The Impact of Liquidity Shocks on Capital

by

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Abstract

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This thesis sheds some light on factors that affect the level and stability of investment in economies with undeveloped capital markets. In particular, we focus on shocks to the demand for bank liabilities (checks). Banks are the major source of finance for firms, and fiat money and checks are the main media of exchange for households. To what extent is investment stimulated by liquidity shocks that increase the demand for bank liabilities? Our analysis is less relevant for economies with developed capital markets. However, to understand how the evolution of capital markets affects the stability and growth of investment, we focus on economies where banks are still important and capital markets are undeveloped.

The thesis is composed of an empirical and a theoretical analysis. We first show with linear regression estimates that the effect of the variance of demand deposits on that of investment is statistically significant in countries with undeveloped capital markets. Such is not the case for developed countries.

We then proceed to provide a theoretical analysis of how liquidity shocks affect investment in developing countries. Throughout our thesis, we model liquidity shocks as resulting from changes in the amounts of relocation of spatially separated agents searching for trading partners. Spatial separation is a trading friction which endoge-
nously gives rise to fiat money as a medium of exchange. The relocation of agents has the effect of changing the demand for liquidity. This changes the demand for investment-backed inside money and therefore impacts investment. This contrasts with the Clower constraints/infinite horizon models in the literature where the real sectors of an economy, and hence capital, are insulated from monetary effects.

We provide two spatial separation/overlapping generations models. We show that equilibrium capital is affected by liquidity shocks due to the overlapping generations (OLG) structure. OLG is analogous to the case of households borrowing constraints, a feature of economies with undeveloped capital markets.

Specifically how investment responds to liquidity shocks depends on the timing of the shock. In the first model, we show that when the young are hit with a liquidity shock, capital is a non-monotonic function of the shock. In the second model, old agents are subject to a liquidity shock. In this case, capital is an increasing function of the liquidity shock.
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Chapter 1

Introduction

This thesis analyzes the impact of shocks to the demand for banks liabilities that provide liquidity services to households on investment. We consider an economy with undeveloped capital markets. Banks are the major source of finance for firms. Moreover, fiat money and bank liabilities (checks) are the main media of exchange. Given undeveloped capital markets, households face borrowing constraints which preclude purchases with credit cards. The demand for bank deposits allows banks to finance investment. By considering an economy with undeveloped capital markets, we try to shed light on factors that affect the level and stability of investment in developing countries. To what extent is investment stimulated by factors, in particular liquidity shocks, that increase the demand for bank liabilities? How are the average level and stability of investment affected? In developed economies with sophisticated capital markets, firms rely less on banks to finance their investment decisions and households use credit cards to finance their purchases. Our analysis is less relevant for such economies. However, to understand how the evolution of capital markets affects the stability and growth of investment, we focus on economies where banks are still important and capital markets are undeveloped.

Chapter 2 presents empirical results showing how the stability of investment is affected by shocks to the demand for banks deposits serving as a medium of exchange. We estimate a simple regression linking the variance of investment to that of demand deposits for a sample of countries with undeveloped capital markets and another for countries with advanced capital markets. We find that in developing countries
investment is significantly more sensitive to fluctuations in demand deposits than in countries with developed capital markets.

In chapters 3 and 4, we provide two theoretical models explaining the effects of liquidity shocks on aggregate investment in economies with undeveloped capital markets. We show how shifts in the demand for liquidity (outside money) caused by liquidity shocks will impact investment via the change in the demand for investment-backed inside money. This contrasts with the Clower constraints/infinite horizon models where capital is insulated from monetary effects.

Specifically, our models are based on Hartley’s (1994) and (1998) models that show that the level and stability of investment are insulated from shocks to the demand for bank liabilities only if capital markets are sufficiently developed to allow household borrowing. Hartley (1998) examines the effect of households purchasing goods with bank checks on investment. Households are infinitely lived and face cash-in-advance constraints and check-in-advance constraints when financing purchases of goods. Households are also hit with a liquidity shock every time period. Fiat money is assumed to be universally acceptable. Checks are accepted as means of payment for only a subset of goods but earn a higher rate of return than fiat. Hartley shows, contrary to intuition, that economies with inside money do not necessarily support a higher equilibrium level of capital than economies without banks. Unless constraints on household borrowing against future labor income exist, the equilibrium capital is the same in both economies: households engage in inter temporal arbitrage until the real interest rate equals the rate of time preference. In other words, firms face a horizontal supply of loans at the rate of time preference and is independent of liquidity shocks. The equilibrium level of capital is then determined by the marginal productivity of capital and is insulated from fluctuations in the demand for inside money caused by liquidity shocks. Thus, in economies with developed borrowing and
lending markets, liquidity shocks to the demand for cash and checks do not affect the inter temporal smoothing of consumption and the capital level. A dichotomy between real and monetary sectors of an economy exists.

Borrowing constraints on households hamper inter temporal arbitrage. As households hold deposits to finance their purchases of goods, bank loans increase and lower interest rates below the rate of time preference. Households have an incentive to borrow until the interest rate rises back to the rate of time preference. However, with borrowing constraints on households, bank loans can only be channeled to firms that end up with a higher level of capital at an interest rate lower than the rate of time preference. Unlike the free borrowing and lending case, firms face a positively rising supply of loans in interest rates as multipliers from credit and liquidity constraints are a function of interest rates. Liquidity shocks shift the supply of loans thereby affecting the equilibrium level of capital. The latter is therefore determined and is sensitive to fluctuations in both firms' demand for capital (the marginal productivity of capital function) and the demand for inside money (the supply of loans).

In addition to providing empirical evidence supporting Hartley's results (chapter 2), chapters 3 and 4 extend Hartley's (1994) and (1998) models in two ways. First, we provide micro foundations for the demand for fiat money and checks as media of exchange. We explicitly spell out frictions in the trading environment that impede barter and the honoring of contracts and therefore support the existence of fiat money and the role of banks in providing checking services. This is in contrast to Clower constraints models that exogenously impose a finance constraint on purchases: money buys goods, goods buy money but goods do not buy goods. We consider a spatial separation/overlapping generation (OLG) model in which agents are subject to relocation shocks that determine the type of medium of exchange they will hold. Spatial separation, a concept derived from Townsend (1980), reflects credit market
fragmentation. We assume some trading partners are geographically isolated and only meet once. Others are spatially close and ‘linked’ by banks that own a record keeping and contract enforcement technology. Bank activity is assumed to be limited to certain regions of the economy. This can be interpreted either as a lack of coordination in credit markets as is the case in developing countries. Spatially close agents can thus use checks when financing their purchases and earn a rate of return higher than that on fiat money, while spatially separated agents use fiat money which is assumed to be accepted on all sites. Liquidity shocks, shocks to the demand for cash and checks, result from changes in the probability of relocation of spatially separated agents searching for trading partners.

Our second extension is adopting an overlapping generations (OLG) structure in contrast to Hartley’s infinite horizon model. OLG is not essential for the existence of fiat money once we assume spatial separation that impedes the writing of contracts among agents. However, the OLG structure causes capital to be sensitive to liquidity shocks. OLG is analogous to the case of borrowing constraints in Hartley (1994) and (1998). When agents are prevented from borrowing against their future income to smooth consumption, an upward sloping supply of capital in interest rates results. The equilibrium level of capital is sensitive to not only changes in firms’ demand for capital but also to liquidity shocks that change households’ capital supply. In the OLG setting, each generation lives for only two periods and naturally cannot borrow against their future endowment as no future endowment exists. If endowed only in their first year of life, agents save in the form of bank deposits net of checks used in transactions. Their saving increases with interest rates (assuming that the substitution effect dominates the income effect) and is a function of liquidity shocks. Therefore, the equilibrium level of capital (the intermediated deposits) is determined and vulnerable to liquidity shocks.
Specifically how investment responds to liquidity shocks depends on the timing of the shock. In the first model (chapter 3), we show that when the young are hit with a relocation shock, capital serving the dual role of consumption smoothing and backing checks is a non-monotonic function of the shock. This is the outcome of two opposing effects. A higher probability of using checks in exchange raises the demand for the check good as well as checks in circulation. Deposits therefore have to increase to accommodate the increase in check writing. As a result, capital, composed of deposits net of checks, increases. On the other hand, however, the higher check level spent on goods erodes capital.

In the second model in chapter 4, old agents are hit with a liquidity shock. In this case, capital is only composed of deposits. Checks do not erode capital since only old agents use them to finance trade; that is, checks are written against deposits only after investment has matured. Any liquidity shock favoring checks will only raise deposits to accommodate the increase in check circulation. Capital is an increasing function of the liquidity shock.

The rest of the thesis is outlined as follows. Chapter 2 provides empirical results as well as a theoretical background. The contribution of chapters 3 and 4 is to show with two spatial separation/OLG models how shocks to the demands for outside versus inside money can translate into changes in equilibrium capital levels. Chapter 5 concludes.
Chapter 2

Empirical Findings and Theoretical Background

2.1 Introduction

This chapter provides empirical findings that motivate our theoretical models. We test and confirm the hypothesis that aggregate investment is more vulnerable to liquidity shocks in countries with developing capital markets than countries with sophisticated capital markets. In addition, we provide a theoretical background for our thesis. We show examples based on Bryant (1985) and Hartley (1994) and (1998) of how Clower constraints do not affect the inter temporal allocation of resources given infinite horizon models and smoothly functioning borrowing and lending markets. We then contrast these results with those of Townsend (1983), Bryant (1989) and Mitsui and Watanabe (1989) who all consider spatial separation models to show that monetary and real phenomena are not independent. Spatial separation hampers the writing and enforcement of contracts which gives a role for money; it allows inter temporal trade to occur and hence affects capital levels.

2.2 Empirical Findings

To determine whether shocks to demand deposits affect the stability of aggregate investment in countries with undeveloped capital markets, we estimate a linear regression linking the variance of aggregate investment ($VARINV$) to the variance of demand deposits ($VARDEP$) for 49 countries with undeveloped capital markets.*

*Countries with undeveloped capital market considered are: Algeria, Antigua and Barbuda, Argentina, Bahrain, Belize, Benin, Bhutan, Bolivia, Botswana, Brazil, Bulgaria, Burkina Faso,
For each country, the variances of the dollar value of demand deposits and of the dollar value of aggregate investment are calculated for the time period ranging from 1992 till 2000 using *International Financial Statistics* data. The regression results are

\[
VARINV = -6.02 + 44.28 \cdot VARDEP
\]

with an R-squared of 0.8006 and an F value of 188.7506. The values in parentheses are standard errors of the estimated coefficients. The effect of the variance of demand deposits on that of investment is statistically significant in countries with undeveloped capital markets. The variance of investment is positively correlated to that of demand deposits suggesting that shocks to demand deposits translate into changes in investment.

The same regression estimated for 16 countries with developed capital markets shows that shocks to demand deposits do not affect aggregate investment. The estimated regression is

\[
VARINV = 71.95 - 0.022 \cdot VARDEP
\]

with an R-squared value of 0.0007 and an F value of 0.0099.\(^1\) The values in parentheses are standard errors. These results confirm Hartley’s (1998) theoretical analysis discussed below.

---

\(^1\)The list of countries with developed capital markets are: Australia, Canada, Hong Kong, Denmark, Germany, Finland, Italy, Japan, New Zealand, Netherlands, Norway, Spain, Sweden, Switzerland, United Kingdom, and United States.
2.3 Theoretical Background

We survey several models that show the interaction between real and monetary sectors. This section is composed of two main parts. In the first part, we demonstrate through a series of simple examples based on Bryant (1985) and Hartley (1994) and (1998) the effect of Clower constraints on consumption smoothing and capital formation. When households face only cash-in-advance constraints on all goods purchased and household borrowing and lending are not constrained, inflation does not affect neither equilibrium consumption nor capital and the laissez faire equilibrium is Pareto optimal: money is a veil. When cash-in-advance constraints apply to some goods and check-in-advance constraints to other goods and borrowing and lending are frictionless, inflation distorts trade but does not affect the equilibrium level of capital, Bryant (1985). In addition, based on Hartley (1994) and (1998), we show that the existence of a check-in-advance constraint does not cause capital to increase when households are able to borrow and lend. Finally, in this section, we review Hartley (1994) to show the vulnerability of capital and interest rates to shocks to the demand for inside money when borrowing constraints are imposed.

The above mentioned models do not provide micro foundations and impose Clower constraints that do not arise from some underlying structure. We proceed in the second part of this chapter to discuss models by Townsend (1983), Bryant (1989) and Mistsui and Watanabe (1989) that show the existence of fiat money as a response to some friction, in particular spatial separation, that hampers the functioning of private credit markets. From the very outset, borrowing and lending markets do not operate perfectly. Real economic activity and financial development are intimately related. Townsend (1983) establishes the direct relationship between the degree of an economy's spatial interconnectedness and output and welfare. The more interconnected an economy is, the better the operation of credit markets which allows for more pro-
duction, higher consumption of market goods as opposed to home produced goods and a higher level of welfare. With spatial separation, fiat money replaces credit in trade and lower levels of production and welfare result as idle cash balances have to be held between periods before trade transactions can occur. Townsend, however, does not include capital in his model.

Bryant (1989) analyzes the effects of different monetary institutions on the intertemporal allocation of resources. In a Townsend turnpike model, Bryant shows that either a positive fiat money or a negative fiat money regime (i.e., banks with positive net worth) can be supported. In the former regime, a monetary equilibrium fails to exist while in the latter, the monetary equilibrium results in the golden rule of capital which is in excess of the optimal planner’s modified golden rule of capital. The optimal monetary policy that produces the modified golden rule of capital is Friedman’s money supply rule: deflating the money supply by the time discount factor.

