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MASTER OF SCIENCE

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ABSTRACT


by

Zhongqing Ji

This thesis will discuss the principles, techniques and applications of the Radio Frequency Single Electron Transistor (RF-SET). In the first part, the operating principles of Single Electron Transistors (SETs) in the normal and superconducting states will be introduced. The general techniques of fabricating and calibrating SETs will also be introduced. In the second part, two of our recent experiments are reviewed. One is related to the sensitivity and linearity of superconducting RF-SETs. We found that the RF-SET achieves the best balance of charge sensitivity and linearity in the subgap regime, as opposed to the usual preferred working point in the above-gap regime. The second experiment relates to the real-time counting of single electrons. We demonstrated that the RF-SET can be used as a fast and ultra-sensitive electrometer which can even detect tunneling of a single electron inside a tunable quantum dot (QD) formed in a two dimensional electron gas (2DEG).
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Contents

Abstract ii
Acknowledgments iii
List of Figures vi

1 Introduction 1

2 The Theory of the Single Electron Transistor 4
2.1 Principles of the Normal State SET 4
2.2 Superconducting SET (S-SET) 10
2.3 Principles of the Radio Frequency Single Electron Transistor (RF-SET) 15
2.4 Quantum Dots (QDs) 18

3 Experimental Techniques 20
3.1 Electron Beam Lithography 20
3.2 Sample Fabrication 22
3.3 Dilution Refrigeration 30

4 SET and Quantum Dot Characterization 32
4.1 I – V Curve Measurement for the SET 32
4.2 Determining the Resonant Frequency 34
4.3 SET Gate Modulation 36
4.4 Measurement of the $I(V_{bias}, Q_0)$ Surface of the SET .......................... 38

5 Study of the Sensitivity and Linearity of Superconducting RF-SET 41
  5.1 Introduction .............................................................. 41
  5.2 Experimental Setup .................................................... 42
  5.3 DC Data Analysis ...................................................... 44
  5.4 RF Results ............................................................... 46
  5.5 Conclusion ............................................................... 48

6 Real-time Electron Counting Experiment ............................ 52
  6.1 Introduction .............................................................. 52
  6.2 System Design ........................................................... 53
  6.3 Vertical Coupling Scheme ............................................. 54
  6.4 Operating Point of the SET ........................................... 57
  6.5 Characterization of the RF-SET ...................................... 57
  6.6 Real-time Measurement for a QD System ............................. 58

7 Conclusion ............................................................... 62

References ................................................................. 64
List of Figures

2.1 Simplified structure of (a) MOSFET and (b) SET. .................................... 5

2.2 Equivalent circuit of the SET. ................................................................. 6

2.3 Energy diagram of the SET before and after modifying the gate voltage. 8

2.4 Stability diagram of a normal state SET. The inset is the corresponding
   SET current as a function of $V_g$. .......................................................... 9

2.5 Three tunneling processes through a single superconductor-insulator-
   superconductor junction. ................................................................. 11

2.6 Contour plot of a represented diamond diagram (from [6]). Solid lines
   are the thresholds for single-electron tunneling through junction 1 and
   2. Dotted lines are the thresholds for Andreev tunneling. Dashed lines
   are where Cooper pair tunneling is resonant. Alternating J-e processes
   form the 3e peak at the intersection of the Cooper pair resonance curves
   at $V = 2E_C/e$. ................................................................. 13

2.7 Various JQP cycles. The symbol ⇔ denotes resonant tunneling and
   the symbol → denotes dissipative quasiparticle tunneling. The number
   within circle represents the number of excess electrons on the SET. ...................................................... 14

2.8 The Modulation of the RF-SET. ............................................................. 16

2.9 The Synchronous demodulation of amplitude modulation (AM). . . 17
2.10 Formation of the quantum dot. .................................. 19

3.1 Simplified schematic of the Leo S440 SEM/NPGS nanolithography sys-
tem. ................................................................. 21

3.2 General processes of sample fabrication. .......................... 22

3.3 Processes of making sample. ....................................... 23

3.4 Shadow evaporation. ............................................... 28

3.5 Sample schematic and micrograph. ................................ 29

3.6 Simplified mixing chamber schematic diagram. .................. 30

4.1 Schematic of the SET $I - V$ curve measurement. ............... 32

4.2 Representative SET $I - V$ curve. ................................ 33

4.3 Schematic of circuit for determining the resonant frequency. .... 34

4.4 Representative reflected power spectra. .......................... 36

4.5 Schematic for Coulomb blockade oscillations measurement. ...... 37

4.6 Coulomb blockade oscillations of the SET. ....................... 38

4.7 Voltage scan circuit for image plot measurement. ............... 39

4.8 A series of $I - V$ curves of the SET at different $V_{gate}$. ........... 39

4.9 Representative false color image plot of $I(V_{bias}, V_{gate})$. Different colors represent different amplitudes of SET current. .................. 40

5.1 Schematic diagram of the RF-SET. ............................... 43
5.2 Al-AlOx-Al SET fabricated on GaAs substrate. 43
5.3 (a) Sequential tunneling simulation. (b) False color image plot of measured current for S2 at T=20mK. 46
5.4 $I-V$ characteristics of (a)S1 (b)S2 (c)S3 50
5.5 Charge sensitivity and SNR as the function of offset charge. (a) S1 above gap (b) S1 subgap (c) S3 subgap 51
6.1 The design of a sample with a single quantum dot strongly coupled to an RF-SET. 53
6.2 SEM Photo of the System 54
6.3 Vertical coupling scheme of our system 55
6.4 Doubly Periodic Behavior for Strongly Coupled Case($C_{gs} \ll C_{gd}$) 56
6.5 Current through SET as a function of gate voltage when the dot is (a) relatively open and (b) relatively closed. (c) Conductance of quantum dot measured simultaneously. 59
6.6 Optimal working point of SET. 60
6.7 Power spectrum of the reflected signal with 0.05 $e$ excitation at 100kHz. 60
6.8 Demodulated sine charge excitation (0.05$e$ rms, 100kHz) recorded by digital oscilloscope. 61
6.9 Representative electron tunneling events captured by digital scope. 61
Chapter 1
Introduction

Since the invention of the transistor by Bardeen and Shockley in 1948, the size of transistors has become smaller and smaller while the operating speed has become faster and faster. For example, in the integrated circuit (IC) industry, 90nm processes (measured by gate length) are becoming more and more popular and a new 65nm technique is already being planned. Until now, the development of the IC industry has complied with Moore's Law, which states the number of the components per IC doubles roughly once every 18 months. But Moore's law is being challenged as the size of transistors is getting close to the scale of atoms and molecules. As mentioned in a recent review [1], even before the size of transistor reaches the ultimate limit, i.e., the size of an atom or a molecule, several quantum effects must be taken into account.

Recently much attention has been paid to the study of mesoscopic transport, which happens when the device size is comparable with the electronic coherence length. In this case, quantum effects can play an important role. Many devices have been invented for exploiting the quantum effects of tunneling and measuring the movement of single electrons. The single electron transistor (SET) [2, 3] is one of them. The principle of an SET in the normal state is surprisingly simple. It can
be explained within the classical regime without too much concern about quantum physics since the only quantized parameter is the charge of one electron. A SET can be an extremely precise electrometer. Its ability to detect a single electron’s movement makes it a possible read-out device of future quantum computers. But there are drawbacks that hinder its use in applications, the most important of which is its operating speed. Although the intrinsic speed of the SET is very high, standard cabling schemes and its high output impedance limit its bandwidth and operating speed. Other problems include the $1/f$ noise due to the fluctuation of background charges in the substrate. To solve these problems, Schoelkopf invented the so-called radio frequency single electron transistor (RF-SET) [4] which works dynamically. An LC resonant circuit containing the SET is modulated with a gate voltage, producing a signal that is later demodulated. The RF-SET is an ultra-fast and ultra-sensitive electrometer which can detect real-time tunneling events in mesoscopic electronics.

