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RICE UNIVERSITY

ANALYSIS, DESIGN, AND CONSTRUCTION
OF A SHAKING TABLE FACILITY

by

MATTHEW J. MUHLENKAMP

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE
MASTER OF SCIENCE

APPROVED, THESIS COMMITTEE

J. P. CONTE, CHAIRMAN
ASSOCIATE PROFESSOR OF CIVIL ENGINEERING

A. J. DURRANI
PROFESSOR AND CHAIR OF CIVIL ENGINEERING

R. P. NORDGREN
HERMAN AND GEORGE R. BROWN PROFESSOR OF CIVIL ENGINEERING

HOUSTON, TEXAS

APRIL, 1997
Analysis, Design, and Construction of a Shaking Table Facility

Abstract

by

Matthew J. Muhlenkamp

Rice University’s Department of Civil Engineering recently added an electro-hydraulic shaking table facility to their testing laboratory. The purpose of the shaking table is to evaluate the response of scaled model structures subjected to base excitation. Every component of the shaking table was carefully designed or sized to perform optimally at targeted levels. The performance curves for the shaking table which define the maximum load, velocity, and displacements that the table can attain were developed from the specifications of the hydraulic system. Interaction between the shaking table system and the foundation mass was analyzed. The dynamic characteristics of the slip plate and its interaction with model structures were studied to determine the optimum size of the slip plate. The transfer function between the command displacement signal sent to the table and the table displacement was developed analytically for the system, encompassing servovalve hydraulics and the PIDF control system.
Acknowledgments

The research for this thesis was performed at Rice University in Houston, Texas, under the direction of Dr. Joel P. Conte, Associate Professor of Civil Engineering, and Dr. Ahmad J. Durrani, Chair and Professor of Civil Engineering. Special thanks are given to the members of my thesis committee, Dr. Conte, Dr. Durrani, and Dr. Ronald P. Nordgren. Thomas Hudgings and Dr. Conte also deserve special praise for their work in the Shaketable design and development. Funding for the Shaketable project was generously provided by the Civil Engineering Department at Rice University and by the National Science Foundation.
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CHAPTER 1
INTRODUCTION

Shaking tables have become an important tool for conducting experimental research in the emerging areas of structural identification, assessment, and control. Recently, a state-of-the-art shaking table was added to the large-scale pseudo-static testing facility in the Civil Engineering Department at Rice University. The primary purpose of a shaking table is to simulate earthquake ground motions to study their effects on reduced scale structural models such as buildings and towers and on vibration sensitive mechanical equipment. Two years of planning and development went into the building of the Rice University shaking table before any construction was begun.

1.1 Rationale for Building a Shaking Table at Rice

The focus of much of the research done by the Civil Engineering Department at Rice University has been and continues to be on structural dynamics. The Civil Engineering Department already utilizes a state-of-the-art large-scale pseudo-static testing facility, capable of testing full-sized structural components under pseudo-static cyclic loading. The shaking table complements the existing facility by allowing the testing of complete structural systems under true dynamic conditions. Additionally, the introduction of a shaking table facility is aligned with the future research directions of the structural engineering community, such as structural assessment, non-destructive damage assessment, on-line monitoring and field evaluation. The Civil Engineering Department is also dedicated to continued excellence as an educational institution. The shaking table is an ideal tool for
illustrating the basic principles of structural dynamics and for conducting correlation studies of analytically predicted with measured dynamic response of structures. The shaking table system, consisting of high-performance mechanical and electrical equipment, creates a multi-disciplinary environment for research in the fields of Civil, Mechanical, and Electrical Engineering. The shaking table has already been used to field test equipment that would be subjected to high accelerations aboard an aircraft.

1.2 Concept Development

The shaking table project was conceived several years before the actual construction of the facility began. Accounting for the financial constraints and the available human resources, the working concept was created: a uni-directional shaking table powered by a hydraulic actuator. The shaking table would be mounted on rails, and the whole system would be mounted on a concrete reaction mass. The reaction mass would be made of concrete slabs, stacked and postensioned together to provide modularity such that additional mass could be added (or removed) as needed. High performance control, data acquisition and processing equipment would complete the facility. Figures 1.1 and 1.2 illustrate the working concept that was developed.
Figure 1.1 Conceptual Shaking Table Facility

Figure 1.2 Shaking Table Setup
1.3 Organization

This work is divided into eight major components, each a chapter of this thesis. Chapter 2, "Design of the Shaking Table" describes the shaking table in terms of its major components, and their respective designs. Exceptions are the slip table and base isolation system, which are covered in Chapters 4 and 5. Chapter 3, "Hydraulic Power Supply and Flow Characteristics" develops the performance envelopes of the shaking table based on the characteristics of the hydraulic system. Chapter 4, "Base Isolation System" details the analysis of the shaking table as a dynamic system and the design of the base isolation pads required for the foundation mass. The similarity conditions used in dynamic modeling are discussed in this chapter. Chapter 5, "Analysis and Design of the Slip Table" deals with the problem of slip table design. Table material, table vibration, and structure-table interaction are examined in determining the final design. Chapter 6, "Shaking Table Construction" details the process of building the reaction mass, and connecting the various table components, such as the rails, actuator, and slip table. In Chapter 7, "Mechanical and Control Model of the Servo-Hydraulic System", the analytical expression for the transfer function of the shaking table system is developed by modeling the servovalve as a mechanical system. The physical and control parameters that affect table performance are discussed. The summation of the project and future directions are discussed in Chapter 8, "Conclusions".
CHAPTER 2
DESIGN OF SHAKEING TABLE

2.1 Introduction

The electro-hydraulic shaking table system is comprised of several components which must be designed to work smoothly and effectively together. To this end, every component of the shaking table system was designed based on the requirements of the entire system and the cost. The goal of the design process was to create a cost-effective, state-of-the-art uni-directional shaking table facility.

2.2 Shaking Table Components

The following figure is a diagram of the various components of the shaking table facility in the Ryon Laboratory at Rice University. The major components of the system are divided into five categories: (1) Program and Data Acquisition, (2) Control, (3) Hydraulic Power System, (4) Servo Valve and Actuator, and (5) Slip Table and Foundation. The shaking table is an electro-hydraulic system, meaning that electrical signals control and regulate hydraulic fluid under pressure to move the table. An electrical signal representing the desired (or commanded) table motion is sent by the personal computer to the controller, which evaluates the difference between the desired position of the table and the actual position. The controller then opens or closes the servovalve appropriately to port a portion of the constant flow of hydraulic fluid that is being provided by the pump into the actuator. This fluid forces an actuator arm, which is connected to the slip table, to move in the desired direction. Test specimens, such as scaled models of structures, are fixed to the slip
Figure 2.1 Diagram of the Shaking Table Components

table. Feedback from the various components can be fed back to the personal computer for the purposes of control or data acquisition. A brief description is provided to give an overview of the function of each component.
2.2.1 Hydraulic Power System

The hydraulic power that creates the force to move the slip table is provided by the pump (hydraulic power supply, or HPS) shown in Figure 2.2 (a). The pump is only able to provide a steady flow and this flow is subject to pressure fluctuations. The service manifold, see Figure 2.2 (b), equipped with nitrogen-charged accumulators, reduces the pressure fluctuations and can help provide extra flow when needed. The properties of the hydraulic power system are discussed in Chapter 3.

2.2.2 Servovalve and Actuator

The servovalve shown in Figure 2.3 (a) ports the fluid provided by the hydraulic power system into the appropriate side of the actuator’s piston to cause the actuator arm to move in the desired direction. The servovalve is essentially a system of smaller valves and spools. A detailed discussion of the servovalve is found in Chapter 7.

The main component of the actuator is the piston rod or “arm” which moves the slip table back and forth. As shown in Figure 2.3 (b), the piston rod is partially enclosed in the
actuator body or actuator chamber, and fluid is forced into the proper port to "push" the piston in the desired direction. The joint between the piston rod and the actuator body is sealed. The fluid pressure and effective piston area determine the force that can be produced by the actuator, and the maximum flow rate available determines the maximum table velocity that can be achieved.

(a) Photograph of the servovalve mounted on the actuator
(b) Cross-section of the actuator piston and chamber

**Figure 2.3 Actuator and Servovalve**

2.2.3 Slip Table

The slip table, see Figure 2.4, is a platform which acts as the moving base for the model structures fixed to its surface. It is mounted, through two pairs of trucks (with linear roller bearings), upon two high precision rails lying in the direction of shaking, see Figure 2.5, and its motion is controlled by the movement of the actuator arm fixed to one end of the table. The details of the design of the slip table are discussed in Chapter 5.
2.2.4 Rails and Steel Base Plate

The rails and trucks provide the sliding surface for the slip table. The trucks ride along the precision milled rails with minimum friction. As shown in Figure 2.5, these rails are mounted to a steel base plate to assure their precise placement.
2.2.5 Reaction Mass

The entire slip table system (table, rails, steel base plate, actuator, and servovalve) is fixed to a concrete reaction mass measuring 12 feet by 12 feet by 3 feet thick. The purpose of the reaction mass is to provide an inertial base to prevent excessive deformation and vibration of the base should base isolation be implemented.

2.2.6 Base Isolation

The proposed shaking table included base isolation in the form of seismic isolation bearings. These bearings would be placed between the reaction mass and the laboratory floor. The study of the base isolation system is found in Chapter 4.

2.2.7 Controller

The controller, see Figure 2.6, is the interface between the command signal and the operation of the servovalve and actuator. Through two levels of control, the controller regulates
the displacement of the actuator arm. First, the controller regulates the porting of fluid by
the servovalve (the "inner loop"), which is the lowest level of control. Secondly, the con-
troller takes information from the actuator mounted LVDT to evaluate and control the
position of the actuator arm. This is the "outer loop", and requires repeated adjustments in
the state of the inner loop (valve openings) to achieve the target actuator position, and is
the higher level of control. The controller employs a proportional-integral-derivative-feed-
forward (PIDF) control algorithm to regulate and monitor the state of the system. Chapter
7 includes a detailed discussion of the PIDF Controller.

2.2.8 Program, Data Acquisition, and Data Processing

External processing of shaking table data is necessary, and is handled by a Gateway 2000
Pentium personal computer, see Figure 2.6, equipped with data acquisition (I/O) and sig-
nal processing boards. This allows for external signal generation and the implementation
of another level of control, using feedback data from the shaking table and test structure.

2.3 Sizing of System Components

In determining the appropriate size for the various system components, realistic perfor-
mance targets were first established by examining typical earthquake records. The process
of sizing the hydraulic components, such as the servovalve, actuator, and pump, was iter-
ative, based on the available sizes of these components from manufacturers, and their com-
patibility with each other. It was determined prior to construction that the structures that
would be tested would typically be one-fifth scaled models of actual structures.
2.3.1 Statistics of Earthquake Ground Motions

The primary motivation for building a shaking table facility is to study the effects of seismic ground motions on reduced scale structures. Therefore, it is essential that the table be able to faithfully reproduce, in scaled version, actual earthquake ground motions. Table 2.1 below shows the peak values of ground acceleration, velocity, and displacement for several recorded earthquakes. Table 2.2 shows the ratios of peak ground velocity to peak ground acceleration and peak ground displacement to peak ground acceleration for the same earthquake records. Table 2.3 gives relative values of maximum ground acceleration, velocity and displacement for a "standard" and typical earthquake. Table 2.4 provides statistical data on the ratios presented in Tables 2.2 and 2.3. Examination of Table 2.4 reveals that typical values for the ratio of peak velocity to peak acceleration are 52 in/sec/g and 25 in/sec/g for soil and rock sites, respectively. Typical values for the ratio of peak displacement to peak acceleration are 26 in/g and 9 in/g for soil and rock sites, respectively.
### Table 2.1  Peak Values of Recorded Earthquake Ground Motions

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Comp.</th>
<th>Peak Ground Accel. (g)</th>
<th>Peak Ground Veloc. (in/sec)</th>
<th>Peak Ground Disp. (in)</th>
<th>Site</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northwest California, 10/7/51</td>
<td>S44W</td>
<td>0.104</td>
<td>1.89</td>
<td>0.94</td>
<td>Deep Cohesionless Soil, 500 ft.</td>
</tr>
<tr>
<td>Ferndale City Hall</td>
<td>N46W</td>
<td>0.112</td>
<td>2.91</td>
<td>1.08</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vert</td>
<td>0.027</td>
<td>0.87</td>
<td>0.64</td>
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<td>Eureka, 12/21/54</td>
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<td>0.159</td>
<td>14.04</td>
<td>5.58</td>
<td>Deep Cohesionless Soil, 500 ft.</td>
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<tr>
<td>Ferndale City Hall</td>
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<td>0.201</td>
<td>10.25</td>
<td>3.79</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Vert</td>
<td>0.043</td>
<td>2.99</td>
<td>1.54</td>
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<td>San Fernando, 2/9/71</td>
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<td>1.66</td>
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<td></td>
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<td>1.38</td>
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<tr>
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<td>0.89</td>
<td>Rock</td>
</tr>
<tr>
<td>San Francisco Golden Gate Park</td>
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<td>1.82</td>
<td>0.33</td>
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<td></td>
<td>Vert</td>
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<td>0.56</td>
<td>Rock</td>
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<td>Helena, Montana</td>
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<td>1.47</td>
<td></td>
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<tr>
<td>Carroll College</td>
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<td>1.11</td>
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<td>Imperial Valley, 5/18/40</td>
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<td>0.348</td>
<td>13.17</td>
<td>4.28</td>
<td>Alluvium, several thousand ft.</td>
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<td>Vert</td>
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<td>4.27</td>
<td>2.19</td>
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<td>Kern County, 7/21/52</td>
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<td>6.19</td>
<td>2.64</td>
<td>40 ft. Alluvium over poorly cemented sandstone</td>
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<td></td>
<td>Vert</td>
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<td>2.63</td>
<td>1.98</td>
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<th>Earthquake</th>
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<th>PGD/PGA (in/g)</th>
<th>Site</th>
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<td>S44W N46W</td>
<td>18.2 (S)&lt;sup&gt;a&lt;/sup&gt; 26.0 (L)</td>
<td>9.04 9.64</td>
<td>Deep Cohesionless Soil, 500 ft.</td>
</tr>
<tr>
<td>Eureka, 12/21/54 Ferndale City Hall</td>
<td>N44E N46W</td>
<td>88.3 (S) 51.0 (L)</td>
<td>35.1 18.9</td>
<td>Deep Cohesionless Soil, 500 ft.</td>
</tr>
<tr>
<td>San Fernando, 2/9/71 Castaic Old Ridge Route</td>
<td>N21E N69W</td>
<td>21.5 (L) 40.4 (S)</td>
<td>5.27 13.8</td>
<td>Sandstone</td>
</tr>
<tr>
<td>San Francisco, 3/22/57 San Francisco Golden Gate Park</td>
<td>N10E S80E</td>
<td>23.4 (S) 17.3 (L)</td>
<td>10.7 3.14</td>
<td>Rock</td>
</tr>
<tr>
<td>Helena, 10/31/35 Helena, Montana Carroll College</td>
<td>S00W S90W</td>
<td>19.8 (L) 36.2 (S)</td>
<td>3.84 10.1</td>
<td>Rock</td>
</tr>
<tr>
<td>Imperial Valley, 5/18/40 El Centro Site</td>
<td>S00E S90W</td>
<td>37.8 (L) 67.9 (S)</td>
<td>12.3 36.4</td>
<td>Alluvium, several thousand ft.</td>
</tr>
<tr>
<td>Kern County, 7/21/52 Taft Lincoln School Tunnel</td>
<td>N21E S69E</td>
<td>39.7 (S) 38.9 (L)</td>
<td>16.9 20.1</td>
<td>40 ft. Alluvium over poorly cemented sandstone</td>
</tr>
<tr>
<td>San Fernando, 2/9/71 8244 Orion Blvd. - 1st Floor</td>
<td>N00W S90W</td>
<td>52.5 (L) 70.3 (S)</td>
<td>23.0 40.7</td>
<td>Alluvium</td>
</tr>
</tbody>
</table>

a. L: Horizontal component with the larger of the two peak accelerations; S: Horizontal component with the smaller of the two peak accelerations.

<table>
<thead>
<tr>
<th>Condition</th>
<th>Maximum Values of Ground Motion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Acceleration (g)</td>
</tr>
<tr>
<td>&quot;Standard&quot; Relative Values</td>
<td>0.50</td>
</tr>
<tr>
<td>Typical Maxima</td>
<td></td>
</tr>
<tr>
<td>El Centro, 1940, horizontal</td>
<td>0.33</td>
</tr>
<tr>
<td>El Centro, 1940, vertical</td>
<td>0.22</td>
</tr>
<tr>
<td>Very Intense Earthquake</td>
<td>0.75</td>
</tr>
</tbody>
</table>

a. After Newmark and Hall (1969)
Table 2.4  Statistics of Peak Value Ratios\textsuperscript{a} (Lognormal Distribution)

<table>
<thead>
<tr>
<th>Soil Category</th>
<th>Group</th>
<th>PGV/PGA (in/sec/g)</th>
<th>PGD/PGA (in/g)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Percentile</td>
<td>Percentile</td>
</tr>
<tr>
<td></td>
<td></td>
<td>50</td>
<td>84.1</td>
</tr>
<tr>
<td>Rock</td>
<td>L</td>
<td>24</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>27</td>
<td>44</td>
</tr>
<tr>
<td>(&lt; 30 \text{ ft. of alluvium underlain by rock})</td>
<td>L</td>
<td>30</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>39</td>
<td>62</td>
</tr>
<tr>
<td>(30 - 200 \text{ ft. of alluvium underlain by rock})</td>
<td>L</td>
<td>30</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>36</td>
<td>58</td>
</tr>
<tr>
<td>Alluvium</td>
<td>L</td>
<td>48</td>
<td>69</td>
</tr>
<tr>
<td></td>
<td>S</td>
<td>57</td>
<td>85</td>
</tr>
</tbody>
</table>

\(\text{a. After Mohraz (1976)}\)

2.3.2 Scaling of Prototype Ground Motions

When performing an experiment on the earthquake response of a scaled test structure, two types of scaling are applied to the actual earthquake ground motion time histories used for the experiment. The first consists of scaling the actual base acceleration, velocity, and displacement time histories such that when applied to the model structure, they are equivalent to the actual base acceleration, velocity and displacement time histories applied to the prototype (full scale) structure. This type of scaling is known as scaling for similitude. The geometric scaling factor, \(\lambda_L\), is defined as

\[
\lambda_L = \frac{L_{\text{prototype}}}{L_{\text{model}}} \quad (2.1)
\]

where \(L\) denotes a geometric length (or dimension). Therefore, \(\lambda_L = 5\) indicates that the model is 1/5th the size of the prototype structure. Similitude is discussed in detail in Section 4.4.1 entitled Modeling Considerations. In the scaling for similitude, the geometric,
mass (or force) and time parameters (or dimensions) must be scaled. The scaling factor for
the time dimension in the case of "same material similitude" is (see Section 4.4.1):
\[ \lambda_T = \lambda_L \]  \hspace{1cm} (2.2)

Therefore, it follows that, for acceleration and velocity, the scaling factors are
\[ \lambda_A = \frac{\lambda_L}{(\lambda_T)^2} = \frac{1}{\lambda_L} \]  \hspace{1cm} (2.3)
\[ \lambda_V = \frac{\lambda_L}{\lambda_T} = 1 \]  \hspace{1cm} (2.4)

The second type of scaling used is simply an amplitude adjustment of the given time
histories, without a change in the time axis. The scaling factor, K, is applied to the base
acceleration, velocity and displacement records. This scaling preserves the ground motion
ratios given in Table 2.4. The factor K is the same for acceleration, velocity, and displace-
ment, since the constant K is carried through differentiation and integration. This scaling
will be referred to as magnitude scaling. It is useful when a magnitude change is desired
for a given set of ground acceleration, velocity and displacement time histories.

