history response for Semiactive control-Newhall FN; (a) Base displacement, (b) Total base shear (normalized by weight of the structure), and (c) 5th floor acceleration.
Figure 7.6  Total Force at the Isolation Level vs. Base Displacement for (a) Rinaldi - FN; (b) Erzinkan - FP; (c) Jiji - FN and (d) Kobe - FP.
investigated. The results of the simulations show that the controller is sufficiently robust and the magnitude of reduction in the response quantities is comparable to the case when there is no stiffness uncertainty. Reductions similar to the case when there is no uncertainty in the stiffness are obtained compared to the passive damping case.

7.7 Conclusions

A new semiactive variable damping device and a new control algorithm based on $H_\infty$ are developed and shown to be effective in reducing the response of base isolated buildings in near fault earthquakes. The new variable damping device called SAIVD device that is capable of varying the level of damping force smoothly is introduced in this chapter. Experimental results have been presented to verify the analytical model of the semiactive device. Passive damping generally leads to increased floor accelerations and inter-story drifts in base isolated buildings. The new control strategy is shown to be effective in reducing structural responses such as base displacements and inter-story drifts without increasing either base shear or structural shear. The nature of the new semiactive device eliminates the disadvantages associated with some abrupt switching devices. Results of the simulations also show that the new semiactive controller is effective in reducing the response of the structure even in the presence of uncertainty in the stiffness.
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### Table 7.2  Results for the $H_\infty$ Semiactive Control case

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(％Reductions compared to passive case are given inside the brackets)
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<td>0.44</td>
<td>0.26</td>
<td>0.83</td>
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<tr>
<td></td>
<td>Fault-Parallel</td>
<td>0.70</td>
<td>0.77</td>
<td>0.68</td>
<td>0.77</td>
<td>0.41</td>
<td>0.23</td>
<td>0.59</td>
<td>0.35</td>
<td>0.83</td>
</tr>
</tbody>
</table>
Table 7.5 Results for the $H_\infty$ Semiactive Control case with -10% Uncertainty

<table>
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<tr>
<th>Earthquake</th>
<th>Case</th>
<th>J1</th>
<th>J2</th>
<th>J3</th>
<th>J4</th>
<th>J5</th>
<th>J6</th>
<th>J7</th>
<th>J8</th>
<th>J9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newhall</td>
<td>FN</td>
<td>0.76 (-7.0)</td>
<td>0.90 (-4.0)</td>
<td>0.75 (-8.5)</td>
<td>0.90 (-4.1)</td>
<td>0.41 (-1.1)</td>
<td>0.37</td>
<td>0.74</td>
<td>0.45</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.42 (-9.0)</td>
<td>0.49 (-3.1)</td>
<td>0.40 (-12.1)</td>
<td>0.49 (-3.1)</td>
<td>0.32 (-10.9)</td>
<td>0.44</td>
<td>0.42</td>
<td>0.27</td>
<td>0.72</td>
</tr>
<tr>
<td>Sylmar</td>
<td>FN</td>
<td>0.54 (-7.4)</td>
<td>0.62 (-5.9)</td>
<td>0.53 (-9.9)</td>
<td>0.62 (-5.9)</td>
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<td>0.33</td>
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<td>0.25</td>
<td>0.71</td>
</tr>
<tr>
<td></td>
<td>FP</td>
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<td>0.49 (-10.1)</td>
<td>0.59 (-5.3)</td>
<td>0.35 (-0.5)</td>
<td>0.35</td>
<td>0.40</td>
<td>0.26</td>
<td>0.72</td>
</tr>
<tr>
<td>Rinaldi</td>
<td>FN</td>
<td>0.74 (-11.6)</td>
<td>0.95 (-10.0)</td>
<td>0.77 (-6.8)</td>
<td>0.94 (-10.0)</td>
<td>0.43 (-10.2)</td>
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<td>0.62</td>
<td>0.39</td>
<td>0.72</td>
</tr>
<tr>
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<td>FP</td>
<td>0.54 (-5.1)</td>
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<td>0.60 (-4.1)</td>
<td>0.35 (-2.9)</td>
<td>0.31</td>
<td>0.40</td>
<td>0.25</td>
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<td>Kobe</td>
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<td>0.64 (-10.3)</td>
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<td>0.63 (-12.6)</td>
<td>0.74 (-9.4)</td>
<td>0.44 (-7.4)</td>
<td>0.31</td>
<td>0.65</td>
<td>0.38</td>
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</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.58 (-7.7)</td>
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<td>0.55 (-9.5)</td>
<td>0.66 (-7.2)</td>
<td>0.43 (0.9)</td>
<td>0.45</td>
<td>0.55</td>
<td>0.36</td>
<td>0.73</td>
</tr>
<tr>
<td>Jiji</td>
<td>FN</td>
<td>0.43 (-9.1)</td>
<td>0.48 (-8.8)</td>
<td>0.42 (-11.0)</td>
<td>0.48 (-8.8)</td>
<td>0.27 (-8.5)</td>
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<td>0.35</td>
<td>0.22</td>
<td>0.63</td>
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<tr>
<td></td>
<td>FP</td>
<td>0.56 (-6.5)</td>
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<td>0.56 (-7.7)</td>
<td>0.61 (-6.5)</td>
<td>0.36 (-6.9)</td>
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<td>0.54</td>
<td>0.34</td>
<td>0.48</td>
</tr>
<tr>
<td>Erzinkan</td>
<td>FN</td>
<td>0.60 (-6.1)</td>
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<td>0.58 (-8.7)</td>
<td>0.67 (-5.9)</td>
<td>0.39 (-4.2)</td>
<td>0.29</td>
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<td>0.71</td>
</tr>
<tr>
<td></td>
<td>FP</td>
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<td>0.79 (-3.4)</td>
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<td>0.79 (-3.4)</td>
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<td>0.27</td>
<td>0.38</td>
<td>0.24</td>
<td>0.70</td>
</tr>
</tbody>
</table>

