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A Probabilistic Approach to the Frequency Domain Analysis of Drill-String Lateral Vibrations

by

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A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree Master of Science

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Abstract

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Drill-string failure is recognized by the drilling community as one of its most costly and frequently encountered problems. In order to diminish the frequency of such failures, the drilling community has been focusing on the modeling of the static and dynamic behaviors of drill-strings, towards increasing efficiency. Drill-string dynamics involves complex phenomena with excitations in axial, lateral and torsional directions, and coupling of these three and other factors such as mass imbalance, whirling, or stick-slip motion of the bit.

The present study focuses on one aspect which has not been studied in detail by the drilling community, namely the consideration of random loads on the BHA studied by a frequency domain approach. A comprehensive literature survey is attempted and elucidating numerical results are provided.
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Contents

Abstract ii
Acknowledgments iii
List of Illustrations vii

1 Introduction 1
  1.1 Drill-String Presentation .................................. 3
  1.2 The Drilling Fluid ........................................... 3
  1.3 Evolution of Drill-Strings: the Role of Drill-Collars .......... 4

2 Theoretical Background 7
  2.1 Preliminary Remarks ....................................... 7
    2.1.1 Friction ................................................. 9
    2.1.2 Damping Models ....................................... 12
  2.2 Longitudinal Motion ...................................... 12
    2.2.1 Preliminary Remarks .................................. 12
    2.2.2 Problems .............................................. 13
    2.2.3 Models ............................................... 14
    2.2.4 Boundary Conditions .................................. 15
  2.3 Lateral Motion .......................................... 16
    2.3.1 Preliminary Remarks .................................. 16
    2.3.2 Problems .............................................. 17
    2.3.3 Models ............................................... 20
    2.3.4 Boundary Conditions .................................. 29
  2.4 Torsional Motion ......................................... 30
2.4.1 Preliminary Remarks ........................................ 30
2.4.2 Problems ..................................................... 30
2.4.3 Models ......................................................... 36
2.5 Coupling ........................................................ 43
  2.5.1 Longitudinal-Lateral Coupling .............................. 44
  2.5.2 Longitudinal-Torsional Coupling ........................... 48
  2.5.3 Lateral-Torsional Coupling ................................. 50
2.6 Further Considerations .......................................... 52
  2.6.1 Influence of Mass Imbalance ............................... 52
  2.6.2 Additional Problems ...................................... 53
  2.6.3 Importance of Non-linearities ............................. 53
2.7 Probabilistic Approaches ...................................... 54
  2.7.1 Coupled Axial and Rotational Motions ..................... 54
  2.7.2 Longitudinal Motion ...................................... 55
  2.7.3 Coupled Longitudinal and Lateral Motions ................ 56

3 Dynamic BHA Behavior Analysis ............................... 57
  3.1 Preliminary Remarks ......................................... 57
  3.2 Equation of Motion ......................................... 57
  3.3 The Finite Element Model ................................... 57
    3.3.1 Element Matrices ...................................... 59
  3.4 Boundary Conditions ......................................... 60
  3.5 Static Condensation ......................................... 60
  3.6 Solving the Eigenproblem ................................... 61
  3.7 Mode Superposition Method .................................. 62
  3.8 Steady-State Harmonic Response ............................. 62
3.9 Transfer Function ........................................... 64
3.10 Using the Transfer Function .............................. 65
  3.10.1 Using the Power Spectral Density .................. 66

4 Numerical Results ............................................ 68
  4.1 First Example: "Maintain" BHA Configuration ...... 68
    4.1.1 Mode Shapes and Natural Frequencies ............ 68
    4.1.2 Grid Refinement ................................... 71
    4.1.3 Complex Frequency Response Function .......... 73
    4.1.4 Damping Influence ................................ 78
    4.1.5 Lateral Displacement of the String ............. 79
  4.2 Random Excitation Model ............................... 80
    4.2.1 White-Noise Excitation .......................... 80
    4.2.2 Spectral Densities ............................... 82
  4.3 Second Example: "Build-Up" BHA Configuration ...... 84
    4.3.1 Complex Frequency Response Function .......... 86
    4.3.2 Power Spectral Densities for a White-Noise Excitation 87

5 Concluding Remarks ........................................ 90

Bibliography ................................................... 94

A Spectral Analysis ........................................... 101
# Illustrations

1.1 The Rotary Drilling Process ........................................ 2
1.2 Common Modern BHA Configurations ............................... 6

3.1 Main Steps of the Study ............................................. 58

4.1 BHA Geometry for Numerical Results ............................. 69
4.2 Natural Vibrations Modes 1-3 for the “Maintain” BHA Geometry . 70
4.3 Natural Vibrations modes 4-6 for the “Maintain” BHA Geometry . 71
4.4 Relative Errors for Different Element Lengths .................. 72
4.5 Magnitude of the Transfer Function ............................... 74
4.6 Magnitude of the Transfer Function from the Frequency Axis .... 76
4.7 Magnitude of the Transfer Function of the String at the Bit .... 77
4.8 Transfer Function at the Bit as a Function of the Frequency of Excitation for Several Modal Damping Factor ..................... 78
4.9 Displacement of the String for an Excitation of 5 Hz Frequency . 79
4.10 Synthesized Time-History of the Lateral Force Applied at the Bit. 81
4.11 PSD of the Excitation, Magnitude Square of the Transfer Function and PSD of the Response for a White-Noise Excitation on the “Maintain” BHA Geometry ........................................ 83
4.12 BHA Geometry for Numerical Results ............................. 85
4.13 Transfer Function at the Bit for the “Build-Up” BHA Configuration 86
4.14 PSD of the Excitation, Magnitude Square of the Transfer Function
and PSD of the Response for a White-Noise Excitation on the
"Build-Up" BHA Geometry ........................................ 89
Chapter 1

Introduction

Standard rotary drilling equipment can be separated in two distinct parts, having components in the bore-hole and others at the surface. The rotary table, the kelly and the rig are located at the top of the drill, whereas the drill-pipes, the drill-collars and the drill-bit are immersed in the bore-hole.

In order of construction, from top to bottom, the kelly, the drill-pipes, the drill-collars and the drill-bit together constitute what is referred to as the "drill-string". The lowest part of the string, namely the drill-collars and the drill-bit, is referred to as the bottomhole assembly or BHA. The kelly is submitted to motor torque and tension force exerted by the rig and the rotary table above it. The drill-pipe, which is under tension, supports the weight of the usually heavy drill-collars, which are for the most part under compression, supplying the weight on bit (WOB), and the torque transmission necessary for drilling. The drilling fluid, also called mud, is circulated down the hole inside the drill-string and up in the annulus between the well-bore and the drill-string. At the end of each drill-pipe or drill-collar one finds the tool joints; the purpose of which is to enable the addition of more drill-pipes or drill-collars, which when added allow the drill-string to be up to several kilometers long, with a common BHA of 100 m length, and an average diameter of approximately 15 cm. The common rotary speed ranges between 60 and 120 rpm.

Payne [43] gives a description of the drilling process and the different components involved (see Figure (1.1) from [43]).
Figure 1.1 The Rotary Drilling Process
1.1 Drill-String Presentation

The steel tube which extends from the surface of a rig down to the bottom of the bore-hole, namely the "drill-string", comprises two major components: the drill-pipes, and the BHA. The drill-pipes are steel tubes located on the upper part of the string. The pipe section constitutes most of the length of the drill-string since the BHA length is usually of the order of fifty meters. This BHA is located at the very bottom of the string and defines a section made of more drill-pipes, of drill-collars and the drill-bit. Drill-collars are heavy-weight steel tubes the purpose of which is to provide weight-on-bit. The drill-bit is located at the extreme bottom of the string and its function is to crush the formation. The specific design of the bit depends on the type of formation one is drilling into. The purpose of the stabilizers is to maintain the string as much as possible in the center of the hole. They are found all along the string especially as part of the BHA. Their placement defines the direction of drilling.

The continuous evolution of drilling techniques since their inception in 1859, has resulted in what is now a fairly universal form of rotary drilling. This standard drilling technique relies on the combined application of rotary-downward movement on the drill-bit, thus providing the necessary force needed to crush the rock. As the formation is being crushed by the bit, the small fragments of rock are transported upward between the drill-string and the bore-hole by use of the mud.

1.2 The Drilling Fluid

The drilling fluid, or as it is better-known the "mud", is a water-based chemical fluid which is moved down the hole inside the drill-pipe and upwards in the annulus between the string and the wall. The mud has several functions and plays
an important role on the behavior of the whole string. Its primary purpose is to lubricate the bit since as it arrives at the drill-bit from inside the pipe, the mud is “projected” outside of the string at some defined pressure by the use of nozzles. Then its function is to prevent it from failure by helping the crushing of the formation. It is further used as a coolant since the drilling process produces high temperatures. Another purpose of the mud is to remove the rock fragments as the formation is crushed by the bit. The debris is transported back to the surface by the drilling mud using the rotational movement of the string.

At the surface, the mud is first filtered to separate the rock fragments from the fluid, cleaned-up, and eventually modified before being sent back down the hole.

1.3 Evolution of Drill-Strings: the Role of Drill-Collars

In 1997, Wilson [53] recalled that failure of drill-strings has always been part of the drilling process. In attempts to dig deeper and apply higher WOB, this problem became more and more frequent, since the drill-pipes in these cases are rotated in compression, causing cyclic stress reversals, ultimately leading to rapid fatigue failures. As a solution to this problem, drilling-crews started to use heavy bars of steel, known as drill-collars, which were placed on the drill-string between the drill-bit and the drill-pipes with purpose of increasing the WOB. At that point another problem arose, namely the insufficient amount of drill-collar weight placed on the bit, which allowed the bit to bounce.

Additional difficulties started to be encountered when directional drilling started to be of interest, due to the fact that all the weight of the collars was not applied on the bit (see also Chen and Géradin [13]).

The placement of stabilizers depends partly on the kind of formation one is drilling into and partly on the particular trajectory one selects to drill at. Several
typical BHA geometries can be recognized for one to achieve a “build-up” (increase of the angle between the axis of the string and the vertical), a “drop-off” (decrease of the same angle) or a “maintain” drilling pattern. Figure (1.2) shows three modern BHA configurations to achieve (a) a “build-up”, (b) a “maintain” or (c) a “drop-off” respectively. It is important to recognize that the shape of drill-strings as they exist today is the result of years of experiments, measurements and cumulative experience. The placement of drill-pipes, drill-collars, stabilizers, shock absorbers and other string components is critical in defining the specific performance of a given drill-string. The idea of using both drill-pipes and drill-collars to prevent buckling was previously studied in 1950 by Lubinsky [39]. Additional considerations will be introduced in an ensuing section.
Figure 1.2 Common Modern BHA Configurations
Chapter 2

Theoretical Background

2.1 Preliminary Remarks

The dynamic behavior of drill-strings is dependent on phenomena such as whirling (Vandiver et al. [33], Shyu [48], Den Hartog [32]), stick-slip motion of the bit (Kyllingstad and Halsey [36], Lin and Wang [38], Dawson et al. [19], Bo and Pavelescu [16], Spanos et al. [49]), bit-bouncing (Spanos et al. [49]), over-critical wear (Lubinsky [39], Huang [28]), well enlargement, and lateral instabilities. These phenomena are very common and make the task of modeling the behavior of drill-strings a complex one. The occurrences of these phenomena cause frequent drill-string failures and also reduce drilling efficiency by slowing the rate of penetration (ROP) (see also Halsey et al. [25]). An ideal analysis of drill-string behavior would consider the effects of tools such as stabilizers, together with the influence of the mud, the three dimensional curved geometry of the well-bore, and the changes in the geometry of the string (Berlioz et al. [3]).

In 1988, Aarrestad and Kyllingstad [1] re-introduced the concept of down-hole vibration and its possible influence on the lowering of the ROP and the fatigue failures of strings.

In 1993, Van der Heijden [27] gave a more precise idea of the different types of vibrations which occur in a drill-string and their influence on its behavior. Transverse vibrations are usually due to mass out-of-balance, whereas axial vibrations generally find their origin in bit-bounce and kelly motion (see also Halsey et al. [25]). Note that friction plays an important part in the genesis of tor-
sional vibrations, and that vibrations are in general found to contribute to collar degeneration and stabilizer fatigue.

In 1986, Halsey et al. [25] recalled that the lateral vibrations of drill-strings are associated with buckling in the well-bore. These vibrations, together with axial vibrations, can be observed from the rig floor, whereas torsional vibrations because of their low traveling speed are more difficult to observe. Halsey emphasizes the importance of considering torsional vibrations as a cause of failure by fatigue, and drill-string problems in general. This is because vibrations with high torsional amplitudes cause the drill-string to be unceasingly loaded and unloaded, which ultimately diminishes the drill-string strength. In 1987, Zamudio et al. [55] pointed out two kinds of vibration responses: forced and self-excited vibrations.

Forced vibrations is the term used to describe a time-varying, external force which is not correlated with the motion it is the cause of. As an example, the axial vibrations induced by roller-cone drill-bit are forced vibrations since the axial modes in the drill-strings are excited by the three-lobed pattern in the rock, independently of the vibration response. The most important point when studying the influence of forced vibrations on the behavior of drill-strings is the tuning of frequency which could exist between the frequency of the external excitation, and one of the natural frequency of the string.

In the case of self-excited vibrations, the forcing function is dependent on the reaction that is produced in the drill-string itself. For instance, cessation of the response would imply cessation of the driving force.

In 1987, Skaugen [22] argued that an ideal model of a drill-string would probably involve a “full, three-dimensional, non-linear” model which would consider the contact between the drill-bit and the rock. A complete model such as this, although
attempted by a few (cf. Birades [5]), has not yet been achieved with any adequate degree of success.

In 1996, Yigit and Christoforou [54] stated that a full model of the dynamic behavior of drill-strings is quite impractical and therefore favored a more classical approach, that is, to examine the vibration mechanisms individually. Van der Heijden [27] also defended this position, pointing out that even if it has been demonstrated that some correlation does exist between the different vibration mechanisms, they also occur separately, as it can be noticed over long periods of time.

Based on these considerations, this section is separated into different sections, each related to a different kind of model found in the literature. Special attention is devoted to the stick-slip and the lateral excitation models.

