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Modeling, Optimization and Synthesis for Fully Integrated Spiral Inductors

by

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ABSTRACT

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Accurate and efficient modeling, optimization, and synthesis of integrated spiral inductors continue to hinder the automated design of mixed-signal circuits in system-on-chip technology. In this thesis, we develop a modeling and automated design methodology for integrated spiral inductors. We have created a wideband inductor model based on closed-form analytical expressions to capture a plethora of resistive, inductive, and capacitive parasitic effects. Leveraging the speed of the inductor model, we have developed a variability-aware automated design methodology that efficiently generates Pareto-optimal inductors based on application requirements. At its core the automated design methodology employs a scalable multi-level single-objective optimization engine that integrates the flexibility of deterministic pattern search optimization with the rapid convergence of local nonlinear convex optimization. The results demonstrate that the inductor modeling, optimization, and synthesis methodology accurately locates and characterizes near-optimal inductor designs with orders of magnitude speed improvement when compared with existing modeling and optimization techniques.
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Chapter 1

Introduction

1.1 Spiral Inductors for Integrated Mixed-Signal Systems

The growing demand for integrated wireless and other mixed-signal system-on-chip (SoC) applications have spurred the need for innovative design techniques to improve chip-performance, reliability and time-to-market. The advancement and miniaturization of semiconductor devices, microprocessors, analog-to-digital converters and low-power wireless transceivers have enabled the development of fully integrated mixed-signal systems in SoC technology. By integrating digital, analog and RF components on the same chip, SoC solutions can offer increased performance and reliability with substantially less power consumption. Furthermore, single-chip solutions decrease overall physical size and manufacturing cost, which enable many innovative applications [1,2].

In order to successfully realize increasingly complex and integrated mixed-signal systems, robust design automation techniques in the analog domain must be developed to rapidly generate reliable systems that meet application requirements. However, analog design automation continues to trail behind its digital counterpart in both
sophistication and utilization [3, 4]. Analog components by nature are highly sensitive to external noise and radiation and require precisely characterized components in order to function properly. Consequently, the analog design space is extensive, and innovative techniques are required to model, optimize and synthesize analog circuits. By overcoming these challenges and leveraging the power of design automation, analog and mixed-signal systems in system-on-chip (SoC) technology will deliver greater functionality, increased performance and improved reliability [5–8].

Within the analog realm, spiral inductor design and synthesis continues to hinder the automated design of mixed-signal systems. Spiral inductors suffer from complex loss mechanisms in the metal and substrate, and consume large chip area, making them difficult to characterize and expensive to implement [9]. For many on-chip analog circuits, the performance of on-chip spiral inductors is a limiting factor [4]. The spiral inductor design space is complex, and innovative optimization and synthesis techniques are required to maximize performance. Numerous analog circuits such as low noise amplifiers (LNA) [10–31], voltage controlled oscillators (VCO) [32–37], and on-chip filters [38–42], depend on inductors with optimized design parameters. Consequently, the modeling and automated design of spiral inductors is critical to the successful realization of mixed-signal systems [43–49].
1.2 Thesis Contributions

In this thesis, we develop a wideband modeling and variability-aware multi-level automated design methodology for fully integrated spiral inductors in mixed-signal systems. Accurate and efficient modeling, optimization, and synthesis of integrated spiral inductors are critical to the successful and cost-effective realization of analog circuits in fully integrated mixed-signal and RF systems.

1.2.1 Wideband Integrated Spiral Inductor Modeling

We have created an analytical wideband spiral inductor modeling methodology in order to quickly and accurately optimize and synthesize integrated inductors [46-49]. To account for the resistance and physical inductance in the spiral inductor’s conductors due to skin and proximity effects, we have developed new closed-form expressions that exploit the inductor’s geometry to significantly reduce computational overhead [46, 48, 49]. To efficiently model the impact of magnetically induced substrate eddy currents on the spiral inductor, we have utilized modeling methods based on complex image theory [47]. For wideband analytical resistance and inductance modeling for both the conductors and substrate, we have created a technique for creating frequency-independent ladder circuit models [49]. Our analytical inductor models provide up to 5 orders of magnitude average performance improvement over field solver-based approaches with typical errors of less than 3 percent [46–49]. Furthermore, our model demonstrates excellent agreement with measured data from
inductors fabricated in a TSMC 0.18 $\mu$m process [46,48]. By providing an efficient, accurate, and scalable wideband modeling methodology, we have enabled efficient spiral inductor synthesis.

1.2.2 Multi-Level Inductor Optimization and Variability-Aware Pareto-Optimal Synthesis

We have developed a robust and flexible automated synthesis methodology for integrated spiral inductors to efficiently generate Pareto-optimal designs based on application requirements [43–45]. At the core of the synthesis methodology, we have created a scalable multi-level single-objective optimization engine for integrated spiral inductors that supports any generalized inductor modeling technique [43,45]. The multi-level optimization approach integrates the flexibility of deterministic pattern search optimization with the rapid convergence of local convex optimization to exploit design space characteristics. Our multi-level optimization engine yielded optimal quality factors for four inductor design examples with an average of 457 function evaluations, resulting in up to a 40000x speedup over existing techniques. Using the multi-level optimization engine, we combine multi-objective optimization techniques with surrogate functions that approximate the Pareto surfaces in the design space [44,45]. The surrogate functions can be reused to design multiple Pareto-optimal inductors in higher level circuits. We also demonstrate how to utilize our methodology to generate inductors that are less susceptible to process variability to improve overall yield. Our results indicate that the synthesis methodology efficiently optimizes inductor designs
with an improvement of up to 51 percent in key design constraints while reducing the impact of process variation. The spiral inductor synthesis methodology has the speed and flexibility necessary to generate optimal designs and provide reliable inductor implementations in mixed-signal SoC systems [43–45].

1.3 Thesis Organization

The thesis is organized in the following manner. Chapter 2 summarizes previously proposed modeling and optimization methods for integrated spiral inductors, and motivates and contextualizes the inductor modeling, optimization, and synthesis research presented in this thesis. In Chapter 3, we describe the analytical modeling approach for integrated spiral inductors and compare the model's results with those obtained from previously proposed modeling techniques, electromagnetic field solvers, and fabricated inductors. Chapter 4 presents the multi-level optimization and variability-aware synthesis methodology including a sensitivity analysis of the inductor design space. We demonstrate that the methodology locates near-optimal inductor designs and can be used to conserve design resources while reducing the impact of variability. Chapter 5 summarizes the key contributions of the thesis and presents future research directions.
Chapter 2

Background and Motivation

In this chapter, we describe the challenges and motivations behind spiral inductor modeling, optimization, and synthesis. Critical RF/analog circuits depend on inductors with optimized design parameters to enhance performance and reliability. The performance of integrated spiral inductors is maximized in the 100 MHz to 10 GHz range, where the low frequency quality factor ($\omega L/R$) of the inductor is large and the inductor is below its self-resonant frequency, which is described in further detail in Section 2.1. For higher frequency applications, alternative inductor technologies must be used [50]. Critical applications in this frequency band include wireless ethernet (IEEE 802.11.(a,b,g)) over the unlicensed bands at 2.4 and 5.4 GHz and other applications that adhere to the newly developed ultrawideband standard (802.15.3a) [51], which has been proposed as a wireless communications medium for a number of consumer applications [16]. For most applications that will implement the ultrawideband standard, the RF front-end will need to simultaneously receive signals across a broad spectrum of frequencies ranging from 3.1 GHz to 10.6 GHz.

Consequently, developing circuits and passive components that meet performance
requirements across the entire frequency range is a crucial challenge that must be addressed. Narrow [10–15] and wideband [16–31] LNAs require precisely characterized and optimized spiral inductors to ensure input and output impedance matching, which facilitates maximum power transfer, low noise, and sufficiently flat gain across the entire range of operating frequencies. For integrated VCOs, high quality inductors are required to lower the phase noise of the oscillator [32–37]. Integrated filters require high quality inductors to ensure a relatively flat response and lower resistive losses in the circuit [38–42]. Consequently, accurate design and optimization of spiral inductors are critical to a successful and cost-effective realization of analog circuits in fully integrated mixed-signal and RF systems [6].

The rest of this chapter is organized in the following manner. Section 2.1 discusses challenges in inductor characterization and previous modeling techniques. Section 2.2 explains the benefits of inductor optimization and Pareto-optimal synthesis and describes previously proposed inductor optimization techniques. This chapter provides the context and motivation for the modeling and synthesis research presented in Chapters 3 and 4.

2.1 Spiral Inductor Modeling

In order to enable rapid spiral inductor design, modeling techniques must provide the accuracy, speed, and scalability necessary to explore the inductor design space over a wide range of process technologies, inductor geometries, and operating
**Figure 2.1:** Electromagnetic affects that must be captured for accurate spiral inductor modeling.

frequencies. The plethora of loss mechanisms affecting integrated spiral inductors makes their characterization particularly challenging. Figure 2.1 displays the electromagnetic effects in the inductor’s conductors and in the substrate below the inductor. The physical inductance is determined based on the current flowing in the conductors (self-inductance) and reinforced by the current flowing in the same direction in adjacent turns (mutual inductance) in the spiral inductor. Resistive losses in the
inductor's conductors are increased at high frequencies due to non-uniform current distribution resulting from skin and proximity effects. Magnetically induced eddy currents produced in the substrate below the inductor affect the inductance and resistance of the inductor, especially on high conductivity substrates. Furthermore, capacitive coupling between the conductors and substrate leakage currents decrease the energy stored in the inductor. Each loss mechanism must be modeled to accurately determine the performance of the inductor for a variety of geometries and process technologies.

Important figures of merit for spiral inductors in mixed-signal circuit applications include the inductor's quality factor, effective inductance value, and self-resonant frequency. The quality factor is defined as

\[ Q = \frac{E_{\text{Stored}}}{E_{\text{Dissipated}}} = \frac{Im(Z)}{Re(Z)} \]  

(2.1)

where \( E_{\text{Stored}} \) is the energy stored in the inductor, \( E_{\text{Dissipated}} \) is the energy dissipated by the inductor parasitics, and \( Z \) is the input impedance of the inductor [9]. Note that \( Q \) depends on the operating frequency of the inductor. For a fixed inductor design at low frequencies, the quality factor increases as the operating frequency is increased since the effective inductance \( (Im(Z) \approx j\omega L) \) is primarily determined by the physical inductance. The effective inductance is defined as

\[ L_{\text{eff}} = \frac{Im(Z)}{\omega} \]  

(2.2)
where \( \omega \) is the operating frequency of the inductor in radians per second. The effective resistance \( (Re(Z)) \) of the inductor remains relatively constant at low frequencies. At higher frequencies, capacitive coupling begins to slow the increase in \( Im(Z) \) and \( Re(Z) \) begins to increase due to skin and proximity effects in the conductors. For inductors fabricated over substrates with high conductivities, magnetically induced substrate eddy currents also decrease \( Im(Z) \) and increase \( Re(Z) \). At the frequency that the maximum quality factor is achieved, capacitive and resistive effects begin to dominate the inductance. For frequencies above the maximum quality factor frequency, the quality factor decreases and eventually reaches 0, which is defined as the inductor’s self-resonant frequency. The inductor should have a self-resonant frequency well above its maximum frequency of operation. At frequencies higher than the self-resonant frequency, \( Im(Z) < 0 \), and therefore, the inductor behaves as a capacitor. The value of the maximum quality factor and its frequency are important for many circuit applications such as low noise amplifiers [10–15].

Modeling techniques spiral inductors fall into two major categories: field-solver based modeling and analytical compact modeling. Full-wave electromagnetic simulators provide the most accurate means of analysis [52–54]. However, since each simulation generally requires several hours to complete, design adjustments and optimization are impractical. Quasi-static field solvers based on the Partial Element Equivalent Circuit (PEEC) method reduce simulation times when compared to full-wave analysis with minimal loss of accuracy but still typically require several minutes
to evaluate an inductor design [55–63]. Furthermore, to accurately model the impact of high conductivity Si substrates, densely discretized structures are required that significantly increase simulation times [64, 65]. To improve simulation time, several special purpose field solver based modeling tools have been developed specifically for spiral inductors [9, 66–71]. One popular tool, ASITIC [70], speeds the modeling of substrate effects using 2-D approximations and exploits spiral inductor symmetry to further reduce computational complexity. While these tools are fairly accurate and faster than using general full-wave or quasi-static field solvers, they still require significant simulation time to explore multiple designs. Overall, the above mentioned design strategies are appropriate for modeling and comparing several different inductor designs. Spiral inductor optimization and synthesis over the entire design space, however, demands faster solutions.

