RICE UNIVERSITY

Risk Adjusted Rate of Return: Directional Distance Function Approach

by

Ick Jin

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE

Doctor of Philosophy

APPROVED, THESIS COMMITTEE:

Robin C. Sickles, Chair
Professor of Economics

Peter Mieszkowski
Professor of Economics

F. Albert Wang, Assistant Professor
Jesse H. Jones Graduate School of Management

HOUSTON, TEXAS

AUGUST, 2003
Copyright

Ick Jin

2003
Abstract

Risk Adjusted Rate of Return: Directional Distance Function Approach

by

Ick Jin

In this dissertation, the risk adjusted rate of return (RAROR) that utilizes the directional distance function (DDF) approaches is developed to integrate conventional RAROR in a consistent manner. The sensitivity and the probabilistic analysis for DDF-RAROR are also illustrated. The DDF-RAROR is used to evaluate security performance of media stocks (1997-2001). Conglomeration in media industry has attracted public concern for a century. The results indicate that stock investors prefer conglomerate stocks, and this preference is explained by the market sentiment rather than by the underlying business prospect. This observation is confirmed through both nonparametric ranking test and nonparametric regression technique. Especially, the underlying return on equity (ROE) significantly influences the corresponding security performance.
Acknowledgments

God, thank you for your guidance. Thank you also for your mysterious power to overcome any circumstances in my life. I praise you, Father, for in midst of even the most difficult times in my life you have always been with me. Even when I didn't feel your presence, even when I felt alone and oppressed, you never left me. When I look back at the past, I see your hand upon me. As I face tough times now and in the future, help me to trust you and to focus not on the darkness of the path, but on the light that your word brings. Hold me tight, Father. In Jesus' name, Amen.

I owe much to my advisor, Professor Robin Sickles, for his guidance, patience and faith in me during the past four years. Working with him has been a unique experience and in the process I have grasped the necessary econometric tools. I benefited greatly from discussions with Professor Peter Mieszkowski, and thank for his instructive comments. I would like to thank Professor F. Albert Wang for providing me with a sound understanding of finance and his numerous insightful comments.

I am much obliged to my friends at Rice for their moral support. Teaching at Rice has been a rewarding experience, and I hope my Econ 211 students learnt from and enjoyed these classes as much as I did teaching them. A special thanks to Margie and Liz for helping in numerous ways that made dealing with the Rice bureaucracy a somewhat pleasant experience.

Finally, I would like to thank my family (parents, brothers, and sisters), for their support, believing in me, and more importantly for putting up with my doings.
Abstract ii
Acknowledgments iii
List of Figures vi
List of Tables vii
Preface viii

1. RAROR and Issue .......................................................................................... 1
1.1 RAROR .......................................................................................................... 3
1.1.1 Conventional RAROR .............................................................................. 4
1.1.2 DPEI ......................................................................................................... 9
1.2 Media Conglomeration ................................................................................ 11
1.3 Data ............................................................................................................. 14
1.4 Concluding Remark .................................................................................... 16

2. DDF-RAROR .................................................................................................. 17
2.1 Definition and Notation ............................................................................. 18
2.2 Application .................................................................................................. 26
2.2.1 Normality Test ....................................................................................... 28
2.2.2 How different is the ranking based on $\tau_1$ that $\tau_2$? ......................... 29
2.2.3 Do two groups share the same ranking distribution? ......................... 31
2.2.4 Do stock investors prefer media conglomerates? .................................. 33
Figures

[Figure 1] Directional Distance ................................................................. 21
[Figure 2] Impact of Diversification on DDF-RAROR .................................. 40
[Figure 3] Considering Market Price of Risk .............................................. 43
[Figure 4] Efficient Frontier of Year 1999 .................................................. 53
[Figure 5] Impact of Diversification ............................................................. 54
[Figure 6] Impact of Price of Risk ............................................................... 56
[Figure 7] Impact of ROE ........................................................................ 58
[Figure 8] Mean of $\eta$ ............................................................................. 76
[Figure 9] Comparison among methods ...................................................... 77
[Figure 10] Significance Interval of $\eta$ ......................................................... 79
[Figure 11] Effect of Conditional Sampling ................................................ 80
Tables

[Table 1] Conventional RAROR ................................................................. 5
[Table 2] Normality Test ........................................................................... 29
[Table 3] Coefficient of Rank Correlation .................................................. 31
[Table 4] Kruskal-Wallis Test ..................................................................... 32
[Table 5] Wilcoxon Rank-Sum Test ............................................................ 34
[Table 6] Specifications of DDF Estimation ................................................ 48
[Table 7] Mean of DDF-RAROR ................................................................. 52
[Table 8] Mean of $\theta$ ............................................................................. 53
[Table 9] Interval of $S$ ............................................................................ 55
[Table 10] Mean of $\lambda$ ......................................................................... 55
[Table 11] Interval of $T$ ........................................................................... 57
[Table 12] Mean of $\eta$ ............................................................................ 57
[Table 13] Interval of ROE ....................................................................... 59
[Table 14] Number of Significant $\eta$ ......................................................... 80
[Table 15] Number of Rejecting Normality ................................................ 82
Preface

The main goal of this dissertation is to develop an alternative risk adjusted rate of return (RAROR) that utilizes the directional distance function (DDF) approach. The DDF-RAROR provides a consistent underpinning to integrate various conventional RAROR. In this study, we have a finite number of securities and select some of them as winners. To be winners, securities must provide more returns even after its return is adjusted for corresponding risk. Thus RAROR is the baseline of this study. Relevant questions might be what is the best way to adjust the return for the risk, and how we should adjust the return for other factors more than the risk once investors are willing to do so. We propose the DDF-RAROR to answer these questions.

In the literature, there exist various approaches -- Sharpe Index, Treynor Index, Jensen's Alpha, Appraisal Ratio, M-square Index, and DPEI (Data Envelopment Analysis Portfolio Efficiency Index), according to evaluation criteria. These approaches may be categorized into three families. The first family is based on moments of return distribution. For instance, the Sharpe Index uses the first two moments of return distribution. As an extension, Arditti (1971) suggests that the third moment should also be considered when securities are evaluated. This argument is supported by the empirical results that some investors prefer securities with positive skewness. The second family adopts a benchmark portfolio against which systematic risk parameter beta and risk premium are calculated, following capital asset pricing model (CAPM). The market index like S&P500 is a popular proxy for the benchmark. There exist a couple of
variations like Treynor Index and Jensen’s Alpha, depending on how to take into account the systematic risk. The third family pays attention to the whole feature of return distribution, rather than moments or the linear formulation between individual securities and the benchmark. When two alternatives are compared to each other, stochastic dominance rule is a useful evaluation criterion.

Even though each of families has its own appeal and is intuitively straightforward, it also suffers long-standing shortcomings. The moment based measures often fail to provide sufficient information about abnormal patterns of return distribution like asymmetry, fat tail, or time-varying moments. On the other hand, CAPM based measures are exposed to the benchmark error, i.e., a ranking reversion is likely to occur whenever a different benchmark is employed. Normality assumption about return distribution and the linear formation between individual securities and the benchmark are also problematic. Last, stochastic dominance approach is nonparametric in nature. Thus, it is relatively flexible, but it is restricted to a pair-wise comparison. In addition, all of three families do not take into account other factors like transaction cost more than the risk.

The objective of this study is to suggest an alternative RAROR to satisfy desirable features. First, we want the new measure to have high capacity in order to easily combine information from higher order moments, underlying business prospects, or time-varying parameters. Next, it is desirable to maintain the nonparametric feature to avoid the benchmark error. We would like also to drop normality assumption about return distribution and the linear formation between individual securities and the benchmark. Last, the new measure is supposed to apply to peer-group comparison or trend analysis and pair-wise comparison as well.
To achieve this objective, this dissertation is organized in the following manner. We begin with introducing empirical issues related to media conglomeration, and with describing data to be used in following chapters for empirical study. Chapter 1 introduces the DDF-RAROR as an alternative evaluation scheme, and employs it for peer group comparison in the context of relative value analysis. Chapter 2 performs a sensitivity analysis on the DDF-RAROR, in order to show the high capacity of DDF-RAROR. Fundamental factors – diversification, price of risk, and underlying Return on Equity (ROE) – are incorporated in security evaluation, and their impacts on evaluation score are investigated. Chapter 3 extends the DDF-RAROR framework into a probabilistic formation. While the statistical inference on DDF-RAROR is illustrated, its nonparametric feature is emphasized.

The new evaluation scheme, DDF-RAROR, is applied to empirical tests to verify the conjecture that investors pay extra premium for conglomerate stocks. The results indicate that stock investors prefer conglomerate stocks. Also, this preference is not explained by the superior performance in the underlying business prospect, but by the favorable market sentiment. This observation is confirmed through both nonparametric ranking test and nonparametric regression technique. Especially, the result that underlying return on equity (ROE) significantly influences the corresponding security performance bridges between fundamental analysis and technical analysis, in security evaluation.
Chapter 1

RAROR and Issue

For security evaluation, academic studies as well as professional evaluators have traditionally compared the risk-adjusted rate of return (RAROR) on a security to the RAROR on a benchmark. RAROR allows securities selected as winners, when they provide the higher return after it is adjusted for their corresponding risk. Conventional RAROR using mean-variance analysis came on stage simultaneously with the capital asset pricing model (CAPM).

In finance literature, conventional RAROR has broad variations according to evaluation criteria -- Sharpe Index (Sharpe, 1966), Treynor Index (Treynor, 1966), Jensen's Alpha (Jensen, 1968), Appraisal Ratio (Treynor and Black, 1973), and M-square Index (Modigliani and Modigliani, 1997). Conventional RAROR have advantages: they assess security performance with data readily available, are easy to calculate, and appeal to those who focus on screening securities within comparison universe sharing the same characteristics.

These approaches may be categorized into three families. The first uses moments of return distribution. For instance, the Sharpe Index uses the first two moments of return distribution. As an extension, Arditti (1971) suggests that the third moment should also be considered when securities are evaluated. This argument is supported by the empirical observation that some investors prefer securities showing positive skewness. Based on CAPM, the second adopts a benchmark portfolio against which systematic risk
parameters (beta and risk premium) are calculated. The market index like S&P500 is a popular proxy for the benchmark. Treynor Index and Jensen's Alpha fall into this category. The third pays attention to the whole feature of return distribution, rather than moments or the linear formulation between individual securities and the benchmark. Sengupta (1991) and Sengupta and Park (1993) belong to this family. When two alternatives are compared to each other, stochastic dominance rule is a useful evaluation criterion.

Despite its wide use, however, the conventional RAROR suffers long-standing shortcomings. The moment based measures often fail to provide sufficient information about abnormal patterns of return distribution like asymmetry, fat tail, or time-varying moments. Next, CAPM based measures require that an optimal market portfolio, which contains all risky assets and is completely diversified, must exist on the efficient frontier. The selection of an appropriate proxy for the market index is critical for the validity of CAPM based measures.\(^1\) A benchmark error is likely to occur with an inappropriate proxy, as emphasized in Roll (1978). The implication is the possibility of ranking reversals: the rank ordering of RAROR can be significantly altered because of incorrect identification of the benchmark.\(^2\) Normality assumption about return distribution and the linear formation between individual securities and the benchmark are also problematic.

---

\(^1\) By default, most analysis of this sort relies on the S&P 500 as the market proxy. However, the S&P 500, while well diversified, certainly does not contain all risky securities. In addition to the selection of appropriate benchmark for comparison, two other major shortcomings have been highlighted in the literature: the role of market timing and the endogeneity of transaction costs. For relevant discussion, see Roll (1978), Green (1986), Lehman and Modes (1987), Elton et al. (1993) and Choi (1995).

\(^2\) This occurs primarily in the analysis of Jensen's Alpha. It has been argued that the Jensen's alpha is sensitive to the choice of the benchmark model that is employed for comparison. One would obtain different indices based on whether they employed the CAPM or the APT model as a benchmark. Relative to Jensen's alpha, Sharpe index is more robust to the specification bias of the benchmark since it uses standard deviation as a risk measure, not beta. However, the performance measurement based on Sharpe
Last, stochastic dominance approach is relatively flexible since it is nonparametric in nature. But it is restricted to a pair-wise comparison. In addition, all of three families do not take into account other factors like transaction cost more than the risk.

In this dissertation, we propose a more general full nonparametric approach that overcomes most of the drawbacks mentioned above. As for a preparatory work, we summarize relevant discussions from the finance literature here. The chapter is organized as follows. Section 1.1 examines various conventional RAROR for security evaluation. Notorious flaws of them are discussed there. In section 1.2, we introduce issues about media conglomeration that is the focus of our empirical study in following chapters. In section 1.3 describes data which will be used for empirical study. Section 1.4 offers concluding remarks.

1.1 RAROR

RAROR is now a common criterion for tracking performance of securities. The development of RAROR relies on three principles in portfolio theory, all centering on risk. First, investors avoid risk and demand a reward for engaging in risky investments. The reward is taken as a risk premium, an expected rate of return higher than that available on alternative risk-free investments. Second, it is possible to summarize and quantify investors' personal trade-off between portfolio risk and expected return. For this, the utility function may be introduced, assuming that investors can assign utility to any

---

ratio is not free from benchmark bias in the sense that the Sharpe index of a portfolio should be finally compared to the Sharpe index of the market portfolio.
risk-return pair. Third, it is incorrect to evaluate the risk of an asset separate from the portfolio of which it is a part. The proper way to measure the risk of an individual asset is to assess its impact on the volatility of the entire portfolio, since seemingly risky securities may be portfolio stabilizers and actually low-risk assets.

Under these principles, market model provides the bottom line for security evaluation, as described in Treynor and Black (1973). While conventional RAROR are introduced, it will turn out that conventional RAROR is not as straightforward to measure, as it might seem. After identifying the problems, we briefly introduce some promising developments in security evaluation.

1.1.1 Conventional RAROR

Assuming that all securities are fairly priced, and using the market model as a guideline for the rate of return on fairly priced securities, the rate of return on the security \( i \) is given by

\[
r_i = r_f + \beta_i (r_M - r_f) + e_i
\]

where \( e_i \) is the zero mean, firm-specific disturbance. For simplicity, assume that the nonsystematic components of returns, \( e_i \), are independent across securities.\(^4\) When

\(^3\) The market model assumes that there are two sources of risk, unanticipated macroeconomic events and firm-specific events. We use the return on the market portfolio as a proxy for the macroeconomic factor and assume all stocks have varying degrees of sensitivity to this macro factor. In addition, each stock’s returns are uniquely affected by firm-specific events uncorrelated across stocks and with the macro events. \(^4\) It is also assumed that the market portfolio \( M \) is the efficient portfolio. The forecast for the passive portfolio already has been made, so that the expected return on the market index, \( E(r_M) \), as well as the variance, \( \sigma_M^2 \), has been assessed. From the beta and macro forecast, \( E(r_M) - r_f \), the required rate of return of the security is determined. Bodie, et al (1999).
security analysis is ignored, the market model represents the rate of return on all securities.

Given the degree of mispricing of each security \( i \), its expected abnormal return \( \alpha_i \) is determined. Security analysts form an active portfolio of positions by mixing the security with the index portfolio. For each security \( i \), its rate of return may be written as

\[
r_i = r_f + \beta_i (r_M - r_f) + e_i + \alpha_i
\]

where \( \alpha_i \) represents the expected abnormal return, attributable to any perceived mispricing of the security. If all the \( \alpha_i \) turn out to be zero, there would be no reason to perform security evaluation. However, in general, there will be a significant number of nonzero alpha values, some positive and some negative.