2.1.1 Friction

Before examining any model, the friction applied to the drill-string itself has to be considered. Rao [46] gave an account of the different kinds of damping one can encounter. The two most important types of damping in BHA modeling, that are viscous and Coulomb damping, along with some additional forms of friction, are introduced below.

Viscous Damping

Viscous damping considers the action of the fluid surrounding the pipe. The fact that the pipe and the mud are moving with respect to each other implies that the former creates a force which is opposite to the motion of the latter. This force is proportional to the vibration velocity of the string.
In 1976, Chen et al. [14] pointed out that when a structural component such as a drill-string vibrates in a viscous fluid, the presence of the fluid creates a reaction which can be modeled as an added mass, and a damping influencing the dynamic response of the string to the excitation. From experimental results, an added mass correction factor was introduced, which was to be multiplied by the mass of the component immersed in the drilling mud. The problem was further considered from an energy point of view and energy losses were assumed due to the vibration of the drill-string in the mud, the internal friction of the string, and the viscosity of the mud which implies various fluid drag forces and non-perfect support location. Finally, it was found in most of the cases, that the internal and support energy losses were small or negligible compared to the fluid viscosity effects.

From a study they conducted in 1995, Chen and Géradin [13] were lead to the conclusion that in most cases, it is very difficult to model an accurate added fluid dampening, and thus took it as an equivalent damping value.

**Coulomb Damping**

Dry or Coulomb friction is the name given to the friction factor coming from the rubbing of two objects moving with respect to each other. In the case of drilling engineering, the contact between the drill-string and the wall or the bit and the formation creates an added damping which is constant in magnitude and directed in the opposite direction of the motion of the string.

It should be noted however that in most of the existing BHA models, it is either exclusively the viscous damping or the dry damping which is considered, since a consideration of both generally results in a non-linear system.
More Types of Friction

The previous two sections dealt with the most important friction models involved in BHA dynamics. However, some additional factors dealing with friction also influence the string behavior.

Elsayed et al. [23] introduced in 1994 a process of damping caused by the interference between the drill-bit and the previously cut formation. This phenomenon was assumed to cause an increase of stability, especially at low speeds and high frequencies.

In 1991, based on a set of experimental field data, Dufeyte and Henneuse [21] noticed that the lubricant found in the mud has more effect on the nature of the friction than on its value. It was concluded that the use of a given type of lubricant enables one to overcome the problem of very strong vibrations, which ultimately relieves the mast of the string. From a "whirling point-of-view", Lichuan and Sen [37] noticed that if the rate of whirling of the drill-collar is not the same as the rate of rotation of the drill-pipe, an internal damping of the collar material appears which induces an unstable motion. In order to consider this internal damping without having too complex a model, it was assumed to be a viscous damping. In 1995, Spanos et al. [49] mentioned the existence of friction due to internal damping but assumed it negligible.

Skaugen [22] also mentioned the existence of an internal damping mostly due to wave radiation from the joint offset and the difference in diameter between each component of the drill-pipe. This was also assumed to be insignificant.
2.1.2 Damping Models

Lin and Wang [38] examined the two fundamental differences between viscous and linear damping. Viscous damping shows a static friction which is greater than its kinetic friction. Furthermore, as the relative speed between the moving bodies increases, the damping friction gradually tends toward a constant value.

The consideration of a given type of damping relies on its ability to fit the requirements needed for a given type of model (longitudinal, lateral, torsional or coupled aspects). These more specific considerations are treated in corresponding sections in which the particular models are discussed.

2.2 Longitudinal Motion

2.2.1 Preliminary Remarks

Primarily, three longitudinal forces are applied on a string: the WOB, the reaction of the formation at the bit, and the fraction of the weight which is supported by the kelly. The WOB is induced mostly by gravity acting on the collar section of the string, whereas the reaction of the formation at the bit is opposed to the WOB direction.

For drilling to be efficient one must be able to balance these three forces. That is, efficient drilling must avoid buckling of the string while maintaining a sufficient ROP by means of a balanced WOB.

In an historical review of the modeling of drill-strings axial behavior, Wilson [53] observed that when drilling-crews started to reach deeper targets or drill in harder formations, they were confronted with more drill-string failures. From this very fact, it was learned that driving a drill-string in a rotational motion under com-
pression induces some cyclic stress reversals which lead to fast fatigue failures. To counter this problem, drill-collars were introduced and drill-strings were no longer driven in compression but rather, for the most part, in traction.

One of the sources of axial vibrations pointed out by many authors is due to the multi-lobe pattern of the bottom of the well when drilling with a three-cone drill-bit. This was given particular attention by Skaugen [22] in 1987 and was distinguished as the most important source of longitudinal vibrations.

2.2.2 Problems

In 1995, Spanos et al. [49] noticed the wide and frequent fluctuations of the WOB while drilling with roller-cone bits. The WOB was frequently observed to suddenly drop to a zero-value which was assumed to be due to a drill-bit lift-off. At the time when the drill-bit was off the formation, it was noticed that the torque also dropped to zero, and this phenomenon was referred to as a "bit-bounce".

In a study based on experimental data in 1968, Cunningham [17] attributed bit-bounce to two main factors, namely the existence of lobes in the formation surface at the point of contact with the drill-bit, and the fluctuations in the drilling-mud pressure.

From these considerations, it was suggested that a tuning of such excitations with the axial natural frequencies of the drill-string would lead to large amplitudes of the longitudinal vibrations of the string, and thus in drill-bit lift-off. From experimental data, it was also found that when the weight fluctuations were small, the axial accelerations of the drill-string seemed to occur at a higher frequency independent from that of the weight. As a concluding remark, it was stated that the "WOB is synchronous with the axial motion and that the bit is bouncing
appreciably". Taking into account these axial accelerations, a vertical estimation of the possible maximum displacement of the bit was derived. Data provided that the axial displacements are sometimes transitory in nature as are many of the torque and weight fluctuations.

2.2.3 Models

The axial vibrations due to intermittent contact of bit teeth were first studied by Paslay and Bogy [42] in 1963. In 1986, Yigit and Christoforou [54] had noted that tension in the string increases the free vibration frequencies, whereas compression decreases them; buckling occurs when the lowest natural frequency is null. Given that in drilling engineering the amplitude of the lateral motions of the string is usually restrained by the presence of the well walls, it is significant to consider compressive loads higher than the critical load of the string. In other cases where the lateral motion of the considered beam is not limited this is purely of "academic interest". In fact, an applied load on the drill-string higher than its critical load implies some contacts and impacts between the string components and the wellbore. This factor must also be considered in an ideal model (this will be dealt with in a ensuing section).

In 1987, Skaugen [22] introduced a model to investigate the behavior of the drill-string under longitudinal forcing functions. As a first approximation, these axial vibrations were treated separately from any coupling with torsional or lateral vibrations. This approach is strengthened by the fact that the different vibration modes usually occur at the end of the drill-string, near the drill-bit, at least in linear models, and that they must therefore be included in the axial vibration forcing function.
A friction model which included both Coulomb and viscous damping was introduced. The dry friction is induced by the interactions between the drill-string and the wall. The viscous friction is mostly due to the presence of the drilling mud. In dealing with viscous damping, the total friction was shown to be dependent upon the former behavior of the drill-string, together with a dependence on its velocity with respect to the medium of friction. This lead to the discovery that the mud was also moving axially, and that it was partly dragged around the drill-string. Therefore, a friction model for the viscous damping was used which took into account a dynamic friction force arising from the interaction of the string with the mud. An added friction coefficient coming from the wave radiations was also considered, due to the variations of the diameter of the drill-string (breathing effects) and the joint offsets. However, it was noted that these latter effects were of minor importance compared to that of the pure viscous friction induced by the mud. The main vibration frequency was found to be generally a multiple of the rate of rotation of the string and when strong resonance was measured at the top of the rig, it was usually inducing pronounced resonance at the drill-bit (see also Spanos et al. [49]).

2.2.4 Boundary Conditions

In order to accurately describe the axial motion of the bit, Spanos et al. [49] considered the boundary condition at the bottom of the string to be of two kinds. The first possibility was to consider the external force to be defined, whereas the second assumed the displacement of the bit to be known.

As interest focused on modeling the lift-off of the drill-bit, the boundary condition was considered to be of the first kind with an external force of zero-value when the bit was off-bottom. The end of the string was then treated as a free
end. On the other hand, when the drill-bit was in contact with the formation, the boundary condition was assumed to be of the second kind, and a model in which its displacement was forced by the profile of the formation was provided. It was noticed that the alternation of the two kinds of boundary conditions tended to couple axial and torsional vibrations of the string.

In 1987, Skaugen [22] modeled the drill-string as having fixed upper boundary conditions where it is attached to the draw-works. He further stated that because of its considerable mass, this equipment had to be added as a factor in the model, and thus considered an induced elasticity, together with a friction damping attributed to the hanging of the string in the draw-works.

2.3 Lateral Motion

2.3.1 Preliminary Remarks

In 1995, Chen and Gérardin [13] noted that while axial and torsional vibrations appear up-hole and can be studied there, it is not the case for lateral vibrations. This perhaps contributed to the delay of their consideration as these can only be studied down-hole. The reason for this is the low traveling speed of bending waves, whose nature implies that it is damped by the contact with the wall and by the viscous and dry damping along the drill-string.

In 1993, Van der Heijden [27] observed that lateral vibrations can be caused by mass out-of-balance. This structural flaw can be attributed to several factors, among which the presence of measurement-while-drilling (MWD) tools can be
highlighted. Since most of the currently used drill-strings have such equipment, it is quite common to encounter the mass out-of-balance phenomenon.

In 1991, Jansen [30] approached the problem of lateral vibrations of drill-strings using elements of rotor dynamics. He recalled the importance of the consideration of such a factor, emphasizing the fact that in some cases the rotation of the drill-string by itself induces lateral vibrations of the collars.

2.3.2 Problems

Jansen [30] noticed that lateral vibrations can be caused by a slight misalignment of the drill-string components and/or the fact that drill-collars sometimes contain electronic components which imply a mass out-of-balance. These two characteristics induce some lateral vibrations with amplitude at maximum when the rotary speed of the string approaches a natural frequency of the collars. Since the study was based on a rotor dynamics background, it was recalled that the analogy between this topic and drill-string dynamics is unfortunately hampered by the presence of non-linear terms due, amongst other things, to the presence of the drilling-mud, the stabilizers clearance, the stabilizer frictions and the bore-hole contacts. The importance of considering contacts of the drill-collars with the bore-hole between two stabilizers was pointed out and this phenomenon was accounted for. The effect of wall contact itself was however approached only in descriptive terms. The main focus of the study was on whirl-induced deflections which was believed to lead to these kinds of wall contacts.

In 1993, Van der Heijden [27] enlarged the study previously undertaken by Jansen [30] and considered alternative possibilities for a forward-whirl of a drill-collar to
be transformed in a backward-whirl. The model, the basis of which is known under the name of Jeffcott rotor in rotor dynamics, is in its consideration of non-linearities pushed further than the one it is inspired of. Non-linearities were due to the presence of the mud, and the bearing forces. Special emphasis was given to determining the vector field associated with the dynamic system as usually treated in contact dynamics. The techniques used in the solving of his problem included numerical continuation and bifurcation.

Buckling

In 1995, Suryanarayana and McCann [51] gave a description of the buckling phenomenon of drill-strings. Buckling occurs on a shaft when a critical value of the compressive load is exceeded. This critical load is a value beyond which the equilibrium shape of the shaft becomes suddenly unstable, which with each little increase in the load can result in a large lateral deflection. In the case of drilling engineering, the load corresponds to the WOB or the slack-off friction. The peculiarity of drill-strings resides in the fact that their lateral motion is constrained by the presence of the bore-hole, making it impossible for them to buckle freely when submitted to load values exceeding their critical value. Therefore, the buckling progresses in a different manner to that of the most common case known as the Euler buckling bars. In drilling operations, when the load applied on the string exceeds its critical value, the bar starts to buckle in a sinusoidal shape along the lower part of the hole. This is referred to as snaking or sinusoidal buckling. As the load is further increased, part of the string starts to lose contact with the bore-hole, and the amplitude of the sinusoidal buckle increases. Eventually, the load can reach a second critical value which contorts the string into an helix shape and
into total contact with the bore-hole. In these cases the drill-string is said to be *helically buckled*.

Between these two critical values of the compressive load, most of the change in the WOB is found to distort the drill-string from a straight to a helical shape. Once it is helical, an increasing part of any subsequent added load is radially directed and increases the contact force between the drill-string and the well-bore, the effect of which is to increase the drag. If the lag happens to be large enough, it results in a situation whereby any further increase of the load at one end of the string remains undetected at the other end. This predicament is referred to as a *locked-up* situation. Thus, buckling sets a limit on the maximum load carried by drill-strings, especially when drilling reaches deep targets. On the other hand, post-buckling phenomenon (lock-up) limits the reach of the drill-string in extended reach wells.

In 1950, Lubinsky [39] was driven to several conclusions dealing with buckling of drill-strings. First, it was found that only a very small WOB is required to make a drill-string only composed of drill-pipes buckle. On the other hand, a drill-string composed of drill-collars only can withstand a much higher WOB. However, they are commonly found to be buckled once, sometimes twice or even three times in normal use. Finally, the critical WOB decreases as the density of the mud increases. However, in most of the cases encountered, except when heavy drill-collars are in heavy mud, the influence of the mud is negligible.

As a means to avoid buckling of drill-strings, Lubinsky gave several alternatives. The simplest way is to have a WOB less than the first critical value of the compressive load. Nevertheless, such a weight is usually too low to have a satisfying ROP and should be applied, in order to meet economical requirements,
only when particular care is taken in order to have a straight hole or to avoid a
cave to enlarge. Increasing the parameters of the drill-string, such as using larger
drill-collars, is also an efficient way to increase the critical value of the compressive
load. For instance, larger drill-collars induce a larger moment of inertia and higher
weight per foot, both leading to a higher critical WOB. Lubinsky’s work showed
that a slight increase of the diameter of the drill-collars induces larger amounts
of gained weight before reaching the first critical load. As a consequence it is of
interest to consider the use of larger drill-collars while keeping the size of the bit
constant. Therefore, decreasing the clearance between the collars and the wall
increases the maximum value of the admissible WOB.