While not as accurate as field solvers, closed-form analytical models provide the speed necessary for automated design. These models fall into two categories: frequency-dependent and frequency-independent. Frequency-dependent models characterize the inductor near the specified frequency with closed-form expressions for lumped circuit elements in a particular compact model topology. These narrow-band models have either ignored substrate eddy currents [72–76] or rely upon technology specific fitted parameters that are not generally applicable to current and future process technology [77–80]. Furthermore, narrow-band inductor models can produce inaccurate results in time domain simulations, which are necessary in the circuit-
level characterization and design of RF circuits. Wideband inductor models provide frequency-independent circuit elements, typically ladder circuits, for frequency-dependent resistive and inductive effects. However, ladder networks with additional lumped circuit elements are difficult to compute without resorting to curve-fitting techniques based on electromagnetic simulations or measured data to model high frequency resistive and inductive effects [71,81–83]. This also fails to provide a scalable modeling solution across multiple processes. To enable accurate and efficient inductor design space exploration, optimization, and synthesis, systematic wideband inductor modeling techniques that model substrate eddy currents and other frequency dependent effects without resorting to technology specific parameters are required.

2.2 Spiral Inductor Optimization and Synthesis

Spiral inductor optimization and synthesis has several fundamental advantages over equation-based or ad hoc design techniques. Numerical optimization and synthesis provides the accuracy, speed, and variability-awareness necessary to account for statistical variations in deep sub-micron (DSM) effects to improve manufacturing yield for high volume applications. By efficiently exploring the complex design space of fully integrated spiral inductors, numerical optimization and synthesis techniques can produce inductor designs that meet performance and area requirements that are not possible to obtain using conventional equation-based design techniques. Finally, numerical optimization and synthesis provides orders of magnitude enhancement in
the design cycle time over manual design techniques and promotes the re-use of inductor designs across multiple systems. Scalable, accurate, and efficient inductor optimization and synthesis is necessary to successfully design fully integrated mixed-signal circuits in integrated wireless and other SoC applications [6, 43–45].

Spiral inductor optimization techniques typically focus on maximizing the quality factor for a particular operating frequency and inductance value. Traditionally, spiral inductor optimization was primarily a manual process using either pre-characterized inductor designs or *ad hoc* optimization techniques. Several single-objective automated spiral inductor optimization techniques have been proposed. The most common methods for inductor optimization include enumeration and binary search algorithms, which may be intractable for computationally intensive spiral inductor models based on field solvers [68, 80, 84]. Other proposed optimization techniques based on geometric programming [85, 86] or analytical expressions [76, 87, 88] require specific model formulations that can potentially limit their usefulness as modeling techniques are enhanced. In [89], nonlinear constrained convex optimization improved performance over enumeration, but substrate effects were neglected. Furthermore, objective and constraint functions were never proven to be convex, which may prevent convergence to optimum designs [71, 89]. Stochastic global spiral inductor optimization techniques using simulated annealing [90] and genetic algorithms [91, 92] typically employ penalty functions for constrained optimization problems that may require tens of thousands of function evaluations to converge [93]. Optimization based on ar-

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Figure 2.2: Single-objective optimization can produce wasteful designs with respect to other design objectives. In contrast, multi-objective optimization techniques locate Pareto-optimal solutions.

Artificial neural networks requires an extensive training phase that may not be practical for inductor design space exploration [92, 94].

While single-objective optimization is an important tool for the automated design of integrated spiral inductors, inductor synthesis methods should provide a flexible framework to explore design space trade-offs in order to maximize resources and improve performance. Mixed-signal synthesis techniques explore design space trade-offs by constructing and analyzing Pareto-optimal surfaces in order to make decisions that maximize design resources and account for potential design problems [95, 96].

As depicted in Figure 2.2, maximizing a single spiral inductor performance objective can produce wasteful design with respect to other design objectives. Placing arbitrary constraints on the other design objectives can also lead to sub-optimal designs. In contrast, multi-objective optimization techniques locate Pareto-optimal solutions. Pareto-optimal surfaces linking the maximum quality factor to key spiral inductor
design constraints such as operating frequency and inductance are vital in circuits such as VCOs and LNAs where synthesis tools can choose several possible inductor values over a range of frequencies [10,97]. Few studies have considered spiral inductor Pareto optimization. In [85], the maximum quality factor and inductance value were compared while [9] explored numerous inductor design trade-offs using Monte Carlo analysis of random inductor geometries to identify the Pareto front between design parameters, which may not be practical for computationally intensive numerical inductor models.

Pareto optimization is also an important tool for designing spiral inductors that are less susceptible to process variability and modeling error. As technology scales and minimum geometric dimensions become smaller, conductors fabricated in current and future technologies are impacted by both inter and intra-die process technology variations [98,99]. Spiral inductor designs also face variation in characterization due to modeling errors. Therefore, in addition to the Pareto optimality of inductor design constraints, automated spiral inductor synthesis techniques must also take into account sources of process variability and modeling error to design reliable mixed-signal systems in SoC technologies. Robust and flexible multi-objective optimization and synthesis techniques for integrated spiral inductors are necessary to efficiently generate Pareto-optimal designs to enhance the performance of mixed-signal systems in SoC technology.
Chapter 3

Wideband Spiral Inductor Modeling Techniques for Efficient Design Space Exploration

In this chapter, we introduce a wideband model for integrated spiral inductors. The modeling approach is centered around new analytical expressions for the inductor's high frequency series resistance, series inductance and magnetically induced substrate eddy currents [46–49]. Based on the resistance and inductance formulations, we have developed a systematic technique for creating wideband circuit models, which are necessary for accurate time domain simulation [49]. We compare the model with numerical field solvers, current analytical modeling techniques, and fabricated inductors. The model provides orders of magnitude performance improvement over numerical field solver-based approaches with typical errors of less than 4 percent when compared with field solvers and less than 10 percent error when compared with measured data from spiral inductors fabricated in the TSMC 0.18 micron 6-metal RF-CMOS process. The model also yields significantly better accuracy than several other analytical resistance modeling techniques. The wideband circuit generation technique captures the frequency-dependent resistance and inductance of the
inductor with typical errors of less than 3.2 percent. The accuracy and substantial improvement in performance attained using the model makes inductor optimization and synthesis feasible, which will unlock the potential for high performance integrated spiral inductors in mixed-signal systems.

In Section 3.1, we present the overall spiral inductor model. Section 3.2 describes the modeling techniques that we have developed for high frequency resistance, physical inductance, substrate eddy currents, and wideband characterization. Section 3.3 describes the results obtained from our model when compared with numerical field solvers, current analytical modeling techniques, and fabricated inductors.

### 3.1 Overall Analytical Inductor Model

In order to characterize the inductor’s design space and evaluate the proposed synthesis methodology, we utilize analytical spiral inductor modeling techniques that have been verified using numerical field solvers and measurements from fabricated inductors [46–49]. Key inductor design parameters such as the number of turns (n), inductor diameter (d), conductor width (w) and conductor spacing (s) are depicted in Figure 3.1. Previous spiral inductor optimization studies have considered n to be a discrete parameter with quarter turn multiples [68, 85, 89]. In contrast, we assume n is a continuous variable in the optimization problem and therefore avoid resorting to mixed-integer optimization techniques that are typically less efficient than their continuous counterparts [93]. Furthermore, by not restricting n to discrete values, we
Figure 3.1: Typical spiral inductor design parameters include the number of turns \((n)\), inductor diameter \((d)\), conductor width \((w)\) and conductor spacing \((s)\).

have the flexibility to generate designs with higher quality factors.

To evaluate the model for a given set of process and geometric parameters, we determine the appropriate component values for each element in the eleven-element \(\pi\)-model depicted in Figure 3.2. \(R_s\) and \(L_s\) model the inductor’s free-space resistance and physical inductance, which are calculated using the methods presented in Sections 3.2.1 and 3.2.2. \(C_s\) models the impact of sidewall capacitance between the conductors, which is calculated using the distributed capacitance model presented in [100]. The oxide capacitance \((C_{ox})\), substrate capacitance \((C_{sub})\) and conductance \((G_{sub})\) account for substrate leakage currents and are modeled using [82] and [72]. Inductive and resistive losses due to magnetically induced substrate eddy currents \((I_{eddy}, R_{eddy}, \) and \(M_{eddy})\) are determined using the methodology presented in Section 3.2.3.

Patterned ground shields can be employed to reduce the impact of substrate eddy currents \((R_{eddy}, L_{eddy})\) and leakage currents \((R_{sel})\) on the spiral inductor at the cost
Figure 3.2: Eleven-element π-model used for spiral inductor modeling.

of increased capacitance due to the decreased distance between the inductor and the ground shield [101, 102]. To model the ground shield, we replace the substrate elements $C_{ox}$, $C_{si}$ and $R_{si}$ with $R_p = \infty$ and $C_p = C_{ox} t_{ox}/t_{spiral,poly}$ where $t_{ox}$ is the distance between the conductors and the substrate, $t_{spiral,poly}$ is the distance between the conductors and the polysilicon ground shield, and $C_p$ and $R_p$ are the equivalent series substrate resistance and capacitance combining $C_{ox}$, $C_{sub}$ and $R_{sub}$ [85]. We also set $M_{eddy}$ to 0 since the patterned ground shield prevents substantial eddy currents from forming on the substrate.

We utilize the eleven-element π-model in this optimization study primarily because of its speed. However, the inductor optimization and synthesis approach presented in Chapter 4 is general enough to be applied to most inductor geometries using many different inductor modeling techniques. More complex analytical spiral inductor mod-
els based on the 2-$\pi$ configuration will yield similar results to the single $\pi$-model for narrow-band applications [82, 83]. Since our inductor synthesis methodology optimizes inductor designs operating at particular frequencies as described in Section 4.2, the wideband properties of the 2-$\pi$ model are not necessary during the optimization process. However, if the model evaluation time is acceptable to the user, our methodology is general enough to be utilized with 2-$\pi$ analytical models or electromagnetic field solvers. Furthermore, once a design is synthesized, a more complex model can be utilized for final characterization and inclusion in time-domain simulations for higher level designs.

In addition to the square planar geometry depicted in Figure 3.1, integrated inductors can also be fabricated in octagonal, circular, and stacked configurations in some manufacturing processes. For fixed values of $n$, $d$, $w$, and $s$, the difference between the quality factors of circular, octagonal, and square spiral inductors is typically on the order of 10\%, with octagonal and circular inductors providing slightly better per unit area inductance [9, 66, 82]. Stacked spiral inductors, which are fabricated on multiple metal layers and connected in series, provide significantly larger inductance values per unit area. However, they suffer from larger capacitive losses due to the decreased distance between the inductor and the substrate and the larger inter-layer capacitive coupling. This results in a lower self-resonant frequency [9, 103]. The eleven-element $\pi$-model depicted in Figure 3.2 is general enough to model octagonal, circular, and stacked inductor configurations, but some of the formulations for the individual model
elements must be modified. These modifications are described for each element in the following sections.

### 3.2 Characterizing Individual Compact Model Elements

In this section, we discuss the computation of each circuit element value in the compact inductor model displayed in Figure 3.2. We also discuss our wideband modeling techniques for high frequency resistance and inductance.

#### 3.2.1 High Frequency Series Resistance Model

To calculate the resistance of the inductor, we have developed a model based on physics-based formulations and linear least squares regression. The physical resistance is defined as

\[
R_s = R_{\text{cond}} + R_{\text{prox}}
\]  

where \( R_{\text{cond}} \) represents the resistance of the conductors including skin effect losses and \( R_{\text{prox}} \) models the contribution from proximity effect between the conductors. Proximity effect is a major source of resistive loss, which can become a factor at frequencies as low as 500 MHz [104]. The contribution of proximity effect to \( R_s \) is difficult to characterize since it depends on the overall geometry of the inductor. Approaches for modeling proximity effect include field-solver based tools [68, 105], curve-fitted formulas [82, 83], and physics-based approximations [75, 104]. In [104], an approximate closed for formula for resistance due to proximity effect is developed.
based on a linear approximation of the magnetic field profile from the center to the outermost turn of the inductor. However, [104] contains several assumptions that limit its use to frequencies well-below the self-resonant frequency. An alternative approach, presented in [75], approximates the increased resistance due to proximity effect by splitting the conductor into several filaments and individually solving for the increased resistance due to the magnetic fields produced by internal and external currents. We model the proximity effect based on the expressions for the added resistance due to the external magnetic fields in [75].

