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Bounce-Resonant
Ion Interaction
with Hydromagnetic Waves

by

Karen M. Klamczynski

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

Master of Science

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Abstract

Bounce-Resonant Ion Interaction with Hydromagnetic Waves

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Wave-particle interactions between hydromagnetic waves and bounce-resonant ring current ions may cause ion precipitation observed during geomagnetic storms. Uncovering mechanisms of ion loss is important to understanding the recovery phases of these storms. A computer model was developed to numerically solve Hamiltonian guiding center equations of motion for a test particle in a three-dimensional time-dependent electromagnetic field model. The background magnetic field is a simple dipole and hydromagnetic waves are modeled by time-dependent electromagnetic perturbations.

Specifically, a single compressional Pc 5 wave well below the ion-cyclotron frequency was used in simulations; the amplitude of the perturbation has been varied up to the maximum observed. Bounce-resonant ring current ions near L=3 undergo pitch angle scattering and ions are moved along the field line toward the loss cone. Wave perturbations which are superpositions of several different wave modes are also considered. Although bounce-resonant interactions alone cannot account for the observed precipitation, they may be an important part of a multi-step precipitation process.
Acknowledgments

A number of people have made this thesis possible. I am very grateful to my advisor, Dr. Anthony Chan, for his patience, encouragement, and copious help throughout this project. I also appreciate the support of my committee members, Drs. John Freeman and Paul Cloutier, and the Space Physics and Astronomy Department.

I owe a debt of gratitude to Dr. Richard F. Martin, Jr., a professor in the Illinois State University Physics Department, who introduced me to space physics research. I would not have been half as prepared for graduate school without this experience (nor would college have been as rewarding without such a friend).

For support, encouragement, grief and guilt, I thank many friends. But especially, I thank my Mom, for always asking “when?” and always accepting the reply, “soon,” and I dedicate this thesis to my Dad, Eugene Florian Klamczynski, whose memory always inspires me.
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Chapter 1

Introduction

1.1 Motivation for This Study

Ever since the first rockets began exploration of the Earth’s upper atmosphere in the 1940s, physicists have been attempting to explain and predict the activities of charged particles in the near-Earth space environment [Van Allen 1956]. The world’s increasing use of the space environment for satellite communication and science, as well as the curiosity of scientists to understand complex phenomena, ensure that the inner magnetosphere remains an important and well-studied region of space.

This research uses test particle calculations to study subauroral precipitation seen during the recovery phases of geomagnetic storms. An introduction to subauroral precipitation is included in the next section, and wave-particle interactions, which are suspected to play a role in the precipitation of ions during the recovery phases of storms, are discussed in Section 1.2.3. Such interactions have already been associated with radial diffusion of ring current particles [Chan 1991], and in a more recent study Kozyra, et al [1994], have suggested that bounce-resonant interaction with magnetospheric waves may be a primary precipitation mechanism of ring current particles. This work has directly served as motivation and groundwork for this project.

Kozyra and her co-investigators modeled the recovery phase of the February 7-10, 1986 magnetic storm. They used a dipole magnetic field to model the magnetosphere between $R_E = 2$ and $R_E = 6$. The numerical model calculated the elevated ion precipitation from the ring current. Charge exchange and Coulomb interactions were included in the calculations; wave-particle interactions were not considered. These researchers found that their model did not recover fast enough: the observed precipi-
tation of $H^+$ and $O^+$ ions during the magnetic storm occurred faster than the model's predictions.

Further investigation of the February 7-10, 1986 magnetic storm revealed hydromagnetic waves in the inner magnetosphere and peaks in the precipitation ion energy flux. Antisymmetric waves with frequency near 2 mHz were identified as close as $R_E = 2.5$. Peaks in the precipitation spectra were also identified in DMSP data. These peaks were observed near $R_E = 3$ and appear to correspond to bounce-resonant energies between $H^+$ and $O^+$ ions and hydromagnetic waves.

These results suggest that wave-particle interactions may be responsible for energetic particle loss from the ring current. In this research, the capability of wave-particle interactions to cause subauroral particle precipitation from the ring current is studied.

1.2 Subauroral Precipitation

1.2.1 Definition and Mechanisms

As defined in this thesis, precipitation is the loss of particles from the magnetosphere to the atmosphere. Precipitation generally occurs at high- and mid-latitudes; charged particles from the magnetotail and plasmasphere follow magnetic field lines toward the Earth, where they might be precipitated if they reach the atmosphere. Auroral precipitation occurs around ovals that are roughly centered around the magnetic poles; POLAR and DMSP satellites continuously monitor the energies of precipitating ions in these regions. Particles which precipitate into the auroral oval do not come from the ring current; instead they generally come from the magnetotail. These particles, when energetic, cause the northern and southern lights to be seen from the ground (and space).

Subauroral precipitation occurs below the auroral oval. These particles come from the ring current. Precipitation of ring current ions, most notably protons and singly-
ionized oxygen atoms, is noticed in magnetic quiet times. During the recovery phases of geomagnetic storms, precipitation increases. This increase may be due to a loss mechanism which is enhanced during the magnetic storm, or it may be due to a mechanism which is only present during these storms.

Charge exchange is considered the main process by which ring current ions are lost during the recovery phase of geomagnetic storms [Kistler et al. 1989]. During charge exchange, an ion or electron from the ring current collides with a particle in the upper atmosphere. These collisions usually occur at an altitude between 60km and 1000km [Gurevich 1978]. The ring current particle either gains or loses an electron to make its net charge zero. The charge exchange process occurs rapidly in the atmosphere, freeing trapped particles from the Earth’s magnetic field lines [Bransden and McDowell 1992].