Lubinsky also attached some importance to the shape of buckled drill-strings.
It was concluded that it is of almost no interest to consider a drill-string made
only with drill-collars or with drill-pipes since the shapes in both cases are very
much the same. The notion of tangency point was also introduced, that is, the
location at which the buckled drill-string touches the well-bore. It was noticed
that the part of the drill-string located between this tangency point and the drill-
bit is progressively deflected while the portion of the string situated upper this
tangency point is progressively straightened. Also noted was that the position of
the tangency point moves slowly towards the drill-bit as the WOB is increased
between the first and the second critical loads. As the weight is increased beyond
the second order critical value this downward displacement of the tangency point
is accelerated.

2.3.3 Models

In 1984, Bernitsas and Kokkinis [4] considered the buckling of strings where special
cases of boundary conditions were treated. A drill-string was modeled with a long,
slender, open-ended, heavy tubular column with a tension exerted at its top where both internal and external static pressure forces due to fluids in the gravity field were included.

In 1991, Jansen [30] determined that the drilling-fluid surrounding the drill-collars introduced a complicated pattern of forces. Both an added-mass term and a velocity-squared dependent drag force were included. The effect of the clearance between the string and the bore-hole, and the friction between the stabilizers and the wall were considered. It was assumed that the hydrodynamic bearing forces were developing in a negligible way. From the observation that the rotary speed of a drill-string is often found to be close to the lowest natural bending frequency of its stabilized part, it was assumed that the dynamic behavior of the string could be simplified by considering only the lowest bending moment. The analysis was further simplified by focusing on a constant rotary speed only, and the consideration of a single span of drill-collars supported by stabilizers at the ends.

The drill-collars were modeled as a rotating shaft supported by two bearings and the first bending mode of the collar section only was studied, thus reducing the problem to a mass-spring system whose motion is constrained in a plane perpendicular to the bore-hole axis. By using a virtual work approach, the equivalent mass, damping and stiffness of his system were derived from the characteristics of the drill-collars. Assuming a constant drill-string speed, the problem was then reduced to the discovery and the analysis of the two translational motions of the system.

Inertia forces resulting from the acceleration of the collars, and the acceleration of the drilling-fluid were also detected, where the former is acting at the center of mass of each drill-collar section, while the latter acts at the geometric center.
It was further assumed that the contact between the stabilizers and the bore-hole wall lead to the introduction of a restoring force which was directed towards the center of the bore-hole. The exact direction of the force was unknown due to the presence of friction. However, it was understood that the stabilizers were slipping on the wall, and as result were eliminating the friction. The restoring force was found to be equal and opposite in sign to the stabilizer contact force, and the system was considered in polar coordinates.

The model lead to the conclusion that if the stabilizer clearance was exceeding the mass eccentricity of the drill-collar, no stable forward whirl was able to occur for rotary speeds of the drill-string above its natural frequency. Assuming that due to the usual roughness of the wall, the friction between stabilizers and the wall was usually high, the occurrence of stable whirl was found unlikely as long as the mass eccentricity was of a value below that of the stabilizer clearance.

Beside this forward motion of drill-collars another periodic effect, namely a self-excited backward rolling of the stabilizers, can result from the friction between the stabilizers and the bore-hole. This phenomenon lead to the conclusion that it is the friction between the stabilizers and the bore-hole wall that leads to backward whirl. In a case in which the friction is very low, or the damping is very high, it was found that no backward whirling could occur since the contact force angle is higher than the friction angle. Finally, it was concluded that when considering a bore-hole section, the chaotic drill-collar motion was highly sensitive to the choice of the boundary conditions, and that the unstable motion of the collars did not converge towards a circular steady-state motion but remained in an annular region.

A further result was that whenever the bearing clearance was of a higher value than the mass eccentricity, it lead to large destabilizing effects of the motion of the collars. The limitation of the two-degrees-of-freedom (DOF) study was also
noticed, and it was stated that a more important number of DOFs should be considered in order to have an accurate model, especially wherever the response of the system includes high-frequency components. Finally, a more complete model would involve the effect of gravity, wall contact, and coupling between the three modes of vibrations. The presented model lead to several observations; the first of which being that the velocity-squared fluid damping drag force reduced the whirl amplitude of the drill-collars, whereas the considered added fluid mass reduced the speed at which the whirl amplitude was at maximum: the critical rotary speed. Secondly, the critical speed was also reduced with stabilizer clearance, and the motion of the string became unstable when this clearance exceeded the mass-eccentricity of the collars. It was possible for forward whirl to develop only at speeds below the rotary critical speed. The whirl amplitude was decreased by stabilizer friction, which also lead to the reduction of the frequency range of the stable forward whirl. Furthermore, as long as the stabilizer clearance exceeded the mass eccentricity of the drill-collars, stable forward whirl was unlikely to happen. The stabilizer friction could also be a cause of self-excited backward whirl, the frequency of which was usually approximately equivalent to the natural frequency of the section of the given collar. Finally, large collar deflections could lead to wall contact during forward or backward whirl. In the case of a backward whirl, this contact could result in a “drill-collar precession” where the collar had a backward motion along the bore-hole.

Further consideration of the whirling process of drill-strings is developed in an ensuing section.

Van der Heijden [27] considered a two DOFs system as a model of drill-strings, assuming that the drill-collars were in their first bending mode. Velocity-squared
fluid damping force and drill-collars mass eccentricity were included in the equations of motion. The mass out-of-balance was assumed to be the driving force of the system by having an acentrifugal force, and a restoring force resulting from a small bearing clearance of the stabilizers on the wall was considered. Taking the path first studied by Jansen, a Coulomb friction was assumed in order to model the rubbing of the stabilizers on the wall; the restoring force was both equal and opposite in sign to this stabilizer contact and was directed towards the center of the stabilizer.

A conservative model was first studied which neglected dissipation. This was then accounted for in a full system. The study of the complete system involved features such as mass out-of-balance, bearing clearance, and stabilizer friction. This lead to the conclusion that different instabilities and motions were likely to occur, and especially that forward whirl coupled with the slipping of the stabilizers along the wall could be observed for a relatively wide region in parameters space. This motion is most likely to occur when mass eccentricity is higher than the bearing clearance with relatively little friction.

When confronted with a case in which there was a mass eccentricity less than the bearing clearance and resonant driving frequencies, it was noticed that several types of quasi-periodic motions dominated, leading to free motion at large frequencies. Next to these, chaotic motions were also found, thus confirming Jansen’s results ([30]). However, the model had two DOFs and was based on only the first bending moment. Since a real drill-string has an infinite number of those moments, the limitations of the presented study for very large frequencies were accounted for. Friction was found to cause synchronous forward whirl at certain driving frequencies, which could lead to a non-synchronous whirl. The whirl presented a cyclic loading of the drill-string, and transitions between forward and backward
whirl were likely to occur. A perfect backward whirl was found when the zero mass eccentricity limit was reached. It was therefore concluded that such a motion couples a rolling and a slipping of the stabilizer on the wall.

When dealing with forward whirl, the BHA was found to have a rigid rotational motion around the bore-hole center. In these cases the bending moments of the deflected drill-string were found to be constant, inducing the stabilizers to slide against the wall without wearing.

A more serious problem for the security of the BHA was found in friction-induced backward motion, since this type of motion might bring into influence strong fluctuations of the bending moments of the string. This could ultimately lead to a possible connection fatigue. As a final observation, it was found that the chaotic motion was predominantly forward. However, in cases where friction was accounted for, results presented several ways for the whirl to switch from forward to backward.

In 1995, Chen and Géradin [13] pointed out that both finite element method and finite difference technique are convenient ways to study the lateral vibrations of drill-strings. However, both require large computer memory to be implemented for long BHA system. This presents practical difficulties when used in the field. As an alternative way of predicting such vibrations of drill-strings on site, a transfer matrix method was introduced and referred to as “the simplest method in the analysis of shaft and beam systems”. Further, a modified transfer matrix method was also introduced which offered the option of modeling the drill-string as a continuous system instead of a lumped-element system. A second modification of the basic method was presented which accounted for synchronous elliptical orbit and non-synchronous multi-lobed whirling orbit; this work did not however consider axial
load. The analysis was based on several assumptions. First, strong non-linearity was not included in the model, which lead to neglect the bit-formation contact, the stabilizer-hole contact and the BHA-hole contact. Recalling that Shyu [48] provided such a model using the same transfer matrix technique, it was nevertheless stated that the best way to consider these contacts was to approach the problem with a finite element model. The decision to disregard torque was made in the light of Shyu's hypothesis and motivated by the relatively small influence of the torque.

A first approach was to model the string as a uniform shaft on which a constant compressive longitudinal load was applied and where the stabilizers were modeled as spring support. The modeling of the complex drill-string/mud interaction was simplified by considering only viscous damping and an added mass coefficient. However, complex flows occur as the mud is compressed against the wall. This phenomenon was accounted for by an increase of the added-mass coefficient as the clearance decreased.

The rotating string was studied as a static beam, which lead to neglect the relatively insignificant gyroscopic effects. It was found that as the WOB increased the natural frequencies of the string decreased. This influence of the WOB was demonstrated to be significant since the first three natural frequencies of the string were found near its rotating speed. From the same set of data, it was also noticed that the eigenfrequencies of the string were lowered by the mud, a phenomenon which was particularly significant for the first natural frequencies. In order to model the effects of the bit induced motion on the BHA dynamics, the whirling motion of the bit was considered as an excitation source. This was made possible by the integration of the forward and backward motion of the bit.
Finally, the placement of stabilizers was emphasized as an indispensable contribution to the avoidance of string buckling. From numerical results, the transfer matrix method was revealed to accurately model the behavior of linear BHA response.

In 1995, Vaz and Patel [52] used an algorithm based on a Galerkin method. This algorithm was chosen for its ability to provide accurate results when submitted to a problem where the string pendulum length is large and the applied WOB small. However, it was stated that in cases where the BHA is more complex, such as having differences in its geometry or in material, and in cases where the shape of the hole and the stabilizers arrangements are accounted for, a finite element model would be more appropriate. In such a complex model other factors such as inertia, damping forces and applied torque are often incorporated.

The study was based on a uniform drill-pipe with a cylindrical bore-hole cross-section. The pipe material was assumed to be homogeneous elastic. Small displacements were considered, which enabled to solve the eigenvalue/eigenvector problem for lateral vibrations. It was found that short drill-strings usually do not have a static mode, and do not buckle in second and higher modes, even when their weights are totally applied on the drill-bit. Furthermore, short drill-strings usually present high natural frequencies, and require quite a substantial WOB in order to buckle. A statically stable drill-string may start to vibrate when excited by a forcing function in one of its natural frequencies. Finally, an increase of the WOB applied on a string induces a decrease of its stiffness and of its natural frequencies. The model lead to the conclusion that the Galerkin approximation method can successfully be applied to the treatment of drill-string behavior both in vertical and deviated bore-holes. The dynamic analysis of vertical drill-strings highlighted
the importance of factors such as inertia forces in order to have an accurate study of the string stability. This study also showed that the Galerkin approximation method was superior to other methods when applied to cases which involved a deviated bore-hole.

In the same year, Suryanarayana and McCann [51] conducted experiments on rods and beams. These rods were loaded to a maximum compressive load, then unloaded back to their original state in order to study their buckling. The experimental set-up involved inserting the rod into a clear extruded acrylic tube which was attached to adjustable blocks. The far end of the modeled drill-string, also called the top end, was displaced by the use of a linear actuator. Load and displacement were recorded by a computer data acquisition system.

It was assumed and later confirmed that rods underwent only elastic deformation during experiments. High-frequency, small-amplitude transverse vibrations were imparted to the outer tube by a vibrating device which, when needed, enabled a reduction of the vibrations. By repeating the test many times, the effect of friction on the response of the rod were studied.

It was demonstrated that the critical buckling load was significantly higher when friction was applied. Accounting for the friction lead to notice hysteresis post-buckling behavior. A direct implication of this hysteresis behavior is that each unbuckling load is always less than the corresponding buckling loads. The mitigation of the friction reduced the hysteresis behavior so much that the friction was believed to be the dominant, if not the only, cause of hysteresis. Another result found with non-negligible friction was that the equations used in the industry were predicting unbuckling rather than buckling loads. The friction was also found to be a cause of post-buckling snapping and reversals in the direction of the helix. It
was finally demonstrated that when friction is non-negligible, the critical sinusoidal and helical buckling loads in straight well-bore intervals are underestimated by the most common used equations. The theoretical load found by these equations is significantly below the real critical load of the string.

2.3.4 Boundary Conditions

In order to evaluate the free lateral vibrations of a drill-string during horizontal drilling, Chen and Géradin [13] considered a simply supported beam with two internal elastic supports. A second model where the BHA was supported by two internal stabilizers was implemented and demonstrated the importance of the placement of stabilizers in the buckling of strings.

Suryanarayana and McCann [51] considered both fixed and free ends as potentially accurate boundary conditions for the string.

Like Lubinsky [39] and Huang and Dareing [28], Vaz and Patel [52] considered a hinged boundary condition for the string at both ends.

Bernitsas and Kokkinis [4] defined two sets of boundary conditions. The first considered rods with a movable boundary condition at their tops (or bottoms) and the second assumed non-movable boundary conditions. Both cases involved rotational and linear springs at the ends of the rods.

The movable boundary conditions included hinged and clamped columns at their upper and lower ends. A further result was that the asymptotic behavior of the stability boundary was of an exponential and geometric form and independent
of the stiffness of the rotational spring at the top. Fast convergence of the stability boundaries to their asymptotic form was noticed.

The non-movable boundary conditions included a restrained top support of the string in the horizontal direction. Imperfections of the rods were accounted for with flaws such as misalignment of the ends. However, this did not affect the boundary conditions.

These sets of movable and non-movable boundary conditions lead to results which were of better accuracy than the ones achieved by classical numerical methods.

2.4 Torsional Motion

2.4.1 Preliminary Remarks

Torsional vibrations are found to be harmful to drilling equipment. In 1991, Dufeyte and Henneuse [21] pointed out that they are a potential source of fatigue or wear of components and therefore induce energy losses and/or low ROP. To simplify the study of torsional vibrations, Halsey et al. [25] formulated the hypothesis that only nominal drill-string parameters need to be considered.