(\% Reductions compared to passive case are given inside the brackets)
<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Case</th>
<th>J1</th>
<th>J2</th>
<th>J3</th>
<th>J4</th>
<th>J5</th>
<th>J6</th>
<th>J7</th>
<th>J8</th>
<th>J9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newhall</td>
<td>FN</td>
<td>0.76 (-2.1)</td>
<td>0.90 (-2.1)</td>
<td>0.70 (-7.8)</td>
<td>0.90 (-2.1)</td>
<td>0.36 (-0.2)</td>
<td>0.34</td>
<td>0.66</td>
<td>0.38</td>
<td>0.79</td>
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<td>0.47 (-9.3)</td>
<td>0.54 (-3.1)</td>
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<td>0.44</td>
<td>0.42</td>
<td>0.26</td>
<td>0.82</td>
</tr>
<tr>
<td>Sylmar</td>
<td>FN</td>
<td>0.56 (-5.8)</td>
<td>0.60 (-6.7)</td>
<td>0.55 (-8.2)</td>
<td>0.60 (-6.7)</td>
<td>0.33 (-3.9)</td>
<td>0.35</td>
<td>0.37</td>
<td>0.23</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>FP</td>
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<td>0.57 (-8.1)</td>
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<td>0.36</td>
<td>0.36</td>
<td>0.22</td>
<td>0.84</td>
</tr>
<tr>
<td>Rinaldi</td>
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<td>0.41</td>
<td>0.25</td>
<td>0.82</td>
</tr>
<tr>
<td>Kobe</td>
<td>FN</td>
<td>0.67 (-7.8)</td>
<td>0.73 (-7.9)</td>
<td>0.67 (-10.1)</td>
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<tr>
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<td>0.49</td>
<td>0.57</td>
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<td>0.80</td>
</tr>
<tr>
<td>Jiji</td>
<td>FN</td>
<td>0.44 (-11.4)</td>
<td>0.47 (-11.5)</td>
<td>0.44 (-12.1)</td>
<td>0.47 (-11.5)</td>
<td>0.27 (-8.5)</td>
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<td>0.37</td>
<td>0.22</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.63 (-7.1)</td>
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<td>0.63 (-9.2)</td>
<td>0.67 (-7.5)</td>
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<td>0.31</td>
<td>0.56</td>
<td>0.34</td>
<td>0.60</td>
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<tr>
<td>Erzinkan</td>
<td>FN</td>
<td>0.61 (-6.1)</td>
<td>0.66 (-5.9)</td>
<td>0.58 (-9.2)</td>
<td>0.66 (-5.9)</td>
<td>0.37 (-1.6)</td>
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<td>0.42</td>
<td>0.26</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>FP</td>
<td>0.67 (-4.8)</td>
<td>0.73 (-5.1)</td>
<td>0.62 (-8.6)</td>
<td>0.73 (-5.1)</td>
<td>0.38 (-5.7)</td>
<td>0.25</td>
<td>0.57</td>
<td>0.35</td>
<td>0.83</td>
</tr>
</tbody>
</table>

(\% Reductions compared to passive case are given inside the brackets)
Chapter 8
Smart Base Isolated Buildings with Variable Stiffness Isolation Systems: Time-Frequency Controller and Novel SAIVS Device

In the previous chapters methods to incorporate the frequency content of earthquake signals into optimal control methods were introduced. It has been shown that the frequency content representation of the excitation is extremely important in achieving the control objectives. However, the earthquakes were modeled as the output of stationary filters excited by a white noise. In reality, earthquakes are non-stationary and simplifying them using the aforementioned stationary filters may not be the optimal way of representing them. The alternative is to use the time-frequency content of the earthquakes in designing the $H_2$ or the $H_{\infty}$ controller with time-varying gains. Such a proposition may not be easy for practical implementation, as the optimal controller and the associated gains may have to be updated frequently (even at each time step) and this leads to very cumbersome calculations. In this chapter, a novel way of incorporating the time-frequency content of the earthquake signals is presented. This method does not involve the calculation of the $H_2$ or the $H_{\infty}$ controller; but uses the time-frequency information of the earthquake directly for achieving control objectives in base isolated structures.