Both similitude and magnitude scaling can be used together to create a model ground
motion. For example, a 1/5 scale model (\(\lambda_L = 5\)) of a structure is to be tested on the shaking
table. A specific earthquake ground acceleration record is chosen because it possesses a
desired characteristic, such as a specific frequency content. It may be desirable to know at
what magnitude (base acceleration, velocity, or displacement) the structure will yield. The
ground motion time histories are scaled for similitude, increasing the acceleration magni-
tude by a factor of 5 (since \(\lambda_A = 1/5\)), leaving the velocity magnitude intact (since
\( \lambda_V = 1 \), decreasing the displacement magnitude by a factor of 5 (since \( \lambda_D = \lambda_L = 5 \)), and compressing the time axis by a factor of 5 (since \( \lambda_T = 5 \)). Furthermore, the time histories can be scaled by a magnitude scaling factor to simulate different levels of magnitude of the same seismic motions. In summary, for this particular case, the model base acceleration, velocity, and displacement time histories would be given by:

\[
\begin{align*}
A_{model}(t_{model}) &= K\lambda_L A_{prototype}(t_{prototype}/\lambda_L) \\
V_{model}(t_{model}) &= KV_{prototype}(t_{prototype}/\lambda_L) \\
D_{model}(t_{model}) &= \frac{K}{\lambda_L}D_{prototype}(t_{prototype}/\lambda_L)
\end{align*}
\]

(2.5)

2.3.3 Payload

The maximum design payload of the shaking table depends on the desired maximum base acceleration, \( A_{\text{max}} \). With a limit on the force that can be applied by the actuator, \( F_{\text{max}} \), the maximum weight of the test structure plus the slip table, \( W_{\text{max}} \), is

\[
W_{\text{max}} = \frac{F_{\text{max}}}{A_{\text{max}}} g
\]

(2.6)

in which \( g \) denotes the acceleration of gravity. Physical limits of the pump and servo valve also play a major role in the acceleration that can actually be produced by the actuator (see Chapter 3). Analyses of the shaking table system were performed for model structures weighting from 0 to 2000 lbs, which represents prototype structures weighing up to 125 tons (with \( \lambda_L \) equal to five and “same material similitude”). The figure below shows the maximum force required by the actuator to reproduce three representative earthquake ground motions. This analysis was done using a shear beam model of a three-story one-bay
steel moment resisting frame with story masses weighting from 0 to 500 lbs. The prototype ground motions were magnitude scaled to 1g peak ground acceleration, and then scaled for similitude with a scaling factor $\lambda_L = 5$. Based on the results shown in Figure 2.7 with a safety factor of 1.5, along with information about available actuators, a 36 kips maximum actuator force was chosen. Knowing that the hydraulic pressure available

![Graph showing Maximum Actuator Force for Three Representative Earthquake Records](image_url)

Figure 2.7 Maximum Actuator Force for Three Representative Earthquake Records

would be 3000 psi (standard), the required effective actuator piston area was determined to be

$$A = \frac{F}{P} = \frac{36000 \text{ lbs}}{3000 \text{ psi}} = 12 \text{ in}^2$$  \hspace{1cm} (2.7)
The actuator that was selected was an MTS Model 244 Hydraulic actuator rated at 35 kips with an effective piston area of 12.73 in\(^2\) and a stroke of +/- 3 inches.

2.3.4 Sizing of Hydraulic System

The pump, servovalve, and actuator were sized to be able to reproduce typical seismic motions, such as those listed in Tables 2.1 to 2.3. The target value for the prototype peak ground acceleration was chosen as 1 g, the prototype peak ground velocity as 36 in/sec, and the prototype peak ground displacement as 15 inches. The value of the similitude geometric scaling factor, \(\lambda_L\), was chosen to be approximately 5 for a typical model structure. Therefore, the maximum values of the model base acceleration, velocity, and displacement are 5g, 36 in/sec, and 3 inches, respectively. The decision for the maximum values was based, in part, on available equipment sizes.

Since the piston effective area was known to be 12.73 in\(^2\), the maximum required flow into the actuator, \(Q_{\text{max}}\), was determined to be

\[
Q_{\text{max}} = AV_{\text{max}} = (12.73)(36) \frac{\text{in}^3}{\text{s}} = 458 \frac{\text{in}^3}{\text{s}} = 118.8 \text{ gpm}
\]  

(2.8)

The servovalve selected was an MTS Model 256 Three-Stage Servovalve, rated at 180 gpm maximum flow. For random flow conditions, which is the case when simulating earthquake motions, the pump must be able to provide an average sustained flow equal to

\[
Q_{\text{pump}} = \frac{Q_{\text{max}}}{\frac{3\pi}{2}} = 25.2 \text{ gpm}
\]

(2.9)

The pump selected, an MTS Model 510.30 pump, is rated at 30 gpm of steady flow.
2.3.5 Maximum Magnitude Scaling Factors for Typical Earthquake Records

Examination of the earthquake records listed in Tables 2.1 to 2.4 reveals that actual earthquake induced ground motions can have a wide range of characteristics as measured in terms of peak ground acceleration, peak ground velocity, and peak ground displacement. For a shaking table with limited displacement, velocity, and acceleration capacities, the scaling factors for the model structure and ground motions are controlled by the physical limitations of the table. Table 2.5 shows, for the cases \( \lambda_L = 5 \) and 6, the maximum value of the magnitude scaling factor, \( K \), considering a maximum table acceleration of 6g's, a maximum table velocity of 42.6 in/sec, and a maximum table displacement of 3 inches, which correspond to the actual characteristics of the shaking table. The symbol in parenthesis next to the value of the scaling constant represents which limit is first violated: d for displacement or v for velocity. Shown also is the peak prototype ground acceleration based on the maximum value of \( K \).
<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Comp</th>
<th>PGA (g)</th>
<th>PGV (in/sec)</th>
<th>PGD (in)</th>
<th>$K_{\text{max}}$ ($\lambda_2=5$)</th>
<th>$K_{\text{max}}$ x PGA ($\lambda_2=5$)</th>
<th>$K_{\text{max}}$ ($\lambda_2=6$)</th>
<th>$K_{\text{max}}$ x PGA ($\lambda_2=6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Northwest California, 10/7/51 Ferndale City Hall</td>
<td>S44W</td>
<td>0.104</td>
<td>1.89</td>
<td>0.94</td>
<td>15.96 (d)</td>
<td>1.66</td>
<td>19.14 (d)</td>
<td>1.99</td>
</tr>
<tr>
<td></td>
<td>N46W</td>
<td>0.112</td>
<td>2.91</td>
<td>1.08</td>
<td>13.89 (d)</td>
<td>1.56</td>
<td>14.64 (v)</td>
<td>1.64</td>
</tr>
<tr>
<td>Eureka, 12/21/54 Ferndale City Hall</td>
<td>N44E</td>
<td>0.159</td>
<td>14.04</td>
<td>5.58</td>
<td>2.69 (d)</td>
<td>0.43</td>
<td>3.03 (v)</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>N46W</td>
<td>0.201</td>
<td>10.25</td>
<td>3.79</td>
<td>3.96 (d)</td>
<td>0.80</td>
<td>4.16 (v)</td>
<td>0.84</td>
</tr>
<tr>
<td>San Fernando, 2/9/71 Castaic Old Ridge Route</td>
<td>N21E</td>
<td>0.315</td>
<td>6.76</td>
<td>1.66</td>
<td>6.30 (v)</td>
<td>1.99</td>
<td>6.30 (v)</td>
<td>1.99</td>
</tr>
<tr>
<td></td>
<td>N69W</td>
<td>0.217</td>
<td>10.95</td>
<td>3.74</td>
<td>3.89 (v)</td>
<td>0.84</td>
<td>3.89 (v)</td>
<td>0.84</td>
</tr>
<tr>
<td>San Francisco, 3/22/57 San Francisco Golden Gate Park</td>
<td>N10E</td>
<td>0.083</td>
<td>1.94</td>
<td>0.89</td>
<td>16.85 (d)</td>
<td>1.40</td>
<td>20.22 (v)</td>
<td>1.68</td>
</tr>
<tr>
<td></td>
<td>S80E</td>
<td>0.105</td>
<td>1.82</td>
<td>0.33</td>
<td>23.41 (v)</td>
<td>2.46</td>
<td>23.41 (v)</td>
<td>2.46</td>
</tr>
<tr>
<td>Helena, 10/31/35 Helena, Montana Carroll College</td>
<td>S00W</td>
<td>0.146</td>
<td>2.89</td>
<td>0.56</td>
<td>14.74 (v)</td>
<td>2.15</td>
<td>14.74 (v)</td>
<td>2.15</td>
</tr>
<tr>
<td></td>
<td>S90W</td>
<td>0.145</td>
<td>5.25</td>
<td>1.47</td>
<td>8.11 (v)</td>
<td>1.18</td>
<td>8.11 (v)</td>
<td>1.18</td>
</tr>
<tr>
<td>Imperial Valley, 5/18/40 El Centro Site</td>
<td>S00E</td>
<td>0.348</td>
<td>13.17</td>
<td>4.28</td>
<td>3.23 (v)</td>
<td>1.13</td>
<td>3.23 (v)</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>S90W</td>
<td>0.214</td>
<td>14.54</td>
<td>7.79</td>
<td>1.93 (d)</td>
<td>0.41</td>
<td>2.31 (d)</td>
<td>0.49</td>
</tr>
<tr>
<td>Kern County, 7/21/52 Taft Lincoln School Tunnel</td>
<td>N21E</td>
<td>0.156</td>
<td>6.19</td>
<td>2.64</td>
<td>5.68 (d)</td>
<td>0.89</td>
<td>6.82 (d)</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>S69E</td>
<td>0.179</td>
<td>6.97</td>
<td>3.60</td>
<td>4.17 (d)</td>
<td>0.75</td>
<td>5.00 (d)</td>
<td>0.90</td>
</tr>
<tr>
<td>San Fernando, 2/9/71 8244 Orion Blvd. 1st Floor</td>
<td>N00W</td>
<td>0.255</td>
<td>11.81</td>
<td>5.87</td>
<td>2.56 (d)</td>
<td>0.65</td>
<td>3.07 (v)</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>S90W</td>
<td>0.134</td>
<td>9.42</td>
<td>5.45</td>
<td>2.75 (d)</td>
<td>0.37</td>
<td>3.30 (d)</td>
<td>0.44</td>
</tr>
</tbody>
</table>
2.3.6 Sizing of the Slip Table

The slip table was designed to be five feet by five feet. The detailed design of the slip table is the subject of Chapter 5.

2.4 Controller Specifications

The purpose of the controller is to regulate the position of the actuator arm. The controller that was chosen was the MTS Model 407 Controller, equipped with displacement and pressure feedback (Δ-P). Though the optimal setup for the shaking table system would be one where the acceleration of the table is the quantity that is directly regulated, the displacement controller had some practical advantages over an acceleration controller: dramatically lower cost, available differential pressure (Δ-P) feedback for stabilization over a wider range of control gain settings, and it is a well developed technology. The MTS Model 469D Digital Control System provides simultaneous control of displacement, velocity, and acceleration, emphasizing displacement at low frequencies, velocity at middle frequencies, and acceleration at high frequencies. This controller, by providing high fidelity and high bandwidth control, seems ideal for seismic applications, however its cost was too high for the budget available.

The MTS Model 407 Controller is a single channel, digitally-supervised, Proportional, Integral, Derivative, Feedforward (PIDF) servo controller, providing control of one servo-hydraulic channel in an MTS test system. It is capable of internal function generation for simple functions (sine, square, and triangular waves), but can also accept external analog and digital signals for command table displacement and triggering. A PIDF control algorithm is utilized, and the gains can be adjusted for optimum table response for chang-
2.5 Requirements of the Data Acquisition System

The Gateway 2000 Pentium computer is equipped with two National Instruments AT-A2150 data acquisition boards, capable of sampling a total of eight analog input channels simultaneously. Continuous sampling rates can range from 4,000 to 51,200 Hz. The boards have internal filtering and analog-to-digital conversion (A/D) capabilities.

A National Instruments AT-DSP2200 Digital Signal Processing (DSP) board was installed on the personal computer. The DSP board has two analog input channels and two analog output channels, capable of simultaneous data acquisition and signal generation at a sampling rate from 4,000 to 51,200 Hz. The DSP processor is equipped with onboard memory and can be programmed to acquire, process, and generate signals without calling on the resources of the main processor of the host PC. Advanced signal processing, waveform generation, and acquisition functions have been pre-programmed for the DSP board, providing a large library of complex functions (e.g. windowing, transforms, and filtering of signals) that can be utilized.

2.6 Design of Steel Base Plate

The steel plate was designed specifically for the size of the actuator and slip table that were chosen. The figure below shows the layout and the dimensions, with tolerances, that were specified for the steel base plate.

The rectangular groove at the top of the plate serves as the mounting surface for the actuator. Four high-strength bolts fix the actuator's swivel base in place (shown as solid circles
Figure 2.8  Layout of Steel Base Plate (5' x 10' x 2'' symmetric)
in the figure). The rectangular groove in the center of the plate allows clearance for the actuator's swivel head to slide when the table is in motion. The two rail grooves provide a flat and level surface for the rails to be mounted. Holes were added for the rods which were used to position the plate before grouting to the reaction mass. Lengths of threaded rod were embedded in the top concrete slab of the reaction mass, and run through these holes, and the plate was tightened in place by a series of hex nuts. Five holes were drilled tapped to provide injection ports for the grout that fixes the plate rigidly to the reaction mass. Four holes were drilled and tapped for the set screws, which were used to level the plate. Additionally, four larger holes were drilled and tapped where eye-bolts were attached for lifting and moving the 4000 pound plate. Finally, sixty-two holes were tapped along the length of the rails (31 per rail) where the rails were fastened to the steel plate.

2.7 Specifications of the Rails

The rails system upon which the slip table is mounted is composed of two stainless-steel monorails and trucks (with pre-loaded linear roller bearings) which grip the rail and move along its axis, and to which the slip table is attached. A cross section of the rail and truck attachment is shown in Figure 2.9. The rails that were used are Schneeberger MRB 45 rails, with the tightest possible tolerances. Tables 2.6 and 2.7 list the dimensions of the rail system, and the tolerances for its positioning. The rail and truck load specifications are shown in Figure 2.10.
2.8 Conclusions

Each element of the shaking table system was designed and sized for high performance. To achieve optimal performance, this interaction of the many components in this complex system must be understood, as well as the inherent limitations of the various elements and of the entire system. While the shaking table can simulate many motions, the limits imposed by the size of the table, servovalve, actuator, and pump, and those imposed by the displacement controller, create a need for an optimization of the simulation with respect to the model size, model weight, and the controller's gain settings. The next chapter investigates the physical limitations imposed by the pump, servovalve, and actuator.

Figure 2.9  Cross Section of a Rail and Truck
Table 2.6  Dimensions of the Rail System

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total height</td>
<td>60 mm (2.36 in.)</td>
</tr>
<tr>
<td>Height of Rail only</td>
<td>40 mm (1.57 in.)</td>
</tr>
<tr>
<td>Width of Carriage</td>
<td>120 mm (4.72 in.)</td>
</tr>
<tr>
<td>Width of Rail</td>
<td>45 mm (1.77 in.)</td>
</tr>
<tr>
<td>Length of Carriage</td>
<td>172.5 mm (6.79 in.)</td>
</tr>
<tr>
<td>Rail to Rail Distance</td>
<td>914 mm o.c. (36 in.)</td>
</tr>
<tr>
<td>Longitudinal Carriage Spacing</td>
<td>914 mm o.c. (36 in.)</td>
</tr>
</tbody>
</table>

Table 2.7  Tolerances of the Rail System

<table>
<thead>
<tr>
<th>Distance</th>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max. permissible lateral height deviation</td>
<td>0.0914 mm (0.0036 in.)</td>
</tr>
<tr>
<td>Max. permissible longitudinal height deviation</td>
<td>0.0457 mm (0.0018 in.)</td>
</tr>
<tr>
<td>Max. permissible lateral parallelism tolerance</td>
<td>0.0120 mm (0.0005 in.)</td>
</tr>
<tr>
<td>Max. permissible long. parallelism tolerance</td>
<td>0.0069 mm (0.0003 in)</td>
</tr>
</tbody>
</table>
Schneebberger Monorail Type MRB 45

Loading Capacity:
- Side and Normal Load (C): 51,600 lbs (static)
- 28,700 lbs (dynamic)

Permissible Moments:
- Roll ($M_Q$): 2,600 ft-lbs
- Pitch and Yaw ($M_L$): 2,120 ft-lbs

Figure 2.10  Load Specifications of the Rails
CHAPTER 3

HYDRAULIC POWER SUPPLY AND FLOW CHARACTERISTICS

3.1 Introduction

The performance envelope of the shaking table is directly related to the physical limits of the hydraulic power supply (pump), servovalve, and actuator. The ability of the hydraulic system to meet the flow demand required to achieve a target table velocity depends on the nature of the applied signal (e.g., harmonic or random). The physical limits of actuator span and maximum force place other physical restraints on the table motion that can be achieved. An understanding of these limits is essential in determining the performance envelope for the shaking table.