Once the values of alpha, beta, and residual risk of securities, \( \alpha_i, \beta_i, \sigma^2(e_i) \), are estimated by market model, RAROR acts as the criterion for security evaluation. Professional evaluators may search securities providing the ex-post ward-to-variability ratios, through computing the realized RAROR. Or investors can use statistical methods to draw inferences from realized RAROR about ex-ante reward-to-variability ratios.\(^5\)

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Conventional RAROR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measure</td>
<td>Formula</td>
</tr>
<tr>
<td>Sharpe Ratio</td>
<td>( S_i = \frac{(r_i - r_f)}{\sigma_i} )</td>
</tr>
<tr>
<td>Treynor Ratio</td>
<td>( T_i = \frac{(r_i - r_f)}{\beta_i} )</td>
</tr>
<tr>
<td>Jensen’s Alpha</td>
<td>( \alpha_i = (r_i - r_f) + \beta_i (r_M - r_f) )</td>
</tr>
<tr>
<td>Appraisal Ratio</td>
<td>( A_i = \frac{\alpha_i}{\sigma(e_i)} )</td>
</tr>
<tr>
<td>M-Square</td>
<td>( M_i^2 = \left[ \frac{(\sigma_M}{\sigma_i}(r_i + r_f) - r_M \right] )</td>
</tr>
</tbody>
</table>
[Table 1] introduces conventional RAROR, and the following discussion illustrates circumstances in which each measure might be most relevant are examined. The Sharpe Ratio divides the average excess return over the sample period by the standard deviation of returns over the same period. It measures the reward to total volatility trade-off. When the stock represents an investor's entire investment fund, its Sharpe Ratio is compared with the Sharpe Ratio of a benchmark, either the market index or another portfolio.

Like Sharpe's, the Treynor Ratio gives excess return per unit of risk, but it uses systematic risk instead of total risk. When a stock is part of a large portfolio, one should weigh its excess return against its systematic risk rather than against total risk. Treynor Ratio is appealing in this case.

The Jensen's Alpha is the return over and above that predicted by the CAPM, given the stock's systematic risk \( \beta_i \) and the market return. Since \( \alpha_i \) is the abnormal return of the stock relative to the market portfolio, the benefit of stock \( i \) to the entire diversified fund is gauged by \( \alpha_i \). And \( \alpha_i \) gives us some indication of \( i \)'s potential contribution to the overall portfolio. Jensen's alpha is a good indicator when the stock is included in the well-diversified portfolio.

The Appraisal Ratio divides the alpha of a stock by its nonsystematic risk. It measures the abnormal return per unit of risk that in principle could be diversified away from holding a market portfolio. Holding asset \( i \) in addition to the market portfolio thus brings a reward of \( \alpha_i \) against the nonsystematic risk voluntarily incurred, \( \sigma(e_i) \). The

\[ 5 \text{ William Sharpe's assessment of mutual fund performance is the seminal work in the area of portfolio performance evaluation. Sharpe, W. (1966).} \]
Appraisal Ratio is proper when the stock is actively managed along with the passive portfolio.

Like the Sharpe Ratio,6 the $M^2$ focuses on total volatility, but it has the easy interpretation of a differential return relative to the benchmark index. A stock is adjusted so that it would have the same risk as the benchmark does. To compute $M^2$, a stock $i$ is mixed with a position in T-bills so that the adjusted stock $i'$ matches the volatility of a market index S&P 500.7 If the stock had lower standard deviation than the index, borrowing money and investing the proceeds in the portfolio would leverage it. The market index and risk-adjusted asset are compared directly as they have the same standard deviation.

In spite of their wide use, conventional measures have been subject to considerable criticism. The literature highlights three major shortcomings: the selection of appropriate benchmark for comparison, the role of market timing and the endogeneity of transaction costs. Here, we summarize these problems briefly.

First, conventional measures suffer the benchmark error. The Jensen's alpha is sensitive to the choice of the benchmark model that is employed for comparative valuation.8 One would obtain different values based on whether they employed the CAPM or the APT model as a benchmark. Relative to Jensen's alpha, The Sharpe Ratio is more robust to the specification bias of the benchmark since it uses standard deviation as a risk measure, not beta. However, the evaluation based on Sharpe Ratio is not free from

---

6 As a variant of Sharpe's measure, $M^2$ was recently introduced by Modigliani and Modigliani (1997).
7 For example, if the stock has 1.5 times the standard deviation of the index, the adjusted asset would be two-thirds invested in the managed asset and one-third invested in bills.
benchmark bias in the sense that the Sharpe Ratio of a stock should be finally compared with the Sharpe Ratio of the market portfolio.

A second, mean-variance criterion has intrinsically suffered two market-timing problems. First, many observations are required for significant results even when market condition is stable and individual stock shows consistent characteristics. Next, shifting parameters make accurate evaluation elusive when market condition actively evolves. Investors dynamically adjust the tolerable risk for a stock by reallocating funds among different vehicles. The capacity of controlling the target risk is one of required features to be a desirable evaluation scheme. But the error is introduced when the estimation of risk is based on an ad-hoc process. For example, Jensen's alpha could be biased due to market timing if a beta coefficient is improperly assumed to be constant over time.

Third, transaction costs are ignored in conventional measures when the stocks are evaluated. Fees are charged to recover the costs of financial transactions and they are beared by investors. The close relation between transaction costs and rate of return has suggested including transaction costs in performance measure. Researchers have showed a connection between transaction costs and portfolio performance. If the collection or usage of information is costly, informed investors should obtain higher returns relative to the uninformed investors. Likewise, if professional investors have superior ability, they are able to expropriate the economic rents by charging higher fees. But none of conventional measures takes into account the transaction costs or the investment expenses.

---

1.1.2 DPEI

In order to avoid Roll's criticism, it is suggested using more comprehensive index what is called a normal portfolio. As an alternative to market indexes, a normal portfolio is a specialized portfolio that is specific to the evaluated security. Sophisticated optimization programming models may be used in order to establish a normal portfolio: first, the universe of securities attained, and second an optimal weight on securities in investment opportunity set is determined. In this manner, the normal portfolio satisfies the appropriate benchmark properties.\(^\text{11}\)

On the other hand, some believe that problems previously mentioned make the benchmark approach ineffective. In the context of evaluation of the performance of mutual funds, Grinblatt and Titman (1993) propose a new performance measure that does not require the use of a benchmark.\(^\text{12}\) They measure the correlation between the return and the changes in portfolio weights and use the correlation as an indicator of management performance.

Murthi et al. (1997) propose another performance measure, DPEI (Data Envelopment Analysis Portfolio Efficiency Index), that is based on data envelopment analysis (DEA)\(^\text{13}\) and does not require the specification of a benchmark. DPEI is defined

---

\(^{11}\) The qualities possessed by a useful benchmark are unambiguous, investable, measurable, appropriate, reflective of current investment opinions, and specified in advance. See Bailey, Richards, and Tierney (1990).

\(^{12}\) They measure the correlation between the return and the changes in portfolio weights and use the correlation as an indicator of management performance. They apply their method in the context of evaluation of the performance of mutual funds.

\(^{13}\) DEA is a non-parametric analysis technique that was proposed by Charnes et al. (1978) to measure the performance of educational institutions. DEA is a linear programming formulation that defines a
as the ratio of the excess return to the level of risk and transaction cost. That is, DPEI is computed as

\[
\max \quad DPEI = \frac{r_0}{\sum_{i=1}^{m} w_i x_{i0} + v\sigma_0}
\]

s.t. \[\frac{r_j}{\sum_{i=1}^{m} w_i x_{ij} + v\sigma_j} \leq 1 \quad j = 1, \ldots, n\]

It represents the surplus return excess to market return after controlling for the risk level of investment and the expenses incurred in conducting the transactions.\(^{14}\)

DPEI has some advantages over conventional RAROR. First, the DPEI is a non-parametric analysis that does not require any theoretical models as measurement benchmarks. Instead DPEI measures how well a stock performs relative to the best set of stocks within comparison universe. Second, it can deal with the endogeneity of transaction costs by incorporating the transaction costs into index. And it is so flexible as to accommodate multiple input simultaneously. If manager’s skills were responsible for performance, their contribution can also be evaluated using the DPEI. Third, it is possible to discuss relative importance among the inputs. It can return the marginal contribution of each input in affecting returns. This makes it possible to discuss optimal resource allocation to generate return and the implications for management.

The DPEI extends Sharpe Ratio by incorporating transaction costs into the denominator. Thus, it represents the surplus return excess to market return after controlling for the risk level, and for the transaction expenses as well. It is also

---

\(^{14}\) An interesting result obtained by them indicates that the mutual funds are all approximately mean variance efficient. This result is consistent with the well known mean variance theory of Markowitz, and
interpreted as an efficiency measure indicating how well the investors utilize the resources to obtain the maximum rate of return.\textsuperscript{15}

1.2 Media Conglomeration

Over last two decades, media companies have integrated vertically by performing different stages of production, distribution or exhibition in-house, or integrated horizontally by moving into multiple forms of media such as film, publishing, radio, etc. Because of the integration, media companies in fields that were previously separate are now required to compete with each other.\textsuperscript{16}

Integrating holdings across various media segments, major media companies have become conglomerates, promoting their products across multiple media, and marketing their wares worldwide at varying degrees. Media conglomeration is a business strategy to pursue synergy or economies of scope under drastically evolving environment, as argued in Croteau et al. (2001).\textsuperscript{17} To operate effectively in an integrated environment, media

\textsuperscript{15} A main result obtained by them indicates that the mutual funds are all approximately mean variance efficient. This result is consistent with the well known mean variance theory of Markowitz, and Sharpe-Lintner in which market portfolios are efficient in the sense that they have the maximum expected return for a given level of variance.

\textsuperscript{16} On the delivery side, telephone companies now offer Internet access as well, while cable companies enter the telephone or Internet business. On the content side, companies that had traditionally been focused in one medium now branch out to work in multiple media such as films, television, print, Internet, etc.

\textsuperscript{17} The media conglomeration may be a natural outcome as an industry matures. Young startups or older, under-performing firms are consolidated into mature but innovative companies. However, the conglomerates strategically employ cross promotion, branding, specialized targeting etc., in order to take advantage of unique characteristics of media product, technological progress, and policy change, Croteau et al. (2001).
companies should draw on the large capital resources, which are available only to major conglomerates.

The media conglomeration has been accelerated by deregulation: the expiration of the financial interest and syndication (fin-syn) rules of 1995 and the Telecommunication Act of 1996 by the Federal Communication Committee (FCC). Theoretically, media integration increases competition, and provides consumers with a multiplicity of choices at a lower average cost. For this reason, FCC has allowed consolidations between major companies: AOL-Time Warner, Viacom-CBS, and Disney-ABC.

As the conglomeration proceeds, however, the ownership of mainstream media has become increasingly concentrated. According to Bagdikian (2000), just 6 conglomerates have a forceful grip on mass media and are now supplying most of the media fare. The concentration of ownership raises issues regarding monopoly power and reminds us of market efficiency concern. According to traditional antitrust criteria, recent mergers and acquisitions should have been rejected because those consolidations might substantially increase the market concentration in terms of market shares.

Unfortunately, it is difficult to investigate the implication of media conglomeration based on the traditional criteria of monopolies. Recent mergers and acquisitions have often occurred across different forms of media, and thus have often brought together companies that were not been direct competitors in the past. As argued in Albarran and

---


19 Also, some scholars have argued that conglomeration leads to a decline in diversity of expression and homogeneous content products. The media conglomerates, they criticize, seem untouchable and therefore few people demand effective anti-trust efforts against the monopoly. For example, it is unlikely that we will see a discussion of media antitrust issues on national television, since the conglomerates own all of major TV news sources, Croteau, et al. (2001).
Dimmick (1996), there is no significant evidence that recent horizontal integrations across media have increased market concentration, when the traditional criteria such as the concentration ratio (CR) or the Herfindahl-Hirschman Index (HHI) are used.\(^{20}\) Moreover, it is difficult to show that rivalry could suffer where none exists, as with a merger between companies that have never competed against each other.

In order to understand the significance of conglomeration, we need to value a media conglomeration. Since it is often intractable to quantify the value of synergy or economies of scope resulting from conglomeration, we investigate publicly traded securities of media companies instead of companies themselves.\(^{21}\) The stock market is assumed to be the final arbiter of how the conglomeration affects the stock prices of media companies, and of whether stock investors favor the media conglomeration or not. Hence, the performance of media stocks assess intangible assets like the synergy or economies of scope generated from conglomeration, which cannot be quantified by available business prospects data.\(^{22}\)

---

\(^{20}\) Albarran and Dimmick (1996) point out that CR or HHI, which originally were developed to gauge the within segment concentration, does not properly measure inter-segments concentration. There are questions that the focus on market shares effectively obscures. Can the media conglomeration be justified because of the technological change and the growth in the number of media outlets? Could it be allowed for just a few conglomerates to exploit the blurring of boundaries among media? Should the deregulation be called for, even though various outlets are linked to a growing concentration in ownership? This is one of reasons that policy-makers seem unwilling to examine the significance of media conglomeration. Croteau, et al. (2001).

\(^{21}\) The value of conglomeration can be computed by converting the expected enhancement of revenues from the conglomeration into present value. Or it is obtained by gauging the synergy after consolidating assets and liabilities of companies involved in the conglomeration. However, it is usually intractable to quantify the value of conglomeration in this way, since the synergy or economies of scope is relevant to the effects of changing management and restructuring combined firms. In particular, it is so when the media conglomeration has proceeded across multiple media in an unprecedented way.

\(^{22}\) Stock investors collect information regarding business prospects from accounting data that reveal the fundamental value of conglomerates. Then they assess intangible assets, and combine unobservable them with available quantitative data. The synergy or economies of scope, which is considered being generated from the conglomeration but is not directly observed, belongs to the qualitative aspect. The residual, after netting out the part explained by observable variables from the overall performance, represents the part of performance that is able to attribute to investors' sentiment.
Although the popularity of conglomeration is obvious, its effect on stock market still remains to be analyzed. The goal of this empirical study is to evaluate the impact of media conglomeration on stock performance. We facilitate the DDF-RAROR of media stocks to address the response of stock market to the media conglomeration.

1.3 Data

We have collected data on 100 media companies included in Veronis Suhler Stevenson (VSS)'s Media 100 index. The 100 companies included in the VSS Media 100 were chosen by senior professionals at VSS and are intended to be representative of the media and communications segments as defined by VSS. The companies generate a majority of their revenues in North America and are publicly listed on U.S. stock exchanges.

Our study concentrates on three types of firms; conglomerates, media companies, and advertising/marketing companies. Following Ben Bagdikian (2002), six media giants - AOL Time Warner, Viacom, Disney, News Corporation, General Electronics, Bertelsmann - are classified as conglomerates. Included companies are those that fall into the following segments of VSS classification: Television, Cable and Satellite Broadcasting, Entertainment, Radio Broadcasting, Consumer Internet, Newspaper

23 The selection of companies used in the VSS Media 100 is based on weighted criteria whereby each communications segment is represented by a percentage of companies that, in VSS's assessment, reflect the segment's share of the communications industry. The weighted criteria were determined by VSS proprietary methodology that factors in spending data from the VSS Communication Industry Forecast and revenue data from the VSS Communications Industry Report. The VSS Media 100 may be adjusted as necessary without further notice to account for future developments including, but not limited to, mergers, acquisitions, bankruptcies, or other market developments, and will be reviewed periodically.

Empirical tests are performed using financial data for the period 1997-2001. This time horizon is chosen for two reasons. First, the expiration of the financial interest and syndication (fin-syn) rules of 1995 and the revision of Telecommunication Act of 1996 by the Federal Communication Committee (FCC) had a big impact on media industry in terms of ownership restrictions and business practice limits. It seems that there was a structural break at that time even though media conglomeration has much longer history. Second, 5 years period is consistent with conventional time horizon in financial analysis. Security evaluation history in the industry conventionally utilizes 5 years business prospects and price. Historical stock price data are collected from Yahoo! Finance site. In order to combine underlying business prospects with the security evaluation, typical financial ratio, Return on Equity (ROE), for media companies are collected from Wharton Research Data Services (WRDS).

To be included in the final sample, each observation must satisfy the following criteria. (I) The firm belongs to VSS Media 100 index. (II) The stock of a company is listed on NYSE, AMEX or NASDAQ and it has an offering date before January 1, 1997. (III) The company's ROE is accessible at WRDS. (IV) Computing the DDF-RAROR is feasible. The final sample contains overall 300 company-year observations: 25 for conglomerates $G_1$, 275 for non-conglomerates $G_2$ (165 for media companies, and 110 for advertising/marketing companies).
1.4 Concluding Remark

Each of conventional RAROR does not necessarily provide consistent assessments of performance, since its risk parameter used differs substantially. In addition, its numerical value is not easy to interpret from time to time.\textsuperscript{24} This is one of reasons why the ranking of stocks within the comparison universe, rather than ratio itself, becomes the focus of empirical study.