In this thesis, I will introduce the principles and techniques of the SET and its variant the RF-SET in Chapter 2. The methods of making and calibrating the SET are discussed in detail in Chapters 3 and 4. Then two experiments will be reviewed in sequence. I will discuss the influence of tunneling resistance on the charge sensitivity and linearity of a superconducting RF-SET and the choice of the superconducting RF-SET working point in Chapter 5. I then briefly review the first real-time electron counting experiment for a quantum dot with a strongly coupled RF-SET in Chapter
Chapter 2
The Theory of the Single Electron Transistor

In this chapter I will review the theory of the SET. The discussion is organized as follows: Section 2.1 reviews the principles of the SET and compares the normal state SET with the conventional metal-oxide semiconductor field-effect transistor (MOSFET). Section 2.2 discusses the superconducting single electron transistor (S-SET), which is more common in real experiments and has much more complicated transport mechanisms than the normal state SET. Section 2.3 introduces the principle of the radio frequency single electron transistor (RF-SET). Then I briefly introduce quantum dots (QDs) in section 2.4 because in the real-time electron counting experiment we use a QD as the device under test.

2.1 Principles of the Normal State SET

The operating principles of the normal state SET are surprisingly easy to understand. The normal state SET operates within the classical regime except that the electronic charges through it are quantized. It is natural to compare it with its larger sibling the metal-oxide-semiconductor field-effect transistor (MOSFET).

The MOSFET consists of two conducting electrodes (source and drain) connected by a semiconducting channel (Fig 2.1(a)). The carrier density of the channel is controlled by the voltage applied on the third electrode, or gate. In other words,
the gate voltage controls the conductance of the MOSFET, and therefore the current through the source and the drain.

The SET has a similar structure (Fig 2.1(b)) and functions similarly too. The channel is replaced by a sandwich structure. Two conducting electrodes (source and drain) are connected to a nanoscale central island through two ultra-thin tunnel junctions. A third electrode, the gate, is capacitively coupled to the central island.

![Diagram of MOSFET and SET](image)

**Figure 2.1** Simplified structure of (a)MOSFET and (b)SET.

For the normal state SET to operate, two assumptions must be valid. The first requirement concerns the temperature and capacitance of the junctions. Because the central island is ultra-small, transfer of even a single electron can cause a large change in the electrostatic energy of the island. Let us denote the total capacitance of the central island as $C_\Sigma$, which usually is the sum of the two tunnel junction capacitances $C_{J1}$, $C_{J2}$ and the gate capacitance $C_g$. The charging energy, which is needed to add
a single electron to the island, is

\[ E_C = \frac{e^2}{2C_\Sigma} \]

where \( C_\Sigma = C_{J1} + C_{J2} + C_g \). This energy must be higher than the ambient thermal energy \( k_B T \), where \( k_B \) is the Boltzmann constant and \( T \) is the temperature, for the total number of electrons on the island not to be subject to thermal fluctuations. This typically requires both ultra-small dimension for the SET, which results in small capacitance \( C_\Sigma \), and ultra-low temperature.

Another requirement sets the minimum total tunneling resistance \( R_K = \frac{h}{e^2} \approx \)
25.8kΩ. Assume a current $I$ flows through the SET while a voltage $V$ is applied on both ends of the SET. The average tunneling time $\tau$ can be calculated as
\[
\tau = \frac{e}{I} = \frac{eR}{V}
\]
To treat the electrons as single electrons, the wave function for successive electrons should not overlap. So the tunneling time $\tau$ should be much longer than the uncertainty time $\Delta t$, which can be estimated using Heisenberg's energy-time uncertainty relation
\[
\Delta t = \frac{\hbar}{\Delta E} = \frac{\hbar}{eV}
\]
where $\Delta E = eV$ is the energy change of a single electron due to the tunneling. So,
\[
\Delta t = \frac{\hbar}{eV} \ll \tau = \frac{eR}{V} \implies R >> \hbar/e^2 = R_K \approx 25.8k\Omega
\]
In our system, the size of the central island is a few hundred nanometers and the experiment is performed at a few tens of mK in a dilution refrigerator.

Because of the charging energy, current can flow only when the electrical energy generated by the bias voltage between source and drain is higher than the charging energy, otherwise the current is blocked, as shown in Fig 2.3. This is the well-known Coulomb blockade effect.

When an electron tunnels onto the island, the net charge on the island is increased. Because the island is very small, the incoming electron lifts potential of the island significantly by $e/C_E$, which prevents subsequent tunneling of other electrons. For
simplicity, we can assume that the background charge for zero gate voltage vanishes since we can always compensate the additional background charges by an offset charge $Q_0 = C_g \Delta V_g$. So when an electron tunnels in or out the island, i.e., the number of electrons inside the island changes from $n$ to $n \pm 1$, the corresponding energy differences are

$$
\Delta E[-ne, \pm e(-1, 1)] = \frac{e}{C_{J1} + C_{J2}} \left[ \frac{e}{2} \pm C_{J1}V_{bias} \pm ne \mp C_g V_g \right]
$$

$$
\Delta E[-ne, \pm e(-1, 0)] = \frac{e}{C_{J1} + C_{J2}} \left[ \frac{e}{2} \pm C_{J2}V_{bias} \pm ne \mp C_g V_g \right]
$$

where the first equation is for tunneling through junction 2 ($C_{J2}$ and $R_{J2}$) and the second is for tunneling through junction 1 ($C_{J1}$ and $R_{J1}$) [5]. For each $n$, these two formulas will define a diamond shaped region in the $V_g$ and $V_{bias}$ plane, which is called the stability diagram of a normal state SET, as shown in Fig 2.4. The four bounding
conditions are given by $\Delta E = 0$, giving

$$V_{bias} = \frac{C_g V_g}{C_{J1}} - \frac{e(n \pm 1/2)}{C_{J1}}$$

$$V_{bias} = \frac{C_g V_g}{C_{J2}} - \frac{e(n \pm 1/2)}{C_{J2}}$$

Inside each Coulomb diamond (dark area), Coulomb blockade is established, so

![Stability diagram of a normal state SET](image)

**Figure 2.4** Stability diagram of a normal state SET. The inset is the corresponding SET current as a function of $V_g$.

that the number of electrons is constant and there is no current through the SET.

Outside the region, a current can flow between the source and drain. Assume the
SET is in the Coulomb blockade regime. To make electrons lift the blockade, we must either increase the bias voltage $V_{bias}$ or lower the tunnel barrier by changing the gate voltage $V_g$. In practice, we usually choose the second way. So the SET current (or conductance) can be modulated by the gate voltage. This is how a SET should work. An equivalent expression for changing the potential of the central island is that the gate voltage changes the offset charge of the central island by $C_g \Delta V_g$, which can compensate the existing real charge of the central island. The inset of Fig 2.4 shows the current through the SET as a function of gate voltage $V_g$ for a specific bias voltage $V_{bias}$, illustrating the famous Coulomb blockade oscillations. Each current peak corresponds the tunneling of a single electron.

So if the SET gate voltage is biased to a point which is very close to the threshold voltage, even a tiny change of gate voltage can cause rapid variation of conductance of the SET. This characteristic makes the SET a very sensitive electrometer that can even detect less than $e$ charge variation in the gate signal.

