INFORMATION TO USERS

This manuscript has been reproduced from the microfilm master. UMI films the text directly from the original or copy submitted. Thus, some thesis and dissertation copies are in typewriter face, while others may be from any type of computer printer.

The quality of this reproduction is dependent upon the quality of the copy submitted. Broken or indistinct print, colored or poor quality illustrations and photographs, print bleedthrough, substandard margins, and improper alignment can adversely affect reproduction.

In the unlikely event that the author did not send UMI a complete manuscript and there are missing pages, these will be noted. Also, if unauthorized copyright material had to be removed, a note will indicate the deletion.

Oversize materials (e.g., maps, drawings, charts) are reproduced by sectioning the original, beginning at the upper left-hand corner and continuing from left to right in equal sections with small overlaps.

Photographs included in the original manuscript have been reproduced xerographically in this copy. Higher quality 6" x 9" black and white photographic prints are available for any photographs or illustrations appearing in this copy for an additional charge. Contact UMI directly to order.

ProQuest Information and Learning
300 North Zeeb Road, Ann Arbor, MI 48106-1346 USA
800-521-0600

UMI
RICE UNIVERSITY

A Stochastic Approach for Estimating Fatigue Life of Equipment Located at Topside of FPSO Offshore Systems

by

Juan Wang

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE Master Of Science

APPROVED, THESIS COMMITTEE:

Dr. Pol D. Spanos, Lewis B. Ryon Professor of Mechanical Engineering and Civil Engineering, Chair

Dr. Andrew J. Meade, Associate Professor of Mechanical Engineering

Dr. Satish Nagarajaiah, Associate Professor of Civil and Environmental Engineering

HOUSTON, TEXAS

April, 2002
April, 2002

ABSTRACT

A Stochastic Approach for Estimating Fatigue Life of Equipment Located at Topside of FPSO Offshore Systems

by

Juan Wang

Floating Production, Storage and Offloading (FPSO) Systems are subjected to stochastic wave loads. In this context, an approach for stochastic fatigue analysis of FPSO topside equipment is developed. Proper FPSO transfer functions, and the Ochi-Hubble sea wave elevation spectrum are combined to provide the design spectrum at the topside of the FPSO. The equipment response is simulated by a time series model; it is approximated as the output of digital filters to a band-limited white noise input. The rainflow cycle counting method is applied to the equipment response time history to identify significant cycles that produce fatigue damage. By using a S-N fatigue life curve, and Miner’s linear damage accumulation rule, the fatigue life is estimated for a generic piece of equipment. The results of the rainflow cycle counting method are supplemented by results from a power spectrum based, exclusively, approach.
Acknowledgements

It is difficult to make a complete list for acknowledging everyone who has been involved in helping me in my research work. So I would like to say “Thank you” to all of my friends and colleagues.

However, there are still some special ones to whom I want to express my gratitude. First is my advisor Dr. Spanos. I appreciate all the support and guidance provided by him. It is his always-challenging questions and insightful comments that lead me in the right direction throughout this investigation. And, I am also grateful to my thesis committee members, Dr. Andrew J. Meade and Dr. Satish Nagarajaiah for spending their precious time in revising my thesis, and giving me recommendations on my research work.

Appreciation is expressed also to Berry Peng and Shelby Song of the Fluor Daniel Co., for providing the requisite data, and helping me to evaluate certain aspects of the derived results.

I want to extend my thanks to all the group mates and office mates. It is always a good feeling to work with so many nice persons, Gokturk, Petros, Oguzhan, Nikolaos, Jale, G. Failla, Lea Nicole. I give my special thanks to my Chinese friends who gave me quite a lot support, Xu guang, Yan, Ji wei, Jian xin, Guo quan, Zhi young.

Thanks MEMS graduate coordinator Judith, Don for their help to make my research go on smoothly.

Last, but the most important of all, I want to express my deepest thanks to my family for their spiritual support. It is their love and support that inspires me and helps me to sustain my course.
Table of Contents

Abstract ii
Acknowledgments iii
List of Tables vi
List of Figures vii
Nomenclature ix

Chapter 1. Introduction 1

Chapter 2. Stochastic Wave Loading Modeling 5

2.1 A Brief on Stochastic Process 5

2.2 Stochastic Description of Deep Sea Waves and Sea States 9

2.2.1 Sea Waves Parameters and the Swell 9

2.2.2 Spectral Properties of Sea Waves 10

2.3 Design Spectrum for Further Fatigue Analysis 13

Chapter 3. Wave Loading Time Series Generation 19

3.1 Time Series Models for Stochastic Process 19

3.2 Auto Regressive (AR) Algorithm and Numerical Results 20

3.3 Moving Average (MA) Algorithm and Numerical Results 24

Chapter 4. Rainflow Counting Analysis of Time Series Data 29

4.1 Rainflow Counting and Related Methods 29
4.2 Rainflow Counting Procedure 30

4.2.1 Peak-Valley Sequence and Range Filter 31

4.2.2 Rainflow Counting Method 32

4.2.2.1 Classical Rainflow Algorithm 32

4.2.2.2 Rychlik Rainflow Algorithm 33

4.2.3 Cycle Count Matrix and Level Crossings 36

4.3 Rainflow Counted Equipment Time Series Results 38

Chapter 5. Damage Calculation and Fatigue Life 42

5.1 S-N curves and Palmgren-Miner's rule 42

5.1.1 S-N curve 42

5.1.2 Miner Linear Damage Accumulation Rule 43

5.2 Damage Calculation for Rainflow Counted Data 46

5.2.1 Stationary and Ergodic loads Case 46

5.2.2 Fatigue Life Distribution 47

5.3 Extreme Value Analysis 49

Chapter 6. Concluding Remarks 53

References 55
List of Tables

Table 2-1  Coordinates of ten locations for transfer function  14
Table 3-1  MA filter coefficients: filter order 20  26
List of Figures

Fig. 1.1  FEM model of FPSO hull  
Fig. 1.2  FPSO topside modules  
Fig. 2.1  A realization of a stochastic process  
Fig. 2.2  Stochastic process $x(t)$, and its energy spectrum $S_x$  
Fig. 2.3  Ochi-Hubble sea wave elevation for various peak enhancement factors  
Fig. 2.4  Coordinate system for ten locations used in the calculation  
Fig. 2.5  Acceleration spectra for ten locations  
Fig. 2.6  The maximum and mean acceleration spectrum  
Fig. 3.1  AR model autocorrelation function for the wave loading spectrum  
Fig. 3.2  AR simulation of wave induced FPSO topside acceleration spectrum, compared with the target spectrum  
Fig. 3.3  AR simulation of wave induced FPSO topside acceleration time series  
Fig. 3.4  MA simulation of wave height and acceleration spectrum  
Fig. 3.5  MA simulation of wave induced FPSO topside acceleration time series  
Fig. 3.6  Acceleration spectrum for a generic particular equipment  
Fig. 3.7  MA simulation of wave induced response acceleration time series of FPSO topside located equipment  
Fig. 4.1  Stress hysteresis loop for a fatigue cycle
<table>
<thead>
<tr>
<th>Fig. 4.2</th>
<th>Rainflow counting method</th>
<th>32</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fig. 4.3</td>
<td>Full cycles and half cycles in rainflow counting method</td>
<td>32</td>
</tr>
<tr>
<td>Fig. 4.4</td>
<td>Turning points sequence</td>
<td>33</td>
</tr>
<tr>
<td>Fig. 4.5</td>
<td>Rychlik definition of rainflow cycle</td>
<td>35</td>
</tr>
<tr>
<td>Fig. 4.6</td>
<td>The amplitude, range, and mean of a cycle</td>
<td>35</td>
</tr>
<tr>
<td>Fig. 4.7</td>
<td>Rainflow cycle $F^{RCC}$, min-max $F$, max-min $\hat{F}$ for equipment time history</td>
<td>37</td>
</tr>
<tr>
<td>Fig. 4.8</td>
<td>Rainflow cycle count matrix for the equipment stresses series</td>
<td>38</td>
</tr>
<tr>
<td>Fig. 4.9</td>
<td>Min-max cycle count matrix for the equipment stresses series</td>
<td>39</td>
</tr>
<tr>
<td>Fig. 4.10</td>
<td>Amplitude histogram for rainflow cycle matrix; equipment acceleration</td>
<td>39</td>
</tr>
<tr>
<td>Fig. 4.11</td>
<td>Amplitude histogram for min-max cycle matrix; equipment acceleration</td>
<td>40</td>
</tr>
<tr>
<td>Fig. 4.12</td>
<td>Level crossing intensity for equipment data; equipment acceleration</td>
<td>40</td>
</tr>
</tbody>
</table>
Nomenclature