3.2 Hydraulic Power Supply (HPS)

The MTS Model 510.30 pump provides the hydraulic power to the servovalve and actuator. This pump is capable of providing 30 gallons per minute of hydraulic fluid flow at both low and high pressures, 150 psi (1 MPa) and 3000 psi (21 MPa), respectively. The pump filters the hydraulic fluid (Mobil DTE 25 Hydraulic Oil) to 3 microns absolute. A fluid-to-water heat exchanger regulates the temperature of the hydraulic fluid, and a limit detector provides automatic shut-off if the fluid temperature exceeds a preset level. The table below lists the technical specifications for the MTS model 510.30 Hydraulic Power Supply unit.
### Table 3.1  MTS Model 510.30 HPS Specifications

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Model 510.30 Specs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reservoir Capacity, l (gal)</td>
<td>305 (80.5)</td>
</tr>
<tr>
<td>Flow Rate at 21 MPa (3000 psi) at 60 Hz, l/min (gpm)</td>
<td>113.6 (30)</td>
</tr>
<tr>
<td>Filtration (microns)</td>
<td></td>
</tr>
<tr>
<td>Nominal</td>
<td>0.45</td>
</tr>
<tr>
<td>Absolute (Beta₃ = 200)</td>
<td>3.0</td>
</tr>
<tr>
<td>Hydraulic Fluid</td>
<td>Mobil DTE 25</td>
</tr>
<tr>
<td>Pump motor at 60 Hz</td>
<td></td>
</tr>
<tr>
<td>kW (hp)</td>
<td>44 (60)</td>
</tr>
<tr>
<td>Starter (3-Phase, 380 V, 60 Hz)</td>
<td>Y-Δ</td>
</tr>
<tr>
<td>Continuous amps</td>
<td>77</td>
</tr>
<tr>
<td>Max. ambient operating temperature</td>
<td>40°C (104°F)</td>
</tr>
<tr>
<td>Min. ambient operating temperature</td>
<td>4°C (40°F)</td>
</tr>
<tr>
<td>Water flow</td>
<td></td>
</tr>
<tr>
<td>34 l/min (9 gpm) at 18.3°C (65°F)</td>
<td></td>
</tr>
<tr>
<td>61 l/min (16 gpm) at 26.7°C (85°F)</td>
<td></td>
</tr>
<tr>
<td>Length. mm (in)</td>
<td>1473 (58)</td>
</tr>
<tr>
<td>Width. mm (in)</td>
<td>1042 (41)</td>
</tr>
<tr>
<td>Height. mm (in)</td>
<td>1194 (47)</td>
</tr>
<tr>
<td>Weight, with fluid, kg (lbs)</td>
<td>975 (2150)</td>
</tr>
</tbody>
</table>

### 3.3 Flow Limits

The sizing of the hydraulic system (Section 2.3.4: Sizing of Hydraulic System) allowed for the reproduction of the peak values of many typical seismic records. However, the performance of the hydraulic system is dependent on the frequency content of the commanded motion. Two types of command signals are of interest here: harmonic and random (or seismic). The performance envelope of the hydraulic system under harmonic conditions is analyzed, from which the performance under random conditions can be derived.
3.3.1 Flow Limits for Harmonic Actuator Motion

Consider a commanded harmonic displacement signal sent to the actuator of the form

$$x_c(t) = C \sin(\omega t)$$  \hspace{1cm} (3.1)

The corresponding table velocity and acceleration are then

$$\dot{x}_c(t) = C \omega \cos(\omega t)$$  \hspace{1cm} (3.2)

and

$$\ddot{x}_c(t) = -C \omega^2 \sin(\omega t)$$  \hspace{1cm} (3.3)

The expression for the required actuator flow is found to be

$$q(t) = v(t)A = AC \omega \cos(\omega t)$$  \hspace{1cm} (3.4)

where \(v(t)\) is the piston velocity and \(A\) is the effective piston area. The value of \(q(t)\) in Eq. (3.4) may take on positive or negative values, representing the direction of the actuator piston. The pump provides flow in only one direction, and the servovalve ports the fluid into the appropriate side of the actuator piston. Therefore, we are only interested in the magnitude of fluid flow, and the above equation can be written as

$$q(t) = |v(t)|A = AC \omega |\cos(\omega t)|$$  \hspace{1cm} (3.5)

The average flow can be found by integrating over a half cycle, and is

$$q_{avg} = \left(\frac{1}{\pi/\omega}\right) \int_{\frac{\pi}{2\omega}}^{\frac{\pi}{2\omega}} AC \omega \cos(\omega t) dt = AC \sin(\omega t) \bigg|_{\frac{\pi}{2\omega}}^{\frac{\pi}{2\omega}}$$  \hspace{1cm} (3.6)

which evaluates to

$$q_{avg} = \frac{2AC \omega}{\pi}$$  \hspace{1cm} (3.7)

The peak flow is simply
\[ q_{\text{peak}} = |\dot{x}_{c, \text{max}}(t)|A = AC\omega \]

(3.8)

The ratio of the peak flow to the average flow is

\[ \frac{q_{\text{peak}}}{q_{\text{avg}}} = \frac{\pi}{2} = 1.571 \]

(3.9)

The pump is designed to provide only the steady or average part of the fluid flow, see Figure 3.1. The nitrogen charged accumulators mounted on both the servovalve and in the hydraulic manifold are designed to provide extra flow above the steady flow provided by the pump itself. The above equation illustrates that the transient flow that can be achieved with the pump and accumulators combined is higher than that achievable by the pump alone. Provided that the accumulators can produce the extra volume of fluid required, the actual peak flow for harmonic flow conditions is 57 percent higher than the maximum steady flow from the pump. Figure 3.2 illustrates the relationship described in Eq. (3.9).

The MTS Model 510.30 HPS pump can provide a steady flow of 30 gpm. With the help of the accumulators, the actual peak flow that can be delivered to the actuator is 47 gpm. In addition to the required flow rate, the accumulators must be able to provide the total required volume of fluid shown in Figure 3.1 as the unshaded area under the flow curve. Figure 3.2 shows the flow limits of the Model 510.30 HPS for steady and harmonic flow conditions. This figure depicts a steady 30 gpm flow for all frequencies, and neglects the roll-off frequency of the servovalve at about 100 Hz where a pronounced drop in flow occurs. It also neglects the force and span limits that will be discussed later. That is, for an actuator without a limitation on its displacement and force, these flows are hypothetically realizable, though as Eq. (3.8) shows, for low frequencies, the associated displacements for these flows are very high.
Figure 3.1  Fluid Flow under Harmonic Conditions

Figure 3.2  Theoretical Maximum Flow for Pump and Actuator (for Harmonic Conditions) Assuming Infinite Reach of Actuator Arm
3.3.2 Flow Limits for Random or Seismic Loading

Figure 3.3 shows a typical flow rate time history when the commanded signal for the table motion corresponds to an earthquake ground motion, characterized by a very peaked (jagged) and random behavior. As a rule-of-thumb based on experience, MTS specifies that with accumulators the peak flow for random loading is about three times the peak flow for harmonic loading, or

\[ q_{\text{seismic}}^{\text{max}} = 3q_{\text{harmonic}}^{\text{max}} = \frac{3\pi}{2} q_{\text{pump}}^{\text{max}} = 4.71 q_{\text{pump}}^{\text{max}} \quad (3.10) \]

Therefore, in the case of random table motion, the accumulators can help reach an actuator peak flow equal to 4.7 times the pump maximum steady flow.

![Graph showing flow rate for conditions of seismic or random loading](image)

**Figure 3.3** Flow Rate for Conditions of Seismic or Random Loading
3.3.3 Accumulator Capacity

The nitrogen charged accumulators store fluid during periods of low actuator flow, and then release fluid when flow is needed above the provision of the pump. The accumulators must be sized so that they can not only provide the hydraulic fluid at the proper flow rate, but also provide a sufficient volume of fluid for the duration when the needed flow exceeds the pump steady flow. For harmonic flow conditions, this volume of fluid is the unshaded region in Figure 3.1, that is,

\[ V_{\text{accum}} = \frac{0.421 q_{\text{peak}}}{\omega} = 0.421 AC \]  \hspace{1cm} (3.11)

3.3.4 Maximum Table Velocity

Maximum actuator flow can be converted to maximum table velocity by the simple relation

\[ q(t) = v(t)A \]  \hspace{1cm} (3.12)

where \( q(t) \) is the fluid flow rate, \( v(t) \) is the table velocity, and \( A \) is the effective piston area.

For a peak flow of 47 gpm and an effective piston area of 12.73 in\(^2\), the maximum table velocity for harmonic motion is 14.2 in/sec.

3.4 Span and Force Limits

Two other physical limitations play a major role in determining the performance envelope of the shaking table: span and force. The span limit is the maximum displacement, or "reach" of the actuator piston. Span is defined as the maximum distance that the piston can travel, which is 6 inches (\(+/-\) 3 inches from the center position) for the present design. Eq.
(3.8) above shows that, for harmonic flow conditions, span and frequency are inversely proportional. Therefore, the maximum span decreases for increasing frequency.

The maximum force that the actuator can produce is 35,000 pounds (35 kips). For a known table mass, this limit can be easily translated into a maximum table acceleration. The maximum acceleration of the bare table is given by

\[
a_{\text{max}}^{\text{bare}} = \frac{F_{\text{max}}}{W_{\text{table}}/g} = \frac{F_{\text{max}}}{W_{\text{table}}} g
\]  

(3.13)

where \(a_{\text{max}}^{\text{bare}}\) represents the maximum bare table acceleration, \(F_{\text{max}}\) represents the maximum actuator force (35,000 lbs), \(W_{\text{table}}\) represents the weight of the bare table, and \(g\) is the gravitational acceleration constant, 386.4 in/sec\(^2\). The weight of the bare slip table, \(W_{\text{table}}\), is 1260 lbs. Therefore, according to Eq. (3.13), the maximum bare table acceleration, \(a_{\text{max}}^{\text{bare}}\), is 27.8 g's, or 10,700 in/sec\(^2\). However, when loaded with a flexible test specimen, such as a building model, the maximum table acceleration decreases significantly. In the case of a 1500 lbs specimen, and assuming a dynamic amplification factor of 3, the total effective weight becomes 5760 lbs. The maximum table acceleration reduces to 6.08 g's or 2,350 in/sec\(^2\).

### 3.5 Performance Envelopes

The plots in Figs. 3.4 through 3.6 show the relationships between frequency and maximum span, maximum velocity, and maximum acceleration for harmonic conditions. Five different cases were considered for the total effective table mass; they are defined in Table 3.2 and labeled on each graph. In Figure 3.4 it can be seen that the maximum span of 6
inches is the controlling factor for frequencies less than 0.75 Hz. In this range, the maximum table velocity that can be imparted to the table is less than the absolute maximum velocity of 14.2 in/sec dictated by the pump capacity. In other words, the flow required in

Table 3.2  Table Weight Condition in Figs. 3.4 through 3.6

<table>
<thead>
<tr>
<th>Load Number</th>
<th>Loading Description</th>
<th>Dynamic Amplification Factor</th>
<th>Total Effective Weight (lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Bare Table</td>
<td>1</td>
<td>1260</td>
</tr>
<tr>
<td>2</td>
<td>Bare Table + 500 lbs of Rigid Mass</td>
<td>1</td>
<td>1760</td>
</tr>
<tr>
<td>3</td>
<td>Bare Table + 1000 lbs of Rigid Mass</td>
<td>1</td>
<td>2260</td>
</tr>
<tr>
<td>4</td>
<td>Bare Table + 1000 lbs of Flexible Mass</td>
<td>3</td>
<td>4260</td>
</tr>
<tr>
<td>5</td>
<td>Bare Table + 1500 lbs of Flexible Mass</td>
<td>3</td>
<td>5760</td>
</tr>
</tbody>
</table>

this low frequency range is less than the actuator flow capacity assuming no span limit. As shown in Figure 3.5, in the intermediate frequency range (0.8 to 25 Hz), the flow capacity of the hydraulic system controls. In the high frequency range (above 25 Hz), the maximum force of the actuator controls, as shown in Figure 3.6. Figure 3.7 shows the relationship between the envelopes of maximum actuator flow volume assuming no span limit, maximum actuator flow volume considering the span limitation, flow volume provided by the pump, and flow volume provided by the accumulators over a half cycle of harmonic flow.

The curves in Figs. 3.4 through 3.6 can be combined into a single curve by utilizing the following relationship for harmonic conditions:

\[ V_{\text{max}} = \frac{A_{\text{max}}}{\omega} = \frac{S_{\text{max}}}{2\omega} \quad (3.14) \]

where \( V_{\text{max}} \) is the maximum table velocity, \( A_{\text{max}} \) is the maximum table acceleration, \( S_{\text{max}} \)
is the maximum table span (2 times the maximum displacement amplitude), and \( \omega \) is the frequency of the harmonic table motion, in rad/sec. This tri-partite curve offers a compact representation of the shake table performance envelope, and is shown in Figure 3.8. The dashed line represents the instantaneous (seismic) capability of the hydraulic system, and is shown for comparison.

Figure 3.4 Maximum Span vs. Frequency for Harmonic Conditions
Figure 3.5  Maximum Table Velocity vs. Frequency for Harmonic Conditions

Figure 3.6  Maximum Table Acceleration vs. Frequency for Harmonic Conditions
Figure 3.7  Volume of Flow over a Half Cycle of Harmonic Motion 
(corresponds to Fig. 3.2)

3.6 MTS Flow Curve

The manufacturer of the pump, MTS, provided the flow diagram shown in Figure 3.9. 
Notice that this is different from the flow diagram in Figure 3.2, where it was assumed that 
the flow was constant at 30 gpm. In the above figure, the pump flow capacity is 22 gpm, 
and the underlying dynamics of the hydraulic system (line accumulator) causes the 
notched flow behavior below 2 Hz. Following the analysis for the 30 gpm case, a tri-partite 
curve for this flow behavior can be derived (see Figure 3.10). The maximum span and 
acceleration limits are unchanged, as these quantities are related to the actuator reach and 
maximum force, not to the flow capacity. The envelopes for the harmonic and instantaneous velocities, however, are approximately 27 percent less than the envelopes based on 
30 gpm constant pump flow capacity.
Figure 3.8  Tri-Partite Graph Theoretical Performance Envelope
3.7 Conclusions

The physical limits imposed by the pump, servovalve, and actuator determine the scale of the simulation that is realizable for a given target earthquake ground motion record. The ratios between the actual peak ground displacement, peak ground velocity, and peak ground acceleration, and the ceiling defined by the hydraulic system’s limits define the minimum amount of scaling that must be applied to achieve the physical simulation. Scaling factors (from prototype to model quantities) are discussed in the following chapter in the discussion of sizing the isolation system for typical model structures.
Figure 3.10  Tri-Partite Graph of Theoretical Performance Envelope Using MTS Flow Diagram
CHAPTER 4
BASE ISOLATION SYSTEM

4.1 Introduction

The proposed shaking table system is shown in Figure 4.1. The model structure to be tested is fixed to the slip table. A base acceleration is applied to the structure by means of an actuator, which applies a motion to the slip table. The actuator is fixed to a concrete foundation acting as a reacting mass. The foundation mass rests upon base isolation pads, which are typically made of alternating laminations of steel and rubber. The purpose of the base isolation system is to reduce the energy transmitted to the laboratory floor beneath the shaking table by the vibration induced on the table.

4.2 Problem Statement

Properly designed, the base isolation pads reduce the shear force transmitted to the floor. The vertical compliance of the isolation pads is not being considered in the following analysis because the actuator introduces a horizontal excitation, and provided that the mass of the structure is not large compared to the foundation mass, the rocking motion will be negligible. However, if the shear stiffness of the isolation pads causes the natural frequency of the foundation mass to be close to the frequency of the actuator force, a resonance condition will exist, and the foundation will experience large displacements. This condition may lead to the bearings becoming unstable due to buckling or roll-out.

The purpose of this analysis is to determine an optimum value of the isolation stiffness ($k_b$ in Figure 4.1) for a range of test models and table acceleration records.
4.3 Equations of Motion

The equation of motion of the coupled actuator-foundation-structure system shown in the figure above will be derived. The solution to the resulting system of equations will lead to expressions for the base shear and transmissibility of the shaking table system.

4.3.1 Formulation of the Equations of Motion

Consider the multi-degree of freedom system shown in Figure 4.1, where

\[
M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \quad C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}, \quad \text{and} \quad K = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \tag{4.1}
\]

define the mass, damping, and stiffness matrices, respectively, of the test structure in the direction of shaking. The absolute acceleration applied to the base of the structure (\(m_b\)) is
defined as $\ddot{u}_g(t)$, and the corresponding displacement relative to the laboratory floor as $u_g(t)$. Likewise, the test structure floor displacements relative to the slip table are defined as $v_1(t)$, $v_2(t)$, and $v_3(t)$. Hence, the vector of floor displacements relative to the base of the test structure can be defined as:

$$V(t) = \begin{cases} v_1(t) \\ v_2(t) \\ v_3(t) \end{cases}$$

(4.2)

The displacement of the foundation mass ($m_f$) relative to the floor is defined as $v_b(t)$. Furthermore, the following vectors are introduced:

$$1 = \begin{cases} 1 \\ 1 \\ 1 \end{cases} \quad \text{and} \quad 0 = \begin{cases} 0 \\ 0 \\ 0 \end{cases}$$

(4.3)

The system of equations defining the motion of the superstructure (or test structure) can be written as

$$M(\ddot{V}(t) + 1\ddot{u}_g(t)) + C\dot{V}(t) + KV(t) = 0$$

(4.4)

where $\ddot{V}(t)$ and $\ddot{V}(t)$ denote the accelerations and velocities of the floor masses relative to the moving base ($m_b$). The quantity ($\ddot{V}(t) + 1\ddot{u}_g(t)$) in Eq. (4.4) represents the vector of absolute accelerations of the floor masses (i.e., relative to the laboratory floor). The force that the actuator applies to the slip table ($m_b$) is referred to as $F_a(t)$\(^1\). The free-body

---

1. Sign convention: the actuator force $F_a(t)$ is positive when the piston rod is in tension.
Figure 4.2  Free Body Diagrams

diagram in Figure 4.2 (a) below reveals the following relation:

\[
F_a(t) = 1^T M(\ddot{V}(t) + \ddot{U}_g(t)) + m_b \ddot{U}_g(t) \quad (4.5)
\]

The actuator force, \(F_a(t)\), is applied equally and oppositely to the actuator and the foundation mass (\(m_f\)), as the free-body diagram in Figure 4.2 (b) indicates. Summation of forces yields the following equation:

\[
-F_a(t) = m_f \dddot{V}_b(t) + c_b \dddot{V}_b(t) + k_b \dddot{V}_b(t) \quad (4.6)
\]

Combining Eqs. (4.5) and (4.6) yields,

\[
1^T M(\dddot{V}(t) + \dddot{U}_g(t)) + m_b \dddot{U}_g(t) + m_f \dddot{V}_b(t) + c_b \dddot{V}_b(t) + k_b \dddot{V}_b(t) = 0 \quad (4.7)
\]

The system of equations defined by Eqs. (4.4) and (4.7) can be written as:
\[
\begin{bmatrix}
m_f (1^T M) \\
0 & M
\end{bmatrix}
\begin{bmatrix}
\ddot{v}_b(t) \\
\ddot{V}(t)
\end{bmatrix}
+ \begin{bmatrix}
c_b & 0^T \\
0 & C
\end{bmatrix}
\begin{bmatrix}
\dot{v}_b(t) \\
\dot{V}(t)
\end{bmatrix}
+ \begin{bmatrix}
k_b & 0^T \\
0 & K
\end{bmatrix}
\begin{bmatrix}
 v_b(t) \\
 V(t)
\end{bmatrix}
= \\
\begin{bmatrix}
(-1^T M1) - m_b & 1 \\
(-M1) & 0
\end{bmatrix}
\ddot{u}_g(t)
\]

(4.8)

The above system of equations is a general expression for the equations of motion governing the system shown in Figure 4.1 when the following assumptions are made:

1. The frictional forces between moving elements are negligible. For example, the rollers shown between the slip table and the foundation mass (see Figure 4.1) are assumed to allow the relative motion of the two masses without the generation of significant frictional forces. Therefore, these forces are ignored in the free-body diagrams in Figure 4.2.

2. The values of the stiffness and damping coefficients remain constant through the range of displacements and velocities experienced by the system. This assumption can generally be considered to be valid for the superstructure, provided that the structural displacements remain small (i.e., the material remains in the elastic range). However, this assumption does not hold when testing is performed in the failure range of the structural model, in which case the structural material is expected to yield.

