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Modeling the Dynamics of Outer Radiation Belt Electrons

by

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in Partial Fulfillment of the
Requirements for the Degree

Master of Science

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Abstract

Modeling the Dynamics of Outer Radiation Belt Electrons

by

Stephen Naehr

A computer model has been built to simulate the dynamic evolution of relativistic electrons in the outer radiation belt. The model calculates changes in electron flux due to three mechanisms: (1) fully-adiabatic response of electrons to variations in the magnetic field, (2) time-dependent radial diffusion, parameterized by overall magnetospheric activity, and (3) penetration of new particles into the model via a time-dependent outer boundary condition. Data from Los Alamos geosynchronous satellites, the CRRESELE statistical electron flux model, the Kp index, and the Tofoletto-Hill-Ding magnetic field model are all used to provide realistic, time-dependent inputs to the model. To evaluate the model, a simulation of the radiation belts during the November 3-12, 1993 magnetic storm was generated. Comparison of results to Global Positioning System (GPS) radiation dosimeter data indicates that the model can accurately predict storm-time flux variations for electrons with energies less than 600 keV. Modeled fluxes for higher energy electrons show insufficient
enhancement during the recovery phase of the storm, suggesting the existence of an acceleration mechanism other than fully adiabatic variations and radial diffusion.
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Chapter 1

The Radiation Belts

1.1 Introduction

When James Van Allen discovered the radiation belts in 1958, the field of magnetospheric physics was born. The old picture of near-Earth space as a cold, empty void was eroded away by dozens of new observations which emerged in the following years. In its stead developed a far more dynamic scenario: a maelstrom of particles from the Earth, Sun, and beyond, tracing out elaborate trajectories under the influence of the Earth's magnetic field. It was soon realized that the radiation belts—composed of energetic protons and electrons magnetically trapped in near-Earth space—are a small subset of this complex system. By the mid-seventies most researchers had branched out to newer areas of magnetospheric physics, leaving the study of the radiation belts in a state of torpor. Only within the past decade have new developments led to a re-examination of the radiation belts. This renewed interest has been fueled by a growing body of observational data, rapidly increasing computing power, improvements in magnetic field modeling, and especially a growing concern over the adverse effects of radiation belts particles on spacecraft and astronauts.

The importance of radiation belt particles is greater than their numbers would suggest. The energetic electrons and protons which compose the belts are far less
numerous than the colder populations which occupy the same region of space, and their effect on the structure of the local magnetic field is negligible. However, radiation belts particles can attain such high energies that they can penetrate the outer layers of spacecraft. Relativistic electrons have been linked to the failure of several spacecraft in recent years. They can interfere with the functioning of exposed devices, such as photodetectors and solar cells, or cause greater harm by penetrating deep into the spacecraft and depositing charge directly onto internal electronic components. This deep dielectric charging can create anomalous signals leading to loss of communication with the spacecraft or other malfunctions. Relativistic particles can also pose a radiation hazard to astronauts, particularly during missions of extended duration or those involving extra-vehicular activity. Concern over these hazardous effects has inspired new efforts to accurately model the radiation belts under dynamic conditions, in the sub-field of space weather modeling.

Radiation belt models have traditionally been successful only in representing long-term, average conditions in the radiation belts. Such models provide adequate simulations of the innermost portion of the belts, where the particle distribution is relatively static over periods of weeks to months. However the outer radiation belt is highly dynamic, undergoing dramatic changes during periods of intense magnetic activity. During a magnetic storm, electron fluxes in the outer belts can vary by several orders of magnitude on time scales of days to minutes. This rapid fluctuation is caused by global changes in the magnetic field configuration. Unfortunately the early gener-
ations of radiation belt models assumed a time-independent model of the magnetic field, and so the dynamic storm-time variations were lost.

Another key limitation to early models was an incomplete description of wave-particle interactions in the radiation belts. These interactions can perturb the orbits of radiation belt particles in a random and irreversible manner. Identification and representation of these time-dependent, diffusive processes has proved a difficult challenge to radiation belt modeling efforts.

Recent modeling efforts have incorporated improved models of the time-dependent magnetospheric fields and/or particle interactions with these fields to develop more precise descriptions of dynamics in the radiation belts. Among the first in the new generation of radiation belt studies was that of Li. et. al. [1993], which successfully modeled the formation of a new electron belt during the massive geomagnetic storm of March, 1991. This study used test-particle simulations to show the response of the radiation belts to rapid, large-scale variations in the magnetospheric electric and magnetic fields. More recently, Kim and Chan [1997] studied fully-adiabatic variations in radiation belt electrons during a magnetic storm, using modern, time-dependent theoretical field models. Their treatment did not examine diffusive effects or variations caused by very rapid field changes, but represents a comprehensive examination of fully-adiabatic variations in a realistic field model. In another approach Beutier and Boscher [1995] and Bourdarie. et. al. [1995] developed the Salammbô code to explore the effects of three-dimensional, multi-modal diffusion on the radiation belts. The
Salammbô code uses fully time-dependent source, loss, and diffusion mechanisms to describe dynamics in a simplified (axisymmetric dipole) magnetic field model. Yet another approach was developed by Brautigam [1997], who examined electron flux variations during a magnetic storm with a model that incorporates both time-dependent magnetic fields (from the Tsygankov [1989] field model) and time-dependent radial diffusion terms. Brautigam also simulated an external source of electron flux through the use of a time-dependent outer boundary condition at a fixed boundary.

The model presented in this thesis is a step toward a fully dynamic radiation belt model. The algorithm is based on a simple physical model of the radiation belts, but makes use of real observations and a realistic, time-dependent magnetospheric field model to accurately simulate magnetically active periods. Time-dependent radial diffusion and outer boundary conditions are used to evolve the particle distribution, while pitch-angle diffusion and other mechanisms are neglected for the sake of simplicity and computational efficiency. The chief advance made in this work lies in the sophistication of the magnetic field model used, which allows for independent adjustment of multiple parameters affecting the magnetospheric field. The algorithm developed here is ultimately intended for use with a three-dimensional magnetohydrodynamic (MHD) code which can accurately simulate the dynamic magnetosphere.

The rest of this chapter will provide a short introduction to the physical structure of the radiation belts, defining the terminology and outlining the fundamental principles underlying the model.
1.2 The Magnetosphere

![Diagram of the Earth's magnetosphere]

Figure 1.1 Schematic representation of Earth's magnetosphere indicating the locations and shapes of the various features (from Wall [1994]).

In order to understand the structure and dynamics of the radiation belts, a brief survey of their magnetic environment is necessary. Figure 1.1 is a cartoon representation of the Earth's magnetosphere—the region of space dominated by the Earth's magnetic field. At the far left of the diagram, a supersonic flow of charged particles known as the solar wind radiates away from the Sun and toward the Earth's magnetic field. These particles carry frozen-in magnetic field lines of solar origin, which form the interplanetary magnetic field (IMF). The IMF becomes distorted when the super-sonic
solar wind flow encounters the Earth's magnetic field. Ahead of this obstacle a bow shock forms to convert some of the supersonic flow into randomized thermal motion, creating a region of subsonic solar wind plasma—known as the magnetosheath—which envelops the Earth's magnetosphere. The region of magnetosheath oriented field lines is separated from field lines of magnetospheric orientation by a thin boundary known as the magnetopause.

