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Structures of Agency:
contradiction, parallel, paradox

by
Elizabeth Burns McQuitty

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ABSTRACT

STRUCTURES OF AGENCY:
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‘Structures of agency’ takes as its model the expanding cone of simultaneous pathways the photon travels through as it propagates through space. That image, together with the expanding cones of Richard Serra’s Clara-Clara (1983) provides a framework from which a discourse on form, spectatorship, geometry and mathematics expands. Clara-Clara produces two sets of contradictory views: the conflicting apperceptions never intersect but rather propagate through the force of their conflict. In so doing, they forge a cognitive structure that is parallel. A review of the history of parallelism yields a multiplied reading of the sculpture itself and the cognitive structure it produces. Finally, the notion of contradiction as paradox reveals how formal systems – aesthetic or mathematic – give rise to alternative propositions that extend beyond the system that created them. Conscious experience recognizes, develops, these extensions even as the systems that created them break down in the emergence.
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introduction to a structure

As the limits of formalism were discovered and a multiplicity of geometries developed during the late 19th and early 20th Centuries, a new paradigm emerged from within the study of physics. Quantum theory suggests a compelling and peculiar structure to describe the agency that emerges from within the simplest of physical systems. When a photon emits from a source it moves through any number of alternative pathways as it travels to its destination. These pathways propagate outward from their source, superposed on each other to produce the wave-like features of light as it travels through space. Yet as soon as the photon encounters a barrier that permits passage through only a single route the wave front of possibilities collapses, the wave-like features of light disappear and the photon acts as a particle.
The photon develops the agency to respond to its environment even as the environment changes. Correlated to this first instance of agency is the decision by the scientist to monitor or to measure a photon’s movement. This decision itself radically disrupts the system, implicating incredible agency within the human acts of observation and measurement. The photon, a tiny particle otherwise presumed utterly inanimate, and the act of observation, until recently assumed a passive, removed and non-interfering view on to physical systems, both emerge within quantum understanding as quite powerful and real sources of agency in the world.

The structure of their agency suggests that contradiction, or multiple mutually exclusive possibilities, expressed in parallel, gives rise to propagation, expansion, development. Quantum theory visualizes a growing sphere that marks the bounds of possible location of the photon. As the sphere grows in time its geometry expands like a cone. The cone’s propagation forward produces what classical physics refers to as the wave function of the light. When the photon’s location becomes known – when it materializes on a screen or in a ‘black box’ – the cone of possibility disappears and the quantum-mechanical particle remains. The cone contains parallel alternative pathways; its structure compels the parallels to move forward, never intersecting but rather propagating through time, growing and developing through the multiplicity they maintain.

Roger Penrose, a mathematician and physicist, develops a quantum theory of mind in his 1989 work entitled *The Emperor's New Mind*. He suggests the notion of superposition

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accounts not only for normal cognitive and apperceptive processes – Gestalt-like vision, for example – but also for the moments where the ordinary selection of a single image, a single idea, either breaks down into multiplicity or emerges from countless possibilities. Penrose takes as his starting point Kurt Gödel’s Incompleteness Theorem of 1931. Gödel’s theorem divorces the meaning of a mathematical statement from its formal structure so that we can see the statement as both correct mathematical formulation and as self-contradicting assertion. Penrose argues that reasonable thought can ‘see beyond’ the formal statement, which is to say thought can acknowledge simultaneously the truth value of two mutually exclusive alternatives. A neurological model of the mind must account for this capacity to deal with parallel propositions, or parallel meanings of a single proposition. For this reason, Penrose suggests superposition as a model for thought, quantum theory as a theory of mind.

Applied to a theory of mind, the quantum model depends on the ability of the neuron to grow and reconfigure its dendritic stubs any number of times through the lifespan of a person. In normal thought patterns, any number of alternative understandings of an event develop along hundreds or millions of possible neurological pathways, each corresponding to an alternative resolution. All are activated, superposed linearly until a few, those that meet certain criteria, surface into conscious thought as an ‘idea’ or a ‘solution’, ranging from the banal to the ‘aha!’ revelation of the creative act. This emergence of a single ‘answer’ can be likened to the notion of Gestalt psychology. However, where Gestalt theory is limited by the insistence that apperception resolve out the single most stable image of several alternatives and that this occur as an algorithmic process, Penrose accounts for the tendency toward a single resolution while leaving open the possibility that sometimes more than a single
solution will emerge. The relation between viable, contradicting alternatives develops over time.

The quantum theoretical notion of superposition provides a powerful visual model for how a set of mutually exclusive possibilities might develop, propagate, grow as they extend through time and space. We can imagine superposed possibilities as a kind of parallel structure, and the space within the parallel as a place for development and growth. Just as the growing sphere of possible locations of a photon expands geometrically so too does the space of the contradictories as they develop. The expanding cone provides a model for development as it occurs within the structure of parallels. The multiple readings we might have of an observed phenomenon, all the models for geometry and paradoxes of mathematics, indeed produce a growing set of paths for a discourse on structures of agency.
Clara-Clara

Richard Serra’s *Clara-Clara* tantalizes with contradiction. The contradiction grows from two cones, one expanding downward from 875 feet above the ground to a circle 300 feet wide, the other upward from as far beneath the surface of the earth. Yet we see only fragments of the cones. These sweep through 86 degrees of the cones’ curvature along twelve feet of vertical rise. The fragments appear as arcs, two Cor-Ten steel sheets posing in mirror symmetry convexly about their center. Disrupting that symmetry the conic geometry causes the arcs to lean, one to the inside of its curvature, the other outward. The cones are too large, the arcs too small a fragment of them, for us to recognize the conic geometry. Instead, a growing set of contradictions develops between the arcs, the ways of seeing they produce, their relationship to site. These, superposed, expand outward from the work. *Clara-Clara* develops its contradictions across the space of the sculpture and leaves a passage through that space, between the mirrored arcs, into which the spectator may enter.
When *Clara-Clara* was located in the Jardin des Tuileries, from October 26, 1983, to late April, 1984, on occasion of a retrospective of Serra’s work at the Pompidou Center, the passageway aligned with Paris’s great Baroque/Neo-Classical axis, the ‘Triumphal Route’ that leads from the Arche de la Defense to the Arc de Triomph, the Place de la Concorde and into the Tuileries, where *Clara-Clara* hovered, and on to the Grand Louvre. Situated here, the sculpture produces two kinds of spectatorship. One view acts to limit *Clara-Clara* as a self-contained sculpture, the product of looking from slightly above and from a distance, as did viewers located on the terraces of the Jeu de Paume and the Orangerie or from a distance within the park. A second view emerges through *Clara-Clara*’s effects on the axis which it joins: passage through the sculpture causes the obelisk at the center of Place de la Concorde – the next monument along the urban axis – to jump behind the sculpture’s leaning walls into and out of sight even as the
spectator remains on course — on axis — with the obelisk. In so doing, the sculpture displaces the notion of the urban axis from the world outside and prior to experience into the spectator and upon the choice to enter *Clara-Clara*’s central axis.

