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STUDY OF MODE LOCKING PROCESS
AND
TIME TAILORING OF SHORT PULSES

By

Shreehari G. Marathe

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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TIME TAILORING OF SHORT PULSES

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ABSTRACT

In this thesis, the analysis of mode locking process is formulated in terms of an eigenequation for the pulse envelope. The solutions of the eigenequation are found to be an infinite, denumerable set of supermodes, consisting of Hermite polynomials modulated by a Gaussian envelope. The kinetics of pulse evolution from spontaneous noise is discussed in close analogy with the quantum statistical description. With the understanding of mode locked pulses, a new method to time tailor short pulses is presented. The method consists of the realization of a recursive filter by means of a suitable combination of high quality mirrors of different reflectivities. An algorithm to design a recursive filter for transforming a given pulse into a desired pulse is presented. As an example, a three mirror filter is designed to tailor a Gaussian pulse to a pulse required for efficient laser induced controlled thermonuclear fusion.
"You must remember what Schiller said about Kant and his interpreters:

'When kings go a-building, wagoners have more work.'

At first none of us are anything but wagoners. But you will see that you, too, will get pleasure from performing minor tasks carefully and consciously and, let us hope, from achieving decent results."

- Sommerfield to Heisenberg
in *Physics and Beyond* by Werner Heisenberg
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My family has been very understanding in accepting my adventures and supported me throughout them.

I dedicate this thesis to my father Ganesh, who in his own peculiar ways, has taught me many many things.
Warning

I absolutely forbid anyone to use the ideas in this thesis for construction of any instruments of world destruction. I promise to take all the necessary action to prevent such an occurrence.
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Chapter 1

INTRODUCTION

One of the earliest generation of short pulses of radiation was accomplished by Cutler (1) in 1955. Cutler produced short microwave pulses by using an expander (a nonlinear element) in a feedback amplifier. The emission of trains of short pulses by modelocking of lasers was first observed by DeMaria et al. (2), (3) by means of a saturable absorber. It is well known that a lasing cavity has several axial modes. These modes oscillate normally with random phases with respect to each other. By introducing in the cavity a saturable absorber or an amplitude/phase modulator, it is possible to correlate the phases of these modes so that they yield short pulses. This has been demonstrated also by DiDomenico et al. (4) for solid state Nd:YAG lasers using amplitude modulators and by Osterink and Foster (5) using a phase modulator; and in solid state ruby laser using an internal amplitude modulator by Deutsch, Pantell and Kohn (6), (7). The applied physicists took advantage of the new phenomena and produced a host of explanations. Some of these are discussed ahead:

1.1. PREVIOUS APPROACHES

Harris and McDuff (8), (9) made a nonlinear analysis of FM as well as AM type mode locking, using Lamb's (10) coupled-mode equation approach. Haken and Pauthier (11) extended the calculation of Harris and McDuff by transforming in an appropriate way the dif-
ference equation governing the axial modes of the field into a
differential equation of a displaced harmonic oscillator. Thus,
for a homogeneously broadened medium they showed analytically
that the steady state mode locked field configuration can be des-
cribed by any element of a harmonic oscillator eigenfunction.
Conspicuously lacking in their calculation, however, is the inves-
tigation of gain factors associated with all the possible pulse
configurations. Hence, the problem of predicting which pulse mode
is actually generated or how the actual pulse output evolves from
the spontaneous emission of radiation from a lasing medium was beyond
the scope of the analysis of Haken and Pauthier. Recently Nelson
(12) solved "directly" the Harris-McDuff coupled-mode equation for
homogeneously broadened lasers and showed that the steady state
laser pulse is described by a Gaussian mode as well as by "super-
modes." Nelson also pointed out that under practical operating
conditions these polynomials are closely approximated by Hermite
polynomials. Also, Nelson carried out explicit calculations for
the associated gain factors for a few supermodes and showed that
the Gaussian mode has the largest gain factor, followed by slowly
decreasing values for higher order supermodes. Clearly, the ana-
lysis of Nelson is a substantial improvement over that of Haken
and Pauthier, but it still does not describe the kinetic evolution
of an actually generated pulse from the spontaneous laser noise.

Recently, adopting the kind of analysis previously done by
Cutler (1) for microwave pulses, Kuizenga and Siegman (13), (14)
introduced "a totally different approach to the homogenously
broadened system, working completely in time domain, rather than the frequency or coupled-mode domain." Using one reasonable approximation that the bandwidth of the generated pulse is small compared with the atomic line width, they assumed that there is a Gaussian pulse inside the cavity and followed it as it goes through the mode locking configuration. Then imposing the self consistency condition after one complete round trip of the pulse, they showed that the Gaussian pulse is indeed capable of being in the steady state. The temporal behavior of the pulse is explicitly determined by the system characteristics such as the atomic line width, gain factor, and modulation depth as well as frequency, detuning, etc. Subsequently, Nelson (12) criticized the time domain analysis of Kuizenga and Siegman as follows: "This procedure is incomplete because it gives only one solution for the steady state oscillation whereas one expects that the modulation couples the countable infinity of axial modes into a new set of modes, also infinite, called supermodes. The self-consistent approach is heavily dependent upon the purely mathematical properties of Gaussians and so the conditions under which this functional form is really appropriate need to be determined."