Within the magnetosphere, the magnetic field is determined principally by a dipolar field—created by currents deep within the Earth—which is distorted by several extraterrestrial current systems. Of primary interest to the present work are the ring current, tail current, and Chapman-Ferraro (or magnetopause) currents (shown in Figure 1.1). The ring current is formed by a band of westward traveling ions and eastward traveling electrons which circle the Earth, creating a perturbation magnetic field which is parallel to the dipole field near the magnetopause and anti-parallel at the Earth's equator. The ring current thus tends to inflate the inner-magnetospheric field, and its fluctuation during a magnetic storm have a major effect on the field line configuration. The tail current is a westward directed current which flows across the tailward region of the magnetosphere (downstream from the Earth, as the solar wind flows), effectively 'stretching' dipolar field lines away from the Earth in that region. The tail current forms a closed circuit loop by connecting with an eastward current which flows along the magnetopause, thus merging with the Chapman-Ferraro currents. The latter current system flows along the entire magnetopause, partially
shielding the interior magnetic field from the IMF. It is the interaction of these current systems and the magnetosphere's convection electric field with the solar wind and interplanetary magnetic field which drives the dynamic magnetosphere, and causes large-scale fluctuations in the radiation belts.

Several plasma populations fill the magnetosphere, distinguished from one another by the regions they inhabit, the average energy of their particles, and/or the species of particles present. Three are of particular relevance to this thesis: the ring current (discussed above), the plasmasphere, and the radiation belts. These populations occupy essentially the same region of space (within 8 Earth radii, or $R_E$, of the Earth) and are separated mainly by the energies of their particles. The plasmasphere is the densest and coldest of the three, with energies of the order of 1 electron volt (eV). There are considerably fewer ring current particles, whose energies lie in the tens to hundreds of kilo-electron volt (keV) range, but these contribute the highest energy density to the region. The radiation belts contain trapped particles of the highest kinetic energies, often up to several MeV for electrons, and GeV for protons. These populations are by no means sharply divided, as there is no universally accepted cutoff energy separating one from the other. In this thesis the term 'radiation belts' will refer specifically to trapped electrons with kinetic energies higher than 100 keV, and generally to those high-energy protons which share the region.

1.3 Structure of the Radiation Belts
Figure 1.2 Illustration of the Earth’s radiation belts. Reproduced from [Meudt 1996].

Figure 1.2 shows a typical picture of the radiation belts, in which higher particle fluxes are represented by darker shading. The belts are customarily represented as two concentric toroidal regions circling the Earth—roughly following the shape of dipole field lines—separated into an inner and outer zone. The inner zone extends from the upper reaches of the atmosphere (at 200 - 1000 km) to \( L \approx 2.0 \), where the parameter \( L \) approximately denotes a magnetic field line’s geocentric distance in \( R_E \) in the magnetic equatorial plane (the precise definition of the \( L \)-parameter, created by McIlwain, is more complicated, and will be discussed below). The outer zone begins at \( L \approx 2.5 \) and extends outward to a variable boundary of stable trapping, beyond which particles escape the magnetosphere before completing a drift cycle around the Earth. The distinction between inner and outer zones originates from early satellite
observations of a 'slot region', or local minimum in counting rates, at \( L \approx 2.0 - 2.5. \) The slot, where number densities of relativistic electrons are typically lower than in neighboring regions by several orders of magnitude, is created by wave-particle interactions which scatter relativistic electrons out of the radiation belts and into the Earth's atmosphere. These wave-particle interactions are the main relativistic electron precipitation mechanism throughout most of the belts. They work most efficiently in the slot region. The location of the region is poorly defined, however, because the local minimum in counting rates depends strongly upon the species and energy range of the particles detected [Brautigam 1997].

Figures 1.3 and 1.4 depict omnidirectional, integral fluxes of energetic protons and electrons averaged over time and longitude, determined from a large number of measurements compiled by NASA. The plots are shown in \( r-\lambda \) coordinates, where \( r \) is the distance from the Earth's center in \( R_E \) and \( \lambda \) is the latitude from the magnetic equatorial plane. Maxima in the contours appear in the equatorial plane, at a geocentric distance which decreases with increasing particle energy. Fluxes tend to die off gradually at high latitude, and more rapidly very near the Earth's surface, where interactions with the atmosphere remove the particles from the belts. These averaged plots are reasonable representations of the inner zone, which exhibits little time variation except in the most extreme conditions. However the outer zone fluxes at a given location may—as noted before—vary wildly during active periods such as magnetic storms and substorms.
Figure 1.3 Omnidirectional, integral proton flux ($\#$/cm$^2$/s), for protons with energy $\geq 10$ MeV. Adapted from [Walt 1994], based on National Space Science Data Center Data.

1.4 Magnetic Storms

Magnetic storms are events which rapidly restructure the magnetosphere on a global scale. They are the primary cause of large fluctuations in the outer radiation belts, and are believed to provide a source of accelerated particles to continually replenish the belts [Roederer 1970]. A magnetic storm is defined by a major increase in the strength of the ring current. This is often preceded by a marked increase in the dynamic ram pressure of the solar wind, causing the Chapman-Ferraro currents to
Figure 1.4 Omnidirectional, integral electron flux (#/cm²/s) for electrons with energy ≥ 1MeV. Adapted from [Walt 1994]. based on National Space Science Data Center Data.

rapidly intensify in an event known as Storm Sudden Commencement (SSC). The ring current strength then increases over a several hour period in the Main Phase of the storm. Eventually the ring current strength levels off and begins a slow (several day) return to pre-storm levels in the Recovery Phase.

The most common indicator of the occurrence and magnitude of a magnetic storm is the $Dst$ (for Storm-Time Disturbance) index (Figure 1.5). $Dst$ is ideally a measure of the North-South perturbation of the magnetic field at the Earth's equator caused by the changing strength of the ring current, but in practice it reflects changes in the
Figure 1.5  Dst index for an idealized magnetic storm.

Chapman-Ferraro and tail currents as well. It is determined by a complex assimilation of ground-magnetometer data from eight low-latitude observing stations, averaged over each hour and normalized to the quietest (least perturbed) days of a given month. Plots of Dst as a function of time during a magnetic storm show a positive (Northward) perturbation at the SSC, if one occurs, followed by a larger negative perturbation—often reaching -100 nanoteslas (nT) or more—during the Main Phase.

A more global indicator of geomagnetic activity is the Kp index. Kp is a three-hour index of activity determined from the compiled observations of several mid-latitude ground magnetometer stations. The data is normalized to a quasi-logarithmic scale ranging from 0 to 9, providing a sort of Richter-scale for geomagnetic activity. Several other indices in common use are derived from Kp. While Kp and its related
indices have no simple theoretical connection to any particular physical process, they provide a good indication of general magnetospheric activity and have been empirically correlated to many phenomena.
Chapter 2

Radiation Belt Theory

2.1 Introduction

The radiation belt model presented in this thesis is based on two components of basic theory. The first is the theory of adiabatic invariants, which allows the representation of complex particle motions by a set of approximately constant quantities. The second component, diffusion theory, provides a means of incorporating the interactions of trapped particles with other particles and random field fluctuations in a numerical simulation. This chapter will survey the key elements of both approaches, with emphasis on results of direct relevance to computer models of the radiation belts. Complete derivations of the adiabatic invariants and the diffusion equations will be omitted for the sake of brevity, but a thorough discussion of them can be found in the texts of [Roederer 1970] and [Walt 1994].

2.2 Adiabatic Invariants

The motion of a single charged particle in an electromagnetic field is given by the Newton-Lorentz equation:

$$\frac{dp}{dt} = q(E + v \times B)$$  \hspace{1cm} (2.1)
Figure 2.1 Trajectory of trapped electrons and protons experiencing magnetic mirroring and gradient and curvature drifts in the geomagnetic field (reproduced from [Wall 1991]).

