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Tests of the OPEN-GGCM and BATS-R-US Global MHD Codes

by

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Abstract

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In this thesis, we present the results from tests of two global magnetohydrodynamic (MHD) codes: the University of New Hampshire’s OPEN-GGCM model and the University of Michigan’s BATS-R-US model. We investigate their consistency with the theory of adiabatic particle drift in the inner and middle magnetosphere. Since the Rice Convection Model (RCM) uses the adiabatic particle drift as one of its basic assumptions, positive results of the tests will help us to establish a better physical model of the magnetosphere and ionosphere by coupling the RCM with one of these global MHD models. An introduction to the two models, the theory for the adiabatic particle drift, and the results from tests of OPEN-GGCM and BATS-R-US models are presented. By tracking individual magnetic tubes and comparing the quantities such as particle number N and theoretical invariant $P V^{5/3}$ integrated along these flux tubes in different times, we find results of both the OPEN-GGCM and BATS-R-US models suggest that the conservation of the adiabatic invariant is very poor when using low-resolution simulation grid. Another test of the OPEN-GGCM simulation with higher grid resolution shows some improvement in $P V^{5/3}$ conservation.
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Chapter 1

Introduction

1.1 The Magnetosphere

![Diagram of the Earth's magnetosphere]

**Figure 1.1.1** Structure of the Earth’s magnetosphere, plasma populations and some of the currents. Adapted from the picture in a web lecture of the magnetosphere [Dr. James Green, NASA Goddard Space Flight Center, http://ssdoo.gsfc.nasa.gov/education/lectures/magnetosphere/Figure_4.jpg, access date: June 28, 2004]

The magnetosphere is the region of space where the motion of charged particles is controlled by the Earth’s magnetic field (Figure 1.1.1). The magnetic field also acts as an
obstacle to the supersonic solar wind, which is a continuous flow of charged particles that originated at the Sun and carries the frozen-in solar magnetic field out into the heliosphere. This interplanetary medium distorts the Earth’s magnetic field, which is approximately a dipole, to a comet-like configuration with the tail extending downstream from the solar wind. We call the part of the Sun's magnetic field that is carried into the interplanetary space by the solar wind the Interplanetary Magnetic Field (IMF).

There are several plasma populations and intricate large-scale current systems inside the magnetosphere. These plasma populations are distinguished from each other by their energy, species and location. The following is an introduction to the three plasma populations in the inner magnetosphere: plasmasphere, radiation belts and plasma sheet.

The plasmasphere is composed of relatively dense (tens to thousands of particles per cubic centimeter), cold (~1 electron volt, or eV) plasma, which has moved along the closed magnetic field lines from the ionosphere and has been being trapped on those field lines. The ionosphere is the region where the Earth's upper atmosphere is partly (0.1% or less) ionized because of the Sun's UV radiation. This region is coupled to both the magnetosphere and the neutral atmosphere. The plasmasphere contains most of the mass of the magnetosphere. But due to its low energy density, it doesn't significantly affect the magnetic field of magnetosphere.

The radiation belts overlap with the plasmasphere (with in 8 Earth radii, or R_E). The differences between them are the density and the energy of the collection of particles. The plasma population in the radiation belts is several orders of magnitude less dense than the plasmasphere. But the kinetic energy per particle in radiation belts is much higher, typically hundreds of keV or more, and up to GeV for protons.
The plasma sheet is the region of closed field lines in the central part of the magnetotail, which is near the magnetic equatorial plane. It contains slowly flowing, high-energy (keV) charged particles. Plasma density in this region is about 0.1 - 1 cm$^{-3}$, varying slowly by time. It correlates with the solar wind density (Wolf, [1995]). These particles can be described by an elegant and well-developed basic theory of adiabatic particle motion (Northrop, [1963]).

There are also other plasma populations in the magnetosphere. For example, the plasma mantle lies on the open field lines near the high-latitude boundary of the magnetotail. It contains low-energy ions that flow antisunward.

The plasma and electromagnetic fields in the magnetosphere build up a current system composed by several kinds of large-scale currents, these include the ring current, field-aligned current (or Birkeland current), tail current, magnetopause current (or Chapman-Ferraro current).

The ring current is a major current system. It is caused by the longitudinal drift of energetic charged particles trapped by the magnetic field between L=2−7. It circles the Earth in the equatorial plane. The ring current produces a perturbation magnetic field in opposition to the Earth's magnetic field at the equator. The energy of the ions and electrons composing the ring current is in the 10−200 keV range. In addition, the ring current carries a large fraction of the particle energy of the magnetosphere and enough current to have a major effect on the magnetic field configuration, especially during a magnetic storm. A magnetic storm is defined as a major increase in the total strength of the ring current, which is indicated by a decrease in the average northward magnetic field observed at low latitudes on the Earth’s surface.
Under certain situations, there is also a more complicated current loop in the inner magnetosphere known as the partial ring current, which connects to the field-aligned current to flow along field lines to and from the ionosphere.

The magnetopause current is caused by the sharp change of magnetic field in the magnetopause. Similarly, the highly stretched magnetic field of the Earth’s magnetotail causes a sheet current near the equatorial plane known as the tail current.

In general, part of the current density perpendicular to the magnetic field is not divergence-free. To avoid a buildup of charge, the current flows along the magnetic field, to and from the ionosphere, where ohmic currents perpendicular to the magnetic field close the circuit.

The understanding and predicting of the physical processes that control this region has significant practical benefits. Since the magnetosphere protects the Earth from potentially harmful high-energy particles and cosmic rays, a disturbed magnetosphere may interfere with human activities. For example, in March 1989, the ionospheric disturbances resulting from a major magnetic storm disabled the power grids of one-third of Canada and part of upstate New York. The space-based assets located in the inner magnetosphere and the ionosphere are also the victims of such disturbances. For instance, on January 11, 1997, the Telstar 401 communication satellite was damaged by the radiation produced by a major magnetic storm. In addition, energetic particles that result from dynamic processes in a disturbed magnetosphere can also threaten the astronauts in this region.

After years of effort, many of the physical processes in the magnetosphere are well understood, and this understanding has become the basis of magnetospheric simulation
codes. It is hoped that these simulation codes can recreate the structure of the magnetic field and other physical characteristics of the Earth’s magnetosphere, simulate the physical processes in this region, and eventually be used for space weather forecasting.

1.2 Global MHD Models

Global MHD simulations have been under development for over twenty years. For the ideal MHD formalism, the properties of the plasma are simplified, such as: the plasma is one-fluid; the velocity distribution is isotropic; particles are ‘frozen’ to the magnetic field since the electrical conductivity is infinite; heat conductivity is neglected. This simple formalism also permits a tractable solution on current computational resources. Despite this simplification, Global MHD models have proven to be very useful for understanding many aspects of the large-scale magnetosphere. With the current computational power, the MHD formalism is a more practical approach than a full-scale kinetic approach. However, the first crude large-scale kinetic models are starting to appear (Karimabadi et al., [2002]; Reddy et al., [2002]).

LeBoeuf et al. [1978], Lyon et al. [1981] presented the first global MHD models of the magnetosphere. These were two-dimensional models and were far from adequate to realistically model the magnetosphere. In the early 80’s, several three-dimensional MHD codes were developed (e.g. Brecht et al., [1982]; Ogino, [1986]). In the late 80’s and early 90’s, more refined models appeared. These models included more sophisticated numerical methods and an electrodynamic ionosphere model, which included the connections between the magnetosphere and ionosphere (e.g. Fedder et al., [1987]; Janhunen et al., [1995]; Raeder et al., [1995]). These models indicated that the
ionosphere could play a significant role in controlling magnetospheric convection. They reasonably reproduced gross features such as the size and shape of the magnetosphere, but it was unclear if other more detailed features, such as the strength of the magnetic field, were correct. The next advance was the comparisons of the models’ output with observations (e.g. Fedder et al., [1995]; Frank et al., [1995]); these model-data comparisons allowed the modelors to improve their codes. In recent years, global MHD models became more efficient and sophisticated. While at the same time, these models have become integrated to many experimental studies. The current major global MHD models are the University of New Hampshire’s Open Global Geospace Circulation Model (OPEN-GGCM) (Raeder, [2003]), University of Michigan’s Block Adaptive Tree Solar-wind Roe Upwind Scheme (BATS-R-US) (Powell et al., [1999]; Gombosi et al., [2003]), and Lyon-Fedder-Mobarry global magnetosphere model (LFM) (Lyon et al., [1998]).