1.2. THESIS CONTRIBUTIONS

In this thesis, it will be shown that the criticism voiced by Nelson regarding this direct time domain approach is not valid. It will be shown that the existence of infinite, denumerable set of supermodes follows as a natural consequence of this time domain
approach by regarding the self-consistency condition as an eigen-equation for the pulse envelope, viz. a Fredholm integral equation of the second kind.

The kinetics of pulse evolution from spontaneous noise will be described in close analogy with the usual quantum statistical description. Other workers hitherto have not considered how the pulses evolve in time. This is the first time, to the writer's knowledge, that this problem has been considered.

Finally with our understanding of pulses, a method to time tailor short pulses will be presented. The method involves the use of a combination of mirrors with different reflectivities, specifically designed to shape a pulse. The approach borrows heavily from the field of digital filters to realize a recursive filter to time tailor pulses. The results are simple and demonstrate unambiguously the feasibility of pulse shaping using three or four mirrors.

1.3. POSSIBLE APPLICATIONS OF THE THESIS

It is well known that there is an energy crisis in the world. One of the classical solutions has been production of thermonuclear fusion. The recent availability of high energy lasers (around 1000 Joules) has encouraged efforts to achieve fusion by bombarding deuterium targets with high energy laser pulses of nanosecond duration. Nuckolls et al. (15) of Lawrence Radiation Laboratories have pointed out that by properly shaping the pulse, it is possible to compress deuterium to high densities (thousand times normal density)
and achieve fusion in an economical manner. Clarke et al. (16) from Los Almos state, "Optimised timing of the laser pulse can significantly improve the quality of implosions . . . the laser exposure profile can be tailored to keep the subsequent compression of target core adiabatic to a maximal degree." Hence this is a valuable area in which some of the ideas for time-tailoring can be used.

1.4. PLAN OF TESIS

The time domain analysis is presented in chapter 2. The kinetic evolution is discussed in chapter 3. In chapter 4 we present the time-tailoring methods.
Chapter 2

TIME DOMAIN ANALYSIS OF PULSE ENVELOPE

In the macroscopic analysis of a large class of phenomena, the concept of a feedback system is very useful. A quantity of interest \(x\), which may be a number or a function, gets modified by a system in the form of \(S \cdot x\) where \(S\) is the system operator. The new quantity is fed back into the system by a feedback connection \(F\) and is represented by \(F \cdot S \cdot x\). For instance, the feedback is provided by mirrors in the laser mode locking configuration (see Fig. 2.1). We are often interested in knowing the quantity \(x\) in steady state. Transient behavior will be covered later. Here we are interested in the electromagnetic field in a mode locked laser as a function of time. The field gets modified by the lasing medium and active modulator and is brought back by the reflecting mirrors.

The steady state solution to the system equations is given by the eigenfunctions of that system. The class of functions which, when operated on by the system do not change (except for multiplication by a constant), is called the class of eigenfunctions of the operator \(F \cdot S\) with \(\lambda\) as the associated eigenvalue, e.g., \(E_n(t) \cdot F \cdot S = \lambda_n E_n(t)\). If we can find the set of functions \(E_n(t)\), then we know that the solution can be represented by a linear combination of the \(E_n(t)\), the particular summation being dictated by the bounding conditions. In this chapter the eigenfunctions of the mode locking system will be found.
Figure 2.1
2.1. INTEGRAL EIGENVALUE EQUATION FOR PULSE ENVELOPE

The mode locking configuration considered here is sketched in Fig. 2.1. It consists of a gain $G(t)$, a time varying modulator $m(t)$, and a time delay loop. The input pulse experiences gain while traversing through the active medium; it is modulated and comes back to its starting position after a time delay. Thus, if $E(t) = E(t)e^{-i\omega_0 t}$ is the pulse entering the active medium, the amplified pulse $E_1(t)$ is given by $E_1(t) = G(t)*E(t)$, where * denotes convolution. Because the laser is a homogeneously broadened medium and thus a time invariant linear system, multiplication in the frequency domain corresponds to convolution in the time domain. On the other hand, the modulator is a time varying system and multiplies the electric field by a factor proportional to the depth of modulation (14). Thus the effect of modulator is $E_2(t) = m(t)E_1(t)$. Finally, the round trip of the pulse through the cavity is completed by including a time delay $2L/c$, $L$ being the cavity length and $c$ the velocity of light. The pulse after one round trip is given by $E_3(t) = rE_2(t - 2L/c)$ with $r$ equal to the reflectivity of the mirror.