To determine the high frequency resistance due to skin effect, which is included in $R_{cond}$, consider a semi-infinite conductor in the z-direction with current only flowing in the x-direction. The current flowing in the conductor is

$$I_x = \int_0^t \int_0^w J_0 e^{-x/w} dy dx \longrightarrow$$

$$I_x = J_0 (1 - e^{-t/\delta}) \delta w$$

(3.2)

where $w$, $t$, and $\delta$ are the width, thickness, and skin depth of the conductor, respectively [72]. Therefore, the resistance due to skin effect is

$$R_{cond} = \frac{\rho l}{w \min(t, t_{eff})}$$

(3.3)

where $l$ and $\rho$ are the total length and resistivity, respectively, and $t_{eff}$ is an effective
conductor thickness due to skin effect, which is typically defined as [72]

\[ t_{\text{eff}} = \delta \left( 1 - e^{-\frac{1}{\delta}} \right). \]  (3.4)

We call (3.4) the \textit{current sheet formulation}.

The current sheet formulation can be inaccurate due to the finite, rectangular geometry of the conductors. Therefore, we have developed a more accurate formulation for \( t_{\text{eff}} \) that accounts for the finite geometry of the conductors

\[ t_{\text{eff}} = \delta_{\text{eff}} \left( 1 - e^{-\frac{1}{\delta_{\text{eff}}}} \right) \left( 1.087 + 2.74r - 1.203r^2 \right) \]  (3.5)

\[ \delta_{\text{eff}} = \sqrt{\frac{\rho}{\pi \mu_0 f_{\text{eff}}}} \]  (3.6)

where \( r = t/w \) and \( \delta_{\text{eff}} \) is the effective skin depth of the conductor with an effective frequency defined as

\[ f_{\text{eff}} = f \left( 0.971 + 0.096r - 0.281r^2 \right) \]  (3.7)

where \( f \) is the operating frequency. Note that in the expressions (3.5) - (3.7), if \( w < t \), then reverse \( w \) and \( t \) to find an effective width for the conductor. The polynomials of \( r \) in \( t_{\text{eff}} \) and \( f_{\text{eff}} \) represent the dependence of the current distribution on the cross-sectional geometry of the conductors. The formula parameters were fitted to 700,000 conductor configurations simulated using the field solver, FastHenry [55].
The parameters were first determined using a linear least squares approach to reduce the mean error followed by a minimization of the maximum error using a simplex-based search algorithm. The formula for $R_{cond}$ is valid for conductors with widths ranging from 0.1 to 50 $\mu$m, thicknesses ranging from 0.1 to 5 $\mu$m, and conductivities ranging from 15 to 60 $(\Omega \cdot \mu m)^{-1}$ provided that the width to thickness ratio does not exceed 50. This range of conductivities includes the values for standard aluminum and copper interconnect technologies. The formula is accurate for frequencies where the magneto-quasistatic assumption employed in FastHenry is valid. Since the skin effect resistance formulation depends only on the length of the inductor's conductors, the modeling technique is valid for circular, octagonal, and stacked configurations.

### 3.2.2 Analytical Inductance Model

To calculate the physical inductance, $L_s$, we developed a formulation that exploits the symmetry of the inductor to significantly reduce computational cost over approaches based on field solvers. The physical inductance can be expressed as

$$L_s = L_{self} + L_m$$  \hspace{1cm} (3.8)

where $L_{self}$ is the partial self inductance of the conductors,

$$L_{self} = 200l \left( \ln \frac{2l}{w+t} + 0.5 + \frac{w+t}{3l} \right),$$  \hspace{1cm} (3.9)
where \( l, w, \) and \( t \) are the length, width and thickness of the conductors, respectively. Similarly, the mutual inductance, \( L_m \), between two conductors of the same length can be expressed as

\[
L_m = 200l \left( \ln \left( r + \sqrt{1 + r^2} \right) - \sqrt{1 + \left( \frac{1}{r} \right)^2 + \frac{1}{r}} \right)
\]

(3.10)

where \( r = l/d \), \( l \) is the length of the conductors, and \( d \) is the distance between the conductors [72]. Formulas for the inductance between two conductors of difference lengths are presented in [106].

By assuming the inductor is symmetric, the total inductance can be calculated from the inductance contributed from one of the 4 sides of the spiral inductor, which significantly reduces computational complexity. To further simplify the formulation, we approximate the average mutual inductance between a given conductor and conductors on both adjacent and opposite sides of the inductor as depicted in Figure 3.3. Therefore, based on (3.10), the mutual inductance depends on the weighted average length and distance between conductors with current flowing in the same direction \((l_+ \text{ and } d_+)\) and the weighted average length and distance between conductors with current flowing in opposite directions \((l_- \text{ and } d_-)\). Consequently, the physical inductance of the inductor is accurately captured based on the interaction between only 2 pairs of virtual conductors with lengths and distances defined by \( l_+, l_-, d_+, \) and \( d_- \). The average length between all possible combinations of conductors on the same side
of the inductor is

\[ l_+ = \frac{1}{n^2 - n} \sum_{i \neq j}^n \left( \frac{l_i + l_j}{2} \right) \]  \hspace{1cm} (3.11)

where \( l_i \) and \( l_j \) are the lengths of the longest conductors on the \( i^{th} \) and \( j^{th} \) turns of the inductor as depicted in Figure 3.3. By expanding the sum in (3.11), \( l_+ \) is simplified to yield

\[ l_+ = d - \frac{(n - 1)^2 p}{n} \]  \hspace{1cm} (3.12)

where \( d \) is the diameter of the inductor as depicted in Figure 3.3 and \( p = w + s \). The average length between pairs of conductors on opposite sides of the inductor is

\[ l_- = \frac{1}{n^2} \sum_{i \neq j}^n \left( \frac{l_i + l_j}{2} \right). \]  \hspace{1cm} (3.13)

Expanding the sum in (3.13) reveals that \( l_- = l_+ \), which we refer to as \( l_{avg} \).

Similar to the calculation of \( l_{avg} \), the average distance between pairs of conductors with currents flowing the same direction can be expressed as

\[ d_+ = \frac{1}{l_{avg}(n^2 - n)} \sum_{i \neq j}^n \left( \frac{l_i + l_j}{2} d_{+(i,j)} \right) \]  \hspace{1cm} (3.14)

where \( d_+ \) is the distance between the conductors in the \( i^{th} \) and \( j^{th} \) turns on the same side of the inductor. The contribution of each distance to the average distance has been weighted by the average length to reflect the proportionality of mutual inductance to the length in (3.10). By expanding the sum in (3.14), \( d_+ \) is simplified
Figure 3.3: The model for physical inductance depends on the interaction between only 2 pairs of virtual conductors.

to yield

$$d_+ = \frac{p(2d(n+1) + p - 2np(n-1))}{6l_{avg}}.$$  \hspace{1cm} (3.15)$$

Similarly, the average distance between pairs of conductors on opposite sides of the conductor can be expressed as

$$d_- = \frac{1}{l_{avg}n^2} \sum_{i,j}^n \left( \frac{l_i + l_j}{2} d_{-(i,j)} \right)$$  \hspace{1cm} (3.16)$$

where $d_-$ is the distance between the conductors in the $i^{th}$ and $j^{th}$ turns on opposite
sides of the inductor. Expanding the summation yields

\[ d_- = \frac{6d^2n - 6dp(n - 1)(2n - 1)}{6n l_{avg}} + \frac{p^2(n - 1)(3 + n(7n - 11))}{6n l_{avg}}. \] (3.17)

Using the average length \( l_{avg} \) and distance of the conductors on the same \( d_+ \) and opposite \( d_- \) sides of the inductor, the overall mutual inductance is

\[ L_{mut} = 4 \left( (n^2 - n)L_m(l_{avg}, d_+) - n^2L_m(l_{avg}, d_-) \right) \] (3.18)

where \( L_m(l, d) \) is given by (3.10). When the inductor contains a fractional number of turns, we linearly interpolate between the inductance results for \( n = [n] \) and \( n = [n] + 1 \). Note that inductance formulation is physics-based and can be easily calculated using the model's simple, closed-form expressions. For circular and octagonal inductor geometries, the formulation in [106] can be used to calculate the series inductance. The physical inductance of stacked configurations can be calculated using the techniques presented in [103].

### 3.2.3 Substrate Eddy Current Model

Spiral inductors fabricated on highly conductive substrates experience significant losses due to the magnetically induced substrate eddy currents. We utilize complex image theory to capture substrate eddy currents [107]. Analogous to the standard
Figure 3.4: Spiral inductor conductor and its associated complex image used to capture the impact of substrate eddy currents.

Method of images, the substrate can be replaced with a mirror image of the spiral inductor at a complex depth to approximate the fields reflected from the substrate as depicted Figure 3.4.

To determine the necessary complex depth, consider an infinitely long conductor that lies at a height $z_o$ above the substrate. The electric field in the region between the substrate and the conductor can be expressed as [107]

$$E = \frac{-j\mu_o\omega I}{2\pi} \int_0^\infty \frac{1}{\lambda} (e^{-\lambda(z-z_o)} + R(\lambda)e^{-\lambda(z+z_o)}) \cos(\lambda x) d\lambda \quad (3.19)$$

where $I$ is the current in the conductor, $z_o$ is the height of the conductors above the substrate, and $R(\lambda)$ is a recursive relation for the substrate reflection coefficient,
which is defined in [108]. The first term of (3.19) is the contribution to the electric field from the conductor and the second term is the reflected electric field from the substrate.

While (3.19) does not have a closed-form solution, it can be manipulated and approximated so that the contribution from the substrate is represented by a complex image at \( z_o + \alpha \). In order to approximate the reflected contribution of the substrate to the electric field with an image conductor, \( R(\lambda) \) in (3.19) can be replaced with

\[
R(\lambda) = f(\lambda)e^{-\alpha\lambda} \rightarrow f(\lambda) = R(\lambda)e^{\alpha\lambda}
\]  

(3.20)

where \( f(\lambda) \) is an unknown function. The function \( f(\lambda) \) can be expanded using a Taylor series about \( \lambda = 0 \) [109]. Since \( f(\lambda) \) is an exponential function,

\[
\frac{df}{d\lambda} = f(\lambda)g(\lambda, \alpha)
\]  

(3.21)

where \( g(\lambda, \alpha) \) is a function of the argument in the exponential of \( f(\lambda) \). The complex image depth, \( \alpha \), is then determined by solving the equation

\[
g(\lambda, \alpha) = 0
\]  

(3.22)

in order to eliminate the first and second order terms in the Taylor series expansion of \( f(\lambda) \). Using the constant term in the Taylor series expansion of \( f(\lambda) \), the field can
now be expressed as

\[
E = \frac{-j\mu_0\omega l}{2\pi} \int_0^\infty \frac{1}{\lambda} (e^{-\lambda(z-z_0)} + e^{-\lambda(z+z_0+\alpha)}) \cos(\lambda x) d\lambda
\]  

(3.23)

Therefore, the reflected field contribution from the substrate can be approximated with a mirror image of the spiral inductor at a complex depth of \(z_0 + \alpha\) below the substrate.

Using complex image theory, we have verified the expression for the complex image depth for a single layer floating substrate presented in [107],

\[
\alpha_1 = \frac{2}{\gamma_1 A},
\]  

(3.24)

and derived the following new explicit expressions for the complex image depths for 2 and 3 layer floating substrates:

\[
\alpha_2 = \frac{2(\gamma_1 + \gamma_2AB)}{\gamma_1(\gamma_1 A + \gamma_2 B)}
\]  

(3.25)

\[
\alpha_3 = \frac{2\gamma_2 A(\gamma_3 B + \gamma_3 C) + 2\gamma_1(\gamma_2 + \gamma_3 BC)}{\gamma_1 \gamma_2 (\gamma_1 A + \gamma_2 B) + \gamma_1 \gamma_3 (\gamma_2 + \gamma_1 ABC)}
\]  

(3.26)

where

\[
A = \tanh(\gamma_1 h_1), \quad B = \tanh(\gamma_2 h_2), \quad C = \tanh(\gamma_3 h_3)
\]

\[
\gamma_1 = \frac{1 + j}{\delta_1}, \quad \gamma_2 = \frac{1 + j}{\delta_2}, \quad \gamma_3 = \frac{1 + j}{\delta_3}
\]
and $h_1$, $\delta_1$, $h_2$, $\delta_2$, $h_3$, and $\delta_3$ are the heights and skin depths of substrate layers 1, 2 and 3, respectively, where layer 1 is the topmost layer of the substrate.