The work described in this thesis uses the OPEN-GGCM model and the BATS-R-US model.

1.3 Adiabatic Invariants

Consider a plasma population on a magnetic flux tube with flux $\Phi_0$ under the ideal MHD approximation. If we combine the perfect conductivity Ohm’s law

$$\vec{E} + \vec{v} \times \vec{B} = 0$$  \hspace{1cm} (1.3.1)

with the Faraday’s law,

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$  \hspace{1cm} (1.3.2)

we then get
\[
\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B})
\]  \hfill (1.3.3)

After integrating the Equation 1.3.3 by using Gauss' divergence theorem and Stokes' theorem, it can be shown that the magnetic flux of the flux tube that contains the plasma is constant during the movement of the plasma:

\[
\frac{d\Phi}{dt} = 0
\]  \hfill (1.3.4)

where \( \frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \)

So we can infer that the magnetic flux moves with the fluid, or that the magnetic field line is frozen into the plasma. This is an important characteristic of ideal MHD plasma and says that the number of charged particles in the flux tube is constant.

We define \( V_c \) as the confining volume of the population of the plasma, or the volume into which the plasma is confined.

The definition of magnetic volume is

\[
V_m = \int \frac{ds(\vec{x})}{B(\vec{x})}
\]  \hfill (1.3.5)

the integration path is along the magnetic line, from some initial position \( i \) to the final position \( f \).

For the plasma in the inner or middle magnetosphere, the description of motion of a whole flux tube is able to represent that of the motion of the plasma under the assumption of an isotropic distribution (Toffoletto et al., [2003]), which is also applicable for ideal MHD plasma. If we treat the plasma as an ideal monatomic gas, gas law of an adiabatic process implies that along a flux tube trajectory,

\[
P_s V_c^{5/3} = \text{Constant}
\]  \hfill (1.3.6)
where \( P_s \) is the plasma pressure.

From Equation 1.3.4 we know that, for any time, the magnetic flux of the flux tube containing the plasma is \( \Phi_0 \). So by Equation 1.3.5 we get

\[
V_m = \int \frac{ds(\vec{x})}{B(\vec{x})} = \int \frac{ds(\vec{x})}{\Phi_0 / S(\vec{x})} = \int \frac{S(\vec{x}) \cdot ds(\vec{x})}{\Phi_0} = V_c / \Phi_0 \tag{1.3.7}
\]

where the \( S(\vec{x}) \) is the intersection area of the flux tube at the point of \( \vec{x} \).

Since \( \Phi_0 \) is a constant for a certain flux tube we discussed here, Equation 1.3.7 can be written as

\[
V_m = V_c / \text{Constant} \tag{1.3.8}
\]

According to the Equations 1.3.6 and 1.3.8, the adiabatic plasma under the ideal MHD condition should satisfy the following conservation:

\[
P_s V_m^{5/3} = \text{Constant} \tag{1.3.10}
\]

Theoretically, the MHD models should then ensure the conservation of \( P_s V_m^{5/3} \).

### 1.4 The Rice Convection Model

The Rice Convection Model (RCM) is a sophisticated convection model of the inner and middle magnetosphere (Harel et al., [1981]). Unlike global MHD models, the RCM is a multi-fluid model that represents adiabatically drifting isotropic particle distributions with various values of energy invariant. This model also includes an ionosphere with a realistic conductance. The RCM model is specifically designed to treat the physics of the inner magnetosphere where energy-dependent gradient and curvature drifts become important. While the RCM can compute the electric field self-consistently, it cannot specify the magnetic field and the particle source/sink boundary conditions through
which the RCM can connect the outer magnetosphere to the ionosphere. It typically uses a pre-computed, time-dependent magnetic field and boundary conditions as input.

The Global MHD models can supply the input for the RCM model. The coupling of RCM and the Global MHD models can combine the advantages of both codes and describe the magnetosphere more accurately. The coupled model should be able to achieve more than either model has been able to do separately.

When the RCM model only uses a set of pre-computed time-dependent magnetic field and boundary conditions calculated by Global MHD model as input, but doesn’t pass any information back to the global MHD simulation, the coupling is called one-way coupling. If the large-scale simulations of the global MHD codes provides the magnetic field and the particle source/sink boundary conditions to the RCM model, and the RCM model computes and passes back plasma parameters such as pressure and density to the global MHD simulations for calculating the next time step’s inputs for the RCM model, this coupling is called two-way coupling (Toffoletto et al., [2004]). Two-way coupling establishes an interactive computational loop and requires concurrent execution of both the RCM model and Global MHD model (De Zeeuw et al., [2004]).

The conservation of $PV_m^{5/3}$ is one of the key tests to determine if the global MHD codes can be coupled to the RCM. In the RCM, the flux tube content $\eta_k$, which is the number of particles per unit magnetic flux, is related to thermodynamic pressure $P$ via the relation (Toffoletto et al., [2003])

$$PV^{5/3} = \frac{2}{3} \sum_k \eta_k |\lambda_k|$$  \hspace{1cm} (1.4.1)

where $V$ is the flux tube volume, and $\lambda_k$ is an energy invariant (Please see Section 2.1 for details). Flux tube content $\eta_k$ should be conserved along a drift path when there are no
sources or sinks to the flux tube. So $PV^{5/3}$ should also be conserved along the drift path when there’s no source or sink for all the species.

It would be ideal if the Global MHD models can ensure the conservation of $PV^{5/3}$, otherwise there will be some problems when we perform one-way coupling between the RCM model and the global MHD models. For one-way coupling, if the new magnetic field distribution computed by the global MHD models cannot ensure the conservation of the quantity $\eta_n$, it will lead to the appearance of fake sources or sinks to the flux tube (Toffoletto et al., [2003]) and make the simulation inaccurate.

For two-way coupling, where the RCM model returns the plasma parameters back to the global MHD models, the influence brought by the deviation from $PV^{5/3}$ conservation may be decreased because the plasma parameters that feed back to the global MHD model will correct the $PV^{5/3}$ distribution in global MHD model [Richard Wolf, Rice University, private communication].

Detailed discussion about the necessity of conservation of $PV^{5/3}$ will be presented in chapter 2. In chapter 3, I’ll show the method and results of the test. The conclusions and discussion will be presented in Chapter 4.
Chapter 2

$PV^{5/3}$, Conserved or Not

2.1 Adiabatic Motions, $PV^{5/3}$ Conservation, and the RCM

The theory for the adiabatic particle motion in the inner and middle magnetosphere has been well developed (Northrop, [1963]; Wolf, [1983]). It is applicable for the region of the magnetosphere within about 10 Earth Radii ($R_E$). The plasmasphere, radiation belts and ring current coexist in this region of the magnetosphere. The adiabatic theory can also be applied to the regions out to about 20 to 30 $R_E$. Most of the important particle populations of the magnetosphere, including those that carry most of the pressure, energy, and current, can be described by the theory. This adiabatic drift theory forms the theoretical basis of the RCM.