Now, if the total closed loop system delay is $T_m$, then the self consistency condition requires $E(t - T_m) = E_3(t)$. Thus

$$E_n(t - T_m) = \frac{\lambda}{n} r \int_{-\infty}^{\infty} dt_o m(t - \frac{2L}{c}) G(t - 2L/c - t_o) E_n(t_o) \quad (1)$$

Kuizenga and Siegman (13), in their analysis, did not arrive at this equation. They intuitively realized that a guassian electric
field would be unaltered in form by the experimental configuration and demonstrated the unalteration in their work. We, on the other hand, do not assume any specific form for \( E_n(t) \) but regard it as an unknown quantity satisfying the above eigenvector equation (1) associated with the corresponding eigenvalue \( \lambda_n \). Physically, \( \lambda_n \) represents the gain factor for a specific eigensolution \( E_n(t) \).

Note that Eq. (1) belongs to the class of a homogeneous Fredholm integral equation of the second kind with the kernel \( K(t|t_0) = \text{re}^{i \phi} m(t) G(t - t_0) \). The determination of various steady state pulse envelopes reduces to the solutions of this Fredholm equation.

2.2. SUPERMODES AS EIGENSOLUTIONS

In a homogeneously broadened lasing medium, the gain including etalon and modulation of this closed loop system reads

\[
G(t) = 2 \Delta \Omega \exp[g - i \omega_0 t - \Delta \Omega^2 (t - \tau)^2] \quad (2a)
\]

\[
\Delta \Omega = \Delta \omega / \{4[g(1 + \chi'') + (\Delta \omega / \Delta \omega_e)^2]^{1/2}\} \quad (2b)
\]

\[
\tau = (2g / \Delta \omega)[1 + \chi' - (\sqrt{2R} \Delta \omega / g \Delta \omega_e)] \quad (2c)
\]

\[
m(t) = \exp(-\alpha t^2) \quad (3)
\]

where \( g \) represents the saturated amplitude gain, \( \omega_0 \) the atomic transition or carrier frequency. \( \Delta \omega, \Delta \omega_e \) are the bandwidths of gain and etalon, respectively. \( \chi', \chi'' \) are the modification factors arising from the first and second order Taylor expansion of the index of refraction in the medium and \( R \) is effective reflectivity of the etalon.
The detailed derivation of $G(t)$ and $m(t)$ is given in ref. 7. $\alpha$ assumes the value $j\delta \omega_m^2$, $2\delta \omega_m^2$ for phase and amplitude modulators. ($\delta$ is the depth of modulation, and $\omega_m$ the modulation frequency.)

There are two underlying assumptions in the above expressions. First, the bandwidth of the pulse is assumed to be much less than the natural linewidth of the lasing medium. In other words, we are considering pulses much longer than the bandwidth limited pulses. Secondly, the experimental fact (14) that pulses occur at the peak of the depth of modulation is used, and hence the arguments in the expressions $m(t) = \exp(-j2\delta \cos \omega_m t)$ for phase modulator, $m(t) = \exp(-2\delta \sin^2 \omega_m t)$ (see Eq. 10,15,Ref.13) are Taylor expanded at the peak points of the functions to the first order.

On inserting Eqs. (2) and (3) in Eq. (1) we obtain the equation of the pulse envelope $E_n(t)$ as

$$E_n(t-T_m) = \lambda_n 2\Delta \Omega e^{\int_{-\infty}^\infty dt_0 e^{-2\Delta \Omega^2 [(t-t_0-2L/c-t_0)^2+\epsilon(t_0-2L/c)^2]}} E_n(t_0)$$

where

$$\epsilon = \alpha/\Delta \Omega$$

represents the strength of modulation compared with the effective bandwidth $\Delta \Omega$ of the medium and is much less than unity for typical mode locking configurations.

It is difficult to find the solutions of Fredholm integral equations for arbitrary kernels. The work involves either a great deal of numerical computation or the intuitive feelings of clever resident mathematicians. The solution of Eq. (4a) was accomplished with the help of
the latter, namely Dr. J. C. Polking. To the order of $\sqrt{\varepsilon}$ the solution reads

$$
\mathcal{E}_n(t) = \mathcal{E}_0 e^{-\frac{1}{2} \gamma \Delta \Omega^2 t^2} H_n(\sqrt{\gamma} \Delta \Omega t) \quad (5a)
$$

$$
\gamma = \varepsilon + [\varepsilon^2 + 4 \varepsilon]^{1/2} = 2\sqrt{\varepsilon} \quad (5b)
$$

$$
T_m = \tau + 2L/c \quad (5c)
$$

with the corresponding eigenvalue $\lambda_n$ given by

$$
\lambda_n = \lambda_0 [(1 - \sqrt{\varepsilon})/(1 + \sqrt{\varepsilon})]^{n/2} \quad (6a)
$$

$$
\lambda_0 = \left[\frac{\pi}{(1+\gamma)}\right]^{1/2} \exp\left\{ g - \Delta \Omega^2 [(\varepsilon (2L/c)^2 + (\tau + 2L/c)^2 + \frac{1}{2} \gamma T_m^2 - (\tau + 2L/c + \frac{1}{2} \gamma T_m)^2] / (1+\gamma) \right\} \quad (6b)
$$

Here, $H_n$ denotes the n-th order Hermite polynomial. The solution may be verified by substitution in (4a). The basic features of the generated fields are contained in $\gamma$, $\lambda_n$, and $T_m$, which in turn have been explicitly determined by the characteristics of the mode-locking configuration such as $g$, $\Delta \Omega$, $\alpha$, etc. Note in particular that the time delay factor $T_m$ is shared by the entire eigensolutions, viz. each eigenpulse is generated in synchronization with each other. Note also that $T_m$ is greater than the usual round trip time $2L/c$ by an amount given in Eq. (5c). This is due to the total dispersive effect of the system.