The central idea of the new algorithm is to achieve the response minimization
in base isolated structures using a non-resonant criteria. The newly developed and patented variable stiffness device (Nagarajaiah 2000), called SAIVS (Semi Active Independently Variable Stiffness) is used to achieve the desired objectives. The SAIVS device is capable of varying its stiffness smoothly, thereby eliminating some of the disadvantages associated with abrupt switching devices. The time-frequency content of the earthquake is measured in real-time using Short Term Fourier Transform (STFT). The central idea of STFT is to break up the signal into small time segments and Fourier analyze each time segment to ascertain the frequencies that exist in it. For each different time a different spectrum is obtained and the totality of these spectra is the time-frequency distribution. STFT is used to determine the energy spectrum and time-frequency distribution of the earthquake excitation signal. Of particular importance is the tracking of the energy of the earthquake excitation corresponding to the fundamental period of the base isolated building. When the energy of the excitation exceeds a predetermined threshold value the STFT controller varies the stiffness of the isolation system smoothly between minimum and maximum values to achieve response reduction. The main reason for the response reduction is the variation of the fundamental frequency of the base isolated building. Additionally, the STFT control algorithm ensures passivity and energy dissipation during the smooth variation of stiffness. The STFT algorithm is implemented analytically on a five story base isolated reinforced concrete building with linear elastomeric isolation bearings.
and variable stiffness system located at the isolation level. Several recent near fault earthquakes are considered. It is shown that the controller is effective in reducing the base displacements and interstory drifts without increasing floor accelerations. The novelty of the STFT controller lies in its effective variation of stiffness only a few times to achieve response reduction, which makes it suitable for practical implementation.

8.1 Structure with Variable Stiffness System: Formulation

The structure used in this study is a modified version of the benchmark structure, called structure B in chapter 3. The eight story benchmark building is reduced to 5 stories by retaining only the bottom five stories. The mass, stiffness and damping properties for the five floors are identical to the benchmark structure. The equations of motion for the five story base isolated structure are developed with three degrees of freedom at the center of mass of each floor and the base. The state space equations for the superstructure and the base were developed and described in detail in Chapter 3. The equations of motion are repeated here for the sake of completeness.

\[
\dot{x}(t) = Ax(t) + Bu(t) + Ea_g(t) = g(x, u, a_g) \tag{8.1}
\]

where \(X = \{U^T \ U_b^T \ \dot{U}^T \ \dot{U}_b^T\}^T\)

\[A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ -M^{-1} \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ -M^{-1} \{ \begin{array}{c} MR \\ R^TMR + M_b \end{array} \} \end{bmatrix} \]
\[
\bar{M} = \begin{bmatrix}
M & MR \\ R^T M & R^T MR + M_b
\end{bmatrix}, \quad \bar{C} = \begin{bmatrix}
C & 0 \\ 0 & C_b
\end{bmatrix},
\]
\[
\bar{K} = \begin{bmatrix}
K & 0 \\ 0 & K_b
\end{bmatrix}, \quad u = \begin{bmatrix}
0 \\ f_d
\end{bmatrix}
\]

In the above equations, \( A, B \) and \( E \) are system matrices. \( M \) is the superstructure mass matrix, \( C \) is the superstructure damping matrix in the fixed base case, \( K \) is the superstructure stiffness matrix in the fixed base case, \( M_b \) is the mass of the rigid base, \( C_b \) is the damping of isolation system, \( K_b \) is the total stiffness of elastic isolation elements and \( f_d \) is the vector of force from the control devices. \( R \) is the matrix of earthquake influence coefficients consisting of zeros corresponding to rotational degree of freedom and ones corresponding to the two lateral degrees of freedom. Furthermore, \( \ddot{U}, \ddot{U} \) and \( U \) represent the floor acceleration, velocity and displacement vectors relative to the base, \( \ddot{U}_b \) is the base acceleration relative to the ground and \( a_g \) is the ground acceleration in two perpendicular directions \( x \) and \( y \).

### 8.2 Semiactive Variable Stiffness Device (SAIVS)

The semiactive variable stiffness device (SAIVS) is capable of providing smooth stiffness variation along \( x \) or \( y \) direction depending on its orientation. In the current study the earthquake excitation is considered in the \( x \) direction only (uniaxial case). Hence, the SAIVS device is oriented to provide variable stiffness in the \( x \) direction
only. The analytical model for a single SAVIS device is shown in Fig. 8.1. It consists of four sets of spring elements arranged in a rhombus configuration as shown. Each of the four spring elements in the device are located at an angle, $\theta$ to the horizontal. The four springs are connected to joints 1 to 4 as shown in Fig. 8.1. Joint 1 is fixed in the $x$ direction and can be positioned at any desired position in the $y$ direction by a linear electromechanical actuator and controller. Joint 2 is free to move in both the $x$ and $y$ directions. Joints 3 and 4 are free to move in the $x$ direction only. The
ends of the guide rail, on which joint 2 moves, are attached to the base slab. The ends of the guide rail, on which joints 3 and 4 move, are attached to the ground. The electromechanical actuator is fixed to the ground and can actuate in the $y$ direction thus moving joint 1 to the required position. The force developed at any time in the device for a specific position is given by

$$ f_{dx}(t) = \left\{ k_e \cos^2 \theta(t) \right\} y_{dz}(t) $$

(8.2)

where, $y_{dz}(t)$ is the relative displacement (between base and ground) at joint 2 in the $x$ direction, $k_e$ is the stiffness of single spring, $\theta(t)$ is the time varying angle of the spring elements with the horizontal for any given device position. The angle of the spring is a function of the controller output command voltage, $I_{dz}$, to the linear actuator of the device as in Eq. 8.8. The device generates certain $f_{dz}$ (eq. 11) for a particular angle. For example, when $\theta = 20.28^0$, $k_{max} = 61,600kN/m$ (passive on) and when $\theta = 67.2^0$, $k_{min} = 10,512kN/m$ (passive off). For intermediate $\theta$, the stiffness, $k_i$ varies between $k_{max}$ and $k_{min}$ (refer to fig. 8.4). Although the device is linear, as the device angle is changed, stiffness varies, resulting in hysteretic behavior leading to additional energy dissipation.

