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THE SCATTERING OF PROTONS
BY CALCIUM-40

by

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A THESIS
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THE ELASTIC SCATTERING OF PROTONS

BY CALCIUM-40
INTRODUCTION

The elastic scattering of charged particles by atomic nuclei provides a powerful method for investigating nuclear structure. Previous scattering experiments may generally be placed into two classes, distinguished by the energy range in which they were carried out and by their objectives. At low bombarding energies (less than about 6 Mev) and with good resolution of the incident and scattered particles, elastic scattering has been used to obtain the spins, parities, resonant energies, and level widths of states in the compound nucleus. Such measurements have been used extensively in studying low Z elements where the density of states is low, using proton and alpha particle beams. \(^1\) At higher bombarding energies (the cyclotron range) and with poor energy resolution of the incident and scattered particles, angular distributions of the scattered particles have been measured for a wide range of nuclear mass values. Anomalies observed in the angular distributions have been interpreted successfully in terms of the complex potential well model of the nucleus. \(^2\)

The present work represents the first extensive high
resolution elastic scattering study that has been made on any nucleus of \( A \geq 40 \). This work is part of a program undertaken to study the structure of medium weight nuclei by means of elastic proton scattering up to proton energies of about 5.5 Mev.

Of the medium weight nuclei, \(^{40}\text{Ca}\) was selected for study first because:

1. The separation energy of a proton from \(^{41}\text{Sc}\) is only \( 1.63 \) Mev. This allows formation of compound states in \(^{40}\text{Ca} + \text{p}\) at low excitation energies \((E_p + 1.63)\); thus one might expect a lower density of states and less experimental difficulty with resolution.

2. The ground state spin in \(^{40}\text{Ca}\) is zero, which simplifies the analysis of the results, as will be seen later. The fact that no reaction processes compete with elastic scattering below bombarding energies of about 5 Mev provides a further simplification of the analysis.

3. \(^{41}\text{Sc}\) presents an excellent opportunity for testing the applicability of the shell model theory of the nucleus. In its ground state \(^{41}\text{Sc}\) may be described on the basis of this theory as being a single particle in an \( f_{7/2} \) shell outside a doubly closed shell \(^{40}\text{Ca}\) core - the \( 1\,s, 2\,p, 1\,d, \) and \( 2\,s \) shells being filled for both protons and neutrons. Thus one might expect the existence of single particle states in
this nucleus.

Exploratory elastic scattering measurements carried out on Ca\(^{40}\) revealed a large number of states, many of which had very small experimental widths. This result indicated that a phase shift analysis for each level, based on angular distribution measurements, would be impossible. Thus it was decided to make measurements simultaneously at only two angles, 90\(^{\circ}\) and 150\(^{\circ}\), and attempt to fit the excitation function at 150\(^{\circ}\) by a phase shift analysis.

The initial effort was to use the dispersion theory analysis for a single level as outlined by Blatt and Biedenharn; however, the final analysis was made using a formulation, outlined in Appendix A, in which all the contributions of unlike resonances were considered.

Since it was necessary to make measurements at more than one angle, it was desirable to find a method of detecting the scattered protons without resorting to magnetic or electrostatic analysis. It was then decided to attempt the measurements with a crystal scintillation spectrometer and rely on the energy resolution which it provides to separate the proton group in question from any others that might be present. As will be see below, this simple method has proven satisfactory.
EXPERIMENTAL METHOD AND PROCEDURE

Target Preparation

In order to achieve the desired energy resolution of the elastically scattered protons, the calcium targets could be no thicker than 2 to 3 Kev. Since a calcium film this thick is not self-supporting, it was necessary to find a suitable backing material onto which the calcium could be deposited. In order that the protons elastically scattered from the backing foil be resolved from those scattered from calcium, the elements in the foil needed to have mass values greatly different from that of calcium, since the energy of the scattered protons is dependent on the mass of the scatterer. A simple calculation shows that the difference in recoil energies of protons scattered from the backing material and from calcium is greatest when the elements of the backing material are of low mass. A survey of lighter backing materials indicated that carbon foils would be most suitable. These foils were produced as described in Appendix B. Fig. 1 shows the energy separation of the protons scattered from calcium and carbon.
at 150° and 90° as a function of the incident proton energy.

The calcium targets were prepared by evaporating calcium metal (97% Ca⁴⁰) onto 100 to 300 microgms/cm² thick carbon foils. Since calcium metal is so reactive, the evaporated films oxidized immediately upon exposure to air and the bombarded targets were, in fact, calcium oxide on carbon. The thicknesses of the evaporated calcium targets were determined by observing at a low bombarding energy the yield of elastically scattered protons in a known solid angle and using the calculated Rutherford cross section.

Scattering Chamber and Detection System

The scattering chamber used in the experiment, as shown in Fig. 2, was 5 inches in diameter and 2 1/4 inches deep. The targets were held in the center of the chamber on a mount that could be rotated to any desired angle. The beam spot on the target was a rectangle approximately 1 mm square and was defined by a set of adjustable tantalum slits about 18 inches from the entrance to the chamber. A halo of scattered protons in the incident beam was reduced by centering a pair of 1/8 inch diameter tantalum collimating apertures 2 inches apart in the entrance to the chamber. The beam from the accelerator which passed through the target was collected in an insulated Faraday cup directly across from the beam entrance.
Crystal spectrometers were mounted on the chamber at 3 fixed laboratory angles: 150°, 125°, and 90°, measured with respect to the forward direction of the beam. In each spectrometer a CsI (TlI) crystal (.25 inches in diameter and 50 mils thick) was optically sealed directly to the photomultiplier tube and a .01 mil nickel foil covered the crystal face to increase the light reflection into the photomultiplier tube.

The scattered beam was defined by a removable two slit collimator with an 83 mil diameter aperture in front and a 65 mil aperture in the rear, giving an angular resolution of approximately .025 degrees. The distance from the rear aperture to the crystal was approximately 1/4 inch. This allowed the crystal a view of an area 1/2 inch in diameter centered on the beam spot. The effective solid angle was measured by scattering protons from a nickel foil approximately .02 mils thick at 1.67 Mev, and comparing the yield of elastic protons to that calculated from the Rutherford cross section. The value of 5.2 x 10^{-4} steradians given by this method agreed with the geometrical solid angle.

The signals from the spectrometers were fed to preamplifiers mounted on the base of the 6291 Dumont photomultiplier tubes and then sent into linear amplifiers at the remote console. The signal from the output of the linear amplifier was split so that it might be sent into a single channel pulse height analyser as well.
as a twenty channel pulse height analyser. The discriminator of
the single channel analyser was set to accept only those pulses
above a certain channel of the twenty channel analyser. The gain
of the linear amplifier was then adjusted to maintain the valley
between the resolved calcium and carbon proton groups in a certain
channel of the twenty channel analyser. Thus the twenty channel
analyser was used simply as a monitor of the pulse height spectrum
from a spectrometer, and the single channel analyser was used for
actually recording the data. This procedure greatly increased
the efficiency for taking the data since the monitoring could be
done during a run with no time lost between points in reading
several channels or in resetting the twenty channels.

Pulse height spectra were taken from time to time on the
twenty channel analyser in order to make any necessary background
corrections caused by the carbon-calcium overlap.

At high incident beam currents counts were noted above the
calcium proton group due to piling up of counts from the very high
yield of elastic protons from carbon. This pile-up was reduced
by restricting the beam current to a maximum value of 0.5 micro-
amperes.