Coulomb collisions are also an important loss mechanism [Fok et al. 1993]. When a particle from the ring current interacts with another charged particle, a Coulomb collision occurs. The paths of both charged particles deviate due to the attraction or repulsion of the charges. The effect of such a collision is to change the pitch angle of the ring current ion. Coulomb collisions are likely to occur near the limit of the upper atmosphere, and ring current particles may be scattered and lost from the ring current.

Once in the atmosphere, most charged particles continue to undergo Coulomb collisions and charge exchange processes, and they remain in the atmosphere. Other processes which cause pitch-angle scattering of ring current particles may also contribute to particle loss, including interactions of charged particles with magnetospheric waves.

1.2.2 Low-Frequency Magnetospheric Waves

Interactions between Earth’s intrinsic magnetic field and the Interplanetary Magnetic Field often lead to geomagnetic micropulsations, or small perturbations in the Earth’s field [Jacobs 1970]. Changes in the Interplanetary Magnetic Field cause dynamical
changes in the Earth's field, which in turn causes Alfvén waves to occur and propagate. These hydromagnetic waves may be the direct results of IMF interaction, or they may be propagated by plasma instabilities.

In this study, Ultra-Low Frequency (ULF) waves are used as perturbations to the Earth's background magnetic field, modeled as a dipole. The addition of ULF perturbations to a dipole field as a geomagnetic model has been studied by [Li et al. 1993] and [Chen et al. 1993].

Many different ULF waves, measured by a variety of satellites, propagate in the magnetosphere [Takahashi 1988]. Table 1.1 lists magnetospheric ULF signals classified as continuous geomagnetic micropulsations, or Pc waves [Schulz and Lanzerotti 1974]. The frequencies of Pc 5 waves range from 1.7 to 6.7 mHz. Consistent with the findings of Kozyra, Pc 5 waves with frequencies of 2 mHz are used in the numerical model in this study.

The presence of ULF waves in the magnetosphere is measured by a number of satellites. For example, elliptically orbiting satellites ISEE-1 and ISEE-2 measure the radial structure of these waves, which are not strongly localized in radial distance from the Earth. Azimuthal structure may be determined by two geostationary satellites, such as GOES2 and GOES3. At geosynchronous orbit, the azimuthal wave mode number of Pc 5 waves is approximately 50 to 150, and the compressional component

<table>
<thead>
<tr>
<th>Name</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pc 1</td>
<td>0.2 - 5.0 sec</td>
</tr>
<tr>
<td>Pc 2</td>
<td>5 - 10 sec</td>
</tr>
<tr>
<td>Pc 3</td>
<td>10 - 45 sec</td>
</tr>
<tr>
<td>Pc 4</td>
<td>45 - 150 sec</td>
</tr>
<tr>
<td>Pc 5</td>
<td>150 - 600 sec</td>
</tr>
</tbody>
</table>

Table 1.1 Classifications of continuous ULF magnetospheric signals.
of these waves during the recovery phases of magnetic storms is typically 10 to 20% of the background magnetic field [Takahashi 1988, and references therein].

1.2.3 Wave-Particle Interactions

Non-collisional mechanisms for pitch-angle scattering are generally classified as wave-particle interactions. These interactions involve a charged particle independently interacting with one or more waves.

Wave-particle interactions may cause changes in a particle's gyration, bounce and drift motion in the magnetosphere. Of particular interest in this research is a bounce-resonant interaction. When wave frequencies are on the order of particle bounce frequencies, the second adiabatic invariant may be broken, and particles may be lost from the ring current.

Wave-particle interactions are known to not be a major precipitation mechanism for most magnetic storms. However, as suggested by [Kistler et al. 1989], bounce-resonant wave-particle interactions may be important for the February, 1986 storm. Also, the role of these interactions in the precipitation of ring current ions is least understood of all the precipitation mechanisms [Kozyra et al. 1994]. A major motivation in this research is to quantify the effect of bounce-resonant wave-particle interactions on particles trapped in the magnetosphere near $R_E = 3$.

1.3 Relevant Previous Work

In this section, we describe two publications relevant to our work. The first shows how wave-particle interactions may cause the loss of ring current particles from the region near the equatorial plane. The second is an investigation of resonant interaction between ring current protons and hydromagnetic waves. Each of these studies focuses on an important aspect of this study.

Manju Prakash [1989] numerically modeled pitch-angle scattering of particles with large initial pitch angles using wave-particle interactions. Particles with large initial
pitch angles remain near the equatorial plane, where the dipolar field lines of the magnetosphere may be modeled using a paraxial, or parabolic, approximation. Prakash showed that particles could be lost from the equatorial region through interaction with a single hydromagnetic wave.

The results from Prakash's study appear to support the hypothesis that wave-particle interactions cause precipitation of ring current particles; however, the approximation of a parabolic potential breaks down away from the equatorial plane. The results from this work may not be relevant to particles that have small initial pitch angles—that is, particles which mirror at higher latitudes.

Chan [1991] studied drift-bounce resonant interaction of protons in the ring current with hydromagnetic waves which are similar to the waves considered in this thesis. The radial transport of particles which mirror near the equatorial plane (i.e. at latitudes less than 20°) was the focus of this work. By modeling particles in drift-bounce resonance with the wave perturbations, Chan was able to quantify the wave-induced radial diffusion of ring current ions.

