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THE ELASTIC SCATTERING OF PROTONS BY He^4,
AND THE ELASTIC SCATTERING OF He^3 BY He^4

by

Philip D. Miller

A THESIS
SUBMITTED TO THE FACULTY
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

Houston, Texas
April, 1958
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I INTRODUCTION

The study of the energy levels of light nuclei has been one of the most fruitful fields of research in nuclear physics. For example, the energy levels of light nuclei often are well enough separated that explicit level parameters may be extracted by means of suitable experiments. The observable level parameters desired are: a more or less exact energy of the level, its angular momentum and parity, its various partial widths for decay, and its isobaric spin.

One of the most powerful tools for obtaining the above information is the elastic scattering of charged particles. In certain simple cases, the phase shift analysis of experiments of this type provides all of the level parameters mentioned above. In addition, the behaviour of the phase shifts provides considerable qualitative information as to the form of the potential interaction between the scattered particle and the target nucleus.

In addition to static potential information, the scattering of particles with spin provides information on the magnitude of the coupling between the spin of the incident particle and its orbital angular momentum. This is provided by the amount of splitting between levels of the same orbital quantum number, \( l \), and \( J \)'s of \( l + 1/2 \), and \( l - 1/2 \). In agreement with the shell model, all of the observed single particle doublets have the higher \( J \) level, lying lower in energy (In53). One consequence of spin-orbit coupling is that the scattered particles are generally polarized to some degree, and are very strongly polarized at certain angles in the neighborhood of resonances.

The difficulty of fitting the data to phase shift analyses
is greatly reduced for small channel spins; yet, even in the case of spin 1/2, if no narrow resonances are present there is a lack of uniqueness which may only be resolved by the determination of the spin polarization of the scattered particles (Cr49).

Most of the experiments in the light nuclei, which meet this requirement of small channel spins, involve targets which are obtainable as gases. It was this fact that led to the construction of the Rice Institute large volume scattering chamber. The use of gas targets has two other very important experimental advantages. First, cross sections may be very accurately determined, as the number of gas molecules per unit volume is accurately known from the pressure and temperature of the target gas. Secondly, the target thickness is easily adjusted by changing the gas pressure. In this manner thicker targets may be obtained for use with very small beams, and where cross sections are low, and much thinner targets may be used when better resolution is desired, as in the observation of narrow resonances.

In the present work two nuclei have been investigated by the elastic scattering of charged particles of spin 1/2 from spin 0 nuclei. In the first experiment protons were scattered from He$^4$ yielding information on the compound nucleus Li$^5$. A phase shift analysis of the angular distributions has been done. The results of experiments from 1 to 18 mev of proton bombarding energy have been collected, and the resulting phase shifts have been interpreted in the light of the dispersion theory. In the second experiment He$^3$ nuclei were scattered from He$^4$ giving information about the compound nucleus Be$^7$. The second excited state of Be$^7$ has been
investigated, and the level parameters have been determined. The non-resonant phase shifts show a somewhat anomalous behaviour in the energy range investigated, and these phase shifts have been qualitatively interpreted in terms of other levels in Be$^7$.

The phase shift analysis of these experiments has been done on an IBM 650 computer, and as a by-product of the analysis, the angular distributions of spin polarization are generated as a function of energy by the computer. Contour maps of spin polarization will be presented for each of the two experiments considered.
II THE EQUIPMENT, EXPERIMENTAL PROCEDURE, AND ERROR

DISCUSSION

A General Description of the Equipment

The features of the large volume scattering chamber have been described in previous theses (Ru54), (Re54), (Ru56), (Re56), (Re56), so they will be only outlined here. The chamber consists of a welded cylinder 30 inches in diameter, and 13 inches high constructed of 1/2 inch rolled aluminum plate. The two ends have been turned to carry an O-ring which seals to the top and bottom plates made from 3/4 inch aluminum. A differential pumping tube is re-entrant on one wall of the chamber, and extends approximately 3/4 of the distance from the wall to the center of the chamber.

The use of a differential pumping tube allows the beam to enter the chamber without having to pass through a foil. There are two pumping stages along the differential pumping tube. The first stage is pumped by a large mechanical pump, and the second stage is pumped by a fast, oil diffusion pump. Even with 1.5 cm. Hg of helium in the chamber, the vacuum at the analyzing magnet of the Van de Graaff accelerator is not appreciably affected.

Opposite the differential pumping tube is a Faraday cup which has both magnetic and electrostatic electron suppression, and has been described in the PhD thesis of J. L. Russel (Ru56). There are two detectors in the chamber. Both counters use scintillation crystals and photomultipliers. The original detector (which will be called detector number 1), uses a Du Mont 6292 photomultiplier, and the newer detector (number 2), uses a 6291 photomultiplier. In all of the present experiments the scintillators
were CsI(Tl) with thicknesses between 20 and 50 mils. The detectors are mounted on aluminum channels which are supported by concentric shafts which come out through the bottom of the chamber through vacuum seals. These shafts are attached to vernier equipped divided circles so that the detector angles may be set with an accuracy of 5' from outside the chamber. The space between the two concentric shafts is at atmospheric pressure, and provides room for the electrical cables to the preamplifiers and the photomultipliers. The paths of these cables are both vacuum sealed from the chamber, so that all of the electronic apparatus is at atmospheric pressure. In both detectors, lucite light pipes provide the vacuum seal between the scintillators and the photomultiplier tubes.

A small electric motor and a wire which may be connected to either of two position indicator contacts are available in the chamber. These contacts may be used either for a circular foil holder on detector number 1, or for the variable slit system on the new detector when the detectors are used in coincidence.

Since the above mentioned theses were written, both detectors have been rebuilt to allow access to the preamplifiers and photomultipliers without disturbing the alignment of the slits. Both detectors' housings were split in the horizontal plane, and flanges with O-rings added. The preamplifiers were then rebuilt with plug-in bases for their electrical connections. Finally, the cap over the outlet in the center of the chamber, which houses the electrical cables to the second detector, was
rebuilt with a removable top to allow easier access to the
cables. Figure 1 is a photograph of the chamber, with the top
removed, showing the two detectors. The number 1 detector is on
the right, and the number 2 detector is on the left. To the rear
of the chamber may be seen the Faraday cup, and in the foreground
is the exit of the differential pumping tube. Figure 2 shows
the preamplifier and photomultiplier-light pipe assembly for
the number 1 detector beside its housing, and the preamplifier
for the number 2 detector beside its housing.

The gas flow system is essentially the same as previously
described by Reich (Re54), (Re56), except that the gas is now
delivered through an all-metal line from the tank through a
standard pressure reducer, through a controlled leak, and into
the chamber. The pressure in the chamber is measured by means
of a manometer backed by the same oil diffusion pump which
evacuates the Faraday cup. The manometer is read with an Eberbach
cathetometer which is accurate to 0.1 mm. The manometer oil
used for most of these experiments was butyl phthalate, whose
specific gravity is 1.04578.

B The Current Integrator and Capacitor Calibration.

The current integrator was built by R. R. Henry, but
has not been previously described. The principle of operation
is as follows. A capacitor is charged to an accurately known
voltage. When the start switch is depressed, the capacitor is
disconnected from the charging voltage, and is connected to a
wire coming from the Faraday cup on the chamber. The positive
current flow from the Faraday cup discharges the capacitor, and
as the voltage across the capacitor falls through zero, a DC amplifier throws a relay which controls other relays that in turn stop the scalers, and reapply the charging voltage to the capacitor.

This same apparatus is used for calibrating the capacitor. The bias controls on the DC amplifier in this case are set to throw the relay before the capacitor voltage reaches zero. The voltage at which the relay does throw is then measured by means of a two volt wet cell, a helipot voltage divider, and a type K potentiometer. An accurately known resistance is now connected from the Faraday cup lead to ground, and a scaler, scaling sixty cycles per second, is controlled by one of the relays, so as to start counting when the start switch is depressed and the capacitor starts to discharge. When the voltage across the capacitor reaches the previously measured final voltage, the relay throws, stopping the scaler. The elapsed time, the initial voltage, the final voltage, and the discharging resistance are known; since the circuit is a simple RC circuit, the capacitance may be easily determined. This method of calibration has a distinct advantage over other methods, in that, the effective capacity in the circuit is measured in the same way that the circuit is used, so that relay throw times, and the leakage are the same in the calibration as in the experiment itself. This method leads to results which are reproducible to 0.2%, and the overall accuracy is estimated to be 0.5%.

Figure 3 shows a circuit diagram of the current integrator, and Figure 4 shows the circuit diagram of the charging supply. In its present form, the charging supply gives two selectable
CURRENT INTEGRATOR

Figure 3
AC SWITCH  6.3 VAC 3 AMP FLEX FORMER

I AMP FUSE  GALV. LIGHT

12 VOLT CAR BATT.  LEEDS & NORTHROP GALVANOMETER .022amps/mm

OUTPUT PLUG  ZERO PL

STANDARD 200 V=1.010

FIGURE 4
voltages, 10.2202 and 5.1057. The Goodall capacitors used for all of the present experiments have been calibrated several times with the results listed in Table I.

<table>
<thead>
<tr>
<th>Date</th>
<th>Capacity</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-3-57</td>
<td>10.549</td>
<td>------</td>
</tr>
<tr>
<td>4-2-57</td>
<td>10.424</td>
<td>- 1.19 %</td>
</tr>
<tr>
<td>7-22-57</td>
<td>10.379</td>
<td>- 0.43 %</td>
</tr>
</tbody>
</table>

The Cross Section Measurements and Discussion of Errors

The formula for the number of counts recorded is:

\[(II-1) \quad N = \sigma(\theta_{lab}) d Q t \Delta \Omega.\]

In this formula,

- \(N\) is the number of counts recorded at a laboratory angle \(\theta_{lab}\) for \(Q\) incident particles,
- \(\sigma(\theta_{lab})\) is the laboratory differential cross section in \(\text{cm}^2\) per steradian,
- \(d\) is the density of the target gas in atoms per \(\text{cm}^3\),
- \(t\) is the target thickness in \(\text{cm}\), and
- \(\Delta \Omega\) is the detector solid angle in steradians.