The proton beam current was integrated by discharging a
6
standard capacitor, across which a known voltage had been applied.
(This was done with the actual proton beam current as it was
collected in the Faraday cup. A 300 volt battery was placed in series in the current integration circuit to suppress emission of secondary electrons from the target and Faraday cup by striking protons. With the proton beam current off, there was no measurable current above .001 microamperes due to the presence of the battery in the circuit, so that for beam currents of the order of .5 amperes, as were used throughout the experiment, the error due to this effect is small.

Energy Calibration

In order to make energy assignments to the observed levels, it was necessary to establish an energy calibration for the excitation function. First, careful attention was given to the Van de Graaff slit system and 90° analysing magnet shims. The slits above the 90° magnet were adjusted to approximately 1.5 mm and the ones below the magnet were set to scrape the beam on the top and bottom (approximately 1 mm). The shims were positioned so that the entire energy spectrum available might be scanned without further adjustment. The beam position was further defined in the vertical plane by a set of tantalum slits (about 1 mm apart) 15 feet from the magnet. These settings produce an energy resolution of approximately .03% and an energy reproducibility of .1%.

The energy of the bombarding proton beam from the
Van de Graaff was measured in terms of the Larmor frequency of a proton nuclear magnetic resonance probe in the magnetic field. This frequency is a direct measure of the magnetic field strength, so that the proton energy $E$ is proportional to $f^2$. The constant of proportionality, $k$, was determined by measuring four neutron thresholds of known energy corresponding to a proton energy interval from 1.881 Mev to 7.524 Mev, using the counter ratio method as described by Bonner and Cook. The neutron thresholds measured were:

1. $^7\text{Li}^7 (p, n)$ \hspace{1cm} $E_{th} = 1.881 \pm 0.005$ Mev
2. $^8\text{C}^{13} (p, n)$ \hspace{1cm} $E_{th} = 3.236 \pm 0.002$ Mev
3. $^9\text{F}^{19} (p, n)$ \hspace{1cm} $E_{th} = 4.240 \pm 0.005$ Mev
4. $^7\text{Li}^7 (H_2^+, n)$ \hspace{1cm} $E_{th} = 1.881 \pm 0.005$ Mev

A plot of the "k" factor, where $E$ (Mev) = $k f^2$ (mc), is shown in Fig. 3.

Immediately following the threshold measurements, several resonances distributed throughout the energy range of the $^{40}\text{Ca} (p, p)^{40}\text{Ca}$ excitation function were remeasured. These resonances then served as standards for establishing the energy scale of the entire excitation curve. The estimated accuracy for the energy scale is $\pm 5$ Kev.

Performance of the Detecting System

It is well known that the resolution of a CsI crystal improves
with proton energy in the range from 1 to 6 Mev. Fig. 1 shows the energy resolution as a function of proton energy of the crystal spectrometers used in this experiment. Comparing this curve with the energy separation curve of the proton groups from calcium and the carbon backing as shown in Fig. 1, one sees that the detection efficiency of the calcium group deteriorates rapidly as one goes to lower energies. This effect can be reduced somewhat if one takes advantage of the fact that the stopping power of protons in matter increases with decreasing energy. Nickel foils were inserted in the path of the scattered beam, and the protons scattered from carbon being of lower energy were slowed down more than those from calcium, thereby effecting a higher separation of the groups when they reached the crystal. Of course, the nickel foil also spread the protons in energy, thus decreasing the resolution of the system. A foil of 4.5 mg/cm² was found to make a marked improvement in group separations. An example of this is shown for the 90° counter at 1.9 Mev in Fig. 1, and a plot of the separation in energy of the proton group from calcium and carbon, as a function of energy at 150° and 90°, with the nickel foil in place is shown in Fig. 4. A group of representative pulse height distributions at 150° and 90° with their respective target thicknesses and bombarding energies are shown in Fig. 5. These curves were taken with the 4.5 mg/cm² nickel foils over the crystals.
ENERGY SEPARATION OF PROTON GROUPS ELASTICALLY SCATTERED FROM CA AND C WITH 4.5 MG/CM$^2$ NICKEL FOIL IN PLACE
EXPERIMENTAL RESULTS

Excitation curves for the $^{40}\text{Ca} (p, p) ^{40}\text{Ca}$ reaction were measured at $150^\circ$ and $90^\circ$ in approximately 1 Kev steps from 1.3 to 5.5 Mev. The data was necessarily taken in several different sections since the number of points and the time per point (approximately 1.5 minutes) made covering the energy range in a single three day run impossible. Each resonance was observed on the average three times at $150^\circ$ and at least once at $90^\circ$. A complete spectrum of the data at $150^\circ$ and a large portion of the data at $90^\circ$ is shown in Fig. 6. A detailed set of curves is shown later in Figs. 11 through 19. The reason for picking the angle $90^\circ$ and the backward angle $150^\circ$ becomes quite clear when one examines the theoretical resonance shapes at these two angles, as is shown in Fig. 8. As will be discussed later, the data at $90^\circ$ compliments the data at $150^\circ$ in such a way that one may immediately determine the $\lambda$ -value of the scattered proton.

Since the data was taken in more than one set run and with more than one target, it was necessary to establish a normalization between various runs and targets. This was done using the data
acquired during the energy calibration measurements mentioned above. The calibration target was approximately 63 microgms/cm² thick of calcium on an etched carbon foil (Appendix B) approximately 100 microgms/cm² thick. Since the data using this target essentially spanned the entire range of the Ca⁴⁰ (p, p) Ca⁴⁰ excitation curve, all other targets used in this experiment were normalized to it in thickness. Table I lists the principal targets used in the measurements, the approximate carbon backing thickness, the calcium thickness, and the energy range and angles at which they were used.

Data at 90° below 3.6 Mev was taken in several sections with several different targets. These are not listed in Table I.

Accuracy of the Data

There was some concern as to whether a thin calcium or calcium oxide layer might deteriorate under bombardment with the proton beam, since there was little opportunity for heat to be carried away. This effect was studied by submitting the target to bombardment for at least 5 hours by a proton beam of approximately .5 microamperes while measuring the excitation curve from 2 to 3 Mev. At the end of this period the energy of the incident protons was adjusted to the value at the first of the period and the number of scattered protons per unit charge of incident protons was again measured. The values were found to be identical within 5%. The same target was treated to a similar test in the range from
<table>
<thead>
<tr>
<th>Target Number</th>
<th>Carbon Thickness $\mu$gms/cm$^2$</th>
<th>Calcium Thickness $\mu$gms/cm$^2$</th>
<th>Use (Mev)</th>
<th>Angles Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration</td>
<td>100</td>
<td>63</td>
<td>1.5-4.5</td>
<td>150$^\circ$</td>
</tr>
<tr>
<td>14</td>
<td>100</td>
<td>10</td>
<td>1.358-1.368</td>
<td>150$^\circ$-90$^\circ$</td>
</tr>
<tr>
<td>112</td>
<td>300</td>
<td>24</td>
<td>1.5-2.4</td>
<td>150$^\circ$-90$^\circ$</td>
</tr>
<tr>
<td>122</td>
<td>300</td>
<td>24</td>
<td>3.6-5.5</td>
<td>150$^\circ$-90$^\circ$</td>
</tr>
<tr>
<td>110</td>
<td>300</td>
<td>21</td>
<td>2.2-4.5</td>
<td>150$^\circ$</td>
</tr>
</tbody>
</table>
2.8 to 4 Mev and again no significant change in yield was observed. Thus errors in the general trend of the data due to target deterioration are small. An additional check on the general trend of the continuum was to normalize at one energy the excitation functions taken with different targets and see if the curves matched at all energies. This was done for three different targets and the general trend of the data was found to be the same within 5%.

