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Two Nucleon Stripping Reactions
And The Role Of The Tensor Force

by

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A THESIS
SUBMITTED TO THE FACULTY
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
Doctor of Philosophy

Houston, Texas
June, 1962
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Acknowledgements

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Introduction

In recent years an increasing number of nuclear reactions have been studied experimentally in which more than one nucleon is transferred from the bombarding particle to the target nucleus. The reactions studied are primarily those in which two nucleons are transferred from an incident three or four nucleon complex, for example $(\text{He}^3, p)$, $(\alpha, d)$, etc. The structure of the angular distributions seem to indicate that at high enough energies "direct" reaction mechanisms may be the predominant reaction process.

H. C. Newns, N. Glenndenning, and M. el Nadi have published theoretical treatments of the two-nucleon stripping process considering it as a direct reaction. Recently W. Tobocman in some unpublished work has considered a different approach, using a generalization of the method of Daitch and French which enables one to investigate the importance, for these reactions, of tensor force and finite range effects. These effects are known to be small for deuteron induced reactions (see Chapter III) but the same general arguments do not apply to two nucleon stripping reactions.

In section I and II we formulate a plane wave Born Approximation direct reaction treatment of the general stripping reaction based on the work of W. Tobocman. Section III is devoted to reviewing the deuteron induced reactions using this formalism to make clear why tensor force and finite range effects are small in these reactions. The formalism is applied to the two nucleon
stripping reaction, specifically the \((\text{He}^3, P)\) reaction in
Chapter IV. To investigate the importance of tensor forces we
need to know the properties of the 3-nucleon ground state wave
function and of the residual nucleus wave function. A discussion
of the 3-nucleon ground state is given in Chapter V and Appendix I.
A simple L-S coupling shell model for the residual nucleus is
used to give explicit expressions for the cross section in
Chapter VI. Finally a discussion of the importance of the tensor
force and the ramifications of our formalism are given in Chapter VII,
for the reaction \(\text{He}^4(\text{He}^3, P)\text{Li}^6\).
I. General Stripping Reaction in Plane Wave Born Approximation.

We consider the general stripping reaction $I (H,P) F$. $H$ is a composite particle which has a part $D$ stripped from it by the initial nucleus $I$ leaving in the final state a part $P$ and the residual nucleus $F = I + D$. Though the labels $H$, $D$, $P$ will have obvious significance for the (He$^3$,P) reaction, the formalism to be presented in this section is not specialized to that reaction alone, but is applicable to any nuclides $I$, $H$, $P$, $F$ ($H = P + D$, $F = I + D$) capable of the reaction $I (H,P) F$.

The center of mass differential cross section for the $I (H,P) F$ reaction is

$$
\frac{d\sigma}{d\Omega} = \frac{M_{HF}}{(2\pi F^2)^2} \frac{k_F}{K_H} \frac{1}{(2J_{H+1})(2J_{F+1})} \sum_{\ell_l} |A_{\mu\nu}^{[j][\ell]}|^2
$$

where

- $M_{HF}$ = reduced mass in initial channel
- $M_{PF}$ = reduced mass in final channel
- $k_I$, $k_F$ = relative momentum in initial channel
- $k_H$, $k_P$ = relative momentum in final channel
- $J_A$, $M_A$ = total angular momentum quantum numbers of particle $A$
- $[J]$ = the set $J_I$, $J_F$, $J_H$, $J_P$
- $[\ell]$ = the set $M_I$, $M_F$, $M_H$, $M_P$
- $A_{\mu\nu}^{[j][\ell]}$ = transition amplitude

The basic postulate of the direct reaction model is that the transition amplitude $A_{\mu\nu}^{[j][\ell]}$ can be well approximated by the matrix element of the interaction in the exit (or entrance) channel. Therefore in the plane wave Born approximation
where \( \tilde{r}_A \) denotes the internal coordinates of nuclide A, \( \tilde{r}_{AB} \) is the displacement of the center of mass of nuclide A relative to the center of mass of nuclide B, and \( \varphi_A^{\tilde{r}_{AB}} \) is the internal state function of nuclide A with spin quantum numbers \( J_A, M_A \). It will be useful to consider the label A as a variable (or set of variables) to distinguish the different internal states of nuclide A which have the same total angular momentum quantum numbers \( J_A, M_A \). The \( \varphi_A^{\tilde{r}_{AB}} \) will be assumed to be a complete orthonormal set of state functions for the internal structure of nuclide A, that is

\[
\langle \varphi_A^{\tilde{r}_{AB}} | \varphi_A^{\tilde{r}_{AB}'} \rangle = \delta_{AA'} \delta_{J_A J_A'} \delta_{M_A M_A'}
\]

\[
\sum_{\tilde{r}_{AB}} \langle \varphi_A^{\tilde{r}_{AB}} | = \delta(\tilde{r}_{AB})
\]

\( V_{PF} \) is the interaction of nuclide P with nuclide F (while \( V_{HI} \) is the interaction between nuclides H and I). Since F is the complex D + I (and H the complex P + D) we have \( V_{PF} = V_{PD} + V_{PI} \) (\( V_{HI} = V_{DI} + V_{PI} \)).

We will use \( V_{PF} \). Then we note that due to the presence of the (bound) state function \( \varphi_A^{\tilde{r}_{AB}} \) in the matrix element (2), those regions of configuration space for which P and D are close together will be favored. On the other hand there is no factor in the matrix element that favors regions in configuration space in which P and I are close together. Consequently, in evaluating the matrix element, we make the simplifying approximation of neglecting \( V_{PI} \) in favor of \( V_{PD} \). The neglect of \( V_{PI} \) will be seen to lead to the consequence that
the internal degrees of freedom of nuclides I and P play no active role in the interaction. (See discussion following equation (15)).

Thus the matrix element we will consider is

\[
\mathcal{M}_{\mu P} = \langle e^{-i k_\mu \rho_{\mu P}} \phi_P(\mathbf{q}_P) \phi_I(\mathbf{q}_I) \mathcal{V}_{DP} | \phi_P(\mathbf{q}_P) \phi_I(\mathbf{q}_I) \mathcal{V}_{DP} \rangle e^{-i k_\mu \rho_{\mu P}}
\]

where the matrix element implies in addition to spin sums and integrations over internal spatial coordinates of P, D, I, integration over the two independent vectors \( \rho_{PD} \) and \( \rho_{PI} \). Since it is \( \rho_{PD} \) and \( \rho_{PI} \) that occur in the bound state functions \( \phi_H^{\rho_{PH}} \) and \( \phi_P^{\rho_{PH}} \) we make a change of coordinates from \( \rho_{HI} \), \( \rho_{PP} \) to \( \rho_{PI} \), \( \rho_{PD} \) (the Jacobian of the transformation is unity; see Figure 1). Then writing

\[
\begin{align*}
\frac{k_H}{\rho_{HI}} &= \frac{k_P}{\rho_{PP}} = \frac{\rho_{PD}}{
\frac{\rho_{HI}}{\rho_{PP}}} \\
\frac{k_H}{\rho_{HI}} &= \frac{k_P}{\rho_{PP}} = \frac{\rho_{PD}}{
\frac{\rho_{HI}}{\rho_{PP}}}
\end{align*}
\]

where now in addition to spin sums and integration over the internal spatial coordinates of P, D, I, integrations over \( \rho_{PD} \) and \( \rho_{PI} \) are implied.

\[
\begin{align*}
\rho_{HI} &= \rho_{PI} + \frac{M_P}{M_H} \rho_{PD} \\
\rho_{PD} &= \varepsilon \left( \rho_{PP} - \frac{M_P}{M_H} \rho_{HI} \right) \\
\rho_{PI} &= \varepsilon \left( \rho_{PI} - \frac{M_P}{M_H} \rho_{PD} \right)
\end{align*}
\]

*Figure 1*
In the previous work on the \(^{3}\text{He}, p\) reaction, the matrix element (4) is evaluated by introducing the further simplifying assumption of setting \(R_{N} = R_{p} = R\), where \(N\) and \(p\) denote the two captured nucleons and \(R\) is the nuclear radius. This approach is similar to that used by A. B. Bhatia and K. Huang in their discussion of the \((d, p)\) reaction, and has the advantage that it makes the multiple integral more tractable. At the same time it introduces into the treatment the limitation that the stripping occurs primarily right at the nuclear surface, and, for the \(^{3}\text{He}, p\) reaction in particular it introduces the limitation that the stripping occurs primarily when both stripped particles are right at the nuclear surface.

We use an alternative procedure modeled after that due to P. B. Daitch and J. B. French. From the Schrödinger equation for \(\Phi_{M}^{T_{P} M_{P}}\), we have

\[
\langle V_{D} R \Phi_{M}^{T_{P} M_{P}} = (\epsilon_{M} - T_{P} - H_{D} - H_{P}) \Phi_{M}^{T_{P} M_{P}}
\]

(5)

where \(\epsilon_{M}\) is the internal energy of the particle \(M\), \(T_{P} = -\frac{\hbar^{2}}{2\Delta_{P}} \frac{\partial^{2}}{\partial \rho^{2}}\) is the kinetic energy operator of relative motion of nuclides \(P\) and \(D\); and \(H_{D}, H_{P}\) are the Hamiltonian operators of the internal structure of nuclides \(D\) and \(P\) respectively

\[
(\epsilon_{D} - H_{D}) \Phi_{D}^{T_{P} M_{P}} = 0
\]

(6a)

\[
(\epsilon_{P} - H_{P}) \Phi_{P}^{T_{P} M_{P}} = 0
\]

(6b)

Incorporating (5) and (6b') into the matrix element (4) and applying Green's theorem to the coordinate \(R_{PD}\), so that \(T_{PD}\) operates on \(\Phi_{M}^{T_{P} M_{P}}\), (the surface term vanishes due to \(\Phi_{M}^{T_{P} M_{P}}\)) we obtain:
\[ A_{\nu \rho}^{I_1, J_2, J_3, J_4, M_1, M_2, M_3, M_4} = \langle \eta_{\alpha \beta}^L | \phi_p (\beta, \gamma) \phi_n (\xi_1, \xi_2, \xi_3) | \epsilon_{r} - \epsilon_{p} - \epsilon_{n} - \frac{k^2 l^2}{2 \hbar^2} | \phi_{\nu} (\omega, \nu, \rho, \sigma) \phi_{\rho} (\delta, \rho, \gamma, \delta) \rangle \] (7)

The state functions \( \phi_p \) may be expanded in terms of the \( \phi_{\nu} \) as

\[ \phi_p^{T_{p M_p}} = \sum_{J_{p M_p}} C_{J_{p M_p} \nu M_{p}}^{\ell M_{p}} \phi_{\nu}^{\ell M_{p}} \phi_{p}^{T_{p M_p}} \] (8a)

\[ \phi_{\nu}^{T_{p M_p}} = \sum_{J_{p M_p}} C_{J_{p M_p} M_{p} \nu}^{\ell M_{p}} \phi_{p}^{T_{p M_p}} \phi_{\nu}^{\ell M_{p}} \] (8b)

where

\[ f_{\ell M_{p} \nu M_{p}}^{J_{p M_p}} = f_{\ell M_{p} \nu M_{p}}^{J_{p M_p}}(\hat{r}_{p \nu}) Y_{\ell M_{p}}(\hat{r}_{p \nu}) \] (8c)

\[ C_{J_{p M_p} \nu M_{p} \nu M_{p}}^{\ell M_{p} J_{p M_p}} \]

denotes the vector coupling or Clebsch-Gordan coefficient \( (J_1 J_2 J_3 | J_1 J_2 J_3) \) and \( Y_{\ell M_{p}}(\hat{r}) \) denotes the spherical harmonic as defined by Condon and Shortley. Then the amplitude (7) becomes

\[ A_{\nu \rho}^{I_1, J_2, J_3, J_4, M_1, M_2, M_3, M_4} = \sum_{J_{p M_p} \nu M_{p} \nu M_{p}} C_{J_{p M_p} \nu M_{p} \nu M_{p}}^{\ell M_{p} J_{p M_p}} \phi_{\nu}^{\ell M_{p} J_{p M_p}} \phi_{p}^{T_{p M_p}} \] (9a)

with

\[ A_{\nu \rho}^{\ell M_{p} J_{p M_p}} = \sum_{D} \langle f_{\ell M_{p} \nu M_{p}}^{J_{p M_p}} | \epsilon_{r} - \epsilon_{p} - \epsilon_{n} - \frac{k^2 l^2}{2 \hbar^2} | f_{\ell M_{p} \nu M_{p}}^{J_{p M_p}} \rangle \] (9b)

where the orthonormality of the \( \phi_{\nu}^{T_{p M_p}} \) and the \( \phi_{p}^{T_{p M_p}} \) plus equation (6a) have been used. Inserting (9a) into (1) and performing the sums on the quantum numbers \( J_{p M_p} \) we obtain...
\[
\frac{d\sigma}{d\Omega} = \frac{M_{p'p} M_{m'\pm m}}{(2\pi \hbar)^2} \frac{k_p}{k_n} \frac{2 J_{p+1}}{2 J_{p}} \sum_{J_f} A_{J_f}^{\ell \phi \pm m} \left| A_{J_f}^{\ell \phi \pm m} \right|^2
\]

(10)

where

\[
A_{J_f}^{\ell \phi \pm m} = \sum_{m' \in \{0, \pm 1, \pm 2, \pm 3, \ldots\}} C_{m' \in \{0, \pm 1, \pm 2, \pm 3, \ldots\}} \sqrt{\frac{(2J_f+1)}{2J_{m'}}} (-1)^{m'+J_{m'}} \frac{\ell}{2 J_{f+\ell}} A_{J=J_f}^{\ell \phi \pm m}
\]

(11)

and \(\{a b e \}\) is the Wigner 6-j symbol, related to the Racah coefficient \(\{d c f \}\) by a phase factor: \(\{a b e \} = (-1)^{a+b-c+d} W(ab,cd;ef)\).

In expression (9b) the Fourier transforms of the functions

\[
\mathcal{F}_{J_f}(a) \quad \text{and} \quad \mathcal{F}_{J_f}(b)
\]

occurred; recalling relations (8c) we can immediately perform the angular integrations in these integrals so that

\[
\langle \mathcal{F}_{J_f}(a) | e^{ik \cdot n} \rangle = 4\pi i^{J_f} \mathcal{F}_{J_f}(a) \quad \text{and} \quad \mathcal{F}_{J_f}(b)
\]

(12a)

\[
\langle e^{ik \cdot n} | \mathcal{F}_{J_f}(b) \rangle = 4\pi i^{-J_f} \mathcal{F}_{J_f}(b) \quad \text{and} \quad \mathcal{F}_{J_f}(a)
\]

(12b)

where \((f(n), g(n))\) stands for \(\int d\alpha n^2 f(n) g(n)\) and \(\mathcal{J}_{J_f}(x)\) is the regular spherical Bessel function. Inserting expressions (12) into equation (9b) we may then perform the sums on \(\ell, \phi, m, m'\) in relations (10) and (11) with the result:

\[
\frac{d\sigma}{d\Omega} = \frac{M_{p'p} M_{m'\pm m}}{(2\pi \hbar)^2} \frac{k_p}{k_n} \frac{2 J_{p+1}}{2 J_{p}} \sum_{J_f} \mathcal{B}_{J_f}^{\ell \phi \pm m} \mathcal{P}_{J_f}^{\ell \phi \pm m} A_{J_f}^{\ell \phi \pm m} \left| A_{J_f}^{\ell \phi \pm m} \right|^2
\]

(13)

where

\[
\mathcal{B}_{J_f}^{\ell \phi \pm m} = (-1)^m C_{\ell \phi \pm m} \mathcal{C}_{\ell \phi \pm m} \left[ \begin{array}{c} \ell \phi \pm m \\ J_f J_f \end{array} \right] \left[ \begin{array}{c} \ell \phi \pm m \\ J_f J_f \end{array} \right] \left[ \begin{array}{c} \ell \phi \pm m \\ J_f J_f \end{array} \right] \left[ \begin{array}{c} \ell \phi \pm m \\ J_f J_f \end{array} \right]
\]

(14)
\[ A_{\tilde{L}L'J_J} = \sum_D (\epsilon_n - \epsilon_p - k^2/2m_0) (F_{\tilde{L}}(n) J_L(kn)) (f_{\tilde{L}}(kn)) f_{\tilde{L}'L'}(n) \]

(15)

and \( P_L(x) \) is the Legendre polynomial.

