RICE UNIVERSITY

Essays in Intrafamily Distribution and Taxation

by

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Abstract

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This thesis consists of three essays. In the first two essays we assume that the wife earns a lower wage rate than the husband and we analyze intrafamily distributional effects if environmental parameters such as the tax system or divorce regulations change. The third essay evaluates the efficiency of a production tax when a public service is provided to business.

The first essay considers a tax reform from individual to joint taxation for couples in a one-period model. An expansion of the utility possibility set due to tax reform makes both spouses better off, if spouses apply a bargaining rule with a disagreement point outside marriage. Alternatively, if spouses receive family resources proportional to their contribution to family full income, the husband prefers joint taxation and the wife prefers individual taxation.

In the model of the second essay, spouses bargain in each of two periods over the resource allocation between them using divorce as the disagreement outcome. Since each spouse's first period labor supply influences his or her second period bargaining power, the couple's labor supply decisions are no longer efficient. (1) A reduction of the tax rate of the wife when divorced increases inefficiency within marriage, but raises the wife's intertemporal utility. (2) An equal split of income between former spouses equalizes
utility shares between spouses and it is less distortionary than a divorce law granting each
former spouse his or her stand alone income. (3) Whenever the wife benefits from a tax
reform from joint towards individual taxation of the family, inequality between spouses
decreases but the husband might be worse off.

In the third essay, local governments finance a public service to firms either with a tax on
capital, the mobile factor, or with a tax on production. We find that in the Zodrow-
Mieszkowski model a production tax is inefficient except for the case of a Cobb-Douglas
technology. Using a CES production function, we show that a production tax is more
efficient than a capital tax in the Zodrow-Mieszkowski model.
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# Table of Contents

## Chapter 1

**Joint vs. Individual Taxation and Intrafamily Distribution**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Literature</td>
<td>5</td>
</tr>
<tr>
<td>The Model</td>
<td>8</td>
</tr>
<tr>
<td>The Utility Possibility Set</td>
<td>10</td>
</tr>
<tr>
<td>Household Behavior 1: The Bargaining Approach</td>
<td>13</td>
</tr>
<tr>
<td>Household Behavior 2: The Competitive Approach</td>
<td>19</td>
</tr>
<tr>
<td>Conclusion</td>
<td>22</td>
</tr>
<tr>
<td>Appendix</td>
<td>26</td>
</tr>
<tr>
<td>References</td>
<td>38</td>
</tr>
<tr>
<td>Figures</td>
<td>42</td>
</tr>
</tbody>
</table>

## Chapter 2

**A Two-Period Bargaining Model of the Family**

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Introduction</td>
<td>47</td>
</tr>
<tr>
<td>Related Literature</td>
<td>49</td>
</tr>
<tr>
<td>The Model</td>
<td>52</td>
</tr>
<tr>
<td>Equilibrium Time Allocation of Spouses</td>
<td>56</td>
</tr>
<tr>
<td>Applications</td>
<td>60</td>
</tr>
<tr>
<td>Conclusion</td>
<td>68</td>
</tr>
<tr>
<td>Appendix</td>
<td>69</td>
</tr>
</tbody>
</table>
References 83
Figures 86

CHAPTER 3

TAX COMPETITION AND THE EFFICIENCY

OF "BENEFIT-RELATED" TAXES ON BUSINESS 87

Introduction 87
The Model 89
Zodrow and Mieszkowski (1986) 90
Oates and Schwab (1991) 98
Sinn (1997) 102
Examples and Simulations for the ZM Model 108
Conclusion 113
Appendix 114
References 121
Chapter 1

Joint vs. Individual Taxation and Intrafamily Distribution

1.1 Introduction

This essay analyzes the effects of family tax reform on intrafamily distribution when one spouse earns a lower wage rate than the other. Joint taxation taxes both spouses at the same rate, while individual taxation taxes the spouse with lower wage rate at a lower rate. For the rest of the chapter, we will refer to the low wage spouse as wife and the high wage spouse as husband. A change from individual to joint taxation of the household does not only change the net-wage ratio between spouses, it also affects the utility possibility set of the household (see Apps and Rees, 1999a and b; Piggott and Whalley, 1996). To fix ideas, we assume that a change from individual to joint taxation of the family results in an outward shift of the utility possibility frontier (UPF). We determine whether both spouses
benefit from such an expansion of the utility possibility set.

In order to evaluate the utility of each spouse before and after tax reform, we need a model of how husband and wife share family resources. If families were to maximize some household utility function subject to family resource constraints, the composition of family income would not affect the distribution of utility shares between spouses. If household utility increases both spouses benefit from tax reform. Increasing empirical evidence, however, rejects the income pooling hypothesis suggesting that a change in the composition of family income has an impact on intrafamily distribution.\(^1\) The recent literature on economics of the family often suggests that cooperative conflict between spouses can explain the empirical findings: While both spouses cooperate to maximize the size of the pie (being on the Pareto frontier of the utility possibility set), husband and wife have conflicting ideas of how to share it (selecting a point on the utility possibility frontier).\(^2\)

Our work is unique in the sense that it addresses two distinctive ways of modelling intrafamily resource allocation simultaneously. In the bargaining approach, spouses bargain with each other over final utility shares. The disagreement utility of each spouse is given by how much each of them can guarantee him or herself when single. While the Nash Bargaining Solution (NBS) is often proposed to solve this problem, we also focus on a less applied solution concept, the Kalai-Smorodinsky solution (KSS).\(^3\) Alternatively, we model the resource allocation problem using a competitive approach. Family full income

\(^1\) See e.g. Bourguignon et al. (1993), Browning et al. (1994), Lundberg et al. (1997), Rubalcava and Thomas (2000), Woolley (2000), and Attanasio and Lechner (2002).


\(^3\) Manser and Brown in their seminal paper in 1980 considered both the NBS and KSS, but in more recent papers in the literature the focus is clearly on NBS.
is divided among spouses according to some rule. Each spouse then maximizes his or her utility, given the share of family full income and household prices. We propose a simple income sharing rule. In Separate Accounts, the income share of a spouse depends only on his or her earning potential when married, i.e. income shares are equal to the stand alone income of each spouse and the husband's share of family full income is therefore larger than the wife's share.

Spouses consume two goods, a market good and a household good. Both goods are private. Each spouse is endowed with the same amount of time that can be used in market labor or household production. Spouses' utility functions are identical and homogenous of some degree \( r, 0 < r \leq 1 \). These assumptions imply a simple functional form of the utility possibility frontier (UPF); the UPF is symmetric, and a change in wage rates or tax rates will result in a homothetic shift of the UPF.

In the bargaining approach our result depends crucially on the direction in which the UPF shifts when we move from individual to joint taxation. Both the Nash Bargaining solution and the Kalai-Smorodinsky solution satisfy the homothetic solidarity property, that is, both spouses benefit from an outward shift of the UPF (Proposition 1.1). Inequality within the household decreases; gains of cooperation as well as utility shares assigned to each spouse are distributed more equally. This result does not depend on the specifics of

---

4Family full income is the income a family would receive if both spouses spent all their time in market labor.

5The assumption that the household good is private is not the only possibility. Household public goods are often emphasized in the literature. As public goods play a bigger role in families with children, our model is more relevant for households only consisting of a husband and a wife. Alternative treatments of household production are discussed in section 1.7.

6The class of bargaining problems with the same functional form of the UPF as considered here also allows for comparison between different bargaining rules along the UPF. Most recently, Ambardi (2002) shows that KSS assigns a higher utility to the person with lower disagreement utility than NBS, if the disagreement utility of this person is set to be zero. See also Lemma 1.2.
the tax reform.\textsuperscript{7} Take any two tax schedules for the couple. If tax reform from one tax schedule to the other results in an outward shift of the UPF, both spouses obtain a higher utility and inequality between spouses decreases.

With Separate Accounts the result critically depends on how net-wage rates change with tax reform, but the result does not depend on the direction in which the UPF shifts. The wife's utility decreases while the husband's utility increases after a switch from individual to joint taxation no matter whether tax reform expands or shrinks the utility possibility set of the household (Proposition 1.2). Inequality between spouses increases, because the gap between net-wage rates of spouses increases under joint taxation leaving the wife with a lower and the husband with a higher stand alone income than before tax reform.

This chapter provides a guideline for evaluating a tax reform if policy makers want to reduce inequality between spouses. Apps and Rees (1999a and b) make a strong argument that a change from individual to joint taxation results in an inward shift of the UPF. In this case our results state that under any resource allocation rule discussed above joint taxation increases inequality between spouses.\textsuperscript{8}

The chapter is organized as follows. We first discuss related literature (section 1.2). In section 1.3, we introduce the formal framework, followed by section 1.4 that stresses the cooperative features of the model. Section 1.5 deals with the bargaining approach. Section 1.6 introduces the competitive approach. Section 1.7 discusses extensions to the basic model and concludes. All proofs can be found in the appendix, and all figures at the end of the chapter.

\textsuperscript{7}Provided that tax rates for singles are the same before and after the reform.

\textsuperscript{8}See also Nelson (1996) who argues in favor of individual taxation because of its intrafamily distributional effects.
1.2 Literature

1.2.1 Household Production and Family Income Taxation

Household production and tax reform in our model is based on Apps and Rees (1999a). Our model distinguishes itself from Apps and Rees' by specifying how a couple shares resources. Apps and Rees do not model any specific allocation rule since their focus is on whether the utility possibility set expands with tax reform, but not how the welfare of an individual household member changes. They suggest, however, that their model naturally lends itself to the analysis of intrafamily distributional effects.

The argument for individual taxation of couples on efficiency grounds is based on the secondary worker hypothesis. While the primary worker has a very inelastic labor supply, the labor supply of the secondary worker is very high. In the absence of household production, "standard Ramsey rule considerations would argue for taxing primary and secondary workers at different marginal rates (Apps and Rees, 1999a, p. 394)." With household production, however, different tax rates on the income of the wife and the income of the husband distort the shadow price of home production. This distortion would not occur under joint taxation. Piggott and Whalley (1996) suggest that this distortion outweighs the benefits of individual taxation, Apps and Rees (1999a and b) argue the opposite.

If Apps and Rees are correct, then this essay puts forth a new argument in favor of individual taxation. It argues that inequality between household members is reduced compared to joint taxation under all of our allocation rules.

---

9In his seminal work Becker (1981) emphasized the role of household production in a multi-person household.
1.2.2 The Resource Allocation Problem of the Household

While the assumption that a multi-person household maximizes a household utility function is still common in the literature, solution concepts from cooperative bargaining theory are increasingly applied to model resource allocation between household members. Family bargaining models stress the importance of bargaining power between spouses and suggest that own income (both from market labor as well as labor-independent wealth) plays a crucial part in determining the fall-back position for each spouse (e.g. Manser and Brown, 1980; McElroy and Horney, 1981; McElroy, 1997; and Lundberg and Pollak, 1993 and 1996). While most authors in the literature of family bargaining apply the Nash Bargaining Solution (NBS) we focus also on the Kalai-Smorodinsky solution (KSS).

Our competitive approach to intrafamily distribution is inspired by the collective approach taken by Chiappori and others (Chiappori, 1997). The collective approach requires that the family is on the Pareto frontier of the utility possibility set which is achieved by letting spouses separately maximize their utility given a share of family full income. The collective approach leaves the specific rule of allocating income open, while we define a specific income sharing rule for the spouses.

1.2.3 Cooperative Game Theory and Distributive Justice

An allocation rule that never makes a person worse off when the utility possibility set expands in an arbitrary way, is said to satisfy the solidarity axiom (e.g. Chun and Thomson, 1988; and Keiding and Moulin, 1991). This axiom is the strongest in the class of monotonicity axioms and only the egalitarian solution satisfies it (Chun and Thomson,
1988). Weaker versions of monotonicity require that an agent be no worse off after restricting
the way in which the utility possibility set expands (e.g. Kalai and Smorodinsky, 1975; Chun
and Thomson, 1988; and Nicolò and Perea, 2002).

Our assumptions about production and consumption guarantee that the UPF can
only shift homothetically thus limiting the way in which the utility possibility set can
expand. An allocation rule is said to satisfy the *homothetic solidarity property* if no agent
is made worse off when the utility possibility set expands homothetically. We show that
both NBS and KSS satisfy the homothetic solidarity property (Proposition 1.1).

Roemer (1986) and others have argued that by focussing on problems of distribu-
tive justice one should consider economic environments and distribute resources rather than
converting everything into utility. In the competitive approach, spouses distribute family
full income and then trade with each other. Sen (1996) points out that the resource al-
location within the household does not only depend on outside options, but also on what
is perceived as the contribution of each household member to the family well-being right
then and there. He states “given other things, if in the accounting of the respective out-
comes, a person is perceived as making a larger contribution to the overall well-being of
the group, then the chosen collusive solution will become more favorable to that person
(p.68).” The sharing rule of family full income that we propose in this essay captures the
idea that spouses evaluate their time at market prices. Hence the contribution of the wife
is perceived as being less than the contribution of the husband.
1.3 The Model

This section is divided into subsections each emphasizing one building block of the model. The reader is referred to section 1.7 for a discussion of the more controversial assumptions.

1.3.1 Household Production and Household Income

Wife ($i = 1$) and husband ($i = 2$) divide their time $T$ between market labor ($l_i$) and work in household production ($t_i$). The couple does not consume leisure. The wife earns a lower wage rate before taxes than the husband, i.e. $w_1 < w_2$.

There are two private goods, one market good ($x$) and one household good ($y$). The market good is taken as the numeraire good. The household good is produced with both spouses' time inputs, $t_1$ and $t_2$. The production function is twice differentiable, and satisfies the following properties

\[ CRS : ay = f(at_1, at_2), a > 0, \]
\[ \frac{\partial f}{\partial t_i} > 0, \quad \frac{\partial^2 f}{\partial t_1 \partial t_2} \geq 0, \quad \frac{\partial^2 f}{\partial t_i^2} \leq 0. \]

Both spouses are equally productive in the household: Consider the output produced with $t_1 = p$ and $t_2 = q$ and the output produced with $t_1 = q$ and $t_2 = p$. For any pair $(p, q) \in [0, \infty)^2$,

\[ f(p, q) = f(q, p). \]

Moreover, the household good can be produced by one person alone,

\[ f(t_i, 0) > 0 \text{ for } t_i > 0. \]
Tax Schedule. Let \( \tau_i \) be the marginal tax rate on the wage rate of a spouse. The hourly net-wage after paying \( \tau_i w_i \) in taxes on hourly wages is \((1 - \tau_i) w_i\). Under individual taxation \( \tau_i = \beta_i \) with \( \beta_1 < \beta_2 \), and joint taxation taxes wage rates of both spouses at the same rate \( \tau_i = \alpha, i = 1, 2 \) where \( \beta_1 < \alpha < \beta_2 \). The tax schedule under individual taxation guarantees people with higher wage rates a higher net-wage rate, \((1 - \beta_1) w_1 < (1 - \beta_2) w_2\), but it narrows the gap compared to the before-tax wage rate ratio, \( \frac{w_1}{w_2} < \frac{(1-\beta_1)w_1}{(1-\beta_2)w_2} \) while joint taxation maintains the same net-wage ratio as the ratio of before-tax wage rates.\(^{10}\)

The couple's feasibility constraints consist of the following. Expenditure on the market good cannot exceed household labor income after taxes, and the couple must consume an amount of the household good that is less or equal to the amount they produced. Denote \( x_i \) the quantity of the market good consumed by spouse \( i \) and denote \( y_i \) the quantity of the household good consumed by spouse \( i \), then

\[
\begin{align*}
x &= x_1 + x_2 \leq \sum_{i=1}^{2} (1 - \tau_i) w_i l_i, \quad (1.3) \\
y &= y_1 + y_2 \leq f(t_1, t_2).
\end{align*}
\]

The time constraint for each spouse is given by

\[
l_i + t_i = T. \quad (1.5)
\]

1.3.2 Utility Functions

Utility functions are identical, quasiconcave, homogeneous of some degree \( r \) where \( 0 < r \leq 1 \), twice differentiable, and non-decreasing.

\[
u(ax_i, ay_i) = a^r u(x_i, y_i).
\]

\(^{10}\)This is the crucial point for Piggott and Whalley's 1996 argument in favor of joint taxation.
In order to compare utility shares, we also need a well-defined lower bound on utility which is implied by the assumption of homogeneity, $u(0, 0) = 0$.

We find the utility possibility set for the household in the next section.

### 1.4 The Utility Possibility Set

We derive the UPF in two steps. We first solve for production efficiency. We then maximize the utility-vector of the spouses subject to the constraint that total consumption of both goods selects a point on the production possibility frontier.

#### 1.4.1 Production Efficiency

Given the technologies for producing the household good and household income, we find that the production possibility set is convex, but it will have a kink.

**Definition** The production possibility set $Y \subseteq \mathbb{R}^2_+$ is the set of all product pairs $(x, y) \geq 0$ satisfying the household's feasibility constraints (1.3), (1.4) and (1.5).

**Definition** Denote the frontier of the production possibility set $\partial Y$ where $\partial Y \subset Y$. For any pair $(a, b) \in \partial Y$, there exists no such pair $(a + \varepsilon, b) \in Y$ and no such pair $(a, b + \varepsilon) \in Y$ for $\varepsilon > 0$. $\partial Y$ is the set of production efficient product pairs.

**Observation 1.1** (1) The production possibility frontier $\partial Y$ has a kink denoted by $(x', y')$. For $0 < y \leq y'$, the slope of $\partial Y$ is constant $\frac{dy}{dx} = -\frac{p_y}{p_x} \frac{\partial f}{\partial x} = 0$. For $y > y'$, if $\frac{\partial^2 f}{\partial x \partial y} \geq 0$, the slope of $\partial Y$ becomes steeper $\frac{\partial f}{\partial y} \leq 0$. (2) Due to the difference in productivity of household income, $(1 - \tau_1)w_1 < (1 - \tau_2)w_2$, there is a unique time
allocation of spouses \((l_1, t_1), (l_2, t_2)\) associated with any \((x, y) \in \partial Y\). (3) The production possibility set \(Y\) is convex, compact and comprehensive.

See Figure 1.1 a. Figure 1.1b shows the special case in which time inputs of the spouses are perfect substitutes. Then we have a Ricardian Model of Trade between spouses with \(f(t_1, t_2) = b(t_1 + t_2)\) and \(b > 0\). Define \(a_i = (1 - \tau_i) w_i\), the PPF is piecewise linear with opportunity cost of \(\frac{\alpha_i}{b}\) up to the point \(y' = bT\); beyond this point, \(y > y'\), the opportunity cost for the household good is \(\frac{\alpha_i}{b}\).

**Definition** The opportunity cost or the shadow price of household production is equal to the absolute value of the slope of the PPF, \(|\frac{dx}{dy}|\), evaluated at \((x, y)\) for all \((x, y) \in \partial Y\).

Hence the shadow price for good \(y\) is independent of the particular point on the PPF for \(0 < y \leq y'\).

### 1.4.2 Household Efficiency

**Definition** The utility possibility set \(U \in \mathbb{R}^2_+\) is the set of all utility pairs \((u_1, u_2)\) satisfying the household’s feasibility constraints (1.3), (1.4) and (1.5).

**Definition** Denote the frontier of the utility possibility set \(\partial U\) where \(\partial U \subset U\). For any pair \((v_1, v_2) \in \partial U\), there exists no such pair \((v_1 + \varepsilon, v_2) \in U\) and no such pair \((v_1, v_2 + \varepsilon) \in U\) for \(\varepsilon > 0\). \(\partial U\) is the set of household efficient utility pairs.

**Definition** Denote \(\lambda_i\) the maximal utility a spouse can get if the other spouse does not consume anything, \(\lambda_i = \max \{u_i | u \in U\}\).
By symmetry the maximal utility for each spouse must be the same, \( \lambda_1 = \lambda_2 \), we refer to it simply as \( \lambda \). Clearly \((\lambda, 0) \in \partial U\) and \((0, \lambda) \in \partial U\).

We can now describe the utility possibility set for the couple.

**Observation 1.2** If \( u(x_i, y_i) \) is homogenous of degree \( 0 < \tau \leq 1 \) in \( x_i \) and \( y_i \) and both goods are private, the utility possibility set takes the form \( U = \left\{ u \geq 0 | u_1^\frac{1}{\tau} + u_2^\frac{1}{\tau} \leq \lambda^\frac{1}{\tau} \right\} \).

As a consequence,

- a change in the couple's tax rates results in a homothetic shift of \( \partial U \);
- for any \( u \in \partial U \), total household consumption \((x, y)\) is the same;
- \( U \) is compact, comprehensive and convex.

Figure 1.2 captures the idea of deriving the UPF.

By Observation 1.2, tax reform can shift the UPF homothetically inwards or outwards. We assume that a change from individual to joint taxation results in an outward shift.

Suppose that preferences are such that the spouses consume an amount of the household good that does not require full specialization of the wife. That is, the family produces on the linear part of the production possibility frontier. We can then find the following condition stating when the household benefits from tax reform.

**Observation 1.3** A tax reform is household efficient if the change in family net wages is positive.
1.5 Household Behavior 1: The Bargaining Approach

The bargaining problem of the couple is composed of two elements: the utility possibility set $U \subset R_+^2$ and a disagreement point $d \in U$. As stated in Observation 1.2, our assumptions about utility functions and production guarantee that $U$ is compact, comprehensive and convex.