Now there are other factors to be considered which affect the accuracy of the data at a given energy:

(i) Although the number of counts due to protons scattered from calcium per data point varied by as much as a factor of 10, the counting statistics were usually less than 2%.

(ii) The regulation of the incident beam energy could be maintained only to approximately 1/4 Kev even with constant monitoring of the 90° analysing magnet current. Thus data points on very narrow resonances (width less than 1 Kev) were subject to large fluctuations.

(iii) Since the groups of protons from Ca and C are not completely separated, there must be some choice as to where the bias of the single channel analyser should be set. Of course, there is a definite minimum point in the valley between the two peaks and one may count all protons above this point. However, as the incident proton energy
is increased, the position of the minimum moves so that one must continually adjust either the single channel bias or the amplifier gain from the spectrometer. The latter was done in this experiment. Errors in positioning the minimum depend on the number of counts in the minimum, and the percent error depends on the peak (of the calcium group) to minimum ratio. Considering the 2.8 Mev pulse height distribution of Fig. 5 to be an average 150° spectrum, if one missed the minimum by one-half a channel, the error would be from 1 to 2%. Now at 90°, considering the 1.9 Mev pulse height distribution of Fig. 1, errors as great as 5% or more are possible. It was for this reason that the data at 90° for the level at 1.36 Mev was obtained by taking pulse height distributions for each point and subtracting the background. At higher energies, as illustrated in the 3.4 and 4.5 Mev pulse height distributions of Fig. 5, the 90° spectra improve and the minima selection error is reduced to 3 to 4%.

(4) Another source of possible error occurs in subtracting the high energy carbon tail that overlaps the calcium group. Two examples of the method of subtraction are shown in Fig. 1. It should be noted that the carbon backgrounds at 2.80 and 3.95 Mev are equal, as is expected since the
$C^{12} (p, p) C^{12}$ cross section at this angle is constant in this energy region. Thus the carbon background subtraction was essentially a constant throughout the energy range, except at the two $C^{12} (p, p) C^{12}$ resonances at 1.57 and 4.8 Mev. Since this background subtraction was of the order of 5 to 10%, it is thought that fluctuations in the data due to this effect are 1 to 2%.

(5) The effect of target impurities of medium or high $Z$ is difficult to evaluate. Before evaporating calcium onto the carbon foils, elastic scattering was observed from the carbon foils alone to determine their purity. Only those foils showing less than 1% (at approximately 2 Mev) of the yield expected from a 2 Kev calcium target were used. Impurities in the calcium itself, either present initially or acquired during evaporation or in the vacuum system, could be detected by magnetic analysis of the elastically scattered protons from targets handled identically as those used in this experiment. However, since this has not yet been done, it is necessary to estimate the importance of these impurities by another means. The author was associated with a previous measurement at 180° of the $Ca^{40} (p, p) Ca^{40}$ excitation curve, Fig. 7, using a 180° annular magnet. Comparison of Fig. 7 with the 150° excitation curve, Fig. 6, of the present
measurement indicates identical structure in both curves. Since the 180° magnet data is assumed to be virtually free of contaminants and the resonance structure is the same, one may conclude that the effect of the contaminants of medium and high Z in the targets of the present experiment was small.

Considering all the possible sources of error discussed above, it is believed that the accuracy of each data point is approximately ± 5%.
ANALYSIS

At the outset of this work it was believed that resonances appearing in the excitation function could be analysed using the single level scattering matrix formalism to obtain the resonance energy $E$, the resonance width $\Gamma$, and the total angular momentum $J$. However, since this formalism applies to an isolated resonance only, difficulties occurred in matching the cross sections in going from resonance to resonance, and for cases where resonances were closer than the level width, the composite shape was impossible to predict by this method. This difficulty was resolved by extending the analysis to include resonances with different $\ell$, and resonances with the same $\ell$ and different $J$. From Appendix A, which contains a formulation of the expression of the differential cross section for the interaction of two particles, one may write the scattering amplitude:

\[ (1) \, \mathcal{M}_{m^5}^{m_5}(\theta) = \frac{\gamma \epsilon}{2} \cos^2 \frac{\theta}{2} \sum_{m_5} \chi_{m_5} \chi_{m_5}^* \chi_{m^5}^* \chi_{m^5} \sum_{m_5, \ell, J} \left( \frac{m_5 m^5}{2 \ell m_5} \frac{m^5 m_5}{\ell m_5} \right) \left( \frac{\ell \ell}{\ell} \right) \]

\[ \times \left( e^{2i\ell \phi} \sum_{\ell_1} \delta_{\ell_1 \ell} - \sum_{\ell_1} e^{2i\ell_1 \phi} \right) \]
where
\[ \eta = \frac{Z_1 Z_2 e^3}{\hbar^2} \]
where \( Z_1 \) and \( Z_2 \) are the atomic numbers of the colliding particles,
ev is the electronic charge,
\( v \) is the relative velocity of the particles at large separations,
\( \chi^m \) are the normalized spin wave functions.
\( \gamma^m \) are the normalized spherical harmonics.
The terms in brackets are the Clebsch-Gordan vector addition coefficients
\(^{10}\)
and
\[ \alpha_\ell = \frac{2}{S_1} \tan^{-1} \frac{\eta}{S} \]
For the special case of a spin 1/2 particle on a spin 0 nucleus, as in the case of proton bombardment of \( ^{40}\text{Ca} \), the incoming and outgoing orbital angular momenta must be the same; that is, \( \ell = \ell' \), and the scattering matrix as it appears in the appendix may be written:
\[ e^{-2i\sigma_\ell S^\ell_{\ell'}} = e^{-2i\sigma_\ell S^\ell_\ell} = e^{2i(\alpha_\ell + \phi_\ell)} \left\{ 1 + \frac{i Q^2}{E_0 - E - i \Gamma_\ell} \right\} \]
where for purely elastic scattering:
\[ Q^2 = \Gamma_\ell \]
In the present experiment only elastic scattering may occur below 5 Mev.
Thus
\[ \Psi_3^{m_5} = -\frac{\gamma}{2} \cos^2 \frac{\theta}{2} e^{i \gamma \ln \cos^2 \frac{\theta}{2} + 2i \phi} |S m_5> 
+ \sum_{m_5', m_5''} (\frac{2}{3} \alpha_1) [\frac{2}{3} \alpha_2] e^{\frac{3}{2}i \phi} (e^{i \phi} - e^{-i \phi}) \]

Now \( e^{2i \phi} \) may be dropped from the equation since it is a common factor and \( f(\phi) \) is to be squared.

From the appendix:
\[ \frac{i \Gamma}{E_0 - E - i \Gamma/2} = (e^{2i \beta} - 1) \]
so that
\[ e^{2i \phi} S^J_{\phi} = e^{2i(\phi_0 + \phi_1 + \phi_2)} \]

If one writes:
\[ \phi_0 + \phi_1 + \phi_2 = \delta \]
then
\[ S^J_{\phi} e^{-2i \phi} = e^{2i \phi_0} \]
and
\[ \Psi_3^{m_5} = -\frac{\gamma}{2} \cos^2 \frac{\theta}{2} e^{i \gamma \ln \cos^2 \frac{\theta}{2} + 2i \phi} |S m_5> 
+ \sum_{m_5', m_5''} (\frac{2}{3} \alpha_1) [\frac{2}{3} \alpha_2] e^{\frac{3}{2}i \phi} (1 - e^{2i \delta}) \]

One may see from equation (3) that the scattering amplitude is made up of a coulomb term and a nuclear term, where the nuclear term includes phase shifts \( \phi_0 \) due to scattering from a hard sphere of radius \( R \), and phase shifts \( \beta \) due to the nuclear resonance scattering. Since the analysis is to be used to fit an excitation function rather than an angular distribution, it is of interest to note that the energy dependence appears as \( \eta \) in the coulomb part of
the scattering amplitude and in the energy dependent phase shifts $\alpha_2, \beta_2$ and $\phi_2$ in the nuclear part of the scattering amplitude.

