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An Investigation of Laser-Welded Corrugated-Core Sandwich Beams and Plates Stiffened with Concrete

by

Shawn R. McCullough

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

Master of Science

APPROVED, THESIS COMMITTEE:

Dr. R. P. Nordgren, Chairman
Professor of Civil Engineering

Dr. A. J. D. Dror, Co-Chairman
Professor and Chairman of Civil Engineering

Dr. M. Terk
Assistant Professor of Civil Engineering

Houston, Texas
January, 2000
Abstract

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This thesis focuses on the behavior of a corrugated-core sandwich panel with a concrete top layer under normal loads applied to the concrete face. This sandwich panel is composed of two steel face plates separated by a corrugated sheet welded to them at its crests and troughs. A concrete layer is placed on the top face of the sandwich panel, utilizing shear connectors to ensure composite action. The objective of this study is to examine the structural behavior of these composite panels. This thesis intends to provide design capabilities for applications in which this type of sandwich panel is well suited, e.g., emergency bridge repair, building floors, or fire walls.

The panels are analyzed using both elementary beam theory (for narrow panels) and the classical theory of orthotropic plates. In order to complete the theory, the bending stiffnesses in the various directions are determined by structural analysis. To verify the theory, extensive experimental testing has been performed on the sandwich panels. It is found that compression of the core accounts for a majority of the deflection in the relatively thick specimens tested here. Measured deflections are compared with those obtained theoretically, and after corrections are made for core compression, they are in fair agreement.
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Chapter 1

Introduction

Sandwich type composite construction is becoming increasingly prevalent in today's world. The focus of this thesis is on a particular type of sandwich construction in which a corrugated metal sheet is fastened, at its crests and troughs, to two metal sheets. This configuration is advantageous in the fact that the corrugated core separates the faces and thus allows a high second area moment and flexural stiffness to be achieved. The main purpose of sandwich construction is to have a core rigid enough such that one face works principally in compression and the other in tension to resist bending.

The sandwich panels considered in this thesis, as shown in Fig. 1.1, were constructed at Stardyne, Inc. by performing full penetration fusion welding with a laser beam from the outer face sheets. Laser welding is preferable in this particular type of sandwich construction because conventional welding cannot be performed due to limited access. However, spot fusion welding may be a suitable alternative. In this study, the sandwich panels are topped with a layer of concrete, as shown in Fig. 1.2. This novel idea has not been investigated previously for a sandwich plate, although concrete layers on other forms of stiffened plates are commonly used in structures.
1.1 Applications

The corrugated-core sandwich panel has been used in many applications in recent years due to its very desirable characteristics such as it being lightweight, and fire, blast, and impact resistant, in addition to its high strength and stiffness. It has been utilized in several airplane structures, such as wings. These lightweight panels have a wide variety of potential naval applications such as decks, bulkheads, double hull structures, and elevator and hangar bay doors. These particular panels have already been successfully utilized as antenna platforms on the USS Mt. Whitney in February 1994, with significant overall cost savings due to weight reduction. Similar sandwich panels are also being manufactured as decks, doors, and walls.

This thesis focuses on the application to bridge decks, especially for emergency repairs. The corrugated-core sandwich panels are topped with concrete in order to provide a wear surface. In addition, the concrete stiffens the top plate. Concrete provides extra strength without a great increase in cost. Composite bridges also have several inherent advantages. Concrete decks can be post-tensioned or prestressed for durability, secondary protection systems can extend their service life, and the deck can be readily replaced without expensive shoring systems. A discussion of composite bridge design has been included in Appendix A.

The light weight of these sandwich panels makes them particularly suitable to long span bridges where structure weight is a very important factor. These sandwich panels would be extremely useful in bridge deck repair where a minimal level of service is maintained during deck rehabilitation or replacement. With its light weight, sandwich plate bridge decking could be easily installed in the field with minimal equipment and
personnel in a matter of hours. Thus, traffic would be minimally disrupted. It may be viable to avoid the concrete layer in the field by use of plates precoated with a skid resistant surface.

1.2 Background

Plate theories have been developed in the past which are applicable to the symmetrical type of corrugated-core sandwich. They are essentially orthotropic plate theories extended to include the transverse shear deflections, which can be significant for the corrugated-core sandwich panel because of its relatively flexible core. A substantial amount of the research in solving orthotropic plates, either by classical plate theory or the finite element method, involves the core being a much softer material than the faces. Little work has been done on unsymmetrical types of corrugated-core sandwich panels, such as the present one with a concrete layer.

Both Timoshenko [14] and Ugural [16] have presented the solution of the governing differential equation for rectangular orthotropic plates under certain boundary conditions and loadings. Hussain [5] has investigated this particular sandwich panel without concrete. Substantial work was done on sandwich construction including the corrugated core in the 1960’s when it was utilized in aerospace applications [2]. Techniques by workers at the U.S. Forest Products Laboratory have been developed that were extensions of earlier studies at the Royal Aircraft Establishment [1]. Until a recent resurgence, its use and investigation of its behavior appears to have diminished.
1.3 Objectives

It is the intent of this research to investigate the behavior of corrugated-core sandwich panels stiffened with concrete. The investigation is directed towards the following in order to reach this objective:

1. Development of the equations for the composite plate stiffnesses.
2. Derivation of the solution for the composite panels using both elementary beam theory and classical plate theory of orthotropic plates.
3. Comparison of experimental results obtained for certain types of loading with theoretical solutions.

1.4 Scope of Work

The first part of this thesis, presented in Chapter 2, determines several elastic constants needed to complete the theory. The stiffnesses in various directions are derived.

The second part of this thesis, presented in Chapter 3, deals with the solution of both corrugated-core sandwich beams and plates stiffened with concrete. Elementary beam theory and classical plate theory solutions are presented. The solution for the governing equations for orthotropic plates for the case of small deflections is found using Levy’s method and an alternative method adopted by Libove [1].

The third part of this thesis, presented in Chapter 4, discusses the experimental work performed on the sandwich panels. The obtained results are presented and compared with the theoretical results to ensure that a satisfactory agreement is reached.
Fatigue was not considered in this research. Typically in bridge decks, reinforcement fails in fatigue. Dead load stresses account for a significant portion of the service load and stresses in most concrete structures. Thus, the change in stress is excessive and fatigue failure of reinforced concrete structures is rare. In the case of the concrete used to stiffen the sandwich panels considered in this study, reinforcement is not used, thus a progression of microcracks would be seen when the panels are subjected to fatigue. Future investigations should address fatigue.

Economics of these composite plates was also beyond the scope of this thesis, although there is reason to believe that their use would result in substantial weight savings, which may justify the higher cost in certain applications. This has already been displayed by the Navy's use of them as antenna platforms.

1.5 Notation

$\alpha$ The angle of twist per unit length

$\Delta L$ Length of the section used in determining $D_{xy}$, defined in Eq. 2.87

$\delta_s$ Parameter used in Libove and Batdorf's approach, defined by Eq. 3.39

$\kappa$ Curvature used in Castigliano's Theorem

$\lambda_1$ Constant defined in Eq. 3.26

$\lambda_3$ Constant defined in Eq. 3.27

$\mu$ Shear modulus for the material in question

$\theta$ Angle of rotation of the section used in determining $D_x$

$\sigma$ Stress
\[ \Psi \quad \text{A stress function used in determining } D_{xy} \]

\[ \nu_c \quad \text{Poisson's ratio of concrete} \]

\[ \nu_s \quad \text{Poisson's ratio of the steel} \]

\[ \nabla^2 \quad \text{Laplacian of a function} \]

\[ A \quad \text{Area of each hole in the corrugation used in determining } D_{xy} \]

\[ a \quad \text{Distance to the applied load from the edge of a beam in beam theory} \]

\[ a_0 \quad \text{Constant defined by Eq. 2.47} \]

\[ a_1 \quad \text{Constant defined by Eq. 2.49} \]

\[ A_d \quad \text{The area of the diagonal part of the corrugated-core} \]

\[ B_1 \quad \text{Constant defined by Eq. 2.79} \]

\[ B_2 \quad \text{Constant defined by Eq. 2.80} \]

\[ B_{Qx} \quad \text{Modified shear stiffness of the sandwich plate in the } x \text{ direction} \]

\[ B_{Qy} \quad \text{Modified shear stiffness of the sandwich plate in the } y \text{ direction} \]

\[ c \quad \text{Distance to the neutral axis from the topmost fiber} \]

\[ C_1 \quad \text{A coefficient in the solution of the plate governing differential equation} \]

\[ C_3 \quad \text{A coefficient in the solution of the plate governing differential equation} \]

\[ C_i \quad \text{Constant for each } i^{th} \text{ hole used in determining } D_{xy} \]

\[ d \quad \text{Depth of concrete} \]

\[ D_1 \quad \text{The cross rigidity of a plate per unit distance} \]

\[ D_{11} \quad \text{Bending stiffness of top or bottom plate used in determining } D_x \]

\[ D_{22} \quad \text{Bending stiffness of the corrugation used in determining } D_x \]