### 2.2 Superconducting SET (S-SET)

In practice, the junction of the SET is usually a sandwich structure of Al/AlOx/Al made with a shadow evaporation technique since the high quality native oxide of Al can act as the tunnel barrier. Because the measurement is always performed at a few tens of mK reduce ambient thermal energy, at such low temperatures the aluminum becomes superconducting. So the carriers inside the SET are not electrons anymore
but can be either coherent Cooper pairs (charge 2e) or dissipative quasiparticles (charge e). In this case, transport becomes much more complicated. The S-SET displays both charge quantization and macroscopic coherent quantum effects.

![Diagram of tunneling processes]

**Figure 2.5** Three tunneling processes through a single superconductor-insulator-superconductor junction.

For superconductor-insulator-superconductor (SIS) junctions, the tunneling processes could be dissipative quasiparticle tunneling, resonant Cooper pair tunneling or Andreev reflection (AR), in which a quasielectron is reflected from the junction as a quasihole and a Cooper pair is emitted on the other side. The main difference between AR and two quasiparticle sequential tunneling is that AR requires 2\(\Delta\) (energy to break a Cooper pair) less energy than the energy needed for tunneling of two quasiparticles [6].

So, for two-junction structures such as the S-SET, combinations of these three
processes, which could happen sequentially or simultaneously, result in very complicated current-producing cycles. To date, study of the transport mechanisms in the S-SET is still incomplete. Depending on the relative size of the bias voltage $V_{bias}$ and the superconducting energy gap $\Delta$, the electronic transport can be divided into several regimes. Within each regime one mechanism typically dominates.

For example, in the above gap regime ($eV_{bias} > 4\Delta$), the transport is dominated by the Coulomb blockade of quasiparticle tunneling [7], which is similar to electron tunneling in the normal state SET. By BCS theory, the minimum energy required to break a Cooper pair is $2\Delta$ [8]. So for a two-junction system, the existence of nonvanishing quasiparticle current requires $V_{bias} > 4\Delta/e$. The record of charge sensitivity for an S-SET was found in this regime [9].

In the subgap regime ($eV_{bias} < 4\Delta$), transport is carried out by several complicated mechanisms, which include resonant tunneling of Cooper pairs [10], the Josephson-quasiparticle (JQP) cycle [3, 11, 12], 3e tunneling processes [13], AR [6] and singularity matching [19, 11]. Fig 2.6 is from Fitzgerald's experiment [6]. It clearly shows the existence of several tunneling processes.

Among them, the Josephson-quasiparticle (JQP) cycles, combining resonant Cooper pair and dissipative quasiparticle tunneling, play the most important role. The current peaks caused by JQP cycles are much higher than those caused by other mechanisms in this regime. Even the JQP cycles have several possible variations. In the
Figure 2.6  Contour plot of a represented diamond diagram (from [6]). Solid lines are the thresholds for single-electron tunneling through junction 1 and 2. Dotted lines are the thresholds for Andreev tunneling. Dashed lines are where Copper pair tunneling is resonant. Alternating J-e processes form the 3e peak at the intersection of the Cooper pair resonance curves at $V = 2E_C/e$.

most general case, as shown in Fig 2.7(a), Josephson tunneling (Cooper pair resonant tunneling) only occurs through one junction. Let’s follow a cycle in Fig 2.7(a), beginning in the initial state $|0\rangle$ (or $|1\rangle$). The cycle can occur only when the transition $|0\rangle \rightarrow |1\rangle$ ($|1\rangle \rightarrow |0\rangle$) is allowed, i.e., for $eV_{bias} > E_C + 2\Delta$. The transition from $|1\rangle$ to $|\rightarrow 1\rangle$ (or $|0\rangle$ to $|2\rangle$) is resonant Cooper pair tunneling. Then from state $|1\rangle$ (or $|2\rangle$) the transition is back to $|0\rangle$ (or $|1\rangle$) by quasiparticle tunneling through opposite junction. The cycle is then completed.

While the JQP cycle is forbidden at a lower bias ($|0\rangle \rightarrow |1\rangle$ or $|1\rangle \rightarrow |0\rangle$
is forbidden), at $eV_{\text{bias}} = 2E_C$ Cooper pair tunneling is resonant at both junctions and the double JQP (DJQP) cycle becomes possible, as shown in Fig. 2.7(b). The transition sequence can be $|0\rangle \leftrightarrow |2\rangle \rightarrow |1\rangle \leftrightarrow |-1\rangle \rightarrow |0\rangle$.

Another possible process that could allow transport along the Cooper pair resonance lines between the JQP and DJQP features is shown in Fig. 2.7(c). Even below the threshold for the transition $|1\rangle \rightarrow |0\rangle \rightarrow \langle 0\rangle |1\rangle$, it may still occur virtually, while the transitions $|0\rangle \leftrightarrow |2\rangle$ and $|2\rangle \rightarrow |1\rangle \rightarrow (|1\rangle \rightarrow |-1\rangle \leftrightarrow |1\rangle \rightarrow |0\rangle)$ are allowed, completing what we call the virtual JQP (VJQP) cycle [14].

In summary, for the S-SET, electronic carriers are usually Cooper pairs or quasiparticles, leading to much more complicated transport mechanisms. Thus many results from the normal state SET can not be directly used for the S-SET. For example, in...
our system, the resistance of SETs can be less than $R_k = 25.8\,\text{k}\Omega$ and the working point of the S-SET can be located within the subgap regime, where the S-SET has slightly downgraded charge sensitivity but achieves much better linearity. Such behavior can not be well explained by the theory for the normal state SET.

### 2.3 Principles of the Radio Frequency Single Electron Transistor (RF-SET)

As described above, theoretically the SET is an ultra-sensitive electrometer. But there are several technical problems that have limited applications of the SET. First of all, the speed of the conventional SET is typically limited by its cabling, although the intrinsic operating speed of an SET is as high as several GHz. The SET output signal goes into an amplifier through $50\,\Omega$ transmission cable. The amplifier sees only the capacitance of the cable $C_{\text{cable}}$ and the resistance of the SET $R_{\text{SET}}$. The capacitance of the SET is neglectable compared to $C_{\text{cable}}$. So the operating speed of the SET is limited by the low pass filter consisted with $R_{\text{SET}}$ and $C_{\text{cable}}$. For a typical SET with $100k\Omega$ and $1\mu\text{F}$ cable resistance, the cut off frequency is only about 10 KHz. The usable bandwidth is reduced by the cabling, and the high SET impedance makes it unsuitable for fast measurement. Also, charge sensitivity of the SET at low frequencies is limited by $1/f$ noise generated by the motion of background charges in the substrate.

The RF-SET was invented by Schoelkopf in 2001 [4], primarily to improve the
operating speed of the conventional SET. The operation of the RF-SET is exactly like an amplitude modulation (AM) radio. As shown in Fig 2.8, the RF-SET works at a high frequency, which is determined by a resonant circuit consisting of an external inductor $L$, stray capacitance from bonding pads $C_p$ and the resistance of the SET $R_{SET}$. The SET junction capacitance usually is much smaller than $C_p$. In our system, we use a Niobium superconducting chip inductor with $L \sim 80$ nH and estimate that $C_p \sim 0.5$ pF. The resonant frequency $\sim 1$ GHz is used as the carrier wave frequency. The reflection coefficient of the resonant circuit is changed by changes of $R_{SET}$. Thus, the amplitude of the reflected signal is modulated by the SET gate signal. By monitoring the damping of the resonant circuit, we can get the information from the gate signal.