The basic theoretical model used in this 1991 study is the same used in this thesis. The focus of the model is now bounce resonant interaction between ring current ions and Pc 5 waves. Instead of radial motion, the effects of the waves on ion motion along magnetic field lines will determine if the ions are precipitated; wave-particle interactions will cause precipitation only if they can effectively move particles along the magnetic field lines and into the atmosphere.

1.4 Synopsis of This Study

In this thesis the interaction of bounce-resonant protons with Pc 5 waves at $R_E = 3$ is investigated as a precipitation mechanism of ring current ions. Compressional hydromagnetic waves consistent with AMPTE-CCE data are modeled. Wave-particle interactions with ring current protons are analyzed as a possible loss mechanism of
these ions during the recovery phases of geomagnetic storms, when Pc 5 waves are commonly observed and increased ion precipitation is seen in subauroral regions.

This study begins with the formulation of the precipitation model. The dynamics of ions and electrons in a dipolar magnetic field, modified by the addition of appropriate Pc 5 waves, are modeled with single-particle simulations. A complete description of the model used in this study is given in Chapter 2. The dynamics of ring current particles with energies that satisfy a bounce-resonant condition with Pc 5 waves were studied, first in the unperturbed dipole field (Chapter 3) and then with the addition of waves (Chapter 4). Chapter 5 is devoted to a summary of the main analytical results, as well as conclusions and a look ahead to future work on subauroral precipitation.
Chapter 2

Formulation of the Theoretical Model

2.1 Definition and Approximations of the Model

Motivated by the observations of Kozyra [Kozyra et al. 1994], the interaction of ring current particles with hydromagnetic waves within three Earth radii is the main focus of this project. Orbits of ions and electrons are numerically calculated to assess the possible precipitation effects of hydromagnetic waves. The background, or unperturbed, magnetic field is modeled by a dipole. Dipolar fields as geomagnetic models have been used since 1907, when Störmer began his study of auroral events and cosmic rays [Dragt and Finn 1976].

Our model numerically integrates the orbit of a single test particle; thus the model is collisionless and not self-consistent. All dynamical results are due to electromagnetic interactions between charged particles, the geomagnetic field, and electromagnetic perturbations. In Chapter 3, the geomagnetic field is a simple dipole, and no perturbations are present. In Chapter 4, hydromagnetic waves are added to the dipole as perturbations.

Precipitation is defined here as the loss of ions and electrons to the Earth’s atmosphere; precipitation occurs when particles undergo collisions and other processes that exert a stronger influence on their motion than the Earth’s magnetic field lines. For the purposes of this study, which is concerned with whether particles can reach the atmosphere due to wave-particle interactions, a complex model of the atmosphere is undesirable. This model effectively ignores the Earth’s atmosphere by defining a precipitation boundary at one Earth radii. Particles must reach \( R_E = 1 \) to be precipitated.
A more complicated, or realistic, atmospheric model is beyond the scope of this study. The goal of this research project is to determine if wave-particle interactions can cause ring current particles to precipitate. If a particle that nearly reaches the precipitation boundary can be shown to surpass the boundary, then it follows that wave-particle interactions will be able to cause precipitation for any atmospheric model. Further discussion of the precipitation boundary is the subject of Section 3.2.

The equations of motion which define the dynamics of the ring current particles are derived in Section 2.3 using gyroaveraged equations of motion. These equations are integrated numerically to obtain orbits of test particles. The method used to choose appropriate bounce-resonant test particles is described in Section 2.5. The results from the numerical simulations are then used in the following chapters to assess the effects of magnetospheric hydromagnetic waves on precipitation of ring current ions and electrons; the organization of results is provided at the end of this chapter in Section 2.6.

2.2 The Magnetic Coordinate System

Throughout this thesis, a magnetic dipole coordinate system is used. A point is identified by its position, \( \mathbf{X} = (\chi, \psi, \zeta) \), where \( \chi \), \( \psi \), and \( \zeta \) are three spatial coordinates. These coordinates identify the position of a point along a specific field line. Dipolar field lines are defined by their azimuthal angle, \( \zeta \), and radial distance from the dipole axis. The azimuthal angle is measured in degrees, and the radial distance, in magnetic coordinates, is specified with a magnetic flux variable, \( \psi \), which has units of magnetic flux: \( \psi = \frac{B_E R_E^2}{L} \), where \( B_E = .31 \) Gauss, the magnetic field strength at the surface of the Earth, \( R_E = 6.38 \times 10^6 \) meters is the radius of the Earth, and \( L \) is the distance of the field line from the dipole axis, measured in \( R_E \) in the equatorial plane.
The position along a field line is designated by the third coordinate, \( \chi \), which is defined as
\[
d\chi = \frac{ds}{|\mathbf{B}|}
\]  
(2.1)
where \( ds \) is the differential distance along the field line, \(|\mathbf{B}|\) is the magnitude of the magnetic field, and \( \chi \) is measured with respect to the equatorial plane. Thus, \( \chi = 0 \) defines the equatorial plane. The units of \( \chi \) are kilometers per Tesla in SI units.