The cross section may then be written:

$$\Sigma(\theta) = \sum_{S}^{S+1} \frac{\Sigma_{L+1}}{m_s} \left| f_{S}^{m_s}(\theta) \right|^2$$

With

$$S = \frac{1}{2}, m_s \text{ may be } \frac{1}{2} \text{ or } -\frac{1}{2}.$$

$$\Sigma(\theta) = \frac{1}{2} \left| f_{\frac{1}{2}}^{\frac{1}{2}}(\theta) \right|^2 + \frac{1}{2} \left| f_{\frac{1}{2}}^{-\frac{1}{2}}(\theta) \right|^2$$

Since there is no nuclear orientation or preferred direction for the scattering:

$$f_{\frac{1}{2}}^{\frac{1}{2}}(\theta) = f_{\frac{1}{2}}^{-\frac{1}{2}}(\theta)$$

and

$$\Sigma(\theta) = \left| f_{\frac{1}{2}}^{\frac{1}{2}}(\theta) \right|^2$$

Equation (5) was then programmed for the Shell Development Company I.B.M. 650 computer through terms in the sum up to $L = 4$. (Terms involving $L$ greater than 4 were omitted due to the very low penetrability for protons of these $L$-values.) One sees that the sum over $J$ produces 2 separate terms for each value of $L$ greater than 0 for the special case of $1/2$ on 0 collisions. There are then $1 + 4 \times 2$ terms available in the sum over $L$ and $J$ for $L$ up to 4. The sum over $m_s$ increases the total number of terms by 2 since it may have a value of 1/2 or -1/2 in each case. Thus there is a total number of eighteen terms available which may contribute to the summation.
The above discussion illustrates how one may combine the contributions to the scattering amplitude of resonances, each with either a different $l$ or $J$ for $l \leq 4$. One must now consider the problem of resonances of the same $l$ and $J$. In this experiment resonances of similar character are usually separated by many times their resonant widths. Since the nuclear phase is described by the dispersion theory form, ie \( \beta = \tan^{-1} \frac{\gamma}{E_0 - E} \), the contribution of one level is essentially zero at the next similar level; thus, for programming simplicity, the similar levels were introduced as a sum of betas, ie \( \beta = \sum \tan^{-1} \frac{\gamma}{E_0 - E} \), where: \( N \) is the number of similar resonances and \( \gamma \) and \( E_0 \) take on the appropriate values for each level in the sum. The program was designed to include the parameters of five resonances of each type or a total possibility of forty-five levels. This limit was imposed by the size of the computer.

The first step in using the analysis was to compute one at a time a set of resonance curves corresponding to the nine possible $J, L, (\pi)$ values for an arbitrary \( \gamma \) at both a low and a high energy at $150^\circ$ and $90^\circ$ in order to become acquainted with the possible single level shapes. Since the shapes vary little in this energy range, a set of these curves is shown at one energy in Fig. 8. Upon inspecting these curves it is apparent that comparison of the resonance shapes at $150^\circ$ and $90^\circ$ will determine the $l$ value of a
given level, since resonances that are similar at 150° have opposite character at 90°. For example, the resonances at 3.54 and 3.64 Mev in Fig. 9 show a definite peak at 90°. These resonances at 150° in Figs. 15 and 16 show a dip and then a rise. From the single level examples of Fig. 8 the level assignment must be \( \ell = 1 \). The levels at 3.51 and 3.79 Mev in Fig. 9 show a dip at 90° while at 150° in Figs. 15 and 16 the resonances rise before the dip which is characteristic of \( \ell = 2 \) levels.

To obtain the \( J \) value of the resonances, a detailed fit to the data must be made using the appropriate values of the parameters \( E_r \) and \( \Gamma \), the resonant energy and the level width respectively. For resonances whose widths are large with respect to the experimental energy resolution reliable \( J \) values may be assigned; however, for narrow resonances energy resolution has a very large effect. This effect was taken into account by integrating the theoretical cross section over the resolution function:

\[
\Delta E = \sqrt{\Delta E_t^2 + \Delta E_b^2}
\]

where \( \Delta E_t \) is the target thickness and \( \Delta E_b \) is the energy spread in the beam. Thus a level of 1 Kev or less is sizeably distorted by a 1 or 2 Kev thick target. One encouraging result, however, is that this distortion does not make shape analysis impossible for \( \ell \) -value assignments, but does make the experiment insensitive to different \( J \)-values of a given \( \ell \).
The next step in performing the analysis was to determine the parameters necessary to fit the continuum under the set of resonances in the data. Most of the continuum was accounted for by the Rutherford or coulomb contribution, as is indicated in Fig. 6, where the line corresponding to the Rutherford cross section is drawn through the data at 90° and 150°. It was assumed at energies below about 1.8 Mev that all the continuum was due to coulomb scattering. The parameters available for fitting the continuum above 1.8 Mev are the hard sphere phase shifts, $\varphi_k$, as defined in Appendix A. As stated above, these phase shifts depend on a nuclear radius $R$ and one might hope to determine a radius which would fit the continuum. However, since the fit was to be made at one angle (150°), it was decided to use only the phase shift $\varphi_0$ to fit the continuum.

The final step of the analysis involved estimating values of $E_r$ and $\gamma$ from the data and using these as initial guesses in computing the cross section. Values of $\mathcal{L}$ were established by shape analysis as described above, and tentative values of $J$ were inserted for as many resonances as could be handled by the program. Of course, ideally one would like to include in the summation of equation (2) the contributions of all levels in the compound nucleus, even those beyond the range of this experiment. Since this is not possible, it was necessary to examine the change in cross section
at a given energy when only the influence of two or three of the nearest neighboring states on either side were considered—as opposed to when these plus as many as ten more distant neighboring states were present. The effect on the cross section was always found to be less than one percent, provided that the distant states were narrow.

After many small adjustments to the various level parameters to improve the theoretical fit to the data and with all "large" experimentally observed resonances included in the program, the hard sphere phase shift $\delta_0$ was readjusted for a closer fit to the continuum. (Some fluctuations in the experimental data greater than the stated 5% experimental error and yet too small to be identified in terms of a particular $\ell$ or $J$ were ignored. Although these fluctuations are no doubt real, it is felt that the widths of the levels they represent are so small that they have very little effect on the analysis.) The hard sphere phase shift $\delta_0$ is shown in Fig. 10 as a function of energy. The theoretical excitation function, using parameters which best fit the data, is shown in the upper portions of Figs. 11 through 19. For comparison, the lower portions of these figures show a plot of the data and the theoretical cross section, which has been altered to include the effect of the experimental energy resolution.

Table II gives values of the parameters used in the fits
HARD SPHERE PHASE SHIFT

$\phi_0$ IN DEGREES

$E_p$ LAB (MEV)

Fig. 10
Fig. 11
Fig. 12
Fig. 13
Fig. 14
Fig. 17
Fig. 18
$C_{40}^{40}(p, p)C_{40}^{40}$

$150^\circ$

$\sigma$ (150°) MB/STER

$E_{p} \text{ LAB (MEV)}$

Fig. 19
and also a tabulation of the reduced widths $\gamma^2$ in the form of a ratio to the Wigner limit,

$$\gamma^2 = \frac{1}{2 \pi \frac{R^2}{\mu}}$$

where

$$\gamma^2 = \frac{3A^2}{2 \mu R^2}$$

where $p_2$ is the penetrability;

$R$ is the nuclear radius (taken to be $5.13 \times 10^{-13}$ cm);

$\mu$ is the reduced mass for the system.