Two types of demodulation are widely used everywhere. The most common demodulation is the diode envelope detector. The modulated signal goes through a
Modulated Signal

\[
\begin{align*}
\text{Split Carrier Wave} & \quad \rightarrow \quad \text{Mixer} \quad \rightarrow \quad \text{Gate Signal} \\
\text{Phase delay} &
\end{align*}
\]

Figure 2.9 The Synchronous demodulation of amplitude modulation (AM).

diode, which picks up the envelope signal generated by the gate modulation, and is then amplified by a lock-in amplifier. This circuit has some advantages because it is very simple and gives adequate performance in some applications. We use this method to find the resonant frequency of the \( LC \) circuit. But diodes for very high frequency are not easy to find since the output signal attenuates rapidly as the frequency increases due to the intrinsic capacitance of diodes. Another disadvantage is that the demodulated signal is usually somewhat distorted because of the nonlinearity of the diode. So a more advantageous synchronous demodulation is used for our real-time electron counting experiment. As illustrated in Fig 2.9, a signal of the same frequency, which is usually split from the original carrier wave and then attenuated to an amplitude similar to that of the modulated wave, enters the mixer with the modulated wave after adequate phase delay. The carrier wave can be compensated
and only the modulation signal (envelope signal) is left.

A significant advantage of the RF-SET is that the speed and bandwidth of the SET is tremendously improved. Also because the RF-SET works at very high frequencies, the $1/f$ noise due to the background charge motion is totally negligible.

2.4 Quantum Dots (QDs)

Quantum Dots are tiny devices which contain a few electrons and show some physical similarity to atoms. Unlike atoms, QDs can be easily fabricated using electrodes and the number of electrons trapped inside the QDs can be finely tuned. These characteristics make QDs perfect tools to study atomic-like behavior.

There are many varieties of quantum dots. But in this thesis we are only concerned with QDs formed by lateral confinement in a two dimensional electron gas (2DEG). The 2DEG material used in our experiments contains a GaAs/Al$_x$Ga$_{1-x}$As heterostructure, created by growing layers of Al$_x$Ga$_{1-x}$As on top of a GaAs substrate using molecular beam epitaxy. The free electrons are trapped at a GaAs/Al$_x$Ga$_{1-x}$As heterointerface. In our sample, the depth of the 2DEG is around 200nm below the surface. By applying negative voltages on metal gates, which are fabricated on the surface of the substrate, the 2DEG beneath the gates can be depleted, forming an isolated quantum dot, as in Fig 2.10. We can finely tune the number of electron inside the QD by applying different voltages on the surrounding gates. From Fig 2.10 (Top view), two pairs of gates control the communication between the QD and the outside
reservoirs. If the gap between the gates is small enough, the 2DEG in the channel will split into a series of one dimensional subbands, forming a so called *quantum point contact* (QPC), which can also act as a tunnel barrier. So the QD shown in Fig 2.10 is also a double junction device like the SET, and can actually also perform like an electrometer as the SET does. In our experiments, however, the QD works as the device under test (DUT).

![Diagram of a quantum dot]

**Figure 2.10** Formation of the quantum dot.
Chapter 3
Experimental Techniques

In this chapter some general techniques involved in our sample fabrication and measurement are discussed. Section 3.1 reviews standard electron-beam lithography. Section 3.2 tracks the whole procedure of making a real sample. Detailed recipes are provided for each step. In section 3.3 the principle of dilution refrigeration is briefly introduced.

3.1 Electron Beam Lithography

Electron-beam lithography is widely used as a technique for sample fabrication in mesoscopic electronics. Compared with traditional optical lithography, e-beam lithography can easily make components with line widths of a few tens of nanometers. But its high cost and slow throughput make it impractical for industrial applications.

The basic idea of e-beam lithography is to use software to control the electron beam of a scanning electron microscope to draw a predesigned pattern on resist. The operations involve tilting the electronic beam, shifting the sample stage and switching the beam on or off (controlled by a beam blanker). Fig 3.1 is a simplified schematic of Leo S440 SEM/NPGS Nanolithography System used in our experiments.

Electron-beam lithography requires a resist that is sensitive to ionizing radiation. In our system, we use positive resist, i.e. the bonds between the molecules of resist
are easy to break if the resist is exposed to the electron beam. After exposure and immersion of the sample into a specific developer, only the resist which is hit by the electron beam is removed. Any pattern drawn using the electron beam then appears (Fig 3.2(a), (b)).

![Leo 440 SEM](image)

**Figure 3.1** Simplified schematic of the Leo S440 SEM/NPGS nanolithography system.

For most applications we use a lower-sensitivity resist as the top layer and higher-sensitivity layer as the bottom layer (Fig 3.2(a)). Using this resist bilayer, it is easy to get a large undercut that forms a "cave" under the resist with a small mouth and a big belly (Fig 3.2(b)). The shape of the pattern is determined by the upper layer. The large undercut makes shadow evaporation possible and lift-off of unwanted metal
easy. After evaporating metal (Fig 3.2(c)) and then immersing into a strong solvent (Fig 3.2(d)), excess metal has been removed.

![Diagram showing process steps](image)

- a) Exposure
- b) Development
- c) Evaporation
- d) Lift-off

**Figure 3.2** General processes of sample fabrication.

### 3.2 Sample Fabrication

Now let's follow the whole process of making a sample which includes double quantum dots and an embedded SET. It involves several steps as described below
and the whole process usually takes 2 - 3 days.

Figure 3.3  Processes of making sample.

1. Cut and clean the substrate

Cleave a 2x3 mm$^2$ chip from a 2DEG (for a real sample) or GaAs wafer (for a test or dummy sample). Cut two notches as permanent marks for alignment purposes (Fig 3.3(a)). Ultrasonicate the sample surface with acetone to clean it.
2. **Prepare the resist for alignment marks**

Apply PMMA-MAA as glue on a smooth square copper piece. Stick the chip to the copper piece for safe handling. Bake the copper piece 10 min at 90°C until the glue is totally dry. Then apply PMMA-495K as resist to cover the whole chip. Bake the chip 30 min at 165 °C.

3. **Exposure for the gold alignment marks**

Put a small amount of silver paint on the chip surface for focusing under electron microscope. Record a series of pictures of the two notches cut in step 1 and choose two small and easily distinguished geometric structures for future reference. Use their average coordinate as the position of the starting point of exposure. Draw the pattern of gold alignment marks.

4. **Evaporating gold alignment mark**

Evaporate 2nm Cr to promote adhesion, and then evaporate 18nm Au.

5. **Lift off gold alignment mark**

Immerse sample in acetone for a couple of hours. Ultrasonication can accelerate lift-off. After lift-off, the chip should look as shown in Fig 3.3(b).

6. **Prepare the resist for etching**

Use the same process as in step 2, but with a different resist recipe. First spin on HMDS, at 4000 rpm for 30 sec, and then spin on PMMA-495K at 3000 rpm and bake 30min at 180°C.
7. *Exposure for etching*

Use saved pictures of the two notches to find the two fine structures. Calculate their average coordinate as the rough starting point of exposure. Use the alignment marks and NPGS alignment run file to adjust the position of the stage and the electron beam to precisely locate the starting point. Then start e-beam lithography for the etching. The new pattern should have the same starting point as the first exposure.

8. *Chemical etching*

This step should be done as soon as possible after cutting the chip because future high temperature baking or U-V exposure cleaning can permanently change the properties of the substrate surface and totally invalidate the etching recipe. The etchant we use is a mixture of citric acid (50% solid powder of citric acid and 50% pure water by weight) and 30% H$_2$O$_2$. Mix 15 ml citric acid solution and 0.5 ml H$_2$O$_2$ together. Immerse the chip into the mixture for 1 min at 24.5°C. Then rinse the chip 30 sec with pure water.