$R_x(\tau)$ Autocorrelation function of stochastic process $x(t)$

$S_x(\omega)$ The spectral density or power spectrum of stochastic process $x(t)$

$m_n$ The $n$th order moments of power spectrum $S_x(\omega)$

$\sigma_x$ Standard deviation of stochastic process $x(t)$

$p(x)$ Probability density function for stochastic process $x(t)$

$a$ Amplitude of stochastic process $x(t)$

$p(a)$ Probability density function of the amplitude $a$

$\varepsilon$ Spectral width parameter

$H(\omega)$ Frequency transfer function

$S_{\text{wav}}(\omega)$ Sea wave elevation spectrum

$S_{\text{sn}}(\omega)$ Sea spectrum, the high frequency part of $S_{\text{wav}}(\omega)$

$S_{\text{swell}}(\omega)$ Sea spectrum, the low frequency part of $S_{\text{wav}}(\omega)$

$H_s$ Significant Wave, the average of the highest $1/3$ th wave population

$\omega$ Radian frequency

$\omega_p$ Peak radian frequency

$\Gamma$ Gamma function.

$\lambda$ Peak enhancement factor

$H_o(\omega, \theta)$ Elementary wave direction transfer functions

$H_m(\omega, \bar{\theta})$ Main wave direction transfer functions
$D(\omega, \theta_i)$  
Spreading function

$S_v$  
Velocity spectrum

$S_a$  
Acceleration spectrum

$S_{mean}(\omega)$  
Mean of the acceleration spectrum

$\sigma_{S_a(\omega)}$  
Standard deviation of acceleration spectrum

$a_k, G$  
AR filter coefficients

$H_{AR}(\omega)$  
Transfer function of the AR difference equation

$W(n)$  
Stationary zero-mean white noise process with power spectrum $S_w(\omega) = 1$

$P_{AR}(\omega)$  
Simulated acceleration or displacement spectrum through AR model

$p$  
AR filter order

$q$  
MA filter order

$b_c$  
MA filter coefficient

$TP(Y, \cdot)$  
Sequence of turning points of stress process $Y(t)$

$m_k, M_k$  
Local minimum, and local maximum

$(m_k^{RFC}, M_k)$  
$k$th rainflow cycle pair

$F^{RFC}$  
Rainflow cycles matrix

$F$  
Min-max matrix

$\hat{F}$  
Max-min matrix

$N(s)$  
Number of cycles at stress amplitudes $s$ when failure occurs obtained from the S-N curve.
\[ K \quad \text{Material dependent random variable} \]

\[ \varepsilon, \beta \quad \text{Material constant} \]

\[ D_i \quad \text{The fraction of damage suffered by the structure component due to } n_i \text{ cycle at stress level } s_i \]

\[ T_f \quad \text{Service lifetime of the component} \]

\[ \Delta D_i \quad \text{Damage rate associated with a sample time } t \]

\[ N^*(s; T) \quad \text{The expected number of crossings of the level } s \text{ with positive slope in the interval } [t, t + T]. \]
Chapter 1

Introduction

The discovery of oil at deep sea regions around the world has stimulated design and erection of a wide range of offshore platforms. As a result, Floating Production, Storage and Offloading (FPSO) Systems are gaining growing importance in the development of offshore oil fields. They have already been installed in about 50 offshore fields, and it is expected that about 5 to 10 FPSO systems will be put into use each year in the near future [1].

FPSO systems are ship shaped floating structures, which are coupled by means of rotary bearings to a cylindrical structure (the turret) permanently constrained to the sea bottom by means of a mooring line spread. They can support the crude oil treatment facilities, and provide the oil storage capacity. The structure is capable of weathervane around the turret in order to minimize the environmental loads acting on it. A system of flexible risers connects the turret to the sea bottom equipment, and a system for transferring fluids, energy, and control between the turret and the structure for all its possible headings.

Being a relative new concept for production of oil and gas and breaking new grounds in terms of the environmental conditions, a very thorough design process is required for floating production systems.

FPSO systems operate in a very complex and hostile environment, and are subjected to a number of loads during its lifetime. There are mainly two kinds of loads acting on these platforms: functional loads, and environmental loads. Functional loads are quasi-
static, which vary slowly over time, due to the steel weight, the ballast, the desk loads, and the reaction forces. Environmental loads are caused by waves, currents, and wind, which at most cases are variable. The wave loads in particular are dominating for the main part of the offshore structure. The waves and the associated structural response have a stochastic nature. The long-term behavior of loads is non-stationary, and non-Gaussian. In a short time interval (hours), the statistical properties of the sea state may be considered stationary. Thus, in this study, the theory of stationary stochastic processes is extensively used for their description.

All loads that are time-varying in magnitude and/or direction will cause stress variations in the structure, and may lead to fatigue damage, which is a very important issue in the offshore industry. Fatigue is the phenomenon that materials, and structure, are "aging" or "degrading" due to the cumulative effect of many time-varying external loading cycles. It is a quite complicated process, and the fatigue life is often divided to crack initiation and crack growth period. For safety design purpose, the fatigue problem considered in this study is within the crack initiation period.

The most catastrophic scenario of a FPSO includes structural failure of hull girders due to extreme bending moments. A methodology for the time-variant reliability assessment of FPSO hull girders subjected to degradations because of corrosion and fatigue has been presented in [2-3]. Some industrial soft wares have integrated the hydrodynamic analysis of the hull, and the load predictions for design and fatigue analysis based upon a 3D finite element model, as shown in Fig. 1.1. Coupled hull-mooring-riser systems analysis based on non-linear response in the time domain has also been considered in these soft wares. But, there is a lack of a comprehensive mathematical
formulation dealing with wave induced fatigue for the topside modules. These topside facilities consist of several super-modules (processing, wellhead, mud, utilities, and accommodation), as well as some topside mounted structures (helideck, flareboom, piperack, main and auxiliary lifeboat stations, and two drilling modules). Some of them are two-story steel truss structures, and some are one-story, as shown in Fig. 1.2. This study is devoted to formulating a methodology to estimate wave induced fatigue life of equipment located on the FPSO topside.

![FEM model of FPSO hull and FPSO topside modules](#)

Fig. 1.1 FEM model of FPSO hull  Fig. 1.2 FPSO topside modules

The wave loading time series is obtained either from prototype measurements or computer simulation. In this context, discrete on-site transfer function values at different locations (modules) in the topside of the FPSO for various spreading directions are obtained directly from industry, and incorporated in this analysis. Further, the Ochi-Hubble sea wave spectrum, which can represent various sea conditions, is used, and is multiplied by the modulus of proper transfer functions to derive a design spectrum for
FPSO topside systems. Also, the design spectrum is treated as a target spectrum for generating a stochastic process, which has a compatible spectrum. In this context, a rainflow cycle counting process and a spectral approach are developed to estimate the fatigue life of a generic piece of equipment located at the FPSO topside.

Chapter 1 provides an overview of the scope of the thesis. Chapter 2 focuses on an approach for stochastic modeling of sea wave induced loads. Chapter 3 addresses the synthesis of spectrum compatible realizations of acceleration on the FPSO topside, and of response of typical topside located equipment. Chapter 4 discusses rainflow cycles counting procedures and algorithms. Chapter 5 provides damage calculations and fatigue life estimations of a typical piece of equipment presumably located at the FPSO topside by using either the rainflow cycle counting procedure or a spectral approach. Finally, Chapter 6 provides a synopsis of the work and some concluding remarks for future work.
Chapter 2

Stochastic Wave Loading Modeling

Fatigue analysis is an integral part of the design cycle for systems operating in a sustained vibration environment. The stochastic modeling and simulation of random processes provide a powerful tool for random vibration analysis of systems. Therefore, a brief introduction to stochastic processes is given in this chapter. Later, the stochastic processes theory is used to describe the stochastic wave loading on FPSO topside modules. A design acceleration spectrum is calculated for further fatigue analysis by combining the Ochi-Hubble sea wave elevation spectrum, and the transfer function data obtained for a particular FPSO system.