Once the complex image depth is known, the impact of substrate eddy currents can be determined by placing a mirror image of the spiral inductor at a complex distance,

$$D = 2t_{oz} + \alpha$$  \hspace{1cm} (3.27)

below the substrate where $t_{oz}$ is the height of the inductor above the substrate. The impact of substrate eddy currents on the inductor are then captured using

$$L_{eddy} = L_{image}(\text{Re}(D))$$  \hspace{1cm} (3.28)

$$R_{eddy} = -\omega \text{Im}(L_{image}(D))$$  \hspace{1cm} (3.29)

$$M_{eddy} = \sqrt{\frac{R_{eddy}^2}{\omega^2} + L_{eddy}^2}$$  \hspace{1cm} (3.30)

where $L_{image}$ is the mutual inductance between each segment of the inductor and the image conductors separated by a complex distance $D$. $M_{eddy}$ was derived based on the model presented in [101]. $L_{eddy}$ and $R_{eddy}$ are effectively added to the series resistance and inductance in the compact inductor model. Note that the complex image formulation is valid for circular, octagonal, and stacked inductor configurations if the appropriate inductance expressions are used in (3.28)-(3.30).
Figure 3.5: Wideband circuit topologies use to approximate the response of the frequency-dependent spiral inductor resistance and inductance.

3.2.4 Wideband Model Realization

For wideband spiral inductor characterization, which is needed for time-domain simulation, systematic methods are required to produce models that are accurate across a broad range of frequencies. Frequency-dependent circuit elements must be replaced by wideband, frequency-independent circuit networks. Figure 3.5 depicts the frequency-dependent and equivalent wideband circuits used to capture the resistance and inductance due to the conductors and substrate eddy currents.

To generate the wideband circuit models, we first deterministically sample the response of the frequency dependent elements of the inductor model. We then formulate
and solve the following linear least squares problem:

\[
\text{Minimize} \quad \| H_{\text{sampled}}(\tilde{s}) - H_{\text{circuit}}(\tilde{s}, R_1 \cdots R_M, L_1 \cdots L_N) \|_2^2 \\
\text{Subject to} \quad R_1 \cdots R_M, L_1 \cdots L_N > 0
\]  

(3.31)

where \( H_{\text{sampled}}(\tilde{s}) \) is the response of the frequency-dependent model sampled at \( \tilde{s} \) frequency points and \( H_{\text{circuit}}(\tilde{s}, R_1 \cdots R_M, L_1 \cdots L_N) \) is the response of the wideband circuit consisting of \( M \) resistors with values of \( R_1 \cdots R_M \) and \( N \) inductors with values of \( L_1 \cdots L_N \). The optimization problem described by (3.31) determines the values of the circuit elements \( R_1 \cdots R_M \) and \( L_1 \cdots L_N \). As depicted in Figure 3.5, the two frequency-independent circuit topologies we consider are the PEEC configuration and the First Cauer Form configuration, which can equivalently represent the frequency response since the optimal circuit topology is not necessarily unique [110]. Therefore, to find a wideband circuit that closely approximates the frequency-dependent inductor model, we solve a series of problems described by (3.31) for the two circuit topologies with varying numbers of resistors \( (M) \) and inductors \( (N) \).

By determining the passive element values in a fixed circuit topology directly, we guarantee that the wideband circuit model is both stable and passive since the formulation in (3.31) places constraints on the allowed resistor and inductor values. In contrast, other common techniques may approximate the frequency response of the system using linear least squares regression to obtain the rational polynomial
transfer function coefficients [110]. This requires the solution to a polynomial fitting problem with the Vandermonde matrix as its basis, which can become ill-conditioned for wideband rational interpolation problems [111]. Using our modeling approach, we can generate wideband circuit models for high frequency resistance and inductance in integrated spiral inductors that are stable, passive and well-conditioned numerically.

3.3 Inductor Modeling Results

To validate the new analytical inductance model for integrated spiral inductors, the model is compared with both current closed-form inductance modeling techniques and simulated results. The model is also compared with measurements taken from inductors fabricated in the TSMC 0.18 micron RF-CMOS process, which is described in further detail in Section 3.3.5.

3.3.1 High Frequency Resistance Results

In order to evaluate the accuracy of the analytical high frequency resistance model, we simulated 700,000 design examples using the field solver, FastHenry [55]. The simulated conductors have geometries and resistivity values distributed across the range of values for which the formulation is valid. The closed-form model provides up to three orders of magnitude performance improvement over the field solver with an average error of 2.6% and 99% of the cases having errors below 8.6%. In Figure 3.6, we compare the error of our method with the current sheet formulation [72]
Figure 3.6: Error distribution for the high frequency analytical resistance model, the current sheet formulation, the first order approximation of the current sheet formulation, and the direct application of the skin depth compared with FastHenry for 700,000 different conductor geometries and resistivity values.

and a standard first order approximation of current sheet formulation [112]. We also compare the accuracy of our analytical resistance model with the direct application of skin depth to the cross-sectional area of the conductor [113],

\[ R_{skin} = \frac{\rho l}{wt - \max(0, w - 2\delta) \max(0, t - 2\delta)}. \]  \hspace{1cm} (3.32)

The current sheet formulation, its first order approximation, and the direct skin depth formulation have mean errors of 43.2%, 20.1%, and 15.6% with 99% of the cases having errors below 213.5%, 49.8%, and 45.4%, respectively. Therefore, the high frequency resistance model provides an order of magnitude improvement in error.
3.3.2 Inductance Results

The series inductance model presented in Section 3.2.2 is compared with the field solver, FastHenry, for 1,700,000 inductor designs with the following design parameters: $1 \mu m \leq w \leq 33 \mu m$, $0.5 \mu m \leq s \leq 3.1 \mu m$, $50 \mu m \leq d \leq 620 \mu m$, $0.5 \mu m \leq t \leq 2 \mu m$ and $2 \leq n \leq 10$ including fractional turns. Our method has an average error of 2.5% with 99% of the cases having less than 6.0% error. The cases with the highest error occurred when $d$ is small or the inductor has large center fill. The physical inductance model is also compared with other existing closed-form expressions [114,115] whose error distributions are depicted in Figure 3.7. When compared with the inductor designs simulated in FastHenry, the expressions presented in [114,115] have mean errors of 6.5% and 2.8% and with 99% of the cases having less than 10.5% and 15.0%, respectively. Finally, our inductance model has a maximum error of 12% while the expressions presented in [114,115] have maximum errors of 23% and 43%, respectively. With smaller mean and maximum errors, the model for $L_s$ provides a reliable means for quickly determining the physical inductance with two orders of magnitude performance improvement over using the field-solver, FastHenry.

3.3.3 Substrate Eddy Currents Results

In order to evaluate the accuracy of the eddy current formulation and its performance improvement over field solver-based modeling techniques, we simulated 650 spiral inductor designs over highly conductive one, two, or three layer substrates using

37
Figure 3.7: Error compared with FastHenry for the inductance formula described by (3.8) and several other existing formulations.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Turns</td>
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<td>8</td>
</tr>
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<td>Conductor Width (μm)</td>
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<td>20</td>
</tr>
<tr>
<td>Conductor Spacing (μm)</td>
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<td>3</td>
</tr>
<tr>
<td>Inductor Diameter (μm)</td>
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<td>400</td>
</tr>
<tr>
<td>Substrate Layers</td>
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<td>3</td>
</tr>
<tr>
<td>Oxide Thickness - t_{ox} (μm)</td>
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<td>10</td>
</tr>
<tr>
<td>Layer 1 Thickness - h_{1} (μm)</td>
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<td>10</td>
</tr>
<tr>
<td>Layer 2 Thickness - h_{2} (μm)</td>
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<td>10</td>
</tr>
<tr>
<td>Layer 3 Thickness - h_{3} (μm)</td>
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</tr>
<tr>
<td>Layer 1 Conductivity - σ_{1} (μm Ω)^{-1}</td>
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<td>0.005</td>
</tr>
<tr>
<td>Layer 2 Conductivity - σ_{2} (μm Ω)^{-1}</td>
<td>10^{-5}</td>
<td>0.005</td>
</tr>
<tr>
<td>Layer 3 Conductivity - σ_{3} (μm Ω)^{-1}</td>
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<td>0.005</td>
</tr>
<tr>
<td>Operating Frequency (GHz)</td>
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<td>5</td>
</tr>
</tbody>
</table>

Table 3.1: Designs parameters for eddy current simulations.

both complex image theory and the field solver, FastHenry [55]. The design parameters were randomly selected for each simulation from the values listed in Table 3.1,
Figure 3.8: Average error in spiral inductor inductance and resistance when substrate eddy currents are ignored for a typical inductor geometry.

which represent a wide range of possible inductor geometries and substrate configurations. Note that if less than three substrate layers are present, then the design parameters from the substrate layers with the lower subscripts are used. The bottom (bulk) substrate layer always has a thickness of 500 $\mu m$ in our simulations.

Figure 3.8 depicts the average error in series resistance and inductance for various inductor geometries at an operating frequency of 2.4 GHz when substrate eddy currents are neglected. At this frequency, the resistive losses become significant when the substrate conductivity is as low as $10^{-4}$ ($\mu m \cdot \Omega)^{-1}$, while higher substrate conductivity is required for eddy current inductive losses to become a factor. As the operating frequency increases, the minimum substrate conductivity required for significant losses
Figure 3.9: Percentage error in inductance using the analytical substrate modeling technique when compared with the field solver decreases, which further necessitates substrate eddy current modeling.

For inductance, the average error was 0.1% with a maximum error of 0.6% as depicted in Figure 3.9. Given the importance of inductance values in many analog designs, the analytical modeling approach provides inductance values with the accuracy necessary for synthesizing reliable circuits. The average error for the inductor’s resistance was 3.9% in resistance when compared with the field solver, FastHenry, with 93% of the cases experiencing less than a 10% error as depicted in Figure 3.10. Cases where the inductor is modeled on top of a substrate with a thin, highly conductive top layer and a low conductivity bulk substrate layer experienced the most error due to the error contributed by the higher order terms in the Taylor series approximation of $f(\lambda)$ in (3.21), which can be mathematically demonstrated [109].
Figure 3.10: Percentage error in resistance using the analytical substrate modeling technique when compared with the field solver

The analytical substrate modeling approach provides an average performance improvement of 340,000x over the field solver, FastHenry, due to the fine substrate discretization required by the field solver [55, 64]. Figure 3.11 depicts the average performance improvement for the substrate modeling approach over the field solver as a function of inductor area. As the size the inductor increases, the analytical modeling technique performs in constant time while the field solver experiences significantly decreased performance due to the additional filaments required to model the increased substrate area.

For spiral inductor design space exploration, optimization, and automated syn-
Figure 3.11: Average performance improvement for the analytical substrate modeling technique over the field solver

thesis, the proposed substrate modeling technique provides a tractable and scalable solution. Using current spiral inductor optimization techniques, the synthesis of a spiral inductor over the entire design space can require up to 500 model evaluations [45]. Given our observed average 188.6 second runtime for field solver-based substrate modeling, the inductor optimization process will require 26.2 hours of substrate characterization time. In contrast, our modeling technique will only require approximately 0.3 seconds to model the substrate throughout the optimization process. Therefore, the substrate modeling approach provides a practical solution for spiral inductor design space exploration and synthesis.
3.3.4 Wideband Modeling Results

For the wideband circuit generation technique, we simulated 500 inductor designs with the following design parameter ranges: $3 \leq n \leq 9$, $100 \, \mu m \leq d \leq 300 \, \mu m$, $0.5 \, \mu m \leq w \leq 25 \, \mu m$, and $0.5 \, \mu m \leq t \leq 5 \, \mu m$. Figure 3.12 displays the distribution of the maximum error in resistance and inductance compared to the frequency-dependent analytical model over a range of frequencies up to 20 GHz for the 500 design examples. The wideband models have an average error of 3.2% with 99% of the cases having errors below 9.6%. Figure 3.13 displays the resistance obtained from our narrow-band model and its wideband equivalent circuit for an inductor with 4 turns, 250 $\mu m$ diameter, 10 $\mu m$ conductor width and 2 $\mu m$ conductor thickness. The analytical resistance formulation and its wideband model closely match across the entire frequency range with a maximum error of 2.8%.