9. *Resist removal after etching*

If needed, heat the acetone to accelerate resist removal. Check the etch depth with an atomic force microscope (AFM). The correct depth should be around 50 nm which is sufficient to change the band bending in the heterostructure and destroy the 2DEG beneath (Fig 3.3(c)).

10. *Making Ohmic Contacts for 2DEG*
Apply small indium spots on the surface of the substrate with a soldering iron at temperature 350°C (Fig 3.3(d)). Shape the spots with tweezers. Remove the excess indium over the edge of the chip. Place the chip inside a small chamber with flowing reducing gas (20% H₂ in Argon). Bake the chip at 110°C for 1 min to remove the water vapor and at 400°C for 3 min. The reducing gas reacts with oxygen at high temperature and prevents oxidation of the indium. The indium diffuses into the substrate and connects the conducting layer of the 2DEG to the indium spots on the surface. When the sample cools down, take sample out of the chamber.

11. Bake resist for gold

Use a bilayer resist. The bottom layer is PMMA-495K. Spin 40 sec at 6000rpm, then bake 1 hr at 180°C. The upper layer is PMMA-950K. Spin 40 sec at 6000rpm, then bake 1 hr at 180°C.

12. Lithography for gold

In this step, we are going to make the gold gates, bonding pads for both quantum dots and the SET, and a new set of alignment mark and finder for the final exposure. Again, use two notches as the rough locators to find 2 sets of alignment marks. Do lithography which includes a new sets of alignement marks and finders which can be used for the final lithography for the SET.

13. Evaporate gold

Same recipe as step 4.
14. *Lift off for gold*

Because the two layers are thin and dense, there is no large undercut, and lift-off can take a long time. We usually keep the chip in acetone over night. If the lift-off is still not completed, one can ultrasonicate the sample for a few seconds, which is usually good enough to remove the excess gold (Fig 3.3(e)).

15. *Clean the surface and bake resist for the SET*

Place the chip under a UV lamp for a couple of hours. This can effectively remove the organic contamination on the surface of the chip. Here is the resist recipe for the Al SET. The bottom layer is PMMA-MAA, spun 30 sec at 3000 rpm, and baked 1 hr at 150°C. The upper layer is PMMA-950K, spun 30 sec at 3000 rpm, and baked 1 hr at 150°C.

16. *Exposure for the SET*

Use the finders we made last time to find the 3 sets of alignment marks. Do lithography at 20kV, which promotes a larger undercut due to the stronger secondary electron scattering.

17. *Shadow evaporation and oxidation*

To form the ultra-small sandwich junctions, we use a technique called shadow evaporation. The idea is to tilt the sample mount to evaporate metal at two different angles. We can then use a single mask to generate two sets of patterns with a small displacement. Parts of the two shadows will overlap. By oxidizing the Al between
the two evaporations, we can get the \( \text{Al} - \text{Al}_2\text{O}_3 - \text{Al} \) sandwich junction. This step is shown in Fig 3.4.

![Diagram](image)

**Figure 3.4** Shadow evaporation.

18. *Lift off Al part*

Because of the large undercut, lift-off usually takes only an hour. Now the sample
is ready to wire for measurement.

Above is the whole process of making a sample. It involves numerous steps, and a single mistake can waste all the work. The most difficult part is to control the tunneling resistance of the SET, which is affected by various factors, such as the oxidation time and junction area.

Fig 3.5(a) is the schematic diagram of a sample with two loosely coupled quantum dots and a SET strongly coupled to one of them. Fig 3.5(b) is the picture of a real sample taken by JEOL 6500 electron microscope. The SET is located on the etched part to reduce the pad capacitance. A branch of the central island enters the 2DEG area to detect the tunneling events happening in the QD beneath it.

![Figure 3.5](image)

**Figure 3.5** Sample schematic and micrograph.
3.3 Dilution Refrigeration

![Diagram of dilution refrigerator]

**Figure 3.6** Simplified mixing chamber schematic diagram.

The $^3\text{He}/^4\text{He}$ dilution refrigerator in our laboratory is an Oxford Instruments Kelvinox 100 dilution refrigerator. It is widely used in current research for producing ultra low temperature. Fig 3.6 shows what happens inside the mixing chamber, onto which our sample is attached and cooled. There are two phases for the liquid $^3\text{He}/^4\text{He}$ mixture, the lighter (dark grey in Fig 3.6) concentrated phase which is rich in $^3\text{He}$, and the heavier dilute phase (light grey in Fig 3.6) which is rich in $^4\text{He}$. Because the transition of $^3\text{He}$ atoms from the concentrated phase to dilution phase absorbs heat, the mixing chamber and attached sample will be cooled down. The dilute phase mixture is pumped into the still where the $^3\text{He}$ gas evaporates. The $^3\text{He}$ is liquidized again into the concentrated phase at the condenser (not shown in Fig 3.6). The fraction of $^3\text{He}$ in the dilute phase remains non-zero even at zero temperature,
allowing this process to continue to remove heat at very low temperatures. In our system, the base temperature can reach 20-30mK.
Chapter 4
SET and Quantum Dot Characterization

In this chapter, some basic techniques for characterizing the SET and quantum dot are introduced. Schematic diagrams and representative results are given for each measurement.

4.1 $I - V$ Curve Measurement for the SET

The performance of the SET depends strongly on its operating point. Scanning the $I - V$ curve of the SET is the first step in determining the proper bias voltage.

![Diagram of SET I-V curve measurement](image)

**Figure 4.1** Schematic of the SET $I - V$ curve measurement.

In the experiment, $V_{bias}$ (0 to 12mV) is provided by a homemade voltage sweep box. The voltage amplifier here is a homemade high precision amplifier with a Burr-Brown OPA128 as the input stage with typical of gain setting 1000. The current amplifier transforms the ultra-small current to a voltage with typical gain setting of
$10^8$ V/A. The $I - V$ curve is recorded by the computer and plotted with Igor software.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{iv_curve}
\caption{Representative SET $I - V$ curve.}
\end{figure}

Fig 4.2 is a representative $I - V$ curve, which is strongly non-linear close to its center. The curve is not symmetric with respect to zero bias because of the thermal voltage of about 7mV between the SET and the room temperature leads. The center of the $I - V$ curve is a flat high impedance area which corresponds the Coulomb blockade area of the SET. There are usually two easily recognized peaks corresponding to the JQP and DJQP features.

We only need a rough scan in this step, because accurately determining the correct bias voltage of the SET requires a more complex measurement. We use the I-V curve
measurement to verify that SET is operating and to measure its resistance. Another important result from this measurement is the value of the thermal offset voltage.

4.2 Determining the Resonant Frequency

In this step, we are going to scan the frequency of the input signal and measure the power of the reflected signal for the SET in both low resistance (usually at high bias voltage) and blockade (at the center of the $I-V$ curve) states.

Figure 4.3 Schematic of circuit for determining the resonant frequency.

Fig 4.3 shows the circuit we use to find the resonance frequency of the SET. It includes both an RF part (Fig 4.3(a)) and a DC part (Fig 4.3(b)).

The DC part is almost as same as the I-V curve measurement except the bias
voltage is provided by a homemade precision voltage reference. The voltage is monitored by a digital multimeter. The purpose of DC part is to provide a stable bias voltage for the SET.