If \(W\) is the width of the front slits, \(A\) is the area of the rear slits, and \(S\) is the separation between the front and rear slits, then reference to Figure 5 shows that

\[(II-2) \quad t \Delta \Omega = \frac{RW}{S \sin \theta} \quad \frac{A}{R^2} = \frac{AW}{RS} \quad \frac{1}{\sin \theta} = \frac{G}{\sin \theta} .\]
\[ \Delta \Omega = \frac{A}{R^2} \]

\[ \frac{T \sin \theta}{R} = \frac{w}{S} \]

**SLIT SYSTEM**

**FIGURE 5**
Formula (II-1) may now be solved for $c(\theta_{\text{lab}})$ to give:

$$\text{II-3} \quad c(\theta_{\text{lab}}) = \frac{N \sin \theta}{d \cdot Q \cdot d}$$

The estimated accuracy of the above quantities is as follows. $N$ contains two sources of error; one is the detector efficiency which is considered to be $100 + 0, -1 \%$, second is the statistical error. The angle of the detector in the laboratory may be set to an accuracy of $5'$, so that the maximum error in $\sin \theta$ is about $0.2\%$. $d$, the number of target nuclei per unit volume, is determined by the assumption of the perfect gas law for the target, and by the measurement of the pressure of the gas in cm. of butyl phthalate, and the measurement of the temperature of the walls of the chamber, which are assumed to be in equilibrium with the gas. The error due to the assumption of the perfect gas law is completely negligible at the pressures used. The pressure of the gas is measured to $0.1 \text{ mm}$. For a typical pressure of $5.0 \text{ cm. of oil}$, the pressure measurement is accurate to $0.2\%$. The temperature was measured with a mercury thermometer laid on top of the chamber. The thermometer was accurate to $1$ degree centigrade, which at usual laboratory temperatures amounts to $0.3\%$. The additional error in $d$ due to convection inside the chamber in the region of the beam, and other causes is estimated to be $0.1\%$. The estimated RMS error in $d$ is thus $1.1\%$.

The quantity $Q$ depends on the knowledge of three quantities, the charging voltage, the current integrator capacity, and the average charge per ion collected in the Faraday cup. As discussed in section II-B, the capacity is estimated to be accurate to
0.5%. The charging voltage has been determined to better than 0.1%. For protons, the average charge per ion is taken to be exactly 1e. Experiments with an aluminum foil and an analyzing magnet to separate the charge 1 protons from the neutral protons indicate that above 1 mev this assumption is valid to about 0.2%, and the error becomes rapidly smaller as the energy gets higher. The RMS error in Q is thus estimated to be less than 0.6% for protons. For He$^3$ ions, the average charge per ion is taken to be exactly 2e. For He$^3$ ions of energy 3.0 mev or above, this approximation is accurate to better than 1% as determined by experiments with an aluminum foil and an analyzing magnet to separate the beams of different charge states. The RMS error in Q is thus estimated to be 1.2% for the He$^3$ experiment.

The measurements of A and W were done with a traveling stage microscope. The results of several measurements of the same slit systems, measured at different times are listed in table II below. The fluctuations are probably a good measure of the error.

<table>
<thead>
<tr>
<th>Date</th>
<th>Counter</th>
<th>AW</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-25-55</td>
<td>2</td>
<td>0.042049</td>
</tr>
<tr>
<td>11-12-55</td>
<td>1</td>
<td>0.081720</td>
</tr>
<tr>
<td>12-28-55</td>
<td>1</td>
<td>0.081658</td>
</tr>
<tr>
<td>12-29-56</td>
<td>2</td>
<td>0.04120</td>
</tr>
<tr>
<td>6-18-57</td>
<td>1</td>
<td>0.081685</td>
</tr>
<tr>
<td>4-2-57</td>
<td>1</td>
<td>0.081680</td>
</tr>
</tbody>
</table>

The RMS deviation for counter number 2 is 1.6%, while that for number 1 is 0.1%. The measurements of R and S were made with a
meter stick taped across the top of the chamber, and a plumb bob. These measurements are estimated to be good to .05 cm., and the distances measured were at least 19 cm., so that the estimated accuracy is 0.2%. The total error in G is taken to be 1.7%.

The RMS total of the errors in $\sigma(\theta_{lab})$ is 2.4% plus counting statistics for proton scattering, and 2.6% plus counting statistics for $^3$He scattering.

The expression for the cross section given above is the laboratory cross section. In order to analyze the data in terms of phase shifts, it is necessary to express the cross section in terms of the center of mass system of coordinates. The conversion to the center of mass is given by formula (II-4), and is derived in appendix C.

\[(\text{II-4}) \quad \sigma_{cm}(\theta) = C(\theta) \sigma(\theta), \]

where

\[
C(\theta) = \frac{1 + m_1/m_2}{[1 + 2(m_1/m_2)\cos \theta_{cm} + (m_1/m_2)^2]^{3/2}}.
\]
III THE $^4\text{He}(p,p)^4\text{He}$ EXPERIMENT

A Introduction

The nucleus $^5\text{Li}$ has been investigated by the elastic scattering of protons from $^4\text{He}$. The range of bombarding energies from 2.0 to 5.5 mev has been studied experimentally, and phase shifts have been extracted from the data. The phase shifts from this and other experiments have been interpreted in the light of the Wigner and Eisenbud dispersion theory. The level parameters of the ground and first excited states of $^5\text{Li}$ have been determined from this analysis, and they show that these two states are very good single particle states of a proton and an $\alpha$ particle. Assuming a P-wave radius of 2.60 Fermis (1 Fermi = $10^{-13}$ cm.) the P3/2 ground state of $^5\text{Li}$ is observed to occur at 2.62 mev laboratory bombarding energy, or 2.09 mev in the center of mass. This leads to a mass defect for $^5\text{Li}$ of 13.28 mev. This is .29 mev larger than the mean value given by Ajzenberg and Lauritzen (Aj55). The discrepancy probably lies in the method of choosing the ground state energy. The criterion used for the above value from elastic scattering is that the resonant portion of P3/2 phase shift must equal 90° at the resonant energy. The reduced width of the ground state as determined by the best fit to the experimental data is 12 mev·Fermis, as compared to 30 mev·Fermis for the Wigner limit. The P1/2 first excited state of $^5\text{Li}$ occurs at an excitation energy of 8.59 mev. Its reduced width is 30 mev·Fermis, and is consequently equal to the Wigner limit within the experimental errors.

Figure 6 shows the energy levels of $^5\text{Li}$, and the energy levels of $^5\text{He}$, the mirror nucleus, for comparison.

Energy levels in $^5\text{Li}$ have been studied by many others at a
\[
\begin{align*}
\text{HE}^6 & \quad 3/2^- & \quad 16.69 & \quad 3/2^+ \\
\text{D} + \text{T} & \quad 16.63 \\
\text{HE}^3 & \quad 3/2^- & \quad 16.80 & \quad 3/2^+ \\
\text{HE}^4 + \text{N} & \quad -0.95 & \quad \text{Li}^5 & \quad -2.09 \\
2.6 & \quad 1/2^- & \quad 8.62 & \quad 1/2^- \\
3/2^- & \quad 3/2^- &
\end{align*}
\]
variety of energies by the scattering of protons from He$^4$. The earliest experiment of any significance was done by the Minnesota group (Fr49). This experiment covered the bombarding energy range from 0.95 to 3.58 mev, and the data was phase shift analyzed by Critchfield and Dodder (Cr49). Isolated angular distributions have been taken at 5.78 mev (Kr54), at 7.5 mev (Pu56), at 9.73 mev (Co54), at 9.76 mev (Wi55), and very recently from 11.4 to 18 mev (Br57). In Putnam's paper (Pu56), the results of their experiments, and the results of some earlier experiments at 9.48 mev are phase shift analyzed.

B The Rice Experimental Data

The present experiment covers the range from 2.0 to 5.5 mev bombarding energy. The experiment was done twice. The first time was in May of 1956. This experiment consisted of excitation curves at 39°14', 54°44', 70°71', 90°, 125°16', 149°26½', and 168°45' in the center of mass. These excitation curves are shown in Figures 7, 8, and 9. The data were taken using an Atomic five channel integral bias pulse height analyzer. It was later discovered that at forward angles, some recoil He$^4$ nuclei were being counted, so that the cross sections were too large. The reason for the inclusion of these curves here is to show that there are no narrow resonances in the energy range under consideration.

Having discovered the above error, it was decided to do the experiment again at all angles, and to use the Atomic twenty channel differential pulse height analyzer to pulse height analyze the scattered proton and recoil α scintillation pulses. Since the excitation curves previously taken show no narrow anomalies, it was decided
He$^4$(p,p)He$^4$

EXCITATION CURVES

- EXPERIMENT

![Graph showing excitation curves with angular momentum values $\Theta_{GM} = 149^\circ 26' 11''$ and $\Theta_{GM} = 125^\circ 16'$, with energy $E_1$ on the x-axis and barns/steradian on the y-axis.]

**Figure 9**
to take angular distributions at approximately 500 kev intervals starting at 3 mev. The angular distributions taken were at 3.027, 3.514, 4.016, 4.500, and at 5.000 mev. These data are presented in Figures 10, 11, 12, 13, and 14 respectively. The fifteen angles chosen, were the multiples of ten degrees in the center of mass from 30° through 160° plus 168°. The last angle was the most backward even degree angle which could be attained with the present counters. The statistics were at worst 2%, and at all but the lowest cross sections, they were better than this ranging down to better than 1% at the backward and forward angles. The energy scale was derived from the 2.660 mev resonance from protons on O16. The energies are accurate to ± 10 kev. Because of the lack of structure in the excitation curves, no effort was made to obtain any more accurate energy scale than this.  

C The Phase Shift Analysis of the Data

The formula for the differential cross section in the center of mass is

\[(III-1) \quad \sigma(\theta) = \left| f_0(\theta) \right|^2 + \left| f_1(\theta) \right|^2,\]

where

\[kf_0(\theta) = -\left( \eta/2 \right) \csc^2(\theta/2)e^{i\pi \ln \csc^2(\theta/2)}\]

\[+ \sum_{l=0}^{\infty} e^{i\alpha_l} P_l(\cos \theta)[\left( l + 1 \right) e^{i\delta_l^+} \sin \delta_l^+ + l e^{i\delta_l^-} \sin \delta_l^-]\]

and

\[kf_1(\theta) = \sum_{l=1}^{\infty} e^{i\alpha_l} P_l(\cos \theta) \sin \theta [e^{i\delta_l^+} \sin \delta_l^+ - e^{i\delta_l^-} \sin \delta_l^-].\]

---

1 If the derivatives of the phases with respect to energy are measured graphically on Figures 15 and 16, the largest turns out to be \( \frac{d\delta_1^+}{dE} = 19^o/\text{mev} \). A ten kilovolt error would then introduce an error of approximately 0.2° in \( \delta_1^+ \), and less than half of this error in \( \delta_1^- \) and \( \delta_0 \).
$\text{He}^4(p,p)\text{He}^4$

$E_{\text{LAB}} = 3.027$

- DATA
- PHASES

$\delta_0 = -27.5^\circ$

$\delta_1^+ = 99.3^\circ$

$\delta_1^- = 10.2^\circ$

RMS DEV = 1.75%

- PHASES

$\delta_2 = -27.3^\circ$

$\delta_1^+ = 99.3^\circ$

$\delta_1^- = 10.3^\circ$

$\delta_2^+ = 0.1^\circ$

$\delta_2^- = 0.7^\circ$

RMS DEV = 1.64%

FIGURE 10
$^{4}\text{He}(p,p)^{4}\text{He}$

- DATA
  $E_{\text{LAB}}=4.016$
- PHASES
  \[ \delta_0 = -3.45^\circ \]
  \[ \delta_1^- = 112.2^\circ \]
  \[ \delta_1^- = 23.6^\circ \]
  RMS DEV=33.9%
- PHASES
  \[ \delta_0 = -32.9^\circ \]
  \[ \delta_1^+ = 112.4^\circ \]
  \[ \delta_1^+ = 23.6^\circ \]
  \[ \delta_2^- = -1.7^\circ \]
  \[ \delta_2^- = 2.1^\circ \]
  RMS DEV=1.89%