3.3.5 Comparison to Fabricated Inductors

Two square, hollow-core inductors were fabricated in the TSMC 0.18 micron RF-CMOS process with a 2 $\mu m$ thick top metal layer. The TSMC 0.18 micron mixed-mode CMOS process has two additional features that improve the performance of integrated inductors. The process features an extra-thick 2 $\mu m$ top-layer with aluminum conductors of $\rho = 28.3 \, n\Omega \cdot m$. The thick top metal layer decreases the series resistance of the fabricated inductors, which results in increased quality factors. The process also uses non-epitaxial wafers that have significantly lower substrate conduc-
Figure 3.12: Distribution of the maximum percentage error in resistance and inductance for the wideband circuit generation technique for 500 inductor design examples.

Figure 3.13: Comparison between the analytical resistance model and its equivalent wide-band circuit representation for a typical inductor design.

tivities, and therefore, reduce the impact of substrate eddy currents. The process has six metal layers with a substrate to metal-6 distance ($z_o$) of 8.15 µm and the
<table>
<thead>
<tr>
<th></th>
<th>Turns (n)</th>
<th>Width (w)</th>
<th>Diameter (d)</th>
<th>Spacing (s)</th>
<th>Thickness (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 nH Inductor</td>
<td>4.5</td>
<td>15 μm</td>
<td>250.5 μm</td>
<td>1.5 μm</td>
<td>2.0 μm</td>
</tr>
<tr>
<td>8 nH Inductor</td>
<td>4</td>
<td>12 μm</td>
<td>293.0 μm</td>
<td>1.5 μm</td>
<td>2.0 μm</td>
</tr>
</tbody>
</table>

**Table 3.2:** Geometric design parameters for fabricated inductors.

**Figure 3.14:** Die micrograph of the inductor test chip fabricated in the TSMC 0.18 micron RF-CMOS process.

standard silicon dioxide dielectric of $\varepsilon_r = 3.9$ [116]. Table 3.2 summarizes the design parameters for the inductors, which have typical dimensions and inductance values for physical validation. A die micrograph is shown in Figure 3.14.

Fabricated inductor test measurements were acquired by probing the fabricated die using a microwave probe station and 40 GHz air coplanar probes with a 100 μm pitch ground-signal ground configuration. A vector network analyzer was used to acquire the data. The short-open-load technique was used to calibrate the probes and the corresponding correction factor was applied internally to the network analyzer.
All measurements were acquired at a sample rate of 1601 data points per 600 ns. An open probe pad structure was measured in order to de-embed the scattering parameters of the device under test from the electrical parasitics associated with the probe pads [46, 48]. The scattering parameters were the converted to admittance parameters using

$$Y_r = \frac{1 - S_{11}}{Z_o(1 + S_{11})}. \quad (3.33)$$

Once converted, the parasitic admittance values are subtracted from the measured admittance parameters [46, 48].

Figures 3.15 and 3.16 display the measured and simulated quality factor and
<table>
<thead>
<tr>
<th></th>
<th>Simulation Time for Analytical Model</th>
<th>Simulation Time for Field Solver</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 nH Inductor</td>
<td>1.23 s</td>
<td>25477.59 s</td>
</tr>
<tr>
<td>8 nH Inductor</td>
<td>0.93 s</td>
<td>23110.04 s</td>
</tr>
</tbody>
</table>

**Table 3.3:** Simulation times (1601 frequency points) for modeling the fabricated inductors.

effective inductance for each inductor. 1601 data points were simulated corresponding to the frequencies up to 6 GHz. For the 6 nH inductor, the mean error for the quality factor was 4.1%, while the mean error for the effective inductance was 3.8%. Similarly for the 8 nH inductor, the mean quality factor error was 9.9% and the mean effective inductance error was 7.5%. The mean error does not include frequencies above 5 GHz, where the error formula, \( \frac{|Q_{measured} - Q_{modeled}|}{Q_{measured}} \), approaches infinity. For the 6 nH inductor, the maximum quality factor has 6.2% error and the maximum quality factor frequency has 9.0% error when compared with the measured data. For the 8 nH inductor, the maximum quality factor has an error of 9.0% while the maximum quality factor frequency has an error of 7.3%. The analytical model accurately determines the quality factor and effective inductance.

As shown in Table 3.3, our spiral inductor model simulates 1601 frequency points in approximately 1 second for both inductors. In contrast, when using the field solver, FastHenry [55], extraction takes approximately 40,000 s. With up to 4 orders of magnitude improvement in performance, the model enables efficient design space exploration to rapidly optimize and synthesize spiral inductors for mixed-signal SoC applications.
Chapter 4

Multi-Level Variability-Aware Integrated Spiral Inductor Synthesis

In this chapter, we develop a robust and flexible automated synthesis methodology for integrated spiral inductors to efficiently generate Pareto-optimal designs based on application requirements. At the core of the synthesis methodology, we have created a scalable multi-level single-objective optimization engine for integrated spiral inductors that supports any generalized inductor modeling technique. The multi-level optimization approach integrates the flexibility of deterministic pattern search optimization with the rapid convergence of local convex optimization to exploit design space characteristics. Using the multi-level optimization engine, we combine multi-objective optimization techniques with surrogate functions that approximate the Pareto surfaces in the design space to efficiently identify the Pareto optimal points defined by application specific design requirements. The surrogate functions can then be reused to design multiple Pareto-optimal inductors in higher level circuits. The results demonstrate the speed and accuracy of both the single-objective multi-level optimization engine and the overall multi-objective inductor synthesis methodology.
We also demonstrate how to utilize the synthesis methodology to generate inductor designs that are less susceptible to process variability and modeling error.

In Section 4.1, we characterize the inductor design space to demonstrate that the spiral inductor optimization problem potentially employs non-convex objective and constraint functions, which can limit the effectiveness of convex inductor optimization techniques. We also analyze the impact of process variability and modeling error on spiral inductor designs. Section 4.2 presents both the multi-level inductor optimization engine and the overall inductor synthesis methodology. In Section 4.3, we compare the multi-level optimization engine with previous optimization techniques and apply the overall inductor synthesis methodology to several spiral inductor design problems. We also demonstrate how to reduce the impact of process variation and modeling error using the synthesis methodology.

4.1 Spiral Inductor Design Space Characterization and Sensitivity Analysis

In this section, we characterize the inductor design space to demonstrate that the spiral inductor optimization problem potentially employs non-convex objective and constraint functions, which can limit the effectiveness of convex inductor optimization techniques. We also analyze the sensitivity of spiral inductor designs on variations in process and model parameters.
4.1.1 Inductor Design Space Analysis

To develop efficient optimization techniques, the spiral inductor design space must be analyzed to determine what properties can be exploited. Design problems solved using gradient-based nonlinear constrained optimization techniques must have convex objective functions for minimization problems or concave objective functions for maximization problems to guarantee that the algorithms will converge to the global solution since they use the gradient of the objective function to determine the search direction [93]. Similarly, the nonlinear constraint functions must be convex for negative inequality constraints \((C \leq 0)\) or concave for positive inequality constraints \((C \geq 0)\). A function is convex if its Hessian is positive semidefinite,

\[
H \geq 0 \forall \overline{x}
\]  \hspace{1cm} (4.1)

for all possible design parameter values, \(\overline{x}\). Similarly, a function is concave if its Hessian is negative semidefinite. If the design space is not convex, gradient-based nonlinear constrained optimization techniques will only converge to a global minimum if the algorithm's start point lies in the convex set containing the global minimum. Consequently, if the function contains multiple local minima or maxima, gradient-based nonlinear constrained optimization techniques will not always converge to the global minimum value or locate feasible solutions.

To explore the convexity of the quality factor, we examined the quality factor as a function of inductor diameter and conductor width for a range of typical inductor
Figure 4.1: Non-convex quality factor function for inductors with varying conductor width and diameter, 6 turns, and 0.5 μm conductor spacing.

generics. As displayed in Figure 4.1, the quality factor has a local minimum value along the 16 nH physical inductance contour due to the trade-off between the series resistance of the conductors, the capacitance between the inductor and the substrate, and the inductance per unit area as both conductor width and inductor diameter are increased. In this case, gradient-based optimization methods will converge to two different function values depending on the algorithm’s initial start point for an inductance constraint of 16 nH. For instance, if the optimization routine starts with a conductor width of 8 μm and a conductor diameter of 300 μm, the optimization algorithm will converge to a geometry with a conductor width of 17.5 μm and an
Figure 4.2: Non-convex effective inductance constraint function for inductors with varying diameter and number of turns, 5 $\mu m$ conductor width, and 0.5 $\mu m$ conductor spacing.

An inductor diameter of 400 $\mu m$, which has a quality factor of 0.2. In contrast, if the gradient-based optimization algorithm starts with a conductor width of 4 $\mu m$ and an inductor diameter of 225 $\mu m$, the optimization routine will converge to a geometry with a conductor width of 1 $\mu m$ and an inductor diameter of 180 $\mu m$, which has a quality factor of 2. Therefore, gradient-based optimization routines alone may fail to provide the optimal value during inductor optimization when using a non-convex inductor model.

Standard gradient-based constrained optimization techniques also require the constraint functions to be convex. If the constraint functions are non-convex, a feasible solution may not be found. Figure 4.2 displays effective inductance as a function of
inductor diameter and number of turns for a typical conductor geometry and process technology. The effective inductance function depicted in Figure 4.2 has three distinct convex sets due to its behavior at the self-resonant frequency of the inductor. Therefore, gradient-based constrained optimization techniques may potentially converge to sub-optimal or infeasible points when the optimization algorithm starts at certain locations in the design space. Consequently, gradient-based optimization techniques alone may fail to provide optimal solutions.

4.1.2 Sensitivity to Inductor Geometry Variation

In order to improve the reliability of spiral inductor designs, we examined the sensitivity to variation in geometric and model parameters on the following design criteria: maximum quality factor, maximum quality factor frequency, effective inductance at the maximum quality factor frequency, and self-resonant frequency. By examining the impact of different sources of variation, we gain insight into what factors are significant for inductor synthesis.

For spiral inductors fabricated in the SoC environment, variation can exist in conductor geometry parameters. Multi-conductor pattern erosion and dishing within individual conductors due to chemical-mechanical polishing can have a significant impact on conductor thickness, with 3-sigma variations of up to 30% for current process technologies [99]. Conductor line width may also vary, especially for conductors with small line width where 3-sigma variation can be up to 20% for wires with minimum pitch [99].

53
<table>
<thead>
<tr>
<th>Percentage Variation</th>
<th>Max. QF</th>
<th>Max. QF Freq.</th>
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<tbody>
<tr>
<td></td>
<td>Mean (%)</td>
<td>3 · σ (%)</td>
</tr>
<tr>
<td>Thickness (t)</td>
<td>Width (w)</td>
<td>$H_{up}$</td>
</tr>
<tr>
<td>±20%</td>
<td>0%</td>
<td>0%</td>
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<table>
<thead>
<tr>
<th>Percentage Variation</th>
<th>Effective Inductance</th>
<th>Self-Resonant Freq.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean (%)</td>
<td>3 · σ (%)</td>
</tr>
<tr>
<td>Thickness (t)</td>
<td>Width (w)</td>
<td>$H_{up}$</td>
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<td>±20%</td>
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Table 4.1: Sensitivity to variations in conductor thickness ($t$), conductor width ($w$) and underpass dielectric thickness ($H_{up}$) for maximum quality factor (QF), the frequency of the maximum quality factor, effective inductance, and self-resonant frequency.

To analyze the impact of conductor geometry variation on spiral inductors, we simulated 50,000 different inductor geometries with geometry and process parameters the following parameter ranges: $2 \leq n \leq 10$, $1 \mu m \leq w \leq 30 \mu m$, $0.5 \mu m \leq s \leq 2 \mu m$, $60 \mu m \leq d \leq 500 \mu m$, $0.5 \mu m \leq t \leq 2 \mu m$ and $0.5 \mu m \leq h_{up} \leq 1.5 \mu m$ where $h_{up}$ is the dielectric thickness between the inductor’s conductors and the underpass leaving the inductor. These geometric parameter ranges represent a broad range of typical configurations used in integrated spiral inductors.