2.1 A Brief on Stochastic Process

The theory of stochastic processes has been extensively studied in a number of books [4-6]. A brief introduction of some basic terms is given in the following.

The basic nature of a stochastic process may be understood by considering a typical history realization of such a process shown in Fig. 2.1.

![Fig. 2.1 A realization of a stochastic process](image)
The value of the process at time $t$ is denoted by $x(t)$. The process is commonly described by its statistical properties, such as mean value, standard deviation etc. A process is said to be stationary if these statistical properties do not vary with time. Many processes maybe considered as stationary if the time interval is thought short enough. In this study, in a short time interval (hours), the statistical properties of the sea state may be considered stationary.

The autocorrelation function of a stationary stochastic process $x(t)$ is defined as:

$$R_x(\tau) = E\left[ x(t) \cdot x(t + \tau) \right]. \quad (2.1)$$

Here, $\tau$ is any time interval, and $E$ denotes the operator of mathematical expectations. Since the sea surface variation about the mean water level is zero, the autocorrelation function for $\tau = 0$ is equal to the variance of the process:

$$R_x(0) = E\left[ x(t)^2 \right] = \sigma_x^2. \quad (2.2)$$

The one-sided spectral density or power spectrum is related to the autocorrelation by the equation

$$S_x(\omega) = \frac{1}{\pi} \int_{0}^{\infty} R_x(\tau)e^{-j\omega\tau} d\tau. \quad (2.3)$$
Here, $\omega$ is angular frequency. The energy spectrum shows how the energy is distributed over the various frequencies, as shown in Fig. 2.2.

![Stochastic process and energy spectrum](image)

**Fig. 2.2** Stochastic process $x(t)$, and its energy spectrum $S_\omega$.

The moments of the energy spectrum are defined as

$$m_n = \int_0^\infty \omega^n S_\omega (\omega) d\omega,$$  \hspace{1cm} (2.4)

where, $m_n$ denotes the $n$th order moments. The zero-order moments gives the area under the spectral curve. It represents the total energy of the process. The zero order moment is also equal to the variance of the process. Specifically,

$$m_0 = \int_0^\infty S_\omega (\omega) d\omega = \sigma^2,$$  \hspace{1cm} (2.5)
where, $m_0$ is the zero order moments, and $\sigma_z$ is the standard deviation.

In many engineering problems, the processes may be assumed to be Gaussian with zero mean. The probability density function of the process is then expressed as

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma_z} \cdot e^{-\frac{x^2}{2\sigma_z^2}}. \quad (2.6)$$

The amplitude of this process is distributed according to the Rice distribution, and the probability density function is

$$p(a) = \frac{1}{\sqrt{2\pi}\sigma_z} \cdot \left[ \varepsilon \cdot \exp\left(-\frac{a^2}{2\sigma_z^2}\varepsilon^2 \right) + \sqrt{1-\varepsilon^2} \cdot \frac{a^2}{\sigma_z^2} \cdot \left[ \int e^{\varepsilon \sigma_z - \frac{a^2}{2\sigma_z^2} \cdot \varepsilon^2} \cdot \exp\left(-\frac{1}{2} y^2\right) \cdot dy \right] \right] \right]. \quad (2.7)$$

Where, $a$ is the amplitude, and $\varepsilon$ is the spectral width parameter defined by the equation

$$\varepsilon = \left( 1 - \frac{m_z^2}{m_0 \cdot m_4} \right)^{1/2}. \quad (2.8)$$

Many engineering problems in industry can be treated as linear systems, where the relation between the input or excitation $x(t)$ and the output or response $y(t)$ is described by a linear differential equation with constant coefficients. An example of such a system is the wave loading on the offshore structures, where the ocean wave forces may be regarded as the excitation and the stresses in a point of the structures as the response. The
amplitudes of the excitation and the response are related through the transfer function $H(\omega)$. The energy spectrum of the response process is given by the equation

$$S_r = |H(\omega)|^2 \cdot S_s(\omega).$$  

(2.9)

2.2 Stochastic Description of Deep Sea Waves and Sea States

Sea is a large body of water bounded by irregular shorelines, an undulating permeable bottom, and a wavy free surface. Waves at the sea surface are generated primarily by the wind blowing over the water surface, but continue to exist after the wind has ceased to affect them. And they are one of the most complex and ever-changing phenomena in nature. In deep sea [7]($d/L > 0.5$), where FPSO systems are operated, the sea bottom does not influence the surface waves, the symbol $L$ denotes the wave length or horizontal distance between successive crests, and $d$ is the distance from the mud line to the mean wave line.

2.2.1 Sea Waves Parameters and the Swell

A wave relates to the profile between two successive up (or down) crossings of the zero line. The wave period, $T_u$, is the time interval between the two successive zero-up-crossings. The wave height, $H$, is the vertical distance between the highest and the lowest points between the two successive zero-up-crossings. Significant Wave ($H_s, T_s$) is the average of the highest $1/3$ th of the wave population. Usually, the sea state is defined in terms of the significant wave height and the mean zero crossing rates.
When the waves move out of the generation area into an area of calm winds or if the wind ceases to blow, they change to swells, start to decay, and slowly change the shape. A swell travels a very long distance without losing its identity, although the life of an individual wave is limited to a much shorter period. Thus, swells are very important in a consideration of the wave climate [8].

2.2.2 Spectral Properties of Sea Waves

Waves of different periods and heights are superimposed on another to form the wave spectrum. The wave spectrum throughout this study is the energy density computed in terms of the water surface elevation. Gross observations of the sea have, generally, confirmed a Gaussian distribution for the water surface elevation. In general, zero-mean stationary Gaussian process can be completely described by a power spectrum.

In this study, the wave spectrum is of the Ochi-Hubble spectrum type [8]. It is a modified Bretschneider spectrum with the parameter $\lambda$ being a peak enhancement factor. The Ochi-Hubble spectrum decomposes the sea states into a high and low frequency component each with the form

$$S_{\text{spec}}(\omega) = \frac{1}{4} \left[ \frac{4\lambda + 1}{\Gamma(\lambda)} \right]^{\frac{1}{2}} \frac{H_s^2}{\omega^{2\lambda+1}} \exp \left[ -\left( \frac{4\lambda + 1}{4} \right) \left( \frac{\omega_p}{\omega} \right)^4 \right]. \quad (2.10)$$

where, $\omega$, $\omega_p$ denote the radian frequency, and the peak radian frequency, respectively.

Further, $\Gamma$ is the gamma function. The parameters $\omega_p$, $\lambda$ and $H_s$ are both for the swell
(low frequency), and the sea (high frequency). Adding the individual sea and swell spectrum together one can obtain the total wave spectrum:

\[ S_{\text{swell}}(\omega) = S_{\text{sea}}(\omega) + S_{\text{swell}}(\omega). \]  \hspace{1cm} (2.11)

If the amount of energy in the two spectrums is comparable, the resulting total spectrum will have two distinct peaks, which is termed as bimodal spectrum. For the Kizomba FPSO studied here, which operates at a relatively mild environment, the \( S_{\text{sea}}(\omega) \) part is so small that it is neglected. Fig. 2.3 shows Ochi-Hubble spectrum for various values of the parameters \( \lambda \).

![Graph showing Ochi-Hubble sea wave elevation for various peak enhancement factors](image)

Fig. 2.3 Ochi-Hubble sea wave elevation for various peak enhancement factors

In this study, the parameters used for Ochi-Hubble spectrum are:
$T_p = 10.36 \text{ s, } \omega_p = 2\pi / T_p \quad \lambda = 6$ and $H_T = 1.5 \text{ meters.}$ The frequency interval used is $[\omega_1, \omega_{33}]$ with $\omega_1 = 2\pi / 35$ and $\omega_{33} = 2\pi / 3$.

A confused sea surface forms a complex wave pattern made up of small and large wavelets moving in many directions. The elementary wave direction transfer function collected from offshore software SESAM for each spreading direction is given at frequency $\omega_1, \omega_2 \ldots, \omega_{33}$. The squared moduli of elementary wave direction transfer functions $H_s(\omega, \theta_i)$ are multiplied with spreading function weights to generate the modulus of the wave height transfer function for each main wave direction $H_s(\omega, \bar{\theta})$. Specifically,

$$|H_s(\omega, \bar{\theta})|^2 = \sum_{i=1}^{n} D(\omega, \theta_i)|H_s(\omega, \theta_i)|^2.$$  \hfill (2.12)

To ensure that the total energy represented by the directional spreading is correct, the spreading function much satisfy the normalization condition

$$\sum_{i=1}^{n} D(\omega, \theta_i) = 1.$$ \hfill (2.13)

There are various forms of the wave spreading function in use \cite{9}. The spreading function used in this study is

$$D(\omega, \theta) = \begin{cases} \frac{2}{\pi} \cos^2 \theta & |\theta| \leq \frac{\pi}{2} \\ 0 & |\theta| > \frac{\pi}{2} \end{cases}.$$ \hfill (2.14)
It is employed in many applications because of its simple form.