The form of the amplitude \( A_{\tilde{L}L'J_J} \) (equation 15) is reminiscent of what one obtains in the plane wave Born approximation treatment of deuteron induced stripping reactions, the difference being that here the complex structure of the transferred particle is reflected in the sum over all its states of internal excitation \( D \).

In the case of one particle stripping where the transferred particle has no internal degrees of freedom (other than its spin), or if in the general stripping reaction we had neglected the complex structure of the transferred particle, the sum over \( D \) in the amplitude \( A_{\tilde{L}L'J_J} \) would degenerate to a single term.

We also see that the internal structure of nuclides I and P in no way enter the amplitude (15). This result is a direct consequence of our neglecting the interaction \( V_{PI} \) in the matrix element (2). By doing so we were left with the matrix element of \( V_{PD} \) which, because of the presence of \( \Phi_{p_n}^{J_nM_n} \), acted like a diagonal operator with respect to the state functions \( \Phi_{p_p}^{J_pM_p} \) and \( \Phi_{n}^{J_nM_n} \). This is not to say that in this model the final nucleus state function \( \Phi_P^{J_pM_p} \) has non-zero projection only on that state function constructed from \( \Phi_{p_p}^{J_pM_p} \) and \( \Phi_{n}^{J_nM_n} \) in which nuclide I is in its initial state; and that the incident particle state function \( \Phi_n^{J_nM_n} \) has non-zero projection only
on that state composed of $\varphi_{F}^{T_{F}M_{F}}$ and $\varphi_{D}^{T_{D}M_{D}}$ in which nuclide P is in its final state. Rather, it is only these projections of $\varphi_{F}^{T_{F}M_{F}}$ and $\varphi_{M}^{T_{M}M_{M}}$ that can contribute to the stripping cross section. On the other hand $V_{\mathcal{P}I}$ can connect different internal states of $I$ and of $P$, and therefore it will allow other projections of $\varphi_{F}^{T_{F}M_{F}}$ and $\varphi_{M}^{T_{M}M_{M}}$ to contribute to the cross section.

For later discussion it will be useful to rewrite the amplitude (15) in terms of a slightly more general expansion of the state functions $\varphi_{F}^{T_{F}M_{F}}$, $\varphi_{M}^{T_{M}M_{M}}$. Instead of expanding in eigenstates of $H_{D}$ we take arbitrary functions of $\tilde{y}_{D}$:

\[
\varphi_{F}^{T_{F}M_{F}} = \sum_{T_{F}M_{F}J_{F}} \sum_{N_{T}L_{T}M_{N_{T}}} C_{F}^{T_{F}M_{F}} C_{L_{T}}^{T_{F}J_{F}} D_{N_{T}L_{T}}^{T_{F}J_{F}} G_{N_{T}L_{T}}^{M_{N_{T}}M_{F}} \varphi_{F}^{T_{F}M_{F}} \varphi_{I}^{T_{I}M_{I}} \tag{16a}
\]

\[
\varphi_{M}^{T_{M}M_{M}} = \sum_{T_{M}M_{M}J_{M}} \sum_{N_{T}L_{T}M_{N_{T}}} C_{M}^{T_{M}M_{M}} C_{L_{T}}^{T_{M}J_{M}} D_{N_{T}L_{T}}^{T_{M}J_{M}} G_{N_{T}L_{T}}^{M_{N_{T}}M_{M}} \varphi_{F}^{T_{F}M_{F}} \varphi_{I}^{T_{I}M_{I}} \tag{16b}
\]

where again

\[
G^{l,n}_{N_{T}L_{T}}(\tilde{y}_{D}) = G^{l,n}_{N_{T}L_{T}}(\tilde{y}_{D}) \tag{16c}
\]

Then from equations (8) and (16)

\[
F_{T_{F}D}^{T_{F}J_{F}}(\eta_{D}) = \sum_{N_{T}} D_{N_{T}L_{T}}^{T_{F}J_{F}} G_{N_{T}L_{T}}^{l,n}(\eta_{D}) \langle \varphi_{D}^{T_{D}M_{D}} | H_{T}^{T_{F}} \rangle \tag{17a}
\]

\[
f_{T_{I}D}^{l}(\eta_{D}) = \sum_{N_{T}} D_{N_{T}L_{T}}^{T_{I}J_{I}} G_{N_{T}L_{T}}^{l,n}(\eta_{D}) \langle \varphi_{D}^{T_{D}M_{D}} | H_{I}^{T_{I}} \rangle \tag{17b}
\]

so that the amplitude (15) becomes by using the completeness of the

\[
A_{T_{F}D}^{T_{F}J_{F}} = \sum_{N_{T}} D_{N_{T}L_{T}}^{T_{F}J_{F}} D_{N_{T}L_{T}}^{T_{I}J_{I}} \langle G_{N_{T}L_{T}}^{l,n}(\eta_{D}) | H_{T}^{T_{F}}(\eta_{D}) | \varphi_{D}^{T_{D}M_{D}} \rangle \times \langle \varphi_{D}^{T_{D}M_{D}} | (\varepsilon_{n} - \varepsilon_{p} - \hbar k_{p}^{2} \cdot k_{p}^{2} - H_{D}) | \varphi_{D}^{T_{D}M_{D}} \rangle \tag{18}
\]
II. The Stripping Cross Section.

The cross section's angular dependence arises from the presence of the vectors $\vec{k}$ and $\vec{k}'$ in the formulas. In addition to the dependence on these vectors in the Legendre polynomial in expression (13), the amplitude $A^{\ell \ell'}_{\sigma_1 \sigma_2} k_1 k_2$ depends on the magnitudes of $\vec{k}$ and $\vec{k}'$. The specific form of the latter dependence stems directly from the form of the state functions with which we represent the final nucleus F and incident particle H.

At this point we make a further postulate about the mechanism of the stripping reaction in keeping with the plane wave assumption of neglecting the nuclear and Coulomb interactions in the entrance and exit channel. We postulate that the reaction primarily takes place when the center of mass of particle D is at a distance $\tau_{D,E} = R_s$ from the center of mass of nucleus I, and call this the surface interaction assumption. This means that the main contribution to the first integral in (18) will come from a region around $R_s$, so we set

$$ (G^\ell_{\sigma_1}(\tau), J_L(\kappa R_s)) = R_s^2 \Delta R \ G^\ell_{\sigma_1}(R_s) J_L(\kappa R_s) $$

(19)

where $4\pi R_s^2 \Delta R$ is the volume in which we assume most of the stripping to take place. This postulate is analogous to that used in the plane wave treatments of the (d,p) (d,n) reactions by A. B. Bhatia and K. Huang.\(^{(3)}\)

Alternatively we might postulate that the stripping takes place primarily when the distance between the centers of mass of particles D and I is larger than $R_A$. Then $R_A$ is taken to be the
lower limit in the radial integral \( (G^L_{\lambda\sigma}(\gamma), \bar{J}_L(k\gamma)) \).

Generally \( R_A \) is taken to be equal to or larger than the "radius" of the initial nucleus. We then further assume that under these conditions the wave function \( G^L_{\lambda\sigma}(\gamma) \) can be taken to have its asymptotic form. We will call this the asymptotic interaction assumption. In this case

\[
(G^L_{\lambda\sigma}(\gamma), \bar{J}_L(k\gamma)) = \mathcal{G}^L_{\lambda\sigma}(R_A) \int_{R_A}^{\infty} \frac{d\gamma}{\gamma^2} \frac{\tilde{G}^L_{\lambda\sigma}(\gamma)}{\tilde{G}^L_{\lambda\sigma}(R_A)} \bar{J}_L(k\gamma)
\]

where \( \tilde{G}^L_{\lambda\sigma}(\gamma) \) is the asymptotic form of \( G^L_{\lambda\sigma}(\gamma) \).

For example if \( D \) is a neutral particle then \( G^L_{\lambda\sigma}(\gamma) \) is the hyperbolic spherical Hankel function

\[
\tilde{h}^{(i)}_{L}(\pm k\gamma) = \bar{J}_L(\pm k\gamma) + i \eta_L(\pm k\gamma)
\]

where

\[
\frac{\hbar^2 k^2}{2\hbar_p D} = \varepsilon_1 - \varepsilon_2 + \sum_{\ell, l, \mu} D_{N, \ell, \mu} \frac{d \bar{J}_{L}\bar{J}_{L}^{*}}{d \varepsilon_{N, \ell, \mu}} \langle \bar{\tilde{h}}_{L}^{(i)} | H_{\sigma} | \tilde{h}_{L}^{(i)} \rangle
\]

In this case

\[
(G^L_{\lambda\sigma}(\gamma), \bar{J}_L(k\gamma)) = \mathcal{G}^L_{\lambda\sigma}(R_A) \left\{ \int \frac{R_A^2}{k^2 + k^2_0} \left[ \bar{J}(k\gamma) \frac{d \bar{J}_{L}}{d \gamma} \tilde{h}^{(i)}_{L}(c k \gamma) \right] \right\}_{R = R_A}
\]

If nuclide \( D \) is charged then the asymptotic form of \( G^L_{\lambda\sigma}(\gamma) \) will be the Coulomb analog to the Hankel function.

Usually the remaining factors in (18) will have a weaker angular dependence than \( (G^L_{\lambda\sigma}(\gamma), \bar{J}_L(k\gamma)) \). Then it is convenient to write the amplitude (18) as

\[
A^{\sigma \ell \ell}_{x_i \bar{J}, \hat{x}_j} = \sum N \mathcal{W}^L_N(k) \mathcal{A}^{\sigma \ell \ell}_{x_i \bar{J}, \hat{x}_j}(k)
\]

where

\[
\mathcal{W}^L_N(k) = R_s^2 \Delta R \bar{J}_L(k R_s) \quad \text{(using the surface interaction assumption)}
\]
or

\[ W^L_N(k) = \sum_{n=1}^{\infty} \frac{C^L_N(n)}{E_n} J^L(kn) \text{(using the asymptotic interaction assumption)} \]  

(23)

and

\[ \alpha^L_{\sigma_{p_j} \sigma_{f_j}} = \sum_{\sigma_{\tau_1 \tau_2 \tau_3 \tau_4}} D_{\sigma_{\tau_1 \tau_2}} D_{\sigma_{\tau_2 \tau_3}} G^{L\tau_3}_{\tau_3 \tau_4} (g^L_{\sigma_{\tau_1}}, g^L_{\sigma_{\tau_2}}) \langle \chi^L_{\tau_3} | (\varepsilon^L_{\sigma_{\tau_1}} - \varepsilon^L_{\sigma_{\tau_2}} - i \eta) | \chi^L_{\tau_4} \rangle \]  

(24)

which is weakly angularly dependent relative to \( W^L_N(k) \). In all following discussions we will exploit the surface interaction assumption, so that \( W^L_N(k) \) depends only on L and the sum on N may be performed on expression (24) to give

\[ \alpha^L_{\sigma_{p_j} \sigma_{f_j}} = \sum_{N} \alpha^L_{N} \]  

If we had considered each of the three complexes I, P, and D, as indivisible and structureless then the surface interaction assumption above would be the direct generalization of the assumption used in the plane wave treatment of the (d, p) (d, n) reaction due to A. B. Bhatia and K. Huang; while the asymptotic interaction assumptions would be the direct analog of the assumption used in the Butler treatment of (d, p) (d, n) reactions.

The angular dependence of the amplitude \( \alpha^{HP} \) is therefore very sensitive to the selection rules for L. From (14) it is clear that the selection rules for L play a large role in determining which \( \mathcal{P}_L(\hat{k} \cdot \hat{k}) \) contribute to the cross section (13). The selection rules on L follow from conservation of angular momentum and parity. Denoting by \( \Delta (J_1, J_2, J_3) \) the set of three triangle
inequalities obtained from \[ |J_2 - J_3| \leq J_1 \leq |J_2 + J_3| \]
by cyclic permutation of the label 1, 2, 3 the selection rules for \( L \) are

\[
\Delta (L J_D) \quad \Delta (J_F J_L)
\]

\[
(-1)^L = \pi_F \pi_L \pi_D
\]

where \( \pi_A = +1 \) if the parity of nuclide \( A \) is even,

\( \pi_A = -1 \) if it is odd. For \( \ell \) we have

\[
\Delta (\ell J_D) \quad \Delta (J_F J_L)
\]

\[
(-1)^\ell = \pi_F \pi_L \pi_D
\]
III. Deuteron Induced Stripping Reactions.

Before examining the two nucleon stripping reaction with the formalism developed above, it will be instructive to review the deuteron induced single nucleon stripping cross section in terms of our formalism. As already mentioned, the sum over internal states of the transferred particle in expression (15) reduces to a single term in this case. Making the obvious changes in notation (H now stands for the incident deuteron, D for the transferred nucleon, etc.) and dropping superfluous labels the cross section becomes:

\[
\frac{d\sigma}{d\Omega} = G^p_{Fe} G^\pi_{EX} \frac{k_p}{k_0} \frac{2J_{Fe+1}}{2J_{EX}+1} \sum_{L,L',J,J',I} B_{L,I,J',I}^L D_{L,I,J} P^L_{(I,J)} \mathcal{A}^{L,I,J}_{J',I} \mathcal{A}^{L',I,J}_{J',I} (27a)
\]

where

\[
\mathcal{A}^{L,I,J}_{J',I} = (e_d - \frac{k^2}{2m_p}) (F^{L}_{E,J}, J_I(kn))(F^{L}_{E,J}, J_I(kn)) \quad (27b)
\]

Now \( l \) is the relative orbital angular momentum of the neutron and proton in the deuteron, \( L \) the orbital angular momentum of the captured nucleon, \( f_{l(n)} \) the deuteron radial wave function, and \( F^{L}_{E,J}(y) \) the radial wave function of the captured nucleon.