The assumption of constant stiffness and damping coefficients is not generally valid for the isolation material. The elastomeric material used in vibration isolation typically exhibits very non-linear force-deflection behavior even at small displacements (Kelly 1987, 1992).

The solution of the system of differential equations in Eq. (4.8) for any input table
motion, \( \dot{u}_g(t) \), yields the time history of displacements for all the lumped masses, from which the velocities and accelerations can be calculated. Either Eq. (4.5) or Eq. (4.6) can then be used to solve for the time history of the actuator force, \( F_a(t) \). Knowing the base displacement, \( v_b(t) \), the expression for the base shear is then

\[
V_b(t) = k_b v_b(t),
\]

and the expression for the transmissibility is simply

\[
TR = \frac{V_{b,\text{max}}}{F_{a,\text{max}}}.
\]

### 4.3.2 Solving the Equations of Motion

An exact piecewise linear integration algorithm was used to integrate the modal equations of motion. This algorithm is explained in detail in Appendix A. The solution method did not consist of solving the system of equations (4.8) directly, but rather, the problem was uncoupled. First, Eq. (4.4) was solved for \( V(t) \), from which the actuator force \( F_a(t) \) was obtained using Eq. (4.5), and then Eq (4.6) was solved for the displacement \( v_b(t) \) of the SDOF foundation mass.

### 4.4 Design of Base Isolation System

The design problem consists of determining the appropriate stiffness of the base isolators such that:

(1) the force transmitted to the floor is kept to an acceptable level, and

(2) the displacement of the foundation mass is kept to an acceptable level,

for a "typical" table dynamic excitation. Implicit in the two conditions above is that under
dynamic loading, the table will not experience resonance or near-resonance conditions. Consider the one-degree-of-freedom system shown in Figure 4.3 below. The natural frequency of the system is simply

\[
f = \frac{1}{2\pi} \sqrt{\frac{K_s}{M}}
\]  

(4.11)

Consider that the system is excited with a harmonic loading of amplitude \( P \) and circular frequency \( \omega \). Figure 4.4 shows the relationship between the stiffness of the spring \( (K_s) \) and the maximum SDOF displacement response \( (x) \), for a given mass \( M \), harmonic load amplitude \( P \), and circular frequency \( \omega \) (see Table 4.1). Figure 4.5 shows the relationship between the spring stiffness and the force in the spring, and therefore the force transmitted to the laboratory floor. The ratio of the amplitude of the transmitted force to the amplitude of the harmonic input force is called the transmissibility of the system. In both cases, the maximum response occurs when the spring stiffness is such that the natural frequency of the SDOF system coincides with the exciting frequency. Notice that the maximum displacement response for very low spring stiffness approaches a constant, but non-zero value, and
it approaches zero as $K_s$ increases to infinity. Conversely, the amplitude of the transmitted force of the system starts out at zero for very low spring stiffness, and approaches the amplitude of the exciting force for high spring stiffness.

Table 4.1 Parameters Used for SDOF Foundation Mass System in Figs. 4.3 to 4.5

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass, $M$</td>
<td>0.168 kips-s$^2$/in</td>
</tr>
<tr>
<td>Forcing Amplitude, $P$</td>
<td>1.0 kip</td>
</tr>
<tr>
<td>Forcing Frequency, $\omega$</td>
<td>1 Hz or $2\pi$ rad/sec</td>
</tr>
<tr>
<td>Stiffness at Resonance, $K_s$</td>
<td>6.63 kips/in</td>
</tr>
</tbody>
</table>

Figure 4.4 Maximum Displacement Response versus Spring Stiffness

The foundation mass of the shaking table can be modeled as a single-degree-of-freedom system, such as the one shown in Figure 4.3. However, the forcing function, which is the actuator force, will contain a range of frequencies of varying amplitudes as it models earthquake activity. One would expect the force and displacement responses of this system
Figure 4.5 Transmissibility of Force versus Spring Stiffness

to look similar to the ones shown in Figs. 4.4 and 4.5, but with multiple irregular peaks corresponding to the dominant frequencies of the simulated ground motion.

The next step was to develop these curves for typical earthquake ground motion records that would be simulated on the shaking table. Ground acceleration time histories have been gathered for a large number of earthquakes that have occurred in this century. One of the most widely used earthquake record is that of the Imperial Valley earthquake of 1940 recorded at the El Centro site. Three earthquakes records were considered in this study: El Centro, 1940 (Figure 4.6), Kern County (Taft Lincoln School Tunnel), 1952 (Figure 4.7), and San Fernando (Orion Blvd.), 1971 (Figure 4.8). These records, after being scaled both in time and amplitude to satisfy the similitude requirements discussed below, were used as the target table accelerations, \( \ddot{u}_g(t) \), in solving Eq. (4.8).
Figure 4.6  Imperial Valley Earthquake, California, May 18, 1940, El Centro Site, Component S00E (N-S)

Figure 4.7  Kern County California Earthquake, July 21, 1952, Taft Lincoln School Tunnel Site, Component N21E
4.4.1 Modeling and Similitude Considerations

The purpose of small scale modeling is:

(1) to accurately recreate the behavior of a full scale prototype structure using smaller or more manageable physical models, and

(2) to simulate real events within the constraints of available laboratory equipment.

In order to achieve similar response behavior between the model and prototype structures, certain relationships must be maintained between the physical variables that describe the model environment and those that describe the prototype environment. The first step is to determine the number of physical dimensions required to describe the variables of the prototype (or model) environment. For example, consider the system described in Eq. (4.4) above. The physical variables involved are: mass variables (M), damping variables (C), stiffness variables (K), displacement variables (V), ground acceleration (\(\ddot{u}_g\)), and time (t).
These six physical variables completely define the problem. Note that once the structural displacement variables (V) are known, the corresponding velocities and accelerations are obtained by simple differentiation with respect to time. The number of physical dimensions required to describe these variables is three: mass (M), length (L) and time (T). Table 4.2 shows the physical dimensions of each physical variable. The number of similitude invariants (i.e., independent dimensionless groups of the variables) is equal to the number of physical variables describing the problem minus the number of physical dimensions of these variables, namely 6 - 3 = 3. Since three physical dimensions are involved here, three similitude invariants can be satisfied (imposed) between the prototype and model environments. In alternate terms, the scaling factors of only three physical variables can be selected arbitrarily: the scaling factors of all other physical variables are then automatically set.

<table>
<thead>
<tr>
<th>Physical Variable</th>
<th>Physical Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass</td>
<td>M</td>
</tr>
<tr>
<td>Distance (Displacement)</td>
<td>L</td>
</tr>
<tr>
<td>Damping</td>
<td>M/T</td>
</tr>
<tr>
<td>Stiffness</td>
<td>M/T^2</td>
</tr>
<tr>
<td>Ground Acceleration</td>
<td>L/T^2</td>
</tr>
<tr>
<td>Time</td>
<td>T</td>
</tr>
</tbody>
</table>

To find the three similitude invariants, first a generic dimensionless scaling factor is defined as:
\[ \lambda_X = \frac{X_{\text{prototype}}}{X_{\text{model}}} \]  
\text{(4.12)}

where \( X \) denotes any physical variable, such as mass, force, stress, or energy. Note that, in the case of a matrix quantity, it is assumed that a scaling factor constant to all the matrix components applies. For example, in the case of the (3x3) mass matrix in Eq. (4.4),

\[ M_{\text{prototype}} = \lambda_M M_{\text{model}} = \lambda_M \begin{bmatrix} m_1^{\text{model}} & 0 & 0 \\ 0 & m_2^{\text{model}} & 0 \\ 0 & 0 & m_3^{\text{model}} \end{bmatrix} \]  
\text{(4.13)}

Consider the matrix equation of motion of a multi-degree-of-freedom system subjected to ground acceleration, \( \ddot{u}_g(t) \):

\[ M(\ddot{V}(t) + \dot{1}\ddot{u}_g(t)) + C\dot{V}(t) + KV(t) = 0 \]  
\text{(4.14)}

Assuming that Eq. (4.14) represents the equations of motion of the prototype (full scale) structure, we can write the equivalent equations of motion for the model structure using the dimensionless coefficients defined in Eq. (4.12)

\[ \left( \frac{1}{\lambda_M^{\text{V}}} \right) M \left( \frac{\lambda_t^2}{\lambda_V} \right) \ddot{V} + \left( \frac{1}{\lambda_M^{\ddot{u}_g}} \right) \dot{1} \ddot{u}_g + \left( \frac{1}{\lambda_C^{\dot{\lambda}}} \right) C \left( \frac{\lambda_t}{\lambda_V} \right) \dot{V} + \left( \frac{1}{\lambda_K^{\lambda_V}} \right) K \left( \frac{1}{\lambda_V} \right) V = 0 \]  
\text{(4.15)}

Enforcing equivalence between Eqs. (4.14) and (4.15) yields the relationships

\[ \frac{\lambda_t^2}{\lambda_M^2 \lambda_V} = \frac{1}{\lambda_M \lambda_{\ddot{u}_g}} = \frac{\lambda_t}{\lambda_C \lambda_V} = \frac{1}{\lambda_K \lambda_V} \]  
\text{(4.16)}

Dividing through by the third term yields

\[ \frac{\lambda_M \lambda_C}{\lambda_M^{\lambda_{\ddot{u}_g} \lambda_t}} = \frac{\lambda_C \lambda_V}{\lambda_M \lambda_{\ddot{u}_g} \lambda_t} = 1 = \frac{\lambda_C}{\lambda_K \lambda_t} \]  
\text{(4.17)}
Evaluating the first term
\[
\frac{\lambda_n \lambda_c}{\lambda_m} = 1 \iff \left( \frac{M_{ij}}{t C_{ij}} \right)_{\text{model}} = \left( \frac{M_{ij}}{t C_{ij}} \right)_{\text{prototype}} \tag{4.18}
\]
Likewise, evaluating the rest of the terms in Eq. (4.17), we find the three similitude invariants:
\[
\Pi_1 = \left( \frac{M_{ij}}{t C_{ij}} \right), \quad \Pi_2 = \left( \frac{K_{ij} t}{C_{ij}} \right), \quad \Pi_3 = \left( \frac{M_{ij} u_{ij} t}{C_{ij} V_k} \right) \tag{4.19}
\]
The invariants $\Pi_1$, $\Pi_2$, and $\Pi_3$ in Eq. (4.19) must take the same value in both the prototype and model worlds. Therefore, three independent scaling factors can be chosen, and the rest of the scaling factors must follow from those three choices. However, the invariants need not be explicitly calculated to determine appropriate scaling factors. The relationships between the scaling factors, in terms of the three chosen factors can be determined from dimensional analysis.

### 4.4.2 Example

The prototype structure is a steel moment resisting frame with known properties. A reduced-scaled model of it will be tested to determine the behavior of the frame when a certain ground acceleration is applied to its base. The prototype ground acceleration record is known.

The choice is made to build the model frame from the same material (steel). Therefore, we can immediately consider two scaling factors, $\lambda_E$ and $\lambda_p$, to be set, where $E$ and $\rho$ are the Young’s Modulus and mass density of the material, respectively. The two known parameters have the dimensions
\[
E = \frac{\text{Force}}{\text{Area}} = \frac{(\text{Mass})(\text{Accel})}{(\text{Length})^2} = \frac{(\text{Mass})(\text{Length})}{(\text{Length})^2(\text{Time})^2} = \frac{\text{Mass}}{(\text{Length})(\text{Time})^2}
\]

and

\[
\rho = \frac{\text{Mass}}{\text{Volume}} = \frac{\text{Mass}}{\text{Length}^3}
\]

We have chosen the scaling factors

\[
\lambda_E = 1, \quad \lambda_p = 1
\]

A third scaling factor must be chosen, and then all the other factors can be determined.

If it is known that the acceleration that the test equipment is capable of is five times that of the prototype ground acceleration, we may wish to choose

\[
\lambda_a = \frac{1}{5}
\]

At this point, all other scaling factors can now be derived from the first three. To determine the scaling factor for length and displacement, \(\lambda_L\), for example, we can make use of the above dimensional analysis as follows. From Eqs. (4.20) and (4.21) we can see that

\[
\lambda_E = \frac{\lambda_m \lambda_a}{\lambda_L^2} = \lambda_p \lambda_L \lambda_a
\]

Substituting in the values in Eqs. (4.22) and (4.23), we determine the value of \(\lambda_L\)

\[
\lambda_L = \frac{\lambda_E}{\lambda_p \lambda_a} = 5
\]

Hence, the appropriate model frame is a one-fifth scale model of the actual frame.
4.4.3 Similitude in Shake Table Analysis

For ease of fabrication, it is convenient to build test models out of the same material as the prototype structure. Hence, the scaling factors for Young's elastic modulus (E) and the material mass density (ρ) are set to unity. It was also determined for the shaking table that the geometric dimensions of the test models would approximately correspond to one-fifth of the prototype dimensions (1/5th scale models). With this information, the scaling factors for all the other physical variables were determined and are summarized in Table 4.3.

4.4.4 Table Parameters

Foundation Mass:

The foundation mass consists of three 12' x 12' x 1' thick reinforced concrete slabs, post-tensioned together to form a 432 cubic feet reacting mass, weighing approximately 64,800 lbs or, in consistent mass units, \( m_r = 0.168 \text{ kips-sec}^2/\text{in} \).

Slip Table:

The slip table is a solid aluminum plate sliding on two high precision rails through four carriage units satisfying the requirements of high running smoothness and low rolling friction. The plate weighs approximately 1250 lbs or, in consistent mass units, \( m_p = 0.00324 \text{ kips-sec}^2/\text{in} \).
Table 4.3  Scaling Factors For Model Based on Given $\lambda_L$, $\lambda_E$, $\lambda_p$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Formulation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (L), or Displacement (x)</td>
<td>$\lambda_L = \text{given}$</td>
<td>5</td>
</tr>
<tr>
<td>Elastic Modulus (E)</td>
<td>$\lambda_E = \text{given}$</td>
<td>1</td>
</tr>
<tr>
<td>Mass Density ($\rho$)</td>
<td>$\lambda_p = \text{given}$</td>
<td>1</td>
</tr>
<tr>
<td>Acceleration (a)</td>
<td>$\lambda_a = \lambda_E / (\lambda_p \lambda_L)$</td>
<td>1/5</td>
</tr>
<tr>
<td>Time (t)</td>
<td>$\lambda_t = (\lambda_L / \lambda_a)^{1/2}$</td>
<td>5</td>
</tr>
<tr>
<td>Velocity (v)</td>
<td>$\lambda_v = (\lambda_L / \lambda_a)^{1/2}$</td>
<td>1</td>
</tr>
<tr>
<td>Force (f)</td>
<td>$\lambda_f = \lambda_E \lambda_L$</td>
<td>25</td>
</tr>
<tr>
<td>Area (A)</td>
<td>$\lambda_A = \lambda_L^2$</td>
<td>25</td>
</tr>
<tr>
<td>Stress ($\sigma$)</td>
<td>$\lambda_{\sigma} = \lambda_E$</td>
<td>1</td>
</tr>
<tr>
<td>Strain ($\varepsilon$)</td>
<td>$\lambda_{\varepsilon} = 1$</td>
<td>1</td>
</tr>
<tr>
<td>Volume (V)</td>
<td>$\lambda_V = \lambda_L^3$</td>
<td>125</td>
</tr>
<tr>
<td>Area Moment of Inertia (I)</td>
<td>$\lambda_I = \lambda_L^4$</td>
<td>625</td>
</tr>
<tr>
<td>Mass (m)</td>
<td>$\lambda_m = \lambda_p \lambda_L^3$</td>
<td>125</td>
</tr>
<tr>
<td>Impulse (i)</td>
<td>$\lambda_i = \lambda_L^3 (\lambda_p \lambda_E)^{1/2}$</td>
<td>125</td>
</tr>
<tr>
<td>Energy (e)</td>
<td>$\lambda_e = \lambda_E \lambda_L^3 \lambda$</td>
<td>125</td>
</tr>
<tr>
<td>Frequency ($\omega$)</td>
<td>$\lambda_\omega = (1 / \lambda_L) (\lambda_E / \lambda_p)^{1/2}$</td>
<td>1/5</td>
</tr>
<tr>
<td>Gravitational Accel. (g)</td>
<td>$\lambda_{g} = 1$</td>
<td>1</td>
</tr>
<tr>
<td>Gravitational Force ($f_g$)</td>
<td>$\lambda_{f_g} = \lambda_m = \lambda_p \lambda_L^3$</td>
<td>125</td>
</tr>
<tr>
<td>Critical Damping ($\xi$)</td>
<td>$\lambda_{\xi} = 1$</td>
<td>1</td>
</tr>
</tbody>
</table>
Model Structure:

The model structure parameters were based on the three-story steel frame shown in Figure 4.9. This frame was modeled using the SAP90 (Wilson and Habibullah 1989) finite element analysis program. Using SAP90, the (3x3) condensed flexibility matrix was determined for the frame in the direction of table motion, from which the corresponding stiffness matrix (K) was determined by matrix inversion. The resultant model is an equivalent stick model of the structure. The floor added masses were varied from zero to 500 lbs per floor. The modal damping coefficients were taken to be one percent for each of the three modes.

Input Acceleration Time Histories:

The input acceleration time histories used in the analysis were the El Centro, 1940 (S00E component), Taft, 1952 (N21E component), and Orion Blvd, 1971 (N00W component) earthquake records (Figs. 4.6 through 4.8), with the amplitude scaled by a factor of 5 and the time scaled by a factor of 1/5 to satisfy similitude requirements.

Base Isolation:

The value of the isolation stiffness, k_b, was varied to study its influence on the response of the system. A damping coefficient varying from 5% to 30% with increments of 5% was considered in the analysis. Table 4.4 summarizes the system parameter values used in the analysis.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foundation Mass</td>
<td>168 lbs-sec²/in</td>
</tr>
<tr>
<td>Table Mass</td>
<td>3.24 lbs-sec²/in</td>
</tr>
<tr>
<td>Floor Added Masses</td>
<td>0 - 500 lbs/floor</td>
</tr>
<tr>
<td>Structural Modal Damping</td>
<td>1% for all modes</td>
</tr>
<tr>
<td>Base Isolation Stiffness</td>
<td>1 - 350 kips/in</td>
</tr>
<tr>
<td>Base Isolation Damping</td>
<td>0 - 30%</td>
</tr>
<tr>
<td>Scaling Factor for Length, $\lambda_L$</td>
<td>5</td>
</tr>
</tbody>
</table>

Figure 4.9  Model Structure
4.5 Results

The analysis performed consisted of two parts: examination of the performance of the table and structural model for a single case corresponding to a given earthquake and given base isolation stiffness and damping; and an examination of the foundation (reaction) mass displacement, actuator force, and base shear (force transmitted to the laboratory floor) with changing values of isolation stiffness and damping.

4.5.1 Results from Analyses of Single Cases

The analysis program that was written utilized the Matlab software package. Single case output consisted of the response time histories for the displacements (test structure, actuator, and foundation mass), the actuator force, and the base shear. The analysis program that was developed, as the output of these single cases illustrates, is a useful tool to analyze the behavior of the test structure and foundation (reaction) mass of the shaking table system for a given commanded input acceleration time history. However, for purposes of isolation design, the peak values of foundation displacement, actuator force, and base shear remain the important quantities extracted from these time history analyses.