\[ D_{33} \quad \text{Bending stiffness of the concrete used in determining } D_x \]
$D_{Qx}$  Shear stiffness of the sandwich plate in the $x$ direction
$D_{Qy}$  Shear stiffness of the sandwich plate in the $y$ direction
$D_x$  The flexural rigidity of a plate in the $x$-direction per unit distance
$D_{xy}$  The torsional/twisting rigidity of a plate per unit distance
$D_y$  The flexural rigidity of a plate in the $y$-direction per unit distance
$e$  Strain in the plate or concrete section used in determining $D_x$
$E_c$  Young's modulus for concrete
$E_s$  Young's modulus for steel
$F_1$  Normal force carried by either the top or bottom plate used in determining $D_x$
$F_2$  Normal force carried by the concrete used in determining $D_x$
$f_c'$  Compressive strength of concrete
$f_m$  Coefficient of the sine function in the Fourier expansion of the deflection
$g$  Parameter used in Libove and Batdorf's approach, defined by Eq. 3.41
$H$  Term relating $D_1$ and $D_{xy}$ as defined by Eq. 3.12
$h$  The height measured between midplanes of the face plates in the sandwich panel
$I_y$  The moment of inertia of the "symmetry element" cross section normalized by the width of this element
$I_{y1}$  Moment of inertia per unit width of top plate section w.r.t. NA of plate.
$I_{y2}$  Moment of inertia per unit width of bottom plate section w.r.t. NA of plate.
$I_{y3}$  Moment of inertia per unit width of top corrugated section touching plate w.r.t. NA of plate
$I_{y4}$  Moment of inertia per unit width of bottom corrugated section touching plate w.r.t. NA of plate
$I_{y5}$  Moment of inertia per unit width of concrete section w.r.t. NA of plate
\( I_{y6} \) Moment of inertia per unit width of diagonal corrugated section w.r.t. NA of plate

\( K \) Constant defined by Eq. 2.65

\( L \) Constant defined by Eq. 2.66

\( l \) Length of the section used in Castigliano's Theorem

\( L_1 \) Length of section considered in determining \( D_y \)

\( L_2 \) Length of the diagonal part of the corrugation

\( l_p \) The length of the open section of corrugation of the sandwich panel

\( l_{pc} \) The length of the flat part of the corrugation welded to the face plates of the sandwich panel

\( M \) Constant defined by Eq. 2.67

\( m \) Number of the term in the Fourier expansion

\( M_x \) Bending moment per unit distance on the \( x \)-plane

\( M_{xy} \) Twisting moment per unit distance on the \( xy \)-plane

\( M_y \) Bending moment per unit distance on the \( y \)-plane

\( N \) Constant defined by Eq. 2.68

\( n \) Number of repeated parts of the sandwich plate

\( P \) Lateral point load

\( P_{cr} \) Euler's buckling load

\( q \) The lateral load applied to the plate

\( q_m \) Coefficient used when expanding \( q \) in terms of a single Fourier Sine Series

\( Q_x \) Shear force per unit distance on the \( x \)-plane

\( Q_y \) Shear force per unit distance on the \( y \)-plane
\( R \)  \hspace{1cm} \text{Radius of the curvature of the midplane of the sandwich plate used in determining } \text{D}_y \\
\( s \)  \hspace{1cm} \text{Length of the section used in Castigliano's Theorem} \\
\( T \)  \hspace{1cm} \text{Torque applied to the sandwich plate} \\
\( t_c \)  \hspace{1cm} \text{Thickness of corrugated sheet used to produce core of sandwich panel} \\
\( t_p \)  \hspace{1cm} \text{Thickness of either top or bottom face plate of sandwich panel} \\
\( t_{pl} \)  \hspace{1cm} \text{The thickness of the face plate and the corrugation together} \\
\( V_c \)  \hspace{1cm} \text{Complimentary energy used in Castigliano's Theorem} \\
\( w \)  \hspace{1cm} \text{The lateral plate deflection} \\
\( z \)  \hspace{1cm} \text{Distance across diagonal part of corrugation used in determining } \text{D}_x, \text{ defined by Eq. 2.33} \\
\( z_l \)  \hspace{1cm} \text{Distance to neutral axis in determining } \text{D}_x
Figure (1.1)  Sandwich plate considered in this study.

Figure (1.2)  Sandwich plate stiffened with concrete.
Chapter 2

Plate Bending Stiffness

2.1 Introduction

The form of the constitutive (moment-curvature) relations for orthotropic plates are the same as for isotropic plates. The stresses are related to strains by the generalized form of Hooke’s Law, and the strains are related to curvatures through the normality assumption, under which there is no shear deformation. Then by integrating, the stresses across the thickness and the corresponding moments per unit length are found as

$$M_x = -\left( D_x \frac{\partial^2 w}{\partial x^2} + D_y \frac{\partial^2 w}{\partial y^2} \right),$$  \hspace{1cm} (2.1)

$$M_y = -\left( D_1 \frac{\partial^2 w}{\partial x^2} + D_y \frac{\partial^2 w}{\partial y^2} \right),$$  \hspace{1cm} (2.2)

$$M_{xy} = -2D_{xy} \frac{\partial^2 w}{\partial x \partial y},$$  \hspace{1cm} (2.3)

where \( w \) is the deflection of the plate and \( D_x, D_y, D_1, \) and \( D_{xy} \) are the bending stiffnesses. \( D_x \) and \( D_y \) are the bending stiffnesses in the \( x \) and \( y \) directions, respectively. \( D_1 \) is the cross stiffness term that is a function of the material’s Poisson ratio. \( D_{xy} \) is the torsional stiffness of the plate. Thus it can be seen that four stiffnesses will need to be computed for the orthotropic plate. Shear connectors between the panel and concrete are assumed to make a negligible contribution to the stiffnesses. In addition to the constitutive
moment-curvature relations, plate theory provides equations of equilibrium and boundary conditions, which will be discussed later. Throughout this thesis, corrugations will be considered to run (as waves) in the x-direction, as shown in Figs. 1.1 and 1.2.

2.2 Determination of $D_x$

This is the bending stiffness parallel to the corrugation, and the corrugation does not contribute a great deal of stiffness in this case as will be shown. To demonstrate this, the stiffness of the corrugation is accounted for separately by Castigliano’s theorem and shown to be very small.

The section considered when determining this stiffness is shown in Fig. 2.1. $F_1$ is the normal force carried by the bottom plate. $F_2$ is the normal force carried by the top plate, and $F_3$ is the normal force carried by the concrete. The normal force carried by the small part of the corrugation touching the plates is considered negligible. $M_x$ is defined to be the total moment carried by the section. $M_1$, $M_2$, and $M_3$ are the moments carried by either the top or bottom plate, and the concrete, respectively. Moments are taken about the right-hand center of the top plate and for moment equilibrium

$$M_x = 2M_1 + M_2 + M_3 + F_1 h + F_3 \left( \frac{d}{2} + \frac{t_c}{2} \right).$$

(2.4)

Introduced are $D_{11}$, $D_{22}$, and $D_{33}$, which are the bending stiffnesses of the plate, the corrugation, and the concrete, respectively.

$$D_{11} = \frac{E_s t_p^3}{12(1 - \nu_s^2)} , \quad D_{22} = \frac{E_s t_c^3}{12(1 - \nu_c^2)} , \quad D_{33} = \frac{E_c d^3}{12(1 - \nu_c^2)} .$$

(2.5)

It then follows that
\[ M_1 = \frac{D_{11}}{L_1} \theta, \quad M_2 = \frac{D_{22}}{L_2} \theta, \quad M_3 = \frac{D_{33}}{L_3} \theta. \quad (2.6) \]

and

\[ F_1 = \frac{E_s t_p}{1 - \nu_s^2} e_1, \quad (2.7) \]

\[ F_2 = \frac{E_s t_p}{1 - \nu_s^2} e_2, \quad (2.8) \]

\[ F_3 = \frac{E_c d}{1 - \nu_c^2} e_3, \quad (2.9) \]

in which \( e_1, e_2, \) and \( e_3 \) are the strains in the bottom plate, top plate, and concrete, respectively, and \( \theta \) is the angle of rotation due to the applied moment. The strains are given as

\[ e_1 = e_0 - \frac{h \theta}{2L_1}, \quad (2.10) \]

\[ e_2 = e_0 + \frac{h \theta}{2L_1}, \quad (2.11) \]

\[ e_3 = e_0 + \frac{\theta}{L_1} \left( \frac{h}{2} + \frac{t_p}{2} + \frac{d}{2} \right). \quad (2.12) \]

Thus the forces \( F_1, F_2, \) and \( F_3 \) become

\[ F_1 = \frac{E_s t_p}{1 - \nu_s^2} \left( e_0 - \frac{h \theta}{2L_1} \right), \quad (2.13) \]

\[ F_2 = \frac{E_s t_p}{1 - \nu_s^2} \left( e_0 + \frac{h \theta}{2L_1} \right), \quad (2.14) \]
\[ F_3 = \frac{E_c d}{1 - \nu_c^2} \left[ e_0 + \frac{\theta}{L_1} \left( \frac{h + t_p + d}{2} \right) \right] \quad (2.15) \]

Using force equilibrium, it is known that
\[ F_1 + F_2 + F_3 = 0 \quad (2.16) \]
from which the strain \( e_0 \) can be found as
\[ e_0 = -\frac{1}{2} \left( \nu_s^2 - 1 \right) E_c d \theta \frac{h + t_p + d}{L_1 \left( 2E_s t_p \nu_c^2 - 2E_s t_p - E_c d + E_c d \nu_c^2 \right)} \quad (2.17) \]