Invoking the zero range hypothesis for the \( n-p \) interaction allows only \( l = 0 \). Further the last factor exactly cancels the first in equation (27b), so

\[
\left( \frac{d\sigma}{d\Omega} \right)_{\text{zero range}} = G^p_{Fe} G^\pi_{EX} \frac{k_p}{k_0} \frac{2J_{Fe+1}}{2J_{EX}+1} \sum_{J} \frac{1}{2} (F^{L}_{E,J}, J_I(kn))^2 \quad (28)
\]
where we have used the property

\[ B_{\ell_1,\ell_2, l_1, l_2}^{L,2} = \delta_\ell_0 \delta_{\ell,\ell_2} \delta_{\ell_1, j_1} \delta_{\ell_2, j_2} \frac{I}{2 j_2 + 1} \]  

(29)

In expression (28) we have not specified the interaction assumption as it is not important for the discussion below. The restrictions on \( L \) are

\[ \Delta (L, J^z_L) \quad \Delta (J^z_F, J^z_T) \]

\[ (-1)^L = \Pi_L \quad \Pi_T \]  

(30)

In we had not used the zero range approximation and had allowed for tensor interaction between nucleons, then both \( L = 0 \) and \( J = 2 \) are present. However due to the fact that we are transferring a spin 1/2 particle (the captured neutron or proton) from an even parity bombarding particle (the deuteron), the coefficient

\[ B_{\ell_1,\ell_2, l_1, l_2}^{L,2} \]

still only allows \( L = 0 \) and requires that \( L_1 = L_2 \), \( j_1 = j_2 \). Consequently the cross section becomes an incoherent sum of the contribution due to the S and D states separately, that is

\[ \frac{d\sigma}{d\Omega} = \frac{M_{PF} M_{pF}}{(2\pi)^2} \frac{k_p}{k_0} \frac{2 J_{F+1}}{2 J_{T+1}} \sum_{J} \frac{|A_{J}^{L=0}|^2 + |A_{J}^{L=2}|^2}{2} \]

Upon substituting relation (27b) and again leaving unspecified the interaction assumption we have (7)

\[ \frac{d\sigma}{d\Omega} = \frac{M_{PF} M_{pF}}{(2\pi)^2} \frac{k_p}{k_0} \frac{2 J_{F+1}}{2 J_{T+1}} (m_0^2 + m_1^2) \sum_{J} \left( \frac{\gamma_{L}(k,n)}{2} \right)^2 \]

(31)

where \( \gamma_{L} = (\epsilon_{P} - \frac{k^2 n^2}{2m_{PN}}) \left( g_{L}(k,n), f_{L}(k,n) \right) \)
Consequently for deuteron induced stripping the sole effect of allowing finite range n-p forces and the inclusion of the deuteron D state is to multiply the zero-range cross section \((28)\) by the factor \(M_0^2 + M_2^2\). If we had neglected the D state contribution this factor would have been simply \(M_0^2\). Therefore \(M_0^2\) represents the finite range effect for central forces, and \(M_2^2\) the effect of including tensor forces. Since the binding energy of the deuteron is small, the expectation value of the neutron-proton separation is larger than the range of the central interaction and we would not expect the central potential finite range effect to be appreciable. As far as \(M_2^2\) is concerned we expect it to be significantly smaller than \(M_0^2\) because the amount of D state is small compared to the amount of S state in the deuteron and we expect the overlap of \(\psi_0(\eta)\) with \(\varphi_2(\kappa \eta)\) to be no better than that of \(\psi_0(\eta)\) with \(\varphi_0(\kappa \eta)\). Therefore for deuteron induced reactions the tensor force can not appreciably affect the stripping cross section.

The simplicity of the stripping cross section for deuteron induced reactions is therefore seen to arise from two different aspects of the reaction. First the particle transferred in the reaction can have a total angular momentum of 1/2 only and it comes from a complex in which it can exist only in states of even orbital angular momentum with respect to the rest of the complex. Consequently the stripping cross section is an incoherent sum of the contributions from each value of \(l\) (and of each value of \(L\)). No interference terms giving rise to terms proportional to \(P_z(\hat{k} \cdot \hat{k})\) can occur.

Second, the quantitative properties of the deuteron structure insure
that the predominant contribution to the stripping process is from $\ell = 0$, and that the finite range of the central part of the n-p interaction will have no appreciable effect.
IV. Two Nucleon Stripping Reactions.

The stripping reactions next in complexity to the \( (d,p) \), \( (d,n) \) reactions are those reactions in which two nucleons are stripped from an incident three nucleon complex. Specifically these are the reactions \( (\text{He}^3,P) \), \( (\text{He}^3,n) \), \( (t,p) \), \( (t,n) \). Let us apply the formalism developed in the first two sections, using the surface interaction assumption when needed. To make the discussion clearer we will deal explicitly with the \( (\text{He}^3,P) \) reaction so \( D \) will now stand for a complex of a neutron and a proton; \( P \) the emergent proton; and \( \varepsilon_P = 0 \).

The functions \( \mathcal{H}^{L,M_0}_{\ell} \), \( \mathcal{H}^{J,M_0}_{\ell} \) appear explicitly as:

\[
\mathcal{H}^{J,M_0}_{\ell} = \mathcal{H}^{J,M_0}_{\ell,\lambda} (n, \nu_P) \Phi_{\ell,\lambda}^{J,M_0} \tag{32a}
\]

\[
\mathcal{H}^{J,M_0}_{\ell} = \mathcal{H}^{J,M_0}_{\ell,\lambda} (n, \nu_P) \Phi_{\ell,\lambda}^{J,M_0} \tag{32b}
\]

where the general label \( T \) \((t)\) has been replaced by the set \( T = (t, \lambda) \) to show the orbital and spin angular momentum quantum numbers explicitly. Here the \( \Phi_{\ell,\lambda}^{J,M_0} \) are the orthonormal spin angle functions for the two body system:

\[
\Phi_{\ell,\lambda}^{J,M_0} = \sum_{\rho, \sigma} \sum_{\varphi, \lambda'} C_{\lambda \rho \sigma} \Phi_{\ell,\lambda}^{J,M_0} \Phi_{\rho, \lambda'}^{J,M_0} \langle \rho | \sigma | \nu_P \rangle \tag{33}
\]

where \( \Phi_{\rho, \lambda'}^{J,M_0} \) denote now simply the spin state functions of nucleons \( n \) and \( p \) respectively.
The Hamiltonian operator $H_D$ appearing in expression (24) is now that for the n-p system. Since we will be interested in the effect of including tensor forces, and not primarily in detailed agreement with experiment we will assume as simple a phenomenological form for the two nucleon interaction as is consistent with the main features of two nucleon system at modest energies. We will neglect exchange effects, and take a combination of tensor and spin dependent central potentials. Explicitly then we take

$$V_{n-p} = -V_0 \left\{ V(a_n, n) \frac{1 + P_\perp}{2} + \mu \cdot V(a_n, n) \frac{1 - P_\perp}{2} + \nu \cdot V(a_n, n) \cdot S_{n-p} \right\}$$  

(34)

where $S_{ab}$ is the tensor operator, $S_{ab} = 3 \left( \frac{\sigma_a \cdot \sigma_b}{2} \right) \left( \sigma_a \cdot \sigma_b \right) - \frac{1}{2} \sigma_a \sigma_b$ in terms of the Pauli spin operators $\sigma_i$. $V(x)$ is the radial form factor for the nuclear part of the nucleon-nucleon interaction which we will take to be of Gaussian shape $V(x) = e^{-x^2}$. The parameters $V_0, \mu, \nu, a_{1n}, a_{1p}, a_{2n}, a_{2p}$ will be those determined by L. C. Biedenharn, J. M. Blatt, and M. H. Kalos to fit the deuteron binding energy and quadrupole moment, and the low energy n-p singlet and triplet effective ranges.

The last factor in the amplitude (24) becomes

$$\langle H_T^b | (\epsilon_n - \epsilon_p - \frac{p^2}{2M_{np}} - H_0) | \phi^b \rangle \rightarrow \sum \theta_{T^b \lambda^b \lambda'^b}$$

$$\theta_{T^b \lambda^b \lambda'^b} = \delta_{\lambda^b \lambda'^b} \int d \lambda^b H_{T^b \lambda^b} \left\{ \epsilon_n - \frac{p^2}{2M_{np}} + \frac{k^2}{2M_{np}} \left( \frac{1}{a_n^2} - \frac{1}{a_p^2} \right) \right\} \gamma^b H_{\lambda^b}^a(n)$$

$$+ \nu \int d \lambda^b H_{T^b \lambda^b} \left( V(a_n, n) \frac{1 + P_\perp}{2} + \mu \cdot V(a_n, n) \frac{1 - P_\perp}{2} + \nu \cdot V(a_n, n) \cdot S_{n-p} \right) \gamma^b H_{\lambda^b}^a(n)$$

(35)
where we have anticipated the reality of the functions $H$, $h$.

$S_{\lambda \lambda'}^{J}$ is the matrix element of the tensor operator between two spin angle functions:

$$S_{\lambda \lambda'}^{J} = \langle \Phi_{\lambda}^{J} | S_{ab} | \Phi_{\lambda'}^{J} \rangle$$

$$= \delta_{J' J} \delta_{\lambda \lambda'}^{2} \delta_{a a'} (-1)^{J'} 6^{J/2} \sqrt{\frac{(2J+1)(2J'+1)}{(2J+1)(2J'+1)}} c_{\lambda \lambda'}^{2} \{ \lambda \lambda' J \}$$

The selection rules (25), (26) become

$$\Delta (\ell J_{D}) \quad \Delta (\pi \lambda \lambda) \quad \Delta (J_{F} \pi_{L}) \quad \lambda = 0, 1$$

$$(-1)^{\ell + \lambda} = \pi_{F} \pi_{L}$$

(37)

and

$$\Delta (\ell J_{D}) \quad \Delta (\ell \lambda \lambda) \quad \Delta (J_{F} \pi_{L}) \quad \lambda = 0, 1$$

$$(-1)^{\ell + \lambda} = 1$$

(38)

The second relation in (38) is a consequence of the incident three nucleon complex being in its ground state which is of positive parity.

In the absence of tensor forces between nucleons, the ground state of the three nucleon system would be one in which all three particles are in relative $S$ states. In this case both $\ell$ and $\lambda$ in (38) are zero, thereby only allowing $\lambda = 0$ in equations (37) since central interactions can not couple different values of orbital angular momentum. Then the selection rules become:

$$\Delta (\ell J_{D}) \quad \Delta (J_{F} \pi_{L}) \quad (-1)^{\ell} = \pi_{F} \pi_{L}$$

(39)

Using equation (29) we have
\[
\frac{d\sigma}{d\Omega} = \frac{M_{nF} M_{nX}}{(2\pi)^2} \frac{k_F}{k_u} \frac{2J_F+1}{2J_u+1} \sum_{j=0}^{J_u} |A^{2LO}_{J_J}|^2 \frac{1}{2J_F+1}
\]  
(40a)

where

\[
A^{2LO}_{J_J} = R^2 n \Delta R J_{2F} (kR_p) Q^{2LO}_{2J_F} (k)
\]  
(40b)

\[
Q^{2LO}_{2J_F} (k) = \sum_{\ell=1}^{N_T} D_{2F}^{J_J} \ast D_{2F}^{J_F} \left( \int d\xi n^2 J_{0} (k n) J_{0} (n) \right)
\]  
(40c)

\[
x \int d\xi n^2 H_{toj}^j (n) \left[ \frac{\xi^2}{2M_p} - \frac{\xi^2}{2M_p} \frac{\partial^2}{\partial n^2} \right] + v_0 \left[ \xi^2 \nu (a, n) + \xi^2 \mu \nu (a, n) \right] \right] \frac{1}{2J_F+1}
\]

and to be specific we have invoked the surface interaction assumption.

From equation (40a) we see that the contributions due to different \(L\) values add incoherently as they do in deuteron induced stripping reactions. However here this is a consequence of assuming that only central nucleon-nucleon forces are present, while for the deuteron induced stripping cross section this result holds even when tensor forces are present. Note also that if either \(J_F\) or \(J_I\) is zero, the selection rules (39) allow the contribution
of at most only two values of $L$ to the cross section.

If we include tensor forces between nucleons, values of $J$ and $\lambda$ greater than zero will be present in the three nucleon ground state. Since the total angular momentum of the two transferred particles can never be $1/2$, values of $\lambda$ greater than zero can occur in the cross section (13). Consequently there will in general be interference terms, the importance of which will depend primarily on the details of the three nucleon ground state. Furthermore the selection rules are not as restrictive as (39) so that for example in the situation where either $J_1$ or $J_F$ is zero more than two values of $L$ may now contribute. Therefore to proceed further in investigating the effects of the tensor force we must say something more specific about the structure of the three nucleon ground state and the final nucleus.
V. The Three Nucleon State Function.

The classification of the allowed configurations in the ground state of the three nucleon system has been considered \( (q,\mu) \) (10) for some time. Sachs introduced isotopic spin into the classification (11), while Blatt and Derrick gave a more thorough and systematic discussion of the admissible symmetries (with respect to the Permutation Group on 3 objects). The spatial coordinates used by Blatt and Derrick, while being well suited to a discussion of permutation symmetry, are very inconvenient for our stripping formalism. The coordinates used by Gerjuoy and Schwinger and Sachs, however, are more easily related to those which we need.

While the introduction of the isotopic spin formalism leads to a formally more elegant classification, it in no way introduces new physics and an equivalent classification can be given without recourse to it. Such a classification is given in Appendix 1. The result is that there are ten types of permissible configurations in a classification according to orbital angular momentum and symmetry (under the permutation group on three objects \( S_3 \)) of the space part of the wave function. Of these ten states the most important will be the space symmetric \( S \) state for in the presence of purely central spin independent forces this will be the only configuration in the ground state. The inclusion of tensor forces will mix in the three \( D \) states in first order, the \( P \) states only in second order. Therefore the configurations which we expect to have the largest amplitudes
in the three nucleon ground state are the space symmetric S and
the three D states.

The simplest three-nucleon wave function then will be of
the form

$$\Phi_{\mu}^{M} = A_1 \Phi_{\mu}(n_1, R_1) \Phi_{\mu}^{M}$$

$$+ \sum_{M_3} \left\{ A_2 \Phi_{\mu}(n_3, R_3) \frac{T_{13}}{2, 1} \right. + A_3 \Phi_{\mu}(n_3, R_3) \left( \frac{T_{13}}{2, 1} + \frac{3}{2} \frac{T_{23}}{2, 2} \right)$$

$$\left. + A_4 \Phi_{\mu}(n_3, R_3) \left( \frac{T_{13}}{2, 1} - \frac{3}{4} \frac{T_{23}}{2, 2} \right) \right\} \frac{1}{2} \frac{M}{M_1, M_2} \Phi_{\mu}$$

where the $f_1(r_3, R_3)$ are functions completely symmetric in the
positions of the three nucleons. We have taken nucleons 1 and
2 to be the two identical ones; $\Delta R_{13}$ is the vector displacement
of nucleon 1 from nucleon 2, $R_{3}$ the displacement of nucleon 3
from the center of mass of nucleons 1 and 2 (see figure 2). The
spherical tensors $T_{LL_1, L_2}^{M} (n_1 R)$ are those defined in Appendix I.

In all calculations we use Gaussian forms for the functions

$$f_1(r_3, R_3) \text{ -- namely } f_1(r_3, R_3) = N_1^{-\frac{1}{2}} e^{-\frac{1}{2} \kappa^2 (n_1^2 + n_2^2 + n_3^2)}$$

$$(n_1^{-1/2} \text{ being a normalization factor}.$$

We should expect that the first D state in (41) should
occur with the largest amplitude since it is proportional to
the product of two $L=1$ solid harmonics -- the other two are
proportional to the product of an $L=1$ and $L=3$ solid harmonic.
The last D state should be least important since, referring to
expression (A7), we see that $T_{213} -3/4 T_{231}$ is proportional to a
product of the components of two mixed symmetry functions while
\( T_{213} + \frac{3}{4} T_{231} \) is proportional to a product of a component of a mixed symmetry function with a symmetric function. This is not to say that the second two D states will be unimportant for a calculation of the properties of the three nucleon ground state (for example the binding energy) but the amplitudes will be smaller than that of the first D state.