It should be mentioned here that the results of this experiment confirm the energy assignments made for the levels in $\text{Sc}^{41}$ using a magnetic spectrometer at $180^\circ$. Also, at low energies (below 1.8 Mev) where the crystal resolution was poorest, the magnetic spectrometer data proved a valuable aid in interpreting the present data.

Since the targets used in the experiment were made from natural abundance calcium metal, there was some concern that resonances in the 2% abundant $\text{Ca}^{44}$ isotope might actually produce resonances in the excitation curve. Of course, one cannot be absolutely certain that this is not so until a 100% $\text{Ca}^{40}$ target has been studied. However, it is evident that even if $\text{Ca}^{44}$ were to have a resonance as great as the largest $\text{Ca}^{40}$ resonance (250 mb),
TABLE II

Summary of Assignments of Levels in Sc$^{41}$

<table>
<thead>
<tr>
<th>E(lab) (Mev)</th>
<th>E$^m$ in Sc$^{41}$ (Mev)</th>
<th>J, $\pi$</th>
<th>$\Gamma$ (Kev)</th>
<th>$\chi^2/\chi^2_w$ (percent)</th>
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<tbody>
<tr>
<td>1.360</td>
<td>2.96</td>
<td>1</td>
<td>$1/2^-$</td>
<td>0.2</td>
</tr>
<tr>
<td>1.672</td>
<td>3.26</td>
<td>0</td>
<td>$1/2^+$</td>
<td>0.4</td>
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<tr>
<td>2.388</td>
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</tr>
<tr>
<td>2.447</td>
<td>4.02</td>
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<td>$1/2^-$</td>
<td>60</td>
</tr>
<tr>
<td>2.722</td>
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<td>$1/2^-$</td>
<td>12</td>
</tr>
<tr>
<td>2.754</td>
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<td>0</td>
<td>$1/2^+$</td>
<td>1</td>
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<tr>
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<td>2.956</td>
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<tr>
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<td>5.05</td>
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<td>$(3/2^-, 1/2^-)$</td>
<td>2</td>
</tr>
<tr>
<td>E(lab) (Mev)</td>
<td>E_{ex} in Sc$^{41}$ (Mev)</td>
<td>$\ell$</td>
<td>j, $\Pi$</td>
<td>$\gamma$ (Kev)</td>
</tr>
<tr>
<td>-------------</td>
<td>--------------------------</td>
<td>------</td>
<td>----------</td>
<td>----------------</td>
</tr>
<tr>
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<td>12</td>
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<td>10</td>
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<tr>
<td>4.554</td>
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<tr>
<td>5.000</td>
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<td>$1/2^+$</td>
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</tr>
<tr>
<td>5.26</td>
<td>6.76</td>
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<td>$(7/2^-, 5/2^-)$</td>
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<tr>
<td>5.28</td>
<td>6.78</td>
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<tr>
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<td>5.41</td>
<td>6.91</td>
<td>2</td>
<td>$5/2^+, 3/2^+$</td>
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</tr>
<tr>
<td>5.43</td>
<td>6.93</td>
<td>3</td>
<td>$(7/2^-, 5/2^-)$</td>
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</tr>
<tr>
<td>5.45</td>
<td>6.95</td>
<td>2</td>
<td>$5/2^+, 3/2^+$</td>
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</table>
it would still only be a 5% fluctuation in the continuum, which averages about 100 mb. On this basis no Ca^{44} resonances would appear outside of the expected data fluctuations.
DISCUSSION AND CONCLUSIONS

Evidence of Single Particle States in Sc$^{41}$

Since the nuclear properties of a large number of states in Sc$^{41}$ have been determined, as summarized in Table II, it would be of interest to attempt to interpret the observations in terms of the nuclear shell model. As mentioned in the introduction, it was believed possible that some of the states of Sc$^{41}$ would show a single particle nature, especially since evidence for single particle character does exist for two states in its mirror nucleus Ca$^{41}$. In this nucleus levels at 1.947 and 3.950 Mev have been tentatively identified as the predicted $p_{3/2}$ and $p_{1/2}$ single particle states on the basis of the Ca$^{40}$ (d, p) Ca$^{41}$ stripping reaction. These same levels are predicted in Sc$^{41}$ along with others at higher energies identified as $f_{5/2}$, $g_{9/2}$, $d_{5/2}$, and $g_{7/2}$. In addition, states of more complex character are expected due to the excitation of nucleons from the Ca$^{40}$ core. Although there has been no direct experimental evidence for single particle states in Sc$^{41}$, levels formed by $\ell = 1$ protons in the Ca$^{40}$ (d, n) Sc$^{41}$ reaction have been reported.
at excitation energies of 1.86 and 2.07 Mev\textsuperscript{13}, one of which, by analogy to Ca\textsuperscript{41}, is probably the \( p_{3/2} \) single particle state. Other levels in Sc\textsuperscript{41} at 2.25 and 3.44 Mev are known from Ca\textsuperscript{40} \( (p,\gamma) \) Sc\textsuperscript{41} measurements\textsuperscript{14}, and at energies above 6 Mev a number of states have been located by the study of the Ca\textsuperscript{40} \( (p, p', e^+, e^-) \) Ca\textsuperscript{40} reaction.

A direct method of testing the applicability of the single particle shell model to Sc\textsuperscript{42} is to examine the ratio of the reduced width \( \gamma^2 \) of a state to the Wigner limit \( \gamma^2_w \), since this ratio is a measure of the single particle character of a state. A plot of this ratio, for a particular \( L \), versus the excitation energy in Sc\textsuperscript{41} is shown in Fig. 20. The pronounced peaks in \( \gamma^2 / \gamma^2_w \) for the different \( L(J) \) values suggest the influence of single particle states predicted by the shell model at these energies, although they appear in a somewhat different order than predicted.\textsuperscript{12}

Of course, the order of the states is dependent upon the choice of potential used in calculating the positions of the states. It is of special interest to note that the s-states observed have significantly smaller reduced widths than those states predicted by the shell model. This would seem to imply that the s-states in this energy region involve an excitation of the Ca\textsuperscript{40} core.

Although the number of states in each peak is small, the character of the \( \gamma^2 / \gamma^2_w \) curves for each \( L(J) \) is suggestive at
Fig. 20

$\frac{Y^2}{Y_{W}^2} \%$

$l = 1, P_{1/2} (\times 1/3)$

$l = 4, (G_{9/2}) (\times 1/2)$

$l = 3 (F_{5/2})$

$l = 2$

$E_{EX}$ in $^{41}$Sc (MeV)

$S_{1/2}$
relatively low energies of the giant resonances predicted by the intermediate coupling model of Lane, Thomas, and Wigner.\(^{16}\)

If one assumes that the groups of states of a given \(\mathcal{L}(J)\) value are identifiable with a single particle state of that \(\mathcal{L}(J)\), then one may extract values of the splitting of levels of the same orbital angular momentum, \(\mathcal{L}\), which provides information on the magnitude of the coupling between the spin of the proton and its angular momentum. In cases where only the \(\mathcal{L}\)-values of the levels were definite, an assumption was made for the \(J\) values on the basis of the shell model prediction that for two levels with the same total and orbital quantum numbers, the one with larger total angular momentum should be lower in energy.\(^{17}\)