Note that \( \chi \) is essentially the distance along a specific dipole field line, modified by the factor \( \frac{1}{|\mathbf{B}|} \). Figure 2.1 demonstrates the relation between this magnetic coordinate and geographic latitude, \( \theta_l \). The formula \( \chi = M \sin \theta / r^2 \) [Chan 1991], where \( M \) is the Earth’s magnetic dipole moment and \( r \) and \( \theta \) are spherical-polar coordinates (\( \theta \) is co-latitude), was used to calculate the relation in Figure 2.1 for a dipole field line that intersects the equatorial plane at three Earth radii (i.e. at L=3). The distance \( L = 3 \) is chosen to match the study of Kozyra, et al [1993]. Using the definition of a dipole field line, \( r = R_E L \cos^2 \theta \), the relation between latitude and \( \chi \) is
\[
\chi = \frac{\sin \theta_l}{L^2 \cos^4 \theta_l}
\]  
(2.2)
where \( L \) is the dipole L-value, \( \theta_l \) is measured in degrees with respect to the equatorial plane, and \( \chi \) is shown normalized without units. The general shape of this relation is the same for all L-values, though large L-values yield curves which intersect the Earth at latitudes approaching 90°. The lower limit of \( L = 1 \) corresponds to a point in the equatorial plane on the surface of the earth. This case yields a trivial solution: \( \chi = \theta_l = 0 \).

In magnetic coordinates, a particle’s motion can be fully expressed as a function of three spatial variables \( (\chi, \psi, \zeta) \) and three associated time-dependent velocities \( (v_\chi, v_\psi, v_\zeta) \). Instead of using this set of velocities, however, guiding-center phase-space coordinates are used, which include the three spatial variables plus \( \rho_{\parallel}, \mu \) and \( \Xi \). These three variables are associated with the velocity of the guiding center parallel to the magnetic field, the first adiabatic invariant, and the gyrophase angle, respectively.
Figure 2.1 Values of $\chi$ are compared to the more familiar quantity, latitude.

The parameter $L$, which is essentially equivalent to the coordinate $\psi$, is often used to designate the surface on which a bouncing and azimuthally-drifting particle's gyrocenter is confined. This surface is called an $L$-shell, or drift shell. Since dipoles are azimuthally symmetric, the drift shell for ions on a particular dipole field line is simply the surface found by rotating that field line about the dipole axis. In general, the drift shell is not so easily defined when time-dependent hydromagnetic perturbations are added to the dipole. Such perturbations may break the symmetry of the drift shell. Also, hydromagnetic waves may cause the phenomenon of shell splitting to occur: particles with different equatorial pitch angles that are initially on the same field line will not generally share the same field line at a different $\zeta$ [Schulz and Lanzerotti 1974].
2.3 Equations of Motion

In this section, the derivation of equations of motion for a charged particle moving in a dipole magnetic field subject to hydromagnetic waves is outlined. The phase-space Lagrangian for a time-independent magnetic field is given, which completely describes the motion of the guiding center. The time-dependent perturbation to this Lagrangian is then given. Finally, the appropriate phase-space Lagrangian, in magnetic coordinates, is presented along with the equations of motion.

The background field, a magnetic dipole, is designated as $B_0$, and the small parameter, $\varepsilon_0 = O \left( \frac{L_B}{L_B} \right)$, is defined as the guiding center small parameter; $L_B$ is the scale length of the magnetic field. The following phase-space Lagrangian describes the motion of a particle's guiding center in a magnetic field [Littlejohn 1983]:

$$L_0 = \left( \frac{A_0}{\varepsilon_o} + \rho||B_0 \right) \cdot \dot{X} + \varepsilon_o \mu \dot{\xi} - H_0 \tag{2.3}$$

$$H_0 = \frac{1}{2} \rho||^2 B_0^2 + \mu B_0 + O(\varepsilon_o^2) \tag{2.4}$$

where $B_0 = \nabla \times A_0$, measured at the guiding center's position, $X$; also, $\rho||$ is the parallel velocity of the guiding center divided by $B_0$, $\mu$ is the magnetic moment, and $\xi$ is the gyrophase.

Equations (2.3) and (2.4) describe the guiding-center motion of a charged particle in a magnetic field. The parallel velocity of the guiding center is the same as the particle, and the perpendicular velocity of the guiding center is the drift velocity of the particle.

The electromagnetic wave perturbations are space and time dependent, and the treatment of these perturbations is aided by the definition of another small parameter, $\varepsilon_1 = O \left( \frac{\delta B}{B_0} \right)$. This parameter is used to order the wave perturbations, and the value of $\varepsilon_1$ is directly related to the amplitude of the perturbation wave. Hydromagnetic perturbations are treated as a correction to the guiding center motion; in other words, $\varepsilon_0 \ll \varepsilon_1$. In this thesis, $\varepsilon_0$ is typically on the order of $10^{-4}$ and $0.1 \leq \varepsilon_1 \leq 0.4$. 
Hydromagnetic waves, $\delta \mathbf{B}(x,t) = \nabla \times \delta \mathbf{A}$ and $\delta \mathbf{E}(x,t) = -\nabla \delta \Phi - \partial \delta \mathbf{A}/\partial t$, are added to the equations as a perturbation to the Lagrangian. In guiding-center phase-space coordinates, the perturbation terms $L_1$ and $H_1$ [Chan 1994] are

$$L_1 = \delta \mathbf{A} \cdot \dot{\mathbf{X}} + \epsilon_o \left( \frac{\rho \cdot \delta \mathbf{A}}{2\mu} \right) \dot{\mu} + \epsilon_o \left( \frac{v_\perp \cdot \delta \mathbf{A}}{B_o} \right) \dot{\xi} - H_1$$

$$H_1 = \delta \Phi$$

where $\rho = (\hat{b} \times \mathbf{v})/B_o$ is the momentum of the particle and $v_\perp$ is the perpendicular velocity of the particle.