Here $p$ represents the particle’s relativistic momentum, $q$ its charge, $v$ its velocity, and $E$ and $B$ are the electric and magnetic field vectors, respectively. In the real magnetospheric field it is often not feasible to solve for a particle’s trajectory by direct integration of the Newton-Lorentz equation. Fortunately a simplified description of the motion is possible due to three approximate periodicities which appear in most cases of relevance to the radiation belts. These periodicities, the cyclotron, bounce, and drift motions, are depicted in Figure 2.1.

The fastest periodicity is known as cyclotron motion, which describes the spiral trajectory of a particle along a line of magnetic flux. The component of motion perpendicular to the magnetic field vector is periodic and nearly circular, with frequency
\( \omega_c \) and radius \( \rho_L \) (called the Larmor radius) given respectively by:

\[
\omega = \frac{qB}{\gamma m} = \frac{qBe^2}{\mathcal{E}} \tag{2.2}
\]

\[
\rho_L = \frac{\gamma m v_\perp}{qB} = \frac{p_\perp}{qB} \tag{2.3}
\]

where \( v_\perp \) and \( p_\perp \) are the particle's velocity and momentum perpendicular to the direction of \( \mathbf{B} \). The angle between \( \mathbf{v} \) and \( \mathbf{B} \) is called the pitch angle \( \alpha \), defined by

\[
\alpha = \arcsin \frac{v_\perp}{v} = \arcsin \frac{p_\perp}{p} \tag{2.1}
\]

The relatively fast cyclotron motion is separated from other components of the motion through the guiding center approximation, which follows the center of the particle's (nearly) circular motion rather than the particle itself.

The second type of periodic motion is the 'bounce' (or 'mirror') motion of a particle, parallel to the magnetic field line on which it spirals. On either end of the particle's path, a gradient in the magnetic field drives the particle back toward the weak field region in the center, causing the particle to oscillate along the field line, between two mirror points. The period of bounce motion for a particle of velocity \( v \) is of the order

\[
\tau_b \sim S/v \tag{2.5}
\]

where \( S \) is the arc-length of the field line. This period is always much greater than the period of cyclotron motion \( \tau_c \).

The third type of periodic motion is gradient-curvature drift, which causes a bouncing, gyrating particle to slowly (with a period on the order of tens of min-
utes, for 1 MeV electrons] drift around the Earth in a quasi-circular path (Figure 2.1). As its name implies, gradient-curvature drift is caused by an Earthward gradient and curvature of magnetic field lines. The guiding center of a particle undergoing gradient-curvature drift moves with a velocity given by:

\[ V_G = \frac{p_i^2}{2 \gamma m q} \frac{\mathbf{B} \times \nabla B}{B^3} \]  

\[ V_C = -\frac{p_i^2}{\gamma m q} \frac{\mathbf{B} \times \mathbf{R_C}}{B^2 R_C} \]  

where \( \mathbf{R_C} \) is the radius of curvature vector. The two drifts have the same direction and order of magnitude in a quasi-dipole field, and so are often considered in tandem. For all particles of interest here, the period of gradient-curvature drift \( \tau_G \) is much greater than the bounce period, such that

\[ \tau_G \ll \tau_B \ll \tau_i \]

This fact allows separation of the motion into three distinct regimes.

Each of the three quasi-periodic motions of a particle can be represented by an adiabatic invariant, which is approximately conserved under slow changes in the field configuration. Here the term ‘slow’ refers to processes which take place over many periods of the cyclic motion. The adiabatic invariants are derived from the classical action variables \( J_i \) of Hamilton-Jacobi theory:

\[ J_i = \oint p_i \, dq_i \]  

where the \( p_i \) and \( q_i \) are the canonical momentum and position variables of Hamilton’s equations, and the integral is performed over one cycle of the periodic motion. In the
case of a charged particle in an electromagnetic field, the action variables are defined by:

$$J_i = \int_{\gamma} \left[ p + q A \right] \cdot dl$$  \hspace{1cm} (2.9)

where $p$ is the particle's momentum vector, $A$ the magnetic vector potential, and $dl$ an element of distance along the cyclic path of the particle. By construction the action variables $\frac{d}{dt}$, together with their respective phase angles $\phi$, form a complete set of canonical variables in which the $\phi_i$ are cyclic (that is, do not appear in the Hamiltonian). The action variables $J_i$ are therefore constants of the motion [Goldstein 1950].

The first of these action variables, $J_1$, represents the cyclotron motion of a particle (of mass $m$ and momentum $p$) in a magnetic field $B$:

$$J_1 = \frac{\pi p^2_{\perp}}{qB}$$  \hspace{1cm} (2.10)

By convention the first adiabatic invariant $\mu$ is defined in relation to $J_1$ by

$$\mu \equiv \frac{p^2_{\perp}}{2mB} = \frac{q}{2\pi m} J_1$$  \hspace{1cm} (2.11)

In the non-relativistic limit the value of $\mu$ is equal to the magnetic moment of the current loop generated by the particle as it circles the field line.

If a particle conserves both energy and the first adiabatic invariant, a useful relationship can be drawn between the particle's pitch angle and the magnetic field magnitude at any point along its bounce path. Combining Equations (2.4) and (2.11), it
is evident that
\[
\frac{\sin^2 \alpha}{B} = \frac{p_{\perp}^2}{p^2 B} = \frac{2m \mu}{p^2} = \text{constant}
\] (2.12)

The second adiabatic invariant \(J\) is identical to the action variable \(J_2\), and is found by integrating the field-aligned momentum of a particle along its bounce path:
\[
J = \oint p_{\parallel} \, ds = 2p \int_{s_{m1}}^{s_{m2}} \sqrt{B_{\text{mir}} - B(s)} \, ds = 2p \, I
\] (2.13)

where \(B_{\text{mir}}\) is the magnetic field strength at the points \(s_{m1}, s_{m2}\), at which the guiding center of the particle reverses direction. The integral term \(I\) is a pure field-geometric quantity, which can be determined numerically in a model field without reference to the characteristics of the particle.

If a particle mirrors very near a minimum in the magnetic field along a given field line, the value of \(J\) approaches zero. In the geomagnetic field these minima are usually found in the magnetic equatorial plane, so particles with \(J \approx 0\) are often labeled 'equatorially mirroring'. This class of particles is simpler to manage in computer simulations, and is often representative of the full population of particles within a flux tube. For this reason the simulations performed for this thesis examine \(J = 0\) electrons exclusively.

The third adiabatic invariant \(\Phi\) is determined from the particle's gradient-curvature drift path around the Earth. It is equal to the magnetic flux enclosed by the drift path of the particle, and is related to the third action variable \(J_3\) by
\[
J_3 = \oint_{\text{drift}} p \, dq = q \iint \mathbf{B} \cdot d\mathbf{A} = q \Phi
\] (2.14)
where $d\mathbf{A}$ is the differential area within the drift path in a plane perpendicular to $\mathbf{B}$. In practice the calculation of $\Phi$ in a dipolar field requires integration of the flux *outside* the drift path toward infinity, to avoid the magnetic singularity at the center of the dipole. The divergence-free nature of magnetic fields ensures the equivalence of the two methods. If the drift shell can be mapped to the ionosphere, where the accurately-determined internal field of the Earth dominates, it is preferable to integrate poleward of the drift shell, thereby avoiding the problem of representing the magnetic field far from the Earth.