The Ochi-Hubble sea wave spectrum is multiplied by the modulus of the transfer function of the main wave direction to provide the combined spectrum.

\[ S_d = |H_\theta(\omega, \bar{\theta})|^2 \cdot S_{\text{swr}}(\omega) \]  \hspace{1cm} (2.15)

Dynamic analyses of offshore structures require the determination of flow induced excitation forces. The classical method for the calculation of these forces is based on the Morison Equation. This approach requires the knowledge of the component of water velocity and acceleration \( S_v \) and \( S_a \). The velocity and accelerations can be simulated using the relationship in the following:

\[ S_v = \omega^2 \cdot S_d(\omega) \]  \hspace{1cm} (2.16)

and,

\[ S_a = \omega^4 \cdot S_d(\omega) \]  \hspace{1cm} (2.17)

2.3 Design Spectrum for Further Fatigue Analysis
The on site transfer functions at ten different locations in the FPSO topside for various spreading directions have been obtained directly from the SESAM software.

The coordinates of the ten locations are listed in Tab. 2.1, and the coordinates axes are shown in Fig.2.4.
The acceleration spectra of the ten locations are shown in Fig. 2.5. The results discussed here, pertain to the surge direction.

**Table 2.1 Coordinates of ten locations for transfer function**

<table>
<thead>
<tr>
<th>Joint number</th>
<th>Module Number</th>
<th>x</th>
<th>y</th>
<th>z</th>
<th>story</th>
</tr>
</thead>
<tbody>
<tr>
<td>21098</td>
<td>a</td>
<td>187.3</td>
<td>12.6</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>14490</td>
<td>e</td>
<td>120</td>
<td>15.75</td>
<td>24.25</td>
<td>2</td>
</tr>
<tr>
<td>15489</td>
<td>f</td>
<td>90</td>
<td>47.25</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>20137</td>
<td>k</td>
<td>190</td>
<td>47.25</td>
<td>24.25</td>
<td>2</td>
</tr>
<tr>
<td>21055</td>
<td>l</td>
<td>155</td>
<td>15.75</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>23090</td>
<td>n</td>
<td>160</td>
<td>47.25</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>28069</td>
<td>r</td>
<td>80</td>
<td>31.5</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>28065</td>
<td>s</td>
<td>120</td>
<td>31.5</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>30626</td>
<td>u</td>
<td>150</td>
<td>31.5</td>
<td>48.5</td>
<td>2</td>
</tr>
<tr>
<td>30065</td>
<td>v</td>
<td>195</td>
<td>31.5</td>
<td>48.5</td>
<td>2</td>
</tr>
</tbody>
</table>

The dynamic analysis that is carried out as part of the design for maximum forces, stresses and deflections is based on the maximum sea state. For each location, the stress (acceleration) spectrum is calculated using Eq. (2.2.16). Furthermore, the statistical properties of the wave, the mean and standard deviation spectrum, are obtained by averaging these stresses spectra. Thus, the maximum spectrum for general dynamic analysis, as shown in Fig. 2.6, is

\[
S_{\text{max}} = S_{\text{mean}}(\omega) + 3 \cdot \sigma_{S_{\omega}}
\]

where, the \( \sigma_{S_{\omega}} \) is the standard deviation of the spectral values corresponding to the frequency \( \omega \).
Fig. 2.4 Coordinate system for ten locations used in the calculation
Acceleration spectra for ten various locations, $x(\text{rad/sec})$, $y(\text{m}^2/\text{sec}^3)$

Fig. 2.5 Acceleration spectra for ten locations

Fig. 2.6 The maximum and mean acceleration spectrum
Since the maximum acceleration spectrum is the one, which generally, produces the most unfavorable condition of the structure, it gives too large an amount of added mass to the structure. For the ensuing fatigue analysis, it is decided to use the mean acceleration spectrum as the design spectrum.

Obviously, this input acceleration can be considered in conjunction with the response of a dynamic model for a generic piece of equipment located on the topside of FPSO, as shown in Fig.2.7, with mass $m$, stiffness $K_{eq}$, and cross section area $A$.

![Fig. 2.7 A generic piece of equipment](image)

Attention is next focused on the significant natural modes of the equipment. Therefore, the equipment motion will be represented by the equation

$$\ddot{y} + 2\xi \omega_n \dot{y} + \omega_n^2 y = -\ddot{x}(t).$$

(2.19)

Here, $x$ is the input wave loading. $y$ is the response displacement of a generic piece of equipment, and $\xi$ and $\omega_n$ are the damping coefficient, and the natural frequency of the equipment, respectively. By applying Eq. (2.9), the response spectrum is expressed as
\[ S_{d_e} = \frac{S_{\text{mean}}}{\left(-\omega^2 + \omega_n^2\right) + (2\zeta\omega\omega_n)^2}, \]  

(2.20)

where \( S_{d_e}(\omega) \) is the equipment displacement spectrum. Applying Eq.(2.17), the equipment acceleration spectrum \( S_{a_e}(\omega) \) is written as

\[ S_{a_e}(\omega) = \omega^2 \cdot S_{d_e}(\omega). \]  

(2.21)

Clearly, the stresses at the equipment cross section area is given by the equation:

\[ \sigma_{\text{equipment}} = \frac{ma}{A}. \]  

(2.22)

Therefore, the spectrum of stress at the interface of the equipment is given by the equation

\[ S_{\sigma}(\omega) = \frac{m}{A} S_{a_e}(\omega). \]  

(2.23)

The next significant step becomes the digital generation of a time series, which is compatible with the target power spectrum \( S_{\text{mean}} \) and \( S_{a_e} \), for further fatigue analysis.
Chapter 3

Wave Loading Time Series Generation

In this chapter, the ocean wave data are treated as realizations of a stationary random process. It is sought to approximate the target spectrum by the power spectrum $P(\omega)$ of the output of digital filters to band-limited white noise input.

3.1 Time Series Models for Stochastic Process

Time series modeling seeks to formulate an algorithm by which a sequence of values are generated representing possible observations of a stochastic process at discrete values of the indexing parameter usually time. The model parameters are then estimated on the basis of a comparison of estimated statistics of the generated sequence with those of the actual observations of the stochastic process, which are treated as sample functions drawn out of an ensemble.

The fundamentals of time series analysis and modeling have been treated in texts such as [11-12], in which the basic characteristics of the various models, their identification, estimation, and validation procedures are covered in considerable detail. Of particular importance are the stationary time series models: Auto Regressive (AR), Moving Average (MA), and Auto Regressive Moving Average (ARMA).

Application of time series modeling and simulation techniques to random vibration, of which fatigue analysis is one part, is only of relatively recent origin. Gersh [13] was perhaps the first to systematically extend the concepts of time series modeling to the application of mechanical and structural engineering problems. Later, they developed a
multivariate model in a series papers. Reed and Scanlan [14] made use of scalar ARMA models to characterize and simulate fluctuating wind velocity time histories. An AR model has been used by Wyatt and May [15] for fluctuating velocity component. Spanos [16,17] used scalar AR, MA, ARMA models to simulate ocean wave surface elevation. They proposed a least square procedure, and the parameters of the model were determined to minimize the sum of the square of the difference between the spectrum of the time series derived from the ARMA models, and that of the target spectrum. Further, Spanos and Mignolet [18,19] presented a unified approach for the development of simulation algorithms based on an approximation of a target spectral matrix by the response of an AR discrete dynamic system to white noise excitation. The ARMA procedures were exemplified by application to spectra encountered in various technical areas, such as earthquake engineering (Kanai-Tajimi spectrum), ocean engineering (Pierson-Muskowitz spectrum), and wind engineering.