The complete perturbed Lagrangian is then $L = L_0 + \epsilon_i L_1$. After transformation from guiding center to gyrocenter magnetic coordinates $[(X, \rho||, \mu, \xi) \rightarrow (\bar{X}, \bar{\rho||}, \bar{\mu}, \bar{\xi})]$, which removes the gyrophase dependence of the equations, the gyroaveraged phase space Lagrangian turns out to be

$$\mathcal{L} = \left( \frac{\psi}{\epsilon_o} + \epsilon J_o \gamma \right) \dot{\xi} + \left( \bar{\rho||} + \epsilon J_o \alpha \right) \dot{\chi} + \epsilon J_o \beta \psi + \epsilon \bar{\mu} \bar{\xi} - \mathcal{H}$$

$$\mathcal{H} = \frac{1}{2} \bar{\rho||}^2 B_o^2 + \bar{\mu} B_o + \epsilon \left( J_o \delta \Phi + \frac{2 J_1}{\chi} \bar{\mu} \delta \bar{B||} \right)$$

where $J_o$ and $J_1$ are Bessel functions which result from assuming an eikonal form of perturbation; also, $\delta \mathbf{A} = \gamma \nabla \chi + \beta \nabla \psi + \epsilon^{-1} \gamma \nabla \zeta$ where $\alpha$, $\beta$ and $\gamma$ are constants. The assumption is made that the wave numbers in the $\zeta$ direction are much greater than those in the $\psi$ direction.

Equations (2.7) and (2.8) describe the motion of a particle’s gyrocenter in a magnetic field with electromagnetic perturbations as defined above. The equations of motion for the gyrocenter completely describe the motion in a magnetic dipole field with electromagnetic perturbations. These equations are obtained directly from Equations (2.7) and (2.8) using Euler-Lagrange equations. The results are [Chan 1994]

$$\ddot{\chi} = \bar{\rho||} B_o^2$$

$$\dot{\psi} = \epsilon_i \left( \bar{\rho||} B_o^2 \bar{b}_\perp + \epsilon \zeta - \bar{\mu} \delta \zeta \bar{B||} \right) \frac{1}{1 + \epsilon_i \bar{b}_\parallel}$$
\[
\dot{\zeta} = \epsilon_o \left[ \left( \bar{\rho}_\parallel^2 B_o + \bar{\mu} \right) \delta_\psi B_o + \epsilon_1 \left( \bar{\rho}_\parallel^2 b_\zeta - \epsilon_\psi + \bar{\mu} \delta_\psi \bar{B}_\parallel \right) \right] \frac{1}{1 + \epsilon_1 \bar{b}_\parallel} \tag{2.11}
\]

\[
\dot{\rho}_\parallel = - \left( \bar{\rho}_\parallel^2 B_o + \bar{\mu} \right) \delta_\chi B_o + \epsilon_1 \left( \bar{\epsilon}_\parallel - \bar{\mu} \delta_\chi \bar{B}_\parallel \right) + \epsilon_1 \delta_\zeta \dot{\psi} - \epsilon_1 \bar{b}_\psi \dot{\zeta} \tag{2.12}
\]

Equation (2.9) describes the parallel velocity of the gyrocenter (that is, velocity along a field line), Equations (2.10) and (2.11) describe radial and azimuthal drift of the gyrocenter, respectively, and Equation (2.12) describes the parallel acceleration.

Note that when the amplitude of the wave perturbations goes to zero (that is, when \(\epsilon_1 \to 0\)), Equations 2.9 through 2.12 simplify to those of a guiding center in a magnetic dipole field:

\[
\dot{\chi} = \bar{\rho}_\parallel^2 B_o^2 \tag{2.13}
\]

\[
\dot{\psi} = 0 \tag{2.14}
\]

\[
\dot{\zeta} = \epsilon_o \left( \bar{\rho}_\parallel^2 B_o + \bar{\mu} \right) \delta_\psi B_o \tag{2.15}
\]

\[
\dot{\rho}_\parallel = - \left( \bar{\rho}_\parallel^2 B_o + \bar{\mu} \right) \delta_\chi B_o \tag{2.16}
\]

Equation (2.14) confirms the statement on page 11 that a particle’s gyrocenter is confined to a single drift shell in a dipole field. The higher-order perturbation terms in Equations (2.9) through (2.12) appear in these equations to preserve the Hamiltonian properties of the system.


2.4 Computer Solutions of the Equations of Motion

Equations (2.9) through (2.12) describe particles in a magnetic field which consists of a magnetic dipole plus hydromagnetic wave perturbations. The computer program DOC (Dipole Orbit Code) [Chan 1991] was developed to solve equations of the previous section to find the motion of a charged particle in this system.

The Dipole Orbit Code was adapted for this study to produce \(\chi - \rho_\parallel\) phase-space plots for test particles in (1) a simple dipole [in order to fully understand the unperturbed system] and (2) a dipole with hydromagnetic wave perturbations.
Routines were also created to allow DOC to calculate appropriate bounce-resonant energies for test particles.