Table 4.1 displays the mean and 3-standard deviations of percentage error for a given deterministic geometric variation in each of the design parameters. Figure 4.3 displays the distribution of the four design criteria for a ±20% variation of the geometric parameters. We assumed that the width variation cannot exceed 0.25s to
Figure 4.3: Sensitivity to ±20% variation in underpass dielectric thickness ($H_{up}$), conductor width ($w$), and conductor thickness ($t$). Note that the variation in the effective inductance ($L_{eff}$) is less than 1.5% for 70% of the geometries simulated.

ensure that the results for inductors with larger widths are realistic. The variation in thickness primarily impacts $R_s$, which greatly affects the maximum quality factor. The thickness variation also has a secondary effect on the other design parameters due to its impact on $R_s$ and $C_s$, with 3-sigma errors of approximately half of the percentage variation in $t$. The variation in conductor width affects every element in the $\pi$-model with 3-sigma errors in design parameters proportional to the percentage variation. The impact of variations in the underpass dielectric thickness ($H_{up}$) was minor when compared with variations of $t$ and $w$ since $C_s \ll C_{ox}$ for most inductor geometries. The maximum quality factor, maximum quality factor frequency and self-resonant frequency were the most greatly influenced by variations in conductor geometry. The effective inductance experienced significantly less variation since the
mutual inductance between the conductors is primarily determined by the length and distance between the conductors, which were unchanged.

### 4.1.3 Sensitivity to Inductor Model Element Variation

Modeling error can also significantly impact the key design criteria of the spiral inductor. Sources of modeling error can include the previously discussed geometric variations, the impact of nearby structures on the spiral inductor in SoC designs, environmental factors, and errors in the model itself. To analyze the impact of variation in the model, we simulated 198,000 different inductor geometries with the geometric parameter ranges discussed in the previous section and substrate conductivities ranging from $0.1 \, (\Omega \cdot m)^{-1}$ to $7000 \, (\Omega \cdot m)^{-1}$.

Table 4.2 displays the mean and 3-standard deviations of the percentage error due to a given fixed deterministic model element variation for the four inductor design criteria. Figure 4.4 displays the distribution of the four design criteria for a ±20% variation of the model elements. The error caused by variation in $R_s + R_{eddy}$ primarily impacts the maximum quality factor while the other design parameters can experience significant error in certain cases as indicated by the approximately 20% 3-sigma error. Variations in the substrate parameter $R_p$ had a small effect on the inductor design, while changes in $C_p + C_s$ and $L_s - L_{eddy}$ significantly impacted all 4 design criteria. In Section 4.3.5, we describe how to synthesize the inductor to reduce the potential influence of model and geometric variations.
<table>
<thead>
<tr>
<th>Percentage Variation</th>
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<th>Max. QF Freq.</th>
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<tr>
<td></td>
<td>Mean (%)</td>
<td>3 · σ (%)</td>
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<tr>
<td>$R_s$</td>
<td>$R_p$</td>
<td>$C_p + C_s$</td>
</tr>
<tr>
<td>±20%</td>
<td>0%</td>
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**Table 4.2:** Sensitivity to variations in the spiral inductor model elements shown in Figure 3.2 for maximum quality factor (QF), the frequency of the maximum quality factor, effective inductance, and self-resonant frequency.

**Figure 4.4:** Sensitivity to ±20% variation in $R_s, R_p, C_s + C_p$ and $L_s$. 

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4.2 Spiral Inductor Synthesis

To maximize critical design resources in mixed-signal systems, we have developed a spiral inductor synthesis methodology that analyzes and optimizes design trade-offs in order to generate robust inductor designs that meet stringent requirements. At the core of the synthesis methodology, we have created a multi-level single-objective optimization engine that combines the flexibility of deterministic pattern search optimization with the rapid convergence of local convex optimization techniques. The synthesis methodology then leverages the power of several multi-objective optimization techniques to capture inductor design trade-offs and to reduce the impact of process variation and modeling error.

4.2.1 Inductor Synthesis Problem Formulation

In general, the multi-objective optimization problem underlying spiral inductor synthesis has a vector valued objective function with elements that cannot be optimized simultaneously. Therefore, metrics must be developed in order to evaluate the relative importance of each design objective. Formally, the multi-objective spiral inductor optimization problem is defined as

Minimize \( \overrightarrow{F}(G) \)

Subject to \( \overrightarrow{C}(G) \leq \overrightarrow{C}_{\text{wanted}} \)

\[ G_{\text{min}} \leq G \leq G_{\text{max}} \]  \( (4.2) \)

58
where \( \vec{F} \) are design metrics that are a function of the inductor's geometric parameters \( (G) \), and \( \vec{C} \) are constraint functions based on application design requirements. Potential design trade-offs can include quality factor, inductance, operating frequency, self-resonant frequency, area, patterned ground shield utilization, and worst-case designs due to process variations and modeling error. Typically, the objective functions do not have the same minima, and therefore, application specific metrics must be employed to determine the Pareto-optimal solution. Since the multi-objective optimization problem requires the solution to the single-objective optimization problem for different combinations of design constraints, computational requirements can be significantly greater than standard single-objective problems. However, the number of function evaluations can be significantly reduced by using the knowledge obtained during previous optimization runs when constructing the Pareto surfaces.

4.2.2 Multi-Level Inductor Optimization Engine

To optimize the non-convex quality factor function with non-convex constraint functions, we utilize multi-level optimization. Multi-level optimization employs several different optimization techniques in tandem to compensate for the weaknesses of each technique individually. Multi-level optimization provides the speed and flexibility necessary to efficiently search the complex spiral inductor design space.
This optimization problem is expressed mathematically as

Maximize \( Q(n, s, w, d) \)

Subject to

\[
\begin{align*}
L(n, s, w, d) &\leq L_{\text{wanted}}(1 + \text{tol}) \\
L(n, s, w, d) &\geq L_{\text{wanted}}(1 - \text{tol}) \\
\vdots &\vdots \\
[n_{\text{min}}, s_{\text{min}}, w_{\text{min}}, d_{\text{min}}] &\leq [n, s, w, d] \\
[n_{\text{max}}, s_{\text{max}}, w_{\text{max}}, d_{\text{max}}] &\geq [n, s, w, d]
\end{align*}
\]

where \( L_{\text{wanted}} \) is the desired inductance value, \( \text{tol} \) is the tolerance on the allowed inductance values, \( Q(n, s, w, d) \) is the quality factor, and \( L(n, s, w, d) \) defines the inductance. Depending on the application, \( L_{\text{wanted}} \) and \( L(n, s, w, d) \) can either correspond to physical inductance, \( L_s \) depicted in Figure 3.2, or effective inductance, \( L_{\text{eff}} \). Other inequality constraints such as area and self-resonant frequency can be imposed to optimize inductor designs for specific applications. Since most optimization techniques seek to minimize a given objective function, we recast "Maximize \( Q(n, s, w, d) \)" as "Minimize \(- Q(n, s, w, d)\)."

Note that the proposed synthesis methodology also provides the capability to design both circular and octagonal inductors since these geometries have the same design parameters as square spiral inductors \((n, d, w, \text{and } s)\). For fixed values of \( n, d, w, \text{and } s \), the difference between the quality factors of circular, octagonal, and square
Figure 4.5: Multi-level single-objective inductor optimization engine.

spiral inductors is typically on the order of 10% [9]. When optimizing multi-layer stacked spiral inductors, the only additional degree of freedom is the number metal layers on which the inductor resides [50], which can be handled by performing Pareto optimization using the number of spiral inductor metal layers as a variable constraint parameter. This is similar to the strategy that we employ for optimizing patterned ground shields, which is described in Section 4.2.2.

The multi-level spiral inductor optimization methodology, which is depicted in Figure 4.5, consists of two distinct phases: the global optimization phase and the local optimization phase. Initially, objective and constraint functions are defined for the spiral inductor optimization problem based on application requirements. We utilize
the Mesh Adaptive Direct Search (MADS) algorithm to globally search the design space. Stochastic global optimization techniques such as simulated annealing and genetic algorithms typically employ penalty functions for constrained optimization problems that may require tens of thousands of function evaluations to converge [93]. In contrast, MADS is specifically designed for rapid convergence on nonlinear constrained optimization problems with many local minima [117].

Since MADS is a pattern search algorithm, the convergence rate depends on the initial start point in the design space. In order to improve convergence, we randomly sample the design space until an inductor with a positive quality factor is located, which typically requires fewer than 10 function evaluations. We then run MADS for 100 function evaluations or until the optimizer finds a feasible solution with a quality factor that meets certain criteria. If a feasible solution is found, the output of the MADS optimizer is used as the start point for the local optimization phase. Otherwise, we choose another random start point and repeat the MADS optimization process. In practice, we have observed that a maximum of 3 MADS optimization runs are typically required to locate a suitable start point for the local optimization phase.

During the local optimization phase, we utilize Sequential Quadratic Programming (SQP), a nonlinear constrained convex optimization technique, to exploit the gradients of the objective and constraint functions at each iteration in order to quickly converge to the optimal inductor geometry. During each iteration SQP approximates the objective function using a quadratic model and solves the quadratic program-
ming (QP) problem locally. The algorithm employs the BFGS approximation for the Hessian at each step to significantly reduce computational cost. Once the new search direction is determined from the QP, a merit function determines the appropriate step size based on the objective and constraint function values. SQP has been successfully applied to many nonlinear constrained convex optimization problems [118].

Once optimization is complete, spiral inductor layout can be synthesized. To determine if a patterned ground shield will enhance the inductor’s performance, two successive runs of the algorithm for inductors both with and without the ground shield can be implemented with the second iteration using the result of the first as a start point in order to accelerate convergence. Note that evaluating the use of patterned ground shields requires two optimization runs since ground shield utilization is a discrete binary input to the model, and therefore cannot be used directly with gradient-based optimization. Our methodology exploits both the global search capabilities of MADS and the rapid convergence of the local convex optimization to locate optimal designs in the inductor’s non-convex design space.

4.2.3 Inductor Synthesis Methodology

In order to synthesize inductor designs, we leverage several multi-objective optimization techniques. Figure 4.6 depicts our methodology for spiral inductor synthesis. Initially, inductor design requirements and synthesis goals are defined based on the circuit application and design constraints. Based on the design goals underlying the particular spiral inductor synthesis problem, we utilize either the $\epsilon$-constraint method
or the weighted $L_p$-norm approach in order to analyze and optimize Pareto-optimal trade-offs in the inductor design space [119].

The $\epsilon$-constraint method reformulates the multi-objective optimization problem
into a series of single objective optimization problems by transforming design parameters into a series of constraints:

\[
\begin{align*}
\text{Minimize} & \quad F_i(G) \text{ for each } i \in [1, I] \\
\text{Subject to} & \quad C_j(G) \leq \varepsilon_k \forall \ j \in [1, J], \ k \in [1, K] 
\end{align*}
\]

(4.3)

where \( I = K^J \) single-objective optimization problems are solved for \( K \) distinct values \( (\varepsilon_k) \) of \( J \) design constraints \( (C_j) \) [119]. The number of function evaluations for each single-objective optimization problem is substantially reduced when the algorithm utilizes previous results to determine the start point for the current optimization problem. The Pareto optimal point can then be found by solving

\[
\begin{align*}
\text{Minimize} & \quad M(\overrightarrow{C}(G)) \\
\text{Subject to} & \quad \overrightarrow{C}(G) \leq \overrightarrow{C}_{\text{bound}}
\end{align*}
\]

(4.4)

where \( M \) is a design metric that determines how to trade-off the different design constraints.

Two different approaches can be utilized to determine the Pareto optimal point defined by \( \min(M(\cdot)) \) in (4.4). The Pareto surface can be implicitly constructed by
solving

\[ \text{Minimize } M(\min F(G) : \overrightarrow{C}(G) \leq \overrightarrow{C}_{\text{part}}) \]
\[ \text{Subject to } \overrightarrow{C}_{\text{part}} \leq \overrightarrow{C}_{\text{bound}} \quad (4.5) \]

where \( C_{\text{part}} \) is the particular set of constraints used as the optimization variables for \( M \) and \( C_{\text{bound}} \) defines the range of acceptable constraint variables. Since each evaluation of \( M \) requires the solution to a single-objective spiral inductor optimization problem, this procedure can be costly.

To reduce the overall required computational cost of computing (4.5), the Pareto surface can be explicitly constructed by solving a series of optimization problems with design constraints spanning the Pareto surface. Once a set of points on the Pareto surface have been generated, we create a surrogate function to approximate the Pareto surface. Using the surrogate function, the Pareto optimization problem defined in (4.5) becomes

\[ \text{Minimize } M(S(C_{\text{part}})) \]
\[ \text{Subject to } \overrightarrow{C}_{\text{part}} \leq \overrightarrow{C}_{\text{bound}} \quad (4.6) \]

where \( S(C_{\text{part}}) \) is a surrogate function fitted to the Pareto surface. To create the surrogate functions, we fit the Pareto surface to a library of possible functions using linear least squares minimization or spline interpolation. Since only a few points
are typically needed to fit the Pareto surface [95], the time it takes to generate the surrogate function is relatively small compared to the time required to optimize an inductor with a single set of design constraints. If we cannot generate a surrogate function that meets the specified error tolerance, we simulate more points on the Pareto surface near the estimated optimal value. Otherwise, using the surrogate function, we optimize the design constraints based on application design requirements. The surrogate function can be reused to design multiple Pareto-optimal inductors in higher level circuits without any additional inductor optimization.