To test this point and also to determine values of the parameters of the function (41) (the amplitudes \( A_i \) and size-parameters \( \gamma_i^2 \)) we performed a variational calculation of the energy using two body potentials with Gaussian radial dependence. The parameters of the potential were taken from the paper by Blatt, Biedenharn and Kalos. Also a set of parameters giving a spin independent central potential was employed.\(^{(12)}\)

For all these sets of potential parameters the amplitude of the second D state was 1/3 to 1/4 of that of the first D state, while the third D state amplitude was approximately 1/2 of the amplitude of the second D state. The resultant values of \( \gamma_2^2 \) were quite insensitive to the potential parameters, the values being

\[
\gamma_1^2 \approx 0.1 f^{-2} \\
\gamma_2^2 \approx \gamma_3^2 \approx \gamma_4^2 \approx 2 \gamma_1^2
\]

\(^{(12)}\)

However all these potentials give a large percentage D state (ranging from 7 to 25 percent) compared to the 4% one deduces from the magnetic moment. The potentials giving a binding
energy near the experimental value of $-8.48$ Mev. (the triton binding energy, since we are not including the coulomb repulsion) give a percent D state of approximately 13%. The total amount of D state is quite insensitive to the presence or absence of the last two D states, but the contribution of these two states to the energy could be appreciable (together they contributed 2 to 3 Mev. binding in those cases where the binding energy was approximately $-8.48$ Mev.). Finally the signs of $A_1$ and $A_4$ are always the same and opposite to the signs of $A_2$ and $A_3$.

Since the variational calculation for the energy indicates that the amplitudes $A_3$ and $A_4$ are at most $1/3$ of $A_2$ we will neglect the last two D states and investigate the role of the first D state in the $(He^3, P)$ reaction. Then we will have a 3 body wave function of the simpler form:

$$
\Phi_{\ell M \ell'} = A_1 \Psi_{\ell} \left( r_{3}^{\ell} + \frac{2}{3} r_{3}^{\ell} \right) \Phi_{0}^{\ell M \ell'} + A_2 \Psi_{\ell} \left( r_{3}^{\ell} + \frac{2}{3} r_{3}^{\ell} \right) \sum_{M_{l}, M_{l'}} \Phi_{\ell M_{l} M_{l'}}^{\ell M_{l} M_{l'}} \Phi_{\ell M_{l} M_{l'}}^{\ell M_{l} M_{l'}} \Phi_{\ell M_{l} M_{l'}}^{\ell M_{l} M_{l'}}
$$

(43)

While $r_{3}, R_{3}$ are convenient coordinates to discuss the classification of the three nucleon ground state function and are the appropriate coordinates for our stripping formalism if we wished to discuss $(T, P)$ or $(He^3, N)$ reactions, they are not the set to discuss the $(He^3, P)$ reaction. For the $(He^3, P)$ reaction we must rewrite our wave function (41) in terms of the set $(r_{1}, R_{1})$ or $(r_{2}, R_{2})$. Let us say 2 and 3 are the captured nucleons so we use $(r_{1}, R_{1})$. Now
\[
R_3 = -\frac{1}{2} (R_1 - \frac{3}{2} R_2) \\
\eta_3 = \pi_1 - \frac{1}{2} \pi_2
\]

and \( R_3^2 + \frac{3}{4} \frac{r_3^2}{3} = R_1^2 + \frac{3}{4} \frac{r_1^2}{1} \).

The tensor \( \sum_{L L_1 L_2}^{(L)} (r_3, R_3) \) may be written as a sum of tensors of rank \( L \) of \( (r_1, R_1) \) (see Appendix II). Using the results in Appendix II we may write (43) in the form needed for our stripping theory (see equations (16), (32), (33)):

\[
\Phi_{\ell}^{M} = \sum_{L, M_1, M_2} C^{M}_{L, M_1, M_2} C^{\ell}_{L, M_2} \Phi_{\ell}^{M_1, M_2} \psi \left( \frac{n}{R_2} \right) \lambda \left( \frac{n}{R_1} \right) g^\ell_n (n) h^\lambda_n (n)
\]

where

\[
g^\ell_n (n) = n^{\ell} e^{-\frac{1}{2} n^2 \frac{2}{R_2}}
\]

\[
h^\lambda_n (n) = n^\lambda e^{-\frac{1}{2} n^2 \frac{2}{R_1}}
\]

and

\[
d_{\ell M_1, M_2}^{\ell M_1, M_2} = \delta_{\ell M_1} \delta_{\lambda, 2(n-1)} d_{\ell M_2}^{\ell M_2}
\]

the non-zero values of \( d_{\ell M_1}^{\ell M_1} \) being

\[
d_{10}^{10} = -\frac{1}{2} A_1 \\
d_{20}^{20} = -\frac{1}{4} A_2 \\
d_{21}^{21} = -\frac{A_2}{6}
\]

\[
A_1 = A_1 N_1^{-\frac{5}{2}} \\
A_2 = A_2 N_2^{-\frac{5}{2}}
\]

\[
N_1 = \pi \frac{\xi}{6} 3^{\frac{2}{3}} 2^{-\frac{7}{3}} \\
N_2 = \pi \frac{\xi}{6} 3^{\frac{2}{3}} 2^{-\frac{11}{3}}
\]
VI. Stripping Cross Section for L-S Coupling Shell Model

Residual Nucleus.

In order to introduce as little complication as possible into the discussion we will take a simple model for the residual nucleus. We assume a simple IS coupling shell model picture using harmonic oscillator wave functions and suppose that the two captured particles are in the first P shell. Then the final nucleus function will be written as:

\[ \Phi_{F}^{\tau P M_F} = \mathcal{N}^{-\frac{1}{2}} \sum_{\lambda, m_1, m_2, m_3} C_{\lambda \tau}^{M_F} C_{\lambda \sigma}^{M_F} C_{\lambda m_1}^{M_1} C_{\lambda m_2}^{M_2} C_{\lambda m_3}^{M_3} \times \psi_{01}^{m_2} (\beta \alpha_{2}) \psi_{01}^{m_3} (\beta \alpha_{3}) \phi_{1}^{\pm M_1} \phi_{3}^{\pm M_2} \phi_{-}^{\pm M_3}. \]

\( \psi_{nl}(\alpha \beta) \) are the orthonormal harmonic oscillator functions:

\[ \psi_{nl}(\alpha \beta) = \mathcal{N}_{nl}(\beta) \frac{f_{nl}(\beta \alpha)}{\sqrt{\Gamma(n+\ell+\frac{3}{2})}} \]

\[ \mathcal{N}_{nl}(\beta) = \frac{\beta^\ell}{\Gamma(\ell+\frac{3}{2})} \sqrt{\frac{2^{n+\ell} \beta^3 \Gamma(n+\ell+\frac{3}{2})}{n!}} \]

\[ f_{nl}(\beta \alpha) = n^\ell e^{-\beta^2 \alpha^2} \Gamma(-n, \ell+\frac{3}{2}; \beta^2 \alpha^2) \]

(45)

\( \Gamma(x) \) being the Gamma function and \( \text{F}_1 (a, b; x) \) the confluent hypergeometric function.

Using the transformation coefficients of Brody and Moshinsky we may rewrite \( \Phi_{F}^{\tau P M_F} \) in terms of a sum of
products, each product containing a harmonic oscillator function of the relative coordinate of nucleons 2 and 3 and an oscillator function of the displacement of the center of mass of nucleons 2 and 3 from the initial nucleus I (i.e. the Talmi transformation).

Then:

$$
\varphi_F^{J=0} = N^{-\frac{1}{2}} \sum_{J=0}^{\infty} \sum_{\Lambda=-J}^{J} C_{J_{24}M_2J_4}^{J_{13}M_{13}J_{13}} C_{LM_{1b}M_{1b}}^{J_{13}M_{13}J_{13}} \Phi_{\alpha \Lambda}^{(23)}(23) \Phi_{\beta \Lambda}^{(12)}(12) Y_L^M(\hat{n}^b) \times \frac{f_{\Lambda}(\hat{n}_1)}{f_{\Lambda}(\hat{n}_2)} \frac{f_{\Lambda}(\hat{n}_3)}{f_{\Lambda}(\hat{n}_4)}
$$

$$
\times \left\{ N_0^{(23)} \left( \frac{\beta}{\alpha} \right) N_0^{(12)} \left( \frac{\alpha}{\beta} \right) \left( \frac{1}{\sqrt{2J_1+1}}(2J+1) \right) \times \left\{ \frac{C_{LM_{1b}M_{1b}}^{J_{13}M_{13}J_{13}}}{C_{J_{24}M_2J_4}^{J_{13}M_{13}J_{13}}} \langle \nu \lambda, \nu \lambda, \tilde{L}_1 | 01, 01, \tilde{L}_1 \rangle \right\} \right\} (46)
$$

The non-zero values of the transformation coefficients

$$
\langle \nu \lambda, \nu \lambda, \tilde{L}_1 | 01, 01, \tilde{L}_1 \rangle
$$

are:

$$
\langle 10, 00, 0 | 01, 01, 0 \rangle = -\langle 00, 10, 0 | 01, 01, 0 \rangle = \frac{\sqrt{2}}{2}
$$

$$
\langle 01, 01, 1 | 01, 01, 1 \rangle = 1
$$

$$
\langle 02, 00, 2 | 01, 01, 2 \rangle = -\langle 00, 02, 2 | 01, 01, 2 \rangle = \frac{\sqrt{2}}{2}
$$

and the overall normalization constant $N$ occurring in (46) is simply the number of $J$ values allowed.

In expression (46) we have recoupled some angular momenta in order to express $\varphi_F^{J=0}$ in the form (16),

$$
\varphi_F^{J=M} = \sum_{J=0}^{\infty} \sum_{\Lambda=-J}^{J} C_{J_{24}M_2J_4}^{J_{13}M_{13}J_{13}} C_{LM_{1b}M_{1b}}^{J_{13}M_{13}J_{13}} \Phi_{\alpha \Lambda}^{(23)}(23) \Phi_{\beta \Lambda}^{(12)}(12) Y_L^M(\hat{n}^b) \times H_{\Lambda}(\hat{n}^b) \Phi_{\alpha \Lambda}^{(23)}(23) \Phi_{\beta \Lambda}^{(12)}(12) \varphi_F^{J=M} (48a)
$$
with

\[ G_n^L(n) = \tilde{f}_{\lambda \nu} (\sqrt{2} \beta n) \]

\[ H_{\lambda \alpha} (n) = \tilde{f}_{\lambda \alpha} (\frac{\beta}{\rho} n) \]

\[ D_{\nu \nu + 1} = AN^{-1} e^{i n + \frac{1}{2} \sqrt{2 \nu + 1}(2 \nu + 1)} \left\{ \begin{array}{ll} L \lambda \nu \nu+1 \\ \frac{1}{2} \nu \end{array} \right. \]

\[ \times N_{\nu} \left( \frac{\beta}{\rho} n \right) N_{\nu} \left( \frac{\beta}{\rho} \right) \left\langle \nu \lambda, \nu \lambda, L \mid 01, 01, L \right\rangle \]  

(48c)

Using Gaussian shapes for the n-p potential (34) the two types of integrals appearing in the stripping amplitude (24) (with expression (35) and the wave functions (44), (46)) are

\[ J_1(k) = \int_0^\infty d\beta \phi_{\lambda \nu}(\beta \nu) e^{-\frac{k}{2} \beta^2} = \frac{1}{2} \frac{k^2}{(2 \nu + 1) \nu + 1} \left( \begin{array}{ll} 2 \nu + 1 \\ \nu \end{array} \right) \]

and

\[ F_{\lambda \nu}(\alpha, \beta) = \int_0^\infty d\nu n \nu e^{-\alpha^2 \nu^2} \int_1 (-\nu \lambda + \frac{3}{2}, \beta^2 \nu^2) \]

\[ = \frac{\pi \left( \frac{\beta^2}{2} \right)}{2 \alpha^{\frac{\alpha^2}{2}}} \int_1 (-\nu \lambda + \frac{3}{2}, \beta^2 \nu^2) \]

(50)

where \( \frac{\beta^2}{2} \) is the hypergeometric function. In terms of these integrals the amplitude \( \varphi_{\lambda \nu}^{J_0 \lambda \lambda} \) can be written (where we have suppressed superfluous subscripts and superscripts):
\[
A_{j,j} = \sum_{N_{TL}} D_{N_{TL}}(\mathcal{L}) \delta_{n_{j},N_{TL}} \delta_{j,j} \sum_{e_{j}} G_{N_{TL}}^{e}(R_{e_{j}}) \mathcal{J}_{j}(4) \Theta^{T_{0},LL}_{N_{TL}}(k)
\]

(51a)

where as a consequence of (47) we have written

\[
\frac{\sum_{T_{0}} D_{N_{TL}}(\mathcal{L})}{D_{N_{TL}}(\mathcal{L})} = \delta_{\lambda,2(N+T-1)-L} \frac{\sum_{T_{0}} D_{N_{TL}}(\mathcal{L})}{D_{N_{TL}}(\mathcal{L})}
\]

(51b)

and

\[
\Theta^{T_{0},LL}_{N_{TL}}(k) = p^{T_{0},LL}_{N_{TL}} + k^{2} Q^{T_{0},LL}_{N_{TL}}
\]

(51c)

\[
\overline{p}^{T_{0},LL}_{N_{TL}} = \delta_{\lambda,2(N+T-1)-L} \delta_{\lambda',2(N+T-1)-L} \times \left\{ \delta_{\lambda,1} \left[ \left( \frac{k^{2}}{m_{L}} - \frac{\hbar^{2} n^{2}}{m_{L}^{2}} \right) \frac{F^{2(A+1)}}{T_{1,1}}(\alpha_{n},\beta_{0}) \right. \\
+ \frac{\hbar^{2} n^{2}}{m_{L}} g^{2} k^{2} \frac{F^{2(A+2)}}{T_{1,1}}(\alpha_{n},\beta_{0}) \\
+ \frac{v_{0}^{2(\lambda+1)}}{T_{1,1}}(\alpha_{n},\beta_{0}) + \mu^{2} \frac{\overline{Q}^{2(A+1)}}{T_{1,1}}(\alpha_{n},\beta_{0}) \left. \right] \\
+ \delta_{\lambda,1} \sqrt{v_{0}^{2 \lambda+1+2}} \frac{F^{\lambda+1+2}}{T_{1,1}}(\alpha_{n},\beta_{0}) \right\}
\]

(51d)

\[
\overline{Q}^{T_{0},LL}_{N_{TL}} = -\frac{3}{4} \frac{k^{2}}{m_{L}} \frac{F^{2(A+1)}}{T_{1,1}}(\alpha_{n},\beta_{0})
\]

(51e)

with

\[
\alpha_{n} = \pm \sqrt{\beta^{2} + 6 \delta_{n}^{2}}
\]

\[
\alpha_{n_{0}} = \pm \sqrt{\beta^{2} + 6 \delta_{n}^{2} + 4 q_{0}^{2}}
\]

\[
\alpha_{n_{1}} = \pm \sqrt{\beta^{2} + 6 \delta_{n}^{2} + 4 q_{1}^{2}}
\]

\[
\alpha_{n_{2}} = \pm \sqrt{\beta^{2} + 6 \delta_{n}^{2} + 4 q_{2}^{2}}
\]
where \( a_0^2, a_1^2, a_2^2 \) are the central singlet, central triplet, and tensor range parameters of the Gaussian \( e^{-\alpha^2 r^2} \) n-p potential. \( M \) is the nucleon mass. The stripping cross section (using the surface interaction assumption) is then:

\[
\frac{d\sigma}{d\Omega} = \mathcal{K} \sum_{j_f j_i T_f T_i} B_{j_f T_f, j_i T_i} P_L(k \pm \hbar) A_{j_f j_i}^T A_{j_f j_i}^{T_2} \quad (52)
\]

where

\[
\mathcal{K} = \frac{M_{t F} M_{n x}}{(2\pi \hbar^2)^2} \frac{2J_F + 1}{2J_L + 1} \frac{K_F}{K_F} \left( R_s^2 \Delta R \right)^2 \quad (53)
\]

and

\[
A_{j_f j_i}^T = j_L(k R_s) \quad A_{j_f j_i}^{T_2} \quad (54)
\]

The reaction to which our approximations may be most valid is the \( ^4\text{He}^4(\text{He}^3, p) \text{Li}^6 \) reaction. Of all p shell nuclei \( \text{Li}^6 \) is perhaps the closest to being described by L-S coupling. Then since the initial nucleus (\( \alpha \) particle) has zero spin, the description of \( \text{Li}^6 \) states as pure L-S coupled states will greatly reduce the possible angular momenta entering the stripping reaction so that the effects of finite range and tensor potentials can be examined with the minimum of complications.
VII. The $^{\text{He}}_4(^{\text{He}}_3, p) ^{\text{Li}}_6$ reaction.

