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Measurement of the Interactions in a Paired Zero Temperature $^6$Li Gas Throughout the BEC-BCS Crossover

by

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ABSTRACT

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Understanding the effects of interactions on the ground-state of a paired degenerate Fermi gas throughout the BEC-BCS crossover has been a long standing physical quest for which countless papers have been written. The crossover describes the smooth transition of the physics of a BEC of tightly bound dimers to that of a paired BCS superfluid. A Feshbach resonance is used to tune the interactions necessary to study the crossover. Right around resonance the interactions are expected to be parameterized by a single universal parameter $\beta$ [1–3]. This thesis describes a measurement of the axial size of paired, $^6$Li clouds in the BEC-BCS crossover and provides a comparison with theory. In the BEC regime, absolute measurements of the molecular scattering length are compared with the atomic scattering length and the ratio is compared with theory. Finally, a measurement of $\beta$ is made and compared with theory.
Acknowledgments

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Chapter 1
Introduction

This thesis presents quantitative measurements of the interactions in a paired, dual species, $^6$Li Fermi gas. Sources of systematic and random experimental errors are an integral part of the thesis. Chapter 2 discusses the physics in the different regimes of the Feshbach resonance, along with the relevant formulas needed for calculating the scattering length, $\beta$, and density profiles. In chapter 3, a detailed description is given on how all the relevant experimental quantities are measured. Integrated in the discussion is a careful explanation of sources of systematic and random error. Finally, results of the measurements are given in chapter 4 including the molecular scattering length $a_m$ as a function of the detuning from resonance and a measurement of $\beta$ on resonance. Also included in chapter 4 is a comparison of the measured axial size of the clouds to the size one would expect from a non-interacting Fermi gas from which one may infer information about the interactions.
Chapter 2
Theoretical Background

This chapter will give an account of the specific physics in the BEC-BCS crossover that we wish to study. The physics applies to the case of a Fermi gas containing two different spin components so that the gas is interacting. The crossover is generally parameterized by the dimensionless number $k_Fa$, where $k_F$ is the Fermi wave vector, and $a$ is the $s$-wave scattering length. To probe the different regimes of the crossover, we can tune $a$ by utilizing a Feshbach resonance between the two lowest hyperfine levels in $^6\text{Li}$ [4]. A Feshbach resonance in atomic physics occurs when the energy of two free atoms in an open channel is magnetically tuned near the energy of a bound molecular state in a closed channel [5]. In this context, a channel is closed if the total relative kinetic energy of the colliding atoms is positive relative their energy at large separation. The position of the Feshbach resonance can be roughly estimated by setting the 1.6 $GHz$ binding energy of the last bound state in the closed-channel singlet potential equal to the amount the kinetic energy of the atoms would have to tune to become resonant with the bound state. The continuum threshold of the triplet potential tunes like $-2\mu_B B$ where $\mu_B$ is the Bohr magneton. This corresponds to a Feshbach resonance position of 570 $G$ close to the actual position of a resonance at 543 $G$ [6]. However, the position of the Feshbach resonance used in this thesis is given by a more subtle reason and is due to the existence of a quasi-bound state in the triplet potential which also causes the Feshbach resonance to be so broad (see Fig. 3.1). A two body picture of the crossover is depicted in Fig. 2.1. The Feshbach resonance forms weakly bound (dressed) molecules that are superpositions of the free atom pairs in the open channel and the bare molecules in the closed channel [4]. The many-body ground state is a BEC of these dressed molecules, and they even persist on the BCS side [4]. Far from resonance on the BEC side, where $k_Fa \ll 1$ and
Figure 2.1 Pictorial Representation of Crossover. The binding energy of the dressed molecules (red curve) is $\hbar^2/ma^2$ near resonance and approaches the energy of the bare molecular state (nearly vertical black line). The strongly coupled regime is the region where $k_F|a| \gtrsim 1$. More specifically, the unitarity regime is a small region right around the resonance position where $k_F|a| \gg 1$. Here the scattering length is limited by quantum mechanical unitarity [7].

$a > 0$, the ground state is a Bose-condensate of tightly bound molecules [8, 9]. Far from resonance on the BCS side, where $k_F|a| \ll 1$ and $a < 0$, the ground state is a BCS superfluid containing Cooper pairs [10]. Intermediate to the two regimes is the strongly interacting region where $k_F|a| \gg 1$. It is here where the physics “crosses over” from BEC to BCS. To investigate the interactions in the BEC-BCS crossover, we will measure the spatial size of a trapped dual species Fermi gas at $k_Fa$ that span the entire interaction range of the Feshbach resonance.
2.1 BEC Regime

In the BEC regime, where the energy of the closed channel is lower than that of the open channel, it is energetically favorable for pairs of unlike fermions to form bound molecules. At $T = 0$ and far from resonance, where interactions are weak, one recovers the mean field description of a weakly interacting BEC which is well described by the Gross-Pitaevskii (GP) equation [9,11,12]. Closer to resonance, where the interactions between fermions are stronger, the molecules are hybridizations of true bosons and fermionic atom pairs, and hence the gas cannot be thought of as a BEC of true bosons as described by the Gross-Pitaevskii equation. Here, the size of the pairs ($\sim a$) is comparable to the inter-particle spacing, and the gas starts to behave more and more like a Fermi gas as resonance is approached.

We use the Thomas-Fermi approximation as a tool to describe the ground-state distributions for both bosons and fermions as done by Ref.s [13,14]. The Thomas-Fermi approximation in the case of bosons neglects the kinetic energy in comparison to the mean field energy and trapping potential. In the Thomas-Fermi approximation the properties of the gas at any point are those of a uniform gas having equal density to the local density. At equilibrium, this amounts to saying that the energy to add a particle to the cloud is the same everywhere. For bosons we start with the GP equation

$$\frac{-\hbar^2}{2m} \nabla^2 \psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}) + U_o|\psi(\mathbf{r})|^2\psi(\mathbf{r}) = \mu\psi(\mathbf{r})$$

(2.1)

where $\mu$ is the chemical potential, $U_o = 4\pi\hbar^2a_m/m$ is the mean-field interaction, and $V(\mathbf{r})$ is the trapping potential. Here $m$ is the mass of a molecule and hence is twice the fermion mass. For a condensate with a large number of atoms and positive interactions, the first term (representing the kinetic energy) can be neglected in comparison to the interaction and potential energies giving

$$[V(\mathbf{r}) + U_o|\psi(\mathbf{r})|^2]\psi(\mathbf{r}) = \mu\psi(\mathbf{r}).$$

(2.2)
This gives a solution for the Thomas-Fermi particle density of

$$n(r) = [\mu - V(r)]/U_o$$ \hspace{1cm} (2.3)

everywhere the right hand side is positive; everywhere else $n(r) = 0$. Using our cylindrically symmetric trap potential

$$V(r) = \frac{1}{2}m\omega_r^2(x^2 + y^2) + \frac{1}{2}m\omega_z^2z^2$$ \hspace{1cm} (2.4)

and that the boundary of the cloud is given by $V(r) = \mu$ in the Thomas-Fermi approximation, we obtain

$$R_i^2 = \frac{2\mu}{m\omega_i^2}$$ \hspace{1cm} (2.5)

where $i$ labels the axes of the trap, and $R_i$ is the spatial extent of the cloud called the Thomas-Fermi radius. Reference [13] gives a relation for the chemical potential using the normalization condition on $\psi$

$$\mu = \frac{15^{2/5}}{2} \left( \frac{Na_m}{\bar{a}} \right)^{2/5} \hbar \bar{\omega}$$ \hspace{1cm} (2.6)

where $N$ is the number of molecules, $\bar{a} = (\hbar/m\bar{\omega})^{1/2}$, and $\bar{\omega} = (\omega_r^2 \omega_z^2)^{1/3}$ is the geometric mean of the trap frequencies. Inserting 2.5 into 2.6 we can solve for $a_m$ in terms of measurable quantities

$$a_m = \frac{m^2}{15\hbar^2} \frac{R_x^5 \omega_x^4}{N \omega_z^2}.$$ \hspace{1cm} (2.7)

### 2.2 BCS Regime

In the BCS regime, where $a < 0$, there is an attractive interaction between fermions. Here the energy of the closed channel is above that of the open channel and so bound molecules do not exist in the two body picture. However, in the case of a dual-species Fermi-gas, large Feshbach induced dressed molecules can exist because they are stabilized by many body effects near resonance [12]. Far from resonance in the BCS regime ($k_F|a| \ll 1$), the ground state becomes a BCS superfluid
much like that in a conventional superconductor. The attractive interactions allow for the formation of Cooper pairs. Arguably more interesting is the strongly coupled region where the physics is expected to be more similar to that of the high $T_c$ superconductors.