A charged particle in a magnetic field $\vec{B}$ and electric field $\vec{E}$ has three motions: cyclotron motion around the magnetic field line; bounce motion along the field line; and drift perpendicular to the magnetic field. For the second and third motion, the drift formula is:

$$\vec{V}_{\text{drift}} = \frac{\vec{E} \times \vec{B}}{B^2} + \frac{\vec{F}_{\text{ext}} \times \vec{B}}{qB^2} + \frac{\gamma_r W_{\perp} \vec{B} \times \nabla B}{qB^3} + \frac{2\gamma_r W_{\parallel} \vec{B} \times \vec{K}}{qB^2}$$  \hspace{1cm} (2.1.1)

where $\vec{F}_{\text{ext}}$ is an external force, $\gamma_r$ is Lorentz's coefficient. $W_{\perp}$ is the kinetic energy that is associated with the component of velocity that is perpendicular to the field line, $W_{\parallel}$ is the kinetic energy that is associated with the component of velocity that is parallel to the field line. $\vec{K}$ is the curvature vector, defined by
\[ \vec{k} = (\hat{b} \cdot \nabla) \hat{b} = -\frac{\hat{R}_c}{R_c} \]  

(2.1.2)

where \( \hat{b} \) is the unit vector of the magnetic field line, \( R_c \) is the radius of curvature of the magnetic field line, and \( \hat{R}_c \) is a unit vector outward from the center of curvature. The four terms in the Equation 2.1.1 are called \( \vec{E} \times \vec{B} \)-drift, external-force drift, gradient drift, and curvature drift in turn.

It’s not convenient to directly use Equation 2.1.1 to track a drifting particle in the magnetosphere. The drift velocities are different with different positions along the particle’s bounce path. So it’s more practical to use the bounce-averaged drift velocity. Based on a simple conservation-of-energy argument, the velocity of bounced-averaged gradient/curvature drift is (Wolf, [2000]):

\[ \vec{V}_{GC} = \frac{\vec{B} \times \nabla K(\mu, J, \bar{x})}{qB^2} \]  

(2.1.3)

where \( K \) is the particle kinetic energy, and

\[ \mu = \frac{mv_z^2}{2B} \]  

(2.1.4)

is the magnetic moment of the particle’s cyclotron motion, also known as the first adiabatic invariant.

Equation 2.1.3 is derived time-independently. But for the case of slow time variation of the magnetic and electric fields, so that the first and second adiabatic invariants are conserved, Equation 2.1.3 is valid for a time-dependent magnetic field configuration.

However, without further assumptions, it is difficult to express \( K \) as an analytic function. So we introduce the assumption that the distribution of particles in the plasma sheet is isotropic. Under this assumption, the motion of a whole flux tube can represent
the motion of the plasma within it (Wolf, [1983]). After a long derivation beginning with Equation 2.1.3, the equation of gradient/curvature drift is written as:

$$\bar{V}_{gc} = \frac{\bar{B} \times (m v_k^2 \bar{k} + \frac{mv_k^2}{2} \nabla B)}{q B^2}$$  \hspace{1cm} (2.1.5)$$

we find (Wolf, [2000])

$$\frac{dW_k}{W_k} = -\frac{2}{3} \frac{dV}{V}$$  \hspace{1cm} (2.1.6)$$

where $W_k$ is the kinetic energy of plasma, and

$$V = V_m = \int^{dS} \frac{ds}{B}$$

$$\hspace{1cm} (2.1.7)$$

is defined as the flux tube volume. After integrating the Equation 2.1.6, we get

$$W_k V^{2/3} = \lambda$$  \hspace{1cm} (2.1.8)$$

where $\lambda$ is called the energy invariant. $W_k$ can be expressed as

$$W_k = W_k(\lambda, \bar{x}) = \lambda V(\bar{x})^{-2/3}$$  \hspace{1cm} (2.1.9)$$

In the flux tube filled with an isotropic particle distribution, Equation 2.1.3 holds for all the particles. And the drift velocity should have the same form (Wolf, [2000]). We write it as

$$\bar{V}_{gc} = \frac{\bar{B} \times \nabla W_k}{q B^2}$$  \hspace{1cm} (2.1.10)$$

Considering Equation 2.1.9, we get the drift law for an isotropic particle distribution (Wolf, [1983]) as

$$\bar{V}_{gc} = \frac{\bar{B} \times \nabla W_k(\lambda, \bar{x})}{q B^2}$$  \hspace{1cm} (2.1.11)$$
The expression for the kinetic energy \( W_k \) is in the form of an analytic function of \( \lambda \) and \( \bar{x} \) under the assumption of isotropic particle distribution.

Equation 2.1.9 describes a similar situation to the thermodynamics of a monatomic ideal gas, whose temperature is proportional to the \(-2/3\) power of the volume \( V_c \) that confines the gas; the pressure of the monatomic gas has the following relation to the confining volume \( V_c \) for an adiabatic process:

\[
P V_c^\gamma = \text{Constant} \tag{2.1.12}
\]

with \( \gamma = 5/3 \) as the adiabatic exponent.

Since thermodynamics can be applied for the particles in the flux tube, the confining volume \( V_c \) can be associated with the flux tube volume \( V \) for the case the ideal MHD (Wolf, [2000]). So we get

\[
P_s V^{5/3} = \text{Constant} \tag{2.1.13}
\]

The Rice Convection Model uses the assumption of the isotropic particle distribution. It defines energy invariant \( \lambda \) as

\[
|\lambda_k| = W(\lambda_k, \bar{x}, t)V^3 \tag{2.1.14}
\]

and defines flux tube content \( \eta_k(\bar{x}, t) \) as the number of particles per unit magnetic flux. Flux tube content \( \eta_k \) is related to thermodynamic pressure \( P \) by the equation (Toffolotto et al., [2003])

\[
P V^{5/3} = \frac{2}{3} \sum_k \eta_k |\lambda_k| \tag{2.1.15}
\]

Flux tube content \( \eta_k \) follows the conservation law (Wolf, [1983])

\[
\left( \frac{\partial}{\partial t} + \bar{v}_k (\lambda_k, \bar{x}, t) \cdot \nabla \right) \eta_k = S(\eta_k) - L(\eta_k) \tag{2.1.16}
\]
where \( S(\eta_k) \) and \( L(\eta_k) \) represent sources and sinks. When there are no sources or sinks, \( \eta_k \) is conserved. And \( P_k V^{5/3} \) is also conserved according to Equation 2.1.15.

### 2.2 Global MHD

The simplest representation of magnetized, collisionless plasma is to use the ideal MHD equations.

#### Continuity equation

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \tag{2.2.1}
\]

#### Momentum equation

\[
\rho \frac{d\vec{v}}{dt} = \vec{j} \times \vec{B} - \nabla P \tag{2.2.2}
\]

#### Energy equation

\[
\left( \frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) (P \rho^{-5/3}) = 0 \tag{2.2.3}
\]

In addition, Maxwell’s equations

\[
\nabla \cdot \vec{B} = 0 \tag{2.2.4}
\]

\[
\nabla \times \vec{B} = \mu_0 \vec{j} \tag{2.2.5}
\]

\[
\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{2.2.6}
\]

Displacement term \( \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} \) is neglected here in Equation 2.2.5, because the characteristic speed of plasma is much less than the speed of light.

Under the assumption of infinite electrical conductivity, the Ohm’s law is

\[
\vec{E} + \vec{v} \times \vec{B} = 0 \tag{2.2.7}
\]
The energy equation of ideal MHD (Equation 2.2.3) requires

\[ P \rho^{-5/3} = \text{Constant} \quad (2.2.8) \]

along a fluid trajectory.

Equation 2.2.7 indicates the magnetic field is frozen in the particles. Then the confining volume \( V_e \) equals to the flux tube volume \( V \),

\[ P(N/V)^{-5/3} = \text{Constant} \quad (2.2.9) \]

where \( N \) is the number of particles in the flux tube. \( N \) is also constant in this case, so

\[ PV^{5/3} = \text{Constant} \quad (2.2.10) \]

Both OPEN-GGCM and BATS-R-US use the conservation equations that are in different forms than the ideal ones. For numerical reasons, the OPEN-GGCM code uses the gas-dynamic conservation form in which the gasdynamic terms are in divergence form, and the electromagnetic terms in the momentum and energy equations are treated as source terms ([Raeder et al., 2003]).