The RF part is somewhat complicated. An RF signal of -80dB is generated by an Agilent signal generator. The frequency of the RF signal varies from 500MHz to 1.5GHz which covers the range of interest. A 100KHz amplitude modulation is also included in the input signal. The RF signal is input to the sample through a directional coupler and the reflected signal is amplified by a low temperature HEMT amplifier. The output signal from the HEMT amplifier is amplified 45dB by a GaAs FET amplifier and goes into a lock-in amplifier through a diode which picks up the small 100kHz AC modulation signal. The lock-in amplifier is locked at 100kHz. We compare the two reflected power spectra recorded in the above gap region (with low resistance) and in the Coulomb blockade region (with high resistance) and choose the frequency at which we obtained the maximum difference between the two spectra as the resonance frequency. At that frequency, the SET will provide the maximum modulation when its conductance changes from maximum to minimum.

Another way to determine the resonant frequency is by using a network analyzer (NA). Because the lowest output signal of our NA is around -35dB, a 50dB attenuator is used to protect the SET. We connect the RF output of the NA to the input for the SET and connect the amplified reflected signal of SET to the RF input of the NA. By
measuring the transmission attenuation we can directly obtain the reflected power spectrum of the SET over a wide frequency range.

![Graph showing reflectance spectra with marked resonant frequency](image)

**Figure 4.4** Representative reflected power spectra.

Fig 4.4 shows representative reflected power spectra recorded by the circuit shown in Fig 4.3. We choose the marked frequency, for which the difference between two curves is maximum, as the resonance frequency.

### 4.3 SET Gate Modulation

Coulomb blockade oscillations (CBO) are an important phenomenon in double junction mesoscopic structures, such as a SETs or quantum dots with two barriers. By measuring the cycle of the oscillations, we can calculate the gate charging energy and then the gate capacitance of the SET, which is important for calculating the
charge sensitivity. The schematic of the circuit is shown below. The bias voltage $V_b$ is provided by a precision voltage reference and the voltage applied to the SET gate $V_g$ is provided by the voltage ramp. $V_{gate}$, $V_{bias}$ and $I_{bias}$ are the real readings through the amplifiers. Because of thermal voltages, they may slightly be different from the applied gate $V_g$ and bias voltage $V_b$. The data is collected by a LabView program and drawn by Igor (Fig 4.6).

![Schematic for Coulomb blockade oscillations measurement.](image)

**Figure 4.5** Schematic for Coulomb blockade oscillations measurement.

The oscillations are caused by charging effects in the SET. When an electron goes onto the central island, it lifts its voltage by $e/C_g$. The next electron can then only pass through the SET by obtaining enough charging energy or if the Fermi level of the central island is lowered by the same value (the second case in this experiment). That means the each peak of the oscillation corresponds a change in the SET charge
of one electron, and \( C_g = e/V_{cycle} \), where \( V_{cycle} \) is the period of the oscillations.

![Figure 4.6](image) Coulomb blockade oscillations of the SET.

4.4 Measurement of the \( I(V_{bias}, Q_0) \) Surface of the SET

A measurement of \( I \) versus both \( V_{bias} \) and \( V_{gate} \) can reveal more information of the SET; here we vary both to record a series \( I-V \) curves at different gate voltages. Fig 4.7 shows part of the circuit for measuring \( I(V_{bias}, Q_0) \) Surfaces. The complete circuit is almost as same as that in Fig 4.5 for measuring CBO, but the gate and bias voltages are provided by the computer because the the voltage \( V_{bias} \) must scan forth and back after the gate voltage \( V_{gate} \) is changed. Fig 4.7 only shows the computer controlled voltage scan circuit. To attenuate the 0-10V output voltages generated by
Figure 4.7  Voltage scan circuit for image plot measurement.

The DAC in the computer, dividers are used for both outputs.

Figure 4.8  A series of $I - V$ curves of the SET at different $V_{gate}$.

The schematic diagrams in this chapter basically are all the circuits needed to perform the measurement. Finally, the grounding scheme is also very important for measuring ultra-small signals.
Figure 4.9  Representative false color image plot of $I(V_{bias}, V_{gate})$. Different colors represent different amplitudes of SET current.
Chapter 5
Study of the Sensitivity and Linearity of Superconducting RF-SET

5.1 Introduction

Although the RF-SET has great potential to be a quantum limited linear amplifier, which is suitable for quantum measurements of individual electronic charges, there are still some technical problems that need to be solved, such as the optimal tunneling resistance and operating point, i.e., the bias voltage and the SET offset charge. Both optimizations have great impact on the performance of the RF-SET. This chapter is based on our recent work [15] which discussed optimization of the sensitivity and linearity of superconducting RF-SETs.

The tunneling resistance $R_d$ is a very important parameter for the SET. For a normal state SET, the requirement is that the total tunneling resistance $R_d > R_K = h/e^2 \sim 25.8\text{K}\Omega$. For a superconducting SET, the situation is much more complicated [16]. As discussed in Chapter 2, electrical transport in the S-SET can be divided into two regimes: the above gap regime $(eV_{\text{bias}} > 4\Delta)$ in which quasiparticle tunneling dominates, and the subgap regime $(eV_{\text{bias}} < 4\Delta)$ in which various JQP cycles dominate.

To obtain a good charge sensitivity, we usually prefer higher $R_d$ because the $I-V$ curves of the S-SET with higher resistances have larger modulation with gate voltage
in the above-gap regime. But since the RF-SET is connected to the HEMT amplifier through a low impedance (50Ω) transmission line, impedance matching is especially important for RF measurements. As shown in Fig 5.1, for a $f=1$GHz resonance frequency, as used in our experiment, the inductance $L = 1/(2\pi f)^2 C \sim 100$nH, where $C \sim 0.2$pF is the stray capacitance of the SET pads. Then the total impedance of the LC circuit at resonance is $Z = L/CR \sim 6\Omega$ for $R=50$KΩ. Although generally 50KΩ is already a compromise, it is still not good for impedance matching. For this reason, a lower $R_d$ is favored.

Another important requirement is linear response to a large charge shift. The higher the linear response range, the wider useful bandwidth of the RF-SET since increased coupling to the device under test will lead to an increased signal. For the real-time tunneling event measurement, because the actual switching events happen randomly, strong coupling is essential.

5.2 Experimental Setup

The sample design for this experiment is a simplified version of the design in our real-time tunneling counting experiment. The whole sample is fabricated on a GaAs substrate using e-beam lithography and shadow evaporation. It only has a single SET with two separate gates G1 and G2 (as shown in Fig 5.2). One gate is to set the DC offset charge of the RF-SET and the other, which is connected to a coax cable, is used to provide fast gate signals. We use it to simulate the device under test. The
measurement was done in a dilution refrigerator at a base temperature of \( \sim 30\text{mK} \).

A Nb chip inductor with \( L \sim 120\text{nH} \) was used to form the LC circuit with resonant frequency \( f_{LC} \sim 1\text{GHz} \).

\[ v_r = \Gamma v_{in} \quad Z_0 \]
\[ v_{in} = V_{dc} + v_{rf} \cos \omega t \]

**Figure 5.1** Schematic diagram of the RF-SET.

**Figure 5.2** Al-AlOx-Al SET fabricated on GaAs substrate.
5.3 DC Data Analysis

The measurements and schematics were introduced in chapter 4. We have fabricated three samples with total tunneling resistance $R_d = 58$, $38$ and $24$ kΩ and identified them as S1, S2 and S3. We first measured the $I - V$ characteristics of the three samples for different SET gate voltages, as shown in Fig 5.4.

Fig 5.4 shows the representative I-V characteristics of samples with different offset charge $Q_0$. Note that the $I - V$ curves in each graph have two peaks even in the conventional CB regime. Because the SETs are superconducting at the base temperature of the dilution refrigerator, much more complicated transport mechanisms play roles, such as quasiparticle tunneling, JQP cycles, DJQP cycles and VJQP cycles, which were discussed in Chapter 3. The right peak is the so-called JQP peak while the left peak is the DJQP peak for which the bias voltage is lower and the the JQP cycle is forbidden.