For the $^{\text{He}}_4(^{\text{He}}_3, p) ^{\text{Li}}_6$ reaction, $J_- = 0$, therefore $J = J_\gamma$. The ten possible L-S coupling states of $^{\text{Li}}_6$ can then be labelled by $(J \quad L \quad S)$; the stripping cross sections to these states, calculated from equations (49)-(54), are given in Appendix III. They have been written in a form to explicitly show the Bessel functions of $KR_\lambda$ which are the dominant factors in the structure of the angular distributions. Also the factors of $P_2(K \cdot k)$ which arise from the coherent addition of different $\lambda$ contributions are shown explicitly. The remaining $k$ dependence is lumped in the functions $B_\lambda$. The basic $k$ dependence of these functions is of the form

\[(ka e^{-bk^2})(c + k^2d)(a = 0, 1, 2)\]

(55)

The first factor comes from the integral $\int r^2 n_\ell$ of equation (49) which is the radial part of the Fourier transform (12b) of that part of the three nucleon wave function describing the motion of the proton relative to the center of mass of the transferred deuteron. The second factor arises from our use of the bound state Schrodinger equation satisfied by the incident $^{\text{He}}_3$ particle.

If we had not included tensor forces (i.e. had only included the $^{\text{He}}_3 S$ state) the $k$ dependence of the non-zero $B_\lambda$ would be of the form (55) with $a = 0$ only. Now $k$ is a monotonically increasing function of the angle between $K_P$ and $K_H$, so that the factor $e^{-bk^2}$ tends to damp out the angular distribution.
at large angles. For a negative Q value reaction (which is the case for the He\(^4\)(He\(^3\),p)Li\(^6\) reaction, Q being \(-\frac{4}{3}\) Mev.) this damping is especially effective. The factor \((\frac{C + k^2 d}{1})\) which is not present in previous work on two nucleon stripping will on the other hand tend to counteract this damping and will be most effective if \(b\) is small and \(d \gg c\).

The same remarks apply when the He\(^3\) D state is included, except that some of the additional terms of the \(B_4\) will have the \(k^a\) factor \((a \neq 0)\) which will further augment the back angle cross section.

While the \(L=1\) cross sections show the most pronounced effect of the interference among \(l\) values, particularly the (011) cross section, these cross sections arise purely from the He\(^3\) D state and therefore are very small in magnitude. The (121) and (221) cross section however, show some effect of coherent contributions of the S and D state and are not small.

To proceed further, we must insert numbers to see the relative magnitudes of the important factors. We fix the strength parameter, \(\beta^2 = \frac{M \omega}{\hbar}\) of the harmonic oscillator functions to fit the r.m.s. charge radius of Li\(^6\) as determined from electron scattering experiments. This gives \(\beta^2 = \left(\frac{1}{1.8}\right)^2 \approx 0.309\). For the He\(^3\) wave function parameters \(\gamma_1^2\), \(\gamma_2^2\) we use the values obtained from the variational calculation (see Chapter V), namely 0.1, 0.2 \(r^{-2}\) respectively. We will use the potential
parameters of reference (12) which are (note by definition, equation (34), \( V_0 \) is positive)

\[
V_0 = 44.3 \text{ MeV}, \quad \mu = 1, \quad \nu = 0.693
\]

\[
A_0^2 = A_1^2 = 0.493 \text{ f}^{-2}, \quad A_2^2 = 0.290 \text{ f}^{-2}
\]

Focusing our attention on the cross section to the (221) state, we obtain for the relevant \( B_i \)'s:

\[
B_\theta = -\sqrt{3} (\beta R^3) \left\{ \chi_1(k^2) \left(0.671 + k^2 \right) + \frac{\sqrt{3}}{\pi} (\xi_6 \cdot q) \chi_2(k^2) \right\} \\
B_q = -\sqrt{3} (\beta R^2) \frac{\sqrt{3}}{\sqrt{2}} k \cdot \chi_2(k^2) \left(0.396 + k^2 \cdot 2.91\right)
\]

For small \( k^2 \), \( \chi_1 \) and \( \chi_2 \) (defined by (A12), Appendix III) are essentially equal to the S and D state amplitudes respectively.

Even with the large percent D state (13%) in the He\(^3\) wave functions of Chapter V, \( \chi_2 \approx \frac{1}{3} \chi_1 \). Thus we see immediately that \( B_q \) will be less than \( 1/20 B_\theta \) and hence the \( P_2 \left( \hat{k} \cdot \hat{k} \right) \) term in the cross section will be negligible.

However the second term in \( B_\theta \), which arises from the He\(^3\) D state, is comparable to the first term. Thus while the D state contribution will not appreciably affect the structure of the (221) angular distribution it will make a sizable contribution to the magnitude of the cross section.

We also note that in the first term in \( B_\theta \) the factor \( (0.671 + k^2 \cdot 6.23) \) will be quite effective in counteracting the damping factor \( \chi_1(k^2) \).
For completeness and to illustrate that this last remark is a general feature for the model we have chosen, we give below the other $B_4$ functions using the above parameters (somewhat arbitrarily we have chosen $R_5 = 4f$ in $B_1, B_2, B_3$; while the magnitudes of $B_1, B_2,$ and $B_3$ are sensitive to $R_5$, the ratio of $d/c$ in the factors $(c + k^2 d)$ are relatively insensitive.)

$$\begin{align*}
B_1 &= \chi_1(k^2) \left( 4.63 + k^2 36.8 \right) \\
B_2 &= -\sqrt{3} \left( \pm 8.9 + \frac{\sqrt{10}}{5} \right) 9.9 \chi_2(k^2) \\
B_3 &= -\sqrt{3} \frac{\sqrt{10}}{20} \left( \frac{1}{2} \right) \chi_2(k^2) \left( 3.88 + k^2 18.0 \right) \\
B_4 &= \frac{\sqrt{3}}{3} 9 k \chi_2(k^2) \left( 0.722 + k^2 3.57 \right) \\
B_5 &= -\frac{\sqrt{3}}{9} 4 k \chi_2(k^2) \left( 2.74 + k^2 3.57 \right) \\
B_6 &= -(\beta R_5)^2 \chi_1(k^2) \left( 0.671 + k^2 6.23 \right) \\
B_7 &= \frac{\sqrt{3}}{4} \left[ 4 \chi_1(k^2) + \frac{\sqrt{10}}{2} \chi_2(k^2) \right] 23.7 \left( 1 + k^2 \right) \\
B_{10} &= \frac{\sqrt{3}}{5} \frac{\sqrt{10}}{20} (\beta R_5)^2 \left( \frac{k^2}{2} \right) \chi_2(k^2) 15.9
\end{align*}$$
VIII. Summary

We have shown that, contrary to the case of deuteron stripping, the role of the tensor force cannot in general be neglected for two nucleon stripping reactions. By considering a specific example, we have demonstrated that the tensor force effects can be appreciable. While for this particular example, only the magnitude of the cross section was altered and not the structure of the angular distribution, this example does not exclude the possibility that in some reactions the shape of the angular distributions may be significantly changed. Further we have demonstrated that the ubiquitous damping factor $e^{-\frac{b^2}{4r^2}}$ occurring in the angular distributions of previous theories, is effectively cancelled in our formulation.
Appendix I. Classification of Allowed Configurations for 3-Nucleon Ground State.

The three nucleon ground state function can be written as a sum of products, each product containing two factors: (1) a spin function of the three nucleons and (2) a factor depending on the relative spatial positions of the 3 nucleons - the space wave function. We will label the two identical particles 1 and 2 and the third particle by 3.

The possible spin wave functions are of the form

$$\Phi_{\lambda} = \sum C^{\lambda}_{\pi_{1} M_{1}} \pi_{1} M_{1} \Phi_{1} C^{\lambda}_{\pi_{2} M_{2}} \pi_{2} M_{2} \Phi_{2} C^{\lambda}_{\pi_{3} M_{3}} \pi_{3} M_{3} \Phi_{3}$$

(AL)

Thus there are three spin functions: two \( \left( \Phi_{i}^{j M_{i}}, \Phi_{i}^{j+ M_{i}} \right) \) which are symmetric in the identical particles and one \( \left( \Phi_{o}^{j M_{i}} \right) \) which is antisymmetric.

The space wave function for a three body system is a function of 6 independent space variables. For our analysis of stripping reactions the natural set of coordinates consists of the two independent vectors (hence 6 spatial variables) \( \underline{r}, \underline{R} \), where \( \underline{r} \) is the vector displacement of nucleon a from nucleon b and \( \underline{R} \) is the vector displacement of nucleon c from the center of mass of a and b, if a and b are the nucleons transferred in the reaction. If the transferred di-nucleon consists of the two identical particles then \( \underline{r} = \underline{r}_{3} \), \( \underline{R} = \underline{R}_{3} \) (see figure 2). For the classification of the 3-nucleon ground state function \( \left( \underline{r}_{3}, \underline{R}_{3} \right) \) is more natural than either \( \left( \underline{r}_{1}, \underline{R}_{3} \right) \)
or ($\mathbf{\tau}_3, \mathbf{\tau}_2$) due to the ease of taking into account the exclusion principle when the wave function is a function of ($\mathbf{\tau}_3, \mathbf{\tau}_2$). Since we will eventually use simple analytic forms, the transformation to the appropriate ($\mathbf{\tau}_2, \mathbf{\tau}_2$) for the stripping reaction being considered will cause no difficulty.

First let us classify our space wave functions by total orbital angular momentum (i.e. by their behavior under space rotations). Since the three nucleon ground state is known to have spin 1/2 and since from equation (A1) the only possible values of total spin angular momentum are 1/2 and 3/2 the possible values of total orbital angular momentum are restricted to $L=0, 1, 2$. That is, under space rotations the 3 possible behaviors of the space wave function are of tensors of rank 0, 1, or 2. These tensors are to be formed from the vectors $\mathbf{r}_3, \mathbf{r}_3$. In fact it is only the angular parts of $\mathbf{r}_3, \mathbf{r}_3$ that determine the rotational properties so it is natural to deal with the spherical tensors

$$T_{L, L}^M(\hat{A}, \hat{B}) = \sum_{\mathbf{A}_L, \mathbf{B}_L} C^{LM}_{\mathbf{A}_L, \mathbf{B}_L} \hat{A}^M_L \hat{B}^M_L$$

(A2)

where $A$ is a vector with spherical coordinates ($A, \theta_A, \phi_A$) and

$$\hat{A}^M_L \equiv \sqrt{\frac{2L+1}{2L+1}} \gamma^L_M(\theta_A, \phi_A)$$

(A3)

$\gamma^L_M(\theta_A, \phi_A)$ being the spherical harmonic defined by Condon and Shortley (5).

Now using the definition (A2) and the Clebsch-Gordan series for the spherical harmonics it is easily seen that $T_{L, L}^M(\hat{A}, \hat{A}) = \delta_{L, L} \frac{(-1)^L}{\sqrt{2L+1}}$ and $T_{L, L}^M(\hat{A}, \hat{B}) = 0$. Further the parity of $T_{L, L}^M(\hat{A}, \hat{B})$ is given by $(-1)^{L+1}$ since the 3 nucleon ground state is known
to be even parity we may only have tensors such that \( L_1 + L_2 \) is even. The \( L = 2 \) tensor can be combined only with the spin function \( \Phi_{1}^{\pm \frac{3}{2}} \) to give total angular momentum \( \frac{1}{2} \) which is known to be the spin of the 3-nucleon ground state. Since \( \Phi_{1}^{\pm \frac{3}{2}} \) is symmetric in the spin coordinates of the identical nucleons 1 and 2 it can only be combined with second rank tensors antisymmetric in the spatial coordinates of these nucleons. Of the three types of tensors \( T_{L_1 L_2} (\vec{r}_1, \vec{r}_2, \vec{r}_3) \), \( T_{L_1 L_2} (\vec{r}_3, \vec{r}_3) \), and \( T_{L_1 L_2} (\vec{r}_1, \vec{r}_2) \) only the first type can be even parity and antisymmetric in nucleons 1 and 2 (interchange of the spatial coordinates of 1 and 2 being equivalent to reversing the direction of \( \vec{r}_3 \)). The consequence of the results in this paragraph is that in the 3-nucleon ground state space wave function only the tensors of type \( T_{L_1 L_2} (\vec{r}_1, \vec{r}_2) \) (with \( L = 0, 1, \) or 2) can occur.