**FIGURE 12**

$\theta_{\text{CM}}$
\[ \text{He}^4(p,p)\text{He}^4 \]

**DATA**

\[ E_{\text{LAB}} = 4.500 \]

**PHASES**

\[ \delta_0 = -43.1^\circ \]
\[ \delta_1^+ = 111.6^\circ \]
\[ \delta_1^- = 29.1^\circ \]
\[ \delta_2^+ = -0.5^\circ \]
\[ \delta_2^- = -3.1 \]

**RMS DEV:** 4.32%
Figure 14

\[ \text{He}^4(p,p)\text{He}^4 \]

- **DATA**
  - \( E_{\text{LAB}} = 500 \text{ MEV} \)

- **PHASES**
  - \( \delta_0 = -51.8^\circ \)
  - \( \delta_1^+ = 109.9^\circ \)
  - \( \delta_1^- = 35.5^\circ \)
  - \( \delta_2^+ = 0.1^\circ \)
  - \( \delta_2^- = -4.7^\circ \)

**RMS DEV:** 2.65%
In this formula,
\[ \eta = \frac{Z_1Z_2e^2}{k}, \]
\[ a_0 = 0, \]
\[ a_1 = 2 \sum_{s=1}^{\ell} \tan^{-1}(\eta/s), \] and
\[ k = \frac{\mu v}{\hbar}. \]

\( Z_1 \) and \( Z_2 \) are the charges of the incident and target particles respectively; \( v \) is the relative velocity of the incident and target nuclei at infinity, and \( \mu \) is the reduced mass of the system. This formula is discussed by Reich (Re56). The use of an IBM 650 computer for the fitting of angular distributions is described in appendix B.

There are two solutions to the problem of the phase shifts. These two are distinguished physically as implying, in one case, a normal doublet, and in the other case, an inverted doublet for the ground state, and the first excited state of \( \text{Li}^5 \). This ambiguity may be resolved by a determination of the spin polarization at a single energy. The phase shifts can then be followed by continuity from this energy. Such measurements have been done by the Minnesota group (Re52), and by the Los Alamos group (Ro57). The results indicate an inverted doublet. Only this branch of the solution has been followed in the present analysis.

The three lowest angular distributions were first fit using only \( S \) and \( P \) wave phase shifts. The fits obtained were not noticeably improved by the addition of \( D \)-wave phase shifts.

Figures 15 and 16 show the results of all of the \( \text{He}^4(p,p)\text{He}^4 \) experiments including the present one. The references to the other
\[ \phi_0 = -\tan^{-1}(F_0/G_0) \big|_{R=1.99 F} \]

**Figure 15**
$\delta_1^+ \quad \gamma_\lambda^2 = 12 \text{ MEV.F}$
$E_\lambda = 3.96 \text{ MEV}$
$E_{R_LAB} = 2.62 \text{ MEV}$
$\theta_P^2 = 0.40$

$\delta_1^- \quad \gamma_\lambda^2 = 30 \text{ MEV.F}$
$E_\lambda = 16.77 \text{ MEV}$
$E_{R_LAB} = 10.75 \text{ MEV}$
$\theta_P^2 = 1.0$

R = 2.60 F
X Rice data
• Other data

Figure 16
data were given in section III-A.

It is very difficult to estimate the errors to be assigned to the phases, because of the many angles involved, and because of the non-linear dependence of $\sigma(\theta)$ on the phase shifts. From the dispersion of the values of the phase shifts from a smooth curve in Figures 15 and 16 however, it appears that $\pm 5^\circ$ would be a reasonable estimate of the error.

Bonner, (Bo58) has recently seen an indication of a new state in He$^5$, the mirror nucleus to Li$^5$, by the elastic scattering of neutrons from He$^4$ using a recoil ionization chamber. This state appears at a laboratory bombarding energy of 15.6 $\pm$ .2 mev. Its width is of the order of 1 mev, and its spin and parity have not been determined.

Brockman (Br57) observed angular distributions of protons scattered by helium, using the Princeton cyclotron, at nine energies between 11 and 18 mev. He formed excitation curves from these angular distributions and drew smooth curves through the nine resulting points. From these smoothed excitation curves, he formed angular distributions at even mev energies, and phase shift analyzed these constructed angular distributions. It was thought that he might have smoothed over the mirror state to the one observed in He$^5$, so his original data was phase shift analyzed. It is these points which are plotted in Figures 15 and 16.

D The Interpretation of the Phase Shifts from 1 Through 18 Mev

S Wave: The S-wave phase shift is fit reasonably well at low energies by a hard sphere phase shift with an interaction
radius of 2.0 Fermis. At the high energy end, as can be seen from Figure 15, there is some deviation in the positive direction from the computed hard sphere value. This is most likely the result of a very high, broad 2S state.

**P-Waves:** The P-wave phase shifts will be discussed in terms of the dispersion theory of Wigner and Eisenbud (Wi47). The pertinent expression from the dispersion theory is:

\[(\text{III-2}) \quad \delta_k^{\pm} = -\varphi_k + \tan^{-1} \frac{\Gamma_{\lambda}/2}{E_{\lambda} + \Lambda_{\lambda} - E} = -\varphi_k + \beta_k^{\pm}.\]

In the above expression, \(-\varphi_k\) is the potential contribution to the phase shift, and in addition, contains all of the effects of distant resonances of the same spin and parity. In the present experiment it will be assumed that there are no distant resonances contributing to \(-\varphi_k\), so that it is given by:

\[(\text{III-3}) \quad \varphi_k = \tan^{-1} \frac{F_k}{G_k} \bigg|_{\rho = kR}.\]

This assumption will be justified by the results. \(R\) is the assumed nuclear radius, and \(F_k\) and \(G_k\) are the regular and irregular Coulomb wave functions as defined by Bloch (Bl51). The second term of (III-2) contains the resonant portion of the phase shift, and the quantities are defined as follows:

\[\Gamma_{\lambda}/2 = k\gamma_{\lambda}^2/\Lambda_{\lambda}^2,\]

where \(\gamma_{\lambda}^2\) is the external width of the state in the center of mass, and \(\gamma_{\lambda}^2\) is the intrinsic width of the level with both the Coulomb and
centripetal barriers removed, so that it is related to the properties of the wave function at the nuclear surface. If \( k \) has the dimensions of reciprocal Fermi, then \( \gamma_\lambda^2 \) has the dimensions of mev · Fermis.

\( A_i^2 \) is the reciprocal of the penetrability, and is given by

\[
A_i^2 = F_i^2 + G_i^2.
\]

\( \Delta_\lambda \) is called the level shift, and is given by the following expression:

\[
(\text{III-3}) \quad \Delta_\lambda = - \left[ \frac{k \gamma_\lambda^2}{\rho} \left( \frac{\rho}{A_i} \frac{dA_i}{dp} + 1 \right) \right].
\]

This term arises from the effect of the centripetal and Coulomb barriers on the resonant energy. \( E_\lambda \) is defined as that energy necessary to make the denominator of the resonant portion of (III-2) vanish at the value of \( E \), where \( \beta_\lambda^\pm \) is experimentally observed to be 90°. Notice that \( E \) is expressed in the center of mass.

The method of finding \( \gamma_\lambda^2 \) and \( R \) suitable for the present problem was as follows:

1. An interaction radius was chosen more or less arbitrarily.

(The first value tried was 3.24 F.) For this radius, values of \( \varphi_1 \) were computed from the tables of Bloch (B151). From these values of \( \varphi_1 \), and the experimentally determined values of \( \beta_1^\pm \), values of \( \beta_1^\pm \) were determined.

2. The quantities \( \Delta_\lambda \gamma_\lambda^2 \) and \( \Gamma \gamma_\lambda^2 \) were computed from the Bloch tables.

3. Various values of \( \gamma_\lambda^2 \) were then tried in order to get a best fit to the experimentally determined \( \beta_1^* \) as given by step 1.

4. A new value of the radius \( R \) was chosen, and steps 1, 2, and 3 were repeated until a satisfactory fit was obtained.

The only serious difficulties encountered were in step 2.
The tables of Bloch (Bl51) are primarily designed for use on heavier nuclei than He^4, and they are most useful for lower energies than 10 mev when light nuclei are concerned. More precisely, there are no entries for values of ν less than .1585, and the entries for values of ρ less than 1 are very scattered. For this reason, a linear interpolation in ν was used, at high energies, between the various functions listed in the tables, and the corresponding neutron functions for ν = 0. This suffices to cover the high energy region where ν is small, and ρ is large. The other region of difficulty is the low energy region, where ν is well within the scope of the tables, but ρ is very small. The worst difficulty was encountered with the energy shift Δλ, and the trouble lies in the singularity of A_1 and dA_1/dρ for small ρ. This difficulty was resolved however with the aid of the Bessel function expansion of the irregular Coulomb G_1 for small ρ (Pr58). With the aid of this expansion, the behaviour of A_1 and Δλ was satisfactorily determined in the low energy region.

Figure 17 shows the behaviour of the shift function Δλ/γ^2 versus laboratory bombarding energy for three different values of the nuclear radius, R.

Figures 18, 19, and 20 show the behaviour of β^±_1 for radii of 2.20, 2.60, and 3.24 Fermis respectively, and for various assumed values of the reduced width γ^2_λ. An interesting number with which to compare the experimentally determined reduced width of a level is the so-called Wigner limit. This is given by (γ^2_λ)_w = 3ω^2/2μR. This reduced width corresponds physically to the single particle
\[- \frac{\Delta \lambda}{\gamma^2} \]

Figure 17

\(- \frac{\Delta \lambda}{\gamma^2}\) vs. \(E_{\text{LAB}}\) in MeV for different values of \(R\): R = 2.20 F., R = 2.60 F., R = 3.24 F.
\[ \beta_{1}^{\pm} \]
\[ \frac{3 \hbar^2}{2 \mu R} = 35.5 \text{ MEV F.} \]
\[ \chi_{\lambda}^2 = 20 \text{ MEV F.} \]
\[ E_{\lambda} = 4.07 \text{ MEV} \]

\[ \beta_{1}^{+} \]
\[ \beta_{1}^{-} \]

\[ \chi_{\lambda}^2 = 35 \text{ MEV F.} \]
\[ E_{\lambda} = 22.38 \text{ MEV} \]

**Figure 18**
\[ \beta_1 \pm \]

\[ R = 2.60 \text{ F.} \]

\[ \frac{3k^2}{2\mu R} = 30.0 \text{ MEV F.} \]

\[ \chi^2 = 10 \text{ MEV F.} \quad E_\lambda = 3.65 \text{ MEV} \]

\[ \chi^2 = 12 \text{ MEV F.} \quad E_\lambda = 3.96 \text{ MEV} \]

\[ \chi^2 = 30 \text{ MEV F.} \quad E_\lambda = 16.77 \text{ MEV} \]