Near resonance ($k_F|a| > 1$), the strong interactions cause a squeezing of the size of the cloud [2]. As one moves towards smaller $k_F|a|$, the size becomes closer to that of the free Fermi-gas. To obtain the size, we will fit the gas to a Thomas-Fermi distribution for fermions. A calculation similar to that in the previous section gives the density distribution for fermions [13]. In the case of fermions the interaction energy can be neglected in comparison to the kinetic energy because of Fermi-statistics (no $s$-wave scattering for fermions). This gives for the density distribution

$$n(r) = \frac{1}{6\pi^2} \left( \frac{2m}{\hbar^2} [\mu - V(r)] \right)^{3/2}$$

(2.8)

if $\mu - V(r)$ is positive, and zero otherwise. For fermions the chemical potential is

$$\mu = (6N)^{1/3} \hbar \omega.$$  

(2.9)

The axial Thomas-Fermi radius of a noninteracting free Fermi gas is then

$$R_{TF} = \sqrt{\frac{2\mu}{m\omega}} = \sqrt{\frac{\hbar \omega^{1/3} (48N)^{1/6}}{m \omega^{5/6}}}.$$  

(2.10)

In this experiment, we measure the Thomas-Fermi radius of an interacting gas, $R_z$, and compare it to $R_{TF}$. We can compare our results to those of Ref. [2] which calculates the ratio of the total energy per particle of an interacting gas to the energy of a free Fermi gas, $E/N\epsilon_{FG}$, as a function of $-1/k_Fa$ using Monte Carlo methods.

### 2.3 Unitarity Regime

Right around resonance, where $k_F|a| \sim \infty$, an accurate theory cannot be obtained because there is no obvious small parameter with which to do perturbation theory. In
this region, called the unitarity regime, the physics is expected to be universal with applications to other strongly coupled systems such as high $T_c$ superconductors, the properties of low density neutron gasses [15], and dense quark matter in the cores of neutron stars [16]. Right on resonance the properties of the gas have been predicted to be characterized by the universal energy parameter $\beta$ which has been calculated from Monte Carlo methods by [2,17] to be $\beta = -0.58 \pm 0.01$ and obtained from a theory by [3] to be $\beta = -0.545$. For a harmonically trapped gas in the unitarity limit, the chemical potential takes the universal form $\mu = E_F(1 + \beta)^{1/2}$ where $E_F$ is the Fermi energy. For a cloud profile measured on resonance, the value of $\beta$ can be calculated from [3] 

$$\beta = (R_x/R_{TF})^4 - 1.$$  

(2.11)

For this thesis, this measurement will be performed at 830 $G$ very near the predicted resonance position of 834 $G$ [18, 19]
Chapter 3
Measurements, Calculations, and Uncertainties

The quantities that we wish to measure, $\beta$ and $a_m$, rely on absolute measurements of $N$, $\omega_z$, $\omega_r$, and $R_z$. For this reason it is important to carefully assess our measurement method and our sources of error. The following sections outline the methods in which these quantities are measured along with the experimental uncertainties in quantity. We begin with a brief description of how we create a degenerate gas.

3.1 Recipe for a Molecular BEC

In this section, I will briefly describe how we produce a dual-species, degenerate Fermi gas of $^6\text{Li}$. Much of the procedure and apparatus has been described previously [20, 21]. A thermal beam of $^6\text{Li}$ and $^7\text{Li}$ sourced from an oven is slowed using laser radiation pressure, and are confined in a Magneto-Optical Trap (MOT). From the MOT, where the atoms have a temperature on the order of 150 – 300 $\mu K$, the $^6\text{Li}$ and $^7\text{Li}$ are transferred to a magnetic trap and dual-evaporatively cooled for 55 s to a temperature of a few $\mu K$. The $^6\text{Li}$ atoms are then transferred to an optical trap at which point they are in the $|F = 3/2, m_F = 3/2\rangle$ hyperfine state. A magnetic bias field (formed from Helmholtz configuration coils) is then ramped to 754 G (near the Feshbach resonance) followed by a microwave transfer to the $|F = 1/2, m_F = 1/2\rangle$ ground state. From here, an incoherent 50 – 50 spin mixture of the two lowest energy Zeeman sublevels, $|F = 1/2, m_F = \pm 1/2\rangle$ is created using a “sawtooth” series of 100 frequency ramps. Now that the fermionic atoms are in different spin-states near the Feshbach resonance, they can interact and form dressed molecules. The atoms are further evaporatively cooled by reducing the power in the optical trap from about 2.5 $W$ to 10 $mW$, or lower, forming a molecular BEC. The interactions in the gas are tuned using the Feshbach resonance by changing the magnetic field to the desired
Figure 3.1 **Feshbach Resonance.** Feshbach resonance is between $|F = 1/2, m_F = -1/2\rangle$ and $|F = 1/2, m_F = 1/2\rangle$ hyperfine ground states. Most up to date information places resonance at 834 G \[18\].

value (see Fig. 3.1).

### 3.2 Imaging

Much of the information we need is obtained from absorption images of the column density recorded onto a CCD camera. The CCD camera records two pictures for every shot: one of the imaging probe with no atoms, which we call the no-atoms shot $I_0(x, z)$, and one of the probe where the cloud has absorbed some of the light which we call the atoms shot $I(x, z)$. For our imaging probe, we use fiber coupled light from a Spectra Physics dye laser. The probe has a wavelength of 671 nm corresponding to the $2S_{1/2}$ to $2P_{3/2}$ transition in $^6$Li and a linear polarization perpendicular to the Feshbach field. The no-atoms shot is divided by the atoms shot and the natural log is taken pixel by pixel. The result is the measured optical depth

$$OD(x, z) = \ln \left( \frac{I_0(x, z)}{I(x, z)} \right) \quad (3.1)$$

and can be related to the column density $\tilde{n}(x, z)$. The column density is the line integral of the 3D density $n(r, z)$ along the camera imaging direction:

$$\tilde{n}(x, z) = \int n(r, z)dy \quad (3.2)$$
where \( r = (x^2 + y^2)^{1/2} \) for our cylindrically symmetric trap. A calculation following from Beer’s law gives us the column density in the general case where the probe intensity is not small compared to \( I_s = 10.2 \, mW/cm^2 \) (see appendix for a derivation):

\[
\bar{n}(x, z) = \frac{2I_o \left( 1 - e^{-OD(x,z)} \right) + \left( 1 + 4\frac{\Delta^2}{\Gamma^2} \right) OD(x, z)}{\sigma_o} \tag{3.3}
\]

where \( \sigma_o \) is the peak absorption cross-section, \( \Delta \) is the detuning, \( I_o \) is the average probe intensity hitting the cloud, \( I_s \) is the saturation intensity, and \( \Gamma = 2\pi \left( 5.9 \, MHz \right) \) is the transition linewidth. In principle \( I_o(x, z) \) should be used in the place of \( I_o \) in equation 3.3, however in practice we used the average intensity over the cloud. The reason this was done is that \( I_o(x, z) \) may have spatial noise on it and we were afraid that using it in Eq. 3.3 might introduce that noise into \( \bar{n}(x, z) \). Using \( I_o(x, z) \) in the optical depth \( OD \) was not a worry because it is divided by \( I(x, z) \) which divides out the structure on the probe beam common to both shots. One might very reasonably assume that this assumption would depend on the comparison of the probe beam waist \( W_o \) with the spatial extent of the cloud \( R_z \), with the validity of the assumption becoming worse as the cloud size increases. For example, at 754 \( G \) the average \( R_z \) is \( \sim 400 \, \mu m \) which is \( \sim .28 \, W_o \). This would imply that the probe intensity, being gaussian, would drop by 14% i.e. a factor of \( e^{-2.28^2} \) at the edge of the cloud. This, however, is not the case in our situation. At all fields, the difference in using \( I_o(x, z) \) instead of \( I_o \) to calculate \( N \) (see section 3.4) was less than 1% except at 620 \( G \) where the difference was around 2%. This insensitivity occurs because the actual light field of our probe beam is relatively flat with spatial variations that are large compared with those expected from the spatial dependence of a gaussian beam. In the future, it is probably best that we use \( I_o(x, z) \) instead of \( I_o \) since it is in principle more robust. Furthermore, using \( I_o(x, z) \) did not result in the addition of any extra noise in the profiles as originally expected.
Figure 3.2  Coordinate System Definition. The $x'$ and $y'$ axes form a plane perpendicular to the optical trap laser beam propagation direction. The $x$ and $z$ axes define the coordinate system of the CCD camera with $y$ being the imaging direction.