Figures 4.10 through 4.17 show the typical output for a single run of the analysis program. For this run, the story masses were taken as 300 lbs per floor and the isolation stiffness was taken to be 30 kips/inch. The imposed table acceleration was given by the first seven seconds of the El Centro ground acceleration record scaled for similitude, corresponding to the first 35 seconds of the actual (prototype) ground acceleration time history. Modal damping is taken as one percent (0.01) for the three modes. The plots in Figures 4.11 through 4.13 show the time histories of various response quantities of the model test
structure, while the plots in Figure 4.14 through 4.17 give the time histories related to the actuator and foundation mass.

Figure 4.10 shows the prototype ground acceleration scaled for similitude and used as commanded acceleration signal for the slip table. The velocity and displacement records are simply the prototype ground velocity and displacement time histories scaled for similitude. Direct integrations of the acceleration record with constants of integration corresponding to the initial ground velocity and initial ground displacement will produce the velocity and displacement time histories shown.

Figure 4.11 shows the time histories of the relative displacement responses of the three floors and the displacement response of the foundation mass. All displacement quantities are normalized with respect to the absolute maximum displacement of the slip table ($u_g$ in Figure 4.1). The floor displacements ($v_1$, $v_2$ and $v_3$) are relative to the slip table. The foundation displacement ($v_b$) is relative to the laboratory floor. Figure 4.12 shows the time histories of the interstory drifts, normalized with respect to the maximum absolute table displacement (see Figure 4.10-c). By definition, an interstory drift is the relative displacement of the top floor with respect to the bottom floor of the story under consideration. Figure 4.13 depicts the absolute acceleration response time histories of the three floors, normalized by the maximum absolute table acceleration (see Figure 4.10-a).

Figure 4.14 displays the actuator force (solid line) and base shear (dotted line) time histories. The transmissibility of the base isolation system, defined here as the maximum base shear force divided by the maximum actuator force, can be deduced immediately from this plot. Figure 4.15 shows the absolute foundation displacement, while Figure 4.16
shows the base shear transmitted to the laboratory floor through the elastomeric bearings.

The relative motion between the foundation mass and the moving slip table represents the actual motion of the actuator arm. An important result of the above analyses was that, for all the cases considered, foundation mass absolute acceleration was found very small compared to the absolute table acceleration as shown in Figure 4.17. This indicates that, even when the system is on flexible supports, the actuator arm relative acceleration nearly equals the absolute target table acceleration. Therefore, the transfer function between the signal sent to the table (which manifests itself as the table relative acceleration) and the actual absolute table acceleration will not be significantly affected by the flexibility of the foundation.

Figure 4.10  Input Table Acceleration with Baseline Corrected Velocity and Displacement Time Histories
Figure 4.11 Normalized Displacement Response of Model and Foundation Mass

Figure 4.12 Normalized Interstory Drift Response Time Histories
Figure 4.13  Normalized Floor Absolute Acceleration Response Time Histories

Figure 4.14  Actuator Force and Base Shear Time Histories
Figure 4.15  Foundation Mass Displacement

Figure 4.16  Base Shear (Force Transmitted to Laboratory Floor)
4.5.2 Foundation Displacement, Base Shear and Transmissibility

To determine the optimal stiffness for the base isolation system, the above analysis was carried out repeatedly for varying isolation stiffness ($k_b$), isolation damping ratio ($\xi_b$), where

$$\xi_b = \frac{c_b}{2\sqrt{k_b m_f}} \quad (4.26)$$

and for varied floor masses and ground acceleration records. The plots in Figures 4.18 through 4.23 show the results of this parametric study. The Taft earthquake ground acceleration record never produced the maximum foundation displacement response or force transmissibility, and therefore is not discussed.
Figure 4.18 shows the maximum displacement of the foundation mass versus the stiffness of the isolation system, $k_b$. This case corresponds to the El Centro ground acceleration record and a test specimen with 400 lbs mass per floor. The different lines represent the response for damping values of 5%, 10%, 15%, 20%, 25% and 30 percent of critical damping. For this value of the floor mass, the resonance condition was near $k_b = 150$ kips/in. As the story mass decreased, the resonance position moved farther to the right to approximately 200 ~ 250 kips/in, as shown in Figure 4.19 which corresponds to a test specimen with 300 lbs mass per floor.

Figure 4.20 shows the force transmissibility of the system versus isolation stiffness. As expected, the transmissibility is zero at the origin, and it approaches unity as $k_b$ gets very large. The case presented in Figure 4.20 corresponds to a test specimen with 400 lbs mass per floor. When the floor masses are decreased to 300 lbs per floor (see Figure 4.21), the resonance region shifts to the right, i.e., to larger isolation stiffness. Figures 4.22 and 4.23 show the maximum base shear values in terms of the isolation stiffness for the 400 lbs and 300 lbs mass per floor cases, respectively.

The full analysis, including the three earthquake records, reveals that resonance may occur when $k_b$ takes on values between 75 and 350 kips/in. When $k_b$ is larger than 350 kips/in, foundation displacements are very low, and force transmissibility is below 1.5, and approaches 1.0 for greater amounts of damping. For $k_b$ less than 75 kips/in, foundation displacements remain below 0.20 inches, while force transmissibility is less than 1.0 even for small isolation damping (5 percent).
4.6 Bearing Design

The relationship between required bearing area and bearing shear stiffness is

\[ A = \frac{K_b h}{G} \]  \hspace{1cm} (4.27)

in which \( A \) is the required bearing area, \( K_b \) is the bearing shear stiffness, \( h \) is the total height (thickness) of the elastomeric material in the bearing (does not include the net thickness of the laminated steel), and \( G \) is the shear modulus of the elastomeric material.

A comfortable clearance of six inches must be provided for the anchor heads of the transversal post-tensioning bars. Thus \( h \) can take values from about one to six inches, the difference to six inches being taken by the bottom and top steel plates. The shear modulus for elastomeric materials ranges from 100 to 350 psi, 150 psi being typical.

For values of \( k_b \) greater than 350 kips/in, the required area of the isolation bearings becomes very large and their use looses any economical advantage, as well as defeats the purpose of base isolation. For this case, a rigid connection to the floor is preferable because it is cheaper and gives a smaller force transmissibility.

A shear stiffness of \( k_b = 30 \) kips/in was selected in order to provide a safe margin from entering the resonance region. For this stiffness, and a value of \( G \) equal to 125 psi, the required bearing area for 4 inches thick elastomeric material would be:

\[ A = \frac{30(4)}{0.125} = 960 \text{ in}^2 \]  \hspace{1cm} (4.28)

which corresponds to five bearings of diameter 15.64 inches, or four bearings of diameter 17.48 inches.
4.7 Conclusions

The design of the base isolation system based on the linear dynamic analysis presented in this chapter can be refined by including the effects of non-linear behavior of elastomeric materials. Non-linearity in the force-deformation behavior of the flexible bearings may cause excess deformations under high loads. The value of the shear modulus, \( G \), is also dependent on the deformation of the bearing, as well as on the compressive axial load (Kelly et al. 1987, 1992). Additional knowledge of the bearing material properties may, in some cases, be necessary in order to achieve an optimal design of the base isolation system.

Ultimately, isolation bearings were not used in the Rice Shaking Table System because of cost considerations. A rigid connection was opted for, which is described in detail in Chapter 6.
Figure 4.18 Maximum Foundation Displacement vs. Isolation Stiffness

Figure 4.19 Maximum Foundation Displacement vs. Isolation Stiffness
Figure 4.20  Force Transmissibility vs. Isolation Stiffness

Figure 4.21  Force Transmissibility vs. Isolation Stiffness
Figure 4.22 Maximum Base Shear vs. Isolation Stiffness

Figure 4.23 Maximum Base Shear vs. Isolation Stiffness
CHAPTER 5
DESIGN AND ANALYSIS OF SLIP TABLE

5.1 Introduction

The slip plate must satisfy the following three constraints: (1) it must be as light as possible to minimize the actuator force required to move it; (2) its fundamental frequency must be three to four times the highest excitation frequency to be tested; and (3) it must be sufficiently stiff to minimize the dynamic table-structure interaction. Therefore, the material of the slip table was selected to minimize the weight-to-stiffness ratio. The design problem consisted of finding the geometry and material of the slip plate minimizing the fundamental natural period of the plate (slip table vibration analysis) and table-structure interaction. Material and fabrication costs also played an important role in determining the final design.

5.2 Slip Table Material

Three materials were compared as possible alternatives for the slip table: steel, aluminum, and magnesium. Magnesium was considered because the machine shop of the Mechanical Engineering Department at Rice had surplus supplies of magnesium plates. The plate material that was finally selected was aluminum because of its favorable weight-to-stiffness ratio and ease of fabrication.

The expression for the flexural stiffness of a solid plate of uniform thickness, h, is

$$D = \frac{Eh^3}{12(1 - \nu^2)}$$  (5.1)
where \( E \) and \( \nu \) denote the Young's elastic modulus and Poisson's ratio of the material, respectively. From Eq. (5.1), it follows that the ratio of plate thicknesses for two plates of equal flexural stiffness is

\[
\frac{h_1}{h_2} = \left( \frac{E_2(1 - \nu_1^2)}{E_1(1 - \nu_2^2)} \right)^{\frac{1}{3}}
\]  

(5.2)

The ratio of plate masses of two plates of equal flexural stiffness is then

\[
\frac{m_1}{m_2} = \frac{\rho_1 h_1}{\rho_2 h_2}
\]

(5.3)

The mass density and elastic properties of the three materials considered are reported in Table 4.1.

<table>
<thead>
<tr>
<th>Table 5.1 Alternative Slip Plate Material Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E )</td>
</tr>
<tr>
<td>Steel</td>
</tr>
<tr>
<td>Aluminum</td>
</tr>
<tr>
<td>Magnesium</td>
</tr>
</tbody>
</table>

Applying Eqs. (5.1) and (5.2) to aluminum and steel reveals that the ratio of plate thicknesses, \( h_{al}/h_{st} \), equals 1.42, indicating that an aluminum plate would need to be nearly one and a half times thicker than a steel plate for equal flexural stiffness. However, the ratio of the plate masses, \( m_{al}/m_{st} \), equals only 0.489. Therefore, although aluminum requires a thicker plate for a given flexural stiffness, its weight is less than half of that of an equally stiff steel plate. Comparing aluminum to magnesium, the thickness and mass ratios are 0.867 and 1.20, respectively. That is, a magnesium plate would need to be even thicker.
than an aluminum plate, but would weigh only 83 percent as much. Therefore, among the
three materials considered, magnesium is the most efficient material. Table 4.2 summar-
izes the comparisons of the three materials.

<table>
<thead>
<tr>
<th>Comparison</th>
<th>thickness ratio</th>
<th>mass ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum to Steel</td>
<td>1.42</td>
<td>0.489</td>
</tr>
<tr>
<td>Aluminum to Magnesium</td>
<td>0.867</td>
<td>1.20</td>
</tr>
<tr>
<td>Magnesium to Steel</td>
<td>1.637</td>
<td>0.406</td>
</tr>
</tbody>
</table>

While magnesium was the most efficient material, working with this material introduces
other concerns, such as safety and workability. Magnesium is reactive, and therefore, was
unsuitable for the amount of milling work that would be required. Aluminum was chosen
as the material for the slip table.

5.3 Finite Element Model of Slip Table

The dynamic analysis of the slip table was performed using the SAP90 structural analysis
program (Wilson and Habibullah 1990). The slip plate was modeled as a 30 by 30 grid of
2" square plate bending elements, with quadrilateral plate elements forming the trapezo-
dal region at the mounting end of the plate (Figure 5.1). Connections to the trucks and rails
were modeled by restraining the nodes at locations corresponding to the center of the
trucks. These support points were fully restrained against rotation and against translation
all directions except the direction of the rails (Y axis). The connection to the actuator arm
was modeled by restraining translation of the center node on the attachment edge in the axial direction.

For the case where the effect of stiffeners was studied, the stiffening elements were modeled as a series of short beam elements. To model the stiffeners' eccentricity with respect to the neutral surface of the plate, fictitious stiff frame elements were used.

Figure 5.1  Finite Element Model of Slip Plate

5.4 Slip Table Vibration Analysis

The first analysis performed on the finite element model of the slip plate was a vibration analysis. The goal of this study was to find an optimum plate design that will minimize the fundamental natural period of the plate as well as the plate thickness. The fundamental
natural period of the plate was required to be as low as possible so that the plate's primary modes of vibration are not excited by the operation of the shaking table. The frequency range of operation of the table is approximately 0-60 Hz. MTS recommends that the table's fundamental natural frequency be at least three times the maximum operating frequency, or about 180 Hz.

The parameters that were varied in this study were the plate thickness, the truck location, and the stiffening element properties. The plate thickness was varied from 1 to 5 inches. Truck placement was varied from 4 to 16 inches from the edge of the plate. Three stiffening configurations were considered: no stiffeners, rectangular steel tubes, and solid steel bars. Figure 5.2 shows the first three mode shapes of the unstiffened aluminum plate when the thickness is 3 inches and the trucks are placed 12 inches from the plate edges. In this case, the fundamental natural frequency (205.8 Hz) is more than three times larger than the highest excitation frequency (~ 60 Hz).

5.4.1 Effect of Truck Placement

The distance of the trucks from the plate edge was found to have a significant effect on the dynamic properties of the aluminum plate. The natural frequencies were very sensitive to the unsupported span length between the rails, especially the fundamental natural frequency. The relationships between the truck-to-edge distance and the natural periods of the plate are shown in Figure 5.3. For plate thicknesses of 1 inches and 3 inches, the fundamental natural period was minimized for a truck-to-edge distance of approximately 12 inches, considering that this distance needed to be an even number of inches to satisfy the grid layout requirement.
"Cylindrical" Mode:

\[ T_1 = 0.004860 \text{ sec} \]
\[ f_1 = 205.8 \text{ Hz} \]

"Oil Can" Mode:

\[ T_2 = 0.004085 \text{ sec} \]
\[ f_2 = 244.8 \text{ Hz} \]

"Warp" Mode:

\[ T_3 = 0.003429 \text{ sec} \]
\[ f_3 = 391.6 \text{ Hz} \]

Figure 5.2  First Three Modes of Slip Table (3" Thick and Trucks at 12")
Figure 5.3  Bare Table Natural Periods versus Truck Placement
5.4.2 Effect of Plate Thickness

Figure 5.4 shows the effect of plate thickness on the first three natural periods of the slip table. The rate of change of the table natural frequencies diminishes as the plate becomes thicker. This is especially true for thicknesses above three inches, where the curves in Figure 5.4 tend to flatten out.

![Graph showing the effect of plate thickness on natural periods](image)

Figure 5.4 First Three Natural Periods vs. Plate Thickness

5.4.3 Effect of Stiffeners

Three stiffening configurations were analyzed:

- no stiffening elements;
- 2"x1"x 0.188" steel tubes;
- 2"x 1" solid steel bars.
The configuration of the stiffeners is shown in Figure 5.5. Figure 5.6 shows the relationship between the plate thickness and the fundamental natural period of the plate for each of the three stiffening strategies. Truck distance was held constant at 12" from the plate edge. For plate thickness above two inches, the marginal benefit of adding stiffeners becomes minimal.

Figure 5.5 Stiffener Configuration
Figure 5.6 Influence of Stiffeners on the Fundamental Natural Period of the Slip Table

5.5 Dynamic Table-Structure Interaction

The presence of a flexible test structure mounted on the slip table can dramatically change the dynamic properties of the system. In this study, the goal was to determine the slip table thickness required to minimize the effects of the table flexibility on the dynamic response of a test structure mounted on the table. Comparisons were made between the acceleration response at the top of a model structure mounted to a rigid base, and the acceleration response of the same structure mounted to the "flexible" aluminum slip plate. The plate thickness was varied between 1 and 5 inches, and the truck locations were varied between 6 and 14 inches from the plate edge. The floor masses of the test structures were also varied from zero mass to 500 lbs per floor. Plate stiffeners were disregarded for most of this
analysis, and would have only been seriously considered if the results had revealed a required plate thickness of less than two inches (see Figure 5.6).

5.5.1 Model Structures

Three model structures were considered to study the dynamic plate-structure interaction:

(1) a three story steel frame (24"x36"x90") with rigid connections and variable floor mass (Figure 5.7);

(2) a three story steel frame (24"x52"x90") with rigid connections and variable story mass (Figure 5.7);

(3) a simple three-degree-of-freedom stick model.

All members of the frame models were steel T-sections with dimensions 2"x2"x0.25". The floor masses were modeled by applying one-quarter of the total floor mass at each of the corner nodes of each floor. The stiffness matrix of the stick model was selected so that it corresponds to a condensed version of the global stiffness of the larger frame in the weak direction. The dynamic analysis assumed the direction of shaking to be parallel to the shorter dimension (24") of the test frame.

5.5.2 Natural Periods of Table-Structure System

Vibration analysis of the combined table-structure system revealed that the first three natural periods of the combined system approached those of the structure on the fixed base, (Figure 5.8), for a plate thickness exceeding 2 inches (see Figure 5.9). The natural periods converged to the fixed base values faster for the smaller frame (36") than for the larger frame (52"), which rests on the cantilevered portion of the slip table. For a table thickness
of 5 inches or larger, in the case of both frames, the combined system has practically the same dynamic characteristics as the frame on the fixed base for the first three modes. The trucks were positioned at 12 inches from the plate edges for the plots shown in Figure 5.9.

Figure 5.7  Model Structure (Frame) Used in Dynamic Analysis
First bending mode about weak axis:

<table>
<thead>
<tr>
<th></th>
<th>52&quot; frame</th>
<th>36&quot; frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$</td>
<td>0.184 sec</td>
<td>0.182 sec</td>
</tr>
<tr>
<td>$f_1$</td>
<td>5.44 Hz</td>
<td>5.505 Hz</td>
</tr>
</tbody>
</table>

First torsional mode:

<table>
<thead>
<tr>
<th></th>
<th>52&quot; frame</th>
<th>36&quot; frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_2$</td>
<td>0.181 sec</td>
<td>0.172 sec</td>
</tr>
<tr>
<td>$f_2$</td>
<td>5.52 Hz</td>
<td>5.82 Hz</td>
</tr>
</tbody>
</table>

First bending mode about strong axis:

<table>
<thead>
<tr>
<th></th>
<th>52&quot; frame</th>
<th>36&quot; frame</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_3$</td>
<td>0.178 sec</td>
<td>0.160 sec</td>
</tr>
<tr>
<td>$f_3$</td>
<td>5.62 Hz</td>
<td>6.25 Hz</td>
</tr>
</tbody>
</table>

Figure 5.8 First Three Modes of Model Structure on Fixed Base (300 lbs per floor)
Figure 5.9  Natural Periods of Table-Structure System vs. Plate Thickness
Figure 5.10 Acceleration Time History of Imperial Valley Earthquake, 1940, El Centro Site, scaled for $\lambda_L=5$, $\lambda_B=\lambda_P=1$.

5.5.3 Response Time History Analysis for Model Frames

A base acceleration was applied to both the rigid base and table-mounted systems. The acceleration time history considered was the scaled for similitude El Centro, 1940, record, already used in the analysis of the base isolation system. The scaling factors used were 1/5 for length ($\lambda_L = 5$), and unity for Young's modulus and material mass density (same material similitude). The scaled acceleration time history is shown in Figure 5.10. The dynamic response of a node on the third floor (roof) of the frame on fixed base was compared to the dynamic response of the same node of the table-structure system, which includes the effect of table flexibility.