2.3. REMARKS

At this point, it may be noted that these time domain results agree with the previous results obtained by Haken and Pauthier (3)
using a coupled mode approach. Clearly the approximation procedure introduced by Haken and Pauthier of transforming a difference equation to a differential equation corresponds to neglecting terms of the order of $\epsilon$ or higher in our analysis. It should be pointed out that the polynomials derived by Nelson (4) can be easily recovered from Eq. (4) by using exponentially modulated $n$-th order polynomials for $\mathcal{E}_n(t)$ and by determining the associated coefficients up to the order to $\epsilon$. Since the parameter $\epsilon$ is, for instance, of the order of $10^{-4}$ for Nd:YAG and $10^{-7}$ for Nd:Glass, these exponentially modulated polynomials are well approximated by the harmonic oscillator eigenfunctions.

The physical significance of these results is rather easy to see. They mean that the gaussian pulse, as assumed by Kuizenga and Siegman, is not the only pulse which would retain its form after one round trip. The pulse can assume any one of the forms of Eq. (5a). This fact enables one to consider the kinetic evolution of pulses discussed in the next chapter.
Chapter 3

KINETIC EVOLUTION OF PULSES FROM SPONTANEOUS NOISE

In Chapter 2, we have found for a given time varying system a complete set of eigenvectors, whose evolution after one round trip in the cavity is given by $\mathcal{E}_n(t+T_m) = \lambda_n \mathcal{E}_n(t)$. Thus, if one expands the total electric field inside the cavity as

$$\mathcal{E}_{\text{Total}}(t) = \sum_n a_n \mathcal{E}_n(t)$$

(1)

at any given instant of time $t$, then

$$\mathcal{E}_{\text{Total}}(t+mT_m) = \sum_n a_n \lambda^m_n \mathcal{E}(t),$$

(2)

where $m$ is the number of round trips. Next, we have to determine the coefficients $a_n$ leading up to that particular instant of time.

3.1. FORMULATION OF THE PROBLEM

There now exists a strong similarity between the above approach and the usual quantum statistical description. We have treated the saturated gain medium and modulator interacting with the laser field as a strongly coupled system, whose diagonal representation has been found. In this vector space of "dressed" normal modes, the coefficients, $\{a_n\}$ are not coupled with each other. Rather, once the ensemble averaged distribution of $\{a_n\}$ is determined, the coefficients evolve in time according to the net cumulative gain factors that arise from the number of round trips the field makes until the time of detection.
Since the coefficients \( \{a_n\} \) are random variables due to the stochastic nature of spontaneous fluorescence radiation, one cannot determine the values of \( a_n \), whose expectation value, \( \langle a_n \rangle \) can be taken to zero. However, one can find \( \langle |a_n|^2 \rangle \), which can be interpreted as the partition of noise energy into the corresponding pulse eigenvectors.

We are thus expanding the electric field in the cavity in terms of the orthonormal eigenfunction in Eq. (2) in the form

\[
\mathcal{E}(t) = \Sigma_{n} a_n \mathcal{E}_n(t)
\]

the \( \sim \) denoting a random variable. Since the functions \( \mathcal{E}_n(t) \) are orthonormal, we use the Karhunen–Loève theorem to obtain

\[
\langle |a_n|^2 \rangle = \int_{-\infty}^{\infty} ds \, G(s) \int_{-\infty}^{\infty} dt \, \mathcal{E}_n(t) \, \mathcal{E}_n^*(t+s)
\]

where \( G(s) \) is the autocorrelation function of spontaneous noise.

3.2. NOISE CHARACTERISTICS AND COMPUTATION OF \( \langle |a_n|^2 \rangle \)

The spontaneous emission in the lasing cavity is characterized by a Gaussian line shape. That is to say that the power spectrum of the laser is given by

\[
G(\omega) = K \exp \left( -\left( \omega - \omega_o \right)/\Delta \Omega \right)^2
\]

where \( \omega_o \) is the laser center frequency and \( \Delta \Omega \) the bandwidth of the lasing line. From a well-known theorem (17) in Fourier analysis the autocorrelation function is given by

\[
G(s) = K' \exp \left( - (s \Delta \Omega)^2 \right)
\]
This would be used to compute the values of $\langle |a_n|^2 \rangle$.

The eigenfunctions $\xi_n(t)$ are given by

$$\xi_n(t) = \xi_0 e^{-\sqrt{\varepsilon} \Delta n^2 t^2} H_n(2\sqrt{\varepsilon} \Delta n t)$$

(7)

from equations 2-5a and 2-5b.