The spacings observed in the present measurements are summarized in Table III; and for comparison, previous estimates of these spacings are included when available. If one assumes an attractive spin-orbit force of the type\(^{17}\):

\[
U(\mathcal{L})(\mathbf{s} \cdot \mathcal{L})
\]

where \(s\) is the proton spin, \(\mathcal{L}\) is its orbital angular momentum, and \(U(r)\) is a function of radial distance and is mainly negative; and if the spin orbit force is small, it follows that the level splitting should be roughly proportional to \((2\mathcal{L} + 1)\). From Table III the \(f\)-level splitting is 6.4 Mev so that 6.4/(\(2\mathcal{L} + 1\)) is approximately 900 Kev; and the \(p\)-level splitting is 2.2 Mev so that 2.2/(\(2\mathcal{L} + 1\)) is
TABLE III

Experimental Single Particle Level Spacings

<table>
<thead>
<tr>
<th>Levels</th>
<th>$\Delta E$ (Mev) (present work)</th>
<th>$\Delta E$ (Mev) (previous estimate)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^1 f_{7/2} - ^1 f_{5/2}$</td>
<td>6.4</td>
<td>$\geq$ 4</td>
</tr>
<tr>
<td>$^2 p_{3/2} - ^2 p_{1/2}$</td>
<td>2.2</td>
<td>$\sim$ 1.8</td>
</tr>
<tr>
<td>$^1 f_{5/2} - ^2 p_{3/2}$</td>
<td>4.5</td>
<td>$\leq$ 1</td>
</tr>
<tr>
<td>$^2 p_{1/2} - ^1 g_{9/2}$</td>
<td>1.5</td>
<td>---</td>
</tr>
<tr>
<td>$^2 p_{1/2} - ^2 d_{5/2}$</td>
<td>1.9</td>
<td>---</td>
</tr>
</tbody>
</table>
approximately 700 Kev— the near equality of the two ratios is in
fair agreement with the theory.

Conclusions

There are a number of conclusions that may be drawn
from the present work:

(1) The results of this experiment demonstrate the use-
fullness of elastic proton scattering from medium weight
nuclei for determining nuclear parameters.

(2) It has been shown that not only are the widths of the
nuclear levels sufficiently large to allow them to be
distinguished above the continuum, but that the levels are
sufficiently separated in this particular nucleus to allow
them to be experimentally resolved.

(3) It has been demonstrated that the single level formula
allows reliable assignments of \( l \) for a particular level
and the extension of the formulation to include contributions
from all levels gives good agreement with the data, even
for so complicated an excitation function as was measured
in the present experiment.

(4) The statement made in the introduction that Sc\(^{41}\)
presented an excellent opportunity for testing the appli-
cability of the shell model theory of the nucleus has been
particularly borne out in the results of the $\chi^2/\nu$ plot of

Fig. 20, in that evidence for the existence of shell model
states has been established.
THE INELASTIC SCATTERING OF PROTONS
BY CALCIUM-40
THE Ca\(^{40}\) (p, p') Ca\(^{40}\) REACTION

The inelastic proton groups from the proton bombardment of Ca\(^{40}\) were studied in the energy range from 4.5 to 5.5 Mev using an experimental arrangement as described in the preceding work. This study was made in order to locate the states in Sc\(^{41}\) which decay inelastically to states in Ca\(^{40}\) and to identify the states in Ca\(^{40}\) involved. This reaction had been studied previously by Bent and Kruse who observed the occurrence of inelastic scattering by means of the pairs emitted from the first excited state of Ca\(^{40}\), which has a spin of 0\(^+\).

The calcium metal targets used were 3 to 5 KeV thick on 100 micrograms/cm\(^2\) carbon foils. Fig. 21 shows a pulse height distribution of the scattered protons at \(E_p = 5.45\) Mev illustrating the separation of the inelastic proton groups from the elastic proton groups using the crystal spectrometer. The identification of the various proton groups in going from higher energies to lower energies is as follows:

1. Ca\(^{40}\) (p, p): The known energy of this peak was used as a calibration point for the other groups after being corrected
for small energy losses in the target and reflecting foil
over the detecting crystal.

(2) $^{12}\text{C} (p, p)$ : The known energy of this group was used as
a second calibration point.

(3) The next three groups of protons are due to transitions
to the first, second, and third excited states of $^{40}\text{Ca}$. The
energies assigned to these groups from the calibration
curve agreed within $\pm 10$ Kev to the calculated energy for
these transitions.

Excitation curves were measured using single channel
analysers set to count the protons in the regions corresponding to
1.1 to 1.6 Mev and 1.6 to 2 Mev of Fig. 21. The excitation curve
for transitions to the first excited state of $^{40}\text{Ca}$ is shown in Fig. 22,
where the energies of the states in $^{41}\text{Sc}$ formed in this reaction are
given by the bombarding energy of the resonances (when corrected
for center of mass motion) added to the binding energy of the proton
which is 1.63 Mev. The resonance labeled $^{12}\text{C} (p, p)$ is due to an
abrupt rise in the background associated with a resonance in the
elastic scattering of protons from the carbon backing at this energy.
The scatter of points at 4.9 Mev is a background fluctuation due to
a very strong resonance in the elastic protons from $^{40}\text{Ca}$ as shown
in Fig. 6 of the elastic scattering section. Resonances labeled
4 to 10 are due to the $^{40}\text{Ca} (p, p')^{40}\text{Ca}$ reaction.
Fig. 22
The insert of Fig. 22 is a portion of the excitation curve for transitions to the 3.73 and 3.90 Mev second and third excited states of Ca$^{40}$. One new peak occurred in this curve at 5.43 Mev which did not appear in transitions to the first excited state.

An energy level diagram of Sc$^{41}$ and Ca$^{40}$ is presented in Fig. 23 showing the decay scheme from each state, with spin values suggested by the data for the participating states in Ca$^{40}$. The spin assignments for the states in Sc$^{41}$ are tentative ones made on the basis of elastic proton measurements in this region and final assignments are pending further calculations of the type discussed in the elastic scattering section. However, the information gained from the preliminary analysis is used as a guide for constructing arguments for the Ca$^{40}$ spin assignments. In support of the spin assignment of 3$^-$ for the second excited state of Ca$^{40}$,

Fig. 24 shows the pulse height distribution of protons from the level at $E_P = 5.43$ Mev in Sc$^{41}$. This level, which from the Ca$^{40}$ ($p, p$) Ca$^{40}$ data at 150$^0$ has a probable $L$-value assignment of 3, decays almost exclusively to the second excited state of Ca$^{40}$. Thus the 3$^-$ assignment is reasonable and agrees with electron scattering experiments.

Now the third excited state of Ca$^{40}$ at 3.90 Mev is a more difficult problem. On a simple shell model picture for Ca$^{40}$ one has available the states $0^+, 2^+, 4^+, \ldots$ and $1^-, 3^-, 5^-, \ldots$. 

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Since γ-rays have been observed to the 0+ ground state from this level, one may rule out the 0+ assignment. And since this level is populated by transitions from \( \mathcal{L} = 0 \) states in \( \text{Sc}^{41} \), as shown in Fig. 23, one may rule out spin assignments higher than 3 on the basis of penetrabilities. The \( \text{Sc}^{41} \) state at \( E_p = 5.43 \) Mev (\( \mathcal{L} = 3 \)) populated this level only weakly, as illustrated in Fig. 24, which would rule out a 3− assignment. Thus one is left with the possibility of a 1− or 2+ assignment for this third excited state of \( \text{Ca}^{40} \). Table IV shows the relative intensities of the transitions at 150° and 90°. The penetrabilities for transitions to the third excited state of \( \text{Ca}^{40} \) with a tentative spin assignment of 2+ are labeled (2) and penetrabilities with an assignment of 1− are designated (1).