Following the Serra retrospective, the Paris Mayoralty decided to re-install *Clara-Clara* at a permanent site in the Square de Choisy, located inside the parc de Choisy in Paris’s 13th arrondissement. Here, the two arcs sit on a bed of grass between the busy avenue de Choisy and the less traveled rue Charles Moureu; passage between the arcs centers on the Donation Eastman, a red brick exemplar of 1930s social architecture which today houses a dental school. *Clara-Clara* overcomes this otherwise dismal context. Here, distortions appear across the space of the piece rather than reconfiguring what lies beyond. Here also the spectator enacts more fully the choice of approach for, where the choice to enter the passageway was so heavily endorsed by the axis of the Tuileries site there is nothing about the parc de Choisy to direct the spectator one way or another. The newer site weakens *Clara-Clara*’s mirror symmetric reading as well, as the spectator, upon entering the square, sees the piece from the side first, rather than on axis, and only from above when standing on the steps of the Donation. Yet two kinds of seeing persist. *Clara-Clara* shows itself as mirror symmetric totality at a glance from a distance and as a series of distortions, dynamic spatial modulations, the spectator will experience over time within the space between the arcs.
The effects of the piece — and the kinds of spectatorship associated with them — align curiously with several of the primary art theoretical models of the Modern era. At the Tuileries location, \textit{Clara-Clara} disrupts the notion of Baroque axiality, acting as a Minimalist work by its reconfiguration of site through the spectator's engagement with the piece. Also in the Tuileries, \textit{Clara-Clara} produces a very strong reading as self-referential, a mirror symmetric sculptural object. Viewed from a distance the self-evidence of \textit{Clara-Clara}'s mirror symmetry avails instantaneous perception of the piece in terms of its limits, a hallmark of Modernist notions of vision. Modernist spectatorship persists with \textit{Clara-Clara}'s removal to Choisy, but here the Minimalist reconfiguration of site inverts in a mirror symmetry of its own to produce spatial modulation within and upon the sculpture itself. The sculpture's effects preexist the spectator whom the sculpture invites to enter. Such spatial modulation occurring within a self-referring and rigorously mathematic structure can only be described as Baroque.

At both sites, \textit{Clara-Clara} produces for the spectator who enters the passage innumerable anomalies that distort the notion of mirror symmetry that nevertheless seems to structure the piece. The two arcs sweep in toward the passageway at remarkably different rates and then diverge in an equally asymmetric manner. From up close the tilt of the two arcs is particularly precarious and their divergence from the presumed mirror symmetry uncanny. Since no spectator can extrapolate the work's geometry\footnote{Yves-Alain Bois describes the geometry of several of Serra's pieces as defying apprehension of their regulating geometry. He describes his own survey of \textit{Spin Out}, from which he concluded there was no such geometry whereas Serra describes the piece as having steel slabs at twelve, four and eight o'clock. Bois suggests \textit{Night Point} functions similarly, and Clara Weyergraf demonstrates the same effect in \textit{Terminal}. See Bois, "A Picturesque Stroll Around \textit{Clara-Clara}." \textit{Richard Serra}, exhibition catalog, Paris: Centre Georges Pompidou.,} the tilt is perceived as distortion, the
suggestion of symmetry at once supported and denied. *Clara-Clara* develops this conflict and so forces the issue of how exactly the perceptual experience is transformed to comprehension of the perceived object.

The term 'apperception' conjoins 'apprehension' with 'perception'. It refers to the mental process of assimilating an idea – particularly one that is newly perceived – into a body of ideas that already exists: we understand this as comprehension. The term reveals the improbability of distinguishing (passive) reception of perceptual data from the (active) formulation of a coherent apperceived image. In effect, *Clara-Clara* evades comprehension: emerging from the passageway, the spectator finds the two ways of perceiving *Clara-Clara* – as mirror symmetric from a distance and as untold distortions from up close – do not collapse in a single conception of the piece but rather counter each other indefinitely. The two ways of seeing the sculpture run parallel, never fully meeting just as the two arcs of *Clara-Clara* never intersect. The spectator upon entering the space between the arcs also enters the space between these ways of seeing.

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apperception as algorithmic

While theories of perception date to ancient Greece the notion that perception coincides with a pre-conscious ordering of the perceived world was first seriously developed by a group of psychologists working in Germany in the 1930s. They theorized that perceptual stimuli are processed automatically and instantaneously into a single unified representation that is then presented to conscious thought. Max Wertheimer proposed the idea as early as 1912 but did not develop it until 20 years later, when Kurt Koffka and Wolfgang Kohler joined him to propose that human perception operates through the recognition of a Gestalt – 'pattern' or 'configuration' – and that this recognition occurs at a pre-cognitive level. Their theory, Gestalt psychology, insists we perceive well-organized wholes rather than the fragments of sensory stimuli as they arrive at the sense organ. Applied to vision, Gestalt theory suggests all the confusion of the visible world arrives through saccadic vision on the surface of the retina where it is converted automatically and unconsciously into the organized patterns, shapes and configurations that then become available to conscious thought.

A parallel between Gestalt notions and Modernist vision – as attributed above to Clara-Clara's view from a distance – should be evident. Modernism in its formulation by Clement Greenberg emerges with the application of the characteristic methods of a discipline toward

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self-criticism within a given medium. His ideas were best developed in relation to the distinction between painting and sculpture. Greenberg posited the self-critical practices of Modern painting as a concern with flatness or two dimensionality, the primary condition painting shared with no other art. "The frank acknowledgment of surface becomes the condition to which the self-critical modernist painting must tend." Within painting this 'frank acknowledgement' was supposed to lead to instantaneous perception of the formal qualities of a piece, to the flatness of the surface as in Manet's *Olympia* (1863), the work to which is often attributed the emergence of Modern painting. The self-evidence of formal qualities, coupled with their instantaneous perception, perception very much like the Gestalt notion that perception tends to occur as a reading of a 'whole', became hallmark features of Modernist vision in its need to distinguish painting from the sculptural.

Ironically, Gestalt vision also was fundamental to the development of Minimalism, itself a movement away from the formalism of Modernist aesthetic theory. The strictly geometric sculptures of Minimalism were supposed to empty the aesthetic object of all narrative and external reference: the viewing subject was intended to encounter nothing other than what s/he saw. A preoccupation with the conditions of spectatorship grew directly from this formulation, and formalist concerns were relegated as subsidiary to the spectator's encounter with the piece. The Minimalists' departure from Modernist concerns enacted an

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5 Ibid, p. 146.