3.3 Experimental Parameters for Measuring Number

This section will discuss the procedure for measuring the quantities that are relevant to calculating number. Also included is a discussion of the sources of systematic and random errors associated with their measurements. Of the following quantities in this section, only the measurement of the magnification has use other than that in calculating number.

3.3.1 Measurement of CCD Pixel Size

To obtain the size of a CCD pixel referenced to the object plane, we must measure the magnification. To get the magnification, the optical trap is moved in the $x'$ and $y'$ directions with a translator, and the corresponding amount the cloud moves up and down in the $x$ direction on the camera is measured. Position along the camera is plotted vs. position of the optical trap for both the $x'$ and $y'$ directions. The coordinate system defining $x'$ and $y'$ is shown in 3.2. For a given displacement $\Delta x'$ in the
Figure 3.3 Determination of Magnification. Shown are plots of the measured location of the cloud in the plane of the camera versus position of the focusing lens for the optical trapping beam. Also shown is a least squares fit to a line for each data set from which we extract slopes $M_{y'}$ and $M_{x'}$.

$x'$ direction, the cloud moves a distance $\Delta X_{x'}$ in the object plane perpendicular to the imaging direction, and $M \Delta X_{x'}$ in the image plane where $M$ is the magnification. Analogously, for a given displacement, $\Delta y'$, in the $y'$ direction, the cloud moves a distance $\Delta X_{y'}$ in the object plane and $M \Delta X_{y'}$ in the image plane. In each direction a slope is measured in units of (pixels along $x$ direction)/(distance moved with translator in $x'$ or $y'$ direction) by fitting to a line. The data is shown in Fig. 3.3. From these two slopes, $M_{x'}$ and $M_{y'}$, the magnification and camera angle $\theta$ can be simultaneously determined:

\[
M = 13 \frac{\mu m}{pixel} (M_{x'}^2 + M_{y'}^2)
\]

and

\[
\theta = \arctan (M_{y'}/M_{x'})
\]

(3.4)

where $13 \mu m/pixel$ is the actual size of a pixel on the camera. The slopes are $M_{x'} = -264.3 \pm 3.9 \text{ pixels/mm}$, and $M_{y'} = -152.8 \pm 1.1 \text{ pixels/mm}$ where the uncertainty in the slopes is determined from the linear fit. These give a magnification of $M = 3.97$
± 0.04 or \( \ell = L/M = 3.28 \pm 0.04 \mu m/pixel \) where \( L = 13 \mu m \) is the actual size of a CCD pixel. The reason the uncertainty in \( M_{\nu} \) is larger than the uncertainty in \( M_{\nu'} \) is that the fit to find \( M_{\nu} \) includes a statistical outlier. These slopes imply \( \theta = 30^\circ \) (see figure 3.2) which is consistent with the known geometry of the chamber.

### 3.3.2 Peak Cross Section

The peak cross-section \( \sigma_\nu \) is calculated as follows. The atoms are probed on a \( \sigma^- \) transition from a \( j = 1/2, m_j = -1/2 \) lower state to a \( j = 3/2, m_j = -3/2 \) upper state. Using the Wigner-Eckart theorem,

\[
\sigma_\nu = \frac{3\lambda^2}{2\pi} (2j_u + 1) |\hat{\varepsilon}_q \cdot \hat{\varepsilon}| \left( \begin{array}{ccc} j_u & \ 1 & j_l \\ -m_u & q & m_l \end{array} \right)^2
\]

where the \( u \) and \( l \) subscripts correspond to the upper or lower state respectively. \( \hat{\varepsilon}_q = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{y}) \) is the polarization unit vector for driving a \( \sigma^- \) transition, and \( \hat{\varepsilon} = \hat{x} \) is the unit vector representing the linear polarization of the probing laser which propagates in the \( \hat{k} = \hat{y} \) direction giving \( \hat{\varepsilon}_q \cdot \hat{\varepsilon} = \frac{1}{\sqrt{2}} \). Plugging in these values and evaluating the 3-j symbol gives \( \sigma_\nu = 3\lambda^2/4\pi \). From this cross-section we can calculate the saturation intensity to be \( I_s = 10.2 \, mW/cm^2 \) from the definition \( \Gamma = \frac{\sigma_\nu I_s}{\hbar \omega} \) where \( \omega = 2\pi c/\lambda \). For the lithium \( 2S_{1/2} - 2P_{3/2} \) transition \( \lambda = 671 \, nm \).

### 3.3.3 Measuring Probe Detuning

The frequency of our probing laser is measured as a beat-note on a spectrum analyzer with another reference laser which is locked to a \( ^6\text{Li} \) atomic transition via a heat pipe lock. The heat pipe feature we lock to is the crossover between the transitions between the \( 2S_{1/2} F = 1/2 \) and \( F = 3/2 \) ground states to the \( 2P_{3/2} \) excited state. The detuning is calculated from \( \Delta = 2\pi (\nu - \nu_\nu) \) where \( \nu \) is the probing beat-note frequency and \( \nu_\nu \) is the resonance beat-note frequency. By measuring the slope of the error signal at the center of the heat pipe feature, and by measuring the amplitude of the noise on the heat pipe error signal when locked, we estimate that
Figure 3.4 Example of Resonance Spectrum. Resonance was obtained at 620 G, with a probe intensity of 2.7 mW/cm². The fitted FWHM of the spectrum is 7.3 MHz. This is in agreement with what one would expect from the saturated width $\Gamma (1 + 2I/I_s)^{1/2}$.

the lock allows for a 600 kHz jitter in the frequency of the reference laser. However, we suspect that this is an overestimate because some of this noise is still present when the reference laser is dialed to a frequency where the slope of the error signal on the heat pipe lock is zero. Because the probing laser is not frequency locked to the reference laser, this noise does not appear on the probing laser. We estimate that we can measure the frequency in which we probe to within a $\delta \nu = 1.5$ MHz statistical uncertainty. To measure the position of resonance at a specific magnetic field, we take a spectrum as shown in 3.4. In most cases, the position of resonance is determined to within an uncertainty of $\delta \nu_\circ = 0.5$ MHz which we take as our systematic uncertainty in $\Delta$. 
3.3.4 Camera Calibration

We have calibrated our CCD camera in order to determine how many photons hitting the plane of the atoms corresponds to one count on the CCD array. This is accomplished by focusing the probe beam on the camera so that the entire beam is in the field of view (The field of view is 4 times twice the waist). The number of counts C on the camera is measured for several pulse durations \( \Delta t \), and the power \( P \) in each pulse is measured. For each \( \Delta t \) several measurements of \( C \) are performed and averaged. The counts are plotted versus \( \tau \) with error bars representing the standard error in the mean. The plot fits well to a line with an offset \( \eta \). This offset is subtracted from each counting measurement so that \( C' = C - \eta \), and the error in \( \eta \) is propagated through to \( C' \). For each pulse duration the photons per count \( E_f \) is calculated from

\[
E_f = \frac{\lambda}{hc} \frac{P \Delta t}{C'}
\]

and the statistical uncertainty in \( P \) is propagated through to \( E_f \). The final value for \( E_f \) is the average of the \( E_f \)'s for each \( \Delta t \) weighted by the statistical error bars (see Fig. 3.5). The final error in \( E_f \) is

\[
\delta E_f = E_f \sqrt{\left( \frac{\delta E_f}{E_f} \right)_{\text{stat.}}^2 + \left( \frac{\delta P}{P} \right)_{\text{sys.}}^2}
\]

where \( \frac{\delta P}{P} \) \(_{\text{sys.}} \) is the systematic error in the power meter calibration which we estimate to be about 10% and \( \frac{\delta E_f}{E_f} \) \(_{\text{stat.}} \) = 2.5%. We obtain \( E_f = 4.36 \pm 0.45 \) photons/count.

The 10% uncertainty in the power meter accuracy is determined from a comparison of the readings from two other power meters (a UDT power meter and another Newport power meter) when measuring the power in a test beam with approximately the same power used to calibrate the camera. The power in the test beam is measured 4 times on each power meter in the ensemble. It was found that over the 4 trials the power meter used to calibrate the camera differed from the UDT power meter by about 10% and was in agreement with the second Newport power meter, although on any one of
Figure 3.5  **Camera Calibration.** Average $E_f = 4.36 \pm 0.11$ photons/count determined by the weighted mean. The error bar is the weighted uncertainty in the mean.

The 4 trials the two Newport power meters disagreed by $1 - 3\%$. It should be noted that because we are calibrating the counts on the camera to the power in the plane of the atoms, we do not need to take into account loss off the imaging optics because they occur after the atoms. The only optical elements that occur after the point where the power is measured and before the atoms are a mirror and the chamber window. The chamber window has negligible loss because it is AR coated for 671 nm, and the dielectric mirror has a loss of $< 0.5\%$ and can also be neglected.