Continuity equation

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0 \quad (2.2.11) \]

Momentum equation

\[ \frac{\partial (\rho \mathbf{v})}{\partial t} = -\nabla \cdot (\rho \mathbf{v} \mathbf{v} + P \mathbf{I}) + \mathbf{j} \times \mathbf{B} \quad (2.2.12) \]

Energy equation

\[ \frac{\partial e}{\partial t} = \nabla \cdot ((e + P) \mathbf{v}) + \mathbf{j} \cdot \mathbf{E} \quad (2.2.13) \]

where

\[ e = \frac{1}{2} \rho v^2 + \frac{P}{\gamma - 1} \quad (2.2.14) \]
is an approximation to the plasma energy, and $\gamma$ is the ratio of specific heats.

And equation

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

(2.2.15)

The formulation that BATS-R-US used is called symmetrizable formulation, which is also semi-conservative formalism (Powell et al., [1999], Gombosi et al., [2003]).

$$\frac{\partial}{\partial t} \begin{pmatrix} \rho \\ \rho \vec{v} \\ \vec{B} \\ E_{mhd} \end{pmatrix} + \nabla \cdot \begin{pmatrix} \rho \vec{v} \\ \rho \vec{v} + (P + \frac{1}{2 \mu_0} B^2) \vec{I} - \frac{1}{\mu_0} \vec{B} \vec{B} \\ \vec{v} \vec{B} - \vec{B} \vec{v} \\ \vec{v}(E_{mhd} + P + \frac{1}{2 \mu_0} B^2) - \frac{1}{\mu_0} (\vec{v} \cdot \vec{B}) \vec{B} \end{pmatrix} = -\nabla \cdot \vec{B} \begin{pmatrix} 0 \\ \frac{1}{\mu_0} \vec{B} \\ \frac{1}{\mu_0} \vec{v} \\ \frac{1}{\mu_0} \vec{v} \cdot \vec{B} \end{pmatrix}$$

(2.2.16)

where the magnetohydrodynamic energy $E_{mhd} = e + \frac{1}{2 \mu_0} B^2$. The factor of constraint $\nabla \cdot \vec{B} = 0$ is added to each of the equations to render them a symmetrizable system (Powell et al., [1999]).

It can be shown that the $PV^{5/3}$ should still be conserved even for the gasdynamic conservation form of MHD equations. Unlike the ideal MHD equations, some global MHD codes, such as OPEN-GGCM, may introduce an explicit numerical resistivity to enable magnetic reconnection (Raeder, [1999]). In contrast, BATS-R-US does not use an explicit numerical resistivity. For the purpose of further discussion about numerical resistivity, we add the resistivity term to the Equation 2.2.7, it becomes

$$\vec{E} = -\vec{v} \times \vec{B} + \vec{\eta} \vec{j}$$

(2.2.17)

where $\eta$ is the resistivity.
Substituting Equation 2.2.17 into the energy conservation Equation 2.2.13 yields
\[
\frac{\partial e}{\partial t} + \nabla \cdot [(e + P)v] = \eta j^2 + \bar{v} \cdot \bar{j} \times \bar{B}
\]  \(2.2.18\)

Eliminating \(e\) and \(\bar{j} \times \bar{B}\) by using the Equation 2.2.14 and Equation 2.2.12 in turn
\[
\frac{1}{\gamma - 1} \frac{\partial P}{\partial t} + \frac{1}{2} \frac{\partial (\rho v^2)}{\partial t} + \nabla \cdot \left[ \left( \frac{\gamma P}{\gamma - 1} + \frac{1}{2} \rho v^2 \right) \bar{v} \right] = \eta j^2 + \bar{v} \cdot \frac{\partial (\rho \bar{v})}{\partial t} + \bar{v} \cdot \left[ \nabla \cdot (\rho \bar{v} \bar{v} + p \bar{I}) \right]
\]  \(2.2.19\)

Simplify Equation 2.2.19 to
\[
\frac{1}{\gamma - 1} \frac{\partial P}{\partial t} + \bar{v} \cdot \nabla P - \frac{v^2}{2} \nabla \rho + \frac{\gamma P}{\gamma - 1} \nabla \cdot \bar{v} - \frac{\rho v^2}{2} \nabla \cdot \bar{v} - \frac{v^2}{2} \frac{\partial \rho}{\partial t} = \eta j^2
\]  \(2.2.20\)

Eliminate \(\nabla \cdot \bar{v}\) by using Equation 2.2.11, we finally get the resistive MHD energy equation:
\[
\left( \frac{\partial}{\partial t} + \bar{v} \cdot \nabla \right) \ln \left( \frac{P}{\rho^\gamma} \right) = \frac{(\gamma - 1) \eta j^2}{P}
\]  \(2.2.21\)

If \(\eta = 0\), Equation 2.2.21 describe the ideal MHD and implies that
\[
\frac{P}{\rho^\gamma} = \text{Constant.}
\]  \(2.2.22\)

Consider a flux tube that moves very slowly compared to the ion sound speed so that the pressure along the flux tube can reach uniform distribution (Wolf, [1983]). Under the ideal MHD condition, the mass in the flux tube is constant. We denote the total mass in the flux tube as \(M\), and use a variable \(m\) as the coordinate: \(m=0\) represent the southern-ionosphere end of the tube and \(m = M\) the northern-ionosphere end. So
\[ P = P(m) \]  \hspace{1cm} (2.2.23)

According to the definition of \( m \) here, using the \( i \) and \( f \) as the notation of initial and final status of the flux tube, we get

\[
\frac{dV_f}{dm} = \frac{\rho_f(m)}{\rho_i(m)} = \text{independent of } m = \frac{V_f}{V_i}
\]  \hspace{1cm} (2.2.24)

where \( V_i \) and \( V_f \) are the total initial and final volumes. We use the last three equations to conclude that

\[
\frac{P_f}{P_i} = \frac{P_f(m)}{P_i(m)} = \left( \frac{\rho_f(m)}{\rho_i(m)} \right)^{5/3} = \left( \frac{V_i}{V_f} \right)^{5/3}
\]  \hspace{1cm} (2.2.25)

So that

\[ PV^{5/3} = \text{Constant} \]  \hspace{1cm} (2.2.26)

But for the case \( \eta \neq 0 \), the plasma will not be frozen in the magnetic field. This means the confining volume \( V_c \) of the plasma doesn’t keep a fixed ratio with the magnetic volume of flux tube \( V \). Therefore, for the given pressure distribution, if \( PV_c^{5/3} \) is still conserved along a flux tube trajectory, then \( PV^{5/3} \) will not be conserved. We’ll discuss this case in chapter 4.
Chapter 3

Tests of $PV^{5/3}$ Conservation and More in Global MHD Codes

3.1 Procedure

For the purpose of testing the global MHD codes, we use several sets of simulated data that cover a certain length of time with uniform temporal spacing. These data provide values such as magnetic field, pressure, density, and velocity at the simulation grid.

To test the conservation of $PV^{5/3}$, we must first determine the pressure $P$ and flux tube volume $V$. The flux tube volume $V$ is defined as

$$V = \int_{sh}^{nh} \frac{ds}{B(\vec{x}, t)}$$

(3.1.1)

which is integrated along the flux tube from its end point at the southern hemisphere ($sh$) to the end point at the northern hemisphere ($nh$).

The local value on the integration path is calculated by trilinear interpolation from the values of eight nearby grid points. The local point is located in the cuboid whose vertexes are the eight grid points. The flux tube volume is computed as

$$V = \int_{eq}^{nh} \frac{ds}{B(\vec{x}, t)} + \int_{eq}^{sh} \frac{ds}{B(\vec{x}, t)}$$

(3.1.2)

where the $eq$ notates the crossing point of flux tube at the equatorial plane.

The magnetic flux tubes are tracked from the equatorial plane for the following three reasons:

Firstly, we can select the flux tubes that are in the region of interest. Setting the starting and ending points inside the plasma sheet and inner region of the magnetosphere...
helps ensure that the theory of adiabatic particle motion is applicable for the plasma in the flux tubes.

Secondly, the density of magnetic field lines is much lower in the equatorial plane than those near the ionosphere. And so tracking the flux tubes based on the velocity distribution in the equatorial plane is more accurate.