6 Michael Fried, for example, criticizes the Minimalists – he calls them Literalists – for the theatricality of their work, which he attributes to a concern with the situation of encounter, the importance of presence and the perceived hollowness of Literalist sculpture. By contrast, he locates modern art's success in the defeat by the aesthetic work of its status as object and in its availability to instantaneous perception. see Fried, Michael. "Art
acknowledgment of that Modernist notions of vision had grown out of a self-critical approach to painting – in opposition to all features it shared with the sculptural. The transferal had transgressed the very notion of self-criticality. While certain sculptors from the same period – particularly David Smith and, later, Anthony Caro – were accepted by formalist criticism as producing Modernist works, the course Minimalism took, from Gestalt notions to phenomenalist concerns, indicates the contradiction of attributing Modernist theory as articulated about vision in painting to aesthetic practice in other media.

Perhaps the appeal of Gestalt perceptual theory lies in that common sense seems on its side. Intuitively, we know the minutiæ of perceptual experience are not sorted in any conscious way but rather arrive pre-processed, as coherent images of the world. Gestalt theory rests on the notion that an automatic process selects the best and most organized image from any number of alternatives. This in turn suggests the act of perception enacts an algorithm that selects the stablest and most coherent image of all the possible images formed in the perceptual event. In other words, Gestalt theory posits that algorithmic function underlie the very act of perception.

The term algorithm refers to any formalized systematic method for arriving at the correct answer to a given problem starting at any arbitrary point. It derives from Euclid's method (c. 300 BC) for finding the highest common factor for two numbers (divide the larger number by the smaller, take the remainder and divide the smaller of the original numbers by it, and so on). The word itself refers to the name of a 9th Century Persian mathematician Abu Ja'far

Mohammed ibn Mūsā al-Khowārizm. The systematic application of explicit rules of procedure to discrete units of information typifies the notion of algorithmic function and of mathematical formalism in general. Algorithmic structure should be applicable to any process amenable to automation and certainly ought to reproduce the selection mechanisms described in Gestalt notions of perception. The gestaltists could only prove their theory with the successful development of a series of rules – algorithms – to model the selection processes they had so aptly described.

In 1935, Kurt Koffka formulated what was to be chief among these, the law of Pragnanz: “Of several geometrically possible organizations the one will actually occur which possesses the best, simplest and most stable shape.” This was followed by a whole series of rules to prop it up: laws of proximity, similarity, good continuation, closure, and common fate (applied to moving objects). An accumulation of these was supposed to determine perception independent of any conscious configuration of the perceptive event – and many of which are challenged by Clara-Clara.

But these rules consistently fell short of coding for selection as described in the law of Pragnanz and they proved exceedingly cumbersome given the requirement that selection occur instantaneously with on-going perception. Gestalt theory suffered further from the fact that experimental research relied on description by the subject of perceptual experience, a description that cannot possibly be transparent to the very processes the gestaltists posited.

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* Incidentally, al-Khowārizm’s c. 825 textbook, Kitab al jabr w’al-mugabalā – contains the Arabic word from which the modern algebra derives. (Ibid, p. 30).

* Matlin and Foley, p.134.
The failure of an algorithm to reproduce the processes of perception and the inability of subjects to describe their own perceptual processes ultimately led to the rejection of Gestalt theory as a model for perception. The theory's legacy remains however in the description of perception as formulating 'whole' images of the world through patterned mechanisms that rest outside conscious thought and within a structure of formalized logic.

In Clara-Clara's view from a distance we see how Gestalt vision remakes the leaning arcs into upright sections of cylinders. As the spectator moves into the space between Clara-Clara's arcs a second kind of spectatorship takes over, one that is not a Gestalt or instantaneous view at all but rather vision through distortions perceived in time. Whereas the gestaltists assumed the perceptive apparatus always to reject complexity in favor of a single coherent view, Clara-Clara presents two images that compete in the process of apperception. The two ways of seeing preclude the spectator from comprehending the piece, for comprehension occurs only with a single apperception. Instead, the failure to select a single image is matched by the recognition that normal, unchallenged vision overwhelmingly selects a single, stable image of any perceived phenomenon. The sculpture transforms apperception into conscious experience, from its usual passive mode to the active work of image formulation.
apperception and other routes to the sublime

Yves-Alain Bois discusses the tension that develops between two ways of seeing which he terms the pictorial and the phenomenal in “A Picturesque Stroll around Clara-Clara” (1983, exhibition catalog). He characterizes the pictorial in terms of the frontal position one takes in relation to a painting and with perception of the ‘whole’ painting at a glance. He describes the phenomenal in terms of the spectator’s movement through Clara-Clara’s passage at the Tuileries location. Bois associates the phenomenal with the Minimalist aspects of Serra’s work, and he aligns Clara-Clara’s view from a distance with Gestalt perception and Modernist notions of vision. However, he does not interrogate how this mirror symmetric image might actually occur or how its breakdown relates to the structure for formalized structure for comprehension Gestalt vision implies.

Bois wrote “A Picturesque Stroll” when Clara-Clara was destined for the Forum, a large open space at the ground level of the Pompidou Center, weeks before Clara-Clara was installed instead in the Tuileries and months before the sculpture’s relocation to the Choisy site. His discussion is limited to the kinds of spectatorship he anticipated Clara-Clara to produce inside the Pompidou Center, which correlate remarkably well to those at the Tuileries location. The two sites are similar in size and in the paths of approach to the sculpture. For this reason, Bois’s discussion centers on the conflict Clara-Clara develops between a Minimalist and a Modernist spectatorship. The Choisy site however unexpectedly reveals the importance of the effects of the piece as they occur across the space of the sculpture itself. These have no place in the aesthetic theories of either Minimalism or Modernism.
While Bois could not have predicted these effects his argument does lend itself to investigation of *Clara-Clara* in terms of the tension it develops between this third visual experience and the view from a distance: the Baroque and the Modern.

Bois structures his argument on the notion of mutuality to show how the two ways of seeing he discusses – the pictorial and the phenomenal – comprise a complementary structure. He traces the conflict between these kinds of vision to 19th Century theories of the Picturesque. Here the optics of pictorial vision, which is tied to a fixed point of view, were applied to the design of landscape, into which the mobile viewer enters. The incompatibility of these ways of seeing – a fixed versus mobile viewpoint, perspectival vision versus the play of parallax – went unacknowledged in theories of the Picturesque. Bois suggests the contradiction formed a rupture from which Modernist aesthetic theory emerged. The contradiction between ways of seeing was repressed and not resolved with the emergence. Modernist aesthetics took up the pictorial, transformed it into the formal, and censored the phenomenal.