We define the disagreement point $d$ as the utility of each spouse when single. The tax rates for singles are the same as the tax rates they face under individual taxation in marriage. We find $d_i$ by solving the problem

$$\max u(x_i, y_i)$$
$$s.t. \quad x_i = (1 - \beta_i) w_i l_i;$$
$$y_i = f(t, 0);$$
$$t_i + l_i = T.$$

**Observation 1.4** *In the present model $d_1 < d_2$. Moreover, $d \in U$, but $d \notin \partial U$.*

Observation 1.4 guarantees that there are gains from cooperation and that a lower net-wage rate translates into a lower bargaining power for the wife.

**Definition** A bargaining solution is a rule that assigns a solution vector $\varphi(U, d) \in U$ to every bargaining problem $(U, d)$ where $\varphi(U, d) \geq d$ and $\varphi(U, d) \in \partial U$.

Before introducing specific bargaining solutions we need more definitions and notation.

**Definition** Spouse $i$'s bliss utility: Denote $\bar{u}_i$ $i$'s maximal utility if $j$ receives $d_j$: $\bar{u}_i =$
max \{ u_i | u_j = d_j \text{ and } (u_i, d_j) \in U \}. The bliss gain one spouse can receive is given by 
\bar{u}_i - d_i.

**Definition** The point on the UPF where both spouses' utilities are equal is given by \( u^0 \):
\[ u^0 = \{(u_1, u_2) \mid u_1 = u_2 \text{ and } (u_1, u_2) \in \partial U \}. \]

We refer to the vertical and horizontal axes originating in \( d \) as \( d \)-axes. The set of individually rational points on the UPF is given by \( S \subset \partial U \) with \( (d_1, \bar{u}_1) \) and \( (\bar{u}_2, d_2) \) as its endpoints. The area that is contained in the intersection between \( S \) and the \( d \)-axes is denoted \( (S, d) \); it is the set of all individually rational utility pairs \( (u_1, u_2) \). (See Figure 1.3.)

Suppose \( \varphi (U, d) = (u_1^*, u_2^*) \). Then \( (u_i^* - d_i) \) denotes \( i \)'s gains from cooperation under bargaining solution \( \varphi (U, d) \). We will also refer to \( \frac{u_i^*}{u_i} \) as the utility ratio and \( \frac{u_i^* - d_i}{u_i - d_i} \) as the gains ratio under the bargaining solution \( \varphi (U, d) \).

We consider four solutions, Egalitarian (EGAL), Equal Split of Cooperative Gains (ES), Nash Bargaining (NBS) and Kalai-Smorodinsky (KSS) Solution.

**Definition** EGAL is the unique \( u^* \) such that
\[
\begin{cases} 
  u^0 & \text{if } d \leq u^0, \\
  (\bar{u}_i, d_j) & \text{if } d \nleq u^0 \text{ and } d_i < d_j.
\end{cases}
\]

EGAL distributes equal utility to both spouses whenever this is individually rational. Otherwise the wife receives her bliss utility \( \bar{u}_1 \).

**Definition** ES is the unique \( u^* \) that equalizes gains between spouses: \( u_1^* - d_1 = u_2^* - d_2 \).

ES divides gains from cooperation \( (u_i - d_i) \) equally.

Egal and ES are not independent of affine transformations of the utility function, but the next two bargaining solutions are.
NBS is the unique \( u^* \) that maximizes the Nash Product (NP) over \( S \):

\[
NP = (u_1 - d_1)(u_2 - d_2).
\]

Figure 1.4 illustrates the well known construction of NBS. The tangency at \((u_1^*, u_2^*)\) has its midpoint at \((u_1^*, u_2^*)\) when the intercepts of this tangency with the \(d\)-axes are taken as its endpoints.

**Definition** KSS is the unique \( u^* \) that equalizes relative gains between spouses

\[
\frac{u_1^* - d_1}{\bar{u}_1 - d_1} = \frac{u_2^* - d_2}{\bar{u}_2 - d_2}.
\]

Figure 1.5 illustrates the well known construction of KSS.

All four bargaining solutions allocate at least as big a utility share to the husband as to the wife (Lemma 1.1), but EGAL and ES are more extreme than NBS and KSS (Lemma 1.2). EGAL is an extreme solution, because the disagreement point plays only the role of a lower bound. Whenever equal utility shares satisfy individual rationality, EGAL implies that each spouse's utility only depends on family resources, i.e. that the income pooling hypothesis holds. As noted there is substantial empirical work that rejects the income pooling hypothesis, which makes EGAL an unlikely candidate for an allocation rule between spouses.\(^{11}\)

On the other extreme, we have ES. This solution is symmetrically implausible because it preserves entirely the difference between the spouses' utilities in the disagreement outcome. It coincides with NBS and KSS when the UPF is linear (Figure 1.6), but when \(0 < r < 1\), NBS and KSS have the normatively appealing property of distributing a higher share of gains to the spouse with lower disagreement utility than ES while still

\(^{11}\)See e.g. Bourguignon et al. (1993), Browning et al. (1994), Lundberg et al. (1997), Rubalcava and Thomas (2000), Woolley (2000), and Attanasio and Lechene (2002).
assigning a higher absolute utility to the spouse with higher disagreement point (see Figure 1.7).\textsuperscript{12}

**Lemma 1.1** EGAL, ES, NBS and KSS with \( d = (d_2, d_1) : d_2 > d_1 \) and \( U \) specified above, have solutions that satisfy the following two properties.

1. \( u_1^* \leq u_2^* \)

2. \( u_1^* - d_1 \geq u_2^* - d_2 \)

See Figure 1.7.

**Lemma 1.2** The NB and KS solution vectors, \( u^N \) and \( u^{KS} \), lie between the egalitarian and equal split of cooperative gains solution vectors, \( u^E \) and \( u^{ES} \), on the UPF: \( u_1^{ES} \leq u_1^N, u_1^{KS} \leq u_1^E \) with inequalities reversed for the husband.

See Figure 1.7. Lemma 1.2 is closely related to a result very recently obtained by Anbarci (2002), in which he shows that KSS always assigns a larger utility share to the person with lower disagreement utility than NBS if the functional form of the utility possibility frontier is the same as in our model and the disagreement point is given by \( d_1 = 0 \) and \( d_2 > 0 \). Lemma 1.2 does not rank the wife’s utility share under KSS and NBS, because with \( d_1 > 0 \), the wife’s utility share can be larger or smaller under NBS than under KSS.\textsuperscript{13}

\textsuperscript{12}Another argument for focusing on NBS and KSS rather than EGAL and ES is their better non-cooperative foundation.

\textsuperscript{13}For example, \( \partial U : u_1^2 + u_2^2 = 1, \)

\[
\begin{align*}
d_1 &= .2, d_2 = .4 & d_1 &= .8, d_2 = .85 \\
u_1^N &= 0.64072 < u_1^{KS} = 0.64747 & u_1^N &= 0.67643 > u_1^{KS} = 0.67620 \\
u_2^N &= 0.76777 > u_2^{KS} = 0.76209 & u_2^N &= 0.73651 < u_2^{KS} = 0.73672
\end{align*}
\]
1.5.1 Tax Rates within Marriage Change

We are now able to determine the effects of tax reform on intrafamily distribution within the first model of household behavior.

**Question** Suppose that tax reform from individual to joint taxation shifts the UPF outwards. Let \((u_1^*, u_2^*)\) be the solution before tax reform, \((u_1^{**}, u_2^{**})\) after tax reform.

- **Q0**: Is the homothetic solidarity property satisfied?
  
  That is, \((u_1^{**}, u_2^{**}) \geq (u_1^*, u_2^*)\).

- **Q1**: Are gains from cooperation shared more equally?

- **Q2**: Do the utilities of the spouses become more equal?

Note that \(Q_0\) only requires an ordinal utility measure, while \(Q_1\) is only independent of affine transformations of \(u\). Finally, \(Q_2\) is not independent of a change in the zeros of \(u\).

The disagreement point does not depend on the new tax rates. Thus, the question whether spouses are better off under the tax reform, depends on the relative position of the new UPF (joint taxation) to the initial UPF (individual taxation), only.

**Proposition 1.1** Consider Nash and KS bargaining solutions.

- The Homothetic Solidarity Property is satisfied \((u_1^{**}, u_2^{**}) > (u_1^*, u_2^*)\).

- The utility ratio becomes more equal\(^{14}\)
  \[
  \frac{u_1^*}{u_2^*} > \frac{u_1^{**}}{u_2^{**}}.
  \]

- The cooperative gains ratio becomes more equal
  \[
  \frac{u_1^{**} - d_1}{u_2^{**} - d_2} < \frac{u_1^* - d_1}{u_2^* - d_2}.
  \]

On a technical note, this result is of some significance, since it is known that for NBS and KSS an arbitrary expansion of the utility possibility set does not necessarily make

\(^{14}\)The proof in the case of KSS is not complete but numerical evidence strongly supports the statement.
both persons better off (e.g. Chun and Thomson, 1988; and Keiding and Moulin, 1991). In the class of utility possibility sets considered here, the utility of both spouses increases under NBS and KSS if the utility possibility set expands. By restricting our attention to identical and homogenous utility functions we have found a way to summarize the information of the utility possibility set in two parameters, $\lambda$ and $r$. Although it is not equivalent to the problem of splitting only one good between two individuals it bears some similarity to it. It is then intuitive that the results for NBS and KSS obtained in the pie-splitting case carry over to the case of homothetic and symmetric utility possibility sets. Instead of restricting the number of goods to be equal to one but allowing for any concave utility functions, we impose strong restrictions on the utility functions but we can allow for any positive number of goods to be shared.

It has to be stressed that the present model is more likely to apply to couples without children for whom household public goods play a minor role. With the presence of children, however, household public goods become more important. In this case we should not ignore the possibility of a disagreement point within marriage determined by the Nash equilibrium of a private-contribution-to-the-public-good game or a disagreement point within marriage in which the division of labor is chosen according to sanctioned gender-roles.\textsuperscript{15}

\textsuperscript{15}See Lundberg and Pollak (1993).
1.6 Household Behavior 2: The Competitive Approach

Instead of settling on a bargaining solution, spouses could base their share of utility on some distribution of family full income (Chiappori, 1997). Family full income is given by

\[ I = \sum (1 - \tau_i) w_i T. \]

The difference between the bargaining rules proposed in the previous section and the sharing rule here is that while the bargaining rules compare a hypothetical case with the current situation of full cooperation, the sharing rule is based on a measure of current contributions to the output in marriage. Discussions in family law suggest that spouses do not only care about total family income, but it is also essential how much each spouse contributes to it. For example, in Germany lawmakers debated recently whether one spouse should have the right to obtain full information about the income of the other spouse.\(^{16}\) We therefore propose a sharing rule of family full income in which one spouse’s own net-wage rate plays a crucial role. As Sen puts it “…notions of who is ‘contributing’ how much have many causal antecedents that call for closer scrutiny. However, the ‘deal’ that women get vis-à-vis men is clearly not independent of these perception problems regarding contribution (p.69).”

One might argue that since the wife will work more in household production, her net-wage rate should play a lesser role in evaluating her contribution, but as Sen argues “Division between “gainful” and other activities is quite arbitrary; […] But the issue is not whether activities within the household are really less productive, but whether they are perceived as such (p.70).”

\(^{16}\)The issue was referred to the committee of justice in 1999, no final decision has been made.
We call Separate Accounts the sharing rule that allocates the stand alone income to each spouse. Under Separate Accounts, the ratio between the wife’s and the husband’s income share is equal to the ratio of the wife’s and the husband’s net-wage rate.\textsuperscript{17}

Separate Accounts: \[ I_i = (1 - \tau_i) w_i T. \]

Once family full income is distributed, we find the price vector for the competitive equilibrium allocation. It is given by the price of the market good, \( p_x = 1 \), and by the shadow price of the household good, \( p_y \) (Observation 1.1). In order to simplify the theoretical analysis in this section, we assume that preferences and technologies are such that the spouses always choose an allocation above the kink of the PPF (see Figure 1.1a). That is, the optimal amount of the household good requires \( \tau_1^* < T \) and therefore \( \tau_2^* < T \) and the opportunity cost of the household good is constant. Each spouse chooses his or her consumption bundle \((x_i^*, y_i^*)\) by solving the problem

\[
\max_{x_i, y_i} u(x_i, y_i)
\]

subject to the individual budget constraint

\[ x_i + p_y y_i = I_i. \]

\textbf{Observation 1.5} In Separate Accounts the wife consumes more of the market good than her net wage would afford her to buy, i.e. there is an actual money transfer from the husband to the wife.

\textsuperscript{17}Even if spouses have a joint checking account, it might be very well possible that the two of them have kept track in their minds of how much each of them has put into the account and is therefore entitled to spend. Woolley (2000) reports that while wives are often involved or even primarily responsible for managing family finances, men are more likely to make cash withdrawals, which “may reflect a partner’s freedom not to account for expenditures.”
1.6.1 Tax Rates within Marriage Change

Note that the competitive approach does not incorporate a disagreement point. Thus, we cannot compare the gains ratio under the two tax regimes, but we are still able to determine whether the sharing rule satisfies the homothetic solidarity property and whether the inequality between spouses' utilities increases with tax reform.

Question Suppose that tax reform from individual to joint taxation shifts the UPF outwards. Let \((u^*_1, u^*_2)\) be the solution before tax reform, \((u^{**}_1, u^{**}_2)\) after tax reform.

\[ Q_0 : \]
Is the homothetic solidarity property satisfied?
That is, \((u^{**}_1, u^{**}_2) \geq (u^*_1, u^*_2)\).

If so,

\[ Q_1 : \]
Do the utilities of spouses become more equal?

Proposition 1.2 Separate Accounts does not satisfy the homothetic solidarity property.

Thus, inequality between spouses increases

\[ u^{**}_1 < u^*_1; u^{**}_2 > u^*_2. \]

Under the Separate Accounts there are losers and winners in the household, even if tax reform expands the utility possibility set. Our result is suggestive of why family tax reform continues to be a subject of vigorous debate in many countries.
1.7 Conclusion

The driving assumption behind our results is that the wife’s net-wage rate is lower than the husband’s which guarantees the husband a bigger utility share under all allocation rules than the wife. Since the joint tax rate lies between the tax rates under individual taxation, the gap between net-wage rates is wider under joint taxation than it is under individual taxation. This difference plays a role in the competitive approach, because the income sharing rule depends crucially on the stand alone income of each spouse. In the bargaining approach, the change in the net-wages plays a lesser role in intrafamily distribution, because the disagreement utility of spouses depends on their net-wage rates outside marriage and therefore remains the same before and after tax reform. Yet the change in the net-wages of spouses determines whether both spouses benefit or loose from tax reform by expanding or shrinking the utility possibility set.

Empirical evidence supporting a specific allocation rule is weak.\textsuperscript{18} We have given examples of reasonable rules and - in future research - hope to develop empirical tests that will let us distinguish between allocation rules. One immediate testable implication of the present model is the following. The rules under the competitive approach depend only on the net-wage rates within marriage but the bargaining rules depend on both net-wage rates when single and when married. Since the net-wage rates within and outside marriage differ for couples in countries with joint taxation, we should be able to empirically distinguish between bargaining and income sharing rules.\textsuperscript{19}

Several issues arise when characterizing a tax reform. Tax rates in reality depend

\textsuperscript{18}Specifically, Bourguignon et al.(1993) find in their paper evidence against NBS.
\textsuperscript{19}With this test, we cannot rule out bargaining models with a disagreement point within marriage, even if the bargaining models with divorce threat point are rejected by the data.
on earned income and not the wage rate of an individual. It can be argued that we focus on a woman whose earnings potential is sufficiently small, so that she would never face a tax rate higher than $\beta_1$, even if she were single and worked more than in marriage. It would also have to be assumed that even if the husband could work less at a lower marginal tax rate, he would never choose to do so.

Since we dealt with both the bargaining and the competitive approach, the present model imposes substantial restrictions on the nature and production of the household good, and the utility functions. While the bargaining approach is more sensitive to the utility specifications, the competitive approach imposes more severe restrictions on the production side of the model.

*Increasing Returns to Scale.* The analysis under the bargaining approach can be easily extended to incorporate increasing returns to scale in household production, but it will be difficult to deal with increasing returns under the competitive approach. The competitive equilibrium fails to achieve Pareto optimality when the production possibility set is not convex.

*Public Goods.* Housework is often interpreted as a public good. Only if the efficient amount of the public good remains the same along the UPF (e.g., Cobb-Douglas or quasi-linear utility function), will tax reform shift the UPF homothetically. For instance, if preferences are of the Cobb-Douglas form, the UPF is given by

$$\partial U : \lambda^{\frac{1}{\gamma}} = u_1^{\frac{1}{\gamma}} + u_2^{\frac{1}{\gamma}}$$

where $\gamma$ is the taste parameter for the private good. A disagreement point within marriage might then be more plausible than a disagreement point outside marriage and thus the
fall-back position of each spouse will depend on tax reform as well. The disagreement point within marriage will be more asymmetric under joint than under individual taxation. It is conceivable that the wife could be worse off with joint taxation even if the UPF shifts outwards, or if both spouses benefit from tax reform, inequality between husband and wife might increase.

For the sharing rule approach, we can subtract household expenditure on the public good from family full income and then distribute remaining family full income between spouses. Still, the question remains how much of family income each spouse should receive after the public good is paid for. Do spouses divide the cost for the public good between them (share the mortgage on the house equally) or will the spouse with higher net-wage rate contribute more?

More restrictions on the model are necessary to obtain unambiguous effects of tax reform when the model incorporates household public goods.

*Homothetic Utility Function.* Our results under the competitive approach do not depend on homothetic shifts of the UPF, homothetic preferences would do. For the bargaining approach, we conjecture that still both spouses will be better off under NBS and KSS if the UPF shifts outwards, but inequality between spouses might increase or decrease.

*Other Income Sharing Rules.* Sharing rules like the following two might seem reasonable. (1) Proportional Transfer transfers a fraction of the husband’s stand alone income to the wife.

\[ I_1 = (1 - \tau_1) w_1 T + \phi (1 - \tau_2) w_2 T, \]

\[ I_2 = (1 - \phi) (1 - \tau_2) w_2 T, \]
where $0 < \phi < 1$. With a switch to joint taxation, the transfer to the wife increases as the husband’s stand alone income increases. If the fraction is small enough, it will not compensate the wife for the decrease in utility due to the fall in her own stand alone income. (2) A compromise between Separate Accounts and Equal Split of Income will produce the rule

$$I_i = (1 - \theta) \frac{((1 - \tau_1) w_1 + (1 - \tau_2) w_2) T}{2} + \theta (1 - \tau_i) w_i T,$$

where $0 < \theta < 1$. Depending on the size of $\phi$ and $\theta$ in the two income sharing rules, it is possible that both spouses gain under joint taxation if the UPF shifts outwards. At the same time, the husband’s utility increases by a larger percentage than the wife’s utility, increasing inequality between spouses.

Analyzing the implications of tax reform and other family policies on intrafamily distribution is important from different points of view. First, neoclassical theory is built on individual choice and welfare analysis should not stop at the family level but consider individual welfare. Second, empirical evidence suggests that families do not pool their income (Hoddinott et al. 1997). Family policies do not necessarily have the same impact on all family members leading to possibly undesired policy effects. As demonstrated in this essay, under Separate Accounts inequality between spouses increases with joint taxation, even if the utility possibility set expands.

---

$^{20}$Hervé Moulin suggested this rule.
1.8 Appendix

1.8.1 Proof of Observation 1.1

Let \((1 - \tau_i) w_i = a_i\). Then production efficiency requires

\[
\max_{x,y} \quad x = a_1 l_1 + a_2 l_2 \\
\text{s.t.} \\
y = f \left((T - t_1), (T - t_2)\right) \cdot \\
l_1 + t_1 = T
\]

and the condition for an interior solution is

\[
\frac{\partial f}{\partial t_1} = \frac{a_1}{a_2} \quad \text{and} \quad \frac{\partial f}{\partial t_2} = \frac{a_1}{a_2} \quad (1.6)
\]

Since \(a_1 < a_2\), the wife spends more time in household production than the husband. Condition (1.6) is satisfied as long as household production requires less than the full amount of the wife's time, i.e. \(t_1 < T\). By CRS, condition (1.6) also implies that spouses sacrifice the same amount of household income in order to produce an additional unit of \(y\) as they did in order to produce the previous unit. Thus, the PPF is linear up to the point \((x', y')\) where

\[
\frac{f_1(T, t_2)}{f_2(T, t_2)} = \frac{a_1}{a_2}
\]

holds. For \(y > y'\) opportunity cost of increasing the household good further will increase, since only the husbands' time (which has a higher productivity in producing income) can now be used. For each additional unit of the husband's time that we use in household production, we loose \(a_2\) units of income. Since \(\frac{\partial f}{\partial t_1} \geq 0\), each additional unit of household
good will require at least as much time of the husband as the previous. Opportunity cost
of the household good increases. Figure 1.1a gives the idea.

Given this shape of the production possibility frontier and free disposal, the pro-
duction possibility set is compact, comprehensive and convex.