The adiabatic invariants, by virtue of being approximate constants of the motion for a given particle, form a useful coordinate system with which to specify the trapped particle population of the geomagnetic field. Consider a group of particles whose adiabatic invariants have the same value, but are at different phases in their cyclotron, bounce, and drift motions. Although these particles have different positions and momenta at a given time $t_0$, they lie along the same trajectory in position-momentum phase space. If the magnetic field configuration is time-independent and spatially smooth, then each particle will eventually pass through the same phase-space location originally occupied by each other particle at $t_0$. If the magnetic field configuration varies slowly (adiabatically), then the phase-space trajectory will change shape, but the particles will continue to share the same trajectory, and their adiabatic invariants will remain unchanged. The following sections will describe the application of the adiabatic invariants to the evolution of particle populations in a more quanti-
tative manner, and discuss the effects of violating the conservation of the adiabatic invariants.

2.3 Distribution Functions

In order to discuss the behavior of large number of particles, it is useful to introduce the phase space density $f(x, P)$. The phase space density is a distribution function, which defines the number of particles in a volume $d^3x$ about $x$ whose momenta lie in a range $d^3P$ about $P$. Here $P$ is the canonical momentum given by $P = p + qA$.

The total number of particles $N$ in the system is given by:

$$ N = \int f(x, P) d^3x d^3P $$  \hspace{1cm} (2.15)

The particle flux $j(E, \alpha)$, which gives the number of particles per unit energy, pitch angle, area, and solid angle at a given point, is related to the phase space density by:

$$ f(x, P) = \frac{j(E, \alpha)}{\rho^2} $$  \hspace{1cm} (2.16)

By Liouville's theorem, the phase space density $f(x, P)$ is a constant along a particle's dynamic path:

$$ \frac{D}{Dt} f(x, P) = 0 $$  \hspace{1cm} (2.17)

The path of a gyrating, bouncing, and drifting particle encompasses a very complicated domain in cartesian phase space, making it difficult to apply Liouville's theorem directly. However the action-angle variables provide an alternate description of phase
space, in which the distribution function \( \overline{F}(\phi, \mathbf{J}) \) is represented by:

\[
N = \int \overline{F}(\phi, \mathbf{J}) \, d^3\phi \, d^3J
\]

The translation from cartesian variables \((\mathbf{x}, \mathbf{P})\) to action-angle variables \(\phi, \frac{J_i}{2\pi}\) constitutes a canonical transformation. By the theorem of Poincaré, a volume in phase space is invariant under such transformations [Goldstein 1950], and thus

\[
\int d^4x \, d^4P = \left(\frac{1}{2\pi}\right)^3 \int d^3\phi \, d^3J
\]

Combining this result with Equations (2.15) and (2.18), it is evident that

\[
f(\mathbf{x}, \mathbf{P}) = \left(\frac{1}{2\pi}\right)^3 \overline{F}(\phi, \mathbf{J})
\]

If one assumes isotropy in each of the three phase angles, then a simplified distribution function which depends only on the action variables may be used:

\[
F(J_1, J_2, J_3) \equiv \int d^3\phi \, \overline{F}(\phi, \mathbf{J})
= (2\pi)^3 \overline{F}(\phi, \mathbf{J})
= f(\mathbf{x}, \mathbf{P})
\]

The distribution function \(F(J_1, J_2, J_3)\) is a function only of the action variables, which are time-independent in a slowly varying magnetic field configuration. Liouville’s theorem can thus be cast in a more manageable form:

\[
\frac{D}{Dt}F(J_1, J_2, J_3) = \frac{\partial}{\partial t}F(J_1, J_2, J_3) = 0
\]
In practice the distribution function is often written as a function of the adiabatic invariants \( \mu \), \( J \), and \( \Phi \), rather than the action variables \( J_i \). The transformed distribution function is given by:

\[
F(\mu, J, \Phi) = 2\pi m F(J_1, J_2, J_3)
\]

(2.23)

Several equivalent forms of the distribution function can be derived by similar transformations to a new coordinate system. A number of such coordinate systems have been used to represent the action variables in a more intuitive form. For example, Mellwain [1961] developed the parameter \( L \), which associates a particle with the adiabatic invariants \( \mu \) and \( J \) with the radial distance (in \( R_E \)) that its crossing point in the equatorial plane would have in a dipole magnetic field. In a similar manner Roederer [1970] developed the parameter \( L_R \) to relate the third invariant to radial distance. \( L_R \) is given by

\[
L_R = \frac{2\pi d_0}{\Phi R_E}
\]

(2.24)

where \( d_0 \) is the dipole moment of the Earth’s magnetic field. In a pure dipole field \( L_R \) is equal to the geocentric radial distance in the equatorial plane of a drift shell with third invariant \( \Phi \). The computer code written for this thesis performs its calculations on the distribution function \( F(\mu, J, L_R) \), related to \( F(\mu, J, \Phi) \) by:

\[
F(\mu, J, L_R) = \left( \frac{R_E}{2\pi d_0} \right) L_R^2 F(\mu, J, \Phi)
\]

(2.25)

The discussion to this point has assumed that the adiabatic invariants are exactly conserved. This is of course not true in the real geomagnetic field. Where a number
of forces influencing the radiation belts occur on spatial or temporal scales which are small compared to those of the cyclotron, bounce, or drift motions of the particles. For example collisions, geomagnetic micropulsations, and a host of plasma waves can all violate the conservation of one or more adiabatic invariants. When these processes are significant equation (2.22) is no longer valid, and a new description of the distribution function’s evolution must be found. Fortunately the effects of many of these non-adiabatic processes can be represented by a simple diffusion equation, as described in the next section.

2.4 Diffusion

Many of the processes which violate an adiabatic invariant have a time-dependent nature, with a Fourier component at the frequency of cyclic motion corresponding to that invariant. In this circumstance the effects on particles of a given adiabatic invariant can be organized with respect to the phase of the particles. If the particles are represented by a phase-independent distribution function, as in equation (2.21), then these processes may alter the adiabatic invariants in a random manner. This component of randomness allows many invariant-violating processes to be represented by a diffusion equation [Schulz and Lanzerotti 1974]. In terms of the action variables $J_i$, the diffusion equation is given by

$$\frac{\partial F(J_1, J_2, J_3)}{\partial t} = \sum_{ij} \frac{\partial}{\partial J_i} \left[ D_{ij} \frac{\partial}{\partial J_i} F(J_1, J_2, J_3) \right]$$

(2.26)

where $D_{ij} = \frac{\Delta J_i \Delta J_j}{2\Delta t}$ is a diffusion coefficient.
Many but not all violations of the adiabatic invariants can be represented by a diffusion equation. In particular, particle injections from various sources and collisions with ionospheric/atmospheric particles are non-diffusive in nature, as they represent a systematic gain/loss of energy from the radiation belts. In this thesis, particle sources are represented by a time-dependent outer boundary condition on the distribution function, and loss processes are neglected.