2.4.2 Problems

Examining a set of experimental data, Cunningham [17] noticed large variations of the rotary speed of the string. The recorded data also featured a stop-and-go pattern of the motion of the bit. Jansen et al. [31] referred to torsional vibrations as a main cause of reduced ROP, twist-off of the drill-string, overtorqued drill-pipe connections, and premature bit wear. Wilson [53] focused on the stick-slip phenomenon and dealt with its influence on drill-string fatigue. It was concluded
that most of drill-string failures are due to fatigue damage. Correlation between fatigue and the rotational bending of drill-strings was also established. Torsional vibrations were further identified as producing cyclic stress reversals of components of the pipe above their endurance limit.

Stick-slip

Stick-slip motion of the bit is usually recognized on a rig as MWD tools indicate periodic stops of the bit for periods going up to several seconds. When such stops occur, the top of the string keeps rotating, twisting up the string, increasing the torque, and storing energy in the string which starts to act as a torsional spring. When the increasing torque is such that the bit cannot withstand it anymore, it starts to spin and the stored energy is suddenly released. This implies a spin of the bit which is usually so fast that the string unwinds and the torque drops. As a result of this phenomenon, the speed of the bit decreases, eventually to a complete standstill. From that point, the process of winding and unwinding of the string repeats itself. Since the energy and the torque required to keep the bit moving are higher than those which induce this rotation, the torque does not drop out during the stick-slip phenomenon. The energy lost by the string while it is rotating is regained when the bit is stopped. The occurrence of stick-slip motion is therefore based on the fact that the static torque is more significant than the dynamic torque (see also Brett [11]). An analogy can be made between the above process and the behavior of both the static and the dynamic frictions between two sliding bodies. This explains why the phenomenon is called stick-slip. It has been demonstrated that some features of strings are likely to induce stick-slip motion. As an example, polycrystalline compact bits, referred to as “aggressive”, are more likely to induce such a phenomenon than roller-cone bits.
Additional considerations of the stick-slip motion have also been established from numerical or experimental data, as in [31]. However, accurate descriptions of stick-slip motion are more readily found in theoretical works.

In 1982, Bo and Pavelescu [16] pointed out that when two bodies in contact can move with respect to each other in dry or boundary frictional contact, a stick-slip motion can occur. Relative motion between the two bodies occurs when the static friction between them is overcome. If a reduction of the friction force goes together with the relative motion of the two bodies, the sliding body increases its speed until the point where the friction force and the elastic restoring force equalize one another. At this point, the relative speed of the two bodies starts to decrease until a new stick period occurs.

It has been noticed that stick-slip motion can also be induced by the non-linearity of the dry friction between the two bodies moving with respect to each other (Lin and Wang [38]).

In 1988, Halsey et al. [26] defined the stick-slip motion of the BHA from the lowest torsional mode of the string (pendulum mode). Making such an assumption implies a view of this phenomenon as extreme self-sustained oscillations (see also Kyllingstad and Halsey [36]). The small value of the natural damping along the string allows the stick-slip motion of the bit to develop.

Dufeyte and Henneuse [21] referred to the stick-slip motion of the bit as the most well-known self-induced torsional vibrations phenomenon. It has a periodical occurrence and is generated by friction forces on the well walls. It can also be induced by an inadequate driving system which is unable to break out of the torque fluctuations. The stick-slip phenomenon was looked at as a result of excessive friction along the bit or the BHA which causes the bit to stop rotating. The experimental data depicted that this stop of the bit accounts for approximately
50% of the drilling-time. As the bit starts spinning, its velocity can increase and reach values as high as ten times the specified rotary speed of the string induced by the kelly.

In 1992, Brett [11] derived a model for stick-slip vibrations which he compared to experimental data. The study considered the string treated as a torsional spring with inertia and a lumped mass at its end. Matching theoretical results with experimental data, a favorable correlation of the two was noticed. The conclusion was formulated that having the bit on bottom is a necessary condition for torsional vibrations to occur. Furthermore, no torsional vibration was observed as the bit was whirling. Finally, a high WOB, together with a hard formation, a low rotary speed, and a dull bit were found to worsen vibrations.

A second model consisting of two coupled differential equations enabled to look at the non-linearities of the self-excited vibrations. The first equation defined the behavior of the string as a lumped-mass/spring-with-mass system, whereas the second equation described the behavior of the surface driving system. Self-excited drill-string vibrations were defined, both from the model and the experiments, as able to occur without a pure static friction effect. Furthermore, low rotary speeds were noticed to induce a higher rate of decline in torque with rotary speed. The absolute value of the reduction in torque increased with the WOB. Finally, dull bits drilling in hard rock formation induced a higher reduction in torque with rotary speed. The data did not feature a classic static friction effect, that is, where there is a high torque at zero speed and a constant torque for all other rotary speed. However, the dynamic torque was noticed to decrease with an increasing rotary speed.

In 1991, Lin and Wang [38] defined different characteristics of the stick-slip vibra-
tions. First, no stick-slip motion occurs for drill-strings shorter than their critical length; that is, for torsional natural frequencies higher than the critical frequency. For drill-strings longer than their critical length, stick-slip severity increases with the length of the string, given a fixed rotary speed. As the rotary speed comes closer to the critical speed, the frequency of the stick-slip vibrations comes closer to the natural frequency of the string. Finally, during strong stick-slip vibrations, the torque in the string may reach some negative value; the direction of the rotary speed and the one of the torque are then the same.

Dufeyte and Henneuse [21] based their study on the observation of thousands of hours of drilling and introduced a new measuring tool known as “the dynamètre”. The primary purpose of this tool is to detect torsional and longitudinal vibrations along the string. Field data yielded that the torque observed at the surface through its different shapes and its oscillation periods is one of the most accurate indicators of the presence of torsional vibrations. The torsional vibrations were defined to be of two kinds; the transient vibrations correspond to variations in the drilling conditions, whereas the stationary phenomena was due to self-induced vibrations.

Self-induced oscillations last over long periods of time. They could potentially lead to fatigue or failure of components. They also make it apparent that not only the BHA but the entire drill-string is submitted to torsional vibrations, which can be observed from torque and rotary speed oscillations.

Dynamic tension component, torque, longitudinal acceleration for tri-cone bits and rotary speed were found equally useful to predict the occurrence of stick-slip motion. A way to compute the rate of efficiency of the bit and the maximum downhole rotary speed was also provided. It was observed that torsional vibrations were
generated by maintaining a low or a very low rotational speed (less than 15 rpm) regardless of the friction, on or off bottom.

From experimental data, a critical speed was defined, above which no stick-slip motion would occur. Furthermore, it was demonstrated that increasing the WOB in a stick-slip motion situation worsened the vibrations. This coupling was attributed to the bending effect of the BHA. On the other hand, a decrease of the WOB enabled the bit to have a steady motion. Finally, high WOB was observed for high values of the rotation speed, which defined a clear correlation between the two.

Observation of transient phenomena was therefore introduced as an accurate tool to predict the occurrence of stick-slip phenomena. The method was even more effective for changing rotary speeds. However, foreseeing stick-slip motion is better accomplished by the study of the duration of transient phenomena rather than their amplitudes.

Stick-slip motion can lead the bit to be stuck up to 50% of the drilling-time and induces it to rotate at speeds up to ten times faster than the driving rotary speed. Occurrence of such a phenomenon therefore requires the equipment to work in domains above the constructor's limits. This explains the repeated occurrence of the premature wear of the teeth of the bit.

Kyllingstad and Halsey [36] used a simple pendulum model to describe the stick-slip motion of the bit with a non-linear self-excitation of lowest mode of the string. Theoretical work predicted a stick-slip frequency lower than the natural pendulum frequency and this stick-slip frequency decreased with rotary speeds. Finally, the magnitude of the dynamic top torque was found to increase with rotary speed
in a linear manner at normal rotational speeds while being less significant at low speeds.

Halsey et al. [26] complemented Kyllingstad’s and Halsey’s [36] work by presenting a way to avoid stick-slip phenomenon using torque feedback. The fundamental concept behind this theory is the control of the speed that the rotary table drives the string at. An accurate control of this speed leads to a dampening of vibrations. The torque feedback technique is therefore a compromise between two contradictory requirements, namely a constant torque and a steady rotary speed.

If the decay-time related to the impedance of torsional waves is much longer than the pendulum period of the string, a poor damping enables stick-slip oscillations to occur. On the other hand, having a decay-time of the same order or shorter than the pendulum mode makes it impossible for a stick-slip motion to occur. The torque feedback technique is therefore based on the control of the rotary speed with respect to the torque level. When a positive torsional wave is going up the string, the measured torque and the rotary speed both decrease. This process was demonstrated to direct most of the torsional energy back down the hole.

2.4.3 Models

Accurate Friction Model

The length of the BHA is relatively short compared to the one of the drill-pipe section when a full-length drill-string is looked at. Dawson et al. [19] therefore modeled the BHA as a localized mass at the bottom of the string which was treated as a torsional pendulum. The walls were accounted for by restricting the motion of the string. A friction coefficient along the string was postulated to take into account the curvature of the hole. This friction was assumed greater at the BHA
because of its mass and stiffness. Experimental data demonstrated that the torque induced by the rotary table has a significant time-dependence which sometimes leads to erratic contact of the bit and the formation. In these particular cases, the bit is said to be “running rough” or “torqueing up”. However, steady motion of the bit and low oscillations were also noticed. Large oscillations were found as the bit was off-bottom or rotating inside casing. This lead to the hypothesis that the rotational oscillations are related in a fundamental manner to the friction force between the string and the wall.

Lin and Wang [38] were interested in an accurate damping model for the study of stick-slip vibrations; and therefore reviewed existing friction models. In order to accurately model stick-slip vibrations, the kinetic friction of the string has to be less than its static friction; this requirement is not fulfilled by a Coulomb model. A polynomial model can only be applied when a narrow range of rotary speeds is of interest and is therefore not applicable to the study of stick-slip. Furthermore, a piecewise linear model is not accurate enough (see Dawson et al. [19] (1987)). Finally the discontinuous, two-friction model presented by Bo and Pavelescu [16] can be linked with other models. However, it is difficult to implement this model practically since it jumps from one exponential to another when vibration without sticking is considered. Based on these considerations, Lin and Wang introduced a continuous exponential form of the dry friction.

Kyllingstad and Halsey [36] assumed the viscous friction to be negligible and considered only a constant Coulomb friction. The model demonstrated that damping coefficients higher than a particular critical value makes it impossible for the speed of the bit to drop to zero. This implies that once instigated, the slip phase contin-
ues without alternating with a stick phase. Furthermore, as the speed increases, the bit speed amplitude comes closer to a mean speed.

Models

In 1991, Lin and Wang [38] considered the torsional behavior of a string modeled as torsional pendulum with an inertia moment and a torsional stiffness. Only one DOF was considered.

Kyllingstad and Halsey [36] also derived a pendulum model stating that it yields a reasonable approximation of the rotational behavior of the string. The round-trip time of torsional waves was neglected compared to the natural oscillation periods of the string. Theoretical results and experimental data were shown to match each other.

Dawson et al. [19] developed a model in order to analytically study the behavior of a string that has its bit off-bottom. The entire length of the pipes was assumed to be lumped into a torsional spring with a given stiffness and without inertia. Two equations were implemented to model a piecewise linear friction model. This friction model featured the important property of a static friction higher than its dynamic friction. The conclusions included that a necessary condition for stick-slip motion to occur is to have the elastic restoring force below the value of the maximum static friction.

Based on experimental data, Cunningham [17] implemented a torsional model of drill-string. The motivation resulted from field data featuring irregular rotation due to beats in axial, angular and radial accelerations, torque and bending. Two
approaches were used to study the irregular rotation of the string. The first was based on the assumption that the string oscillated at the same period as the one of the beats superimposed on a uniform rotation of the rotary table. Friction was not accounted for, however, its presence was recognized in experimental data. The second approach was to consider the string as a non-rotating torsional pendulum. The pendulum was assumed to be clamped at its top end, and damping was neglected. A correlation between the theoretical results and the experimental data was found and conclusion was made that the string was acting as an unrestrained pendulum. It was demonstrated that these beats occurred more frequently at high rotary speeds. Damping resulting from external sources did not affect the periods of oscillation. However, in approximately 10% of the cases, friction was found to increase the period of oscillation of 20% or more. Finally, friction was demonstrated to potentially distort the normal harmonic motion of the string without seriously affecting its frequency.

Brett [11] introduced a model to study how different types of drill-bits induce torsional vibrations. The study showed that polycrystalline diamond compact bits (PDC) induce a reduction of the torque as the rotary speed increases. PDC were found to induce more torsional vibrations and of a higher amplitude than three-cone bits. Further, higher WOB, dull-bits and low rotary speed were established as parameters that contributed to appearance of torsional vibrations.

In 1986, Halsey et al. [25] modeled a drill-string as different sections interconnected with each other where geometric and physical properties along the same section were invariant. The torsional vibration velocity was assumed to be negligible compared to the rate of rotation of the string. All the points along the drill-string
were considered to rotate in the same direction but with different time-varying speeds. Based on this assumption, only the viscous friction was considered to affect the torsional vibrations. The static friction between the string and the bore-hole wall was overcome by the rotary drive machinery. This lead to model the description of the torsional motion of each section by a partial differential equation which lead to damped harmonic motion. A further observation was that stick-slip motion eventually developed when the magnitude of torsional vibrations increased to the point wherein some sections of the string stopped or reversed direction. Effects of viscous damping were viewed as relatively insignificant. This was also confirmed by the very sharp peaks in the measured torsional vibration power spectra of the analyzed data. Offsets at the tool joints of the pipe had a measurable effect on the torsional resonance frequencies. The tool joint effect was therefore transferred in a correction factor of the wavenumber. The experimental data lead to the conclusion that the frequencies of the torsional resonances were not measurably affected by the rate rotation or the WOB.