At 5.39, 5.41, 5.43, and 5.45 Mev the yields at 150° of protons to the first excited state of \( \text{Ca}^{40} \) at 3.35 and the third excited state at 3.90 Mev are essentially the same, so that a simple comparison of the penetrabilities for transitions to these states indicates the 1− assignment. At 5.39 Mev the \( \text{Sc}^{41} \) state is formed by \( \mathcal{L} = 2 \) protons and since the yields at 150° and 90° do not show an isotropic behavior, the 2+ assignment is discredited. The small yields at 5.41 Mev are inconclusive but the character of the scattering at 5.45 Mev from an \( \mathcal{L} = 2 \) \( \text{Sc}^{41} \) level again is not isotropic. Thus the arguments presented above imply a 1− assignment for the 3.90 Mev third excited state of \( \text{Ca}^{40} \).
<table>
<thead>
<tr>
<th>INCIDENT ENERGY MEV</th>
<th>RELATIVE LEVEL IN Ca-40 POPULATED</th>
<th>RELATIVE PROTON PENETRABILITY 150°</th>
<th>PROTON YIELD 90°</th>
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<td>3.35</td>
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<td>0.10</td>
<td>3.0 x 10^-6.4</td>
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<td>0.08</td>
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<td>5.39 (9)</td>
<td>3.35</td>
<td>0.05</td>
<td>3.0 x 10^-6.1</td>
</tr>
<tr>
<td>5.41 (9-A)</td>
<td>3.35</td>
<td>0.02</td>
<td>3.0 x 10^-6.0</td>
</tr>
<tr>
<td>5.45 (10)</td>
<td>3.35</td>
<td>0.002</td>
<td>3.0 x 10^-6.0</td>
</tr>
</tbody>
</table>
APPENDICES AND REFERENCES
APPENDIX A

Derivation of Differential Elastic Scattering Cross Section, (A-1)

The wave function which represents the collision of two charged particles interacting only through their coulomb field may be written:

\[ \psi^c = \sum_{l=0}^{\infty} i^l (2l+1) P_l(\cos \theta) e^{i \sigma^c} \frac{F_l(\rho)}{\rho} \]

where

\[ P_l(\cos \theta) \] is the Legendre polynomial of order \( l \),
\[ \sigma^c = \arg \Gamma(l+1-i \eta) \] is the \( l \)th partial wave coulomb phase shift,
\[ \rho = \frac{2 m v}{\hbar} \] where \( \mu \) is the reduced mass of the particles and \( v \) is their relative velocity at large separations,
\[ F_l(\rho) \] is the regular coulomb function that satisfies the differential equation

\[ \frac{d^2 F_l}{d \rho^2} + \left( 1 - \frac{2 \eta}{\rho} - \frac{l(l+1)}{\rho^2} \right) F_l = 0 \]

and has the asymptotic behavior

\[ F_l \sim \sin \left( \rho - \frac{2 \pi l}{2} - \eta \ln 2 \rho + \sigma^c \right) \]

where

\[ \eta = \frac{Z_1 Z_2 e^2}{\hbar^2 m} \]
If one includes a definite channel spin, \( S \), for the collision with its \( z \) projection \( m_s \), the relation may be written:

\[
\psi^0 |S, m_s\rangle = \sum_{\ell \geq 0} i^{2\ell} (2\ell+1) \hat{P}_\ell (\cos \theta) e^{i\beta} \frac{F_{\ell}(\theta)}{\ell + \ell} |S, m_s\rangle.
\]

In terms of the Coulomb distorted ingoing and outgoing waves

\[
U^-_\ell = e^{i\beta} (G_\ell - F_\ell),
U^+_\ell = e^{-i\beta} (G_\ell + F_\ell),
\]

one may write (2) in the form:

\[
\psi^0 |S, m_s\rangle = \sum_{\ell \geq 0} i^{2\ell} (2\ell+1) \hat{P}_\ell (\cos \theta) \frac{1}{\ell + \ell} \left( e^{2i\beta} U^+_\ell - U^-_\ell \right) |S, m_s\rangle.
\]

In terms of the normalized spherical harmonics (3) becomes

\[
\psi^0 |S, m_s\rangle = \sum_{\ell \geq 0} i^{2\ell+1} \sqrt{\ell(2\ell+1)} \frac{1}{\ell + \ell} Y^\ell_\ell (\theta, \phi) \left( e^{2i\beta} U^+_\ell - U^-_\ell \right) |S, m_s\rangle
\]

where

\[
Y^m_\ell = \left[ \frac{(2\ell+1)(\ell-m)!}{4\pi (2\ell+m)!} \right]^{\frac{1}{2}} \frac{1}{\ell + \ell} e^{im\phi} (-\sin \theta)^{m} \left( \frac{d}{d(\cos \theta)} \right)^{\ell+m} (\cos^2 \theta - 1)^{\ell-m}.
\]

The collision may now be described in terms of a total angular momentum (\( J \) and the \( z \) projection of \( J \), \( M = m_s \)) wave function. This can be done by use of the Clebsch-Gordan vector addition coefficients for angular momentum, thus

\[
Y^0_\ell |S, m_s\rangle = \sum_{J} \frac{\left[ \begin{array}{c} J \\ m_s_1 m_s_2 \end{array} \right]}{\begin{array}{c} J \\ m_s \end{array}} |(J S) J m_s\rangle.
\]

Then (4) may be written:

\[
\psi^0 |S, m_s\rangle = \sum_{J} \frac{i^{2\ell}}{\sqrt{4\pi (2\ell+1)}} \left[ \begin{array}{c} J \\ m_s_1 m_s_2 \end{array} \right] \left( e^{2i\beta} U^+_\ell - U^-_\ell \right) |(J S) J m_s\rangle.
\]
The nuclear interaction of two particles, that is, an interaction other than the coulomb interaction can be described by a change in the outgoing wave. The Wigner single level dispersion theory form for the scattering matrix as given by Blatt and Biedenharn (A-2) gives the relation of the coefficients of the ingoing wave function to those of the outgoing wave function.

Thus, if one writes a wave function:

\[ \psi_{\text{sa}}^{L_m} = \frac{1}{\sqrt{2\pi\alpha_2}} |(L S)J m_s\rangle (A_{Ls_0}^{Jm_0} U_L^- - B_{Ls_0}^{Jm_0} U_L^+) \]

then the scattering matrix is defined by:

\[ B_{Ls_0}^{Jm_0} = \sum_{L_0s_0} \langle L_0s_0' | L s_0 | L' s_0' \rangle A_{L_0s_0}^{Jm_0} \]

so that the scattering matrix is the relation of the outgoing wave to the ingoing wave.

Considering now the case of the coulomb field, it is convenient to use a closed form for \( \psi^c \) and consider the scattering of the first \( L' \) partial waves, since these are in general the only ones affected by nuclear scattering. First one must subtract these \( L' \) waves from \( \psi^c \) and add them in the scattered form. The analysis is now restricted to elastic scattering; that is, \( s = s' \). It is assumed that channel spin is a good quantum number; that is, \( s = s' \).