8.3 Short Term Fourier Transform (STFT)

The Short-Term Fourier Transform (STFT) is the most widely used method for studying non-stationary signals. The basic idea of STFT is to break up the signal
into small time segments and Fourier analyze each time segment to ascertain the frequencies that existed in that segment. For each different time a different spectrum is obtained and the totality of such spectrum indicates the time-frequency distribution.

The STFT procedure is ideal in many aspects. It is well defined, based on reasonable physical principles, and for many signals and situations it gives an excellent time-frequency structure consistent with intuition (Cohen 1995).

To study the properties of the signal, say \( a_{gx} \), at time \( t \), the signal at that time is emphasized and the signal at other times is suppressed. This is achieved by multiplying the signal by a window function, \( h(t) \) centered at \( t \), to produce a modified signal,

\[
\hat{a}_{gst} (\tau) = a_{gx} (\tau) h(\tau - t)
\]

The modified signal is a function of two times, the fixed time, \( t \), that is of interest, and the running time, \( \tau \). The window function, \( h \) is chosen to leave the signal more or less unaltered around the time \( t \) but to suppress the signals for times distant from the time of interest. The resulting Short Time Fourier Transform reflecting the distribution of frequency around that time is given by,

\[
A_t(\omega) = \frac{1}{\sqrt{2\pi}} \int e^{-j\omega \tau} \hat{a}_{gst} (\tau) d\tau
\]
\[
= \frac{1}{\sqrt{2\pi}} \int e^{-j\omega \tau} \hat{g}_{st}(\tau) h(\tau - t) d\tau \quad (8.5)
\]

The energy density spectrum at time \( t \) is therefore

\[
P_{SP}(t, \omega) = |A_t(\omega)|^2 = \frac{1}{\sqrt{2\pi}} \int e^{-j\omega \tau} \hat{g}_{st}(\tau) h(\tau - t) d\tau \quad (8.6)
\]

For each different time a different spectrum is obtained and the totality of these spectra is the time-frequency distribution, \( P_{SP} \), called the "Spectrogram". The Spectrogram for any signal results in a matrix \( P_{SP} \) of size \( m \times n \) where, \( m \) is the number of frequency intervals and \( n \) the number of time steps. The power computed in linear scale is converted to a logarithmic scale in decibels (dB) for the purposes of algorithm implementation.

### 8.4 STFT Control Algorithm

For simplicity the STFT control algorithm is implemented only in the \( x \) direction; however, it can easily be extended to both \( x \) and \( y \) directions in general. Such an assumption is deemed appropriate in the current study since the excitation is considered only in the \( x \) direction (uniaxial case). The control block diagram is shown in Fig. 8.2.

At time \( t = 0 \) the stiffness value of the SAIVS device is set to its maximum, \( k_{max} \). The control algorithm is as follows:
1. A moving window (advanced at every time step) of \( n \) time steps of signal is chosen. At any time \( t_k \) the signal is multiplied with the 'hanning' window function, \( h \) as in eq. 12.

2. The energy density spectrum at \( t_k \) is computed using STFT in discrete form resulting in a vector \( P_{\text{SP}}^{t_k} \) of size \( m \times 1 \).

3. From the vector of the energy density spectrum, \( P_{\text{SP}}^{t_k} \), the scalar \( P_{f_{max}}^{t_k} \) is selected corresponding to \( f_{max} \) which is the fundamental frequency at \( k_{max} \).

4. Either one of the following steps is implemented next (i) step 5 is implemented if the current stiffness state is \( k_{max} \) and smooth stiffness variation is continued until \( k_{min} \) is reached, or (ii) step 6 is implemented if the current stiffness state is \( k_{min} \) and smooth stiffness variation is continued until \( k_{max} \) is reached, or (iii)
the algorithm is restarted at step 1.

5. The smooth stiffness variation from $k_{\text{max}}$ to $k_{\text{min}}$ begins at the instant, $t_{k_{\text{uc}}}^{\text{uc}}$, when $P_{f_{\text{max}}}^{\text{uc}}$ is increasing and exceeds (termed as upcrossing at $t_{k_{\text{uc}}}^{\text{uc}}$) a pre-determined threshold value, $\zeta$. A smoothing function, $\lambda^+ = \frac{2}{1 + e^{-\alpha \tau_k}}$ is used to vary the stiffness from $k_{\text{max}}$ to $k_{\text{min}}$, where $\alpha$ is a constant (a value of 4 is chosen for the current study) and $\tau_k^l$ is the discrete elapsed local time beginning at the upcrossing instant $t_{k_{\text{uc}}}^{\text{uc}}$. Once $k_{\text{min}}$ is reached, the local time $\tau_k^l$ is continued into step 6, which always follows step 5.