In this dissertation, we propose a more general full nonparametric approach that overcomes most of the drawbacks mentioned above, relying on the directional distance function (DDF) developed in Chung et al. (1997).\textsuperscript{25} The evaluation score based on DDF estimates is interpreted as a comprehensive RAROR and it is called the DDF-RAROR. The DDF-RAROR measures the distance in risk-return space, how far securities are away from the efficient frontier. Whenever it is necessary, the risk on a security is considered as an undesirable output, the return on a security as a desirable output, and underlying business prospects as inputs, following conventions in productivity literature.

\textsuperscript{24} For instance, comparing the ratios for assets M and A, suppose that $S_A = 0.69$ and $S_M = 0.73$. This suggests that asset A under performed the market index. But it is not clear what a difference of 0.04 in the Sharpe ratio means economically.

\textsuperscript{25} The DDF was originally designed to construct the Malmquist-Luenberger productivity index in Chung, Fare, and Grosskopf (1997). The DDF is an extension of the output distance function to explain production technology, which was first defined in Shephard, R. W. (1970).
Chapter 2

DDF-RAROR

Sengupta (1991) and Sengupta and Park (1993) provide links between mean-variance theory and nonparametric estimation of frontiers in Data Envelopment Analysis (DEA). The investment opportunity set of attainable securities, in mean-variance theory by Markowitz (1959), is naturally convex and the efficiency frontier is monotone concave. The investment opportunity set also has the free disposability and convexity. The estimation model for an efficient frontier needs to be nonparametric since no particular functional form is imposed.

In this chapter, we develop the DDF-RAROR as an alternative evaluation score, and show that the DDF-RAROR maintains the nonparametric feature so as to avoid the benchmark error. It is also free from normality assumption about return distribution and the linear formation between individual securities and the benchmark. The new measure applies to peer-group comparison in the context of relative value analysis. The relative analysis on media stocks proceeds as follows. First, media stocks are grouped into a comparison universe. Second, the DDF-RAROR within the universe is computed and ordered. Third, each stock receives a percentile ranking depending on relative DDF-RAROR within the comparison universe.26 Last, we perform a couple of statistical tests using the ranking.

---

26 For example, the stock with the ninth-best performance in a universe of 100 stocks would be the 90th percentile stock: its performance was better than 90% of all competing stocks over the evaluation period.
The chapter is organized as follows. Section 1.1 develops the definition and notation for DDF-RAROR. In section 1.2, the DDF-RAROR is applied to evaluate media stocks. Nonparametric tests are employed to answer three research questions about media conglomeration. Section 1.3 offers concluding remarks.

\section{2.1 Definition and Notation}

We consider an investment opportunity where a security is characterized by a pair of return and risk, and a corresponding set of underlying business prospects: the pair of return $r \in \mathbb{R}^1_+$ and risk $\sigma \in \mathbb{R}^1_+$ can be supported by the business prospects $x \in \mathbb{R}^n_+$ - i.e. earnings, financial ratios or news about them. In this framework, the investment opportunity set is the set of financially feasible combinations of $(x, \sigma, r)$. It is defined as\textsuperscript{27}

$$F = \{(\sigma, r) \mid (x, \sigma, r) \in P\}$$

where $P = \{(x, \sigma, r) \in \mathbb{R}^{2+n}_+ \mid x \text{ can support } (\sigma, r)\}$.

A couple of assumptions are imposed on the investment opportunity set. First, the free disposability of risk and return is assumed, expressed as

$$(\sigma, r) \in F \text{ and } \sigma' \geq \sigma, r' \leq r \Rightarrow (\sigma', r') \in F$$

When a security is financially feasible, other securities having higher risk and lower return are also financially feasible. Second, it is imposed that the reduction of risk can be

\textsuperscript{27} It is obvious that the investment opportunity set depends on the underlying business prospects, and so it is rigorous to denote the investment opportunity set as $F(x)$. However, we use $F$ instead of $F(x)$ for notational brevity. It should not bring into any confusion.
attained by the simultaneous reduction of return when underlying business prospects are
given as fixed.\textsuperscript{28} That is,

\[(\sigma, r) \in F \quad \text{and} \quad 0 \leq k \leq 1 \Rightarrow (k\sigma, kr) \in F\]

It implies that the reduction of risk is costly. Third, we also assume that the return in
investment opportunity set cannot exceed the risk-free rate without taking the positive
risk.\textsuperscript{29} That is,

\[(\sigma, r) \in F \quad \text{and} \quad \sigma = 0 \Rightarrow r = r_f\]

Fourth, the convexity of investment opportunity set \(F\) is also employed.\textsuperscript{30} When two
securities are financially feasible, the linear combination of them is also financially
feasible.

Since the performance in RAROR terms is of concern, distances between a security
and the efficient frontier, which is the boundary of investment opportunity set \(F\), are of
interest. In order to measure the distances, we employ the Directional Distance Function
(DDF) approach developed in Chung, Fare, and Grosskopf (1997). The directional
distance for a security at \((\sigma, r)\) is defined as:

\[\bar{d} = \sup \{d \mid (\sigma + dg_{\sigma}, r + dg_r) \in F\}\]

where \(g_{\sigma}\) (and \(g_r\)) are the element for risk (and return) of the direction vector \(g\),
respectively. When \((\sigma, r)\) is inside \(F\), \(\bar{d} \leq 1\) means the proportionate reduction of risk
and the simultaneous enhancement of return that a security at \((\sigma, r)\) should attain to be

\textsuperscript{28} It means technically the weak disposability of the pair of return and risk. If undesirable risk could be
disposed of freely, we could reduce only undesirable risk as much as it is desired without the change in return.
\textsuperscript{29} This assumption is null-jointness of desirable return and undesirables risk.
\textsuperscript{30} See e.g. Shephard (1970) for a modern formulation of the problem in the context of production.
evaluated efficient. The efficient frontier corresponds to points with \( \tilde{d} = 1 \). When the
direction is chosen as \( g = (-\sigma, r) \), the frontier is then described as the set
\(((1 - \tilde{d})\sigma, (1 + \tilde{d})r) \in F(x) \), where \(((1 - \tilde{d})\sigma, (1 + \tilde{d})r) \) is the projection of \((\sigma, r)\) on the
frontier.

The distance may vary according to the direction along which the performance of a
security is evaluated. If we are looking in the risk-minimizing direction, i.e. \( g = (-\sigma, 0) \),
the directional distance may be computed as:

\[
\tilde{d} = \inf \{ d \mid (d\sigma, r) \in F \}
\]

where \( \tilde{d} \leq 1 \) represents the proportionate reduction of risk for a security at \((\sigma, r)\) to be
evaluated efficient. Alternatively, if we look in the return-maximizing direction, i.e.
\( g = (0, r) \), the directional distance will be:

\[
\tilde{d} = \sup \{ d \mid (\sigma, dr) \in F \}
\]

where \( \tilde{d} \geq 1 \) represents the proportionate enhancement of return that a security at
\((\sigma, r)\) should attain to be evaluated efficient. [Figure 1] compares special cases to
highlight insights behind directional distance scheme.

The inverse of directional distance is referred to the risk-adjusted rate of return based
on the directional distance function (DDF-RAROR). Hence, the DDF-RAROR \( \tau \) for a
security at \((\sigma, r)\) is defined as:

\[
\tau = 1/(1 + \tilde{d})
\]
The rationale of DDF-RAROR is that there exists a trade-off between return and risk: for a security to receive higher evaluation, it should be close to the efficient frontier, and the simultaneous increase of return and decrease of risk should be attained.\footnote{Only the increase of return along with non-increase of risk is considered as a real improvement of investment opportunity set. It is consistent with the logic that the efficiency frontier is refined as the upper part from the minimum variance frontier on risk-return space.}

\textbf{Figure 1] Directional Distance}

Empirically, the investment opportunity set $F$ is unknown and so is the efficient frontier. The DDF-RAROR is therefore estimated by econometric methodology from a random sample of risk-returns $Y_n = \{(\sigma_i, r_i) | i = 1, \ldots, n\}$.\footnote{Only the increase of return along with non-increase of risk is considered as a real improvement of investment opportunity set. It is consistent with the logic that the efficiency frontier is refined as the upper part from the minimum variance frontier on risk-return space.} The nonparametric model is appealing in DDF-RAROR case since no assumption on the Data Generating Process (DGP) of DDF-RAROR is imposed.

As proposed in Deprins, Simar and Tulkens (1984), the Free Disposal Hull (FDH) of the set of the observations to estimate $F$ is

\[
\hat{F}_{FDH} = \{(\sigma, r) \in \mathbb{R}_+^2 | \sigma \geq \sigma_i, r \leq r_i, \ i = 1, \ldots, n \}
\]
The convex hull of $\hat{F}_{FDH}$ provides the DEA estimator of $F$, popularized as linear programming estimator by Charnes, Cooper and Rhodes (1978)

$$\hat{F}_{DEA} = \left\{ (\sigma, r) \in \mathbb{R}^2 \mid \sigma = \sum_{i=1}^{n} w_i \sigma_i, r \leq \sum_{i=1}^{n} w_i r_i, \text{ s.t. } \sum_{i=1}^{n} w_i = 1; w_i \geq 0, i = 1, \ldots, n \right\}$$

It is the smallest free disposal convex set covering all the data. The DDF-RAROR is then obtained by employing $\hat{F}_{DEA}$. The directional distance $\tilde{d}_i$ of security $i$ is estimated by:

$$\tilde{d}_i = \max \beta$$

s.t.

$$(1 + \beta) r_i \leq w'r$$

$$(1 - \beta) \sigma_i = w'\sigma$$

$$x_i \geq w'x$$

$$w'c = 1$$

where $r$ is a $(n \times 1)$ vector of returns, $r_i$ is a return on security $i$, $\sigma$ is a $(n \times 1)$ vector of variances, $\sigma_i$ is the variance of security $i$ and the others, $x$ is a $(n \times 1)$ vector of underlying fundamental factor, $x_i$ is the underlying fundamental factor of security $i$, $w$ is a $(n \times 1)$ weight vector, and $c$ is a $(n \times 1)$ unit vector.

The diversification, which depends on covariance structure of observed returns, implies that it is possible to create portfolios with higher expected returns without increasing risk. When the diversification is taken into account to estimate the DDF-RAROR, the investment opportunity set use in our study is

$$\hat{F}_{DIV} = \left\{ (\sigma, r) \in \mathbb{R}^2 \mid \sigma = \sum_{i=1}^{n} \sum_{j=1}^{n} w_i w_j \sigma_{ij}, r \leq \sum_{i=1}^{n} w_i r_i, \text{ s.t. } \sum_{i=1}^{n} w_i = 1; w_i \geq 0, i = 1, \ldots, n \right\}$$

32 Since the pioneering work of Farrell (1957), the literature has developed a lot of different approaches to achieve this goal. The most popular approaches are based on envelopment estimators in the spirit of Farrell approach.
The corresponding DDF-RAROR is then obtained by employing \( \hat{F} \). The DDF-RAROR \( \tilde{d}_i \) of security \( i \) can be estimated by solving the maximization problems with nonlinear constraints.

\[
\tilde{d}_i = \max \beta \\
\text{s.t.} \quad (1 + \beta)r_i \leq w'r \\
\quad (1 - \beta)\sigma_i = w'\Sigma w \\
\quad x_i \geq w'x \\
\quad w'c = 1
\]

where \( r \) is a \((n \times 1)\) vector of returns, \( \Sigma \) is a \((n \times n)\) variance-covariance matrix, \( \sigma_i \) is the variance of security \( i \), and \( w \) is a \((n \times 1)\) weight vector, \( x \) is a \((n \times 1)\) vector of underlying fundamental factor, \( x_i \) is the underlying fundamental factor of security \( i \), and \( c \) is a \((n \times 1)\) unit vector.

The principal idea behind this optimization problem is that, for each security in the investment opportunity set, the distance between a security itself and the efficient frontier represents the relative value of the security, compared to the alternative securities within the investment opportunity set. Each constraint of the above optimization problem has some economic intuition behind it.

Terms on right hand side of the first and the second constraints are the return and variance of the normal portfolio with weights \( w_i \) in security \( i \). It is the formal presentation of diversification principles, thereby estimating the efficient frontier. Without considering the diversification principles, the second constraint can be replaced with

\[
(1 - \beta)\sigma_i = w'\sigma
\]
Then, the optimization problem degenerates into the usual linear programming model that is employed in conventional productivity analysis. In this case, the DDF-RAROR is based on the distance between the security and the straight Capital Allocation Line (CAL), instead of the distance between the security and concave efficient frontier. Hence, as long as the securities in the investment opportunity set are less than perfectly correlated, the DDF-RAROR $\tau_i$ with diversification will be lower than that the DDF-RAROR $\tau_i$ without diversification. It is because portfolios of less than perfectly correlated securities always offer better risk-return pairs than the individual component securities on their own. The lower is the correlation among securities, the lower $\tau$ or the bigger achievable benefit from diversification is.

In addition to pairs of risk and return, the underlying business prospects $x$, which summarize the performance of underlying company, may be fed into the optimization program. It reflects the fact that investors use underlying business prospects, like dividends and earnings prospects, as inputs into security valuation. A company's business prospects are readily available from its financial report, and used to estimate the intrinsic value of its security.\textsuperscript{33} Very often, analysts use financial ratios to explore the sources of a firm's profitability and evaluate the quality of its earnings in a systematic fashion.\textsuperscript{34}

\textsuperscript{33} The income statement, the balance sheet, and the statement of cash flows are the basic sources of such data. It is obvious that there exists the difference between economic and accounting earnings. Although economic earnings are more important for issues of valuation, whatever their shortcomings, accounting data still are useful in assessing the economic prospects of the firm.

\textsuperscript{34} For example, the impact of debt policy on firm's value can be examined by various financial ratios. Financial ratios also are utilized as a tool in uncovering mispriced securities. In spite of its popularity, the limitation of financial statement analysis is well known. Some of these limitations are due to differences in firms' accounting procedures. Others arise from inflation-induced distortions in accounting numbers.
Using the DDF-RAROR framework, some investors may want to investigate the security performance after netting out the differential in dividend yield across firms. In this case the input list will include a set of dividend yields and the third constraint will ensure that the DDF-RAROR gauges the rate of return after risk and dividend factors are adjusted. In general, the DDF-RAROR with extra inputs will be higher than the DDF-RAROR without them.

In this DDF-RAROR framework, the DDF-RAROR without business prospects represents the overall evaluation score for securities. On the other hand, underlying business prospects (such as assets, earnings and cash flows) reflect the profitability of underlying company. When they are incorporated into the DDF-RAROR, they are regarded as the security performance resulting from supply side. And the change in DDF-RAROR accounts for the security performance resulting from demand side.

The performance from demand side, i.e., the residual after netting out the performance from supply side from the overall performance, captures how the market responds to underlying business performance. Hence, the model that is presented in this section embraces those used by fundamental analysts for it contains information concerning the profitability of a company that the fundamental analysts use. Moreover, the market trend or measures of market conditions, which technical analysts essentially use, may be also employed in our framework.

---

35 This approach is somewhat different from the one that stock market analysts use to uncover mispriced securities. In finding mispriced securities, the residual of overall evaluation, which is not explained by fundamental variables, is just the misalignment of stock price and provides the arbitrage opportunity. As discussions of market efficiency in finance literature indicated, finding undervalued securities is hardly easy. At the same time, however, there are pieces of empirical evidence against the efficient market hypothesis enough that the search for such securities should not be dismissed out of hand. It is the ongoing search for mispriced securities that maintains a nearly efficient market.
The fourth constraint means that the portfolio weights sum to one. The fifth constraint is employed because many investors are prohibited from taking short positions in securities. Thus, the constraint ruling out negative positions during the search for efficient portfolio is added. Note that it is possible for a single stock to be, in and of itself, the normal portfolio. The stock having the highest rate of return stays on the efficient frontier because, without the opportunity of short sales, the only way to obtain that rate of return is to hold the stock as the entire portfolio. The distance of this stock from the efficient frontier is zero, \( d_i = 0 \), and its DDF-RAROR is equal to one, \( \tau_i = 1 \).

### 2.2 Application

In this section, we analyze the relative value analysis for media stocks in terms of the DDF-RAROR, \( \tau \). In security markets, the relative value refers to the ranking of individual securities by sectors, structures, issuers, and issues in terms of their performance over investment horizon. The relative-value analysis refers to the methodologies employed to generate such rankings.\(^{36}\) The simplest and most popular approach is to compare RAROR among securities with similar risk characteristics. After grouping stocks according to industry specification, the DDF-RAROR of each stock within the group are ordered, and each stock receives a percentile ranking depending on relative performance with the comparative group.