Thirdly, to make the contour map in the equatorial plane, we need sufficient and uniform data distribution on it. If we trace the magnetic field lines from the ionosphere of the Earth, there may be no field line crossing the equatorial plane at the desired location.

Here we define $\bar{P}$ as the average effective pressure, which is weighted by the flux tube volume:

$$
\bar{P} = \frac{\int_{q}^{h} P(s) \frac{ds}{B(s)} + \int_{q}^{h} P(s) \frac{ds}{B(s)}}{\int_{q}^{h} \frac{ds}{B(s)} + \int_{q}^{h} \frac{ds}{B(s)}} = \frac{\int_{q}^{h} P(s) \frac{ds}{B(s)}}{\int_{q}^{h} \frac{ds}{B(s)}}
$$

(3.1.3)

After integrating $\bar{P}$ and $V$ along a single flux tube, we can calculate the value of $\bar{P} V^{5/3}$ for this flux tube.

The procedure for testing the conservation of $PV^{5/3}$ of these two MHD codes is as follows:

1. 40 points are selected in the equatorial plane and the values of $P$ and $V$ in the flux tubes that pass through these points are determined. These points are distributed uniformly on the semicircle at the night side of the Earth. The distance from the points to the center of the Earth is $8 R_E$. This ensures the magnetic lines passing through these points are closed ones because these lines are in the quasi-dipolar part of the magnetosphere. Furthermore, this
region is far enough from the magnetic reconnection regions of the runs of these two global MHD models.

2. After computing $PV^{5/3}$ values at the time $T_1$, the problem becomes how to determine the positions of the flux tubes at the next time step $T_2$. Two sets of velocity distribution data for time $T_1$ and $T_2$ are read and interpolated both spatially and temporally to track the points’ movements. If the time interval between the two sets of data is

$$T = T_2 - T_1$$

(3.1.4)

The interval is then divided into $N$ time steps, the sub-time-step

$$t_h = T / N$$

(3.1.5)

In the time series $t_0, t_1, t_2, \ldots, t_N, t_0 = T_1, \text{ and } t_N = T_2$. At the time $t_n$ of this time series, for the point at $\vec{x}_n$, a spatial interpolation is done to get two velocity values $\vec{V}_0(\vec{x}_n)$ and $\vec{V}_N(\vec{x}_n)$ based on the data for time $T_1$ and $T_2$, respectively.

By temporal interpolation, the velocity of the point at position $\vec{x}_n$ and time $t_n$ is inferred to be

$$\vec{V}_n(\vec{x}_n) = \vec{V}_0(\vec{x}_n) + \frac{(t_n - t_0)}{(t_N - t_0)} (\vec{V}_N(\vec{x}_n) - \vec{V}_0(\vec{x}_n))$$

(3.1.6)

The position of the flux tube at equatorial plane is then updated as

$$\vec{x}_{n+1} = \vec{x}_n + \vec{V}_n(\vec{x}_n) \cdot t_h$$

(3.1.7)

By using this method, these points’ positions at time $t_N$ are tracked and then recorded to be used as the start positions for the run from $T_2$ to $T_3$. The steps are repeated until last set of data is read. Thus, the $PV^{5/3}$ values of the
flux tubes in the whole time series can be computed. This method improves the accuracy of the tracking by reducing the influence caused by the limitation of temporal resolution of the data. We also track the flux tubes backward from last set of data in the time series to get the tracks of flux tubes outside of 8R_E from the Earth. In the actual tests, time step is self-adapted so that we can ensure that the results of tracking are stable.

3. After the tracks of these points are plotted, only those points whose tracks are totally inside the region where adiabatic particle motion theory is applicable are selected. These points are the crossing points of the flux tubes in the equatorial plane. By computing the $PV_{SV}$ of the flux tubes associated with these points, a relationship between the $PV_{SV}$ and time is determined, which reveals if the $PV_{SV}$ is conserved during the motion of the flux tubes. In addition, the contours of $PV_{SV}$ along with the velocity vector in the equatorial plane are plotted. If the velocity vectors cross the contours, this would imply a violation of $PV_{SV}$ conservation.

Other than the $PV_{SV}$, some other tests were performed. In chapter 1, it has been indicated that for ideal MHD, magnetic flux is frozen into the plasma. Therefore, the number of charged particles in the flux tube should be constant. The test for the normalized particle number in the flux tubes has been done for the two codes.

For a flux tube where the magnetic flux is $\Phi$, $|\vec{B}(\vec{x})| \cdot S(\vec{x}) = \Phi$ (Figure 3.1.1).

The total mass $M$ of the particles in that flux tube is

$$M_{\text{particle}} = \int \rho(\vec{x}) \cdot dV(\vec{x}) = \int \rho(\vec{x}) \cdot dl(\vec{x}) \cdot S(\vec{x})$$

$$= \int \rho(\vec{x}) \cdot dl(\vec{x}) \cdot \frac{\Phi}{B(\vec{x})} = \Phi \cdot \int \rho(\vec{x}) \cdot \frac{dl(\vec{x})}{B(\vec{x})}$$

(3.1.8)
where $\rho(\vec{x})$ is the density at the point of $\vec{x}$, and $dV(\vec{x})$ is the infinitesimal element of the volume at $\vec{x}$.

![Diagram](image)

**Figure 3.1.1** A volume element of a flux tube. $S(\vec{x})$ is the section area of the flux tube at point $\vec{x}$, $B(\vec{x})$ is the magnetic field at point $\vec{x}$, and $dl(\vec{x})$ is the infinitesimal element of length along the flux tube. The magnetic flux of the flux tube is denoted by $\Phi$.

Since MHD models use one-fluid assumption, the particle number of plasma population is proportional to the total mass of the plasma. And because the magnetic flux of the same flux tube is constant for all the time, the normalized particle number in the flux tube is

$$N_{\text{norm}}(t = t_n) = \frac{N_{\text{particle}}(t = t_n)}{N_{\text{particle}}(t = t_0)} = \frac{M_{\text{particle}}(t = t_n)}{M_{\text{particle}}(t = t_0)} = \frac{\int \rho(\vec{x}) \frac{dl(\vec{x})}{B(\vec{x})} \big|_{t=t_n}}{\int \rho(\vec{x}) \frac{dl(\vec{x})}{B(\vec{x})} \big|_{t=t_0}}$$  (3.1.9)

The time-dependent plots of normalized particle number are presented in Section 3.3 and Section 3.6.
According to the gas-dynamic conservation form of the MHD equations, the
following equation is derived in Section 2.2 (Equation 2.2.21):

\[
\left( \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla \right) \ln \left( \frac{P}{\rho^\gamma} \right) = \frac{(\gamma - 1) \eta^2}{P} \quad (3.1.10)
\]

By investigating if quantity \( \frac{P}{\rho^\gamma} \) is conserved, we can determine if the resistivity \( \eta \) is
zero in these two global MHD models. The time-dependent plots of \( \frac{P}{\rho^\gamma} \) are also
presented in the following chapter. The integration is

\[
\frac{P}{\rho^\gamma} = \left( \frac{\int_{s_h}^{s_m} P(s) \frac{ds}{B(s)}}{\int_{s_h}^{s_m} \frac{ds}{B(s)}} \right) \left/ \left( \frac{\int_{s_h}^{s_m} \rho(s) \frac{ds}{B(s)}}{\int_{s_h}^{s_m} \frac{ds}{B(s)}} \right) \right)^\gamma \quad (3.1.11)
\]

### 3.2 Description of the OPEN-GGCM MHD Data

![Figure 3.2.1](image)

**Figure 3.2.1** Numerical grid of OPEN-GGCM model in the equatorial plane.
OPEN-GGCM uses a stretched Cartesian grid (Figure 3.2.1). The advantage of using this grid is that it has low programming and computational overhead, and is relatively straightforward to parallelize. In the region where we are interested, the grid density can be pre-set higher than other regions. But once the grid is set, it can’t be adjusted during the simulation.