1.8.2 Proof of Observation 1.2

Figure 1.2 captures the idea of deriving the UPF. Take any pair \((x, y) \in \partial Y\) and
divide it efficiently between spouses. Since the utility function is identical for both spouses,
homogenous and quasiconcave, provided \((x, y) > 0\)

\[
\frac{\partial u_1/\partial x_1}{\partial u_1/\partial y_1} = \frac{\partial u_2/\partial x_2}{\partial u_2/\partial y_2}.
\]

With homothetic and identical preferences, efficient division of \((x, y)\) requires that spouses
consume the two goods in the same ratio. We can write

\[
\begin{align*}
u_1 &= u(x_1, y_1) = u(ax, ay) \\
u_2 &= u(x_2, y_2) = u((1 - a)x, (1 - a)y) \\
1 &> a > 0.
\end{align*}
\]

By homogeneity of the utility function

\[
\begin{align*}
u_1 &= a'u(x, y) & (1.7) \\
u_2 &= (1 - a)'u(x, y). & (1.8)
\end{align*}
\]

Solving for \(a\),

\[
u(x, y)^{1/2} = u_1^{1/2} + u_2^{1/2}.
\]
Choosing optimal \((x,y)\) for \(u(x,y)\) is therefore all we need to determine \(\partial U\). Let \(u(x^*,y^*) = \lambda\). Then

\[
\partial U : u_1^{\frac{1}{2}} + u_2^{\frac{1}{2}} = \lambda^{\frac{1}{2}},
\]

where \((x^*,y^*)\) is to be obtained by

\[
\max_{l_1,l_2} u((1 - \tau_1) w_1 l_1 + (1 - \tau_2) w_2 l_2, f((T - l_1), (T - l_2))).
\]

A change in wage rates or tax rates will have the same proportional effect on \(u_1, u_2,\) and \(\lambda\).

We have just determined the Pareto frontier of the utility possibility set. Allowing for free disposal, the utility possibility set is given by

\[
U = \left\{ u \geq 0 | u_1^{\frac{1}{2}} + u_2^{\frac{1}{2}} \leq \lambda^{\frac{1}{2}} \right\},
\]

it is compact, comprehensive and convex.

1.8.3 Proof of Observation 1.3

Assume that we are at \(y < y'\), then the shadow price of household production \(p_y\) is constant and given by \(p_y = c((1 - \tau_1) w_1, (1 - \tau_2) w_2)\), where \(c()\) denotes the constant marginal cost. Using indirect utility, we can write

\[
\lambda = v(p_y, I),
\]

where \(I = \sum_i (1 - \tau_i) w_i\). Differentiating \(\lambda\) with respect to the tax rates \((1 - \tau_1)\) and \((1 - \tau_2)\), we have

\[
d\lambda > 0 \Leftrightarrow \sum_i \left( \frac{\partial v}{\partial p_y} \frac{\partial p_y}{\partial (1 - \tau_i) w_i} + \frac{\partial v}{\partial I} \frac{\partial I}{\partial (1 - \tau_i) w_i} \right) w_i d(1 - \tau_i) > 0.
\]
This proof uses similar arguments as the proof of Proposition 1.2. See Proposition 1.2 for the derivation of \( \frac{\partial v}{\partial p_y}, \frac{\partial p_y}{\partial (1-\tau_i)w_i} \), and \( \frac{\partial l}{\partial (1-\tau_i)w_i} \). After manipulations we find

\[
d\lambda > 0 \iff \sum_i w_i l_i d (1 - \tau_i).
\]

### 1.8.4 Proof of Observation 1.4

We find \( d_i \) by solving the problem

\[
\max_{l_i} u((1 - \beta_i) w_i l_i, f(0, T - l_i)).
\]

Suppose the single female chooses the bundle \((x_1^d, y_1^d)\) which she produces with \((l_1^d, t_1^d)\). With the same division of his time the single male is able to consume more of good \( x \), since \((1 - \beta_1) w_1 l_1^d < (1 - \beta_2) w_2 l_2^d\). Whichever allocation of time the single male chooses he must be better off than the female.

Let \( U(\alpha) \) denote the utility possibility set under joint taxation, and \( U(\beta) \) denote the utility possibility set under individual taxation. By assumption, \( U(\beta) \subset U(\alpha) \). We first show that the disagreement point lies strictly below the Pareto frontier when we have individual taxation, i.e. \( d \not\in \partial U(\beta) \). Clearly, \( d_i > 0 \), and two people when married can choose to do exactly what they have done when they are single, thus \( d \in U(\beta) \). If \( \frac{\partial^2 f}{\partial l_1 \partial l_2} > 0 \),

\[
f\left(t_1^d, t_2^d\right) > f\left(t_1^d, 0\right) + f\left(0, t_2^d\right).
\]

With the same time inputs as singles, spouses produce more of the household good, \( d \not\in \partial U(\beta) \).\(^{21}\) It follows \( d \in U(\alpha) \) and \( d \not\in \partial U(\alpha) \).

\(^{21}\)In the special case where \( \frac{\partial^2 f}{\partial l_1 \partial l_2} = 0 \), we can apply Ricardo’s argument of comparative advantage in international trade. Being single is analogous to autarky, and marriage to international trade.
1.8.5 Proof of Observation 1.5

Under Separate Accounts and with identical and homothetic preferences, spouses consume the market good and the household good in proportion to their net wage rate. That is,

$$\frac{x_1}{x_2} = \frac{(1 - \tau_1) w_1}{(1 - \tau_2) w_2}.$$  

There will be a transfer of actual income from the husband to the wife if

$$x_1 > (1 - \tau_1) \cdot w_1 l_1 \land x_2 < (1 - \tau_2) \cdot w_2 l_2$$

or equivalently if

$$\frac{x_1}{x_2} > \frac{(1 - \tau_1) w_1 l_1}{(1 - \tau_2) w_2 l_2}.$$  

Since the wife spends more time in household production than the husband, $l_1 < l_2$ and above inequality holds.

1.8.6 Proof of Lemmas 1.1 and 1.2

1. EGAL. Whenever the UPF intersects with the $d_2$-axis below or at the 45°-line, $u^*_1 = u^*_2$. If the UPF intersects with $d_2$-axis above the 45°-line and $u^*_1 = \bar{u}_1$, $u^*_2 = d_2$.

2. ES. Obvious.

3. NBS. Let $u^{ES}_{i} - d_i = A_i$, that is $A_1 = A_2 = A$, and $u^{ES}_{i} - d_i = B_i$.$^{22}$ Then $B_2 < A < B_1$.

Let $\sigma (X)$ denote the absolute value of the slope at point $X$ on the UPF. Then $\sigma (A) < 1$, and $\sigma (B) = 1$. First, we show that the Nash Product increases by a small

$^{22}$It is obvious that $u = (\bar{u}_1, d_2)$ cannot maximize the Nash Product.
enough shift to the left of $B$, 

$$dNP = B_2 dB_1 + B_1 dB_2 > 0$$

since we have $dB_1 + dB_2 = 0$ by $\sigma (B) = 1$. Second, we show that the Nash Product increases by a small enough shift to the right of $A$, 

$$dNP = A_2 dA_1 + A_1 dA_2 > 0$$

since $dA_2 = -\lambda dA_1$ by $\sigma (A) = \lambda < 1$. Thus the NBS lies between $A$ and $B$ and therefore $u_1^* < u_2^*$, $u_1^* - d_1 \geq u_2^* - d_2$.

4. KSS. First we show that the bliss-utility point $(\overline{u}_1, \overline{u}_2)$ is above the $45^\circ - line$. For given $\lambda$ and $d$, $\overline{u}_i = \left(\lambda \frac{d_j}{d_j} - d_j \right)^r$. Clearly, $\overline{u}_i$ decreases in $d_j$ and hence $\overline{u}_1 < \overline{u}_2$. Both disagreement point and bliss utility point $\overline{u}$ lie above the $45^\circ - line$, therefore $u^*$ lies above the $45^\circ - line$. Next we show that $\overline{u}_1 - d_1 > \overline{u}_2 - d_2$ which is equivalent to $u_1^* - d_1 > u_2^* - d_2$. Note that $\overline{u}_1 - d_1 > \overline{u}_2 - d_2 \Leftrightarrow \overline{u}_1 + d_2 > \overline{u}_2 + d_1$. We can write

$$\overline{u}_i + d_j = f (d_j) + d_j$$

We need

$$f (d_2) + d_2 > f (d_1) + d_1.$$ 

We know that $d_2 < f (d_1)$ by Observation 1.4. We have to distinguish between two cases. Recall that the point on the UPF where $u_1 = u_2$ is given by $u^0$. By symmetry of $\partial U$, $f (u^0) = u^0$. The slope of $f (u^0)$ is $-1$. Notice that $f (z) + z$ is increasing in $z$ for $0 \leq z \leq u^0$, because the the slope at $f (z)$ is greater than $-1$. But for $u^0 \leq z \leq \lambda$, $f (z) + z$ is decreasing in $z$ because the slope at $f (z)$ is less than $-1$. 


Case 1: \( u^0 < d_2 < f(d_1) \). For this range of \( d_2 \), \( f(d_2) + d_2 \) reaches its lower bound at
\( d_2 = f(d_1) \). In this case

\[
f(f(d_1)) + f(d_1) \geq f(d_1) + d_1.
\]

By symmetry \( f(f(z)) = z \) and hence

\[
d_1 + f(d_1) \geq f(d_1) + d_1.
\]

Case 2: \( d_1 < d_2 < u^0 \). For this range of \( d_2 \), \( f(d_2) + d_2 \) reaches its lower bound at
\( d_2 = d_1 \). In this case

\[
f(d_1) + d_1 \geq f(d_1) + d_1.
\]

This completes the proof.

1.8.7 Proof of Proposition 1.1

Nash Bargaining Solution

See Figure 1.8.

If the utility ratio remains the same, we are at \( \alpha u^* \) on UPF 2. If the gains ratio remains the same, we are at \( u' \) on UPF 2 where

\[
\frac{u^*_2 - d_2}{u^*_1 - d_1} = \frac{u'_2 - d_2}{u'_1 - d_1}.
\]

At \( u' \) a parallel line to the tangency \( \overline{AB} \) must have \( u' \) as its midpoint. But by homothetic expansion we know that the tangency at \( \alpha u^* \) has the same slope as \( \overline{AB} \) and therefore the slope of the tangency at \( u' \) - being to the southeast of \( \alpha u^* \) - must be steeper than the slope of \( \overline{AB} \). Call the points where the tangency at \( u' \) intercepts with the \( d \)-axes \( C \) and \( D \).
respectively. Then $|Cu'| > |u'D|$; the NBS must lie to the northwest of $u'$. In point $\alpha u^*$ the slope of the tangency is the same as in $u^*$. But since this point lies to the northwest of $u'$, $\alpha u^*$ cannot be the midpoint of $EF$. Then $|E\alpha u^*| < |\alpha u^*F|$. Therefore NBS must lie between $\alpha u^*$ and $u'$.

**Kalai-Smorodinsky Solution**

Given $\lambda$ before tax reform, we want to know how the gains ratio changes with tax reform, i.e. as $\lambda$ increases. We use the following equality. By KSS

\[
\frac{\left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^r - d_2}{\left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^r - d_1} = \frac{u_2 - d_2}{u_1 - d_1}.
\]

Taking elasticity instead of the derivative will not change the sign and will give us a less complicated expression.

\[
\frac{\partial \ln \left(\frac{\left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^r - d_2}{\left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^r - d_1}\right)}{\partial \ln \lambda} = \frac{\left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^{r-1} \lambda^r}{\left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^r - d_2} - \frac{\left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^{r-1} \lambda^r}{\left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^r - d_1}.
\]

(1.10)

Below we show that the sign of (1.10) is positive. Since both denominators are positive we just need to know whether

\[
\left(\left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^r - d_2\right) \left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^{1-r} - \left(\left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^r - d_2\right) \left(\frac{\lambda^r}{\lambda^1} - \frac{d_1}{d_2}\right)^{1-r} > 0.
\]

(1.11)

First, note that for $r \to 1$ the left hand side of (1.11) is zero, and for $r \to 0$ inequality (1.11) holds. Inequality (1.11) is also satisfied for $r = \frac{1}{2}$. We now check if for any $0 < r < 1$ we can sign (1.11). Dividing both sides by $\lambda^r$ and after a change in variables, such that $\delta_i = \left(\frac{d_i}{\lambda}\right)^{\frac{1}{r}}$,

\[
\delta_1 - \delta_2 - \delta_1 (1 - \delta_2)^{1-r} + \delta_2 (1 - \delta_1)^{1-r} > 0.
\]

(1.12)
In Observation 1.4, we have established that \( \bar{u}_i - d_i > 0 \) which is equivalent to

\[ (1 - \delta_2)^r - \delta_1^r > 0. \]

Let

\[ \varphi = \delta_2^r (1 - \delta_1)^{1-r} - \delta_1^r (1 - \delta_2)^{1-r} - \delta_2. \]

Inequality (1.11) holds if and only if for \( \delta_1 < \delta_2 < 1 - \delta_1 \) and \( \delta_1 < \frac{1}{2} \),

\[ \varphi > -\delta_1. \]

Observe that at \( \delta_2 = \delta_1 \) and \( \delta_2 = 1 - \delta_1 \)

\[ \varphi = -\delta_1. \tag{1.13} \]

We make the following two arguments that will prove \( \varphi > -\delta_1 \).

1. \( \varphi \) is concave in \( \delta_2 \) on \( [\delta_1, \frac{1}{2}] \) and

2. \( \varphi \) decreases in \( \delta_2 \) on \( \left[ \frac{1}{2}, 1 - \delta_1 \right] \).

Taking first and second order derivatives with respect to \( \delta_2 \) yields

\[ \varphi_2 = r \delta_2^{r-1} (1 - \delta_1)^{1-r} + (1 - r) \delta_1^r (1 - \delta_2)^{-r} - 1, \tag{1.14} \]

\[ \varphi_{22} = r (1 - r) \left( \delta_1^r (1 - \delta_2)^{-r-1} - \delta_2^{r-2} (1 - \delta_1)^{1-r} \right). \tag{1.15} \]

Observe that \( \varphi_{22} \) is increasing in \( \delta_1 \). Therefore we just have to check whether for fixed \( \delta_2 \),

\( \varphi_{22} \leq 0 \) when \( \delta_1 \) is largest. This is the case for \( \delta_1 = \delta_2 \leq \frac{1}{2} \).

For argument 2, we show first that \( \varphi_2 \) is increasing in \( \delta_1 \) for \( \frac{1}{2} < \delta_2 \leq 1 - \delta_1 \). For

\( \delta_2 \in \left[ \frac{1}{2}, 1 - \delta_1 \right] \) it follows that

\[ \delta_1 < 1 - \delta_2 < \delta_2. \tag{1.16} \]
Taking the derivative of $\varphi_2$ with respect to $\delta_1$,

$$\frac{\partial \varphi_2}{\partial \delta_1} = \varphi_{21} = r \left( 1 - r \right) \left[ \frac{1}{1 - \delta_2} \left( \frac{1 - \delta_2}{\delta_1} \right)^{1-r} - \frac{1}{\delta_2} \left( \frac{\delta_2}{1 - \delta_1} \right)^{1-r} \right]. \quad (1.17)$$

The first expression in the square bracket of (1.17) is larger than the second term by constraint (1.16). Hence $\varphi_{21} > 0$. To complete the argument we need $\varphi_2 \leq 0$ at the largest possible value of $\delta_1$. At $\delta_1 = 1 - \delta_2$, in (1.14) we get $\varphi_2 = 0$.

We have thus shown that the husband receives proportionally more gains from cooperation as the UPF expands. Since $d$ does not change, this implies $u_2^* < u_2^r$. Next we show that the wife's utility increases with an expansion of the UPF as well. In order to evaluate the change in $u_1$, we solve a system of two equations

$$\frac{\left( \lambda^d - d_2^{\frac{1}{r}} \right)^{r} - d_1}{\left( \lambda^d - d_1^{\frac{1}{r}} \right)^{r} - d_2} = \frac{u_1 - d_1}{u_2 - d_2},$$

$$u_1^\frac{1}{r} + u_2^\frac{1}{r} - \lambda^\frac{1}{r} = 0.$$  

We change the variables to $\lambda^\frac{1}{r} = a, d_i^{\frac{1}{r}} = \varepsilon_i, u_i^\frac{1}{r} = v_i$. Then we can write above system of equations as

$$((a - \varepsilon_2)^r - \varepsilon_1^r)(v_2^r - \varepsilon_2^r) - ((a - \varepsilon_1)^r - \varepsilon_2^r)(v_1^r - \varepsilon_1^r) = 0$$

$$v_1 + v_2 - a = 0$$

Totally differentiating yields

$$V_1 \frac{dv_1}{da} + V_2 \frac{dv_2}{da} = -A$$

$$\frac{dv_1}{da} + \frac{dv_2}{da} = 1$$

\(^{23}\)Since $a$ and $v_i$ are positive monotonic transformation of $\lambda$ and $u_i$, the sign of the change in $u_1$ when $\lambda$ increases will be the same as the change in $v_1$ as $a$ increases.
where

\[ V_1 = -rv_1^{r-1}((a - \varepsilon_1)^r - \varepsilon_2^r) < 0, \]

\[ V_2 = rv_2^{r-1}((a - \varepsilon_2)^r - \varepsilon_1^r) > 0, \]

\[ -A = -r \left[ (a - \varepsilon_2)^{r-1}(v_2^r - \varepsilon_2^r) - (a - \varepsilon_1)^{r-1}(v_1^r - \varepsilon_1^r) \right]. \]

By Cramer's rule we find

\[ \tilde{\nu}_i = \frac{-A - V_2}{V_1 - V_2} > 0 \text{ if } -A < V_2. \quad (1.18) \]

We divide \( V_1, V_2 \) and \(-A\) by \( a^{2r-1}\) and get

\[ \frac{V_1}{a^{2r-1}} = \Psi_1 = -r z_1^{r-1}((1 - \delta_1)^r - \delta_2^r) < 0, \]

\[ \frac{V_2}{a^{2r-1}} = \Psi_2 = r z_2^{r-1}((1 - \delta_2)^r - \delta_1^r) > 0, \]

\[ -\frac{A}{a^{2r-1}} = \Lambda = -r \left[ (1 - \delta_2)^{r-1}(z_2^r - \delta_2^r) - (1 - \delta_1)^{r-1}(z_1^r - \delta_1^r) \right]. \]

where \( \delta_i = \left( \frac{d_i}{x} \right)^{\frac{1}{r}} \) and \( z_i = \left( \frac{u_i}{x} \right)^{\frac{1}{r}} \). The sign of \( \Lambda \) is the same as the sign of the change in the gains ratio and thus \( \Lambda > 0 \). Inequality (1.18) holds if and only if

\[ \frac{z_1^r - \delta_1^r}{(1 - \delta_1)^{1-r}} - \frac{z_2^r - \delta_2^r}{(1 - \delta_2)^{1-r}} < \left( \frac{(1 - \delta_2)^{r-1} - \delta_1^r}{z_2^{1-r}} \right). \]

Note that \( \left( \frac{(1 - \delta_2)^{r-1} - \delta_1^r}{z_2^{1-r}} \right) > \frac{z_2^r - \delta_2^r}{(1 - \delta_1)^{1-r}} \) because \((1 - \delta_2) > z_1\) and \((1 - \delta_1) > z_2\). Thus we have shown that \( \Lambda < \Psi_2 \) and therefore \( \tilde{\nu}_i > 0 \). As the UPF shifts outwards, the utility assigned to the wife by KSS increases. The homothetic solidarity property is satisfied.

1.8.8 Proof of Proposition 1.2

From Observation 1.1 we know that each unit of the household good is produced with the same amount of each spouse's time inputs \((\tilde{u}_1, \tilde{u}_2)\) as long as \( y \leq y' \). Then the
shadow price of the household good \( p_y \) must satisfy

\[
p_y = \sum_{i=1}^{2} (1 - \tau_i) w_i \tilde{t}_i.
\]

We can now rewrite the household budget constraint in terms of family full income (I)

\[
x + p_y y = \sum (1 - \tau_i) w_i T = I,
\]

where the left hand side is virtual household expenditures and the right hand side denotes family full income. By Shephard’s lemma taking the derivative of the cost function (given by \( p_y y \)) with respect to the price of an input factor yields the demand for the input factor and hence

\[
\frac{\partial p_y}{\partial (1 - \tau_i) w_i} = \frac{t_i}{y} = \tilde{t}_i.\tag{24}
\]

Indirect utility for each spouse is given by \( u(p_y, I_i) \). We can write the individual income share under Separate Accounts as

\[
I_i = (1 - \beta_i) w_i T.
\]

Taking the derivative of the indirect utilities with respect to each spouse’s net-wage rate yields

\[
\frac{\partial v_i}{\partial (1 - \tau_i) w_i} = -\lambda_i y_i \frac{\partial p_y}{\partial (1 - \tau_i) w_i} + \lambda_i T
\]

\[
\frac{\partial v_i}{\partial (1 - \tau_j) w_j} = -\lambda_j y_j \frac{\partial p_y}{\partial (1 - \tau_j) w_j},
\]

where \( \lambda_i \) here denotes the Lagrangian Multiplier, i.e. the marginal utility of the income share.\(^{25}\) We evaluate an infinitesimal change in the tax rates such that \( d(1 - \beta_1) < 0 \)

\(^{24}\) In the special case when spouses’ time inputs are perfect substitutes, \( \frac{\partial^2 f}{\partial t_1 \partial t_2} = 0 \), we have \( \tilde{t}_2 = 0 \), and \( \tilde{t}_1 = \frac{1}{2} \).