Substituting the above values into the moment equilibrium equation results in
\[ M_\chi = 2\frac{E_s t_p^3 \theta}{12 \left( 1 - \nu_s^2 \right) L_1} + \frac{E_s t_c^3 \theta}{12 \left( 1 - \nu_s^2 \right) L_2} + \frac{E_c d^3 \theta}{12 \left( 1 - \nu_s^2 \right) L_1} + \frac{E_s t_p h^2}{1 - \nu_s^2} \left( e_0 - \frac{h \theta}{2 L_1} \right) h \]
\[ + \frac{E_c d}{1 - \nu_c^2} \left[ e_0 + \frac{\theta}{L_1} \left( \frac{h + t_p + d}{2} \right) \right] \left( \frac{d}{2} + \frac{t_p}{2} \right) \quad (2.18) \]

Introduced are \( R \) as the radius of curvature of the midplane of the sandwich plate and \( z_1 \) as the distance to the neutral axis from the top of the concrete, expressed as
\[ R = \frac{L_1}{\theta} \quad (2.19) \]
\[ z_1 = e_0 \frac{L_1}{\theta} \quad (2.20) \]

It then follows that
\[ M_\chi = \frac{1}{R} \left\{ \frac{E_s t_p^3}{6 \left( 1 - \nu_s^2 \right)} + \frac{E_s t_c^3}{12 \left( 1 - \nu_s^2 \right) L_2} + \frac{E_c d^3}{12 \left( 1 - \nu_s^2 \right) L_1} + \frac{E_s t_p h^2}{1 - \nu_s^2} \left( z_1 - \frac{h}{2} \right) h \right\} \]
\[ + \frac{E_c d}{1 - \nu_c^2} \left[ z_1 + \left( \frac{h + t_p + d}{2} \right) \left( \frac{d}{2} + \frac{t_p}{2} \right) \right] \quad (2.21) \]
so then the stiffness per unit length becomes
\[ D_\chi = \frac{E_s t_p^3}{6(1-v_s^2)} + \frac{E_s t_c^3}{12(1-v_s^2)} L_1 + \frac{E_c d^3}{12(1-v_c^2)} + \frac{E_s t_p h^2}{1-v_s^2} \left( z_1 - \frac{h}{2} \right) h \]
\[
+ \frac{E_c d}{1-v_c^2} \left[ z_1 + \left( \frac{h}{2} + \frac{t_p}{2} + \frac{d}{2} \right) \left( \frac{d}{2} + \frac{t_p}{2} \right) \right].
\]

(2.22)

For the corrugation only, as shown in Fig. 2.2, an external moment is placed on a section of corrugation causing a constant internal moment throughout the section, and then the slope at that point is found. Through curvature relations, the stiffness can be found. The applied moment \( M_0 \) is constant throughout the section. It then follows that

\[ V_c = \int_0^l \frac{M^3}{2EI} ds = \frac{M_0^3}{2EI} . \]

(2.23)

where \( V_c \) is the complementary energy, and \( l \) is the length of the corrugation following the assumption that curved beams can be treated as straight beams. It then follows that

\[ M_0 = D\kappa = \frac{E_s I}{l} \theta . \]

(2.24)

where \( \kappa \) is the curvature. Then for plane strain conditions the stiffness becomes

\[ D = \frac{E_s I L_1}{l(1-v_s^2)} . \]

(2.25)

Substituting \( l = t_c^3/12 \), the stiffness becomes

\[ D = \frac{E_s t_c^3 L_1}{12l(1-v_c^2)} = t_c^3 . \]

(2.26)

Thus, it can be seen that the stiffness of the corrugation is very small. As expected, the corrugation provides very little stiffness in this direction.
2.3 Determination of $D_y$

The "symmetry element" that is considered in the determination of the stiffness perpendicular to the corrugation is shown in Fig. 2.3. If one assumes a condition of plane bending strain and transforms the entire section to steel, then the following equation holds:

$$D_y = \frac{E I_y}{1 - v_s^2}, \quad (2.27)$$

in which $I_y$ is the moment of inertia of the "symmetry element" cross-section normalized by the width of this element, which is given in the expression:

$$I_y = I_{y1} + I_{y2} + I_{y3} + I_{y4} + I_{y5} + I_{y6}, \quad (2.28)$$

in which

$I_{y1} = \text{Moment of inertia per unit width of top plate section w.r.t. NA of plate.}$

$I_{y2} = \text{Moment of inertia per unit width of bottom plate section w.r.t. NA of plate.}$

$I_{y3} = \text{Moment of inertia per unit width of top corrugated section touching plate w.r.t. NA of plate}$

$I_{y4} = \text{Moment of inertia per unit width of bottom corrugated section touching plate w.r.t. NA of plate}$

$I_{y5} = \text{Moment of inertia per unit width of concrete section w.r.t. NA of plate}$

$I_{y6} = \text{Moment of inertia per unit width of diagonal corrugated section w.r.t. NA of plate}$

The distance to the neutral axis must then be found. The diagonal part of the corrugation is shown in Fig. 2.4. The following equation results:
\[ + \left( \frac{1}{2} + \frac{1}{2} \right) t_p \left( c - d - \frac{t_p}{2} \right) - \left( \frac{1}{2} + \frac{1}{2} \right) t_p \left( h + \frac{t_p}{2} + d - c \right) \]

\[ + \left( \frac{1}{2} \right) t_c \left( c - d - t_p - \frac{t_c}{2} \right) - \left( \frac{1}{2} \right) t_c \left( h + d - c - \frac{t_c}{2} \right) \]

\[ + \left( \frac{1}{2} \right) \left( \frac{1}{n} \right) \left( \frac{1}{2} \right) D \left( c - \frac{d}{2} \right) + A_d \left( \frac{h}{2} + d - c + \frac{t_p}{2} \right) = 0 \],

(2.29)

in which \( A_d \) is the area of the diagonal part and is found in the following manner with help from Figs. 2.4 and 2.5:

\[ d_y = \frac{h}{2} + \frac{t_p}{2} + d - c . \]

(2.30)

\[ x = \frac{2L_2^2 - 2(h - t_p - t_c)^2}{L_2} , \]

(2.31)

\[ \alpha = \sin^{-1} \left[ \frac{2L_2^2 - 2(h - t_p - t_c)^2}{L_2 \sqrt{L_2 - (h - t_p - t_c)^2}} \right] , \]

(2.32)

\[ z = \frac{t_c}{\cos \alpha} \Rightarrow z = \frac{t_c}{\cos \sin^{-1} \left[ \frac{2L_2^2 - 2(h - t_p - t_c)^2}{L_2 \sqrt{L_2 - (h - t_p - t_c)^2}} \right]} . \]

(2.33)

\[ dA_d = zd\gamma , \]

(2.34)

\[ A_d = \int dA_d = \int_{-(h/2-t_p/2-t_c)}^{h/2-t_p/2-t_c} z \gamma = z(h - t_p - 2t_c) . \]

(2.35)

Thus \( c \) can be solved from Eq. 2.29 and then used to find \( I_y \) resulting in:

\[ I_{y1} = \left( \frac{1}{2} + \frac{1}{2} \right) \left( \frac{t_p^3}{12} \right) + \left( \frac{1}{2} + \frac{1}{2} \right) t_p \left( c - d - \frac{t_p}{2} \right)^2 , \]

(2.36)
\[ I_{y2} = \left( \frac{l_p}{2} + \frac{l_c}{2} \right) \left( \frac{t_p^3}{12} \right) + \left( \frac{l_p}{2} + \frac{l_c}{2} \right) (h + \frac{t_p}{2} + d - c)^2. \] (2.37)

\[ I_{y3} = \left( \frac{l_c}{2} \right) \left( \frac{t_c^3}{12} \right) + \left( \frac{l_c}{2} \right) (c - d - t_p - \frac{t_c}{2})^2. \] (2.38)

\[ I_{y4} = \left( \frac{l_c}{2} \right) \left( \frac{t_c^3}{12} \right) + \left( \frac{l_c}{2} \right) (h + d - c - \frac{t_c}{2})^2. \] (2.39)

\[ I_{y5} = \left( \frac{l_p}{2} + \frac{l_c}{2} \right) \left( \frac{d^3}{12} \right) \left( \frac{1}{n} \right) + \left( \frac{l_p}{2} + \frac{l_c}{2} \right) (d) \left( \frac{1}{n} \right) (c - \frac{d}{2})^2. \] (2.40)

\[ I_{y6} = \int y^2 \, dA + A d_y^2 = z(h - t_p - 2t_c) \left[ \frac{2(h/2 - t_p/2 - t_c)^2}{3} \right] + \left( \frac{h}{2} + \frac{t_p}{2} + d - c \right)^2. \] (2.41)

It is found that the ratio \( D_y / D_x \) is approximately 1.25, which is the value usually obtained for typical corrugated-core sandwich panels.

### 2.4 Determination of \( D_1 \)

This stiffness represents the property that applying curvature in one principal direction of the panel results in moments induced in the other principal directions. This is caused by the Poisson's ratio of the material. To derive this stiffness, a moment is applied in either the \( x \) or \( y \) direction. It will be shown that nearly the same value of \( D_1 \) will be derived regardless of which direction the moment is applied. This is expected as shown in the governing moment equations, Eqs. 2.1 and 2.2.