The hydromagnetic waves affect the magnitude, but not the direction, of the background magnetic field. This is the effect of the compressional wave component; the direction of the magnetic perturbation is along the original field lines. In order for particles to be lost from the model via precipitation, particles must move along a field line until they are lost to the atmosphere via collisional processes. Hence, as stated in Section 1.3, this study is primarily concerned with parallel dynamics (motion in the $\chi$-direction).

Radial motion, such as the diffusive radial loss of ring current ions studied by Chan [1991], is not analyzed in this thesis. Radial diffusion is expected to be minimal.

Radial motion was monitored during all particle simulations. Only test particles interacting with two or more wave modes exhibited any radial motion. In all cases, radial movement of the particle's guiding center, as measured in the equatorial plane, did not exceed 1.8% of the initial value.

### 2.5 Solutions of the Resonance Condition

In Chapter 4, the amplitudes and frequencies of hydromagnetic wave perturbations are treated as parameters, and particle energies are chosen appropriately to model bounce-resonant interaction with the waves. This section describes the method by which test particles in this study were chosen.

Ions are in resonance with a wave when a natural frequency of the ion motion in the magnetic field matches the frequency of the wave. The resonance condition for a low-frequency wave and a trapped particle is [Southwood et al. 1969]

\[
\omega - M\omega_d = \mathcal{K}\omega_b
\]  

(2.17)
where \( \omega \) is the wave frequency, \( \mathcal{K} \) is an integer, \( M \) is the azimuthal mode number, and \( \omega_b \) and \( \tilde{\omega}_d \) are the particle's bounce frequency and bounce-averaged drift frequency, respectively. The convention is used such that \( \tilde{\omega}_d \) is positive for Eastward drift.

To solve this resonance condition, expressions for \( \tilde{\omega}_d \) and \( \omega_b \) are substituted into Equation (2.17). From [Hamlin et al. 1961],

\[
\tilde{\omega}_d = \frac{3\gamma v^2 E(\alpha)}{\Omega_c r_0^2 T(\alpha)} \quad \omega_b = \frac{\pi v}{2r_0 T(\alpha)}
\]

where \( \Omega_c \) is the gyrofrequency, and \( r_0 \) is the distance from the dipole axis measured in the equatorial plane. A particle's initial pitch angle, measured in the equatorial plane, is \( \alpha_i = \arcsin(v_L/v) \), and

\[
\frac{E(\alpha)}{T(\alpha)} = 0.35 + 0.15 \sin \alpha_i \quad T(\alpha) = 1.30 - 0.56 \sin \alpha_i
\]

After a few steps of algebra, the resonance condition is shown to be a quadratic equation in the particle's velocity, \( v \).

\[
v^2 + v \left[ \frac{\pi \Omega_c r_0 \mathcal{K}}{6 M \gamma E(\alpha)} \right] - \left[ \frac{\omega \Omega_c r_0^2 T(\alpha)}{3 M \gamma E(\alpha)} \right] = 0
\]

(2.20)

The two solutions of Equation (2.20) correspond to bounce and drift-bounce resonant energies. The lower-energy solution corresponds to the bounce resonant solution; when a charged particle has this energy, the particle is in bounce resonance with the hydromagnetic wave. The bounce period is an integer multiple of the wave period. A particle is in drift-bounce resonance when the drift period is a multiple of the wave period; this corresponds to the higher-energy solution of Equation (2.20).

For given frequencies of the hydromagnetic wave perturbations, L-value, masses and charges of the particles and initial pitch angles were used as parameters in this study. The solution of Equation (2.20) provided appropriate resonant energies for test particles used in the Dipole Orbit Code.
2.6 Organization of Thesis Results

In the next two chapters, the results of test particle simulations are presented. Chapter 3 is devoted to the dynamics of ring current ions in an unperturbed dipole field. The adiabatic behavior of particles is discussed first, then orbital characteristics are reviewed; these results are consistent with well-known gyrocenter results, and they are presented in the magnetic coordinate system which is used extensively in Chapter 4. A phase space analysis of simulation results provide a basis for analyzing and understanding the effects of hydromagnetic waves on the system, particularly for processes which break the adiabatic invariants.

Chapter 4 contains the results of adding the hydromagnetic perturbations to the magnetic dipole model. Solutions of the resonance condition are discussed, as well as specific particles used in this study. The effects of perturbation waves on adiabatic motion are discussed. Effects of the perturbations on orbital characteristics and a thorough phase space analysis are also presented.
Chapter 3

Results for the Magnetic Dipole

3.1 Adiabatic Motion in a Magnetic Dipole

That charged particles may be trapped in a simple dipole is well known and for most particles in the magnetosphere is well characterized by adiabatic theory [Wolf 1993]. Three adiabatic invariants are relevant to charged particle motion in a magnetic dipole. Cyclotron (or gyro-) motion, bounce motion, and azimuthal drift all have associated quantities that can be constants of the motion in a dipole field. Since the motion of a charged particle in a magnetic dipole is generally insoluble [Dragt and Finn 1976], the adiabatic quantities are actually empirical invariants: there is no rigorous mathematical proof to show why adiabatic theory works as well as it does. In fact, the success of adiabatic theory is a "plausible minor miracle" [Dragt and Finn 1976], yet extremely helpful in the study of charged particles in magnetic fields.