The transport is divided into two regimes: the above gap regime, shown on the right side of dashed line in each I-V curve, where quasiparticle tunneling dominates; and the subgap regime, to the left side of dashed line, where quasiparticle tunneling is forbidden but more complex cycles, such as the JQP and DJQP cycles dominate. As indicated in Fig 5.4, when the tunneling resistance $R_d$ decreases, so does the current modulation in the above gap regime ($eV_{dc} > 4\Delta$). When $R_d$ decreases from $58K\Omega$ (S1) to $38K\Omega$ (S3) then to $24K\Omega$ (S2), the current modulation becomes small
for S2 and almost absent for S3. The mechanism of quasiparticle cotunneling is not straightforward [17], but is comparable with electron transport in the normal state SET. So for samples with resistance $R_d < h/e^2 = 25.8K\Omega$, the Coulomb modulation is destroyed by cotunneling of quasiparticles. The behavior is consistent with previous work, which used samples with high tunneling resistance and chose the bias voltages of SET in the above-gap regime to obtain the best charge sensitivity [9].

Although current modulation caused by quasiparticle tunneling disappears for the low resistance sample S3, the JQP cycles still exist but become less sharp. From the explanation in Chapter 2, the transport in subgap regime occurs by a combination of both quasiparticle and Cooper pair tunneling. If $eV_{dc} > E_C + 2\Delta$, the simplest JQP cycle plays the most important role. It consists of one Cooper pair tunneling through one junction followed by two quasiparticles tunneling through another, as shown in Fig 2.7(a). For a lower bias voltage of $eV_{dc} = 2E_C$, the DJQP cycle (Fig 2.7(b)) dominates. Also there is a candidate process that could allow transport along the Cooper pair resonance line between the JQP and DJQP features, which we call the virtual JQP cycle (VJQP, Fig 2.7(c)). All these JQP cycles are discussed in detail in chapter 2.3.

Fig 5.3(a) shows a false color image plot of a sequential tunneling simulation of $I(V_{bias}, Q_0)$. Comparing with experimental results in Fig 5.3(b), Cooper pair resonance lines $0 \leftrightarrow 2(-1 \leftrightarrow 1)$ are clearly visible in both Fig 5.3(a) and Fig 5.3(b),
Figure 5.3  (a) Sequential tunneling simulation. (b) False color image plot of measured current for S2 at T=20mK.

suggesting that the number of Cooper pairs is well defined. But the quasiparticle tunneling thresholds $1 \rightarrow 0$ and $0 \leftarrow 1$ are less sharp in the experiment than in the simulation, indicating that significant quantum fluctuations for quasiparticles exist.

5.4 RF Results

The charge sensitivity $\delta q$ of the RF-SET is defined as

$$\delta q = \frac{q_0}{\sqrt{BW}} 10^{-SNR/20}$$

where the $q_0$ is the input amplitude and SNR (in dB) is the optimized signal to noise ratio for each input signal. The resolution bandwidth BW is restricted to 1kHz by the spectrum analyzer. The circuit used is the same as for measuring the RF reflected power, except the output is monitored by the spectrum analyzer without a diode. We compare the height of the side peaks versus the noise floor of the power spectrum to get the SNR (Fig 6.7). Typically the input signal is set first and the bias voltage and
offset charge of the SET are adjusted to maximize the SNR.

Fig 5.5 shows the charge sensitivity $\delta q$ and SNR versus input signal $q_0$ in $e$ rms. Charge sensitivity (solid symbols) is plotted on the left axis and the SNR (open symbols) is plotted on the right axis. Linear response is also indicated as the dashed lines for $\delta q$ measured at the smallest $q_0$. The results in Figs 5.5(a) and 5.5(b) are comparable because the same sample S1 with the highest resistance ($58\,\Omega$) was used, but the measurement we performed in different bias voltage regimes. For S1 the best charge sensitivity $\delta q \sim 9 \times 10^{-6} e/\sqrt{\text{Hz}}$ was found in the above-gap regime ($V_{\text{bias}} = 860\,\mu\text{V}$), consistent with previous results. But from Fig 5.5(a), when $q_0$ increases, the measured SNR immediately becomes sublinear, compared with the dashed straight line. The linearity was poor in this regime and sensitivity $\delta q$ worsened quickly. If S1 was operated in the subgap regime as in Fig 5.5(b), its performance was slightly better. We find the best $\delta q \sim 1.3 \times 10^{-5} e/\sqrt{\text{Hz}}$ in this regime, with SNR nearly linear to $q_0 \leq 0.01e$ rms. Figs 5.5(b) and 5.5(c) can be used to compare the effects of different tunneling resistances $R_d$ on the performance of the RF-SET. In Fig 5.5(c), sample S3 with the lowest resistance ($24\,\Omega$) was operated in the subgap regime. Such a low resistance can not be used as tunneling resistance for a normal SET because it is even lower than $R_K \sim 25.8 \,\Omega$. The best operating point occurs at bias voltage $V_{\text{bias}} = 440\,\mu\text{V}$ which is located between the DJQP and JQP features with $\delta q \sim 1.2 \times 10^{-5} e/\sqrt{\text{Hz}}$. Although it is not as good as the best charge sensitivity
we can get from high resistance sample (S1) above the gap, from the Fig 5.5(c),
the linearity is greatly improved. The SNR remains linear and $\delta q$ remains flat until
$q_0 = 0.038 e$ rms.

We can define a parameter

$$\alpha = \frac{\Delta \pi \hbar}{E_C e^2} (R_{J1}^{-1} + R_{J2}^{-1}) = \frac{8E_J}{E_C}$$

to characterize the strength of quantum fluctuations for quasiparticles. After some
calculations, we find that the RF-SET (S3 subgap) with $\alpha \sim 1 - 2$ (strong quantum
fluctuations) shows both good linearity and sensitivity. The RF-SET (S1 above gap)
with $\alpha \sim 1$ (weak quantum fluctuations) shows poor linearity and only slightly better
charge sensitivity.

5.5 Conclusion

We have investigated the influence of the resistance of the SET and bias voltage
regime on the performance of the superconducting RF-SET. While the best charge
sensitivity can be achieved for samples with high tunneling resistance working in the
above-gap regime, the linearity of the SNR is poor. Choosing an operating point
in the subgap regime can improve the linearity with only a small sacrifice in charge
sensitivity. The best balance of sensitivity and linearity can be achieved for low
resistance samples working in the subgap regime, probably because of the influence
of quantum charge fluctuations. The quantum fluctuations likely play important role
in the linearity of the RF-SET, but additional theoretical work is required to answer this question.
Figure 5.4 $I-V$ characteristics of (a)S1 (b)S2 (c)S3
Figure 5.5  Charge sensitivity and SNR as the function of offset charge. (a) S1 above gap (b) S1 subgap (c) S3 subgap
Chapter 6
Real-time Electron Counting Experiment

In this chapter, the first real-time electron counting experiment [18] will be introduced. We use a RF-SET as an ultra sensitive and fast electrometer to detect the tunneling events of a single electron in or out a quantum dot, which is laterally fabricated on 2DEG material and is strongly coupled to the SET.

6.1 Introduction

Ordinary DC measurements are widely used in the study of mesoscopic electronics. In DC measurement what is obtained are average effects: for example, 1 pA current corresponds to 6 million tunneling events per second.