First, a bending moment will be applied in the \( x \)-direction in which case the curvature in the \( y \)-direction will be kept equal to zero, as shown in Fig. 2.6. \( F_1 \) is the force resisted by the top plate, \( F_2 \) is the force resisted by the bottom plates, and \( F_3 \) is the
force resisted by the concrete. These forces were given by Eqs. 2.13, 2.14, and 2.15. For plane strain conditions to hold, and while neglecting the negligible contribution of the flat part of the corrugation in resisting normal forces, the expression for $M_y$ becomes

$$M_y = -\left[ D_1 \frac{\partial^2 w}{\partial x^2} + D_y \frac{\partial^2 w}{\partial y^2} \right] = v_s F_1 h + v_c F_3 \frac{d}{2}. \quad (2.42)$$

Proceeding to solve for the stiffness, it follows that

$$v_s F_1 h + v_c F_3 \frac{d}{2} = D_1 \frac{\Theta}{L_1} + 0, \quad (2.43)$$

$$v_s h \left( \frac{E_s t_p}{1 - v_c^2} \right) \left( e_0 - \frac{h \Theta}{2L_1} \right) + v_c \frac{d}{2} \left( \frac{E_c d}{1 - v_c^2} \right) \left[ e_0 + \frac{\Theta}{L_1} \left( \frac{h}{2} + \frac{t_p}{2} + \frac{d}{2} \right) \right] = D_1 \frac{\Theta}{L_1}. \quad (2.44)$$

$$D_1 = \frac{v_s E_s t_p h}{1 - v_c^2} \left( z_t \right) + \frac{v_c E_c d^2}{2 \left( 1 - v_c^2 \right)} \left( z_t + \frac{h}{2} + \frac{t_p}{2} + \frac{d}{2} \right), \quad (2.45)$$

where $e_0$ and $z_t$ were given in Eqs. 2.19 and 2.20.

To find the stiffness in another way, a bending moment should be applied in the $y$-direction, in which case the curvature in the $x$-direction will be kept zero. If the force in the corrugation is considered negligible, then the same stiffness $D_1$ as given in Eq. 2.45 will be found. The corrugation may give a slight contribution to the stiffness. This can be confirmed by a finite element analysis in future studies.

### 2.5 Determination of $D_{xy}$

The solution for the torsion problem of thin-walled multiply connected sections is considered in order to determine the torsional rigidity $D_{xy}$. The stress function approach,
developed by Sokolnikoff [12], will be used in deriving this rigidity, following Hussain [5]. A stress function $\Psi$ is assumed for each section as follows:

$$\Psi = 0 \text{ on the exterior boundaries.} \quad (2.46)$$

$$\Psi = C_i \text{ on the boundaries of each hole.} \quad (2.47)$$

$$\nabla^2 \Psi = -2 \text{ inside the section,} \quad (2.48)$$

where $C_i$ is a constant for each $i^{th}$ hole. The constants corresponding to the outer boundaries, $C_0$ and $C_n$ for $n-1$ holes, are both equal to zero.

Using the membrane analogy for thin-walled sections, and neglecting the variation of $\Psi$ along the axis perpendicular to the thickness of the section considered, then $\Psi$ takes the form

$$\Psi \equiv -x_2^2 + a_1 x_1 + a_0 \quad (2.49)$$

where boundary conditions are used to determine the constants $a_0$ and $a_1$. The general form that results is

$$\Psi \equiv \left( \frac{t^2}{4} - x_2^2 \right) + C_{\text{outer}} \left( \frac{1}{2} + \frac{x_2}{t} \right) + C_{\text{inner}} \left( \frac{1}{2} - \frac{x_2}{t} \right), \quad (2.50)$$

where $t$ is the thickness of the section and $C_{\text{inner}}$ and $C_{\text{outer}}$ are the values of $C_i$ on the inner and outer sides of the local axis $x_2$.

Considering the section in Fig. 2.7, the concrete again is transformed to steel, this time using the ratio of shear moduli, $m$, rather than the ratio of Young's moduli, $n$. This should be done because when the section is undergoing torsion, the concrete is shearing and the shear modulus takes account of this approximately. Neglecting the higher order
terms of \( t \) in Eq. 2.50, then the following stress functions result with respect to the local axes \( x_1 \) and \( x_2 \):

\[
\Psi_{AB} = C_0 \left( \frac{1}{2} + \frac{x_2}{t_c} \right) + C_1 \left( \frac{1}{2} - \frac{x_2}{t_c} \right),
\]

\[
\Psi_{BC} = C_1 \left( \frac{1}{2} - \frac{x_2}{t_{pc} + d/m} \right),
\]

\[
\Psi_{CD} = C_1 \left( \frac{1}{2} - \frac{x_2}{t_c} \right) + C_2 \left( \frac{1}{2} + \frac{x_2}{t_c} \right),
\]

\[
\Psi_{AD} = C_1 \left( \frac{1}{2} - \frac{x_2}{t_p} \right).
\]

\[
\Psi_{CF} = C_2 \left( \frac{1}{2} + \frac{x_2}{t_p + d/m} \right),
\]

\[
\Psi_{DE} = C_2 \left( \frac{1}{2} + \frac{x_2}{t_{pc}} \right).
\]

\[
\Psi_{EF} = C_2 \left( \frac{1}{2} + \frac{x_2}{t_c} \right) + C_3 \left( \frac{1}{2} - \frac{x_2}{t_c} \right).
\]

For each hole, the following equation applies:

\[
\oint_{C_i} \frac{d\Psi}{dn} ds = 2A_i,
\]

or, to integrate with respect to the local \( x_1 \) direction, or the length of the section considered, then

\[
n_1 = \frac{dx_2}{ds}, \quad n_2 = -\frac{dx_1}{ds},
\]

and
\[
\int c_i \left( \frac{d\Psi}{dx_1} n_1 + \frac{d\Psi}{dx_2} n_2 \right) = 2A .
\] (2.60)

For the first two holes, substituting Eqs. 2.51 through 2.57 into the above equation results in
\[
\int c_i \frac{d\Psi}{dx_1} ds = \int c_i \frac{d\Psi}{dx_2} ds = \frac{l_e}{t_c} (C_1 - C_0) + \frac{l_{pe}}{t_{pc} + d/m} C_1 + \frac{l_e}{t_c} (C_1 - C_0) + \frac{l_p}{t_p} C_1 = 2A .
\] (2.61)

and
\[
\int c_i \frac{d\Psi}{dx_2} ds = \int c_i \frac{d\Psi}{dx_1} ds = \frac{l_e}{t_c} (C_2 - C_1) + \frac{l_{pe}}{t_{pc}} C_2 + \frac{l_e}{t_c} (C_2 - C_3) + \frac{l_p}{t_p + d/m} C_2 = 2A .
\] (2.62)

Simplifying the above results in
\[
\frac{l_e}{t_c} C_1 - \frac{l_e}{t_c} C_0 + \frac{l_{pe}}{t_{pc} + d/n} C_1 + \frac{l_e}{t_c} C_1 - \frac{l_e}{t_c} C_2 + \frac{l_p}{t_p} C_1 = 2A .
\] (2.63)

and
\[
\frac{l_e}{t_c} C_2 - \frac{l_e}{t_c} C_1 + \frac{l_{pe}}{t_{pc}} C_2 + \frac{l_e}{t_c} C_2 - \frac{l_e}{t_c} C_3 + \frac{l_p}{t_p + d/m} C_2 = 2A .
\] (2.64)

where
\[
K = \frac{l_e}{t_c} ,
\] (2.65)
\[
L = 2 \frac{l_e}{t_c} + \frac{l_{pe}}{t_{pc}} + \frac{l_p}{t_p + d/m} ,
\] (2.66)
\[
M = 2 \frac{l_e}{t_c} + \frac{l_{pe}}{t_{pc}} + \frac{l_p}{t_p + d/m} ,
\] (2.67)
\[
N = \frac{l_e}{t_c} .
\] (2.68)

Generalizing the above results in
\[- NC_{i-1} + MC_i - NC_i = 2A \quad (i=1,3,5\ldots) \quad (2.69)\]
\[- KC_i + LC_{i+1} - KC_{i+2} = 2A \quad (i=1,3,5\ldots) \quad (2.70)\]

More simplification leads to the following

\[- C_{i-1} + \frac{M}{N} C_i - C_{i+1} = 0 \quad (i=1,3,5\ldots) \quad (2.71)\]
\[- C_{i-1} + \frac{L}{K} C_i - C_{i+1} = 0 \quad (i=2,4,6\ldots) \quad (2.72)\]

From this it can be seen that

\[C_0 = C_n = 0 \quad ,\quad (2.73)\]
\[C_{n+1} = C_1 = C \quad .\quad (2.74)\]

So then,

\[C_1 = C_3 = C_5 = \ldots = B_1 \quad .\quad (2.75)\]
\[C_2 = C_4 = C_6 = \ldots = B_2 \quad .\quad (2.76)\]

Upon substituting the above into the general form, the following results:

\[MB_1 - 2NB_2 = 2A \quad .\quad (2.77)\]
\[- 2KB_1 - LB_2 = 2A \quad .\quad (2.78)\]