\(^{25}\) By Roy’s Identity \( \frac{\partial y_i}{\partial p_y} = -\lambda_i y_i, \frac{\partial u_i}{\partial I_i} = \lambda_i \).
\[ d(1 - \beta_2) \] \[ d v_1 = \lambda_1 \left( w_1 (y - y_1) \frac{t_1}{y} d(1 - \beta_1) - w_2 y_1 \frac{t_2}{y} d(1 - \beta_2) + w_1 l_1 y (1 - \beta_1) \right) < 0 \]

\[ d v_2 = \lambda_2 \left( -w_1 y_2 \frac{t_1}{y} d(1 - \beta_1) + w_2 (y - y_2) \frac{t_2}{y} d(1 - \beta_2) + w_2 l_2 d(1 - \beta_2) \right) > 0 \]

A further increase in the wife's tax rate and a further decrease in the husband's would again have the same implications as above. Thus we find that the wife is worse off and the husband better with tax reform from individual to joint taxation. Separate Accounts does not satisfy the homothetic solidarity property.

Since our sharing rule always guarantees the husband a bigger share of resources than the wife, utility shares between wife and husband are more equal under individual taxation.

1.9 References


\[ ^{26} \text{We follow Apps and Rees (1999a and b) who use the same method in analyzing the efficiency of tax reform.} \]


1.10 Figures

Figure 1.1a: PPF when inputs are imperfect substitutes.

Figure 1.1b: PPF when inputs are perfect substitutes.
Figure 1.2: Deriving UPF.

Figure 1.3: Notation.
\begin{align*}
|Au^*| &= |Bu^*| \\
\frac{u_2^* - d_2}{u_1^* - d_1} &= \frac{u_2' - d_2}{u_1' - d_1}
\end{align*}

Figure 1.4: NBS.

\begin{align*}
\frac{u_2' - d_2}{u_1' - d_1} &= \frac{u_2^* - d_2}{u_1^* - d_1}
\end{align*}

Figure 1.5: KSS.
Figure 1.6: $r = 1$.

Figure 1.7: NBS and KSS lie between ES and EGAL.
Figure 1.8: Both spouses gain under NBS.
Chapter 2

A Two-Period Bargaining Model of the Family

2.1 Introduction

Cooperative bargaining models of the family are often criticized for being static models (see e.g. Bergstrom, 1997; and Behrman, 1997). We can re-interpret them as dynamic models, but then we would have to argue that spouses can write a complete contract at the beginning of their marriage and that there is no possibility of renegotiation. In this case spouses bargain over intertemporal utilities comparing the gains of life-time cooperation to the utility of never cooperating. An alternative - and probably more realistic - way of addressing a long term relationship is to assume that spouses can renegotiate and that they bargain each period over the resource allocation between them (e.g. Muthoo, 1999).
Most recently, economists have developed dynamic models of family decision making. The models include frameworks in which spouses are Nash-bargainers in each period (e.g. Wells and Maher, 1996; Bolin et al., 2001; and Lich-Tyler, 2001) but also models that are closely related to Chiappori's collective approach\(^1\) (e.g. Aura, 2001; Basu, 2001; Ligon, 2000; and Iygun and Walsh, 2002).

This essay operates on the premise that most of the problems analyzed in multi-period models can also be addressed in a two-period model. This allows us to model the decision of each spouse explicitly and it makes it easier to analyze the effects of policies on the household as a whole and each spouse individually.

In the present model spouses take into account that their current labor supply decisions will effect their future bargaining power. This leads to a misallocation of the spouses' time. Husband and wife oversupply labor and do not spend enough time in household production compared to the intertemporally efficient allocation of time.\(^2\)

Spouses consume two non-durable goods, a household public good and a private good. They produce the household public good with time spent in household production. In order to pay for the expenditure on the private good they need to earn income through employment. The before-tax-wage rate of both spouses increases from the first to the second period with the amount of hours spent on the job. The husband earns a higher wage rate in both periods than the wife. In each period, husband and wife individually decide on their labor supply. Spouses then split utility gains from cooperation equally, where the threatpoint is determined by divorce. In the unique subgame perfect Nash equilibrium the

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\(^{1}\)Chiappori does not consider a dynamic framework, but above authors do.
\(^{2}\)Lich-Tyler (2001) shows that period-by-period bargaining can also lead to inefficiencies if the threat points are exogenously given. When spouses differ in their preferences, bargaining over life-time utility and period-by-period bargaining do not yield the same allocation.
husband works full time while the wife spends part of her time in household production, and the wife's strategic labor supply exceeds the efficient amount.

Several policy changes are examined, among them are the following. (1) If the tax rate of the wife when divorced decreases, her labor supply and her intertemporal utility increase while the intertemporal utility of the husband decreases. (2) A divorce law that splits income between former spouses equally is less distortionary than a divorce law that grants each former spouse his or her stand alone income, and it also equalizes utility shares between spouses. (3) Whenever the wife benefits from a tax reform from joint towards individual taxation of the family, inequality between spouses is also reduced. It is possible that the wife gains but the husband loses from a tax reform from joint towards individual taxation.

2.2 Related Literature

The present model is closely related to the work of Wells and Maher (1996), Lundberg (2002), Lommerund (1989), Konrad and Lommerund (2000), Wrede (2003) and Rainer (2002). Similarly to results in this chapter, all authors find that spouses underinvest in marriage-specific activities or overinvest in marketable skills.

Wells and Maher focus on a multi-period model in which renegotiation is possible. In contrast to our model, both activities, employment and household production increase the human capital in the respective sector. With the assumption that the efficient division of labor between spouses would be full specialization, Wells and Maher show that the spouse who would have to specialize in household production will refuse to do so. The
present model, however, allows us to derive both the efficient allocation and the equilibrium allocation of time. Our work also differs from Wells and Maher in that it looks at the effects of changes in family policies.

Lundberg considers a two-period bargaining model, in which spouses have a utility over a public good and a private good in the first period, while in the second period they only consume a private good. Lundberg solves for the efficient division of labor but does not solve for the equilibrium strategies of the spouses. The main focus of Lundberg's paper is the impact of child care policies on intrafamily distribution and on household efficiency, while the present work analyzes the effects of a change in the tax system of married and single people on the bargaining power of spouses. We also discuss efficiency and equity implications of changes in the probability of divorce and different alimony regulations.

The following papers do not consider renegotiation within marriage but they consider two stages. Lommerund assumes that spouses maximize a welfare function that takes into account two possible states in the second period: continued marriage or divorce with positive probability on both states. The household cooperatively chooses the amount of time each spouse spends in household and market activities. As the probability of divorce goes up, both spouses spend more hours in employment. The present model confirms that the labor supply of spouses increases with an increase in the probability of divorce and shows that if spouses non-cooperatively choose their labor supply an increase in the probability of divorce further increases the bias towards market labor.

In both Konrad and Lommerund's and Wrede's models spouses are able to enter a complete marriage contract, but spouses non-cooperatively choose their investment in
human capital before marriage. In Konrad and Lommerund (2000) spouses are Nash-bargainers within marriage with a non-cooperative marriage as the threatpoint. In a non-cooperative marriage, the utility of the spouse with higher human capital in marketable skills is higher than the utility of the spouse with higher investment in marriage-specific activities. This leads to an over-investment in education in marketable skills. Wrede (2003) analyzes investment in domestic human capital by both spouses prior to marriage and finds that there is underinvestment in family-specific human capital that enhances the future spouses’ capabilities as parents, because future spouses hope to free-ride on the prospective partner’s investment. He finds that there is more specialization during marriage if families are taxed according to the income splitting method rather than individual taxation and the person with higher productivity in rearing children will invest more in family-specific human capital prior to marriage if there is a switch in family taxation towards the income splitting method. This, however, might hurt the spouse who increases family-specific investment and Wrede concludes that young women who decide on their education will benefit from individual taxation while young men and already married couples will be the losers of an abolition of income splitting provided that women have an intrinsic comparative advantage for child care. Our model, in contrast, shows that even if people are already married they might have conflicting preferences with regard to joint or individual taxation.

In Rainer’s model spouses take the divorce law of asset division as given and share resources within marriage cooperatively using divorce as the threatpoint but non-cooperatively choose their level of physical investment.\(^3\) He then discusses changes in the

\(^3\) Rainer deals with a model in which spouses invest in assets but he does not consider the time allocation problem of spouses. See also Fethke (1984) for an early treatment of how divorce laws regarding asset division influence the investment decisions of husbands and wives.
divorce law that might enhance the efficiency of investment during marriage. Our work abstracts from investment in assets but discusses how changes in the divorce law regarding income flows between ex-spouses might impact the equilibrium division of labor within marriage.

2.3 The Model

Consider a couple with a wife ($i = f$) and a husband ($i = m$). There are two periods, $k = 1, 2$. In each period, both spouses are endowed with one unit of time. Spouses can neither borrow nor save and leisure does not generate any utility for the spouses. In each period, spouses can work in the market ($m_k, f_k$) and thereby produce family income or they can spend time in the production of a household public good ($y_k$). Denote $t_k$ total spousal time spent in household production,

$$t_k = 2 - f_k - m_k.$$

The household public good is produced by

$$y_k = h(t_k),$$

where $h(t_k)$ is a mapping from $\mathbb{R}_+$ to $\mathbb{R}_+$. The household production function is strictly concave and its derivative is convex and three times differentiable

$$h' > 0, h'' < 0, h''' \geq 0.$$

We also need the inverse function of $h'$ denoted by $g$ with $g(r) > 0$ and decreasing at a diminishing rate in $r$, i.e. $g' < 0, g'' \geq 0$. 

Income is used to purchase a private composite good in the market \( (x_k^m, x_k^f) \). The price of the composite good is taken as the numeraire.

The husband's first period wage rate before taxes is normalized to one and the wife's wage rate before taxes is given by \( w \) with \( w < 1 \). Husband and wife face a tax rate of \( (1 - \sigma) \) and \( (1 - \tau) \) on their wage rate.\(^4\) Both spouses can increase their second period wage rate by increasing the time spent on the job in the previous period. The percentage increase of the second period wage rate is equal to \( \alpha m_1 \) for the husband and \( \alpha f_1 \) for the wife with \( \alpha > 0 \). The husband's net-wage rate in the second period is therefore \( \sigma (1 + \alpha m_1) \). Denote \( w_2 \) the wife's second period before-tax-wage rate, then

\[
w_2 = w (1 + \alpha f_1),
\]

and her net-wage rate in the second period is given by \( \tau w (1 + \alpha f_1) \). We assume that in the second period the wife cannot reach the husband's first-period net-wage rate, even if she works full time in the first period. Put differently, the husband has an absolute advantage in producing family income\(^5\)

\[
\tau w (1 + \alpha) < \sigma. \tag{2.2}
\]

The family's budget set in each period is given by

\[
x_1^m + x_1^f \leq \tau w f_1 + \sigma m_1, \tag{2.3}
\]

\[
x_2^m + x_2^f \leq \tau w (1 + \alpha f_1) f_2 + \sigma (1 + \alpha m_1) m_2. \tag{2.4}
\]

In each period household production must satisfy

\[
y_k \leq h(t_k). \tag{2.5}
\]

\(^4\)We follow Apps and Rees (1999a and b) in the way family taxation is introduced in the model.

\(^5\)The cause for the absolute advantage is left open. Possible explanations are different educational choices before marriage or discrimination in the labor market.
When married, both spouses' utility functions in each period are identical and given by

\[ u_k^i = x_k^i + y_k. \]  \hspace{1cm} (2.6)

For simplicity, we assume that intertemporal utility is additively separable and that the discount factor is equal to one.\(^6\) Intertemporal utility for each spouse is given by

\[ u^i = u_1^i + u_2^i = x_1^i + x_2^i + y_1 + y_2. \]

The sum of spouses' utilities in a given period is

\[ u_k = u_k^f + u_k^m = x_k + 2y_k. \]

**Observation 2.1** Given the labor supply of both spouses in period k and therefore \((x_k, y_k)\) the utility possibility frontier in each period is linear and of the form

\[ u_k = u_k^m + u_k^f = x_k + 2y_k. \]

If spouses get divorced the household public good is no longer available. A divorcée's utility in each period is given by the amount of the private good consumed and hence divorced people always work full time in the market.

\[ d_1^f = aw, d_2^f = aw(1 + \alpha f_1) \]

\[ d_1^m = b, d_2^m = b(1 + \alpha m_1) \]

where \((1 - a)\) and \((1 - b)\) are the tax rates faced by the wife and the husband when filing as singles. The tax schedule for singles has a progressive rate structure. Since the husband earns a higher wage rate than the wife, he faces a higher tax rate \((1 - b) > (1 - a)\) but

\[ aw < b. \]  \hspace{1cm} (2.7)

\(^6\)Introducing a discount factor \(0 < \delta < 1\) would not change our basic results.
Family income taxation in countries like the US and Germany satisfies the following characteristics.\textsuperscript{7} The spouse with lower wage rate typically faces a higher tax rate when married than when single, while the spouse with higher wage rate typically experiences a reduction in the tax rate upon marriage, i.e.

\[(1 - \tau) \geq (1 - \alpha) \quad \text{and} \quad (1 - \sigma) \leq (1 - b). \quad \text{(2.8)}\]

### 2.3.1 The Bargaining Game

#### Period k

Stage 1: Spouses choose labor supply \((m_k, f_k)\).

Stage 2: Given labor supply of each spouse and therefore family resources \((x_k, y_k)\), spouses split gains from cooperation equally. Each spouse’s utility in period \(k\) is computed according to

\[u_k^i = \frac{u_k + d_k^i - d_k^j}{2} = \frac{x_k + d_k^i - d_k^j}{2} + y_k.\]

We do not allow for negative transfers of the private good and hence the lower bound on the utility of each spouse in marriage is given by

\[u = y_1 + y_2.\]

While both spouses consume the same amount of the household public good, the spouse with higher divorce utility gets a larger amount of the private good than the other spouse. The husband receives a bigger share of the private good than the wife in both periods, because his net-wage rate is higher than the net-wage rate of the wife in both periods,

\[d_k^m > d_k^f.\]

\textsuperscript{7}See e.g. Nelson (1996), Berliant and Rothstein (2001), Apps and Rees (1999a and b) and Chade and Ventura (2002).
Equal Split of cooperative gains is an efficient allocation rule in a one-period model. In the two-period model, however, are the utility possibility set and the divorce utility of the spouses in the second period affected by the first period labor supply. We analyze below the tradeoff between securing a bigger share of the second period pie on one hand and achieving intertemporal efficiency on the other hand.

2.4 Equilibrium Time Allocation of Spouses

2.4.1 Efficient Division of Labor

Observation 2.2 The intertemporal utility possibility frontier is linear and of the form

$$U = \left\{ \left(u^m, u^f \right) > 0 | u^m + u^f = \lambda \right\}$$

where $$\lambda = x_1^* + 2y_1^* + x_2^* + 2y_2^*$$ and $$(x_1^*, y_1^*, x_2^*, y_2^*)$$ are the solution to

$$\max_{x_k, y_k} \sum_{k=1,2} x_k + 2y_k$$

subject to the household constraints (2.3), (2.4), and (2.5).

We therefore solve for the optimal vectors of labor supply in each period by maximizing $$\lambda$$.

$$\max_{m_1, f_1, m_2, f_2} \left[ \begin{array}{c} \sigma m_1 + \tau w f_1 + 2h (2 - m_1 - f_1) \\ + \sigma (1 + \alpha m_1) m_2 + \tau w (1 + \alpha f_1) f_2 + 2h (2 - m_2 - f_2) \end{array} \right]$$

This function is symmetric in $$f_1$$ and $$f_2$$ and in $$m_1$$ and $$m_2$$. However, the function is not concave and we need to check second order conditions (see appendix).

---

8The same division of labor would obtain if we were to maximize the Nash product of the intertemporal utility of a spouse when married minus the intertemporal utility of a spouse when never married. Similarly, Equal Split of Cooperative Gains applied to life-time utility would satisfy efficiency.
To avoid that the wife's labor supply is a corner solution, we need the following condition about the opportunity cost of working in household production and the benefit from household production

\[ 2h'(0) > \sigma (1 + \alpha) > \tau w > 2h'(1). \] (2.9)

Condition (2.9) states that in each period it is worthwhile for one of the spouses to work in household production. On the other hand, (2.9) rules out that any spouse would ever spend his or her whole time in household production.

**Proposition 2.1** Household efficient division of labor. The efficient labor supply of both spouses in each period requires the husband to work full time in the market, \( m_1^e = m_2^e = 1 \), while the wife spends time in both sectors. The efficient labor supply of the wife in each period is given by \( f_1^e = f_2^e = f^e \) where \( f^e \) is the highest possible value of \( f^e \in (0,1) \) that solves

\[ f^e = 1 - g \left( \frac{\tau w (1 + \alpha f^e)}{2} \right). \]

A spouse's labor supply in each period is the same, and spouses consume the same amount of the household public good in each period.

### 2.4.2 Strategic Division of Labor

The following assumption makes the labor supply decision of spouses more interesting. If

\[ 2h'(0) > \sigma (1 + \alpha) + b \alpha > \tau w > 2h'(1) \] (2.10)

a spouse will always find it beneficial to work for some time but not full time in household production if the other spouse refuses to work in household production. Assumptions (2.2),
(2.7) and (2.10) guarantee a unique subgame perfect Nash equilibrium of the two-period bargaining problem. In equilibrium, spouses divide their time between household production and employment in the traditional pattern: the wife works in both sectors, while the husband works full time in the market.

Spouses solve for their optimal strategies by backward induction. In the second period, spouses will choose the efficient amount of household production, given their choice of labor supply in the first period. In the second period, the threat points are determined by first period choices and the threat points will play no role in determining the second period equilibrium strategies. Whenever threat points cannot be manipulated by individuals equal split of cooperative gains is efficient.

\[
f^*_2 = 1 - g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right), \quad (2.11)
\]

\[
m_2 = 1. \quad (2.12)
\]

Given second period equilibrium strategies, each spouse chooses first period labor supply so as to maximize his or her intertemporal utility. The husband’s problem is

\[
\max_{m_1} \left[ \frac{\tau w_1 + \sigma + b - aw}{2} + h (2 - f_1 - m_1) \\
+ \frac{\tau w (1 + \alpha f_1) f^*_2 + (\sigma + h)(1 + \alpha m_1) - aw (1 + \alpha f_1)}{2} + h (1 - f^*_2) \right],
\]

and the wife chooses first period labor supply according to

\[
\max_{f_1} \left[ \frac{\tau w_1 + \sigma - b + aw}{2} + h (2 - f_1 - m_1) \\
+ \frac{\tau w (1 + \alpha f_1) f^*_2 + (\sigma - h)(1 + \alpha m_1) + aw (1 + \alpha f_1)}{2} + h (1 - f^*_2) \right].
\]

**Proposition 2.2** Division of labor in subgame perfect Nash equilibrium. The unique SG-PNE consists of the following strategies: The husband does not participate in housework in either period, the wife works in household production and in the market in
each period. The equilibrium labor supply vector of the wife \((f_1^*, f_2^*)\) selects the vector of highest possible values for \((f_1, f_2) \in (0, 1)^2\) that solve

\[
\begin{align*}
    f_1 &= 1 - g\left(\frac{rw(1 + \alpha f_2) + awa}{2}\right), \\
    f_2 &= 1 - g\left(\frac{rw(1 + \alpha f_1)}{2}\right).
\end{align*}
\]

The wife's strategic labor supply in both periods exceeds the efficient amount, i.e. \(f_k^* > f_k^e\) and \(f_1^* > f_2^*\). Spouses consume a smaller amount of the household public good in the first period than in the second period.

The efficient and the equilibrium labor supplies of the wife are illustrated in Figure 2.1. Strategic allocation of time leads to a level of household production that is too low compared to the efficient amount.

We can also compare the intertemporal utility of each spouse depending on different behavior. Intertemporally efficient bargaining (we again use Equal Split of cooperative gains) would give each spouse a utility of

\[
\begin{align*}
    u^f &= \frac{u^e(1, 1, f_1^e, f_2^e) - (b + b(1 + \alpha)) + (aw + aw(1 + \alpha))}{2}, \\
    u^m &= \frac{u^e(1, 1, f_1^e, f_2^e) + (b + b(1 + \alpha)) - (aw + aw(1 + \alpha))}{2}.
\end{align*}
\]

and with period-by-period bargaining we have

\[
\begin{align*}
    u^f &= \frac{u^*(1, 1, f_1^*, f_2^*) - (b + b(1 + \alpha)) + (aw + aw(1 + \alpha f_2^*))}{2}, \\
    u^m &= \frac{u^*(1, 1, f_1^*, f_2^*) + (b + b(1 + \alpha)) - (aw + aw(1 + \alpha f_1^*))}{2}.
\end{align*}
\]

The comparison of utility shares under lifetime bargaining and under period-by-period bargaining shows that the husband might be tempted to renegotiate the marriage contract.
The wife prefers a complete contract, so that her efficient supply of the household good will not negatively effect her share in the second period, but the husband has an incentive to deviate from the agreed division of goods. Once the second period starts the wife's outside option is no longer $aw(1+\alpha)$ but in fact $aw(1+\alpha f^e)$ and the husband could negotiate a higher share for him. Knowing that this might happen, the spouses therefore anticipate renegotiation and the wife chooses her labor supply according to $(f_1^*, f_2^*)$. The result is a reduction in the intertemporal utility set and the wife's share with renegotiation is lower than her share under full commitment.

The assumption that the husband has an absolute advantage in both periods is crucial for the uniqueness of the equilibrium. If it is possible that the wife can surpass the husband's net wage rate in the second period by refusing to work in household production in the first period, there can be two equilibria in pure strategies. However, simulations suggest that the net wage rates between spouses will have to be almost the same for both equilibria to occur.$^9$

The next section examines how the first period labor supply of the wife changes if policies toward singles or families change.

### 2.5 Applications

#### 2.5.1 Increase in Wage Rate

Suppose the wage rate of the wife increases but by not enough to put wife and husband in the same tax bracket.