\(^{36}\) Classic relative-value analysis has two forms: top-down approach, and bottom-up approach. In top-down scheme, projections related to large-scale economic developments are used to allocate funds to broadly defined corporate asset classes. According to bottom-up approach, undervalued issues are sought that are expected to outperform their peer groups. Superior selection is intended to result in out-performance of a benchmark while maintaining a neutral position in terms of sector allocation within the benchmark.
We are interested in seeking whether conglomerate stocks outperform their peer groups. For this purpose, we compute the DDF-RAROR twice: $\tau_1$ without underlying ROE, and $\tau_2$ including underlying ROE. The directional distance for $\tau_1$ is estimated as:

$$\tilde{d}_{1,j} = \max \beta$$

s.t. \( (1 + \beta)\tau_i \leq w'r \)
\( (1 - \beta)\sigma_i = w\Sigma w \)
\( w'c = 1 \)

and the second estimation problem for $\tau_2$ is:

$$\tilde{d}_{2,j} = \max \beta$$

s.t. \( (1 + \beta)\tau_i \leq w'r \)
\( (1 - \beta)\sigma_i = w\Sigma w \)
\( x_i \geq w'x \)
\( w'c = 1 \)

For these computations, the *fmincons* command of Matlab is used. From estimates of the directional distances, the DDF-RAROR is computed as:

$$\tau_{1,j} = 1/(1 + \tilde{d}_{1,j}) \quad \text{and} \quad \tau_{2,j} = 1/(1 + \tilde{d}_{2,j})$$

Then, rankings among 300 observations are generated according to these DDF-RAROR. Finally, five relevant statistical tests are performed using such rankings.

First, the normality of $\tau_1$ and $\tau_2$ is tested. As the hypothesis that the distributions of $\tau_1$ and $\tau_2$ approximate the normal distribution is rejected, the following empirical tests employ nonparametric ranking tests - coefficient of rank correlation, Kruskal-Wallis test, and Wilcoxon rank-sum test, for which the normality assumption is not required.\(^{37}\)

---

According to the DDF-RAROR ranking, the following four questions are tested.

(Q1) How different is the ranking based on $\tau_1$ that $\tau_2$? (Q2) Does the conglomerate stocks share the same ranking distribution with non-conglomerate stocks? (Q3) Do stock investors prefer conglomerate stocks?

For the hypothesis testing, as usual, the test statistic is computed and the computed figure is compared with the critical value corresponding to the assumed significance level. The test statistic is designed using the differences in two ranking that are dependent observations. The ordinal ranking of DDF-RAROR is of interest, rather than the index value itself.

2.2.1 Normality Test

We first check whether $\tau_1$ (and $\tau_2$) follow the normal distribution or not. One method that has been suggested for testing if the distribution underlying a sample is normal is to refer the statistic

$$L = n \times \left( \frac{1}{6} \hat{S}^2 + \frac{1}{24} \hat{K}^2 \right)$$

where

$$\hat{S} = n \sum z_i^3 / (n-1)(n-2),$$

$$\hat{K} = n(n+1) \sum z_i^4 / (n-1)(n-2)(n-3) - 3(n-1)^2 / (n-2)(n-3),$$

$$z_i = (x_i - \hat{\mu}) / \hat{\sigma}, \quad \hat{\mu} = \sum x_i / n,$$

and $\hat{\sigma}^2 = (\sum x_i^2 - n\hat{\mu}^2) / (n-1).$
The statistic L follows the Chi-squared distribution with 2 degrees of freedom. And \( \hat{S} \) is the Skewness\(^{38}\) and \( \hat{K} \) is the Kurtosis\(^{39}\) of IND distribution, respectively. Also, the null hypothesis is that the distribution underlying a sample is normal.

As shown in [Table 2], for both \( \tau_1 \) and \( \tau_2 \), the formal tests reject the normality of DDF-RAROR distribution. When we take into consideration underlying ROE, the non-normality becomes more obvious. This result implies that the evaluation \( \tau_2 \) resulting from the part after netting out the performance from supply side from the overall performance shows a more abnormal pattern than the evaluation \( \tau_1 \) based on the overall performance does.

<table>
<thead>
<tr>
<th>[Table 2] Normality Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_1 )</td>
</tr>
<tr>
<td>L statistics</td>
</tr>
<tr>
<td>P-value</td>
</tr>
</tbody>
</table>

2.2.2 How different is the ranking based on \( \tau_1 \) that \( \tau_2 \)?

We check whether both \( \tau_1 \) and \( \tau_2 \) rank the same company as the best or the worst.

In order to investigate the association between the ranking based on \( \tau_1 \) and that on \( \tau_2 \),

---

\(^{38}\) Skewness characterizes the degree of asymmetry of a distribution around its mean. Positive Skewness indicates a distribution with an asymmetric tail extending toward more positive values. Negative Skewness indicates a distribution with an asymmetric tail extending toward more negative values.

\(^{39}\) Kurtosis characterizes the relative peakedness or flatness of a distribution compared with the normal distribution. Positive Kurtosis indicates a relatively peaked distribution. Negative Kurtosis indicates a relatively flat distribution.
we test whether the correlation in the ranking is actually zero.\textsuperscript{40} The null and the alternate hypotheses are

\textit{H0:} The rank correlation between $\tau_1$ and $\tau_2$ is zero.

\textit{H1:} The rank correlation between $\tau_1$ and $\tau_2$ is positive.

The Spearman's coefficient of rank correlation, denoted $r_s$, provides a measure of the association between two sets of ranked data.\textsuperscript{41} The coefficient of rank correlation is computed as

$$r_s = 1 - \frac{6}{n(n^2 - 1)} \sum_{i=1}^{n} a_i^2$$

where $a_i$ is the difference between the ranks for observation $i$, and $n$ is the number of paired observations. After computing the coefficient of rank correlation, we conduct a test of significance. For a sample of 10 or more, the significance of $r_s$ is determined by computing $t$ as

$$t = r_s \sqrt{\frac{n - 2}{1 - r_s^2}}$$

where the distribution of the statistics follows the $t$ distribution with $(n - 2)$ degrees of freedom. The decision rule is to reject $H0$ if the computed value of $t$ is greater than the

\textsuperscript{40} For instance, the rank correlation coefficient of 0.726 indicates a rather strong relationship between the two sets of ranks. Is it possible that the correlation of 0.726 is due to chance and that the correlation in the populations is really 0?

\textsuperscript{41} The coefficient of correlation measures the association between two variables. Charles Spearman, a British statistician, introduced a measure of correlation for ordinal-level data. Like the coefficient of correlation, the coefficient of rank correlation can assume any value from -1.00 up to 1.00. A value of -1.00 indicates perfect negative correlation and a value of 1.00 perfect positive correlation among the ranks. A rank correlation of 0 indicates that there is no strong association among the ranks. Rank correlations of -0.84 and 0.84 indicate a strong association, but the former indicates an inverse relationship between the ranks and the latter a direct relationship.
critical value under conditions of $\alpha$ significance level, one-tailed test, and $(n - 2)$ degrees of freedom.

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>T statistics</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.649</td>
<td>14.71</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Empirical result in [Table 3] shows that there exists a significantly strong positive association between the ranking based on $r_1$ and that on $r_2$. The coefficient of rank correlation is 0.649, implying that investors would change their investment decision after incorporating the underlying ROE into security evaluation procedure. In other words, the evaluation $r_2$ resulting from the demand side performance is different from the evaluation $r_1$ based on the overall performance. We contend that investors put weights on information other than underlying ROE. For instance, good companies like conglomerates may be preferred regardless of underlying business performance.

2.2.3 Do two groups share the same ranking distribution?

We investigate whether conglomerates and non-conglomerates perform equally well. The null and the alternate hypotheses are

$H_0$: The distributions of ranking are equal among three groups.

$H_1$: The distributions of ranking are not equal for three groups.
Without the normality assumption, the Kruskal-Wallis one-way analysis of variance by ranks is appropriate. In the Kruskal-Wallis test, the performance of stocks is assumed to be independent. For example, the stock performance of conglomerates does in no way influence the performance of non-conglomerates. All stocks are combined as one universe, the stocks are ordered from low to high according to DDF-RAROR, and the ordered values are replaced by ranks, starting with 1 for the smallest value. The test statistic is

\[ H = \frac{12}{n(n+1)} \sum_{k=1}^{K} \frac{(SR_k)^2}{n_k} - 3(n+1) \]

where \( K \) is the number of groups, where \( SR_k \) is the rank sums of stocks belonging to group \( k \), \( n_k \) is the number of stocks in group \( k \), and \( n \) is the combined number of stocks for all groups.

The distribution of \( H \) statistic is very close to the Chi-square distribution with \((K - 1)\) degrees of freedom if every group contains more than 5 stocks. Therefore, we use the Chi-square in formulating the decision rule.

<table>
<thead>
<tr>
<th>[Table 4] Kruskal-Wallis Test</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>H statistics</td>
</tr>
<tr>
<td>P-value</td>
</tr>
</tbody>
</table>

\(^{42}\) Kruskal-Wallis test does not require the shape of the populations, unlike the analysis of variance (ANOVA) procedure. For ANOVA, it is assumed the populations are normally distributed and the standard deviations of those populations are equal (W. H. Kruskal and W. A. Wallis, 1952). Recall that for the analysis of variance technique to apply, we assume that: (1) the populations are normally distributed, (2) these populations have equal standard deviations, and (3) the samples are selected independently. If these assumptions are met, we use the F-distribution as the test statistic. If these assumptions cannot be met, we apply the distribution-free test by Kruskal-Wallis.
According to the computed value of $H$ and its p-value in [Table 4], the diversity across groups becomes clearer after incorporating information about fundamental variables. That is, three groups share the same ranking distribution in terms of the overall evaluation $\tau_1$, whereas the ranking distribution varies across two groups in terms of the demand side evaluation $\tau_2$. Hence, the selection of the sector becomes an important task as for stock investors who pay attention to the demand side evaluation.

2.2.4 Do stock investors prefer media conglomerates?

We now test whether conglomerates are preferred to non-conglomerates. The null and alternative hypotheses are

$H0$: The ranking distribution is the same for all companies.

$H1$: The ranking distribution is larger for conglomerates.

To determine whether rankings of two independent groups come from equal distribution, the Wilcoxon rank-sum test is employed.\(^{43}\) For purposes of the Wilcoxon rank-sum test, companies are divided into two groups, conglomerates and non-conglomerates. Their DDF-RAROR is ranked as if the observations were part of a single group. If the null hypothesis is true, then the ranks will be about evenly distributed between conglomerates and non-conglomerates, and the average of the ranks for the two groups will be about the same. That is, the low, medium, and high ranks should be equally distributed between the two groups.

\(^{43}\) The Wilcoxon rank-sum test does not require conditions that the two populations be normally distributed and have equal population variances, unlike the two-sample t-test.
Otherwise, if the alternate hypothesis is true, conglomerates will have more of the low ranks and, an overall smaller rank. Non-conglomerates will have more of the higher ranks and, therefore, a larger total. If each of groups contains at least 8 observations, the standard normal distribution is used as the test statistic. The formula is

\[
Z = \frac{W - 0.5n_1(1 + n_1 + n_2)}{\sqrt{n_1n_2(1 + n_1 + n_2)/12}}
\]

where \(n_1\) is the number of conglomerates, \(n_2\) is the number of non-conglomerates, and \(W\) is the sum of the ranks from conglomerates.

As for the Wilcoxon rank-sum test, the two groups may be numbered in either order. If non-conglomerates is identified as the group of interest, \(W\) must be the sum of the ranks from non-conglomerates. Thus, the direction of the alternate hypothesis would be changed, but the absolute value of \(Z\) remains the same.

\(H0\): The ranking distribution is the same for all companies.

\(H1\): The ranking distribution is smaller for conglomerates.

This is a one-tailed test as we check whether conglomerates perform better, or worse, than non-conglomerates.

<table>
<thead>
<tr>
<th>Table 5 Wilcoxon Rank-Sum Test</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z) statistics (\tau_1)</td>
</tr>
<tr>
<td>-------------------------------</td>
</tr>
<tr>
<td>Z statistics</td>
</tr>
<tr>
<td>P-value</td>
</tr>
</tbody>
</table>

When the underlying ROE is incorporated, investors prefer conglomerates to non-conglomerates as shown in [Table 5]. Interestingly, this preference is obscured without underlying ROE. There is no evidence that stock investors have a preference for
conglomerates in terms of the overall evaluation $\tau_1$. The biased perception of stock investors for conglomerates can be explained by the demand side evaluation $\tau_2$.

2.3 Concluding Remarks

In this chapter, the DDF-RAROR is proposed to overcome problems contained in conventional RAROR such as the Sharpe Ratio or the Jensen's Alpha. The DDF-RAROR facilitates a non-parametric analysis that does not require any theoretical models to identify the benchmark portfolio. Instead, the DDF-RAROR measures the attractiveness of a security relative to the most preferable and potentially achievable portfolio. And the distance between the efficient frontier and the security of interest presents the attractiveness of the security. The rationale behind DDF-RAROR is that these distances reflect the trade-off between risk and return in flexible ways.

In order to show the applicability of DDF-RAROR, we do empirical study on media stocks. Using the ranking according to DDF-RAROR, it is shown that stock investors positively react to media conglomeration. From the normality test, the pure investors' sentiment captured by $\tau_2$ shows a more abnormal pattern than the overall evaluation by $\tau_1$ does. The coefficient of rank correlation between $\tau_2$ and $\tau_1$ implies that good companies such as media conglomerates may be preferred regardless of business performance since investors put some weight on information other than business performance. It seems by the result of Kruskal-Wallis test that market sentiment quite varies over sectors, and so sector selection becomes an important task as for stock
investors. According to the result of Wilcoxon Rank Sum test, there exists a biased perception of stock investors for media conglomerates.

The DDF-RAROR framework is quite flexible. It may address the relation between security performance and underlying business performance. Also, the framework may decompose the overall security performance into two parts: the evaluation on underlying business performance and the evaluation on market sentiment. At the beginning, stock investors collect information about the underlying value of securities, which is usually obtained from accounting data. Then they augment their values through taking into account qualitative factors like value of conglomerations. The residual, after accounting for the part explained by fundamental variables, may capture the response of market sentiment to synergy generated from the media conglomerations. It seems the value of conglomerations, in nature, is quite sensitive to the market sentiment. We investigate these issues in the next chapter.
Chapter 3

Sensitivity Analysis on DDF-RAROR

As described in the previous chapter, the directional distance function (DDF) method can apply to calculating the risk adjusted rate of return (RAROR). The directional distance estimate in risk-return space, which measures how far a security is away from the efficient frontier, is used as an evaluation score. The DDF-RAROR is analogous to models of production in Chung, Fare, and Grosskopf (1997).\footnote{The DDF was originally designed to construct the Malmquist-Luenberger productivity index in Chung, Fare, and Grosskopf (1997). The DDF is an extension of the output distance function to explain production technology, which was first defined in Shephard, R. W. (1970).} regarding the risk on a security as an undesirable output, and the return on a security as a desirable output. Underlying business prospects may be considered as inputs.

The DDF-RAROR framework is extended to explore the reasons of different level of DDF-RAROR according to fundamental factors like underlying business prospects. Some of these fundamental factors may be priced at market, and the return on a security is not adjusted for them. But, the influence of fundamental factors on the DDF-RAROR is indeed a relevant issue related to the explanations of security return, the identification of fundamental conditions that create sentiment, and finally to the improvement of investment outcome.

Fundamental factors may contribute to define the investment opportunity set, but they may not be active in the optimization process for estimating the DDF-RAROR. A fundamental factor could be advantageous to the DDF-RAROR or damaging to the DDF-
RAROR, depending on what is the role of the fundamental factor on the DDF-RAROR. For instance, the underlying return on equity (ROE) appears to be beneficial to the DDF-RAROR.\footnote{The drawback of this approach is twofold: For relevant issues, see Banker and Morey (1986), Fare, Grosskopf, Lovell and Pasurka (1989), Fare, Grosskopf, and Lovell (1994).}

In this chapter, we propose an approach that allows the impact of fundamental factors on DDF-RAROR easily analyzed. The basic idea is to compute two sets of DDF-RAROR (ignoring a fundamental factor for one and considering it for the other), and to construct an index defined as a ratio of two DDF-RAROR. The DDF-RAROR conditioning on a fundamental factor is referred as the constrained DDF-RAROR. This adapts the case analyzed in Daraio and Simar (2003), in which environmental variables are considered in production frontier model.