Finally, we discuss Mitsui and Watanabe’s (1989) spatial separation model with agents being constantly and randomly relocated on different sites. Spatially close agents can use private credit markets to invest in high return investment (good project) while agents that end up on remote locations end up with a low return investment (bad project). By holding fiat money that has a higher return than the bad project, the composition of saving is improved. Money elevates the economy to a higher growth rate and to a Pareto superior path.

2.3.1 The effect of Clower constraints on general equilibrium

We consider infinitely lived households who prefer to consume two types of goods $c_{1t}$ and $c_{2t}$ every time period $t$. Their utility is $U(c_{1t}, c_{2t})$ and is separable. Households can invest $k_t$ units of capital each period. Firms produce goods according to the
production function \( k_t^\alpha \), where \( 0 < \alpha < 1 \). We follow closely Hartley’s (1994) and (1998) notation all throughout the subsections below.

The planner problem

\[
\max_{c_{1t}, c_{2t}, k_{t+1}} \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t})
\]

such that

\[
c_{1t} + c_{2t} + k_{t+1} = k_t^\alpha.
\]

The Lagrangean then is

\[
\max_{c_{1t}, c_{2t}, k_{t+1}} \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t}) + \lambda_t[k_t^\alpha - c_{1t} - c_{2t} - k_{t+1}]
\]

where \( \lambda_t \) is the Lagrangean multiplier. The first order conditions are:

\[
\begin{align*}
    c_{1t} : & \quad U_1(c_{1t}) - \lambda_t = 0 \quad (2.3) \\
    c_{2t} : & \quad U_2(c_{2t}) - \lambda_t = 0 \quad (2.4) \\
    k_{t+1} : & \quad \alpha \beta \lambda_t k_{t+1}^{\alpha - 1} - \lambda_t = 0 \quad (2.5) \\
    \lambda_t : & \quad c_{1t} + c_{2t} + k_{t+1} = k_t^\alpha \quad (2.6)
\end{align*}
\]

From (2.3) and (2.4), \( \frac{U_1(c_{1t})}{U_2(c_{2t})} = 1 \) and \( \frac{U_1(c_{1t})}{U_1(c_{1t+1})} = \frac{U_2(c_{2t})}{U_2(c_{2t+1})} = \frac{\lambda_t}{\lambda_{t+1}}. \)

From (2.5), \( \frac{\lambda_{t+1}}{\lambda_t} = \frac{1}{\alpha \beta k_{t+1}^{\alpha - 1}}. \)

Assuming stationarity: \( c_{1t} = c_1, c_{2t} = c_2, k_t = k \) and \( \lambda_t = \lambda; \forall t \), then \( \frac{1}{\beta} = \alpha k^{\alpha - 1}. \)

The stationary equilibrium level of capital is determined by the rate of time preference \( \frac{1}{\beta} \) and the marginal productivity of capital \( \alpha k^{\alpha - 1}. \)
Laissez faire with cash-in-advance constraints and unconstrained borrowing and lending

When households face cash in advance constraints when purchasing $c_1$ and $c_2$, we show that the laissez faire is Pareto optimal and money is a veil. Let $m_t = \frac{M_t}{P_t}$ be the real value per capita money balances in period $t$, where $M_t$ is the per capita nominal money balances held and $P_t$ is the dollar price of one unit of good. Let $y_t = \pi m_t$ be the real per capita lump sum money transfers received from the government every period $t$ and $\Pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ be the period $t$ inflation rate. Finally, let $s_t$ be the amount households borrow or lend to firms in period $t$ and $i_t$ the real interest rate on loans. Positive values of $s_t$ is lending to firms, borrowing otherwise. We follow Hartley's (1994) sequence of transactions:

\[
\begin{array}{c|c|c}
  t & \text{Goods market trade} & \text{Capital market trade} \\
  \hline
  t & & t+1 \\
\end{array}
\]

At period $t$, firms enter with a capital stock and loans from households in period $t-1$. Households enter with cash and loans from period $t-1$. In the goods market, firms produce goods with their capital and purchase capital for the next period but do not pay until capital market trade. Households use cash to purchase consumption goods (Clower constraint). In the capital market trade, firms pay back loans from previous with interest and arrange new loans to pay of their capital purchases this period. Government provides real per capita cash transfers to households. Households choose cash and loans for the following period.

*Households' problem:*

\[
\max_{c_{1t}, c_{2t}} \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t})
\]
such that

\[ y_t + s_t(1 + i_t) + m_t - c_{1t} - c_{2t} = (1 + \Pi_{t+1})(m_{t+1} + s_{t+1}) \]  
\[ m_t \geq c_{1t} + c_{2t} \]  

or

\[
\max_{c_{1t}, c_{2t}, m_{t+1}, s_{t+1}} \sum_{t=0}^{\infty} \beta^t \{ U(c_{1t}, c_{2t}) + \lambda_t[y_t + s_t(1 + i_t) + m_t - c_{1t} - c_{2t} - (1 + \Pi_{t+1})(m_{t+1} + s_{t+1})] + \phi_t[m_t - c_{1t} - c_{2t}] \}
\]

where \( \lambda_t \) and \( \phi_t \) are the Lagrangean multipliers. The first order conditions are:

\[ c_{1t} : \quad U_1(c_{1t}) = \lambda_t + \phi_t \]  
\[ c_{2t} : \quad U_2(c_{2t}) = \lambda_t + \phi_t \]  
\[ s_{t+1} : \quad \beta \lambda_{t+1}(1 + i_{t+1}) - \lambda_t(1 + \Pi_{t+1}) = 0 \]  
\[ m_{t+1} : \quad \beta \lambda_{t+1} - \lambda_t(1 + \Pi_{t+1}) + \beta \phi_{t+1} = 0 \]  
\[ \lambda_t : \quad y_t + s_t(1 + i_t) + m_t - c_{1t} - c_{2t} = (1 + \Pi_{t+1})(m_{t+1} + s_{t+1}) \]  
\[ \phi_t : \quad \phi_t[m_t - c_{1t} - c_{2t}] = 0, \phi_t \geq 0, m_t \geq c_{1t} + c_{2t}. \]

From (2.9) and (2.10),

\[ \frac{U_1(c_{1t})}{U_2(c_{2t})} = 1, \]  

and

\[ \frac{U_1(c_{1t})}{U_1(c_{1t+1})} = \frac{U_2(c_{2t})}{U_2(c_{2t+1})} = \frac{\lambda_t + \phi_t}{\lambda_{t+1} + \phi_{t+1}}. \]
From (2.9), \( \lambda_t + \phi_t > 0 \) and from (2.11) and (2.12), \( i_{t+1} \lambda_{t+1} = \phi_{t+1} > 0 \), which implies that the cash constraint is binding.

When stationary equilibrium is considered (2.15) becomes

\[
\frac{U_1(c_1)}{U_2(c_2)} = 1, \tag{2.17}
\]

and (2.16),

\[
\frac{U_1(c_1)}{U_1(c_1)} = \frac{U_2(c_2)}{U_2(c_2)} = \frac{\lambda + \phi}{\lambda + \phi} = 1. \tag{2.18}
\]

The marginal rates of substitution are equal to those in the planner’s solution.

The households’ supply of capital is obtained from (2.11) with \( \lambda_{t+1} = \lambda_t \),

\[
\frac{(1 + i)}{1 + \Pi} = \frac{1}{\beta}. \tag{2.19}
\]

*The firms’ problem:*

Firms are perfectly competitive. They maximize profits:

\[
\max_{k_t, s_t} k_t - (1 + i_t)s_t
\]

subject to: \( k_t = s_t(1 + \Pi_t) \)

f.o.c.

\[
\alpha k_t^{a-1} = \frac{(1 + i_t)}{(1 + \Pi_t)}. \tag{2.20}
\]

Stationary equilibrium:

\[
\alpha k_t^{a-1} = \frac{(1 + i)}{(1 + \Pi)}, \quad \text{firms’ demand for capital.} \tag{2.21}
\]

*Money market clearing:* Money supply = Money demand.
At time $t$, the nominal demand for money is $M_t = P_t(c_{1t} + c_{2t})$.

$$
(1 + \Pi_{t+1}) = \frac{P_{t+1}}{P_t} = \frac{M_{t+1}}{M_t} \cdot \frac{c_{1t} + c_{2t}}{c_{1t+1} + c_{2t+1}} = (1 + \pi) \cdot \frac{c_{1t} + c_{2t}}{c_{1t+1} + c_{2t+1}}.
$$

Given stationarity: $(1 + \Pi) = (1 + \pi)$. The inflation rate is equal to the growth rate of money $\pi$.

Goods market clearing:

$c_{1t} + c_{2t} + k_{t+1} = k_t^\alpha$

or

$c_1 + c_2 + k = k^\alpha$.

In equilibrium, the supply of capital is equal to the demand for capital: $\alpha k^{\alpha-1} = \frac{1}{\beta}$, which is exactly the same as the planner's condition for capital.

The equilibrium level of capital is the planner's solution for capital and is unaffected by the inflation rate $\Pi$. The Clower constraint does not affect capital formation. Consumption $c_1$ and $c_2$ are optimal as (2.17) is the same as in the planner's problem. Moreover, $c_1$ and $c_2$ are independent of inflation. A dichotomy therefore exists between the real and monetary sides of this model.

Laissez faire with cash-in-advance constraints, check-in-advance constraints and unconstrained borrowing and lending

We show Bryant's (1985) results that inflation distorts trade but does not affect the equilibrium level of capital. Although Bryant assumes that $c_{1t}$ is the market good purchased with cash and $c_{2t}$ is the home produced good and can be consumed directly, the basic results are unchanged when we assume that $c_{2t}$ is purchased with checks and households borrow and lend to firms instead of investing directly in home production. The reason we consider this setup, which is close to Hartley (1994) and (1998), is to
illustrate Hartley's (1998) point that checks used as media of exchange do not increase the equilibrium level of capital. Any increase in bank loans due to households' use of checks in trade transactions will be absorbed by increased household borrowing.

We keep the above sequence of trade the same but add banks that accept household deposits and issue loans to firms and households. The banking industry is perfectly competitive. The interest rate on bank loans is $i_t$ and interest on deposits is $r_t$. Zero profits imply that $i_t = r_t + \chi$ where $\chi$ is the per unit of deposits cost that banks face in providing loan and checking services.

**Households' problem:**

$$\max_{c_{1t}, c_{2t}} \sum_{t=0}^{\infty} \beta^t U(c_{1t}, c_{2t})$$

such that

$$m_t \geq c_{1t}, \text{ cash-in-advance constraint}$$

$$(1 + r_t)b_t \geq c_{2t}, \text{ check-in-advance constraint}$$

$$y_t + s_t (1 + i_t) + (1 + r_t)b_t + m_t - c_{1t} - c_{2t} =$$

$$(1 + \Pi_{t+1})(m_{t+1} + b_{t+1} + s_{t+1}), \text{ where } b_t \text{ is the real check balances.}$$

Or,

$$\max_{c_{1t}, c_{2t}, m_{t+1}, b_{t+1}, s_{t+1}} \sum_{t=0}^{\infty} \beta^t \{U(c_{1t}, c_{2t}) + \lambda_t [y_t + s_t (1 + i_t) + (1 + r_t)b_t +$$

$$m_t - c_{1t} - c_{2t} - (1 + \Pi_{t+1})(m_{t+1} + s_{t+1} + b_{t+1})]$$

$$+ \phi_t [m_t - c_{1t}] + \gamma_t [(1 + r_t)b_t - c_{2t}])}$$
where \(\lambda_t\) and \(\phi_t\) and \(\gamma_t\) are the Lagrangean multipliers. The first order conditions are:

\[
\begin{align*}
\lambda_t & : \quad U_1(c_{1t}) = \lambda_t + \phi_t \quad (2.22) \\
\phi_t & : \quad U_2(c_{2t}) = \lambda_t + \gamma_t \quad (2.23) \\
b_{t+1} & : \quad \beta \lambda_{t+1}(1 + r_{t+1}) - \lambda_t(1 + \Pi_{t+1}) + \beta \gamma_{t+1}(1 + r_{t+1}) = 0 \quad (2.24) \\
m_{t+1} & : \quad \beta \lambda_{t+1} - \lambda_t (1 + \Pi_{t+1}) + \beta \phi_{t+1} = 0 \quad (2.25) \\
s_{t+1} & : \quad \beta \lambda_{t+1}(1 + i_{t+1}) - \lambda_t(1 + \Pi_{t+1}) = 0 \quad (2.26) \\
\gamma_t & : \quad \gamma_t[(1 + r_t)b_t - c_{2t}] = 0, \quad \gamma_t \geq 0, \quad b_t(1 + r_t) \geq c_{2t} \quad (2.27) \\
\phi_t & : \quad \phi_t[m_t - c_{1t}] = 0, \quad \phi_t \geq 0, \quad m_t \geq c_{1t} \quad (2.28)
\end{align*}
\]

\[
\begin{align*}
\lambda_t : \quad y_t + s_t(1 + i_t) + (1 + r_t)b_t + m_t - c_{1t} - \\
c_{2t} = (1 + \Pi_{t+1})(m_{t+1} + s_{t+1} + b_{t+1}). \quad (2.29)
\end{align*}
\]

From (2.22) and (2.23),

\[
\lambda_t + \phi_t > 0, \quad (2.30)
\]

and

\[
\lambda_t + \gamma_t > 0. \quad (2.31)
\]

From (2.26), \(\frac{\lambda_{t+1}}{\lambda_t} = \frac{1 + \Pi_{t+1}}{\beta(1 + i_{t+1})}\).