Diffusive processes in the radiation belts are customarily separated into pitch-angle diffusion, which violates the first and/or second adiabatic invariants, and radial diffusion, which violates the third invariant while conserving $\mu$ and $J$. The present work focuses on the latter type, and it is therefore assumed that $\mu$ and $J$ are conserved (with $J = 0$ for equatorially mirroring particles). Under these conditions Equation (2.26) simplifies to a one-dimensional diffusion equation in $J_A$, or equivalently the radial parameter $L_R$. For the distribution function $F(\mu, J, L_R)$, the radial diffusion equation has the form

$$
\frac{\partial F(\mu, J, L_R)}{\partial t} = L_R^2 \frac{\partial}{\partial L_R} \left[ \frac{D_{LL}}{L_R^2} \frac{\partial}{\partial L_R} F(\mu, J, L_R) \right]
$$

(2.27)

where $D_{LL} = \frac{(\Delta L)^2}{2\Delta t}$ is the radial diffusion coefficient. $D_{LL}$ represents generically the combined effects of electric and magnetic field variations which violate the third adiabatic invariant, and thus depends on the power spectra of fluctuations in these fields. Numerous empirical and theoretical studies have been dedicated to determining the value of $D_{LL}$, and correlating it with general indices of magnetic activity [Brautigam 1997]. In the present work the value of $D_{LL}$ is taken from the empirical investigation.
of Lanzorotti, et. al. [1978], who used geosynchronous measurements to derive the diffusion coefficient for particles with $\mu = 115\text{MeV/G}$ and $\mu = 500\text{MeV/G}$. The data was plotted against $SKp$, the half-day sum of $Kp$, and a least-squares fit was computed. The resulting $D_{LL}$ is given by:

$$D_{LL}L^{-10} = 10^{2.07(SKp)-9.6}$$ (2.28)

This value is used in the model to provide an indication of the average degree of diffusion for particles of all energies/values of $\mu$. It should be noted that a more realistic diffusion coefficient would depend on the spectral density of field fluctuations at the particle drift frequency, and would therefore depend on the energy of the particles. However it is difficult to obtain the time-dependent power spectra of the electric and magnetic fields, and it has been shown experimentally [Lanzorotti and Morgan 1973] that the energy dependence of the diffusion coefficient is often very weak.
Chapter 3

Model Description

3.1 Introduction

The primary task of this thesis work was the creation of a computer program to simulate the response of the radiation belts to a magnetic storm. The program, titled RDIF (for Radial Diffusion), calculates storm-time flux variations by applying the radial diffusion equation presented above to a realistic, time-dependent magnetospheric environment. This chapter will explain the computational procedure the model uses to achieve this end.

3.2 Coordinate System

RDIF calculates the omni-directional flux of relativistic electrons in six energy bins on a two-dimensional spatial grid. The energy ranges of each bin are (in MeV): 0.2-0.3, 0.3-0.4, 0.4-0.6, 0.6-0.9, 0.9-1.1, and 1.4-2.0. In the equatorial plane the spatial coordinates are linear functions of geocentric distance and local time angle. There are 100 grid points in the radial direction, spanning 1.1 to 10.0 R_E, and 48 grid points covering all local times. The model also keeps track of the positions of the ionospheric foot points of each field line which intersects a grid point in the equatorial plane. This coordinate system is used to manage particle flux and magnetic field information.
The model performs its central calculations by first transforming the input data into a coordinate space defined by the adiabatic invariants. This space is divided into one hundred logarithmically-spaced bins in $\mu$—from 1.0 to $10^4$ MeV/G—and one hundred linearly-spaced bins in $L_R$—from $L_R = 2.0$ to $L_R = 9.0$. The model is restricted to particles which mirror in the equatorial plane, so the second adiabatic invariant is zero for all particles.

Both coordinate spaces use the same discretization in time. In the simulation of the November 1993 magnetic storm, a 15-minute time step was used for the dynamic early portion of the event, from 1200 UT November 3 to 2400 UT November 4. During the unusually slow recovery phase, which lasted a full week, the model switched to a one hour time-step to conserve computer time. This time scale determined how often the magnetic fields and drift shells were computed for this event. The model's radial diffusion algorithm operates on a much shorter time scale, which adjusts itself automatically to ensure stability in its numerical calculations.

### 3.3 Initial Flux Model

RDIF begins with four key inputs which specify the environment in which calculations will be performed: the initial flux configuration, boundary conditions, a time-series of magnetic field models, and the time-dependent radial diffusion coefficient. The initial condition is supplied by an initial flux model which supplies the omni-directional
electron flux at each energy range throughout the modeling region. This distribution is derived from the CRRESELE statistical radiation belt model.

CRESSELE is a model of the outer radiation belts developed by Brautigam and Bell [1995], from observations made by the Combined Release and Radiation Effects Satellite (CRRES), which made measurements of electron fluxes, electric and magnetic fields, and plasma properties in the radiation belts from July 20, 1990 to October 10, 1991. The satellite's elliptical orbit, inclined 18° to the ecliptic plane, brought the satellite from an altitude of 350 km at perigee to nearly 6.3 $R_E$ at apogee, with an orbital period of roughly nine hours. This provided CRRES with data covering almost all of the radiation belts, and led to the compilation of a large body of statistical data over the fourteen-month lifespan of the satellite.

CRRESELE is a statistical model of electron fluxes derived from this data set. It is composed of a series of outer belt electron flux models, which specify flux at a given position for a certain range of $Ap_{15}$. $Ap_{15}$ is a fifteen-day average of the magnetic activity index $Ap$, which is a linearized version of the familiar $Kp$ index. Each of the $Ap_{15}$-specific models gives omni-directional electron fluxes for a given energy range, radial geocentric distance, and equatorial pitch angle. There are nine energy bins whose central energies range from 0.65 MeV to 5.75 MeV, and 81 bins of radial distance from 2.5 $R_E$ to 6.75 $R_E$. The model has no local time dependence, and so must be interpreted as an average of the fluxes over all local times.
To generate an initial flux distribution for RDIF, the CRRESELE model for a quiet-time level of Ap15 was first interpolated and extrapolated onto the RDIF coordinate grid. This was accomplished by fitting the CRRESELE fluxes at each radial grid point to a function of distance \( r \) and energy \( \mathcal{E} \) of the form

\[
j(\mathcal{E}, r) = c_1(r) \mathcal{E}^{-2(r)}
\]  

(3.1)

This series of functions was then evaluated directly at the mid-range values of each energy bin in RDIF, at each value of \( r \) on the grid.

### 3.4 Boundary Conditions

The RDIF coordinate grid covers geocentric distances from \( 1.1R_E \) to \( 10.0R_E \) in the equatorial plane, but the region in which its flux calculations are valid covers a much smaller range. The objective of the model is to represent outer-belt electrons, so it does not simulate the loss processes of significance to the inner belt or slot region. Its results are therefore not valid earthward of \( r \approx 2.75R_E \). Particles are allowed to flow freely through this inner boundary and out of the system.

The outer boundary condition has a much more significant effect on the model, and as such it must be determined with care. The time-dependent flux at the outer boundary is taken from observations made by one of the geosynchronous satellites operated by Los Alamos National Laboratories (LANL), which maintain a constant geocentric distance of \( \approx 6.63R_E \) (geosynchronous orbit) in the geographic equatorial
plane. These fluxes are set everywhere along a drift shell determined in the following manner.

In the model the magnetic dipole and rotational axis of the Earth are aligned in order to simplify the calculations, so geosynchronous orbit is approximated by a circle which lies in the magnetic equatorial plane. Under most circumstances the outer boundary drift shell (for a particle with \( J = 0 \)) is defined to be a closed contour of equatorial magnetic field strength which passes through the instantaneous position of the satellite on this circle. When the magnetosphere is highly compressed, however, the drift shell passing through the satellite’s position may not close within the magnetosphere, or it may do so along a path which breaks conservation of the second adiabatic invariant. The latter can occur if the drift shell passes through a region where the minimum magnetic field strength \( \mathbf{B} \) along a field line is located above or below the equatorial plane. When a drifting particle encounters this region, it follows the minimum in \( \mathbf{B} \) out of the equatorial plane in a process which can break the second adiabatic invariant [Orlolf et. al. 1999], thereby invalidating the assumptions of the model. Therefore when the satellite’s path encounters a drift shell on which the minimum in \( \mathbf{B} \) leaves the equatorial plane, the outer boundary of the model is defined to be the last drift shell which does not exhibit this behavior. The flux \( j_{\text{bnd}} \) at this boundary is determined from the measured flux \( j_{\text{geo}} \) by

\[
j_{\text{bnd}}(\mu) = j_{\text{geo}} \cdot \frac{B_{\text{bnd}}}{B_{\text{geo}}} \tag{3.2}
\]
which assumes that the distribution function is constant between the position of the satellite and that of the boundary (at a given moment), and that particles move from one location to the other in a way that conserves $\mu$.