FIGURE 19
\[ \beta_{1}^{\pm} \]
\[ R = 3.24 \text{ F.} \]
\[ \frac{3\kappa^{2}}{2\mu R} = 24.1 \text{ MEV F.} \]

\[ \gamma_{\lambda}^{2} = 5 \text{ MEV F.} \]
\[ E_{\lambda} = 2.71 \text{ MEV} \]
\[ \gamma_{\lambda}^{2} = 15 \text{ MEV F.} \]
\[ E_{\lambda} = 4.05 \text{ MEV} \]
\[ \gamma_{\lambda}^{2} = 12 \text{ MEV F.} \]
\[ E_{\lambda} = 8.91 \text{ MEV} \]

**FIGURE 20**
limit for the width of a state. The fraction of the Wigner limit experimentally determined is a measure of the amount of single particle parentage of a particular state. The parameters determined by the best fits to the experimental data for the first two states of Li$^5$ are as follows:

\[ R = 2.60 \text{ Fermis} \]
\[ 3\hbar^2 / 2\mu R = 30.0 \text{ mev Fermis} \]

For the ground state,
\[ \gamma_\lambda = 12 \text{ mev Fermis} \]
\[ J = 3/2^- \]
\[ E_\lambda = 3.96 \text{ mev} \]

For the first excited state,
\[ \gamma_\lambda = 30 \text{ mev Fermis} \]
\[ J = 1/2^- \]
\[ E_\lambda = 16.77 \text{ mev} \]

As can be seen, the ground state of Li$^5$ has a reduced width of about 40% of the Wigner limit, and the first excited state is fit very well with a reduced width of 100% of the Wigner limit. Thus in this energy region (0 - 18 mev) is well described in terms of the two body interaction of a proton and an $\alpha$ particle. In figure 16 the above parameters are used with (III-2), and the resulting theoretical values of $\delta_1^+$ are plotted.

The fit to the data, particularly at high energies is very reasonable. In the P1/2 case there might be a slight trend of the experimental data away from the dispersion theory fit, but the trend seems to be less than the scatter in the experimental points. The assumption of only hard sphere potential effects for $\phi_1$ thus seems
to be justified.

**D Waves:** The values of the D-wave phase shifts from the present and other experiments are shown in Figure 15. The inaccuracies of the existing experiments preclude the possibility of getting any quantitative information. Qualitatively it may be said that the D-wave phases appear to be unsplit below 10 mev, and appear to be decreasing with energy. Above 10 mev, one or both of the D-wave phase shifts appear to be moving in the positive direction. From (III-1) it may be seen that for small splittings, and small phase shifts, it is impossible to ascertain which \( \delta \) of a given \( \lambda \) is about to resonate.

*E Spin Polarizations of Protons Scattered by Helium from 1 Through 18 Mev.*

In appendix A, the following expression is derived for the percent spin polarization along the x axis for particles scattered in the y-z plane at a center of mass angle \( \theta \) with respect to the incident beam.

\[
(III-9) \quad SP = 2 \left( \frac{\Re f_1(\theta) - \Re f_n(\theta) \cdot \Im f_n(\theta)}{\sigma(\theta)} \right)
\]

Since in the present experiment, the phase shifts are measured, thus determining both the coherent and incoherent wave functions, predictions of the spin polarization may be made. Angular distributions of spin polarization were calculated using the program for the IBM 650 computer described in appendix B. The results of these calculations are presented in Figure 21, in the form of a contour map of spin polarization. There appear to be three regions in which virtually 100% polarized beams of protons may be obtained. These
He$^4$$(p,p)$He$^4$

PER CENT SPIN POLARIZATION

FIGURE 21
are from 75° to 90° in the center of mass at a bombarding energy of from 1.7 to 2.2 mev, secondly, from 120° to 131° with bombarding energies from 5.7 to 17 mev, and finally at 90° with a bombarding energy of 10.7 mev.

Since the cross section appears in the denominator of the expression for the spin polarization, it is no coincidence that the regions of high spin polarization correspond to regions of low cross section. A few typical cross sections in the neighborhood of points of high spin polarization are also presented in Figure 20, where the points are indicated by X's, and the number beside each point is the cross section in millibarns per steradian. The Los Alamos experimental points at 10 mev are also shown on the figure (Ro57). The number beside each experimental point is the observed per cent spin polarization at that particular angle.
The range of excitation energies in Be\(^7\) from 3.28 to 4.73 mev has been investigated by the scattering of He\(^3\) from He\(^4\). The second excited state of Be\(^7\) has been studied, and its resonant energy has been determined to be 5.171 mev in the laboratory. This corresponds to an excitation energy in Be\(^7\) of 4.532 \(\pm\) .020 mev. The spin and parity of this state have been determined by a phase shift analysis to be \(J = 7/2^-\). The laboratory width of this state is 280 kev. Figure 22 shows the energy levels of Be\(^7\) as given in Ajzenberg and Lauritsen (Aj55) modified by the information of the present experiment, and modified by the inclusion of the work of Marion and Weber (Ma56), on the 6.46 and 7.18 mev states.

A Coincidence Slits and Circuitry

The He\(^4\)(He\(^3\),He\(^3\))He\(^4\) experiment is afflicted with one difficulty, which has not occurred in any of the problems previously done with the scattering chamber. This difficulty is associated with the similarity of the masses of He\(^3\) and He\(^4\). At forward angles, the energy of the recoil He\(^4\) nuclei, and the energy of the scattered He\(^3\) particles are so similar that the two groups of particles entering the scintillation detector may not be satisfactorily resolved using pulse height analysis alone. The fraction of the bombarding energy which the scattered particle retains as a function of the bombarding energy is:

\[
(IV-1) \quad \frac{E_{He^3}}{E_{1 \text{ Lab}}} = 1 - \frac{4m_1m_2\sin^2(\theta_{cm}/2)}{(m_1 + m_2)^2}
\]

in which,

\(m_1\) is the mass of the bombarding, (He\(^3\)), particle;
ENERGY LEVELS IN Be$^7$

\[
\begin{array}{c|c}
5/2^- & 7.18 \\
3/2^+ & 6.46 \\
7/2^- & 4.53 \\
1/2^- & 0.430 \\
5/2^- & -0.863 \\
\end{array}
\]

\[
\begin{array}{c}
5.600 \\
\frac{\text{Li} + p}{\text{Li}^7} \\
1.583 \\
\frac{\text{He}^3 + \alpha}{\text{He}^3 + \alpha} \\
\end{array}
\]

FIGURE 22
\( m_2 \) is the mass of the target nucleus, \( \text{(He}^4) \); and
\( \theta_{\text{cm}} \) is the center of mass angle of the scattered \( \text{He}^3 \) nucleus.

In Figure 23, the fraction of the incident energy imparted to the recoil \( \text{He}^4 \) nucleus is plotted versus the laboratory counter angle, and the same is done for the scattered \( \text{He}^3 \) nucleus. Typical counter pulse height resolutions obtained experimentally were from 8 to 12%, so that it was necessary to count the scattered \( \text{He}^3 \) particles and the recoil \( \text{He}^4 \) particles in coincidence. It would appear from Figure 23, that at angles in back of, perhaps, 60°, that the scattered \( \text{He}^3 \) nuclei might be counted without requiring coincidence. Their energy was so low, however, that it was desirable to count coincidences even at the more backward angles to reduce the background noise. A block diagram of the coincidence electronic apparatus is shown in Figure 24. A typical pulse height distribution, both with the coincidence gate on, and with the coincidence gate off, is shown in Figure 25. As can be seen from this illustration, there were sometimes losses due to the counting of coincidences. In cases such as that shown in Figure 25, the "no gate" pulse height distribution was used for the peak number of counts, and the gated distribution was used for determining the background. At backward angles it was necessary to use the gated distribution entirely for determining the number of scattered \( \text{He}^3 \) particles, as the group could not even be distinguished without the coincidence gate.

The slit system geometry is shown schematically in Appendix C. The calculations of the slit widths necessary in order to collect all of the particles in detector number two, which are coincident with those entering detector number one are also given in Appendix C.
ELECTRONIC APPARATUS BLOCK DIAGRAM

TO INPUT OF TWENTY CHANNEL ANALYZER
2.3μ SEC. DELAY (RG 65)

DET.#1
-2 VOLTS
CATHODE FOLLOWER
PREAMPLIFIERS

AI AMPLIFIERS
0.5μ SEC. RISE
FAST DECAY
+30 VOLTS

6 BNG COINCIDENCE CIRCUIT

-5 VOLTS

DET.#2
-2 VOLTS

+30 VOLTS

SCHMITT TRIGGER CIRCUIT
3μ SEC. LONG

20 VOLT SQUARE PULSE TO GATE 20 CHANNEL PULSE HEIGHT ANALYZER

FIGURE 24
$\text{He}^4(\text{He}^3,\text{He}^3)\text{He}^4$ PULSE HEIGHT DISTRIBUTION WITH AND WITHOUT GATE

$\theta_{\text{CM}} = 100^\circ$
$
\text{He}^3 \quad \theta_{\text{LAB}} = 59^\circ 40'$
$
\text{He}^4 \quad \theta_{\text{LAB}} = 40^\circ$

NUMBER OF COUNTS

GATE

GATE

CHANNEL

FIGURE 25
The description of the code for the IBM 650 computer which was used to make these calculations is given in the same place. Figure 1 in chapter II is a photograph, from the top, of the chamber with the lid removed, showing the variable slit system in position on detector number two. The rear slit widths available are .75, 1.0, 1.25, 1.5, 2.0, and 2.5 cm.

Coincidences losses introduce a new source of error not considered in the case of protons on helium. In addition to the errors introduced by coincidence losses, the high background at backward angles introduces uncertainties which are difficult to estimate. A reasonable estimate of the probable error is 5% from 70° to 90° in the center of mass, and, perhaps, 10% outside of this range.

B Presentation of the Data

Two angular distributions at 2.974, and 3.877 mev laboratory bombarding energy, and six excitation curves at center of mass angles 54°44', 63°26', 90°, 109°52', 116°34', and 125°16' constitute the data for the He^4(He^3,He^3)He^4 experiment. All but the last of the above excitation curves cover the laboratory bombarding energy range from 3.0 to 5.5 mev, and the last excitation curve, at 125°16', covers the range from 4.5 to 5.5 mev.