3.2 Auto Regressive (AR) Algorithm and Numerical Results

Assume that the process $x(n)$ is the output to a white-noise input of a linear, time-invariant system, $p^{th}$ order AR filtered, and represented by the equation

$$\sum_{k=1}^{p} a_k x(n - k) = GW(n) .$$

(3.1)

The symbol $a_k$ and $G$ denote the unknown AR filter coefficients, $W(n)$ is stationary zero-mean white noise process with power spectrum $S_{ww}(\omega)=1$. The transfer function of the AR difference equation is
\[ H_{AR}(\omega) = \frac{G}{1 + \sum_{k=1}^{p} a_k z^{-k}} \]  

(3.2)

Using Eq.(2.9), the relationship between the input and output power spectrum becomes

\[ P_{AR}(\omega) = |H_{AR}(\omega)|^2 \cdot S_w = \left| \frac{G}{1 + \sum_{k=1}^{p} a_k z^{-k}} \right|^2 \]  

(3.3)

To find the unknown coefficients, the condition

\[ \int_{\omega_1}^{\omega_2} \frac{S_{\text{mean}}(\omega)}{P_{AR}(\omega)} d\omega = \text{minimum} \]  

(3.4)

must be satisfied, which produces the following Toeplitz matrix equation

\[
\begin{bmatrix}
R_s(0) & R_s^T(1) & \cdots & R_s^T(p-1) \\
R_s(1) & R_s(0) & \cdots & R_s^T(p-2) \\
\vdots & \vdots & \ddots & \vdots \\
R_s(p-1) & R_s(p-2) & \cdots & R_s(0)
\end{bmatrix}
\begin{bmatrix}
a_1 \\
a_2 \\
\vdots \\
a_p
\end{bmatrix}
= 
\begin{bmatrix}
R_s(1) \\
R_s(2) \\
\vdots \\
R_s(p)
\end{bmatrix}
\]  

(3.5)

where, \( R_s(k) \) denotes the value of the auto correlation function of \( S_{\text{mean}}(\omega) \) calculated at a time lag \( \tau = kT \); it is obtained by the equation

\[ R_s(k) = \int_{-\pi}^{\pi} S_{\text{mean}}(\omega) \cos(k \omega T) d\omega ; \quad k = 0, 1, \ldots \]  

(3.6)
Further, $G$ can be obtained by the equation

$$G^2 = \frac{T}{2\pi} \left[ R_0 + \sum_{k=1}^{p} a_k R_k \right]. \quad (3.7)$$

Also, the time series can be generated by recursive procedure

$$x_n = x(nT) = GW_n - \sum_{k=1}^{p} a_k x_{n-k}. \quad (3.8)$$

The minimum order is decided by the integer part of the number $\tau_{\text{max}} / T$, where $\tau_{\text{max}}$ is the time lag $\tau$, beyond which the autocorrelation function is considered negligible. Fig. 3.1 shows the autocorrelation function for the spectrum. The simulation results are shown Fig. 3.2 and 3.3. The filter order used here is $p = 27$.

Fig. 3.1 AR model autocorrelation function for the wave loading spectrum
Fig. 3.2 AR simulation of wave induced FPSO topside acceleration spectrum, compared with the target spectrum.

Fig. 3.3 AR simulation of wave induced FPSO topside acceleration time series.
Since AR model are best suitable for treating all pole spectra, the numerical results show some predictable erratic features. For the spectra considered in this study, Moving Average (MA) algorithm is more approximate.

### 3.3 Moving Average (MA) Algorithm Numerical Results

Any stationary process can be regarded as the output of an infinite order filter, which has the pulse transfer function expressed in the form

\[ H_{MA}(\omega) = \sum_{v=0}^{\infty} b_v z^{-v} \quad . \]  

where \( q \) is the filter order, and \( b_v \) is the filter coefficient, determined by the equation

\[ b_v = \frac{T}{\pi} \int_{-\pi}^{\pi} \sqrt{S_{\text{meas}}(\omega)} \cos(v\omega) d\omega \quad . \]  

To find the unknown coefficients, the condition

\[ \frac{T}{2\pi} \int_{-\pi}^{\pi} \left| \sqrt{S_{\text{meas}}(\omega)} - H_{MA} \right|^2 d\omega = \text{minimum} \quad (3.11) \]

must be satisfied.
Furthermore, a time series can be synthesized by using recursive numerical procedure

\[ x_n = \sum_{\nu = q}^{q+1} b_\nu W_{n-\nu}, \; q \to \infty. \]  

(H3.12)

Herein, \( x_n \) is the simulated wave loading process, which possesses a power spectrum \( P_{\text{MA}}(\omega) \) identical to the design spectrum. In fact, the stochastic process obtained though MA method can be thought a weighed average of \( 2q + 1 \) white noise deviates moving in time. Fig. 3.4 shows good matching between the simulated wave height spectrum and acceleration spectrum. The filter order used here is \( q = 20 \).

![Wave Height Spectrum](image1)

![Acceleration Spectrum](image2)

Fig. 3.4 MA simulation of wave height and acceleration spectrum
The time series generated with compatible spectrum is also shown in Fig 3.5.

![Graph]

Fig.3.5 MA simulation of wave induced FPSO topside acceleration time series

The filter coefficients $b_n$ are listed in Tab. 3.1.

<table>
<thead>
<tr>
<th>$b_n$</th>
<th>$b_{12}$</th>
<th>$b_{22}$</th>
<th>$b_{32}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
<td>0.0782</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_2$</td>
<td>-0.0218</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_3$</td>
<td>-0.0835</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_4$</td>
<td>-0.054</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_5$</td>
<td>-0.038</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_6$</td>
<td>-0.0426</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_7$</td>
<td>0.0653</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_8$</td>
<td>0.2343</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_9$</td>
<td>0.2343</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{10}$</td>
<td>-0.131</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{11}$</td>
<td>0.3712</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1 MA filter coefficients: filter order 20
The wave loading time series results $x_n$ can be considered as the excitation or input for any specific structure module. The response or the output $y_n$, which is necessary for further fatigue analysis, must be obtained by considering various frequency transfer functions, which include the information about the module natural frequency, damping coefficients etc. Obviously, the time history, which has been produced, can be used for fatigue calculation of both linear and nonlinear structures of mechanical systems. However, for the case of linear system, the previous digital synthesis can be used to produce any time series of linear system response due to the effectiveness of the MA approach. It is used exclusively for the synthesis of a time series, which represents the response of an equipment module.

The pertinent results are shown for a piece of equipment module, shown in Fig. 2.7, with $\zeta = 0.02$ and $\omega_n = 0.5\omega_p$ in Fig. 3.6 and 3.7. It can be seen that the acceleration of the equipment is with in $\pm 1.5 \, g$, which is the gravity acceleration.

Fig. 3.6 Acceleration spectrum for a generic particular equipment
Fig. 3.7 MA simulation of wave induced response acceleration time series of FPSO
topside located equipment

Therefore, the spectrum of stress at the interface of the equipment, shown in Fig. 2.7,
is obtained by using Eq.(2.23).
Chapter 4

Rainflow Counting Analysis of Time Series Data

In many cases the results of an experimental or an analytical investigation of the stress and strains produced in the structure by the loads imposed upon it are plots that follow the stress (strain) as a function of time. These time series are typically generated from an experimental investigation of the structure, or a numerical simulation as the algorithm described in Chap. 3. In this chapter, the necessary time series generated in the preceding chapter will be Rainflow counted for further fatigue life estimation.

4.1 Rainflow Counting and Related Methods

Various counting methods have been developed to reduce cyclic time histories to some simple form of cycle count for analysis and testing purposes. The three basic counting methods are crossing count, peak count, and range count. Numerous variations of each method have evolved [20], mainly to eliminate higher frequency-lower amplitude cycles or to match some preconceived theory of fatigue damage accumulation or crack propagation. All of these methods basically count one parameter and eliminate all knowledge of the loading sequences. Since two independent parameters are needed to define a cycle, assumptions have to be made about the other parameter, and these methods are thus sometimes found to be inadequate. As a result, two-parameter counting methods, such as the range-pair-range, rain-flow and racetrack method [21-24], have been developed. These methods reflect some aspects of loading sequence, and are
extensively used in conjunction with local stress-strain notch fatigue damage calculation schemes. Rainflow counting, which was introduced by Endo [25] in 1968, is perhaps the most widely accepted method for the identification of fatigue critical events and is useful when pursuing a basic understanding of material behavior.