### 3.5 Magnetic Field Model

The most important factor in determining the configuration of the radiation belts within RDIF is the time series of magnetic field models input to the program. These field models were generated by a version of the Toffoletto-Hill-Ding (THD) magnetic field model—an analytic, theoretical model of the magnetosphere, driven by solar wind conditions, the IMF, and $Dst$ [Toffoletto and Hill 1993], [Ding 1994]. The THD model has a number of advantages over other models for the purpose of modeling the radiation belts. Statistical models, such as Tsyganenko [1989], depend only upon the overall magnetic activity at a given time, as determined from the $K_p$ index. The ring current determined by such models cannot be adjusted independent of global magnetospheric changes (such as variations in magnetopause location), and so cannot reproduce the close correlation of measured fluxes with $Dst$. THD has an adjustable ring current, which has been calibrated to the input values of $Dst$ for use in RDIF. Another limitation of many statistical field models (e.g., [Tsyganenko 1996]) is a small range of allowed input parameters. During a magnetic storm, solar wind and IMF values can far exceed the range of quiet-time, average data from which statistical models are derived. This can lead to inconsistent magnetic field models during the
most intense portions of a storm—which are the periods of greatest impact upon
the radiation belts. A theoretical model like THD reduces this problem, since its
theoretical basis ensures reasonable results for any range of inputs (at least within
the limitations of the theory).

At each time step through the event, THD is used to calculate two quantities at
each grid point: the magnitude of magnetic flux $B_{eq}$ in the equatorial plane, and the
ionospheric foot point location of the field line which intersects the grid point.

3.6 Drift Shell Mapping

RDIF greatly simplifies the task of identifying drift shells by limiting its calculations
to particles which mirror in the equatorial plane. In this case a drift shell, along
which $\mu$, $J$, and $\Phi$ are conserved, is simply a contour of the equatorial magnetic
field strength $B_{eq}$. This contour is easily found by interpolation of the grid-point $B_{eq}$
values at each local time. RDIF calculates one hundred such drift shells at each time
step, which are labeled by the value of $L_R$ on the shell.

To calculate $L_R$ for a given drift shell, the contour in the equatorial plane is
mapped to the ionosphere, where the magnetic field is assumed to be purely dipolar.
The third adiabatic invariant $\Phi$ is then calculated by integrating the dipole magnetic
flux $B_{dip}$ over the area poleward of the mapped drift shell:

$$\Phi = \iint B_{dip} \cdot dA$$

$$= \frac{d\phi}{R_E} \int \sin^2 \theta(\phi) \, d\phi$$  \hspace{1cm} (3.3)
RDIF iteratively performs this calculation on drift shells determined by contours of constant $B_{eq}$, varying $B_{eq}$ until it finds the value which corresponds to a grid point value of $L_R$. This value and the interpolated loci of points along the drift shell in the equatorial plane are calculated at each time step in the model, determining a new drift shell mapping for each change in the magnetic field.

### 3.7 Distribution Function and Radial Diffusion Calculations

When the model has determined the initial flux and configuration of drift shells, the initial distribution function in adiabatic invariant coordinates, $F(\mu, L_R, t = 0)$ can be established. The mean flux on each drift shell for each energy range is first determined by averaging the interpolated flux values along the drift shell over all local times. From this the distribution function is determined, by

$$F(\mu, L_R) = \frac{j}{L_R^2 \rho^2} = \frac{j}{2m \mu BL_R^2}$$

(3.4)

The algorithm uses a second-order interpolation/extrapolation routine to compute $F$ at all $\mu$ bins from the six energy bins of the flux array.

In the limit of fully adiabatic particle transport, the distribution function at given values of $\mu$ and $L_R$ remains constant everywhere except at the boundary, i.e.

$$\frac{\partial F(\mu, L_R, t)}{\partial t} = 0$$

The moving outer boundary condition allows changes at a certain drift shell to remain in the model if the boundary is expanding, but no changes can propagate
earthward of the smallest boundary shell by adiabatic effects alone. To describe more
global changes in the distribution function, the radial diffusion equation (Equation
2.27) is employed. This equation is integrated by a standard Crank-Nicholson numerical
method, which is second-order accurate in space and time [Press et. al. 1992].
The value of the diffusion coefficient \( D_{LL} \) is related to \( SKp \) by equation [2.28], which
stated:

\[
D_{LL} L^{-10} = 10^{0.07 (SKp) - 9.6} \tag{3.5}
\]

The value of \( SKp \) is updated every three hours in the model, making the diffusion
coefficient time-dependent. The maximum time step which preserves the stability of
the method is smaller than the time step of the model by a factor \( \approx 100 \), so RDIF
must perform several hundred iterations of the diffusion equation at each time step
before updating the magnetic field and boundary conditions.

3.8 Output

RDIF performs its calculations of the distribution function in adiabatic invariant
space, and converts the distribution function back into fluxes at each grid point for
output purposes only. This process is essentially a reversal of the steps above, call-
ing for a second series of interpolations over energies and spatial coordinates. Flux
changes at a single position which are the result of fully adiabatic transport are re-
vealed only in this last step, which assigns the values on a drift shell to its new position
in space. The outputs of the model are the time-series of flux distributions \( j(\mathcal{E}, \mathbf{x}, t) \).
and the time-dependent distribution function $F(\mu, L_R, t)$ in adiabatic invariant space.

This dual output allows analysis and comparison of the model results with both theoretical work and experimental observations. The next chapter presents an example of such a comparison, for a magnetospheric event which has been thoroughly examined and chronicled.
Chapter 4

November 1993 Magnetic Storm Simulation

4.1 Description of the Storm

In the period spanning November 2 to November 11 of 1993, a major magnetic storm propagated through the magnetosphere. The event was the subject of a coordinated effort within the solar, magnetospheric, and ionospheric physics communities to track the development of a storm from its solar origins to its effects on magnetospheric and ionospheric systems. Knipp, et. al. [1998] have provided a comprehensive summary of the storm’s progress, as witnessed by a host of space- and ground-based observing platforms. Figure 4.1 displays a compilation of activity indices through the storm period, which show that the event satisfies the criteria of a major geomagnetic storm. The event was the result of a high speed solar wind stream emanating from a large coronal hole in the Sun, in which a compact flux rope of particularly high density was embedded. The onset of the main phase of the storm occurred a few hours before 0 UT on November 4. Between 2300 and 2400 UT November 3, the high density flux rope encountered the magnetosphere, driving solar wind density measurements as high as 125 cm$^{-3}$ and momentarily pushing the dayside magnetopause inside of geosynchronous orbit. The $Dst$ index reached its highest magnitude midway through November 4, and recovered very gradually over an atypical 8-day period.
LANL geosynchronous satellite 1984-129 provided the time-dependent outer boundary condition for the model, courtesy of G.D. Reeves. A time series of magnetic field models simulating the storm was developed from the Tofololetto-Hill-Ding magnetic field model, using input from the $D_{st}$ index, and from solar wind and IMF data from the GEOTAIL and IMP-S satellites. The time-dependent radial diffusion coefficient used in RDIF was parameterized by the $K_p$ index. Finally the results of the model were compared to observed energetic electron count rates at various locations in the radiation belts, provided by the radiation dosimeter measurements of several Global Position System (GPS) satellites, courtesy of T. Cayton and G. Reeves at LANL. The input data used in the simulation is shown in Figures 4.2 and 4.3.
Figure 4.2 Model input parameters for the November 3-12, 1993 Magnetic Storm. The solar wind parameters beyond day 4.75 have been approximated by constant values. The $Kp$ data was held constant between reported values, and all other inputs have been interpolated to fill in gaps in the data.
Figure 4.3 Electron directional fluxes (in units of \#/cm^2/sec/sr/keV) during the November 3-12, 1993 magnetic storm, measured by LANL geosynchronous satellite 1984-129 (courtesy G. Reeves, LANL). The data provided the outer boundary condition to the RDIF model simulation of the event.
4.2 GPS Radiation Dosimeter Data