### 3.3.5 Probe Intensity

The probe intensity $I_o$ is measured on every shot, and is determined from

$$I_o = E_f \frac{hc}{\lambda} C_f \frac{1}{\Delta t \ell^2 A_{zz}}$$

where $C_f$ is the sum of the counts in a box around the cloud in the no-atoms shot, $A_{zz}$ is the number of pixels in the box, $\Delta t$ is the probe duration, $\ell = 3.28 \pm 0.04$
\( \mu m/pixel \) is the size of a CCD pixel array referenced to the plane of the atoms, and \( hc/\lambda \) gives the energy of a photon with wavelength \( \lambda = 671 \) nm. The length of the box is adjusted for each magnetic field to account for the dependence of the clouds axial size with increasing interactions. The box’s axial length and radial width are chosen to be \( 2R_z \) and \( 2R_r \) respectively. To get an estimate of the uncertainty in the counts \( \delta C_I \), we first calculate the variance \( \sigma_n^2 \) of the residual noise per pixel in the background region of the subtracted image \( I_o(x, y) - I(x, y) \). So now we can ascribe \( \sigma_n = \sqrt{\sigma_n^2} \) as the uncertainty in the number of counts on a single pixel so that \( \delta C_I = \sqrt{A_{xz} \sigma_n^2} \). The uncertainty in the intensity is

\[
\delta I_o = I_o \sqrt{\left( \frac{\delta E_f}{E_f} \right)^2 + \left( \frac{\delta C_I}{C_I} \right)^2 + 4 \left( \frac{\delta \ell}{\ell} \right)^2}
\]

(3.9)

\( \frac{\delta E_f}{E_f} = 10.3\% \), \( \frac{\delta \ell}{\ell} = 1\% \), and their determination is outlined in sections 3.3.4 and 3.3.1 respectively.

### 3.4 Number Determination from Integrating Optical Depth

One method to obtain \( N \) involves summing the array containing \( \bar{n}(x, z) \) using a Matlab computer program so that:

\[
N = \frac{2\ell^2}{\sigma_o I_s} \sum_{x,z} \left( 1 - e^{-OD(x, z)} \right) + \frac{\ell^2}{\sigma_o} \left( 1 + 4 \frac{\Delta^2}{\Gamma^2} \right) \sum_{x,z} OD(x, z)
\]

(3.10)

where \( \sum_{x,z} \) denotes summing the pixels of the array. The uncertainty in the number \( \delta N \) contains two parts: the uncertainty in \( N \) from \( \delta \Delta \), which is denoted \( \delta N_\Delta \), and the uncertainty in \( N \) from \( \delta I_o \), which I denote \( \delta N_{I_o} \). These are calculated from

\[
\delta N_\Delta = \frac{\partial N}{\partial \Delta} \delta \Delta = \left( \sum_{x,z} OD(x, z) \right) \frac{\ell^2}{\sigma_o} 2|\Delta|\delta \Delta
\]

(3.11)

where \( \Delta \) is from now on written in units of \( \Gamma \), and

\[
\delta N_{I_o} = \frac{\partial N}{\partial I_o} \delta I_o = \frac{2\ell^2 I_o}{\sigma_o I_s} \sum_{x,z} \left( 1 - e^{-OD(x, z)} \right).
\]

(3.12)
One must be careful in applying equation 3.11 to the case where $\Delta = 0$ because one would incorrectly assume that $\delta N_{\Delta} = 0$ regardless of the uncertainty in the detuning. In this case one should instead calculate $\delta N_{\Delta}$ by simply replacing $\Delta$ with $\delta\Delta$ in Eq. 3.10. The uncertainty in $N$ from $\ell$ is given by

$$\delta N_{\ell} = \frac{2\ell}{\sigma_o} (1 + 4\Delta^2) \delta\ell \sum_{x,y} OD(x, y). \quad (3.13)$$

### 3.5 Number Determination from Counting Scattered Photons

An alternative, and somewhat more direct, way to measure the number of molecules is to instead measure the number of photons spontaneously scattered out of the probe beam by the cloud. Assuming we probe on a cycling transition for a duration $\Delta t$, the number of scattered photons is

$$N_\gamma = N \Gamma \rho_{ee} \Delta t \quad (3.14)$$

where $\rho_{ee} = (I_0/I_s)/(1 + 2(I_0/I_s) + 4\Delta^2)$ is the excited state population, and $\Gamma$ is the excited state decay rate. Solving for $N$ gives,

$$N = \frac{N_\gamma}{\Gamma \Delta t} \left( \frac{1 + 2I_0/I_s + 4\Delta^2}{(I_0/I_s)} \right) \quad (3.15)$$

The problem now reduces to measuring $N_\gamma$, which is done by summing the number of CCD counts in a box surrounding the cloud in both the atoms and the no-atoms shot, $C_A$ and $C_{NA}$ respectively. So we have that,

$$N_\gamma = (C_{NA} - C_A) E_f \quad (3.16)$$

where $E_f$ is the number of photons per count calculated in section 3.3.4. The sum box is chosen to be $\sim 20\%$ longer than $2R_s$ to ensure that all the signal is counted. Nothing is gained by making the box larger because you would only sum in more background noise. The uncertainty in $C_{NA}$ and $C_A$ is calculated from the variance $\sigma_n^2$ in the
residual background noise which was delineated in section 3.3.5. The uncertainty in both $C_{NA}$ and $C_A$ is then calculated from $\delta C = \sqrt{\sigma_n^2 A}$ where $A$ is the number of pixels in the sum box. One should not calculate the uncertainty in the number from both $E_f$ and $I_o$ separately because $N$ has $E_f$ dependence in it separate from that in $I_o$ from Eq. 3.16. To make this more clear, $N$ should be written in terms of independent quantities as:

$$N = \frac{(C_{NA} - C_A)(1 + 2E_f \frac{\hbar c}{N_o \Delta t} \frac{1}{\tau A_{xx}} + 4\Delta^2)A_{xx}}{\frac{F}{F_o} \frac{\hbar c}{C_f}}. \quad (3.17)$$

So the uncertainty in $N$ from the systematic error in the power measurement when calibrating the camera is treated correctly by including $(\delta P/P)_{sys}$ in $\delta E_f$ and then calculating the uncertainty in $N$ from $E_f$. The result is

$$\delta N_{E_f} = \frac{2\delta E_f (C_{NA} - C_A)}{\Gamma \Delta t}. \quad (3.18)$$

The uncertainty in $N$ from the detuning is calculated from

$$\delta N_\Delta = \frac{\frac{N8|\Delta|\delta \Delta}{1 + 2(I_o/I_o + 4\Delta^2)}}{1} \quad (3.19)$$

The same care as in section 3.4 should be taken when calculating the uncertainty from detuning when $\Delta < \delta \Delta$. Lastly, the uncertainty in $N$ from $\ell$ is

$$\delta N_\ell = \frac{2I_s(1 + 4\Delta^2)(C_{NA} - C_A)A_{xx}\ell}{C_f \Gamma \hbar c/\lambda} \delta \ell \quad (3.20)$$

3.6 Comparison of Methods for measurement of $N$

At all fields the uncertainty in both methods of calculating $N$ are nearly identical. Furthermore, they never disagree with each other by more than about 10%, and usually the agreement is even better being on the order of a few percent. In some cases there is less than a 1% difference between the two. The discrepancy between
Figure 3.6 Determination of Radial Trapping Frequency. Shown is a plot of the measured signal in the optical trap versus the frequency at which the pointing of the optical trapping beam is modulated. The data is fit to a lorentzian and is normalized by the background value. The uncertainty in the center frequency comes from the fit.

the two methods at all fields is consistent with the uncertainty in N. In all the data, except 830 G, the method of counting the number of scattered photons is used. I feel that this method is preferable because it is conceptually simpler and more direct.

3.7 Measurement of Radial Trapping Frequency

The radial trapping frequency is first measured with $^7$Li at a higher power $P_o$ by modulating the pointing of the optical trap with an AOM and looking for the frequency $\nu_o$ at which a loss of signal is observed. The resonance is shown in Fig. 3.6. This frequency is then scaled to the power $P$ at which the current shot is taken in order to get the radial frequency

$$\omega_r = 2\pi \nu_o \sqrt{\frac{7}{6}} \left(\frac{P}{P_o}\right)^{1/2},$$

(3.21)
where $\nu_o = 3919$ Hz is the measured frequency at $P_o = 1.957$ W, and the factor of $\sqrt{\frac{2}{\hbar}}$ is a mass correction coming from the fact that the beam-shaking resonance was taken with $^7$Li. We pick off a portion of the optical trap beam onto a photodiode and measure the voltage on an oscilloscope plotting $P$ versus $V$. At low powers (less than 10 mW where our data is taken) this curve is fit to a line so that by measuring $V$ on every shot we can determine $P$ on a shot to shot basis. We have also confirmed that the photodiode calibration has not drifted over the few month timescale.