The two most forward angle excitation curves were taken by counting the recoil He^4 particles in the first detector, and counting the coincident He^3 particles in the second detector. This was done so that the second detector, with its much larger solid angle and consequent much larger counting rate, would have the more energetic particles to detect in the presence of a less intense weak group, rather than vice versa.
The data was originally taken in the summer of 1956. When angular distributions were formed from the excitation curves at energies running across the resonance, it appeared that the $125^0$ excitation curve had a cross section which was perhaps too low. This curve was repeated in the fall of 1957 and the difficulty was determined. The 1956 data was taken with beam currents of about one to two microamperes. With the number two counter at the forward angle, this meant that the coincidence losses were very high because the A1 amplifier and the coincidence circuit were being severely overloaded. When this excitation curve was repeated in the fall of 1957, it was found that if the beam current was kept less than 0.2 microamperes, the cross section determined was a maximum, and did not vary with beam current outside of statistics. The excitation curve was therefore repeated with a beam current of less than 0.2 microamperes. This raised the measured cross section on the peak of the resonance from 109 mb./steradian to 151 mb./steradian as shown in Figure 33. Figure 26 shows the angular distribution at 2.974 mev, and Figure 27 shows the angular distribution at 3.377 mev. In the angular distributions, both points observed by counting scattered He$^3$ nuclei and He$^4$ recoils are shown. Figures 28 to 33 show the excitation curves at center of mass angles $54^0 44'$, $63^0 26'$, $90^0$, $109^0 52 1/2'$, $116^0 34'$, and $125^0 16'$ respectively. The energy was determined from the following formula:

$$(IV-2) \quad E = Kf^2 - \Delta E_1 - \Delta E_g.$$ 

$\Delta E_1$ is the correction to the energy due to fringing of the magnetic field of the $90^0$ analyzing magnet. This correction has been determined at a number of different magnetic fields by R. A. Chapman.
He$^3 +$ He$^4$

$E_{\text{LAB}} = 3.877$ MEV

- $\bullet$ He$^3$
- $\times$ He$^4$
- EXCITATION CURVES He$^3$
- EXCITATION CURVES He$^4$
- PHASES
  - $\delta_0 = -22^\circ$
  - $\delta_1^+ = -23^\circ$
  - $\delta_1^- = -13^\circ$
  - $\delta_2^- = -3^\circ$
  - $\delta_2^+ = 0^\circ$
  - $\delta_3^- = 6^\circ$
  - $\delta_3^+ = 0^\circ$

$\sigma_{\text{cm}}$ BARNS/STER.

$\theta_{\text{cm}}$

FIGURE 27
\[ H^4_\text{He}^3(\text{He}^3\text{He}^4)H^4 \]

\[ \theta_{CM} = 54.0^\circ \pm 0.4^\circ \]

Data from phase shifts

Figure 28: \( E_{\text{He}^3} \) vs. barns/steradians
$\text{He}^4(\text{He}^3, \text{He}^3)\text{He}^4$

$\theta_{\text{CM}}$ 63° 26

- DATA

\text{x} FROM PHASE SHIFTS

$\sigma_{\text{CM}} \text{ BARNES/STERADIAN}$

$E_{\text{He}^3 \text{LAB MEV}}$

FIGURE 29
$\text{He}^4(\text{He}^3,\text{He}^3)\text{He}^4$

$\theta_{\text{CM}} = 90^\circ$

* DATA

* FROM PHASE SHIFTS

$\sigma_{\text{CM}}$ BARNS/SERADIAN

$E_{\text{He}^3_{\text{LAB}}}$ MEV

FIGURE 30
$\text{He}^4(\text{He}^3,\text{He}^3)\text{He}^4$

$\theta_{\text{CM}} = 109^\circ 52.5'$

- DATA
- FROM PHASE SHIFTS

**Figure 31**
\[ \text{He}^4 (\text{He}^3, \text{He}^3) \text{He}^4 \]

\[ \theta_{CM} = 125^\circ 16' \]

DATA FROM PHASE SHIFTS

\( \sigma_{CM} \) BARN/S/STERADIAN

\( E_{\text{He}^3 \text{LAB}} \) MEV

FIGURE 33
(Ch 57). $\Delta E_g$ is the energy loss in the gas from the 90° magnet to the target region of the chamber at its center. $K$ has been determined from data supplied by the 180° annular magnet (Yo58), (Sp58). The shim and slit settings used by the annular magnet have also been used by the scattering chamber. The probable error in energy at 5.17 mev, is estimated to be $\pm$ 20 kev. The principle source of this error is the value of $\Delta E_I$.

C The Phase Shift Analysis

The Phase shifts obtained from the 2.974 and 3.877 mev angular distributions are shown in the headings of Figures 26 and 27 respectively. The method of analysis of the excitation curves was as follows. First the data points were plotted. A smooth curve was drawn through the experimental points, and angular distributions were formed from these smoothed excitation curves at convenient intervals. It was these angular distributions which were fitted to (III-1) by the IBM 650 computer. When a rigorous least squares analysis of these data was done, it was found that the non-resonant phase shifts were not smooth in going across the resonance. Smooth curves were drawn through the non-resonant phase shifts, more weight being given to those points at the bottom of the resonance, and at the top, where the cross sections were not subject to quite so much error due to slight differences in the energy scales of the different excitation curves. These phases were then used as starting points and only $\delta_3^+$ was allowed to vary on the final fit. This final fit is shown in Figure 34.

D Theoretical Interpretation of the $He^4 (He^3, He^3) He^4$ Phase Shifts.

S. Wave: The S wave phase shift is shown on an expanded scale in Figure 35 together with the hard-sphere phase shift from a
$^{4}\text{He}(^{3}\text{He},^{3}\text{He})^{4}\text{He}$

PHASE SHIFTS

FIGURE 34
radius of 2.75 F. This radius is anomalously small, just as in
the cases of the p + He⁴, and He⁴ + He⁴ (Ru56) interactions, and
might be interpreted as due to a high lying S state, or in a po-
tential model it might suggest a repulsive core for the He³ + He⁴
interaction. The hard sphere phases for radii of 2.2 F., and of
4.2 F. are shown at 2.974 mev for comparison. The positive inflection
of the high energy S-wave phase shift is very possibly not real,
but might be a result of the smoothing of the non-resonant phase
shifts in the neighborhood of the resonance, as outlined in the
previous section.

P-Waves: The P-Wave phase shifts are shown in Figure 36
together with hard sphere phase shifts for radii of 4.2 F., and
3.5 F. The splitting of δ₉⁺ above the 2.974 mev angular distribu-
tion is definitely real, and is not a property of the method of
analysis. The fact that the two curves are so parallel sug-
ests that this splitting is not due to higher P states, but is more
probably due to the ground state and first excited states of Be⁷.
This possibility was explored using the formalism of Thomas (Th52).

Since nothing is known about the reduced widths of the ground
state and the first excited state of Be⁷, it was assumed that the
reduced widths of these states were equal to the Wigner limit for
the assumed radius of 4.2 F. This reduced width is:

\[ \frac{3 \hbar^2}{2 \mu R} = 8.68 \text{ mev-F.} \]

The resonant portion of the phase shift is given by (III-2), and
is repeated here for reference, namely:

\[ \beta^{\pm}_{\lambda} = \frac{\Gamma_{\lambda}}{2} \frac{E_{\lambda} + \gamma \lambda - E_{cm}}{\hbar^2} \]
The definitions of the terms in this expression are the same as given in Chapter III, except that difficulty arises as to what to use for \( E_\lambda \). Thomas (Th52) has derived an expression for the level shift, \( \Delta_\lambda \),

\[
\Delta_\lambda = -k\gamma^2 \left( \frac{A_1}{A_2} + \frac{b_c}{c} \right).
\]

In order to be consistent with the notation of Wigner and Eisenbud, as given in (III - 3), we must use \( b_c = \sqrt{p} \). (Note that this is not the boundary condition which Thomas adopts. His condition is that the level shift be zero at the resonant energy, so that

\[
b_c = k\gamma^2 \left| \frac{A_1}{A_2} \right| \eta = E_{res}.
\]

The level shift for the case of states which are bound against particle emission is given by (Th52)

\[
\Delta_\lambda = -k\gamma^2 \left( \frac{W'}{W} + \frac{1}{p^2} \right),
\]

where \( W \) is the Whittaker function, \( W = W_{\eta,} + 1/2 (2p) \). Thomas, in the above paper has also shown that the WKB approximation for the Whittaker function gives good results in the ranges of interest for bound states. In the WKB approximation,

\[
\frac{W'}{W} = -\xi + 1/2 [ \rho \eta + (\lambda + 1/2)^2 ] \xi^{-2}, \text{ where} \nonumber \\
\xi = [(\lambda + 1/2)^2 + 2\rho \eta + \rho^2]^{1/2}.
\]

For the case of bound states, the quantities \( \rho, \eta, \) and \( k \) have the following definitions:

\[
(IV-7) \quad k = \sqrt{\left( \frac{2 \mu |E_{cm}|}{\hbar^2} \right)}
\]

\[
\eta = \frac{\mu Z_1 Z_2 e^2}{\hbar^2 k}
\]

\[
\rho = kR
\]
For the ground state $E_{cm} = -1.583$, and for the first excited state of Be$^7$, $E_{cm} = -1.153$. When the level shift is evaluated by formulas (IV-5), (IV-6), and (IV-7), the level shifts at resonance are

$$\Delta_{gs} = +2.704 \text{ mev},$$
$$\Delta_{fes} = +2.392,$$

where the subscript "gs" refers to the ground state, and "fes" refers to the first excited state. Using this result to make the denominator of (IV-3) vanish, leads to:

$$E_{gs} + 2.704 + 1.583 = 0 \text{ or } E_{gs} = -4.287,$$
$$E_{fes} + 2.392 + 1.153 = 0 \text{ or } E_{fes} = -3.545.$$

These values of $E_\lambda$ were used, and the expression for $\Delta_\lambda$ appropriate to the continuum was used with (IV-3), and the values of $\beta_1^\pm$ were computed at 5.0 mev bombarding energy; the results are:

$$\beta_1^+ = 164^\circ 12', \text{ and}$$
$$\beta_1^- = 162^\circ 45'. $$

This disagreement with the experimental results could be resolved by choosing a smaller interaction radius for the He$^3 +$He$^-$ system. The size of the difference between these two phase shifts is about 1.5$^\circ$ as compared to 3.5$^\circ$ splitting for the P-wave phase. It is interesting to note, however, that the sign of the splitting is correct, and that the disagreement in the splitting could be resolved by the assumption of a width which was smaller than the Wigner limit for the P1/2 state. With the accuracy of the present data however, it is doubtful whether very accurate reduced widths for the ground and first excited states could be determined. Two recent experiments should help clarify further analysis of the P-wave phase shifts. In one experiment, the total non-resonant cap-
ture cross section, and branching ratio for the $^{He}_3(a,\gamma)^{Be}_7$ reaction was measured (Ho58). In the other experiment, the lifetime of the first excited state of the mirror nucleus $^{Li}_7$ was measured by using resonance fluorescence techniques (Sw58).