When the Pareto optimization problem is not convenient to reformulate in terms of the constraint functions, we convert the multi-objective optimization problem into a single-objective optimization problem using the weighted $L_p$ norm approach,

\[
\begin{align*}
\text{Minimize} \quad & \sum_{i=1}^{I} \omega_i |F_i(G) - f_{\text{nom}}|^p \\
\text{Subject to} \quad & \overrightarrow{C}(G) \leq \overrightarrow{C}_{\text{wanted}}
\end{align*}
\]

where $f_{\text{nom}}$ is the desired value of the objective function, $\omega_i$ are weights associated with the relative importance of each design parameter and $p$ corresponds to the type of norm being used to evaluate the multi-objective function [119]. If $p = 2$, then (4.7) can be interpreted as minimizing the distance between the design variables and their respective desired values. Once we have located a solution, we solve a final single-objective optimization problem using the multi-level engine with the Pareto-optimal design constraints. If the synthesis problem requires the additional design trade-offs
to be optimized, we repeat the process. The automated inductor design methodology provides a tractable solution for synthesizing inductors to meet circuit requirements.

Multi-objective inductor optimization provides several important advantages over current optimization techniques, which seek to maximize the quality factor at a particular frequency given a fixed set of design constraints [68, 80, 85, 86, 89]. First, our approach allows for considerable flexibility in determining what criteria are considered optimal. For wideband VCOs and inductors in systems with significant load capacitance attached to the inductor, the inductor should have a large quality factor and relatively equal effective inductance values over a range of frequencies, which can be synthesized using our methodology. In addition to flexibility in inductor optimality criteria, the multi-objective synthesis methodology enables the efficient analysis and exploitation of key inductor design parameters in order to maximize design resources and provide more robust designs. Surrogate functions provide an efficient means for optimizing design constraints based on application specific requirements and can be reused to design multiple spiral inductors in higher level circuits. Given the expensive manufacturing cost for today's mixed-signal systems in SoC technology, the added versatility of the proposed inductor synthesis methodology over single objective techniques justifies its additional computational cost in many situations.
<table>
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<th></th>
<th>$L_{\text{wanted}}$ (nH)</th>
<th>$F$ (GHz)</th>
<th>$t$ ($\mu$m)</th>
<th>$\sigma_{\text{sub}}$ ($\Omega \cdot m)^{-1}$</th>
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<tr>
<td>Example 1</td>
<td>16</td>
<td>4</td>
<td>2</td>
<td>0.1</td>
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<td>Example 2</td>
<td>4</td>
<td>1</td>
<td>1</td>
<td>0.1</td>
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<td>Example 3</td>
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</tr>
<tr>
<td>Example 4</td>
<td>6</td>
<td>0.9</td>
<td>0.5</td>
<td>7000</td>
</tr>
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</table>

Table 4.3: Design examples for multi-level optimization engine.

4.3 Inductor Optimization and Synthesis Results

In this section, we compare the multi-level optimization engine with previous optimization techniques and apply the overall inductor synthesis methodology to several spiral inductor design problems. We also demonstrate how to utilize the synthesis methodology to reduce the impact of process variability and modeling error on integrated spiral inductors.

4.3.1 Multi-Level Optimization Engine

In order to demonstrate the performance improvement of the single-objective multi-level spiral inductor optimization engine described in Section 4.2.2 over current techniques, we applied several different methodologies to maximize the quality factors for the four typical design problems listed in Table 4.3 where $L_{\text{wanted}}$, $F$, $t$, and $\sigma_{\text{sub}}$ are effective inductance, operating frequency, conductor thickness, and substrate conductivity, respectively. The geometric parameters have the following bounds: $2 \leq n \leq 10$, $1 \mu m \leq w \leq 30 \mu m$, $0.5 \mu m \leq s \leq 3 \mu m$ and $60 \mu m \leq d \leq 800 \mu m$. Each inductor is assumed to have copper conductors that are $6 \mu m$ above a $500 \mu m$ thick single layer
<table>
<thead>
<tr>
<th>Optimization Method</th>
<th>Example 1</th>
<th>Example 2</th>
</tr>
</thead>
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<tr>
<td></td>
<td>Max. QF</td>
<td>Func. Eval.</td>
</tr>
<tr>
<td>Enumeration - Large</td>
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<td>15323481</td>
</tr>
<tr>
<td>Enumeration - Medium</td>
<td>6.59</td>
<td>179315</td>
</tr>
<tr>
<td>Enumeration - Small</td>
<td>6.59</td>
<td>13548</td>
</tr>
<tr>
<td>SQP - Random SP</td>
<td>7.11</td>
<td>12207</td>
</tr>
<tr>
<td>Enumeration (5) + SQP</td>
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<td>N/A</td>
</tr>
<tr>
<td>Enumeration (10) + SQP</td>
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<td>866</td>
</tr>
<tr>
<td>Enumeration (15) + SQP</td>
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<td>3440</td>
</tr>
<tr>
<td>Enumeration (20) + SQP</td>
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<tr>
<td>MADS</td>
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<td>225</td>
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<tr>
<td><strong>MADS + SQP</strong></td>
<td><strong>7.16</strong></td>
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<table>
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<td>Enumeration (20) + SQP</td>
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<td>200</td>
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<tr>
<td><strong>MADS + SQP</strong></td>
<td><strong>6.39</strong></td>
<td><strong>380</strong></td>
</tr>
</tbody>
</table>

Table 4.4: Comparison of function evaluations and maximum quality factors for single-objective optimization techniques.

substrate. The four design examples represent a broad range of typical problems with varying levels of difficulty for the optimization routines.

The maximum quality factor and the number of function evaluations required for the multi-level optimization approach (MADS + SQP) and other proposed techniques are listed in Table 4.4. Figure 4.7 displays the maximum quality factor and number of
Figure 4.7: Maximum quality factor and number of function evaluations for previous optimization methods and the proposed multi-level approach.

function evaluations for previously proposed optimization methods and the proposed multi-level approach. For the enumeration method, we iterate over each of the four inductor design parameters \((n, w, s, d)\) for the total number of function evaluations listed in Table 4.4. For the SQP - Random Start Point method, we optimize the inductor using SQP with 200 random start points that correspond to feasible inductor geometries with positive quality factors. To determine the average number of function evaluations to obtain an optimal result using SQP, we divide the total number of function evaluations for 200 SQP runs by the number of runs that produce a quality factor value within 5 percent of the optimal quality factor. Similarly, the quality factors listed in Table 4.4 for SQP are the average of the quality factors that are within 5 percent of the optimal value.

To illustrate the multi-level optimization methodology's performance improve-
ment over other potential multi-level approaches, we optimized spiral inductors using a combination of global enumeration with local SQP optimization. The maximum quality factor and number of function evaluations for several different multi-level single-objective optimization methods including our proposed method, which couples MADS with convex optimization, are depicted in Figure 4.8. Furthermore, we applied MADS alone to the inductor optimization problem. Finally, we optimized the quality factor with MADS coupled with SQP using the proposed methodology. We utilized the NOMADm optimization program to implement the MADS algorithm [117]. For the local optimization engine, we utilized a standard version of the SQP algorithm, which is one of the best nonlinear constrained convex optimization techniques available [93]. For both multi-level approaches, enumeration with SQP and MADS with SQP, we only optimized with respect to three design variables \((n, w, d)\) in the global optimization phase since the quality factor is less sensitive to conductor spacing.

In design example 1, enumeration required 15 million function evaluations to achieve optimal results, which is unacceptable even using analytical modeling techniques. SQP performed well on design example 2, where eddy current effects were minimal and a low inductance value was desired. On the other more difficult design problems that either experienced eddy current effects or required large inductance values, the SQP algorithm often either failed to find a feasible solution due to the non-convexity of the effective inductance constraint function or converged to a sub-optimal quality factor due to the non-convexity of the quality factor function. Multi-
level optimization based on enumeration coupled with SQP achieved near optimal results when the enumeration technique discretized the design space into a 20x20x20 grid. However, this required over 6000 function evaluations on average. For lesser discretization during the enumeration step, the design problems either did not obtain a feasible value or achieved sub-optimal results for at least one of the design examples. Since the optimal quality factor values for a particular inductor design problem are typically not known \textit{a priori}, 20x20x20 enumeration is the minimum discretization necessary to achieve consistent results. The inductors optimized with MADS alone only required several hundred function evaluations, but MADS failed to reach an optimal value in design example 3.

In contrast, our methodology yielded near-optimal quality factors for each of the four design examples with an average of 457 function evaluations, which is up to a
40000x speedup over enumeration and up to a 25x speedup over SQP. For instance, on design example 4 the optimization methodology produced an inductor with 4.45 turns, 27.26 μm conductor width, 0.50 μm conductor spacing and an edge-to-edge diameter of 367.5 μm. We verified the predicted inductance and quality factor using the field solver, FastHenry [55], to extract the resistance and inductance of the conductors and substrate. When compared with FastHenry for the extraction of the resistance and inductance of the conductors only, the analytical modeling and multi-level optimization methodology yields errors of 4.1% and 3.3% in quality factor and inductance, respectively. When FastHenry is used to extract the resistance and inductance of the conductors and substrate, the analytical modeling and multi-level optimization methodology produces errors of 6.8% and 3.1% in quality factor and inductance, respectively. These error values are within manufacturing tolerances for spiral inductor realization.

The multi-level approach provides a tractable spiral inductor optimization solution even when expensive field-solver based modeling techniques are employed. For the final inductor design produced by example 4, modeling the conductors only with FastHenry for resistance and inductance takes 3.34 seconds, while extracting resistance and inductance for both the conductors and the substrate required 36.51 seconds of CPU time. Therefore, using the field solver to model the conductors only in the optimization of design example 4 will require approximately 26.3 minutes. Additionally, using the field solver to model the conductors and substrate in the optimization of
design example 4 will require approximately 287.8 minutes. If the field solver based models are used only for the local optimization phase, the required CPU time reduces to 20.8 minutes for optimization with field solver simulations of only the conductors and 227.0 minutes for optimization with field solver simulations of both the conductors and the substrate. By coupling the global search capabilities of MADS with the local optimization strength of SQP, the proposed multi-level spiral inductor optimization methodology provides an efficient means to locate optimal spiral inductor designs.

4.3.2 Inductor Synthesis: Inductance vs. Quality Factor

For circuits where a range of inductance values are permissible, understanding the relationship between inductance and maximum quality factor is crucial. In order to demonstrate our synthesis methodology, we determined the Pareto surface for inductance versus quality factor for an inductor operating at 4 GHz and having 1 \( \mu m \) thick conductors that are located 6 \( \mu m \) over a substrate with 10 \((m \cdot \Omega)^{-1}\) conductivity. Figure 4.9 displays the 5 points that we synthesized on the Pareto surface as well as the surrogate function generated to fit the Pareto surface, which in this case is the third degree polynomial

\[
Q_{max} = -7.43 \cdot 10^{-5}L^3 + 0.0096L^2 - 0.49L + 13.05
\]  

(4.8)
Figure 4.9: Pareto surface for effective inductance versus optimal quality factor. The points in the lower left portion of the Figure represent the results from 500,000 simulated random inductor designs.

where $Q_{\text{max}}$ is the maximum attainable quality factor for a given effective inductance value ($L$). Using (4.8) as the surrogate function $S(C_{\text{part}})$, we solve the optimization problem presented in (4.6). In order to validate the accuracy of our approach, we simulated 500,000 inductor designs with random geometric parameters, which are indicated by the small points in the lower left part of Figure 4.9. Note that the optimized points on the Pareto surface and the surrogate function accurately match the Pareto front in the inductance versus quality factor design space. The generation of the Pareto optimal points required 558 function evaluations with a runtime of 16.6 seconds for the 5 required optimization runs and surrogate function generation. Note
that if the field solver, FastHenry, is used to model the resistance and inductance of the conductors only, the generation of the Pareto surface will require approximately 31.0 minutes. Similarly, if FastHenry is used to model the resistance and inductance of both the conductors and substrate, the generation of the Pareto surface will require approximately 339.5 minutes of CPU time.