Assume stationary equilibrium: the supply of loans by households is

\[
\frac{(1 + i)}{(1 + \Pi)} = \frac{1}{\beta}. \quad (2.32)
\]

**Proposition 1:** The cash and check constraints are binding.
Proof:

From (2.24): \( \lambda = \frac{\beta \gamma (1+r)}{(1+\Pi - \beta (1+r))} \). Substitute for \( \lambda \) in (2.31) to find:

\[
0 < \lambda + \gamma = \frac{(1+\Pi)}{(1+\Pi - \beta (1+r))}. \tag{2.33}
\]

From (2.32): \( (1+\Pi) = \beta (1+i) \), which implies that \( (1+\Pi) > \beta (1+r) \) since \( i = r + \chi \).

From (2.33), \( \gamma > 0 \): the check constraint binds. From (2.25): \( \lambda [1+\Pi - \beta] = \beta \phi \).

Replace this in (2.30): \( \phi [1 + \frac{\beta}{1+\Pi - \beta}] > 0 \) which implies \( \phi > 0 \): the cash-in-advance constraint binds. QED

**Proposition 2:** Inflation distorts trade but does not affect the equilibrium level of capital (Bryant 1985).

**Proof:**

From (2.22) and (2.23),

\[
\frac{U_1(c_1)}{U_2(c_2)} = \frac{\lambda + \phi}{\lambda + \gamma} = (1+r). \tag{2.34}
\]

The marginal rate of substitution between \( c_1 \) and \( c_2 \) is not equal to one as in the planner’s problem. With \( U_1(c_1) > U_2(c_2) \), the consumption of the cash good \( c_1 \) is lower than optimum and/or the check good \( c_2 \) is higher than optimum. The laissez faire with cash-in-advance constraint on only one good and a check-in-advance on another produces this trade distortion.

Moreover, the higher the inflation rate the bigger the distortion is. From (2.32),

\( (1+i) = \frac{1+\Pi}{\beta} \) which implies that \( (1+r) = \frac{1+\Pi}{\beta} - \chi \). Equation (2.34) becomes

\( U_1(c_1) = [\frac{(1+\Pi)}{\beta} - \chi] U_2(c_2) \). With a higher inflation rate \( \Pi \), the consumption of \( c_1 \) decreases and \( c_2 \) increases. QED

**Proposition 3:** Inflation does not affect the inter temporal allocation of resources. Capital is unaffected by the cash in advance constraint, the second result of Bryant
(1985). In addition, inside money $b$ does not lead to higher levels of capital than the planner's as long as borrowing and lending $s$ is unconstrained (Hartley 1998).

**Proof:**

From (2.32), the interest rate is pinned down by the rate of time preference $\frac{1}{\beta}$. The firms' optimization problem is the same as in the above section. They maximize profits:

$$\max_{k_t, s_t} k_t^\alpha - (1 + i_t)(s_t + b_t)$$

subject to: $k_t = (s_t + b_t)(1 + \Pi_t)$

f.o.c.

$$\alpha k_t^{\alpha - 1} = \frac{(1 + i_t)}{(1 + \Pi_t)}.$$  \hspace{1cm} (2.35)

*Stationary equilibrium:* $\alpha k_t^{\alpha - 1} = \frac{(1 + i_t)}{(1 + \Pi_t)}$ firms' demand for capital.

*Goods market clearing:* demand for capital = supply of capital.

We have $\alpha k_t^{\alpha - 1} = \frac{1}{\beta}$ which is same as the in planner's problem. The stationary equilibrium level of capital is independent of inflation and of the level of inside money. It is only determined by the marginal productivity of capital and the rate of time preference. QED

**Hartley’s model**

Up till now, we have paved the way to reviewing Hartley's (1994) model by examining the effects of cash-in-advance constraints and check-in-advance constraints on stationary equilibrium. We stated the main results of Bryant (1985) of the distortionary effect of inflation on trade and capital's independence of inflation. We also
illustrated Hartley’s (1998) point that inside money does not produce higher levels of capital than economies with no banks given unconstrained borrowing and lending. We conclude by laying out Hartley’s (1994) model and showing that shocks to the demand for inside money when borrowing and lending are constrained affect the equilibrium levels of interest rates and capital. Households are subject to preference shock \( \theta_t \) every time period. This shock affects the substitution between fiat money and inside money. The former is universally accepted while the latter can be used in purchases of \( c_2 \) only.

The households’ utility function is

\[
\sum_{t=0}^{\infty} \beta^t \left( \frac{c_{1t}^{1-\gamma}}{1 - \gamma} + \theta_t \frac{c_{2t}^{1-\gamma}}{1 - \gamma} \right),
\]

where

\[
\theta = \begin{cases} 
\theta^* : & \text{with probability } p \\
0 : & \text{with probability } 1 - p.
\end{cases}
\]

State 0 is when \( \theta = 0 \) and state 1 is when \( \theta = \theta^* \). \( c_{10} \) is the consumption of the cash good in period \( t \) and \( W_0 \) is remaining wealth in period \( t + 1 \) when state 0 occurs in period \( t \). \( c_{11} \) is the consumption of the cash good, \( c_{21} \) the consumption of check goods, \( m_t \) the real value of cash spend on check goods in period \( t \) and \( W_1 \) is the remaining wealth in period \( t + 1 \) when state 1 occurs in period \( t \).

**Households maximize:**

\[
\max_{m_t, s} \beta[(1 - p) \max_{c_{10}, W_0} \left\{ \frac{c_{10}^{1-\gamma}}{1 - \gamma} + V(W_0) \right\} \\
+ p \max_{c_{11}, c_{21}, m_t, W_1} \left\{ \frac{c_{11}^{1-\gamma}}{1 - \gamma} + \theta \frac{c_{21}^{1-\gamma}}{1 - \gamma} + V(W_1) \right\}] 
\]

subject to the cash and check in advance constraints:
\[ c_{10} \leq m \]  \hspace{1cm} (2.36)
\[ m_1 + c_{11} \leq m \]  \hspace{1cm} (2.37)
\[ c_{21} \leq b + m_1 \]  \hspace{1cm} (2.38)
\[ m_1 \leq c_{21} \]  \hspace{1cm} (2.39)
\[ 0 \leq m_1 \]  \hspace{1cm} (2.40)

and the budget constraints:

\[ W_0 = \frac{1}{1 + \Pi} \left[ m - c_{10} + b(1 + r) + s(1 + i) + y \right] \]  \hspace{1cm} (2.41)
\[ W_1 = \frac{1}{1 + \Pi} \left[ m - c_{11} - m_1 + (b + m_1 - c_{21})(1 + r) + s(1 + i) + y \right] \]  \hspace{1cm} (2.42)
\[ W = m + b + s. \]  \hspace{1cm} (2.43)

Banks are perfectly competitive. Zero profits imply \( i = \frac{r + x}{1 - \rho} \) where \( \rho \) is an exogenously imposed fraction of deposits held by banks as reserves.

Firms maximize profits:

\[ \max k^\alpha + (1 - \delta)k - \frac{(1 + i)k}{1 + \Pi} \]

where \( \delta \) is the depreciation rate and \( \frac{k}{1 + \Pi} \) is the firms' real liabilities which arise from two sources, direct household lending to firms \( s \) and bank loans which are the fraction of deposits not held as reserves, \( (1 - \rho)b \).

Hartley shows that when \( s \) is unconstrained, the interest rate \( i \) can be determined from the households' problem independently of the firms' side. Then, given the solution of interest from the first order conditions of the firms' problem, the equilibrium
level of capital can be pinned down by the marginal productivity of capital schedule. The interest rate \( i \) and the capital level are ‘immune’ to shocks to the demand for inside money, \( \theta \), as unhampered disintermediation \( s \) counteracts the changes in the demand for inside money to produce a flat and unchanging supply of household funds.

When household borrowing constraints are imposed, \( s \geq 0 \), Hartley shows that firms face an upward sloping supply of investment funds in interest rates. The equilibrium interest rate is jointly determined by the households' supply and firms' demand for funds (the marginal productivity of capital). Exogenous disturbances in the demand for inside money (changes in liquidity shock \( \theta \)) shift the supply of funds and alter the equilibrium levels of interest rates and capital. An increase in the demand for inside money results in an increase in the supply of household funds which lowers interest rates. Households have an incentive to increase their borrowing given lower interest rates. However, due to borrowing constraints, the increased supply of deposits is channeled by banks to firms which end up with a higher equilibrium level of capital at lower interest rates.

### 2.3.2 Spatial separation models

The above section analyzes the impact of exogenously imposed Clower constraints and household borrowing constraints on capital and trade. Below we briefly discuss models that provide an environment that naturally produces frictions in the credit market and which gives rise to valued fiat money and interdependence of real and financial/monetary sectors.

**Townsend’s model**

In Townsend (1983), the interconnection between economic growth and financial development is set forth in a choice theoretic model with spatial separation, a proxy for
credit market fragmentation. Townsend notes the following empirical observations that are based from previous studies:

- On a secular and cross sectional basis, the share of agriculture (the home produced good in his model) is inversely proportional to GDP per capita while that of the industrial sector (the market produced good) and the banking sector are positively related.

- The share of financial intermediaries in national assets and the ratio of financial intermediaries' assets to GNP rise over time.

- The velocities of broad money $M_2$ and of private credit increase from less to more developed countries cross sectionally and secularly.

The punch line is that the degree of interconnectedness in an economy affects the amount of goods produced, trade and the type financial assets used. Market fragmentation affects economic and financial development.

Townsend's (1983) model is built upon Lucas' version of the Cass-Yaari circle modified to include a variable supply of labor. Three regimes are compared: i) autarky, the result of complete market fragmentation, islands are completely isolated, ii) decentralized exchange, fiat money links islands and allows trade to occur and iii) centralized exchange through a central clearinghouse that allows credit in trade.

The transition from autarky to fiat to centralized credit results in a reduction in the cost of the market produced good compared to the home produced good. Also, supplied labor, trade, consumption of the market good and welfare increase whereas the consumption of the home produced good decreases. With fiat money, the laissez faire equilibrium is Pareto superior to autarky but not Pareto optimal as fiat money balances are held idle for one period before any purchase of the market goods occurs. In other words, past labor-output decisions finance the present consumption of market goods in the fiat regime, while trade with credit (fully interconnected economy) allows
contemporaneous labor-output decision to finance present consumption of market goods resulting in Pareto optimality.

We briefly sketch Townsend’s model with the three different regimes below. There is a countable set of infinitely lived household types. Household of type $i$ has preferences:

$$\sum \beta^t U(c_{it}(i), c_{i+1,t}(i), n_{it}(i)),$$

where $c_{it}(i)$ is the consumption of commodity $i$ produced by household $i$. One unit of labor $n_{it}(i)$ at time period $t$, produces one unit of commodity $i$. $c_{i+1,t}(i)$ is household $i$’s consumption of commodity $i+1$ produced by household $i+1$.

*Autarky:* Each household type lives on an island with no communication with other households on other islands. The household’s problem is:

$$\max \sum_{t=0}^{\infty} \beta^t U(c_{it}(i), c_{i+1,t}(i), n_{it}(i))$$

subject to $c_{it}(i) = n_{it}(i)$ and $c_{i+1,t}(i) = 0$.

*Decentralized exchange regime with fiat money:*

Townsend bases this on Lucas’ version of the Cass-Yaari circle. Each household of type $i$ is located on some real line at point $i$. These are countably infinite parallel real lines. Household $i$ consists of a pair of agents. Each agent could travel every time
period $t$ half way to meet either household $i-1$ or $i+1$ in spatially separated markets $(i-1,i)$ and $(i+1,i)$. Between periods, every even $i$ moves vertically downward to the next line. Every odd $i$ remains in the same location. This precludes debt issuance. Each household of type $i$ would have to use fiat money in transactions since the pairing with $i-1$ and $i+1$ does not result in double coincidence of wants.

The household problem is:

$$\max \sum_{t=0}^{\infty} \beta^t U(c_{i,t}(i), c_{i+1,t}(i), n_{i,t}(i))$$

subject to $P_{i+1,t}c_{i+1,t}(i) + M_{i+1}(i) \leq M_t(i) + P_{it}[n_{it}(i) - c_{it}(i)]$ and $P_{i+1,t}c_{i+1,t}(i) \leq M_t(i)$,

where $M_t$ is the nominal holding of money by household $i$ in period $t$, $P_{i+1,t}$ is the dollar price of good $i+1$ in period $t$ and $P_{it}$ is the dollar price of good $i$ in period $t$.

The market clearing condition is $c_{it}(i-1) \leq n_{it}(i) - c_{it}(i)$.