3.1.1 Cyclotron Motion

Cyclotron motion is the component of motion perpendicular to the magnetic field direction. Centripetal acceleration associated with the \( (q\vec{v} \times \vec{B}) \) force causes a charged particle to orbit field lines in a circular path. When the magnetic field is approximately constant over a gyro orbit, an invariant quantity is associated with this motion. This first adiabatic invariant, \( I_1 \), is defined as

\[
I_1 = \oint p_1 ds
\]  

(3.1)

where the integral is taken over one gyration. An evaluation of this integral yields the familiar nonrelativistic result

\[
I_1 = \mu = \frac{1}{2} q\Omega_c a_c^2
\]  

(3.2)
where $\Omega_c$ and $a_c$ are the frequency and radius of the cyclotron motion, respectively, and $\mu$ is the magnetic moment. The conservation of this invariant is interpreted as the conservation of magnetic flux through the current loop formed by the gyrating particle.

The first invariant is approximately conserved as long as changes in the electric and magnetic fields are slow compared to the cyclotron period and the cyclotron radius is small compared to the scale length of the magnetic field. Empirical formulas to estimate the accuracy of using $I_1$ as an invariant are available [Murakami and Sato 1990]. Commonly, the adiabaticity of $I_1$ is tested using a parameter, $\epsilon$:

$$\epsilon = \frac{a_c}{R_E L} \tag{3.3}$$

where $a_c$ is the particle's Larmor radius. In order for $I_1$ to be conserved in a dipole field, $\epsilon$ must be very small; that is, $\epsilon \ll 1$. Murakami and Sato [1990] adjusted this criterion for testing the invariance of $I_1$ by adding a dependence on pitch angle to the formula. In order for $I_1$ to be invariant in a dipole field,

$$\epsilon^* = \frac{a_c}{R_E L \sin^2 \alpha_i} \ll 1 \tag{3.4}$$

and the cyclotron period must be much smaller than the timescale of the electric and magnetic field perturbations.

### 3.1.2 Bounce Motion

Periodic bounce motion is due to the geometry of the dipole field, which is a magnetic bottle. The adiabatic quantity conserved is

$$I_2 = \oint p_\parallel ds \tag{3.5}$$

The limits of integration encompass an entire bounce path. An equivalent form [Wolf] is

$$I_2 = 2\sqrt{2m\mu} \int \sqrt{B_m - B(s)} ds \tag{3.6}$$
where $B(s)$ is the magnitude of the magnetic field along the field line and $B_m$ is the magnitude of the magnetic field at the mirror point. $B_m$ is a function of a particle's equatorial pitch angle $\alpha_i$ and the magnetic field strength in the equatorial plane at a certain $L$, $B_o$. It can easily be shown that

$$B_m = \frac{B_0}{\sin \alpha_i^2}$$  \hspace{1cm} (3.7)

### 3.1.3 Drift Motion

The third invariant relevant to motion in a magnetic dipole is associated with azimuthal drift. As a particle undergoes cyclotron and bounce motion, its center of gyration will drift azimuthally due to curvature of the field lines and radial gradient of magnetic field strength. Designating this third invariant as $I_3$,

$$I_3 = \int mW_\perp d\zeta$$  \hspace{1cm} (3.8)

where $W_\perp$ is the guiding center drift velocity perpendicular to the field (i.e. in the azimuthal ($\zeta$) direction). The limits of the integration encompass one complete drift path, $\zeta = 0$ to $2\pi$. Particles trapped in a dipole conserve $I_3$, which represents the total magnetic flux within the guiding center's drift path.

For an equatorially-mirroring 10 KeV proton at $L = 3$, the drift period is approximately 22 hours, while the bounce period of the same particle is about 40 seconds. The cyclotron period of this proton is 9 milliseconds. Thus, the third invariant is generally the least likely to be conserved under a system perturbation; more specific discussion of these invariants when waves are added to the dipole is given in the next chapter.

### 3.2 The Loss Cone and Precipitation

The presence of a precipitation boundary prevents particles with very small pitch angles from reaching their bounce points. Since these particles never bounce, they
are considered lost. The ions and electrons which are precipitated are said to be in the loss cone, an imaginary geometric structure which describes the range of pitch angles that will reach the precipitation boundary and be lost from the magnetic field. An illustration of the loss cone is provided in Figure 3.1, followed by a quantitative description of the precipitation criteria.

![Diagram of the loss cone in a dipole system.](image)

**Figure 3.1** A qualitative illustration of the loss cone in a dipole system.

Every field line contains two critical points which correspond to the intersection of the field line and the precipitation boundary at the Earth’s surface. The intersection of a dipole field line with this surface is where

\[ R_E = R_E L \cos^2 \theta_{\text{cr}} \]  \hspace{1cm} (3.9)

This yields the critical latitude for which particles on a particular field line are lost:

\[ \theta_{\text{cr}} = \pm \arccos \sqrt{\frac{1}{L}} \]  \hspace{1cm} (3.10)

The critical latitude for \( L = 3 \) is 54.74°.