5.5.4 Effect of Truck Position

Figures 5.11 and 5.12 show the acceleration response time histories of the 36 inch and 52 inch frames, respectively, for the case of 300 lbs mass per floor and variable truck location.
Figure 5.11 Acceleration Response Time History for the 36" Frame vs. Truck Position
Figure 5.12  Acceleration Response Time History for the 52” Frame vs. Truck Position
The plate thickness was kept constant at 3 inches. Different trends were realized for the 36 inch and 52 inch frames. The acceleration response time history for the combined system approached that of the fixed base frame in the case of the smaller frame. For increasing truck distance from the edge (i.e., the trucks closer together), up to 14 inches, the mean-square-error of the acceleration response decreased (Figure 5.13). However, in the case of the 52 inch frame, the minimum mean-square error was realized when the trucks were only 8 inches from the edge, and the error increased as the truck locations moved further to the center of the plate.

![Mean Square Error vs. Truck Position](image)

Figure 5.13 Mean Square Error of Top Node Acceleration vs. Truck Position
5.5.5 Effect of Plate Thickness

Figures 5.14 and 5.15 show the acceleration response time histories of the 36 inch and 52 inch frames, respectively, with 300 lbs mass per floor, for variable plate thickness. A fixed truck location of 12 inches from the plate edge was used in generating these figures. In the case of the smaller frame, the response of the fixed base frame nearly coincides with that of the frame on flexible base for a table thicker than three inches. Figure 5.16 illustrates the convergence of the two frame models by plotting the mean-square-error of the top node acceleration response time histories versus plate thickness.

5.5.6 Stick Model Analysis

A reduced stick model (Figure 5.17) of the larger frame was developed by choosing the value of the story bending stiffness moduli, EI, which render the stiffness of the stick model similar to that of the larger frame in the weak direction. The equivalent stick model was then analytically mounted on the slip table in the center position, and the resulting system was subjected to the El Centro record shown in Figure 5.10. The results (Figure 5.18) showed that the plate thickness needed to be nearly 8 inches before the acceleration response time history of the combined system closely resembled that of the stick model on fixed base. Therefore, very narrow structures placed in the center of the plate will tend to interact much more with the plate than would a wider structure.
5.6 Conclusions

Based on the analysis presented in this chapter, the slip table that was fabricated for the Rice Shaking Table Facility was an aluminum plate, three inches thick. The rails were placed such that the center of the trucks were twelve inches from the side edges of the plate. Sixty-four holes with helicoil inserts were placed into the top surface of the plate at even intervals of 6 inches to provide attachments for test specimens. The final layout of the aluminum plate is shown in Figure 5.19.
Figure 5.14 Acceleration Response Time History of 36" Frame vs. Plate Thickness
Figure 5.15  Acceleration Response Time History of 52" Frame vs. Plate Thickness
Figure 5.16  Mean Square Error of Top Node Acceleration vs. Plate Thickness

Figure 5.17  Stick Model
Figure 5.18  Acceleration Response Time History of Stick Model with Varying Plate Thickness
Figure 5.19  Final Layout of Aluminum Slip Table
CHAPTER 6
SHAKING TABLE CONSTRUCTION

6.1 Introduction

While some of the shaking table components were manufactured out-of-house, most of the individual components were designed by the shaking table research team. With the exception of the hydraulic power equipment and the electronic control system, manufactured by MTS corporation, the shaking table facility was constructed in-house. Precision milling of the steel and aluminum plates was done by local companies to the precise specifications set by the design team. The construction process was divided into 5 parts:

(1) Construction of concrete slab
(2) Building of connection to Ryon Laboratory strong floor
(3) Leveling and grouting of steel plate
(4) Mounting of aluminum slip table
(5) Connection of hydraulic and control systems.

6.2 Concrete Slab Construction

The three concrete slabs which form the reaction mass of the table were poured separately using a single formwork of size 12'x12'x1'. Before pouring each slab, the forms were prepared and the hardware was placed into the form. First, the formwork seams were sealed with a silicon-based caulk. Then, any hardware which was to be placed into the concrete, such as eyebolt sleeves, were tacked into place. The forms were then coated with oil to aid in separating them from the concrete after curing. Finally, the steel reinforcement was
placed and tied. The specific layouts and hardware of each slab were different.

6.2.1 Layout of Bottom Slab

Each of the three slabs that made up the reaction mass of the system were designed and constructed differently. The three slabs were designed to be of overall dimensions 12’x12’x1’ each. A plan view of the layout of the first (bottom) slab is shown in Figure 6.1 below, and a cross-section of the slab is shown in Figure 6.2. The reinforcing steel was placed in two layers (top and bottom), with the top bars in each layer running parallel to the direction of shaking. Two inches of cover was provided for both top and bottom layers.

Common to all three slabs were the post-tensioning sleeves and the eyebolt sleeves. The eyebolt sleeves had threaded inserts, so that eyebolts could be screwed into the

![Figure 6.1 Layout of the Bottom Slab](image)

Figure 6.1 Layout of the Bottom Slab
sleeves, by which the finished slab was hoisted and moved. The round sleeves were welded to steel base plates 8"x8"x0.5", such that the total height was 12 inches. The base plates provided protection from the sleeves pulling out under the heavy weight of the slab. Figure 6.3 details the eyebolt sleeves.

Figure 6.3  Sleeves Mounted in the Concrete Slabs
(a) Eyebolt Sleeve and (b) Post-Tensioning Sleeve

The post-tensioning sleeves provided the voids through the concrete slab where the post-tensioning steel bars were run to tie the slabs together. The sleeves were made of
2"x2"x1/16" steel tubing, welded to 4"x4"x1/4" steel plates on both top and bottom.

6.2.2 Layout of Middle Slab

With the exception of the reinforcing bars, the layout of the middle slab was the same as that of the bottom slab. The reinforcing bars were placed on 12 inch centers, starting at 6 inches from the edge of the slab. Figures 6.4 and 6.5 show the plan and cross-sectional views of this slab.

Figure 6.4 Layout of the Middle Slab
6.2.3 Layout of Top Slab

The layout of the top slab was more complex because the attachments for the steel plate were required to be embedded in this slab. The two inch thick steel base plate was connected to the slab by a combination of threaded tie-down bars and four set screws. Figure 6.6 illustrates the fastening of the steel plate. Once the plate was leveled, the void space was filled with grout to permanently fix the plate position. For this reason, the plate needed to be partially embedded below the top surface of the slab. The steel layout was the same as for the middle slab, however, the bottom and top layers of steel were lowered 1/2 inch to allow for the cut-out region where the steel plate was placed while still providing cover for the reinforcing bars.

Sixteen 0.75 inch diameter threaded bars were welded to thin strips of flat steel bar so that they could be placed on the floor of the form and would extend up, matching exactly the hole pattern of the steel plate.

A smaller form was constructed to create the void region in the concrete slab. This smaller form was an open box approximately 122"x62"x4", with a plywood floor and sev-
eral cross beams for support. The length and width of this form allowed approximately one inch of void space on all sides of the embedded plate, which was filled later with grout. The exact hole pattern from the steel base plate was transferred to the floor of the inner form. This hole pattern served to stabilized the threaded bars at the top so that they couldn't move as the concrete was being poured. It was very important to precisely place these holes because the holes in the steel plate allowed only 1/8 inch deviation. It was also necessary to make sure that the form itself would not shift during pouring, as a small shift would cause the threaded bars to move from their vertical positions.

Four small seats were made to hold the small formwork ten inches above the floor of the larger form. The secondary purpose of these seats was to provide a flat and even surface directly under the set-screws that was used to level the plate. Small openings were cut into the flooring of the smaller form so that the wet concrete could flow freely into these and under this form, reducing the chance of large air pockets becoming trapped by the inner form. Nuts were tightened on the threaded bars to keep the smaller form from float-

Figure 6.6 Connection of Steel Plate to Top Concrete Slab
ing upward under the pressure of the concrete. Figure 6.7 shows a cross-section of the formwork.

![Cross-Section of Formwork for Top Slab](image)

**Figure 6.7** Cross-Section of Formwork for Top Slab

The eyebolt and post-tensioning sleeves were placed as before. A three inch thick beveled steel washer was placed at the top end of each post-tensioning sleeve to accommodate the nut (anchoring head) of the steel post-tensioned bars. The washer was 4"x6". A detail of the washer is shown in Figure 6.8.

![Beveled Washer for Post-Tensioning Sleeves](image)

**Figure 6.8** Beveled Washer for Post-Tensioning Sleeves
6.2.4 Pouring of Concrete

The slabs were poured in three pours. The specified concrete compressive strength was 3000 psi. All cylinder specimens for the three pours had at least this minimum strength at 28 days.

6.3 Connection to the Strong Floor

The decision was made to have the reaction mass rigidly connected to the strong floor of the Ryon Laboratory at Rice. The strong floor connections required 1-1/2 inch coarse threaded bolts, and were spaced on a grid at approximately three feet on centers. A series of wide-flange sections (W8x35 and W8x18) were bolted to the floor and welded together to provide the base for the concrete slabs. Figure 6.9 shows the layout of the flange sections. Holes were cut into the top flange of the outer sections to accommodate the post-tensioning steel bars. Beveled washers (Figure 6.8) were welded to the underside of the flanges. The three concrete slabs were stacked on the base (Figure 6.10), and the post-tension steel bars were inserted in the six sleeves. The post-tension steel bars were one inch diameter, grade 150 Dywidag "Threadbars". Post-tensioning nuts were attached to each end of the bars. Figure 6.11 shows a cross-section of the slabs and base.
Figure 6.9 Layout of Wide-Flange Sections for the Shaking Table Base

Figure 6.10 Placing the Slabs on the Ryon Laboratory Floor
6.3.1 Post-Tensioning of the Slabs

Figure 6.11 illustrates the post-tensioning apparatus that was used to apply the force to the embedded post-tension bars. A coupling nut was attached to the portion of the embedded post-tension bar that extended above the slab. An additional bar was placed into the open end of the coupling nut and run through the center of a hydraulic jack. The free end of the bar extension was fixed with a beveled washer and nut, and was tightened in place atop the jack. The jack was raised until approximately 10,000 lbs of force was applied to the bar. The post-tensioning nut at slab level was tightened until the pressure gauge on the jack revealed that the tensile force had been taken up by the embedded post-tension rod. This event occurred when the reading on the pressure gauge suddenly dropped. Alternating rods were tensioned, moving clockwise around the slab. This process was repeated and the final post-tension force applied was 20,000 lbs.
Figure 6.11 Cross-Section of Post-Tensioned Slabs Fixed to WF Base and Tensioning Apparatus
6.4 Steel Plate Leveling and Grouting

The next step in the construction process was to attach the steel plate to the concrete reaction mass. The steel plate was lowered onto the concrete base, into the void space provided for the plate. The plate serves as the base to which all the moving components of the shaking table are fixed.

6.4.1 Leveling of the Steel Base Plate

Once the steel plate was placed onto the concrete mass, the plate was leveled. While the unfinished surface of the plate was inherently uneven, the precision milled grooves for the rails were specified to be level and in the same plane. It was important that the rails be as level as possible. The tolerance specified for the rails themselves allowed 0.003 inches in height deviation between the two rails which were placed 36 inches apart.

The grooves for the rails were cleaned and the rails were placed into the grooves and tightened down. The rails were fixed to run straight and parallel, with a maximum absolute deviation of the rail-to-rail distance of 0.006 inches. The plate was leveled with the rails attached because variability in the torquing of the rail's bolts could lead to unlevel rails on an otherwise level plate. A precision tilting level was used to measure elevations on the surface of the plate, and could determine relative elevations to 0.001 inch (Figure 6.12).

The set screws were used to adjust the plate elevation so that the four corners of the steel plate were roughly at the same elevation. It was discovered that the four set screws were not adequate to level the plate because the plate was bending significantly under its own weight. A small study was done to determine how the addition of new set screws could reduce the bending deformation of the steel plate (Figures 6.13 and 6.14). Eight more set
screws were added to the original four in the positions indicated in Figure 6.15, making a total of twelve. The two positions midway along the length of the rails were added to make the leveling of the rail sections more precise. The swivel connection on the actuator arm makes the precise leveling of the actuator end of the plate less critical.

![Figure 6.12 Precision Leveling of the Rails](image)

The rails were leveled to 0.002 inches rail to rail height. Figure 6.16 shows the final position of the rails before grouting. Each rail was level to 0.001 inches along the path of the trucks.
6.4.2 Grouting the Steel Plate in Place

Once the steel plate was properly leveled, nuts were tightened on all the threaded bars to keep the steel plate from floating under pressure from the grout being injected into the void space. The grout was pumped into the void space through the five 1-1/2" grouting holes fitted with pipe threads. The grout was very quick to set, so a final leveling check was performed to make sure that the plate did not move during the grouting operation.

The grout that was used was a no-shrink grout with a high fatigue tolerance, essential for continued performance under the dynamic loadings it would be subjected to.
Figure 6.14 Improvement of Steel Plate Bending with Additional Set Screws

(a) Deflection along the length of the rails
(b) Plate with 4 original set screws
(c) Plate with 8 set screws
(d) Plate with 10 set screws
Figure 6.15 Steel Plate Set Screw Layout

Figure 6.16 Elevation Profiles of the Rails
6.5 Mounting the Aluminum Slip Plate

The aluminum plate was lowered onto the rails, and bolted down to the trucks. A steel yoke was made to fit over one end of the plate, and attached to the swivel base of the actuator. The detail of the attachment is shown in Figure 6.17.

![Figure 6.17 Detail of Slip Table Yoke](image)

6.6 Connecting the Hydraulic and Control Systems

Once the slip table was mounted to the rails, the actuator was fastened to the steel plate with high strength bolts. The swivel base of the actuator arm was then connected to the
yoke. It was important that the high pressure hoses that ran from the service manifold to the servovalve/actuator were short and ran in a straight line. The manifold was mounted on a separate base immediately to the rear of the actuator and five feet away. The hydraulic power supply (HPS), on the other hand, was located in a separate room. High pressure hoses were run over sixty feet from the pump to the service manifold. Before operation, the system needed to be flushed of any debris before connecting the hoses to the servovalve. The hoses from the manifold were connected to each other (pressure lines to return lines), and the pump was run to circulate the fluid. The main and pilot pressure and return lines were then connected to the servovalve.

After the hydraulic system was properly connected, the control system was installed. The MTS 407 controller regulates or monitors all time-varying processes of the hydraulic system: high and low pressure control of the service manifold, servovalve valve driver, servovalve LVDT, actuator piston LVDT, and piston differential pressure (ΔP). Cables were run from the appropriate modules on the hydraulic system to the controller, which was set up at a monitoring station along with the personal computer and data acquisition equipment.

### 6.7 Conclusions

The construction of the shaking table marked a major milestone in the effort to build a real-time dynamic testing facility to Rice University. After assembly, the control system of the shaking table needed to be tuned, which involved the adjustment of the control parameters, or gain factors, thus completing the optimization of the system.
CHAPTER 7
MECHANICAL AND CONTROL MODEL OF SERVO-HYDRAULIC SYSTEM

7.1 Introduction

The objective in designing a shaking table facility is to faithfully reproduce a specified or target motion (acceleration, velocity, and displacement) on the table. Because the servo-hydraulic system is a real, physical system, the relationships between its input and output is non-linear. Physical phenomena such as leakage and compressibility of the hydraulic fluid cause these non-linearities in the servovalve and actuator response. The proportional-integral-derivative-feedforward (PIDF) controller applies corrective signals to the servo-hydraulic system to correct errors between the desired and actual position of the actuator. Corrections are also made based on the actuator force being applied to the table.

The servovalve, actuator, and slip table system can be modeled as an equivalent mechanical system. In this chapter, the transfer function of this system is derived based on the physical parameters of the servovalve, actuator, and table. By coupling the PIDF controller with the mechanical servomodel, the transfer function between commanded signal and table response is derived.

7.2 Control Hardware

The hardware used to control the actuator movement consists of an MTS 407 displacement controller, MTS series 244 hydraulic actuator, MTS series 256 three stage servovalve, and a Gateway 2000, 90 MHz pentium personal computer.
7.2.1 MTS 407 Controller

The servocontroller is a single channel, Proportional, Integral, Derivative, Feedforward (PIDF) controller that can provide control for a single channel in a hydraulic system. Components include AC and DC transducer conditioners, a three-stage servovalve driver, signal monitor, function generator, system interlocks, external analog and digital input and output, and hydraulic pressure control. Figure 7.1 shows the connectivity between the various components.

![Diagram](image)

Figure 7.1 Shaking Table Mechanical System and Control System Components

7.2.2 MTS Series 244 Hydraulic Actuator

The hydraulic actuator consists of the actuator piston rod, high pressure fluid ports, cush-
ions, and swivel end connections. The piston rod is made of heat-treated steel alloy, is hard-chrome plated, and is equipped with a core mounted linear-variable-differential-transducer (LVDT). The cushions protect from impact of high-speed and high-force loads.

7.2.3 MTS 256 Series Three-Stage Servovalve

The hydraulic servovalve is a three-stage servo-mechanism, with an MTS Series 252 servovalve (pilot stage servovalve) providing pilot flow to an MTS Series 256 servovalve (main stage servovalve), which ports the main hydraulic flow to the actuator piston. Figure 7.2 illustrates the function of the pilot and main stage servovalves.

Figure 7.2 Cross-section of Three-Stage Servovalve
7.2.4 Gateway 2000 Pentium Computer

The Gateway personal computer is equipped with a 90 MHz Pentium processor, a one gigabyte hard drive, and both ISA and PCA slots for additional components. Two National Instruments data acquisition boards (AT-2150) and one DSP board (DSP-2200) have been installed on the PC for data acquisition, signal generation, and real-time data processing. The data acquisition boards are each capable of sampling up to 4 channels simultaneously at sampling rates from 4 kHz to 51.2 kHz. The DSP board has numerous on-board functions for signal processing (e.g., Fourier transforms, windowing, PSD estimations), as well as for data acquisition and signal generation. These operations can be performed completely by the board’s DSP processing chip and on-board memory, without calling upon any of the PC’s memory or CPU resources. The PC acts only as the host computer for the data acquisition and DSP boards.