The total systematic error in the radial frequency is 1.4%, and has two contributions

$$
(\delta \omega_r)_{\text{sys.}} = \omega_r \sqrt{\left(\frac{\delta \nu_o}{\nu_o}\right)^2 + \frac{1}{4} \left(\frac{\delta P_o}{P_o}\right)^2}.
$$

The first contribution, $\delta \nu_o/\nu_o = 1.35\%$, comes from the uncertainty in the fit shown in Fig. 3.6 and is the dominant contribution. The second contribution, $\delta P_o/P_o = 1\%$, comes from the statistical uncertainty in determining the power at which the beam modulating resonance was taken and could have been neglected in comparison to $\delta \nu_o/\nu_o$. There is a 3% systematic uncertainty (quoted by company) in the Ophir power meter calibration accuracy which should enter into $\delta P_o$, however this does not enter into the uncertainty in $\omega_r$ because we are only interested in relative powers. The statistical error in $\omega_r$ comes from the statistical error in determining the lower power $P$ for which a specific shot is taken, and is given by

$$
(\delta \omega_r)_{\text{stat.}} = \omega_r \frac{1}{2} \frac{\delta P}{P}.
$$

$\delta \omega_r/\omega_r$ ranges from 2.5 - 5% over the range of our data depending on the power in the optical trap. Although done with a different optical trap (constructed using a laser from Crystal Technology as opposed to the IPG laser used for this thesis) we have previously checked that this method of scaling is valid by measuring the trapping frequency at two different powers and comparing the measured frequency to the scaled frequency. The trapping frequency was first measured at $P_1 = 335$ mW.
and the resonance fit to a frequency of $\nu_1 = 1694 \pm 6 \text{ Hz}$. The trapping frequency was also measured at $P_2 = 128.6 \text{ mW}$ where we obtained $\nu_2 = 1035 \pm 25 \text{ Hz}$ which is consistent with the value, 1050 Hz, we would expect from scaling.

### 3.8 Measurement of Axial Trapping Frequency

The axial trapping frequency $\omega_z = 2\pi \nu_z$ is measured directly by exciting a trap oscillation and plotting the position versus time. This is then fit to a sinusoid to extract $\nu_z$. The measured values of $\nu_z$ do not match what one would expect from the aspect ratio $\xi$ of the trap. One would expect $\nu_z$ to be related to the radial frequency by

$$\nu_z(\text{opt}) = \frac{\nu_r}{\xi}$$

where the aspect ratio is given by $\xi = \sqrt{2\pi w_o/\lambda_L} = 107.6$ where $w_o = 26.15 \pm .27 \mu m$ is the waist of the optical trap and $\lambda_L = 1080 \text{ nm}$ is the wavelength of our trapping laser. The waist is most precisely determined from the measured power $P_o$ and radial frequency $\nu_o$ referred to in the previous section using the expression

$$w_o = \left( \frac{2\Gamma^2 P_o h}{\Delta \nu I_s \pi m (2\pi \nu_o)^2} \right)^{1/4}$$

where $\Delta \nu$ is the detuning of the optical trap from the lithium resonance, $m$ is the mass of a $^7\text{Li}$ atom, and $I_s = 5.1 \text{ mW/cm}^2$ is the correct saturation intensity for a far off resonance optical dipole trap with linear polarization perpendicular to the bias field. This is because the laser detuning is large compared with the $2P$ state fine-structure splitting, and hence we must sum the cross-sections to the $(2P_{1/2}, m_j = -1/2), (2P_{3/2}, m_j = -1/2)$, and $(2P_{3/2}, m_j = -3/2)$ states. This value of the waist is in reasonable agreement to the value measured by looking at the optical trapping beam on a CCD camera for which we obtained 25.7 $\mu m$. The uncertainty in the waist is calculated from

$$\delta w_o = w_o \sqrt{\frac{1}{4} \left( \frac{\delta \nu_o}{\nu_o} \right)^2 + \frac{1}{16} \left( \frac{\delta P_o}{P_o} \right)^2}$$
Figure 3.7 Measured and Predicted Axial Frequency vs. $B$. Because of the short lifetime of the molecules at 620 G, a reliable measurement of $\nu_z$ was not possible. For 620 G only the theoretical model is shown. At 860 G, $\omega_z$ is measured twice, at two different trap depths, and the values are in agreement to better than 1.2% when scaled to a common power.

where $\delta \nu_o / \nu_o = 1.35\%$ and $\delta P_o / P_o = 3.1\%$ as outlined in section 3.7, however since the measurement of the waist depends on an absolute measurement of the power, we must in this case include the 3% uncertainty in the Ophir power meter accuracy.

The discrepancy in the measured values of $\omega_z$ led us to conclude that something else was adding to the axial potential of the optical trap. We have found that the axial frequency is modified by added curvature from the magnetic bias field which is described by

$$\nu_z = (\nu_{z(\text{opt})}^2 + \alpha B)^{1/2}$$ \hspace{1cm} (3.27)

where $B$ is the magnetic bias field, $\nu_{z(\text{opt})}$ is given by equation 3.24, and $\alpha$ is a constant to be determined. This has been checked by plotting $(\nu_z / \nu_{z(\text{opt})})^2 - 1$ versus
$B/\nu_{z(\text{opt})}^2$, and observing that the data fit well to a line through the origin. From the fit we determine that $\alpha = 0.0503 \pm 0.0015$ $Hz^2/G$. There is a 2\% scatter of the measured points about the points predicted by the theoretical model as shown in figure 3.7.

This is attributed to the uncertainty in extracting a frequency from sinusoidal fits to the oscillations as well as uncertainty in the optical trap depth. This gives a systematic uncertainty of $\delta \omega_z/\omega_z = 2\%$ in using the model. Statistical error in $\omega_z$ due to power is reduced because $(\alpha B)^{1/2}$ is comparable to $\nu_{z(\text{opt})}$, and is given by

$$\frac{\delta \nu_z}{\nu_z} = \frac{1}{2} \left( \frac{\nu_{z(\text{opt})}}{\nu_z} \right)^2 \left( \frac{\delta P}{P} \right).$$

(3.28)

This error is only about 0.4\%.

### 3.9 Measurement of Thomas-Fermi Radius

There are four choices of distributions from which we can use to extract $R_z$ from an absorption image on the CCD camera. For clouds on the BCS side one can fit to the column density for fermions. Alternatively, one can choose to integrate the signal along $x$ and fit to an integrated column density distribution for fermions. To obtain the column density $\bar{n}(x, z)$ for fermions we integrate Eq. 2.8 along $y$,

$$\bar{n}(x, z) = \int n(y)dy$$

(3.29)

and obtain,

$$\bar{n}(x, z) = \frac{1}{16\pi} \left( \frac{m}{\hbar^2} \right)^{3/2} \left( \frac{1}{\mu_\pi} \right)^{1/2} (2\mu - m\omega_r^2 x^2 - m\omega_z^2 z^2)^2.$$  

(3.30)

We now take the $x = 0$ cut and write the answer in the form

$$\bar{n}(x = 0, z) = \bar{n}_o(1 - z^2/R_z^2)^2.$$  

(3.31)

The $x$-integrated column density is

$$n_{xy}(z) = \frac{1}{15\pi} \left( \frac{m}{\hbar^2} \right)^{3/2} \frac{1}{m\omega_r^2} (2\mu - m\omega_r^2 x^2 - m\omega_z^2 z^2)^{3/2}$$  

(3.32)
which we rewrite in the form we can use for fitting as

\[
n_{xy}(z) = n_{xy}(0)(1 - z^2/R_z^2)^{5/2}.
\]  

(3.33)

For the BEC side we perform the same integrations on Eq. 2.3. These are the listed results:

\[
\bar{n}(x, z) = \frac{2}{3U_o} \frac{(2\mu - m\omega_x^2 x^2 - m\omega_z^2 z^2)^{3/2}}{\sqrt{m\omega_z^2}}
\]

and for fitting,

\[
\bar{n}(x = 0, z) = \bar{n}_o(1 - z^2/R_z^2)^{3/2}.
\]  

(3.35)

For the integrated column density we get

\[
n_{xy}(z) = \frac{\pi}{4U_o m\omega_x^2} (2\mu - m\omega_z^2 z^2)^2.
\]  

(3.36)

The form used for fitting is,

\[
n_{xy}(z) = n_{xy}(0)(1 - z^2/R_z^2)^2.
\]  

(3.37)

We feel that it is preferable to use the \( x \)-integrated distribution because of effects that cause radial spreading of the cloud leading to systematic uncertainties in the peak column density. Radial spreading is caused by imperfect resolution, probe induced heating [22], and distortions caused by dispersive phase shifts of the probe beam upon passing through the cloud [23]. If we are only interested in determining the axial radius, we in principle could use the column density. However, then we would be susceptible to irregularities in the cloud’s edge as well as uncertainty in the determination of the cloud’s center, the first of which integrating reduces, and the second of which integrating eliminates.