The total wave function may then be written:

\[ \psi_{\text{total}} = \psi_c |LSm_s\rangle \sum_{L s_0} \frac{1}{\sqrt{2\pi\alpha_2}} \langle L_0s_0 | L s_0 | L' s_0' \rangle \left( e^{i\pi s_0} U_L^+ - U_L^- \right) |(LS)Jm_s\rangle \]

\[ + \sum_{L s_0} \frac{1}{\sqrt{2\pi\alpha_2}} |(LS)Jm_s\rangle \left( A_{Ls_0}^{Jm_0} U_L^- - B_{Ls_0}^{Jm_0} U_L^+ \right) \]
Since the scattering does not affect the ingoing wave, the coefficients of \( U^+_l \) must vanish, so:

\[
A^+_l = -i \frac{\ell}{E} \sqrt{4\pi(2\ell+1)} \frac{1}{2\ell+1} \sqrt{\frac{J_{m_5}^s}{L_{m_5}^s}}
\]

and

\[
B^+_l = \sum_{l' \ell} S_{l \ell} \, i^{2 \ell+1} \frac{\ell \alpha}{2 \ell+1} \sqrt{4\pi(2\ell+1)} \left[ \frac{J_{m_5}^s}{L_{m_5}^s} \right]
\]

from which one gets

\[
\psi_{\text{total}} = \psi^0 |S_{m_5}\rangle + \sum_{l \ell} \frac{U^+_l}{\sqrt{4\pi(2\ell+1)}} \, i^{2 \ell+1} \left[ \frac{J_{m_5}^s}{L_{m_5}^s} \right] \left( \frac{e^{2i\alpha}}{2\ell+1} S_{l \ell} - S_{l \ell}' \right)
\]

where

\[
S_{l \ell} = 0 \quad l \neq l'
\]

\[
S_{l \ell} = 1 \quad l = l'
\]

The wave function \( \langle(lS) | J_{m_5} \rangle \) is now decomposed into \( |l m_5\rangle \) and \(|S m_5\rangle \) using:

\[
\langle(lS) | J_{m_5} \rangle = \sum_{m_5'} \left[ \frac{J_{m_5}^s}{L_{m_5}^s} \right] \, Y_{l}^{m_5-m_5'} \langle S m_5' \rangle
\]

Then (7) may be written:

\[
\psi_{\text{total}} = \psi^0 |S_{m_5}\rangle + \sum_{l \ell} \frac{U^+_l}{\sqrt{4\pi(2\ell+1)}} \, Y_{l}^{m_5-m_5'} \, \chi_{m_5'}^{m_5} \, i^{2 \ell+1} \sqrt{4\pi(2\ell+1)} \left[ \frac{J_{m_5}^s}{L_{m_5}^s} \right] \left( \frac{e^{2i\alpha}}{2\ell+1} S_{l \ell} - S_{l \ell}' \right)
\]

If a wave function \( \chi \) which represents an incident wave and a scattered wave is asymptotically of the form \( (A-3) \):

\[
\chi \sim e^{i \frac{\ell \alpha}{2}} + f(\theta) \, \frac{e^{i \frac{\ell \alpha}{2}}}{\alpha}
\]

then

\[
\sigma(\theta) = |f(\theta)|^2
\]

But

\[
U^+_l \sim e^{i (\frac{\ell \alpha}{2} - \frac{\ell \pi}{2} + \eta \ell n 2 \ell \alpha)}
\]
\[
\psi^c \sim e^{ikz + i\theta_0 \sqrt{1 - z^2}} + \frac{e^{ik_{11} - i\theta_0 \sqrt{1 - z^2}}}{\frac{2}{\zeta}} \text{cosec}^2 \frac{\eta}{2} \frac{\eta}{2} \eta
\]

so

\[
(9) \Phi_{s}^{m_{s}}(\theta) = \sum_{m_{s}, m_{s'}} \left[ -\frac{\eta}{2} \text{cosec}^2 \frac{\eta}{2} \cdot e^{i\eta \text{ncosec}^2 \frac{\eta}{2}} \right]_{m_{s}} \left. \chi_{s}^{m_{s}} \right)
\]

\[
+ \sum_{m_{s}, m_{s'}} i^{L - L' + 1} \chi_{s}^{m_{s}} \left. \left[ \frac{\eta}{2} \text{cosec}^2 \frac{\eta}{2} \right] \left. \chi_{s}^{m_{s}} \left[ \frac{\eta}{2} \text{cosec}^2 \frac{\eta}{2} \right] \right]_{m_{s}} \left. \chi_{s}^{m_{s}} \right)
\]

\[
\times \left( e^{i\alpha \Delta} \left. S_{L} \right| e^{i\alpha \Delta} \left. S_{L} \right| - S_{L} \right| e^{i\alpha \Delta} \left. S_{L} \right| e^{i\alpha \Delta} \left. S_{L} \right|
\]

where

\[
e^{i\alpha \Delta} \left. S_{L} \right| = e^{i(\Delta \theta_{L} + \alpha \theta_{L} + \phi_{L} + \phi_{L}')} \left( \delta_{L} \left. \right| + \frac{\Delta \theta_{L} + \phi_{L} + \phi_{L}'}{E_{0} + E_{L} - E_{0}} \right)
\]

\[
\phi_{L} = -\tan^{-1} \frac{E_{0}}{Q_{L}}
\]

\[
\alpha_{L} = \frac{\Delta \theta_{L}}{S_{L}} \tan^{-1} \frac{S_{L}}{Q_{L}}
\]

and \( Q_{L} = \pm \sqrt{\frac{Q}{2}} \)

where \( Q_{L} \) is the elastic scattering width of the nuclear level,

\( \Gamma \) is the total width of the level of angular momentum \( J \) formed by \( L \) -wave protons,

\( E_{0} \) is the resonant energy of the level.

The relation for the differential cross section is then given by:

\[
(10) \sigma(\theta) = \sum_{S m_{s}} \frac{1}{2L + 1} \frac{1}{2J + 1} |\Phi_{s}^{m_{s}}(\theta)|^2
\]

where \( i \) is the spin of the incident particle, and \( I \) is the spin of the target nucleus. Finally, it is convenient to write the
quantity:
\[ \frac{i g_q g_{q'}}{E_0 - \varepsilon - i\varepsilon} \]

(which appears in the scattering matrix)
as:
\[ \frac{g_q g_{q'}}{\varepsilon^J} \left( e^{2i\delta_{q,q'}^J} - 1 \right) \]

where
\[ \delta_{q,q'}^J = \tan^{-1} \frac{\varepsilon^J}{E_0 - \varepsilon} \]
APPENDIX B

Preparation of Carbon Foils

The carbon foils were prepared by cracking the carbon out of methyl iodide onto thin (.1 to .5 mil thick) nickel filaments. The nickel filaments (.5 inches by 1.5 inches) were mounted between two electrodes in a vacuum system. The pressure was reduced using a mechanical vacuum pump to approximately 50 microns of mercury. Then 1 to 3 inches of mercury of methyl iodide were admitted to the chamber and the system was closed. A current was then passed through the filament until the nickel took on a bright red glow. As the methyl iodide cracked, the carbon adhered to the filament and the iodine coated the chamber walls. The temperature of the filament was then abruptly lowered by turning off the current. As the nickel contracted, the carbon foils simply lifted up in large bubbles. These delicate foils (50 to 150 micrograms/cm² thick) were then removed from the filament and mounted on a target holder by means of a quick drying cement. The carbon backing foils were then mounted in a second evacuated chamber where the Ca metal was evaporated onto them.

A technique for producing carbon backing foils less than 50 micrograms/cm² was also employed with some success. The
carbon was cracked out of methyl iodide as above onto .1 mil thick nickel, except in smaller quantities. These thinner layers did not separate from the nickel filament. The nickel was removed over an area approximately .25 inches in diameter by etching it away with a sulfuric acid and nitric acid solution. The residual salts were then removed by repeated water rinsing. The final carbon foils by this method were quite thin (10 to 30 micrograms/cm² thick) but also likely to contain heavy contaminants such as nickel and sulfur.
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APPENDIX A