We apply the constrained DDF-RAROR to a sample of media stocks over the period 1997-2001. The purpose of this analysis is to see how fundamental factors (such as the diversification, the price of risk, and the sentiment) influence DDF-RAROR. The influence of fundamental factors might be different across categorical groups like conglomerate. The impact of fundamental factors on DDF-RAROR for conglomerate stocks is compared with that for other stocks.\footnote{An alternative approach may be based on a two-stage approach. Here the estimated DDF-RAROR are regressed, in an appropriated limited dependent variable parametric regression model like truncated normal regression on the fundamental factors. Three-stage and four-stage analysis might also be proposed as extension of the two-stage approach. For relevant issues, see Fried, Schmidt, and Yaisawarng (1999), Fried, Lovell, Schmidt, and Yaisawarng (2002).}

This chapter is organized as follows. Section 2.1 introduces basic concepts for constrained DDF-RAROR. We propose the DDF-RAROR constrained by fundamental factors: the diversification, the price of risk, and the underlying ROE. An empirical
application to media stocks is given in section 2.2. And conclusions about the sensitivity analysis on DDF-RAROR are discussed in section 2.3.

3.1 Constrained DDF-RAROR

The DDF-RAROR can easily be extended to the case conditioning on a restriction \( s \in S \). The basic idea for constrained DDF-RAROR is to impose a set of restrictions on the evaluation procedure. The definition of DDF-RAROR has to be adapted to the state \( S = s \) and we obtain the constrained DDF-RAROR \( \tau_s \) as follows:

\[
\tau_s = \frac{1}{1 + \tilde{d}_s}
\]

where

\[
\tilde{d}_s = \sup \{ d | (\sigma + dg, r + dg) \in F_s \}
\]

and \( F_s = \{(\sigma, r) | (x, \sigma, r) \in P_s \} \) is a conditional investment opportunity set.

The comparison of \( \tau_s \) with \( \tau \) is certainly of interest for analyzing the global influence of \( S \) on the DDF-RAROR. When \( S \) is a single trait, the ratios \( \tau / \tau_s = (1 + \tilde{d}_s) / (1 + \tilde{d}) \) would be helpful to describe the influence of \( S \) on the DDF-RAROR.\(^{48}\) If \( \tau / \tau_s > 1 \), it indicates that \( S \) is detrimental (unfavorable) to DDF-RAROR and when \( \tau / \tau_s < 1 \), it specifies a \( S \) factor conducive (favorable) to DDF-RAROR.

\(^{47}\) In this situation, testing issues for comparing group evaluation scores can be proposed using appropriate bootstrap algorithms, in the spirit of Simar and Wilson (2002).

\(^{48}\) We can do the same with the differences \( \tilde{d}_s - \tilde{d} \), but since evaluation scores are proportions, ratios seem very natural.
The restriction $S$ might be a fundamental factor, which needs to be incorporated during the evaluation process, in addition to the risk. Or, it may be exogenous to the evaluation process itself, but explain part of the effect of unobservable economic factors on DDF-RAROR. For instance, the diversification, the market price of risk, and the underlying ROE are taken into account for security evaluation in our empirical study.

3.1.1 Diversification

[Figure 2] illustrates the role of diversification in security evaluation. Security $b$ is preferable to security $a$ according to the DDF-RAROR, whereas $a$ is considered as a better asset by the Sharpe Ratio. This reversal in rankings comes from the fact that the DDF-RAROR does not appraise $a$ only by its risk-return, but compares $a$ with its benchmark $a^*$, a potentially achievable optimal portfolio. As for $b$, which stays on the efficient frontier already, the diversification benefit is unachievable under the current

[Figure 2] Impact of Diversification on DDF-RAROR

---

49 This feature makes DDF-RAROR advantageous over any other conventional RAROR - like Sharpe Ratio, Treynor Ratio, Jensen’s Alpha, Appraisal Ratio, and M-square measure. None of them does utter the diversification benefit as precisely as DDF-RAROR does.
investment opportunity set.

The diversification pushes the efficient frontier to the northwest, compared with the case where the diversification is ignored, because the risk can be reduced without a loss of the return. The magnitude of this shift depends on the covariance structure of securities in the investment opportunity set. When the efficient frontier is moved, the DDF-RAROR seems to change correspondingly. The response of DDF-RAROR to the movement of efficient frontier summarizes the potential benefits from diversification, which might be fluctuating across securities.

In order to investigate the impact of the diversification on the DDF-RAROR formally, we compute two sets of DDF-RAROR. The first set of directional distance \( \{ \tilde{d}_i \mid i = 1, \ldots, n \} \) is estimated under the diversification, i.e. considering \( \{ \sigma_y \mid i \neq j \} \), as follows:

\[
\begin{align*}
\tilde{d}_i &= \max \beta \\
\text{s.t.} \quad & (1 + \beta) r_i \leq w'r \\
& (1 - \beta) \sigma_i = w \Sigma w \\
& w'c = 1
\end{align*}
\]

On the other hand, the second set of directional distances \( \{ \tilde{d}_{s,i} \mid i = 1, \ldots, n \} \) is computed, with the assumption that the performance of securities is irrelevant to relations among securities \( \{ \sigma_y \mid i \neq j \} \), as follows:

\[
\begin{align*}
\tilde{d}_{s,i} &= \max \beta \\
\text{s.t.} \quad & (1 + \beta) r_i \leq w'r \\
& (1 - \beta) \sigma_i = w'\sigma \\
& w'c = 1
\end{align*}
\]
Now, we introduce the index $\theta$ to denote the influence of market price of risk on DDF-RAROR. For a security $\theta_i$,

$$\theta_i = \frac{\tau_i}{\tau_{s,i}} = \frac{1 + \tilde{d}_{s,i}}{1 + \tilde{d}_i}$$

By removing the diversification out of security evaluation, it is possible to improve DDF-RAROR because it is possible to reduce risk without decreasing returns through the diversification. It implies, mathematically, $\tilde{d}_s < \tilde{d}$, i.e. $\theta < 1$ for each security. When $\theta$ is low, the diversification effect big. We may conclude that the security with higher $\theta$ has a stable DDF-RAROR without regard to the diversification.

3.1.2 Market Price of Risk

As it is seen in [Figure 3], the ranking based on DDF-RAROR is reversed when the direction changes. The direction represents the market consensus about how much additional return is required for taking additional risk. The DDF-RAROR framework provides security evaluators with a systematic tool taking into account the market price of risk. If the one-to-one tradeoffs between return and risk are supposed, then security $a$ beats security $b$. But $b$ can become the winner if the market condition evolves and the market price of risk, i.e. the slope of direction, turns bigger.

The market price of risk quantifies the extra return that investors demand to bear a unit of risk, which tells us how much extra return must be earned per unit of risk. Under equilibrium, the RAROR (or reward-to-variability ratio) of any security must be equal to
[Figure 3] Considering Market Price of Risk

market price of risk.\(^\text{50}\)

\[ k = \frac{r_i}{\sigma_i}, \forall i \]

If not, there exists an arbitrage opportunity: buy a security with higher RAROR and sell a security with lower RAROR in order to get extra return after risk adjusting. Hence, the market price of risk measures the trade-offs between risk and return that are present in current market condition.

The market price of risk has a significant implication in security evaluation since it rules how much a security's return has to be discounted against its embedded risk. The magnitude of discount directly relies on the market price of risk: the same level of risk may be regarded less aggressive when the market price of risk is lower. The DDF-RAROR explicitly incorporates the information about the market price of risk into its evaluation process.\(^\text{51}\)

\(^{50}\) We open ourselves to ambiguity in using this term, because the risk premium to variability ratio \((r_i - r_f)/\sigma_i\) is generally referred to as the market price of risk. If we assume that \(r_i\) denotes an excess return, the consistency in terminology will be restored.

\(^{51}\) Note that a security has the same value in terms of the conventional RAROR like Sharpe Ratio, regardless of the market price of risk.
In order to investigate the impact of the market price of risk on the DDF-RAROR formally, we compute two sets of DDF-RAROR. The first set of directional distance \[ \{ \overrightarrow{d}_i | i = 1, \ldots, n \} \] is estimated under the assumption that \( g = (-\sigma, kr) \) for \( k = 1 \), i.e. one-to-one tradeoff between the risk and the return, as follows:

\[
\begin{align*}
\overrightarrow{d}_i &= \max \beta \\
\text{s.t.} \quad (1 + \beta)r_i &\leq w'r \\
(1 - \beta)s_i &= w'\Sigma w \\
w'c &= 1
\end{align*}
\]

On the other hand, the second set of directional distances \( \{ \overrightarrow{d}_{s,i} | i = 1, \ldots, n \} \) is computed with the direction set along the market price of risk, i.e. \( g = (-\sigma, kr) \) for \( k = (\prod r_i / \sigma_i)^{1/n} \), as follows:

\[
\begin{align*}
\overrightarrow{d}_i &= \max \beta \\
\text{s.t.} \quad (1 + k\beta)r_i &\leq w'r \\
(1 - \beta)s_i &= w'\Sigma w \\
w'c &= 1
\end{align*}
\]

Now, we introduce the index \( \lambda \) to denote the influence of market price of risk on DDF-RAROR. For a security \( \lambda_i \),

\[
\lambda_i = \frac{\tau_i}{\tau_{s,i}} = \frac{1 + \overrightarrow{d}_{s,i}}{1 + \overrightarrow{d}_i}
\]

By incorporation market price of risk into security evaluation, it is possible to improve DDF-RAROR because \( \overrightarrow{d}_i \) makes use of the right price of risk. It implies, mathematically, \( \overrightarrow{d}_i < \overrightarrow{d}_s \), i.e. \( \lambda < 1 \) for each security. When \( \lambda \) is lower, the Sharpe ratio of a security
deviates further from the market price of risk. We may conclude that the security with higher $\lambda$ is priced appropriately with respect to the market price of risk.

3.1.3 Market Sentiment

Sources of excess return on securities are sound underlying business prospects, good news, and favorable market sentiment. Security evaluation requires analysts to understand how each of them converts into the return on securities, and how they are interrelated.

The baseline of security evaluation is fundamental model: the underlying business prospects (e.g., ROE) combined with expected growth determine current level of return on stocks. Relative returns on stocks are determined by relative business prospects across companies. It is incomplete for security evaluation, however, to implement the fundamental model in isolation, due to the uncertainty in the inputs: current business prospects and expectations of future growth.

Under uncertainty, news about business prospects, the differences between actual figures of business prospects and their expected value to be realized (e.g., earnings surprise), could be useful to explain parts of extra return. News announcements appear to affect security returns, but they are effective for a time period of no longer than a day or two.\(^{52}\)

---

\(^{52}\) Beyond a couple of days after release of news, there seems to be little relationship between news and security returns. Hence, if news were to be used in trading, you would have to make sure trading activity happens before other participants have a chance to trade. Once news hits the market, positions will adjust quickly.
Abnormal returns, which are often persistent over longer period, can be explained by market sentiment. Market participants expect future return to be a certain level. Their buy or sell transactions, depending on their expected returns, essentially cause security returns to move to expected levels. It is essentially a self-fulfilling prophecy.

Business prospects and news are important since they influence market sentiment, that is, market participants use business prospects and news when market sentiment is formed. But it is hard to get which particular business prospect are importantly used, due to inconsistent treatment of the business prospects.\(^{53}\) Without knowledge about verified relations among factors, a nonparametric method is suggested to estimate the impact of them on security returns.

Market participants collect information regarding business prospects from accounting data and relevant news. Then they assess intangible assets like synergy or economies of scope, which contribute returns but are not represented by quantitative data. In this scenario, the stock market is the final arbiter of how market participants value the underlying business prospects and relevant news. The residual, after netting out the part explained by business prospects and new, is assumed to represent the part attributable to market sentiment.

In order to investigate the impact of underlying ROE on the DDF-RAROR formally, we compute two sets of DDF-RAROR. The first set of directional distance \(\left\{ d_i , i = 1, \ldots , n \right\}\) is estimated under the assumption that \(X = \phi\), i.e. without underlying ROE, as follows:

\(^{53}\) One year, dividend yield may receive a lot of attention by market participants. The next year, the focus is on earnings surprise. The psychological characteristics of market participants dynamically evolve over time.
\[
\tilde{d}_i = \max \beta \\
\text{s.t.} \\
(1 + \beta)r_i \leq w'r \\
(1 - \beta)s_i = w\Sigma_w \\
w'c = 1
\]

On the other hand, the second set of directional distances \(\{\tilde{d}_{s,i} \mid i = 1, \ldots, n\}\) is computed with underlying ROE, i.e. \(X \neq \phi\), as follows:

\[
\tilde{d}_i = \max \beta \\
\text{s.t.} \\
(1 + \beta)r_i \leq w'r \\
(1 - \beta)s_i = w\Sigma_w \\
x_i \geq w'x \\
w'c = 1
\]

Now, we introduce the index \(\eta\) to denote the influence of ROE on DDF-RAROR.

For a security \(\eta_i\),

\[
\eta_i = \frac{\tau_i}{\tau_{s,i}} = \frac{1 + \tilde{d}_{s,i}}{1 + \tilde{d}_i}
\]

By incorporation ROE into security evaluation, it is possible to improve DDF-RAROR because \(\tilde{d}_s\) looks at the remaining part, after netting out the contribution of ROE to RAROR of the security. It implies, mathematically, \(\tilde{d}_s < \tilde{d}\), i.e. \(\eta < 1\) for each security. The lower is \(\eta\), the less portion of RAROR can be explained by underlying ROE. In other words, a security with low \(\eta\) has big exposure to the market sentiment. We may conclude that a high \(\eta\) security is well priced according to a fundamental model.

### 3.2 Application
In order to investigate the impact of market factors – like diversification, price of risk, and underlying ROE – on media stocks, we estimate DDF-RAROR in several different settings, each of which represents an investment opportunity set. We estimate eight sets of DDF-RAROR for 425 observations over years 1997-2001. [Table 6] summarizes specifications of DDF-RAROR estimation. The asterisk means that a related factor is taken into account during an estimation process.

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Specifications of DDF Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \tilde{d}_1 )</td>
</tr>
<tr>
<td>Diversification Benefit</td>
<td>*</td>
</tr>
<tr>
<td>Price of risk</td>
<td></td>
</tr>
<tr>
<td>Business Prospects</td>
<td>*</td>
</tr>
</tbody>
</table>

The default DDF-RAROR, ignoring all factors, is estimated by

\[
\tilde{d}_{1,i} = \max \beta
\]

\[s.t. \quad (1 + \beta)r_i \leq w'r
\]
\[ (1 - \beta)\sigma_i = w\Sigma w
\]
\[w'c = 1\]

where \( r \) is the \((n \times 1)\) vector of returns, \( r_i \) is the return of security \( i \), \( \Sigma \) is the \((n \times n)\) covariance matrix of securities, \( \sigma_i \) is the variance of asset \( i \), \( w \) is a \((n \times 1)\) weight vector, and \( c \) is a \((n \times 1)\) unit vector.

As for the impact of underlying ROE, the DDF-RAROR is estimated under an alternative set of assumptions, incorporating ROE into optimization procedure. The alternative estimation problem is
\[ \tilde{d}_{2,i} = \max \beta \]

s.t. \[
\begin{align*}
(1 + \beta)r_i & \leq w'r \\
(1 - \beta)\sigma_i & = w\Sigma w \\
x_i & \geq w'x \\
w'c & = 1
\end{align*}
\]

where \( x \) is the \((n \times 1)\) vector of ROE of the company corresponding to securities, and \( x_i \) is the ROE of security \( i \). Note that an additional constraint regarding to ROE is added to the default estimation problem.

As for the price of risk, the DDF-RAROR is estimated under an alternative set of assumptions, setting the direction at the price of risk, i.e. \( k = (\prod_i r_i / \sigma_i)^{1/n} \). The alternative estimation problem is

\[ \tilde{d}_{3,i} = \max \beta \]

s.t. \[
\begin{align*}
(1 + k\beta)r_i & \leq w'r \\
(1 - \beta)\sigma_i & = w\Sigma w \\
w'c & = 1
\end{align*}
\]

Note that a marginal increase of distance \( \beta \) influences the risk constraint by one unit, but impacts the return constraint by \( k \) units. In this way, the estimation process reflects the current price of risk \( k \).