Solving for \(B_1\) and \(B_2\), the following expressions result:

\[B_1 = \frac{2A(L + 2N)}{LM - 4KN} \quad ,\quad (2.79)\]
\[B_2 = \frac{2A(M + 2K)}{LM - 4KN} \quad .\quad (2.80)\]

This results in
\[ C_i = B_1 \quad \text{for } i = \text{odd} \]  
(2.81)

\[ C_i = B_2 \quad \text{for } i = \text{even} \]  
(2.82)

The torsional rigidity \( D \) is defined by the formula

\[ T = D\alpha \]  
(2.83)

where \( T \) is the total torque applied over a length \( L \), and \( \alpha \) is the angle of twist per unit length. The torsional rigidity is then found as the following expression after substituting for \( C_i \):

\[ D = 2\mu A \sum_{i=1}^{n} C_i = 2\mu A(nB_1 + nB_2) \]  
(2.84)

Furthermore,

\[ \frac{D}{n} = 2\mu A(B_1 + B_2) \]  
(2.85)

If the total length of the plate is considered as

\[ L = n\Delta L \]  
(2.86)

where

\[ \Delta L = \frac{l_p}{2} + l_{pc} \]  
(2.87)

and then comparing Eqs. 2.84 and 2.3, an expression for the torsional rigidity results as

\[ D_{xy} = \frac{D}{2L} = \frac{2\mu A(B_1 + B_2)}{2\Delta L} = \frac{\mu A}{\Delta L} \left[ \frac{2A(L + 2N) + 2A(M + 2K)}{LM - 4KN} \right] \]  
(2.88)

Simplifying the above results in
\[ D_{sv} = \frac{2\mu A^2}{\Delta L} \left[ \frac{L + 2N + M + 2K}{LM - 4KN} \right]. \]  

(2.89)

which is the stiffness required to solve the governing differential equation for the sandwich plate. It should be noted that the concrete causes the section to defy the membrane analogy since it is not thin-walled, thus the foregoing analysis is approximate. This approximation is considered acceptable since the concrete does not contribute much torsional stiffness.

All results of this Chapter are summarized in graphical format, in Table 2.1 and Figs. 2.8 through 2.11.
Table 2.1: Cross-sectional dimensions and material constants for sandwich plate considered in this study.

<table>
<thead>
<tr>
<th>Entity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_1$</td>
<td>1.401 in.</td>
</tr>
<tr>
<td>$L_2$</td>
<td>2.998 in.</td>
</tr>
<tr>
<td>$l_p$</td>
<td>2.386 in.</td>
</tr>
<tr>
<td>$l_{pc}$</td>
<td>0.416 in.</td>
</tr>
<tr>
<td>$t_p$</td>
<td>0.080 in.</td>
</tr>
<tr>
<td>$t_c$</td>
<td>0.045 in.</td>
</tr>
<tr>
<td>$t_{pc}$</td>
<td>0.125 in.</td>
</tr>
<tr>
<td>$h$</td>
<td>2.830 in.</td>
</tr>
<tr>
<td>$d$</td>
<td>0, 1.5&quot;, or 2.5&quot;</td>
</tr>
<tr>
<td>$E_s$</td>
<td>29,000,000 lb/in$^2$</td>
</tr>
<tr>
<td>$E_c$</td>
<td>57000 $\sqrt{E_s}$ lb/in$^2$</td>
</tr>
<tr>
<td>$\nu_s$</td>
<td>0.30</td>
</tr>
<tr>
<td>$\nu_c$</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Figure (2.1) Section used in calculating $D_x$.

Figure (2.2) Section used for Castigliano's Theorem in calculating $D_x$. 
Figure (2.3)  Symmetry element used in calculating $D_y$.

Figure (2.4)  Diagonal part of corrugation used in calculating $D_y$. 

\[ \sqrt{L_z^2 - (h - t_p - t_c)^2} \]
\[ \sqrt{L_2^2 - (h - t_p - t_c)^2} \]

Figure (2.5) Used to determine area of diagonal section of corrugation in calculating \( D_y \).

Figure (2.6) Section used in calculating \( D_1 \).
Figure (2.7) Section used in calculating $D_{xy}$.

$D_x$ vs. Depth of Concrete ($f_c' = 6000$ psi)

Figure (2.8) $D_x$ vs. depth of concrete.
Figure (2.9) $D_y$ vs. depth of concrete.

Figure (2.10) $D_1$ vs. depth of concrete.
Figure (2.11) $D_{xy}$ vs. depth of concrete.
Chapter 3

Analytical Solution using Elementary Beam Theory and Theory of Plates

3.1 Introduction

In this chapter, the stiffnesses derived in the previous chapter will be used in finding the solution of the governing differential equation by utilizing elementary beam theory and the classical theory of plates. The governing equation is obtained via both Levy’s method and an alternative method introduced by Libove and Batdorf [1], for a variety of loading and support conditions. All of these solutions will be verified experimentally, and some of these solutions may be applied to design of bridge decks.

3.2 Structural System

Both bare and concrete-stiffened beams and plates, with a cross-section as shown in Figs 1.1 and 1.2, are studied in this thesis. In associated experiments, beams are loaded in a four-point configuration as shown in Figure 3.1. This type of loading is ideal due to the fact that the shear is zero and the moment is constant on the middle of the beam. Beams were also tested with a three-point loading. Plates were simply supported along opposite edges and subjected to line loads to represent a situation of a bridge deck, in which the deck is supported on two sides by the girders, and an automobile drives down the center, causing bending in the strong direction. Other configurations also tested
were with two sides simply supported but with bending in the weak direction, and all four sides simply supported with a point load in the center.

3.3 Beam Theory

Elementary beam theory is used to find the deflections at the midpoint of the beam. Bending is about the y-axis, thus $D_x$ is the only stiffness needed for the solution. The bending stiffness for the beam is found using the formula

$$D_x = \frac{EI_x}{1 - v^2} \Rightarrow EI_x = D_x (1 - v^2).$$  \hspace{1cm} (3.1)

For a three-point loading configuration, the expression for the deflection is

$$w = \frac{PL^3}{48EI_x},$$  \hspace{1cm} (3.2)

where $P$ is the load applied at the center and $L$ is the length of the beam. For a four-point loading configuration, the expression for the deflection is

$$w = \frac{Pa}{24EI_x} \left(3L^2 - 4a^2 \right),$$  \hspace{1cm} (3.3)

where $P$ is the load applied at each point, $L$ is again the length of the beam, and $a$ is the distance from the end of the beam to the point of load application.

Deflections of the four-point loading configuration have also been computed using a truss analysis, which is shown in detail in Appendix B. Additionally, deflections were computed using Visual Analysis, a structural frame analysis computer program from Integrated Engineering Software. Results from this analysis are included in Appendix C.
Both of these alternative methods verify the results obtained from elementary beam theory.

It is worthwhile to compute the failure loads and stresses of each of the components of the sandwich beam. For the four-point loading configuration shown in Fig. 3.1, stresses and failure loads are approximated and graphed in Figs. 3.2 through 3.4. The concrete is in compression, the bottom plate is in tension, and the top plate changes from compression to tension as the depth of concrete increases. At the point were the top plate has zero stress, the failure load reaches infinity, as is seen in Fig. 3.4. The stresses were found using the formula

\[ \sigma = \frac{Mc}{I}, \]  

(3.4)

and the failure loads were found by equating this stress to either the yield strength of steel, 50,000 psi, or the strength of concrete, 6000 psi. The buckling load of the corrugation can also be approximated, assuming hinged ends, for a lower bound. From the truss analysis shown in Appendix B, the highest force in a truss member is 1.1083P. occurring over a support in the first mode. Thus the buckling load can be approximated using Euler's formula as follows:

\[ P_{cr} = \frac{\pi^2 EI}{L^2} = \frac{\pi^2 29000000( t_c^3 \times 3)}{L^2}. \]  

(3.5)

Thus, the corrugation would buckle with a load applied of approximately \( P_{cr}/1.1 \).
3.4 Levy's Method

Navier's method is a technique that applies to a plate with all four edges simply supported. Levy's method is a more general technique than Navier's method in which two opposite edges are simply supported and the remaining two have arbitrary boundary conditions. This greatly simplifies the solution of the sandwich plate studied here by reducing a double sine series to a single one. The following analysis is based on the analysis done by Ugural [16]. Any loading can be considered provided that it can be expanded into a Fourier sine series as

\[ q(x,y) = \sum_{n} q_n \sin \left( \frac{n \pi x}{a} \right). \]

(3.6)

In addition, the deflections must be small enough such that the small deflection theory of bending applies.