Two sets simulation data are used for this study, they are labeled b153, and jxr1605. Among these, data set b153 is a simulation using a low density grid (Figure 3.2.2), while data set jxr1605 is a simulation using high density grid (Figure 3.2.3). The distributions of grid data are set for numerical reason (Jimmy Raeder, UNH, private communication).

![Figure 3.2.2](image)

**Figure 3.2.2**  Ideal grid resolution in x-direction for data set b153. The x value of a point is the position of a simulation grid in x-direction, and y value is its grid resolution in x-direction.

The data set b153 has 21 effective data files with a time interval between every file of 180 seconds, which covered a total time of 1 hour. The simulation’s grid size is 100 points in x-direction, and 60 points in both the y-and z-directions. The range of simulation box is from $-300 \ R_E$ to $18 \ R_E$ in x-direction, $-40 \ R_E$ to $40 \ R_E$ in y-direction,
and $-40 \, R_E$ to $40 \, R_E$ in $z$-direction. The highest resolution is about $0.5 \, R_E$ near the Earth, the lowest resolution is about $16 \, R_E$ in $x$-direction at $x = -300 \, R_E$.

The solar wind inputs used in this simulation are a velocity of the solar wind which is a steady $-400$ km/s in $x$-direction, $0$ m/s in $y$- and $z$-directions, a density of $5.0$ cm$^{-3}$, and a thermal pressure of $4.0$ pPa. And the IMF (Interplanetary Magnetic Field) is also held steady at $-5.0$ nT, southward.

![Graph]

**Figure 3.2.3** Ideal grid resolution in $x$-direction for data set jxr1605. The $x$ value of a point is the position of a simulation grid in $x$-direction, and $y$ value is its grid resolution in $x$-direction.

The data set jxr1605 has 85 data files with a time interval between each file of 40 seconds. The total time of the data coverage is about 1 hour. The model grid size is 360 points in $x$-direction, and 120 points in both $y$- and $z$-direction. The range of simulation box is from $-300 \, R_E$ to $19 \, R_E$ in $x$-direction, $-36 \, R_E$ to $36 \, R_E$ in $y$-direction, and $-36 \, R_E$ to $36 \, R_E$ in $z$-direction. The solar wind condition is same as the case of b153. The highest resolution is about $0.15 \, R_E$ near the Earth, the lowest resolution is about $4.6 \, R_E$ in $x$-direction near $x = -300 \, R_E$. 
3.3 Results with the OPEN-GGCM MHD Code (Low Resolution Grid)

In this section, the data set b153, which is the simulation using the low resolution grid, is used in both track-forward and track-backward method.

After tracking the 40 flux tubes forward for 1 hour, some of the tubes moved out of the simulation box (Figure 3.3.1). The 32 remaining flux tubes are used as samples to check the conservation of $PV^{5/3}$ as well as other values (Figure 3.3.2). Since the positions of these flux tubes are recorded for each time step, the pressure, density and magnetic volume of the flux tubes are calculated and stored in a time series. Figure 3.3.3 and Figure 3.3.4 show the flux tube volume and effective pressure as a function of time, respectively. The pressure and flux volume of flux tubes both drop rapidly along the trajectories. Logarithm of $PV^{5/3}$ is plotted as the function of time in Figure 3.3.5. The result shows that $PV^{5/3}$ is not conserved in this 1-hour timescale. The values of $PV^{5/3}$ decreased 1~6 orders for these flux tubes.

Figure 3.3.6 is the plot of the normalized particle number inside flux tubes as a function of time. This result suggests significant deviation of this set of OPEN-GGCM simulation data from the ideal MHD theory. The field lines are not frozen-in the plasma, and particles move out from the flux tubes rapidly.

The next test is for $\frac{P}{\rho^y}$. Figure 3.3.7 shows that the $\frac{P}{\rho^y}$ of this model is not conserved either. According to the Equation 2.2.21, the non-conservation may imply that some kind of resistivity exists in this global MHD simulation. Further discussion about this can be found in the next chapter.

We also investigate the situation when the flux tubes are more than 8 $R_E$ from the Earth. 40 flux tubes start at about $-11R_E$ in x-direction and end at the semicircle centered
on the Earth of radius $8 \, R_E$ (Figure 3.3.8). Figure 3.3.9 and Figure 3.3.10 show the flux tube volume and effective pressure as a function of time, respectively. Figure 3.3.11 is the plot of $\log_{10}(PV^{5/3})$ as a function of time. Though the $PV^{5/3}$ seems to be conserved for quite a long time, almost every flux tube stays the same radial distance during that time (Figure 3.3.12). When the flux tubes move toward the Earth, the $PV^{5/3}$ drops rapidly (Figure 3.3.13). The values of $PV^{5/3}$ of all the flux tubes decrease 0.5 ~ 1 orders of magnitude in about 12 minutes.

Figure 3.3.14 also illustrates that $PV^{5/3}$ is not conserved in the case of data set b153. The figure shows the contours of the $PV^{5/3}$ plotted with the velocity vectors in the equatorial plane at the time 5400s. As shown, the vectors of velocity are not parallel to the contours. Some of the vectors are perpendicular to the contours of $PV^{5/3}$. When flux tubes travel across the contours of $PV^{5/3}$, they break the conservation of $PV^{5/3}$.
Figure 3.3.1  Tracks of flux tubes in the equatorial plane based on the data set b153. The figure shows some flux tubes move out of the simulation box we set during the 1 hour.
Figure 3.3.2  Tracks of the selected flux tubes in the equatorial plane. 32 flux tubes are selected to check the conservation of $PV^{2/3}$. They are notated as tube 1 to tube 32, as shown in the figure.
Figure 3.3.3  The flux tube volume of the 32 flux tubes as a function of time, based on the OPEN-GGCM data b153.
Figure 3.3.4  The average pressure of the 32 flux tubes as a function of time, based on the OPEN-GGCM data b153.
Figure 3.3.5 \( \log_{10}(PV^{5/3}) \) of the 32 flux tubes as a function of time, based on the OPEN-GGCM data b153. The unit of \( \log_{10}(PV^{5/3}) \) is \( \log_{10}(\text{nPa}(R_\odot/nT)^{5/3}) \).
Figure 3.3.6  Normalized particle number inside the flux tubes as a function of time, based on the OPEN-GGCM data b153.
Figure 3.3.7 \( \frac{P}{\rho^2} \) of the flux tubes shown as a function of time, based on the OPENGGCM data b153.
Figure 3.3.8  Tracks (tracked backward) of flux tubes in the equatorial plane, based on the OPEN-GGCM data b153. 40 flux tubes are notated.
Figure 3.3.9  The flux tube volume of the 40 flux tubes as a function of time, based on the OPEN-GGCM data b153.
Figure 3.3.10 The effective pressure of the 40 flux tubes as a function of time, based on the OPEN-GGCM data b153.
Figure 3.3.11 \( \log_{10}(P V^{5/3}) \) of the 40 flux tubes as a function of time, based on the OPEN-GGCM data b153. The unit of \( \log_{10}(P V^{5/3}) \) is \( \log_{10}(\text{nPa}(R_\oplus/\text{nT})^{5/3}) \).
Figure 3.3.12  y-axis indicates the distance from the points of flux tubes across the equatorial plane to the Earth, based on the OPEN-GGCM data b153.
Figure 3.3.13  \( \log_{10}(PV^{5/3}) \) of the 40 flux tubes as a function of distance to the Earth, based on the OPEN-GGCM data b153. The unit of \( \log_{10}(PV^{5/3}) \) is \( \log_{10}(nPa(R_E/NT)^{5/3}) \).
Figure 3.3.14 Contours of $PV^{5/3}$ plotted with the velocity vectors in the equatorial plane at time = 5400s. The unit of $PV^{5/3}$ is nPa*($R_TS/NT$)$^{5/3}$. 
3.4 Results with the OPEN-GGCM MHD Code (High Resolution Grid)

The test of data set jxr1605 will be presented in this section. This data set is the result of simulation that uses higher grid resolution than the one from data set b153. It shows more detailed structure than the b153 set, and the plasma movement of this set is more turbulent. 40 flux tubes were tracked and 28 flux tubes were selected to test the \( PV^{5/3} \) (Figure 3.4.1). Other flux tubes either move out of the interesting region or become open field lines.