D Waves: The D-wave phase shifts are shown in Figure 37 on an expanded scale. The solid curve is the hard sphere phase shift resulting from a radius of 4.2 $F$. An attempt was made to analyze the D3/2 phase shift in terms of hard sphere scattering, and a resonant contribution from the $6.46$ mev, 3/2$^+$ state investigated by Marion (Ma56), and by Bashkin and Richards (Ba51). The contribution to the center of mass width $\Gamma_\lambda$ due to the $^{Li}_6 + p$ channel is zero below its threshold, (which would occur for $^{He}_3 + ^{He}_4$ at 7.044 mev laboratory bombarding energy). The level shift, however, is the sum of the level shift due to the $^{Li}_6 + p$ channel, and the shift due to the $^{He}_3 + ^{He}_4$ channel. As a first approximation it was decided to ignore the level shift due to the $^{He}_3 + ^{He}_4$ channel, since it was known that the 6.46 state has a reduced width which is a large fraction of the Wigner limit for the $^{Li}_6 + p$ channel (Ba51). The level shifts due to the $^{Li}_6 + p$ channel, at energies corresponding to $^{He}_3$ bombarding energies of 4.5 and 5.5 mev, were calculated by (IV-6). The denominator of (IV-3) was thus determined, using the parameters $\xi_\lambda = 6.4$ mev, and $\gamma^2_p = 9$ mev-$F$, as given by Bashkin and Richards (Ba51). The $^{He}_3 + ^{He}_4$ reduced width was then solved for from the experimental value of $\beta^2_2 = 8^0$ at 5.5 mev. The resulting reduced width, $\gamma_{He_3}$, is 3.6 mev-$F$. This value seems very large considering the known single particle $^{Li}_6 + p$ configuration for the 6.46 mev state, but the inherent inaccuracies of the data of the $^{Li}_6 + p$ experiment, and the small
He$^4$(He$^3$,He$^3$)He$^4$

- $\phi_2$ (R = 4.2 F.)

$\delta_2^+$

$\delta_2^-$

$\Delta$ - $\phi_2^+ \beta_2^-$ (R = 4.2 F., $\gamma_{He^3} = 3.6$ MEV)

$E_{He^3_{LAB}}$ MEV

FIGURE 37
size of the D-wave phase shifts in the present experiment must be
considered. It would not be inconsistent with the present experi-
mental data for the reduced width to be a factor of ten smaller.
However, the splitting of the D-wave phase shifts as shown in Figure
36 is qualitatively consistent with the broad 3/2^+ state at an
excitation energy of 6.46 mev. The D-wave phase shift is so small,
at even the highest bombarding energies of the present experiment,
that, perhaps, nothing quantitative may be said about the reduced
width of this state for the emission of He^3 + He^4.

F Waves: The F 7/2 phase shift is shown in Figure 38. The
best dispersion theory fit which has been obtained is also shown.
The level parameters which have been extracted are: R = 4.4 F,
\gamma_{He^3}^2 = 3.0 mev F., E_\lambda = 3.433 mev, and \theta_{He^3}^2 = .362. The last par-
parameter is the ratio of the reduced width of the state to the
Wigner limit. It is perhaps remarkable that this state which
agrees in spin, parity, and energy with the shell model prediction
of Inglis (In53) should be such a good single particle state of He^3 +
He^4.

E Spin Polarizations of Scattered He^3 Nuclei.

Figure 38 shows a contour map of the per cent spin polariza-
tion for the scattered He^3 nuclei as a function of center of mass
angle, and as a function of bombarding energy for the energy range
of the present experiment. The per cent spin polarizations are
again seen to be high in the neighborhood of the observed resonance,
and particularly they are high in the region of low cross sections
as in the He^4 (p,p)He^4 experiment.
\[ \text{He}^4(\text{He}^3,\text{He}^3)\text{He}^4 \]
\[ \delta_3^+ \]
\[ R = 4.4 \text{ F.} \]
\[ \gamma_{\text{He}^3}^2 = 3.0 \text{ MEV.F.} \]
\[ \theta_{\text{He}^3}^2 = .362 \]
\[ E_{\text{R LAB}} = 5.17 \text{ MEV} \]

* EXPERIMENT

**FIGURE 38**
V SUMMARY AND CONCLUSIONS

Two problems, concerning the elastic scattering of spin 1/2, charged particles from He$^4$, have been investigated.

The compound nucleus Li$^5$ was investigated experimentally by the elastic scattering of protons from He$^4$ in the energy range from 3.0 to 5.5 mev. Excitation curves at bombarding energies from 3.0 to 5.5 mev show no unexpected anomalies. Angular distributions, taken every 500 kev from 3.0 to 5.0 mev, were phase shift analyzed, and phase shift analyses were taken from the literature. A fairly complete picture of the broad structure of the S- and P-wave phase shifts now exists from 1 through 18 mev.

The S-wave phase shift was interpreted in terms of the phase shift due to a hard charged sphere of radius 1.99 Fermis.

The P-wave phase shifts were interpreted in terms of the Wigner and Eisenbud dispersion theory. The level parameters extracted for the ground and first excited states of Li$^5$ are:

For the ground state,

$E_{\text{res lab}} = 2.62$ mev,

$\gamma_p^2 = 12$ mev·Fermis,

$\Theta_p^2 = 0.40$, and

$J = 3/2^-$. 
For the first excited state,

\[ E_{\text{res lab}} = 10.75 \text{ mev}, \]
\[ \gamma_p^2 = 30 \text{ mev} \cdot \text{Fermis}, \]
\[ \theta_p^2 = 1.00, \text{ and} \]
\[ J = \frac{1}{2}^- . \]

The above parameters were derived using an interaction radius of 2.60 Fermis.

The situation concerning the D-wave phase shifts is not at all clear. However, good resolution \( \text{He}^4(p,p)\text{He}^4 \) experiments above 10 mev would help clarify their behaviour.

The 16.80 mev state in \( \text{Li}^5 \) has been investigated by the \( \text{He}^3(d,p)\text{He}^4 \) reaction, (Bo52), but the state has never been observed by the elastic scattering of protons from \( \text{He}^4 \). The level parameters for this state were determined by the above authors to be:

\[ \gamma_d^2 = 5.0 \text{ mev} \cdot \text{Fermis}, \]
\[ \theta_d = .21, \]
\[ \gamma_p^2 = .097 \text{ mev} \cdot \text{Fermis}, \]
\[ \theta_p^2 = .004, \text{ and} \]
\[ J = \frac{3}{2}^+. \]

Thus the three lowest known states of \( \text{Li}^5 \), covering an excitation energy range of nearly 17 mev, are all very good single particle states. The 16.80 mev state is very well described by almost pure \( d + \text{He}^3 \) parentage, while the ground and first excited states of \( \text{Li}^5 \) are very well described by almost pure \( \text{He}^4 + p \) parentage.
The compound nucleus $^7\text{Be}$ was investigated by the elastic scattering of $^3\text{He}$ from $^4\text{He}$. The phase shift analysis of the data determines the second excited state of $^7\text{Be}$ to have the following parameters:

$$E_{\text{res \ lab}} = 5.17 \text{ mev},$$

$$\gamma_{^3\text{He}} = 3.0 \text{ mev.Fermis},$$

$$\theta_{^3\text{He}} = .362, \text{ and}$$

$$J = 7/2^{-}.$$ 

The F-wave interaction radius used was $4.4$ Fermis.

The P- and D-wave phase shifts show splittings which are in qualitative agreement with states in $^7\text{Be}$ which lie outside the energy range of the present experiment.

The S-wave phase shift was fit to a hard sphere radius of $2.75$ Fermis. This hard sphere radius for S waves is again anomalously small.

It seems very significant that in the three systems: $p + ^4\text{He}$, $^3\text{He} + ^4\text{He}$, and $^4\text{He} + ^4\text{He}$, the S-wave phase shift always implies a very small interaction radius. On a potential model, these small radii would imply a less attractive well for $l = 0$, which would imply a repulsive core. The case of $^7\text{Be}$ would be particularly favorable for investigation as both the S, and P waves seem to have small radii, the P-wave radius being intermediate between the S- and D-wave radii.

In conclusion, the following additional, related problems would bear investigation.

1. Enough data is now available to make a new investigation
of potential well calculations for the $p + \text{He}^4$ and $n + \text{He}^4$ systems very desirable. These calculations could be done in a manner similar to those of Sack, Biedenharn, and Breit (Sa54).

2. The splitting and magnitude of the P-wave phase shifts for the $\text{He}^3 + \text{He}^4$ system, together with the $\text{He}^3(\alpha, \gamma)\text{Be}^7$ non-resonant capture experiments, should give moderately accurate reduced widths for the ground and first excited states of $\text{Be}^7$.

3. The $\text{He}^4(t, t)\text{He}^4$ experiment should be done at high enough energies to observe the state in $\text{Li}^7$, which is presumably the mirror to the 4.53 mev, F7/2 state in $\text{Be}^7$. Since the laboratory bombarding energy at resonance would be only 3.77 mev, the region above the 4.61 mev state in $\text{Li}^7$ could be investigated, and the existence of the very doubtful 5.5 mev state could be confirmed or denied.

4. Finally, the fundamental question, of what causes an aggregate of several nucleons to assume different two particle configurations at different energies needs investigation.
VI APPENDIX A

SPIN POLARIZATION OF ELASTICALLY SCATTERED, SPIN 1/2 PARTICLES ON SPIN 0 NUCLEI

The asymptotic form of the scattered wave, for the case of spin 1/2 charged particles on spin 0 nuclei, with LS coupling present, has been given by Critchfield and Dodder (Cr49). For the case of an incident beam with spin along the z-axis,

\[ \psi_{\text{scatt}}^{1/2} \sim e^{i(kr - \eta \ln 2kr)} \left[ f_c(\theta) \kappa^+ + e^{i\Phi} f_1(\theta) \kappa^- \right] \]

\[ = e^{i(kr - \eta \ln 2kr)} A_\frac{1}{2}. \]

\( \psi_{\text{scatt}} \), and \( f_1(\theta) \) are defined in (III-1); \( \Phi \) is the azimuthal angle, and the \( \kappa^\pm \) are normalized spin 1/2 functions relative to a z-axis defined by the incident beam. (Note, that (III-1) ought to include the phase factor \( e^{i\Phi} \) in the definition of \( f_1(\theta) \), but, since the differential cross section contains only \( |f_1(\theta)|^2 \), the phase factor was dropped as is customary.) Similarly, for the case of an incident beam with spin against the z-axis,

\[ \psi_{\text{scatt}}^{-1/2} \sim e^{i(kr - \eta \ln 2kr)} \left[ f_c(\theta) \kappa^- - e^{-i\Phi} f_1(\theta) \kappa^+ \right] \]

\[ = e^{i(kr - \eta \ln 2kr)} A_{-\frac{1}{2}}. \]

For an unpolarized incident beam, the cross section for scattering with spin along the x-axis is given by,

\[ \frac{1}{2} \left[ \langle A_{\frac{1}{2}} | \sigma_x | A_{\frac{1}{2}} \rangle + \langle A_{-\frac{1}{2}} | \sigma_x | A_{-\frac{1}{2}} \rangle \right]. \]

Finally, the per cent spin polarization along the x-axis, for
particles scattered in the \((\theta, \Phi)\) direction, is obtained by dividing the cross section for scattering with spin along the \(x\)-axis, by the cross section for scattering.