As for the impact of diversification, another DDF-RAROR is estimated, allowing the efficient frontier to neglect the diversification. The alternative estimation problem is

\[ \tilde{d}_{5,i} = \max \beta \]

s.t. \[
\begin{align*}
(1 + \beta)r_i & \leq w'r \\
(1 - \beta)\sigma_i & = w'\sigma \\
w'c & = 1
\end{align*}
\]
where $\sigma$ is the $(n \times 1)$ variances of securities, and $\sigma_i$ is the variance of security $i$. Note that the second constraint is linear in this case. Other specifications are various combinations of four basic formats.

As described in section 3, a ratio of two DDF-RAROR acts as an index capturing the impact of unobservable market factors on DDF-RAROR. The impact of diversification is measured by

$$\theta_{15} = \frac{1 + \tilde{d}_5}{1 + \tilde{d}_1}$$

$\theta_{51} < 1$ is expected in general since the potential benefit from diversification is positive. Likewise, $\theta > 1$ implies that the potential benefit from diversification is negative, and so the DDF-RAROR is positively affected by ignoring diversification. In order to investigate interrelations among factors, we also compute variations of $\theta$ as:

$$\theta_{26} = \frac{1 + \tilde{d}_6}{1 + \tilde{d}_2}; \quad \theta_{37} = \frac{1 + \tilde{d}_7}{1 + \tilde{d}_3}; \quad \theta_{48} = \frac{1 + \tilde{d}_8}{1 + \tilde{d}_4}$$

where $\theta_{26}$ is calculated with ROE, $\theta_{37}$ is along the price of risk, and $\theta_{48}$ adopts both ROE and the price of risk, whereas $\theta_{15}$ considers none of them.

In order to explain the impact of price of risk, we use the ratio

$$\lambda_{13} = \frac{1 + \tilde{d}_3}{1 + \tilde{d}_1}$$

$\lambda_{13} \leq 1$ is expected in general. High $\lambda$ means that the DDF-RAROR is insensitive to incorporating the price of risk. In this case, the current price of risk is cheap relative to the reward-to-variability ratio of the security. For considering interrelations among factors, we also compute variations of $\lambda$ as:
\[ \lambda_{24} = \frac{1 + \tilde{d}_4}{1 + \tilde{d}_2} ; \quad \lambda_{57} = \frac{1 + \tilde{d}_7}{1 + \tilde{d}_5} ; \quad \lambda_{68} = \frac{1 + \tilde{d}_8}{1 + \tilde{d}_6} \]

where \( \lambda_{24} \) is estimated under both diversification and ROE, \( \lambda_{57} \) is with neither diversification nor ROE, and \( \lambda_{68} \) is with ROE without diversification, and whereas only diversification is considered in \( \lambda_{13} \).

As for the impact of underlying ROE, the following ratio is employed

\[ \eta_{12} = \frac{1 + \tilde{d}_2}{1 + \tilde{d}_1} \]

\( \eta_{21} \leq 1 \) is expected in general. High \( \eta \) means that the large portion of DDF-RAROR attributes to the sentiment, rather than underlying ROE. To investigate interrelations among factors, we also compute variations of \( \eta \) as:

\[ \eta_{34} = \frac{1 + \tilde{d}_4}{1 + \tilde{d}_3} ; \quad \eta_{56} = \frac{1 + \tilde{d}_6}{1 + \tilde{d}_5} ; \quad \eta_{78} = \frac{1 + \tilde{d}_8}{1 + \tilde{d}_7} \]

where \( \eta_{34} \) is calculated along the price of risk under diversification allowed, \( \eta_{56} \) is without either diversification or price of risk, and \( \eta_{78} \) adopts only price of risk, whereas only diversification is taken into account in \( \eta_{12} \).

### 3.2.1 DDF-RAROR Estimates

Under the DDF-RAROR framework, an efficient frontier is a set of points, which are projections of observations on the frontier. As expected, the estimates of efficient frontier are monotone concave. [Figure 4] reports the result for year 1999. Note that the efficient
frontiers shift downward (or to southwest) as additional factors are included in estimation procedure. Especially, the efficient frontier without diversification is piecewise linear rather than a curve.

The estimates are geometrically averaged for media conglomerate stocks, and non-conglomerate stocks. For each specification, the mean of DDF-RAROR \( \tau \) for conglomerate stocks is higher than that for non-conglomerate stocks. We conclude that conglomerate stocks show better performance in terms of DDF-RAROR. And this conclusion is robust against change in assumptions as seen in [Table 7].

<table>
<thead>
<tr>
<th>Table 7</th>
<th>Mean of DDF-RAROR</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \tau_1 )</td>
</tr>
<tr>
<td>Con.</td>
<td>0.799</td>
</tr>
<tr>
<td>Non-Con.</td>
<td>0.790</td>
</tr>
</tbody>
</table>

We need to point out a few things from the summary statistics. First, the mean value of DDF-RAROR improves as fundamental factors (diversification, price of risk, or underlying ROE) are added. Second, the impact of price of risk dominates the impact of diversification or underlying ROE.\(^{54}\) Third, the DDF-RAROR deteriorates when the diversification is taken into account exactly like what we expected.

\(^{54}\) The means of DDF-RAROR with price of risk are close to 1 for both groups, and its effects dominate other factors. To demonstrate interrelation among factors, we report the case with \( k = 2 \) instead of the case with price of risk. The pattern is robust against the size of \( k \).
3.2.2 Impact of Diversification

As shown in [Table 8], the mean of $\theta$ for conglomerates is higher than that for non-conglomerates: it means the DDF-RAROR for conglomerate stocks are less affected by the diversification. Note that the differential is magnified when underlying ROE is also included.

<table>
<thead>
<tr>
<th>[Table 8] Mean of $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>Con.</td>
</tr>
<tr>
<td>Non-Con.</td>
</tr>
</tbody>
</table>
It is interesting to see the relation of $\theta$ with the reward-to-variability ratio relative to its median $S$, which is computed as

$$S_i = \frac{r_i / \sigma_i}{\text{Median}_{j=1,\ldots,n}(r_j / \sigma_j)}$$

The point, which is around the center of reward-to-variability ratio, tends to be projected to the right end of efficient frontier. At the right end, the efficient frontier with diversification converges to the efficient frontier without diversification. Hence, the effect of diversification on DDF-RAROR is negligible over there. It is expected that $\theta$ goes high around the point $S = 1$.

[Figure 5] Impact of Diversification
[Figure 5] illustrate how the nonparametric regression of $\theta$ on $S$ captures the effect of diversification on the DDF-RAROR. We recover what we expect: $\theta$ is maximized as the reward-to-variability ratio relative to its median approaches to one.

The pattern of reward-to-variability ratio relative to the median $S$ is consistent with the pattern of $\theta$, in a sense that the group having $S$ closer to one is projected to the outward part of efficient frontier, and is less sensitive to the inclusion of diversification. Moreover, [Table 9] shows that the one standard deviation interval of $S$ for conglomerates is narrow around one, whereas the interval $S$ for non-conglomerates is wide.

<table>
<thead>
<tr>
<th>[Table 9] Interval of $S$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Conglomerates</td>
</tr>
<tr>
<td>Low</td>
</tr>
<tr>
<td>High</td>
</tr>
</tbody>
</table>

3.2.3 Impact of Price of risk

As shown in [Table 10], the mean of $\lambda$ for conglomerates is higher than that for non-conglomerates: it means the DDF-RAROR for conglomerate stocks are less sensitive

<table>
<thead>
<tr>
<th>[Table 10] Mean of $\lambda$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
</tr>
<tr>
<td>Con.</td>
</tr>
<tr>
<td>Non-Con.</td>
</tr>
</tbody>
</table>

to changes in price of risk. Note that the differential is magnified when underlying ROE is also included.
It is interesting to see the relation of $\lambda$ with the individual price relative to the market price of risk $T$, which is computed as

$$T_i = (r_i / \sigma_i) / (\prod_{j=1}^n r_j / \sigma_j)^{1/n}$$

The market price of risk is estimated by the mean of individual price $(\prod_{j=1}^n r_j / \sigma_j)^{1/n}$. The security having higher price than the market price keeps a buffer to absorb expensive price, and is less sensitive to the inclusion of price of risk. Hence, the effect of price of risk on DDF-RAROR $\lambda$ gets bigger as individual prices inflate. It is expected that $\lambda$ goes high around the point $T > 1$.

[Figure 6] illustrates how the nonparametric regression of $\lambda$ on $T$ captures the effect of including the price of risk on the DDF-RAROR of security. We recover exactly

[Figure 6] Impact of Price of Risk
what we expect: the price of risk set at a relatively cheap level is favorable to the DDF-RAROR.

The pattern of individual price to market price ratio $T$ is consistent with the pattern of $\lambda$, in a sense that the group having higher price is less sensitive to the inclusion of price of risk. [Table 11] shows that the one standard deviation interval of $T$ for conglomerates is covered with the interval for non-conglomerates. But the effect of including price of risk is more significant at the left tail of $\lambda$. Thus, the mean of $\lambda$ for conglomerates is higher than that for non-conglomerates.

<table>
<thead>
<tr>
<th>[Table 11] Interval of $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Low</td>
</tr>
<tr>
<td>High</td>
</tr>
</tbody>
</table>

**3.2.4 Impact of Underlying ROE**

As shown in [Table 12], the mean of $\eta$ for conglomerates is lower than that for non-conglomerates: it means the DDF-RAROR for conglomerate stocks are more sensitive to including underlying ROE in estimation procedure. Note that the differential is magnified with diversification allowed.

<table>
<thead>
<tr>
<th>[Table 12] Mean of $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Con.</td>
</tr>
<tr>
<td>Non-Con.</td>
</tr>
</tbody>
</table>
It is interesting to see the relation of $\eta$ with the mean of underlying ROE. The mean of ROE for conglomerates is lower than that for non-conglomerates. Since the level of $\eta$ indicates the portion explained by fundamental value out of overall DDF-RAROR, the sentiment may deviate further away from the underlying value as $\eta$ lowers.

[Figure 7] illustrates how the nonparametric regression of $\eta$ on ROE captures the effect of underlying ROE on the DDF-RAROR. We find what we expect: a high ROE makes the DDF-RAROR less sensitive to including underlying ROE.

[Figure 7] Impact of ROE

[Table 13] shows that the one standard deviation interval of underlying ROE for conglomerates lies on the lower range. Hence, the stocks of conglomerates relies less on fundamental value, but depends more on market sentiment. In other words, the DDF-
RAROR for conglomerates is more independent to underlying ROE, while the DDF-RAROR is closely binding to the business performance in non-conglomerate stocks.

<table>
<thead>
<tr>
<th></th>
<th>Conglomerates</th>
<th>Non-Conglomerates</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low</td>
<td>0.499</td>
<td>0.685</td>
</tr>
<tr>
<td>High</td>
<td>1.544</td>
<td>2.268</td>
</tr>
</tbody>
</table>

### 3.3 Concluding Remark

In this paper, extending ideas developed in the previous chapter, we provide a conditional formulation of the DDF-RAROR. This formulation allows the introduction in security evaluation according to the DDF-RAROR of fundamental factors which influence the security performance but that is not clear how they have impacts on the security return.

The presentation allows general multiple factors to be incorporated into the DDF-RAROR and provides a practical way for evaluating the nonparametric impact of fundamental factors. We also propose a useful graphical tool for highlighting the eventual influences of factors on the DDF-RAROR. This method will show whether the fundamental factor is conducive or detrimental to the security evaluation.

The approach is illustrated with a set on media stocks, where the influence of three fundamental factors is investigated: diversification, price of risk, and underlying ROE. The diversification shows a negative influence on the DDF-RAROR, while the price of risk and the underlying ROE have positive influence on the DDF-RAROR.
Moreover, the sensitivity of DDF-RAROR to excluding diversification is demonstrated by a reversed U-shape curve. On the other hand, the curves showing the sensitivity to including price of risk or underlying ROE are monotonically increasing. The DDF-RAROR for conglomerates is less sensitive to excluding diversification or including price of risk, but more sensitive to including underlying ROE than the DDF-RAROR for non-conglomerates. This pattern confirms our previous finding that conglomerate stocks are influenced in higher degree by the market sentiment.
Chapter 4

Probabilistic Analysis on DDF-RAROR

The risk-adjusted rate of return based on the directional distance function (DDF-RAROR) is estimated by nonparametric programming, as described in the previous chapters. Under nonparametric programming, the efficient frontier is estimated as the boundary of the convex, conical, or free-disposal hull of the observed data. The estimated number invariably presents point estimates, and estimates of DDF-RAROR are deterministic in this sense.

Yet, estimates of DDF-RAROR are subject to uncertainty due to sampling variation since they are measured relative to an estimate of the underlying true efficient frontier. If one views risk-return data as having been generated from a distribution with bounded support over the true investment opportunity set, then DDF-RAROR is always measured conditional on observed risk-return data resulting from the underlying and unobserved data generating process. Hence, the observed efficient frontier is pushed outward although not beyond underlying boundary of the true investment opportunity set when an additional observation is obtained.

In spite of its potential appeal, the sampling distributions of DDF-RAROR are not easily available, due to the complexity and multidimensional nature.\textsuperscript{55} In this chapter, the DDF-RAROR framework is extended to give a statistical interpretation about whether

\textsuperscript{55} Lovell (1993) and others have labeled DEA and similar approaches to efficiency measurement as deterministic, as if to suggest that DEA models have no statistical underpinnings. But, the theory of statistical consistency in DEA models has been extended to the general multi-input and multi-output case for both input- and output-oriented efficiency measures in Kneip, Park, and Simar (1998).
fundamental factors significantly influence the DDF-RAROR on securities in a statistical sense. In other words, we determine whether indicated changes in DDF-RAROR are real, or merely artifacts of the fact that we do not know the true efficient frontiers and must estimate them from finite samples.

The bootstrap methodology, proposed in Simar and Wilson (1998, 1999, and 2000), is a way to investigate sampling properties of distance function estimators. The bootstrapping procedure presented in Simar and Wilson (2000) is easily adapted to the present DDF-RAROR case: analyzing the sensitivity of the DDF-RAROR to sampling variation, confidence intervals of DDF-RAROR estimates, and corrections for the bias inherent to the estimation procedure. Now, usual inference on the estimates of DDF-RAROR is available. For application of the bootstrapping method to DDF-RAROR, securities are assumed to deviate from the underlying true efficient frontier in a risk-reducing and return-enhancing direction. These random deviations from the efficient frontier, measured by DDF-RAROR, may be further assumed to result from various fundamental factors other than the risk.

Alternatively, the partial frontier method originally developed in Cazals, Florens and Simar (2002), and extended in Daraio and Simar (2003), provides another probabilistic formulation of a nonparametric frontier model. The estimation procedure presented in Daraio and Simar (2003) is adapted to the present DDF-RAROR case: the idea about m-order efficiency score and conditional efficiency score is extended to the directional

---

56 Simar and Wilson (1998) proposed a bootstrap strategy for analyzing the sensitivity of the efficiency measures to sampling variation providing confidence intervals and corrections for the bias inherent to the DEA procedure. The method has been applied to time dependence structures for estimating Malmquist indices (Simar and Wilson, 1999). However, the methodology in both of the papers relies on some restrictive homogeneity assumptions on the distribution of efficiency among firms. As pointed out in Simar
distance function method, which is based on the Data Envelopment Analysis (DEA) technically. This partial frontier method is used to investigate the influence of underlying fundamental factors which on the security evaluation process.

The chapter proceeds as follows. Section 3.1 analyzes the underlying statistical model of DDF-RAROR by both the bootstrapping method and the partial frontier method. Section 3.2 presents the influence of underlying ROE on DDF-RAROR for media stocks, and provides its statistical inference. In section 3.3 we conclude.

4.1 Significance of DDF-RAROR

The DDF-RAROR $\tau$ and its relevant index $\eta$ outlined in the previous section provide us with point estimates. Clearly, there is sampling variability and thus statistical uncertainty about these estimates. We address this issue by turning to two probabilistic formulation of DDF-RAROR: the bootstrapping method and the partial frontier method. As for the bootstrapping approach, the simulation procedure described in Simar and Wilson (2000) is adapted to our DDF-RAROR framework. On the other hand, the partial frontier approach relies on the simulation procedure suggested in Daraio and Simar (2003).