The industry has drawn upon their techniques for converting time series into rainflow cycle count matrices. A number of numerical techniques are available from these previous studies. The technique typically used is the rainflow counting algorithm presented in Downing and Socie [26]. A complete description of this algorithm is also provided in Rice et al. [27]. Wu and Kammula [28] have developed a real-time algorithm for rainflow counting techniques that reduces memory requirements and speeds computations. A new definition of the rainflow cycle equivalent to the original one was given by Rychlik [29-51]. In this study, the rainflow counting method is based on Rychlik's definition.

4.2 Rainflow Counting Procedure

After the pre-processor and filter, the input to the rainflow counting algorithm is a simple series of peaks and valleys (troughs), local maxima and minima, that form hysteresis loops, see Fig. 4.1. The rainflow counting algorithm proceeds by matching peaks and valleys to form closed hysteresis loops. Each local maximum is used as the maximum of a hysteresis loop with amplitude, which is computed by the rainflow algorithm.
4.2.1. Peak-Valley Preprocessor and Range Filter

The first step in data reduction for detailed fatigue analysis is to determine the sequence of peaks and valleys in the loading history. A peak is defined as the point at which the first derivative of the load history changes from positive to negative. A valley is defined as the point at which the first derivative of the load history changes from negative to positive. A valley is also known as a trough. All these peaks and valleys can also be called turning points.

To avoid counting the many small cycles that result from numerical jitter in the time series, and/or to eliminate the many small cycles that do not contribute significantly to the damage of the structure, a range filter is typically used. This filter requires that successive local extremes must differ by a minimum value, typically called the threshold, before they are considered to be extremes that should be retained by the filter. The overall sequence of loading is still retained after the filter [21]. Various filter algorithms have been proposed for processing time series data.
4.2.2 Rainflow Counting Method

4.2.2.1 Classical Rainflow Algorithm

The Classical rainflow algorithm is illustrated in Fig. 4.2. The stress time history is plotted as the shape that the axis is vertically downward, and the lines connecting the stress peaks are imagined to be a series of roofs. The rainflow drips down these roofs according to some rule to capture the complete cycles and half cycles. As a rule, rainflow begins sequentially at the inside of a stress peak. If the rainflow initiates at a local minimum, the rain starting at each peak is allowed to drip down and continue, until it meets opposite another local minimum smaller than the minimum from which it initiated, the rain must

![Fig. 4.2 Rainflow counting method](image)

![Fig. 4.3 Full cycles and half cycles in rainflow counting method](image)
stop. On the contrary, if the rainflow initiates at a local maximum, the rain stops until it meets opposite another local maximum larger than the maximum from which it initiated. For example shown in Fig. 4.2, the rain begins at peak 1 and stops opposite peak 9, just because peak 9 smaller than peak 1. And a half cycle is counted between peak 1 and peak 8. Fig 4.3 shows the cycles and half cycles for the rainflow counting example in Fig. 4.2.

This algorithm is a recursive procedure.

4.2.2.2 Rychlik Rainflow Algorithm

In this study, the algorithm proposed by Rechlik is used, which is a non-recursive algorithm. And, it is more tractable for mathematical and statistical analysis. Moreover, it is easy to see the connection between rainflow cycles and crossings of intervals.

Suppose there is a process \( \{Y_t\}, \ t \geq 0 \), with a finite number of local extremes occurring at time points \( t_1, t_2, \ldots \). For simplicity, it is assumed that the first local extreme is a minimum, therefore, the sequence of turning points is denoted by

\[
TP(Y_t) = \{Y_{t_1}, Y_{t_2}, Y_{t_3}, \ldots\} = \{m_0, M_0, m_1, M_1, \ldots\}.
\]

(4.1)

\( m_t \) and \( M_t \) denote the local minimum, and local maximum, respectively as in Fig. 4.4.

![Fig. 4.4 Turning points sequence](image-url)
For \( t < T \), \( \{ y(t) \} \) is a function with finite local maxima of magnitude \( M_k \) occurring at \( t_k \), such as the stress time history, and \( T \) is the loading time. For the \( k \)th maximum at time \( t_k \), the following left minima \( m_k^- \), and right minima \( m_k^+ \) are defined in the following:

\[
m_k^- = \min\{ Y(t) : t_k^- < t < t_k^+ \};
\]

and

\[
m_k^+ = \min\{ Y(t) : t_k^- < t < t_k^+ \}.
\]

here,

\[
t_k^- = \begin{cases} 
\max\{ t \in [0, t_k) : Y(t) > Y(t_k) \} & \text{if } t_k^- < T \\
0 & \text{if } t_k^- = T
\end{cases};
\]

\[
t_k^+ = \begin{cases} 
\min\{ t \in (t_k, T] : Y(t) \geq Y(t_k) \} & \text{if } t_k^+ < T \\
0 & \text{if } t_k^+ = T
\end{cases}.
\]

Therefore, the \( k \)th rainflow cycle is defined as \( (m_k^{RFC}, M_k) \), with

\[
m_k^{RFC} = \begin{cases} 
\max(m_k^-, m_k^+) & \text{if } t_k^+ < T \\
m_k^- & \text{if } t_k^+ = T
\end{cases}.
\]

The definition is best described in Fig 4.5.
Fig. 4.5 Rychlik definition of rainflow cycle

The rainflow cycle is a pair consisting of a minimum $m_k^{\text{med}}$, and a maximum $M_k$. The amplitude is the most important characteristic for fatigue evaluation. Often in fatigue analysis applications, a cycle is represented as a range-mean pair. The definition of the amplitude is shown in Fig. 4.6.

Fig. 4.6 The amplitude, range and mean of a cycle

When the algorithm reaches the end of time-series data record, a series of unmatched peaks and valleys remains unclosed and, therefore, are not counted by the
algorithm. These so-called "half-cycles" typically include the largest peak and valley in
the record, and they may also include other large events. Therefore, the potentially most
damaging events (the largest cycles) contained in the time series are not counted by the
classical formulation of the rainflow algorithm. Various researchers have proposed
several techniques for handling these half-cycles in fatigue applications. Some ignore
them, some count them as half of a complete cycle, and others count them as full cycles.
The second one is the most conservative approach. In this study, for safety reason, all the
half cycles are counted as full cycles.

4.2.3 Cycle Count Matrix and Level Crossings
The output of the rainflow counting algorithm is a characterization of the stress cycle by
its maximum and its minimum value. After the rainflow counting, this characterization of
those fatigue cycles must be converted to practical use by the fatigue analysis. The output
file can take many forms, depending on the analysis and numerical techniques being used
to determine the damage. From the discretized turning points, min-max and max-min
cycles can also be extracted. Furthermore, the cycle count can be summarized in a two-
dimensional histogram and be represented by a matrix. The rainflow cycles matrix,
$F^{RFC}$, the min-max matrix $F$, and the max-min matrix $\tilde{F}$ is summarized in the
following:

$$
F^{RFC} = \left( f_{y}^{RFC} \right)_{i,j=1}^{p},
$$

(4.7)

$$
F = \left( f_{y} \right)_{i,j=1}^{p},
$$

(4.8)
\[
\hat{F} = \left( \hat{f}_{g} \right)_{i, j = 1}^{n}.
\]

(4.9)

And all the terms are defined as:

\[
f_{0}^{RFC} = \text{number of } \{ m_{k}^{RFC} = u_{i}, M_{k} = u_{j} \};
\]

(4.10)

\[
f_{g} = \text{number of } \{ m_{k} = u_{i}, M_{k} = u_{j} \};
\]

(4.11)

\[
f_{g} = \text{number of } \{ M_{k} = u_{i}, m_{k+1} = u_{j} \}.
\]

(4.12)

In Fig. 4.7, the matrices \( F^{RFC} \), \( F \), and \( \hat{F} \) are illustrated for a discrete load.

Fig. 4.7 Rainflow cycle \( F^{RFC} \), min-max \( F \), max-min \( \hat{F} \) for equipment time history

For most cycle counts, rainflow and min-max cycles, the counting distribution contains information about level crossings. The stress time series (the sequence of turning
points) is discretized by the levels \( u_1 < u_2 < \ldots < u_n \). The number of up-crossings is given by:

\[
N_T(u) = \text{number of } \{ t \in [0,T] : t \text{ is a up-crossing of } Y(t) \} = N_r(u,u). \tag{4.13}
\]

The crossing intensity function is the number of times per time unit that the stress up-crosses the level \( u \).