The effect of the spin operator \(\sigma_x\) is to interchange the two spin functions characteristic of spin along and against the \(z\)-axis:

\[
(A-4) \quad \sigma_x^z \chi^\pm = \chi^\mp.
\]

Using \((A-1), (A-2), (A-3), (A-4)\), and the definition of the spin polarization given above leads to

\[
(A-5) \quad P_x = \frac{\left[ f_c^* \chi^+ + e^{-i\Phi_f} f_1^* \chi^- \right] \left( f_c \chi^- - e^{i\Phi_f} f_1 \chi^+ \right)}{|f_d|^2 + |f_1|^2},
\]

or, using the orthogonality property of the spin functions,

\[
(A-6) \quad P_x = \frac{2\left[ \Re f_1 \Im f_c - \Re f_c \Im f_1 \right] \sin \Phi}{\sigma(\theta)}.
\]
Programming of the Spin 1/2

Problem for Computation Using an IBM 650 Computer

A program is described for fitting angular distributions of elastically scattered, charged particles on an IBM 650 computer in the case of channel spin 1/2. Two options are available within the program. Either, (a), data may be least squares fitted from initial guesses of the phase shifts, or, (b), angular distributions may be calculated from given phase shifts. In either case, the first five partial waves are taken into account. Angular distributions with as many as forty data points may be processed.

The formula for the elastic scattering cross section for charged particles of spin 1/2, in terms of the phase shifts is given in (III-1), and is repeated here for reference:

\[
\frac{d\sigma}{d\Omega} = \left| f_c \right|^2 + \left| f_1 \right|^2
\]

where,

\[
k_f = -\left( \eta/2 \right) \csc^2(\theta/2) \sin \ln \csc^2(\theta/2)
\]

\[
\text{B-2)} \quad k_f = -\left( \eta/2 \right) \csc^2(\theta/2) \sin \ln \csc^2(\theta/2)
\]

\[
+ \sum_{l=0}^{\infty} e^{i\Delta_l} P_l[\cos \theta] \left[ (l+1)e^{i\delta^+} \sin \delta^+ + \right.  
\]

\[
\left. \left. \left( l^+ \right) \sin \delta^- \right] P_l \left( \cos \theta \right) \sin \theta.
\]

39
In the above formulae,
\[ \eta = \frac{Z_1 Z_2 e^2}{\hbar \nu} \]
\[ a_0 = 0 \]
\[ a_1 = 2 \sum_{s=1}^{8} \tan^{-1}(\eta/s) \]
\[ k = \mu v/\hbar \]

\[ \theta \] is the center of mass angle of the scattered particle with respect to the incident beam.

This problem has been programmed for an IBM 650 computer for the first five partial waves, and the general features of this code will follow. The problem may be broken down into two types of desired operations. First, it is desired to compute cross sections and spin polarizations from given phase shifts. Secondly, it is desired to, in some sense, fit the formula to experimental angular distributions in order to derive phase shifts from experimental data.

The criterion for goodness of fit used here, is to minimize the following expression.

\[ \text{ERROR} = \sum_{\theta} \frac{(\sigma_{\text{exp}} - \sigma_{\text{phases}})^2}{\nu(\theta) \sigma_{\text{exp}}} \]

\( v \) is a statistical weight factor and is used as follows: If all of the cross sections from one angular distribution are obtained from the same quantity of integrated charge, then, aside from center of mass corrections, and target thickness corrections, all values of \( v \) should should be the same. For the analysis of data where an effort has been made to keep approximately the same statistics for all points, the \( \sigma_{\text{exp}} \) in the denominator of the above expression should be removed, and provision has been made for this. In (B-5),

40
and in equations to follow, underlined expressions such as ERROR are the symbolic locations of the corresponding expressions in the code itself. The method of minimization of ERROR is as follows: Initial guesses of phases are put into the computer, and a number (DLTCT), is put in whose digits specify the order in which phases, are to be inspected to find minima in ERROR. The increment, (DEL), with which the phases are to be adjusted is built into the code to be one degree, but of course, it is easily modified. The course of the problem, in the event that a least squares analysis is called for, is then to take the digits of DLTCT in order, and to increment the corresponding phase shift by DEL, first in the positive direction, then in the negative direction until a minimum is found. At the minimum a set of phase shifts is punched out, together with ERROR, the next digit of DLTCT is taken, and the process repeated with the new phase shift. The 650 being a ten digit machine, this process may be repeated up to ten times without interrupting the machine.

In order to put (B-2) and (B-3) into a form amenable to calculation, they are separated into real and imaginary parts as follows:

\[ \text{Re}(k_f c) = RKFC = (-\eta/2)\csc^2(\theta/2) \cos[\eta \ln \csc^2(\theta/2)] \]

\[ \text{Re}(k_f c) = RKFC = (-\eta/2)\csc^2(\theta/2) \cos[\eta \ln \csc^2(\theta/2)] + \sum_{l=0}^{4} F_k (\cos \theta) [\sin \delta^+_l (l + 1)(\cos \alpha \cos \delta^+_l - \sin \alpha \sin \delta^+_l) \]

\[ + l \sin \delta^-_l (\cos \alpha \cos \delta^-_l - \sin \alpha \sin \delta^-_l)], \]
\[ \text{Im}(k_f) = \text{IKFC} = (-\eta/2) \csc^2(\theta/2) \sin[\eta \ln \csc^2(\theta/2)] \]

\[ + \sum_{l=0}^{L} P_l(\cos\theta)[\sin\delta_l^+(1 + 1)(\sin a_l \cos\delta_l^+ + \cos a_l \sin\delta_l^-) + \sin\delta_l^-(\sin a_l \cos\delta_l^- + \cos a_l \sin\delta_l^-)], \]

\[ \text{Re}(k_f) = \text{RKFC} = \sum_{l=1}^{L} P_l'(\cos\theta) \sin\theta[\sin\delta_l^+(\sin a_l \cos\delta_l^- - \sin a_l \sin\delta_l^-) - \sin\delta_l^-(\cos a_l \cos\delta_l^- - \sin a_l \sin\delta_l^-)], \]

\[ \text{Im}(k_f) = \text{IKFC} = \sum_{l=1}^{L} P_l'(\cos\theta) \sin\theta[\sin\delta_l^+(\sin a_l \cos\delta_l^- + \cos a_l \sin\delta_l^+) - \sin\delta_l^-(\sin a_l \cos\delta_l^- - \cos a_l \sin\delta_l^-)]. \]

The coefficients of the Legendre polynomials and their derivatives in the above expressions are evaluated within a subroutine, (entrance DCOEP), since the need for them arises in several different parts of the problem.

The only other quantity computed which is of interest is the spin polarization in the x direction, and this is punched out when calculated cross sections are punched.

As can be seen from the above formulas, the problem is very easy to scale, so it was decided to use IBM's Symbolic Optimum Assembly Program (SOAP II), and to do the calculation in fixed point arithmetic.

To clarify the problem, the input output will next be described. Card 1 of the input is the fixed data card, (FDC):

Word1 = CNTRL. The first nine digits of this control are irrelevant. If the tenth digit is an eight, the process is a least squares analysis. If it is a nine, the course of the problem is to punch out an angular distribution. Any other digit results in an error stop of the machine.
Word 2 = \( Z_1 \), the charge number of the incident particle, in the form \((2,8)\). The notation, \((a,b)\), means \(a\) digits before the decimal point, and \(b\) digits after. For example if the incident particle were a proton, \(Z_1\) would equal 01\(00000000\). By using \(Z_1=0000000000\), the code may be used to fit neutron data with no difficulty.

Word 3 = \( Z_2 \), the charge number of the target nucleus, also in the form \((2,8)\).

Word 4 = \( M_1 \), the mass of the incident particle in atomic mass units in the form \((1,9)\).

Word 5 = \( DLTCT \), the sequence of phases to be "wiggled" as described above. The correspondence between digits and phases is as follows.

\[
\begin{align*}
0 &= 0 \\
1 &= 1^+ \\
2 &= 2^+ \\
3 &= 3^+ \\
4 &= 4^+ \\
5 &= 5^- \\
6 &= 6^- \\
7 &= 7^- \\
8 &= 8^- \\
\end{align*}
\]

For example, the word \( DLTCT = 1015381267 \) would indicate, first "wiggles" \( 8^+ \), then \( 0^- \); then \( 1^+ \), etc. Note that the digit nine is never used.

Word 6 = \( M_2 \), the mass of the target nucleus in atomic mass units in the form \((2,8)\). Words 7, and 8 are not used, and need not.
be filled in.

The second input card is the \(N\)' card, which specifies the number of \(\theta\) cards to follow, that is, it specifies the number of points on the angular distribution to be punched out or fit. This card contains only word 1, and that is \(N\) in the form \((6,4)\).

Following the \(N\) card in the data deck are \(N\), \(\theta\) cards, whose format is:

Word 1 = \textit{Theta}, the center of mass angle in degrees and decimal fractions thereof in the form \((3,7)\).

Word 2 = \textit{SIGEX}, the experimental center of mass cross section in barns per steradian in the form \((1,9)\).

Word 3 = \textit{NU}, the statistical weight assigned to this particular data point in the form \((2,8)\). The permissible range of \(NU\) is \(1 \leq NU < 100\). As can be seen from \((5)\), cross sections with small \(NU\) are heavily weighted, while those with large \(NU\) are less heavily weighted. In practice it has been found convenient to use values of \(NU\) of around 0.50000000.

Words 4 through 8 of the 8 cards are unused by the computer, and may be left blank. Usually though, the energy has been filled in, in word 4 simply to identify the card completely.

Following the \(N\), \(\theta\) cards in the data deck are the two \(\delta\) cards. The format of the two is similar. In both cards,

Word 1 = \textit{SERAL}, a serial number. In the event of a least squares analysis, the output is further pairs of \(\delta\) cards which may be then used as future input \(\delta\) cards for further fitting of an angular distribution. In each succeeding pair of cards, one in the last digit is added to \textit{SERAL}. One caution, the last two digits
must not go over 99 within the machine. (This is a property of the
FOUR system, an interpretive system which is used for many of the
logical operations in the code. The other eight digits of SERAL
are arbitrary and may be filled in with any convenient numbers such
as the date of the run etc. In the first 8 card,

Word 2 = EI, the laboratory bombarding energy in mev in the form
(2,3). In the second 8 card,

Word 2 = ERROR, when the card is an output card. When it is an
input card, this word is unused. In the first card,

Word 3 = J0001 = $\delta_0$, the S-wave phase shift in degrees and
decimal fractions thereof in the form (3,7).

Word 3 of the second card is unused, and is usually set to
ZERO, this being its contents when it is an output card. In the
first card,

Word 4 = J0002 = $\delta_1^+$ Degrees (3,7)
Word 5 = J0003 = $\delta_2^+$ " "
Word 6 = J0004 = $\delta_3^-$ " "
Word 7 = J0005 = $\delta_4^-$ " "

When the 8 cards are output cards, Word 8 of both cards is LAMDA,
the digit corresponding to the phase shift which has just been
"wiggled", in the form (6,4). In the second card,

Word 4 = J0006 = $\delta_1^-$ degrees (3,7)
Word 5 = J0007 = $\delta_2^-$ " "
Word 6 = J0008 = $\delta_3^-$ " "
Word 7 = J0009 = $\delta_4^-$ " "

This completes the description of the data deck except
for a word about stacking successive angular distributions.
When the course of the problem is to be one or more least squares analyses, after the tenth $\delta$ is "wiggled", control of the problem is returned to START, where an instruction is given to read a fixed data card. After the last output card has been punched in the construction of an angular distribution, ($\text{CNTRL} = 0000000009$), the control of the problem is returned to the point where another pair of $\delta$ cards is read in, so that another angular distribution at a new energy and/or set of phases is punched at the same set of angles. This is convenient for punching excitation curves, where $N = 0000010000$, is followed by one $\theta$ card, and then a series of pairs of $\delta$ cards.