Substituting for $G(z)$ and $\xi_n(t)$ in Eq. (4) we get

$$\langle |a_n|^2 \rangle = 1 - n\sqrt{\varepsilon}/16g + (\varepsilon)$$

Thus the resulting electric field can be written as

$$\xi_n(t+mT) = Z^{-1} \sum_n (\langle |a_n|^2 \rangle)^{1/2} \lambda_n^{m} \xi_n(t)$$

(8)

where $Z$ is a normalization constant and $\lambda_n$ is given by Eq. 2-6a.

Eq. (8) contains the kinetic nature of light pulses generated by actively modulating laser radiation emanating from homogenously broadened gain medium. The basic features of steady state eigenpulses plus their relative gain factors (with respect to ground state) are explicitly determined in terms of system characteristics. In addition, the net expansion coefficients in Eq. (8) change with $m$, making thereby the resulting field to depend on the build up time. The number of round trips before the pulse is detected depends on gain factors, the sensitivity of detectors, etc. and can be considered as another parameter.

3.3. TYPICAL RESULTS

Eq. (8) has been applied in two cases of interest as shown in Fig. 3.1. For the case of Nd:YAG with etalon, Fig. 3.1a shows the evolution of pulse intensity as a function of $m$, using typical values.
Evolution of pulse for Nd:YAG

Figure 3.1a

Evolution of pulse for Nd:Glass

Figure 3.1b
for the system used by Kuizenga and Siegman (14). The parameters are \( \Delta \omega = 7.5 \times 10^{11} \text{sec}^{-1} \), \( \Delta \omega / \Delta \omega_{e} = 3.75 \), \( g = .1 \), \( \alpha = \frac{2 \omega_{m}^{2} \delta}{m \gamma} = 3.8 \times 10^{18} \), \( \chi_{1} = 0 \). Clearly the pulse shape shows a strong asymmetry at the initial stage, and converges very fast to a Gaussian form at \( m = 10 \), with its peak shifting toward \( t = 0 \) with increasing \( m \), at which the maximum modulation occurs. Since the transient buildup time is estimated to be of the order to 200 ns, \( (m = 30) \), the string of pulses are essentially Gaussian nature, in agreement with the experiment.

On the other hand, the parameters for Nd:Glass are \( \Delta \omega = 6 \times 10^{13} \text{sec}^{-1} \), \( \Delta \omega_{e} = 0 \), \( g = .1 \), \( \alpha = \frac{2 \omega_{m}^{2} \delta}{m \gamma} = 1.28 \times 10^{20} \), \( \chi_{1} = 0 \), \( \chi_{2} = -5 \). For the case of Nd:Glass (Fig. 3.1b) which has much broader \( \Delta \omega \), the change in pulse shape is slow, reaching to a Gaussian form at \( m = 100 \). This means that if one starts detecting pulses at \( m \) less than 100, there exists a substantial difference in nature of pulses at the beginning and end of the pulse train, where the net difference in total round trips, \( \Delta m = 30 \).
Chapter 4

RECURSIVE TIME TAILORING OF SHORT PULSES

In this chapter a new pulse shaping scheme is described whereby one can transform a single input pulse \( \mathcal{E}_{\text{in}}(t) \) into a shape that fits optimally into a desired output pulse \( \mathcal{E}_{\text{op}}(t) \). The approach is based on the use of a recursive filter that can be physically realized by a set of parallel, specially coated mirrors separated by equal distances (Fig. 4.1a). Specifically, the input pulse will be the output of an AM mode-locked laser, which assumes a Gaussian time envelope. Its duration ranges from one to several hundred picoseconds (Fig. 4.1b). This input pulse when incident normally on, and traversing through the mirror system will split, due to multiple reflections and transmissions, into an infinite number of time delayed and overlapping pulses. By adjusting the characteristics of the mirror system, e.g., delay factor and the reflectivity, it is possible to have the transmitted pulse optimally fit a desired output pulse shape \( \mathcal{E}_{\text{op}}(t) \). For the transmitted pulse, regarded here as a signal, the set of mirrors plays the role of a recursive filter. Thus, the analysis lies in designing an optimal filter which is capable of producing the desired signal, i.e., \( \mathcal{E}_{\text{op}}(t) \). The time domain design of digital filters has been extensively investigated by many workers in communication and information theory. Some of the formulations developed in these areas are applied in this thesis to generate time tailored optical laser pulses.
Recursive Filter Realized by Mirrors

Figure 4.1a

Laser System Configuration for Pulse Shaping

Figure 4.1b
4.1. OPTICAL TRANSFER FUNCTION IN Z-DOMAIN

The optics of a light transmission through the multiple layered mirror system can be compactly described by a feedback flow diagram technique, which is depicted in Fig. 4.2a. Here, each mirror is described by a two port network (17) and the transmitted pulse is represented by a forward flow path. Attached to this path are various feedback loops, each of which corresponds to an infinite geometrical pulse train. By incorporating all the possible feedback loops and invoking Mason's rule (18), one can readily find the exact expression for the transmission coefficient of the pulse through this mirror system, namely its filter function. For convenience, the design analysis or the physics of this filter function is presented in the language of the z-transform. It reads

\[
G(z^{-1}) = (\prod_{j=1}^{M} t_j) \frac{z^{-m/2}}{[1 + \sum_{j=1}^{M} b_j z^{-j}]} (1)
\]