That Modernist art is structured on repression is hardly a new argument; the structure of repression nevertheless provides a foil to the functional structure of *Clara-Clara*. In her article entitled “Grids” Rosalind Krauss asserts the grid of Modernism functioned to cover over, to repress and therefore to repeat, a set of contradictions that Modernist aesthetics pretended to have resolved.” Specifically, she traces the grid’s repression of the tension between a spiritual versus materialist approach to art, its aggression toward representation versus its overwhelming presence in the treatises on optics and perspective, it’s a-historicity
versus its lineage not to mention through religious symbology to the gridded windows of Symbolist art.

Krauss suggests the grid acts in accordance with Claude Levi-Strauss's understanding of myth by permitting contradictions to coexist, unresolved, at the price of development. In Krauss's analysis is Structuralist: she considers mythic structure in terms of its ability to contain contradictory beliefs while its narrative covers over these binary oppositions within the primary belief system of a culture. The narrative allows contradictory beliefs to coexist but at the same time resists development of those beliefs. Mythic structure — the grid of Modernism — does not challenge conflicting beliefs but rather holds them in abeyance. The grid appears to neutralize certain fundamental and contradictory notions about art — its content, its relationship to representation and to history — yet in fact it merely submerges these beneath the very image of neutrality. Like the neurotic patient who is doomed to repeat the symptom until the repressed conflict is acknowledged the grid repeats with resounding success but never can develop.

If the structural repression of conflict dooms the myth, the grid, the patient, to repetition without development, Clara-Clara by contrast appears to express, to effuse, to force the issue of the conflicts it produces. Clara-Clara develops through the structure of contradiction, a structure that propels itself forward through the conflict its contradictories produce. The arcs themselves contradict one another as each arc's radius of curvature acts against the

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curvature of the other. Conflict emerges between *Clara-Clara*’s self-professed axially and the axis it disrupts at the Tuileries site. The piece produces conflicting kinds of spectatorship at each site and contradiction develops further between the two binary visual models as they differ at the two sites. The contradictory kinds of spectatorship in turn produce conflicting apperceptions for the spectator, and these continue to counter each other in a cognitive structure even after the spectator has left *Clara-Clara*. The sculpture’s success lies precisely in the extent to which it produces and develops these very conflicts. Where Modernism sought a static and instantaneous vision *Clara-Clara* prolongs the act of vision and propels development with the force of contradiction.

Bois proposes his own explanation for why *Clara-Clara* ‘develops’. He recalls that Modernist aesthetic theory, particularly as articulated by Clement Greenberg and Michael Fried, crystallized with their articulation of the first book of the first division of Immanuel Kant’s third critique, the *Critique of Judgment* (1790, 1793). Their singularly partial aesthetics stems from Kant’s ‘Analytic of the Beautiful’ a theory of the aesthetic object in terms of its limits, its shape, its availability to instantaneous comprehension. However, the second book of the first division – composed in full mutuality with the ‘Analytic of the Beautiful’ – constructs an alternative aesthetic theory dealing with experience as it develops in time. The ‘Analytic of the Sublime’ describes the satisfaction derived from the experience of that which is available at once to apprehension (and so apperceived in a moment) and that which exceeds comprehension. The sublime emerges from the tension a single object develops between those aspects of itself that are ‘beautiful’ and those that go beyond comprehension. Kant’s

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sublime consists not of a single kind of vision but in the simultaneous production of two mutually incompatible types of vision. These in turn produce a cognitive structure where the contradiction between the two develops through the very conflict they sustain. Whereas repression of contradiction according to the ‘beautiful’ produces repetition – as with the grids of Modernism – the tension produced by the sublime – *Clara-Clara* – permits the aesthetic object, and experience, to develop.

But there is more at stake than Bois indicates. He shows how the sublime emerges within a tension that develops between two ways of seeing as the aesthetic object produces them. But he interrogates neither the mechanisms that produce these two images nor the structure the images produce. Normally, the mechanisms of apperception select the single image of an observed phenomenon we see: these mechanisms’ failure to select just one – *Clara-Clara*’s production of two apperceptions at each site – compels cognitive function to run in parallel. The experience of this cognitive structure is the experience Bois associates with Kant’s ‘Aesthetic of the Sublime’. Rather than cover over conflict with its form, *Clara-Clara*’s non-intersecting arcs produce conflict by developing, exacerbating, a whole range of incompatibility. These, the conflicts that never intersect, occur as cognitive structure. Their effect, the Kantian sublime, propagates independent of the object: the sublime develops as cognitive structure. There must exist other paths of approach, parallels to the aesthetic object, that also produce this structure of the sublime.
contradiction in parallel – part 1

_Claro-Claro_ succeeds to the extent that it maintains its binaries whose multiplicity exceeds any dialectical structure we might assume. There are two arcs, two sites, two sets of two ways of seeing: they all collide as contradiction yet the many contradictories multiply each other. The sculpture was installed at two sites yet was intended for a third, the two ways of seeing evolve into three alternatives divided into two sets at each site. When two or more alternatives – apperceptions, in _Claro-Claro’s_ case – never intersect but rather propel side by side, indefinitely juxtaposed, the structure they produce is parallel.

Yet the very notion of parallelism is itself rift with contradiction and has been since Antiquity. The Classical notion of parallelism derives from Euclid’s parallel postulate which rests on assumptions about infinite space. These assumptions are impossible to prove and so in conflict with the notion that geometry comprises a set of testable propositions about spatial relationships observable in the world. Attempts through the centuries to resolve the problem of the parallel postulate merely repeated for twenty centuries the original misstep of assuming a purely intuitive notion to hold in an otherwise empirical formal system. It was not until the 19th Century that certain mathematicians re-introduced empiricism to the study of geometry. They opened the way for development of their field by allowing observation to contradict the long-guarded and fundamentally problematic theories of geometry.
Classical, or Euclidean, geometry emerges from a set of definitions, ‘common notions’ and postulates and the system deduced by them in Euclid’s *Elements* (c. 300 BC). The definitions describe the properties of the geometric elements – points, lines and planes – and the relationships that obtain between them – convergence, intersection, parallel, for example. Five common notions denote self-evident assumptions about the relationships between elements, such as “things which are equal to the same thing are also equal to each other” and five postulates state rules by which relationships between the elements may be constructed. Euclid’s is an extraordinary work and was seminal to centuries – millennia – of scientific development within a static spatial model. Euclid did not invent this system so much as compile the accumulated ‘science of geometry’ of his day. Ironically, the one postulate he is credited as having formulated is by far the most problematic of so-called Euclidean geometry.