6. The smooth stiffness variation from $k_{\text{min}}$ to $k_{\text{max}}$ begins when the two conditions (i) $f_{d_{k_{\text{max}}}} y_{d_{k_{\text{max}}}} > 0$ and (ii) $\tau_k^l \geq \delta$ are satisfied, where $f_{d_{k_{\text{max}}}}$ is the force in the SAIVS device, $y_{d_{k_{\text{max}}}}$ is the relative velocity at the device location, $\tau_k^l$ is the discrete elapsed local time, and $\delta$ is a constant. The first condition ($f_{d_{k_{\text{max}}}} y_{d_{k_{\text{max}}}} > 0$) is checked before the stiffness is varied from $k_{\text{min}}$ to $k_{\text{max}}$, in order to ensure that the stiffness change is always dissipative in nature. The second condition ($\tau_k^l \geq \delta$) is checked to ensure adequate time for the stiffness recovery from $k_{\text{min}}$ to $k_{\text{max}}$ and to allow for the smooth variation of the stiffness. The stiffness is varied from $k_{\text{min}}$ to $k_{\text{max}}$ using the smoothing function $\lambda^- = \frac{2}{1 + e^{-\alpha \tau_k}}$. Once $k_{\text{max}}$ is reached, the local time $\tau_k^l$ is reset to zero.
8.5 Application of STFT Controller: Numerical Example

The structure considered is a symmetric base-isolated five-story building of length, \( L = 55 \text{-m} \) and width, \( W = 55 \text{-m} \). The superstructure bracing is located at the building perimeter. Metal decking and a grid of steel beams support all concrete floor slabs. The steel superstructure is supported on a reinforced concrete base slab, which is integral with concrete beams below, and drop panels below each column location. The isolators are connected between these drop panels and the footings below as shown in Fig. 8.3. The superstructure is modeled as a three dimensional linear elastic system. The superstructure members, such as beam, column, bracing, and floor slab are modeled in detail. Floor slabs and the base are assumed to be rigid in plane. The superstructure and the base are modeled using three master degrees of freedom (DOF) per floor at the center of mass. The combined model of the superstructure (15 DOF) and isolation system (3 DOF) consists of 18 degrees of freedom. The superstructure damping ratio is assumed to be 5% in all fixed base modes and 2% critical for the elastomeric bearings. First nine modes are used for modeling the superstructure. The computed periods \( T_n \) for the nine modes in the fixed-base condition (along the three DOF) are 0.73, 0.57, 0.43, 0.29, 0.23, 0.17, 0.14, 0.12 and 0.10 seconds. The eccentricities at all floors are chosen to be zero. The total weight of the structure is 142,575 kN. There are 61 linear elastomeric bearings each with a stiffness of 1435.5 kN/m (total stiffness of 87,565 kN/m). The variable stiffness device is assumed to be
connected at the center of mass of the base only in the $x$ direction, as shown in Figs. 8.3 and 8.4 (the restoring force is assumed to be coincident with the center of mass). The SAIVS device shown in Fig. 4 has been magnified for clarity. The stiffness $k_{x} = 70,000 \text{ kN/m}$ of the SAIVS device is chosen so as to provide a fundamental period of 2 seconds ($f_{\text{max}} = 0.5 \text{Hz}$) in the passive on case ($k_{\text{max}} = 61,600 \text{ kN/m}$) and 2.5 seconds ($f_{\text{min}} = 0.4 \text{Hz}$) in the passive off case ($k_{\text{min}} = 10,512 \text{ kN/m}$). The equations of motion are solved using constant average acceleration method inside the S function block named Linear Time Variant Analysis as shown in the Simulink block diagram in Fig. 8.5. The STFT computation is performed using the standard blocks available within the Simulink toolbox.

The structure considered here is excited by the following set of near-fault ground
Figure 8.4  Detail of the SAIVS Device with Four Surrounding Elastomeric Bearings

Figure 8.5  Simulink Block Diagram for Simulation
motions in the uniaxial x direction as shown in Figs. 3 and 4.

- **Newhall** - Fault normal Newhall component of the 1994 Northridge earthquake, with a peak ground acceleration of 0.736g.

- **Sylmar** - Fault parallel component of the 1994 Northridge earthquake recorded in Sylmar county, with a peak ground acceleration of 0.605g.

- **Kobe** - East-West component of the Kobe earthquake (JMA), with a peak ground acceleration of 0.631g.

8.5.1 STFT Spectrogram of Earthquakes

The Spectrogram for three excitations, Newhall, Sylmar and Kobe are shown in Fig. 8.6, Fig. 8.7 and Fig. 8.8 with time plotted on the x axis, frequency on the y axis and the STFT power (in decibels, dB) on the z-axis. Also shown in these figures is a threshold level that intersects the surface of the STFT spectrogram at all frequencies.

The value of the threshold parameter, $\zeta$, described in the earlier sections is determined based on a parametric study of variables in the algorithm. This parametric study is conducted for the set ground motions considered, namely, Newhall, Sylmar and Kobe. For each windowing operation, 256 earthquake data points are taken and zero-padded to length 2048. To this zero-padded data, a Hanning window of time duration $\Delta t = 10.24$ sec (frequency bandwidth of $\Delta \omega = 0.098$ Hz) is applied for STFT computation. The results of this study are shown in Fig. 8.9. The results are plotted
Figure 8.6  STFT Spectrogram for Newhall Earthquake

Figure 8.7  STFT Spectrogram for Sylmar Earthquake
Figure 8.8  STFT Spectrogram for Kobe Earthquake

Figure 8.9  Determination of Threshold, $\zeta$
in terms of the percentage reduction of story drifts achieved for various values of $\zeta$ that result in positive drift reductions. Based on the performance of threshold level $\zeta$ that is optimal over the three ground excitations, $\zeta = 135$ dB is chosen as the design value for simulations.