The study was accomplished for the lowest vibrational mode. The string was found to behave as a torsional pendulum with the maximum torsional displacement and velocity occurring at the bit. Considered separately and at zero frequency, it was observed that both the pipe and the collar were resonating. When these two sections were coupled, the two resonances split. The geometric parameters of both the pipes and the collars sections had a significant influence on this resonance. It was demonstrated that as the drill-string was rotating off-bottom, it displayed very well defined torsional oscillations. Furthermore, the rate of rotation of the string, its WOB and its damping effects were found to have almost no effect on the frequencies of its torsional resonance. It was suggested that the nominal geometric and physical parameters of the string could be used in order to calculate
its torsional resonance frequencies. Finally, the properties of both the pipes and the collars were proven to sensitively influence the lowest torsional resonance of a string.

**Solutions to Stick-Slip**

Among other solutions to avoid stick-slip vibrations, Lin and Wang [38] proposed an increase of the viscous damping between the string and the wall. This increase also resulted in a decrease of the amplitude of the stick-slip vibration, which would eventually vanish if the critical damping was reached. The principal drawback of such a technique is that an increase of the viscous damping implies an increase of the energy which has to be furnished to the rotary driving system.

Jansen *et al.* [31] introduced an active-damping system to eliminate stick-slip torsional vibrations. The system controls the energy flow through a hydraulic top drive and makes it react as a tuned vibration damper. An increase of the damping of the vibrations that are due to viscous friction is obtained by an increase of the rotary speed at the surface. Having the string rotating faster than its critical speed implies that the damping is high enough for the energy stored in the string, while it is stuck, to be insufficient to balance the energy lost during the “slip” part of the stick-slip behavior. In this particular case of high damping, the moving phase continues and the stick-slip pattern is broken. Jansen *et al.* pointed out the fact that a dynamic adjustment of the damping parameters of the string is a key to diminishing or avoiding stick-slip. Furthermore, it allows the natural frequencies of the total system to coincide. When setting these parameters in a particular way,
the vibrational energy resulting from the bit/formation contact can be directly directed to the surface. There, it is dissipated in the active-damping system.

Dufeyte and Henneuse [21] introduced a set of remedies to avoid stick-slip. The better way to counteract imminent stick-slip motion is to increase the rotary speed. This should be accomplished as long as the level of existing torque stays in the range specified by the constructor (see also Lin and Wang [38]). If the speed cannot be increased, the WOB has to be reduced. As a final means to counteract stick-slip motion, the mud properties have to be changed. However, it was pointed out that increasing the rotary speed of the string or decreasing the value of the WOB are “first-aid” behaviors, whereas the control of the driving system and of the lubricant parameters were referred to as being the real “shock treatments”.

Kyllingstad and Halsey [36] pointed out that the stick-slip motion of strings could potentially be avoided by decreasing the static friction down-hole or by controlling the rotary table speed in a way that dampens torsional oscillations.

Dawson et al. [19] noticed that an increase of the drilling rotary speed produces an increase of the oscillation frequency and amplitude. Elimination of the string vibrations can also be accomplished by exceeding the critical rotary speed. Finally, lowering the static friction was presented as another way to avoid or diminish stick-slip.

**Boundary Conditions**

Spanos et al. [49] introduced a set of boundary conditions which defined the force acting on the bit and the velocity of the string at the bottom of the hole. The
displacement, the velocity and the acceleration at the top were also defined. The force at the bottom of the string and the torque-on-bit were defined as functions of the WOB, whereas the displacement at the top was governed by the motion at the kelly bushing.

Halsey et al. [26] modeled the string section as a transmission line for torsional waves. Cases in which a torsional wave generated by change in the down-hole friction torque traveled upwards from the bottom of the string were studied. The rotary table was introduced as a fixed end.

Assuming that the string could be modeled by two different sections, one of pipes and the other of collars, Halsey et al. [25] considered the drill-pipe section to be fixed at both ends, whereas the drill-collars were assumed to be free at both ends.

2.5 Coupling

The WOB applied to the string is a longitudinal excitation. Nevertheless, when deriving a lateral model of the string, one has to consider it together with coupling between lateral and torsional modes. Berioz et al. [3] pointed out that the equations governing the lateral dynamic behavior of the string contain periodic coefficients. These are due to axial excitations generated by the bit and couplings between vibration modes. It is therefore important to consider not only the excitation of the string in one direction but also the coupling of the excitation of the string in longitudinal-lateral, longitudinal-torsional and lateral-torsional ways.
2.5.1 Longitudinal-Lateral Coupling

Motivation

As pointed out by Yigit and Christoferou [54] in 1996, most engineering applications neglect the coupling between longitudinal and lateral modes. This comes from the fact that in most cases the natural frequencies of longitudinal vibrations are much higher than that of flexural vibrations. However, in the case of a loaded beam such as a drill-string, the frequencies of vibrations of these modes are of the same order of magnitude.

Yigit and Christoferou used a uniform beam submitted to axial and transverse deformations as a model of lateral vibrations propagation in a string submitted to a given WOB. Nonlinear terms were took into account which lead to fully coupled differential equations. Contacts of the string and the wall were also accounted for.

In 1990, Vandersloot et al. [33] demonstrated that the coupling between bending and axial vibrations could lead to additional axial shortening of the string. The behavior of an unbalanced whirling string was studied. In addition to whirling, experimental data provided with the information that transverse vibrations are coupled with longitudinal vibrations through linear or parametric coupling mechanisms. In the case of linear coupling, time-varying changes in the bending moment appear between the longitudinal force and the bending deflection. Unfortunately, since curvature is an essential condition for linear coupling to occur, and since whirling induces curvature, whirling and linear coupling are likely to occur simultaneously. Beside this aspect, both linear coupling along axial and bending vibrations are found to play an important role in fatigue of connection components in the string. Finally, as the drill-collar adjacent to the drill-bit is whirling,
the direction of drilling, the bit side force and the bit tilt are affected. Vandiver et al. stated that linear coupling would not occur on a perfectly straight beam excited by an axial load less than the critical buckling load. However, any initial curvature coupled with a longitudinal load causes a lateral deflection. For small amounts of curvature the lateral deflection increases with the initial curvature.

In 1994, Eustes et al. [29] determined the nature of the core rod vibrations and characterized their vibratory spectrum in order to define an optimal core rod size. Combining three vibration theories, a vibration model of the core rod was first defined. The axial compressional and torsional wave transmission theories were used in one dimension. These were derived from the work of Dareing and Livesay [18]. A third theory, which accounted for helical buckling and the attendant bending stresses, was combined with the first two theories. Following Dareing and Livesay, the wave equation was solved in the complex plane, rather than through trigonometry. This enabled to apply more complex boundary conditions.

To avoid non-linearity in the equations of motion, a viscous damping was chosen as the sole form of friction. Longitudinal and torsional stress equations were solved for a series of frequencies based on the rotary speed capabilities of the coring rig. Resonant frequencies were determined by examination of the fast fluctuations of the stress over short frequency ranges. The helical buckling of the string was referred to as a transient buckle wave. Helical buckling appeared independent of any effect of torsional stress.

**Drill-Collar Whirling**

Whirling is a consequence of the centrifugally induced bowing of drill-collars. As imperfections or MWD tools are present in collars, their center of gravity is not
necessarily located on the centerline of the hole. Additional parameters such as high WOB, and curvature of the hole also induce eccentricity. Therefore, as the string is rotated, a centrifugal force appears at the collars center of gravity causing them to bend. The magnitude of this force is a function of the mass of the collar, the square of the rotation rate and the original eccentricity.

As a string is rotating at its critical speed, whirling does not usually induce collar failure since the deflection of the string is limited by the presence of the well-bore. However, two types of whirling can be differentiated. The forward synchronous whirl is a motion in which collars always present the same side to the wall. This implies a more or less serious abrasion, which can eventually lead to failure.

In the case of backward whirl, the collar rolls without slipping along the perimeter of the bore-hole wall in the opposite direction to the rotary displacement. In pure backward whirling, the center of the collar has a motion around the hole the frequency of which is much higher than the rotation rate of the string (see also Vandiver et al. [33]).

Model and Boundary Conditions

In 1996, Yigit and Christoforou [54] assumed that the lower portion of the beam was under compression which was provided by the upper part of the string. The BHA was modeled as simply supported. Combined longitudinal and lateral vibrations were applied on the drill-collar, while the upper section of the collars underwent longitudinal vibrations only. The model was derived for a non-rotating string, and all the deformations were considered to occur in a single plan.

As the bending motion of the string is confined by the presence of the bore-hole wall, the Euler-Bernoulli beam theory was chosen. By accounting for the effect
of rotation in the longitudinal strain, coupling between longitudinal and lateral deformations was observed.

The vibration frequency of the coupled case was found to be lower than that of the uncoupled case. This lead to the assumption that non-linear coupling was likely to reduce the critical buckling load.

Finally, it was noticed that as a periodic response was predicted by the classical uncoupled equations, accounting for coupling lead the motion to feature chaotic properties.

Using a substantial part of rotor dynamics theory, Berlio, et al. [3] developed a finite element model. The behavior of the string was modeled by a two nodes system, resulting in six degrees of freedom. Stabilizers were accounted for as bearing elements, whereas the bore-hole was considered as a radial gap element. The properties of the mud located between the string and the bore-hole wall were accounted for modeling it as a mass factor and a stiffness matrix. The drilling fluid circulating inside the string was modeled with a mass matrix only. The study accounted for the effects of gravity and buoyancy, and elements such as mass, stiffness and damping could potentially be added to the model. Particular elements such as rig components could also be considered. The string was subjected to motor torque, frictional force, tension resulting from the rig, WOB, drill-torque, gravity, buoyancy, and force exerted on the bit. The friction factor accounted for both the contact with the fluid and the wall.

Experimental data obtained from a clamped-clamped beam subjected to tension load were also provided. Analysis of lateral modes of a beam excited by a lateral impact gave the first few lateral frequencies. The study of the fourth natural frequency showed a weakness in the boundary conditions. Additional tests
revealed that at constant torque, the natural frequencies were increasing with the tension of the beam. Moreover, a decrease of the compression force induced an increase of the natural lateral frequencies. Density and viscosity of the drilling mud were found to decrease the lateral frequency of the axial force and reduce the vibrations levels. It was demonstrated that parametric excitation, and thus lateral instabilities of the clamped-clamped beam at specific frequencies, were induced by the forcing frequency of the longitudinal force.

In 1994, Eustes et al. [29] studied buckling of drill-strings. The top boundary condition was modeled with a mass and spring system. The top driver and the swivel were considered as a point mass for longitudinal vibrations, whereas for torsional vibrations, they were modeled by a mass moment of inertia. The components of the rig, such as the mast and the chain, were considered together and modeled as axial and torsional spring constants. The mass moment and spring were included in the torsional case.

2.5.2 Longitudinal-Torsional Coupling

Motivation

In 1987, Skaugen [22] stated that vibrations at the bit have large longitudinal and rotational quasi-random components. From experimental data, it was noticed that the large torsional flexibility of the string induced non-uniform rate of rotation at the bit. These variations of the rate of rotation were shown to be induced by the excitation of torsional modes. Therefore, accounting for the coupling of random
oscillations and torsional vibrations introduces complicated axial vibration. The stochastic oscillations were induced by the unsteady breakage of the rock.

In 1995, Spanos et al. [49] introduced a model accounting for the lift off the formation of the bit. Taking into account such an event implies a change in the boundary condition at the bit which considers the time when the bit is off-bottom and when it is in contact with the rock. Taking into account the change in the boundary condition introduces a coupling between longitudinal and torsional vibrations.

Model and Boundary Conditions

In 1988, Aarrestad and Kyllingstad [1] also considered coupling between axial and torsional vibrations of drill-strings. A model based on the consideration of the axial motion of the bit using an elevation function was derived. In a first linear approximation, it was assumed that the dynamic variations of force and bit speed was negligible. As a consequence, longitudinal and torsional vibrations were uncoupled. The dynamic perturbation of the angular bit speed was assumed to be harmonic, having a cycle of frequency equal to the main vibration frequency. However, it was recalled that during a real drilling operation, or when the down-hole exciter is used, a more stochastic nature of the torque provided to the string would be found. It was demonstrated that the lowest torsional resonance frequency of the string, its pendulum mode, was often dominating the dynamic motion of the bit. Considering only the pendulum mode was justified by the fact that both pipe and collar sections were acting as torsional mass/spring systems.

Results were obtained both from the model and experiments. It was shown that the frequency spectra of down-hole parameters such as torque-on-bit, axial acceleration, WOB, and longitudinal acceleration often contained a dominating
vibration frequency which was three times the rotary speed. This dominating frequency clearly indicates the tendency of the bit to form a three-lobed pattern in the formation. The fact that the spectra often contained higher harmonics of the fundamental frequency components was explained by the nonlinear coupling between torsional and longitudinal vibrations. Finally, the frequency spectra was often noticed to contain small side lobes near the most dominant frequency components. Calculated and measured frequencies for torsional oscillations of the collar section were found to yield results in close correlation with the frequency spacing between side lobes and the center frequency.

2.5.3 Lateral-Torsional Coupling

Motivation

A problem of transverse vibrations induced by whirling of a rotating unbalanced string was approached in 1990 by Vandiver et al. [33]. Beside the whirling of the string, experimental measurements were also provided which showed that transverse vibrations are coupled with axial vibrations, through a linear coupling mechanism.

In 1988, Halsey et al. [26] attempted to smoothen the motion of the string. The rotary speed was controlled by incorporating the torque signal in addition to the speed signal. Tested on a full-scale research drilling rig, this system demonstrated that it could stop phenomena such as stick-slip motions and even prevent them from beginning. Another benefit of using such a tool was showed by observing a more uniform motion of the bit.
Model and Boundary Conditions

In 1992, Choi et al. [15] studied the dynamic behavior of shafts submitted to flexural forces when rotating about their longitudinal axis. The coupling between flexural and torsional vibrations was assumed to come only from the gyroscopic moments and the coupling due to mass eccentricity was thus neglected. Such a study was dedicated to rotating drill-strings, since non-rotating shafts do not present the features introduced as underlying assumptions.