*Centralized exchange regime with credit:* Suppose on each line, there is a centralized market with a Walrasian auctioneer operating a credit-debit exchange system. Households then maximize

$$\max U(c_{it}(i), c_{i+1,t}(i), n_{i,t}(i))$$

subject to

$$r_{i+1,t}c_{i+1,t}(i) \leq r_{it}[n_{it}(i) - c_{it}(i)],$$

where $r_{i+1,t}$ is the per unit price of good $i+1$ in terms of some abstract unit of account, $r_{i+1,t}c_{i+1,t}(i)$ is the intra period debt incurred by household $i$ and demanded from the intermediary or auctioneer and $r_{it}[n_{it}(i) - c_{it}(i)]$ is the intra period credit.
extend by the intermediary to household $i$ given units of output $n_u(i) - c_u(i)$ supplied. A shortcoming of this model, as Townsend notes, is the absence of capital as a factor of production and a main determinant of economic growth. The following two models remedy this defect.

**Bryant’s model**

Unlike Bryant (1985) where capital is independent of money and inflation, the spatial separation setting in Bryant (1989) obstructs investment activity and renders it vulnerable to the available type of monetary regime. The model supports either a positive fiat money regime, a system of indirectly backed notes like banknotes circulating in exchange, or a negative fiat money regime. The latter has banks with positive net worth: each period, banks own claims to goods in the next period without offsetting liabilities.

Given a constant money supply, monetary equilibrium fails to exist because real wealth grows without bound. Capital is zero in this case. However, Friedman’s money supply rule, deflating the money supply by the time discount rate $\beta$, results in a monetary equilibrium which is Pareto optimal with the capital level at the modified golden rule. When the negative fiat money regime is imposed, the laissez faire is inefficient as it produces the golden rule of capital which is in excess of the planner’s modified golden rule.

Bryant considers three types of agents: a central authority that decides the type of monetary institution to impose, bankers and mobile infinitely lived agents. Mobility precludes writing of contracts and therefore a need for money in transactions. There are $V$ islands. Agents move from one site to another between time periods and remain on one site within a period. They possess the following technology: an agent at one site can produce $\alpha$ units at time $t$ and $(1 - \alpha)^{\frac{3}{2}}$ units at time $t + 1$ at the same site.
\( \alpha \) is a choice variable and belongs to the set \([0, 1]\). However, an agent would have left the site by the time \((1 - \alpha)^{\frac{1}{2}}\) is produced. Goods cannot be transported between sites.

Agents maximize

\[
\max \sum_{t=0}^{\infty} \beta^t U(c_t).
\]

The laissez faire non monetary equilibrium is \( c_t = \alpha = 1 \). Investment is zero and equilibrium is not optimal.

**Constant positive fiat money supply:**

Currency is issued in period 1 by the central authority at each site and sells them for promises of ‘tomorrow’ goods at that same site. The central authority then turns those promises to banks who promise to sell all delivered goods for currency. Agents, after selling promises to the central authority for currency in period 1, leave their produced good \((1 - \alpha)^{\frac{1}{2}}\) with the banks and carry with them currency to the new island and purchase the goods on the new island which are the production of another agent. Agents then sell promises of ‘tomorrow’ goods in return for currency from banks, and so on.

Agents maximize:

\[
\max \sum_{t=0}^{\infty} \beta^t U(c_t)
\]

subject to: \( c_t = E_t + \alpha_t + P_t\left[\frac{(1-\alpha)^{\frac{1}{2}}}{W_t}\right] + M_{t-1} - M_t \),

where \( E_t \) is the endowment at time \( t \) and \( E_1 = \frac{\theta}{2} \) and \( E_2 = E_3 = \ldots = 0 \). \( M_t \) is money holding in period \( t \), \( P_t \) is goods price of currency in period \( t \) and \( \frac{1}{W_t} \) is the money price of a promise of a ‘tomorrow’ good.
Bryant shows that the only perfect foresight equilibrium is the non monetary outcome $\alpha = 1$ and $P_t = 0 \forall t$. Another candidate solution is the golden rule $\alpha_t = \frac{3}{4}$ and $P_t = \frac{\beta^{t-1}}{2M}$. But this implies that real money balances grow without bound. This candidate solution is therefore rejected.

*Negative fiat money:*

Under this regime, agents do not carry currency between sites. Rather, the central authority in period 1 taxes agents the amount $\bar{T}$ goods and sells those goods to agents for promises for ‘tomorrow’ goods at the same site. The central authority hands those promises to banks who in turn promise to forever sell goods to agents for ‘tomorrow’ goods promises. Banks in this case are creditors and no currency is carried between sites. Banks have positive net worth as they own promises to goods next period without offsetting liabilities. In other words, when individuals arrive at a new site, banks hand them goods produced by other individuals in return for their promise of producing a ‘tomorrow’ good and leaving it at the banks before they move on to another site.

Agents’ problem is:

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to: $c_t = E_t + \alpha_t + q_t(1 - \alpha_t)^{\frac{1}{2}} - T_t$, where $q_t$ is the period $t$ goods price of a promise of a good ‘tomorrow’ and $T_1 = \bar{T}$ and zero thereafter.

Bryant shows that the perfect foresight equilibrium solution is $\alpha = \frac{3}{4}$ which is the golden rule of capital and is not Pareto optimal. There is capital overaccumulation
compared to the modified golden rule of capital. Capital is equal to $1 - \alpha = \frac{1}{4}$ which is greater than the planner’s level of capital $\frac{\beta}{4}$. 

Finally, Bryant shows that Friedman’s money supply rule is the optimum monetary policy which produces the planner’s modified golden rule of capital. The money supply is deflated at the rate $\beta$ per period.

**Mitsui and Watanabe’s model**

Finally, Mitsui and Watanabe (1989) show that in a Townsend turnpike economy money eliminates investment in projects with low returns, therefore improving the quality of investment as well as increasing the growth of the capital stock. In an economy with fragmented credit markets represented by spatial separation, agents randomly move around. With probability $p$ they remain on the same site and $(1 - p)$ they move to another site every time period. Those remaining on the same island are capable of using credit markets to invest in a good investment project that yields a high return $g$ while those with a relocation shock end up with a bad investment project with a low rate of return $b < g$. Relocated agents have an incentive to hold money with rate of return $r$, $b < r < g$, which elevates the economy to a Pareto superior path. In a non monetary economy agents maximize:

$$\max \sum_{t=0}^{\infty} \beta^t U(c_t)$$

subject to $0 \leq c_t \leq w_t$ and $w_{t+1} = \theta(w_t - c_t),$

---

This is analogous to the case where inside money produces higher levels of capital than the planner’s capital when borrowing constraints exist in Hartley (1994). Here, the only way to smooth consumption is through borrowing from banks which leads to capital over accumulation.
where
\[
\theta = \begin{cases} 
g & : \text{with probability } p \\
b & : \text{with probability } 1 - p,
\end{cases}
\]
c_t is per capita consumption in period t, w_t is time t endowment, and \( \theta \) is the return on investment.

In a monetary economy, given a constant money supply with return \( r \) on money holdings, agents solve the same problem with the only difference being:
\[
\theta = \begin{cases} 
g & : \text{with probability } p \\
r & : \text{with probability } 1 - p.
\end{cases}
\]
Mitsui and Watanabe show that the growth rate of endowment \( \frac{w_{t+1}}{w_t} \) is the same as the growth rate of the capital stock, which is the sum of
\[
(1 - p) \ast \text{savings of relocated agents} + p \ast \text{savings of remaining agents},
\]
and increases as money is introduced in an economy. Growth and welfare increase in a monetary economy.

\section{2.4 Conclusion}

This chapter provided an empirical and theoretical background. We first showed with regression estimates that shocks to demand deposits affect the stability of aggregate investment in countries with undeveloped capital markets but not in countries with mature capital markets. These findings support Hartley’s hypothesis (1998). We then discussed the dichotomy between the real and monetary sectors of an economy that arose when Clower constraints were imposed on purchases and household borrowing was frictionless: capital and economic growth were largely unaffected by monetary changes and Clower constraints did not affect the inter temporal allocation of resources. These Clower constraints did not arise from any explicit trading frictions.
We also covered some of the literature on spatial separation, a form of trading friction that hampered the writing of contracts needed in trade. Fiat money overcame the spatial separation obstacle to inter temporal trade. If a banking system existed to link all agents together, credit would have replaced fiat money in trade. The type of medium of exchange that prevailed had a direct effect on the levels of capital, output and welfare. Monetary and real phenomena were not independent.
Chapter 3

A spatial separation/OLG model with the young subject to liquidity shocks

3.1 Introduction

This chapter analyzes the impact on capital formation of liquidity shocks to the demand for cash versus checks in an overlapping generations model (OLG). Unlike the infinite horizon models with no borrowing constraints, we show that liquidity shocks have a direct influence on the level of capital. The OLG environment is analogous to Hartley’s (1998) borrowing constraint case with infinitely lived households. Such is the case in countries with undeveloped capital markets, the focus of our study. With OLG, households are finitely lived and are naturally constrained from borrowing against their future endowment, which is absent when households are old. In this case, household saving in the form of demand deposits is an increasing function of interest rates (provided that the substitution effect outweighs the income effect of an increase in interest rates), and is directly affected by liquidity shocks. § Liquidity shocks change households’ holdings of cash and deposits, thus changing their saving level. This is not neutralized by changes in household borrowing since agents are naturally constrained from borrowing. A shift in household saving brought about by liquidity shocks therefore changes the equilibrium level of capital.

We find that equilibrium capital is a non-monotonic function of the liquidity shock favoring the use of checks in trade. There are two offsetting forces in effect. On the one

§This is in contrast to the infinite horizon/no borrowing constraints case where the supply of loanable funds is flat at the rate of time preference and household borrowing activity keeps it insulated from liquidity shocks.
hand, a liquidity shock favoring the use of checks in trade raises the transactions value of inside money and increases the demand for it. Deposits increase to accommodate check writing and thus bank loans which finance investment increase. On the other hand, raising the liquidity of inside money makes it harder for banks to invest in longer term assets as they need to keep more of their assets in liquid reserves.

Unlike the infinite horizon models that impose exogenous cash-in-advance constraints, we provide micro foundations for the demand for fiat money and checks which endogenously emerges from within our model’s structure. Specifically, the transaction demands for fiat money and checks are generated by random relocation of households in search of market goods to spatially separated sites. Each site has a competitive banking system that owns a record keeping and contract enforcement technology. Banks also channel demand deposits to finance default free investment. Spatial separation in the model prevents the connection of the different bank systems. Spatial separation can be interpreted as credit market fragmentation. In the case of developing countries, such fragmentation could result from inadequate coordination, or a weak legal framework.

The implication of spatial separation is that when households travel to different sites of an economy, contracts issued in trade cannot be honored. Spatial separation prevents banks at one site from clearing checks drawn against deposits in banks at a different site. Relocated households can only use fiat money instead of checks to finance their purchases. Fiat money provides more liquidity services than checks in the sense that it is a universally accepted medium of exchange.

Households remaining at the same site can use either cash or checks in trade. Whenever buyers and sellers are close enough to permit checks to be cleared, sellers are indifferent to the type of medium of exchange they receive. The assumption that banks are default free makes local checks perfect substitutes for fiat money from the
sellers' point of view. Hence, the nominal price of goods sold is independent of the means of payment presented to the sellers.

Liquidity shocks to the demand for cash balances and deposits stem from consumption uncertainty over two types of goods. The first good is purchased at a remote site and requires cash to effect trade. The second good can be purchased with either cash or checks from the household’s native island, but from a specific trading partner. This good yields a higher level of utility than the cash only good. The consumption uncertainty arises from the possibility of not finding the suitable trading partner on the native island. Changes in the probability of finding the specific trading partner, the liquidity shock, affects the households' use of cash and checks in trade. A higher probability of finding the specific trade partner results in a higher demand for the native good with checks supplementing cash purchases of this good. Banks ensure households against liquidity shocks by allowing them to withdraw the minimum cash amount needed to finance purchases without unnecessarily forgoing interest on their deposits.

We argue that capital is affected by changes in the transaction demand for cash and checks. The latter, in turn, is a function of the liquidity shock. Specifically, we show that capital is a non-monotonic function of the liquidity shock. A positive change in the probability of finding a trade partner on the native island increases the use checks in transactions. As a result, more deposits would be held at the beginning of the period to enable the likely increase in check writing. Banks, therefore, have more loanable funds that can finance investment. Hence, the resulting increase in deposits positively contributes to capital formation. However, as more checks circulate, banks have to put more of their assets in liquid reserves. Capital can either increase or decrease above its prevailing level depending on the relative magnitudes of these two
offsetting forces. Below, we describe the environment of our model and show the effects of liquidity shocks on equilibrium capital.

3.2 The model

The economy consists of two islands each inhabited by four types of agents: households consisting of shoppers, bankers, sellers, and a government which issues fiat money. Households can travel between the two islands while sellers and bankers remain stationary. Both islands are perfectly symmetric: all activities occurring on one island are mirrored on the other island. The banking system on each island is perfectly competitive with spatial separation preventing inter-location bank communication. The banking system owns a record keeping and contract enforcement technology. It also accepts real goods demand deposits $d$ and channels them to liquid reserves and capital $k$. To simplify the analysis, merchants are assumed to be agents of the bankers. Merchants sell the real goods bank reserves for fiat money and checks. Finally, banks ensure households against liquidity shocks. Below, we describe the trading environment, preferences and technology.