7.3 Mechanical Model of Servovalve

In order to construct an equivalent mechanical model for the servovalve control system, we consider the three stage servovalve shown in Figure 7.2. Following the derivation in Rinawi and Clough (1991), we assume a linear relationship between the displacement of the main stage spool and the pressure drop induced across the pilot stage spool,

\[ x_{ms} = k_1 \Delta P_p \]  

(7.1)

in which

- \( x_{ms} \) = main stage spool displacement
- \( \Delta P_p \) = pressure drop across the pilot stage spool
\( k_1 = \text{gain factor.} \)

Assuming a linear relationship between the fluid flow to the actuator and the main spool displacement for small valve openings (Rea et al. 1977), the fluid flow from the main stage to the actuator can be expressed as:

\[
q = k_{xm} x_{ms} - k_{ls} F_a = q_s - q_{ls} \tag{7.2}
\]

where

\( q = \text{fluid flow from the main stage (or flow leaving the servovalve)} \)

\( k_{xm} = \text{flow-gain coefficient} \)

\( x_{ms} = \text{main stage spool displacement} \)

\( k_{ls} = \text{force-flow coefficient, or leakage coefficient for leakage in the servovalve and at the servovalve-actuator interface} \)

\( F_a = \text{force in the actuator} \)

\( q_s = \text{component of flow due to spool movement} \)

\( q_{ls} = \text{component of flow due to leakage in the servovalve and at the servovalve-actuator interface} \)

Leakage within the servovalve may occur across the main stage spool valves and at the interface between the servovalve and the actuator. Ideally, this leakage is negligible. The sign convention used in Eq. (7.2) assumes that movement of the main stage spool in the positive direction results in a positive actuator force. The flow into the actuator can be expressed in terms of the piston movement and fluid compressibility as

\[
q = x_t A + \frac{V}{4\beta A} F_a + k_{la} F_a = q_p + q_F + q_{la} \tag{7.3}
\]
where a dot over a variable denotes the time derivative, and

\[ q = \text{fluid flow into the actuator chamber from the main stage} \]

\[ x_t = \text{piston displacement, or equivalently, table displacement} \]

\[ \dot{x}_t = \text{velocity of the actuator piston, or equivalently, table velocity} \]

\[ A = \text{effective piston area} \]

\[ V = \text{volume of oil in the actuator chamber} \]

\[ \beta = \text{bulk modulus of the oil} \]

\[ F_a = \text{force in the actuator} \]

\[ k_{la} = \text{force-flow coefficient, or leakage coefficient, for leakage within the actuator} \]

\[ q_p = \text{flow component due to piston movement} \]

\[ q_F = \text{flow component due to fluid compressibility} \]

\[ q_l = \text{flow component due to leakage in the actuator} \]

Leakage within the actuator can occur at the "sealed" joints within the actuator chamber, like across the piston and where the actuator arm (or piston rod) extends from the actuator to the slip table (see Figure 7.2).

The pressure differential in Eq. (7.1) is induced by a command signal to produce the appropriate table displacement, \( x_t \), in Eq. (7.3). The command signal is given in terms of a target table displacement, and then converted into a differential pressure needed to produce the target displacement. The differential pressure is determined from the commanded displacement and the current state of the system (displacement and force feedbacks). In our case, for which displacement and force feedbacks are available, this feedback relation-
ship is expressed as

\[ \Delta P_p = k_2(x_c - k_f F_a - k_d x_t) = k_2 x_{fb} \]  \hspace{1cm} (7.4)

where

- \( \Delta P_p \) = differential pressure induced in the pilot spool
- \( k_2 \) = feedback gain
- \( x_c \) = command displacement, or equivalently, target table displacement
- \( k_f \) = force feedback gain
- \( k_d \) = displacement feedback gain
- \( F_a \) = actuator force
- \( x_t \) = actuator piston, or equivalently, table displacement
- \( x_{fb} \) = feedback-adjusted displacement

Assuming no friction between the slip table and the rails, the relationship between table motion and actuator force is simply described by the second-order differential equation:

\[ F_a = m_t \ddot{x}_t \]  \hspace{1cm} (7.5)

where \( m_t \) denotes the mass of the table. It is convenient to use the Laplace transform notation in determining the transfer function from the command signal to the actual table displacement. Combining Eqs. (7.1) through (7.5), we arrive at the expression

\[ \left[ s^3 \frac{V_{mt}}{4\beta A} + s^2 m_t (k_t k_f + k_1) + s A + k_t k_d \right] x_t(s) = k_t x_c(s) \]  \hspace{1cm} (7.6)

in which \( k_t \) denotes the table gain factor defined as
\[ k_t = k_2 k_l k_{xm}, \]

and \( k_l \) represents the total leakage coefficient, which is simply the combination of the leakage between the servovalve and actuator and the leakage within the actuator. The expression for this term is

\[ k_l = k_{ls} + k_{la} \]

The relations expressed by Eqs. (7.1) to (7.5) are represented graphically by the block diagram in Figure 7.3.

![Figure 7.3: Control Block Diagram of Servovalve](image)

Thus, the closed loop transfer function for the servovalve is given by:

\[ S_{\text{closed}}(s) = \frac{x_t(s)}{x_c(s)} = \frac{k_t}{s^3 \left( \frac{V_m}{4 \beta A} \right) + s^2 m_t (k_t k_f + k_l) + s A + k_t k_d} \quad (7.7) \]

The open loop transfer function is obtained by setting the feedback parameters, \( k_d \) and \( k_f \), to zero in the expression for the closed loop transfer function, which gives
\[ S_{\text{open}}(s) = \frac{x_t(s)}{x_c(s)} = \frac{k_t}{s^3 \left( \frac{V m_t}{4 \beta A} \right) + s^2 m_t k_l + s A} \]  

(7.8)

or, after rearranging terms,

\[ S_{\text{open}}(s) = \frac{k_t}{A} \frac{1}{s \left[ s^2 \left( \frac{V m_t}{4 \beta A^2} \right) + s \frac{m_t k_l}{A} + 1 \right]} \]  

(7.9)

Now, it is shown that the servo-hydraulic system whose transfer function is given in Eq. (7.9) can be physically interpreted as a single-degree-of-freedom mass/spring/damper system. Consider the actuator and oil system shown in Figure 7.4. The relationship between fluid pressure and volumetric strain is

\[ \Delta P = \beta \Delta \epsilon_v \]  

(7.10)

where \( \Delta P \) is the fluid pressure change, \( \beta \) is the bulk modulus of the fluid, and \( \Delta \epsilon_v \) is the corresponding change in volumetric strain. For a constant cross-sectional area (the

![Diagram](image.png)

Figure 7.4 (a) Actuator Oil Column, and (b) Equivalent Spring
hydraulic cylinder is considered to be rigid), \( \varepsilon_v \) reduces to

\[
\varepsilon_v = \varepsilon_{11}
\]  

(7.11)

where \( \varepsilon_{11} \) is the longitudinal strain along the cylinder axis. As shown in Figure 7.4, the oil column acts on the piston as two mechanical springs, one for each half of the oil column. The piston is assumed to be originally at rest and in the center position. The stiffness of each of these equivalent springs is given by the force required to compress or decompress half of the oil column as the piston moves by one unit. Thus,

\[
\frac{k}{2} = F = \beta \varepsilon_{11} A = \beta A \left( \frac{1}{L/2} \right)
\]  

(7.12)

Rearranging terms in Eq. (7.12), and multiplying numerator and denominator by \( A \), Eq. (7.12) becomes

\[
k = \frac{4\beta A^2}{V}
\]  

(7.13)

Finally, the oil column in the actuator acts as a single spring of stiffness \( k \). Therefore, the oil column frequency can be written as

\[
\omega_{oil}^2 = \frac{k}{m} = \frac{4\beta A^2}{Vm}
\]  

(7.14)

Continuing with the single-degree-of-freedom analogy, consider the system shown in Figure 7.5. The natural frequency of the system is \( \omega \) with damping ratio \( \xi \). Consider the command signal, \( x_c \), to act as a base displacement. The equation of motion for this system is, in the Laplace domain:

\[
s^2 x_t + s2\xi \omega x_t + \omega^2 x_t = \omega^2 x_c
\]  

(7.15)
The transfer function between the input, \( x_c \), and the output, \( x_t \), is

\[
H(s) = \frac{x_t}{x_c} = \frac{\omega^2}{s^2 + s2\xi\omega + \omega^2} = \frac{1}{\left[s^2 + s\frac{2\xi}{\omega} + 1\right]} \quad (7.16)
\]

Equation (7.9) can be interpreted in the light of Eq. (7.16) and the following relationships are obtained:

\[
\omega_{\text{open}}^2 = \frac{4\beta A^2}{V m_t} = \omega_{\text{oil}}^2 \quad (7.17)
\]

\[
\xi_{\text{open}} = \frac{\omega_{\text{open}} m_t k_l}{2A} = \frac{k_l}{2} \sqrt{\frac{4\beta m_t}{V}} \quad (7.18)
\]

\[
C = \frac{k_l}{A} \quad (7.19)
\]

Notice, however, that the second-order polynomial in \( s \) in the denominator of Eq. (7.9) is multiplied by the term \( s \), which does not appear in Eq. (7.16). This difference arises from
the fact that the command force (not including damping) applied to the mass in the single-degree-of-freedom model is directly proportional to the command signal, namely \( F_c = k x_c \). On the other hand, in the more complicated servo-hydraulic model shown in Figure 7.3, the command signal is proportional to the derivative of the applied force, as apparent from the block diagram in Figure 7.3.

The open-loop transfer function in Eq. (7.9) can be rewritten as

\[
S_{\text{open}}(s) = \frac{C}{s \left[ \frac{1}{\omega_{\text{open}}^2} + s \left( \frac{2\xi_{\text{open}}}{\omega_{\text{open}}} + 1 \right) \right]} \tag{7.20}
\]

7.4 PIDF Controller Model

The servovalve model previously developed can be modified to include the PIDF control provided by the MTS 407 controller. The block diagram in Figure 7.6 shows the configuration using the PIDF controller. In this system, the open-loop servovalve model, \( S_{\text{open}}(s) \), in Eq. (7.9) is used, and the feedback is looped back to the PIDF controller for processing.

7.4.1 PID Transfer function, \( P_{\text{PID}}(s) \)

The command signal, \( \bar{x}_c \), minus the feedback signal results in an error signal, \( e \), known as the DC error. This error is processed by the PID controller as shown in Figure 7.7. A proportional, integral, derivative (PID) controller has a transfer function of the form

\[
P_{\text{PID}}(s) = \frac{\epsilon(s)}{e(s)} = K_{\text{pro}} + \frac{1}{s}K_{\text{int}} + sK_{\text{der}} \tag{7.21}
\]

in which
Figure 7.6 Control Block Diagram of Servovalve with PIDF Controller and Conditioned Displacement and Load Feedback

e(s) = error signal (input)
\( \varepsilon(s) = \) output signal or PID adjusted error signal

\( K_{pro} = \) proportional gain

\( K_{int} = \) integral gain

\( K_{der} = \) derivative gain

The error signal is multiplied directly by the proportional gain. The integral of the error over time, or accumulated error, is multiplied by the integral gain. The time derivative of the error, or rate of error change, is multiplied by the derivative gain.
7.4.2 Feedforward Transfer Function, $P_{FF}(s)$

The feedforward transfer function has the form

$$P_{FF} = \frac{x_f(s)}{\bar{x}_c(s)} = sK_F$$  \hspace{1cm} (7.22)

The derivative of the command signal is multiplied by the feedforward gain, $K_F$, and added directly to the servovalve command signal.

7.4.3 Servovalve Transfer Function, $S(s)$

The servovalve transfer function, $S(s)$ in Figure 7.6, is the open-loop transfer function in Eq. (7.9), with the command signal, $x_c$, being the PIDF adjusted error signal augmented by the delta-P feedback. This transfer function is

$$S(s) = \left(\frac{k_i}{A}\right) \frac{1}{s\left[s^2\left(Vm_t\right) + s\frac{m_t}{A} + 1\right]}$$  \hspace{1cm} (7.23)
7.4.4 System Transfer Function, H(s)

The relationship between the command signal and the adjusted command signal due to PIDF adjustment and delta-P feedback is

\[ x_c(s) = \bar{x}_c(s)P_{FF}(s) + (\bar{x}_c(s) - x_A(s))P_{PID}(s) - x_D(s) \]  

(7.24)

as shown in Figure 7.6. The relationship between the servovalve command signal, \( x_c \), and the table displacement, \( x_t \), is simply

\[ x_t(s) = S(s)x_c(s) \]  

(7.25)

The feedbacks consist of the piston displacement feedback and the differential pressure across the actuator piston. In the case of the displacement feedback, the actual piston or table displacement is converted to a voltage by the linear variable differential transformer (LVDT) mounted on the actuator, and is sent to the 407 controller’s AC conditioner, see Figure 7.6, which multiplies the voltage by a preset gain value. The transfer function, \( A(s) \), is taken to be unity for displacement control. Thus,

\[ A(s) = \frac{x_A(s)}{x_t(s)} = 1 \]  

(7.26)

In the case of the differential pressure feedback, an actuator mounted delta-P cell monitors the difference in pressure across the piston, which is directly related to the force applied by the actuator. The delta-P cell converts the pressure difference into a voltage, which is then converted by the DC conditioner into a feedback signal to be subtracted from the command signal coming out of the PID controller. The relationship between the table displacement and the differential pressure is, in Laplace notation,
\[ \Delta P(s) = \frac{F_{\text{net}}}{A} = \frac{m_t \dot{x}_t(s)}{A} = \frac{s^2 m_t x_t(s)}{A} \]  

(7.27)

If we define the operator

\[ R(s) = \frac{s^2 m_t}{A} \]  

(7.28)

then

\[ \Delta P(s) = R(s)x_t(s) \]  

(7.29)

If we denote by \( D(s) \) the transfer function between the pressure differential, \( \Delta P \), and the delta-P feedback signal, \( x_D \), then the delta-P feedback signal can be expressed as

\[ x_D = [D(s)R(s)]x_t \]  

(7.30)

By combining Eqs. (7.24), (7.25), (7.26), and (7.30), we obtain

\[ x_t(s) = S(s)[\bar{x}_c(s)P_{\text{FF}}(s) + [\bar{x}_c(s) - A(s)x_t(s)]P_{\text{PID}}(s) - D(s)R(s)x_t(s)] \]  

(7.31)

Rearranging terms, Eq. (7.31) can be expressed as

\[ \{1 + S(s)[A(s)P_{\text{PID}}(s) + D(s)R(s)]\}x_t(s) = S(s)[P_{\text{FF}}(s) + P_{\text{PID}}(s)]\bar{x}_c(s) \]  

(7.32)

and the closed-loop transfer function, \( H(s) \), between the command displacement signal and the actual table displacement is

\[ H_{\text{closed}}(s) = \frac{x_t(s)}{\bar{x}_c(s)} = \frac{S(s)[P_{\text{FF}}(s) + P_{\text{PID}}(s)]}{1 + S(s)[A(s)P_{\text{PID}}(s) + D(s)R(s)]} \]  

(7.33)

or

\[ H_{\text{closed}}(s) = \frac{S(s)P_o(s)}{1 + S(s)[A(s)P_{\text{PID}}(s) + D(s)R(s)]} \]  

(7.34)

in which \( P_o(s) = [P_{\text{FF}}(s) + P_{\text{PID}}(s)] \).

The open loop transfer function between the command displacement and the table dis-
placement can be found by setting the feedback terms to zero. Eq. (7.24) becomes

\[ x_c(s) = [P_{FF}(s) + P_{PID}(s)]\bar{x}_e(s) = P_o(s)\bar{x}_c(s) \]  \hspace{1cm} (7.35)

and combining with Eq. (7.25)

\[ x_l(s) = S(s)P_o(s)\bar{x}_c(s) \]  \hspace{1cm} (7.36)

The open loop transfer function is then

\[ H_{open}(s) = S(s)P_o(s) \]  \hspace{1cm} (7.37)

### 7.5 Parameter Study of the Transfer Function

There are seven parameters that must be determined to optimize the performance of the servo-hydraulic system: the PIDF control parameters, \( K_{pro}, K_{int}, K_{der}, \) and \( K_F \); the delta-P conditioner gain, \( K_{DP} \); the table gain factor, \( k_t \), and the leakage coefficient, \( k_l \). The leakage coefficient, \( k_l \), cannot be neglected in this case, since it is the only factor which controls the damping of the open loop servovalve transfer function. The first five parameters (\( K_{pro}, K_{int}, K_{der}, K_F, \) and \( K_{DP} \)) can be set by the shake table user, while the table gain factor, \( k_t \), and leakage coefficient, \( k_l \), are properties of the servovalve which must be determined experimentally. Figures 7.8 through 7.11 show the magnitude and phase of the servovalve transfer function, \( S(s) \), the PID transfer function, \( P_{PID}(s) \), the open-loop system transfer function, \( H_{open}(s) \), and the closed-loop system transfer function, \( H_{closed}(s) \), for a particular set of parameter values (reported in Tables 7.1 and 7.2). In these plots, the unknown parameter \( k_t \) is taken to be 300 in\(^2\)/sec and the leakage coefficient is taken to be 0.001 in\(^3\)/(lbs-sec).

The oil column frequency for this set of parameters is 89 Hz. The transfer functions
shown below have a prominent peak at this frequency.

Figure 7.12 shows the Nyquist plot for the seismic table. This representation of the open loop transfer function is useful in studying the stability of the closed loop system. The *Nyquist Criterion* for stability says that a closed loop system is stable if the number of counterclockwise encirclements of the point (-1 + i0) on the open loop Nyquist plot is equal to the number of poles of the open loop system in the right half of the s-plane. Eq. (7.34) above shows the expression for the closed loop transfer function as

\[
H_{\text{closed}}(s) = \frac{S(s)P_0(s)}{1 + \tilde{S}(s)} \quad (7.38)
\]

where

\[
\tilde{S}(s) = S(s)[A(s)P_{\text{PID}}(s) - D(s)R(s)] \quad (7.39)
\]

<table>
<thead>
<tr>
<th>Table 7.1 Physical Parameters of Servovalve and Actuator System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>Effective piston area, A</td>
</tr>
<tr>
<td>Bulk modulus of oil, (\beta)</td>
</tr>
<tr>
<td>Volume of oil in actuator chamber, (V)</td>
</tr>
<tr>
<td>Mass of table, (m_t)</td>
</tr>
<tr>
<td>Table gain factor, (k_t)</td>
</tr>
<tr>
<td>Leakage coefficient, (k_l)</td>
</tr>
</tbody>
</table>

If we express the transfer functions in terms of their poles and zeros, it can be shown that the poles of \(\tilde{S}(s)\) are equal to the poles of \(S(s)\). Therefore, for a stable system, the Nyquist
plot of $\tilde{S}(s)$ should encircle the point $(-1 + 0i)$ as many times as there exist poles of $S(s)$ in the right-hand-plane (RHP) of the $s$-space. If $S(s)$ has no poles in the RHP, as is the case with the physical servo valve, any encirclement(s) means that the closed loop transfer function has unstable pole(s) due to inappropriate control parameter values or unreasonable estimates of the two physical parameters. The Nyquist plots shown in this chapter are the plots of the open loop transfer function $\tilde{S}(s)$. 
### Table 7.2 PIDF Control Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_{pro}$</td>
<td>Proportional gain</td>
</tr>
<tr>
<td>$K_{der}$</td>
<td>Derivative gain</td>
</tr>
<tr>
<td>$K_{int}$</td>
<td>Integral gain</td>
</tr>
<tr>
<td>$K_F$</td>
<td>Feedforward gain</td>
</tr>
<tr>
<td>$K_{DP}$</td>
<td>Delta-P gain</td>
</tr>
</tbody>
</table>

![Graph](image)  

**Figure 7.8** Transfer Function of Servo Valve, $S(f)$
Figure 7.9 Transfer Function of PID controller, $P_{PID}(f)$

Figure 7.10 Open-Loop System Transfer Function, $H_{open}(f)$
Figure 7.11  Closed-Loop System Transfer Function, $H_{\text{closed}}(f)$

Figure 7.12  Nyquist Plot of the System Transfer Function
7.5.1 Proportional Gain

The error signal is multiplied by the proportional gain, $K_{pro}$, and this corrected signal is sent to the servo valve as shown in Figures 7.6 and 7.7. The figures below show that increasing proportional gain tends to stabilize the system, flattening out the peak at the oil column frequency and flatening the phase of the transfer function. However, too much proportional gain can cause "overshooting" as sketched in Figure 7.13 and possibly drive the system into an unstable state.