A bending motion of a shaft changes its angular momentum and thus induces gyroscopic moments. It was recalled that most of the models that derive the equations of motion treat gyroscopic moments either as external moments, such as in a Newtonian approach, or as some external work terms, as found in a Lagrangian framework. The objective of the presented model was to derive a set of mathematically consistent equations that described the three-dimensional dynamics of a flexible beam, and to examine the floating frame approach. In the floating frame approach, a locally attached frame moves with the considered system. This type of moving frame is also known as a shadow beam. Five types of floating frames were defined, namely locally attached, principal axis, Tisserand, Buckens and rigid-body mode frames. The floating frame approach enables to model small strains. However, nonlinear inertia terms due to the rotation of the frame were found in the resulting equations of motion. A finite strain beam theory was presented as an alternative approach. It accounted for large rotation and strains. Based on fully non-linear finite and geometrically exact strain measures, this approach considers shearing strains, twist, longitudinal strain and curvature.

The flexural vibrations in two orthogonal planes and the torsional vibration of a straight beam were considered. The effects of rotary inertia, gyroscopic moments, and shear deformation were included in the formulation. Several hypotheses were
put forward. First, the shaft was considered to have a uniform cross-section along its length which had two axes of symmetry. The plane sections perpendicular to the centroidal line of the shaft in the undeformed geometry was assumed to remain plain in the deformed configuration. Poisson’s effects were not considered. Further, no mass eccentricity was considered. One end of the shaft was submitted to an external, constant-direction torque along the undeformed centroidal line. A compressive longitudinal load with constant magnitude and direction was applied on both sides of the shaft. Longitudinal deformations and damping were neglected.

Based on the equations of Dokumaci [20], Bishop et al. [7] derived the equation of motion of a uniform beam submitted to coupled free bending and torsional vibrations. The boundary conditions were supposed to be either free, clamped or pinned ends. Harmonic vibrations for each given frequency were assumed, and results were provided for a free-free beam.

2.6 Further Considerations

2.6.1 Influence of Mass Imbalance

The mass imbalance phenomenon has several potential sources in drill-strings, such as conception flaws or measurement-while-drilling tools integrated in the drill-collars. Additional factors such as curvature of the hole or rotation of the bended collar can also produce a mass-out-of-balance phenomenon. An increasing number of models available in the literature examine this property of strings.

In 1993, Lichuan and Sen [37] introduced collar sections as submitted to two main sources of vibration. The first is the super-imposition of random variation initial conditions due to lateral impact between drill-collars and the well-bore wall
along with the bending free vibrations of the string. The second source of vibration comes from the imbalance mass inducing a synchronous whirl.

Accounting for the mass imbalance of drill-collars implies considering phenomena such as chocks on the wall, hole deviation, etc.. These phenomena significantly modify the dynamic behavior of the string.

### 2.6.2 Additional Problems

Based on experimental results, Brett [11] demonstrated the significance of dull-bits which change the behavior of a string by modifying parameters such as ROP. The additional consideration of bottomhole cleaning also influences the down-hole dynamics by reducing the bit torque with rotary speed. Accounting for other string components can also lead to modifications in theoretical results. Shock absorbers, for instance, lead to a decrease of vibrations in the string (see [40] and [24]) and can also lead to instabilities when used with specific kinds of drill-bits.

### 2.6.3 Importance of Non-linearities

In 1995, Noah and Sundararajan [41] demonstrated the significance of non-linear terms in rotating machinery. It was pointed out that it is specially localized non-linearities which are of interest since they influence the behavior of rotors locally and globally.

Non-linearities are consequences of several drilling properties, such as clearance between the string and the wall, the rubbing of the string on the bore-hole wall, the built-up rotating assemblies, the interaction with the drilling mud, and damping properties. In rotor dynamics, these sources of non-linearity have lead to sub- and super-harmonic, quasi-periodic, and chaotic behaviors. Studies usually do not include non-linearities, or consider these factor via linearized models.
The non-linear behavior of fluid bearing was accounted for, and several additional parameters were also considered. First, the gyroscopic effects were found to induce both backward and forward whirling motions. It can also induce both a synchronous backward whirl with a sub-synchronous forward whirl in close locations. The oblique impact of the string on the wall, known as the super-ball effect was also accounted for. In non-linear cases, cross-coupling can lead to non-synchronous or random response, together with undesirable jump behavior. Finally, when this behavior becomes chaotic, it can lead to contact fatigue and degradation of the bearing assemblies.

Harmonic balance procedure can be used to study rotor with bearing clearances. In these cases, both the displacement and the support forces are described by different harmonic series expansions, and then used in linking the coefficients of displacements and forces.

2.7 Probabilistic Approaches

The consideration of random factors in drilling has been part of models for several decades ([8], [9]). However, the available literature accounting for stochastic parameters is still fairly limited. The following section introduces the most accessible models available in the literature.

2.7.1 Coupled Axial and Rotational Motions

In 1961, Bogdanoff and Goldberg [9] considered the axial and torsional motions of a string assuming that the forces acting on the bit and on the sides of the string were random in nature. Based on previous hypothesis ([8]), it was pointed out that a deterministic description of the forces acting at the bit and on the sides of the string was not accurate. Through a statistical analysis, the influence of axial
force and torque resulting from the lack of straightness of the hole, the buckling of
the pipe under compression and the whipping at various points were shown.

Axial and rotational displacements only were considered. The viscous damping
for each type of displacement followed a uniform distribution along the pipe. The
axial load was considered to be a zero-mean random process.

The study lead to the conclusion that knowledge of the power spectral density
of the applied force is necessary. However, knowledge of the various probability
distributions is not required to derived accurate results. It was demonstrated that
if the bit forces are assumed to be Gaussian, then so are the axial force, torque,
shear stress, and normal stress.

Finally, the assumption of random forces applied to the string was pointed out
to lead to less sensitive uncertainty in boundary conditions than with a determ-

\textbf{2.7.2 Longitudinal Motion}

In 1987, Skaugen [22] introduced a model in which he investigated the behavior of
a string under longitudinal forcing functions.

A friction which was including both Coulomb and viscous factors was intro-
duced. The viscous damping was found to be dependent upon the former behavior
of the drill-string along with a dependence on its velocity with respect to the
medium of friction. The increase of contact forces was assumed to be proportional
to the weight of the string, and it was shown that the main vibration frequency
was generally a multiple of the rate of rotation.

A dynamic model operating in the time domain was introduced. In order to
obtain a numerical solution, the drill-string was divided into sections and the state
of motion was calculated at discrete times. Results demonstrated that the pre-
dictions obtained from deterministic drill-string models did not accurately define the dynamic behavior of the string. This was particularly the case when these deterministic models predicted sharply increased vibration amplitudes as the rotary speed was a given sub-harmonic of resonance frequencies. On the other hand, these resonance peaks are usually strongly reduced and smoothed out when the quasi-random nature of the vibrations of the bit are considered.

2.7.3 Coupled Longitudinal and Lateral Motions

In 1994, Kotsonis [35] developed a two-dimensional model of a drill-string involving stochastic factors. The model was based on the effects of longitudinal forces (mainly WOB) on the lateral vibrations of a section of BHA. Several kinds of WOB were applied: static, periodic and stochastic. Due to the complexity of the system behavior (high degree of non-linearity and coupling of the state variables), statistical linearization could not model the system accurately. It was found that the consideration of coupling between axial force and lateral vibrations was accurate when applying a static WOB. Conversely, the consideration of randomness in the WOB did not seem to significantly modify the results.
Chapter 3

Dynamic BHA Behavior Analysis

3.1 Preliminary Remarks

In the ensuing sections, a dynamic study of the lateral vibrations of a drill-string is conducted. A brief review of the theoretical concepts is provided. Figure (3.1) illustrates the summary of the study.

3.2 Equation of Motion

Consider the classic equation of motion for a linear model

\[ M \ddot{u}(t) + C \dot{u}(t) + K u(t) = p(t), \tag{3.1} \]

where \( M, C, \) and \( K \) are the mass, damping, and stiffness system matrices, respectively, \( u(t) \) is the displacement vector, \( p(t) \) is the excitation vector applied on the system, and \( \dot{u} = \frac{du}{dt} \).

Equation (3.1) must be solved for the lateral displacement of the BHA system. The latter is modeled by a finite element method.

3.3 The Finite Element Model

Since only small displacements are expected, the Euler-Bernoulli beam theory is assumed to yield accurate results. In this regard, the mass and stiffness matrices were computed using the Euler-Bernoulli theory.
Figure 3.1 Main Steps of the Study
3.3.1 Element Matrices

The formulation of the element properties is as follows, according to Przemieniecki [45]. The beam element is assumed to be a straight bar of uniform cross-section and two nodes per element are considered. Since there is no coupling between transverse and longitudinal displacements, only transverse displacement and rotation are used in defining the equations of motion. The corresponding stiffness and mass element matrices are $4 \times 4$ matrices. The transverse displacement within the element is assumed to be a cubic function of the axial coordinate.

Stiffness Matrix

The element stiffness matrix is computed for lateral displacement accounting for the weight-on-bit $P$. A positive value of $P$ corresponds to a tensile load in the beam.

$$
\mathbf{k}_e = \begin{bmatrix}
\frac{12EI}{L_e^2} + \frac{6P}{5L_e} & \frac{6EI}{L_e^2} + \frac{P}{10} & -\frac{12EI}{L_e^2} - \frac{6P}{5L_e} & \frac{6EI}{L_e^2} + \frac{P}{10} \\
\frac{6EI}{L_e^2} + \frac{P}{10} & \frac{4EI}{L_e^2} + \frac{2PL_e}{15} & -\frac{6EI}{L_e^2} - \frac{P}{10} & \frac{2EI}{L_e^2} - \frac{PL_e}{30} \\
-\frac{12EI}{L_e^2} - \frac{6P}{5L_e} & -\frac{8EI}{L_e^2} - \frac{P}{10} & \frac{12EI}{L_e^2} + \frac{6P}{5L_e} & -\frac{6EI}{L_e^2} - \frac{P}{15} \\
\frac{6EI}{L_e^2} + \frac{P}{10} & \frac{2EI}{L_e} - \frac{PL_e}{30} & -\frac{6EI}{L_e^2} - \frac{P}{10} & \frac{4EI}{L_e^2} + \frac{2PL_e}{15}
\end{bmatrix} \tag{3.2}
$$

Mass Matrix

According to the above assumption, the element mass matrix can be derived and yields Equation (3.3).

$$
\mathbf{m}_e = \begin{bmatrix}
\frac{13M_t}{35} + \frac{6dI}{L_e} & \frac{11L_eM_t}{210} + \frac{dI}{10} & \frac{9M_t}{70} - \frac{6dI}{5L_e} & -\frac{13M_tL_e}{420} + \frac{dI}{10} \\
\frac{11L_eM_t}{210} + \frac{dI}{10} & \frac{M_tL_e^2}{105} + \frac{2dI}{15} & \frac{13M_tL_e}{420} - \frac{dI}{10} & -\frac{M_tL_e^2}{140} - \frac{dI}{30} \\
\frac{9M_t}{70} - \frac{6dI}{5L_e} & \frac{13M_tL_e}{420} - \frac{dI}{10} & \frac{13M_t}{35} + \frac{6dI}{5L_e} & -\frac{11L_eM_t}{210} - \frac{dI}{10} \\
-\frac{13M_tL_e}{420} + \frac{dI}{10} & \frac{-M_tL_e^2}{140} - \frac{dI}{30} & -\frac{11L_eM_t}{210} - \frac{dI}{10} & \frac{M_tL_e^2}{105} + \frac{2dI}{15}
\end{bmatrix} \tag{3.3}
$$
The matrix (3.3) includes a mass term $M_i$ which considers the added masses of the fluid inside and outside the string.

### 3.4 Boundary Conditions

Two kinds of boundary conditions are of interest in the present problem: at the top of the BHA (supposed to model the junction with the upper part of the drill-string) and at the lateral stabilizers. In this regard, the top boundary condition is modeled as a clamped end; restricting both lateral displacement and rotation. Stabilizers are modeled as pinned nodes, thus allowing rotation but not lateral displacement.

The lateral force applied on the bit will be discussed in detail in section 4.2.

### 3.5 Static Condensation

Since the load applied on the string is defined to be a lateral force, the system is not excited in its degrees of freedom corresponding to rotation. Therefore, it is possible to reduce the size of the system by using static condensation where only the lateral displacements are computed (even so rotation is still considered).

Static condensation is a well-known method by structural analysts; however it is not the goal of this thesis to describe it in detail (additional information can be found in [2], [44]). Conceptually, in the present case, the method consists of reducing the size of the total system by expressing the vector of rotational displacements in terms of the vector of lateral displacements.
The first step in accomplishing static condensation is to express the system stiffness matrix in the following form [50].

\[
[K] = \begin{bmatrix}
[K]_{tt} & [K]_{tr} \\
[K]_{rt} & [K]_{rr}
\end{bmatrix},
\]

(3.4)

where the sub-matrix indices "tt, rt, tr, rr" denote terms dealing with translation only, rotation-translation, translation-rotation and rotation only, respectively.

Once the stiffness matrix is expressed in that form, the statically condensed system stiffness matrix is derived using Equation (3.5)

\[
[K] = [K]_{tt} - [K]_{tr} [K]_{rr}^{-1} [K]_{rt}.
\]

(3.5)

It is then assumed that both the system mass and damping matrices can be modified in the same way.

3.6 Solving the Eigenproblem

Beside being the first step of the mode superposition method, solving the eigenproblem associated with the system provides one with valuable information such as the values of the natural frequencies of the system.

Solving the eigenproblem corresponds to finding the eigenfrequency/eigenvector couples of the undamped drill-string. In this regard, the following equation has to be solved for both \( \omega \) and \( \phi \).

\[
[-M\omega^2 + K]\phi = 0,
\]

(3.6)

where \( \omega \) is the matrix of which the diagonal terms are the natural frequencies of the system and \( \phi \) is the matrix of the corresponding eigenvectors.

The knowledge of the natural frequencies of the string is of significant importance in a dynamic study. However, it should be recalled that solving the
eigenproblem also provides one with information about the buckling state of the string. Since the WOB $P$ has been incorporated in the expression of the stiffness of the system, solving the eigenproblem for different values of $P$ can yield zero or negative eigenfrequencies. Such null values of the natural frequency would mean that the string has statically buckled, and thus a qualitative study on $P$ can be conducted to find the maximum admissible WOB.