3.2.1 Households

Households are overlapping generations each living for two periods. The number of households in each generation is fixed and normalized to 1. When young, households are endowed with $x$ units of a good that can be invested in capital. They receive no endowment when old. Since they desire to consume in both periods, households have a demand for capital as a means of smoothing consumption over their lifetime.
We assume that capital $k$ is financed through banks and yields a positive net return of $r$ after one time period. To smooth consumption, households at time $t$, after consuming part of their endowment when young ($c_{1t}$), deposit the rest $(x - c_{1t})$ at their native island bank.

$$x = c_{1t} + d_t.$$  \hspace{1cm} (3.1)

Deposits also allow households to purchase goods with checks. Young households search for a specific market good ($c_{2t}$) sold by local merchants. If the young households' search is unsuccessful, they travel to the neighboring island and purchase an easily found good ($c_{2t}^*$) which yields a lower utility than ($c_{2t}$). The probability of finding ($c_{2t}$) on the native island is $p$. There is no aggregate uncertainty: a fraction $p$ of young households each time period $t$ finds ($c_{2t}$) on the native island while $(1 - p)$ of young households leave to the neighboring island. The implication of this pattern of trade for the means of payment used in trade is as follows. Young households traveling to the neighboring island to purchase $c_{2t}^*$ find that they can only use fiat money as a medium of exchange. If the market good is found on the native island ($c_{2t}$), then either cash or checks may be used to finance purchases of $c_{2t}$.

These market goods, $c_{2t}$ and $c_{2t}^*$, are the fraction of real deposits that banks hold as reserves and merchants, the agents of banks, sell for fiat money and checks. The aggregate resource constraint is then

$$x = c_{1t} + pc_{2t} + (1 - p)c_{2t}^* + k_t.$$  \hspace{1cm} (3.2)

\* All upper (lower) case symbols correspond to nominal (real) values.
\*\* Direct consumption of the home good or endowment $x$ is needed to pin down the effective rate of time discount.
Prior to their search, young households withdraw fiat money $M_t$ from their banks (see sequence of events summarized in table 3.1 below). At this point, young households only know the probability $p$ of having a successful trade search on the native island, not the outcome of their search. This creates uncertainty in the level and composition of the market good consumption, which in turn affects the level of cash balances and checks used in transactions. We will show below how equilibrium capital $k$ varies with the liquidity shock defined as a change in the probability $p$.

Let $P_t$ be the nominal price of the good purchased on the native island $c_{2t}$, and $P^o_t$ the nominal price of $c^o_{2t}$, the neighboring island market good.

\[ c_{2t} = \frac{M_t}{P_t} + \frac{B_t}{P_t} \quad \text{with probability } p, \quad (3.3) \]

where $M_t$ and $B_t$ are the nominal values of fiat and checks respectively spent on $c_{2t}$. We also use $m_t$ and $b_t$ to denote the corresponding real values. Also,

\[ c^*_{2t} = \frac{M_t}{P^o_t} \quad \text{with probability } 1 - p. \quad (3.4) \]

Barter is ruled out by spatial separation. As households travel, it is prohibitively costly to carry the endowment good along to barter it for either $c_{2t}$ or $c^*_{2t}$.

When old, all households return to their native island. If they find $c_{2t}$ when young, households born at time $t$ consume $c_{1,t+1}$, the return on their bank deposits net of the cash withdrawal $M_t$ and checks spent $B_t$:

\[ c_{1,t+1} = (1 + i)(d_t - \frac{M_t}{P_t} - \frac{B_t}{P_t}) \quad \text{with probability } p, \quad (3.5) \]
Table 3.1 Sequence of events

<table>
<thead>
<tr>
<th>Households born at time $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. learn the probability $p$ of finding the specific good $c_{2t}$, not the outcome of their market goods search</td>
</tr>
<tr>
<td>2. consume their home good $c_{1t}$</td>
</tr>
<tr>
<td>3. deposit goods $d_t$ at banks</td>
</tr>
<tr>
<td>4. withdraw cash $M_t$ from banks</td>
</tr>
<tr>
<td>5. search for market good $c_{2t}$ on native island</td>
</tr>
<tr>
<td>6. travel to neighboring island to purchase $c^*<em>{2t}$ if the search for $c</em>{2t}$ is unsuccessful</td>
</tr>
</tbody>
</table>

where $(1 + i)$ is the return on bank deposits.

If a young household does not find $c_{2t}$ on the native island, such a household when old consumes $c^*_{1,t+1}$, the return on the deposits net of cash withdrawal $M_t$:

\[ c^*_{1,t+1} = (1 + i) \left( d_t - \frac{M_t}{P_t} \right) \quad \text{with probability } 1 - p. \]  \hspace{1cm} (3.6)

At time $t$, young households maximize the following expected utility:

\[ \ln(c_{1t}) + p \theta \ln(c_{2t}) + p \beta \ln(c^*_{1,t+1}) + (1 - p) \ln(c^*_{2t}) + (1 - p) \beta \ln(c^*_{1,t+1}) \]  \hspace{1cm} (3.7)

(where $\theta$ is a utility parameter and $\beta$ is the time discount factor) subject to (3.1), (3.3), (3.4), (3.5), (3.6), as well the non-negativity constraint $b_t \geq 0$. Due to the logarithmic form of the utility function which tends to $-\infty$ as consumption tends to zero, a small $\epsilon > 0$ consumption would yield an infinite gain in utility. Hence, consumption $c_{1t}$, $c_{2t}$, $c^*_{2t}$, $c^*_{1,t+1}$ and $c^*_{1,t+1}$ are strictly positive.

We restrict $\theta$ to be greater than one. As will be shown below, this is a necessary condition for checks to be used in trade to supplement cash spending on $c_{2t}$, the purchased good on the native island. Although the spatial proximity of the households’
and sellers' banks allows the use of checks as means of payment, it is not a sufficient condition for checks to be drawn against deposits. We show that if $\theta$ is not greater than one, checks will not circulate and only cash will be used to purchase $c_{2t}$. When $\theta$ is greater than one, the money held by the households who do not relocate is insufficient to purchase $c_2$ (i.e. $c_2 > c^*_2$). They have to supplement their purchases of $c_2$ (paid with fiat money) with checks $b$ drawn against their deposits $d$. The reason that $c_2 > c^*_2$ is that with a higher $\theta$, the marginal utility of $c_2$ increases thereby increasing the demand for $c_2$.

### 3.2.2 Banks

Bankers and sellers are infinitely lived and remain stationary. To simplify the analysis, we assume that merchants are agents of the bankers. Spatial separation prevents inter-location bank communication. Banks can only clear local checks.

Banks provide insurance by allowing households to supplement their fiat purchases of $c_{2t}$ with checks. Since it is uncertain whether households can buy $c_{2t}$, they withdraw the minimum cash amount needed to purchase $c^*_2$, so as not to forgo the interest on their deposits. If they end up buying $c_{2t}$ instead, and since $c_{2t} > c^*_2$, households can supplement their cash purchases of $c_{2t}$ with checks.

At time $t = 0$, the government issues a stock of fiat money $\bar{M}$ on each island and gives it to the banks. Banks use this fiat money to meet households' cash withdrawals. At the end of each time period, banks have to maintain the same level of cash reserves to make good on their promise that agents can withdraw cash.

At time $t$, banks accept real goods deposits ($d_t$) and channel a fraction of them ($\alpha$) to capital ($k_t$) which yields a rate of return $(1 + r)$ at time $t + 1$. The remaining fraction of real goods deposits $(1 - \alpha)$ is set aside as reserves to be sold by merchants during the same time period $t$ (See table 3.2). These reserves may be regarded as the
liquid assets of the banks (available for current consumption), while capital \( k_t \), which matures after one time period, is less liquid.

At time \( t \), young households withdraw fiat money \( M_t \) from banks to conduct trade (table 3.3). Banks in their role as sellers sell goods to households for fiat money, thereby replenishing their cash reserves, and for checks (table 3.4).

Since capital earns the rate \( (1 + r) \), banks will only set aside reserve \( \alpha d \) equals to \( \frac{M_t}{P_t} + p \frac{B_t}{P_t} \). Therefore, the equilibrium level of capital is

\[
k_t = (1 - \alpha)d = d - \frac{M_t}{P_t} - p \frac{B_t}{P_t},
\]

the total level of deposits net of cash withdrawals and checks spent on goods.

Given pure competition, zero profits imply that the rate of return \( 1 + i \) on deposits is equal to the rate of return on capital \( 1 + r \).

### 3.2.3 Market clearing conditions

The money market clearing condition is the demand for money \( M_t \) should be equal to the stock of fiat money \( M \) every time period \( t \).

\[
M_t = M.
\]

### Table 3.2 Bank balance sheet 1

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>reserves ( \alpha d_t + \frac{M}{P_t} )</td>
<td>deposits ( d_t )</td>
</tr>
<tr>
<td>capital ( (1 - \alpha)d_t )</td>
<td>net worth ( \frac{M}{P_t} )</td>
</tr>
</tbody>
</table>
Table 3.3 Bank balance sheet 2

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>reserves = ( \alpha d_t )</td>
<td>deposits = ( d_t - \frac{M}{P_t} )</td>
</tr>
<tr>
<td>capital = ( (1 - \alpha) d_t )</td>
<td>net worth = ( \frac{M}{P_t} )</td>
</tr>
</tbody>
</table>

Table 3.4 Bank balance sheet 3

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>reserves = ( \alpha d_t - \frac{M}{P_t} - pb_t )</td>
<td>deposits = ( d_t - \frac{M}{P_t} - pb_t )</td>
</tr>
<tr>
<td>capital = ( (1 - \alpha) d_t )</td>
<td>net worth = ( \frac{M}{P_t} )</td>
</tr>
<tr>
<td>cash reserves = ( \frac{M}{P_t} )</td>
<td>net worth = ( \frac{M}{P_t} )</td>
</tr>
</tbody>
</table>

From equations (3.3) and (3.4), the demand for money is composed of the demand for money by households who travel to the neighboring island \( ((1-p)M_t = (1-p)P_t c_{2t}) \) and by households who remain stationary \( (pM_t = pP_t (c_{2t} - \frac{B_t}{P_t})) \):

\[
M_t = pP_t (c_{2t} - \frac{B_t}{P_t}) + (1-p)P_t c_{2t}^*.
\]  

(3.10)

The aggregate resource constraint is:

\[
x - c_{1t} = pc_{2t} + (1-p)c_{2t}^* + k_t.
\]  

(3.11)

From equations (3.10) and (3.11), the nominal price level is:

\[
P_t = \frac{\overline{M} + pB_t}{x - c_{1t} - k_t}.
\]  

(3.12)

3.3 Results

We present our results with three propositions. The first two propositions show the boundary values of equilibrium capital when \( p = 0 \) and \( p = 1 \). The third proposition
shows how, when $0 < p < 1$, capital varies with a liquidity shock via changes in the demand for cash and checks. We consider stationary equilibrium where the money supply and prices are constant over time with $P_t = P$ for all $t$. We drop the subscripts $t$ and denote $c_{1,t+1}$ by $c_1'$ and $c_{1,t+1}'$ by $c_1''$. Also, symmetry implies that prices on both islands are equal, $P_t = P_t^c = P$.

**Proposition 1:** If all households know with certainty that the market good can be found on their native island; i.e. $p = 1$, a household will be indifferent between using checks or fiat money as means of payment. Checks and fiat money are perfect substitutes since, given that households do not relocate, sellers will accept either as a medium of exchange. The equilibrium price level is:

$$P = \frac{\bar{M} + B}{x - c_1 - k}. \quad (3.13)$$

The equilibrium level of capital is independent of the demand for checks and is equal to:

$$k = \frac{\beta x}{1 + \theta + \beta}. \quad (3.14)$$

**Proof:**

When $p = 1$, households maximize:

$$\max_{c_1, c_2, c_1'} \ln(c_1) + \theta \ln(c_2) + \beta \ln(c_1')$$

subject to:
\[ c_1 = x - d \quad (3.15) \]
\[ c_2 = m + b \quad (3.16) \]
\[ c'_1 = (1 + i)(d - m - b) \quad (3.17) \]
\[ b \geq 0 \quad (3.18) \]
\[ m \geq 0. \quad (3.19) \]

The Lagrangean \( \mathcal{L} \) is

\[
\max_{d,b,m} \quad \mathcal{L} = \ln(x - d) + \theta \ln(m + b) + \beta \ln((1 + i)(d - m - b)) + \lambda_1(b) + \lambda_2 m
\]

where \( \lambda_1 \) and \( \lambda_2 \) are the Lagrangean multipliers for the non-negativity constraints. The resource constraint (3.15) holds with equality since utility is monotonically increasing in \( c_1 \). If there were any resources left at period \( t \), consumption could be raised without affecting anything at \( t + 1 \). This would contradict the assumption that the original allocation was maximizing. A similar argument implies that equations (3.16) and (3.17) hold with equality. These equalities justify replacing \( c_1, c_2 \) and \( c'_1 \) in the objective function. Due to the logarithmic form of the utility function which tends to \( -\infty \) as consumption tends to zero, a small \( \epsilon > 0 \) consumption would yield an infinite gain in utility. Hence, consumption \( c_1, c_2 \) and \( c'_1 \) are strictly positive.

The first order conditions are:

\[
d : \quad \frac{-1}{x - d} + \frac{\beta}{d - m - b} = 0 \quad (3.20)
\]
\[
b : \quad \frac{\theta}{m + b} - \frac{\beta}{d - m - b} + \lambda_1 = 0 \quad (3.21)
\]
\[
\begin{align*}
  m : & \quad \frac{\theta}{m + b} - \frac{\beta}{d - m - b} + \lambda_2 = 0 \quad (3.22) \\
  \lambda_1 : & \quad \lambda_1 \geq 0, b \geq 0, \lambda_1 b = 0 \quad (3.23) \\
  \lambda_2 : & \quad \lambda_2 \geq 0, m \geq 0, \lambda_2 m = 0. \quad (3.24)
\end{align*}
\]

From equations (3.21) and (3.22), \( \lambda_1 = \lambda_2 \). If both \( \lambda_1 \) and \( \lambda_2 \) are strictly positive, then \( b = m = 0 \) and \( c_2 \) would be zero. This would yield a \(-\infty\) utility level which is not optimal. Therefore, \( \lambda_1 = \lambda_2 = 0 \) and agents are indifferent between cash and checks.