The fifth, or ‘parallel’ postulate is paraphrased “Through a given point can be drawn only one parallel to a given line.” The statement assumes that lines must be either parallel or not – there is no third category – and that they extend, equidistant, to infinity in both directions on a perfectly flat plane. These assumptions could not be proved because Euclid’s infinite homogeneous space was impossible to see and measure, and certainly to record. To the extent that the parallel postulate could not be subject to any rigorous observational tests it transgressed the empirical basis of geometry. Here in this evasion of even the possibility of contradiction we see emerge the very grid, the assumption of its neutrality and its infinite extension, that Krauss shows prevails twenty-three centuries later.

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covering over and neutralizing, restraining, undeveloped, any number of conflicted notions about the representation of space.

Geometers since the time of Euclid worked to ‘solve’ the problem of the parallel postulate from the point of view of re-defining parallelism and re-postulating the relationship that must obtain between parallel lines. Because they worked within Euclid’s assumption these attempts merely repeated the original problematic statement. In 1824, Karl Friedrich Gauss, the dominant figure in European mathematics at the time, asserted publicly after having spent decades on the problem that geometries alternative to the Euclidean system must be possible and that they must be based in some alternative notion about the extension of parallel lines. Evidence suggests Gauss was the first to formulate such a geometry, and he is recognized as having coined the term ‘non-Euclidean’ but he never published his results.\(^{14}\) By 1829, Nikolai Ivanovich Lobachevsky, a Russian, and Janos Bolyai of Hungary independently formulated and published (Bolyai in 1832) a coherent system of geometry contradictory to Euclid’s. All three mathematicians tested a single assumption about parallel lines contradictory to what Euclid had assumed and found the contradiction yielded a strikingly different but internally coherent new geometry.

Where Euclid’s postulate assumes only one parallel can be drawn through a point not on a line the Lobachevsky-Bolyai model asserts more than one parallel to a line will pass through a given point. Euclid’s assumption produces the unprovable notion that parallel lines must extend, equidistant, to infinity: the Lobachevsky-Bolyai assertion permits and requires parallels to approach and diverge from each other as they extend through space.

Furthermore, if two parallels to a line cross through a point not on the line there must by implication be infinitely many lines that pass through the point within the angle inscribed by the first two. The two lines that bound the infinitely many interior lines are called the parallels to the line; the infinitely many lines that pass within their angle are referred to as non-intersecting lines.

As the point through which the parallels and non-intersecting lines pass moves away from the line they never intersect, the angle the parallels inscribe enlarges. With this, the infinite number of non-intersecting lines increases. Where Classical mathematics considers infinity as a single, homogeneously large number that can exist only potentially, Lobachevsky and Bolyai demonstrate how infinitely many different sizes of infinity can and must exist. In this way, Gauss, Lobachevsky and Bolyai disrupted not only Euclidean assumptions about space as homogeneous and static but also the very notion of infinity and infinite extension as the transparent and linear continuation of local spatial conditions.

Their geometry induced a paradigm shift as much philosophical as mathematical. Gauss’s self-proclaimed reasons for not coming forward with his own research on non-Euclidean geometry were based in fear, on the one hand of ridicule by his associates and on the other, fear of the personal consequences he might pay for defending a spatial model so in conflict with the prevailing worldview.15 This world view and its cultural force stemmed from the overwhelming coincidence of Kantian and Euclidean notions about space: both models intuit three-dimensional space as static and homogeneous, deterministic and computable in

15 Wolfe reproduces at length several of Gauss’s letters on the topic. Wolfe himself comments on the import to the prevailing Enlightenment-Kantian worldview brought about by non-Euclidean geometries. (Ibid., p. 44-48).
its extension. Gauss, Bolyai, Lobachevsky, and later Riemann all reconfigured the study of geometry as an empirical endeavor, using testable hypotheses and observed data to demonstrate the dynamics and uncomputability of spatial conditions.

Ironically, Kant likened his own project within philosophy to Newton’s approach to science. He accorded primary importance to observation rather than to the isolated refinement of predetermined definitions and theories. Specifically, Kant attempted to shift the primary concern of philosophy from a preoccupation with the search for complete definitions of concepts – not unlike Euclid’s definitions, common notions and postulates – to focus instead on “those characteristic marks that are certainly to be found in the concept of any general property”. However, like Newton, Kant did not grant the conditions of space the same agency he did the structure of concepts.

This conflict between the Kantian-Euclidean model of space as intuitive, homogeneous and static and the empirically based model of dynamic and modulated space developed by Gauss, Lobachevsky and Bolyai, re-emerges with Clara-Clara’s presence in the Tuileries. Here, monuments, trees and buildings pose equidistant about a straight line that extends from the center of the Louvre through the Arc de Triomphe and the Arche de la Défense. The monuments frame a vanishing point, a presumed view to infinity that the axis constructs. The equidistant rectilinear walls and trees naturalize the agency of conic perspectival vision by framing a view to infinity which is exactly not that of infinite vision but rather the farthest reach of a strictly limited capacity to see. Clara-Clara disrupts the notion that we see to infinity along this axis; the sculpture reproduces vision as sight that is limited in its capacity to see through space. Passing through the piece, the spectator reenacts the paradigm shift –
the conflict – that leads from a spatial model that frames the subject's view to infinity to one that admits the limits and situation of the subject. *Clara-Clara* produces, rather than a command of infinity, infinitely many possibilities, contradictory alternative ways of seeing, that grow a space for development in the cognitive structure of the subject.

Furthermore, the Lobachevsky-Bolyai model for parallel lines permits us to imagine the cognitive space *Clara-Clara*'s two conflicting apperceptions must construct. If parallels, lines that never intersect, must approach and diverge as they extend forward so too must apperceptions that never intersect. The spectator never correlates the view from a distance to the view within the piece, yet at moments the two must almost join as a single image or else diverge radically apart. As they propel inward and outward and always forward through the conflict they produce the parallels in effect compress and expand the cognitive structure in which they move. Even if Kant never envisaged a spatial model so dynamic surely this is exactly the modulation through conflict coincides with – develops – an aesthetic of the sublime.
contradiction in parallel – part 2

Georg Friedrich Bernhard Riemann, one of Gauss’s students, formulated a geometry even more empirical – and so more contradictory to the Euclidean scheme – with his observation that unbounded space need not be infinite. He first presented this reformulation in 1854, in a lecture to the Philosophical Faculty at the University of Göttingen. His observation inverted the finding of previous non-Euclidean geometers, who had suggested a bound infinity where parallels form the outer limits of infinitely many non-intersecting lines. Riemann on the other hand showed how unbounded space, as on the surface of a sphere, can be finite.

Within this space, extension of any line forms a great circle.\textsuperscript{16} By implication, all lines must intersect at the poles of the sphere and so no line can ever be drawn parallel to any other. Riemann’s observation and hypothesis – that parallelism is impossible on the space of a sphere produced an entirely new geometry that describes qualities of space with remarkable accuracy even as it defies intuition.

Among other departures, Riemann’s formulation of lines as great circles reconfigures the Euclidean notion of the line – a breadthless length – so that it contains space, a power reserved in Classical geometry for no fewer than six lines. The Riemannian line passes first

\textsuperscript{16} Extension must occur perpendicular to the tangent of the surface on which it travels, and any line must connect two points through the shortest possible route. On a spherical surface this shortest route will always form an arc of a great circle, and all great circles will intersect twice. Lines of latitude are not true lines and so do not contradict Riemann’s hypothesis.
through one pole then back to the equator, to its second pole and back to the starting point. However, Riemann discovered there was no predetermined number of poles through which a given line should pass, and he found no predetermined radius of curvature for the sphere on which lines travel. He found lines might pass through a single pole to form a single-sided surface like that of the Möbius strip, which reconfigures notions of inside and outside, front and back with its single continuous twisting surface. Or lines might pass through infinitely many poles, forming a geometry far more complicated than the three-dimensional space presumed by Euclidean mathematics. Furthermore, the radius of curvature might vary from a very small radius, a tightly curving surface to a radius that approaches infinity, in which case lines along the surface straighten out and Euclidean geometry holds perfectly well. In order to accommodate this spatial flexibility Riemann abandoned the study of spherical surfaces and developed instead an elliptical geometry. Here, lines pass through any number of poles and they curve through an infinite variety of radii.

Study of elliptical surfaces permitted Riemann to construct multiple coherent geometries which he proposed must occur along a single continuum. He had predicted even in his earliest public statement “It will follow from this [the assertion that all lines must intersect] that a multiply extended magnitude is capable of different measure-relations, and consequently that space is only a particular case of a triply extended magnitude.” The multiple measure-relations he describes in this statement he later would name manifolds, a reference to the multiple folds that occur along a single surface as changes in the polarity of lines and in their radius of curvature construct and modulate the surface they move across.
Riemann and his successors developed a metrical geometry to account for the differential measures of space as they occur along a manifold surface. Where lines pass through any number of poles and curve about an infinite range of radii, vectors provide a consistent set of units for measuring change across space. For example, they used vectors to map the changes of an object’s shape as it moves along a curving surface. A vector-based highly mathematic system permits the points of one fold to map onto the points of another along a single manifold surface and from one manifold to the next. In this way, the vector system describes qualitative properties of figures as they move through space, a task Euclidean geometry could never achieve due to its static conception of space and of the objects within it. Riemann’s success at describing these qualitative changes was twofold: he was able to map the topological changes of objects as they move and of the surfaces which their movement affects.

*Clara-Clara’s* modulation of space and of a spectatorship that moves through it is obvious in certain ways. The points along the base of each arc map along the surface of the arc to a corresponding point along its top plane. Each vector describes its own plane; the accumulated planes crowd into each other, compressing the space between the arcs and then releasing it with incredible expansion. The vectors that make the arcs form a spatial fold through which we pass as we move through the passageway between the arcs. The stasis of the urban setting, whether in the Tuileries or in the Square de Choisy, sheds its neutrality, its

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17 Quoted from Riemann’s 1854 *Über die Hypothesen welche der Geometrie zu Grunde liegen*. Ibid, p. 61.

18 While Riemann is credited with the salient features of a geometry of manifolds in fact his memoir dealt almost entirely in generalities about this geometry; it was left to Cayley, Klein, Clifford and others to work out the details of how such a geometry might work.

19 Riemann’s description of spatial curvature proved incredibly useful to Einstein’s later description of space-time curvature in his *Theory of General Relativity*, 1911.
presumed passivity, as the spectator enters a space whose dynamism transforms the spectator into the object that compresses and expands as it, he, she, moves along the manifold surface. Here, Clara-Clara’s apperceptions become true lines in the Riemannian sense. They pass through a pole that Clara-Clara produces even as the spectator passes through a fold the arcs produce.

The lesson of these geometries is precisely that contradiction, once permitted and pursued, compels unimagined development. For Gauss, Lobachevsky and Bolyai, the choice to test an assumption, to open theory to empirical contradiction, propelled development of a spatial model that ended twenty-four centuries of seizure within the grid. Riemann recreated geometry as contradiction, using the incompatibilities of changing rates of curvature and of the metric polarity of lines to describe his manifold surface, where conflict develops and expresses across its own curvature and modulations. Yet mathematicians at the time interpreted the sudden development of multiple geometries as seriously undermining their field of study. The notion that multiple contradictory models account equally for observable spatial qualities produced the structure of paradox, the ancient concept of a contradiction that never resolves but rather turns over on itself through endless iteration. It seemed one of the geometries should resolve out as the correct one – as it seems a single way of seeing should resolve perception of Clara-Clara – yet each system, each apperception, proves internally coherent even as the systems remain incompatible.

Just as Clara-Clara offers different ways of approach, each tied to a kind of spectatorship, mathematicians sought an approach to their field that would offer a single reliable point of departure – a point of view – on the study of number and measure. Because geometry had
proved untenable as a source for a unified system of proof mathematicians turned inward to pure mathematics in their search for a foundational structure for their field. The goal of constructing a complete system of rules and procedures — algorithmic in structure and Euclidean in its stasis — to derive all mathematical truth became the primary activity of mathematicians working in the late 19th and early 20th Century.
paradox reconfigured as parallel

The notion that a single system of patterned reason will yield all mathematical truth is the fundamental stance of formalist mathematics. It assumes reason, or at least mathematical reason, to be patterned, linear, deterministic, algorithmic. That such a system might be able to produce all mathematical truths and only mathematical truths in a manner free from contradiction was not even questioned by these mathematicians even as they encountered time and again the structure of paradox within their systems. The recognition that the systems actually produce paradox, like the recognition that Gestalt mechanisms of vision sometimes produce contradictory readings, these mathematicians perceived as the failure of a particular system. The acknowledgment that paradox is the inherent product of formal systems and the effect this would have on the field of mathematics came as a shocking, unsettling yet incredibly productive development.

Georg Cantor today is considered the father of Modern mathematics. His theory of sets, developed in the 1880s in a desperate attempt to resist the multiplicity offered by non-Euclidean geometries, contributed enormously to the establishment of a system of proofs for mathematics.3 While his work ultimately led to the development of a mathematics that builds on multiplicity and contradiction he intended exactly the opposite, to weed contradiction out of a mathematics of infinity. Like the non-Euclideans, he developed a

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radical new mathematics by reconfiguring Classical notions of infinity and infinite extension. His theory develops the notion of a set, a collection of things of any kind, the set itself defined by a rule that determines what is or is not included in it.

Sets subsume whole collections of numbers, objects and principles, which then act collectively as mathematical entities rather than as a combination of members. Set theory was particularly helpful in that it permitted mathematicians to treat numbers and groups of numbers on equal footing. This permitted mathematicians to work with infinity, or a set with infinitely many members, as though it were any other number rather than one that only potentially existed as it did in Classical mathematics. It also permitted mathematicians to distinguish between kinds of infinity. Where Lobachevsky and Bolyai related different sizes of infinity to the angle formed by the outermost parallels to a given line, and where Riemann distinguished between unbounded and infinite space, Cantor developed a mathematical method for describing larger and smaller infinite sets.

He formulated the theory of sets by pursuing to its logical end the effect of adding an extra number, $\omega$, to the set of natural numbers. $0, 1, 2, 3, 4, \ldots \omega$. The addition of $\omega+1, \omega+2, \omega+3, \ldots$ immediately follows and leads directly to the series $0, 1, 2, \ldots \omega, \omega+1, \omega+2, \ldots 2\omega \ldots 3\omega \ldots \omega^2 \ldots \omega^3 \ldots \omega^4 \ldots \omega^\omega$. The effect is dizzying. At the point where $\omega$ is raised to the $\omega$ an infinite number of times, indicating an infinitely large infinite number, Cantor suggests we refer to the number as epsilon-nought, $\varepsilon_0$.

Cantor used these numbers, which he calls the ordinals, to create a different set of infinite numbers, the cardinals. Where ordinal numbers describe positions in infinite lists, $0, 1, 2, \ldots$
\( \omega, \omega+1, \omega+2, \ldots 2\omega \ldots 3\omega \ldots \omega^\omega \), cardinal numbers measure the size of infinite sets. The first cardinal number, aleph-nought, measures the set of all natural numbers. Aleph-1 measures the set of all subsets of the natural numbers, such as \( \pi=3.1415926 \). Aleph-2 measures the set of all subsets of the set of all subsets of the set of all natural numbers. Cantor pursued this progression to produce Aleph-\( \omega \), the set of all subsets of the set of all subsets of the set of \( \ldots \) and so on. Because the subscripts of all the cardinal numbers are in fact ordinal numbers, the system eventually produces the cardinal Aleph-\( \varepsilon_0 \). In other words, his system for adding \( \omega \) to the set of natural numbers yields a set of ordinals to organize the cardinal numbers of his system. In so doing, the system generates the capacity to provide qualitative information through the systematic application of a single proposition to its logical conclusion at – or near – infinity. It gives mathematical substance to the notion of properties.

The use of a numbering system to configure qualitative information can be used to describe any kind of quality. For example, the theory of sets can be used to define the qualitative property of redness as the set of all red things. Redness ceases to be a property and instead functions as an object, the set of all red things. This ability to account mathematically for qualitative properties solves several basic problems of mathematical formalism. Just as non-Euclidean models of modulated space described qualitative change and so exceeded by far the capacity of traditional geometry. Further, Cantor’s use of set theory to deal with quality was greatly expanded in 1884 by Gottlob Frege, the mathematician turned philosopher, who applied set theoretical thinking to the definition of numbers themselves. The number three, for example, can be defined as the set of all sets demonstrating the property of threeness.
Cantor's formalist system accounted for quality from within mathematics. This ability, and Frege's application of it to account for the properties of number, wildly expanded the project of formalist mathematics. But the circularity of set theoretical reasoning, the common-sense problem of defining redness as the set of all red things, for example, itself yields paradox, most powerfully demonstrated by Bertrand Russell in 1901.

Russell's paradox poses the proposition '\( R \) is the set of all sets which are not members of themselves.' This proposition immediately produces the question of whether \( R \) is a member of itself, and the answer is both 'yes' and 'no': a paradox. The self-referential structure of the proposition is exactly such that it produces two equally viable yet irresolvable answers. \( R \) cannot be a member of itself if it is to be true to its definition yet it becomes a member of itself exactly to the extent that it is not a member of itself. \( R \) defines itself through a function of negative self-reference. Its assertion of negative self-identity moves the hierarchical method of defining sets into a circular, alternating, irresolvable contradiction. This structure defines the set \( R \) even as it renders determination of \( R \) in terms of itself impossible. \( R \)'s structure produces these parallel answers, and the thinker is restrained in a back-and-forth loop between two alternatives answers.

While this loop might be one of development through its own conflict the structure of paradox usually acquires the functional status of myth. It gives the impression that its contradictories must exist indefinitely as they are, undeveloped rather than modulating through their contradiction. For this reason, paradox, once discovered, generally is left to repeat, undeveloped, for centuries. While Russell he clearly 'saw' how the circularity of Cantor's set theory produces contradiction he did not look to that conflict in terms of how it
might develop the very notion of mathematical proof. He accepted the paradox as a failure of set theory rather than its active product.

Russell, along with Alfred North Whitehead set out to compile a formalized mathematical system of axioms and rules of procedure into which all correct methods of mathematical reasoning should be included and which would produce all the correct statements of mathematical proof. They worked in response to the development of non-Euclidean geometries and the paradox Russell had discovered in Cantor's set theory. Their goal was to excise the possibility of contradiction from formalist mathematics. In other words, they disallowed the possibility of an experimental approach to mathematics. Their work, the *Principia Mathematica*, was published in three volumes appearing in 1910, 1912 and 1913.

The *Principia Mathematica* – like the algorithms proposed by the gestaltists – proved extremely cumbersome and accounted only for a very limited scope of mathematical reasoning. David Hilbert, who had worked to conserve Euclidean geometry against the challenges alternative geometries posed, then attempted what was to be a more workable and a more comprehensive formalization of mathematical thought. He included in it even an attempt to prove his system free of contradiction.21

But contradiction prevailed when, in 1931, Hilbert's project still underway, Kurt Gödel, a 25-year old Viennese mathematician, demonstrated in his paper “On Formally Undecidable Propositions in *Principia Mathematica* and Related Systems I” the impossibility that any such system account for all its propositions and still be free of contradiction. While Gödel

responded directly to Russell and Whitehead’s *Principia Mathematica*, he was able to show that any formal system sufficiently complex to account merely for the rules of linear arithmetic must produce paradox. Gödel’s work, commonly referred to as his *Incompleteness Theorem*, reconfigured formalist mathematics forever. His theorem established the impossibility of its primary goal yet it opened the way for stunning advance as formalists learned to work within the limits of computational systems.

The genius of Gödel’s argument lay in the idea to code the statements of number theory—the rules of the system—into mathematical operations themselves. He achieved this by classifying mathematical functions by the number of variables on which they are dependent and then ordering them by length within each classification. For example, the functions \( f(x) = -1 + y \) and \( f(x) = .5y \) both are functions whose solution for \( x \) depends only on the value of \( y \). By extension, solution of a function such as \( f(x) = y + z \) is dependent on two variables, \( f(x) = w + y + z \) on three, and so on. Within each group, functions were labeled according to the length of their statement, so \( f(x) = .5y \) would be assigned a lower code number than \( f(x) = -1 + y \) within the two-variable category. Gödel also coded the proofs of the system in a similar manner so that all the statements of a formalized mathematical system might be assigned a code number.

This coding permits the system to act on numbers both as numbers and as statements about number. In this way, the system renders all the statements of number theory subject to the functions of the very system the statements constitute. The structure compels formalized mathematics to act on itself, to run the propositions and proofs that make up the system through the very functions they describe. Clearly Gödel’s coding draws on Cantor’s use of
ordinal numbers to order his cardinals in that an invented system for numbering works to organize a second level of the same system. Gödel’s choice to code number theoretical statements necessitates that his findings extend to all systemized mathematical thought.

His coding complete, Gödel turned his number theory system to question the possibility that a mathematical system such as Hilbert’s or Russell and Whitehead’s ever be free of contradiction. More precisely, he constructed a coded statement that pushed into contradiction the notion that any system might generate all the true statements of mathematics as syntactically correct statements produced by the rules of the system. He formed a proposition that states there can be no proof within the system for a certain function. "There does not exist any number x such that the xth proof of the system proves the wth function as applied to the number w and coded it as \( P_x(w) \). Mathematically, this is recorded as

\[
P_x(w) = \exists x[\Pi x \text{ proves } P_x(w)]
\]

Then Gödel tested what the function might say if \( w = k \).

\[
P_x(k) = \exists x[\Pi x \text{ proves } P_x(k)]
\]

With the substitution, the proposition \( P_x(k) \), or ‘proposition k applied to the number k’ states ‘there does not exist any number x such that the xth proof of the system proves the kth function as applied to the number k’. In other words, the proposition states there cannot be any proof of itself within the system. If the statement is false and there is a proof of it in the system then the system has produced a false assertion. Otherwise the statement must be true, in which case the system is incomplete as it evidently holds and produces statements it cannot prove. Either way, the formalist notion of a complete, unerringly mathematical system is revealed impossible.
A statement of negative self-identity produces not so much a contradiction (as the notion of paradox is generally understood) but two irresolvable alternatives. With the substitution, a proposition of negative identity ("there exists no \( x \) such that . . .") is turned over on itself to produce contradiction: \( P(\hat{e}) = \neg P(\hat{e}) \)." The contradiction emerges as a well formed (syntactically correct) statement from within a closed system of formal reasoning: the paradox it gives rise to must also be a product of the system.

Gödel's paradox is reminiscent of Russell's: a proposition of negative self-assertion produces them both. But Gödel's overwhelming success lay in his demonstration that all formal systems generate statements whose proofs cannot be held within the system that makes them. Any system must produce contradiction or else remain incomplete. Gödel's ability to extend Russell's paradox to all of formalist mathematics lay in divorcing the meaning of mathematical propositions from their formal expression. He revealed paradox as structural where Russell presupposed it semantic. The negative self-assertion \( \text{"R is the set of all sets that are not members of themselves\"} \) produces the mutually exclusive structure of paradox, but that structure appears to emerge from the meaning(s) of the statement rather than from the structure of the statement itself. Gödel, on the other hand, shows how, while a system produces the truth-value of any statement produced according to its rules, some statements that are well-formed will nevertheless produce more than one truth-value.

Contradiction enters the system with negative self-assertion, and cognitive reason "sees" the alternative meanings of the statement while the system that produces these alternatives can only cope with its syntax. In other words, while formalist systems as confined within their
linear restrictions they produce alternative pathways that propel beyond the system itself. Reason ‘sees’ what the agency of the system produces. Just as Clara-Clara produces two apperceptions that in turn produce the effects of the sublime, formalized logic produces alternative cognitions that activate a structure for reason. In both cases it is the agency of the object, the system, that produces conflict in the form of self-contradiction: the structure of the contradiction compels cognitive response within a parallel structure.

The double truths that reason ‘sees’ recall the double apperceptions that develop with Clara-Clara. The similarity of the two – the structure of the sublime and the structure produced with Gödel’s paradox – returns us to Kant, this time on his account of the properties of mathematics. Kant invented the category of the synthetic a priori specifically to account for the properties of mathematics. Synthetic statements are those in which the predicate is not already present in the subject. Rather than the structure ‘A equals not-A’, what he considered a true contradiction, synthetic statements formulate as ‘A is not B’. Kant argues in the Critique of Pure Reason that statements such as ‘7 + 5 = 12’ are synthetic in that the predicate ‘12’ is not already contained in the subject ‘7 + 5’. Two sides of the equation, even if mutually incompatible, can continue to exist side by side across the space of the equal sign, which incidentally is formed by two parallel lines. Kant’s notion of mathematics was that it was not merely synthetic but rather synthetic a priori, its truths pre-existing their formulation and so precluding the possibility of any fundamentally disruptive – or paradoxical – proposition.

Kant both allows and does not allow contradiction, and his formulation of the synthetic a priori admits, even as it tries to evade the centrality of contradiction and paradox within the
study, the structure, of mathematics. Yet it is exactly from within this structure that mathematics produces its own development. Like aesthetics, math demonstrates agency and develops exactly through the juxtaposition of contradictories just as Gödel’s statement propagates beyond its statement of equation. *Clara-Clara*, like Gödel’s statement, reformulates Kant’s binary of mathematical equation into a multiplicity of inequation. Innumerable values and formulations of the piece hold in parallel. For example, if we attempt to map all the points through which the two arcs pass within a single coordinate system a series of six inequalities emerges.

\[
\begin{align*}
\zeta_\sigma &= b - m \sqrt{x_\sigma^2 + y_\sigma^2} \\
0 &= b - mr: b = 875, r = 175 \\
m &= \frac{b}{r} = \frac{875}{175} = 5 \\
\zeta_\sigma &= 875 - 5 \sqrt{x_\sigma^2 + y_\sigma^2} \\
such\ that \\
\begin{cases}
-119 \leq \sigma \leq 119 \\
-175 \leq y \leq 127 \\
0 \leq \zeta \leq 12
\end{cases}
\end{align*}
\]

\[
\begin{align*}
\zeta_\sigma + S &= -b + m \sqrt{x^2 + (y - c)^2} \\
\zeta_\sigma + 12 &= 875 + 5 \sqrt{x^2 + (y - c)^2} \\
such\ that \\
\begin{cases}
-119 \leq \sigma \leq 119 \\
-183 \leq y - 356 \leq 230 \\
12 \leq \zeta + 12 \leq 0
\end{cases}
\end{align*}
\]

And the power of the sculpture is that it stops here, leaving the two sides of the equation unresolved, and the possibility of the work, the experience, the sublime to develop.
WORKS CONSULTED

on aesthetic theory, and Clara-Clara


on mathematics and quantum theory


on perception

other