8.5.2 Structural Responses

Figure 8.10 shows the base displacement responses of the base isolated building due to Newhall, Sylmar and Kobe earthquake excitations in the uniaxial direction. Three cases considered are (i) passive off, where the SAIVS device develops its minimum stiffness, (ii) passive on, where the SAIVS device develops its maximum stiffness and (iii) controlled, where the new STFT algorithm described in the earlier sections is implemented. The stiffness variation time histories are shown in Figure 11 for the three excitations. Also shown in Fig. 8.11 is a slice of STFT spectrogram at 2 sec (fundamental period of passive on case) as a function of time. In the controlled case when the magnitude of STFT exceeds the threshold value during upcrossing, the stiffness is varied smoothly from the passive on to the passive off position and after the passivity and elapsed time considerations are satisfied it is varied smoothly from passive off to passive on position. The threshold level is fixed at 135 dB for all the three excitations and the parameter $\delta=0.25$ sec ($T_n/8$) is chosen. The parameter, $\alpha = 4.0$ for the current study. The ± sign for $\alpha$ is determined by the direction of switching; from maximum to minimum or vice versa.
Figure 8.10  Base Displacement Responses for Newhall, Sylmar and Kobe Excitations.

Fig. 8.12 shows the peak base displacement as a function of peak ground acceleration in passive on, passive off and controlled cases for the three earthquakes. From the results we can see that the reductions in the peak base displacements for the controlled case compared to the passive off case is 14% for Newhall, 11% for Sylmar and 26% for Kobe. The corresponding reductions for the controlled case compared to the passive on case is 4% and 6% for Newhall and Sylmar respectively. The peak base
displacements for Kobe excitation is the same as the passive on case. The controlled case reduces the peak base displacement response in all three earthquakes. Particularly in the case of Newhall and Sylmar with peak base displacement greater than 0.25 m the controlled case reduces the response further than the passive on case. It can be observed from Fig. 8.10 that the controlled case remains bounded between the passive on and passive off cases. Also, it is evident in Fig. 8.10 that the stiffness and frequency varies from passive on to passive off or vice versa to avoid peak displacement response during peak STFT power in Newhall and Sylmar earthquakes. In the case of Kobe earthquake the controlled case remains at the passive on state most of the time since least displacement response occurs in that state.

Fig. 8.13 shows the peak inter-story drifts at various floor levels for all the three excitations considered. From Fig. 8.13 it is evident that the story drifts are substantially reduced for the controlled case as compared to the passive on case in all three earthquakes for all the floors. Maximum drift reductions in the controlled case for Newhall, Sylmar, and Kobe earthquakes are 18%, 49%, and 15%, respectively, when compared to the passive on case, and nearly the same when compared to the passive off case. Comparing Fig. 12 and Fig. 8.13 it is evident that the greatest reductions in the inter-story drifts (49%) is achieved where the magnitude of the base displacement is the highest (Sylmar excitation).

It is also worth noting from Fig. 8.12 and Fig. 8.13 that in the passive on case the
base displacements are smaller due to a stiffer isolation system; however, the interstory drifts are larger. Alternatively, in the passive off case the base displacements are larger; however, the interstory drifts are smaller. In the controlled case both the base displacements and interstory drifts are smaller which clearly indicates the advantage of the new STFT controller.

The peak floor accelerations shown in Fig. 8.14 are maintained at the same levels as the passive off case. Only in the case of Newhall the first floor acceleration for the controlled case is reduced by nearly 10%. This shows that the STFT controller with smooth stiffness variation does not introduce the additional accelerations in the lower stories, when compared to on-off stiffness switching algorithms developed by other researchers (Yang et al. 1996, Singh et. al. 1997). The force displacement plots are shown in Fig. 15. The magnitude of the SAIVS force is normalized by the total weight of the structure. The stiffness variation is clearly evident in Fig. 8.15. It is the stiffness or frequency variation which leads to drift response reduction. This can be inferred from the response in Kobe - drift response is reduced by nearly 15% with stiffness variation of the isolation system and with minimal additional energy dissipation in the isolation system (see Fig. 15(c)).

The estimated time delay in the online implementation of this algorithm is investigated through simulations; only the summary of the results are reported here for brevity. The time delay for STFT implementation is nearly 5 milliseconds (mS).
However, for evaluation purposes, delays of 25 mS and 50 mS are introduced and the simulations are performed. The greatest effect of the time delay is on the magnitude of base displacement. For the case of Kobe earthquake, by introducing a delay of 50mS, the base displacements increased by 5 % and by 3.7 % for a delay of 25 mS, compared to the base line case of 5 mS. By comparison, for 50 mS delay, the base displacements increased by 2% for Sylmar and 2.27% for Newhall. Similarly, increases in base displacement for 25 mS delay for Sylmar and Newhall are 1.5% and 1.88% respectively. The effect of time delay on inter-story drifts and accelerations is less than 2% for 50mS delay and less than 1.3 % for 25 mS delay in all the three earthquakes.