3.7 Mode Superposition Method

Solving Equation (3.1) under its current form for a $N$ degrees-of-freedom (N-DOFs) system would lead to a $N \times N$ system of coupled equations for a N-DOFs system. The idea behind the mode superposition method, as it is presented by Craig [47], is to rewrite this system in modal coordinates $\eta_r$ where $r = 1, 2, \ldots, N$. By choosing an appropriate type of damping the equation of motion is uncoupled; its solution is equivalent to the solution of a single degree-of-freedom oscillator problem at each node. The total displacement of the string is then the sum of the displacements at each node.

3.8 Steady-State Harmonic Response

The solution to the second-order ordinary differential equation of motion (3.1) for a single degree-of-freedom problem is given by the Duhamel integral (convolution integral)

$$\eta_r(t) = \left(\frac{1}{M_r \omega_{dr}}\right) \int_0^t P_r(\tau) e^{-\zeta_r \omega_{dr}(t-\tau)} \sin(\omega_{dr}(t-\tau)) d\tau +$$

$$\eta_r(0)e^{-\zeta_r \omega_{dr}t} \cos(\omega_{dr}t) + \left(\frac{1}{\omega_{dr}}\right)[\dot{\eta}_r(0) + \zeta_r \omega_{dr} \eta_r(0)]e^{-\zeta_r \omega_{dr}t} \sin(\omega_{dr}t), \quad (3.7)$$
where $\omega_{dr} = \omega_r \sqrt{1 - \zeta_r^2}$ is the damped circular natural frequency and the initial conditions $\eta_r(t = 0) = \eta_r(0)$ and $\dot{\eta}_r(0) = \frac{\eta_r}{dt}|_{t=0}$ denote the displacement and the velocity at time $t = 0$ respectively.

Recalling that $\eta_r$ is the expression of the displacement of the $r$-th node in the modal coordinates system and that the physical set of coordinates $u$ can be expressed in terms of $\eta$ via the relation $u = \phi \eta$, one can express the displacement in the physical (or generalized) coordinates

$$u(t) = \sum_{r=1}^{N} \phi_r \eta_r(t). \quad (3.8)$$

The steady-state response of the system to a harmonic excitation $p$ at frequency $\omega$ can be found and can be expressed by

$$u(t) = \sum_{r=1}^{\hat{N}} \left( \phi_r \phi_r^t \frac{p}{K_r} \right) \left[ \frac{1}{\sqrt{(1 - r_r^2)^2 + (2 \zeta_r r_r)^2}} \right] \cos(\Omega t - \alpha_r). \quad (3.9)$$

Several remarks can be addressed with regards to Equation (3.9). First, \( \hat{N} \) represents the number of nodes considered in the truncated mode superposition method. The term $\phi_r^t p$ accounts for the degree of alignment between the excitation vector and the mode shape. This term is zero for modes that do not contribute to the response of the system. Further, the term $K_r$ in the denominator converts the modal force in the parenthesis into a static deflection. This static deflection has to be multiplied by the term in the square brackets corresponding to a dynamic amplification factor. Multiplying the deflection factor by the eigenvector $\phi_r$ and summing at each node yields the response of the system in the physical coordinates. The dynamic amplification factor, the term in the square brackets, is a function of the frequency ratio $r_r = \omega/\omega_r$ and the modal damping factor $\zeta_r$ at each node. If no damping were considered, the steady-state displacement would blow up whenever the frequency of excitation would be equal to a natural frequency of the system.
Finally, the cosine term represents the time variance of the response, including the phase shift, and is not of importance if one is seeking only the magnitude of the response.

3.9 Transfer Function

The first step of an approach for frequency domain deterministic analysis or random vibration analysis is to obtain the frequency response function of the system. This is achieved by setting \( P_r(t) = P_r e^{i\omega t} \) and \( \eta_r(t) = H(\omega) e^{i\omega t} \) which leads to the following equation

\[
(-\omega^2 + 2i\zeta_r\omega_r\omega + \omega_r^2)H(\omega) = \frac{P_r}{M_r},
\]

which can be expressed as

\[
H(\omega) = \frac{\frac{P_r}{M_r}}{-\omega^2 + \omega_r^2 + 2i\zeta_r\omega_r\omega}.
\]

Recall \( r_r = \omega/\omega_r \). The previous equation then becomes

\[
H(\omega) = \frac{\frac{1}{K_r}}{(1 - r_r^2) + 2i\zeta_r r_r}.
\]

The elements of the complex frequency response matrix for the system of Equation (3.1) can then be expressed as

\[
H_{ij}(\Omega) = \sum_{r=1}^{N} \left( \frac{\phi_{ir}\phi_{jr}}{K_r} \right) \left[ \frac{1}{(1 - r_r^2) + i(2\zeta_r r_r)} \right].
\]

This complex frequency response function, or transfer function, yields the response of the system at the coordinate \( u_i \) due to a unit amplitude harmonic excitation at \( p_j \) in physical coordinates. By observing Equation (3.13) the comparison with the expression of the steady-state displacement and the remarks developed for Equation (3.9) is facilitated.
It is useful at this point to recall that in order for the frequency domain study to be meaningful, the system has to be stable; mathematically this is expressed as

$$\int_{-\infty}^{\infty} |h(t)| \, dt < \infty ,$$  \hspace{1cm} (3.14)

where \( h(t) \) denotes the inverse Fourier transform of the complex frequency response function.

### 3.10 Using the Transfer Function

Consider the case where the excitation is given by a power spectral density. In other words, the frequency content of the excitation is given; this yields a constant spectrum in the case of a white-noise excitation. This first model can be readily modified by incorporating a filter between the white-noise generator and the system, which enables the study of the response of the system to any kind of stochastic excitation.

Acknowledging the versatility of this model, one might be interested in finding the relationship between the spectral densities of the excitation and response. This can be achieved by using the frequency response function

$$S_{yy}(\omega) = H^*(\omega)H(\omega)S_{xx}(\omega) ,$$  \hspace{1cm} (3.15)

where the star denotes the complex conjugate. Equation (3.15) for a single degree-of-freedom is equivalent to

$$S_{yy}(\omega) = |H(\omega)|^2 S_{xx}(\omega) ,$$  \hspace{1cm} (3.16)

which depends on the magnitude of the frequency response function only.

Equations (3.15) and (3.16) are valid for a single excitation. When considering the case of \( M \) uncorrelated inputs for which all cross-spectral density terms are
equal to zero, one derives

\[ S_{yy}(\omega) = \sum_{r=1}^{M} |H(\omega)|^2 S_{xx}(\omega) . \] (3.17)

Equation (3.17) could be used in a study of drill-string vibration where not only the lateral force acting on the bit is considered, but also forces acting on the stabilizers along the string. Conceptually, Equation (3.17) states that if uncorrelated forces acting on the string are considered, the response at each node is yielded by the sum of the response of the given node to each excitation. The mean square response can then be derived by

\[ E[y^2] = \int_{-\infty}^{+\infty} S_{yy}(\omega) d\omega . \] (3.18)

Equation (3.18) expresses the fact that the area under the power spectral density curve of the response describes the total power of the process, its variance.

3.10.1 Using the Power Spectral Density

Once the power spectral density of the response of the string is calculated, several ways exist to present this information.

A first choice is to work in the time domain. This can be achieved by using an auto-regressive filter (for finite length signals) or an auto-regressive moving average filter (for infinite length signals) to generate time-histories of the displacement which are compatible with the power spectral density (PSD) of the displacement. Running Monte-Carlo simulations; i.e. generating enough compatible time-histories, the statistical properties of the response (such as its moments) are obtained.

An alternative is to use Equation (3.18). The variance, or the total power of the response, can be computed from the PSD of the latter. Recall that the standard
deviation $\sigma$ or RMS value, of a process $y$ is computed from the variance of $y$ by using the equation

$$\sigma = \sqrt{\text{var}(y)}.$$ \hspace{1cm} (3.19)

The standard deviation gives a sense of how far the process is likely to be from its mean value. In the present study, the standard deviation of the displacement of the bit represents the magnitude of this displacement with respect to its zero-position (the center of the hole).
Chapter 4

Numerical Results

A dynamic study of a drill-string with the following parameters has been conducted.

Drill-pipe:
External diameter $D_o = 0.17 \ m$, Internal diameter $D_i = 0.07 \ m$
Young's modulus $E = 2.1 \times 10^{11} \ Pa$, Density $\rho = 7833 \ kg/m^3$

Drill-collar:
External diameter $D_o = 0.20 \ m$, Internal diameter $D_i = 0.07 \ m$
Young's modulus $E = 2.1 \times 10^{11} \ Pa$, Density $\rho = 7833 \ kg/m^3$

The respective lengths of the pipe and collar sections differ for the two numerical examples considered and will be introduced in the corresponding sections.

The geometry of the studied BHA for the first numerical example is shown in Figure (4.1). It is a “maintain” configuration consisting of 5 stabilizers of which the objective is to keep the angle between the axis of the string and the vertical constant.

4.1 First Example: “Maintain” BHA Configuration

4.1.1 Mode Shapes and Natural Frequencies

Figures (4.2) and (4.3) depict the 6 first mode shapes and the values of the corresponding natural frequencies of the string calculated with a finite element model of 272 elements of size 18.75 cm each. The mode shapes are plotted as normalized lateral deflections with respect to the distance from the surface.
Figure 4.1 BHA Geometry for Numerical Results
The eigenvectors (mode shapes) are defined with respect to a constant and need to be normalized through the use of one of several techniques [47]. In Figures (4.2) and (4.3), eigenvectors were normalized by setting either their highest component to 1 or their lowest value to -1. Squares denote the locations of the stabilizers along the string. The shape of the modes after the last stabilizer is of significance, since

Figure 4.2 Natural Vibrations Modes 1-3 for the “Maintain” BHA Geometry

for the first 3 modes, this shape does not really separate from the axis of the string. On the other hand, for the three following modes the value of the lateral deflection at the bit is quite important, especially for modes 5 and 6 which have corresponding
numerically close natural frequencies. This phenomenon is important with respect to the transfer matrix, to be studied later; the correlation between the shapes of these two close modes and the shape of the complex frequency response around these frequencies will be established.

![Graph showing normalized lateral deflection vs position in BHA from surface](image)

Figure 4.3  Natural Vibrations modes 4-6 for the "Maintain" BHA Geometry

4.1.2 Grid Refinement

Once the system mass and stiffness matrices have been defined, boundary conditions and static condensation have been applied, and the natural frequencies of
the system have been computed, it is of interest to determine the optimal finite element model which can accurately describe the dynamic behavior of the system.

Originally, a model with elements of 75 cm length was chosen and the eigenproblem was solved. Then, several models were developed with 1.5 m, 37 cm, 19 cm and 9 cm long elements, respectively. The last model was chosen as a reference and was assumed to yield the exact solution for the values of the natural frequencies. The results provided by the other models were compared to those obtained by the reference model and Figure (4.4) illustrates the values of the relative errors of the different models plotted with respect to the values of the natural frequencies expressed in Herz up to 150 Hz.

![Figure 4.4 Relative Errors for Different Element Lengths](image-url)
It is clear from Figure (4.4) that neither the 1.5 m nor the 0.75 m element model produce satisfying results for the values of the natural frequencies of the string, whereas both the 37 cm and the 19 cm long elements models yield accurate results for frequencies up to approximately 100 Hz. As will be discussed in detail later, frequencies higher than 100 Hz are also of significance. Therefore, the 19 cm model was chosen, being the only one to yield results with accuracy below the engineering error of 10%.

4.1.3 Complex Frequency Response Function

Figure (4.5) shows the transfer function of the string calculated with the 19 cm element-length finite element model; that is, the absolute value of the complex frequency response function (CFRF) plotted versus the frequency of excitation and the position along the string. The modal damping factor $\zeta_r$ was assumed to be equal to 5% at each node and the first 30 nodes (that accounts for frequencies up to 170 Hz) were considered in the truncated mode superposition method.

The transfer matrix presented in Figure (4.5) represents the response of each node of the string to an unit amplitude harmonic excitation at the bit (the last node of the string). The fact that the excitation is of unit magnitude explains the low value of the magnitude of the CFRF.

The circles on the frequency axis denote the value of the first 13 natural frequencies of the string, and the squares on the distance axis represent the positions of the stabilizers.

From Figure (4.5), it can be concluded that only a narrow range of frequencies of excitation is of significance (frequencies less than 20 Hz approximately).
Figure 4.5  Magnitude of the Transfer Function
The CFRF has also been computed using the direct inversion matrix method as part of the present thesis. In that method, the CFRF is given by

\[ H(\omega) = \frac{1}{-M\omega^2 + iC\omega + K}. \] (4.1)

Both methods produced similar results.

Figure (4.6) shows the CFRF from the frequency axis. The CFRF of the entire string is plotted versus the frequency of excitation, and the circles on the frequency axis denote the natural frequencies of the string. Figure (4.6) illustrates the resonance phenomenon and the influence of the damping as the frequency of excitation approaches the natural frequencies of the string.

It is clear by studying Figure (4.6) that the highest value of the magnitude of the transfer function at the bit appears around 10 Hz, which corresponds to the value of the two close modes of the string (modes 5 and 6). The shape of the modes and the transfer function at that frequency can be linked recalling that eigenvectors 5 and 6 represent the highest displacement at the bit.

By observing Figures (4.5) and (4.6), it can be concluded that in the case of the BHA geometry, the response at the bit represents the most interesting pattern.

A spectral analysis involving the magnitude of the CFRF of the string at the bit is to be performed. The behavior of the string at its bit is therefore the only interesting one and Figure (4.7) illustrates this magnitude of the CFRF at the bit plotted with respect to the frequency of excitation. In Figure (4.7), the circles on the frequency axis denote the values of the natural frequencies of the string, and
Figure 4.6  Magnitude of the Transfer Function from the Frequency Axis
observation of Figure (4.7) yields clearly that only the first few eigenfrequencies are of interest, especially the fifth and the sixth frequencies.

![Graph](image)

**Figure 4.7** Magnitude of the Transfer Function of the String at the Bit

By comparing Figures (4.6) and (4.7), it can be noticed that, for instance at approximately 30 Hz the transfer function does not reach its maximum at the bit. This property is even more obvious for smaller damping values. However, the magnitude of the displacement reaches its maximum at the bit and since the contact between the string and the wall has not been considered in this thesis, only the displacement at the bit is studied.
4.1.4 Damping Influence

Figure (4.8) shows the magnitude of the transfer function at the bit plotted versus the frequency of excitation for three values of the modal damping factor $\zeta_r$.