4.1.1 Bootstrapping

and Wilson (1998), the key to statistically consistent estimation of confidence intervals lies in the replication of the unobserved data-generating process.
Assume a DGP wherein securities randomly deviate from the underlying true efficient frontier. The random deviations are measured by the distance function, as described in Simar and Wilson's (1999, 2000). Following Kneip, Park, and Simar (1998), we employ the following assumptions to characterize the data generating process (DGP).

[A1] \( \{(x_i, r_i, \sigma_i), i = 1, ..., n\} \) are i.i.d. random variables on the convex investment opportunity set, \( F = \mathbb{R}_+^3 \).

[A2] Underlying ROE \( X \) possesses a density \( f(\cdot) \) whose bounded support \( X \subseteq \mathbb{R}_+ \) is compact.

[A3] For all \( x \in X \), a polar coordinate \( \nu \) has a conditional density \( f(\nu \mid x) \) on \([0, \pi/2]\) and conditional on \((x, \nu)\), a distance \( d \) has a density \( f(d \mid x, \nu) \).

[A4] For all \( x \in X \) and for all \( \nu \in [0, \pi/2] \), there exist constants \( (\varepsilon_1, \varepsilon_2) > 0 \) such that \( f(d \mid x, \nu) \geq \varepsilon_1 \) for all \( d \in [0, \varepsilon_2] \).

[A5] The distance function \( d \) is differentiable in its argument.

Under these assumptions, for the fixed point \((x, r, \sigma)\),

\[
\hat{d} - d = O_p\left(n^{-1/2}\right)
\]

where \( \hat{d} \) is a consistent estimator of \( d \). For our empirical analysis, the rate of convergence is equal to the typical rate of 1/2.

---

\(^{57}\) In their study, the bootstrapping method is used to provide a statistical interpretation to the Malmquist/Malmquist-Luenberger index. Since our ratios are constructed from two correlated DDF-RAROR's just like ML index, their bootstrapping approach can directly be applied to our situation.  

\(^{58}\) For notational brevity, we use a simpler notation without upper arrow for directional distance function measure from now on.  

\(^{59}\) This is a simple application of the general result \( \hat{d} - d = O_p\left(n^{-1/2}\right) \) proved in Kneip et al. (2001).
In order to implement the bootstrapping methods, we first suppose that a random sample \( Z = \{(x_i, r_i, \sigma_i), i = 1, \ldots, n\} \) is drawn by a DGP under assumptions [A1]-[A5]. Bootstrapping process involves replicating this DGP.\(^{60}\) It generates an appropriately large number \( B \) of pseudo samples \( Z^b = \{(x^b_i, r^b_i, \sigma^b_i), i = 1, \ldots, n\} \) (or \( Z^b \) constrained DDF-RAROR), where \( b = 1, \ldots, B \) and applies the original estimators to these pseudo samples. For each bootstrap replication \( b = 1, \ldots, B \), we measure the distance from each observation in the original sample \( Z \) to the efficient frontiers estimated from the pseudo sample \( Z^b \) (or \( Z^b_i \)). The distance function based on pseudo data can be estimated by solving:

\[
\begin{align*}
    d^b_i &= \max \beta \\
    s.t. \quad (1 + \beta)r_i \leq w'r^b \\
    &\quad (1 - \beta)w'\sigma_i = w'\sigma^b \\
    &\quad w'c = 1
\end{align*}
\]

and for constrained DDF-RAROR:

\[
\begin{align*}
    d^b_{r,s} &= \max \beta \\
    s.t. \quad (1 + \beta)r_i \leq w'r^b \\
    &\quad (1 - \beta)w'\sigma_i = w'\sigma^b \\
    &\quad x_i \geq w'x \\
    &\quad w'c = 1
\end{align*}
\]

This process yields bootstrap estimates \( \{d^b, d^b_{r,s}\} \) for each security. These estimates can then be used to construct bootstrap estimates \( \tau^b \) and \( \eta^b \).

\(^{60}\) The bootstrap method is based on the idea that if the \( DGP^b \) is a consistent estimator of \( DGP \) and \( \partial Q(\hat{f}^b, \hat{f}) / \partial \hat{f}^b \) exists continuously for \( (\hat{f}^b, \hat{f}) \) in an open neighborhood of \( (f, f) \) then the bootstrap distribution of \( \sqrt{K}Q(\hat{d}^b, \hat{d}) \), given \( \hat{d} \), is asymptotically equivalent to the sampling distribution of \( \sqrt{K}Q(\hat{d}, d) \) given the true probability distribution \( d \). (Efron, 1979, pp22-23 Remark G).
As suggested by Simar and Wilson (2000), we follow the 11-step bootstrapping algorithm.

[Step1] From the original data set \(Z = \{(x_i, r_i, \sigma_i), i = 1, \ldots, n\}\), estimate \(\{\hat{d}, \hat{d}_s\}\) for all securities.

[Step2] Form the augmented matrix \(A\). To ensure consistency of our estimator, we use the reflection method proposed by Silverman (1986). From \((n \times 1)\) matrix \(P_1 = [\hat{d}_1, \ldots, \hat{d}_n]'\) and \(P_2 = [\hat{d}_{s1}, \ldots, \hat{d}_{sn}]'\) where \(n\) is the number of securities, construct the augmented \((4n \times 2)\) matrix \(A\) by reflection since the values in \(P_1\) and \(P_2\) are bounded from above at zero.

\[
A = \begin{bmatrix}
P_1 & P_2 \\
-P_1 & P_2 \\
P_1 & -P_2 \\
-P_1 & -P_2
\end{bmatrix}
\]

where \(A\) contains \(4n\) pairs of values corresponding to the unconditional and conditional estimates.

---

61 Note that \(d \in [0,1]\). Since we will likely have many values of \(\hat{d} = 0\) in a given sample of size \(n\), the boundary condition is binding. The boundary conditions \(x \geq 0\) and \(\nu \in [0, \pi / 2]\) are not taken into account here for the sake of simplicity and because in practice, these constraints are seldom binding. Because of boundary problem, i.e., \(x \in \mathcal{R}_+\) and \(\eta \in [0, \pi / 2]\), and \(d \in [0,1]\), the naïve bootstrap yields inconsistent estimates (see Simar, L., and P. Wilson, 2000). One way to avoid this problem is to use a smoothed bootstrap. This method draws i.i.d. pseudo samples \((x_i^b, r_i^b, \sigma_i^b), i = 1, \ldots, n\) from a density \(\tilde{f}(x, r, \sigma)\) that is a smooth, consistent estimator of the joint density \(f(x, r, \sigma)\) on the investment opportunity set \(\mathcal{F}\). In terms of polar coordinates, this is equivalent to estimating the density \(f(x, \nu, d)\) and drawing bootstrap samples \((x^b, \nu^b, d^b)\) from the estimated density. Unfortunately, \(f(x, \nu, d)\) has bounded support and ordinary kernel density estimates are inconsistent near boundaries. Since the use of boundary kernels is not known for higher dimensional spaces, we adopt the reflection method proposed by Silverman (1986) to estimate \(f(x, \nu, d)\).
[Step3] Compute the estimated covariance matrix $\hat{\Sigma}_i$ from the original data $[P_1 \ P_2]$ (or the reflected data $[-P_1 \ -P_2]$), and $\hat{\Sigma}_2$ from the data $[P_1 \ -P_2]$ (or $[-P_1 \ P_2]$). That is,

$$\hat{\Sigma}_1 = \text{Cov}(P_1, P_2) = \text{Cov}(-P_1, -P_2)$$

$$\hat{\Sigma}_2 = \text{Cov}(P_1, -P_2) = \text{Cov}(-P_1, P_2)$$

Next, obtain the lower triangular matrices $L_1$ and $L_2$ such that $\hat{\Sigma}_1 = L_1L_1'$ and $\hat{\Sigma}_2 = L_2L_2'$ via Cholesky decomposition.

[Step4] Draw $n$ rows randomly with replacement from $A$, and denote the result by the $(n \times 2)$ matrix $A^b$. Then compute $\bar{A}^b$, which is the $(1 \times 2)$ row vector containing the means of each column of $A^b$.

[Step5] Use a random number generator to generate an $(n \times 2)$ i.i.d. matrix $\varepsilon$ and construct $\varepsilon^b$ so that

$$\varepsilon_i^b = \varepsilon_j L_j', \quad j = 1, 2$$

where $\varepsilon_i^b \sim N(0, \hat{\Sigma}_1)$ or $N(0, \hat{\Sigma}_2)$. If $i \in \{1, \ldots, n, 3n+1, \ldots, 4n\}$ then the covariance matrix is $\hat{\Sigma}_1$, whereas $i \in \{n+1, \ldots, 2n, 2n+1, \ldots, 3n\}$ then the covariance matrix is $\hat{\Sigma}_2$.

[Step6] Compute the $(n \times 2)$ random deviates needed for the bootstrap by the $\Delta$ function, as in Silverman (1986),

$$\Delta = \left(1 + h^2\right)^{-0.5} \left(M \cdot A^b + he^b\right) + c_n \otimes \bar{A}^b$$

where $M = I_n - c_n c_n' / n$, $I_n$ is a $(n \times n)$ identity matrix, and $c_n$ is a $(n \times 1)$ unit vector.

[Step 7] Set $h = (4/5n)^{1/6}$, as an appropriate band width $h$ of bivariate kernel density estimator for the previous step, as suggested in Silverman's (1986).
[Step 8] Define the \((n \times 2)\) matrix of bootstrap pseudo data \(D^b\)

\[ d^b = |\delta^b| \quad \text{where} \quad \Delta = (\delta^b) \]

[Step 9] Construct the pseudo sample \(Y^b = \{(x^b_i, r^b_i, \sigma^b_i), i = 1, \ldots, n\}\) by setting:

\[ r^b_i = r_i (1 + \hat{d}_i) / (1 + d^b_i), \quad \text{and} \quad \sigma^b_i = \sigma_i (1 - \hat{d}_i) / (1 - d^b_i) \]

[Step 10] Compute \(\hat{d}^b, \hat{d}^b_i\). This can be obtained by solving the programming problem under the pseudo investment opportunity set consisting of the pseudo sample \(Z^b\) (and \(Z^b_s\)).

[Step 11] Repeat steps [5]-[10] \(B\) times to get a set of bootstrap estimates \(\{\hat{d}^b, \hat{d}^b_i | b = 1, \ldots, B\}\). We can then compute the \(\hat{\eta}^b\) for \(b = 1, \ldots, B\). Confidence interval and the bias of the estimator can be derived.

The confidence interval of the estimator then can be estimated by noting that the bootstrap approximates the unknown distribution of \((\hat{\eta} - \eta)\) by the distribution of \((\hat{\eta}^b - \eta)\) conditioned on the original data set where \(b\) denotes the estimate on bootstrapped data. Therefore, we can find critical values of the distribution, \(c_{a/2}, c_{100-a/2}\) by simply sorting the value \((\hat{\eta}^b - \hat{\eta})\) for \(b = 1, \ldots, B\) and then finding the \(\alpha/2\) percentile and \((100 - \alpha/2)\) percentile values. This critical value provides us with the following confidence interval:

\[ \hat{\eta} - c_{100-a/2} \leq \eta \leq \hat{\eta} - c_{a/2} \]

If this interval covers 1, i.e., no change in DDF-RAROR, then we cannot reject the null hypothesis that the underlying ROE does not affect the DDF-RAROR.
Because of the deterministic nature of the distance function, we have $0 \leq \hat{d} \leq d$, and $0 \leq \hat{d}^b \leq \hat{d}$ for $b = 1, \ldots, B$. Since $\eta$ is a function of the directional distance function, the downward biased estimator $\hat{d}$ yields the biased ratio $\hat{\eta}$. We can correct finite-sample bias of the estimators using the bootstrap estimates. The bootstrap bias estimate for the estimator $\hat{\eta}$ is

$$bias(\hat{\eta}) = \frac{1}{B} \sum_{b=1}^{B} \hat{\eta}^b - \hat{\eta}$$

The bias corrected estimator of $\eta$ will be

$$\tilde{\eta} = \hat{\eta} - bias(\hat{\eta}) = 2\hat{\eta} - \frac{1}{B} \sum_{b=1}^{B} \hat{\eta}^b$$

### 4.1.2 Partial Frontier Method

The rationale behind partial frontier approach is to assess uncertainty about distance to the true efficient frontier from a relatively small number of securities in the investment opportunity set, cleverly chosen to reflect the location of most of the observation, or of at least the most interesting part of securities.

---

62 For observations where this results in infeasible solutions, repeat steps 5-10.

63 Once $c_{100-\alpha/2} < 0$, the estimated confidence interval will not include the original estimate $\hat{\eta}$ from the inequality of critical values, $c_{\alpha/2} < c_{100-\alpha/2}$.

64 The variance of bias corrected estimator will be $4\text{Var}(\hat{\eta})$ as $B \rightarrow \infty$. According to Efron and Tibshirani (1993), the bias corrected estimator can have higher mean square error than the original estimator $\hat{\eta}$. To obtain minimum mean square error estimator, we compare the mean squared error of $\tilde{\eta}$, $4\text{Var}(\tilde{\eta})$, with the mean squared error of the original estimator $\hat{\eta}$, $\text{Var}(\hat{\eta}) + bias(\hat{\eta})^2$. The variance of $\tilde{\eta}$ can be estimated using bootstrapped data, i.e., the sample variance of the bootstrap estimators $\{\tilde{\eta}^b | b = 1, \ldots, B\}$. The bias corrected estimator will have higher mean squared error if $\text{Var}(\tilde{\eta}) > bias(\hat{\eta})^2 / 3$. 


The evaluation process is here described by the joint probability measure $G(y)$ of $y = (\sigma, r)$ on $Y = R^2$. The support of $y$ is the investment opportunity set $F$. In terms of the joint probability measure $G(y)$, the directional distance can be characterized as: for a given point $y_o = (\sigma_o, r_o)$,

$$d(y_o) = \sup \{d \mid G(g' y \in F \mid y_o) > 0\}$$

where

$$g' y = ((1 - d)\sigma, (1 + d)r),$$

$$G(y \in F \mid y_o) = \Pr(y \in F \mid y \in Y_o),$$

$$Y_o = \{y \in R^2 \mid \sigma \leq \sigma_o, r \geq r_o\},$$

and $g$ is a direction vector. A nonparametric estimator of the $d(y_o)$ is given by plugging the empirical version of $G(y \in F \mid y_o)$ given by

$$\hat{G}(y \in F \mid y_o) = \frac{\sum_{i=1}^{n} 1(y_i \in F, y_i \in Y_o)}{\sum_{i=1}^{n} 1(y_i \in Y_o)}$$

where $1(\cdot)$ is the indicator function. Then, the estimator of directional distance for a given point $y_o$ is the solution of

$$d(y_o) = \sup \{d \mid \hat{G}(g' y \in F \mid y_o) > 0\}$$

where $g' y = ((1 - d)\sigma, (1 + d)r)$.

This coincides to the DEA estimator of $d(y_o)$ given by

$$d_o = \max \beta$$

$$s.t. \quad (1 + \beta)r_o \leq w' r^p$$

$$\quad (1 - \beta)w' \sigma_o = w' \sigma^p$$

$$\quad w'c = 1$$
where $r^p$ is a vector of returns included in $Y_0$, and $\sigma^p$ is a vector of standard deviations included in $Y_0$.

As pointed in Cazals, Florens and Simar (2002), the estimator $\hat{\mathbf{F}}$ of investment opportunity set is an estimator of the full-frontier, enveloping all the cloud of points $y$. For a probabilistic formulation, we estimate a partial frontier, which corresponds to another definition of the benchmark against which securities will be compared. The idea can be summarized as follows, adapting the formulation in Cazals, Florens and Simar (2002) to the security evaluation.