Here, \( t_j \) denotes the transmission coefficient of the \( j \)-th mirror and the coefficients \( b_j \) in the denominator are related to the reflectivity \( r_j \) and transmittance \( t_j \) according to Mason's rule. Note: \( r_j^2 + t_j^2 = 1 \). \( z^{-1} = \exp(i (2Dw/c)) \) represents the time delay associated with the optical path difference \( 2D \), \( D \) being the uniform spacing between two adjacent mirrors. The order of polynomial in \( z^{-1} \) in the denominator of Eq. (1) is equal to the number of layers in the mirror system. As a specific example (Fig. 4.2b), the transfer function of a two layer (or three mirror) system is given by
Feedback Flow Diagram for the Filter

**Figure 4.2a**

Transfer Function \( G(z^{-1}) = \frac{\sum P_i \Delta_i}{\Delta} \)

where \( P_i \) = the \( i \)-th forward path gain

\( P_{jk} = j \)-th possible product of \( k \)-nontouching loop gains

\( \Delta = 1 - \sum P_{j1} + \sum P_{j2} - \sum P_{j3} \ldots \)

\( \Delta_i = \Delta \) evaluated with all loops touching \( P_i \) eliminated

Here \( i=1 \); \( P_{1i} = t_1 t_2 t_3 z^{-1} \); \( P_{11} = r_1 r_2 z^{-1} \); \( P_{21} = r_2 r_3 z^{-1} \)

\( P_{31} = r_1 r_2 r_3 z^{-2} \); \( P_{12} = r_1 r_2 r_3 z^{-2} \)

Mason's Rule applied to Three Mirror Filter

**Figure 4.2b**
\[ G_2(z^{-1}) = \frac{t_1t_2t_3z^{-1}}{1-r_1r_2z^{-1}r_2r_3z^{-1}r_1t_2r_3t_2z^{-2}r_1r_2r_2r_3z^{-2}} \]

4.2. FILTER SPECIFICATIONS

Next, regarding the delay time \( \frac{2D}{c} \) as the time unit, both the desired \( E_{op}(t) \) and the input pulse are sampled when we introduce their \( z \)-transforms:

\[ E_{op}(z^{-1}) = \mathcal{E}_z^{-m/2} \sum_{n=0}^{K-1} h_n z^{-n}; h_n = E_{op}(2Dn/c) \]  

(2)

\[ E_{in}(z^{-1}) = \mathcal{E}_z^{'-N/2} \sum_{n=0}^{N-1} a_n z^{-n}; a_n = E_{in}(2Dn/c) \]  

(3)

2D should be short enough to smoothly sample out the two pulse envelopes. For instance if \( E_{op}(t) \) and \( E_{in}(t) \) have 3 and .3 ns. time durations respectively, the choice of the sampling time to be of the order of 50 ps., thus the mirror spacing \( D = .74 \) cm should be adequate.

The coefficients, \( h_n \), \( a_n \) denote the time domain heights of two envelopes at the \( n \)-th point. Note that the first sampling point should be taken at a suitable rising edge of each pulse. The optimal filter function of the mirror system is specified by

\[ G_{op}(z^{-1}) = \frac{E_{op}(z^{-1})}{E_{in}(z^{-1})} \]  

or substituting from Eq. (1), (2), (3)

\[ \begin{align*}
\sum_{j=1}^{m} \frac{t_j}{1 + \sum_{j=1}^{m} b_j z^{-j}} & = \sum_{n=0}^{K-1} h_n z^{-n} \\
\sum_{n=0}^{N-1} a_n z^{-n} & = \end{align*} \]

(4)
The ratio of powers $E/E'$ is equal to $\sum_{j=1}^{m} t_j$, if the coefficients $a_n$ and $h_n$ are normalized viz.:

$$\sum_{i=0}^{N-1} a_i^2 = \sum_{n=0}^{K-1} h_n^2 = 1.$$ This will be assumed so in the further analysis. Thus to construct the desired filter, the solution of equation for $b_j$'s given by

$$\frac{1}{\sum_{j=1}^{M} b_j z^{-j}} = \left( \sum_{n=0}^{K-1} h_n z^{-n} \right)\left( \sum_{n=0}^{N-1} a_n z^{-n} \right)$$

is required and is discussed in the next section.