Three satellites of the GPS constellation provided the observed electron flux variation for this study. The right side of Figure 4.4 shows count rates from dosimeter readings of these satellites for six $L$-shells. The geometric factor relating counting rate to flux for these detectors was unavailable, so the data was normalized by setting the average count rate at each $L$-shell equal to the average of the corresponding flux calculated by RDIF (on the left side of Figure 4.4).

The GPS satellites have nearly circular orbits of $1.2R_E$ radius, which do not lie in the magnetic equatorial plane. The data at $L > 4.2$ were produced tracing field lines from the satellite position to the magnetic equatorial plane, using a quiet-time statistical magnetic field model (Tsyganenko 1989). Count rates are plotted for $L$-shells calculated by this method which fall within $\Delta L \leq 0.1$ of the listed value. The data at $L > 4.2$ become increasingly unsuitable for comparison to the model for two reasons. First, the GPS data at higher $L$-shells reflect electrons encountered at higher latitudes, which have lower equatorial pitch angles. Since RDIF models only electrons which mirror in the equatorial plane, the similarity of its results to GPS-detected count rates is expected to break down with increasing $L$-shell. Second, the determination of the GPS satellite’s $L$-shell becomes increasingly dependent on the magnetic field model as the satellite goes to higher latitudes. The field model used to produce the right side of Figure 4.4 has no time-dependence, but rather assumes uniformly quiet conditions. As the magnetospheric activity increases in response to
**Figure 4.4** Comparison of GPS count rates (right) to local time averaged model electron fluxes (left), at various $L$-shells. The GPS data has been normalized to the average flux at each $L$ as computed by the model. GPS data courtesy of T. Cayton, LANL.
the storm, the field model becomes less accurate, leading to miscalculation of the $L$-shell which intersects the satellite’s position. For these reasons, the fluxes computed by RDIF will only be compared to GPS data at $L = 4.2$, where the count rates include equatorially-mirroring electrons and the data is least dependent on the magnetic field model.

4.3 Magnetic Field Model Considerations

Figure 4.1 reveals a troubling feature in the model-computed fluxes. At $L = 4.2$ and 4.5, the computed fluxes show an unusually sharp drop and recovery just prior to 0 UT November 4. This feature appears for approximately one hour in the computed fluxes, but no such variation is seen in the GPS data. The discrepancy is most likely caused by an inaccuracy in the magnetic field models generated by THD for this time-period, during which the input parameters were driven to extreme values by the storm. Figure 4.5 shows a comparision of the input $Dst$ index to the deviation in magnetic field intensity $\Delta B$ very near the Earth’s equator, as computed by THD. If the model had correctly calculated the magnetic field throughout the storm, then the two quantities should be nearly identical. The figure shows that $Dst$ matches $\Delta B$ closely for most of the event, but just prior to 0 UT November 4 the two briefly diverge. The lower two plots in Figure 4.5 show IMF $B_z$ and solar wind ram pressure, which are input to THD. While the precise reason for the inaccuracy in THD is
unclear, the figure suggests that exceptional values of IMF $B_z$ and/or solar wind ram pressure drive the model beyond the range in which it is valid.

The apparent error in the magnetic field model is very localized in time, and does not obviously affect the magnetic field computations far from the Earth. The latter point is supported by the agreement of the computed magnetopause standoff distance with observed magnetopause crossings at geosynchronous orbit during this period [Knipp et. al. 1998], and by the similarity of electron flux at higher L-shells with geosynchronous observations (Figures 4.3 and 4.4). Based on these considerations, a correction to the time-series of THD field models was made. The outer boundary condition of the model was computed as before, under the assumption that the errors in the magnetic field model were small at large distances from the Earth. Then the magnetic field models which showed strong disagreement with $Dst$ were replaced with a series of field models identical to the one computed just before the discrepancy appeared. The magnetic field was thus held constant for approximately one hour, and then returned to time-dependence. The short duration of the correction limits the errors produced by this process, and ensures that computed fluxes are minimally affected in this period.

### 4.4 Model Results

Figure 4.6 shows a comparison of (corrected) electron fluxes to GPS data at an equatorial geocentric distance of 4.2 $R_E$, for various energies. These plots were calculated
Figure 4.5  Comparison $Dst$ to $\Delta B$ at $r = 1.1 R_E$, as computed by the THD magnetic field model (top plot). The strong disagreement at 0 UT November 4 is associated with the extreme values of IMF $B_z$ and solar wind ram pressure (lower plots) input to the model.
Figure 4.6  RDIF computed omni-directional fluxes (solid) compared to GPS data (dotted) at $L = 4.2$, with no radial diffusion. The GPS data has been normalized to match pre-storm levels to computed pre-storm fluxes.
with a radial diffusion coefficient $D_{LL} = 0$, to illustrate the case of pure adiabatic transport. The figure shows that the adiabatic effects can create the correct magnitude of flux decrease in the early part of the storm's main phase (within a few hours of 0 UT November 4), at all energy levels shown. This result is in contrast to [Li et al. 1997], in which the flux drop caused by adiabatic variations was estimated to be insufficient to account for the observed dropout in the November 1993 event. However, Li's estimate was calculated by assuming a simple, linear relationship between variations in the magnetic field magnitude and $Dst$ given by $\Delta B = 1.2 Dst - 43.4$ (nT), and it was noted that the flux decrease is expected to depend strongly on the actual change in the magnetic field. The THD model provides a much more sophisticated representation of the magnetic field variations through the storm, and allows a more quantitative examination of adiabatic effects. The results shown in Figure 4.6 suggest that these effects can, in fact, produce the approximate level of flux decrease observed by the GPS satellites.

The calculated dropout is caused by a rapid inflation of the inner-magnetospheric field, due to the strengthening of the ring current in the main phase. In order to conserve the third adiabatic invariant, drifting electrons move out to larger geocentric distances and experience a decrease the magnitude of the local magnetic field. This causes the electrons to lose kinetic energy by conservation of $\mu$, and this appears as a decrease in the flux at a given energy. This purely adiabatic transport is the only
mechanism affecting the fluxes at 4.2 $R_E$, since radial diffusion has been turned off and the outer boundary never penetrates this close to the Earth.