Figure 3.4.2 and Figure 3.4.3 show the flux tube volume and effective pressure of as a function of time, respectively. Figure 3.4.4 shows the \( \log_{10}(PV^{5/3}) \) as the function of time, and Figure 3.4.6 shows the \( \log_{10}(PV^{5/3}) \) as the function of distance to the Earth. The \( PV^{5/3} \) of the tested flux tubes varies in a range of 0.1 ~ 0.5 orders of magnitude in one hour (Figure 3.4.5).

\( PV^{5/3} \) is still not conserved in this case. However, compared with the result of data set b153, which gives out the result that \( PV^{5/3} \) of all the flux tubes decrease 0.5 ~ 1 orders in about 12 minutes (Figure 3.3.11, Figure 3.3.13), there is some improvement in \( PV^{5/3} \) conservation.

Considering the significant difference in the resolution of simulation grids of these two sets of data (Figure3.2.2, Figure3.2.3), this suggests that using higher resolution of simulation can make Global MHD models better conserve \( PV^{5/3} \).
Figure 3.4.1 Tracks of the selected flux tubes in the equatorial plane, based on the data set jxr1605. 28 flux tubes are selected to check the conservation of $PV^M$. They are notated as tube 1 to tube 28, as shown in the figure.
Figure 3.4.2  The flux tube volume of the 28 flux tubes as a function of time, based on the OPEN-GGCM data jxr1605.
Figure 3.4.3 The effective pressure of the 28 flux tubes as a function of time, based on the OPEN-GGCM data jxr1605.
Figure 3.4.4  \( \log_{10}(PV^{5/3}) \) of the 28 flux tubes as a function of time, based on the OPEN-GGCM data jxr1605. The unit of \( \log_{10}(PV^{5/3}) \) is \( \log_{10}(\text{nPa}(\text{R}_E/\text{nT})^{5/3}) \).
Figure 3.4.5 $y$-axis indicates the distance from the points of flux tubes across the equatorial plane to the Earth, based on the OPEN-GGCM data jxr1605.
Figure 3.4.6 \( \log_{10}(P V^{5/3}) \) of the 28 flux tubes as a function of distance to the Earth, based on the OPEN-GGCM data jxr1605. The unit of \( \log_{10}(P V^{5/3}) \) is \( \log_{10}(\text{nPa}(R_E/nT)^{5/3}) \).
3.5 Description of the BATS-R-US MHD Data

The grid in the BATS-R-US MHD code uses a technique known as Structured Adaptive Mesh Refinement (SAMR). SAMR automatically adapts the computational grid to the solution of the governing equations, and it avoids under-resolving the solution in regions of interest and over-resolving in the solution in the regions of less interest. Therefore, this technique can provide the most efficient and accurate solutions for a given amount of grid cells compared to other grids (Powell et al. [1999], Gombosi et al. [2003]).

![Diagram of grid blocks]

**Figure 3.5.1** An example of neighboring refined and unrefined blocks in the BATS-R-US MHD code (Powell et al. [1999]).

The self–similar blocks compose the computational grids of the BATS-R-US code. These blocks may be of different size. In the region that requires higher resolution, a
parent block is divided into eight children blocks. Each of the eight octants of the parent block becomes a new block, thus the cell resolution doubles. On the other hand, when the region is over-resolved, for example, the gradient in this region turns from high to low, eight neighboring children blocks are coarsened and coalesced into a single parent block. This is the refinement process. Figure 3.5.1 is an example of neighboring refined and unrefined blocks.

The data set used in this test has 47 effective files in total with a 300 seconds time interval between every file. The total time coverage of the data is 3 hours and 50 minutes. Because the BATS-R-US MHD code doesn’t set fixed grid points, the result of the simulation is interpolated on a selected grid (Stanislav Sazykin, Rice University, private communication). The output grid size is 80 points in x-direction, and 60 points in both y- and z-direction. The range of output data is from $-25 \, R_E$ to $15 \, R_E$ in x-direction, $-15 \, R_E$ to $15 \, R_E$ in both y- and z-direction, which includes the region of interest. The resolution of the simulation grid is about $0.25 \, R_E$ inside $10 \, R_E$ from the Earth. The resolution turns to be $0.5 \, R_E$ in the region of $10 \, R_E \sim 20 \, R_E$ from the Earth.

The solar wind inputs used in this simulation are a velocity of the solar wind a steady $-400 \, \text{km/s}$ in x-direction, $0 \, \text{m/s}$ in y- and z-direction, a density of $5.0 \, \text{cm}^{-3}$, and a thermal pressure of $20 \, \text{pPa}$. And the IMF (Interplanetary Magnetic Field) is a steady $-5.0 \, \text{nT}$, southward.
3.6 Results with the BATS-R-US MHD Code

After tracking 40 flux tubes from the starting points by using nearly 4 hours simulation data, 32 of these tubes moved out of the simulation box (Figure 3.6.1). The other 8 remaining flux tubes are used as samples for checking the conservation of $PV^{5/3}$ and other values (Figure 3.6.2). The tracks are asymmetric because the BATS-R-US model considers the effect caused by the rotation of the Earth. Since the positions of these flux tubes are recorded for each step, the pressure, density and magnetic volume of the flux tubes are calculated and stored as a time series. Figure 3.6.3 and Figure 3.6.4 show the flux tube volume $V$ and effective pressure $P$ respectively as a function of time. $\log_{10}(PV^{5/3})$ of the flux tubes and the position of the flux tubes are plotted as functions of time in Figure 3.6.5 and Figure 3.6.6, respectively. Figure 3.6.7 shows the relationship between $\log_{10}(PV^{5/3})$ and the distance from the Earth. The values of $PV^{5/3}$ decrease about 3 orders for the flux tubes. The result shows that $PV^{5/3}$ is not conserved along the trajectories for the BATS-R-US either.

Figure 3.6.8 is the plot of the normalized particle number inside flux tubes as a function of time. This plot suggests deviation of the BATS-R-US model from the ideal MHD equations. The field lines are not frozen-in the plasma, and particles move out from the flux tube rapidly. The next test is for $\frac{P}{\rho^y}$. Figure 3.6.9 shows that the $\frac{P}{\rho^y}$ of this model is not conserved either. According to the Equation 2.2.21, the non-conservation may imply that some kind of resistivity exists in this global MHD simulation.

The case for those flux tubes that move out of the range of $8R_E$ is also tested. 40 flux tubes start at about $-14R_E$ on x-axis and end their travel at the semicircle of $8R_E$ from the Earth (Figure 3.6.10). Figure 3.6.11 and Figure 3.6.12 show the flux tube volume and
effective pressure as a function of time, respectively. Figure 3.6.13 is the plot of $\log_{10}(PV^{5/3})$ as the function of time. The $PV^{5/3}$ seems to be conserved for most of the time, but the actual reason for such conservation is that the flux tubes almost stay at the same radial distance during that time (Figure 3.6.14). When the flux tubes start to move toward the Earth, the $PV^{5/3}$ drops rapidly (Figure 3.6.15). The values of $PV^{5/3}$ of all the flux tubes decrease about 1 order of magnitude in half an hour.