For a given $y_i$ in the interior of the support $Y$, consider $I$ i.i.d. random variable $y_j$ generated by the conditional distribution function $G(y \in F \mid y_i)$ and define the set:

$$F_i^p = \{y_j \mid y_j \in F, y_j \in Y_i, \ j = 1, ..., I\}$$

Then, for any $y$, we may define

$$\hat{d}_i^p = \sup \{d \mid g'y \in F_i^p\}$$

where $g'y = ((1-d)\sigma, (1+d)r)$. Note that $\hat{d}_i^p$ may be computed by the following formula:

$$d_i^p = \max \beta$$

s.t.  
$$(1 + \beta)\eta_i \leq w'r^p$$
$$(1 - \beta)w'\sigma_i = w'\sigma^p$$
$$w'c = 1$$
where $r^p$ is a vector of returns included in $Y_i$, and $\sigma^p$ is a vector of standard deviations included in $Y_i$. The directional distance using partial frontiers is defined as follows. For any $y_i \in R^2_+$, the directional distance $\hat{d}_i$, is defined as:

$$\hat{d}_i = E\left(\hat{d}_i^p \mid y \in Y_i\right),$$

where we assume the existence of the expectation. Therefore, for any $y \in R^2_+$, the expected efficient frontier derived from the partial frontier method consists of the set

$$(\hat{\sigma}, \hat{r}) = ((1-\hat{d})\sigma, (1+\hat{d})r).$$

The analysis can easily be extended to the case where underlying ROE is provided by other variable $x \in R_+$, which may be the part of evaluation process or may be exogenous to the evaluation process itself, but which may explain part of it. The basic idea is to condition the evaluation process to a given value of $x = x_0$.

The joint distribution $G(y)$ conditional on $x = x_0$ defines the evaluation process if $x = x_0$. In particular the DDF-RAROR can be adapted to the condition $x = x_0$, under convexity, as:

$$d(y_0, x_0) = \sup \{d \mid G(g'y \in F \mid y_0, x_0) > 0\}$$

where

$$g'y = ((1-d)\sigma, (1+d)r),$$

and $G(y \in F \mid y_0, x_0) = Pr(y \in F \mid y \in Y_0, x = x_0)$.

---

65 Note that the asymptotic properties of $\hat{d}$ have not yet been derived in the literature, but we might expect that the asymptotic properties of the conditional order-m input efficiency measure, obtained in Cazals, Florens, and Simar (2002), can apply to our DEA estimator.
A nonparametric estimator of the conditional full-frontier DDF-RAROR \( d(y_0, x_0) \) is given by plugging a nonparametric estimator of \( G(y \in F \mid y_0, x_0) \). This requires some smoothing techniques in \( x \). At this purpose we use a kernel estimator of \( G(y \in F \mid y_0, x_0) \) defined as:

\[
\hat{G}(y \in F \mid y_0, x_0) = \frac{\sum_{i=1}^{n} 1(y_i \in F, y_i \in Y)K((x_i - x_0)/h)}{\sum_{i=1}^{n} 1(y_i \in Y)K((x_i - x_0)/h)}
\]

where \( K(\cdot) \) is the kernel, \( 1(\cdot) \) is the indicator function, and \( h \) is the bandwidth of appropriate size. Hence we obtain the constrained DDF-RAROR as follows:

\[
\hat{d}(y_0, x_0) = \text{sup}\{d \mid \hat{G}(g'y \in F \mid y_0, x_0) > 0\}
\]

where \( g'y = ((1-d)\sigma, (1+d)r) \).

For a given level of return \( y_j \) in the interior of the support of \( Y \), consider the \( l \) i.i.d. random variables \( y_j, \ j = 1, \ldots, l \) generated by the conditional distribution function \( G(g'y \in F \mid y_i, x_i) \) and define the set:

\[
\hat{F}_i^p = \left\{ y \in R^2 \mid \sigma = \sum_{j=1}^{p} w_j \sigma_j, r \leq \sum_{j=1}^{p} w_j r_j, \text{ s.t. } \sum_{j=1}^{p} w_j = 1; w_j \geq 0, j = 1, \ldots, l \right\}.
\]

Note that this estimate of investment opportunity set depends on the value \( y_j \) since the \( y_j \) is generated through \( G(g'y \in F \mid y_i, x_i) \). Then, for any \((\sigma, r)\), we may define

\[
\hat{d}_i^p = \text{sup}\{d \mid g'y \in \hat{F}_i^p\}
\]

where \( g'y = ((1-d)\sigma, (1+d)r) \).

Note that \( \hat{d}_i^p \) may be computed by the following formula:
\[ d_i^p = \max \beta \]
\[ \text{s.t.} \quad (1 + \beta) r_i \leq w'r^p \]
\[ (1 - \beta) w'\sigma_i = w'\sigma^p \]
\[ w'c = 1 \]

For the partial frontier DDF-RAROR, we employ the four steps Monte-Carlo algorithm suggested in Daraio and Simar (2003). Suppose that \( h \) is the chosen bandwidth for a particular kernel \( K(\cdot) \).

[Step 1] For a given \((\sigma_i, r_i)\), draw a sample of size \( l \) with replacement, and with a probability
\[ K((x_i - x_0)/h) / \sum_{i=1}^{n} K((x_i - x_0)/h), \] among those \((\sigma_j, r_j)\) satisfying one of following conditions:

(a) \( Y_i = \{ y_j \in R^2_i | r_j \geq r_i \} \),

(b) \( Y_i = \{ y_j \in R^2_i | \sigma_j \leq \sigma_i \} \),

(c) \( Y_i = \{ y_j \in R^2_i | \sigma_j \leq \sigma_i \wedge r_j \geq r_i \} \), or

(d) \( Y_i = \{ y_j \in R^2_i | \sigma_j \leq \sigma_i \vee r_j \geq r_i \} \).

Denote this sample by \((\sigma^p, r^p)\).

[Step 2] Compute \(\{\tilde{d}^p, \tilde{d}^p_i\} \) through the problem
\[ d_i^p = \max \beta \]
\[ \text{s.t.} \quad (1 + \beta) r_i \leq w'r^p \]
\[ (1 - \beta) w'\sigma_i = w'\sigma^p \]
\[ w'c = 1 \]

[Step 3] Redo [1]-[2] for \( p = 1, \ldots, P \) where \( P \) is large.
[Step 4] Finally, calculate $\hat{d}_i = 1/P \sum_{p=1}^{P} \hat{d}_i^p$.

The quality of the approximation can be tuned by increasing $P$ but in our application, say $P = 200$, seems to be a reasonable choice.

4.2 Application

In order to derive the empirical distribution of $\eta$, and to perform its statistical inference, we employ two probabilistic approaches in nonparametric frontier model: the bootstrapping method and the partial frontier method. As for the partial frontier method, four variations of sampling to construct subset of investment opportunity set are tried conditioning on

(a) lower risk and higher return securities, $Y_{1,i} = \{y_j \in R^2_+ \mid \sigma_j \leq \sigma_i \wedge r_j \geq r_i\}$

(b) lower risk securities, $Y_{2,i} = \{y_j \in R^2_+ \mid \sigma_j \leq \sigma_i\}$

(c) higher return securities, $Y_{3,i} = \{y_j \in R^2_+ \mid r_j \geq r_i\}$

(d) lower risk or higher return securities, $Y_{4,i} = \{y_j \in R^2_+ \mid \sigma_j \leq \sigma_i \vee r_j \geq r_i\}$.

Here, we focus on the statistical significance of the influence of underlying ROE on the DDF-RAROR for media stocks.

4.2.1 Bias of $\eta$
Looking at Figure 1, we see that $\eta$ is less than one, i.e. including ROE has the positive effect on the DDF-RAROR for media securities. The original estimate $\eta$ is compared with the mean $\eta_b$ of bootstrapping estimates, and with the mean $\eta_p$ of partial frontier estimates. $\eta_b$ (or $\eta_p$) is computed over 200 bootstrapped (or partially sampled) estimates, respectively. The original estimate $\eta$ is higher than $\eta_b$, but lower than $\eta_p$.

As mentioned before, the estimate $\hat{d}$ of directional distance is biased downward, which results in the upward bias in estimates of the DDF-RAROR $\tau$. In Figure 1, the mean $\eta_b$ of bootstrapping estimates suggests that the original estimate $\eta$ is upward biased, implying that the denominator $\tau_x$ is less biased than the nominator $\tau$. Thus, the inclusion of ROE partially corrects the upward bias of $\tau$ estimates. Furthermore, the magnitude of bias rises as ROE increases. On the other hand, the mean $\eta_p$ of estimates

[Figure 8] Mean of $\eta$
stays above the original estimate \( \eta \). [Figure 8] shows that the original estimate \( \eta \) is severely downward-biased according to \( \eta_{\rho} \), in particular over the range of low ROE.

The statistical analysis, based on both the bootstrapping and the partial frontier method, confirms the previously observed pattern between ROE and DDF-RAROR. The mean of \( \eta \) increases along with ROE, and shows that the ROE has bigger effects on the DDF-RAROR at the left tail. In terms of both the mean of \( \eta \) and the mean of \( \tau \), the partial frontier method provides higher evaluation score, as shown in [Figure 9]. It is because of the structure that the partial frontier uses only the superior parts of total observations, in order to estimate efficient frontiers: higher return securities and/or lower risk securities. The partial frontier produces the higher DDF-RAROR than the full

[Figure 9] Comparison among methods
frontier with bootstrapping, when the risk is used as the criterion for selecting partial investment opportunity set, i.e., in $Y_{i,j}$ or $Y_{2,j}$. The effect of ROE on DDF-RAROR at the left tail is mitigated under the partial frontier method in $Y_{i,j}$ or $Y_{2,j}$.

### 4.2.2 Significance of $\eta$

The 200 estimates of DDF-RAROR $\tau$ and sensitivity index $\eta$ are sorted by algebraic value. After deleting 5-percent of the elements at both ends of this sorted array, we set the low and the high critical value equal to the endpoints of the resulting sorted array. Their smoothed nonparametric regression lines are presented in [Figure 10]. The 90-percent of 200 estimates fall into the bend. The confidence interval and bias corrected estimate are computed as:

$$2\hat{\eta} - \hat{\eta}_9^b \leq \eta \leq 2\hat{\eta} - \hat{\eta}_5^b$$

$$\bar{\eta} = 2\hat{\eta} - \frac{1}{B} \sum_{b=1}^{B} \hat{\eta}_b$$

The length of significant interval looks independent of underlying ROE, when it is derived from the bootstrapping method. On the other hand, the significant interval collapses to a point as the underlying ROE increases, when it is derived from the partial frontier method. It results from the fact that the number of securities belonging to partial investment opportunity set becomes few as ROE goes high. At the left tail, however, the partial frontier method provides a broader significant interval than the bootstrapping method.
[Table 14] shows the result about significance of $\eta$ under 5-percent level. The null and the alternate hypotheses are stated:

$$H_0: \eta \geq 1.$$  
$$H_1: \eta < 1.$$  

When $2\hat{\eta} - \hat{\eta}_S^b < 1$, we reject the null hypothesis at 5-percent significance level. The null hypothesis is rejected 91 out of 300 times: 12 out of 25 times for conglomerates, and 79 out of 275 times for non-conglomerates, according to bootstrapping results $B$. The deviations of $\eta$ from 1 occur more often at 5-percent significance for conglomerates, which implies that the underlying ROE has a significant positive effect on DDF-RAROR for conglomerates. The result of significance test varies across statistical methods: values
of $\eta$ are more significant under partial frontier methods. When the risk is used as the criterion for selecting partial securities, i.e., in $Y_{1,i}$ or $Y_{2,i}$, the result $P1$ or $P2$ looks more significant than the result $B$ of bootstrapping.

<table>
<thead>
<tr>
<th>Year</th>
<th>$B$</th>
<th>$P1$</th>
<th>$P2$</th>
<th>$P3$</th>
<th>$P4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G1$</td>
<td>12</td>
<td>15</td>
<td>15</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>$G2$</td>
<td>79</td>
<td>120</td>
<td>114</td>
<td>101</td>
<td>91</td>
</tr>
</tbody>
</table>

4.2.3 Unconditional vs. Conditional $\eta$

[Figure 11] compares the impact of conditional sampling on $\eta$ with the counterpart under unconditional sampling. When the sampling depends on information about

[Figure 11] Effect of Conditional Sampling
underlying ROE, the impact of including ROE on DDF-RAROR is mitigated. And the curve representing $\eta$ is flattened, meaning that the sensitivity of DDF-RAROR to ROE is more independent on the level of ROE. When the risk is used as the criterion for selecting partial investment opportunity set, i.e., in $Y_{1,i}$ or $Y_{2,i}$, the curvature of unconditional $\eta$ is much smaller than in $Y_{3,i}$ or $Y_{4,i}$. Correspondingly, the influence of conditional sampling is larger in $Y_{3,i}$ or $Y_{4,i}$.

4.2.4 Normality Test of $\eta$

Since there is no parametric specification about the underlying distribution of $\eta$, it becomes our interest to check whether the empirical distribution of $\eta$ conforms to the normal distribution. The goodness-of-fit test is used to show if the observed values in a frequency distribution coincide with the expected values based on a normal distribution.\(^{66}\) The null and the alternate hypotheses are stated:

$H_0$: $\eta$ is normally distributed.

$H_1$: $\eta$ is not normally distributed.

If the null is rejected, it means that $\eta$ is not normally distributed. We select the 5-percent significance level. The test statistic, which follows the chi-square distribution, is

$$\chi^2_{k-3} = \sum \frac{(f_o - f_e)^2}{f_e}$$

---

\(^{66}\) The goodness-of-fit test is one of the most commonly used nonparametric tests. Developed by Karl Pearson in the early 1900s, it can be used for any level of data. See chapter 14 of Mason et al. (1999).
where \( k \) is the number of categories (in this case 10), and \( f_e \) is an empirical frequency in a decile, and \( f_n \) is an expected frequency from normal distribution in the decile. Note that we lose a degree of freedom for each estimate for mean and standard deviation when we estimate population parameters from sample data, and that the degree of freedom is \((k - 3)\) in our case.\(^{67}\)

By construction, the normal frequency \( f_n \) is 20 for each category because the category is designed as deciles. The empirical frequency \( f_e \) is found through counting the observation belonging to the area between two deciles of the normal distribution curve, out of total number of estimates 200.

<table>
<thead>
<tr>
<th>Year</th>
<th>( B )</th>
<th>( P1 )</th>
<th>( P2 )</th>
<th>( P3 )</th>
<th>( P4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G1 )</td>
<td>9</td>
<td>25</td>
<td>25</td>
<td>23</td>
<td>25</td>
</tr>
<tr>
<td>( G2 )</td>
<td>85</td>
<td>275</td>
<td>275</td>
<td>274</td>
<td>272</td>
</tr>
</tbody>
</table>

[Table 15] shows the result about number of rejecting normality hypothesis under 5-percent significance level. The normality test is rejected 94 out of 300 times: 9 out of 25 times for conglomerates, and 85 out of 275 times for non-conglomerates. Over half of empirical distributions of \( \eta \) confirm to the normal distribution, according to results \( B \) from bootstrapping. The result of normality test varies across statistical methods: almost all empirical distributions of \( \eta \) deviate from the normal distribution under the partial frontier methods. This is consistent with the result about confidence intervals.

\(^{67}\) Suppose we know the mean and standard deviation of a population but wish to find whether some sample information conformed to the normal distribution. In this case the degrees of freedom is \( k \).
4.3 Concluding Remark

In this chapter, we investigate the statistical properties of DDF-RAROR developed in previous chapters. As noted in the introduction, the DDF-RAROR examines security performance in a variety of evaluation setting. Yet, the uncertainty surrounding DDF-RAROR estimates can lead to erroneous conclusions unless the sampling noise in the resulting efficient frontiers is properly taken care of.

In order to provide a general, computationally tractable way for statistical properties of DDF-RAROR, we adapt two methods from the literature on DEA estimators: the bootstrapping approach (Simar and Wilson, 2000), and the partial frontier approach (Daraio and Simar, 2003). The statistical results confirm the pattern between ROE and DDF-RAROR observed in the previous chapter. The mean of \( \eta \) increases along with ROE, and shows the bigger effect of ROE on DDF-RAROR at the left tail. Interestingly, two probabilistic approaches return contradictory outcomes. First, the original estimate \( \eta \) is upward biased from the mean \( \eta_b \) of bootstrapping estimates, while it is downward biased according to the mean \( \eta_p \) of partial frontier estimates. Second, the length of significant interval looks independent of underlying ROE under the bootstrapping method, whereas it narrows with the rise of ROE under the partial frontier method. Third, about one third of \( \eta \) is significantly lower than one by the bootstrapping, but the significance of \( \eta \) is stronger by the partial frontier. Fourth, the normality hypothesis is rejected for about half of the empirical distribution of \( \eta \), with
bootstrapping. On the other hand, almost all empirical distributions are different from the normal distribution, with partial frontier.
References