The solution for deposits, cash and checks and equilibrium capital is as follows:

\[
\begin{align*}
  d &= \frac{(\theta + \beta) x}{1 + \theta + \beta} \\
  m + b &= \frac{\theta x}{1 + \theta + \beta} \\
  k &= d - m - b = \frac{\beta x}{1 + \theta + \beta}.
\end{align*}
\]

Since checks and fiat money are perfect substitutes, the total demand for money is \( M + B \) which in equilibrium is equal to \( \bar{M} + B \). From equation (3.16), and the money market clearing condition, we get

\[
Pc_2 = \bar{M} + B,
\]

which together with the aggregate resource constraint (3.11), determines the equilibrium price level

\[
P = \frac{\bar{M} + B}{x - c_1 - k} \quad \text{QED.}
\]
**Proposition 2:** When households know with certainty that the market good will not be found on their native island, \((p = 0)\), the equilibrium level of capital is equal to \(\frac{gx}{2 + \beta}\).

**Proof:**

When \(p = 0\), households maximize:

\[
\max_{c_1, c_2, c_1^*} \ln(c_1) + \ln(c_2^*) + \beta \ln(c_1^*)
\]

subject to:

\[
c_1 = x - d \quad (3.25)
\]

\[
c_2^* = m \quad (3.26)
\]

\[
c_1^* = (1 + \iota)(d - m). \quad (3.27)
\]

The Lagrangean \(\mathcal{L}\) is

\[
\max_{d, m} \mathcal{L} = \ln(x - d) + \ln(m) + \beta \ln((1 + \iota)(d - m))
\]

The first order conditions are:

\[
d : \quad \frac{-1}{x - d} + \frac{\beta}{d - m} = 0 \quad (3.28)
\]

\[
m : \quad \frac{1}{m} - \frac{\beta}{d - m} = 0. \quad (3.29)
\]

Solving for \(d\) and \(m\):

\[
d = \frac{(1 + \beta)x}{2 + \beta}
\]

\[
m = \frac{x}{2 + \beta}
\]
The equilibrium level of capital is:

\[ k = d - m = \frac{\beta x}{2 + \beta} \quad \text{QED.} \quad (3.30) \]

**Proposition 3:**

a) Even though the spatial proximity of households’ and sellers’ banks may allow the use of checks in trade, it is not a sufficient condition for checks to circulate, only a necessary one. When \( \theta \leq 1 \), checks will not circulate and only cash will be used to purchase \( c_{2x} \). When \( p \in (0, 1) \), if the utility parameter \( \theta \) is greater than 1, then the households who do not travel to the neighboring island supplement their cash purchases of \( c_2 \) with positive checks \( b \).

- When \( 0 < \theta \leq 1 \), and \( 0 < p < 1 \), then \( c_2 = c_2^* = m \), and the equilibrium value of checks is the corner solution \( b = 0 \).

- When \( \theta > 1 \), and \( 0 < p < 1 \), then \( c_2 > c_2^* \) and \( b > 0 \).

- The equilibrium solution for checks, cash, deposits and capital are:

\[
\begin{align*}
    b &= \frac{\beta(\theta - 1)(1 + \beta + p(\theta - 1)) x}{(1 + \beta)(\theta + \beta)(2 + \beta + p(\theta - 1))} \\
    m &= \frac{(1 + \beta + p(\theta - 1))}{(1 + \beta)(2 + \beta + p(\theta - 1))} \\
    d &= \frac{(1 + \beta + p(\theta - 1)) x}{2 + \beta + p(\theta - 1)} \\
    k &= \frac{\beta(\theta + \beta - p(\theta - 1))(1 + \beta + p(\theta - 1)) x}{(1 + \beta)(\theta + \beta)(2 + \beta + p(\theta - 1))}
\end{align*}
\]

b) As a result of two opposing force, capital is a non-monotonic function of the liquidity shock \( p \). An increase in \( p \) leads to an increase in deposits \( d \) that are held for
consumption smoothing and transactions purposes. With more deposits, more goods are available for banks to channel to capital. However, a higher $p$ also raises the level of checks $b$ used in transactions and forces banks to keep more of their assets in liquid reserves.

**Proof a:**

Households maximize the expected utility

$$\max_{c_1, c_2, c_2^*, c_1'} \quad \ln(c_1) + p\theta \ln(c_2) + p\beta \ln(c_1') + (1 - p)\ln(c_2^*) + (1 - p)\beta \ln(c_1'^*)$$

subject to:

$$c_1 = x - d$$
$$c_2 = m + b$$
$$c_2^* = m$$
$$c_1' = (1 + i)(d - m - b)$$
$$c_1'^* = (1 + i)(d - m).$$

Given the logarithmic utility function, $c_1$, $c_2$, $c_2^*$, $c_1'$ and $c_1'^*$ are strictly positive.

The Lagrangean $\mathcal{L}$ is

$$\max_{d, b, m} \quad \mathcal{L} = \ln(x - d) + p\theta \ln(m + b) + p\beta \ln((1 + i)(d - m - b)) + (1 - p)\ln(m)$$
$$+ (1 - p)\beta \ln((1 + i)(d - m))$$

The first order conditions are:

$$d : \quad \frac{-1}{x - d} + \frac{p\beta}{d - m - b} + \frac{(1 - p)\beta}{d - m} = 0 \quad (3.31)$$
\[ \begin{align*}
\frac{b}{m + b} - \frac{p^\theta}{d - m - b} + \frac{p^\beta}{d - m} &= 0, \\
\frac{m}{m + b} - \frac{p^\beta}{d - m - b} + \frac{1 - p}{m} - \frac{(1 - p)\beta}{d - m} &= 0.
\end{align*} \tag{3.32} \tag{3.33} \]

Solving for \( d, m, b \) and \( k \) we get:

\[ 
\begin{align*}
b &= \frac{\beta(\theta - 1)(1 + \beta + p(\theta - 1))}{(1 + \beta)(\theta + \beta)(2 + \beta + p(\theta - 1))} \tag{3.34} \\
m &= \frac{(1 + \beta + p(\theta - 1))}{(1 + \beta)(2 + \beta + p(\theta - 1))} \tag{3.35} \\
d &= \frac{((1 + \beta + p(\theta - 1))x}{2 + \beta + p(\theta - 1)} \tag{3.36} \\
k &= \frac{\beta(\theta + \beta - p(\theta - 1))(1 + \beta + p(\theta - 1))x}{(1 + \beta)(\theta + \beta)(2 + \beta + p(\theta - 1))} \tag{3.37}.
\end{align*} \]

When \( 0 < \theta \leq 1 \), then \( c_2 = c_2^* = m \), and the equilibrium value of checks is the corner solution \( b = 0 \). When \( \theta > 1 \), the money held by the households who do not relocate is insufficient to purchase \( c_2 \) (i.e. \( c_2 > c_2^* \)). They have to supplement their purchases of \( c_2 \) (paid with fiat money) with checks \( b \). QED

**Proof b:**

We now show how checks, deposits and equilibrium capital vary with a change in the probability \( p \), the liquidity shock. We have

\[ 
\begin{align*}
\frac{\partial b}{\partial p} &= \frac{\beta(\theta - 1)}{\beta + \theta} \frac{\partial m}{\partial p}, \\
\frac{\partial d}{\partial p} &= (1 + \beta) \frac{\partial m}{\partial p}, \\
\frac{\partial m}{\partial p} &= \frac{(\theta - 1)x}{((1 + \beta)(2 + \beta + p(\theta - 1))^2} > 0.
\end{align*} \tag{3.38} \tag{3.39} \tag{3.40} \]

If \( \theta > 1 \), then \( \frac{\partial m}{\partial p} \) is strictly positive which implies that \( \frac{\partial b}{\partial p} \) and \( \frac{\partial m}{\partial p} \) are also strictly positive. Therefore, a positive change in the probability \( p \) leads to an increase in deposits as well as checks used in trade.
Deposits, transferred to capital by banks, not only serve the purpose of consumption smoothing but also provide transactions services. The consumption smoothing role of deposits can be seen as follows. As \( p \) increases, the marginal rate of substitution between first and second period consumption decreases. This can be observed in equation (3.31). When \( p \) is in the neighborhood of 1, the marginal rate of substitution (MRS) between \( c_1 \) and \( c'_1 \) is approximately equal to 1. An increase in \( p \) then lowers the MRS between \( c_1 \) and \( c'_1 \). This leads to an increase in the second period consumption \( c'_1 \) and a decrease in \( c_1 \). To increase \( c'_1 \), households increase their deposits \( d \).

Deposits also are held for transaction purposes. As \( p \) increases, since \( c'_1 \) increases, then from equation (3.32), the MRS between the check good \( c_2 \) and \( c'_1 \) increases. Therefore, \( c_2 \) increases as well as the checks \( b \) that purchase this good. The level of deposits has to increase to accommodate the increased check writing.

As shown previously in the banks section, the level of capital \( k \) is equal to \( d - m - pb \). There are two forces affecting capital when \( p \) rises. On the one hand, deposits increase for consumption smoothing and transactions purposes, and therefore, more goods are available for banks to invest in capital. On the other hand, check writing increases and erodes the capital level as banks need to keep more of their assets in liquid reserves \( (m + pb) \). Capital is therefore a non-monotonic function of the liquidity shock as shown below.

When \( p = 0 \), capital is \( k = \frac{\beta x}{2 + \beta} \), and when \( p = 1 \), capital is \( k = \frac{\beta x}{1 + \beta + \theta} \). So, over the interval \((0, 1)\) of \( p \), capital should definitely decrease over some values of \( p \). This can, however, be obtained by differentiating equation (3.37) with respect to \( p \):

\[
\frac{\partial k}{\partial p} = A(\theta - 2(1 + \beta) - \beta^2 - 2p(\theta - 1) - p^2(\theta - 1)^2)
\]

where \( A = \frac{\partial x(\theta - 1)}{(\beta + \theta)(1 + \beta)(2 + \beta + p(\theta - 1))^2} > 0 \).
To get the sign of \( \frac{\partial k}{\partial p} \), we need to sign \((\theta - 2(1 + \beta) - \beta^2 - 2p(\theta - 1) - p^2(\theta - 1)^2)\)
which is a quadratic with two roots for \( p \):

\[
\begin{align*}
p_0 &= \frac{-2 - \beta - \sqrt{2 + 2\beta + \theta}}{\theta - 1} < 0 \quad \text{reject } p_0 \\
p_1 &= \frac{-2 - \beta + \sqrt{2 + 2\beta + \theta}}{\theta - 1}
\end{align*}
\]

The value of \( \theta \) that maximizes the root \( p_1 \) is \( \theta_0 = 7 + 10\beta + 4\beta^2 \). Hence, the maximum value of \( p_1 \) is equal to \( \frac{1}{2(\beta + 2\beta)} = p_g \); i.e., \( p_g \) is the value of \( p_1 \) beyond which \( \frac{\partial k}{\partial p} \) is definitely negative for any value of \( \theta \).

For \( p < p_g \), \( \frac{\partial k}{\partial p} \) could be either positive or negative. Specifically, at \( p = 0, \frac{\partial k}{\partial p} < 0 \)
if \( \theta < \bar{\theta} = 2 + 2\beta + \beta^2 \) which implies that \( \frac{\partial k}{\partial p} < 0 \) over the entire interval of \( p \). If \( \theta > \bar{\theta} \), at \( p = 0 \) the slope \( \frac{\partial k}{\partial p} > 0 \) and therefore \( k \) increases as \( p \) increases to \( p_1 \) and then decreases thereafter. QED

The combination of a high level of \( \theta \) and a low level of \( p \) results in a higher level of capital as \( p \) increases. A high value of \( \theta \) raises the demand for deposits for transactions purposes. In addition, a low value of \( p \) means that there is a low probability of households spending checks.

### 3.4 Conclusion

We show how capital varies with a liquidity shock favoring the use of checks in financing transactions. When households know with certainty whether they will shop on their native or neighboring island, deposits will be held for inter temporal purposes. When the probability of relocating to the neighboring island is one, there is no demand for checks for transactions purposes. When all agents remain on their native island with certainty, they are indifferent between cash and checks and capital is independent of households' choice of cash and checks.
Figure 3.1  Capital as a function of the probability $p$
With uncertainty, a liquidity shock in the form of an increase in the probability of finding the market good on the native island increases deposits needed to smooth consumption and to accommodate checks. On the one hand, the MRS between consuming the home good in the first period and the second period decreases leading to an increase in future consumption as well as deposits. Thus, banks can allocate more goods to capital assets. However, a liquidity shock causes the MRS between the consumption of the home good in the future and the check good to decrease, leading to an increase in the consumption of the check good. The increased use of checks for transactions purposes forces banks to raise their liquid reserves. Hence, capital is a non-monotonic function of the liquidity shock.