Figure 3.6.16 and Figure 3.6.17 are the plots that show the contours of the $PV^{5/3}$ overlapping the velocity vectors in the equatorial plane at the time 3600s and 7200s, respectively. In the most of area as shown, the vectors of velocity are parallel to the contours. But there are still some vectors that are perpendicular to the contours of $PV^{5/3}$, implying that the plasma associated with these velocity vectors may break conservation of $PV^{5/3}$. 
Figure 3.6.1 Tracks of the flux tubes in the equatorial plane. Some tubes go out of data box output from the BATS-R-US model.
Figure 3.6.2  Tracks of the selected flux tubes in the equatorial plane. 8 flux tubes are selected to check the conservation of $PV^{2/3}$. They are notated as tube 1 to tube 8, as shown in the figure.
Figure 3.6.3  The relationship between the time and the flux tube volume of the 8 tubes, based on the BATS-R-US data.
Figure 3.6.4  The relationship between the time and the effective pressure of the 8 tubes, based on the BATS-R-US data.
Figure 3.6.5  The relationship between the time and the $PV^{5/3}$ of the 8 tubes, based on the BATS-R-US data. Obviously, $PV^{5/3}$ is not conserved.
Figure 3.6.6 y-axis indicates the distance from the points of flux tubes across the equatorial plane to the Earth, based on the BATS-R-US data.
Figure 3.6.7  \( \log_{10}(PV^{5/3}) \) of the 8 flux tubes as a function of distance to the Earth, based on the BATS-R-US data. The unit of \( \log_{10}(PV^{5/3}) \) is \( \log_{10}(n\text{Pa}(R_E/nT)^{5/3}) \).
Figure 3.6.8  Normalized particle number inside the flux tubes as a function of time, based on the BATS-R-US data.
Figure 3.6.9 \( \frac{P}{\rho^y} \) of the flux tubes shown as a function of time, based on the BATS-R-US data.
Figure 3.6.10 Tracks (tracked backward) of flux tubes in the equatorial plane, based on the BATS-R-US data. 40 flux tubes are notated.
Figure 3.6.11 The flux tube volume of the 40 flux tubes as a function of time, based on the BATS-R-US data.
**Figure 3.6.12** The effective pressure of the 40 flux tubes as a function of time, based on the BATS-R-US data.
Figure 3.6.13  $\log_{10}(PV^{1/3})$ of the 40 flux tubes as a function of time, based on the BATS-R-US data. The unit of $\log_{10}(PV^{1/3})$ is $\log_{10}(\text{nPa}(\text{Re}/\text{nT})^{1/3})$. 
Figure 3.6.14 y-axis indicates the distance from the points of flux tubes across the equatorial plane to the Earth, based on the BATS-R-US data.
**Figure 3.6.15** \( \log_{10}(PV^{5/3}) \) of the 40 flux tubes as a function of distance to the Earth, based on the BATS-R-US data. The unit of \( \log_{10}(PV^{5/3}) \) is \( \log_{10}(n\text{Pa}(R_E/nT)^{5/3}) \).
Figure 3.6.16 Contours of $PV^{5/3}$ plotted with the velocity vectors in the equatorial plane at time = 3600s. The unit of $PV^{5/3}$ is nPa*(R_E/nT)^{5/3}.
Figure 3.6.17 Contours of $PV^{5/3}$ plotted with the velocity vectors in the equatorial plane at time = 7200s. The unit of $PV^{5/3}$ is nPa*(R_E/nT)^{5/3}.
Chapter 4

Conclusions and Discussion

This thesis reports the tests of the conservation of $PV^{5/3}$ and other invariants for two global MHD codes. The formalisms of these models imply that these invariants should be conserved.

However, after testing the two global MHD codes, OPEN-GGCM and BATS-R-US, it has been found that neither of them can ensure the conservation of $PV^{5/3}$. Thus the current version of these two codes may not be appropriate to perform one-way coupling with the RCM model. However, for the case of the OPEN-GGCM, it is found that using higher density of simulation grid can make some improvement in how well the simulation keeps $PV^{5/3}$ conserved. A similar grid refinement experiment has not been done with the BATS-R-US code.

As shown in Chapter 3, Equation 3.1.10 shows the effect of finite resistivity on the conservation of $PV^{5/3}$. There are two possible causes of finite resistivity in global MHD models. The first is that the resistivity is introduced to the simulation for the purpose of enabling magnetic reconnection. The other is caused by numerical diffusion.

The BATS-R-US model doesn’t have any explicit resistivity. It enables magnetic reconnection by numerical diffusion that occurs in the region where the magnetic field has sharp gradients. The region studied in this thesis is not applicable to this situation. OPEN-GGCM does introduce the electrical resistivity to enable the reconnection. In order to determine if this resistivity occurs in the test region, we have plotted the
numerical resistivity in the OPEN-GGCM code. According to Raeder [1999], the artificial resistivity $\eta$ is given by:

$$
\begin{align*}
\eta &= \alpha j^2 & \text{if} & \quad j \geq \delta \\
\eta &= 0 & \text{if} & \quad j < \delta
\end{align*}
$$

(4.1)

where $\alpha$ is the anomalous resistivity constant, and has a value of 0.2 for the tested simulation run; $\delta$ is the anomalous resistivity threshold, which has value of 0.75 for the tested simulation. The normalized current density is $j' = \frac{|\vec{j}|}{|\vec{B}| + \varepsilon} \Delta$, where $\vec{j}$ is the local current density, $\vec{B}$ is the local magnetic field, $\Delta$ is the grid spacing, and $\varepsilon$ is a very small number ($10^{-8}$) introduced to avoid division by zero.

Because the resistivity is most likely to appear in regions of magnetic reconnection and magnetic reconnection typically occurs near the equatorial plane, the resistivity distribution in the equatorial plane is plotted. Plots Figure 4.1 ~ Figure 4.3 show the distributions of magnetic field, current density, and resistivity in the equatorial plane at the time 180s, 3600s and 7200s, respectively. Because only a resistivity of $10^4 \, \Omega$ or larger can cause significant magnetic diffusion (Raeder [1999]), the artificial resistivity less than that amount is ignorable. These plots suggest that there is no artificial resistivity in the region where $PV^{5/3}$ conservation is tested.

The first possibility for the occurrence of resistivity can be eliminated for both the BATS-R-US model and OPEN-GGCM model. The other possibility that leads to the appearance of resistivity in the tested region is numerical diffusion, which may be determined by the schemes used in the OPEN-GGCM and BATS-R-US. Those schemes whose error terms associate with even derivatives tend to have numerical diffusion. Conversely, if the error terms associate with odd derivatives, it causes the dispersion.
Figure 4.1 Distributions of magnetic field, current density, and resistivity in the equatorial plane at the time 180s. Resistivity distribution is shown in two different scales.
Figure 4.2  Distributions of magnetic field, current density, and resistivity in the equatorial plane at the time 3600s. Resistivity distribution is shown in two different scales.
Figure 4.3  Distributions of magnetic field, current density, and resistivity in the equatorial plane at the time 7200s. Resistivity distribution is shown in two different scales.
Numerical dispersion is even worse than the diffusion since it can lead to the
nonphysical solutions such as negative density or pressure.

Equation 3.1.10
\[
\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \ln \left( \frac{P}{\rho^\gamma} \right) = \frac{\gamma - 1}{\gamma} \eta^2
\]
(4.2)

can be rewritten in the following form [Wolf, Rice University, personal communication]:
\[
\frac{d \ln(PV^\gamma)}{dt} = \frac{\gamma}{V} \int \nabla \times (\eta \hat{b}) \cdot \hat{b} ds + \frac{1}{V} \left( \int ds \right) \frac{\gamma - 1}{\gamma} \eta^2
\]
(4.3)

where \( \hat{b} \) is the unit vector of the magnetic field. The second term of right side of
Equation 4.3, associated with Joule heating, is positive. The first term could be negative
and hence could cause the value of \( PV^\gamma \) to decrease with time, as the case in the tests.

Numerical diffusion is presumably the cause of the non-conservation of \( PV^{5/3} \) and
other quantities.

The results also suggest that global MHD models in the current form are not accurate
enough to couple with the RCM model in a one-way coupling mode. However, two-way
coupling may correct the deviation from conservation for the global MHD codes, since
the RCM will periodically correct the \( PV^{5/3} \) of the MHD code.

For future work it may be worthwhile to investigate the dependence of the
conservation of \( PV^{5/3} \) when raising the grid density of simulation and temporal resolution
of global MHD models.

A second test would be to confirm the conservation of \( PV^{5/3} \) when two-way coupling
is used and compare these results with those obtained with the one-way coupling. This
will show how well two-way coupling can adjust the problem of Global MHD models
and achieve reasonable simulation results.
Biography


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