The plot in Figure 7.14 shows, as expected, that changing the proportional gain does not affect the servo valve transfer function. Therefore, any change in the system transfer function must come only from a change in the PID transfer function, as illustrated in Figure 7.15. Increasing the proportional gain results in increasing the magnitude of the PID transfer function along with a correction of the phase spectrum which tends toward zero degrees. Both phase and magnitude responses tend to flatten out over the frequency range. This suggests that while the response signal tends to follow the command signal more

---

![Diagram](image)

**Figure 7.13** Time Domain Effects of Changing Proportional Gain, $K_{pro}$
closely, it is accompanied by substantial overshoot.

Figure 7.16 shows the effect of changing the proportional gain on the system open loop transfer function. The phase of the frequency response tends toward that of the servo-valve alone, as the PID phase contribution tends to zero. While the magnitude of the response tends to increase in the case of the open loop, proportional gain increases tend to flatten out the transfer function in both magnitude and phase in the case of the closed loop for the frequencies below the oil column frequency (Figure 7.17). Increasing proportional gain increases the magnitude of the peak at the oil column frequency.

Increasing the proportional gain tends to lead to a less stable system. As Figure 7.18 indicates, an increase in proportional gain tends toward encirclement of the point (-1+0i).

![Servovalve Transfer Function with Changing Proportional Gain, $K_{pro}$](image)
Figure 7.15  PID Transfer Function with Changing Proportional Gain, $K_{pro}$

Figure 7.16  Open-Loop System Transfer Function with Changing Proportional Gain, $K_{pro}$
Figure 7.17  Closed-Loop System Transfer Function with Changing Proportional Gain, $K_{pro}$

Figure 7.18  Nyquist Plot of System Transfer Function with Changing Proportional Gain, $K_{pro}$
7.5.2 Derivative Gain

The time derivative of the error is multiplied by the derivative gain, $K_{der}$, and the resulting signal is sent to the servovalve, as shown in Figures 7.6 and 7.7. The effect of derivative control is to reduce the overshoot caused by the proportional gain. Figures 7.19 through 7.23 below show that increasing the proportional gain tends to increase the bandwidth of the open loop transfer function. It also tends to flatten the phase of the closed loop transfer function below the oil column frequency, and tends to increase the closed loop resonance frequency, while increasing the magnitude of the peak. The Nyquist plot in Figure 7.23 suggests that increasing derivative gain leads to greater stability, as the trend is away from encirclement of the critical point (-1+0i).

Figure 7.19 Servovalve Transfer Function with Changing Derivative Gain. $K_{der}$
Figure 7.20  PID Transfer Function with Changing Derivative Gain, $K_{der}$

Figure 7.21  System Transfer Function (Open Loop) with Changing Derivative Gain, $K_{der}$
Figure 7.22 System Transfer Function (Closed Loop) with Changing Derivative Gain, $K_{der}$

Figure 7.23 Nyquist Plot of the System Transfer Function (Open Loop) with Changing Derivative Gain, $K_{der}$
7.5.3 Integral Gain

The integral of the error signal over time interval is multiplied by the integral gain, $K_{\text{int}}$, and the resulting signal is sent to the servovalve, as shown in Figures 7.6 and 7.7. The effect is to reduce the "accumulated" error. It is very effective at lower frequencies and compensating for DC error in the system. The figures below illustrate the effects of increasing the integral gain. Notice that the lower frequencies are the ones primarily affected, and the transfer function at these frequencies of both the open and closed loop systems increases in the low frequency region with increasing integral gain. Increases in the integral gain tend to effect stability very little, though they tend toward greater instability.

Figure 7.24 Servovalve Transfer Function with Changing Integral Gain, $K_{\text{int}}$
Figure 7.25  PID Transfer Function with Changing Integral Gain, $K_{int}$

Figure 7.26  System Transfer Function (Open Loop) with Changing Integral Gain, $K_{int}$
Figure 7.27  System Transfer Function (Closed Loop) with Changing Integral Gain, $K_{int}$

Figure 7.28  Nyquist Plot of Open Loop Transfer Function with Changing Integral Gain, $K_{int}$
7.5.4 Feed-Forward Gain

The rate of change of the command signal, rather than the error signal, is multiplied by the feed-forward gain, $K_F$, and the result is sent to the servovalve, as shown in Figure 7.6. The feed-forward gain is employed when more gain is needed, but the proportional gain cannot be increased without causing system instability. Increased feed-forward gain tends to flatten out the phase of the closed-loop transfer function.

Figure 7.29 Servovalve Transfer Function with Changing Feed-Forward Gain, $K_F$
Figure 7.30  System Transfer Function (Open Loop) with Changing Feed-Forward Gain, $K_F$

Figure 7.31  System Transfer Function (Closed Loop) with Changing Feed-Forward Gain, $K_F$
Figure 7.32 Nyquist Plot of the Open Loop Transfer Function with Changing Feed-Forward Gain, $K_F$

7.5.5 Delta-P Gain

The delta-P gain is applied to the feedback signal from the differential pressure cell mounted on the actuator, as shown in Figure 7.6. Delta-P feedback measures the pressure drop across the piston, and is directly related to the piston acceleration. Increased delta-P gain tends to dampen the system, flattening the resonant peak at the oil column frequency. Because this is a feedback gain, only the closed loop transfer function is plotted below.
Figure 7.33  Closed Loop Transfer Function with Increasing Delta-P Gain, $K_{DP}$

Figure 7.34  Nyquist Plot of Open Loop Transfer Function with Increasing Delta-P Gain, $K_{DP}$
7.5.6 Table Gain Factor

The table gain factor of the system, $k_t$, is a physical property of the shaking table system, but is not known explicitly. This gain must therefore be estimated by fitting the analytical transfer function curve to experimental data. The larger the table gain factor, the greater the damping in the system. With increasing table gain, the resonant peak of the closed loop transfer function flattens out, and the resonant frequency increases. However, the phase and gain margin decrease, indicating that the system is becoming less stable with increasing table gain factor.

Figure 7.35 Servovalve Transfer Function with Increasing Table Gain, $k_t$
Figure 7.36  Open Loop Transfer Function with Increasing Table Gain, \( k_t \)

Figure 7.37  Closed Loop Transfer Function with Increasing Table Gain, \( k_t \)
7.5.7 Leakage Coefficient

As the table gain factor, the leakage coefficient, $k_t$, is unknown, and must be determined by matching experimental results with the analytical transfer function. Leakage provides damping in the servovalve transfer function. Note that increased leakage results in higher stability, but at the expense of the transfer function for open and closed loops tending toward zero, which means the system is becoming less responsive to command input.
Figure 7.39 Servovalve Transfer Function with Increasing Leakage Coefficient, $k_l$
Figure 7.41  Closed Loop Transfer Function with Increasing Leakage Coefficient, \( k_l \)

Figure 7.42  Nyquist Plot of Open Loop Transfer Function with Increasing Leakage Coefficient, \( k_l \)
7.5.8 Rigid Payload

The following figures illustrate the effects on the various transfer functions when the mass of the table is changed due to the addition or removal of a rigid payload. As the mass of the rigid payload increases, the oil-column frequency decreases and the oil column peak shifts toward lower frequencies.

The plot in Figure 7.43 shows how the oil-column frequency decreases with an increase in rigid payload mass. The magnitude of the peak is not significantly affected. Figures 7.44 and 7.45 show similar results. The Nyquist plot in Figure 7.46 shows that changing of the rigid payload mass has little effect on the stability of the system.

![Figure 7.43 Servovalve Transfer Function with Changing Mass of Rigid Payload, $m_t$](image)

Figure 7.43 Servovalve Transfer Function with Changing Mass of Rigid Payload, $m_t$
Figure 7.44  System Transfer Function (Open Loop) with Changing Mass of Rigid Payload, $m_i$
Figure 7.45  System Transfer Function (Closed Loop) with Changing Mass of Rigid Payload, $m_t$

Figure 7.46  Nyquist Plot of the System Transfer Function (Open Loop) with Changing Mass of Rigid Payload, $m_t$
7.6 Analytical Results for the Table Transfer Function

The theoretical table displacement, velocity, and acceleration for a given command displacement input was calculated using the transfer function shown in Figure 7.11. A balanced record of the Imperial Valley Earthquake (El Centro site) was used as the prototype ground motion, and a one-fifth scaled model of the displacement of this record was used as the command input. The following figures show the resulting table displacement, velocity, and acceleration for the set of parameters shown in Table 7.2.

Figure 7.47 shows the analytical table displacement for the given commanded signal. The two signals are in very good agreement, due to the content of low frequencies in the commanded displacement. The magnitude of the transfer function is very close to unity at lower frequencies, which results in a nearly identical response.

Figure 7.48 shows the analytical response velocity for the given commanded displacement signal and the "commanded" velocity. The commanded velocity is the time derivative of the commanded displacement signal. The agreement between the two signals is still very good, though the higher frequency content of the velocity signal results in some disparity between the commanded and the response signals. The magnitude of the transfer function at higher frequencies is not unity and the phase is not zero.

Figure 7.49 shows the analytical response acceleration and the commanded acceleration. The commanded acceleration is the second time derivative of the commanded displacement. The table acceleration response is very close to the commanded acceleration. The difference between the two signals at the very beginning of the record is due to the presence of significant high frequency components in the commanded acceleration signal.
As Figure 7.50 illustrates, the effect of the transfer function is to accentuate the magnitude of the higher frequency components. This graph shows the magnitude of the frequency components contained in the commanded and response signals. The transfer function has a peak at around 90 Hz, which amplifies the components of the commanded signal around this peak. To determine the effect of this high frequency amplification, consider the transfer function whose magnitude is shown in Figure 7.51. The high frequency peak has been truncated at unity. The resulting table acceleration response is shown in Figure 7.52. Notice that it contains less high frequency components than the table acceleration response shown in Figure 7.49. The filtering shown in Figure 7.51 is not easily realizable because the location and amplitude of the peak is highly dependent on the control settings and physical parameters of the system, as show in the preceding parameter study. However, through the optimization of the control parameters, $K_{pro}$, $K_{int}$, $K_{der}$, $K_F$ and $K_D$, the influence of these high frequencies can be reduced.
Figure 7.47 Commanded and Computed Table Response Displacements for the Balanced 1940 El Centro Record

Figure 7.48 Commanded and Computed Table Response Velocities for the Balanced 1940 El Centro Record
Figure 7.49 Commanded (a) and Computed (b) Table Acceleration Responses for the Balanced 1940 El Centro Record.
Figure 7.50 Fourier Amplitude Spectra of Commanded (a) and Computed (b) Table Acceleration Responses for the Balanced 1940 El Centro Record

Figure 7.51 Magnitude of Transfer Function with Resonant Peak Truncated
Figure 7.52  Commanded (a) and Computed (b) Table Acceleration Responses using Truncated Transfer Function
7.1 Conclusions

The expression in Eq. (7.34) gives the closed loop transfer function between the commanded displacement and the actual table displacement. The performance of the table is controlled by the seven parameters: $K_{pro}$, $K_{int}$, $K_{der}$, $K_{F}$, $K_{D}$, $k_{t}$, and $k_{l}$. The first five may be set by the operator of the table, while the last two must be determined empirically. Once these unknown physical values are determined, the optimal setting for the five control parameters would ideally be one where the closed loop transfer function has a magnitude very near 1 over the frequency range 0-60 Hz, and a phase of close to zero degrees. Additionally, the mass of the table (and model structure) is an additional variable that affects the closed loop system transfer function. Therefore, the values for the five control parameters must be reset for different table masses.
CHAPTER 8
CONCLUSIONS

The Rice University Shaking Table Facility is a state-of-the-art uni-directional testing facility, capable of testing the behavior of scaled structural models under (real-time) dynamic loadings. The shaking table is fully functional, and on-going work is focused on utilizing the data acquisition and signal processing capabilities to implement acceleration based control strategies, and the determination of the optimal table and control parameters. The completion of the shaking table facility has opened doors for new research. Future applications of the table include passive and active vibration control, structural identification, and nondestructive damage detection based on dynamic signature analysis.

8.1 Project Summary

The first stage of the design was the conceptual development of the shaking table. The working design was a uni-directional shaking table, riding on a pair of high-precision, frictionless monorails, and powered by a hydraulic actuator. The table was designed to be mounted on a concrete reaction mass, and either rigidly connected to the floor, or isolated from it through the use of elastomeric bearings.

The second phase of the design was to size the individual components of the shaking table system. The hydraulic system was sized based on typical earthquake ground motions that would be reproduced by the table. Maximum required actuator force, velocity, and displacements were used to size the actuator, pump, and servovalve. From the specifications for the hydraulic system, theoretical performance envelopes were determined.
The base isolation system was designed based on modeling and analysis of the shaking table system mounted atop isolation bearings, and calculating maximum foundation displacements and force transmissibility for cases of typical model earthquakes being commanded on the table. Eventually, base isolation was not used because of budgetary constraints and the reacting mass was connected rigidly to the laboratory floor.

The slip plate was designed by analyzing the plate for vibration and dynamic interaction with typical test structures, and choosing the optimal plate configuration with respect to size, natural vibration frequencies, and table-structure interaction.

Construction of the table was the third phase of the project. Most of this work was done in-house by the project team. Careful design of the slabs and the steel base plate was required to facilitate the precise placement of the table components.

The final phase was the development of the transfer function between the command signal and the actual table displacement. The servovalve was modeled as an mechanical system, and the effects of the PIDC controller and feedback were added to develop the closed-loop system transfer function.

8.2 Future Applications

With the addition of the shaking table to the existing testing facilities, many areas of potential research will benefit greatly from testing capabilities now available in the Civil Engineering Department at Rice. Future applications include:

(I) Vibration control of structures subjected to dynamic loading environments

(A) Tie-down of vibration sensitive equipment

(B) Earthquake protective systems for structures
(1) Passive systems

(2) Active systems

(3) Hybrid systems

(4) Semi-active systems

(II) Condition assessment and health monitoring of structures

(A) Non-destructive damage detection using on-line or off-line system identification

(III) Experimental verification of analytical methods of structural dynamics used in design

(IV) Modal testing of complex structures and substructures

(V) Component and system reliability and safety

The figure on next page lists examples of passive and active vibration control systems.
Figure 8.1 Vibration Control Systems for Seismic, Wind, Ocean Waves and Hurricane Protection of Civil Structures
Appendix A: Piecewise Exact Integration Method

The Piecewise Exact Integration Method is a quick and easy way to solve for the displacement, velocity, and acceleration time histories of a structure subjected to dynamic loading. Consider the n degree-of-freedom system shown below, subjected to a base acceleration $\ddot{u}_g(t)$. The base acceleration is defined at discrete points in time, $t_0, t_1, ..., t_m$. The base excited system can be expressed as an equivalent force excited system with the effective earthquake load vector

$$ F(t) = M \{1\} \ddot{u}_g(t) \quad (A.1) $$

where $M$ is the mass matrix of the n DOF system, and $\{1\}$ is a vector of ones. Assuming classical (orthogonal) damping and changing the geometric coordinates into modal coordinates, the governing matrix equation of motion can be transformed into a system of uncoupled modal equations of motion. The modal earthquake forces are then given by:

$$ f_j(t) = \phi_j^T F(t), \quad j = 1, 2, ..., n \quad (A.2) $$

![n-DOF Example](image)
Define the quantity

\[ \omega_{dj} = \omega_j \sqrt{1 - \zeta_j^2} \]  

(A.3)

where \( \omega_j \) is the jth circular natural frequency, \( \omega_{dj} \) is the jth damped natural circular frequency, and \( \zeta_j \) is the jth modal damping ratio. Assuming that the ground acceleration time history is piecewise linear (i.e., linear between two consecutive discretized values), each modal equation of motion is integrated exactly. The value of the jth modal displacement, velocity, and acceleration at discrete time \( t_1 \) is found to be:

\[
Y_j(t_1) = A_{0_1} + A_{1_1} \Delta t + A_{2_1} \exp[-\zeta_j \omega_j t_0] \cos(\omega_{dj} \Delta t) + A_{3_1} \exp[-\zeta_j \omega_j t_0] \sin(\omega_{dj} \Delta t) 
\]  

(A.4)

\[
\dot{Y}_j(t_1) = A_{1_1} + (\omega_{dj} A_{3_1} - \zeta_j \omega_j A_{2_1}) \exp[-\zeta_j \omega_j t_0] \cos(\omega_{dj} \Delta t) - (\omega_{dj} A_{2_1} - \zeta_j \omega_j A_{3_1}) \exp[-\zeta_j \omega_j t_0] \sin(\omega_{dj} \Delta t) 
\]  

(A.5)

\[
\ddot{Y}_j(t_1) = \frac{\ddot{x}_j(t_1) - 2 \zeta_j \omega_j \dot{x}_j(t_1) - \omega_j^2 x_j(t_1)}{m_j} 
\]  

(A.6)

where

\[ \Delta t = t_1 - t_0 \]  

(A.7)

and

\[
A_{0_1} = \frac{a_j}{\omega_j^2} - \frac{2 \zeta_j b_j}{\omega_j^3} 
\]

\[
A_{1_1} = \frac{b_j}{\omega_j^2} 
\]

\[
A_{2_1} = x_j(t_0) - A_{0_1} 
\]

\[
A_{3_1} = \frac{1}{\omega_{dj}} [x_j(t_0) + \zeta_j \omega_j A_{2_1} - A_{1_1}] 
\]  

(A.8)
where

\[ a_j = f_j(t_0) \]  \hspace{1cm} (A.9)

and

\[ b_j = \frac{f_j(t_1) - f_j(t_0)}{\Delta t} \]  \hspace{1cm} (A.10)

The procedure consists of evaluating recursively the modal displacement, velocity, and acceleration at each time step (denoted by \(t_1\)), using the corresponding results at the previous time step (denoted by \(t_0\)). The nodal relative displacements, relative velocities, and relative accelerations are then obtained by superimposing all the modal contributions as

\[
\begin{align*}
    u_k(t) &= \phi_{1,k} Y_1(t) + \phi_{2,k} Y_2(t) + \cdots + \phi_{n,k} Y_n(t) \\
    \dot{u}_k(t) &= \phi_{1,k} \dot{Y}_1(t) + \phi_{2,k} \dot{Y}_2(t) + \cdots + \phi_{n,k} \dot{Y}_n(t) \quad , \quad k = 1, 2, \ldots, n \\
    \ddot{u}_k(t) &= \phi_{1,k} \ddot{Y}_1(t) + \phi_{2,k} \ddot{Y}_2(t) + \cdots + \phi_{n,k} \ddot{Y}_n(t)
\end{align*}
\]  \hspace{1cm} (A.11)

where \( \phi_{j,k} \) represents the value of the \(j\)-th mode of vibration at node \(k\).
REFERENCES