### 4.3. Rainflow Counted Equipment Time Series Results

For the acceleration time series generated in Chap 3 for the equipment shown in Fig. 2.7, the Rainflow cycle matrix \( F^{RFC} \), and the min-max matrix \( F \) are shown in Fig. 4.8 & 9, respectively. Fig. 4.10 & 11 show the amplitude histogram for rainflow and min-max cycle, respectively. It can be seen that \( F^{RFC} \) captures more cycles with highest amplitudes, compared to \( F \). The crossing intensity is also plotted in Fig. 4.12.

![Rainflow cycle count matrix for the equipment stresses series](image)
Fig. 4.9 Min-max cycle count matrix for the equipment stresses series

Fig. 4.10 Amplitude histogram for rainflow cycle matrix; equipment acceleration
Fig. 4.11 Amplitude histogram for min-max cycle matrix; equipment acceleration

Fig. 4.12 Level crossing intensity for equipment data; equipment acceleration
The 2-D form for the cycle count matrix may be simplified to a 1-D form for analysis. The 1-D distributions are derived from the 2-D cycle count matrix by holding the range constant and summing over all means.
Chapter 5

Damage Calculation and Fatigue Life Estimation

For variable amplitude loads, the S-N curve is combined with cycle counting method by means of the recognized standard, Miner's linear damage accumulation rule. The equivalent stress cycles generated by the rainflow counting procedure are used for damage calculation and fatigue life prediction. The topic extended to the extreme value analysis, which is of particular interest in wave load analysis.

5.1 S-N curves and Cumulative Damage rule

5.1.1 S-N curve

The fatigue life of a structural component is found to depend in a complex manner on a large number of factors, such as, the material properties, the load sequence, the geometry size and surface finish of the component, and the environmental factors.

The basic theory of fatigue is based on the macroscopic observation of the relationship between the applied stress and the number of cycles to failure. Because of the relative ease of obtaining data in the form, and its easy application to the fatigue design, the bulk of fatigue test data is presented even today in the form of S-N curves, which is also called the Wohler [52] diagrams. The S-N curves exhibit a large scatter which is primarily due to the inherent uncertainties underlying the fatigue phenomenon, as a result of the variability of the microscopic influence-dislocation, lattice defects, grain boundaries.
In laboratory experiments, the structure is often subjected to a constant amplitude load, and the number of cycles (periods) is counted until the specimen breaks. The number of cycles $N(s)$, as well as the stress amplitudes $s$ is recorded. For small amplitudes $s < s_\infty$, the fatigue life is often very large, and is set to infinity $N(s) = \infty$, which means no damage will be observed even during an extended experiment. The amplitude $s_\infty$ is called the fatigue limit or the endurance limit. The relationship between $N(s)$ and $s$ is expressed as

$$N(s) = \begin{cases} K^{-1} s^{-\beta} & s > s_\infty \\ \infty & s \leq s_\infty \end{cases}.$$  

(5.1)

Here, $K$ is a material dependent random variable, usually log-normally distributed, with

$$K^{-1} = E \varepsilon^{-1}.$$  

(5.2)

Here, $\ln E \in N\left(0, \sigma_\varepsilon^2\right)$ and $\varepsilon$, $\beta$ are fixed constants. $\varepsilon$, $\beta$ and $\sigma_\varepsilon$ can be estimated from an S-N experiment, or by fitting a certain given S-N curve.

### 5.1.2 Palmgren-Miner Linear Damage Accumulation Rule

The prediction of fatigue life based on the S-N curves rests on the use of a suitable cumulative damage criterion representing the progressive deterioration of the structure during the cyclic loading. The concept of fatigue damage has been introduced for this
purpose. Basically, it assumes that the structure component under repeated cyclic
loading suffers and amount of damage which can be expressed as

\[ d(D) = f(n, s, N(s)) \]  \hspace{1cm} (5.3)

where \( D \) denotes the damage function, \( n \) is the number of cycles at a constant stress
amplitude \( s \). As mentioned in the preceding part, \( N(s) \) is the number of cycle to failure
at stress amplitude \( s \) obtained from the S-N curve.

It is assumed that the damage occurred is permanent, and further applications of
stress cycles at varying stress levels cause additional cumulative damage. The total
damage is the sum of all damage increments accrued at each stress level, and when the
cumulative damage reaches a critical value, fatigue failure occurs. The cumulative
damage is simple to apply. But considerable difficulty is encountered in practice in
defining a damage function incorporating the stress sequencing and stress interaction
effects.

The first cumulative damage law was proposed by Palmgren [53], and later
developed by Miner [54]. The linear damage theory is still widely used in spite of many
limitations because of its simplicity in application, and because of the fact that many
other theories developed to account for the limitations do not give better and more
accurate estimates of fatigue life.

The first assumption made for Palmgren-Miner damage rule (Miner’s rule) is that
each alternate stress cycle inflicts a certain damage in the material equal to the reciprocal
of the number of the cycles of failure at the stress level given by the S-N curve. This
damage is irrespective of the relative position of the stress cycle in the whole stress history and, and independent of the magnitude of the stress before or after it. Second one is that the total damage due to the whole stress history is the sum of all damages of each individual cycle. The third hypothesizes that the fatigue failure of the material occurs when the cumulative damage reached a value unity.

The fraction of damage suffered by the structure component due to $n_i$ cycles at stress level $s_i$ is expressed by $D_i$, which is computed by the equation

$$D_i = \frac{n_i}{N_i}.$$  \hfill (5.4)

Here, $N_i$ is the number of cycles to failure at $s_i$ given by S-N curve. The structural component failures when the total cumulative damage

$$D = \sum_i D_i = \sum_i \frac{n_i}{N_i}.$$  \hfill (5.5)

reaches the value $D = 1$.

Typically, the fatigue cycles imposed on a structure are analyzed over some fixed period of time. Therefore, Eq.5.5 should be expressed as the damage rate $\Delta D_i$ associated with the sample time $t$. This damage rate can be treated as the average damage rate over the service lifetime of the component $T_f$ ($\Delta D_i = \Delta D_f$) especially for static and ergodic process, and, then the service lifetime of the structure $T_f$ is the reciprocal of $\Delta D_f$, that is
\[ T_f = \frac{1}{\Delta D_r} = \frac{1}{\Delta D_T}. \] (5.6)

Again, Eq. 5.6 is predicated on the assumption that failure will occur when the damage equals one, and that the damage rate computed over time \( t \) is representative of the average damage rate imposed upon the structure during its service lifetime. Namely, the number and distribution of fatigue cycles contained in the sample are essentially identical to the number and distribution over the structure's service lifetime.

5.2 Damage Calculation for Rainflow Counted Data

5.2.1 Stationary and Ergodic loads Case

For the time series in the sample time \( t \), the damage rate is used to calculate the fatigue life. If the \( k \) th cycle has amplitude \( s_k \), then it is assumed that it causes a damage equal to \( 1/N(s_k) \). Therefore, the total damage at time \( t \) is

\[ D(t) = \sum_{k} \frac{1}{N(s_k^{RFC})} = \sum_{k} K\left(S_k^{RFC}\right)^\beta = KD_\beta(t). \] (5.7)

where the sum contains all rainflow cycles which have been completed up to time \( t \), and

\[ S_k^{RFC} = \left(M_k - m_k^{RFC}\right)/2. \] (5.8)
The damage rate $\Delta D_i$ is defined by

$$\Delta D_i = \frac{D(t)}{t}.$$  \hspace{1cm} (5.9)

Therefore, the fatigue life $T_f$ for stationary and ergodic loads can be obtained using Eq. 5.6.

A very simple predictor of $T_f$ is obtained by replacing $K^{-1} = E\varepsilon^{-1}$ in Eq. (5.7) by a constant, for example, the median value of $K$, which is equal to $\varepsilon$.

For the particular steel equipment on topside of FPSO, which is discussed in the previous chapter and shown in Fig. 2.7, the material parameters $\beta = 3.2$, $\varepsilon = 5.5E - 10$, $m = 40kg$, cross section area $A = 100mm^2$ are selected. The fatigue life is calculated by applying Eq. (5.7-5.9), and it is found to be

$$T_f = 8.2\text{years}.$$ \hspace{1cm} (5.10)

5.2.1 Fatigue Life Distribution

For non-ergodic loads, the sampled loading time series after rainflow count is not enough to represent all the damage during the service time. The fatigue life obtained is just a statistical distribution.