![Figure 4.8: Transfer Function at the Bit as a Function of the Frequency of Excitation for Several Modal Damping Factor](image)

Figure (4.8) illustrates the importance of a proper choice of the damping ratio $\zeta_r$; it can be noticed that as the damping ratio decreases from a 10% value to a...
1\% value at each node, the magnitude of the transfer function is multiplied by a factor of approximately 10.

4.1.5 Lateral Displacement of the String

The lateral displacement of the string is plotted in Figure (4.9) versus time and distance from the surface. This plot was obtained using a harmonic excitation and an applied lateral force of 10000 $N$ magnitude. Squares on the distance axis denote the locations of stabilizers. At these points the lateral displacement is equal to zero. As expected, the highest displacement appears near the bit and a harmonic pattern can be recognized.

![Figure 4.9 Displacement of the String for an Excitation of 5 Hz Frequency](image)
Figure (4.9) shows the displacement for an applied lateral force of frequency 5 Hz. The modal damping factor is set at 5% at each node. The magnitude of the maximum lateral displacement is found to be of order 1 cm. However, as the string is submitted to a harmonic excitation and the frequency increases to 10 Hz, so does the magnitude of the displacement at the bit. When the critical frequency of 10 Hz is reached, the displacement along the string is still of the order of 1 cm, whereas at the bit, it reaches 10 cm.

This phenomenon can be explained by looking at the transfer function in Figure (4.5); it exhibits a single peak of narrow range of frequencies located at the bit.

4.2 Random Excitation Model

Based on the results presented in Birades [6], Brakel [10] and other studies ([22]), the assumption of having a random lateral force applied on the bit is considered. Many studies currently available in the literature acknowledge the random nature of forces acting on drill-strings, but only a few include this hypothesis in their models.

4.2.1 White-Noise Excitation

The consideration of random forces in this thesis is accomplished by applying a zero-mean Gaussian white-noise to the string; the PSD of the displacement at the bit is computed using the PSD of the excitation and the transfer function of the string at the bit.

Figure (4.10) shows a synthesized time-history of the lateral force applied on the string, where the magnitude of its amplitude is taken to be of the order of
The random excitation was generated using a sufficient number of points for its autocorrelation function (ACF) to feature an adequate approximation of the theoretical Dirac delta function that should be obtained when considering a white-noise process.

The power spectral density of the excitation was obtained by taking the Fourier transform of the ACF; a constant function of the frequency should be expected as the theoretical prediction for the PSD of the white-noise excitation. However, the results obtained using the basic periodogram method were not satisfying in terms of both the ACF and the PSD of the excitation.
An alternative approach was attempted using Welch's procedure; Figure (4.11 (a)) shows the result obtained for the PSD of the excitation in the case of a white-noise process plotted with respect to the frequency of excitation. Figure (4.11 (a)) was found to be an adequate approximation of the constant theoretical case, having a mean of order $2.5 \times 10^7 \, N^2/Hz$ and a standard deviation of order $8.0 \times 10^5 \, N^2/Hz$.

### 4.2.2 Spectral Densities

Figure (4.11) shows the PSD of the zero-mean white-noise excitation, the square of the magnitude of the transfer function at the bit, and the PSD of the output plotted with respect to the frequency of excitation.

As the square of the CFRF is taken, only a peak at frequency 10 $Hz$ remains from Figure (4.7). Therefore, filtering the PSD of a white-noise excitation by this CFRF yields quite a narrow PSD of the response; the string acts as a band-pass filter allowing only a very narrow range of frequencies to pass.

Equations (3.18) and (3.19) were used to compute the variance and the standard deviation, respectively, of the response of the bit. The variance was found to be $Var = 5.6 \times 10^{-3} \, m^2$ and the RMS value $\sigma = 7.5 \, cm$.

In the present study, the RMS value gives an estimate of the amplitude of the displacement at the bit.
Figure 4.11  PSD of the Excitation, Magnitude Square of the Transfer Function and PSD of the Response for a White-Noise Excitation on the "Maintain" BHA Geometry
4.3 Second Example: "Build-Up" BHA Configuration

As mentioned earlier, BHAs have different typical geometries corresponding to the kind of directional drilling in question. The first example treated in this thesis was corresponding to a "maintain" BHA configuration, where the objective is to keep a constant angle between the string axis and the vertical. Since many physical factors are involved in the constitution of BHAs, it was chosen to consider a second numerical example. The objective here is to establish the differences in the lateral dynamic behavior at the bit when changing the number of stabilizers and the ratio of length between the drill-pipe section and the drill-collar section. The example of the following section refers to a "build-up" geometry and is shown in Figure (4.12). This type of BHA configuration is used to increase the angle between the string axis and the vertical.

Beside the clamped end boundary condition at the top, only one stabilizer is included in the BHA geometry here, and is located very close to the bit. The drill-collar section was chosen to be longer than that of the first example, to have more weight applied on the bit. Both the total length of the BHA and its radius are kept the same as for the first example. The damping used in the mode superposition method is still set at 5 % at each node, and the number of nodes considered when deriving the complex frequency response function via the truncated mode superposition method is still assigned to be equal to 30, resulting in frequencies of up to 140 Hz.
Figure 4.12  BHA Geometry for Numerical Results
4.3.1 Complex Frequency Response Function

Figure (4.13) shows the transfer function of the string at the bit plotted with respect to the frequency of excitation.

![Graph showing the transfer function](image)

**Figure 4.13** Transfer Function at the Bit for the "Build-Up" BHA Configuration

By comparing Figure (4.7) to Figure (4.13) it is appreciated how much the location of the stabilizers and their number influence the lateral behavior of the string. As part of this thesis, several numerical analysis were computed for different BHA geometries. It was understood from these models that the respective lengths
of the pipe and collar sections influence more the value of the magnitude of the complex frequency response function than they change the shape of the response function. Therefore, comparing Figure (4.7) to Figure (4.13) enables one to relate the differences in the shapes of the two transfer functions to the position of the stabilizers only.

The "build-up" BHA geometry cannot be viewed as a system yielding a single frequency band-pass filter as the "maintain" geometry did. The current BHA configuration causes high response peaks for several distinct frequencies of excitation. The maximum amplitude of the CFRF at the bit is less significant than the one of the first numerical example but both are of the same order of magnitude.

The first natural frequencies of the "build-up" BHA configuration were found to be much smaller than those of the "maintain case". Nevertheless, all natural frequencies are positive, meaning that no static buckling has occurred. The low values of the first natural frequencies explain the very tight and intricate behavior of the CFRF for low frequencies of excitation. However, observing the CFRF of the "build-up" case suggests that a first approximation of this of system could be achieved by modeling its transfer function with a low-pass filter, whereas in the case of the "maintain" BHA configuration a band-pass filter is necessary.

4.3.2 Power Spectral Densities for a White-Noise Excitation

Figure (4.14) shows the transfer function of the string at its bit and the PSD of its response when submitted to a zero-mean Gaussian white-noise excitation whose PSD is shown in Figure (4.14 (a)). Although the random load applied to the string is not the same as that of the first numerical study, the statistical properties of the two are identical.
As the CFRFs of the two examples are compared, the one related to the "build-up" BHA configuration (Figure (4.13)) vanishes slightly slower than the one of the "maintain" case (Figure 4.7). This behavior is more obvious as the modal damping factor $\zeta_r$ is lowered to 1%. As the square of the CFRF is considered, both transfer functions vanish in the same range of frequencies. In this regard, the frequency content of the response is bounded by an upper limit of approximately 12 Hz and no higher content should be expected.

Calculations of the variance and the RMS value for this case yields: $Var = 3.3 \times 10^{-3} m^2$ and $\sigma = 5.7 cm$, respectively. These values are of the same order of magnitude as those computed for the "maintain" BHA geometry. However, the estimate of the displacement at the bit provided by the RMS values shows that in the case where a "maintain" BHA geometry is studied, the displacement at the bit is higher than with a "build-up" BHA geometry.
Figure 4.14 PSD of the Excitation, Magnitude Square of the Transfer Function and PSD of the Response for a White-Noise Excitation on the "Build-Up" BHA Geometry
Chapter 5

Concluding Remarks

A study of the dynamic behavior of drill-string bottomhole assemblies is presented. The theoretical background of the subject is introduced by means of a literature review, in which the parameters and phenomena that account for a dynamic study of drill-strings behavior are addressed. Lateral vibrations and specific phenomena such as stick-slip vibrations, whirl of the bit, and mass-out-of-balance phenomenon constitute the main focus. Finally, a review of studies involving a probabilistic approach is attempted.

The numerical analysis of the lateral vibrations of drill-strings bottomhole assemblies using a frequency-domain analysis with consideration of a random load on the string is undertaken. A finite element model, including the effects of weight-on-bit and added-mass coefficients resulting from the presence of the mud is defined. The method of static condensation is used and a reduction of the system size is achieved. A solution of the corresponding eigenproblem is computed. Natural frequencies and mode shapes of the string are derived and the solution to the static buckling of the string is discussed. A qualitative study of the values of the natural frequencies is presented for different element sizes of the finite element model and an optimal model size is defined.

The mode superposition method is applied to the model in order to calculate the time-history of the response when the string is submitted to a harmonic lateral force. The mode superposition method is also used to compute the complex frequency response function of the string.
A time-history of a zero-mean Gaussian white-noise excitation is generated and its power spectral density is computed. A zero-mean Gaussian white-noise excitation has been applied to the system and the power spectral density of the displacement of the string at its bit has been derived. Two numerical cases related to different typical bottomhole assembly geometries are considered and the power spectra are compared for excitations having the same statistical properties. Important differences in the behavior of the bit are addressed as stabilizers are moved along the bottomhole assembly.

The linear model accounts for lateral consideration only, expressed in the frequency domain. Comparisons of the two numerical examples indicate that the stabilizer location plays a significant role in the frequency response of the string. As the choice of this location is limited by the objectives of a drilling-crew with a given string, results should be viewed as defining the range of frequencies producing resonance for a given BHA geometry.

The second numerical example, simulating the “build-up” BHA, yields maximum lateral displacements along the string rather than at the bit. In order to compare the results of the two different geometries, the behavior at the bit was the only one studied.

The value of the displacement at the bit provided by the RMS values exhibits the same order of magnitude for both cases; in the order of a few centimeters. However, the lateral displacement at the bit for the “build-up” geometry is lower than the one of the “maintain” geometry. This result, of limited interest in this study, would be of significant benefit if the presence of the wall was considered.
The current model can be improved in different ways. As previous works indicate, the weight-on-bit is rarely constant with respect to time, that is, it generally fluctuates with a random pattern. To account for time-varying WOB, one needs to consider the coupling between the axial and lateral vibrations of the string. An accurate finite-element model with more degrees of freedom should be derived, where the axial displacement of each node would be taken into consideration. Since the WOB is included in the definition of the stiffness matrix of the string, consideration of a time-varying WOB could be achieved by viewing it either as an axial force applied at the bit or as variations in the stiffness matrix definition. The stiffness matrix of the string would then be time-dependent. A qualitative study of the effect of such a property would need to be conducted, since considering a time-dependent WOB in the stiffness matrix of the string requires a much more expensive model in terms of computational time since the mass matrix would have to be derived at each time-step.

Possible contact between the drill-string and the bore-hole wall has not been accounted for in the present study. It has to be considered in further models since it plays a significant role on the lateral behavior of drill-strings. The commonly accepted value of the clearance between the string and the wall is in the order of a few centimeters. The results provided by the numerical examples suggest that the displacement of the string is in general greater than the clearance. In this regard, clearance and contact considerations should be addressed.

Previously derived models ([43]) have considered frequency-dependent mass matrices. The main objective in the origin of such a consideration is to account for the presence of the mud which is driven in the hole at some frequency inducing pulses. Accounting for the presence of the mud, especially between the string and the hole has been shown to modify the value of the natural frequencies of the
string up to 20 - 30% [43]. The presence of the mud implies a modification of the mass matrix which becomes a frequency-dependent parameter. In such a case, an iterative process needs to be introduced in order to solve the eigenproblem. Other drill-string properties, such as the mass-out-of-balance of some components of the string (coming from either flaws in the conception of the components or more likely from collars including measurement-while-drilling tools), induce frequency-dependent parameters.

The prediction of the behavior of drill-strings is a very challenging engineering problem. A wide range of phenomena must be considered within the scope of a broad analysis. Consideration of coupled excitations is of importance since excitations in the axial, lateral, and torsional directions have been shown to influence a string in the same range of frequencies. Engineering judgment needs to be carefully applied in order to consider all the relevant parameters needed to derive accurate results; models more refined than the one presented in this thesis will certainly expedite this objective.
Bibliography


Appendix A

Spectral Analysis

Several methods exist in order to define the autocorrelation function $R_{xx}(\tau)$ and the power spectral density $S_{xx}(\omega)$ of a random process $x(t)$. Classical methods will be described in the following section.

A first way to derive the PSD of a signal is to compute it as being the Fourier transform of the autocorrelation function of the signal.

This method is referred to as the basic periodogram method. An alternative used to obtain a more accurate estimate of the PSD of a process is Bartlett’s procedure. Mathematically, the concept underlying the present method is

$$\hat{S}_{xx}(\omega) = FT\{\hat{R}_{xx}(\tau)\} = |FT\{x(t)\}|^2,$$  (A.1)

where $\hat{S}_{xx}$ is the approximate PSD of the signal $x(t)$, $\hat{R}_{xx}$ is the approximate ACF of $x(t)$ and $FT$ stands for “Fourier transform”.

Equation (A.1) states that the approximate power spectral density of a signal is the Fourier transform of the autocorrelation function of this signal, and also the square of the magnitude of the Fourier transform of the original signal.

Bartlett’s procedure is based on the concept of separating the given time-history in several intervals on each of which an approximate PSD is computed. The approximate PSD of the process is then found as the mean of the interval PSDs. A more accurate approximation can be obtained using overlapping windows and
computing the PSD in each of them. This method is commonly referred to as Welch's procedure.

Finally, an optimal approximation of the PSD can be obtained by considering more sample points in the time-history of the process, more overlapping windows, or optimized window shapes (rectangular, Hanning, etc.). For additional information on spectral analysis, see [12] and [34].