8.6 Conclusions

A new variable stiffness control algorithm based on STFT has been developed and shown to be effective in reducing the response of base isolated buildings in near fault earthquakes. The variation of stiffness is based on tracking of energy of the earthquake excitation at the fundamental period. In base isolated buildings with stiffer isolation systems generally the base displacements are smaller and interstory drifts are larger. Exactly the opposite response results in the case with softer isolation system. However in the controlled case both the base displacement and interstory drifts are reduced. Simulated results show that the greatest reduction in interstory drifts occur when the magnitude of the base displacement is the largest. No increase in acceleration in the lower stories occurs due to few smooth stiffness variations as
compared to increase in acceleration in the lower stories observed in on-off stiffness algorithms with rapid and abrupt switching. This study also shows that algorithms based on time-frequency content of the ground excitation hold significant promise for use with variable stiffness devices in base isolation applications.
Figure 8.11  STFT Power and Stiffness Time Histories for Newhall, Sylmar and Kobe Excitations.
Figure 8.12  Base Displacement Responses for Newhall, Sylmar and Kobe Excitations.
**Figure 8.13** Story Drifts for Newhall, Sylmar and Kobe Excitations.
Figure 8.14  Peak Floor Accelerations for Newhall, Sylmar and Kobe Excitations.
Figure 8.15  Force Displacement Loops of Variable Stiffness device for Newhall, Sylmar
and Kobe Excitations.
Chapter 9
Conclusions and Recommendations

A comprehensive study in the context of control of smart base isolated buildings has been accomplished in this dissertation. Novel control algorithms and semiactive devices have been developed and shown to be effective in reducing the responses of smart base isolated buildings. The key contributions of this thesis are summarized as follows:

1. A new class of $H_{\infty}$ active and semiactive smooth controllers have been developed for smart base isolated structures.

2. A new class of semiactive variable friction and damping devices have been developed and experimentally verified.

3. A new class of input filters that better represent near fault earthquakes have been developed and adopted to augment the state-space formulation to explicitly include the excitation characteristics.

4. Robust optimal frequency-domain control methods, namely $H_2$ and $H_{\infty}$, have been rigourously introduced in the context of structural control. Relative advantages of these control designs as applicable to base isolated structures have been discussed. Methods of implementing various input and output weighting functions in the frequency domain and augmentation methods have been
introduced through numerical simulation examples. A brief discussion of the robustness of these methods along with its applicability to the current study is also presented.

5. A new smooth clipped optimal controller has been developed based on $H_2/LQG$. This algorithm along with magnetorheological dampers has been implemented on a fully three dimensional base isolated benchmark structure.

6. A new active and semiactive control algorithm based on $H_\infty$ is developed, along with a novel semiactive variable friction device (SAIVF) for smart base isolated structures. The central idea of the control algorithm is vary the level of friction semiactively by calculating the position of the semiactive variable friction device based on the optimal force generated by the $H_\infty$ controller. The $H_\infty$ controller is developed using a set of newly developed weighting filters that have been chosen for optimal performance in reducing near-fault earthquake responses. The level of friction force in the SAIVF device is varied through a smooth variation of the position of the arms of the device. Experimental results have also been presented to verify the proposed analytical model of the device.

7. Another novel semiactive variable damping device (SAIVD) has been developed and implemented on a smart base isolated structure. The level of damping force in the SAIVD device is varied through a smooth variation of the position of the
arms of the device. A new $H_\infty$ controller with a set of optimum weighting filters is developed for this system. Experimental results have also been presented to verify the proposed analytical model of the device.

8. A new time-frequency based control algorithm for smart base isolated buildings with variable stiffness isolation systems has been developed using Short-Term Fourier Transform. The novel variable stiffness device (SAIVS) is capable of smooth stiffness variation through the appropriate positioning of its arms.

9. A general framework for the control of smart base isolated structures in the form of a benchmark problem has been developed with a broad set of carefully chosen performance indices. A variety of smart base isolated buildings with varying levels of complexity have been analytically formulated for algorithm implementation.

9.1 Conclusions

The main conclusions in the dissertation can be summarized as follows:

1. $H_2$ and $H_\infty$ frequency domain methods are well suited for the control of smart base isolated structures; additionally, the problem formulation and controller design incorporating weighting filters to introduce excitation characteristics in the frequency domain is both powerful and flexible. In this thesis it is shown that the performance of the controller in reducing the response of the base isolated
structure is highly dependent on the choice of input excitation weighting filters.

2. The control algorithms developed in this thesis are inherently smooth in nature through a combination of novel device characteristics and introduction of low-pass output filters (at the controller output). Such controllers do not result in bang-bang type behavior during switching. It is shown that such smooth algorithms eliminate many disadvantages associated with rapid switching devices such as increased floor accelerations and inter-story drifts.

3. The new $H_\infty$ semiactive control algorithm is shown to be effective in reducing the response of smart base isolated buildings in near-fault earthquakes. The performance of this controller is enhanced by a good choice of input weighting filters that sufficiently characterizes the energy of the excitation at lower frequencies close to the fundamental frequency of the base isolated structure.

4. It is shown that the novel $H_\infty$ semiactive controller offers superior performance when compared to the active $H_\infty$ controller for both variable damping and variable friction systems.

5. The new semiactive devices developed in this work are capable of varying the level of stiffness, damping, or friction smoothly to achieve response reductions in smart base isolated buildings subjected to a variety of earthquakes. It is shown that these novel semiactive devices, through their inherent smooth nature, can
provide attractive and viable alternatives to existing semiactive devices currently being used in structural control.

6. A novel time-frequency control algorithm, that uses the input excitation time-frequency characteristics directly, has been developed and shown to be effective in response reductions in smart base isolated buildings. The use of time-frequency methods to achieve smooth stiffness variation in control of smart base isolated buildings shows promise in variable stiffness isolation systems.