---

$^9$For example, with $y_k = \gamma \ln (t_k + 1)$, we found that ignoring taxes, i.e. $\sigma = \tau = a = b = 1$, and $a = .2, \gamma = .8$ the wife's wage rate in the first period would have to be $w > .965$. 

---
Proposition 2.3a An increase in the wife's wage rate has a positive effect on the wife's threatpoint and increases her utility share, but it decreases the husband's utility. The effect on total utility is ambiguous.

An increase in the husband's wage rate has the following impact on utilities.

Proposition 2.3b An increase in the husband's wage rate has a non-negative effect on the wife's utility, and it increases the husband's utility.

2.5.2 Progressivity of the Tax Schedules for Singles

How does the tax structure for singles affect married people? Suppose the government contemplates a tax reform for singles that increases the progressivity of the tax schedule for singles.

Proposition 2.4 The wife's strategic labor supply increases (decreases) with a decrease (increase) in her tax rate when single. The change in the sum of utilities is negative (positive) with a decrease (increase) in her tax rate when single.

$$du^f + du^m = -awc \frac{df^*}{da} da.$$

The sum of utilities and the spouses' labor supplies stay the same with a change in the husband's tax rate when single.

The tax rates of singles might change in five different ways.

1. The tax rate of the divorced male goes up.

2. The tax rate of the divorced female goes down.
3. The tax rate of the divorced male goes up and the tax rate of the divorced female goes down.

The effects of the change in tax rates on the wife's utility, the husband's utility and household utility are summarized below.\textsuperscript{10} The last column indicates the change in the difference of utilities. If this change is negative, it implies that inequality between spouses' utilities decreases, because the husband always receives at least as big a share as the wife.

<table>
<thead>
<tr>
<th>Case</th>
<th>$dw^f$</th>
<th>$dw^m$</th>
<th>$dw^m + dw^f$</th>
<th>$d (u^m - u^f)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

4. The tax rate of both divorced male and divorced female go down, but the female's tax rate goes down by more than the male's.

In this case we have to distinguish between two possible outcomes. It is not enough that the wife's tax rate goes down by more than the husband's to guarantee an increase in the utility of the wife. The wife could be better off or worse off after this reform. The wife is better off if (case 4a)

$$\frac{[w (2 + \alpha f^*_1)]}{2 + \alpha} \geq \frac{db}{da}.$$  

The ratio between the intertemporal divorce income of the wife and the husband is bigger than the ratio of the change in tax rates between husband and wife. The wife is worse off

\textsuperscript{10}An ambiguous effect is indicated by "?", and no change is indicated by "0."
otherwise (case 4b),

\[
\frac{[w(2 + \alpha f^*_T)]}{2 + \alpha} < \frac{db}{da}.
\]

5. The tax rate of both divorced male and divorced female go up, but the female's tax rate goes up by less than the male's.

<table>
<thead>
<tr>
<th>Case</th>
<th>(dw^f)</th>
<th>(dw^m)</th>
<th>(du^m + dw^f)</th>
<th>(d(u^m - u^f))</th>
</tr>
</thead>
<tbody>
<tr>
<td>4a</td>
<td>+</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>4b</td>
<td>-</td>
<td>?</td>
<td>-</td>
<td>?</td>
</tr>
<tr>
<td>5</td>
<td>+</td>
<td>?</td>
<td>+</td>
<td>?</td>
</tr>
</tbody>
</table>

This result implies that if tax reform is the all dominating issue in federal elections and one party strongly advocates more progressivity in the tax schedule for singles than the other, married women should be more likely than their husbands to vote for this party. Edlund and Pande (2002) note that there is a political gender gap in the US that has widened over the years. In the 2000 US Presidential election, 54 percent of female voters cast their vote for Gore, while 53 percent of men voted for Bush. The authors argue that with an increase in the probability of divorce it is the increase in divorced women and their demand for redistribution that has increased the political gender gap. Married women, however, have also an incentive to favor a progressive tax system for singles because it would increase their outside options and therefore their bargaining power within marriage.
2.5.3 Divorce

Divorce Law

Next we examine how divorce laws might change the labor supply of the wife. So far we have assumed that after divorce ex-spouses have to make due with their stand alone income, i.e. there is no income transfer between ex-spouses. In some states of the US and many countries in Europe, however, the divorce law provides for alimony payments to the ex-spouse with lower stand alone income. We can think of such a provision in the law as applying the following rule.

\[ d^f_k = (1 - \theta) \frac{aw_k + b}{2} + \theta aw_k, \]
\[ d^m_k = (1 - \theta) \frac{aw_k + b}{2} + \theta b, \]

where \( \theta \) is the weight on the stand alone income of a spouse and depending on the particular law, it will fall somewhere between zero and one, i.e. \( 0 \leq \theta \leq 1 \). If \( \theta = 1 \), we are in the case analyzed in the previous section. As \( \theta \) decreases, stand alone income of each spouse receives less weight, and when \( \theta = 0 \), the ex-spouses split the sum of their incomes equally.

**Proposition 2.5** A divorce law that splits income between former spouses equally equalizes utility shares between spouses and it is less distortionary than a divorce law that grants each former spouse his or her stand alone income.

Chiappori et al. (2002) find that the labor supply of married women increases in states with divorce laws that are unfavorable to the spouse with lower income. Since each spouse consumes leisure in their model and it is assumed that household decisions are efficient, the authors argue that an increase in the labor supply of a spouse indicates a
weaker bargaining position.\textsuperscript{11} Our model provides an alternative explanation why married women work more in states with divorce laws unfavorable to the low-income spouse. Our result suggests that married women will supply more labor in states where divorce laws can be characterized by a high $\theta$, because the wife's bargaining power critically depends on the wife's stand alone income and by increasing the labor supply in the first period the stand alone income in the second period will rise. We would expect to see a lower labor supply of wives in states with a low $\theta$, since the bargaining power of the wife depends less on her own earning potential compared to states with high $\theta$.

Peters (1986) and Parkman (1992) found in their empirical studies that the labor supply of married women increased when states adopted no-fault divorce laws. Previously to that change in law, if one spouse wanted a divorce he or she needed the consent of the other. This gave women more bargaining power in the divorce settlement, provided it was the husband who asked for divorce. With no-fault divorce, courts started to play a major role in the financial settlements of divorce. Parkman argues that the courts' lack of recognition that wives who specialized in household production sacrificed future earning capacity led to an increase in married women's labor supply (Parkman, 1992, p. 672).

In the present model, singles or divorcees always work full time in the market, because we assumed that there is no substitute for the consumption of the private good. While empirical studies show that singles consume much more private goods and there is less household production, there is also some empirical evidence that men reduce their working hours after divorce. If singles and divorcees also consume leisure, then a divorce

\textsuperscript{11}Chiappori et al. do not consider household production. A spouse's utility depends on his or her consumption of a private Hicksian composite good and leisure. The authors do, however, allow for caring preferences, i.e. one spouse's consumption and leisure enters the other spouse's utility function.
law that splits the sum of ex-spouses' income equally is likely to distort the work-leisure choice. Spouses would have to take the reduction in working hours of the ex-spouse into account when deciding on their labor supply during marriage.

**Probability of Divorce**

What happens to first period labor supply if there is an actual chance of divorce? Denote $p$ the probability of the couple staying together in the second period and $(1 - p)$ the probability of getting a divorce. It is obvious that the husband has even more incentive to work full time as he faces a probability of actual divorce. The wife will also increase her labor supply.

**Proposition 2.6** The wife's first period labor supply increases with the probability of divorce.

$$\frac{df^D_1}{dp} = \frac{aw\alpha - \tau w\alpha f^D_2}{\phi'(f^D_1)} > 0.$$

This result is consistent with the empirical finding that married women increased their labor supply up to three years prior to divorce (see e.g. Johnson and Skinner, 1986).

**2.5.4 Tax Reform from Joint Towards Individual Taxation**

In this section we examine the change in the wife's labor supply when the tax rate for each spouse changes. Suppose spouses were taxed at a joint rate, i.e. $\sigma = \tau$. A tax reform towards individual taxation decreases the tax rate of the wife and increases the tax rate of the husband, i.e. $\tau > \sigma$. This tax reform changes the net-wage ratio between spouses, but it will not change the pattern of specialization, since the net-wage of the husband is still higher than the net-wage of the wife in both periods.
Proposition 2.7 Strategic as well as efficient labor supplies increase with a change from joint towards individual taxation. A sufficient condition that the difference between the efficient labor supply and the strategic labor supply decreases with tax reform is that

$$\varphi'(0) < 0 \text{ and } \varphi'(1) > 0.$$  

We now examine the impact of a switch from joint to individual taxation on each spouse’s utility.

Proposition 2.8 (1) The wife’s utility increases with a switch from joint to individual taxation if the ratio between the change in tax rates is smaller than the ratio of the wife’s intertemporal wage income and the husband’s intertemporal wage income.

$$\frac{-d\sigma}{d\tau} < \left(\frac{w f_1^* + w (1 + \alpha f_1^*) f_2^*}{2 + \alpha}\right).$$

(2) Even if the wife’s utility increases with a switch from joint to individual taxation, an increase in the husband’s utility is not guaranteed. Both spouses benefit from tax reform iff $$du^m > 0$$, i.e.

$$\frac{-d\sigma}{d\tau} < \left(\frac{w f_1^* + w (1 + \alpha f_1^*) f_2^* - 2aw\alpha f_1^*}{2 + \alpha}\right).$$

(3) A shift from joint towards individual taxation increases the ratio between the wife’s and the husband’s utility for sure if the utility of the wife increases with tax reform. Even if both spouses’ utilities decrease with tax reform, inequality between spouses decreases as long as

$$\frac{dw^f}{w^f} > \frac{dw^m}{w^m}.$$
the percentage decrease in the wife’s utility is smaller than the percentage decrease in
the husband’s.

In the two-period bargaining model it is possible that one spouse gains while the
other spouse loses with tax reform. A static model of family bargaining with a threat-
point outside marriage, however, yields different results. In this case, both spouses gain if
household utility increases and both spouses lose if household utility decreases (see Gugl,
2002).

2.6 Conclusion

If spouses can enter binding agreements at the beginning of marriage, intertemp-
oral household decisions will be efficient. But when spouses are aware of the possibility
of future renegotiation of the marriage contract, they will evaluate their current decisions
also with respect to the effect on their future bargaining power and decisions are no longer
intertemporally efficient.

The evaluation of the impact of policies on intrafamily distribution in a static
framework often differs from the evaluation in a dynamic framework. For instance, in Gugl
(2002) we analyze the effects of tax reform from individual to joint taxation on intrafamily
distribution in a model with complete marriage contracts. If spouses adopt a bargaining rule
with a threatpoint outside marriage, both spouses will benefit from tax reform if household
utility increases. In the dynamic model presented here, this is no longer true. The wife’s
utility might decrease with joint taxation even if household utility slightly increases. In
addition, the static bargaining model used in Gugl (2002) predicts that a change in the tax
rates of singles would have no effect on the labor supply of the spouses. In the two-period bargaining model, we find that the wife increases her labor supply when her tax rate in the case of divorce goes down. In future research we will exploit these different implications of the two models in empirical tests.

We have made the assumption that the wife can never receive a higher second-period wage rate than the husband independent of the labor supply decisions in the first period. Suppose it is possible for the wife to receive a higher wage rate in the second period than her husband. It could be the case that the wife’s wage rate increases more with first period labor supply than the husband’s, because the husband’s earning profile has already reached its plateau. Then multiple SGPNE are possible and households with non-traditional labor supplies might emerge. This is the focus of work in progress.

We have limited the analysis in this chapter to a specific bargaining rule. Other rules of how families might share resources can also be considered. For example, the Separate Accounts rule in which spouses receive utility shares proportional to their earning potential is a rule that is efficient in a static setup, but it fails to achieve efficiency in a period-by-period framework.

2.7 Appendix

2.7.1 Proof of Observations 2.1 and 2.2

1. The utility of each spouse in period $k$ is equal to

$$ u_k = x_k + y_k. $$
Given \((x_k, y_k)\) and that the utility function of each spouse in each period is linear in the goods of each period the utility vector \((u^f_k, u^m_k)\) is maximized by

\[
u_k = \left\{ u^f_k, u^m_k \geq y_k | u^f_k + u^m_k = x_k + 2y_k \right\}.
\]

Since the utility functions for husband and wife are identical and homogenous of degree 1, the utility possibility frontier is linear in each period and given by the sum of individual utilities (see Gugl, 2002).

2. The intertemporal utility of each spouse is equal to

\[
u^i = u^i_1 + u^i_2
\]

It follows that

\[
u^i = \sum_{k=1}^{2} x^i_k + y_k.
\]

Similarly to 1., the intertemporal utility function of each spouse is linear in the goods of each period. Because the utility functions for husband and wife are identical, this implies a linear intertemporal utility possibility frontier of slope 1 (see Gugl, 2002).

2.7.2 Proof of Proposition 2.1

\[
\max_{m_1, f_1, m_2, f_2} \left[ \begin{array}{c}
\sigma m_1 + \tau w f_1 + 2h (2 - m_1 - f_1) \\
+ \sigma (1 + \alpha m_1) m_2 + \tau w (1 + \alpha f_1) f_2 + 2h (2 - m_2 - f_2)
\end{array} \right]
\]

Note that we can simplify this problem by working backwards. After any history, an efficient allocation of time in the second period must be the solution to the problem

\[
\max_{m_2, f_2} \sigma (1 + \alpha m_1) m_2 + \tau w (1 + \alpha f_1) f_2 + 2h (2 - m_2 - f_2)
\]
and Kuhn-Tucker first order conditions for the second period labor supplies are given by

\[
\tau w (1 + \alpha f_1) = 2h' (2 - f_2 - m_2) \quad \text{and} \quad 0 < f_2 < 1
\]  
\tag{2.13}

\[
\tau w (1 + \alpha f_1) > 2h' (2 - f_2 - m_2) \quad \text{and} \quad f_2 = 1
\]  
\tag{2.14}

\[
\sigma (1 + \alpha m_1) = 2h' (2 - f_2 - m_2) \quad \text{and} \quad 0 < m_2 < 1.
\]  
\tag{2.15}

\[
\sigma (1 + \alpha m_1) > 2h' (2 - f_2 - m_2) \quad \text{and} \quad m_2 = 1.
\]  
\tag{2.16}

Since the husband earns a higher wage rate in the second period than the wife independent of the spouses' labor supply in the first period, it must be the wife who supplies time to household production first. From condition (2.9), we know that it cannot be the case that both spouses work in household production, hence (2.13) is binding and inequality (2.16) holds with \( m_2 = 1 \) for the husband. Next we examine second order sufficient conditions for the wife's labor supply given the husband's labor supply.

\[
2h''(1 - f_2) < 0.
\]

This is true.

The wife's labor supply in the second period is given by

\[
f_2 = 1 - g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right),
\]

where \( g \) is the inverse function of \( h' \). Let \( r = \frac{\tau w (1 + \alpha f_1)}{2} \), then \( g (r) > 0 \) and decreasing at a diminishing rate in \( r \), i.e. \( g' < 0, g'' > 0 \). Note that \( h' (g) = r \).

We now maximize intertemporal utility given the optimal second period labor supplies

\[
\max_{m_1, f_1} \left[ \sigma m_1 + \tau w f_1 + 2h (2 - m_1 - f_1) + \sigma (1 + \alpha m_1) + \tau w (1 + \alpha f_1) \left( 1 - g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) \right) + 2h \left( g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) \right) \right].
\]
First order conditions for the first period labor supplies

\[ \tau w \left( 1 + \alpha \left( 1 - g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) \right) \right) = 2h' (2 - f_1 - m_1) \tag{2.17} \]

and \( 0 < f_1 < 1, \)

\[ \tau w \left( 1 + \alpha \left( 1 - g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) \right) \right) > 2h' (2 - f_1 - m_1) \tag{2.18} \]

and \( f_1 = 1. \)

\[ \sigma (1 + \alpha) = 2h' (2 - f_1 - m_1) \text{ and } 0 < m_1 < 1 \tag{2.19} \]

\[ \sigma (1 + \alpha) > 2h' (2 - f_1 - m_1) \text{ and } m_1 = 1. \tag{2.20} \]

Note that (2.17) and (2.18) can be written as

\[ \tau w (1 + \alpha f_2) = 2h' (2 - f_1 - m_1) \text{ and } 0 < f_1 < 1, \]

\[ \tau w (1 + \alpha f_2) > 2h' (2 - f_1 - m_1) \text{ and } f_1 = 1. \]

The same argument that was used for second period optimal labor supplies applies. Since spousal time inputs in household production are perfect substitutes and the husband earns a higher net-wage rate than the wife in each period, if (2.17) holds with equality and (2.19) cannot be satisfied. The marginal productivity of time in household production is well defined and we can solve for \( f_2 \) and \( f_1 \), given \( m_1 = m_2 = 1. \)

\[ f_2^\circ = 1 - g \left( \frac{\tau w (1 + \alpha f_1^\circ)}{2} \right) \tag{2.21} \]

\[ f_1^\circ = 1 - g \left( \frac{\tau w (1 + \alpha f_2^\circ)}{2} \right). \tag{2.22} \]

The functions (2.22) and (2.21) are symmetric, \( f_1^\circ = f_2^\circ = f^\circ. \)

There could be multiple solutions to (2.22) and (2.21). Define

\[ \varphi (f_1) = 2h' (1 - f_1) + \tau w g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right). \]
Checking the second order condition for (2.17), we need

\[ \varphi'(f_1) > 0. \]

\[ \varphi'(f_1) = -2h''(1 - f_1) + \frac{(\tau w \alpha)^2}{2} g' \left( \frac{\tau w (1 + \alpha f_1)}{2} \right). \]

It could be that \( \varphi'(0) < 0 \), but \( \varphi'(1) > 0 \). We show that \( \varphi'(1) \) is increasing in \( f_1 \):

\[ \varphi''(f_1) = 2h'''(1 - f_1) + \frac{(\tau w \alpha)^3}{4} g'' \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) > 0. \]

This means that we could have two interior solutions \( \left( \hat{f}, \tilde{f} \right) \in (0, 1) \), with \( \hat{f} < \tilde{f} \). Then we have a minimum at \( \hat{f} \) and a maximum at \( \tilde{f} \). Condition (2.9) guarantees that the argmax lies within the time constraint, \( f^* < 1 \). Figure 2.1 shows the case where the minimum is at some \( f < 0 \).

### 2.7.3 Proof of Proposition 2.2

From backward induction we know that the husband will always work full time in the second period. We argue that because of assumption (2.10) the husband's equilibrium strategy is working full time in both periods. We solve for the best response of the wife to the husband's labor supply in both periods. Then we show that given the wife's best response to the husband working full time, the husband indeed chooses to work full time in both periods. The wife's first order condition given the husband works full time is given by

\[
\begin{bmatrix}
\tau w - 2h'(1 - f_1) + \tau w \alpha \left( 1 - g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) \right) \\
-\tau w (1 + \alpha f_1) \frac{\tau w}{2} g' \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) \\
+2 \frac{\tau w \alpha}{2} h' \left( g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) \right) \times g' \left( \frac{\tau w (1 + \alpha f_1)}{2} \right)
\end{bmatrix}
\leq 0.
\]
Note that \( h'(g(r)) = r \), and the last two terms on the left hand side cancel out. We can simplify to obtain

\[
\tau w (1 + \alpha f_2^*) + aw\alpha = 2h'(1 - f_1) \quad \text{and} \quad 0 < f_1 < 1. \tag{2.23}
\]

\[
\tau w (1 + \alpha f_2^*) + aw\alpha > 2h'(1 - f_1) \quad \text{and} \quad f_1 = 1. \tag{2.24}
\]

By assumption (2.10), this implies

\[
\tau w (1 + \alpha f_2^*) + aw\alpha = 2h'(1 - f_1^*). 
\]

Given the wife’s best response to the husband’s full time work, would the husband want to change his strategy in the first period? The best response of the husband to a given labor supply of the wife is given by (remember he will always work full time in the second period)

\[
\sigma (1 + \alpha) + b\alpha = 2h'(2 - f_1 - m_1) \quad \text{and} \quad 0 < m_1 < 1,
\]

\[
\sigma (1 + \alpha) + b\alpha > 2h'(2 - f_1 - m_1) \quad \text{and} \quad m_1 = 1.
\]

Assumptions (2.2) and (2.7) imply

\[
\sigma + b\alpha > \tau w (1 + \alpha f_2^*) + aw\alpha
\]

and hence

\[
\sigma (1 + \alpha) + b\alpha > 2h'(1 - f_1^*).
\]

The husband’s best response to the wife’s first period labor supply chosen at \( m_1 = 1 \) is again \( m_1 = 1 \). There is a unique subgame perfect Nash equilibrium, in which the wife divides her time between the two sectors in the first period, while the husband works full
time. Although the system of equations
\[ \tau w (1 + \alpha f_2^*) + aw\alpha = 2h' (1 - f_1^*) \]
\[ \tau w (1 + \alpha f_1^*) = 2h' (1 - f_2^*) \]
might have two solution vectors in the admissible range of \( f_1 \) and \( f_2 \), the vector with the lower values for \( (f_1, f_2) \) will be a minimum and the vector with the larger values for \( (f_1, f_2) \) will be the maximum by the same argument made in Proposition 2.1.

Next we show that the strategic labor supply of the wife exceeds her efficient labor supply. We can write the strategic labor supply in the first period as
\[ \tau w + \tau w\alpha + aw\alpha = 2h' (1 - f_1) + \tau w\alpha g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) \]  
(2.25)
The left hand side under strategic behavior is larger than under efficiency, yet the right hand side in both cases has the same functional form.