4.3. FILTER DESIGN

The problem of solution to Eq. (5) is well-known in the design of recursive digital filters (19). If we cross multiply Eq. (5), and equate the coefficients of identical powers of $z^{-1}$ on both sides, we get more equations for $b_j$ than the number of unknowns $M$. Thus it cannot be solved exactly. One can obtain approximate solutions which minimize a particular error criterion. In this chapter, an adaptation of the method given by Burrus and Parks (20), is used to solve the problem. The method consists of a matrix-formulation (ref. 19) of the Padé approximation method. Upon crossmultiplying Eq. (5), we get

$$\sum_{n=0}^{N-1} a_n z^{-n} = \left( \sum_{n=0}^{K-1} h_n z^{-n} \right)\left( \sum_{n=1}^{M} b_n z^{-n} \right)$$

(6)
This equation can be rewritten in matrix form as

\[
\begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_{N-1} \\
0 \\
\vdots \\
0
\end{bmatrix}
= 
\begin{bmatrix}
h_0 & 0 & 0 & \cdots & 0 \\
h_1 & h_0 & 0 \\
& h_1 & h_0 & 0 \\
& & \ddots & \ddots & \ddots \\
& & & h_{K-1} & h_{K-2} & \cdots & h_{K-M} \\
& & & & & & b_{M-1}
\end{bmatrix}
\begin{bmatrix}1 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{M-1}\end{bmatrix}
\]

(7)

This equation is partitioned as

\[
\begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_{N-1} \\
0 \\
\vdots \\
0
\end{bmatrix}
= 
\begin{bmatrix}
h_0 & 0 & 0 & \cdots & 0 \\
h_1 & h_0 & 0 \\
& h_1 & h_0 & 0 \\
& & \ddots & \ddots & \ddots \\
& & & h_{K-1} & h_{K-2} & \cdots & h_{K-M} \\
& & & & & & b_{M-1}
\end{bmatrix}
\begin{bmatrix}1 \\ b_1 \\ b_2 \\ b_3 \\ \vdots \\ b_{M-1}\end{bmatrix}
\]

i.e.,

\[
\begin{bmatrix}
a \\
a_0
\end{bmatrix} = 
\begin{bmatrix}
h_1 & H_3 \\
& b^*
\end{bmatrix}
\begin{bmatrix}1 \\ b^*\end{bmatrix}
\]

(8)

We can rewrite this equation for \( b^* \) as

\[
H_3 b^* = \hat{a} - h_1; \hat{a} = \begin{bmatrix} a \\ a_0 \end{bmatrix}
\]

(9)

The exact solution of Eq. (9) is in general not possible as we have more equations than unknowns. So if we define the error

\[
\hat{e} = (\hat{a} - h_1) - H_3 b^*
\]

(10)
then the solution of Eq. (9) which minimizes the sum of squares of residuals $S = \hat{e}^T \hat{e}$ is given by ref. 21.

$$b^* = (H_3^T H_3)^{-1} H_3^T (\hat{a} - h_1)$$  \hspace{1cm} (11)

Once $b^*$ is found from Eq. (11), it is possible to solve for reflectivity $r_j$ and transmittance $t_j$ of the desired filter through Eq. (1). This completes the filter design.

4.4. ERROR RELATIONS

The actual output of the filter using the values $b^*$ from Eq. (11) will in general differ from the desired output $h_1$. In order to develop the measure of this error, we reformulate Eq. (10) as

$$\begin{bmatrix} b_0 & 0 & 0 \cdots 0 \\ b_1 & b_0 & 0 \\ \vdots & \vdots & \ddots & \ddots \\ b_{M-1} & \vdots & & b_0 \end{bmatrix} \begin{bmatrix} a \\ 0 \end{bmatrix} - \varepsilon = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{K-1} \end{bmatrix} = Bh$$  \hspace{1cm} (12)

where $B$ is a $K \times K$ lower triangular matrix which is therefore nonsingular. Next, the difference between the actual and desired output is defined:

$$e(k) = g(k) - h(k)$$  \hspace{1cm} (13)

This error can be written as a $K \times 1$ vector $e$ and becomes

$$e = g - h_1$$  \hspace{1cm} (14)
The filter equation in terms of the actual output $g$ is

$$
\begin{bmatrix}
a \\
0
\end{bmatrix} = Bg
$$

and in terms of the desired output $h$ and the two measures of error is

$$
\begin{bmatrix}
a \\
0
\end{bmatrix} - e = Bh_1 \quad \text{and} \quad \begin{bmatrix}
a \\
0
\end{bmatrix} = B[h_1 + e]
$$

From Eq. (16), we get the desired relation

$$
e = B^{-1}a - h_1
$$

Thus after $b^*$ is found by Eq. (11), we use Eq. (17) to find the error and the actual output of the designed filter.