If we can perform real-time electron counting experiment on Coulomb blockade devices, such as a quantum dot (QD), we can obtain information such as tunneling rates, charge occupational probabilities, temporal electron-electron correlation and other dynamic phenomena much more directly. For real-time detection, however the technical requirements are much more strict, because the electron charge is very small and the time scale of the tunneling events can be very short.

The electrometer must be both very sensitive and very fast since a wide bandwidth is necessary to detect random tunneling events. The RF-SET is perfect for these purposes. In addition, the quantum dot must be strongly coupled with the RF-SET
to increase the signal to noise ratio (SNR). We also need to find a way to slow down the electron tunneling rates within the 1 MHz bandwidth of our system.

6.2 System Design

Fig 6.1 shows the design of our quantum dot/RF-SET system. All leads are fabricated on the surface of a GaAs/AlGaAs heterostructure containing a two dimensional electron gas (2DEG) with standard e-beam lithography and evaporation techniques. The junctions of the SET have the AlO\textsubscript{X}/Al/AlO\textsubscript{X} sandwich structure which is fabricated with shadow evaporation introduced in Chapter 3. Five gold gates are used to laterally define the quantum dot on the 2DEG. A small branch of the central island of the SET extends inside the area surrounded by the gold gates. Another gold gate for the SET is fabricated to allow adjustment of the offset charge on the SET island.

![Figure 6.1](image)

**Figure 6.1** The design of a sample with a single quantum dot strongly coupled to an RF-SET.

In our system, the quantum dot is defined in a 2DEG in an GaAs/AlGaAs heterostructure. When the gates located on the surface of the sample are energized,
the electrons beneath the gold gates are depleted. If the voltage on the gold gates is high enough, a small island of electrons remains in the center, forming a so-called zero-dimensional quantum dot. Fig 5.2 is an electron micrograph of a real sample. Dashed lines define the areas where the electron on the 2DEG will be depleted while applying a negative voltage on the QD gates.

![Figure 6.2 SEM Photo of the System](image)

6.3 Vertical Coupling Scheme

We use a vertical coupling scheme to increase the coupling between the SET and the QD: a portion of the SET island extends directly above the QD. When the dot
Figure 6.3  Vertical coupling scheme of our system

charge changes by one electron, it will effectively charge the SET offset charge by $\Delta q$,

$$\Delta q = \frac{C_c}{C_i + C_c} \equiv \kappa e$$

where the coupling coefficient $\kappa$ depends on the ratio of the coupling capacitance $C_c$ and internal dot capacitance $C_i$. The strongly coupled regime can be complex because of the strong interaction between the SET and the QD. If the dot gate voltage is swept, it will cause both the SET and QD offset charges to change. To solve the problem, in our system, the gate is designed to have much stronger coupling to the QD than to the SET. So doubly periodic behavior is expected in the SET current as a function of the gate voltage, as shown schematically in Fig 6.4, in which the faster oscillations are caused by changes in the dot charge and the slower oscillations, shown as the envelope of the oscillations, are caused by direct coupling of the gate to the SET.
Indeed such doubly periodic oscillations are observed in the SET current, as shown in Figs 6.5(a) and (b). We also show the zero bias dot conductance measured at the same time in Fig 6.5(c). The dot conductance shows singly periodic Coulomb blockade oscillations with the same period as the faster oscillations in Figs 6.5(a) and (b), verifying that the fast oscillations are indeed caused by changes in the dot charge. Each peak corresponds a change in the average dot charge of one electron. From the size of the oscillations we can estimate the coupling between the SET and the QD. The coupling coefficient actually increases when the dot is more closed, as shown in Figs 6.5(a) and (b). When the dot is more closed, the internal capacitance $C_i$ of the QD becomes smaller due to its smaller size. That leads to a larger coupling coefficient $\kappa = \frac{C_C}{C_i + C_C}$ and also offset charge shift $\Delta q = \kappa e$. 

Figure 6.4 Doubly Periodic Behavior for Strongly Coupled Case($C_{gs} \ll C_{gd}$).
6.4 Operating Point of the SET

Although while performing this experiment last year we did not carefully study the influence of the tunneling resistance and operating point systematically, the working points of our SETs are consistent with our later study (described in Chapter 5). We have found when the SET has a normal state resistance of around 25KΩ there is relatively large linear region with respect to the input gate signal at different offset charges without sacrificing charge sensitivity too much, as shown in the SET $I - V$ curves in Fig 6.6.

6.5 Characterization of the RF-SET

We characterize the RF-SET by introducing a small offset charge excitation through one of its gates. Fig 6.7(a) shows the power spectrum of the reflected signal with 0.05\textmu{}rms excitation at 100KHz. This small charge excitation is clearly detected by the RF-SET as the side peak around the main peak, as in Fig 6.7(a). After demodulation, the sine wave at 100KHz is recovered, and then detected by a digital oscilloscope, as shown in Fig 6.8. This test clearly demonstrates that the RF-SET in our system has the ability to detect small and fast offset charge changes. In our system, the bandwidth of the RF-SET was limited to 1MHz to increase the signal to noise ratio, and the charge sensitivity is about $2.2 \times 10^{-5} e/\sqrt{\text{Hz}}$. 
6.6 Real-time Measurement for a QD System

For real time tunneling event measurements, one requirement is that the SET should be fast and sensitive enough. Another requirement is that tunneling events must be slowed down because even a 1nA current corresponds to $\sim 10^{10}$ tunneling events per second, which still too fast for existing RF-SETs. Fortunately, laterally defined QDs in a 2DEG are adjustable devices. By adjusting the voltage applied on the QD gates, we can pinch off the chanel between QD and its leads, i.e., change the conductance the two quantum point contacts. If the voltage is high enough, we can totally isolate the QD or can let an electron pass the QD within a long time scale. Fig 6.9 shows the tunneling events captured by the digital scope. By changing the gate voltage we can observe changes in the speed of tunneling events.
Figure 6.5  Current through SET as a function of gate voltage when the dot is (a) relatively open and (b) relatively closed. (c) Conductance of quantum dot measured simultaneously.
Figure 6.6  Optimal working point of SET.

Figure 6.7  Power spectrum of the reflected signal with 0.05 e excitation at 100kHz.
Figure 6.8  Demodulated sine charge excitation (0.05e rms, 100kHz) recorded by digital oscilloscope.

Figure 6.9  Representative electron tunneling events captured by digital scope.
Chapter 7
Conclusion

In this thesis, I have presented our recent work on study of the RF-SET. Two experiments are discussed, especially our recent work on analysis of the influence of the tunneling resistance and operating region on the performance of the superconducting RF-SET. We were the first group to implement the real-time electron counting experiment with RF-SET. Because of the advantages of real-time measurement over traditional DC measurement, we are hoping to get more information with this new, promising technique.

Other projects currently under investigation in the group include:

1 Double QD system coupled to an RF-SET

We propose a new design to study the back action of the SET, consisting of two tunnel coupled quantum dots and an SET electrostatically coupled to one of the dots.

2 Single charge tunneling oscillations

We are searching the single charge tunneling oscillations in a system containing two QPC arrays and a tunnel barrier. If we can detect the flow of single electrons through the tunnel barrier in the nanostructure, we should be able to see oscillations in the current $I$ owing to the periodic charging and discharging across the barrier at a frequency $f = I/e$. This effect was predicted in 1985 but has not been directly
observed.

3 Dynamics of single quantum dot system

This is the extension of our former work on a single dot system. Because the QD has limited number of electrons, it has atom-like properties. We can drive a single QD atom out of equilibrium by applying a transient voltage signal, and observe the dynamics of the dot as it returns to equilibrium.
References