We have established above that an interior solution under efficiency must have an amount of labor supply where \( \varphi' (f_1) \) is increasing. The same argument applies for the equilibrium labor supply. This implies that \( f_1^* > f_1^\circ \).

### 2.7.4 Proof of Propositions 2.3a and 2.3b

As long as the husband’s wage is still higher than the wife’s and tax rates do not change we have
\[ \tau w + \tau w\alpha + aw\alpha = 2h' (1 - f_1) + \tau w\alpha g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) \]
and therefore a change in \( w \) has the following effect on the wife’s labor supply,
\[ \frac{df_1}{dw} = \frac{2\tau (1 + \alpha f_2) + 2aw\alpha - \tau^2 w\alpha (1 + \alpha f_1) g' (r)}{2\varphi' (f_1)} > 0. \]
The wife’s utility is give by

\[ u^f = \left[ \frac{(\tau f_1^* + a)w + \sigma - b}{2} + h(1 - f_1^*) \right] + \frac{(\tau f_2^* + a)w(1 + \alpha f_1^*) + (\sigma - b)(1 + \alpha)}{2} + h(g(r)) \]

and the effect of an increase in the wage rate on the wife’s utility is

\[ du^f = \frac{\tau f_1^* + a + (\tau f_2^* + a)(1 + \alpha f_1^*)}{2} dw > 0. \]

The husband’s utility is given by

\[ u^m = \left[ \frac{(\tau f_1^* - a)w + \sigma + b}{2} + h(1 - f_1^*) \right] + \frac{(\tau f_2^* - a)w(1 + \alpha f_1^*) + (\sigma + b)(1 + \alpha)}{2} + h(g(r)) \]

and the effect of an increase in the wage rate on the husband’s utility is

\[ du^m = \frac{\tau f_1^* - a + (\tau f_2^* - a)(1 + \alpha f_1^*)}{2} dw - \alpha w \frac{df_1^*}{dw} dw < 0. \]

Household utility changes by

\[ du^f + du^m = (\tau f_1^* + \tau f_2^* (1 + \alpha f_1^*)) dw - \alpha w \frac{df_1^*}{dw} dw \leq 0. \]

Denote \( \omega \) the husband’s first-period wage rate. An increase in the husband’s wage rate does not change the labor supply decisions of the spouses,

\[ \frac{dm_k}{d\omega} = 0, \quad \frac{df_k}{d\omega} = 0. \]

The wife’s utility increases if \( \sigma > b \) or is unaffected if \( \sigma = b \),

\[ u^f = \frac{(\tau f_1^* + a)w + (\sigma - b)w}{2} + h(1 - f_1^*) \]

\[ + \frac{(\tau f_2^* + a)w(1 + \alpha f_1^*) + (\sigma - b)w(1 + \alpha)}{2} + h(g(r)) \]

\[ du^f = \frac{(\sigma - b)(2 + \alpha)}{2} \geq 0. \]

while the husband’s utility increases,

\[ du^m = \frac{(\sigma + b)(2 + \alpha)}{2} > 0. \]
2.7.5 Proof of Proposition 2.4

From (2.25) it is easy to see that a decrease in the divorced female’s tax rate has the following effect on the first period labor supply of the wife

\[
\frac{df^*_1}{da} = \frac{wx}{\varphi'(f^*_1)}.
\]

As noted above \(\varphi'(f^*_1) > 0\) and hence

\[
\frac{df^*_1}{da} > 0.
\]

A change in the tax rate of the divorced male has no effect on the spouses’ labor supplies as long as the net wage rate of the divorced male is still higher than the net-wage rate of the divorced female in both periods. Hence

\[
\frac{dm^*_1}{db} = 0.
\]

We now evaluate the intertemporal utility of each spouse as the tax rates for singles become more progressive. Intertemporal utility for each spouse is given by

\[
\begin{align*}
    u^f &= \left[ \frac{(rf^*_1 + a)w + \sigma - b}{2} + h(1 - f^*_1) \right. \\
    & \quad \left. + \frac{(rf^*_1 - a)w(1 + \alpha f^*_1) + (\sigma - b)(1 + \alpha)}{2} + h(g(r)) \right] \\
    u^m &= \left[ \frac{(rf^*_2 - a)w + \sigma + b}{2} + h(1 - f^*_1) \right. \\
    & \quad \left. + \frac{(rf^*_2 - a)w(1 + \alpha f^*_1) + (\sigma + b)(1 + \alpha)}{2} + h(g(r)) \right]
\end{align*}
\]

and therefore a change in the tax rates of singles changes each spouse’s utility by

\[
\begin{align*}
    du^f &= \frac{1}{2} [w(2 + \alpha f^*_1)] \, da - \frac{1}{2} (2 + \alpha) \, db, \\
    du^m &= -\frac{1}{2} [w(2 + \alpha f^*_1)] \, da - aw\alpha \frac{df^*_1}{da} \, da + \frac{1}{2} (2 + \alpha) \, db.
\end{align*}
\]
Intertemporal utility of the family changes by

\[ du^f + du^m = -aw\alpha \frac{df^*}{da} da. \]

The change in the difference of utilities is

\[ du^m - du^f = -aw\alpha \frac{df^*}{da} da - [w (2 + \alpha f^*_1)] da + (2 + \alpha) db. \]

Note that in case 5, where the tax rates of both ex-spouses go up, but the ex-wife’s tax rate goes up by less than the ex-husband’s, the wife’s utility unambiguously increases because

\[ |da| < |db|, \quad da < 0, \quad db < 0. \]

### 2.7.6 Proof of Proposition 2.5

By the same argument as above, in the unique SGPNE with divorce laws characterized by \( \theta \), the wife works in both sectors in each period and the husband works full time in the market. First period labor supply of the wife is given by

\[ \tau w (1 + \alpha) + \frac{1 + \theta}{2} a w\alpha = \varphi (f_1). \]

A change in \( \theta \) changes the wife’s labor supply by

\[ \frac{df^*_1}{d\theta} = \frac{aw\alpha}{2\varphi' (f_1)} > 0, \]

but as \( \theta \) increases the effect on each spouse’s utility is

\[ du^f = \left( \frac{aw (2 + \alpha f^*_1)}{2} - \frac{b (2 + \alpha)}{2} \right) d\theta < 0, \]

\[ du^m = -\left( \frac{aw (2 + \alpha f^*_1)}{2} - \frac{b (2 + \alpha)}{2} \right) d\theta - a\theta w\alpha \frac{df^*}{d\theta} d\theta \leq 0. \]
The effect on the sum of the utilities is
\[ du^f + du^m = -a\theta w\alpha \frac{df^*}{d\theta} d\theta < 0, \]
and on the difference in utilities is
\[ du^m - du^f = -\left(awl (2 + \alpha f_1^*) - b(2 + \alpha) - a\theta w\alpha \frac{df^*}{d\theta} \right) d\theta > 0. \]
As \( \theta \) decreases, the wife’s utility increases, inequality between spouses decreases and household utility increases.

### 2.7.7 Proof of Proposition 2.6

By the same argument as above, in the unique SGPNE with divorce probability \((1 - p)\), the wife works in both sectors in each period and the husband works full time in the market. The wife’s first period labor supply is therefore given by
\[ \tau w + (2 - p) a w\alpha + prw\alpha = 2h' (1 - f_1) + prwag \left( \frac{\tau w (1 + \alpha f_1)}{2} \right). \]
Let \( \phi (f_1) = 2h' (1 - f_1) + prw\alpha g \left( \frac{\tau w (1 + \alpha f_1)}{2} \right) \). Then
\[ \phi' (f_1) = -2h'' (1 - f_1) + p \frac{(\tau w\alpha)^2}{2} g' \left( \frac{\tau w (1 + \alpha f_1)}{2} \right). \]
where \( \phi' (f_1^D) > 0 \) following the same argument as to why \( \phi' (f_1^*) > 0 \). The wife’s response to a decrease in the probability of divorce is given by
\[ \frac{df_1^D}{dp} = \frac{-aw\alpha + \tau w\alpha f_1^*}{\phi' (f_1^D)} < 0. \]
2.7.8 Proof of Proposition 2.7

Recall

\[ \tau w + \tau wa = \varphi (f_1^x) \]

\[ \tau w + (\tau + a) wa = \varphi (f_1^x) \]

Let \( z = f_1^e, f_1^s \). Then

\[ \frac{\partial z}{\partial r} = \frac{2w (1 + \alpha (1 - g (r))) - \tau w^2 \alpha (1 + \alpha z) \varphi' (r)}{2 \varphi' (z)} \]

or equivalently

\[ \frac{\partial z}{\partial r} = \frac{2w (1 + \alpha f_2) - \tau w^2 \alpha (1 + \alpha z) \varphi' (r)}{2 \varphi' (z)} > 0, \]

since \( \varphi' (z) \) must be greater zero at \( f_1^e \) and \( f_1^s \) and \( g' < 0 \).

The labor supply response changes with higher \( z \)

\[ \frac{\partial (\frac{\partial z}{\partial r})}{\partial z} = \left[ -\frac{\tau (wa)^2 \varphi' (r) - \tau (wa)^4 \tau w (1 + \alpha z) \varphi' (r)}{\varphi' (z)} \right] \]

\[ -\frac{w + wa (1 - g (r)) - \tau wa \frac{\varphi'' (1 + \alpha z)}{2} \varphi' (r)}{\varphi' (z)} \frac{\varphi'' (z)}{\varphi' (z)} \]

where the first term is positive and the second term unambiguously negative. In order to sign the whole expression we need to know the sign of

\[ \tau w a \varphi' (r) \left( \frac{w (1 + \alpha z)}{2} \varphi'' (z) - wa \varphi' (z) \right) - \frac{\tau (wa)^2 \tau w (1 + \alpha z)}{2} \varphi'' (r) \varphi' (z) \]

\[ - (w + wa (1 - g (r))) \varphi'' (z). \]

Note that \( \varphi (z) \) is convex. If \( \varphi' (0) \leq 0 \) then \( z \varphi'' (z) > \varphi' (z) \). A sufficient condition that \( \frac{\partial (\frac{\partial z}{\partial r})}{\partial z} < 0 \) is therefore that the first term is negative. We only need to show that

\[ \frac{1 + az}{2} \geq az \]

\[ \alpha z \leq 1 \]

which is true. Hence the distortion created by strategic behavior decreases as the marginal tax rate of the wife decreases if \( \varphi' (0) \leq 0 \).
2.7.9 Proof of Proposition 2.8

The wife’s intertemporal utility under joint taxation (τ = σ) is given by

\[ u^f = \left[ \frac{(\tau f_1^* + a)w + \sigma - b}{2} + h(1 - f_1^*) \right] \]

\[ + \frac{\tau(1-g(r))a + b}{2} + h(g(r)) \]

Individual taxation would increase the husband’s tax rate and decrease the wife’s tax rate, i.e.

\[ \sigma < \tau. \]

Differentiating \( u^f \) with respect to \( \tau \) and \( \sigma \) we find

\[ du^f = \left[ \left( \frac{w f_1^* + w(1 + \alpha f_1^*) f_2^*}{2} \right) d\tau + \frac{2 + \alpha}{2} d\sigma \right] \]

where the last term is negative and the first term is positive. In order for the wife to benefit from tax reform we need

\[ -d\sigma < \left( \frac{w f_1^* + w(1 + \alpha f_1^*) f_2^*}{2 + \alpha} \right) \]

which proves (1). The husband’s utility changes with tax reform by

\[ du^m = \left[ \left( \frac{w f_1^* + w(1 + \alpha f_1^*) f_2^*}{2} \right) d\tau + \frac{2 + \alpha}{2} d\sigma \right] \]

\[ -a\alpha \frac{df_1^*}{d\tau} d\tau \]

Independent of the sign of the changes in the spouses’ utilities

\[ du^m - du^f = -a\alpha \frac{df_1^*}{d\tau} d\tau < 0. \]

Hence if tax reform increases the wife’s utility by less than \( a\alpha \frac{df_1^*}{d\tau} d\tau \), the husband’s utility decreases which proves the first part of (2). In order to make both spouses better off, we need

\[ du^m > 0, \]
which implies
\[
\frac{-d\sigma}{d\tau} < \frac{w f_1^* + w (1 + \alpha f_1^*) f_2^* - 2awc \alpha f_1^*}{2 + \alpha}.
\]

Next we show that inequality between spouses decreases whenever \(du^f > 0\). First observe that the husband always gets a bigger utility than the wife since the difference in utility between spouses under each tax regime is given by
\[
u^f - u^m = \left[ \begin{array}{c} \frac{u_1 + d_1^f - d_1^m}{2} + \frac{u_2 + d_1^f - d_2^m}{2} \\ -\left( \frac{u_1 - d_1^f + d_1^m}{2} + \frac{u_2 - d_1^f + d_2^m}{2} \right) \end{array} \right] = d_1^f - d_1^m + d_2^f - d_2^m < 0
\]

But the wife's second period divorce utility goes up with first period labor supply and hence second period divorce utility is higher under individual taxation than under joint taxation of the family. The utility ratio with a shift towards individual taxation can be written as \(\frac{u^f + du^f}{u^m + du^m}\). We then want to verify that
\[
\frac{u^f + du^f}{u^m + du^m} > \frac{u^f}{u^m}
\]

Independent of whether \(du^m\) is positive or negative, the denominator of both sides is positive and we can therefore write
\[
u^m du^f > u^f du^m,
\]

which is true if \(du^f \geq 0\). This could even be true if the utility for both spouses goes down. Dividing both sides by \(u^m u^f > 0\) will not change the sign of the inequality and we find
\[
\frac{du^f}{u^f} > \frac{du^m}{u^m}.
\]

As long as the percentage change from tax reform in the wife's utility is greater than the percentage change in the husband's utility, inequality in the household decreases.
2.8 References


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2.9 Figures

Figure 2.1: Efficient and strategic labor supply of the wife.
Chapter 3

Tax Competition and the Efficiency of "Benefit-Related" Taxes on Business

3.1 Introduction

While most of the literature on tax competition focusses on public services or public goods provided to households, Zodrow and Mieszkowski (1986) also discuss the case in which local jurisdictions provide a service to firms. More recently, Oates and Schwab (1988 and 1991), Keen and Marchand (1997), Sinn (1997 and 2003) and Bayindir-Upmann (1998) analyze the efficiency implications of capital taxation when a (congested) public good is provided to business. In this essay we compare business-related taxes such as broad-based taxes like a tax on production or a lump-sum tax on firms with taxes on specific inputs in
production and evaluate the efficiency implications of such taxes. Bird (2002) makes the following argument.

The economic—as opposed to the political economy—case for local business taxation is simply as a form of generalized benefit tax. Where possible, specific business enterprises should be paid for by appropriate user charges. Where it is not feasible to recoup the marginal cost of cost-reducing public sector outlays through user charges, some form of broad-based general levy on business activity may well be warranted (Bird, 2002, p. 225).

Even though this is a widespread opinion in the literature, to our knowledge, there does not exist a framework that systematically analyzes this claim. Our paper emphasizes the importance of the interaction between the public service and the production technology employed by the private sector. We use Zodrow and Mieszkowski (ZM)'s model as a starting point in developing an analytical framework in which above issue is addressed. The business public service is treated as an input in the production of a consumption good and firms receive a fixed level of the public service. Oates and Schwab (1991) on the other hand, allow jurisdictions to distribute the public service to firms in a fixed proportion to the capital that they employ. Although Sinn (1997) has developed a model in which the public service produces an intermediate good, his model is quite closely related to the model by Oates and Schwab, because the public service in Sinn's model also directly benefits capital.

Our results suggest that a production tax will be more efficient than a capital tax in cases, where the public service is in a somewhat symmetric relation to the private input factors, because one might interpret a tax on production as a tax that is closer in nature to a head tax than a tax on the mobile factor. If on the other hand the public service directly benefits capital and only indirectly labor, a capital tax is more efficient. Since a tax on labor or a lump-sum tax on firms is efficient in ZM's model, it intuitively makes sense that
a production tax will yield better results than a capital tax. In the special case of a Cobb-Douglas type production function jurisdictions in ZM’s model even achieve efficiency with a production tax. If we use the more general CES type, our results suggest that a production tax outperforms a tax on capital but is not efficient. A capital tax in ZM’s model is always inefficient. In Oates and Schwab’s as well as in Sinn’s model a tax on capital is efficient, but head taxes and a production tax are not.

3.2 The Model

A union consists of $N$ jurisdictions with the same number of residents in each. Residents everywhere have identical preferences and endowments. Individuals work where they live and only consumption generates utility. The labor supply of a jurisdiction, $L$, is therefore fixed.\(^1\) People own an equal share of the union’s capital stock $\bar{K}$. Since capital ($K$) is perfectly mobile across jurisdictions, the return on capital, $r$, is the same in every jurisdiction.

Each jurisdiction produces a single consumption good, $X$, with constant returns to scale (CRS) technology. Labor and capital are private inputs in the production of $X$. The consumption good is taken as the *numeraire*.

The local government provides a public service ($B$) to firms which is used directly or indirectly in the production of $X$. The per unit cost of the public service is equal to 1.

Local governments maximize the income of their residents taking the policies of the competing jurisdictions as given. They also think that their own policies will not affect

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\(^1\)ZM call the fixed factor land.
the union-wide return on capital or the price of the consumption good. Residents' income is equal to the sum of their net wage income, profit of the local firms (if any), and residents' capital income from union-wide investment

\[ I = F(\cdot) - F_K K - \sigma L + r \frac{K}{N}. \]  

(3.1)

In the following sections, we specify the production function of the consumption good, \( F(\cdot) \), for each model separately. The marginal productivity of capital in the production of the consumption good, \( F_K \), is determined by the profit maximizing behavior of firms.

We analyze different taxation environments. Jurisdictions can tax capital (\( \tau \)), impose a tax on labor (\( \sigma \)), and lump-sum tax firms (\( H \)) to balance their budget,

\[ \tau K + \sigma L + H = B. \]  

(3.2)

Alternatively, jurisdictions may impose a production tax (\( t \)) to balance their budget

\[ tX = B. \]  

(3.3)

3.3 Zodrow and Mieszkowski (1986)

In ZM's model, the public service is just another input in the production of the consumption good. A firm's production function is given by\(^2\)

\[ X = F(L, K, B) \]  

(3.4)

\[ CRS : \quad F(L, K, B) = F_L L + F_K K + F_B B \]  

(3.5)

\(^2\)The authors do not state explicitly in the section on business public services that the production function is CRS in \( (L, K, B) \). The authors state, however, in the previous section that the production function is assumed to be CRS in all its inputs. Although Simm (1997 and 2003) interprets this to mean that the production in the case of business public service is not constrained to be CRS in \( (L, K, B) \), I read ZM that the assumptions from the previous section apply.
where

\[ F_K > 0, F_B > 0, F_L > 0 \]  \hspace{1cm} (3.6)

\[ F_{KB} > 0, F_{KL} \geq 0, F_{BL} \geq 0, F_{K} < 0. \]  \hspace{1cm} (3.7)

for \( i = K, B, L \). Once labor is fixed, \( F(B, K) \) is strictly concave.\(^3\)

### 3.3.1 Capital Tax, Labor Tax and Lump-Sum Tax

The local government maximizes the income of its residents subject to firms’ profit maximizing condition

\[ F_K (K, \tau K + \sigma L + H, L) = \tau + \tau, \]  \hspace{1cm} (3.8)

and the jurisdiction’s budget constraint (3.2). The local government’s objective function is

\[ I = F(K, \tau K + \sigma L + H, L) - (\tau + \tau) K - \sigma L + \tau \frac{K}{N}. \]  \hspace{1cm} (3.9)

Local governments need to know how the demand for capital in the jurisdiction changes as they increase the various taxes. Given \( \sigma \) and \( H \), differentiating (3.2) and (3.8) with respect to \( K \) and \( \tau \) yields the response of capital to an increase in the capital tax.

\[ \phi \equiv \frac{dK}{d\tau} = -\frac{1 - F_{KB} K}{F_{KK} + F_{KB} \tau}. \]  \hspace{1cm} (3.10)

Given \( \tau \) and \( H \), differentiating (3.2) and (3.8) with respect to \( K \) and \( \sigma \) yields the response of capital to an increase in the labor tax.

\[ \frac{dK}{d\sigma} = -\frac{F_{KB} L}{F_{KK} + F_{KB} \tau}. \]  \hspace{1cm} (3.11)

---

\(^{3}\)For example, \( F(B, K) = (L^a + bH^a + cK^a)^{\frac{1}{a}} \) with \( 0 < a < 1 \), satisfies all the assumptions about technology.
Given $\tau$ and $\sigma$, differentiating (3.2) and (3.8) with respect to $K$ and $H$ yields the response of capital to an increase in the lump-sum tax.

$$\frac{dK}{dH} = \frac{-F_{KB}}{F_{KK} + F_{KB}\tau}. \quad (3.12)$$

We show below that the denominator of the response of capital to an increase in each of the tax instruments is negative. Hence increasing $H$ or $\sigma$ will cause an increase of capital in the jurisdiction, and at the interior solution an increase in $\tau$ will decrease $K$.