Instead of imposing Clower constraints, we provide micro foundations for the demand for cash and checks which results from a spatial separation structure that prohibits barter as well as the honoring of contracts in trade. Our model’s results are analogous to Hartley’s when household borrowing constraints, a feature of undeveloped capital markets, are considered. In Hartley’s case, borrowing constraints impede consumption smoothing and results in an upward sloping supply of capital. The equilibrium capital in this case would be sensitive to liquidity shocks that shift the supply of capital around. Our OLG structure is a form of borrowing constraints that renders the investment process dependent on liquidity shocks. Our households live only for two periods and naturally cannot borrow against their future endowment as no future endowment exists. When endowed only in their first period of life, households save in the form of positive demand deposits which are an increasing function of interest rates (assuming that substitution effect dominates the income effect) and are also a function of the liquidity shock. Equilibrium capital composed of households’ savings net of checks would therefore be a function of the liquidity shock as well.
The spatial proximity of agents is a necessary but insufficient condition for the demand for checks to be positive. The latter also depends on the utility \( \theta \) derived from the good purchased on the native island. When \( \theta \) is below a certain threshold, checks do not circulate. We move to the next chapter to examine the effect of liquidity shocks on equilibrium capital when the demand for checks solely arises from the spatial structure of the model and is independent of the utility function.
Chapter 4

A spatial separation/OLG model with the old subject to liquidity shocks

4.1 Introduction

To provide more insight into the effects of liquidity shocks on capital formation, we consider a different trading environment from the one analyzed in chapter 3. The latter has young households using both cash and checks to purchase goods on the native island. There, the role of checks as a medium of exchange ultimately depends on the spatial proximity of households (parameterized by $p$) and the utility ($\theta$) derived from consuming the good that can be bought with checks. The higher $\theta$ and $p$, the higher the demand for checks. Households wish to consume more local goods than they have cash balances to pay for. They are willing to supplement their cash balances with checks even though it means they will forgo some future interest receipts.

Moreover, the liquidity shock in chapter 3 hits young households only. A higher likelihood of using checks for transactions purposes has two implications: the young have to hold more deposits but they also draw down these balances to buy goods before the investment matures. These two opposing forces make capital a non-monotonic function of the liquidity shock.

The model analyzed in this chapter differs from the one discussed in chapter 3 in a number of respects. A positive demand for checks is generated from the spatial closeness of individuals and does not depend on the value of the market good's utility parameter. The spatial proximity of trading partners' banks is a necessary and sufficient condition for the acceptance of checks as a means of payment. The
trading environment is such that the economy is composed of spatially separated sites inhabited by overlapping generations that are subject to random relocation shocks. Agents search for two types of market goods to consume. One can be easily located while the other has a high search cost and can only be obtained by a search process that matches trading partners who do not physically meet. Parties who meet as a result of this complicated search process can only trade if they have deposits at banks that can communicate with each other. In particular, the parties cannot exchange goods using barter or cash. Hence, only agents who remain on the same site (that is, their banks can communicate with each other and can clear checks) can purchase this high search cost good. Once a match in trade occurs, buyers pay with checks. Agents who relocate to neighboring sites cannot purchase this good due to the lack of communication between sites which prevents the issuance of private claims backed by assets. Relocated agents end up only purchasing the low search cost good with fiat money.

In contrast to the model of chapter 3, the old (but not the young) are exposed to the possibility of a liquidity, or relocation, shock. The timing of the liquidity shock affects the nature of the response of capital to these shocks. In chapter 3, capital is a non-monotonic function of the liquidity shock when the young relocate. In the model examined in this chapter, we show that when the old are hit with a relocation shock, capital is always an increasing function of the liquidity shock. A higher probability of remaining on the native island when old raises the likelihood of purchasing the high search cost good and the level of checks needed in trade. Since the households need to hold sufficient deposits to cover the checks they write, an increase in the probability of remaining on the home island raises the capital level. However, unlike chapter 3, agents write checks in their second time period after capital matures. Hence, the increase in checks in trade does not erode the capital level which is determined
during the agents' first time period. Capital in this case is a monotonically increasing function of the liquidity shock.

Section 4.2 outlines the trading environment, and the methods of payment that arise from the various trading frictions present in that environment. We solve for a unique stationary equilibrium in section 4.3. In section 4.4, we analyze how the equilibrium responds to variations in the liquidity shock, with a particular focus on how the amount of k varies. We conclude in section 4.5 by contrasting the results in chapters 3 and 4.

4.2 Endowments, preferences, technology and trade

We consider an economy composed of two islands that are perfectly symmetric: all activities on one island occur in the same way on the other island. Each island has overlapping generations (OLG) each living for two periods. The number of agents in each generation is fixed and normalized to 1. When young, an agent is endowed with \( x \) units of a good that can be deposited at a bank. Each island has a competitive banking system that channels deposits to investment yielding a positive net rate of return \( r \). Banks on one island are disconnected from banks on the other island. Checks drawn against deposits in one island, therefore, cannot be cleared by banks on the adjacent island.

When young, agents at time \( t \) consume \( c_{1t} \) (part of their endowment \( x \)), make deposits \( d_t \) and choose cash balances to hold to the next period. Let \( \frac{M_t}{P_t} \) be the real cash balances held, where \( M_t \) is the nominal cash balances demanded and \( P_t \) is nominal price level of \( x \) at time \( t \):

\[
x = c_{1t} + d_t + \frac{M_t}{P_t}
\]  
(4.1)
Total capital \( k \) is the sum of all young agents’ deposits \( d_t \).

When old, agents face a probability \( p \) of remaining on their native island and a probability \( 1 - p \) of relocating to the neighboring island. (See figure 4.1). There is no aggregate uncertainty: a fraction \( 1 - p \) of each generation ends up relocating to the neighboring island while \( p \) remain on their native island when old.

Old agents derive utility from two types of market goods. One has a zero search cost and belongs to the young generation. The remaining old purchase and consume this market good \( c_{2,t+1} \) from the young of their native island with their cash balances:

\[
c_{2,t+1} = \frac{M_t}{P_t^{t+1}}. \tag{4.2}
\]

The relocated old purchase and consume \( c'_{2,t+1} \) the low search cost good from the young of the neighboring island

\[
c'_{2,t+1} = \frac{M_t}{P'_t^{t+1}}, \tag{4.3}
\]

where \( P'_t^{t+1} \) is the nominal price of the endowment good \( x \) sold by the young on the neighboring island.

The second market good belongs to the old and can only be found through a search process. Parties matched through the search process do not physically meet, making it impossible for cash to be used to finance trade. The sellers will not be able to verify that the buyers have positive cash balances. Therefore, for trade to occur, sellers will accept checks from buyers whose banks are in contact with the sellers’ banks. This enables sellers to verify that buyers have the means to pay for their purchases. Therefore, only the remaining old can purchase this good \( c_{3,t+1} \) with checks \( B_{t+1} \) since they share the same banking system.
\[ c_{3,t+1} = \frac{B_{t+1}}{P_{x_0,t+1}}. \]  

(4.4)

\(B_{t+1}\) is the nominal value of the check paid to purchase \(c_{3,t+1}\) and \(P_{x_0,t+1}\) is the nominal price of \(c_{3,t+1}\).

Banks will debit the buyer's bank balance of \((1 + r)d_t\) by the check amount \(B_{t+1}\) leaving a net amount of \((1 + r)d_t - \frac{B_{t+1}}{P_{x_0,t+1}}\) to consume as \(c_{1,t+1}\):

\[ c_{1,t+1} = (1 + r)d_t - \frac{B_{t+1}}{P_{x_0,t+1}}. \]  

(4.5)

The problem with this formulation, however, is that equations 4.4 and 4.5 only represent the buyer's side of the transaction. They do not show from where the buyer obtained the goods and why the seller would accept to give up goods in return for checks. A more explicit trade structure is therefore needed within the group of old
agents that remains on the native island. We assume that they are located around a circle (figure 4.2).

![Diagram](image)

**Figure 4.2** The location of old agents remaining on their native island

Every remaining old \( A \) in figure 4.2 derives utility from consuming the hard to find market good owned by the agent to the right. This market good is the matured deposit \( (1 + r)d_t \). However, the search cost of this good is high and can only be found through a complicated search process. (Agents are close only in the sense that they have the same banking system). The absence of double coincidence of wants resulting from this structure precludes barter between any two trading partners. We also assume that the cost of all traders getting together is infinite. This prevents them signing a joint agreement to redistribute their goods and leads to a centralized accounting system (banks) to redistribute the goods. The banks debit and credit the old agents' deposit accounts such that goods are redistributed counter-clockwise.

Specifically, to purchase this good, an old agent borrows from the bank the needed dollar amount \( B \). However, since the buyer cannot physically meet to exchange cash with the seller, the bank opens a line of credit for the buyer. The seller can contact the buyer’s bank to verify that the buyer has the funds to pay for the good. When trade occurs, the buyer’s bank then transfers the amount \( B \) to the seller’s account. To repay the bank loan, the old agent sells part of his matured deposits \( (1 + r)d_t \).
and receives a transfer of $B$ to his bank account (equation 4.5). Therefore, a seller of this market goods accepts this transfer as a means of payment because it is needed to repay the bank loan she herself took to purchase the high search cost good.

Finally, since banks cannot pay off-island agents on their deposits, the relocated old consume the return on their deposits on their native island ($c^*_{1,t+1}$) prior to relocation:

$$c^*_{1,t+1} = (1 + r)d_t$$  \hspace{1cm} (4.6)

4.3 The existence of a unique stationary equilibrium

When young, each agent maximizes expected utility:

$$\max_{c_{1t}, c_{1,t+1}, c_{2,t+1}, c_{3,t+1}, c^*_{1,t+1}, c^*_{2,t+1}} U(c_{1t}) + p\beta U(c_{1,t+1}) + p\beta U(c_{2,t+1}) + p\beta U(c_{3,t+1}) + (1 - p)\beta U(c^*_{1,t+1}) + (1 - p)\beta U(c^*_{2,t+1})$$  \hspace{1cm} (4.7)

subject to equations (4.1) to (4.6). $U$ is a general utility function with a strictly positive first derivative and a strictly negative second derivative. To rule out corner solutions, we assume that $U(c) \to -\infty$ as $c \to 0$. $\beta$ is the discount factor.

As in a ‘conventional’ OLG model of money demand, the old at time $t=0$ are endowed with the money supply $\bar{M}$ which remains constant over time.

The money market clearing condition requires the money supply $\bar{M}$ to equal the demand for money $M_t$ in every time period $t$. The nominal price level $P_t$ is equal to $rac{\bar{M}}{x - c_{1t} - d_t}$.

The agent’s Lagrangean $\mathcal{L}$ is
\[
\max_{d_t, B_t, M_t} \quad \mathcal{L} = U(x - d_t - \frac{M_t}{P_t}) + p\beta U((1 + r)d_t - \frac{B_t}{P_{x0,t+1}}) + p\beta U\left(\frac{M_t}{P_{t+1}}\right) + p\beta U\left(\frac{B_t}{P_{x0,t+1}}\right) + (1 - p)\beta U((1 + r)d_t) + (1 - p)\beta U\left(\frac{M_t}{P_{t+1}}\right).
\]

The first order conditions are:

\[
d_t : \quad -U'(x - d - \frac{M_t}{P_t}) + p\beta(1 + r)U'((1 + r)d_t - \frac{B_t}{P_{x0,t+1}}) + (1 - p)\beta(1 + r)U'((1 + r)d) = 0 \tag{4.8}
\]

\[
M_t : \quad \frac{-U''(x - d_t - \frac{M_t}{P_t})}{P_t} + \frac{p\beta U'(\frac{M_t}{P_{t+1}})}{P_{t+1}} + \frac{(1 - p)\beta U'(\frac{M_t}{P_{t+1}'})}{P_{t+1}'} = 0 \tag{4.10}
\]

\[
B_t : \quad -U'((1 + r)d_t - \frac{B_t}{P_{x0,t+1}}) + U'\left(\frac{B_t}{P_{x0,t+1}}\right) = 0. \tag{4.11}
\]

\[U'\] is one-to-one and monotonic and therefore equation (4.11) becomes:

\[
\frac{(1 + r)d_t}{2} - \frac{B_t}{P_{x0,t+1}} = 0. \tag{4.12}
\]

We consider stationary equilibrium where \(P_t = P, P_{x0,t} = P_{x0}, d_t = d, M_t = M, B_t = B, c_{1t} = c_1, c_{1,t+1} = c'_1, c_{2,t+1} = c'_2, c_{3,t+1} = c_3, c_{1,t+1} = c'_1, c_{2,t+1} = c'_2\), and \(c_{2,t+1} = c'_2\) for every time period \(t\). Also, given symmetry on the two islands, \(P_t = P_{t}' = P\).