Although adiabatic affects seem to account for the observed flux changes in the early part of the storm, the model results clearly diverge from the observations beyond 600 UT November 4. At this point the GPS data indicates that flux levels rapidly recover to a point equal to or greater than pre-storm fluxes before the end of November 4, while the model shows an extremely gradual recovery which is far from complete by the start of November 12. The slow increase in the calculated fluxes mirrors the gradual recovery of the ring current (as shown by $Dst$ in Figure 4.1). By the end of the storm, the model fluxes have recovered to approximately 1/2 of their pre-storm levels, by purely adiabatic transport. In contrast the GPS count rates showed an increase by a factor of 2 or more in the same time period, strongly indicating the significance of non-adiabatic transport in the recovery phase.

Figure 4.7 shows GPS count rates compared to electron fluxes calculated from the RDIF model, with time-dependent radial diffusion determined by Equation (3.3). The observed period of rapid enhancement which occurs between 600 UT and 1800 UT on November 4 is now well-represented by the model, particularly at the lowest energies. The addition of time-dependent radial diffusion to RDIF clearly improves its performance during this early recovery period, although a number of discrepancies with the observations still exist.
Figure 4.7  Same as previous figure, except the model results were calculated with a time-dependent radial diffusion coefficient.
At energies above 600 keV, the model fails to account for the full degree of flux increase shown by the observations during this period of rapid recovery. By 1800 UT November 4, the calculated flux recovery is approximately a factor of ten lower than observed in the high energy bins. The cause of this disagreement is not known, but two possibilities are likely. The initial flux distribution, supplied to the model by CRRESELE, may inaccurately represent the radiation belts immediately preceding the storm. If the real electron distribution function before the storm showed a much stronger radial dependence—that is, if the gradient in $L_R$ of $F(\mu, L_R)$ was very large at high energies—then radial diffusion would have a much greater impact on the flux levels. This effect would be enhanced when $K_p$ (and therefore $D_{LL}(K_p)$) is also large, as it is throughout most of November 4 (Figure 4.1). A more interesting possibility is that the rapid enhancement in observed fluxes is caused by a non-adiabatic energization mechanism beyond the radial diffusion encoded into the model. Several candidates for such a mechanism have been suggested, including whistler-wave resonance heating [Li et. al. 1997], multi-modal diffusion which couples the third and second adiabatic invariants [Boscher et. al. 1999], and enhanced radial diffusion via drift-resonant ULF wave heating [Hudson et. al. 1999, Elkington et. al. 1999]. Any such mechanism may be consistent with this study, so long as it preferentially accelerates electrons with energies above 600 keV at $L \approx 4.2$. However without greater certainty in the initial flux distribution it is impossible to state whether or not such a source must be present to energize these electrons.
In the days following November 4, the GPS data shown in Figure 4.7 show a very gradual rise in flux, at a rate much slower than the rapid recovery occurring through November 4. The fluxes calculated from RDIF rise at a slightly higher rate. The discrepancy may be due in part to excessive radial diffusion in the model, caused by inaccuracy in the determination of $D_{LL}$. However, the distribution function in the later part of the storm varies weakly with $L_R$ at $L = 4.2$, as is shown in the bottom half of Figure 4.8. Since diffusion is driven by gradients in the distribution function, the effect of varying $D_{LL}$ is muted when $F(\mu, L_R)$ is nearly flat with respect to $L_R$. A more significant factor affecting the rate of flux increase may be electron loss processes, which are not represented in the model. It is known that pitch angle diffusion can drive trapped electrons into the atmosphere, and that this mechanism can have a tangible effect on the distribution function [Lyons et. al. 1972]. The inclusion of such losses in RDIF would increase its level of sophistication, and could improve the calculated rate of flux increases through an extended storm recovery period.

Figure 4.8 shows the distribution function $F(\mu, L_R)$ calculated by RDIF at the end of the model run, on 0 UT November 12, for the cases of no diffusion (left), and time-dependent radial diffusion (right). In the former, the distribution function for $L_R < 4.2$ is identical to the pre-storm distribution. Without diffusion, the only mechanism which can alter $F(\mu, L_R)$ is the time-dependent outer boundary condition, which never penetrates deeper than $L_R = 4.2$. The markedly lower values of $F(\mu, L_R)$
between \( L_R = 4.2 \) and 4.5 are a remnant of the sudden compression in the main phase of the storm, during which the magnetopause reached geosynchronous orbit and the fluxes at the boundary were low. At larger \( L_R \), each discontinuity in the slope of \( F(\mu, L_R) \) represents the next deepest penetration of the outer boundary. The right side of the figure shows the dramatic effect that radial diffusion has on the system. By the end of the storm, all signs of the penetration of the outer boundary to low \( L \)-shells have been smoothed out, and the peak in the distribution function has become steeper and closer to the Earth.

Figure 4.9 displays the entire history of the distribution function \( F(\mu, L_R, t) \) through the storm. The border of the colored region shows the value of \( L_R \) for the outer boundary drift shell as it varies with time. At day 4 the effect of the sudden compression of the magnetosphere on the entire distribution function is prominently displayed. Even within \( L_R < 3 \), the distribution function jumps to a noticeably higher value when the compression occurs. The figure shows that most of the flux increase is caused by high flux values at the outer boundary, which diffuse over time to lower \( L \)-shells. The diffusion occurs most rapidly between November 4 and November 5, when \( Kp \) was at its highest, and gradually abates as the magnetic activity subsides.
Figure 4.8 Electron distribution function $F(\mu, L_R)$ vs. $L_R$ at 0 UT November 12, 1993. The left plot was generated by RDIF with no diffusion, the right with $K_p$-driven radial diffusion.
Figure 4.9  The distribution function $F(\mu, L_R, t)$, plotted against $L_R$ and time $t$ for $\mu = 1905\, MeV/G$.
Chapter 5

Conclusions

This thesis has described the construction of a computer model which calculates energetic electron flux variations in a fully time-dependent magnetospheric field. The model uses $K_p$-dependent radial diffusion coefficients and a time-dependent outer boundary condition, derived from direct observations, to determine non-adiabatic variations in the radiation belt electron population. The principal advance made in this work over similar models, such as that of Brautigam [1997], lies in the use of a carefully constructed, time-dependent magnetic field model, which features a ring current driven by $Dst$ as well as an independently controlled magnetopause, determined by input solar wind parameters. The sophistication of the magnetic field model allows for careful determination of the fully-adiabatic component of electron flux variations, while the diffusion term and boundary condition incorporates non-adiabatic mechanisms into the model.

The comparison of RDIF to independent satellite observations during the November 1993 magnetic storm shows that RDIF can accurately predict variations in flux of electrons with energy less than 600 keV. Fully-adiabatic transport drives a rapid flux dropout in the main phase of the storm, while radial diffusion accounts for much of the subsequent recovery and enhancement. The results for higher energy electrons are less consistent with observations, which may indicate inaccuracy in the initial electron
distribution function or the presence of a non-adiabatic, non-diffusive mechanism. A more rigorous determination of the initial distribution, based on observations, could help determine the existence and magnitude of non-diffusive mechanisms.

The capabilities of the model could be further enhanced by the incorporation of electron loss mechanisms, better determinants of the radial diffusion coefficients, and an improved model of the time-dependent magnetic field. A magnetic field model appropriate to the most intense periods of a magnetic storm would be especially useful, allowing the model to precisely determine the extent of fully-adiabatic variations during a storm.

A complete model of the dynamic outer radiation belts will have to wait for considerable advances in the modeling of the storm-time magnetospheric field, and a deeper understanding of the small-scale effects which lead to violation of the adiabatic invariants. The work described in this thesis is nonetheless a significant step toward a fully-dynamic model, which can efficiently couple observation-based inputs to a theoretical foundation within a realistic, dynamic model of the magnetic field.
Bibliography