Local jurisdictions want to find $\sigma, \tau$ and $H$ as to maximize residents' income. In order to do that, they would also want to find the socially optimal level of $K$. Given (3.8), they can then induce firms to demand the optimal level of capital by taxing capital at the efficient amount. However, $K$ is not a choice variable, we can only use the optimality condition for $K$ as a benchmark for efficiency. If jurisdictions are constrained in their choice of tax instrument this benchmark might not be obtainable. Maximizing (3.9) with respect to $\sigma, \tau, H$ and $K$, yields the following first order conditions

w.r.t. $\sigma$, \quad $L(F_B - 1) + F_B\tau \frac{dK}{d\sigma} = 0 \quad (1)$

w.r.t. $\tau$, \quad $K(F_B - 1) - F_B\tau \phi = 0 \quad (2)$

w.r.t. $H$, \quad $(F_B - 1) + F_B\tau \frac{dK}{dH} = 0 \quad (3)$

w.r.t. $K$, \quad $F_B\tau = 0 \quad (4)$

**Proposition 3.1** If labor or firms can be taxed, $\tau = 0$ and $F_B = 1$.

All equations of (3.13) hold simultaneously if and only if $\tau = 0$. The marginal benefit of the public service is exactly equal to its cost. Together with (3.8), jurisdictions solve for $B$ as a function of $r$ and firms demand $K$ according to (3.8) given $B$. Once $r$ is determined in the whole economy, local governments supply the efficient level of $B$. 
If local governments cannot tax labor or lump-sum tax firms at the efficient amount, they need to impose a tax on capital as well. If $\tau > 0$, conditions (2) and (4) are the only relevant. However, (4) in (3.13) can no longer be satisfied. The best jurisdictions can do is to set

$$\tau = K \frac{F_B - 1}{F_B \phi}. \quad (3.14)$$

We immediately see that the question of whether there will be an over or underprovision of the public service depends critically on the sign of $\phi$. Next we present an intermediate result that helps to determine the denominator of $\phi$.

1. A proportional increase in $B$ and $K$ will cause $F_K$ and $F_B$, respectively to decrease

$$BF_{BK} + KF_{KK} < 0, \quad KF_{BK} + BF_{BB} < 0, \quad (3.15)$$

2. $F_{KB}$ is a decreasing function in $K$ and $B$

$$KF_{KB} < F_B, BF_{KB} < F_K. \quad (3.16)$$

See the appendix for the proof of the two properties.

**Observation 3.1** The denominator of $\phi$ is negative.

See the appendix for the proof.

**Proposition 3.2** If a labor tax or a lump-sum tax on firms is not available, an interior solution with capital tax $\tau > 0$ has the following properties: (1) $F_B > 1$, the business public service is underprovided, and (2) $\phi > 0$, jurisdictions expect to drive out capital if they increase $\tau$. 
See the appendix for the proof.

For an interior solution, we need \( 1 > K F_K B \), since the denominator of \( \phi \) is negative. An interior solution, however, might not always exist.\(^4\)

In a capital tax equilibrium, identical jurisdictions set the same tax rate, with each believing that there will be an outflow of capital. The fixed capital stock in the union and identical capital tax rates leave the level of capital invested in each jurisdiction unchanged relative to the efficient allocation. The marginal productivity of capital declines because the public service is undersupplied. With a lower public service level firms pay a lower price for capital than in the efficient equilibrium, implying that capital owners will bear part of the burden of the tax. The distortionary tax also lowers wages, since marginal productivity of labor decreases with a lower public service level.

### 3.3.2 Production Tax

In this section we examine the efficiency implications if the business public service is financed by a production tax. We take the consumer price of the consumption good to be normalized to 1. We can rewrite (3.3) as

\[
 tF(L, K, B) = B. 
\]  

(3.17)

Since firms receive the consumer price minus the tax for each unit of the consumption good residential income is now given by

\[
 I = (1 - t) F(L, K, B) - rK + \frac{K}{N},
\]
and the profit maximizing condition is

\[(1 - t) F_K (L, K, B) = r. \tag{3.18}\]

In order to find the optimal production tax, we first need to evaluate the impact of capital and the production tax on the amount of \(B\). Implicit differentiation of (3.17) yields

\[
\frac{\partial B}{\partial K} = \frac{tF_K}{(1 - tF_B)},
\]

and

\[
\frac{\partial B}{\partial t} = \frac{F}{(1 - tF_B)}.
\]

Using (3.18) and the budget balance, we can find the reaction of capital to an increase in \(t\).

\[
\frac{dK}{dt} = \frac{(F_K - (1 - t) F_{KB} \frac{\partial B}{\partial K})}{(1 - t) (F_{KK} + F_{KB} \frac{\partial B}{\partial K})} \nonumber
\]

\[
= \frac{F_K (1 - F_B t) - F_{KB} F (1 - t)}{(1 - t) (F_{KK} (1 - F_B t) + F_{KB} F_K t)}.
\]

**Observation 3.2** The denominator of \(\frac{dK}{dt}\) is negative.

See the appendix for the proof.

First order conditions are given by\(^5\)

w.r.t. \(t\), \(F_K t (F_{KB} F - F_B F_K) - F_{KK} F (F_B - 1) = 0 \quad (1) \tag{3.19}\)

w.r.t. \(K\), \(\frac{(1-t)F_{BF} K}{1-F_B t} = 0 \quad (2)\)

**Proposition 3.3** A production tax is inefficient. Only in the special case when the cross derivatives of the log of the production function are zero, i.e. \(F'_{KB} = F_{BF} K\), is a production tax efficient.

---

\(^5\)The derivation of the first order conditions can be found in the appendix.
Whether there is an overprovision or an underprovision of the business public service depends on the sign of \( F_{KB}F - F_B F_K \). This is equivalent to asking whether the cross derivative of the log of \( F (L, K, B) \) is positive (overprovision) or negative (underprovision).\(^6\)

In the special case of a Cobb-Douglas production function \( F = AK^a B^b \)

\[
F_{KB}F - F_B F_K = 0.7
\]

Then an interior solution has

\[
F_B - 1 = 0,
\]

and the public service is supplied efficiently. Of course, the jurisdictions believe also that there will be less capital in their jurisdiction, but as all jurisdictions face the same constraint the price of capital will decrease in the whole economy until local firms employ the same amount of capital as previously. The Cobb-Douglas production function, however, is a special case, and typically \( F_{KB}F - F_B F_K \neq 0 \). For example, if instead the production function is given by

\[
F (L, K, B) = \left( L^a + bK^a + cB^a \right)^\frac{1}{a}
\]

\[
0 < a, b, c < 1
\]

then

\[
F_{KB}F - F_B F_K < 0,
\]

and \( B \) would be undersupplied.\(^8\)

---

\(^6\) \( G = \ln F \), then \( \frac{\partial G}{\partial B} = \frac{F_B}{F} \) and \( \frac{\partial^2 G}{\partial K \partial B} = \frac{F_{KB}F - F_B F_K}{F} \).

\(^7\) Since \( \frac{\partial F}{\partial K} = \alpha AK^{\alpha-1} B^\beta \), \( \frac{\partial F}{\partial B} = \beta AK^\alpha B^{\beta-1} \) and \( \frac{\partial^2 F}{\partial K \partial B} = \alpha \beta AK^{\alpha-1} B^{\beta-1} \).

\(^8\) Since \( F_K = \beta F^{1-a} K^{a-1} \), \( F_B = c F^{1-a} B^{a-1} \), and \( F_{KB} = bc(1-a) F^{1-2a} K^{a-1} B^{a-1} \), we have

\[
ch(1-a) F^{1-a} K^{a-1} B^{a-1} - cb F^{1-s} K^{a-1} B^{a-1} < 0
\]

\[
ch(1-a - 1) < 0.
\]
3.3.3 Comparison between Capital Tax and Production Tax

Note that the condition for an interior solution with a capital tax can be written as

$$\tau (F_{KB} K - F_B) - F_{KK} K (F_B - 1) = 0.$$  

Comparing this with the condition for an interior solution in the case of a production tax

$$F_{Kt} (F_{Kt} F - F_B F_K) - F_{KK} F (F_B - 1) = 0,$$

we can exclude the possibility that both tax regimes will yield the same level of $B$.

**Proposition 3.4** The level of inefficiency of the capital tax and the production tax is never the same.

See appendix for proof.

If the production function is of the CES type we can say even more.

**Proposition 3.5** Suppose that the parameters of the model are such that an interior solution to the capital tax and the production tax problem exist and that $F_B = 1$ is obtainable for $0 < B < \infty$. Under these conditions the production tax is more efficient than the capital tax.

This section shows that the question of how well a production tax does compared to a capital tax depends crucially on the production function of the consumption good. In the case of a Cobb-Douglas production function, the production tax is efficient, but the capital tax is not. In the section on simulations we provide necessary and sufficient conditions for the Cobb-Douglas production function. We also provide simulations with the
more general CES function and find that the production tax outperforms the capital tax if interior solutions in both tax regimes exist.

3.4 Oates and Schwab (1991)

The only difference between ZM's model and Oates and Schwab (OS)'s model is that jurisdictions have one more instrument available: They can provide the public service to business in proportion to the amount of capital that each firm employs. That is

\[ F(K, B, L) = F(K, \alpha K, L), \]

and the budget balance of a jurisdiction is given by

\[ \alpha K = \tau K + \sigma L + H \]

or

\[ \alpha = \frac{\tau K + \sigma L + H}{K}. \]

An increase in \( \tau \) has the following effect on \( \alpha \)

\[
\frac{\partial \alpha}{\partial K} = -\frac{\sigma L + H}{K^2}, \\
\frac{\partial \alpha}{\partial \tau} = 1, \frac{\partial \alpha}{\partial \sigma} = \frac{L}{K}, \frac{\partial \alpha}{\partial H} = \frac{1}{K}.
\]

Because the firms know that they will receive the public service in proportion \( \alpha \) to the capital employed their profit maximization condition becomes

\[ F_K + \alpha F_B = r + \tau. \]

Next we find the response of capital to an increase in each of the tax instruments.
\[
\frac{dK}{d\tau} = \frac{(1 - \alpha F_{BB}K - F_{KB}K - F_B)}{(F_{KK} + F_{KB}\tau + \alpha (F_{BK} + F_{BB}\tau) - F_B \frac{\sigma L + H}{K^2})}.
\]

With respect to \(\sigma\)
\[
\frac{dK}{d\sigma} = \frac{(-\alpha F_{BB} - F_{KB} - F_B \frac{1}{K}) L}{(F_{KK} + F_{KB}\tau + \alpha (F_{BK} + F_{BB}\tau) - F_B \frac{\sigma L + H}{K^2})},
\]

and with respect to \(H\)
\[
\frac{dK}{dH} = \frac{(-\alpha F_{BB} - F_{KB} - F_B \frac{1}{K})}{(F_{KK} + F_{KB}\tau + \alpha (F_{BK} + F_{BB}\tau) - F_B \frac{\sigma L + H}{K^2})}.
\]

**Observation 3.3** The denominator of \(\frac{dK}{d\tau}, \frac{dK}{d\sigma}\) and \(\frac{dK}{dH}\) is negative.

By properties 1 and 2, we can sign the response of capital to an increase in the tax on labor or the lump-sum tax on firms. It is unambiguously positive (both the denominator and the numerator are negative). The response of capital to an increase in \(\tau\) is ambiguous.

For very small values of \(B\) it is likely to be positive and for larger values of \(B\) such that \(F_B \leq 1\) it is for sure negative. The jurisdictions’ maximizing problem becomes

\[
\max_{\tau, \sigma, H, K} F(K, (\tau K + \sigma L + H), L) - (r + \tau) K - \sigma L - H + \frac{\overline{K}}{N},
\]

and first order conditions are given by

w.r.t. \(\tau\)
\[
F_B K - K + (F_K + \tau F_B - (r + \tau)) \frac{dK}{d\tau} = 0 \quad (1)
\]

w.r.t. \(\sigma\)
\[
F_B L - L + (F_K + \tau F_B - (r + \tau)) \frac{dK}{d\sigma} = 0 \quad (2)
\]

w.r.t. \(H\)
\[
F_B - 1 + (F_K + \tau F_B - (r + \tau)) \frac{dK}{dH} = 0 \quad (3)
\]

w.r.t. \(K\)
\[
F_K + \tau F_B - (r + \tau) = 0 \quad (4)
\]

**Proposition 3.6** In the Oates and Schwab model, a tax on capital is a benefit tax and it is efficient. If capital cannot be taxed, a tax on the fixed factor or a lump-sum tax on firms are inefficient.
Note that by our new profit maximizing condition, (4) only holds if \( \alpha = \tau \), which implies that \( F_B = 1 \) and \( \sigma = H = 0 \). Now suppose capital cannot be taxed. This means \( F_K + \frac{aL + H}{K}F_B = r \) and therefore (4) cannot hold. We have \( F_K + \tau F_B - (r + \tau) < 0 \) and find that there is an undersupply of the business public service. If, on the other hand \( \tau \) would be higher than \( \alpha \) we would find that jurisdictions oversupply the public service. This is consistent with the literature on incentives for firms. It is argued that if firms pay more than the benefit tax, tax competition among jurisdictions causes jurisdictions to give firms tax incentives or non-tax incentives to lower the burden on firms and move closer to a benefit taxation (see Zodrow, 2003).

3.4.1 Production Tax

If jurisdictions rely exclusively on a production tax in the OS model, budget balance must satisfy

\[
\alpha K = t F(K, \alpha K, L),
\]

(3.20)

and profit maximization requires that

\[
(1 - t)(F_K + \alpha F_B) = r.
\]

(3.21)

Similarly to the ZM model with production tax, we need to find the reaction of \( \alpha \) to an increase in \( K \) or \( t \). Using (3.20) we find

\[
\frac{\partial \alpha}{\partial K} = \frac{t (F_K + \alpha F_B) - \alpha}{K (1 - t F_B)} < 0,
\]

\[
\frac{\partial \alpha}{\partial t} = \frac{F}{K (1 - t F_B)} > 0.
\]
The denominator is unambiguously positive since

\[ 1 - tF_B = \frac{F - BF_B}{F} > 0. \]

The numerator of \( \frac{dK}{dt} \) is negative since

\[ \frac{B (F_K K + \alpha K F_B) - \alpha K F}{KF} = \frac{B (F_K K + BF_B - F)}{KF} < 0. \]

Next we find the response of \( K \) to an increase in \( t \).

\[ (1 - t) (F_K + \alpha F_B) = r \]

\[ \frac{dK}{dt} = \frac{(F_K + \alpha F_B) - (1 - t) (F_K K + \alpha F_B + K + F_B) \frac{\partial \alpha}{\partial K}}{(1 - t) (F_K K + \alpha F_B + (F_K K + F_B + \alpha F_B) \frac{\partial \alpha}{\partial K})} \]

**Observation 3.4** The denominator of \( \frac{dK}{dt} \) is unambiguously negative.

**Proposition 3.7** In the Oates and Schwab model a tax on production is inefficient.

Maximizing residents' income

\[ \max_{t,K} (1 - t) F (K, \alpha K, L) - r K + \frac{K}{N}. \]

We find the first order conditions

w.r.t. \( t \)

\[ (1 - t) F_B K \frac{\partial \alpha}{\partial t} - F + (1 - t) F_B K \frac{\partial \alpha}{\partial K} \frac{dK}{dt} = 0 \quad (1) \]

w.r.t. \( K \)

\[ (1 - t) (F_K + \alpha F_B + F_B K \frac{\partial \alpha}{\partial K}) - r = 0 \quad (2) \]

Substituting for \( \frac{\partial \alpha}{\partial t} \) and \( \frac{\partial \alpha}{\partial K} \) in (1) of (3.22) we obtain

\[ F - \frac{F_B - 1}{(1 - tF_B)} + (1 - t) F_B \frac{t (F_K + \alpha F_B) - \alpha dK}{(1 - tF_B)} = 0. \]

If \( \frac{dK}{dt} < 0 \), jurisdictions provide an inefficiently high level of the business public service, and there is an undersupply of the public service if \( \frac{dK}{dt} > 0 \). Since a lump-sum tax on firms was
inefficient it is intuitive that a tax on production is also inefficient. In the next section we present Sinn’s model which can be seen as a special case of the Oates and Schwab model. In Sinn’s model as well as in the OS model, capital directly benefits from the public service. It is therefore not surprising that the results obtained in Sinn’s model very closely resemble those in the OS model.

3.5 Sinn (1997)

Sinn’s model bears close resemblance to Oates and Schwab’s model in that it also ties the public service to one input only, namely mobile capital. By separating the production of the final good and the public service, Sinn incorporates the idea of economies of scale in the public service. If the public service directly enters the production of the final good and causes economies of scale in the production of the final good, we run into difficulties, since production efficiency will now depend on the number of firms in the jurisdiction. As long as the production of the final good is CRS, the production efficiency does not depend on the number of firms. By separating the public service from the production of the final good, Sinn has found a clever way of taking account of the idea of economies of scale without having to deal with the problems just described.

Sinn considers a public good like a highway which does not directly enter in the production of the consumption good. Instead, in order to use capital as an input in the production of the consumption good, firms have to transport capital from one point to another. Each unit of capital has to make the same trip. The total number of trips is therefore equal to the units of capital used in the production of X. Local governments
provide the highway, but firms incur transportation costs which are user fees imposed by the local governments. The governments adopt average cost pricing to determine user fees. Let \( B \) be the width of the highway. The user cost depends on the number of trips as well as the width of the highway. Denote \( c \) the average user cost of the highway, then \( c \) increases with \( K \) and decreases with \( B \).

\[
c = c(K, B),
\]

\[
c_K \geq 0, c_B < 0, c_{BB} > 0.
\]

If \( c_K > 0 \), we have congestion and the marginal private cost differs from the marginal social cost of using the highway.

User fees cover the cost of transportation, but not the provision of the highway. As noted the per unit cost of \( B \) is equal to 1.

The consumption good is produced by transported capital, \( K \), and labor. The public service plays only an indirect role in the production of \( X \) by providing the highway that makes the transport of capital possible.

\[
X = F(K, L),
\]

where \( F(K, L) \) is CRS and marginal products satisfy

\[
F_K > 0, F_L > 0,
\]

\[
F_{KL} > 0, F_{KK} < 0, F_{LL} < 0.
\]
3.5.1 Capital Tax, Labor Tax and Lump-Sum Tax

Firms' profit maximizing condition is given by

\[ F_K(K, L) = r + c + \tau. \]  \hspace{1cm} (3.23)

Local governments maximize the income of their residents \((I)\) subject to (3.2) and (3.23)

\[ \max_{\sigma, \tau, K} F(K, L) - [r + c(K, \sigma L + \tau K + H) + \tau] K - \sigma L + r \frac{K^\rho}{N}. \]  \hspace{1cm} (3.24)

As in ZM's model, jurisdictions need to take into account the response of capital to an increase in the capital tax. This response is obtained by differentiating equations (3.2) and (3.23) with respect to \(\tau\) and \(K\). We find

\[ \phi \equiv -\frac{dK}{d\tau} = \frac{1 + c_B K}{F_{KK} - c_K - c_B \tau}. \]  \hspace{1cm} (3.25)

Note that the numerator of \(\phi\) is independent of the final good production, while in ZM and OS the numerator of the response of capital (3.10) depends on \(F_{KB}\). Similarly, finding the response of capital with respect to an increase in one of the other two taxes,

\[ \frac{dK}{d\sigma} = \frac{c_B L}{F_{KK} - c_K - c_B \tau}, \]
\[ \frac{dK}{dH} = \frac{c_B}{F_{KK} - c_K - c_B \tau}. \]

Maximizing (3.24) with respect to \(\tau, \sigma, H, \) and \(K\) we find

\[ \text{w.r.t. } \sigma, \quad -(c_B K + 1) L - (c_K K + c_B \tau K) \frac{dK}{d\sigma} = 0 \]  \hspace{1cm} (1)
\[ \text{w.r.t. } \tau, \quad F_{KK} K \phi = 0 \]  \hspace{1cm} (2)
\[ \text{w.r.t. } H, \quad -(c_B K + 1) - (c_K K + c_B \tau K) \frac{dK}{dH} = 0 \]  \hspace{1cm} (3)
\[ \text{w.r.t. } K, \quad -c_K K - c_B \tau K = 0 \]  \hspace{1cm} (4)
Since $F_{KK} K < 0$, condition (2) holds if the response of capital to an increase in the capital tax is zero. The marginal benefit of increasing the capacity by one unit summed over all users (equal to the amount of capital) must be equal to the marginal cost of supplying one more unit of capacity. This is the familiar Samuelson condition for the provision of public goods. Local governments get an extra condition from maximizing the income of their residents with respect to $K$, because firms do not consider the externality they impose on others by using the highway.\footnote{In ZM’s model, in contrast, local governments face the same first order condition with respect to capital as firms do because the public service is a publicly provided private good.} This is quite similar to the optimality condition for $K$ in the OS model where firms benefit from an increase in capital with a higher level of the public service and optimality requires that $\tau$ be set equal to this benefit.

**Proposition 3.8** (Sinn, 1997 and 2003) *Local jurisdictions choose a combination of capital tax and tax on labor (lump-sum tax on firms) that satisfies the Samuelson condition, $-c_B K = 1$. In the optimal tax mix, $\tau = c_K K$, the marginal crowding externality.*

What if all jurisdictions are constrained in the use of one of their tax instruments?