The uncertainty in measuring \( R_z \) has a systematic component from the 1% uncertainty in the magnification, \( \frac{\delta R_z}{R_z}_{sys.} = 1\% \), and a statistical component, \( \frac{\delta R_z}{R_z}_{stat.} \), which varies depending on the quality of the fit. For shots with excellent signal to noise (these tend to occur at higher fields where the signals are higher), \( \frac{\delta R_z}{R_z}_{stat.} \) is
around < 1 − 2%. For shots of an average signal to noise $\left( \frac{\delta R_z}{R_z} \right)_{\text{stat.}}$ is around 2 − 4%. For poor shots, which can occur at lower fields, $\left( \frac{\delta R_z}{R_z} \right)_{\text{stat.}}$ is 4 − 9%. The statistical uncertainty in $R_z$ is estimated by fitting a few distributions using Microcal Origin’s nonlinear curve-fitting feature which generates uncertainties in fitting parameters. As a test of whether the uncertainties produced by Origin are grossly too large or too small, simulations were conducted wherein random noise of fixed amplitude, produced with a random number generator in Matlab, is added to a Bose Thomas-Fermi distribution generated from a given $N$, $\omega_r$, and $\omega_z$, and uses $a_m = 0.6a$. The same distribution is fit many times, but each time Matlab randomly sums in a new noise distribution of the same amplitude. The signal to noise was chosen to be about the same as that of an average shot. The standard deviation of the fitted $R_z$'s was 3% in reasonable agreement to the uncertainty produced with Origin. Upon increasing the amplitude of the noise, the standard deviation of $R_z$ increased as expected.
Chapter 4
Results

Now that we have the ingredients to calculate the number, the trapping frequencies, and the radial size, we can compare the properties of the gas to theory. Below resonance the molecular scattering length is analyzed from 620 \( G \) to 790 \( G \). Throughout the crossover we plot \( R_z/R_{TF} \) from 620 \( G \) to 920 \( G \).

4.1 Axial Size Throughout Crossover

At every field, the measured axial size normalized to the size one would expect for a non-interacting Fermi gas is plotted versus \( B \) and versus \( 1/k_F a \) in Figs. 4.1 and 4.2 respectively. The plots demonstrate the smoothness of the crossover by the lack of any critical behavior at resonance. This smooth behavior is consistent with a previous measurement of the axial size by [24], however our values of \( R_z/R_{TF} \) in the BCS regime are somewhat larger. The observed smooth behavior is also consistent with that predicted in Ref. [2] for the behavior of the energy per particle minus the binding energy in the BEC-BCS crossover.

For fields \( \leq 754 \ G \), \( R_z \) is obtained from a boson Thomas-Fermi fit, whereas for fields \( \geq 790 \ G \), \( R_z \) is obtained from a fermion Thomas-Fermi fit. The measured \( N \), \( \omega_r \), and \( \omega_z \) are used to calculate \( R_{TF} \) at every point using Eq. 2.10. The error in \( R_z/R_{TF} \) is calculated using

\[
\delta \left( \frac{R_z}{R_{TF}} \right) = \left( \frac{R_z}{R_{TF}} \right) \sqrt{\frac{1}{36} \left( \frac{\delta N}{N} \right)^2 + \frac{1}{9} \left( \frac{\delta \omega_r}{\omega_r} \right)^2 + \frac{25}{36} \left( \frac{\delta \omega_z}{\omega_z} \right)^2 + \left( \frac{\delta R_z}{R_z} \right)^2}
\]

(4.1)

to obtain the systematic component. The statistical component is obtained from the standard error in all the measurements which is added in quadrature to the systematic component to give the total error in \( R_z/R_{TF} \). A summary of systematic and statistical errors for every field is given in Table 4.1.
Figure 4.1 \( R_z/R_{TF} \) Versus \( B \). Black circles are fit using integrated column density, blue squares fit using non-integrated column density, and red circle is a bimodal fit to integrated column density.

Figure 4.2 \( R_z/R_{TF} \) Versus \( 1/k_Fa \). Black circles are fit using integrated column density, blue squares fit using non-integrated column density, and red circle is a bimodal fit to integrated column density.
Figure 4.3  Thomas-Fermi Fit at 620 $G$. The same shot is shown in both frames using the column density (left) and the integrated column density (right). The integrated column density has more background noise than the non-integrated column density because non-random noise from bad subtractions in the background of the image is summed into the profile in the integrated case. For this shot $N = 4.6 \times 10^3$, $R_z = 140 \mu m$ giving $a_m = 434 a_o$ (left), and $R_z = 178 \mu m$ giving $a_m = 1467 a_o$. The profiles are filtered for clarity.

Example fits at 620 $G$ are shown in Figs. 4.3 and 4.4. At this field the fits were performed with both the integrated and non-integrated forms of the boson column density (Eq.s 3.35 and 3.37). The fits using Eq. 3.37 tended to give values of $R_z$ on average about $\sim 15\%$ higher than those obtained using 3.35. We attribute this to especially noisy backgrounds in the images at this field due to low signals (see Fig. 4.3).

Shots at 650 $G$ had the same background noise problem as at 620 $G$ increasing the scatter in $R_z$. One of the better profiles at 650 $G$ is shown in Fig. 4.5. Another problem at 650 $G$ is that most of the profiles showed evidence of a thermal fraction indicating that we did not ramp the intensity of the optical trap far enough to make a $T = 0$ gas. Background noise only allowed for 3 of the shots to have reasonable fits to bimodal distributions (see Fig. 4.6). This data will be retaken at a lower trap depth so as to obtain a pure BEC.

The data at 695 and 754 $G$ are of better quality. The largest contribution to the
Figure 4.4 Thomas-Fermi Fit at 620 G. Column density (left) and integrated column density (right) is fit to a Thomas-Fermi distribution for bosons. Here $N = 4.7 \times 10^3$, $R_z = 147 \, \mu m$ giving $a_m = 550 \, a_o$ (left), and $R_z = 158 \, \mu m$ giving $a_m = 800 \, a_o$. Several shots at 620 G showed some evidence of a distortion in the axial direction, as can be seen in this profile, which may explain the larger than expected measured axial size. The profiles are filtered for clarity.

Figure 4.5 Thomas-Fermi Fit at 650 G. Column density (left) and integrated column density (right) are fit to respective Thomas-Fermi distributions for bosons. $N = 1.6 \times 10^4$, $R_z = 180 \, \mu m$ giving $a_m = 426 \, a_o$ (left), and $R_z = 187 \, \mu m$ giving $a_m = 520 \, a_o$ (right). For this shot, the measured number and trapping frequencies give $R_{TF} = 557 \, \mu m$, and mean field theory within the T-F approximation (assuming $a_m/a = 0.6$) predicts a T-F radius of 185 $\mu m$. 
Figure 4.6  Bimodal Thomas-Fermi Fit at 650 G. Shown is the integrated column density for the same shot as in Fig. 4.5, but here it is fit to a Thomas-Fermi distribution for bosons plus a gaussian. The gaussian fit is indicated by the dashed line. The fit gives a measured condensate fraction of 46%, giving $N = 7.2 \times 10^3$, $R_z = 140 \mu m$, $R_z/R_{TF} = 0.29$, and $a_m = 267 a_o$. From the width of the thermal portion we estimate $T = 51 nK$, and from the total number and trapping frequencies we get a critical temperature $T_c = 83 nK$ using $T_c \approx 0.94\hbar\omega N^{1/3}$. This predicts a condensate fraction of 77% using $1 - (T/T_c)^3$ which is in disagreement with the measured fraction. The discrepancy can be explained by the reduction of $T_c$ from finite size effects and from interactions [25, 26].