4.5. AN EXAMPLE

As an example of this design procedure, a three mirror filter to produce the pulse shape

$$
\mathcal{E}_{op}(t) = \mathcal{E}_o (1 - t/T)^{-5/6}
$$

(for efficient thermonuclear fusion) from a Gaussian pulse is designed. An APL program to implement the algorithm written for IBM 370/155 given in Appendix .. The transit time $T$ was chosen to be 1 nsec. It is found that by varying the width $a$ and the center $t_o$ of the input Gaussian pulse

$$
\mathcal{E}_{in}(t) = \mathcal{E}_o \exp \left(- \frac{(t - t_o)^2}{a^2} \right)
$$
the error between actual output and desired output can be minimized for a given output pulse. The best results are obtained for $\alpha = .187$ nsec, $t_0 = .9$ nsec. The graph of actual output and desired output is given in Fig. 4.3. The values of $b_1$ and $b_2$ obtained from the algorithm are $b_1 = 0.135$ and $b_2 = 0.376$. The sum of least squares of error is 3.89% of the total energy of the desired output. The values of reflectivity and transmittance for the three mirrors are computed with use of the expression for $G_2(\omega^{-1})$ in section 4.2. They are

<table>
<thead>
<tr>
<th>Mirror</th>
<th>Reflectivity</th>
<th>Transmittance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The distance between the mirrors is $D = 1.5$ cm.

4.6. ENGINEERING CONSIDERATIONS

The practical construction of the filter is easy and feasible with presently available technology. One of the requirements is the availability of mirrors of desired reflectivity. Laser Optics, Inc., in their 1973 catalog on page 11 illustrates the availability of the coated partial mirrors at specific wavelength and reflectivity. The next important requirement is the adjustment of the distance between two mirrors to the accuracy of $\lambda/10$, which is feasible with interferometry.
Desired output

Actual Output

0.0645 0.0945 0.103 0.13 0.181 0.244 0.323 0.428 0.57 0.67 0.392 0.196 0.119 0.131 0.147 0.167 0.194 0.234 0.293 0.417 0.744 0.101 0.0136 0.0037 -0.0443 -0.0369 -0.0134 0.0101 0.025 0.0104 -0.173 0.291

15 k=3 15 k=15 CENTER OF GAUSSIAN AT T=9 VARIANCE=1.67
Chapter 5

CONCLUSIONS

After all has been said and done, there are several unanswered questions.

In the time domain analysis of pulse envelope in Chapter 2, the arguments in the expression for modulator m(t) were Taylor expanded at the peak of modulation. It would be possible to expand them at some other point and derive a new set of eigenfunctions for mode locked laser. In order to explain the experimental fact that pulses occur at the peak of modulation, it would be necessary to show that the eigenvalues associated with the new eigenfunction are much less than unity. Although this can be accomplished, the worth of this project is debatable.

It would be possible to verify experimentally the results of Chapter 3 regarding kinetic evolution of pulses, if improved time-resolving instruments are available. The statistical nature of the spontaneous noise would, though, make the verification rather inconclusive. Another area of investigation could be the cross-correlation and the phase relationships between the expansion coefficients $a_n$, with the ensuing effect on the kinetics of pulses. Again the results of such project will not be commensurate with the efforts.

In the author's mind, recursive pulse shaping, presented in Chapter 4, would open an interesting and fruitful area in the field of laser technology. There are many unanswered theoretical and practical questions. First, the algorithm presented for design
of such filters minimizes $\mathbf{c}^T \mathbf{e}$. It may be possible to find an algorithm which minimizes the real quantity of interest $\mathbf{c}^T \mathbf{e}$. Presently it seems that such an algorithm would involve solutions of non-linear simultaneous equations. Second, it would be worthwhile to analyze the problem in frequency domain. Such analysis would generalize the filters to where the distances between the different mirrors are different. A brief look has revealed that such an analysis would involve the application of a non-linear optimization technique. This makes it a rather formidable problem. Third, the study of the class of pulses that could be obtained from a given pulse using recursive filters implemented by mirrors is desirable. It may be best undertaken by an understanding of the poles of the transfer function $G(z^{-1})$ under the constraints imposed by use of mirrors on the coefficients $b_j$. This would reveal the limitations of this specific pulse shaping method. Fourth, it may be possible to enlarge the above class by the application of two or more recursive filters separated from each other by a distance greater than the output pulsewidth of each one. This process would produce multiple poles in $z$-domain.

On the experimental side, it is necessary to do some simple experiments to verify the results of this thesis. They would really determine the relevance of the above problems. Some minor engineering problems to measure and maintain the distance between two adjacent mirrors may have to be tackled in the above experiments.

If such a scheme works, as the author hopes, it would yield some important benefits. First, it is the author's conjecture
that there exists an optimal manner for energy or momentum transfer between two physical systems. An example is the transfer of laser field energy (or momentum) to a deuterium pellet. If one can find analytically the above optimal manner in the interaction of laser pulses and any other system, one can tailor the pulses by using recursive filters to optimize such a transfer of energy. Second, one can use the coefficients $b_j$'s, the distance $D$ and the input pulse width $a$ to specify a time-tailored pulse. This would ensure their reproducibility for experimental usage in different set-ups.

Convinced of the opening quotation, the author feels that theses, like books, are never finished but merely abandoned.
REFERENCES


Appendix

The appendix contains the copy of an APL program to obtain the values of \( b_j \), for a given input pulse \( a_j \) and the desired output pulse \( h_j \). The comments explain the significance of the variables used.