At this point it appears as if there are an infinite number of such tensors, but we will now show that in fact there are only five basic ones (in addition to the trivial tensor \( T_{000} \)). For this we need the expression for the product of two spherical tensors. This is readily worked out using the Clebsch-Gordan series for the spherical harmonics and the algebra of angular momentum coupling. The general result is

\[
T^{\mathbf{m}}_{L_1 L_2} (\vec{r}_1, \vec{r}_2) T^{\mathbf{m}'}_{L_1' L_2'} (\vec{r}_1, \vec{r}_2) = \sum_{L, L_{3}}^{
abla m} C_{L_{3} L_{3}'}^{L \mathbf{m}} \left[ L \ L_{1} \ L_{2} \ L_{1}' \ L_{2}' \ L_{3}' \ L_{3} \right] T_{L L}^{\mathbf{m}} (\vec{r}_1, \vec{r}_2)
\]

where

\[
\begin{bmatrix}
L_1 & L_{12} & L_{13} \\
L_{12} & L_{22} & L_{23} \\
L_{13} & L_{23} & L_{33}
\end{bmatrix} = [ (2L_{3} + 1)(2L_{32} + 1)(2L_{32} + 1)(2L_{32} + 1)]^L
\]

\[
\times C_{L_{1} \ L_{2} \ L_{12}}^{L_{3}} C_{L_{12} \ L_{32} \ L_{3}}^{L_{3}} \left\{ L_{11} \ L_{12} \ L_{13} \ L_{21} \ L_{22} \ L_{23} \ L_{31} \ L_{32} \ L_{33} \right\}
\]

(A3)
The curly bracket is the Wigner 9-J symbol. The specific products we will need are

\[
T_{0L, L} T_{l' m', m} = \sum_{l_1 l_2} \left[ \begin{array}{ccc} l & l' & l \\ l_1 & l_2 & l \end{array} \right] T_{l_1, l_2, l} \left[ \begin{array}{ccc} 0 & 0 & l' \\ 0 & 0 & l \end{array} \right] T_{l' m', m} = \sum_{L} (-1)^{l + l' + l + L} \left( \frac{(2L + 1)(2l_1 + 1)}{2l + 1} \right) \left[ \begin{array}{ccc} l & l_1 & l \\ 0 & 0 & l \end{array} \right] C_{L, 0, 0}^{l_1, 0} \left[ \begin{array}{ccc} l' & l_2 & l' \\ 0 & 0 & l \end{array} \right] C_{L, 0, 0}^{l_2, 0} \left( \frac{(2L + 1)(2l_2 + 1)}{2l + 1} \right) \left[ \begin{array}{ccc} l' & l & l' \\ 0 & 0 & l \end{array} \right] C_{L, 0, 0}^{l_1, 0} \left( \frac{(2L + 1)(2l_1 + 1)}{2l + 1} \right) \left[ \begin{array}{ccc} l & l_2 & l \\ 0 & 0 & l \end{array} \right] C_{L, 0, 0}^{l_2, 0} \left( A1 \right)
\]

where \{a b c \} is the Wigner 6-J symbol.\(^{(4)}\)

From (A1) we have \(T_{0L, L} T_{0L, L} = a_0^L T_{0L, L}, b_0^L T_{0L, L},\)
with \(a_0^L = 0, b_0^L = 1.\) Therefore in general \(T_{0L, L}\) can be expressed as a power series in \(T_{0L, L} :\)

\[
T_{0L, L} = \sum_{n=0}^{L} C_{L, n, 0} (T_{0L, L})^n
\]

For \(L = 1,\) the even parity requirement allows only \(T_{11, L} \) for \(L \geq 1.\) Again from (A1) we have \(T_{0L, L} T_{11, L} = a_1^L T_{1L, L} + b_1^L T_{1L, L},\)
with \(a_1^L = 0.\) So in general \(T_{1L, L}\) can be expressed as

\[
T_{1L, L} = T_{11, L} \sum_{n=0}^{L} C_{L, n, 0} (T_{0L, L})^n \quad L \geq 1
\]

For \(L = 2,\) there are three types of tensors, \(T_{2L, L}^{11, L}, T_{2L, L}^{21, L}, T_{2L, L}^{1L, L}.\) Now \(T_{0L, L} T_{2L, L} = a_2^L T_{2L, L} + b_2^L T_{2L, L},\)
with \(a_2^L = 0.\) Therefore

\[
T_{2L, L} = T_{21, L} \sum_{n=0}^{L} C_{L, n, 0}^2 (T_{0L, L})^n \quad L \geq 1 \quad (A5)
\]

For \(T_{2L, L}^{21, L} \) and \(T_{2L, L}^{1L, L} \) (A1) gives

\[
T_{0L, L} T_{2L, L} + T_{2L, L} = a_2^{2L} T_{2L, L} + b_2^{2L} T_{2L, L} + c_2^{2L} T_{2L, L} + d_2^{2L} T_{2L, L} + e_2^{2L} T_{2L, L} + f_2^{2L} T_{2L, L} + g_2^{2L} T_{2L, L} + h_2^{2L} T_{2L, L} + i_2^{2L} T_{2L, L} + j_2^{2L} T_{2L, L}
\]

\[
T_{0L, L} T_{2L, L} = a_2^{2L} T_{2L, L} + b_2^{2L} T_{2L, L} + c_2^{2L} T_{2L, L} + d_2^{2L} T_{2L, L} + e_2^{2L} T_{2L, L} + f_2^{2L} T_{2L, L} + g_2^{2L} T_{2L, L} + h_2^{2L} T_{2L, L} + i_2^{2L} T_{2L, L} + j_2^{2L} T_{2L, L}
\]

where \(a_2^{2L} = a_2^{2L} = 0.\)
Therefore from (A6) and (A7) we have

\[ T_{2L,L+1}^M = T_{2L,1}^n \sum_{m=0}^L C_{Ln}^{2+} (T_{01})^m + T_{2L,0}^n \sum_{m=0}^L d_{Lm}^{2+} (T_{01})^m \]

\[ T_{2L,L-1}^M = T_{2L,1}^n \sum_{m=0}^L C_{Ln}^{2-} (T_{01})^m + T_{2L,0}^n \sum_{m=0}^L d_{Lm}^{2-} (T_{01})^m \]

Consequently, with respect to behavior under space rotations, there are five distinct states, one S state, one F state, and three different D states. As we noted earlier the D states must be antisymmetric in nucleons 1 and 2 since they can only combine with the completely symmetric spin 3/2 function. Only the tensors \( T_{2L,1} \), \( T_{2L,1} \), \( T_{2L,1} \) with \( L \) odd will be antisymmetric in nucleons 1 and 2. Therefore instead of the three basic tensors \( T_{20}, T_{21}, T_{22} \) we must take the set \( T_{21}, T_{23}, T_{22} \). (All other second rank tensors antisymmetric in nucleons 1 and 2 may be generated from this set as before).

In general our space wave functions will be of the form

\[ f_L^M(\mathbf{r}_1, \mathbf{r}_3) = \sum_{\ell, \ell_L} g_{\ell, \ell_L}(\mathbf{r}_3, \mathbf{r}_3) T_{\ell, \ell_L}^M(\hat{\mathbf{r}}_3, \hat{\mathbf{r}}_3) \]

or writing \( g_{\ell, \ell_L}(\mathbf{r}_3, \mathbf{r}_3) = g_{\ell, \ell_L}(\mathbf{r}_3, \mathbf{r}_3) \mathbf{N}_{\ell_L} \mathbf{N}_{\ell_L} \)

we may write

\[ f_L^M(\mathbf{r}_3, \mathbf{r}_3) = \sum_{\ell, \ell_L} g_{\ell, \ell_L}(\mathbf{r}_3, \mathbf{r}_3) T_{\ell, \ell_L}^M(\hat{\mathbf{r}}_3, \hat{\mathbf{r}}_3) \]

where \( T_{\ell, \ell_L}(\mathbf{A}, \mathbf{B}) \) is a spherical tensor constructed from the solid harmonics of \( \mathbf{A} \) and \( \mathbf{B} \).

Now let us make a further classification of the functions by symmetry with respect to the Permutation Group on 3 objects \( \mathcal{S}_3^{(3)} \). This classification will be useful since those f's which will occur with the largest amplitude will be the most symmetrical in the three particles.
For three particles there are three types of symmetry (partitions) corresponding to the three irreducible representations of the group—the completely symmetric (s) representation, the completely anti-symmetric (a) representation, and the "mixed" (m) representation. The s and a representations are one dimensional, while the m representation is two dimensional.

In cycle notation the six elements of $S_3$ are (1), (123), (132), (12), (23), and (31), the first three being even (an even number of interchanges) and the last three odd. The matrices of the three irreducible representations are:

<table>
<thead>
<tr>
<th>Group Element</th>
<th>(1)</th>
<th>(123)</th>
<th>(132)</th>
<th>(12)</th>
<th>(23)</th>
<th>(31)</th>
</tr>
</thead>
<tbody>
<tr>
<td>s</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>a</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
</tr>
<tr>
<td>m</td>
<td>$\left[ \begin{array}{c} 1 \ 1 \ \frac{-1}{\sqrt{2}} \end{array} \right]$ $\left[ \begin{array}{c} 1 \ 1 \ \frac{-1}{\sqrt{2}} \end{array} \right]$ $\left[ \begin{array}{c} 1 \ 1 \ \frac{-1}{\sqrt{2}} \end{array} \right]$ $\left[ \begin{array}{c} 1 \ 1 \ \frac{-1}{\sqrt{2}} \end{array} \right]$ $\left[ \begin{array}{c} 1 \ 1 \ \frac{-1}{\sqrt{2}} \end{array} \right]$ $\left[ \begin{array}{c} 1 \ 1 \ \frac{-1}{\sqrt{2}} \end{array} \right]$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table A1.

For examining the permutation symmetry of a given function it is useful to note that due to the multiplication table of $S_3$, the behavior of a function under only two of the group elements (neither being the identity of course) need be examined; the behavior under the remaining elements is given in terms of these two. For example, we will take the two transposition elements (12) and (31). Then clearly (23) = (12) x (13) x (12), (123) = (12) x (13), (132) = (13) x (12) where the product of cycles is taken left to right.

Just as there is a set of addition coefficients in the case of the three-dimensional rotation group (the vector coupling or
Clebsch-Gordan coefficients), there is a unique set of addition
coefficients for combining basis functions of different irreducible
representations \((s, a, m)\) of \(\mathcal{S}_3\) where the basis functions may refer to
different coordinates of the three objects (e.g. for three nucleons one set
might be the space coordinates, the other set the spin coordinates).
Denoting by \(\psi_{K_i}^{P_i}(123)\) and \(\psi_{K_i}^{P_i}(1'2'3')\) a function of the unprimed
(primed) coordinates of three objects belonging to the \(K_i\) \((K_{1} \ldots K_{3})\) row
of the irreducible representation \(P_i\) \((P_{1} \ldots P_{3})\) of \(\mathcal{S}_3\) we can construct
orthonormal linear combinations of products of the form
\[
\psi_{K_i}^{P_i(P_{1}P_{2}P_{3})} = \sum_{K_{1}K_{2}K_{3}} (P_{1}P_{2}P_{3})_{K_{1}K_{2}K_{3}} \psi_{K_i}^{P_i}(123) \psi_{K_i}^{P_i}(1'2'3').
\]

\(\psi_{K_i}^{P_i(P_{1}P_{2}P_{3})}\)
belongs to the \(K\) row of the irreducible representation
\(P\) of \(\mathcal{S}_3\) under joint permutations of the two sets of coordinates
\((123)\) and \((1'2'3')\). The addition coefficients \((P_{1}P_{2}P_{3})_{K_{1}K_{2}K_{3}}\) satisfy
the orthogonality relations
\[
\sum_{K_{1}K_{2}} (P_{1}P_{2}P_{3})_{K_{1}K_{2}K_{3}} (P_{1}P_{2}P_{3})_{K_{1}K_{2}K'_{3}} \delta_{K_{3}K_{3}'} = \delta_{P_{3}P_{3}'} \delta_{K_{3}K_{3}}'
\]
\[
\sum_{P_{1}} (P_{1}P_{2}P_{3})_{K_{1}K_{2}K_{3}} (P_{1}P_{2}P_{3})_{K'_{1}K'_{2}K'_{3}} \delta_{K_{1}K'_{1}} \delta_{K_{2}K'_{2}} \delta_{K_{3}K'_{3}} = \delta_{K_{1}K'_{1}} \delta_{K_{2}K'_{2}} \delta_{K_{3}K'_{3}}'
\]
and the non-zero values are
\[
\begin{align*}
(1111) & = (saq) = (asq) = (asa) = (aas) = 1 \\
(m \bar{m} \bar{m}) & = (m \bar{m} \bar{m}) = \sqrt{2} (m \bar{m} \bar{m}) = \delta_{K_{1}K_{1}} \\
(a \bar{m} \bar{m}) & = (a \bar{m} \bar{m}) = -\sqrt{2} (a \bar{m} \bar{m}) = (0 -1)_{K_{1}K_{1}} \\
(-m \bar{m} \bar{m}) & = (m \bar{m} \bar{m}) = (m \bar{m} \bar{m}) = (m \bar{m} \bar{m}) = \frac{1}{\sqrt{2}}
\end{align*}
\]
All three partitions can occur for \( L = 0, 1 \). We will show this by giving an example for each case. First we note that \( T_{01}(\_R_3, R_3) = R_3 \times R_3 \) and \( T_{111}(\_R_3, R_3) \sim R_3 \times R_3 \). Further one may verify that if \( P_{12} \) and \( P_{13} \) denote the operators that interchange the positions of particles 1 and 2, and 1 and 3 respectively then
\[
\begin{align*}
P_{12} R_3^3 &= - R_3^3 \\
P_{13} R_3^3 &= \frac{1}{2} R_3^3 \\
P_{12} R_3^3 &= R_3^3 \\
P_{13} R_3^3 &= \frac{1}{4} (3 R_3^3 - 2 R_3^1)
\end{align*}
\]
So for \( L = 0 \), an example of a completely symmetric function (other than the identity) is \( g_o^s = R_3^2 + \frac{3}{2} R_3^1 \), a mixed function (which has two components) is \( \left( \begin{array}{c} g_o^{s'} \\ g_o^{m'} \end{array} \right) = \left( \begin{array}{c} R_3^2 - \frac{3}{2} R_3^1 \\ R_3^3 \end{array} \right) \)
while an antisymmetric function is \( g_o^a = \pm R_3^3 \times R_3^3 \) \( \left\{ (R_3^2 - \frac{3}{2} R_3^1)^2 - (R_3^3 \times R_3^3)^2 \right\} \)
For \( L = 1 \), \( g_o^a = R_3 \times R_3 \) is a completely antisymmetric function which can be combined by means of the \( \delta \) addition coefficients with a scalar mixed function to give an \( L = 1 \) mixed function, for example
\[
\left( \begin{array}{c} g_o^{s'} \\ g_o^{m'} \end{array} \right) = g_o^a \left( \begin{array}{c} -g_o^{m2} \\ g_o^{m2} \end{array} \right)
\]
or with a scalar antisymmetric function to give an \( L = 1 \) symmetric function, for example
\( g_i^s = g_i^a g_o^s \).

For \( L = 2 \) we have three distinct second rank tensors \( T_{211}, T_{213} \), and \( T_{213} \). Now \( T_{211} \) is proportional to the dyadic \( \frac{3}{2} R_3 \times R_3 \) and hence is the second component of a mixed symmetry function (the first component being \( \left( T_{202} - \frac{3}{4} T_{110} \right) \) \( \left( \frac{1}{2} \right) \left( R_3^3 \times R_3^3 \right) \)

\[
= \frac{1}{2} \left( R_3^3 - \frac{3}{4} R_3^3 \right) \]
which is symmetric in nucleons 1 and 2 and consequently inadmissible).

From $T_{2/3}$ and $T_{3/4}$, we may form two linearly independent combinations

\[ T_{1/3} + \frac{3}{4} T_{2/3} \propto \alpha T_{0/1} (T_{2/3} + \frac{3}{4} T_{2/3}) + b T_{2/1}, \]

\[ T_{1/3} - \frac{3}{4} T_{2/3} \propto \alpha T_{0/1} (T_{2/3} - \frac{3}{4} T_{2/3}) + b' T_{2/1}. \]

Now $T_{2/3} \propto \frac{R_3}{R_2}$ and $T_{2/3} \propto \frac{R_3}{R_3}$, and $T_{0/1} \propto \frac{R_3}{R_3}$.

Referring to the $O_3$ addition coefficients we see that both of these linear combinations are second components of mixed symmetry functions.