The configuration of the plate in question is shown in Fig. 3.5. In order to develop the governing equation, a plate element that is subjected to a uniformly distributed normal load is considered. Three equations result from the equilibrium of the element:

\[ \frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0, \]

(3.7)

\[ \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = 0, \]

(3.8)

\[ \frac{\partial M_{xy}}{\partial y} + \frac{\partial M_x}{\partial x} - Q_x = 0, \]

(3.9)
in which $Q_x$ and $Q_y$ are the shear forces per unit distance on the $x$ and $y$ planes, respectively. They are found using Eqs. 3.8, 3.9, 2.1, 2.2, and 2.3 as

$$Q_x = -\frac{\partial}{\partial x} \left( D_x \frac{\partial^2 w}{\partial x^2} + H \frac{\partial^2 w}{\partial y^2} \right),$$  \hspace{1cm} (3.10)

$$Q_y = -\frac{\partial}{\partial y} \left( D_y \frac{\partial^2 w}{\partial y^2} + H \frac{\partial^2 w}{\partial x^2} \right),$$  \hspace{1cm} (3.11)

where $w$ is the deflection of the midplane of the plate, and $H$ is a term defined as

$$H = D_t + 2D_{xy}. \hspace{1cm} (3.12)$$

The governing differential equation for the deflection is found by substituting Eqs. 2.1, 2.2, 2.3, 3.10, and 3.11 into Eq. 3.7 resulting in

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} = q(x,y). \hspace{1cm} (3.13)$$

The solution for the governing equation requires several boundary conditions. The following conditions follow from the fact that the plate is simply supported at $x = 0$ and at $x = a$:

$$w|_{x=0} = 0 \hspace{1cm} (3.14)$$

$$w|_{x=a} = 0 \hspace{1cm} (3.15)$$

$$M_x|_{x=0} = -\left( D_x \frac{\partial^2 w}{\partial x^2} + D_{xy} \frac{\partial^2 w}{\partial y^2} \right)|_{x=0} = 0 \hspace{1cm} (3.16)$$

$$M_x|_{x=a} = -\left( D_x \frac{\partial^2 w}{\partial x^2} + D_{xy} \frac{\partial^2 w}{\partial y^2} \right)|_{x=a} = 0 \hspace{1cm} (3.17)$$
The next two conditions are obtained from the fact that the edges \( y = 0 \) and \( y = b \) are free and thus carry no moment, and the equivalent transverse shear resultant is zero:

\[
M_y|_{y=0} = -\left( D_y \frac{\partial^2 w}{\partial y^2} + D_{xy} \frac{\partial^2 w}{\partial x^2} \right)_{y=0} = 0 \quad ,
\]

\[
M_y|_{y=b} = -\left( D_y \frac{\partial^2 w}{\partial y^2} + D_{xy} \frac{\partial^2 w}{\partial x^2} \right)_{y=b} = 0 \quad ,
\]

\[
V_{yz}|_{y=0} = \left( Q_y + \frac{\partial M_{xy}}{\partial x} \right)_{y=0} = \frac{\partial}{\partial y} \left[ D_y \frac{\partial^2 w}{\partial y^2} + \left( H + 2D_{xy} \right) \frac{\partial^2 w}{\partial x^2} \right]_{y=0} = 0 \quad .
\]

\[
V_{yz}|_{y=b} = \left( Q_y + \frac{\partial M_{xy}}{\partial x} \right)_{y=b} = \frac{\partial}{\partial y} \left[ D_y \frac{\partial^2 w}{\partial y^2} + \left( H + 2D_{xy} \right) \frac{\partial^2 w}{\partial x^2} \right]_{y=b} = 0 \quad .
\]

Levy's method gives the homogeneous solution in the form of

\[
w = f_m(y)\sin\left( \frac{m\pi x}{a} \right) ,
\]

where \( f_m(y) \) is a function of \( y \) to be determined. Substituting this assumed solution into the governing equation yields

\[
\frac{d^4 f_m}{dy^4} - 2\left( \frac{m\pi}{a} \right)^2 \frac{d^2 f_m}{dy^2} + \left( \frac{m\pi}{a} \right)^4 f_m = \frac{q_m}{D_x} ,
\]

which has a particular solution of

\[
(f_m)_{\text{part}} = \frac{q_m}{D_x} \left( \frac{a}{m\pi} \right)^4 ,
\]

and a homogeneous solution, being symmetric about the \( y \)-axis, of

\[
(f_m)_{\text{hom}} = C_1 \cosh(\lambda_1 y) + C_3 \cosh(\lambda_3 y) ,
\]

where \( C_1 \) and \( C_3 \) are constants of integration, and \( \lambda_1 \) and \( \lambda_3 \) are defined as
\[
\lambda_1 = \frac{m \pi}{a} \sqrt{\frac{1}{D_y} \left( H + \sqrt{H^2 - D_x D_y} \right)}.
\]

\[
\lambda_3 = \frac{m \pi}{a} \sqrt{\frac{1}{D_y} \left( H - \sqrt{H^2 - D_x D_y} \right)}.
\]

Assembling the homogeneous and particular solutions results in a complete solution of:

\[
w = \sum_{m=1}^{\infty} \left[ C_1 \cosh(\lambda_1 y) + C_3 \cosh(\lambda_3 y) + \frac{q_m}{D_x} \left( \frac{a}{m \pi} \right)^4 \right] \sin \frac{m \pi x}{a}.
\]

Applying the boundary conditions, the constants of integration can be found from a linear system of two equations and two unknowns. Applying the conditions at \(y = b/2\), the following system of equations results:

\[
C_1 \cosh\left( \frac{\lambda_1 b}{2} \right) + C_3 \cosh\left( \frac{\lambda_3 b}{2} \right) + \frac{q_m}{D_x} \left( \frac{a}{m \pi} \right)^4 = 0.
\]

\[
C_1 \lambda_1^3 \cosh\left( \frac{\lambda_1 b}{2} \right) + C_3 \lambda_3^3 \cosh\left( \frac{\lambda_3 b}{2} \right) = 0.
\]

For the case of a line load at the center, the value \(q_m\) becomes

\[
q_m = -\frac{2P}{a} \left[ \sin \left( \frac{\pi m}{2} \right) \right].
\]

Thus, by summing over all values of \(m\), the deflection \(w\) caused by a line loading can be found using Eq. 3.28. The above analysis can be changed for bending in the strong direction rather than the weak by interchanging the flexural stiffnesses.
3.5 Libove and Batdorf's Method

H. G. Allen [1] outlines an alternative approach adopted by Libove and Batdorf. They developed this method as an alternative to energy methods using strains in the core and the faces, and the strain energy associated with these strains. Libove and Batdorf's method is based on differential equations and they derive a strain energy function from bending, twisting, and shearing stiffnesses. The difference between this method and the Levy's method is that shear stiffness is included. They take the shear stiffness in the direction of the corrugations to be large enough to be called infinite. This greatly simplifies the procedure, and only the effect of the shear deformations perpendicular to the corrugations needs to be considered. This method is used here for a plate simply supported on all edges with a concentrated load in the center, as shown in Figure 3.6. The following analysis and equations are found in reference [1].

For a point load in the center, the load written as a Fourier sine series is

$$q_{mn} = -\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin\left(\frac{m\pi}{2} x\right)\sin\left(\frac{n\pi}{2} y\right),$$  \hspace{1cm} (3.32)

where $q_{mn}$ is defined as

$$q_{mn} = \frac{4P}{ab}. \hspace{1cm} (3.33)$$

The core shear stiffness in the x and y directions are included in this approach, due to the core's considerable flexibility. They are given as

$$D_{ox} = \infty, \hspace{1cm} (3.34)$$
\[ D_{Qy} = \frac{\text{ShE}_{\text{core}}}{1 - \nu_{\text{core}} \left( \frac{t_e}{h - t_p - t_e} \right)^3} , \] (3.35)

where \( S \) is a coefficient with a value of 7.5 for the sandwich plate cross-section featured in this study.

The maximum deflection given at the center of the plate is given as

\[ w_{\text{max}} = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{q_{mn}}{\pi^2 D_{x} \theta} (-1)^{(m-1)/2} (-1)^{(n-1)/2} , \] (3.36)

where

\[ \theta = \Phi_0 \pi^2 , \] (3.37)

\[ \Phi_0 = \frac{\delta_s}{\psi} . \] (3.38)

\[ \delta_s = \frac{\pi^2}{2g} \left\{ \frac{m^2 b^2}{B_{Qy}} + \frac{n^2}{B_{Qx}} \right\} \left\{ D_{y} \frac{m^4 b^4}{a^4} + \frac{D_{y} D_{y}}{D_{x}} n^4 + 2m^2 n^2 \frac{b^2}{a^2} \left( D_{y} - \nu_{y} D_{xy} \right) \right\} + \frac{1}{g} \left\{ \frac{m^4 b^4}{a^4} + \frac{D_{x}}{D_{y}} n^4 + 2m^2 n^2 \frac{b^2}{a^2} \left( \nu_{y} + \frac{D_{xy}}{D_{y}} \right) \right\} , \] (3.39)

\[ \psi = \frac{\pi^2}{2g B_{Qx} B_{Qy}} \left\{ D_{xy} D_{x} \frac{m^4 b^4}{a^4} + D_{xy} D_{y} n^4 + 2m^2 n^2 \frac{b^2}{a^2} \left( D_{x} D_{y} - D_{xy} D_{xy} \right) \right\} + \frac{\pi^2}{g} \frac{m^2 b^2}{2 B_{Qy}} \left\{ D_{x} + \frac{g D_{xy}}{2 B_{Qy}} \right\} + \frac{\pi^2}{g} \frac{n^2}{B_{Qx}} \left\{ D_{y} + \frac{g D_{xy}}{2 B_{Qx}} \right\} + 1 , \] (3.40)

\[ g = 1 - \nu_{\text{core}}^2 , \] (3.41)

\[ \nu_{y} = \nu_{\text{core}} \frac{D_{y}}{D_{x}} , \] (3.42)
\[ B_{Qx} = b^2 D_{Qx} = \infty \quad , \] (3.43)

\[ B_{Qy} = b^2 D_{Qy} \quad . \] (3.44)
Figure (3.1) Four-point loading of a corrugated-core sandwich beam stiffened with concrete.