Uncertainty at these fields came from the systematic uncertainty in probe power. A profile at 695 G is shown in Fig. 4.7. At 790 G, where $k_F a = 1.4$, we enter the strongly interacting BEC regime. At 790 G the dominant source of uncertainty is from the systematic uncertainty in the imaging probe detuning. This is because the data were obtained off resonance ($\Delta \approx 1\Gamma$) causing increased sensitivity to $\Delta$. A profile at this field is shown in Fig. 4.8. A discussion of $R_z/R_{TF}$ for 830 G is given in section 4.3. At 860 G, $k_F a = -2.28$ putting the gas in the strongly interacting BCS regime. At this field the uncertainty in $R_z/R_{TF}$ is dominated equally by the systematic uncertainties in $\omega_z$ and $R_z$. The axial size measurement at 920 G, where $k_F a = -1.1$, is just on the border between the weakly interacting and strongly interacting regimes. We obtain $(R_z/R_{TF}) = 0.916 \pm 0.02$ from a total of 13 shots. The largest contributor
Figure 4.7  Thomas-Fermi Fit at 695 G. Shown is a bosonic fit to the integrated column density. For this shot \( N = 1.9 \times 10^5 \) and \( R_s = 331 \, \mu m \) giving \( a_m = 869 \, a_0 \). The measured number and trapping frequencies give \( R_{TF} = 807 \, \mu m \).

Figure 4.8  Thomas-Fermi Fit at 790 G. Shown is a bosonic fit to the integrated column density at 790 G. For this shot \( N = 3 \times 10^5 \) and \( R_s = 605 \, \mu m \). From the measured number and trapping frequencies \( R_{TF} = 896 \, \mu m \).
Figure 4.9  **Thomas-Fermi Fit at 920 G**. Integrated column density is fit to Thomas-Fermi distribution for fermions. For this shot $N = 8.9 \times 10^4$, $R_z = 641 \mu m$, and $R_{TF} = 0.92$.

to the uncertainty at 920 G is from the systematic uncertainty in $\omega_z$. An example profile at 920 G is shown in Fig. 4.9. The measurement of $R_z/R_{TF}$ from 830 – 920 G is a clear demonstration of how interactions reduce the chemical potential of an interacting Fermi gas near resonance causing a squeezing of the cloud.

### 4.2 Molecular Scattering Length Versus Detuning from Feshbach Resonance

We wish to measure the molecular scattering length in both the strongly coupled and weakly coupled BEC regimes by measuring the spatial size. The equation of state obtained from theories that describe the BEC-BCS crossover using a BCS type ground state [27] conclude that the ratio of the molecular to fermionic scattering length in the BEC regime is $a_m/a = 2$. It is now generally agreed among most theorists that this is not the case. The problem of dimer-dimer scattering has been solved exactly by [12] from the solution to a four-body fermion problem. They obtain $a_m/a = 0.6$ with 2% accuracy by fitting their numerically obtained wave function to the form of
the wavefunction in the limit where the distance between molecules is large compared
to the size of the pairs. This same value is obtained by [2] as a fitting parameter to
data generated by a quantum Monte Carlo simulation. Stajic et. al. calculate the
molecular scattering length from a many-body theory based on a generalized BCS
ground state in which they include the two-channel nature of the Feshbach resonance
[11]. They admit that in their theory the boson-boson interactions are only treated in
a mean-field sense, but the theory has the advantage that it includes the many-body
physics. Their theory predicts that $a_m/a$ decreases as one moves away from resonance
towards smaller $k_F|a|$ on the BEC side, but still predicts that $a_m$ is in general less
than $2a$. The method they use for obtaining $a_m$ is to fit the profiles generated by
their theory to that of a Thomas-Fermi distribution for bosons (see Eq. 2.3) using
the GP equation (Eq. 2.2). This is what we will do for our measured profiles.

The molecular scattering length is obtained by fitting bosonic Thomas-Fermi dis-
tributions to the data in all cases except 790 $G$ where a fermionic Thomas-Fermi
distribution was a better fit (see Fig. 4.8). Fits to the data used the integrated
column density (except at 620 and 650 $G$ where both are shown) to remove the possi-
bility of systemically underestimating the Thomas-Fermi radius. This would occur
if the fit did not go through the center of the cloud or if there were irregularities in
the profile. Furthermore, we do not trust the peak column densities obtained on the
camera because of radial heating, radial distortions, and imaging resolution. Using
Eq. 2.7, plots of the molecular scattering length versus $B$ and versus $1/k_Fa$ are shown
in Fig.s 4.10 and 4.11 respectively. The plotted values of $a_m$ are normalized to the s-
wave atomic scattering length at each field which is obtained from a coupled-channels
calculation [18]. Low signal to noise at 620 and 650 $G$ are the cause for ambiguous
results because the fits to the integrated column density systematically fit the the
value of $R_z$ to be larger than those produced from fits to the non-integrated column
density. This is because summing the array in the radial direction to obtain the in-
Figure 4.10  Plot of $a_m/a$ versus $B$. Error bars include statistical and all systematic components in quadrature (Eq. 4.2). See table 4.1 for a summary.

Figure 4.11  Plot of $a_m/a$ versus $1/k_Fa$. Error bars include statistical and all systematic components in quadrature (Eq. 4.2). See table 4.1 for a summary.
tegrated column density summed in residual background noise into the signal. Low signals at these fields are a result of the reduced lifetime of the molecules further away from resonance. Signals at 620 G are especially poor causing increased scatter in $R_z$. Furthermore, at 620 G there was some evidence of an axial distortion as discussed in section 4.1. It is not completely clear why the measured scattering length at 620 G is higher than expected. However one possible cause that would at least partially explain the effect is axial heating from the release of the molecules’ binding energy as well as probe induced heating causing an increased measurement of $R_z$. At 620 G the binding energy $E_b = -\hbar^2/ma^2$ is 109 $\mu K$ where $a = 515 a_o$ is the atomic scattering length and $m$ is the fermion mass. As an estimate of the velocity $V_b$ the atoms would gain upon being dissociated, one can set $E_b$ equal to $(mV_b)^2/2m$ and solve for $V_b$ to obtain $V_b = (2E_b/m)^{1/2} = 55 \text{ cm/s}$. Over the 5 $\mu s$ probe duration used at 620G, this corresponds to a 2.75 $\mu m$ spreading. Furthermore, once dissociated the atoms would experience another amount of spreading $x(t = 5\mu s)$ from probe induced heating which is obtained from the equation $x(t) \simeq \frac{\gamma}{3}(\Gamma \rho_{ee} t^3)^{1/2}/\nu_R$ obtained from Ref. [22] where $\Gamma \rho_{ee}$ is the scattering rate, $t$ is the probe duration, and $\nu_R = \hbar k/m = 9.9 \text{ cm/s}$ is the recoil velocity of a lithium 6 atom. For probing on resonance for 5$\mu s$ with an intensity of $\sim 3mW/cm^2$, this gives an additional 2 $\mu m$ spreading or a total spreading of 4.75 $\mu m$. For the typical number, $N \sim 4000 - 6000$, and measured trapping frequencies at this field, we would expect an $R_z$ of around $\sim 130\mu m$ (assuming $a_m = 0.6a$), and so a 4.75 $\mu m$ spreading would imply an overestimate of $a_m$ of about $\sim 20\%$. This, however, is not enough to explain the discrepancy. If there is an axial distortion as described above, this could also explain why $a_m$ was measured too high. At 650 G most of the shots showed evidence of a thermal fraction. A bimodal distribution is fit to the integrated column density on 3 of the 6 total shots at this field (see Fig. 4.6 for an example). These fits are added to the plots of $a_m$ vs. $B$ and $a_m$ vs. $1/k_Fa$. The results at 695 and 754 G are of better quality. The dominant source of error for
these fields is from the systematic error in the probe power. Finally we measure the
scattering length at 790 $G$. At 790 $G$ the dominant source of uncertainty is system-
atic error in the detuning because the shots were not taken on resonance. Scatter in
the data is consistent with the statistical uncertainty in $N$ and $R_z$. At this field, the
molecular scattering length fits to a value higher than that of the atomic, however in
the strongly interacting region the theory of [12] does not apply. Uncertainty in $a_m$
is determined by performing error analysis on equation 2.7, and is given by

$$\delta a_m = a_m \sqrt{16 \left( \frac{\delta \omega_z}{\omega_z} \right)^2 + \left( \frac{\delta N}{N} \right)^2 + 4 \left( \frac{\delta \omega_r}{\omega_r} \right)^2 + 25 \left( \frac{\delta R_z}{R_z} \right)^2}.$$  \hspace{1cm} (4.2)

The values of $a_m/a$ at 695 and 754 $G$ quantitatively agree well with that predicted
from reference [12], however their theory is valid for $k_F a << 1$.