4.3.3 Patterned Ground Shield Optimization

Patterned ground shields can significantly reduce the impact of substrate eddy currents and other resistive losses due to the substrate. However, the increased capacitance between the ground shield and the inductor can reduce the quality factor and self-resonant frequency. In Figure 4.10, we analyze the benefits of using a patterned ground shield for inductors with effective inductance values of 1 to 15 nH, substrate conductivities of 10 and 1000 (\(\Omega \cdot m\))\(^{-1}\), and operating frequencies of 2.4 and 5.4 GHz. For the inductors fabricated over a substrate with low conductivity, the benefit of the patterned ground shield depends on the desired inductance values. For lower inductance values, the patterned ground shield improves the optimal quality factor since the shield mitigates the impact of the substrate leakage resistance (\(R_{\text{sub}}\)). When large inductance values are required, the increased inductor size creates additional capacitive coupling between the ground shield and the inductor, which dominates the impact of the ground shield on \(R_{\text{sub}}\).

For the high substrate conductivity case, the increase in maximum quality factor due to the reduced impact of substrate eddy currents outweighs the decrease due
Figure 4.10: Effective inductance versus optimal quality factor for designs with (a) low substrate conductivity, high substrate conductivity and a patterned ground shield at 2.4 GHz, and (b) low substrate conductivity and a patterned ground shield at 5.4 GHz

to the increased ground shield capacitance. For the inductors with an operating frequency of 5.4 GHz, the effective inductance value above which the ground shield degrades the quality factor is significantly less than the effective inductance trade-off point in the 2.4 GHz case since the substrate capacitance has a greater impact on the quality factor at higher frequencies.

In Figure 4.11, we display our surrogate function utilized to determine the effective
Figure 4.11: Surrogate function for ground shield optimization. The fitted points were used to create the surrogate function, and the simulated points were used to verify intermediate values.

Inductance value for which using a patterned ground shield is no longer optimal. We define the design metric as $M = Q_{pgs} - Q_{nom}$ where $Q_{pgs}$ is the quality factor obtained using a patterned ground shield and $Q_{nom}$ is the quality factor with no ground shield. We generated the surrogate function using 8 data points. The surrogate function predicts the break-even inductance to be 5.41 nH. When compared with the actual break-even point 5.27 nH, the surrogate function has an error of 2.8%. Our synthesis methodology provides a 75% percent performance improvement over the 14-point linear interpolation and a 388% performance improvement over locating the optimal value to 3 digits of accuracy using a binary search algorithm.
4.3.4 Inductor Area vs. Self-Resonant Frequency vs. Quality Factor

The proposed synthesis methodology can also be used to efficiently balance design constraints such as inductor area and self-resonant frequency (SRF). To illustrate this advantage, we examine Pareto optimization for area and SRF for an inductor with an operating frequency of 5.4 GHz, a desired effective inductance of 10 nH, a substrate conductivity of $10 \ (\Omega \cdot m)^{-1}$, and conductors fabricated 5 $\mu$m above the substrate. For this design problem, we assume that we can decrease the maximum quality factor by up to 5% to reduce the overall area and increase the SRF. While we simultaneously consider area and SRF trade-offs, our methodology can also sequentially handle these design constraints.

In order to determine the Pareto optimal surface, we first optimize the inductor design with no area or SRF constraints. The unconstrained design has a quality factor of 8.12. Using a surrogate function fitted to 25 fixed area and SRF constraint values, we constrain the maximum quality factor to be at least 95% percent of the quality factor of the no area/SRF constrained design and maximize

$$Q_{\text{max}}(\vec{v}) = \| \vec{v} - \vec{v}_{\text{nom}} \|_2$$  \quad (4.9)

where $Q_{\text{max}}$ is the maximum quality for a given constraint vector $\vec{v}$, which for this problem is defined as $\vec{v} = [\text{area}, \text{SRF}]$. The vector $\vec{v}_{\text{nom}}$ contains the area and SRF of the unconstrained inductor design. Note that we normalize the vectors to remove the impact of constraint variable scaling. In this context, the Pareto optimization
objective function in (4.9) attempts to locate the design with the greatest distance from the original design, implying the most savings in area and increase in SRF. Other criteria for the Pareto optimal point can be chosen based on the relative importance of the constraints for a given application.

Using the surrogate objective function in (4.9) and the optimization formulation presented in (4.6), we calculate Pareto optimal inductor constraints of 99.23 μm for diameter and 8.35 GHz for SRF with a predicted maximum quality factor of 7.713. Single-objective optimization using these constraints produces a design with a 7.710 quality factor. Therefore, in this example the error produced by the surrogate function was only 0.04%. The area and SRF constrained optimization yielded a 51.9% percent decrease in area and a 1.9% increase in SRF. The disparity in area and SRF improvement is due to the much steeper slope of the Pareto surface in the SRF dimension as shown in Figure 4.12. The generation of the Pareto surface required 2869 function evaluations and took 84.3 seconds when using the analytical inductor model. Overall, the synthesis methodology successfully determined the Pareto-optimal trade-off that resulted in a 51.8% decrease in area with only a 3% reduction in quality factor.

4.3.5 Synthesis with Process and Model Variability

In order to provide robust spiral inductor designs in the presence of process variation and other sources of modeling error, we apply the weighted norm technique from (4.7). The goal is to maximize the minimum quality factor given either geometric dimension or model parameter variation. To demonstrate our methodology
Figure 4.12: Surrogate Pareto surface for the maximum quality factor versus area and minimum self-resonant frequency. The solid black line indicates the quality factor constraint.

and determine the impact of these sources of variation on the spiral inductor, we applied our multi-objective optimization technique to 570 different inductor design problems with desired effective inductance values ranging from 1 to 20 nH, operating frequencies ranging from 1 to 8 GHz, substrate conductivities ranging from 0.1 to 10000 $(\Omega \cdot m)^{-1}$, and conductor thickness values ranging from 1 to 2 $\mu m$.

We employ 2 different strategies for coping with process and model variation. In the maximize minimum quality factor (MMQF) strategy, we attempt to minimize the worst case quality factor for a given set of process variations, which is equivalent to using an $L_\infty$-norm in the multi-objective optimization formulation defined in (4.7).
Figure 4.13: Illustration of how the synthesis methodology reduces the impact of geometric variability and modeling error.

In the maximize weighted quality factor (MWQF) strategy, we combine each process variation that produces a quality factor less than the nominal (no variation) quality factor using a weighted $L_2$-norm for multi-objective optimization.

For both the MMQF and MWQF strategies, we deterministically sample the distribution of the possible geometric or model variations during each iteration of the optimization process. For instance, if the maximum percentage variation in conductor width is $\pm20\%$, we determine the quality factors for inductor designs of a predetermined spread of variation percentages such as $\pm20\%$, $\pm15\%$, $\pm10\%$ and $\pm5\%$. Note that for certain geometric or model parameters, such as conductor thickness and $R_s$, we can avoid evaluating the model for many of the prescribed percentages.
<table>
<thead>
<tr>
<th>Variation Type</th>
<th>Design Parameters</th>
<th>Percentage Increase in Worst-Case QF</th>
<th>Percentage Decrease in Nominal QF</th>
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<td>Operating Frequency</td>
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<td>6.36</td>
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<td>10 - 20 nH</td>
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Table 4.5: Mean and 3-sigma percentage change in minimum quality factor (QF) and the nominal (no variation) quality factor for 570 different optimization problems using the maximize minimum quality factor (MMQF) strategy for both geometric and model variations.

since the quality factor monotonically decreases or increases with variations in these parameters. For MMQF, the objective function is the worst case design that results from all combinations of deterministic percentage variations in every design parameter. For MWQF, the objective function is the weighted average of all designs that have quality factors less than the design obtained when no geometric or model variations are applied. Figure 4.13 illustrates the basic idea behind both variability-aware optimization strategies – we want to improve the worst-case variation in quality factor at the cost of decreasing the quality factor when no variation is present.

Tables 4.5 and 4.6 display the results for the MMQF and MWQF optimization schemes for geometric variations of ±20% in conductor thickness and ±20% in conductor width as well as the results for model parameter variations of ±20%. For
<table>
<thead>
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<th>Percentage Increase in Worst-Case QF</th>
<th>Percentage Decrease in Nominal QF</th>
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<td>10 - 20 nH</td>
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</table>

Table 4.6: Mean and 3-sigma percentage change in minimum quality factor for geometric and model variation and the percentage impact on the nominal (no variation) quality factor for 570 different inductor optimization problems.

Inductors with desired effective inductances greater than 10 nH or operating frequencies greater than 5 GHz, the MMQF strategy can significantly improve the worst-case optimal quality factor due to geometric variation and modeling errors with average increases in the worst-case quality factor of over 13\% for inductors satisfying both conditions. Note that the 3-sigma variation in percentage improvement can exceed 41\% as depicted in Figure 4.14, which can lead to improvements of over 50\% certain cases. When optimizing inductor designs using the MMQF strategy, the no variation quality factor will only typically decrease by several percent, which is significantly less than the increase obtained in the worst-case quality factor. Therefore, for a slight decrease in the quality factor when no variations are present, we can significantly improve the worst-case quality factor due to variation.

When optimizing the inductor using the less aggressive MWQF strategy, the gains
Figure 4.14: 3-sigma variation in percentage improvement in worst-case quality factor for the 570 different inductor optimization problems.

in worst-case quality factor were less than those obtained using MMQF, but the corresponding decrease in the no variation quality factor was also smaller. This suggests that different weights can be utilized in order to trade-off conservatism in the worst-case quality factor with the decrease in the no variation quality factor. Since many future wireless devices will be operating in 5 GHz frequency band, considering the effects of variation on the spiral inductor will lead to more reliable designs.
Chapter 5

Conclusions and Future Work

5.1 Conclusions

We developed an accurate and efficient modeling, optimization, and synthesis methodology for spiral inductors to analyze and optimize fundamental design trade-offs and to reduce the impact of variability on inductor designs. The wideband model utilizes accurate formulations for high frequency resistance, physical inductance and eddy currents induced in multi-layer substrates that exploit specific inductor geometric properties to quickly characterize spiral inductors. The wideband circuit generation technique creates circuits for time-domain simulation that accurately approximate the results obtained from the analytical resistance model. The model agrees with data from field solvers and fabricated inductors with orders of magnitude overall improvement in performance. The multi-level optimization methodology couples the flexibility of the deterministic pattern search algorithms with the rapid local convergence of nonlinear convex optimization techniques to provide an efficient, model-independent spiral inductor optimization methodology. We have shown that the quality factor and effective inductance functions can be non-convex, which may lead to
the non-convergence of standard convex optimization techniques. The methodology overcomes this difficulty and locates optimal spiral inductor designs with significantly fewer function evaluations than current techniques. To improve the performance of Pareto optimization, we develop surrogate functions that limit the number of required single-objective optimization problem solutions. The surrogate functions can then be reused to design multiple Pareto-optimal inductors in higher level circuits. The results indicate that the synthesis methodology efficiently optimizes inductor design trade-offs based on application design requirements with improvements of up to 51% in key inductor design constraints. The wideband modeling, multi-level optimization, and variability-aware synthesis methodology presented in this thesis will enable the effective realization of higher-level analog circuits in fully-integrated mixed-signal systems with improved system performance, reliability, power consumption, and cost.

5.2 Future Work

In the future, we want to expand the optimization and synthesis techniques presented in this thesis to higher-level analog circuits. Low noise amplifiers are a prime candidate since they employ multiple spiral inductors and fundamentally impact the performance of the entire RF receiver in both narrow and wideband applications. LNA designs must simultaneously balance noise figure, gain, power consumption, and input and output impedance matching while considering circuit element sizing and parasitic effects in the SoC environment [10–15]. The synthesis methodology can
be expanded to synthesize LNA circuits by utilizing multi-level optimization to efficiently combine deterministic global pattern search algorithms, local gradient-based optimization routines, and mixed-integer stochastic optimization algorithms. Numerical optimization techniques can provide the flexibility to generate designs with passive components in fully integrated LNAs that are suitable for SoC integration without resorting to external matching networks. In addition, the optimization and synthesis methodologies presented in this thesis can also be adapted and expanded to engineer many emerging nanophotonic and carbon nanotube-based structures for applications in future integrated circuits [120,121]. Computer aided design through numerical optimization promises to facilitate the realization of fully integrated mixed-signal and nanotechnology-based systems that will ultimately be too complex for manual or ad hoc design techniques.
Bibliography


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