Figure (3.2) Maximum stress in beam vs. depth of concrete for face plates and concrete.
Figure (3.3)  Failure load for beam vs. depth of concrete for bottom face plate and concrete failure.

Figure (3.4)  Failure load for beam vs. depth of concrete for top face plate failure.
Figure (3.5)  Configuration used in solving Levy's method.

Figure (3.6)  Configuration used in solving Libove and Batdorf's method.
Chapter 4

Experimental Results and Comparisons

4.1 Introduction

To verify the theory of Chapter 3, extensive experiments were conducted on the sandwich panels in various loading and support configurations. Deflections were recorded during all tests and are presented here along with results from the different solutions previously obtained. All tests are kept within elastic limits, except for two that were taken to failure.

4.2 Loading and Supporting Systems

Beams, as described in Section 3.2 and shown in Fig. 3.1, were tested. Results and comparisons between theoretical and experimental data are shown graphically in Figs. 4.1 through 4.10. Pictures of actual tests are included in Figures 4.13 through 4.18.

To ensure that composite action between the sandwich panel and the concrete is taking place, shear studs are utilized. High-strength epoxy was used to fasten the studs to all specimens rather than welding because heat may be generated that would warp the specimens. Rectangular ribs were used for the beam specimens and cylindrical studs were used for plate specimens to account for the additional direction of bending. In
addition to preventing separation of the concrete and steel, the shear connectors aid in resisting shear.

The thickness of concrete used to stiffen the sandwich panels was chosen after trying both 1.5" and 2.5" thicknesses. The 2.5" thickness was felt to be too large and disproportionate to the beam, thus 1.5" was settled upon, which gives a considerable increase in strength over a bare panel.

Type III cement was used for all test specimens to expedite the experimental stage. ASTM procedures were followed for all concrete mixing, curing, and testing processes. Standard cylinders were cast and capped, and then tested for strength and modulus of elasticity. Typically, a 7 - day strength of 6000 to 7000 psi was established. The maximum aggregate chosen for the specimens was 3/8" based on sizes of the specimens. No admixtures of any kind were used and the water/cement ratio was typically near 0.40. A slump was consistently obtained between 3 and 4 inches. All specimens were vibrated and allowed to cure in a humidity room. Details of the concrete mix design are given in Appendix D. A batch trial was done with this design which was acceptable, thus the mix was kept.

The Instron 1332 was the testing machine used for all experiments. Specimens were on supports that were placed and fixed on a flat surface. These supports are considered rigid enough under the applied loads that no displacement was assumed for them. Loads were applied using an appropriate load cell, and the load is transmitted to the specimen using a system of rigid cylinders. Loads were applied in ten pound increments. Concrete was filed down at points of loading to ensure proper contact and
distribution. Deflections were automatically recorded by measuring the travel of the actuator. Verifications of these deflections were made with dial gauges read manually.

4.3 Discussion of Experiments

All beams and plates were kept within elastic limits with the exception of one concrete-stiffened beam and one concrete-stiffened plate. The beam exhibited buckling of the core at points above the supports, as can be seen in the photos of Figs. 4.13 and 4.14. Plates exhibited failure of the concrete by cracking and crushing at the location of the loads, and separation at the interface, as can be seen in the photos of Figs. 4.16 through 4.18.

It was found that experimentally recorded deflections were several times greater than those found theoretically. Extensive diagnostic tests were performed with the testing machine. The stiffness of the test setup was analyzed to see if settlement was occurring, which turned out to be negligible. It was found that compression of the corrugated-core was causing all of the excess deflection. Thus, each “wave” of the corrugated core was treated as a spring between two rigid plates and their spring stiffness was found experimentally as shown in Figs. 4.11 and 4.12. This was done by uniformly compressing a bare beam and a bare plate by placing a rigid plate over the specimen and placing a load on it, causing each “wave” of the core to compress equally. The panel is modeled as a system of springs in parallel, which are additive. Thus, after the force-deflection relationship is established for each system of “waves” over the whole panel, then the force-deflection relationship can be found for each “wave” separately. Then this relationship is used to find the compression of the core, which occurs over the supports.
The deflection from the core compression is then superimposed with the deflection found from bending as follows:

$$\Delta_{\text{corrected}} = \Delta_{\text{bending}} + \Delta_{\text{core}} = \Delta_{\text{bending}} + \frac{P}{k},$$

(4.1)

where P is the load at the support and k is the stiffness obtained from the appropriate load-deflection curve.

The deflection from the compression was incorporated into the theoretical solution, thus obtaining fair agreement. There is better agreement for the bare specimens than the concrete-stiffened specimens. The crushing of the core accounts for between 75% and 85% of the total deflection, depending on the stiffness of the particular specimen. Specimens that are concrete-stiffened, and thus stiffer overall, exhibit more core compression, because it is the more flexible component of the specimen.

Any discrepancy between the theoretical and experimental results may be explained by several factors. As was noted earlier, the bare specimens exhibit better agreement between theoretical and experimental values than the concrete-stiffened specimens. This can be explained by the fact that there is local crushing of the concrete at loading points. The compression of the corrugations causes both beam and plate specimens to not behave entirely in bending, thus beam theory and plate theory are not completely accurate. A majority of the specimens tested had visible deformities and imperfect laser-welds. It is also difficult to fully eliminate the local crushing of concrete at the location of the applied loading. Additionally, heat generated from cutting the specimens minimally warped them. It should be noted that the plates studied in this thesis are not very large in size, causing their behavior to be unlike an ideal thin plate.
Additionally, the beams tested were not very long, thus causing them to act as short beams producing shear deformations that are not negligible.
Figure (4.1) Configuration and comparison of theoretical and experimental deflections for three-point loading of a bare beam. (Theory adjusted to include core compression.)
Figure (4.2) Configuration and comparison of theoretical and experimental deflections for four-point loading of a bare beam. (Theory adjusted to include core compression.)
Figure (4.3) Configuration and comparison of theoretical and experimental deflections for four-point loading of a beam with 1.5" concrete. (Theory adjusted to include core compression.)
Figure (4.4) Configuration and comparison of theoretical and experimental deflections for four-point loading of a beam with 2.5" concrete. (Theory adjusted to include core compression.)
Figure (4.5) Configuration and comparison of theoretical and experimental deflections for bare plate simply supported on all sides with a point load in the center. (Theory adjusted to include core compression.)
Figure (4.6) Configuration and comparison of theoretical and experimental deflections for bare plate bending in the weak direction.
(Theory adjusted to include core compression.)
Figure (4.7) Configuration and comparison of theoretical and experimental deflections for bare plate bending in the strong direction. (Theory adjusted to include core compression.)
Figure (4.8) Configuration and comparison of theoretical and experimental deflections for plate with 1.5" concrete simply supported on all sides with a point load in the center. (Theory adjusted to include core compression.)
**Figure (4.9)** Configuration and comparison of theoretical and experimental deflections for plate with 1.5" concrete bending in the weak direction. (Theory adjusted to include core compression.)
1.5" Concrete Plate - 2 Sides SS, LL, strong direction bending

Figure (4.10) Configuration and comparison of theoretical and experimental deflections for plate with 1.5" concrete bending in the strong direction. (Theory adjusted to include core compression.)
Figure (4.11) Load vs. deflection for compression of a beam. (For core-compression adjustment)

Figure (4.12) Load vs. deflection for compression of a plate. (For core-compression adjustment)
Figure (4.13) Photo of four-point loading of concrete-stiffened sandwich beam with buckling of corrugation over support.

Figure (4.14) Photo of buckling of beam featured in Fig. 4.13.
Figure (4.15) Photo of testing of concrete-stiffened plate.

Figure (4.16) Photo of failure of the plate featured in Fig. 4.15.
Figure (4.17) Photo of crushing failure of plate featured in Fig. 4.15.

Figure (4.18) Photo of failure of plate featured in Fig. 4.15 where section of concrete and studs lifted off.
Chapter 5

Summary and Conclusion

Corrugated-core sandwich panels, both bare and stiffened with a top layer of concrete, have been investigated both theoretically and experimentally. Three and four-point bending of sandwich beams and sandwich plates simply supported on two or four edges subjected to normal loads applied to their faces were considered in this study.

In order for theoretical solutions to be carried out, panels were modeled as orthotropic plates. Formulas for the plate bending stffnesses in various directions were developed using moment curvature relationships and stress functions. Elementary beam theory was applied to the beam bending cases. The configuration of the plates allowed either Levy's method or an alternative method as given by Allen [1] to be used in solving the governing differential equation. Thus the deflections at the midplanes of the sandwich panels were obtained. From these deflections, moments and shears can easily be obtained from the equations and, furthermore, stresses and strains can be found.

Extensive experiments were conducted on the sandwich panels to verify the theoretical analyses. Various loadings were applied to sandwich beams and plates. Deflections were recorded and compared with deflections obtained from the theoretical solutions. Initially, theoretical and experimental results were not in good agreement, and it was found that the corrugations were compressing in a direction normal to the plate forces, thus contributing significantly to the overall deflection. After corrections were
made to account for this compression, the theoretical and experimental results are in fair
agreement for all cases. Bare specimens exhibit better agreement than the concrete-
stiffened specimens.