### 4.3 Measurement of $\beta$

We have measured the value of $\beta$ at 830 $G$ by comparing the measured size to
that of a free Fermi gas. The clouds were imaged with $\Delta = 0$ and a probe intensity
of $I_0 \simeq 0.4 ~ I_s$. We fit the clouds to the form of the integrated column density for
fermions, as given in equation 3.33, to extract $R_z$. The value of $R_{TF}$ is obtained from
Eq. 2.10. The average of about 60 measurements of $R_z/R_{TF}$ is used to calculate
$\beta$ from equation 2.11. We obtain the value of $R_z/R_{TF}$ to be $0.825 \pm 0.02$ giving
$\beta = -0.54 \pm 0.05$. The uncertainty in $\beta$ comes from

$$\frac{\Delta \beta}{\beta} = \frac{4 \left( \frac{R_z}{R_{TF}} \right)^4}{\left( \frac{R_z}{R_{TF}} \right)^4 - 1} \left| \frac{\delta (R_z/R_{TF})}{(R_z/R_{TF})} \right|$$  \hspace{1cm} (4.3)

and the systematic uncertainty in $(R_z/R_{TF})$ comes from Eq. 4.1 where only the sys-
tematic component of each quantity is used. The systematic uncertainty in $(R_z/R_{TF})$
of 1.82% is added in quadrature with the statistical uncertainty of 0.4% which is de-
erved from the standard error in the mean of the 60 shots. The largest contributor
Figure 4.12 Scatter Plot of $R_z/R_{TF}$ at 830 G. Data is taken in two separate runs. The standard error of all the shots is 0.4%. The $x$ axis has no meaning and only serves to separate the data for the sake of clarity.

to the uncertainty in $\beta$ comes from uncertainty in $\omega_z$. The data are plotted in Fig. 4.12 This value of $\beta$ is in good agreement with $\beta = 0.58 \pm 0.01$ obtained from Monte Carlo simulations [2,17], and from the theory given in [3] which gives $\beta = -0.545$. We are also in agreement with previously measured values of $\beta$, but have made the measurement with much greater precision [24,28–30].

4.4 Concluding Remarks

Our plot of the axial size normalized to that of a free Fermi gas demonstrates a smooth transition of the physics from that of a BEC of tightly bound dimers to a BCS superfluid. Increasing interactions on the BEC side increases the size of the cloud because in this regime the interactions dominate the physics. On the BEC side the kinetic energy plays a larger role and increasing interactions reduce the chemical potential thus decreasing the size. From the size of the cloud at 830 G, we have been able to extract the value of $\beta$ to be in excellent agreement with theory and previous measurements, but we have made the measurement with better precision. From the
<table>
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<th>( B )</th>
<th>( \frac{\delta \omega_z}{\omega_z} ) sys/stat</th>
<th>( \frac{\delta \omega_T}{\omega_T} ) sys/stat</th>
<th>( \frac{\delta R_z}{R_z} ) sys/stat</th>
<th>( \frac{\delta N}{N} \Delta ) sys/stat</th>
<th>( \frac{\delta N}{N} ) ( E_f )</th>
<th>( \frac{\delta N}{N} ) ( t )</th>
<th>( N_{\text{shots}} )</th>
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<tr>
<td>650</td>
<td>2% 0.4%</td>
<td>1.4% 2.5%</td>
<td>1% 3 - 6%</td>
<td>10.9% 39%</td>
<td>2.2%</td>
<td>1.8%</td>
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<tr>
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<td>1% 0.3%</td>
<td>1.4% 1.8%</td>
<td>1% ( \lesssim ) 3 - 4%</td>
<td>4.1% 11%</td>
<td>8.4%</td>
<td>1.5%</td>
<td>6</td>
</tr>
<tr>
<td>754</td>
<td>1% 0.3%</td>
<td>1.4% 1.8%</td>
<td>1% ( \lesssim ) 3 - 4%</td>
<td>5.7% 15%</td>
<td>7.5%</td>
<td>1.5%</td>
<td></td>
</tr>
<tr>
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<td>1% ( \lesssim ) 3%</td>
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<td>1.6%</td>
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<td>1.9% 17%</td>
<td>3.6%</td>
<td>1.7%</td>
<td>13</td>
</tr>
</tbody>
</table>

**Table 4.1 Summary of Estimated Uncertainties.** This table summarizes the estimated measurement uncertainty in the quantities that go into calculating \( R_z/R_{TF} \) and \( a_m \) at each field. Where relevant, each quantity is separated into a systematic and a statistical component. The quoted estimates of the statistical error in \( R_z \) are approximate and can vary from shot to shot, the typical values are given. Not shown in the table is a statistical uncertainty in number from background noise and bad subtractions in the images which is \( \lesssim 1\% \) for all points except 620 and 650 \( G \). At these fields low signals caused the background noise to give an error ranging from roughly 5 - 9\% for shots at 620 \( G \), and an error ranging from roughly 3 - 30\% for shots at 650 \( G \). At 650 \( G \), only 3 of the 6 shots were able to be fit to bimodal distributions. Data taken at 695 (and 754) \( G \) is an average of 2(3) groups of data taken at different probe detunings, each group containing 3 data points. We noticed no systematic effect of varying the detuning. For these fields, the quoted systematic errors in \( N \) from \( \Delta \) and \( E_f \) are weighted by the number of points at each detuning. Error bars on \( R_z/R_{TF} \) and \( a_m \) in Fig's 4.1, 4.2, 4.10, and 4.11, are a quadrature sum of the standard error of the mean and the estimated systematic errors given in this table.

measured axial size, we have been able to extract \( a_m \) and have demonstrated that \( a_m/a \) is smaller than the result given by BCS mean field theory. Also, some of the results suggest agreement with \( a_m/a = 0.6 \), however more data in the BEC regime is needed to make a conclusive claim.
Chapter 5
Appendix

In this appendix we will derive the column density in equation 3.3 from chapter 3 using Beer’s law. The analysis follows that of W. Demtröder [31]. The general form of Beer’s law states (equation 3.21 in [31])

\[
\frac{dI}{dy} = -\alpha_s I, \tag{5.1}
\]

where for a population of two-level atoms (states labelled 1 and 2), \(\alpha_s = \sigma(n_1 - n_2)\) is the saturated absorption coefficient, and I is the intensity of the probe beam as a function of position \(y\) through the sample. Demtröder rigorously derives in equation 3.88 of section 3.6.2 of [31] that the saturated absorption coefficient \(\alpha_s(\omega)\) is

\[
\alpha_s(\omega) = \frac{\alpha_o(\omega)}{1 + S_\omega}, \tag{5.2}
\]

where \(\alpha_o\) is the absorption coefficient in the limit of low intensity such that all the population is in the ground state, and \(S_\omega\) is the spectral saturation parameter given by equation 3.85 of [31]

\[
S_\omega = S_o \frac{\Gamma^2}{4(\omega - \omega_o)^2 + \Gamma^2} \tag{5.3}
\]

where \(S_o\) is the on resonance saturation parameter. So now we have for the absorption coefficient

\[
\alpha_s(\omega) = \frac{(4\Delta^2 + \Gamma^2) \alpha_o(\omega)}{4\Delta^2 + \Gamma^2(1 + S_o)}, \tag{5.4}
\]

where \(\Delta = (\omega - \omega_o)\). In section 3.6.1 of [31], where Demtröder first defines the saturation parameter, he obtains \(S_o = 2P/(R_1 + R_2)\) where \(P\) is the pumping rate, and \(R_1\) and \(R_2\) are the relaxation probabilities for states 1 and 2 respectively. In our case, spontaneous emission from state 2 is the only relaxation mechanism giving \(R_1 = 0\) and \(R_2 = A_{21} = \Gamma\) where \(A_{21}\) is the Einstein A coefficient. The pumping rate \(P\) for a wave with intensity \(I\) is

\[
P = \sigma I/\hbar \omega. \tag{5.5}
\]
We can insert Eq. 5.5 and $\Gamma = \sigma I_s / \hbar \omega$ into $S_o$ (which gives $S_o = 2I / I_s$) for use in Eq. 5.3, and insert the result into Eq. 5.4. The last piece we need is the absorption coefficient for no saturation $\alpha_o$

$$\alpha_o = n_o \sigma_o \frac{\Gamma^2}{\Gamma^2 + 4\Delta^2}$$  \hspace{1cm} (5.6)

where the peak cross-section $\sigma_o = 3\lambda^2 / 4\pi$ as calculated in section 3.3.2. We can now obtain the useful form of Beer’s law from which we can obtain the column density in terms of measurable quantities,

$$dI/dy = -\sigma_o n_o \frac{I}{1 + 4\Delta^2 + 2(I/I_s)}$$  \hspace{1cm} (5.7)

After moving the $I$ and $y$ dependence to opposite sides and expressing $\Delta$ in units of $\Gamma$, Eq. 5.7 can be integrated

$$-\frac{1}{\sigma_o} \int_{I_o}^I \frac{dI(1 + 4\Delta^2 + 2I/I_s)}{I} = \int n_o dy \equiv \bar{n}.$$  \hspace{1cm} (5.8)

Performing the integral we obtain Eq. 3.3
References