From equations (4.9), (4.10) and (4.12) we get:

\[
\begin{align*}
p(1 + r)U'\left(\frac{(1 + r)d}{2}\right) + (1 - p)(1 + r)U'((1 + r)d) & - U'\left(\frac{M}{P}\right) = 0. \tag{4.13}
\end{align*}
\]

Let \(A = p(1 + r)U'\left(\frac{(1 + r)d}{2}\right) + (1 - p)(1 + r)U'((1 + r)d)\). Then equations (4.9) and (4.13) become:
\[ U'(x - d - \frac{M}{P}) = \beta A \]  \hspace{1cm} (4.14)

\[ U'(\frac{M}{P}) = A. \]  \hspace{1cm} (4.15)

Since \( U' \) is one-to-one monotonically decreasing, equation (4.15) can be expressed as

\[ \frac{M}{P} = U'^{-1}(A). \]  \hspace{1cm} (4.16)

Differentiate equation (4.16) with respect to \( d \) to get:

\[ \frac{\partial M}{\partial d} = \frac{\partial U'^{-1}(A)}{\partial A} \frac{\partial A}{\partial d} \]

\[ = \frac{\partial U'^{-1}(A)}{\partial A} \left( p \frac{(1+r)^2}{2} U''\left( \frac{(1+r)d}{2} \right) + (1-p)(1+r)^2 U''\left( (1+r)d \right) \right), \]  \hspace{1cm} (4.17)

where \( \frac{\partial U'^{-1}(A)}{\partial A} < 0 \) since \( U'' < 0 \). This implies that \( \frac{\partial M}{\partial d} > 0 \). Therefore, from equation (4.16), \( \frac{M}{P} \) and \( d \) are positively related.

Similarly, equation (4.14) can be re-written as:

\[ \frac{M}{P} = x - d - U'^{-1}(\beta A). \]  \hspace{1cm} (4.18)

Differentiate (4.18) with respect to \( d \) to get:

\[ \frac{\partial M}{\partial d} = -1 - \frac{\partial U'^{-1}(\beta A)}{\partial (\beta A)} \frac{\partial (\beta A)}{\partial d} \]

\[ = -1 - \frac{\partial U'^{-1}(\beta A)}{\partial (\beta A)} \left( \beta p \frac{(1+r)^2}{2} U''\left( \frac{(1+r)d}{2} \right) + (1-p)(1+r)^2 U''\left( (1+r)d \right) \right) \]  \hspace{1cm} (4.19)
where \( \frac{\partial M}{\partial d} < 0 \) in equation (4.19). From equation (4.14), \( \frac{M}{P} \) and \( d \) are negatively related. Therefore, both first order conditions (4.14) and (4.15) provide a unique solution for \( \frac{M}{P} \) and \( d \). Given that \( U \) is globally concave and the constraints are linear thus forming a convex set, the first order conditions are necessary and sufficient conditions for the existence of equilibrium.

4.4 Comparative statics

We perform comparative statics analysis on the equilibrium solutions for the demand for cash balances \( M \), deposits \( d \) and the demand for checks \( B \) when the probability \( p \) of remaining on the native island increases. We find that as \( p \) increases, more checks will be used as a medium of exchange. This in turn increases the level of deposits \( d \) as more is needed to accommodate the increase in checks and reduces cash balances \( M \) held as agents move from young to old age. Hence, the total equilibrium level of capital \( k \), which is the sum of all young agents’ deposits, is an increasing function of the liquidity shock.

Proposition:

\[
\frac{\partial M}{\partial p} < 0, \quad \frac{\partial B}{\partial p} > 0, \quad \frac{\partial d}{\partial p} = \frac{\partial k}{\partial p} > 0.
\]

Proof:

Plugging equation (4.12) in equations (4.9) and (4.10) we get

\[
-U'(x - d - \frac{M}{P}) + \beta U'(\frac{M}{P}) = 0 \tag{4.20}
\]

\[
-U'(x - d - \frac{M}{P}) + \beta p(1+r)U'(\frac{(1+r)d}{2}) + \beta (1-p)(1+r)U'((1+r)d) = 0. \tag{4.21}
\]
Differentiate equations (4.20) and (4.21) with respect to $p$ to get:

$$U''[c_1](\frac{\partial d}{\partial p} + \frac{\partial M}{\partial p}) + \beta U''[c_2]\frac{\partial M}{\partial p} = 0,$$

(4.22)

and

$$U''[c_1](\frac{\partial d}{\partial p} + \frac{\partial M}{\partial p}) + \beta(1 + r)U'[c'_1] + \frac{\beta p(1 + r)^2}{2}U''[c'_1]\frac{\partial d}{\partial p} - \beta(1 + r)U'[c'_1^*]$$

$$+ \beta(1 - p)(1 + r)^2U''[c'_1^*]\frac{\partial d}{\partial p} = 0.$$

(4.23)

From equation (4.22) we have

$$\frac{\partial M}{\partial p} = \frac{-U''[c_1]\frac{\partial d}{\partial p}}{U''[c_1] + \beta U''[c'_2]}.$$  

(4.24)

Replacing equation (4.24) in (4.23):

$$\frac{\partial d}{\partial p} \left( \frac{\beta U''[c_1]U''[c'_2]}{U''[c_1] + \beta U''[c'_2]} \right) + \frac{\beta p(1 + r)^2}{2} U''[c'_1] + \beta(1 - p)(1 + r)^2 U''[c'_1^*] =$$

$$\beta(1 + r)(U'(1 + r)d - U'(\frac{(1 + r)d}{2})).$$

(4.25)

From equation (4.25) and the assumption $U'' < 0$, $\frac{\partial M}{\partial p}$ is strictly positive. Hence, from equation (4.24), $\frac{\partial M}{\partial p}$ is strictly negative. Also, since $\frac{B}{R_w} = \frac{(1 + r)d}{2}$ (equation (4.12)), then $\frac{\partial R}{\partial p}$ is strictly positive. QED

As $p \rightarrow 1$, the marginal rate of substitution between consumption when young ($c_1$) and when old ($c'_1$) is approximately equal to the marginal rate of transformation ($1 + r$). Any increase in $p$ will lower this marginal rate of substitution with agents preferring to consume more when old than when young. Therefore, $c_1$ decreases and $c'_1$ increases. Moreover, since $c'_1$ is equal to $c'_3$, the consumption of the hard to find
good increases as well. Since $c'_3$ is purchased with checks, an increase in $p$ results in an increase in spent checks. Therefore, deposits against which checks are drawn have to increase as well. The equilibrium level of capital, the sum of bank deposits, unambiguously increases as $p$ increases.

4.5 Conclusion

We study the effects of liquidity shocks on capital in a spatial separation/OLG trading environment that gives deposits a medium of exchange role. Old agents are subject to relocation shocks affecting their demand for cash and checks. We find that capital is a monotonically increasing function of the liquidity shock.

Agents derive utility from a low search cost market good as well as from a high search cost one. The latter could be located through a search process that matches traders who do not physically meet. This precludes monetary exchange and only checks could settle trade in this market. Therefore, spatially close agents linked to a common banking system could have checks exchanged and obtain the high search cost good. Spatially separated agents, in the sense that their banks have no intercommunication, end up only purchasing the low search cost good with fiat money. Unlike chapter 3, where checks circulation depends on the value of the good's utility parameter as well as relocation shocks, the existence of checks as a medium of exchange in this chapter only depends on the spatial closeness of agents' banks.

The timing of the liquidity shock has a direct effect on how equilibrium capital varies with the liquidity shock. In chapter 3, the young are hit with a relocation shock which results in capital being a non-monotonic function of the shock. A higher probability of not relocating lowers the marginal rate of substitution between present and future consumption which results in a higher future consumption level achieved through higher level of deposits. At the same time, however, a higher probability of
remaining on the same site increases the demand of the good purchased with checks. These are drawn against deposits when agents are young and before capital matures. Therefore, the increase in checks erodes deposits which are transferred to capital and hence capital decreases.

In this chapter, when the old are hit with the relocation shock, an increase in the probability of not relocating leads to a decrease in the marginal rate of substitution between present and future consumption resulting in more future consumption through higher deposits and hence more capital. This higher shock, as in chapter 3, also results in an increase in the demand for the market good purchased with checks. However, since the old are subject to the shock, the increase in checks happens after capital matures and therefore does not erode it. Capital is always an increasing function of the liquidity shock.
Chapter 5

Conclusion

This thesis sheds some light on factors, in particular shocks to the demand for bank liabilities (checks), that affect the level and stability of aggregate investment in economies with undeveloped capital markets. Banks are the major source of finance for firms and fiat money and checks are the main media of exchange for households. Our models are less relevant for economies with developed capital markets where firms rely less on banks to finance their investment decisions, and households use credit cards to finance their purchases. However, to understand how the evolution of capital markets affects the level and stability of investment, we focus on economies with undeveloped capital markets.

The thesis is composed of an empirical and a theoretical analysis. We show with regression estimates that the effect of the variance of demand deposits on that of investment is statistically significant in countries with undeveloped capital markets. Such is not the case for countries with developed capital markets.

We then proceed to provide a theoretical explanation of how liquidity shocks that change the demand for checks affect capital formation in countries with undeveloped capital markets. These liquidity shocks stem from relocation shocks that force agents to change their geographical location in search of trading partners. Fiat money, though dominated in rate of return by bank deposits, is accepted on all sites of an economy in contrast to checks where they can only be used on sites where banks exist. We show that equilibrium capital is sensitive to liquidity shocks that cause agents to switch from checks to fiat money in financing their purchases. This is contrary to
infinite horizon models where a dichotomy between monetary and real sectors of the economy exists.

Our problem is two folds. First, we provide a trading environment which allows fiat money and checks to settle the exchange of goods. This is unlike Clower constraint models which exogenously impose fiat money as a medium of exchange by making the observation that money buys goods, and goods buy money, but goods do not buy goods. Throughout the thesis, the main trading friction that disrupts barter and the honoring of contracts is the spatial separation of agents. Due to the limited inter-site bank communication, which prevents the circulation of privately issued claims against assets, spatially separated agents can only settle trade transactions with fiat money. Spatially close agents, in the sense of being connected by a common banking system, can use checks in trade. Liquidity shocks to the agents’ demand for cash and checks result from relocation shocks that alter the agents’ location in the economy.

Second, the effect of the liquidity shock on capital differs depending on the type of economic agent the shock hits. We specify who is affected by the liquidity shock. We consider an OLG model with agents living for only two periods. The OLG setting is not essential as a friction in trade once spatial separation is introduced, but is crucial for the liquidity shocks to affect capital. Unlike the infinite horizon setting, agents’ saving, consisting of deposits that are transferred to capital, is not infinitely elastic at the rate of time preference. Rather, it is increasing in interest rates, and liquidity shocks to the demand for inside money shift this saving schedule and alter the equilibrium level of capital. The effect of liquidity shocks on saving and capital differs when the shock hits young agents than when it hits old agents. We show that the equilibrium capital level is a non-monotonic function of the liquidity shock affecting young agents’ demand for checks and is increasing in the liquidity shock affecting old agents.
To gain some insight on the effect of liquidity shocks on capital, we provide two models with spatial separation as the main friction giving rise to a positive cash and check demand. Both models involve finitely lived agents in an OLG setting. However, the trading environment and the timing of the liquidity shock differ in each model. Chapter 3 provides a setting where agents are subject to a liquidity shock when young. The trading environment has young agents searching for specific market goods owned by trading partners who are spatially close to the buyers. However, there is a possibility of young agents not finding the specific good. In such case, they settle for a good that yields lower utility and is purchased with fiat money from a remote location. If young agents find their preferred good from the specific spatially close traders, they pay for their good with cash and checks. Checks supplement their cash purchases of the higher yielding utility good.

In chapter 3, we show that capital is a non-monotonic function of the liquidity shock when it hits young agents. An increase in the probability of finding the preferred good owned by specific spatially close trading partners results in a lower marginal rate of substitution between the consumption of the home good and this market good. This increases the demand for the latter and more checks are therefore spent to purchase it. Deposits therefore increase to accommodate the increased check circulation. This has two effects on the equilibrium level of capital which is composed of the aggregate bank deposits net of checks. The increase in checks drawn against deposits erodes the total capital level. However, the resulting increase in deposits has the opposite effect. Capital therefore not only reflects the inter temporal allocation of resources for consumption smoothing but also the households’ liquidity need for checks. This stands in contrast to the certainty case where all households know with certitude whether or not they will find the right trading partner. In this case, capital reflects
only the inter temporal allocation of resources for consumption smoothing and is unaffected by changes in the demand for cash and checks.

In chapter 3, the demand for checks not only depends on relocation shocks but also on the value of the utility parameter attached to the consumption of the good purchased with checks. Checks do not circulate if this parameter is below a certain threshold. Chapter 4 provides a model where the demand for checks is independent of the utility derived from the ‘check’ good and only depends on the relocation shock. The trading environment is such that agents derive utility from two types of market goods. One has a high search cost and the other low search cost. A search process helps agents locate the high search cost and matches up trade partners who do not physically meet. We show that only checks can settle trade in this market with spatially close agents connected to a common banking system purchasing this good. Agents who are forced to relocate can only purchase the low search cost good with fiat money.

In contrast to chapter 3, old agents, rather than the young, are subject to the liquidity shock in chapter 4. A higher probability of not relocating has two effects. First, the marginal rate of substitution between present and future consumption of the home good decreases thereby raising the level of deposits to attain a higher level of future consumption. Since deposits are channeled to investment, capital increases. Second, the higher probability of remaining on the native island implies a higher probability of purchasing the high search cost good. The demand for check good therefore increases and so do checks in circulation. However, since the old are hit with the liquidity shock, the increase in checks is drawn against already matured deposits and therefore does not erode the capital level. Hence, capital is an increasing function of the liquidity shock affecting the old.
In both models, whether the young or old are hit with the shock, the vulnerability of capital to liquidity shocks stems from the OLG setting where each generation is only endowed in the first period of life and only lives for two periods. This structure results in households providing a positive level of deposit saving which is increasing in interest rates and a function of the probability of relocation. Our OLG structure is analogous to the borrowing constraint case in Hartley (1994) and (1998) which is a necessary condition for the vulnerability of capital to liquidity shocks.
Bibliography