With this result, inspection of the addition coefficients tells us that we cannot have D states antisymmetric in nucleons 1 and 2 of symmetry under $O_3$ other than mixed. Thus there are three types of $L=2$ space wave functions (each being the second component of a mixed symmetry function). We will label these $f_{20}^{m_1}$, $f_{21}^{m_2}$, $f_{2-1}^{m_2}$

where

\[ f_{20}^{m_1} \propto T_{2/1}, \]

\[ f_{21}^{m_2} \propto (T_{2/3} + \frac{3}{4} T_{2/3}), \]

and

\[ f_{2-1}^{m_2} \propto (T_{2/3} - \frac{3}{4} T_{2/3}). \]

We must combine our spin and space wave functions such that the resultant wave functions obey the exclusion principal - namely, they must be totally antisymmetric in the coordinates of nucleons 1 and 2 (the two identical nucleons). The completely symmetric space functions $\tilde{f}_L^S$ are clearly symmetric under interchange of 1 and 2 while the completely antisymmetric space functions $f_L^A$ are antisymmetric under interchange of 1 and 2. Finally (as can be seen from the matrix isomorphic to the group element (12), see Table A1) the first component of a mixed space function $f_L^{m_1}$ is symmetric in nucleons 1 and 2, while $f_L^{m_2}$ is antisymmetric in nucleons 1 and 2.
Therefore in the 3-nucleon ground state function (remembering the three body ground state has spin 1/2 and is of even parity) we can have the following types of configurations only:

\[ \begin{align*}
L = 0 & \quad f_0^0 \phi_0^{i_1^m} + f_0^a \phi_1^{i_1^m} + f_0^e \phi_2^{i_1^m} + f_0^w \phi_3^{i_1^m} + f_0^m \phi_4^{i_1^m} - f_0^{w_2} \phi_5^{i_1^m} \\
L = 1 & \quad f_1^0 \phi_0^{i_1^m} + f_1^a \phi_1^{i_1^m} + f_1^e \phi_2^{i_1^m} + f_1^w \phi_3^{i_1^m} + f_1^m \phi_4^{i_1^m} - f_1^{w_2} \phi_5^{i_1^m} \\
L = 2 & \quad f_2^0 \phi_0^{i_1^m} + f_2^a \phi_1^{i_1^m} + f_2^e \phi_2^{i_1^m} + f_2^w \phi_3^{i_1^m} + f_2^m \phi_4^{i_1^m} - f_2^{w_2} \phi_5^{i_1^m}
\end{align*} \]

At this point it appears as if we have four S states, six P states, and 3 D states. However we may make one further demand on our wave functions (which is equivalent to excluding isotopic spin 3/2 if we had used the isotopic spin formalism) and that is that no configuration is to appear which can also be a configuration of three identical nucleons. To see which of the states listed above would exist for three identical nucleons we imagine nucleon 3 to be the same kind of nucleon as 1 and 2. Then to satisfy the exclusion principle, a configuration must be completely antisymmetric in the 3 particles. Each of the first three S and P, the last P, and all the D configurations are clearly second components of mixed functions which can not be admissible configurations for three identical nucleons. The fourth S and P and fifth P configurations however are completely antisymmetric and are therefore configurations which can occur for three identical nucleons.

Thus the configurations unique to the 3-nucleon ground state are ten, three S states, four P states and three D states. The space wave functions of the three D states are second components of functions of mixed symmetry. The three S states are distinguished by spatial
All

symmetry as are the three spin 1/2 F states.

\[ R_{ij} = R_j - \frac{\alpha_j}{2} = - R_k + \frac{\alpha_k}{2} \]

\[ R_{ij} = - \frac{1}{2} (R_j + \frac{3}{2} \alpha_j) = - \frac{1}{2} (R_k - \frac{3}{2} \alpha_k) \]

Figure 2
Appendix II. Two Lemmas

To prove the statement preceeding equation (4.4a) on page 28 we first prove the following useful lemma:

Let \( \mathbf{A}, \mathbf{B}, \) and \( \mathbf{C} \) be three vectors with \( \mathbf{A} = \mathbf{B} + \mathbf{C} \).

Then

\[
A^m_l = \sum \sqrt{\frac{(2l)}{2l+1}} \; T^m_{l, l} \; (B, C)
\]  \hspace{1cm} (A8)

where

\[
A^m_l = \sqrt{\frac{2l}{2l+1}} \; \gamma^m_l (A)
\]

\[
T^m_{l, l} (B, C) = \sum_{M, M'} C^m_{l M, l M'} B^M_l C^{M'}_{l'}
\]  \hspace{1cm} (A9)

and \( \binom{b}{a} \) is the binomial coefficient which we define to be zero if \( b > a \). \( \gamma^m_l (A) \) is the solid harmonic of the vector \( A \).

We may prove (A8) by induction. For \( L = 0 \), it is the trivial identity \( L = l \). For \( L = 1 \), we have

\[
A^M_1 = B^M_1 + C^M_1
\]  \hspace{1cm} (A10)

which is just the relation \( A = B + C \) in terms of the spherical components of the vectors \( \mathbf{A}, \mathbf{B}, \mathbf{C} \). Then all we need show is that if (A8) is true then it is true if we replace \( L \) by \( L + 1 \). This can be done using a special case of the inverse of the Clebsch Gordan series for the spherical harmonics. In terms of the harmonic polynomials (A9) the special case we need is

\[
C^m_{l, l} \; A^m_l = \sum_{\mu, l} C^m_{l \mu, l \mu} A^m_l A^\mu_l
\]  \hspace{1cm} (A11)
We now assume that \((\text{AB})\) is true for \(L=L\) and we use the fact that it is true for \(L=1\) (equation \((\text{Al0})\)). Then the right hand side of (All) can be written

\[
\sum_{l,m,m_0,m_{-1}} C_{l,m,m_0}^{\text{L}} \sqrt{(2l+1)} C_{l-l,m,m_0}^{\text{L}} \tilde{B}_{l,m_0,m_{-1}}^{m_{-1}} \left( B_{l,m_0,m_{-1}}^{m_{-1}} + C_{l,m_0,m_{-1}}^{m_{-1}} \right)
\]

(Alla)

Using the Clebsch-Gordan series and some algebra of angular momentum coupling we may write (Alla) as

\[
\sum_{l} \sqrt{(\frac{2l+1}{2})} C_{l,0,0}^{\text{L}} \left( T_{l+l', l', l', l-l'}^{\text{L}} (g, \zeta) + T_{l+l', l, l-l, l-l'}^{\text{L}} (g, \zeta) \right).
\]

Then since \(C_{l,0,0}^{\text{L}} = \sqrt{\frac{l+1}{2l+1}}\) equation (All) becomes

\[
A_{l+l'}^{\text{M}} = \sum_{l} \sqrt{(\frac{2(l+l')}{2l+1})} \left( T_{l+l', l', l', l-l-l'}^{\text{L}} (g, \zeta) + T_{l+l', l, l-l, l-l'}^{\text{L}} (g, \zeta) \right)
\]

Changing the dummy index \(l\) to \(l-1\) in the first term and to \(l-l\) in the second term gives

\[
A_{l+l'}^{\text{M}} = \sum_{l} \sqrt{(\frac{2(l+l')}{2l+1})} \left( T_{l+l', l', l-l-l'}^{\text{L}} (g, \zeta) \right)
\]

which is equation \((\text{AB})\) with \(L\) replaced by \(L+1\).

Using this lemma we may express a tensor \(T_{L}^{M} (R_1, R_2)\) as a sum of tensors of rank \(L\) of the vectors \(R_1, R_2\). Where the two sets of vectors are related in the following way

\[
\begin{pmatrix}
R_1 \\
R_2
\end{pmatrix} =
\begin{pmatrix}
A_{l, l} & A_{l, l'} \\
A_{l', l} & A_{l', l'}
\end{pmatrix}
\begin{pmatrix}
R_{l} \\
R_{l'}
\end{pmatrix}
\]
By definition
\[ T_{L_1 L_2}^{M_1 M_2} (r_1, s_2) = \sum_{m_1, m_2} C_{m_1, m_2}^{L_1, L_2} (r_1, m_1) (s_2, m_2). \]

Using equation (A8) to write \( (r_1)_{L_1} \) and \( (r_2)_{L_2} \) in terms of tensors of \( R_1 \) and \( R_2 \), we obtain:
\[ T_{L_1 L_2}^{M_1 M_2} (r_1, s_2) = \sum_{m_1, m_2} C_{m_1, m_2}^{L_1, L_2} \sqrt{\frac{2 L_1}{2 L_2}} \frac{2 L_2}{2 L_2} T_{L_1 L_2}^{m_1} (r_1, R_1) T_{L_1 L_2}^{m_2} (R_2, R_2). \]

Recognizing that \( T_{L_1 L_2}^{M_1 M_2} (A, B) = A_{L_1} B_{L_2} T_{L_1 L_2}^{M_1 M_2} (A, B) \)

and using equations (A3) for the product of two \( T \)'s enables us to write:
\[ T_{L_1 L_2}^{M_1 M_2} (r_1, s_2) = \sum_{A, B} \sum_{l_1, l_2} \Delta_{l_1 l_2}^{l_1 l_2} (A) R_{l_1}^{l_1} R_{l_2}^{l_2} T_{L_1 L_2}^{M_1 M_2} (R_1, R_2), \]

where
\[ \Delta_{l_1 l_2}^{l_1 l_2} (A) = A_{l_1} A_{l_2} L_{l_1} L_{l_2} \sqrt{\frac{2 L_1}{2 L_2}} \frac{2 L_2}{2 L_2} \]

the square bracket having been defined by equation (A3).
Appendix III. List of Cross Sections for Chapter VI.

In this appendix we present the expressions for the stripping cross sections, \( \frac{d\sigma}{d\Omega} (J^I\Lambda) \) obtained from equations (49)-(54) for the case \( J^I = 0 \) (therefore \( J_F = J \)). The quantum numbers \( J^I\Lambda \)'s distinguish the ten L-S coupling states of the final nucleus.

\[
\frac{d\sigma}{d\Omega}(000) = C \frac{3}{16} (B_1)^2 J^2 (\kappa_1^2)
\]

\[
\frac{d\sigma}{d\Omega}(101) = C \frac{3}{16} \left( (B_3)^2 + (B_3)^3 \right) J^2 (\kappa_2^2)
\]

\[
\frac{d\sigma}{d\Omega}(110) = 0
\]

\[
\frac{d\sigma}{d\Omega}(011) = C \frac{5}{(18)^2} J^2 (\kappa_3^2) (B_4)^2 \left( 1 - \frac{P_2}{2} (\kappa \cdot k) \right)
\]

\[
\frac{d\sigma}{d\Omega}(111) = C \frac{5}{(18)^2} \frac{3}{4} J^2 (\kappa_3^2) \left\{ (B_4)^2 + (B_5)^2 + \frac{1}{2} \frac{P_2}{2} (\kappa \cdot k) \left[ (B_4)^2 + \frac{2}{5} (B_5)^2 + \frac{8}{5} B_4 B_5 \right] \right\}
\]

\[
\frac{d\sigma}{d\Omega}(211) = C \frac{5}{(18)^2} \frac{7}{5} J^2 (\kappa_3^2) (B_4)^2 \left( 1 - \frac{17}{5} \frac{P_2}{2} (\kappa \cdot k) \right)
\]

\[
\frac{d\sigma}{d\Omega}(120) = C \frac{1}{3} (B_6)^2 J^2 (\kappa_1^2)
\]

\[
\frac{d\sigma}{d\Omega}(121) = C \frac{1}{3} \frac{1}{5} \left\{ J^2 (\kappa_3^2) (B_4)^2 + J^2 (\kappa_3^2) \left( (B_6)^2 + (B_7)^2 \right) \right. \\
+ \frac{1}{2} \frac{P_2}{2} (\kappa \cdot k) \left[ J^2 (\kappa_3^2) B_9 (B_9 + 2 \sqrt{2} B_9) \right] \\
\left. + J_0 (\kappa_3^2) J_2 (\kappa_3^2) \left( 2 \sqrt{2} B_7 B_9 + B_9 (B_9 + 2 \sqrt{2} B_9) \right) \right\}
\]
\[
\frac{d\sigma}{dQ_2} (211) = C \frac{1}{3} \frac{1}{3} f_2^2 (kR_3) \left\{ (B_0)^2 + (B_1)^2 - \frac{1}{2} P_2 (k \cdot b) B_0 (B_0 + 2 \sqrt{2} B_0) \right\}
\]

\[
\frac{d\sigma}{dQ_2} (311) = C \frac{1}{3} \frac{7}{15} f_2^2 (kR_3) \left\{ (B_0)^2 + (B_1)^2 + \frac{1}{2} P_2 (k \cdot b) B_0 (B_0 + 2 \sqrt{2} B_0) \right\}
\]

where 
\[
C = \frac{M_{PF} M_{HI}}{(2\pi h)^2} \frac{k_F}{k_H} \left( 2 \right)^6 (R^2 \Delta R)^3 e^{-2\beta^2 R^2}
\]

and 
\[
B_1 = - X_1 (k^2) (c_1 + k^2 D_1)
\]

\[
B_2 = - \sqrt{3} \left( \frac{1}{3} B_1 + \frac{\sqrt{10}}{\sqrt{3}} X_2 (k^2) C_2 \right)
\]

\[
B_3 = \frac{\sqrt{3}}{2} \frac{\sqrt{10}}{30} \left( \frac{k}{l_z} \right)^2 X_3 (k^2) (c_3 + k^2 D_3)
\]

\[
B_4 = - \frac{\sqrt{3}}{3} k R_1 X_4 (k^2) (c_4 + k^2 D_4)
\]

\[
B_5 = \frac{\sqrt{3}}{9} k R_2 X_5 (k^2) (c_5 + k^2 D_5)
\]

\[
B_6 = (\beta R_3)^2 X_6 (k^2) (c_6 + k^2 D_6)
\]

\[
B_7 = \frac{\sqrt{3}}{4} \left\{ X_7 (k^2) C_7 - \frac{\sqrt{10}}{\sqrt{3}} X_2 (k^2) (c_7 + k^2 D_7) \right\}
\]

\[
B_8 = \sqrt{3} \left\{ B_6 - \frac{\sqrt{10}}{\sqrt{3}} (\beta R_3)^2 X_2 (k^2) C_8 \right\}
\]

\[
B_9 = \frac{\sqrt{3}}{9} \frac{\sqrt{10}}{30} (\beta R_3)^2 (\frac{k}{l_z})^2 X_5 (k^2) (c_9 + k^2 D_9)
\]

\[
B_{10} = \frac{\sqrt{3}}{3} \frac{\sqrt{10}}{30} (\beta R_3)^2 (\frac{k}{l_z})^2 X_5 (k^2) C_8
\]
\[ X_1 (k^2) = A_1 e^{-\frac{k^2}{\theta R_{x_2}^2}} \]
\[ X_2 (k^2) = A_2 e^{-\frac{k^2}{\theta R_{x_1}^2}} \]

\[ C_1 = \beta^3 \{ P_{011}^{000} - P_{101}^{000} (1 - \frac{2}{3} \beta^2 R_{x_2}^2) \} \]
\[ C_2 = \beta^3 R_{x_2}^2 \{ P_{012}^{110} - P_{102}^{110} (1 - \frac{2}{3} \beta^2 R_{x_2}^2) \} \]
\[ C_3 = C_1 (x_1 \rightarrow x_2) \]
\[ C_4 = \beta^5 P_{002}^{1111} \]
\[ C_5 = \beta^5 P_{002}^{1211} \]
\[ C_6 = \beta^3 P_{101}^{0000} \]
\[ C_7 = \beta^5 P_{102}^{1100} (x_2 \rightarrow x_1) \]
\[ C_8 = \beta^5 R_{x_2}^2 P_{002}^{1100} \]
\[ C_9 = \beta^3 P_{101}^{0000} (x_1 \rightarrow x_2) \]

\[ D_1 = \beta^3 \{ Q_{011}^{000} - Q_{101}^{000} (1 - \frac{2}{3} \beta^2 R_{x_2}^2) \} \]
\[ D_2 = D_1 (x_1 \rightarrow x_2) \]
\[ D_3 = \beta^5 Q_{002}^{1111} \]
\[ D_4 = \beta^3 Q_{002}^{0000} \]
\[ D_5 = \beta^3 Q_{101}^{0000} \]
\[ D_6 = \beta^5 Q_{002}^{1100} \]
\[ D_7 = \beta^3 Q_{101}^{0000} (x_1 \rightarrow x_2) \]
\[ D_8 = \beta^3 Q_{101}^{0000} (x_2 \rightarrow x_1) \]
A quantity $S$ with \( \kappa \rightarrow \kappa' \) after it means that in the expression for $S$ (which is given as a function of $\kappa$) change $\kappa'$ everywhere to $\kappa$. The functions $P$, $Q$ above are those of equations (51d) and (51e); for the sake of completeness we give the relevant ones below. $A_1$ and $A_2$ are the amplitudes of the three nucleon $S$ and $D$ state respectively (equation (43)).