**Proposition 3.9** (Sinn, 1997 and 2003) *If jurisdictions can only tax capital, they provide the efficient amount of infrastructure.*

If $\sigma = H = 0$, local governments face a self-financing constraint: users have to pay for the provision of the infrastructure. In this case, setting $\tau = c_K K$ does not generate enough revenues to cover the expenses for $B$ in the case of imperfect congestion. That is, the average user cost function satisfies

$$c_K K + c_B B < 0.$$  \hspace{1cm} (3.27)
But if $\sigma = 0$, we must have $\tau K = B$. Comparing (3.27) with condition (3) in (3.26), they cannot simultaneously hold. The best local governments can do is to set $\tau$ such that the Samuelson condition holds. Condition (2) together with (3.23) lets local governments solve for $c$ and $\tau$ as a function of $r$. If $r$ does not change compared to the situation in which both inputs could be taxed, firms demand less capital. But $r$ changes because every jurisdiction faces the same self-financing constraint and hence offers the same rate structures of $c$ and $\tau$ to its firms. Indeed, “a self-financing constraint will not result in an underprovision of public infrastructure even when there are scale economies (Sinn, 1997, p. 256).” The return on capital decreases by the increase in $\tau$; capital bears the burden of the self-financing constraint, but there is no distortion.

**Proposition 3.10** If jurisdictions can only tax labor or impose a lump-sum tax on firms, they underprovide infrastructure.

If jurisdictions can only tax labor or impose a lump-sum tax on firms, i.e. $\tau = 0$, condition (4) in (3.26) cannot be satisfied. Then the second term in conditions (1) and (3) in (3.26) becomes negative and therefore we must have $-c_B K - 1 > 0$. This implies that there is an undersupply of infrastructure. Although jurisdictions take into account that by increasing $\sigma$ they can attract more capital, if $\tau$ cannot be set equal to the crowding externality and indeed is set to be less than $c_K K$, there will be underprovision. If, on the other hand, $\tau$ is constrained to be higher than the crowding externality but not high enough that it is a self-financing constraint, there will be overprovision of the infrastructure (see Sinn, 2003).

Proposition 3.9 seems counterintuitive at first glance. Why is there no distortion,
i.e. $\phi \neq 0$ if local governments cannot tax labor (or lump-sum tax firms) at the efficient amount? Local governments in Sinn's model do not face the same tradeoff as local governments do in ZM's model, because they do not see any possibility for substitution between the public service and capital. Given the demand for capital, local governments know the exact number of trips that have to be made and can therefore provide the amount of infrastructure that minimizes the cost of capital for firms. The cost of capital for local firms increases the least if local governments choose a tax on capital so as to satisfy the Samuelson condition. In ZM's model, on the other hand, local jurisdictions take into account the possibility of substitution between the public service and capital in the production of the consumption good which causes the distortion.

Sinn's neutrality result also depends critically on the assumption of identical jurisdictions. Although a tax on capital only leads to efficiency in his model, jurisdictions cannot choose any combination between wage tax and capital tax unilaterally. Redistribution from capital to labor and vice versa is possible if and only if all jurisdictions face the same constraint on all tax instruments.

### 3.5.2 Production tax

Each jurisdiction takes the consumer price of the consumption good as given which is equal to 1. Budget balance is given by

$$tF(L,K) = B,$$

and the response of capital to an increase in the production tax is given by

$$\frac{dK}{dt} = \frac{c_BF + F_K}{(1-t)F_{KK} - c_K - c_BtF_K}.$$
Maximizing residents’ income

\[ \max_{t,K} (1 - t)F(L, K) - (r + c)K + r\frac{K}{N}. \]

**Proposition 3.11** A production tax is inefficient in Sinn's model.

See appendix for proof.

The reason why a production tax does worse than a capital tax in Sinn’s model is that the public service is closely tied to capital. A tax on capital should therefore be more efficient than a broader based tax.

### 3.6 Examples and Simulations for the ZM Model

We first derive necessary and sufficient conditions for the Cobb-Douglas function under both tax regimes, the capital and the production tax. We then compare the level of inefficiency that arises with a more general CES function when jurisdictions use a capital tax or a production tax.

#### 3.6.1 Cobb-Douglas

The Cobb-Douglas production function is given by

\[ F = L^aK^bB^c \]

\[ 0 < a, b, c < 1 \]
\[ a + b + c = 1. \]
In the case of a tax on capital we have the following budget constraint and profit maximizing condition.

\[ B = \tau K \]

\[ aL^cK^{\alpha-1}B^b = r + \tau \]

Solving for the response of capital to an increase in the capital tax we obtain

\[
\phi = \frac{1 - abL^cK^\alpha (\tau K)^{b-1}}{a(a-1)L^cK^{\alpha-2} (\tau K)^b + \tau abL^cK^{\alpha-1} (\tau K)^{b-1}} \\
\phi = -\frac{1 - ab\frac{F}{K}}{aFK^2(a+b-1)} = \frac{1 - ab\frac{F}{K}}{aF(1-a-b)}.
\]

The first order condition is given by

\[ F_B (K - \tau \phi) - K = 0, \quad (3.28) \]

after several manipulations, we find

\[ \tau = \left( \frac{a+b}{abL^cK^{\alpha+b-1}} \right)^{\frac{1}{b-1}}. \]

At a \( \tau \) given by (3.28), the second order sufficient condition is satisfied which requires

\[-a < 0.\]

We have already shown that the production tax is efficient in the case of a Cobb-Douglas production function. The sufficient condition

\[ -\frac{b}{\tau^2} < 0 \]

is always satisfied.
The inefficiency of a capital tax in the Cobb-Douglas case can be substantial. Below table reports values\textsuperscript{10} for $B$, under different parameters. $B^*$ denotes the efficient level of public service, and the last column indicates inefficiency of the public service under a capital tax when compared to the efficient level. Note that the inefficiency in all except one case is more than 50%.

<table>
<thead>
<tr>
<th>L</th>
<th>K</th>
<th>a</th>
<th>b</th>
<th>$\tau$</th>
<th>B</th>
<th>$B^*$</th>
<th>1-B/B*</th>
</tr>
</thead>
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<td>2</td>
<td>1/3</td>
<td>1/3</td>
<td>.05</td>
<td>0.96</td>
<td>0.2</td>
<td>52%</td>
</tr>
<tr>
<td>1</td>
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<td>1</td>
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<td>2</td>
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<td>3</td>
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<td>.25</td>
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<td>0.58</td>
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<tr>
<td>5</td>
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<td>.1</td>
<td>.7</td>
<td>.03</td>
<td>0.1</td>
<td>93.98</td>
<td>99.97%</td>
</tr>
</tbody>
</table>

Analytically, we see that

$$1 - \frac{B}{B^*} = 1 - \frac{1}{K} \left(\frac{a}{a+b}\right)^{\frac{1}{2\beta}}$$

and hence the inefficiency increases with an increase in $K$ or $b$, and decreases with an increase in $a$.

\textsuperscript{10}Values are rounded to two decimals.
3.6.2 CES-Function

In order to allow for the possibility that the efficient solution can be reached we need coefficients that guarantee that \( F_B = 1 \) is obtainable for \( B < \infty \). We therefore assume

\[
F = (L^a + bK^a + cB^a)^{\frac{1}{a}}
\]

\[
0 < a, b, c < 1.
\]

We show below that this is unfortunately not enough to ensure an interior solution.

Capital Tax

The derivatives of the CES function in case of a capital tax are given by

\[
F_K = bF^{1-a}K^{a-1}
\]

\[
F_B = cF^{1-a}K^{a-1}r^{a-1}
\]

\[
F_{KB} = bc(1-a)F^{1-2a}K^{2a-2}r^{a-1}
\]

\[
F_{KK} = -b(1-a)F^{1-2a}K^{a-2}(F^a - bK^a)
\]

\[
F_{KK} + F_{KB}\tau = -L^a b(1-a)F^{1-2a}K^{a-2}
\]

Solving for the response of capital to an increase in \( \tau \)

\[
\phi = \frac{1 - F_{KB}K}{F_{KK} + F_{KB}\tau}
\]

\[
= \frac{1 - bc(1-a)F^{1-2a}K^{2a-1}r^{a-1}}{L^a b(1-a)F^{1-2a}K^{a-2}},
\]

and the first order condition is

\[
K (F_B - 1) - F_B \phi \tau = 0,
\]

\[
-abc r^a K^a + (F^a - bK^a) (b(1-a) (F_B - 1) - c\tau^a) = 0.
\]
Production Tax

First, we find an explicit function for $B$

$$B = t \left( L^a + bk^a + cB^a \right)^\frac{1}{a}$$

and the derivatives can be written as

$$F_K = bF^{1-a}K^{a-1}$$

$$F_B = cF^{1-a}B^{a-1} = ct^{a-1}$$

$$F_{KB} = bc(1 - a)F^{-a}K^{a-1}t^{a-1}$$

$$F_{KK} = -b(1 - a)F^{1-2a}K^{a-2}(F^a - bK^a)$$

The reaction of capital to an increase in $t$ is

$$\frac{dK}{dt} = \frac{F^aK\left(1 - ct^{a-1}(1 - a + at)\right)}{-L^a(1 - t)(1 - a)}.$$

The first order condition for the CES production function is given by

$$F_Kt(F_{KB}F - F_BF_K) - F_{KK}F(F_B - 1) = 0$$

$$-abcd^aK^a + (1 - a)(F^a - bK^a)(ct^{a-1} - 1) = 0.$$

There will be undersupply with the CES function since

$$F_{KB}F - F_BF_K < 0$$

$$cb(1 - a)F^{1-a}K^{a-1}t^{a-1} - cbF^{1-a}K^{a-1}t^{a-1} < 0$$

$$cb(1 - a - 1) < 0.$$
Our simulations show that an interior solution does not always exist and it is possible that an interior solution exists under one tax regime but not under the other. However, if an interior solution exists under both tax regimes, our results suggest that the production tax is more efficient. Note also that the Cobb-Douglas function is a limiting case of the CES function as a goes to zero and hence it is not surprising that the production tax is more efficient than the capital tax for small a.

\[
\begin{array}{cccccccc}
L^a & K & a & b & c & \tau & t & \tau K & t^* \\
1 & 2 & .25 & .25 & .25 & .16 & N/A & 0.33 & N/A & 0.89 \\
1 & 2 & .125 & .25 & .25 & N/A & .20 & N/A & 8.77 & 8.85 \\
1 & 2 & .25 & .125 & .125 & .04 & .06 & 0.07 & 0.14027 & 0.14085 \\
1 & 2 & .25 & .125 & .5 & .09 & N/A & 0.18 & N/A & 5.22 \\
1 & 10 & .25 & .5 & .25 & .21 & .15 & 2.14 & 3.77 & 3.98 \\
0.05 & 3 & .1 & .7 & .3 & .17 & .24 & 0.5 & 0.78 & 0.87 \\
\end{array}
\]

3.7 Conclusion

In ZM's model the business public service is efficiently provided if jurisdictions tax labor or impose a lump-sum tax on firms. A capital tax always leads to underprovision. The public service directly enters the production function of the consumption good, and the rate structure of the capital tax depends on the marginal product of capital because the level of business public services has an impact on the cost of capital in the jurisdiction as well as on the marginal product of capital. Although the capital in each jurisdiction does not change in the capital tax equilibrium compared to the labor tax or lump-sum tax equilibrium, there is a distortion and the business public service is underprovided. In the case of a production tax over- or underprovision can occur. If the production function is
of the Cobb-Douglas type a production tax is efficient. If the production function is of the CES type, a production tax leads to underprovision of the public service to business. If an interior solution under both a capital tax regime and a production tax regime exists, the production tax is more efficient than the capital tax for the CES technology. However, an interior solution might not always exist and it is also possible that we can find an interior solution under one tax regime but not under the other.

Both Oates and Schwab's as well as in Sinn's model have built in a close relationship between capital and the public service. As capital directly benefits from the public service a tax on capital is efficient while broader based taxes such as a lump-sum tax on firms or a production tax are always inefficient.

3.8 Appendix

3.8.1 Proof of Properties 1 and 2

Strict concavity of $F(B, K)$ means that the matrix of second order derivatives of $F(B, K)$ is negative definite. Since $F_{KK} < 0$ by assumption, the determinant of the matrix of second order derivatives must be positive.

$$\begin{vmatrix} F_{KK} & F_{KB} \\ F_{BF} & F_{BB} \end{vmatrix} = F_{KK}F_{BB} - F_{KB}^2 > 0.$$ 

Multiply both sides with $KB$ and add the same term $B^2F_{KB}F_{BB} < 0$ on both sides to obtain

$$BF_{BB}(BF_{KB} + KF_{KK}) > BF_{KB}(KF_{KB} + BF_{BB}).$$
Alternatively add $K^2 F_{KB} F_{KK}$ instead of $B^2 F_{KB} F_{BB}$ to obtain

$$KF_{KK} (KF_{KB} + BF_{BB}) > KF_{KB} (BF_{KB} + KF_{KK})$$

where $iF_{ii} < 0, i = B, K$ while $iF_{KB} > 0$ by assumption. Since both inequalities must be true, it cannot be the case that either $(KF_{KB} + BF_{BB})$ or $(BF_{KB} + KF_{KK})$ be positive. This proves property 1. By property 1 we also have

$$BF_{BK} < -KF_{KK}, KF_{BK} < -BF_{BB}.$$  

Since both $B$ and $K$ exhibit diminishing returns, 

$$F_i > -iF_{ii}.$$ 

which proves property 2.

### 3.8.2 Proof of Observation 3.1

We show that

$$F_{KK} + F_{KB} \tau < 0.$$  

(3.29)

Multiply (3.29) by $K$ to obtain

$$F_{KK} K + F_{KB} \tau K.$$ 

By budget balance

$$\tau K = B,$$

and hence

$$F_{KK} K + F_{KB} B.$$ 

By property 1 $BF_{KB} + KF_{KK} < 0.$
3.8.3 Proof of Observation 3.2

We know that $B = tF$. We need to sign

$$(F_{KK} (1 - F_B t) + F_{KB} F_K t)$$

which can be written as

$$= \frac{1}{F_K} (F_{KK} K (F - F_B B) + F_{KB} B F_K K) .$$

By property 1, $-F_{KK} K > F_{KB} B$, and from CRS we know that $F - F_B B > F_K K$, hence

$$(F_{KK} (1 - F_B t) + F_{KB} F_K t) < 0 .$$

3.8.4 Proof of Observation 3.4

By property 1 $F_{KK} + \alpha F_{BK} < 0$ since it can be written as

$$\frac{F_{KK} K + B F_{KB}}{K} < 0 ,$$

and by concavity $F_B > -B F_{BB}$ and hence the coefficient of $\frac{\partial \alpha}{\partial K}$ is positive. But $\frac{\partial \alpha}{\partial K}$ is negative and hence the denominator of $\frac{dK}{dt}$ is negative.

3.8.5 Proof of Proposition 3.2

Substituting for $\phi$ in (3.14)

$$\tau = \frac{K F_{KK} (1 - F_B)}{F_B - K F_{KB}} .$$

Note $KF_{KK} < 0$. Then $\tau > 0$ if and only if

-case 1: $1 - F_B < 0$ and $F_B - K F_{KB} > 0$,

-case 2: $1 - F_B > 0$ and $F_B - K F_{KB} < 0$. 

By property 2, $F_B > KF_B$. This means we are in case 1, and $F_B > 1$. Moreover by (3.14)

$$F_B = \frac{K}{K - \tau \phi},$$

we must have $\phi > 0$ thus concluding the proof.

### 3.8.6 Proof of Proposition 3.3

Maximizing residents’ income with respect to $t$ and $K$, the FOC with respect to $t$

is given by

$$-F + (1 - t) F_B \frac{F}{1 - F_B t} + \left( F_B \frac{F_K t}{1 - F_B t} \right) \frac{F_K (1 - F_B t) - F_{KB} F (1 - t)}{(F_{KK} (1 - F_B t) + F_{KB} F_K t)} = 0,$$

and with respect to $K$

$$(1 - t) \left( F_K + F_B \frac{F_K t}{1 - F_B t} \right) - r = 0$$

$$(1 - t) \left( F_B \frac{F_K t}{1 - F_B t} \right) = 0.$$}

Clearly, the FOC with respect to $K$ can only be satisfied if $t = 0$ or $t = 1$.

We now investigate the conditions for an interior solution with $1 > t > 0$.

Assume $(1 - F_B t) \neq 0$, then

$$-F (1 - F_B t) + (1 - t) F_B F + F_B F_K t \frac{F_K (1 - F_B t) - F_{KB} F (1 - t)}{(F_{KK} (1 - F_B t) + F_{KB} F_K t)} = 0$$

$$F (F_B - 1) + F_B F_K t \frac{F_K (1 - F_B t) - F_{KB} F (1 - t)}{(F_{KK} (1 - F_B t) + F_{KB} F_K t)} = 0$$

Note that an interior solution needs to have the following properties: If there is an undersupply, i.e. $F_B > 1$, then the second term must be negative and hence $\frac{dK}{dt} < 0$. If there is an oversupply, i.e. $F_B < 1$, then the second term must be positive and hence $\frac{dK}{dt} > 0$. If $t$
is efficient, $\frac{dK}{dt} = 0$. At this point we cannot exclude any of these possibilities. Assuming that at the interior solution $F_{KK}(1 - F_B) + F_{KB}F_K t \neq 0$, then

$$F(F_B - 1) (F_{KK}(1 - F_B) + F_{KB}F_K t) + F_B F_K t (F_K (1 - F_B) - F_{KB}F(1 - t)) = 0$$

$$(tF_B - 1)(F_K t (F_{KB}F - F_B F_K) - F_{KK}F(F_B - 1)) = 0$$

$$F_K t (F_{KB}F - F_B F_K) - F_{KK}F(F_B - 1) = 0$$

### 3.8.7 Proof of Proposition 3.4

We prove Proposition 3.4 by contradiction. Multiplying both equations with the respective tax,

$$\tau (F_{KB}B - F_B \tau) - F_{KK}B(F_B - 1) = 0$$

$$F_K t (F_{KB}B - F_B F_K t) - F_{KK}B(F_B - 1) = 0.$$  

Suppose the level of public services is the same under both tax regimes. That is,

$$tF = \tau K$$  \hspace{1cm} (3.30)

and total output and all derivatives must be the same. We need to show that then it must be true that

$$F_K t = \tau$$

Substituting from (3.30) we need

$$F_K \frac{\tau K}{F} = \tau$$

$$F_K K = F$$

Contradiction.
3.8.8 Proof of Proposition 3.5

The optimal capital tax will satisfy

\[-abcr^aK^a + (F^a - bK^a)(b(1 - a)(F_B - 1) - cr^a) = 0\]

\[(F^a - bK^a)(1 - a)(F_B - 1) = \frac{abcr^aK^a + cr^a(F^a - bK^a)}{b}\]  \hspace{1cm} (3.31)

Using the same amount of \(B\) in a production tax regime, we have

\[\tau K = tF\]

and hence

\[t = \frac{\tau K}{F}.\]  \hspace{1cm} (3.32)

The optimality condition for the production tax is given by

\[-abct^aK^a + (1 - a)(F^a - bK^a)(F_B - 1) = 0.\]  \hspace{1cm} (3.33)

We need to show that the optimality condition for \(t\) is not satisfied at this level of \(B\) and moreover the left hand side is positive that is,

\[-abct^aK^a + (1 - a)(F^a - bK^a)(F_B - 1) > 0.\]

Using (3.31) and (3.32) we find

\[-abc \left(\frac{\tau K}{F}\right)^aK^a + \frac{abct^aK^a + cr^a(F^a - bK^a)}{b} > 0.\]

Note that we can factor out \(cr^a\) and we multiply the inequality by \(bF^a\)

\[-ab^2K^aK^a + abK^aF^a + F^a(F^a - bK^a) > 0\]

\[abK^a(F^a - bK^a) + F^a(F^a - bK^a) > 0\]

\[(F^a + abK^a)(F^a - bK^a) > 0\]
which is true. The optimal level of $B$ with a capital tax, does not correspond with the optimal level of $B$ under a production tax. Since the left hand side of (3.33) is positive, we would need to increase $B$ to satisfy the condition. We also know that a production tax still leads to underprovision in the ZM model. Hence a production tax is more efficient than a capital tax. What is left to show is that it is possible that an interior solution exists under both tax regimes.

3.8.9 Proof of Proposition 3.11

The first order condition with respect to $t$ is given by

$$(-F - c_B F K) - (c_K K + c_B t F_K K) \frac{c_B F + F_K}{(1 - t) F_K K - c_K - c_B t F_K} = 0 \quad (3.34)$$

and with respect to $K$

$$-c_K K - c_B t F_K K = 0. \quad (3.35)$$

If it is feasible that

$$t = \frac{-c_K}{c_B F_K},$$

the first order condition with respect to $t$ becomes

$$1 + c_B K = 0.$$

This is the Samuelson condition, and jurisdictions supply the efficient amount of infrastructure. Next we check if $t = \frac{-c_K}{c_B F_K}$ is feasible. With imperfect congestion inequality (3.27) holds. We can rewrite (3.27) as

$$\lambda c = c_K K + c_B B,$$
where $\lambda < 0$.

\[
B = \frac{\lambda c - c_K}{c_B} K \\
B = \frac{\lambda c + tc_B F_K}{c_B} K \\
B = -\lambda c K + tF_K K
\]

Note that if $\lambda = 0$, i.e. average user costs do not change with a proportional increase of $K$ and $B$, we have a contradiction, since $B > tF_K K$. But with $\lambda < 0$, it is certainly true that $B > tF_K K$ since $B = tF$. Furthermore by CRS of the production function

\[
t(F_K K + F_L L) = -\lambda c K + tF_K K \\
tF_L L = -\lambda c K.
\]

This will be true only by coincidence. Hence typically (3.35) won’t hold, and a production tax will be inefficient in Sinn’s model.

### 3.9 References