The Palmgren-Miner hypothesis states that fatigue failure occurs when the damage exceeds one. Thus, the probability for fatigue failure is
\begin{equation}
P(T_f \leq t) = P(D(t) \geq 1) = P(K \leq \varepsilon D_\beta(t)). \tag{5.11}
\end{equation}

Here, \( K \) accounts for the uncertainty in the material. In the previous section, a lognormal distribution has been used for the fluctuation of \( K \) about \( \varepsilon \), by assuming that

\[
\ln K = \ln \varepsilon - \ln E \tag{5.12}
\]

is a normal variable with mean \( \ln \varepsilon \) and standard deviation \( \sigma_K \).

The cycle sum \( D_\beta(t) \) is the sum of a large number of damage terms, only dependent on the cycles. For loads with short memory, it is assumed that \( D_\beta(t) \) is approximately normal, and the following can be defined

\[
d_\beta = \lim_{t \to \infty} \frac{D_\beta(t)}{t} \quad \text{and} \quad \sigma_\beta^2 = \lim_{t \to \infty} \frac{V(D_\beta(t))}{t}. \tag{5.13}
\]

Thus, the normal distribution for \( D_\beta(t) \) can be expressed in the form

\[
D_\beta(t) \sim N(d_\beta t, \sigma_\beta^2 t). \tag{5.14}
\]

The fatigue life distribution can be computed by combining the lognormal distribution \( K \) with the normal distribution for \( D_\beta(t) \).
5.3 Extreme Value Analysis

Of particular interest in wave analysis is how to find extreme quantities and extreme significant values for a wave load time series. Often this implies exceeding the observed or simulated data range, from a limited number of observations, to predict how large the extreme might be. This kind of analysis is commonly known as Weibull analysis, from the names of a familiar extreme value distribution. If the process is Gaussian, the Weibull distribution becomes a Rayleigh distribution giving by the equation

\[ p(a) = \frac{a}{\sigma_y} \exp \left( -\frac{a^2}{2\sigma_y^2} \right) . \] (5.15)

Comparing Eq.(5.15) with Eq.(2.7), it is seen that the Rayleigh distribution is a Rice distribution with a spectrum width parameter equals to zero.

With the help of the Rayleigh distribution, it is convenient to directly apply Miner's rule to the stress time history if the time history is a narrow band process, without resorting to the rainflow procedure. For a narrow band process, the expected number of cycles is nearly equal to the expected number of maxima.

Consider the situation in which the wave load process \( y(t) \) is narrow band. The expected number of peaks occurring between the stress levels \( s \) and \( s + ds \) in the time interval \( [t, t + T] \) is

\[ N^*(s; T) - N^*(s + ds; T). \] (5.16)
where $N^*(s;T)$ denotes the expected number of crossings of the level $s$ with positive slope in the interval $[t, t+T]$.

The expected incremental damage at the stress level $s$, using Miner’s rule, is

$$E[D_s(t, t+T)] = \frac{N^*(s;T) - N^*(s + ds;T)}{N(s)}.$$  \hspace{1cm} (5.17)

where $N(s)$ is the number of cycles for fatigue failure at stress level $s$. The Eq. (5.17) can be written as

$$E[D_s(t, t+T)] = -\frac{1}{N(s)} \frac{\partial N^*(s;T)}{\partial s} ds = -\frac{1}{N(s)} \frac{\partial}{\partial s} \left[ \int_{T}^{T} N^*(s;\tau) d\tau \right] ds.$$  \hspace{1cm} (5.18)

For a stationary process, $N^*(s,t)$ is independent of $t$. Thus, the total expected damage at all stress level in the same interval can be obtained by integral

$$E[D(T)] = -\int_{0}^{T} \frac{\partial}{\partial s} \left[ N^*(s) \right] ds.$$  \hspace{1cm} (5.19)

Eq. (5.18) can be alternatively written by using the probability density of the peaks in the form

$$E[D(T)] = \int_{0}^{T} \frac{T}{N(s)} [N^*(0)\rho(s)] ds.$$  \hspace{1cm} (5.20)
where \( N^*(0) \) is the expected rate of zero crossings with positive slope.

Substituting Eq. (5.1) into Eq. (5.20) obtains

\[
E[D(T)] = N^*(0) \cdot T \cdot \frac{1}{\sigma_i} \int_0^\infty p(s) \cdot s^\beta ds.
\]  

For a narrow band process, the function \( p(s) \) is Rayleigh distribution, therefore, the total expected damage can be written as

\[
E[D(T)] = \frac{N^*(0) \cdot T \cdot K}{\sigma_i^2} \int_0^\infty \exp \left( \frac{-s^2}{2\sigma_i^2} \right) \cdot s^\beta ds = N^*(0) \cdot T \cdot K \left( \sqrt{2\sigma_i} \right)^\beta \Gamma \left( 1 + \frac{\beta}{2} \right).
\]  

According to Miner's rule, the average time to failure \( T_f \) can be calculated by setting the expected fatigue damage equals to 1. That is

\[
E[T_f] = \frac{1}{N^*(0) \cdot K \left( \sqrt{2\sigma_i} \right)^\beta \Gamma \left( 1 + \frac{\beta}{2} \right)}.
\]  

In this equation, the value \( N^*(0) \) can be determined by the expression

\[
N^*(0) = \frac{1}{2\pi} \frac{\sigma_i}{\sigma_s}.
\]  

where \( \sigma_s \) is the standard deviation of the stress spectrum, and is given by the equation
\[ \sigma_i^2 = \int S_\sigma(\omega) d\omega. \]  
(5.25)

where \( S_\sigma(\omega) \) is the stresses spectrum of the equipment, shown in Fig. 2.7.

\[ \sigma_i \] is given by the equation

\[ \sigma_i^2 = \int \omega^2 \cdot S_\sigma(\omega) d\omega. \]  
(5.26)

By applying Eq. (5.23), and selecting the same material parameter \( \beta = 3.2 \), \( \epsilon = 5.5 \times 10^{-10}, \ m = 40kg \), cross section area \( A = 100mm^2 \), the expected fatigue life for the equipment, shown in Fig. 2.7, is

\[ T_f = 14.4 \text{ years}. \]  
(5.27)

where the value of \( N^*(0) \) has been found with \( N^*(0) = 0.11 \) by using Eq. (5.24).

The deviation from the one derived by using the rainflow counting algorithm should be deemed reasonable, since Eq.(5.23) is based on some simplified assumptions, such as the uni-modal, and the narrow band feature of the equipment response.
Chapter 6

Concluding Remarks

The deterioration of engineering structures due to fatigue has been a challenging problem facing engineers for many decades. A Floating Production, Storage and Offloading (FPSO) system is a relative new concept of the oil and gas facilities in deep sea. There is limited literature discussing the fatigue analysis of topside structure. Thus, a detailed solution procedure has been pursued to deal with this kind of problem. Several numerical techniques have been used along with certain digital simulation procedures. All program have been done by using Matlab.

Discrete transfer functions at different locations (modules) in the topside of the FPSO for various spreading directions have been incorporated in the analysis. Combined with the Ochi-Hubble sea wave elevation spectrum, a design spectrum has been determined for fatigue studies. A Moving Average algorithm has been adopted to generate wave loads time history, which provided good matching between the simulated, and the target spectra. Further, using a generic piece of equipment on the topside of FPSO, the related response spectrum has been obtained. Next, the rainflow counting algorithm for identifying significant cycles that produce fatigue damage has been applied. Finally, the fatigue life has been derived using the S-N curve and Miner’s linear damage rule. The particular data used pertain to the Kizomba FPSO.

It is noted that the solution in this study is based on the S-N approach, and has certain shortcomings. One important one is that it cannot be used in assessing the
structural integrity of cracked tubular joints in service. For these kinds of fatigue crack problem, Fracture Mechanics (FM) method is perhaps desirable.

Another point deals with the joint probability density functions of the wave height, and the period. If it is incorporated in the analysis, it can give more accurate results.

Finally, the three directional wave loadings in the three acceleration axes on the FPSO topside has been treated in this study as uncoupled of each other because of the availability of auto power spectra solely. If these directions are considered as coupled, a more complex random multi-axis fatigue problem is formulated. In this context, cross spectrum can be considered in a future study. In this case, it is noted that the parameters, such as, the autocorrelation function in MA algorithm will become matrices. This, of course, will complicate the numerical treatment, but it will hopefully provide more realistic predictions of critical structures and equipment on FPSO topside.
Reference:


