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RICE UNIVERSITY

Automatic Characterization of the Spatial Statistics of Topography
Using Geostatistical Methods

By

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ABSTRACT

Automatic Characterization of the Spatial Statistics of Topography

Using Geostatistical Methods

by

Sunitha Kesavan

An automatic geostatistical method of analyzing spatial characteristics of topography is presented. Anisotropy in spatial continuity is modeled by estimating the range of the directional semivariograms. Spatial characteristics of topography are defined by orientation of the semi-major axis, length of the semi-major axis and length of the semi-minor axis of the anisotropy model ellipse. Orientation of the ellipse gives the average orientation of the linear features. Aspect ratio of the ellipse estimates the degree of lineation of features. Area of the ellipse determines the roughness of the topography. The automatic geostatistical topography characterization method was applied to land topography from Western US and sea-floor topography from the Ross sea, Antarctica. The method was able to characterize the spatial patterns of topography from Western US. The method was used to separate highly lineated regions on the Antarctic sea floor from areas with less prominent or no lineations.
Acknowledgements

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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>LIST OF TABLES</td>
<td>IX</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>X</td>
</tr>
<tr>
<td><strong>INTRODUCTION</strong></td>
<td>1</td>
</tr>
<tr>
<td>Previous Studies on Quantitative Analysis of Topography</td>
<td>3</td>
</tr>
<tr>
<td>Objectives</td>
<td>5</td>
</tr>
<tr>
<td>Geological Setting of Study Area</td>
<td>9</td>
</tr>
<tr>
<td>Western US</td>
<td>9</td>
</tr>
<tr>
<td>Basin and Range Province</td>
<td>9</td>
</tr>
<tr>
<td>Columbia Plateau</td>
<td>11</td>
</tr>
<tr>
<td>Snake River Plain</td>
<td>12</td>
</tr>
<tr>
<td>Colorado Plateau</td>
<td>13</td>
</tr>
<tr>
<td>Ross Sea, Antarctica</td>
<td>14</td>
</tr>
<tr>
<td>Data</td>
<td>15</td>
</tr>
<tr>
<td>Elevation Data, Western US</td>
<td>15</td>
</tr>
<tr>
<td>Bathymetry Data - Ross Sea, Antarctica</td>
<td>16</td>
</tr>
<tr>
<td><strong>PRINCIPLES AND METHODS OF GEOSTATISTICS</strong></td>
<td>17</td>
</tr>
<tr>
<td>Grid/WINDOWS</td>
<td>18</td>
</tr>
<tr>
<td>LAG</td>
<td>18</td>
</tr>
<tr>
<td>Measuring Spatial Continuity Using the Semivariance</td>
<td>19</td>
</tr>
<tr>
<td>Semivariogram</td>
<td>20</td>
</tr>
</tbody>
</table>
STATIONARITY.................................................................21
PREPROCESSING..............................................................21
Lag Mean Method............................................................21
Frequency Filtering Method................................................22
Running Average Method...................................................23
CALCULATING THE SEMIVARIOGRAM MAP FOR
WINDOWS OF TOPOGRAPHY...........................................23
CONVENTIONAL METHOD................................................24
MOVING WINDOW METHOD..............................................26
Conventional Method Applied to Window A............................27
Moving Window Method Applied to Windows A, B and C.............27
CONVENTIONAL METHOD VS. MOVING WINDOW METHOD...........27
DISPLAYING THE SEMIVARIOGRAM MAP...............................28
INTERPOLATING SEMIVARIOGRAMS FROM THE MAP..................29
SEMIVARIOGRAM INTERPOLATION ALGORITHM.........................30
Interpolation Algorithm Applied to Windows A, B and C.............30
MODELING SPATIAL CONTINUITY........................................30
BEHAVIOR OF DIRECTIONAL SEMIVARIOGRAM......................30
Profile A.............................................................................31
Profile C (intentionally out of order) ....................................31
Profile B.............................................................................31
Profile D and E...................................................................32
Comparing Directional Semivariograms with Topography............32
SYNTHETIC TOPOGRAPHY MODEL A – SINUSOIDAL

PROFILE ........................................................................................................................................... 113

Semivariogram Map and Modeling ......................................................................................... 114

Conclusions from this Model ............................................................................................. 115

SYNTHETIC TOPOGRAPHY MODEL B – BASIN AND RANGE
PROFILE UNFILTERED .................................................................................................................. 116

Semivariogram Map and Modeling ......................................................................................... 116

Modeling Anisotropy in Range and Wavelength ................................................................. 118

Conclusions from this Model ............................................................................................. 119

SYNTHETIC TOPOGRAPHY MODEL C - BASIN AND RANGE
PROFILE DETRENDED .................................................................................................................. 119

Semivariogram Map and Modeling ......................................................................................... 120

Modeling Anisotropy in Ranges and Wavelength ................................................................. 121

Conclusions from this Model ............................................................................................. 121

APPLICATION TO TOPOGRAPHY DATA ...................................................................................... 122

WESTERN US TOPOGRAPHY ......................................................................................................... 123

Semivariogram Map and Modeling ......................................................................................... 123

Modeling Anisotropy ............................................................................................................. 125

Inferences from the Anisotropy Models for Different Window Sizes ......................... 125

Modeling Spatial Continuity over Western US................................................................. 128

Spatial Continuity Maps from Model Parameters ......................................................... 129
ANTARCTIC BATHYMETRY – MULTIBEAM DATA EXAMPLES ............ 131

Semivariogram Map and Modeling ........................................ 131
Modeling Anisotropy in Ranges and Wavelength ..................... 132
Conclusions from the Semivariogram Analysis on Example Windows .......... 133
Modeling Spatial Continuity over Grids 049c, 052c and 048c ............. 133

DISCUSSION AND CONCLUSIONS ........................................ 209

Application to Data .............................................................. 210
Window Size of Analysis ....................................................... 211
Effect of Trend on Semivariogram Analysis .............................. 213
Limitations of Semivariogram Modeling ................................... 213
Future Directions of Study ..................................................... 215

REFERENCES ................................................................. 218
LIST OF TABLES

Table 1.1: Antarctic multibeam bathymetry grids. 17
Table 2.1: Model parameters along 4 directional semivariograms for the 400*400 and 200*200 windows. 136
Table 2.2: Parameter of model ellipse for different window sizes. 137
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure Reference</th>
<th>Description</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1.1a</td>
<td>Digital elevation model of the Western US (GTOPO30)</td>
<td>47</td>
</tr>
<tr>
<td>Figure 1.1b</td>
<td>GTOPO30 DEM projected using Lambert Conical Projection</td>
<td>48</td>
</tr>
<tr>
<td>Figure 1.2</td>
<td>Location of the grids 052c, 049c and 048c</td>
<td>50</td>
</tr>
<tr>
<td>Figure 1.3a-c</td>
<td>Antarctic Bathymetry grid – 049c, 052c and 048c</td>
<td>52</td>
</tr>
<tr>
<td>Figure 1.4</td>
<td>Western US grid showing 200*200 window</td>
<td>57</td>
</tr>
<tr>
<td>Figure 1.5</td>
<td>Examples of lags and number of pairs at each lag</td>
<td>59</td>
</tr>
<tr>
<td>Figure 1.6</td>
<td>Semivariogram: Plot of the normalized semivariance with lag for elevation data</td>
<td>60</td>
</tr>
<tr>
<td>Figure 1.7a</td>
<td>Elevation grid of Western US showing two 200*200 windows: window A and window B</td>
<td>62</td>
</tr>
<tr>
<td>Figure 1.7b</td>
<td>Antarctic grid 052c showing 50*50 window C</td>
<td>64</td>
</tr>
<tr>
<td>Figure 1.8</td>
<td>Conventional method of semivariogram calculation</td>
<td>65</td>
</tr>
<tr>
<td>Figure 1.9a</td>
<td>4*4 window of sample elevation values</td>
<td>66</td>
</tr>
<tr>
<td>Figure 1.9b</td>
<td>Examples of pairs used in the semivariance calculation at each lag</td>
<td>67</td>
</tr>
<tr>
<td>Figure 1.10</td>
<td>Moving window method of calculating the semivariogram map</td>
<td>68</td>
</tr>
<tr>
<td>Figure 1.11</td>
<td>Elevation pairs for the 4*4 window at lag h (0,1) and h(0,-1)</td>
<td>69</td>
</tr>
<tr>
<td>Figure 1.12</td>
<td>Lags at which semivariances are calculated for window A, B and C</td>
<td>70</td>
</tr>
<tr>
<td>Figure 1.13</td>
<td>Window A isolated from the larger grid</td>
<td>71</td>
</tr>
<tr>
<td>Figure 1.14</td>
<td>Semivariogram map of window A using the moving window method</td>
<td>72</td>
</tr>
<tr>
<td>Figure 1.15</td>
<td>Semivariogram map of window B</td>
<td>73</td>
</tr>
<tr>
<td>Figure 1.16</td>
<td>Semivariogram map showing the semivariances calculated for window C</td>
<td>74</td>
</tr>
</tbody>
</table>
Figure 1.17: Semivariogram map of window A from the conventional method.................................................................75
Figure 1.18: Directional semivariograms extracted from semivariogram map of window A............................................76
Figure 1.19: Directional semivariograms extracted from the semivariogram map at intervals along radial transects.........................77
Figure 1.20a-e: Directional semivariograms extracted from the semivariogram of window A......................................................78
Figure 1.21a: Spherical model.................................................83
Figure 1.21b: Periodic model.....................................................84
Figure 1.21c: Nested model.......................................................85
Figure 1.22a-d: Behavior of semivariograms of topography....................86
Figure 1.23: Ideal shaped semivariogram of topography modeled by the spherical model...............................................88
Figure 1.24a: Semivariogram showing periodicity fit by a nested model.................89
Figure 1.24b: Above semivariogram fit by a purely spherical model............89
Figure 1.25a: Semivariogram with no periodic component fit by a nested model.....90
Figure 1.25b: Semivariogram in (a) fit by a purely spherical model...............90
Figure 1.26: Semivariogram with no periodicity modeled using the nested model....91
Figure 1.27a: Semivariogram showing periodicity and effects of underlying trend fit by a nested model.................................92
Figure 1.27b: Above semivariogram fit by a spherical model....................92
Figure 1.28a: Semivariogram showing high correlation/trend modeled by a purely spherical component........................................93
Figure 1.28b: Semivariogram showing high correlation/trend fit by a nested model....93
Figure 1.29a: Semivariogram with an ideal shape fit by a nested model............94
Figure 1.29b: Above semivariogram fit by a spherical model........................94
Figure 1.30a: Semivariogram showing periodicity fit by the nested model..........95
Figure 1.30b: Above semivariogram fit by the spherical model....................95
Figure 1.31a: Semivariogram showing trend and periodicity fit by the nested model...96
Figure 1.31b: Above semivariogram fit with a spherical model........................96
Figure 1.32: Directional semivariograms from the semivariogram map of window A ........................................... 97
Figure 1.33: Directional semivariograms from the semivariogram map of window B ........................................... 100
Figure 1.34: Semivariogram modeling algorithm fits spherical model to directional semivariogram from semivariogram map of window B ........................................... 101
Figure 1.35a-c: Directional semivariograms from the semivariogram map for Window C ........................................... 102
Figure 1.36: Rose-diagram of ranges for window A ........................................... 105
Figure 1.37: Rose-diagram of ranges for window B ........................................... 106
Figure 1.38: Rose-diagram of ranges for window C ........................................... 107
Figure 1.39: Ellipse fit to the rose-diagram of ranges of window A ........................................... 108
Figure 1.40: Ellipse fit to the rose-diagram of ranges of window B ........................................... 109
Figure 1.41: Ellipse fit to the rose-diagram of ranges of window C ........................................... 110
Figure 1.42: Variation of wavelength and range with direction for window A ........................................... 111
Figure 1.43: Variation of wavelength and range with direction for window C ........................................... 112
Figure 2.1a: Sinusoid profile used to generate synthetic topographic model A ........................................... 138
Figure 2.2: Synthetic model topography A ........................................... 139
Figure 2.3: Semivariogram map of 400*400 window from the synthetic topography model A ........................................... 140
Figure 2.4a-d: Directional semivariograms of 400*400 window from synthetic topography model A ........................................... 141
Figure 2.5a: Variation of modeled amplitude and wavelength with direction for the 400*400 window from Synthetic topography model A ........................................... 143
Figure 2.5b: Variation of modeled parameter amplitude and wavelength, with direction for a 250*250 window from the model topography ........................................... 144
Figure 2.6b: Semivariograms along N90E for the different window sizes ........................................... 145
Figure 2.7: E-W line from the Basin and Range used to generate synthetic topography model B ........................................... 146
Figure 2.8a: Profile from western US used to build Model topographic surface B ........................................... 147
Figure 2.8b: Synthetic model topography B created from Basin and Range ........................................... 148
Figure 2.9a:  Semivariogram map for a 400*400 window from the synthetic model topography B..................149

Figure 2.9b:  Semivariogram map for a 200*200 window from the synthetic model topography B..................149

Figure 2.10a-d: Directional semivariograms interpolated from the semivariogram map of the 400*400 window of Synthetic topography model B........150

Figure 2.11a-d: Directional semivariograms interpolated from the semivariogram map of the 200*200 window of the synthetic topography model B......152

Figure 2.12a:  Rose diagram of the ranges for the 400*400 window..................154

Figure 2.12b:  Rose diagram of ranges modeled for the 200*200 window.............154

Figure 2.13:  Wavelength and range vs. direction for 200*200 window from synthetic topography model B..........................155

Figure 2.14a:  Model profile used to construct synthetic topography model C........156

Figure 2.14b:  Synthetic model topography C created from Basin and Range profile filtered............................................157

Figure 2.15a-d:  Directional semivariograms from semivariogram map of the 400*400 window of synthetic topography model C.............158

Figure 2.16a-d:  Directional semivariograms from the semivariogram map of the 200*200 window of synthetic topography model C............160

Figure 2.17a:  Rose diagram of the ranges for the 400*400 window of synthetic topography model C.................................162

Figure 2.17b:  Rose diagram of ranges for the 200*200 window........................162

Figure 2.18a:  Wavelength vs. direction for 400*400 window from synthetic topography model B...............................163

Figure 2.18b:  Wavelength vs. direction for 200*200 window from synthetic topography model B.................................164

Figure 2.19:  E-W elevation profile across the Western US before and after filtering..165

Figure 2.20:  E-W topography profile from the Antarctic grid 052c before and after filtering........................................166

Figure 2.21a:  Decreasing window sizes shown on the Western US topography......167
Figure 2.21b: The location of the 400*400 window on the larger grid.................................................................168

Figure 2.22a-g: Semivariogram maps for the different window sizes –
400*400, 350, 300, 200,150,100 and 50*50.......................................................170

Figure 2.23a-g: Anisotropy models of range for different window sizes........171

Figure 2.24: Model ellipses for the different window sizes..............................175

Figure 2.25a: Directional semivariograms along N10E for different window sizes....176

Figure 2.25b: Directional semivariograms along S85E for different window sizes....177

Figure 2.26: Variation of model parameters of ellipse with window size............178

Figure 2.27a-d: Spatial continuity maps of the Western US using 100*100,
200*200, 300*300 and 400*400 windows.......................................................180

Figure 2.27e-h: Anisotropy model parameters for Western US
topography using 100*100 window..............................................................184

Figure 2.28: Window A and B from Antarctic grid 049c.....................................189

Figure 2.29a: Semivariogram map of window A from the Antarctic grid 049c.......190

Figure 2.29b: Semivariogram map of window B from the Antarctic grid 049c.......191

Figure 2.30a-d: Directional semivariograms for window A...............................192

Figure 2.31a-d: Directional semivariograms for the window B........................194

Figure 2.32a: Anisotropy in ranges modeled by fitting an ellipse to
the range values of window A.................................................................196

Figure 2.32b: Anisotropy modeled by fitting an ellipse to the range
values of window A..................................................................................197

Figure 2.33a: Variation of wavelength and range with direction
for the window A......................................................................................198

Figure 2.33b: Variation of wavelength and range with direction
for the window B......................................................................................199

Figure 2.34a-c: Spatial continuity maps of the Antarctic grids
049c, 048c and 052c using 50*50windows.................................................201
INTRODUCTION

Description of topography has largely been a qualitative process. Topography can exhibit immense complexity within a small area and qualitative descriptions do not always explain all of the existing spatial variability. The problem with such descriptions is even more evident when we want to compare the topography from two different study areas. In such cases, topography can be better described by statistical models, which provide quantitative descriptions of parameters like surface roughness and anisotropy. Pattern recognition techniques, namely spectral analysis, fractals, neural networks and geostatistics have been used to separate topography into different classes based on discrimination criteria. Be it a simple classification of the topography in a region, modeling the patterns of different geomorphic features namely drainage, hills and valleys, sea-floor features like abyssal hills or understanding tectonic features like the fractures in the Mid-Atlantic ridge, a statistical study is a good complement to the existing understanding of the geological, geophysical or tectonic process. While we do have these tools, we still need to understand the most robust manner in which they can be applied to a study area. Topography can be observed at varying resolution and scales. Depending on the region and method of data collection we may have to deal with extremely dense or very sparse regular or irregular distributions of data. The emphasis therefore is on quick, cost-efficient and automatic modeling methods that work on the available topography data and produce maps of variations in model parameters. The parameters should be significant and be relatable to the general morphology of the
topography and more usefully interpretable with the geology, geomorphology
and tectonics of the area.

One area of interest for applying the geostatistical topography
characterization is on the seismic data from the Iberia abyssal plain in order to
characterize the sea-floor and basement features. Automatic topography
classification methods will be very useful with the upcoming release of topography
data from the Shuttle Radar Topography Mission data (SRTM). The objective of the
SRTM is to produce digital topographic data for 80% of the Earth's land surface with
data points located every 1-arc second (approximately 30 meters) on a
latitude/longitude grid (EROS data center, 1999). Currently, the best available public
domain elevation data of global coverage is the GTOPO30 dataset, which has a
resolution of 30 arc seconds (1 km). GTOPO30 has been compiled from various
sources of data and therefore varies dramatically in quality over the globe. Using the
technique of interferometry, SRTM collected global radar data during a single, 11-
day Space Shuttle mission. These data will be processed to provide a globally
seamless and high quality dataset at much higher resolution than is presently
available. This high-resolution data increases the scope of using automatic spatial
characterization for regional and global studies where we can compare the spatial
statistics of topography from one region to the other. For various parts of the world,
maps of Earth's topography are limited, inaccurate, or nonexistent. Many mountain
chains, deserts, and tropical rain forests have very poor topographic coverage to date
mainly because of the difficulty in getting to these locations. The automated
topography classification methods using the SRTM data would prove to be extremely useful in initial reconnaissance studies of these regions.

PREVIOUS STUDIES ON QUANTITATIVE ANALYSIS OF TOPOGRAPHY


Krause and Menard (1963) analyzed the depth distribution of the sea floor of the northeastern Pacific Ocean and studied the distribution and classification of seamounts and abyssal hills. Neidell (1966) studied the amplitude spectrum of marine geophysical magnetic and gravity profiles in order to model sea floor features like the mid-ocean ridges, abyssal hills and abyssal plains. Hayes and Conolly (1975) studied the morphology of the Southeast Indian Ridge by studying the spectrum of sea-floor topography profiles.

Bell (1975) modeled the topography of abyssal hills in the North-east Pacific as a random distribution of statistically independent hills on the ocean floor. He computed the amplitude spectrum of several topographic profiles and found that the topographic spectrum exhibits an inverse square law dependence on wave number at large wave numbers and the spectrum flattens out at low wave numbers. Fox and
Hayes (1985) used amplitude spectra to quantify the roughness of the sea floor adjacent to the coast of Oregon. They divided the sea floor into stationary provinces and calculated the amplitude spectra along profiles in each province. The spectra showed a power law form and were fit by a straight line. The slope and intercept was computed for profiles along different directions. The anisotropy of the sea bottom was modeled by studying the variation of the slope and the intercept of the power law model as a function of azimuth (Fox and Hayes, 1985). They identified a general pattern of slope variation corresponding to tectonic and sedimentary provinces. Goff and Jordon (1988) modeled sea-floor morphology, mainly abyssal hills, using an autocovariance function. Their aim was to identify stochastic parameters of small-scale topography. Their model function had five parameters describing the amplitude, orientation, and characteristic wave numbers and fractal dimension of topography. In order to test the results they generated synthetic sea beam data from the model and compared the results with actual profiles.

and amplitude with direction. Mulla (1988) did spectral analysis on the elevation
data by calculating 2-dimensional periodograms of the residual elevation data. From
the periodogram, Mulla (1988) computed the orientation angle and wavelength of
significant periodic features. Herzfeld and Higginson (1996) developed an automated
sea-floor classification method and applied it to bathymetric data from the Mid-
Atlantic Ridge. They made sea-floor classification maps based on distinguishing
features such as sediment ponds, spacing and strike of abyssal hill terrain and
roughness of the sea floor.

None of the previous studies applied geostatistical methods to large-scale
tectonic province characterization or to the automatic detection of lineated
bathymetric features. None of the spectral studies have looked into classifying
topography using irregular data distributions. They are either sea-floor profiles or
data distributed at regular intervals. These are among the objectives of my study.

OBJECTIVES

The purpose of this study is to develop an automated method for
characterizing the spatial statistics of topography. With the aid of this automatic
spatial analysis tool, I should be able to quantitatively study the directional variations
of the statistics of any spatially distributed data. The area of analysis need not
necessarily be a regular grid of points. The method should therefore be able to handle
irregular data distributions. The method should also handle drift/trend that is
characteristic of most spatial phenomena. The spatial continuity should be expressed
quantitatively in terms of model parameters in order to facilitate easy comparison with the topography or with similar spatial statistics from another area.

I propose to use geostatistical methods for the automatic characterization of the statistical properties of topography. I will use the directional semivariogram as the basic measure of spatial variability in elevation or depth data. I will first divide the study area into suitable windows. The geostatistical approach will involve calculating and modeling the semivariogram map of these windows of topography data and finding a suitable model that explains the semivariogram map. This model will quantify the anisotropy in the correlation scales of topography within each window of observation. I will then use the model parameters as a means of associating specific topographic patterns with specific anisotropy models, thereby, possibly delineating topographic provinces showing the same statistics.

I will test the automatic geostatistical topography characterization method I develop on two datasets.

a) A relatively low-resolution topography dataset from the Western US covering approximately 3,060,000 square kilometers. In the Western US, my goal is to study and compare the anisotropy model for different physiographic provinces. I hope to be able to distinguish topographic domains within the Basin and Range Province that may be correlable with other dynamic parameters like crustal thickness, heat-flow and seismicity in extensional settings. I will also study the usefulness of the semivariogram
analysis in mapping the spacing between the periodic/pseudo-periodic features of topography.

b) Three high-resolution bathymetry datasets from the Ross Sea, Antarctica each covering about 1000 square kilometers. My primary goal is to use the geostatistical method for automatically locating lineated features and thereby aiding in robust reconnaissance of topographic patterns on the sea-floor.

As part of achieving the above objectives the study would also focus on addressing the following problems.

1) Given a dataset of known resolution what is the optimum window size for the semivariogram analysis? This will involve studying the effect of varying window size of observation on the anisotropy model.

2) How does the trend/drift in data affect the estimation of robust semivariogram model parameters. This study will aim at understanding the correct application and limitations of semivariogram modeling.

3) Effective ways of mapping the spatial continuity at finer intervals over large datasets and creating a geostatistical map of the study area from the model parameters.

4) Exploring the advantages and disadvantages of geostatistical methods for modeling topography data.

The application of geostatistics lies in spatial mapping and correlation studies of spatial data like topography. Geostatistical analysis can be effectively used as a pattern recognition tool. The semivariogram map and modeling techniques can be
used to find the orientation of lineated features, separate lineated areas from non-lineated areas, identify cross-cutting features in topography and model the length to width ratio of features like abyssal hills. By using geostatistical analysis to model spatial continuity of topography data from different regions with similar geological or tectonic environments, we can quantitatively compare the topographic patterns from the two areas. Irregularities in modeling topography can occur due to the differences in the method of preparing digital elevation models of topographic data. Geostatistical analysis of the SRTM (EROS, 1999) topography data can be used to study the topography patterns on a global scale as the elevation data has been collected on a single mission and processed using the same set of procedures.

An important application of semivariogram analysis lies in being able to model the cross-strike spacing between linear features. The spacing between the linear basin and range topography caused by rifting has been related to the thickness of the crust. By applying geostatistical analysis on topography data from a rifted province, we can spatially model the average spacing between the linear ranges and correlate the spacing between ranges with maps of crustal thickness. The method can be applied to topography data available from other planets in order to quantitatively compare patterns in topography with those on Earth. Another application would be to use geostatistical models to differentiate topography patterns created by tectonics from topography patterns created by more localized erosional processes. Geostatistical modeling using data of different resolutions may show differences in spatial continuity based on the scale of processes. Spatial continuity of high resolution topography data from a particular region may capture local correlation
scales related to erosional processes while spatial continuity modeled using low resolution elevation data may estimate correlation scales related to large scale tectonic or structural processes.

GEOLOGICAL SETTING OF STUDY AREA

WESTERN US

Basin and Range Province

The Basin and Range Province of the western US is characterized by subparallel mountain ranges and intervening alluviated basins formed by high-angle extensional faulting. The region extends from southern Idaho and Oregon through most of Nevada and parts of western Utah and eastern California, western and southern Arizona, southwestern New Mexico and northern Mexico. It forms an irregularly shaped region more than 1500 km long and 500-1000 km wide. The physiography of the Basin and Range is distinguished by a conspicuous pattern of alternating more or less regularly spaced, sublinear ranges and basins. The ranges are abrupt and steeply sloping and deeply dissected, the basins are typically broad, gently sloping and largely undissected. Relief within the uplands generally ranges between 500 m and 1500 m. The Nevada portion shows very high mountain ranges as long as 1180 km and 25 km wide. In contrast ranges of the Mojave Desert are significantly smaller and lower with range lengths of 25 km and width of 6 km. Extension in the Basin and Range was marked by the intitiation of widespread shallow detachment faulting. The exhumed remnants of this detachment faulting are the metamorphic core complexes. These are asymmetric antiformal uplifts of
metamorphic and plutonic rocks that are overlain by unmetamorphosed sedimentary rocks and separated from these cover rocks by one or more gently to moderately dipping zones of detachment faulting (Coney, 1980). The later phase of extension, which is responsible for the classic Basin and Range structure began at approximately 17 Ma. The mountain ranges and intermontane basins of the Great Basin are manifestations of high-angle listric faulting. Faults follow range fronts indicating that faulting has uplifted the range and down-dropped the adjacent basin (horst and graben structure). The ranges show a similarity in trend with northerly and northeasterly strikes. This style of deformation has continued episodically to the present day but had tended to become progressively more concentrated towards the margins of the Great Basin. The Great Basin presently varies from 500-1000 km in width, and has a mean elevation of 1.5-1.8 km above sea-level. The topography within the Basin and Range is bimodal consisting of a short wavelength component associated with individual basins and ranges (20-50 km spacing, with a mean amplitude of about 1 km), and a long -wavelength component associated with regional swells (200-300 km spacing, with a mean amplitude of 700m). (Harry et al 1993). Trend surface analysis of the general range morphometry within the Basin and Range demonstrate general increases in range length, width, relative area and average relief from the Mojave and Sonoran deserts to the Mexican Highlands to the Great Basin (Lustig, 1969). The general correspondence between range, morphometry and tectonic history across the province suggests the profound influence of tectonic environment on landscape development. The Basin and Range province is characterized by widespread seismicity. The heat flow averages 1.5-2.0
times the value for the stable continental areas. The crust is thinned and averages 20-30 km beneath the Basin and Range compared to 40-50 km beneath the Sierra Nevada and the Colorado Plateau.

**Columbia Plateau**

The Columbia Plateau is a basin-like sub-province characterized by late Cenozoic outpourings of basaltic lava (Baker et al, 1991). The Blue Mountains area of southeastern Washington and northeastern Oregon is the southern upwarped part of the province, whereas the regions east of the Cascade Mountains and west of Idaho Rockies make up the less deformed part of the province. Because of the gentle dips on the lava flows in the northern and eastern section of the plain, the term Columbia Plateau has been applied to this region. During the late Cenozoic, the Juan de Fuca Plate was converging obliquely northeastward against the North American plate and subducting beneath it (Atwater, 1970). Regional topography and drainage in the region has been broadly controlled by volcanism and tectonism related to this plate-tectonic convergence along the margin of western North America. The western part of the Columbia Plateau has been deformed into a series of anticlinal ridges, the Yakima folds (Baker et al, 1991). Subsidence during basalt emplacement about 14 Ma initiated the Pasco Basin, which developed into the structural and topographic low of the region. In the Pasco Basin, the basalt is overlain by Pliocene clastic sediments. Most of the eastern Columbia Plateau is relatively undeformed. Its basalt plain is overlain by tens of meters of Pleistocene loess called the “Palouse loess”. In the southeast, the loess is extensively dissected to form the rolling topography of the Palouse Hills. The surface of the Columbia Plateau is saucer shaped—nearly at sea
level at Pasco Basin but 600-2000m higher at its rims. The altitude of the Loess Hills ranges from 200-800m.

**Snake River Plain**

The Snake River Plain is an area of subdued, locally almost featureless, relief surrounded by mountains and highlands. The greater part of the plain falls in Idaho plain and is as much as 70 km wide in the western part, 90-100 km wide in the eastern part and about 600 km long. It surface rises with a gradually increasing gradient from altitudes of less than 700m in the western end to about 2000 m in the eastern end (Malde 1991). The Snake River Plain is a major late Cenozoic tectonic feature. The plain consists of distinctive western and eastern parts, which differ in structural trend and geology. The western Snake River Plain is a northwest trending structural basin, bounded by high angle faults and filled with sedimentary deposits that are 1.7 km thick. Geophysical findings have indicated that the structural basin is an area of crustal extension and that its present low altitude relative to uplands and mountains to the north and south reflects isostatic compensation for a large underlying mass of basalt and an unusually thick lower crust (Mabey, 1982). Altitudes within the western plain range from 600-1100 m. The northeast-trending plain in the east is a volcanic province characterized by undissected lava flows of basalt throughout most of its length and underlain by rhyolitic volcanic rocks. Most of the lava flows accumulated as small low shields, tube flows and fissure flows. The rhyolite and basalt are thought to be products of a mantle plume, or hot spot (Vink and others, 1985), which produced a belt of volcanic terrain extending from southeastern Oregon and adjacent Nevada to the Yellowstone Plateau, as the North
American plate drifted southwestward between 17.5 and 0.6 Ma. The surface of the eastern plain ranges between 1000-2000 m in relief.

**Colorado Plateau**

The Colorado Plateau is a huge roughly kidney shaped region of many high plateaus and isolated mountains that encompass large parts of Utah, Colorado, New Mexico and Arizona. The distinguishing features of Colorado Plateau are: its considerable altitude, nearly all above 1500 m; its near horizontal bedrock and its strong stepped landscapes consisting of cliff like escarpments separated by wide, gentle slopes. On the west, faults and the perimeters of volcanic plateau mark the boundary between the Colorado Plateau and the Basin and Range. At the beginning of and during the early tertiary, the Colorado Plateau region was a basin with interior drainage not far above sea level. Epeirogenic uplift that started in middle Eocene time, and lasted at least into late Miocene time caused the plateau to be a "high" region subject to widespread erosion. Thus the modern physiography of the Colorado Plateau is the product of the continuous erosion throughout late Cenozoic time. Different bedrock stratigraphy and structure have profoundly affected the physiography and geomorphology in the plateau. The northern section consists of a broad structural basin called the Uinta Basin. South of this is the Canyonlands and High Plateau's section. In the Canyonlands, the Colorado River has cut canyons into relatively flat lying Paleozoic and Mesozoic sedimentary strata. The High Plateaus section consists of elongate, linear, north-trending plateaus reaching up to 3500 m in altitude. Further south in the Navajo and Acomi-Zuni sections, structural basins are the principal physiographic features. In the southwest of the province the Grand
Canyon section consists of a relatively flat plateau surfaces, which range from 1500-1700 m in altitude.

**ROSS SEA, ANTARCTICA**

The Antarctic seafloor consists of a variety of subglacial geomorphic features formed beneath the ice sheet. The 1999 cruise of the R/V N.B. Palmer collected multibeam swath bathymetry as well as seismic data, sediment cores and side-scan sonar from the continental shelf of the eastern and central Ross Sea. These suites of data are used to interpret the nature of advance and retreat of the ice sheet and determine ice sheet position during the Last Glacial Maximum. Multibeam bathymetry data was collected with a hullmounted Seabeam 2100 swath bathymetry system. This system provides images of the sea-floor geomorphology at a resolution of 20 horizontal meters. The raw multibeam data are processed and distributed at a suitable spacing on a geographic grid of points.

The dominant large-scale geomorphic features on the shelf consist of glacial troughs caused by advancing ice streams. These troughs can be about 45-65 km wide and reach depths of greater than 600 m on the shelf (Shipp and Anderson, 1999). Multibeam surveys and data collection are focused in the troughs, as they are the locuses of primary drainage of the West Antarctic Ice Sheet flowing into the eastern and central Ross Sea.

Geomorphic bedforms within the troughs include mega-scale glacial lineations, drumlins, iceberg furrows and grooves. Mega-scale glacial lineations are relatively long, straight and parallel and over 40 km long. They range from 8-70 km in length; widths between 200-1300 m and spacings in the order of 300-5000 m.
Cross-cutting patterns are common in lineations and these represent changing flow directions through time. Iceberg furrows are arcuate features carved by iceberg keels impacting and ploughing the sea-floor (Anderson, 1999). These are 10-200 m wide and less than 10 km in length. Grooves are elongated linear depressions formed by erosional processes.

DATA

Elevation Data, Western US

I use GTOPO30 elevation data (USGS EROS Data center, 1993) of the Western United States. This is a global digital elevation model (DEM) with a horizontal grid spacing of 30 arc seconds. GTOPO30 elevation data were compiled from several raster and vector sources of topographic information and projected in geographic coordinates on a WGS84 datum. To facilitate electronic distribution, GTOPO30 has been divided into 33 smaller pieces, or tiles, with each tile covering 50 degrees of latitude and 40 degrees of longitude. I obtained a subset of elevations from tiles w140n40 and w140n90 (USGS EROS Data center, 1993) consisting of 1920*2160 values. The area covers most of the Western US extending from 33°N to 49°N and 106°W to 124°W (Figure 1.1a). High elevations are color coded in red and low elevations are shown in blue. Elevations below 500 m are black. Ocean areas have been given a value of 0 and are recognized as “No Data” areas in the analyses. The Basin and Range Province is characterized by N-S trending mountains separated by flat basins. To the west of the Basin and Range are the Sierra Nevadas. The flat area to the north of the Basin and Range is the Snake River Plain. The highly
elevated region east of the Basin and Range is the Colorado plateau. The 30-arc second grid spacing equates to about 1 kilometer along the latitudinal and longitudinal directions at the equator. However, the distance covered on the ground by 1 grid unit decreases in the east/west (longitudinal) direction as latitude increases. This uneven spacing is undesirable for spatial analysis. In order to construct a uniform grid I projected the elevation data using the Lambert Conical Conformal Projection with standard parallels at 36N and 46N and scaled it to a uniform grid spacing of 1000 m. The projected image (Figure 1.1b) has 1811*1689 elevation values.

**Bathymetry Data - Ross Sea, Antarctica**

I use three multibeam bathymetry grids from the Antarctic that provide images of sea-floor geomorphology. The bathymetry grids have been made after editing and processing raw multibeam data collected from the Ross Sea on the R/V N.B Palmer between February 12, 1999 and ended March 24, 1999 (S. O’ Hara, 1999). The data has been gridded at suitable scales to produce shaded relief maps of the Antarctic sea-floor. I obtained the depth data in the GMT netCDF grid format (GMT cookbook, 1996). The grid projection is geographic and the spacing in latitude and longitude is uniform. The geographic location of the grids is shown in Figure 1.2. The location, size and grid spacing of the three grids are listed in Table 1.1.
Table 1.1: Antarctic Multibeam Bathymetry Grids

<table>
<thead>
<tr>
<th>Grid</th>
<th>Location</th>
<th>Size: Row*column</th>
<th>Grid spacing (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>049c</td>
<td>Wedge front</td>
<td>700*634</td>
<td>35</td>
</tr>
<tr>
<td>052c</td>
<td>Inner trough</td>
<td>1410*325</td>
<td>36</td>
</tr>
<tr>
<td>048c</td>
<td>Inner trough</td>
<td>1600*450</td>
<td>39</td>
</tr>
</tbody>
</table>

The shaded relief maps of the grids 049c, 052c and 048c are shown in Figures 1.3 a, b and c. The Grid 049c shows very prominent grooves, lineations and iceberg furrows. Grid 052c shows a very high density of N30-60W trending lineations but the grid does not cover the entire length of the lineations. The part, north of the grid 048c does not appear to have any prominent patterns. The trackline of the ship is very prominent in the grid 048c. The southern part of grid 048c shows NE-SW linear features.

**PRINCIPLES AND METHODS OF GEOSTATISTICS**

Spatial analysis is the study of data located in space. One of the things that distinguish earth science data sets from most others is that the data belong to some location in space. Elevation, ore grades in a mineral deposit, depth of a geologic formation, concentration of pollutants in a contaminated site are a few examples. One important method of analyzing spatial features is by determining the spatial continuity. Spatial continuity can be defined as the rate of change of a spatial variable along a particular orientation. It determines statistically how the values at two points become different as the separation between these points increases.
Geostatistics is a set of tools that describe and model spatial continuity that exists in most earth science data sets (Isaaks and Srivastava, 1989). Note that geostatistics is not the general application of statistics to geology but rather a specific and limited set of methods applied to any spatial data. Using the semivariogram, which is the basic statistical measure of geostatistics, we can model the spatial continuity of a variable and give a quantitative description of the stochastic process that underlies it.

GRID/WINDOWS

Geostatistical methods can be applied to spatial data located at regular intervals or irregularly distributed in space. Note that this is different from most spectral methods, which require regular data grids. For the present study, I have used elevation and bathymetry data that are distributed regularly on a grid. The grid spacing is uniform and is denoted by the term 1 grid unit. The equivalent distance unit would vary depending on the resolution of the dataset. For the topography grid of the western US 1 grid unit is equal to 1km and for the Antarctic grids 1 grid unit is 35 m, 36 m or 39m. I have used the word “grid” to refer to the entire gridded dataset in a region. A subset of this grid may be called a window. A window is described by its size in rows and columns and location with respect to the larger grid. A 200*200 window will have 40000 elevation values arranged in 200 rows and 200 columns (Figure 1.4). For the present study all geostatistical calculations are done on square windows of data.

LAG

Measuring spatial continuity involves comparing the value on a surface, such
as elevation with a value on that surface at a point offset by a given distance.

The offset distance is called the lag. Each measure of spatial continuity involves comparing many pairs of data values separated by a particular distance in a particular direction. Lag is therefore a vector and is denoted by \( h \) (X-lag, Y-lag) where X-lag is the separation distance in the E direction and Y-Lag is the separation distance in the N direction. A few examples of lags and the pairs of data compared at that lag are illustrated (Figure 1.5) for a 4*4 window of data. The values that contribute to a pair are referred to as the head and tail values. The head value comes from a point on the surface and the tail value comes from a point on the surface offset by a distance \( h \) (X-lag, Y-lag).

**MEASURING SPATIAL CONTINUITY USING THE SEMIVARIANCE**

The semivariance is the standard measure of spatial continuity in geostatistics. It is a measure of the degree of similarity between samples offset by a specific distance along a specific direction. For each lag the semivariance is computed as half the average squared difference between the components of every data pair. The function is

\[
\gamma(h) = \frac{1}{2N(h)\sigma^2} \sum_{i=1}^{N(h)} [Z(u_\alpha) - Z(u_{\alpha + h})]^2
\]

where \( \gamma(h) \) is called the semivariance at lag \( h \).

\( Z(u_\alpha) \) is the value of the spatial variable at location \( u_\alpha \). \( Z(u_{\alpha+h}) \) is the value of the spatial variable at location \( u_{\alpha+h} \). In other words, \( Z(u_\alpha) \) and \( Z(u_{\alpha+h}) \) are the head and tail values of pairs that are separated by lag \( h \).

\( N(h) \) is the number of data pairs used to calculate the semivariance at lag \( h \).

\( \sigma^2 \) is the variance of all the data values in the window.
Without the normalization by the variance, the semivariance its units are the same as for variance. For elevation data, the units of semivariance are \( L^2 \). The normalized semivariance is unitless.

I illustrate how the semivariance is calculated at lag \( h \) (0,1) for a 4x4 window of sample elevation values. The elevation values are in meters and spaced 1 grid unit apart. We first identify all pairs separated by lag \( h \) (0,1). There are 12 pairs separated by lag \( h \) (0,1) i.e. 1 grid unit in the N-S direction. The variance of all the data values in the window is 5. Therefore the semivariance is

\[
\gamma(h(0,1)) = \frac{1}{2 \times 12 \times 5} \left[ (1-6)^2 + (6-8)^2 + (7-3)^2 + (2-3)^2 + (6-1)^2 + (8-3)^2 + (2-4)^2 + (3-5)^2 + (1-2)^2 + (3-3)^2 + (4-6)^2 + (5-7)^2 \right]
\]

Similarly, the semivariance at other lags along the same direction can be calculated. In the present study the lag increment used to calculate the semivariances is 1 grid unit.

**SEMIVARIOGRAM**

The plot of semivariance vs. lag in a particular direction is called the semivariogram (Figure 1.6). The semivariogram gives an effective summary of the spatial continuity with increasing distance along a particular direction. At small lags, sample values of the spatial variable are likely to be highly correlated and the value
of semivariance will be small. As the lag increases the semivariance increases.

At small lags the semivariogram typically shows a linear behavior. At very large lags, in most datasets, the function would be calculating the semivariance between sets of effectively independent samples. At this distance the semivariance would remain relatively constant with lag and the function would reach a plateau near a value of 1. The separation distance at which the semivariogram flattens is called the range or decorrelation lag of the semivariogram. The decorrelation lag is an important estimation parameter in geostatistical analysis.

**STATIONARITY**

Semivariogram analysis assumes that data obey a condition called intrinsic stationarity (Isaaks and Srivastava, 1989, Chiles, 1999). Intrinsic stationarity requires that the local sample mean of the data does not vary with location. For stationary data, the semivariance depends only on the lag. A local mean that varies with location is called as drift or a trend (Chiles, 1999) and data having drift yield semivariograms that increase without bound and are not characteristic of the features of interest. However, data that are not stationary may still be analyzed using semivariograms after appropriate preprocessing to remove drift or trend.

**PRE-PROCESSING**

**Lag Mean Method**

In this method of removing drift, the mean of all head values and the mean of all the tail values is calculated for each lag. The mean $M_{\text{head}}$ is subtracted from all the head values and the mean $M_{\text{tail}}$ is subtracted from all the tail values at that particular lag. The semivariance is calculated using the residual elevations. This process is very
general and works well by removing local variations in the mean from small areas. I have incorporated the lag mean method by default while calculating the semivariograms for this study. However, if the data set is large this method is not completely effective in removing the drift in the mean and I had to resort to more specific processes in addition to the lag mean method to make the area of analysis stationary.

**Frequency Filtering Method**

Topography data distributed over a large grid area consist of features of different wavelengths. A window taken for semivariogram analysis from this larger grid may cover a part of the very-long wavelength features. These long wavelength features are a trend for the window of observation and can result in drift in the local mean. A high-pass frequency filter may be used to remove the wavelengths that are liable to cause drift. One method of filtering the elevation data is by using a 2d Fast Fourier transform algorithm which transforms the raw topography grid into the frequency domain, performs a high pass frequency filter to remove long wavelengths and outputs the filtered data in the space domain. The optimum filter parameters to use would depend on the window size and the resolution of the study area. The semivariogram is then calculated using the filtered elevation. It may take a few trials using different filter parameters and calculating the semivariogram before one decides on the optimum filter parameters. The frequency filtering method works well for data with no gaps. If there are data gaps ("no data values") in the grid, the filtering process produces edge effects around these data gaps. Calculating the semivariances using the filtered elevations close to the data gaps would give
incorrect estimates of the semivariance. Hence one should avoid using the filtering method on grids with data gaps. There is one special case where data with gaps may be detrended using the frequency filtering method; when the data gaps are confined to the boundary of the grid. The detrended grid would show edge effects near its boundaries. However, in such cases, if the grid is large and windows are not lagged close to the data gap areas, the artifacts of filtered elevation do not affect the semivariances.

**Running Average Method**

The frequency filtering works well as long as there are no data gaps within the grid and the window of interest is not very close to the edge of the grid. However, if the data gaps are scattered throughout the grid, and there is a significant trend in the data that causes the mean to drift, I use the running average method for removing long-wavelengths. In this method the filter is a moving window of a user-defined size m*m. The m*m filter is defined around every data value on the grid.

The mean of all the depths that fall within the filter window is calculated. This mean was then subtracted from the center data point and a residual elevation is calculated for that location. The "no data values" were recognized and not used in the running average method and they were left unchanged. This process removed longer wavelength trends without creating significant edge effects in grids with data gaps.

**CALCULATING THE SEMIVARIOGRAM MAP FOR WINDOWS OF TOPOGRAPHY**

Once the major trends are removed, the data are ready for semivariogram analysis. In order to analyze spatial continuity we need to calculate the semivariances
for windows of topography at all possible lags. The distribution of the
semivariances at various lags is called a semivariogram map. I describe below, two
methods for calculating the semivariogram map. First is the conventional method
used in geostatistics, second is the moving window method that I have used in the
present study.

I use 3 windows, A B and C as examples to illustrate the principles and
methods of geostatistical analysis. Window A and B are 200*200 windows from the
Western US grid (Figure 1.7a). The grid spacing is 1 km in the X and Y directions.
Window C is a 50*50 window from Antarctic grid 052c (Figure 1.7b). The grid
spacing for window C is 36 m in the X and Y directions.

Window A shows a ridge-valley-ridge topography with the ridges trending
approximately N-S. Window B shows topographic features oriented with no obvious
orientation dominating. Window C shows NW-SE striking linear features.

CONVENTIONAL METHOD

In order to calculate the semivariogram map of a window in a larger grid
using the conventional method, the window of interest is first separated from the
grid. We then translate this window across itself in increments of 1 grid unit and
compare pairs of elevations in the overlapping portion (Figure 1.8). The elevations
from the original window and the translated window will form the head and tail
values at a particular lag. The effective window area of overlap for calculating the
semivariance gets smaller as the lag increases (Figure 1.8). The semivariance at large
lags would not be a good estimate of the average semivariance for the entire window.
Therefore we stop calculating the semivariance once the window is translated half
way across itself.

I have illustrated the different lags at which the semivariance is calculated for a 4*4 window (Figure 1.9a) located on a larger grid. Semivariances are calculated at all lags from 0 to 2 in increments of 1 grid unit in all directions (Figure 1.9a). A few examples of lags and the pairs that go into the calculation of the semivariance are illustrated (Figure 1.9b). As the lag increases the number of elevation pairs used to calculate the semivariogram decreases. This is because only values within the window that have a lagged data point are used to calculate the semivariance. In the conventional method, the semivariogram calculated for any particular direction will be identical to the semivariogram calculated in the opposite direction i.e. $\gamma(h) = \gamma(-h)$. This is because the elevation pairs used to calculate the semivariogram at $\gamma(h)$ are the same as the elevations used to calculate the semivariogram at $\gamma(-h)$ with the head and tail values reversed (Figure 1.9b).

If the grid has data gaps, then parts of windows for which we need to calculate the semivariogram may fall in a data gap region. Data may be paired with the ‘no-data values’ at small and large lags. Including the ‘no-data values’ as normal depths in the calculation would give us an incorrect spatial summary of the topography by overestimating the semivariance calculated for that lag. This is because we might compare a depth of about 500m with some typical no-data values like ‘0’ or ‘−9999’ of the data-gap at lags of a few grid units. To counter this problem, while calculating the semivariogram values for a window, at each lag, I do not use those pairs that have ‘no-data values’. By excluding pairs that have a ‘no-data value’ in the head or tail value from the calculation, I avoid spuriously high
values of semivariance that will corrupt the semivariogram and give a completely false picture of the spatial continuity. The number of pairs used to calculate the semivariance is therefore different at each lag and the semivariance would thus be characteristic of the number of pairs available for analysis.

**MOVING WINDOW METHOD**

This method of semivariogram calculation for a window from a larger grid is subtly different from the conventional method. In this case the window is not isolated from the larger grid while calculating the semivariance at each lag. The moving window method involves translating the window over the larger grid and calculating the semivariances at each lag. The elevations from the original window and the translated window will form the head and tail values at a particular lag. Since the window moves over the larger grid there is always 100% overlap between the original window and the translated window (Figure 1.10). As lag increases, the area of overlap over which the semivariance is calculated does not decrease as it did in the conventional method. Elevations outside the window are paired with elevations within the window while calculating the semivariance (Figure 1.11). The number of pairs remains the same at each lag and is equal to the number of elevations in the window. Therefore, the farthest one could lag the window is only limited by the boundaries of the larger grid. However, at very large lags we would be comparing elevations in the window with elevations located at a great distance away on the grid. This may not give an accurate estimate of the semivariance of the values within the window we set out to map. Therefore, we calculate the semivariance from a lag of 0 to a maximum lag of half the length of the window as we had done for the
conventional method. The semivariogram values calculated by this method will no longer be point symmetric \([\gamma(h) \neq \gamma(-h)]\) because the pairs that go into the semivariogram calculation at opposite lags are no longer the same (Figure 1.11). Grids with data gaps are treated in the same manner as for the conventional method. ‘No-data values’ are not included in the calculation of the semivariance.

**Conventional Method Applied to Window A**

I calculated the semivariogram map for the window A from the western US (Figure 1.7a) using the conventional method. The window is separated from the larger grid and translated across itself and semivariances were calculated at lag increments of 1 grid unit.

**Moving Window Method Applied to Windows A, B and C**

I calculated the semivariogram map of window A, B and C (Figures 1.7a and 1.7b) by translating the windows over the larger grid and calculating the semivariances at lag increments of 1 grid unit. The windows remain part of the grid during the semivariogram calculation. Semivariogram maps were calculated from lag 0 to lag 100 in all directions for window A and window B (Figure 1.12). A semivariogram map was calculated from lag 0 to lag 25 in all directions for window C.

**CONVENTIONAL METHOD VS. MOVING WINDOW METHOD**

The moving window method gives a more robust estimate of the semivariance at each lag. It also helps map the spatial continuity of long linear N-S features in the Basin and Range Province using much smaller window sizes than the
conventional method would allow. Keeping the window size smaller improves resolution.

The window A shows linear ridges of the Basin and Range extending N-S. By using the conventional method and isolating the window from the grid, (Figure 1.13) I will be assuming that the N-S striking features end at the edge of the window, while this is not so. The window A (Figure 1.7a) on the larger grid shows that linear features extend much beyond the edge of the grid. In the conventional method of calculation, the semivariances along the strike of the linear features may increase unbounded even at very large lags. This is because along the strike of the linear features elevations would be highly correlated even at the maximum lag. Therefore, analysis would have to be done over larger window sizes. A very large window size may pick up a very highly averaged spatial variability that is not of interest.

In the moving window method I assume that the window is part of the larger grid and allow the window to move freely over the larger grid. I compare elevations within the window with elevations outside the edge of the window even at small lags. Therefore, this method gives a better estimate of the semivariance at each lag and gives me a more robust semivariogram map for smaller window sizes.

**DISPLAYING THE SEMIVARIOGRAM MAP**

In order to get a global view of semivariances calculated at all lags, I plot the semivariogram map as X-lag vs. Y-lag (Figures 1.14, 1.15, 1.16). The vertical line passing through lag 0 represents the N-S direction and the horizontal line through lag 0 represents E-W direction. The semivariogram map of window A (Figure 1.7a) calculated by the conventional method is point symmetric about the origin $\gamma(h) = \gamma(-h)$. 
h)]. The first quadrant is a mirror image of the third quadrant and the second quadrant is a mirror image of the fourth quadrant. The semivariogram map of window A calculated by the moving window method (Figure 1.14) is no longer point symmetric about the origin and the map shows that the four quadrants have different semivariance distribution.

**INTERPOLATING SEMIVARIOGRAMS FROM THE MAP**

The distribution of the semivariances on the semivariogram map contains information about the spatial continuity of the topography within a window. The semivariogram map for window A (Figure 1.14) shows semivariances increasing relatively faster along the N-S direction than the E-W direction. The semivariogram map for window B (Figure 1.15) shows semivariances increasing in a similar manner along most directions. The semivariogram map of window C (Figure 1.16) shows the semivariances increasing relatively slower in the NW-SE direction than in the SW-NE direction. In order to model the distribution of the semivariances, we need to model the semivariogram map. Since geostatistics uses semivariograms as a basic measure of spatial continuity, the modeling techniques are geared toward fitting a model to the semivariogram. Therefore, I interpolate the semivariances on the semivariogram map along different directions and obtain the directional-semivariograms for a window. The directional semivariogram is a plot of lag vs. interpolated semivariance (Figure 1.18). It maps the rate at which elevation values decorrelate along a particular direction.
SEMIVARIOGRAM INTERPOLATION ALGORITHM

For this study, I compute the directional semivariograms by interpolating the semivariances from the semivariogram map along radial transects about the lag 0. The input is the semivariogram map and the lag increment at which to interpolate. I use bilinear interpolation and calculate the semivariances along each profile. Along a given orientation, the semivariances are interpolated from 0 to the maximum lag in increments of grid units. The grid increment for interpolation may vary with the grid resolution and the window size. For the present study, I have extracted directional semivariograms at 5-degree intervals from 0° to 360°, the direction being measured counter-clockwise from the East (Figure 1.19).

Interpolation Algorithm Applied to Windows A, B and C

I used the interpolation algorithm to extract directional semivariograms from the semivariogram map of window A (Figure 1.14), window B (Figure 1.15) and window C (Figure 1.16). The semivariances for windows A and B were interpolated every 1 grid unit (1 km) along 5-degree intervals. The semivariances for window C were interpolated every 0.25 grid units (9 m).

MODELING SPATIAL CONTINUITY

BEHAVIOR OF DIRECTIONAL SEMIVARIOGRAM

The semivariogram map for window A (Figure 1.17) calculated using the conventional method is used to illustrate the behavior of the directional semivariograms. Semivariances increase very rapidly along the E-W direction and more slowly along N-S direction. On this semivariogram map I have marked five directional semivariograms of interest. These are Profile A along N70W, Profile B
along N45W, Profile C along N10E, Profile D along N45E and Profile E along N70E (Figure 1.20a-e).

**Profile A**

In the semivariogram for profile A (Figure 1.20a), the semivariance increases rapidly to a value of 1 at a lag of about 15 km. This direction is called the direction of minimum continuity because the elevation values decorrelate at small lags. The semivariance does not level off at a constant value but increases and decreases in cycles about a value of 1. The semivariance at lag of 30 km is less than the semivariance at a lag of 15 km. This indicates that elevation pairs separated by 30 km are more similar to each other than elevation pairs separated by 15 km. This shows a periodic component. Note the high semivariance at small lags on the semivariogram. This has been characteristically observed for the semivariograms of topography in the presence of periodicity.

**Profile C (intentionally out of order)**

Along N10E the semivariogram increases rather steadily over 20 km (Figure 1.20c). The value of semivariance does not increase as quickly as it does in the other profiles. One obvious reason for this would be that along N10E elevation pairs remain correlated for a longer distance suggesting greater spatial continuity. The semivariance becomes constant and reaches a plateau at a decorrelation lag of about 35-40 km. Profile C is in the direction of maximum continuity because elevations show high correlation even at large lags.

**Profile B**

There seems to be a distinct difference in the structure mapped by the
semivariogram in profile A and profile C. In order to verify this, we study the semivariogram along profile B whose direction lies in between the orientation of profile A and C. The decorrelation distance for Profile B (Figure 1.20b) is greater than for profile A and less than profile C. The periodicity in the semivariogram is not as prominent as in profile A.

Profile D and E

The semivariogram along Profile D (Figure 1.20d) shows continuity that is in between that of Profile A and Profile C. The periodicity is almost negligible and the semivariogram is tending toward a flatter shape. Along Profile E (Figure 1.20e), the semivariogram maps a periodic component at a shorter lag and flattens with increasing lag.

Comparing Directional Semivariograms with Topography

We compare the semivariogram profiles (Figure 1.20a-e) with the topography of window A (Figure 1.13). The topography for which we calculated the semivariograms consists of N-S trending linear ridges separated by valleys. The orientation of semivariogram profile C (N10E) is close to the average strike of the linear features and the orientation of semivariogram profile A (N70W) is approximately perpendicular to the strike of the features. Along the strike of an elongated feature like a ridge bound by a valley on either side of it, elevation pairs would remain correlated for a longer distance. Across the same feature, one might expect elevations to rapidly decorrelate. However, in this case since the valley-ridge-valley topography repeats, the semivariance increases and decreases in cycles. Thus, we can infer two important things from observing the different directional
semivariograms. First, we observe a distinct anisotropy in the correlation scales of the semivariograms perpendicular and parallel to the strike of the linear features. Second, a strong periodicity in the topography is mapped by the directional semivariograms perpendicular to the trend of the periodic features.

NECESSITY FOR MODELING THE SPATIAL CONTINUITY

In order to quantify the spatial continuity of topography, it is necessary to obtain the directions of maximum and minimum continuity and the lag at which elevation values decorrelate in both these directions. We can determine the direction of maximum and minimum continuity and the average orientation of linear features qualitatively from the semivariogram map. The semivariogram map (Figure 1.17) of window A shows elliptical contours whose long axis is oriented approximately N-S. This is roughly the orientation of the direction of maximum continuity. Periodicity is observed in the rapid rise and fall of the semivariance along the E-W axis. We are now set to quantify what we observe qualitatively from the semivariogram map. I do that in two steps.

1) I model the directional-semivariogram and estimate the decorrelation lag of semivariograms in all directions. Since the decorrelation lag varies with direction, continuous functions are fitted to each of these directional semivariograms in order to calculate the lag at which each directional semivariogram decorrelates.

2) I model the anisotropy by studying the variation of the decorrelation lag with direction and variation of the wavelength of periodic features with direction. The ratio of the maximum decorrelation distance to the minimum decorrelation distance gives us a measure of anisotropy. The azimuth along the directional semivariogram
with the maximum decorrelation distance represents the average trend of linear features in the sample data. The wavelength modeled from the periodic component in the semivariogram is the wavelength of the dominant periodic features in the topography.

MODELING THE SEMIVARIOGRAM

Range and Sill

Two important parameters that describe the semivariogram model are the range and the sill. Range is the lag at which semivariogram reaches a stable value. In other words, it is the decorrelation distance calculated by fitting a model to the experimental semivariogram. The semivariance at this lag is referred to as the sill. The range limits a region within which sample values are related to each other and defines the zone of influence of the sample (Isaaks and Srivastava, 1989). There are a number of functions that describe the shape of the semivariogram and can be used to model the range and sill of an experimental semivariogram.

SEMIVARIOGRAM MODELS

The Spherical Model

This is the most commonly used semivariogram model. It has a nearly linear behavior at small lags and reaches the sill C at a lag equal to a. At all values of lag greater than a, the value of the function is equal to the sill (Figure 1.21a). The spherical model is given mathematically as
\[
\gamma(h) = C \left( \frac{1.5h}{a} - 0.5 \left[ \frac{h}{a} \right]^3 \right) \quad \text{when } h \leq a
\]

\[
\gamma(h) = C \quad \text{otherwise}
\]

Where \( h \) is the lag.

\( \gamma(h) \) is the semivariance at lag \( h \).

\( C \) is the sill.

\( a \) is the range.

**The Periodic Model**

The semivariogram can exhibit periodicity along certain directions. This periodicity is usually modeled by a sinusoid. The periodic model is a function of the lag, the wavelength and the amplitude of the semivariogram. Additional parameters maybe used in the model depending on the behavior of the semivariogram. One such periodic model is the damped sinusoidal function (Figure 1.21b). It is given as

\[
\gamma(h) = A e^{-hc} \sin \left( \frac{2\pi h}{\lambda} \right)
\]

Where \( h \) is the lag.

\( \gamma(h) \) is the semivariance at lag \( h \).

\( A \) is the amplitude.

\( c \) is the exponential damping constant.

\( \lambda \) is the wavelength.

In this model the amplitude of the sine wave falls off with distance at a rate dependent on the damping constant ‘c’. Note that this model does not have either the range or the sill as one of its parameters.
The Nested Model

The semivariogram models can be a sum of 2 or more basic models. Such models are called nested models (Clark, 1979). The nested model used in this study combination of a spherical and a periodic model (Figure 1.21c).

\[
\gamma(h) = C \left( \frac{1.5h}{a} \right) - 0.5 \left( \frac{h}{a} \right)^{1.5} + A e^{-hr} \sin \left( \frac{2\pi h}{\lambda} \right) \quad \text{when } h \leq a
\]

\[
\gamma(h) = C + A e^{-hr} \sin \left( \frac{2\pi h}{\lambda} \right) \quad \text{when } h > a
\]

Where \( \gamma(h) \) is the semivariance at lag \( h \)

\( a \) is the range and \( C \) is the sill

\( A \) is the amplitude of the periodic component

\( c \) is the damping coefficient

\( \lambda \) is the wavelength

FITTING MODELS TO DIRECTIONAL SEMIVARIOGRAMS

In order to fit the model functions to the semivariograms, I use a non-linear minimization function fminsearch (Math Works Inc, 1999). The functional form of the model, the semivariogram data values and starting values for the model parameters are inputs to fminsearch. The minimization function iteratively varies model parameters to give a best fit of the model function to the semivariogram in a least squares sense.

MODELING SEMIVARIOGRAMS OF TOPOGRAPHY

Semivariograms of topography can show varied structures based on the features they map. The ideal shape (Clark, 1979) of a semivariogram of topography
(Figure 1.22a) can be modeled using the spherical model. However, not all semivariograms have this ideal shape. Some of the directional semivariograms warrant a separate treatment.

1) If the topography has periodic features, directional semivariograms would map this periodicity and a periodic component would be superimposed on the ideal shape of the semivariogram (Figure 1.22b). It has been observed that the periodicity of the semivariogram generally decreases in amplitude with increasing lag. However, one must be certain that there is a physical explanation for this cyclic behavior of the semivariogram.

2) The filtering technique does not necessarily remove the trend completely. In this case the elevation values along some directions can remain correlated for distances larger than the dimensions of the window. The semivariogram along these directions may show an increase in the semivariance from the origin to the maximum lag, never reaching a plateau (Figure 1.22c).

3) If there is an underlying periodicity along the trend direction the amplitude of the periodic component of the semivariogram will increase with lag (Figure 1.22d). In the simplest case, the ideal semivariogram can be modeled using the sill and range of the spherical model (Figure 1.23). However, if the directional semivariogram shows a periodic component, the periodicity needs to be taken into account while modeling the semivariogram. Ignoring the periodicity and modeling these semivariograms with a purely spherical model would result in inconsistent model fit and erroneous range values. The directional semivariogram (Figure 1.22b) that displays a short-range spherical component and a periodic component is modeled
using the nested model (Figure 1.24a). The nested model fits the semivariogram showing periodicity well and gives a range of 18 km and a wavelength of 59 km. Modeling the same directional semivariogram (Figure 1.22b) with a purely spherical component underestimates the value of the range giving a range of 12 km (Figure 1.24b). Not all directional semivariograms map periodicity. Fitting this nested model to an ideal semivariogram with no distinct periodicity (Figure 1.22a) would result in model parameters that are incorrect and do not explain the structure of the semivariogram (Figure 1.25a). The reason behind this is that the plateau region of the experimental semivariogram is modeled as periodicity. The immediate consequence of such a fit is extremely low or negative amplitude and inconsistent range and sill values. The estimate of the range obtained from fitting a purely spherical model to this semivariogram (Figure 1.25b) explains the structure of the semivariogram better than the nested model. In some cases the entire semivariogram is modeled as a periodic component with very high amplitude and a long wavelength (Figure 1.26). The spherical component is very small and the final model fit does not give the right interpretation of the range and sill of the semivariogram. The directional semivariogram with drift is associated with an increase in the amplitude of the periodic component (Figure 1.22d). The nested model would fit the semivariogram with a negative damping coefficient for the periodic component (Figure 1.27a). The short-range spherical component along these directions can be modeled more precisely by fitting a purely spherical model (Figure 1.27b). There are directional semivariograms (Figure 1.22c), which cannot be usefully modeled by either the spherical (Figure 1.28a) or the nested model (Figure 1.28b). The semivariogram
increases and does not reach a plateau. I have not attempted modeling the spatial
continuity along these semivariograms. A few examples of experimental
semivariograms modeled by a nested model and purely spherical model are
illustrated in Figure 1.29, Figure 1.30 and Figure 1.31.

SEMIVARIOGRAM MODELING ALGORITHM

With the directional semivariograms falling into any one of the above categories,
modeling the semivariogram automatically would involve making the modeling
algorithm differentiate the behavior of the directional semivariograms and fit the
optimum model that would explain the structure of the semivariogram. I therefore
present a problem of modeling different directional semivariograms with either a
purely spherical model or a nested model. By default, the modeling algorithm first
fits the nested model to every experimental semivariogram.
The modeling algorithm then takes into account the above limiting factors of the
nested model and the checks for the following conditions-

1) Amplitude less than 0.1 – This indicates that the periodicity is not well-
defined; it could be a semivariogram approaching an ideal spherical shape.

2) Damping coefficient less than 0 – This indicates that there is an increase in
amplitude of the periodic component due to drift. This is not physically
reasonable.

3) Amplitude greater than 0.8 – This indicates that the entire semivariogram is
being modeled as a periodic component.

4) Range greater than half the lag – This indicates that the range is large and
periodicity is poorly mapped at large lags.
If any one of the above conditions is true then the nested model is discarded and the semivariogram is fit with purely spherical model and the range and the sill of the spherical model are determined. After either model has been fit the program checks the magnitude of the range. If the range is greater than the maximum lag allowed for the window, the range and sill along the directional semivariogram are posted as undefined by giving them a value of 0.

I have not illustrated all the different semivariogram behaviors. Semivariograms can be very diverse and complicated in structure and the automatic modeling may not always be robust in fitting the semivariogram. However, I believe that the more complicated semivariograms would probably be along a few directions in a window and the range modeled from a majority of the directional semivariograms where the model fits well can be used to study the spatial continuity.

**Semivariogram Modeling Algorithm Applied to windows A, B and C**

I applied the semivariogram modeling algorithm on the directional semivariograms interpolated from semivariogram map for window A, window B and window C. The program fit the directional semivariograms with the optimum model based on the limiting conditions given in the program. I have illustrated example model fits from window A, window B and window C.

For the semivariogram map of window A (Figure 1.14), the directional semivariogram along S45E (Figure 1.32a) is fit by the nested model. The model calculates a range of about 25.8 km and wavelength of 61 km along this direction. The directional semivariogram along N45E (Figure 1.32b) is modeled using a purely spherical model. The range and sill of this model are 17.6 km and 0.96 respectively.
The semivariogram along N5E (Fig 1.32c) is modeled by the spherical model with a range of 27.8 km and sill of 0.90.

For the semivariogram map of window B (Figure 1.15), the directional semivariograms show a similar structure in different directions (Figure 1.33). The directional semivariograms were modeled using the spherical model (Figure 1.34).

For the semivariogram map of window C (Figure 1.16), the directional semivariogram along N45E (Figure 1.35a) shows a strong periodic component and is modeled using the nested model with a range = 5.5 grid units (198 m) and wavelength = 11.6 grid units (418 m). The directional semivariogram along S45W (Figure 1.35b) shows greater continuity and is modeled by a purely spherical model with range of 17.5 grid units (630 m) and a sill of 1.34. Along the N15W semivariogram (Figure 1.35c) the presence of a drift causes the semivariogram to increase with lag. The spherical model fits the short-range structure and estimates a range of 10.8 grid units (388 m) and sill of 1.01.

**MODELING THE ANISOTROPY IN RANGES**

A data distribution is said to be isotropic when the pattern of spatial variability does not change with direction (Goovaerts, 1997). When the variation is isotropic, the semivariograms increase in a similar manner in every direction and the range modeled from all the sample directional semivariograms will be approximately equal. When the variation is anisotropic, spatial continuity changes with direction and the range modeled from semivariograms will have different values in different directions.
The ranges obtained from modeling the semivariograms from the semivariogram map of window A (Figure 1.14) are plotted along their respective azimuths (Figure 1.36). The rose diagram of the ranges is in the shape of an ellipse. This indicates anisotropy in the spatial continuity, which can be observed qualitatively from the semivariogram map, which shows elliptical contours.

The ranges obtained from modeling the semivariogram from the semivariogram map of window B (Figure 1.15) are plotted along their respective azimuths. The rose diagram of the ranges is closer to the shape of a circle (Figure 1.37). This indicates isotropy in the spatial continuity and this can be observed qualitatively from the semivariogram map, which shows circular contours.

The ranges obtained from modeling the semivariogram from the semivariogram map of window C (Figure 1.16) are plotted along their respective azimuths. The rose diagram of the ranges is in the shape of an ellipse with the long axis oriented along N30-40W (Figure 1.38). This indicates anisotropy in the spatial continuity and this can be observed qualitatively from the semivariogram map, which shows elliptical contours.

Comparing Rose-diagram of Ranges with Topography

Window A has a linear ridge-valley-ridge topography. Window B shows topographic features oriented randomly with no specific orientation dominating. Window C shows lineation oriented N30-40W. The rose diagram for window A and C are elliptical and the rose-diagram for window B is circular. From these observations we can infer that if the topography has no dominant linear or periodic pattern, as in window B, then the spatial continuity remains the same in all directions
and the rose-diagram of the ranges would be close to a circle in shape. If there is a dominant pattern to the topography, as in window A, the ranges are distributed in the shape of an ellipse with the long axis of the ellipse oriented in the direction of the dominant pattern.

ANISOTROPY MODEL

In order to quantify the anisotropy, I fit an ellipse to the range values.

Equation of the ellipse is

\[
\frac{(r \cos \theta)^2}{a^2} - \frac{(r \sin \theta)^2}{b^2} = 1
\]

Where \( r \) is the range

\( a \) is the semi-major axis of the ellipse

\( b \) is the semi-minor axis of the ellipse

\( \theta \) is the direction of the semi-major axis measured counter-clockwise from the east.

The semi-major axis and the semi-minor axis represent the range in the directions of maximum and minimum spatial continuity respectively and are separated by 90 degrees. The direction of maximum continuity in case of anisotropy gives the average orientation of the linear features in the window. The ratio of the semi-major axis to semi-minor axes gives us an aspect ratio that measures the anisotropy in the correlation scales.

ANISOTROPY MODELING ALGORITHM

I used the non-linear minimization function fminsearch (Math Works Inc, 1999) to fit an ellipse to the range values. The function iteratively varies \( a, b \) and \( \theta \) to fit an ellipse to the range data in a least squares sense.

Anisotropy Models of Windows A, B and C
I modeled the anisotropy in spatial continuity for window A and window B and window C by fitting an ellipse to the rose diagram of the range values.

The variation of the ranges with direction for window A was modeled by an ellipse (Figure 1.39) with semi-major axis = 29.3 km, semi-minor axis = 17.0 km and orientation of the semi-major axis = N1W. The aspect ratio of the ellipse = 1.8. The direction of the semi-major axis (axis of maximum continuity) is close to the average trend of the ridge-valley-ridge topography in window A (Figure 1.7a).

The variation of the ranges with direction for window B was modeled by an ellipse (Figure 1.40) with semi-major axis = 18.3 km and semi-minor axis = 16.3 km. The aspect ratio of the ellipse = 1.1. The two axes are almost equal indicating approximate isotropy. In such cases, the direction of maximum or minimum continuity is not significant.

The variation of the ranges with direction for window C was modeled by an ellipse (Figure 1.41) with semi-major axis = 12.8 grid units (448 m) and semi-minor axis = 5 grid units (175 m). The orientation of the axis of maximum continuity = N32W. The aspect ratio of the ellipse = 2.56. The direction of the semi-major axis (axis of maximum continuity) is close to the average trend of the linear features in the topography of window C (Figure 1.7a)

**MODELING THE ANISOTROPY IN WAVELENGTH**

Wavelength of the periodic component of the experimental semivariogram is one of the important parameters of the periodic model. The true wavelength of any feature that is periodic, for example a ridge-trough-ridge topography is always measured perpendicular to the strike of the periodic feature. This is the also the
shortest wavelength. Along other directions we would measure an apparent wavelength, which is longer than the true wavelength. The directional semivariograms for the topography show a similar behavior. The periodic component is shortest and strongest in a semivariogram perpendicular to the linear features. If we plot the wavelengths obtained from the periodic model along the individual semivariogram directions, the minimum wavelength may correspond to the true average wavelength of the periodic features. The orientation of the semivariogram with the minimum wavelength would be perpendicular to the orientation of linear features.

**Wavelength vs. Direction for Windows A, B and C**

The plot of the model wavelength with direction for window A gives a minimum wavelength of 45 km along direction 170° (angle is measured counter-clockwise from East). This corresponds to N80W, which is approximately perpendicular to the average orientation of linear features in the window (Figure 1.42).

The semivariogram map of window B does not show any periodicity. Hence no wavelengths have been modeled for the directional semivariograms/

The graph of variation of wavelength with direction for window C from Antarctic grid 052c (Figure 1.43) gives a minimum wavelength of 400 m along the directional semivariogram at N35E. The semivariogram maps longer wavelengths as we move away from this direction. This is perpendicular to the trend of the lineations which are oriented approximately N30-40E.
SUMMARY OF METHODS

Semivariogram analysis is done on data located in space. Data can be uniformly distributed on a grid or can be irregularly spaced. Initial checks involve identifying trends and removing them using appropriate preprocessing methods. Semivariogram analysis is carried out for windows of data on the detrended grid. A semivariogram map is calculated by lagging the window over itself (conventional method) or by lagging the window over the larger grid (moving window method) from zero to a maximum lag of half the window size. Directional semivariograms are extracted from the semivariogram map along radial transects at suitable intervals. The directional semivariograms are fit with suitable models and the range, sill and periodicity of the semivariograms are estimated. The variation of the ranges and periodicity with direction are studied. The ranges are fit by an ellipse in order to obtain the orientation of the ellipse, magnitudes of the major and minor axes and the aspect ratio of the ellipse.
Figure 1.1a: Digital elevation model of the Western United States (GTOPO30). Grid has 1920*2160 elevation values with a constant grid spacing of 30" in the X and Y directions. Colors represent elevation in meters.
Figure 1.1b: GTOPO30 DEM projected using Lambert Conical Conformal projection. Grid size is 1811*1689 km. Grid spacing is uniform (1 km) in latitude and longitude. Colors represent elevation in meters.
Figure 1.2: Location of the grids 052c, 049c and 048c
Figure 1.3a: Antarctic Bathymetry grid –049c showing high resolution sea-floor features on the continental shelf. Grid size is 700*654. Grid spacing is constant in the X and Y directions and is equal to 35 m. Colors represent depths below sea-level.
Figure 1.3b: Antarctic Bathymetry grid –052c. Grid size is 1420*325. Grid spacing is uniform and equal to 35m.
Figure 1.3c: Antarctic Bathymetry grid –048c. Grid size is 1600*450. Grid spacing is uniform in X and Y axes and is equal to 39m.
Figure 1.4: Western US grid showing 200 * 200 window. Window is centered on (700, 700) on the larger grid.
Figure 1.5 Examples of lags and number of pairs at each lag. X-Lag is the separation distance in E-W direction. Y-Lag is the separation distance in the N-S direction. Negative Y-lag is toward South, Negative X-Lag is toward West. Head value from a surface compared with the tail value h distance away on the same surface.
Figure 1.6: Semivariogram: Plot of the normalized semivariance with lag for elevation data. Semivariance at lag 0 is 0. The semivariogram at small lags is almost linear. At large lags the semivariogram reaches a plateau indicating loss in correlation between sample values.
Figure 1.7a: Elevation grid of Western US showing two 200*200 windows: window A and window B. These windows are used to illustrate aspects of the methods of semivariogram analysis. Elevation is in meters and grid spacing in X and Y axes =1 km. Window A shows N-S striking ridge-basin-ridge topography. Window B does not show any obvious lineated pattern in topography.
Figure 1.7b: Antarctic grid 052c showing 50*50 window C. Grid shows depth below sea-level in meters. Grid spacing in the X and Y axes = 36 m. Window C is used to illustrate the aspects of the methods of semivariogram analysis. Window C shows NW-SE striking mega-scale lineations.
Larger grid of data with 4*4 window

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Window removed from grid

Translated window at different lags

h(0,0)

h(0,1)

h(0,2)

h(1,1)

h(2,2)

Figure 1.8 Conventional method of semivariogram calculation. 4*4 window is separated from grid. Window is translated across itself and semivariance at each lag is calculated by comparing elevations in the overlapping shaded regions. At lag = 0, the window is compared with itself and there is 100% overlap. As lag increases along a particular direction, the area of overlap decreases.
Figure 1.9a: (Top) 4*4 window of sample elevation values. (Bottom) Plot showing the different lags at which the semivariance is calculated for the 4*4 window.
Figure 1.9b Examples of pairs used in the semivariance calculation at each lag. As the lag increases along a particular direction, the number of pairs N used to calculate the semivariance decreases. Semivariance calculated for lag $h(0,1)$ and $h(0,-1)$ are equal because the pairs used for each lag are the same and only head and tail values are reversed. This shows the point symmetric property of the semivariogram about the lag 0.
Figure 1.10 Moving window method of calculating the semivariogram map. (Left a and b): 4*4 window shown in magenta on a larger grid (only the first and last rows of the 4*4 window are colored to define the boundary of the moving window at each lag on the grid). To the right (completely shaded) are the translated windows at each lag. The head and tail values from the overlapping portions of the original and translated window are compared. There is 100% overlap between windows at each lag.
Figure 1.11 Elevation pairs for the 4x4 window at lag h(0,1) and h(0,-1). Elevations within the window (shaded blue) are paired with elevations outside the window. All the elevations values in the window have a lagged data point. Therefore number of pairs, N is constant and equal to number of elevation values in the window. Pairs used to calculate semivariance at h(0,1) are not the same as pairs used to calculate semivariance at h(0,-1). Semivariances are therefore not point symmetric about the origin.
Figure 1.12: Lags at which semivariances are calculated for window A, B and C. X-Lag and Y-Lag are in grid unit.
For window A and B, semivariogram map is calculated for lags 0 to 100 grid units in all directions (1 grid unit = 1km)
For window C, semivariogram map is calculated for lags 0 to 25 grid units in all directions (1 grid unit = 36 m)
Figure 1.13: Window A isolated from the larger grid. Elevation is in meters. X and Y axes are in grid units (1 grid unit = 1km).
Figure 1.14: Semivariogram map of window A using the moving window method. Semivariances are color-coded and displayed as X-Lag vs. Y-Lag. Lag increment is 1 grid unit (1 km). The semivariance values are not point symmetric about the origin. Elliptical contours indicate anisotropy in the distribution of semivariances.
Figure 1.15: Semivariogram map of window B. Semivariances are color-scaled and displayed as X-Lag vs. Y-Lag. Lag increment = 1 grid unit (1 km). Semivariances increase in a similar manner along different directions. Circular contours indicate isotropy in the distribution of semivariances.
Figure 1.16: Semivariogram map showing the semivariances calculated for window C. Semivariances are color-coded and plotted as X-lag vs. Y-Lag. Lag increment = 1 grid unit (1 grid unit = 36 m). The map shows elliptical contours indicating anisotropy in the distribution of semivariances.
Figure 1.17: Semivariogram map of window A from the conventional method. Semivariances are color-coded and displayed as X-Lag vs. Y-Lag. Lag increment is 1 grid unit (1 km). The semivariance values are point symmetric about the origin. First quadrant is a mirror image of the third quadrant and second quadrant is a mirror image of the fourth quadrant. Elliptical contours indicate anisotropy in the distribution of semivariances. A-E lines are directional profiles.
Figure 1.18: Directional semivariograms extracted from semivariogram map of window A (Figure 1.17) along different directions. Y-axis represents the interpolated semivariance. X-axis is lag.
Figure 1.19: Directional semivariograms extracted from the semivariogram map at 5° intervals along radial transects.
Figure 1.20 a-e: Directional semivariograms extracted from the semivariogram map of window A (see Figure 1.17 for profile lines)
a) Directional semivariogram along Profile A (N70W). Semivariogram increases very rapidly and elevation values decorrelate at small lags. At large lags semivariogram increases and decreases indicating periodicity in the spatial continuity. Profile A is in the direction of minimum continuity.
Figure 1.20 (continued.)
b) Directional semivariogram along Profile B (N45W). Decorrelation distance is greater than for the profile a and less than that of the profile C. Periodicity is not very prominent at large lags.
Figure 1.20 (continued)
c) Directional semivariogram along Profile C (N10E). Semivariogram increase is relatively slow. Elevation values remain correlated over a long distance. Semivariogram flattens to a constant value of semivariance around 30-40 km. Periodicity is distinctly absent along this direction. Profile C is the direction of maximum continuity.
(d) Semivariogram along N45E

Figure 1.20 (continued.)
d) Directional semivariogram along N45E. Semivariogram increases rapidly at small lags. Periodicity is mapped but to a smaller extent. Semivariogram is approaching a flatter shape at the decorrelation lag.
Figure 1.20 (continued.)
e) Directional Semivariogram along profile E (N70E). Semivariogram rises rapidly. Periodicity is mapped at initial lags and semivariogram flattens at larger lags.
Figure 1.21a: Spherical model. The parameters of the model are Range \( a \) and Sill \( C \). Spherical model is almost linear at small lags and flattens to a constant value \( C \) for distances \( \geq a \). Range \( a \) is the lag beyond which elevation values are no longer correlated. Sill \( C \) is the value of semivariance at this lag.
Figure 1.21b: Periodic model. Model is a function of lag. Damped sinusoid is defined by an amplitude, a wavelength and a damping coefficient.
Figure 1.21c: Nested model. Model is a function of lag. The nested model in the above figure is a sum of the spherical model and the periodic model. Parameters include range and sill of spherical model and amplitude, wavelength and damping coefficient of the periodic model.
Figure 1.22: Behavior of semivariograms of topography.
(a) Ideal shape of the semivariogram. Almost linear at small lags and flattens at the decorrelation lag.
(b) Periodicity superimposed on the ideal shape. Semivariogram increases rapidly at initial lags. Semivariogram does not flatten completely but increases and decreases in cycles. Amplitude of the periodic component decreases with increasing lag.
Figure 1.22 (continued):

(c) Semivariogram increases without reaching a plateau. Elevation values remain correlated for distances greater than the maximum lag. This could be the result of an underlying trend in the data or a very small window size compared to the size of the features.

(d) Periodic component superimposed on a semivariogram with trend. Amplitude of the periodic component increases with lag.
Figure 1.23: Ideal shaped semivariogram of topography modeled by the spherical model. Model semivariogram estimates a range of 30.3 km and sill of 0.97. Semivariogram was fit by a spherical model using a non-linear minimization function fminsearch.
Figure 1.24a: Semivariogram showing periodicity fit by a nested model. 
\( a = 17.4 \text{ km}, C = 1.07, A = 0.5132, c = 0.1356, \lambda = 59.2 \text{ km} \).

Figure 1.24b: Above semivariogram fit by a purely spherical model. \( a = 12.2 \text{ km} \) and \( C = 1.09 \). Note the difference in the estimates of the range between the nested and spherical model fit to a semivariogram showing periodicity. Nested model appears to fit the structure of the semivariogram better than a purely spherical model.
Figure 1.25a: Semivariogram with no periodic component fit by a nested model. \( a = 24.9 \text{ km} \), \( C = 0.95 \), \( A = -0.4426 \), \( c = 0.0453 \), \( \lambda = 109 \text{ km} \). Range value is inconsistent with the structure of the semivariogram. Amplitude is negative in order to fit the slowly increasing semivariogram.

Figure 1.25b: Semivariogram in (a) fit by a purely spherical model. \( a = 51.9 \text{ km} \) and \( C = 0.98 \). Spherical model interprets the range and sill more accurately than the nested model.
Figure 1.26: Semivariogram with no periodicity modeled using the nested model. Semivariogram is modeled with a large periodic component and a very small spherical component. $a = 17.7 \text{ km}$ $C = 0.68$, $A = 1.8$ $c = 0.0359$ and $\lambda = 416$ km. Model gives a very high wavelength for periodicity that is very inconsistent with the experimental semivariogram.
Figure 1.27a: Semivariogram showing periodicity and effects of underlying trend fit by a nested model. $a = 17.2$ km, $C = 0.97$, $A = 0.0021$, $c = -0.1075$, and $\lambda = 67$ km. Model gives a negative damping coefficient because the amplitude of the directional semivariogram increases with lag.

Figure 1.27b: Above semivariogram fit by a spherical model. The short-range spherical component is modeled by the spherical model. $a = 8.0$ km and $C = 1.01$. 
Figure 1.28a: Semivariogram showing high correlation/trend modeled by a purely spherical component. : $a = 65.0$ km and $C = 1.05$. The spherical model does not explain the high correlation structure of the semivariogram.

Figure 1.28b: Semivariogram showing high correlation/trend fit by a nested model. include $a = 57.3$ km $C = 0.99$, $A = 0.0000$, $c = -0.2570$, $\lambda = 33$ km. Both spherical and nested model fail to explain the almost unbounded increase in the semivariogram with lag.
Figure 1.29a: Semivariogram with an ideal shape fit by a nested model. $a = 25.6$ km, $C = 0.87$, $A = -0.0794$, $c = -0.001$ and $\lambda = 63$ km. Negative amplitude and damping coefficient is due to the plateau region being modeled incorrectly as periodicity.

Figure 1.29b: Above semivariogram fit by a spherical model. $a = 30.2$ km, $C = 0.9$. The spherical model gives a much better fit and estimates the short-range component.
Figure 1.30a: Semivariogram showing periodicity fit by the nested model. $a = 26.2$ km, $C = 0.96$, $A = 0.6094$, $c = 0.0246$ and $\lambda = 66$ km. Nested model gives a good fit to the experimental semivariogram.

Figure 1.30b: Above Semivariogram fit by the spherical model. $a = 13.9$ km and $C = 1.00$. The spherical model underestimates the value of the range.
Figure 1.31a: Semivariogram showing trend and periodicity fit by the nested model. $a = 119.3$ km, $C = 0.98$, $A = 0.4186$, $c = 0.0060$ and $\lambda = 136$ km. Model fit is influenced by the trend. Range modeled is greater than half the lag and inconsistent with the experimental semivariogram.

Figure 1.31b: Above semivariogram fit with a spherical model. $a = 46.0$ km $C = 0.90$. Short-range component in the presence of trend is modeled better by the spherical model.
Figure 1.32: Directional semivariogram from the Window A modeled using the semivariogram modeling algorithm.
(a) Semivariogram along S45E showing prominent periodicity fit by the nested model.
\( a = 25.8 \text{ km}, C = 1.17, A = 0.5760, c = 0.0500 \) and \( \lambda = 61 \text{ km} \).
Figure 1.32 (Continued)
(b) Directional semivariogram along N45E fit by a purely spherical model. 
a = 17.6 km and C = 0.96
Figure 1.32 (continued)
(c) Semivariogram along N5E fit by the spherical model.
\[ a = 27.8 \text{ km and } C = 0.90 \]
Figure 1.33: Directional semivariograms from the semivariogram map of window B. Note that all the semivariograms decorrelate at approximately equal lags.
Figure 1.34: Semivariogram modeling algorithm fits spherical model to directional semivariogram from semivariogram map of window B. $a = 19.9$ km, $C = 1.10$
Figure 1.35: Directional semivariograms from the semivariogram map for window C modeled using the semivariogram modeling algorithm. Lag is in grid units. Semivariances have been interpolated every 0.25 grid units (9m).
(a) Semivariogram along N45E showing periodicity fit by the nested model.
\[ a = 5.5 \text{ grid units (198 m)}, C = 1.20, A = 0.1300, c = 0.0001 \text{ and } \lambda = 11.6 \text{ grid units (418 m)} \]
Figure 1.35 (continued.)
b) Semivariogram along S45W showing a high correlation between data values. Semivariogram is fit by the purely spherical model. \( a = 17.5 \) grid units (630 m), \( C = 1.34 \).
(c) Semivariogram along N15W showing the presence of trend. Modeling algorithm fits a spherical model to the semivariogram and maps the short-range spherical structure of the semivariogram. \( a = 10.7 \) grid units (388 m), \( C = 1.0 \).
Figure 1.36: Rose-diagram of ranges for window A: Ranges modeled for different directional semivariograms are plotted along the direction of the profiles. Elliptical shape indicates anisotropy. Long-axis of the elliptical shape is in the direction of maximum continuity (approximately N-S).
Figure 1.37: Rose-diagram of ranges for window B: Ranges modeled for different directional semivariograms are plotted along the direction of the profiles. Approximately circular shape indicates isotropy. Directions of maximum or minimum continuity are not significant.
Figure 1.38: Rose-diagram of ranges for window C: Ranges modeled for different directional semivariograms plotted along the direction of the profiles. X-Lag and Y-Lag are in grid units (1 grid unit = 36 m). Elliptical shape indicates anisotropy with maximum continuity along N30-40W.
Figure 1.39: Ellipse fit to the rose-diagram of ranges of window A. The X and Y axes represent lag in km. Semi-major axis = 29.3 km, semi-minor axes = 16.6 km and orientation of semi-major axis = N1W, aspect ratio = 1.8. The semi-major and semi-minor axes represent the range in the direction of maximum and minimum continuity respectively. The angle between the direction of maximum and minimum continuity = 90°.
Figure 1.40: Ellipse fit to the rose-diagram of ranges of window B. The X and Y axes represent lag in km. Semi-major axis = 18.6 km, semi-minor axes = 16.5 km, aspect ratio = 1.1. The semi-major and semi-minor axes are almost equal and model ellipse is close to a circle. Directions of maximum and minimum continuity are not significant.
Figure 1.41: Ellipse fit to the rose-diagram of ranges of window C. The X and Y axes represent lag in grid units (1 grid unit = 36 m). Semi-major axis = 12.8 grid units (448 m), semi-minor axes = 5 grid units (175 m) and orientation of semi-major axis = N32W, aspect ratio = 2.6.
Figure 1.42: (Top) Variation of wavelength with direction for window A. The minimum wavelength modeled is 400 m for the semivariogram along direction 170° (N80W). (Bottom) Variation of range with direction. Directional semivariograms with high range values do not have a corresponding wavelength since no periodicity would have been mapped along these semivariograms which are close to the direction of maximum continuity.
Figure 1.43: (Top) Variation of wavelength with direction for window C. The minimum wavelength modeled is 400 m for the semivariogram along direction 40° (N35 E).
(Bottom) Variation of range with direction: Directional semivariograms with high range values do not have a corresponding model wavelength since no periodicity would have been mapped along these semivariograms which are close to the direction of maximum continuity.
RESULTS

The semivariogram of topography quantifies the spatial statistics of topography using two important components. One is the short-range component that is associated with average loss of correlation between elevation values with distance. This directional variation of the decorrelation distance helps us identify orientations and magnitudes of the directions of maximum and minimum continuity. The other is the periodic component of topography, which maps repetitive patterns. The wavelength of the periodic component models the average spacing between linear features. I have attempted to characterize the spatial continuity of topography from different physiographic provinces using the variation in the short-range correlation scale and wavelength of features. I applied the semivariogram methods to 3 synthetic topography surfaces in order to test the semivariogram analysis on simple models. These analyses lead us into understanding the concepts of calculating and modeling semivariograms of real topography data.

SYNTHETIC TOPOGRAPHY MODEL A – SINUSOIDAL PROFILE

I simulated a topographic surface made up of ridges and valleys that were sinusoidal in shape. This topography can be compared to a corrugated tin roof. The building block of this topography is a sine wave (Figure 2.1) of wavelength 125 km. The length of the profile is 1000 km. The topography model is built by extending the sine wave for 1000 km along the second dimension and creating a lineated surface with strike 0° (Figure 2.2). The model topography is geographically distributed on a
1000*1000 grid with a grid spacing of 1 km. The wavelength of the ridge-valley-ridge features is constant and equal to 125 km. Topography with such clear periodicity does not exist on earth but the model gives insight into how periodicity is mapped by the semivariogram.

**Semivariogram Map and Modeling**

A semivariogram map was calculated for a 400*400 window centered at (500,500) on the larger grid of the model topography. The 400*400 window was translated across the larger grid from 0 to 200 grid units and the semivariance was calculated at each lag. The semivariogram map (Figure 2.3) shows equal semivariances along any N-S line. This is because elevations along any N-S trending line on the grid are the same. Therefore, the pairs that we compare at all N-S lags will be the same. Along the E-W direction the semivariance increases and decreases in cycles. Directional semivariograms were interpolated along 5° intervals. The directional semivariograms along azimuth 90, 60, 45 and 15 (Figure 2.4a-d) show that the semivariogram maps a periodic component. The directional semivariograms were fit with a model sinusoid defined by an amplitude and wavelength. The wavelength and amplitude calculated from the model is plotted against direction (Figure 2.5a). The amplitude of the directional semivariograms is constant (Figure 2.5a (top)). The minimum wavelength modeled is 125 km along the direction 0° and 180° (N90E and N90W) semivariograms (Figure 2.5a(bottom)). The wavelength along other directions is longer and this apparent wavelength increases as the orientation of the semivariogram comes closer to the N-S orientation of the ranges. The wavelength along direction 90° is infinite and therefore cannot be modeled.
Semivariograms modeled for a 250*250 window give a similar distribution for
the amplitude and wavelength (Figure 2.5b). The semivariogram analysis was done
for different window dimensions, with all the windows centered at (500, 500) on the
grid. The directional semivariogram along azimuth 90° (perpendicular to the
orientation of the ridges) for each of the windows was graphed and compared (Figure
2.6). As the window size decreases, we observe that the semivariogram maps only
part of the cycle of the sinusoid. The minimum window size necessary to map 1
cycle of the sinusoid is 250*250. This is because we lag our semivariograms by half
the window size. Semivariograms calculated from window dimensions less than 250
pick up only a part of the sine wave. By observing the directional semivariograms at
the same horizontal scale (Figure 2.6), it can be clearly seen that the semivariograms
for the 200*200 and 100*100 have mapped only part of 1 cycle of the sinusoid.

Conclusions from this Model

The semivariogram maps the periodicity of those features within the window
of interest whose wavelength is less than or equal to the maximum lag. Since we
calculate the semivariances only until the lag is equal to half the window size, in
order to map the periodicity the window dimensions for the semivariogram analysis
should be at least twice the average wavelength of the features. If we calculate the
semivariogram using a smaller window size model, parameters of the periodic
component of the semivariogram would not be characteristic of the actual physical
feature. Thus, any window size below 250*250 will not be able to map the
wavelength of the linear features of the above topography model.
SYNTHETIC TOPOGRAPHY MODEL B – BASIN AND RANGE

PROFILE UNFILTERED

Some features noticeably absent in the previous model but present in the semivariograms of real topography are as follows.

a) The short-range correlation scale of topography was not present in the sinusoid topography model, but is an integral part of real topography.

b) Linear features of topography may be characterized by multiple wavelengths as opposed to a single wavelength in the sinusoid model. The repetitive patterns of real topographic data may be pseudo-periodic (Chiles and Delfiner, 1999) i.e. the spacing may not be constant as we had assumed in the previous model.

c) Long wavelength trends may underlie the spatial patterns of interest and these trends can the cause the semivariogram to drift. Semivariogram analysis in the presence of drift is unreliable.

In order to incorporate all these features into a topographic model, I took an E-W profile from the topography data of western US. The profile extends for 1000 km and for the most part runs across the Basin and Range Province (Figure 2.7). Thus, it records the wavelength of the linear basins and ranges along the line (Figure 2.8a). The profile was extended in the second dimension for 1000 km in order to create the 1000*1000 topographic surface on a grid (Figure 2.8b). All the topographic features in this model strike N-S.

Semivariogram Map and Modeling

I calculated the semivariogram for a 400*400 and a 200*200 window centered at (500, 500) on the larger grid. The semivariogram maps of the windows
(Figure 2.9a and Figure 2.9b) show the distribution of the semivariances at all possible lags. Along E-W the semivariances increase and decrease in cycles. Along any N-S line on the map, the semivariances are constant. The interpolated semivariograms were modeled by both a purely spherical model and a nested model using the modeling algorithm. The semivariogram models are fit based on the decision process in the program. The directional semivariograms for the 400*400 window (Figure 2.10 a-d) and 200*200 window (Figure 2.11 a-d) along N90E, N45E, N15E and N45W are given as examples to illustrate the modeling process.

The directional semivariograms for the 400*400 window (Figure 2.10 a b d) show an increase in semivariance in a parabolic fashion. This indicates the presence of a space-varying mean (Chiles and Delfiner, 1993) or drift that is mapped by the semivariogram. Two scales of spatial correlation, one at a lag of 20 km and the other at 80 km can be distinguished from inflexions of the experimental semivariogram along N90E. If we ignore the drift and observe the structure, all three directional semivariograms (Figure 2.10 a b d) show a short-range spherical component. The directional semivariogram along N90E (Figure 2.10a) has been modeled using the nested model. The parameters of the model, specifically the range and the wavelength, are found to be inconsistent with the structure of experimental semivariogram because of the drift. The range modeled by the semivariogram does not fit either of the spatial structures. The directional semivariogram along N45E and N45W have been fit with a spherical model and the short-range correlation scale has been modeled. Along N15E the semivariogram is correlated for a long distance and the range and sill are modeled by the purely spherical model.
The directional semivariograms for the 200*200 window show a slight drift along N90E (Figure 2.11a). The spherical model fit by the program gives a consistent estimate of the short-range correlation. The semivariograms along N45E (Figure 2.11b) and N45W (Figure 2.11d) are modeled using the nested model. The directional semivariogram along N15E is modeled by the spherical model (Figure 2.11c). The semivariograms of the 200*200 window appear to be not affected much by the drift. This is probably because the window size is smaller and has not been lagged far enough.

Table 2.1 summarizes the type of model used and the model parameters of the four example semivariograms of the 400*400 and 200*200 window.

**Modeling Anisotropy in Range and Wavelength**

The rose-diagram of the ranges modeled from the directional semivariograms of the 400*400 window (Figure 2.12a) and the 200*200 windows (Figure 2.12b) indicates a direction of maximum continuity along N-S and a direction of minimum continuity along E-W. The variation of the range with azimuth cannot be modeled accurately with an ellipse because the range along directions close to the N-S strike of the features cannot be modeled from the directional semivariogram. However, it is possible to obtain an estimate of the range along the axis of minimum continuity by observing the variation of range with direction. The ranges modeled for the 400*400 window are completely inconsistent because of the drift. Hence, the variation of these ranges with direction is inconclusive. The ranges modeled for the 200*200 window (Figure 2.13-bottom) is about 20 km along N90E. The 200*200 window is modeled by the periodic component along some directions where the trend is not
very prominent. The variation of the model wavelength of the periodic component with direction (Figure 2.13-top) shows that the nested model fit very few of the directional semivariograms. Hence, it was not been possible to infer details of the wavelength.

**Conclusions from this Model**

The main inferences drawn from this model are that semivariogram analysis is not robust in the presence of a long wavelength trend. When there is a trend in the data the mean changes with position making the assumption of stationarity invalid. As the mean increases along a direction, the semivariance calculated for the lags along that direction also increase. This would result in the sample semivariogram reflecting more the variations in the average value of the data rather than the spatial variability itself (Chiles and Delfiner, 1993). It therefore becomes necessary to remove the long wavelength trend before calculating the semivariogram.

**SYNTHETIC TOPOGRAPHY MODEL C - BASIN AND RANGE PROFILE DETRENDED**

The importance of detrending the area of analysis before applying semivariogram analysis has been established from the previous model. In order to model and infer results from the analysis, data should be sufficiently preprocessed so that we can assume stationarity over the window of analysis. The profile (Figure 2.14) used to construct topography model C is taken along the same line (Figure 2.7) as the previous model. The elevations in this grid have been detrended using a high-pass frequency filter; wavelengths greater than 60 km have been removed. The model topography was constructed by extending the profile along the second
dimension. Semivariogram analysis was done using the window dimensions 400*400 and 200*200.

Semivariogram Map and Modeling

The semivariograms for the 400*400 window (Figure 2.15 a-d) and 200*200 window (Figure 2.16 a-d) are compared with the semivariograms of the synthetic topography model B.

The semivariograms for the 400*400 window along N90E, N45E and N45W (Figure 2.15 a b d) have no characteristics of trend that was observed in the previous model (Figure 2.10 a b d). The semivariograms consist of a short-range spherical component and a periodic component whose amplitude falls with lag. The nested model picks up the range and wavelength of these semivariograms. The semivariogram along azimuth 15 has been modeled using the spherical model (Figure 2.15c).

The semivariograms for the 200*200 window along azimuth N90E, N45E and N90W (Figure 2.16 a b d) are similar to the directional semivariograms along the same orientation for the unfiltered data (Figure 2.11 a b d) and have been fit by a nested model. The trend observed in the structure of the semivariogram along N90E for topography model B (Figure 2.11a) is not present in the N90E semivariogram in this model (Figure 2.16a). Along N15E (Figure 2.16c) the semivariogram shows a high degree of correlation and a spherical model is fit to this semivariogram. The parameters of the model fit to the semivariograms along N45E, N45W and N15E for the 200*200 of the two topographic models are approximately the same.
Table 2.1 summarizes the type of model used and the model parameters of the four example semivariograms of the 400*400 and 200*200 window.

**Modeling Anisotropy in Ranges and Wavelength**

The rose diagram of the ranges for the 400*400 window and the 200*200 window (Fig 3.17a and 3.17b) indicate maximum continuity along N-S and minimum continuity along E-W. The variation of the range with direction for the 400*400 window (Figure 2.18a) and the 200*200 window (Figure 2.18b) are illustrated. The 400*400 window is modeled by a range of 22 km along N90E and the 200*200 window is modeled by a range of 26 km along N90E direction. The detrended semivariograms are fit by the nested model along most of the directions that map periodicity. Hence, the variation of wavelength with direction can be studied. The model wavelength for the 400*400 window is 60 km along N90E. The minimum wavelength for the 200*200 window is also 60 km along the N90E semivariogram.

**Conclusions from this Model**

The main inference that we can make from this model is that it is very important to remove long-wavelength trends from the Basin and Range topography in order to model the spatial continuity. Once the effect of trend is removed the range and wavelength can be modeled from the directional semivariogram by using a suitable function. The direction of maximum continuity modeled from the range values is usually along the dominant orientation of the features in the window of observation. The nested model picks up a minimum wavelength along the semivariogram profile oriented perpendicular to the linear features.
APPLICATION TO TOPOGRAPHY DATA

The synthetic models have illustrated in depth the amenability of topography data to spatial analyses using semivariograms. I now apply the semivariogram analysis on windows of elevation data from the study areas. In order to map the spatial continuity for the entire western US and the Antarctic sea-floor each study area represented on the geographic grid was detrended.

The western US grid was detrended using the frequency filtering method. I used the Fast Fourier transform algorithm grdfit (GMT cookbook, 1996), which transformed the raw topography grid into the frequency domain, performed a high-pass frequency filter to remove long wavelengths and outputed the filtered data in space domain. The filter cuts off wavelengths above 240 km, passes all wavelengths below 60 km and ramps between 60 and 240 km. An E-W elevation profile from the Basin and Range Province before and after the filtering (Figure 2.19) illustrates the long wavelength trend (red line) that has been removed by the filtering process. The Antarctic grid has data-gaps. Hence, I used the moving average method in order to remove the trend. For the grids 049c and 052c, I used an 11*11 unweighted averaging. For the grid 048c, I used a 21*21 unweighted averaging filter. This removed long-wavelength trends without causing significant edge effects. The profile across the Antarctic grid 052c (Figure 2.20) before and after filtering illustrates the trend (red line) that has been removed from the sea-floor profile.

The detrended topography grids were divided into windows of suitable sizes and semivariogram map calculated for all the windows. The directional semivariograms of all the windows were modeled to obtain the range, sill and
periodicity (if any). The ranges for each window were modeled by fitting an ellipse defined by its semi-major axis, semi-minor axis and orientation of semi-major axis. The ellipse modeled for each window was plotted on the grid with the center of each ellipse corresponding to the center of the window on the grid.

WESTERN US TOPOGRAPHY

The previous model results have shown that the window dimension is an important variable in the semivariogram analysis. Analysis over a small region can result in not enough data for calculating the semivariance. A large window would result in averaging over a large number of values and may not represent the statistics of the features of interest. We have already concluded that the periodicity modeled by the semivariogram is dependent on the window size. We now need to understand how window dimensions affect the short-range correlation modeled by the semivariogram. The western US topography data is a large grid with a complete coverage of elevations, which makes it possible to test different window sizes.

Semivariogram Map and Modeling

I calculated the semivariogram map for window sizes from 400*400 to 50*50 at intervals of 10*10 (400*400, 390*390, ....60*60, 50*50). I have illustrated window dimensions- 400, 350, 300, 250, 200, 150, 100 and 50 (Figure 2.21a) centered at a common point (900, 900) on the larger grid (Figure 2.21b). The semivariogram maps of the 7 windows (Figure 2.22a-g) have been plotted using the same scale for the X-lag, Y-lag and semivariance. Only lags extending from -50 to +50 have been plotted on the maps. I observed the following spatial characteristics in the windows since this is the region over which most of the short-range correlation
has been observed. The semivariogram maps of the different windows (Figure 2.22) show the following similarities and contrasts in the spatial characteristics.

1) All the semivariogram maps show elliptical contours indicating the presence of anisotropy in the spatial continuity.

2) The semivariogram maps show similarity in the distribution of semivariances for the window sizes 400, 350, 300, 250 and 200.

3) Semivariances increase slowly along the N-S direction and plateau around a value of 1 (yellow color on map). Along the E-W direction semivariances increase very quickly along the E-W direction to a value of about 1.25. The rate of increase of the semivariance along E-W is higher for windows 150*150, 100*100 and 50*50 windows. This is observed by the elliptical shape of the semivariances bounded by the yellow line becoming narrower in the E-W direction and longer in the N-S direction.

4) High semivariance values (greater than 1) occur on either side of the central low values in all the windows. For the smaller window sizes (150 to 50) the semivariances increase to values as high as 1.5.

5) There is a periodic component mapped along approximately the E-W direction in all the windows. This is observed from alternating high and low values of semivariances and a stronger periodicity is mapped with decreasing window size.

I interpolated directional semivariograms from the semivariogram maps along 5° intervals. The directional semivariograms obtained for each window was modeled to calculate the range.
Modeling Anisotropy

The range values for each of the windows were fit with an ellipse. The model ellipses for the windows (Figure 2.23 a-g) show the direction of maximum continuity and the ranges in the direction of maximum and minimum continuity. Table 2.2 lists the model parameters for the 7 window sizes. I observed that the window sizes 400, 350, 300, 250 and 200 are modeled by ellipse with an aspect ratio \( \equiv 2 \). The orientations of these ellipses vary between N 2 E and N 9 E. Window sizes 150*150, 100*100 and 50*50 give an aspect ratio of 2.5, 3 and 2.9 respectively. The model ellipses of these windows show a decrease in the range along the axis of minimum continuity. I have plotted the model ellipses for the different windows on the same figure in order to compare their shapes (Figure 2.24). We can divide the ellipses into two categories. The blue ellipses are models for window sizes 400*400-200*200 and show a similar orientation, shape and aspect ratio. The magenta, green and red ellipses represent the window sizes 150, 100 and 50 and the aspect ratio for these ellipses is higher. The ellipses for these windows are relatively narrower along the E-W and longer along N-S.

Inferences from the Anisotropy Models for Different Window Sizes

The semivariogram analysis on larger window sizes is calculating the semivariance over a larger area. Hence, the spatial continuity mapped represents the average over the entire area. It appears that the statistical attribute (in this case the semivariance) of the topography is the same when viewed at these different scales (window sizes- 400*400-250*250). This indirectly shows the fractal nature of topography. The spatial variability being independent of the scale of observation.
suggests self-similarity, which is the term for any property that repeats itself at all scales (Chiles and Delfiner, 1993). As the window size decreases the window encompasses only the Basin and Range features, which comprise of alternating areas of high and low elevation values. For the smaller window sizes, the semivariance at each lag is calculated by averaging over fewer elevation pairs. Hence, the semivariogram can increase to extremely high values $\gamma(h) > 1.25$. The elevations within the window of observation are highly correlated along the N-S and decorrelate very rapidly along the E-W direction. As a result the range along the direction of minimum continuity decreases and anisotropy increases.

The orientation of the ellipse for the 50*50 window (Figure 2.23g) is very different from the other windows. The 50*50 window covers an area of ridges and basins (Figure 2.21a). The semivariogram map for the 50*50 window shows that semivariances along N-S have not reached the plateau value of 1.0 probably because the window has not been lagged far enough while calculating the semivariances. I have plotted the semivariograms along N10E and S85E for different window sizes (Figure 2.25a and Figure 2.25b). These directions are close to the average orientations of the maximum and minimum continuity of the topography in the windows. The semivariogram of the 400*400 window increases rapidly and plateaus around a semivariance of 1.25. The periodic component has been averaged over the entire window and is not very prominent. A strong periodicity is mapped by the semivariograms of windows 250, 200 and 100. The 50*50 window size only maps part of a cycle of the periodic component. Modeling this semivariogram would result in an underestimated range value since the semivariogram structure is not complete.
A lag of 25 grid units is not sufficient to map the spatial continuity of the features in the Basin and Range. Along N10E the semivariograms rise slowly and plateau around a sill of 1-1.25. The semivariogram of the 50*50 window is still increasing and has not reached a semivariance close to 1. Modeling this semivariogram will give erroneous range value that does not have any physical explanation in the topography. It is thus conceivable that the 50*50 window is too small and we cannot interpret the spatial continuity in the Basin and Range province at a 1 km scale using a window size of 50*50.

I have plotted the variation of the orientation, aspect ratio and the magnitude of the range in the direction of minimum continuity with window size (Figure 2.26). The reason I plot the variation of the range in the direction of minimum continuity is because it is the change in this range with window size that changes the aspect ratio of the ellipse. This can be observed from the similarity in the variation of the aspect ratio with window size and the variation of the range in the direction of minimum continuity with window size (Figure 2.26). The range in the direction of maximum continuity is relatively constant for all window sizes. The aspect ratio of the anisotropy model ellipses is fairly constant and lies between 1.8 and 2.2 for the window sizes 400-200. The aspect ratio of the anisotropy model for the larger window sizes (400-200) is estimating the overall average spatial continuity of the topography along different directions. The aspect ratio increases with decreasing window size from about a window dimension of 200*200 to 60*60. The increase in the aspect ratio (> 2.) indicates that the semivariogram is mapping the spatial variability of more localized features. The range along the direction of minimum
continuity decreases while the range along the maximum continuity remains the same. This results in a high value for the aspect ratio for window sizes between 200-60. The aspect ratio falls again for the window size 60*60 and lower.

The constant aspect ratio for large window sizes 400-200 is due to the averaging of the semivariances over a larger area. The increase in the aspect ratio at a scale of window size 150*150 and lower in the Basin and Range Province is probably because the semivariogram starts mapping local correlation scales and spatial continuity of the linear basins and ranges. The decrease in the aspect ratio at smaller window sizes (60*60 and 50*50) is probably because the window size becomes too small to map the spatial continuity of features at a 1 km resolution. Incorrect estimates of range values could result from modeling semivariograms maps of small windows that are not lagged far enough. The model ellipse would therefore be unreliable for inferring details of spatial continuity.

Modeling Spatial Continuity over Western US

I have overlaid the anisotropy models for 4 different window sizes on the topography of the Basin and Range. The ellipses are plotted at the center of each window of observation. The semivariogram analysis using 400*400 window (Figure 2.27d) and 300*300 window (Figure 2.27c) gives a similar anisotropy model for all the windows no matter what the location of the window is. The ellipses have a similar shape and orientation. The aspect ratio of the ellipses for the 400*400 window lies between 1.8-2.4. The aspect ratio of the ellipses of the 300*300 window lies between 1.5-2.5. The 200*200 window (Figure 2.27b) begins to show the spatial continuity of the local features within the window especially in the Snake River Plain
and in some areas south of the Basin and Range. But overall the aspect ratio for the 200*200 window in the Basin and Range region is between 1.5-2.22. Thus, the average correlation scale of topography is similar when viewed at the scales of 400*400, 300*300 and in most cases for the 200*200.

The correlation scales mapped by the 100*100 window are different from those of the 200*200 and larger windows. The semivariogram mapped from the 100*100 (Figure 2.27a) window characterizes the correlation scales of the localized features within the window of observation. The anisotropy model for the 100*100 window varies with the window location. For the Basin and Range, the aspect ratio of the model ellipses lie between 2 and 2.6. There is a sharp change in the aspect ratio of the ellipses east of the Basin and Range. The aspect ratio falls to 1.6 and lower in the Colorado Plateau region. Similarly, to the north and northwest of the Basin and Range Province, the Snake River Plain and Columbia Plateau provinces show aspect ratios less than 1.6.

Spatial Continuity Maps from Model Parameters

The model parameters, namely orientation of the semi-major axis of the ellipse, aspect ratio of the ellipse and the area of the ellipse are used to study the spatial statistics of topographic patterns. The size of the arrows (Figure 2.27e) are related to the aspect ratio which determines the shape of the of the model ellipse. The arrows point toward the direction of lineation. The greater the size of the arrow, greater is the aspect ratio of the ellipse and greater the degree of lineation of the features. The Basin and Range Province shows features that are highly lineated (large arrows) when compared to the Colorado Plateau to the east and the Snake
River Plain Province to the north. The arrow size diminishes for the Snake River
Plain Province and Colorado Plateau, the ellipticity decreases and the orientation of
the lineated features becomes less important.

The contours of aspect ratio (Figure 2.27f) clearly separate a zone of high
aspect ratio (> 2) in the Basin and Range Province from the Snake River Plain and
Colorado Plateau. Areas north of the Snake River Plain show low aspect ratios. The
topography here is rough and highly dissected with no prominent direction of
lineation. This highly dissected topography may be related to erosional processes. On
the other hand, the topography in the Basin and Range is rough and lineated. The
lineated topographic pattern may be due to the influence of tectonics in shaping the
landscape in the Basin and Range Province.

The area of the ellipse indicates roughness of the topography. The size of the
circles (Figure 2.27g) is inversely proportional to the area of the ellipse. Therefore
larger circles represent rougher topography. The Basin and Range Province and the
region north of the Snake River Plain show very rough topography. The Snake River
Plain and the Colorado Plateau region show smoother topography as indicated by the
smaller size of the circles. The contours of smoothness (Figure 2.27h) drawn by
contouring the area of the ellipses illustrate regions of very rough and very smooth
topography. Large contour values indicate smoother topography. The model ellipses
for the Snake River Plain and regions of Colorado Plateau show areas of > 1000
square kilometers. The size of the ellipse is very small (area < 500 square kilometers)
in the region with extremely rough topography, north of the Snake River Plain.
ANTARCTIC BATHYMETRY – MULTIBEAM DATA EXAMPLES

The multibeam data from the continental shelf of Ross Sea, Antarctica show geomorphic features that have been carved by the deforming ice sheet. The topography of the sea floor is mapped by multibeam bathymetry along narrow elongated swaths. Thus the regular rectangular grid has incomplete depth data with a lot of data gaps. However, in the areas where the depth data has been collected, the bathymetry has been mapped at a very high resolution. The average grid spacing is 35 m for the Antarctic grids. Therefore, a 50*50 window from the Antarctic grid covers an area of approximately 4 square km when compared to 2500 square km in the western US topography grid. The features of the sea-floor in Antarctica have a relief averaging couple of tens of meters or less. The spatial distribution of the features on the sea-floor is at a completely different scale. In order to illustrate the spatial continuity modeled using the semivariogram analysis and explain how the correlation scales could be used to distinguish different patterns at this large scale, I have used 2 windows from grid 049c (Figure 2.28). The Window A is a 50*50 window from a region on the grid that has features oriented in an apparently random manner. The Window B is a 70*70 window from a region with prominent grooves and lineation.

Semivariogram Map and Modeling

Predictably the semivariogram map of the window A (Fig 3.29a) shows circular contours that indicate isotropy. The semivariogram map of the window B (Figure 2.29b) shows a distinct anisotropy with the direction of maximum continuity
oriented approximately N-S and the direction of minimum continuity oriented E-W. The semivariances map a periodic component along the direction of minimum continuity. The directional semivariograms for window A along N90E, N45E, N15E and N90W (Figure 2.30 a-d) are illustrated. The range modeled from the directional semivariograms is approximately equal in all directions. The directional semivariograms for window B (Figure 2.31 a-d) exhibit periodicity along certain directions and are fit by the spherical model and the nested model.

**Modeling Anisotropy in Ranges and Wavelength**

The rose-diagram of the range values for window A has been modeled with an ellipse (Figure 2.32a). The semi-major axis = 175 m and a semi-minor axis = 157 m. The aspect ratio of the ellipse = 1.1. Thus the ellipse is close to a circle indicating isotropy. The direction of maximum continuity is not clear in this case. The model ellipse for the window B (Figure 2.32b) gives a semi-major axis of 910 m and a semi-minor axis equal to 214 m. The orientation of the direction of maximum continuity for window B is N3E, which is close to the average orientation of the linear features in the window. The aspect ratio for window B is 4.3 indicating a strong anisotropy in the spatial continuity. The variation of the wavelength and range (Figure 2.33a and Figure 2.33b) for the two windows has been plotted. The range for the window A (Figure 2.33a-bottom) is constant with direction indicating isotropy. No significant minimum for the wavelength can be interpreted for the wavelength based on the topography in the window A (Figure 2.33a-top). The variation of model wavelength with direction for window B (Figure 2.33b-top) gives a wavelength of 700m along direction 180 (N90W). This seems plausible as the semivariogram could
be mapping the spacing between the repeating patterns (grooves and lineation) oriented N-S.

**Conclusions from the Semivariogram Analysis on Example windows**

The difference in the aspect ratio for window A (aspect ratio = 1.1) and window B (aspect ratio = 4.6) is clearly due to the difference in the pattern in the topography within the window. The grooves in window A are very elongated in the N-S direction and very narrow in the E-W direction. Therefore, the semivariogram decorrelates much slower in the N-S direction than it does in the E-W direction and the correlation scales show very high directional variation. For the window A, there is no dominant orientation for the features within the window and the semivariance increases in a similar in all directions. The semivariogram for the Antarctic grid is thus mapping the highly localized correlation scale of the seafloor features.

**Modeling Spatial Continuity over Grids 049c, 052c and 048c**

I have mapped the spatial continuity of the topography using 50*50 windows (window dimensions are in grid units for all the three grids. I arrived at the window size of analysis of 50*50 for all the grids after observing the sample profiles from the region, the resolution of the grid and the narrowness of the swath area. The model ellipses are plotted on the grid with the center of the ellipse coinciding with the center of the window on the grid (Figure 2.34 a b c). The ellipses that fall in the no-data areas are windows close to the edge of the grid that encompass > 80% data-gaps and about 20% of depth values. Therefore these ellipses are not robust for any inferences.
The Antarctic grid 049c shows prominent grooves and lineation (Figure 2.34a). The aspect ratio of the model ellipse is high in the region of the large groove like features. The value is anywhere between 4.0 and 5.0. The ratio falls off east, west and north of the large grooves to 3.5. The areas south of the grid show more isotropy in the spatial continuity. These features are oriented in a random manner and the model ellipses have an aspect ratio of < 2. At some parts in the southern part of the grid E-W oriented iceberg furrow are mapped by the semivariograms. The aspect ratio of the ellipses are plotted as arrows with the size of the arrows proportionate to the value of the aspect ratio and the direction of the arrow corresponding to the orientation of the semi-major axis of the ellipse (Figure 2.34a – 1). The region where there are grooves have very large aspect ratios as indicated by the larger arrows when compared to the regions with not prominent lineations. In these regions the size of the arrow is small.

The Antarctic grid 048c does not appear to have any linear features (Figure 2.34b). The spatial continuity is mostly isotropic. In the southern portion of the grid, the semivariogram maps anisotropy but the orientation of the ellipses do not seem to coincide with the trend of the most of the observable linear features (Figure 2.34b-1). This may probably be due to the extremely chaotic pattern in the sea-floor features in the southern part of the grid. The semivariograms of certain windows that encompass the mid-track distortions due to the ship show a pseudo-anisotropy (Figure 2.34b-3). This is not due to the spatial continuity of geomorphic features of the topography. The mid-track distortions can be observed along the prominent straight line that runs through the center of the grid.
The Antarctic grid 052c shows prominent lineations along the NW-SE direction (Figure 2.34c). A strong anisotropy in the correlation scales is observed in the northern and southern parts of the grid. The central part of the grid shows a region almost free of lineation and the model ellipses there have a smaller area and a lower aspect ratio. In the northern part of the grid where the lineations seem to trend N30-40, the semivariogram maps an anisotropy with the axis of maximum continuity trending N30-40E.
Table 2.1 Model parameters along 4 directional semivariograms for the 400*400 and 200*200 windows

<table>
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<tr>
<th>Window size (grid units)</th>
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<th>Model</th>
<th>a (km)</th>
<th>C</th>
<th>A</th>
<th>c</th>
<th>λ (km)</th>
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Table 2.2 Parameters of model ellipse for the different window sizes

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<th>Aspect Ratio</th>
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Figure 2.1: Sinusoid profile used to generate synthetic topographic model A. Wavelength = 125 km. Profile was extended over 1000 km in the Y-direction in order to generate a topographic surface.
Figure 2.2: Synthetic model topography A. Only a part of the surface is shown. Model extends for 1000 km in X and Y directions with all features striking N-S. Semivariogram maps are calculated for 400*400 and 200*200 windows centered at (500,500) on the topographic surface.
Figure 2.3: Semivariogram map of 400x400 window from the synthetic topography model A. Map shows a very high correlation along the N-S direction. Semivariance is constant along any N-S strike because elevations along any N-S line on the model surface are the same. Semivariance increases and decreases in cycles along the E-W direction.
Figure 2.4: Directional semivariograms of 400*400 window from synthetic topography model A fit with a sinusoid. Note that the value of the modeled wavelength increases as we move away from the N90E semivariogram profile toward N0E semivariogram profile. Periodicity decreases for the semivariograms profiles close to the strike of the features.
(a) Semivariogram along azimuth 90 (N90E). Wavelength = 125 km
(b) Semivariogram along azimuth 60 (N60E). Wavelength = 144.9 km
Figure 2.4: (Continued)
(c) Semivariogram along azimuth 45 (N45E). Wavelength = 178 km.
(d) Semivariogram along azimuth 30 (N30E). Wavelength = 256 km.
Figure 2.5a: (Top) Variation of modeled amplitude with direction for the 400*400 window from Synthetic topography model A. Amplitude is constant along all directions. Semivariograms close to N-S could not be modeled and the amplitudes along this direction have been given a value zero. (Bottom) Variation of wavelength with direction. Minimum wavelength of 125 km is estimated along direction 180° (N90W) and 360° (N90E). Apparent wavelength > 125 km is estimated along semivariograms not perpendicular to the N-S strike of the topography. Wavelength is not modeled for profiles close to the N-S direction.
Figure 2.5b: (Top) Variation of model parameter amplitude, with direction for a 250*250 window from the model topography. Observe that the variation of the amplitude and wavelength is the same as for the 400*400 window. (Bottom) Minimum wavelength of 125 km is modeled along direction 180°(N90W) and 360°(N90E). This is the minimum window size that can be used to model the wavelength of periodic features of the sinusoid topography model.
Figure 2.6a: Semivariograms along N90E for the different window sizes are plotted on the same horizontal scale. Directional semivariograms for windows 100*100 and 200*200 map only part of one cycle of the sinusoid.
Figure 2.7. E-W line from the Western US used to generate synthetic topography model B. The line of profile extends for 1000 km and for the most part runs across the Basin and Range region.
Figure 2.8a: Profile from Basin and Range Province. Model topographic surface B was generated by extending the profile along the Y-direction for 1000 km. Profile has not been detrended.
Figure 2.8b: Synthetic model topography B created from Basin and Range profile-unfiltered. Only a part of the surface is shown. Model extends for 1000 km in X and Y directions with all features striking N-S. Semivariogram maps are calculated for 400*400 and 200*200 windows centered at (500,500) on the topographic surface.
Figure 2.9a: Semivariogram map for a 400*400 window from the synthetic model topography B. Along any N-S direction elevation values are highly correlated. Along the E-W direction the semivariogram maps the periodicity in the topography.

Figure 2.9b: Semivariogram map for a 200*200 window from the synthetic model topography B. Along any N-S direction elevation values are highly correlated. Along the E-W direction the semivariogram maps the periodicity in the topography.
Figure 2.10: Directional semivariograms interpolated from the semivariogram map of the 400*400 window of Synthetic topography model B. Experimental semivariograms are modeled using the semivariogram modeling algorithm.

(a) Semivariogram along N90E showing periodicity and an underlying trend. Amplitude of periodic component increases with distance. The semivariogram is fit with a nested model. a = 73.3 km, c= 1.05, A = 0.5833, c= 0.0339 and λ = 68 km.

(b) Semivariogram along N45E showing trend modeled using the spherical model. a = 36.1655, C = 0.96
Figure 2.10 (Continued)

c) Semivariogram along N15E fit by a spherical model. $a = 67.7$ km, $C = 0.743$.

d) Semivariogram along N45W showing periodicity and trend. The spherical model estimates a short-term range from the semivariogram. $a = 32.9$ km, $C = 0.9775$
Figure 2.11: Directional semivariograms interpolated from the semivariogram map of the 200*200 window of the synthetic topography model B. Experimental semivariograms are modeled using the semivariogram modeling algorithm.

a) Semivariogram along N90E showing periodicity and a underlying trend. Amplitude of periodic component increases with distance. Semivariogram is modeled using the spherical model. \( a = 17.0 \) km, \( C = 0.93 \)

b) Semivariogram along N45E showing periodicity modeled using the nested model. The model parameters are \( a = 38.1 \) km, \( C = 0.83 \), \( A = 0.4207 \), \( c = 0.0193 \) and \( \lambda = 93 \) km
Figure 2.11 (Continued)

c) Semivariogram along N15E modeled using the spherical model. $a = 79.7$ km, $C = 1.07$

d) Semivariogram along N45W modeled using the nested model. $a = 32.7$ km, $C = 0.98$, $A = 0.3725$, $c = 0.0061$, $\lambda = 82$ km
Figure 2.12a: Rose diagram of the ranges for the 400*400 window. The range values parallel to the N-S orientation of the topography are undefined. Distribution of range values indicate anisotropy with maximum spatial continuity along N-S and minimum spatial continuity along E-W. Most range values have been modeled with the spherical model for the 400*400 window because of the trend in the directional semivariogram. Range along N90E = 23 km.

Figure 2.12b: Rose diagram of ranges modeled for the 200*200 window. Ranges show anisotropy similar to (a). Some range values are modeled using the nested model and some with the spherical model. Range along N90E = 20 km.
Figure 2.13: (Top) Wavelength vs. direction for 200*200 window from synthetic topography model B. Note that due to the presence of trend not all the directions with periodicity have been mapped using the nested model. Hence the number of directional semivariograms with an estimated wavelength are few. (Bottom) Variation of range with direction. Minimum range is approximately 20 km along the E-W direction.
Figure 2.14a: Model profile used to construct synthetic topography model C. Profile is along the line shown in Figure 2.7. Profile has been detrended using the frequency filtering method. Long wavelengths (> 60 km) have been removed using the filtering process. Profile is extended for 1000 km in the Y-direction in order to create the 1000*1000 synthetic topography model C.
Figure 2.14b: Synthetic model topography C created from Basin and Range profile-filtered. Model extends for 1000 km in X and Y directions with all features striking N-S. Semivariogram maps are calculated for 400*400 and 200*200 windows centered at (500,500) on the topographic surface.
Figure 2.15a-d: Directional semivariograms from semivariogram map of the 400*400 window of synthetic topography model C. Model parameters are calculated using the semivariogram modeling algorithm.

a) Semivariogram along N90E showing periodicity fit by the nested model. $a = 22.1$ km, $C = 1.02$, $A = 0.5132$, $c = 0.0182$ and $\lambda = 64$ km

b) Semivariogram along N45E fit by a nested model. $a = 32.7$ km, $C = 1.01$, $A = 0.5742$, $c = 0.0158$ and $\lambda = 94$ km
Figure 2.15 (Continued):
c) Semivariogram along N15E fit by the spherical model. 
   \( a = 57.8 \text{ km, } C = 1.13 \)
d) Semivariogram along N45W fit by a nested model. 
   \( a = 24.3 \text{ km, } C = 1.09, A = 0.303, c = 0.0123, \)
   \( \lambda = 82 \text{ km} \)
Figure 2.16a-d: Directional semivariograms from the semivariogram map of the 200*200 window of synthetic topography model C. Model parameters are calculated using the semivariogram modeling algorithm.

a) Semivariogram along N90E showing periodicity fit by the nested model. $a = 26.2 \text{ km}, C = 0.96, A = 0.6094, c = 0.0246$ and $\lambda = 66 \text{ km}$

b) Semivariogram along N45E fit by a nested model. $a = 38.5 \text{ km}, C = 0.98, A = 0.6239, c = 0.0177$ and $\lambda = 96 \text{ km}$
Figure 2.16 (Continued):
c) Semivariogram along N15E fit by the spherical model. $a = 79.4$ km, $C = 1.37$
d) Semivariogram along N45W fit by the nested model. $a = 33.6$ km, $C = 1.1$, $A = 0.6830$, $c = 0.0191$, $\lambda = 86$ km.
Figure 2.17a: Rose diagram of the ranges for the 400*400 window of synthetic topography model C. The range values of semivaroags parallel to the N-S orientation of the topography are undefined. Distribution of range values indicate anisotropy in spatial continuity with maximum spatial continuity along N-S and minimum spatial continuity along E-W. Range along N90E = 22 km.

Figure 2.17b: Rose diagram of ranges for the 200*200 window. Ranges show anisotropy in spatial continuity similar to (a). Range value modeled along N90E = 26 km.
Figure 2.18a: (Top) Wavelength vs. direction for 400*400 window from synthetic topography model B. Minimum wavelength is 60 km along N90E. (Bottom) Variation of range with direction. Minimum range is approximately 20 km along the E-W direction.
Figure 2.18b: (Top) Wavelength vs. direction for 200*200 window from synthetic topography model B. Minimum wavelength is 60 km along N90E. (Bottom) Variation of range with direction. Minimum range is approximately 20 km along the E-W direction.
Figure 2.19: (Top) E-W elevation profile across the Western US. Topography was filtered using a high-pass frequency filter. Red line shows the long-wavelength trend that was removed by the filter. (Bottom) Elevation profile after the application of the filter. Semivariogram maps were calculated from residual elevations.
Figure 2.20: (Top) E-W topography profile from the Antarctic grid 052c. 11*11 unweighted averaging filter was used to remove the long wavelength trend of the sloping sea-floor. Red line represents the trend that was removed. (Bottom) Profile after the filter was applied. Distance is given in grid units. Semivariogram maps were made using the residual depth values.
Figure 2.21a: Decreasing window sizes on the Western US topography. Only 8 windows are shown 400*400...50*50. Colors represent elevation in meters. 1 grid unit in X and Y directions = 1km.
Figure 2.21b: The location of the 400*400 window on the larger grid.
Figure 2.22 a-g Semivariogram maps for the different window sizes – 400*400, 350, 300, 200, 150, 100 and 50*50. The semivariance values have been uniformly color-scaled for all windows in order to compare the maps. Only lags from -50 to +50 in the X and Y direction have been plotted.
Figure 2.23: a-g: Anisotropy models of range for different window sizes.  

a) Ranges obtained for the 400*400 window modeled by fitting an ellipse. Axis of maximum continuity ‘a’ = 31.7 km, axis of minimum continuity ‘b’ = 16.6 km, orientation of maximum continuity = $N1.5E$, aspect ratio = 1.9

b) Ranges obtained for the 350*350 window modeled by fitting an ellipse. Axis of maximum continuity 'a' = 30.4 km, axis of minimum continuity 'b' = 15.8 km and orientation of maximum continuity = $N4E$, aspect ratio = 1.9
c) Ranges obtained for the 300*300 window modeled by fitting an ellipse. Axis of maximum continuity = 30.3 km, axis of minimum continuity = 15.2 km, orientation of maximum continuity = N7E, aspect ratio = 2.0

d) Ranges obtained for the 250*250 window modeled by fitting an ellipse. Axis of maximum continuity = 29.8 km, axis of minimum continuity = 14.8 km and orientation of maximum continuity = N9E, aspect ratio = 2.0
e) Ranges obtained for the 200*200 window modeled by fitting an ellipse. Axis of maximum continuity = 31.0 km, axis of minimum continuity = 14.0 km, orientation of maximum continuity = N9E, aspect ratio = 2.2.

f) Ranges obtained for the 150*150 window modeled by fitting an ellipse. Axis of maximum continuity = 32.0 km, axis of minimum continuity = 13.0 km, orientation of maximum continuity = N7E, aspect ratio = 2.5
g) Ranges obtained for the 100*100 window modeled by fitting an ellipse. Axis of maximum continuity = 29.4 km, axis of minimum continuity = 10.3 km, orientation of maximum continuity = N11E, aspect ratio = 3.0

h) Ranges obtained for the 50*50 window modeled by fitting an ellipse. Axis of maximum continuity = 22.8 km, axis of minimum continuity = 8.4 km and orientation of maximum continuity = N19E, aspect ratio= 2.9
Figure 2.24: Model ellipses for the different window sizes, 400*400, 350*350, 300*300, 250*250, 200*200, 150*150, 100*100 and 50*50. Note the difference in the orientation and shape of the 150*150, 100*100 and 50*50 ellipses. Other window sizes show similar model ellipses and it is difficult to differentiate one from the other.
Figure 2.25a: Directional semivariograms along N10E for different window sizes. Semivariograms rise slowly and plateau about a semivariance of 1 and 1.25. The semivariogram for the 50*50 window has not reached a plateau.
Figure 2.25b: Directional semivariograms along S85E for the different window sizes. Note the high semivariance over a short lag in the semivariogram for the 50*50 window. The 400*400 window averages semivariances over a larger area giving a smoother semivariogram.
Figure 2.26: Variation of model parameters of ellipse with window size. (Top) Orientation vs. window size. (Middle) Aspect ratio vs. window size. (Bottom) Magnitude of the range in the direction of minimum continuity (b) vs. window size.
Figure 2.27 a-d: Spatial continuity maps of the Western US using 100*100, 200*200, 300*300 and 400*400 windows. Each anisotropy model ellipse is plotted at the center of the window. Ellipses have been scaled in order to plot them over the topography map.
Figure 2.27a: Window size - 100*100
Figure 2.27b: Window size- 200*200
Figure 2.27c: Window size – 300*300
Figure 2.27d: Window size – 400*400
Figure 2.27e: Spatial continuity maps using 100*100 window for Western US topography. Arrows point in the direction of lineation. Arrow size is related to aspect ratio and reflects degree of lineation of topographic features. Larger the aspect ratio, larger the size of the arrow and greater is the degree of lineation.
Figure 2.27f: Contours of aspect ratio. Note the high aspect ratio (> 2) in the Basin and Range Province and low aspect ratios (< 2) in the Colorado Plateau and Snake River Plain Province.
Figure 2.27g: Size of circles represent roughness. Larger circles indicate rougher topography.
Figure 2.27h: Contours of smoothness. Large contour values indicate smoother topography.
Figure 2.28: Window A and B from Antarctic grid 049c. Window A has topographic features oriented in a random manner. Window B shows N-S striking linear features. The grid spacing is uniform in the X and Y directions. 1 grid unit = 35 m.
Figure 2.29a: Semivariogram map of window A from the Antarctic grid 049c. Semivariances were calculated at lags -25 to +25 in increments of 1 grid unit (1 Grid unit = 35 m). Semivariogram map shows circular contours indicating isotropy in spatial continuity.
Figure 2.29b: Semivariogram map of window B from the Antarctic grid 049c. Semivariances are calculated from -35 to 35 grid units. 1 grid unit = 35 m. Semivariogram map shows a high spatial continuity along approximately N-S. Periodicity is mapped along the E-W direction.
Figure 2.30a-d: Directional semivariograms for window A have been interpolated every 0.25 grid units (9 m) from 0 to maximum lag of 25 grid units. Experimental semivariograms are modeled using the semivariogram modeling algorithm.

a) Directional Semivariogram along N90E fit with the spherical model.
a = 5.1 grid units (178 m), C = 1.03

b) Directional Semivariogram along N45E fit with the spherical model.
a = 3.75 grid units (133 m), C = 0.96
Figure 2.30 (Continued.)
c) Directional Semivariogram along N15E fit with the spherical model. 
   a = 3.8 grid units (135 m), C = 0.87 
d) Directional Semivariogram along N45E fit with the spherical model. 
   a = 4.9 grid units (172 m), C = 1.06
Figure 2.31 a-d: Directional semivariograms for the window B have been interpolated every 0.25 grid unit. Experimental semivariograms are modeled using the semivariogram modeling algorithm.

a) Semivariogram along N90E fit by the nested model. $a = 7.6$ grid units (266 m), $C= 0.88$, $A = 1.42$, $c= 0.3067$ and $\lambda = 22.9$ grid units (809 m).

b) Semivariogram along N45E fit by the nested model. $a = 12.5$ grid units (437 m), $C=0.98$, $A = 0.9123$, $c= 0.1058$, $\lambda= 30.1$ grid units (1054 m)
Figure 2.31 (Continued.)
c) Semivariogram along N15E fit by a purely spherical model. Range = 31.2 grid units (1092 m), C = 1.39
d) Semivariogram along N90W fit by a periodic model. a = 7.63 grid units (267 m), C = 0.99, A = 0.9962, c = 0.2576 and λ = 19.8 grid units (693 m)
Figure 2.32a: Anisotropy in ranges modeled by fitting an ellipse to the range values of window A. Negative sign indicates the direction of lag. The axis of maximum and minimum continuity = 175 m and 157 m respectively. Aspect ratio = 1.1, the model ellipse is close to a circle indicating isotropy. The direction of maximum and minimum continuity is not significant.
Figure 2.32b: Figure 2.28a Anisotropy modeled by fitting an ellipse to the range values of window A. Negative sign indicates the direction of lag. The semi-major axis (axis of maximum continuity) = 26 grid units (910 m), the semi-minor axis (axis of minimum continuity) = 6 grid units (210 m). Orientation of maximum continuity = N3E. Aspect ratio = 4.3
Figure 2.33a: (Top) Variation of wavelength with direction for the window A. (Bottom) Variation of range with direction. Note the almost constant value of range along each directional semivariogram.
Figure 2.33b: (Top) Variation of wavelength with direction for window B. Minimum wavelength is approximately 700m along direction 180 (N90W). (Bottom) Variation of range with direction.
Figure 2.34 a-c: Spatial continuity maps of the Antarctic grids 049c, 048c and 052c using 50*50 windows. Each anisotropy model ellipse is plotted at the center of the respective window. Ellipses have been scaled in order to plot them over the topography map.

(Next 3 figures)
2.34a : 049c
2.34a-1 : 049c
2.34b : 048c (divided into 3 parts in order to see ellipses on topography better)
2.34c : 052c (divided in to 2 parts in order to see the ellipses on topography better)
Figure 2.34a-1: Spatial continuity maps using 50*50 window for Grid 049c topography. Arrows point in the direction of lineation. Arrow size is related to aspect ratio and reflects degree of lineation of topographic features. Larger the aspect ratio, larger the size of the arrow and greater is the degree of lineation.
DISCUSSION AND CONCLUSIONS

I have developed an automatic geostatistical method for characterizing the spatial statistics of topography. I use the directional semivariogram as a basic geostatistical structure and model the spatial anisotropy in topography. I divided the topography in the study area into a number of windows and calculated the semivariogram map of each window by moving the windows over the larger grid and calculating the semivariance at all possible lags. I interpolated directional semivariograms from the semivariogram map and modeled them using spherical and nested models in order to estimate the range, sill and periodicity along a given orientation. I modeled the anisotropy in spatial continuity by fitting an ellipse to the range values.

Results are presented in the form of anisotropy model ellipses for each window on the topography grid. The spatial characteristics of topography in the window of observation are defined by the orientation of the semi-major axis (direction of maximum continuity corresponding to average strike of linear features), the length of the semi-major axis (range in the direction of maximum continuity) and the length of the semi-minor axis (range in the direction of minimum continuity) of the ellipse. The aspect ratio of the ellipse determines the shape of the ellipse and the degree of lineation of the features within the window. The area of the ellipse
estimates the roughness of topography. A small ellipse indicates a relatively rough topography; relatively large ellipses indicate smooth topography.

**Application to Data**

I applied the geostatistical topography characterization method I developed to model the spatial continuity of two datasets having different resolution, scale and type of topographic features – a relatively low resolution Western US topography covering 3 million square kilometers and a very high resolution sea-floor topography from the Ross Sea, Antarctica covering about one thousand square kilometers. I made a comparison of the model parameters with the topography and I am able to associate specific topographic patterns with specific anisotropy models.

The spatial continuity in the Western US topography was best modeled using a 100*100 window size (Figure 2.27a). Most model ellipses gave an orientation that was consistent with the dominant orientation of the features in the windows. For the Basin and Range Province, which is characterized by lineated and dissected topography, model ellipses showed an elongated shape with the orientation of the long axis of the ellipse oriented approximately N0-10 E. This is the approximate orientation of the ranges in the Basin and Range Province. The ellipses yield high aspect ratios (>2) indicating a high degree of lineation in the region. For regions showing lineations but with a smoother topography, model ellipses were elongate and large with a relatively low aspect ratio (<2). This was observed for the topography in the Colorado Plateau and the Columbia Plateau regions. Rough topography had smaller decorrelation distances than smooth topography and this controlled the area of the anisotropy model ellipse. Model ellipses for areas with no
prominent lineations were close to circular with an aspect ratio of approximately 1. The area of the circle was relatively large or small depending on the roughness of topography. For the highly dissected region, north of the Snake River Plain Province there is a combination of either high degree of lineation or no obvious dominant lineation, coupled with very rough topography. The size of the ellipses are relatively much smaller but with a relatively high aspect ratio in the lineated areas and lower aspect ratios (close to 1) in regions with no obvious dominant orientation. The spatial continuity of the sea-floor topography from Ross Sea, Antarctica was modeled using 50*50 window. The semivariogram analysis did a robust job in calculating the spatial continuity from available depth values in the window. I am able to separate regions with lineated features from the sea-floor topography based on the aspect ratio of the ellipses. I identified lineated areas with aspect ratios as high as 4 from the grids 049c (Figure 2.34a) and 052c (Figure 2.34c). The model ellipses pick up the average orientation of the linear features.

Window Size of Analysis

An important question that I address as part of achieving the objective of automatic geostatistical modeling is, given a dataset of known resolution, what is the optimum window size for the semivariogram analysis? When we use a small window size we can obtain a higher resolution with a shorter computing time. A larger window size would give a smoother semivariogram by averaging the noise out of the data. Hence, for better quality of modeling and mapping periodicity, a larger window size is desirable. There is no single answer to the optimum window size to use because the choice is a trade-off between obtaining a high resolution and obtaining a
consistent estimate of the semivariance. The window size needs to be large enough to encompass the major features of interest and small enough to maintain stationarity within the window. This is pretty difficult to achieve in regions with changing topography. Choosing the window size has largely been a trial and error process in this study. For the Western US topography, I started off with the largest window size that I can use for the analysis and calculated the model parameters using successively decreasing window sizes. This study indicated that there is a cutoff below which spatial continuity is not modeled correctly because of averaging over insufficient data points and calculating semivariances over shorter lags. There is a cutoff below which periodicity is not mapped clearly because the window dimensions for the semivariogram analysis should be at least twice the average wavelength of the features within the window in order for the directional semivariogram to model periodicity. The two cut off window sizes may be the same or different. For the Basin and Range Province, I observed that the cutoff for mapping spatial continuity is 100*100 while the cutoff for mapping the periodicity is 200*200. For the Antarctic grids the region of analysis covers small areas of the sea-floor. I arrived at an appropriate window size by studying the sea-floor profiles and obtaining an estimate of the average wavelength of the features. I picked a window size such that the window encompassed sufficient number of the lineated features. This approach gave a very good estimate of the spatial continuity of the topography. I used a window size of 50 grid units (1 grid unit = 35, 36 and 39 m respectively for the three grids) for calculating the semivariogram map of the Antarctic sea-floor topography.
Effect of Trend on Semivariogram Analysis

Semivariogram analysis is not robust in the presence of a long wavelength trend. When there is a trend in the data, the mean changes with position, making the assumption of stationarity invalid. Detrending the data to remove long-wavelengths that cause drift is necessary to ensure stationarity within the window of observation. I used two types of detrending methods, namely, frequency filtering and moving average for the topography grids in this study, depending on the presence or absence of data-gaps. The wavelengths to remove depend on the size of the window for analysis and the wavelength of the features of interest. For the Antarctic grids it is necessary to remove the trend caused by the slope of the sea-floor as most of the high frequency features (lineations on the sea-floor) are masked by long wavelengths of the gently sloping continental shelf (Figure 2.20).

In order to study the effect of trend on semivariogram analysis, I modeled the spatial continuity of semivariograms of 2 synthetic model topography surfaces constructed from unfiltered and filtered profiles from the Western US. The windows from the detrended topography gave much better directional semivariograms than the topography with trend. Estimates of the spherical and nested models, namely range, sill and periodicity were more consistent with the semivariogram structure for the directional semivariograms of the detrended topography.

Limitations of Semivariogram Modeling

The semivariogram map picks up prominent periodicity present in topography. The periodic component of the directional semivariogram is shortest and strongest in a directional semivariogram perpendicular to the linear features. Along
other directions, the directional semivariograms pick up an apparent periodicity with longer wavelengths. Along directions close to the average strike of the lineated features, the semivariogram shows a purely spherical component with almost no periodic component.

I first attempted modeling all the directional semivariograms with a simple spherical model. This did not work for directional semivariograms that show periodicity. In order to obtain the right estimate of the range, it is necessary to take into account the periodic component of the semivariogram. I then attempted using a more complicated model to explain the semivariogram structure. I fit a nested model to account for the spherical and the periodic component in the semivariogram. However, this model also has some problems fitting certain directional semivariograms. One important feature of the directional semivariogram showing periodicity is that the amplitude of the directional semivariogram decreases with lag. The nested model overestimated the wavelength of the periodic features in cases where there was a sharp drop in amplitude over a short lag between the first and second peak of the periodic component (Figure 3.31c).

Mulla (1988) modeled semivariograms using a nested model with a combination of spherical, linear and periodic components. His periodic component was defined by a wavelength and a cosine and sine amplitude in order to allow phase to vary. However, the model did not consider the fall in the amplitude of the semivariograms with lag, which is characteristic of topography showing less than perfect periodicity. Herzfeld and Higginson (1996) did not attempt to fit a model to the entire semivariogram structure. They removed noise from the semivariogram
using suitable filtering techniques and picked the lag of the first minima after the
first maxima from the smoothed residual semivariances in order to quantify the
spacing between abyssal hills. Thus they avoided the complications involved in
modeling the entire directional semivariogram. They obtained the difference between
the first maximum and minimum for the semivariance and the lag. They used the
ratio of the difference in semivariance to the difference in lag in order to quantify the
slope of the abyssal hills. They quantified the size of the abyssal hill by the
difference in the first maximum and minimum semivariances.

The decision-based semivariogram modeling algorithm I use does attempt to
recognize different semivariogram behavior and fit an appropriate model to every
directional semivariogram. However, it is difficult to envision one or two models
fitting all the complex variations of the directional semivariograms. The variation of
the model wavelength with direction has not been clearly interpreted in this study.
For a study focused on a small area, it is possible to apply semivariogram modeling
techniques by manually checking the model fit. This ensures better estimations of
model parameters by suitably incorporating changes in the model depending on the
semivariogram behavior. In this study, I deal with modeling thousands of directional
semivariograms and it was not possible to ensure a good model fit for each of the
semivariograms by automatic modeling.

Future Directions of Study

While this study has made it possible to correlate model ellipses with the
general morphological features, we still need to understand better the relationship
between semivariogram models and the erosional and structural processes that
generate topography. The next step would involve making a map of the model parameters for the entire study area. The semivariogram map of a large number of windows will be required to be modeled, with suitable overlap between the windows so that contouring the parameters would result in a smooth transition of model parameters through the entire study area. These maps can be compared to dynamic geological and geophysical parameters.

In the present study I use a constant window size to map the spatial continuity for the entire study area. Generally, topographic features over a small area that show a relatively constant wavelength can be modeled using a constant window size. However, using a constant window size over large areas like the Western US imposes specific limitations on the spatial continuity that is modeled. A 200*200 window size may be just right to model the linear ridge-valley-ridge topography in the Basin and Range Province. A relatively smaller 50*50 window dimension may be adequate or better to model the spatial continuity in the adjacent Snake River Plain region. In such cases, using a constant window size of 200*200 for the entire area would mean increasing the computation time and cost. Another drawback of using an overestimated window size for a region would be that the real spatial continuity gets obliterated due to averaging of semivariances and periodicities. While it is beyond the scope of this study, I can envisage a method of using spatially varying window sizes to map the spatial continuity. This would involve automatically identifying the optimum window size of analysis.

One of the important features of geostatistical analysis is that it can be used on irregularly spaced data. While I have mentioned this as one of my reasons for
choosing the method, I have not incorporated ungridded data for analysis in the present study. Trying the method on ungridded data would be a useful study.

The semivariogram picks up a strong periodic component perpendicular to the strike of the lineated features. This periodicity drops as we go away from the perpendicularity to this strike. We need to look at a better way to model the periodicity of the directional semivariogram in order to quantify the anisotropy in wavelength.

The significance of the spatial continuity mapped by the semivariogram depends on the scale of observation. We therefore need to analyze the Western US topography at different scales to explore resolution stability.

An important area of application of this method would be on the seismic data from the Iberia margin in order to characterize the sea-floor features of the region. Additionally, the release of the SRTM data will give a very high resolution, gridded, global topography coverage. The above analysis can be applied for useful studies such as comparison of anisotropy models of spatial continuity in regions with common tectonic, structural or sedimentary background.
REFERENCES