The sandwich panel considered herein would be ideal for quick bridge deck repair
or rehabilitation. The panels are lightweight enough so that minimal equipment and
personnel are needed to place the deck quickly with limited traffic disruption. To
expedite the process even further, it may be advisable to not use concrete at all but
instead glue a suitable skid resistant surface to the outer face of the panel. This would
eliminate the time to pour and cure the concrete as well as lowering materials costs and
structure weight. However, it should be noted that concrete significantly increases the
stiffness of the bare sandwich panel. Specifically, there is approximately a 140% increase in stiffness using a 1.5” deep layer of concrete and a 290% increase using a 2.5”
deep layer of concrete for the beam as shown in Fig. 3.1. Additionally, perhaps an
alternative can be found for laser-welding of the sandwich plate due to its high cost.
Spot-welding has been used for this type of sandwich plate, which results in a lower weld
strength.

The techniques used in deriving the rigidities of the sandwich panel considered in
this thesis have proven to be very useful. These techniques can be extended to almost
any type of section used as a core for the sandwich panel as long as the cross section can
be modeled as a thin-walled multiply connected closed section. It remains for a three-
dimensional finite element analysis to be done on these concrete-stiffened panels, which
was beyond the scope of this study.
Bibliography


Appendix A

Composite Bridge Design

The following is a brief discussion of the design of composite bridges prescribed by the American Association of State Highway Officials (AASHTO). A typical composite bridge consists of a reinforced concrete slab on steel girders, in which they interact compositely through the use of shear connectors. This type of construction takes advantage of concrete's compressive properties and steel's tensile properties and is very economic in the fact that less steel will be used. Additional benefits include improved resistance to lateral and dynamic loads.

Loads

Dead loads considered include the weights of all the bridge components and any other permanent equipment attached to the bridge. Snow loads are not considered because normally the presence of snow would cause a reduction in traffic on the bridge. The dead load is assumed to be uniformly distributed.

Vehicle loads constitute the live loads, which are dynamic. AASHTO specifies basic units of load, e.g. a two-axle truck loading is H20-44. Because live loads are dynamic, their "impact" is considered, and AASHTO specifies an impact factor I as

\[ I = \left( \frac{50}{125 + L} \right) , \quad I \leq 0.3 , \quad (A.1) \]
in which $L$ is the loaded length in feet. This impact factor magnifies the live load. Forces from braking and collisions must also be considered. Additionally, if the bridge is curved, centrifugal forces must be considered. The width of the bridge is used to determine the number of design lanes. Depending on whether the design is controlled by fatigue or permanent deformation, whichever of lane loading or truck loading that produces maximum load effects is used.

**Load Effects**

The bridge is analyzed for the longitudinal and transverse moments, and shear. Flexure of longitudinal strips is used to find the longitudinal bending moment, in which their width is equal to the spacing of the longitudinal steel girders. Then these strips are subjected to a fraction of one line of wheel loads of a design truck. The maximum longitudinal moment due to one line of wheel loads of the design truck divided by the actual maximum intensity is given by AASHTO for various structures. The design live load moment can then be calculated by multiplying the fraction of load on the strip by the maximum longitudinal moment due to one line of truck load wheels. The transverse moment is determined by combining the local and global load effects, although AASHTO does not require the global effects to be considered. The shear is independent of the span and width of the composite bridge.

The bridge is treated as a beam in order to find the deflections of the bridge due to the dead load. Average deflections due to the live load are found by applying a fraction of the load to a longitudinal strip and treating this strip as a bending beam. Creep
deflections are treated elastically, and a 15% increase in deflections is used to account for them.

**Design Practice**

A trial section is chosen and is checked for flexure, shear, and deflections and these calculations are compared to those capacities obtained from the load effects. The area of the concrete deck slab is found by the least of: one fourth of the span, center to center distance between the steel girders, or twelve times the least thickness of the concrete deck slab. Then the factored moment of resistance can be derived. The web of the steel girder is to carry the total vertical shear of the composite section. In addition, shear connectors must be designed to resist horizontal shear and to ensure composite action. Studs are the most popular shear connector in bridge construction. Shear connectors are designed against the shear effects from the live load and their shear strength should be at least equal to that of the compressive strength of the concrete deck or the tensile strength of the steel girder, whichever is smaller. The concrete deck slab is designed such that its ultimate moment is equal to or greater than the transverse moment due to the load effects. A thickness for the slab is assumed, and the ultimate moment is determined. A minimum concrete cover is also prescribed.
Appendix B

Truss Analysis

A truss analysis was performed to verify the deflections obtained by elementary beam theory using the derived stiffnesses of Chapter 2. This method is used for a bare 3” wide beam loaded in a four-point configuration, as shown in Fig. 3.1. The process can easily be used for a concrete-stiffened beam by transforming the section to steel. Joints and members are numbered accordingly, then the method of joints is proceeded to be followed at each numbered joint. Virtual forces and actual forces are found and then the displacement at the center of the beam is obtained. The results are summarized in the following table:

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<th>Member</th>
<th>Virtual Force $F_v$ (lb)</th>
<th>Actual Force $F_a$ (lb*P)</th>
<th>Length $L$ (in)</th>
<th>Area $A$ (in$^2$)</th>
<th>$E$ (ksi)</th>
<th>$nNL/AE$</th>
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33 & 1.673 & 2.389 & 2.802 & 0.226 & 29000000 & 1.706E-06 \\
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47 & -1.434 & -1.434 & 2.802 & 0.226 & 29000000 & 8.771E-07 \\
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\sum 2.928E-05
\]

\[
(llb)\Delta_{center} = \sum \frac{nNL}{AE}, \quad (B.1)
\]

\[
\Delta_{center} = \frac{1}{llb} \sum \frac{nNL}{AE} = 2.928 \times 10^{-5} \text{ P in.} \quad (B.2)
\]

which is very close to the value obtained from beam theory.
Appendix C

Visual Analysis Results

*Visual Analysis* is a structural frame analysis program by *Integrated Engineering Software*. Nodes and members are inputted with the proper loading and support configurations. The following results were obtained for the beam shown in Fig. 3.1 with $P = 450$ pounds.

![Diagram](image)

**Figure (C.1)** Unloaded and deformed shapes of bare sandwich beam with four-point loading.

**VisualAnalysis 3.12.STUDENT Report**

*Project: Sandwich Plate*

Default Units: Inches, Pounds, Degrees. °Fahrenheit, Seconds.

**Nodal Displacements**

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<th>Load Case</th>
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<th>$DY$</th>
<th>$RZ$</th>
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Appendix D

Concrete Mix Design

The following worksheet gives details about the mix design. A batch trial done with this design went well and thus was kept.
### Specific Gravities

- Cement = 3.15
- Fine Aggregate = 2.65
- Coarse Aggregate = 2.65

### Unit Weights (Air Voids Neglected)

- Cement = \(3.15 \times 62.4 \text{ pcf} = 196.56 \text{ pcf}\)
- Fine Aggregate = 110 pcf (given)
- Coarse Aggregate = 94 pcf (given)

### Compute Volume of 1 Bag Mix

- Mix will be 7 bags/cy
- \(27/7 = 3.857 \text{ cf of concrete per bag of cement}\)

- Cement volume = \(94/196.56 = 0.478 \text{ cf}\)
- Entrapped air volume = \(0.03 \times 3.857 = 0.116 \text{ cf}\)

- Assumed water for proper workability is 4.8 gallons per sack of cement
  \(4.8 \times 8.32/62.4 = 0.64 \text{ cf}\)

- Remaining volume of mix will be aggregates such that
  \(3.857 - 0.478 - 0.116 = 2.623 \text{ cf}\)

- Assume for workability that 35% of total aggregate is fine
  \(0.35 \times 2.623 = 0.918 \text{ cf of sand}\)

- Remaining volume of mix is coarse aggregate
  \(2.623 - 0.918 = 1.705 \text{ cf}\)

### Batch Weights

- Mix volumes are converted to batch weights by multiplying each constituent volume by its unit weight.

- Cement = \(0.478 \times 196.56 = 94 \text{ lb}\)
- Water = \(0.64 \times 62.4 = 39.94 \text{ lb}\)
- Fine Aggregate = \(0.918 \times 110 = 100.98 \text{ lb}\)
- Coarse Aggregate = \(1.705 \times 94 = 160.27 \text{ lb}\)

### Proportions

- Cement : FA : CA : Water
- 1 : 1.07 : 1.71 : 0.42
The total number of sacks of cement needed for a pour is the calculated volume divided by 3.857. This number of sacks multiplied by the constituent weight per sack gives all the required weights. For example:

Volume of concrete needed: 2 ft³

Cement: \( \frac{94}{3.857} \times 2 \text{ ft}^3 = 48.7 \text{ lb Cement} \)

Water: \( \frac{36.6415}{3.857} \times 2 \text{ ft}^3 = 19.0 \text{ lb Water} \)

Fine Aggregate: \( \frac{100.98}{3.857} \times 2 \text{ ft}^3 = 52.4 \text{ lb FA} \)

Cement: \( \frac{160.27}{3.857} \times 2 \text{ ft}^3 = 83.1 \text{ lb CA} \)