The output when $\text{CNTRL} = 0000000008$ has already been described. The output when $\text{CNTRL} = 0000000009$ is as follows.

Word 1 = E1 mev (2,8)
Word 2 = THETA degrees (3,7)
Word 3 = RKFC (2,8)
Word 4 = IKFC (2,8)
Word 5 = RKFI (2,8)
Word 6 = IKFI (2,8)
Word 7 = SIGMA barns per steradian (1,9)
Word 8 = SP, the per cent spin polarization along the x axis in the form (3,7). The spin polarization is given by,

\[
\text{(B-10)} \quad \text{SP} = \frac{2[\ (\text{RKFI})(\text{IKFC}) - (\text{IKFI})(\text{RKFC})]}{(\text{RKFI})^2 + (\text{IKFI})^2 + (\text{RKFC})^2 + (\text{IKFC})^2}
\]

The denominator of the above expression has the symbolic name $\text{SUM}$, and is just $k^2\sigma(\theta)$.

The flow diagram of the problem is given in Figure 39. Notice that the code resides in four blocks. The block numbers are in time order, however, this is just the opposite of the order so that best
NOTE TO USERS

Oversize maps and charts are microfilmed in sections in the following manner:

LEFT TO RIGHT, TOP TO BOTTOM, WITH SMALL OVERLAPS

This reproduction is the best copy available.

UMI
optimization of the most frequently used parts might be obtained. 
The notation on the flow diagram together with the SOAP listing is fairly self-explanatory.

In case modification of the code is desired, an edition is available with \( N_{\text{max}} = 10 \), which has space reserved for the Shell Laboratory SAM package. This routine is a general purpose debugging aid which includes provision for selective tracing either with or without tracing of subroutines.
VIII  APPENDIX C

COINCIDENCE SLIT CALCULATIONS, CENTER OF MASS CONVERSION, AND
MACHINE COMPUTATION OF SAME

The transformation between laboratory and center of mass coordinates is most easily seen by reference to Figure 40.

Figure 40

The subscripts "cm" refer to center of mass quantities, and the subscripts "l" refer to laboratory quantities. 1 refers to the bombarding particle, and 2 refers to the recoil particle. The definition of the center of mass coordinate is,

\[
(C-1) \quad R_{cm} = \frac{m_1 r_1 + m_2 r_2}{m_1 + m_2}.
\]

Differentiating \( R_{cm} \) with respect to the time, and letting the target nucleus be initially stationary, \( \dot{r}_2 = 0 \), gives:

\[
(C-2) \quad v_{cm} = \frac{m_1 v_b}{m_1 + m_2}, \quad \text{and} \quad v_{1cm} = v_b - v_{cm} = \frac{m_2 v_b}{m_1 + m_2}.
\]
From Figure 40,

\[ v_{1\text{cm}} \cos \varphi_{\text{cm}} + V_{\text{cm}} = v_{1\text{i}} \cos \varphi_{\text{i}}, \text{ and} \]

\[ v_{1\text{cm}} \sin \varphi_{\text{cm}} = v_{1\text{i}} \sin \varphi_{\text{i}}. \]

Dividing (C-4) by (C-3), and using (C-2), gives the relation between between \( \varphi_{\text{i}} \) and \( \varphi_{\text{cm}} \),

\[ \varphi_{\text{i}} = \tan^{-1} \frac{\sin \varphi_{\text{cm}}}{\gamma + \cos \varphi_{\text{cm}}}, \]

where \( \gamma = m_{1}/m_{2} \). By definition of the center of mass,

\[ \theta_{\text{cm}} = \pi - \varphi_{\text{cm}}. \]

Since, the sum of the linear momenta of the two particles with respect to the center of mass must be zero,

\[ m_{1}v_{1\text{cm}} = m_{2}v_{2\text{cm}}, \text{ or} \]

\[ v_{2\text{cm}} = \gamma v_{1\text{cm}}. \]

It can now be seen from (C-2) and (C-7), that,

\[ v_{2\text{cm}} = v_{\text{cm}} = \frac{m_{1}v_{1\text{cm}}}{m_{1} + m_{2}}. \]

Reference to Figure 40 now shows that,

\[ \theta_{\text{i}} = \theta_{\text{cm}}/2. \]

For the purpose of doing coincidence experiments, such as the \( ^{4}\text{He} \left(^{3}\text{He}^{3}, ^{3}\text{He}\right)^{4}\text{He} \) experiment, it is very desirable to know the fraction of the bombarding energy carried off by the scattered particle, and the fraction carried off by the recoil particle. Applying the law of cosines to the lower triangle in Figure 40 gives,

\[ v_{2\text{cm}}^{2} = v_{\text{cm}}^{2} + v_{2\text{i}}^{2} - 2v_{\text{cm}}v_{2\text{i}} \cos \theta_{\text{i}}. \]
(C-2), and (C-7) may then be substituted in (C-10) to give,

\[ v_{2L} = \frac{2m_1 v_b \cos \theta_2}{m_1 + m_2}, \]

or in terms of energies,

\[ \frac{E_2}{E_b} = \frac{4m_1 m_2 \cos^2 \theta_2}{(m_1 + m_2)^2}. \]

By conservation of energy, the fraction of the bombarding energy retained by the scattered particle is,

\[ \frac{E_1}{E_b} = 1 - \frac{E_2}{E_b}. \]

Since the number of particles scattered into a differential solid angle must be independent of whether that solid angle is expressed in the center of mass or laboratory system of coordinates,

\[ \sigma_{cm}(\phi) \sin \varphi_{cm} \, d\varphi_{cm} = \sigma_{i}(\phi) \sin \varphi_{i} \, d\varphi_{i}, \]

or

\[ \sigma_{cm} = \frac{\sin \varphi_{i}}{\sin \varphi_{cm}} \frac{d\varphi_{i}}{d\varphi_{cm}} \sigma_{i} = C(\phi) \sigma_{i}. \]

(C-14) and (C-5) together give for \( C(\phi) \),

\[ C(\phi) = \frac{1 + \gamma \cos \varphi_{cm}}{(1 + 2\gamma \cos \varphi_{cm} + \gamma^2)^{3/2}}. \]

It was pointed out in Chapter IV, that it is sometimes necessary to infer a center of mass cross section from the observation of recoil particles, counted in detector number one, in coincidence with scattered particles in detector number two. For this situation,

\[ C(\theta) = \frac{\sin \theta_{\perp}}{\sin \theta_{cm}} \frac{d\theta_{\perp}}{d\theta_{cm}}, \]

but since \( \theta_{\perp} = \theta_{cm}/2 \),

\[ C(\theta) = \frac{1}{4 \cos \theta_{\perp}}. \]
Consider now, the problem of how wide the slits on the
number two detector should be, in order to collect all of the
of the particles in the second detector, corresponding to those
entering the first detector.

\[ \Delta \theta_1 = \frac{w_{f1} + w_{b1}}{S_1}, \text{ and} \]
\[ t \sin \theta_1 = \left[ R_1 - S_1 + \frac{w_{f1} S_1}{w_{f1} + w_{b1}} \right] \Delta \theta_1, \]
or,
\[ w_2 = \left( \frac{w_{f1} + w_{b1}}{S_1} \right) \left[ R_2 \frac{\partial \theta_2}{\partial \theta_1} + \frac{\sin \theta_2}{\sin \theta_1} ( R_1 - S_1 + \frac{S_1 w_{f1}}{w_{f1} + w_{b1}} ) \right]. \]

For the case where the scattered particle is detected in counter
number 1,
\[ \frac{\partial \theta_2}{\partial \theta_1} = \frac{1 + \gamma^2 - 2 \gamma \cos \theta_i}{2(1 - \gamma \cos \theta_i)} \]
so that combining (C-19), and (C-20)

\[(C-21) \quad W_{2m1} = \left( \frac{W_{r1} + W_{b1}}{S_1} \right) \left[ \frac{R_2(1 + \gamma - 2\gamma \cos 2\theta_k)}{2(1 - \gamma \cos 2\theta_k)} \right] \left\{ \sin \theta_k \left( R_1 - S_1 + \frac{S_1 W_{r1}}{W_{r1} + W_{b1}} \right) \right\} \cdot \sin \vec{\phi}_k \left( R_1 - S_1 + \frac{S_1 W_{r1}}{W_{r1} + W_{b1}} \right). \]

For the case of the recoil particles in detector number one,

\[(C-22) \quad W_{2m2} = \left( \frac{W_{r1} + W_{b1}}{S_1} \right) \left[ \frac{2R_2(1 - \gamma \cos 2\theta_k)}{1 + \gamma^2 - 2\gamma \cos 2\theta_k} \right] \left\{ \sin \theta_k \left( R_1 - S_1 + \frac{S_1 W_{r1}}{W_{r1} + W_{b1}} \right) \right\}. \]

The above relations were programmed for an IBM 650 computer using the Bell Laboratory Interpretive System. The card format is the standard Bell card form (Ga57). The first input card loads into locations 301 through 307 in the memory, and it contains the following information:

- 301 = m_1 (AMU),
- 302 = m_2 (AMU),
- 303 = R_1 (cm.),
- 304 = S_1 (cm.),
- 305 = W_{r1} (cm.),
- 306 = W_{b1} (cm.),
- 307 = R_2 (cm.).

Following this input card are as many cards as desired, having \( \phi_{cm} \) in degrees to load into memory location 401.
The output format is as follows:

**Card 1**

Word 1 = $\varphi_{cm}$ degrees

Word 2 = $\varphi_i$ "

Word 3 = $\theta_{cm}$ "

Word 4 = $\theta_i$ "

Word 5 = $E_1/E_b$ (C-12)

Word 6 = $E_2/E_b$ (C-13)

Word 7 = $\varphi_{cm}$ degrees

**Card 2**

Word 1 = $\varphi_{cm}$ degrees

Word 2 = $\varphi_i$ "

Word 3 = $\theta_i$ "

Word 4 = $C(\phi)$ (C-15)

Word 5 = $W_{2m1}$ cm. (C-21)

Word 6 = $\varphi_{cm}$ degrees

**Card 3**

Word 1 = $\varphi_{cm}$ degrees

Word 2 = $\theta_i$ "

Word 3 = $\varphi_i$ "

Word 4 = $C(\theta)$ (C-17)

Word 5 = $W_{2m2}$ cm. (C-22)

Word 6 = $\varphi_{cm}$ degrees
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