\[
\begin{align*}
\mathcal{P}_{011}^{000} &= (2\alpha_1)^{-3} \left\{ \left( \epsilon_N - 9 \frac{\kappa^2}{\alpha^2} \right) \left( 1 - \frac{5}{3} \frac{\beta^2}{\alpha_1^2} \right) + 9 \frac{\kappa^2}{\alpha^2} \left( 1 - \frac{5}{3} \frac{\beta^2}{\alpha_1^2} \right) \frac{3\kappa^2}{\alpha_1^2} \right\} \\
&\quad + V_0 \left\{ (2\alpha_1)^{-3} \left( 1 - \frac{5}{3} \frac{\beta^2}{\alpha_1^2} \right) + \mu (2\alpha_1)^{-3} \left( 1 - \frac{5}{3} \frac{\beta^2}{\alpha_1^2} \right) \right\}
\end{align*}
\]

\[
\begin{align*}
\mathcal{P}_{101}^{000} &= (2\alpha_1)^{-3} \left\{ \epsilon_N - 9 \frac{\kappa^2}{\alpha^2} \right\} + 9 \frac{\kappa^2}{\alpha^2} \frac{3\kappa^2}{\alpha_1^2} \\
&\quad + V_0 \left\{ (2\alpha_1)^{-3} \right\} + \mu (2\alpha_1)^{-3} \right\}
\end{align*}
\]

\[
\begin{align*}
\mathcal{P}_{012}^{110} &= \mathcal{P}_{102}^{110} \left( 1 - \frac{5}{3} \frac{\beta^2}{\alpha_2^2} \right)
\end{align*}
\]

\[
\begin{align*}
\mathcal{P}_{102}^{110} &= 12 \sqrt{2} \nu V_0 (2\alpha_2)^{-5}
\end{align*}
\]

\[
\begin{align*}
\mathcal{P}_{002}^{111} &= 6 \left\{ (2\alpha_2)^{-5} \left[ \epsilon_N - 15 \frac{\kappa^2}{\alpha^2} + 9 \frac{\kappa^2}{\alpha^2} \left( 1 - \frac{5}{3} \frac{\beta^2}{\alpha_1^2} \right) \right] \right\}
\end{align*}
\]

\[
\begin{align*}
\mathcal{P}_{002}^{111} &= 6 \left\{ (2\alpha_2)^{-5} \left[ \epsilon_N - 15 \frac{\kappa^2}{\alpha^2} + 9 \frac{\kappa^2}{\alpha^2} \left( 1 - \frac{5}{3} \frac{\beta^2}{\alpha_1^2} \right) \right] \right\}
\end{align*}
\]

\[
\begin{align*}
\mathcal{P}_{002}^{110} &= 60 \left\{ (2\alpha_2)^{-7} \left[ \epsilon_N - 21 \frac{\kappa^2}{\alpha^2} + 9 \frac{\kappa^2}{\alpha^2} \left( 1 - \frac{5}{3} \frac{\beta^2}{\alpha_1^2} \right) \right] \right\}
\end{align*}
\]

\[
\begin{align*}
\mathcal{P}_{002}^{110} &= 60 \left\{ (2\alpha_2)^{-7} \left[ \epsilon_N - 21 \frac{\kappa^2}{\alpha^2} + 9 \frac{\kappa^2}{\alpha^2} \left( 1 - \frac{5}{3} \frac{\beta^2}{\alpha_1^2} \right) \right] \right\}
\end{align*}
\]
\[ Q_{101}^{0000} = - \frac{3}{4} \frac{\xi^2}{\Delta} (2\alpha_1)^{-3} \]

\[ Q_{011}^{0000} = Q_{101}^{0000} \left( 1 - \frac{1}{2} \frac{\beta^3}{\alpha_1^2} \right) \]

\[ Q_{002}^{111} = - \frac{3}{4} \frac{\xi^2}{\Delta} 6 (2\alpha_2)^{-5} \]

\[ Q_{002}^{1100} = - \frac{3}{4} \frac{\xi^2}{\Delta} 60 (2\alpha_2)^{-7} \]
Appendix IV. Three Nucleon Ground State Energy Variational Calculation.

The method used to obtain the parameters of the three nucleon wave function discussed in Chapter V (equations (41) (41a)) is a standard variational calculation. The wave function is of the form

$$\Psi = \sum_{i}^n A_i \chi_i \langle \psi_i \rangle$$

where the $\chi_i$ are orthonormal functions. Given a Hamiltonian $H$, for a given set of $\chi_i$, we diagonalize the matrix

$$(\mathcal{H}_{ij}) = \left( \langle \chi_i | H | \chi_j \rangle \right)$$

by a similarity transformation with an orthogonal matrix $(O_{ij})$ (since $(H_{ij})$ is real and symmetric). The smallest eigenvalue, $E_0 (\{\psi_i\})$, will be the lowest energy, while the row $0_{ij}$ of $(O_{ij})$ corresponding to $E_0 (\{\psi_i\})$ will be the eigenvector with respect to the orthonormal basis functions $\chi_i \langle \psi_i \rangle$. Therefore since $(O_{ij})$ is orthogonal, the elements of $0_{ij}$ will be the correct $A_i$ so that $\Psi$ is normalized. We then vary the set of $\chi_i$, repeating the diagonalization of $(H_{ij})$ with each new set, until $E_0 (\{\psi_i\})$ is a minimum. The orthogonal matrix $(O_{ij})$ is generated and the matrix $(H_{ij})$ diagonalized by Jacobi's method.

We list below the matrix elements $H_{ij}$ for a Hamiltonian $H = T + V = \sum_{i=1}^{3} T_i + i \sum_{i<j}^{3} V_{ij}$

$$V_{ij} = -V_0 \left\{ e^{-a_0^2 n_i^2} \frac{P_{ij}}{x} + \mu e^{-a_0^2 n_i^2} \frac{P_{ij}}{x} + \nu e^{-a_0^2 n_i^2} \frac{S_{ij}}{x} \right\}$$

$$P_{ij} = \frac{i}{x} (1 - \sigma_i \cdot \sigma_j) \quad S_{ij} = 3 (\sigma_i \cdot \sigma_j \cdot \sigma_j - \sigma_i \sigma_j)$$
between the functions

\[
\mathcal{X}_1 = \frac{1}{2} \left( n_3, r_3 \right) \phi_0^{\frac{3}{2}} m_3
\]

\[
\mathcal{X}_2 = \frac{1}{2} \left( n_3, r_3 \right) \sum_{\lambda_3, m_3} C_{\lambda_3, m_3}^{rac{3}{2}} \frac{1}{2} m_3, 2 m_3 \ T_{1/3}^{m_3} \ \phi_1^{\frac{3}{2}} m_3
\]

\[
\mathcal{X}_3 = \frac{1}{2} \left( n_3, r_3 \right) \sum_{\lambda_3, m_3} C_{\lambda_3, m_3}^{rac{3}{2}} \frac{1}{2} m_3, 2 m_3 \ \left( T_{1/3}^{m_3} + \frac{3}{4} T_{2/3}^{m_3} \right) \phi_1^{\frac{3}{2}} m_3
\]

\[
\mathcal{X}_4 = \frac{1}{2} \left( n_3, r_3 \right) \sum_{\lambda_3, m_3} C_{\lambda_3, m_3}^{rac{3}{2}} \frac{1}{2} m_3, 2 m_3 \ \left( T_{2/3}^{m_3} - \frac{3}{4} T_{1/3}^{m_3} \right) \phi_1^{\frac{3}{2}} m_3
\]

of equation (41). The \( \phi_\xi \) are given by equation (41a) with

\[
N_1 = \pi \xi^{-6} \ 3^{-\frac{1}{2}} \ 2^{-7}
\]

\[
N_2 = \pi \xi^{-10} \ 3^{-\frac{1}{2}} \ 2^{-7}
\]

\[
N_3 = \pi \xi^{-10} \ 3^{-\frac{1}{2}} \ 2^{-13} \ 5.7
\]

\[
N_4 = \pi \xi^{-14} \ 3^{-\frac{5}{2}} \ 2^{-13} \ 5.7
\]

\[
H_{11} = 3 \left( 3 \frac{\xi_1^2 \xi_2^2}{m} - \frac{3}{2} V_0 \phi_1 \right)
\]

\[
H_{22} = 3 \left( 3 \frac{\xi_1^2 \xi_2^2}{m} - \frac{3}{2} V_0 \phi_2 \right)
\]

\[
H_{33} = 3 \left( 7 \frac{\xi_1^2 \xi_2^2}{m} - \frac{3}{2} V_0 \phi_3 \right)
\]

\[
H_{44} = 3 \left( 7 \frac{\xi_1^2 \xi_2^2}{m} - \frac{3}{2} V_0 \phi_4 \right)
\]

\[
H_{12} = \frac{3}{2} V_0 \nu \frac{\xi_1^3 \xi_2^5}{(\xi_1^2 + \xi_2^2)^7} \phi_{12}
\]

\[
H_{13} = -\frac{3}{2} V_0 \nu \frac{\xi_1^3 \xi_2^7}{(\xi_1^2 + \xi_2^2)^5} \phi_{13}
\]

\[
H_{14} = -\frac{3}{2} V_0 \nu \frac{\xi_1^3 \xi_2^7}{(\xi_1^2 + \xi_2^2)^5} \phi_{14}
\]
\[ H_{23} = -\frac{3}{4} V_0 \frac{k_3^5 k_3^7}{(k_3^2 + k_3^4)^6} P_{23} \]
\[ H_{24} = -\frac{3}{4} V_0 \frac{k_2^5 k_3^7}{(k_2^2 + k_3^4)^6} P_{24} \]
\[ H_{34} = -\frac{3}{4} V_0 \frac{k_3^7 k_4^7}{(k_3^2 + k_4^4)^6} P_{34} \]

\[ P_1 = \mu \omega_1^{3(1)} + \omega_1^{3(1)} \]
\[ P_2 = \frac{1}{2} \omega_2^{3(1)} + \omega_2^{5(1)} + \frac{1}{2} \omega_3^{7(1)} + \nu \left( 1 + \omega_2^{5(1)} \right) \omega_2^{7(1)} \]
\[ P_3 = \frac{3}{5.2} \left\{ \omega_3^{3(1)} - \frac{23}{27} \omega_3^{5(1)} - \frac{70}{27} \omega_3^{7(1)} + \omega_3^{9(1)} + \omega_3^{7(2)} \right\} \]
\[ + 2 \nu \left[ \frac{8}{7} \omega_3^{5(1)} - \frac{23}{7} \omega_3^{7(1)} + \omega_3^{9(1)} \right] \]
\[ P_4 = \frac{3}{5.2} \left\{ \omega_4^{3(1)} + \frac{35}{27} \omega_4^{5(1)} - \frac{80}{27} \omega_4^{7(1)} + \omega_4^{9(1)} + \omega_4^{7(2)} \right\} \]
\[ + 2 \nu \left[ \frac{64}{85} \omega_4^{5(1)} - \frac{3}{7} \omega_4^{7(1)} + \frac{13}{27} \omega_4^{9(1)} \right] \]
\[ P_{12} = \frac{\sqrt{15} \omega_2^{5(1)}}{5} \omega_2^{5(1)} \]
\[ P_{13} = \frac{\sqrt{15} \omega_3^{5(1)}}{5.7} \left\{ 5 \omega_3^{(1)}(1) - 7 \omega_3^{(5)}(1) \right\} \]
\[ P_{14} = \frac{\sqrt{15} \omega_4^{5(1)}}{5.7} \left\{ -5 \omega_4^{(1)}(1) + 7 \omega_4^{(5)}(1) \right\} \]
\[ P_{23} = \frac{\sqrt{15} \omega_3^{5(1)}}{5.7} \left\{ -\omega_3^{3(1)} + \omega_3^{5(1)} + \omega_3^{7(1)} - \omega_3^{9(1)} + 2 \nu \left[ \omega_3^{5(1)}(1) - \omega_3^{9(1)} \right] \right\} \]
\[ P_{24} = \frac{\sqrt{15} \omega_3^{5(1)}}{5} \left\{ -\omega_3^{3(1)} + \omega_3^{5(1)} - 3 \omega_3^{7(1)} + \omega_3^{9(1)} + \omega_4^{7(1)} \right\} \]
\[ + 2 \nu \left[ \omega_3^{5(1)}(1) - \omega_3^{9(1)} \right] \]
\[ P_{34} = \frac{2.3}{5} \left\{ \omega_3^{3(1)}(1) - 2 \omega_3^{5(1)}(1) + 2 \omega_3^{7(1)}(1) - \omega_3^{9(1)}(1) \right\} \]
\[ + 2 \nu \left[ \omega_3^{5(1)}(1) - \omega_3^{9(1)}(1) + \frac{20}{9} \omega_3^{7(1)}(1) \right] \]
\[ \omega_{x}(\omega) = \{ 1 + \frac{3}{5} \frac{a^{2}}{k_{x}^{2} - k_{y}^{2}} \}^{-\frac{1}{2}} \omega_{x}(\omega) = \omega_{x}(\omega) \]
Acknowledgments

The author would like to express his sincere gratitude to Dr. W. Tobocman, who suggested and guided this investigation from its inception. His kindness and patience during the course of this work has been of immeasurable value to the author. Several helpful and stimulating discussions with Dr. L. L. Foldy and Dr. M. K. Banerjee are gratefully acknowledged. This work has been assisted financially by the Atomic Energy Commission and the National Science Foundation.
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This large per cent D state is probably related to the Gaussian potentials, which compared to the exponential and Yukawa shapes favors more percent D state since the Gaussian is less concentrated near the origin where the D state wave function goes to zero while the S state wave function remains finite. The calculation of R. L. Pease and H. Feshbach (Phys. Rev. 88, 945 (1952)) used a Yukawa shape and gave 3.6% D state.

To be consistent we should include tensor forces in the residual nucleus. However this would greatly complicate the discussion, and we are primarily interested here in the effect of the presence of the He\(^3\) D state.


19 These results are due to Derrick and Blatt and are given in an appendix of their paper (Reference 11).

20 These three wave functions are clearly mutually orthogonal. This is one advantage of writing them in terms of the spherical tensors.