9.2 Recommendations for Future Study

Some recommendations for future work which are related to the current work are:

1. Develop new nonlinear active and semiactive control algorithms for base isolated structures.

2. The weighting filters used in the current study have fixed frequency characteristics. The performance of the semiactive control algorithms based on optimal control methods can be investigated when these stationary filters are replaced with time-dependent frequency filters.

3. In the current study, the use of STFT has been investigated on variable stiffness isolation systems. The use of other time-frequency methods such as wavelets could be studied on semiactive control of smart base isolated buildings. The
time-frequency content of both input excitation and structural responses could
be used as feedback to make control decisions.

4. Studies on hybrid systems, i.e., a combination of SAIVS and SAIVD or SAIVS
and SAIVF, can be conducted to combine the advantages of variable damping,
variable friction, and variable stiffness systems.

5. Control algorithms cannot be applied to real structures without experimentation
on scaled models through shake table tests. Though some experimental work
has been undertaken to verify analytical models of proposed new devices, a
thorough experimentation of the algorithms developed in this dissertation would
be of significant value.
Appendix A
Solution Procedures

The following is the procedure to compute the $H_2$ and $H_\infty$ controllers (Doyle et al., 1989):

A.1 Computation of the $H_2$ Controller
The following procedure is adopted for the computation of the $H_2$ controller:

1. Define the Hamiltonian matrices

$$H_2 = \begin{pmatrix} A & -B_2 B_2^T \\ -C_1^T C_1 & -A^T \end{pmatrix}$$

and

$$J_2 = \begin{pmatrix} A^T & -C_2^T C_2 \\ -B_1 B_1^T & -A \end{pmatrix}$$

2. $X_2$ and $Y_2$ are given by the riccati solutions, $X_2 = \text{Ric}(H_2)$ and $Y_2 = \text{Ric}(J_2)$.

3. Calculate $F_2 = -B_2^T X_2$ and $L_2 = -Y_2 C_2^T$.

4. Calculate $\hat{A}_2 = A + B_2 F_2 + L_2 C_2$.

5. The optimal controller is given by

$$K(s) = \begin{pmatrix} A & -L_2 \\ F_2 & 0 \end{pmatrix}$$

(A.1)

The $H_2$ norm of the closed-loop transfer function, $G_{zw}$ is

$$\|G_{zw}\|_2 = \sqrt{\|G_c B_1\|_2^2 + \|F_2 G_f\|_2^2} = \sqrt{\|G_c L_2\|_2^2 + \|C_1 G_f\|_2^2}$$

(A.2)

where,

$$G_c(s) = \begin{pmatrix} A_{F_2} & I \\ C_1 F_2 & 0 \end{pmatrix}$$

$$G_f(s) = \begin{pmatrix} A_{L_2} & B_{1L_2} \\ I & 0 \end{pmatrix}$$
and
\begin{align*}
A_{F_2} &= A + B_2 F_2 \\
A_{L_2} &= A + L_2 C_2 \\
C_{1F_2} &= C_1 + D_{12} F_2 \\
B_{1L_2} &= B_1 + L_2 D_{21}
\end{align*}

### A.2 Computation of the $H_{\infty}$ Controller

1. Pick an arbitrarily large value of $\gamma$. As a starting value of $\gamma$, the $H_2$ problem may be solved and the value of the peak in the resulting closed loop transfer function may be used.

2. Define the Hamiltonian matrices
\[ H_\infty = \begin{pmatrix}
A & B_1 B_1^T - B_2 B_2^T \\
-C_1^T C_1 & \frac{1}{\gamma^2} - A^T
\end{pmatrix} \]
and
\[ J_\infty = \begin{pmatrix}
A^T & C_1^T C_1 - C_2^T C_2 \\
-B_1 B_1^T & -A
\end{pmatrix} \]

3. An admissible controller such that the closed-loop transfer function $\|G_{zw}\|_\infty < \gamma$ exists only if the following three conditions are satisfied:
   
   (a) $H_\infty \in \text{dom}(\text{Ric})$ and $X_\infty = \text{Ric}(H_\infty) \geq 0$
   
   (b) $J_\infty \in \text{dom}(\text{Ric})$ and $Y_\infty = \text{Ric}(J_\infty) \geq 0$
   
   (c) $\rho(X_\infty, Y_\infty) < \gamma^2$

where, $\rho(A)$ denotes the spectral radius of matrix, $A$, which is the largest eigenvalue of $A$. If these conditions are satisfied, then
\begin{align*}
F_\infty &= -B_2^T X_\infty \\
L_\infty &= -Y_\infty C_2^T \\
Z_\infty &= (I - Y_\infty X_\infty / \gamma^2)^{-1}
\end{align*}

4. One of the many $H_\infty$ controllers is then
\[ K_{\text{sub}}(s) = \begin{pmatrix}
\hat{A}_\infty & -Z_\infty L_\infty \\
F_\infty & 0
\end{pmatrix} \quad \text{(A.3)} \]

where, $\hat{A}_\infty = A + B_1 B_1^T X_\infty / \gamma^2 + B_2 F_\infty + Z_\infty L_\infty C_2$
5. The variable, $\gamma$ is reduced and the steps 2-4 are repeated. This procedure is repeated until when $\gamma$ is reduced further, the conditions in step 3 are no longer satisfied. The admissible controller corresponds to the lowest value of $\gamma$ for which all the conditions in step 3 are satisfied.
References