The corresponding critical equatorial pitch angle, \( \alpha_{\text{cr}} \), is simply derived. The critical pitch angle will define the size of the loss cone; all particles with pitch angles equal to or less than \( \alpha_{\text{cr}} \) will be lost. The magnetic moment is

\[ \mu = \frac{mv^2}{2B} \]  \hspace{1cm} (3.11)
This definition is rearranged and used to eliminate \( v_\perp \) from the definition of pitch angle:

\[
\alpha = \arcsin \frac{v_\perp}{v} = \arcsin \sqrt{\frac{2B\mu}{mv}} \tag{3.12}
\]

Now, at the mirror point, the magnetic moment is

\[
\mu = \frac{mv^2}{2B_m} \tag{3.13}
\]

Substitution of this into Equation (3.12) leaves the simple relation

\[
\alpha = \arcsin \sqrt{\frac{B}{B_m}} \tag{3.14}
\]

which reveals that the pitch angle varies along the field line. Now, at the critical latitude, the particle crosses the \( 1R_E \) surface before bouncing; the magnitude of the dipole field at this critical latitude is

\[
B_m(R_E, \theta_{\text{cr}}) = \frac{\mu_0 M \sqrt{(1 + 3\sin^2 \theta_{\text{cr}})}}{4\pi R_E^3} \tag{3.15}
\]

where \( \mu_0 \) is the magnetic permeability of free space and \( M \) is the dipole moment. The magnitude of the dipole field at the critical latitude is approximately \( 4.357 \times 10^{-5} \) Tesla at \( L = 3 \). This yields the critical equatorial pitch angle:

\[
\alpha_{\text{cr}} \approx 9.31^\circ \tag{3.16}
\]

Thus at \( L = 3 \) all particles with equatorial pitch angles equal to or less than \( \alpha = 9.31^\circ \) are lost. This result is independent of mass and energy, so it is true for all particles.

The trapped particles simulated in this study have initial equatorial pitch angles between \( 9.31^\circ \) and \( 90^\circ \). A comparison of Equations (3.3) and (3.4) shows that \( \epsilon^* \) varies from \( \epsilon \) by a factor of \( \frac{1}{\sin^2 \alpha_i} \). So, in this study

\[
\epsilon \leq 38.2\epsilon^* \tag{3.17}
\]

Since \( \epsilon \) is smaller than \( \epsilon^* \), \( \epsilon \) is used as the criterion for testing the adiabaticity of the magnetic moment.
3.3 Time-Series Analysis of Dynamics in a Magnetic Dipole

The first particle modeled in the dipole field is a 107 eV proton started at $L = 3$ with initial pitch angle of $40^\circ$. This particle has a natural bounce frequency in the dipole field of approximately 2 mHz; this frequency corresponds to that of the Pc 5 wave which will be used as a perturbation in Chapter 4. This proton has a gyroradius of 842 meters. From Equation (3.4), the parameter $\epsilon$ is computed:

$$\epsilon = \frac{842}{3R_E \sin^2 40^\circ} \approx 1.1 \times 10^{-4} \quad (3.18)$$

Since this result is on the order of $10^{-4}$, the first adiabatic invariant is expected to be well conserved.

Figure 3.2 illustrates the variables $\chi$ and $\psi$ plotted against time. As expected for a guiding center undergoing normal bounce motion, values of $\chi$ change periodically as a function of time, with the period equal to the bounce time. Figure 3.2 also

![Figure 3.2](image)

$\chi$ and $\psi$ versus time for a proton at $L = 3$. In the dipole field, most variables associated with particle motion are either cyclic or constant.

verifies that the test particle remained at $L = 3$; the gyrocenter of a single particle in a magnetic dipole will remain on the same L-shell.
As the magnitude of the magnetic field varies along the dipole field line, so must the pitch angle and Larmor radius of the test particle. Figure 3.3 shows the evolution of the pitch angle for a test particle. Recall that the pitch angle in the equatorial plane, also referred to as the initial pitch angle, is used to identify particles in this study.

![Figure 3.3](image)

**Figure 3.3** The cyclic variation of pitch angle with time for a proton in a dipole field.

All of the particles simulated in the dipole field follow orbits with qualitatively similar characteristics.

### 3.4 Phase Space Analysis of Unperturbed Dipole

#### 3.4.1 Phase Space Portraits

Though phase portraits are relatively commonplace in the literature, for completeness this section defines the plotting technique used to analyze the research results in this thesis.

Graphing three-dimensional orbits onto two-dimensional plots is cumbersome. In this study, phase portraits are used as simple two-dimensional representations of
three-dimensional motion. In autonomous Hamiltonian systems, like the guiding center motion in a magnetic dipole field (in this chapter), the value of the phase space coordinates "completely defines the state of the system at that time" [Tabor 1989].

The $\chi - \rho_\| \|$ phase space is examined in this study. A guiding center in a magnetic dipole has three degrees of freedom, characterized by three spatial coordinates ($\chi, \psi, \zeta$) and three independent velocities, or energies ($\rho_\|, \mu, W$). Due to the azimuthal symmetry of the dipole field, the motion of the guiding center is independent of the azimuthal variable $\zeta$; also, since the guiding center remains on a single drift shell, motion of the guiding center is also independent of the variable $\psi$. When the first adiabatic invariant, $\mu$, is conserved, and the system is conservative—that is, total energy is a constant of the motion—all of the information about the guiding center's motion is displayed in the phase space of the remaining variables, $\chi$ and $\rho_\|$. Time is the dependent variable.

Generally, particles (or in this case, guiding centers) trace out smooth curves in phase space, called phase curves. Guiding centers with different energies trace out different phase curves, which do not cross due to the uniqueness of solutions to differential equations [Tabor 1989]. However, when adiabatic invariants are broken, key features of these plots are area-filling regions: instead of tracing out smooth curves, guiding centers will eventually visit every allowed area in the phase space.

Phase portraits are plots which contain a set of phase curves; the phase curves and phase portraits in this chapter will serve as a reference for the analysis of results in the next chapter.

3.4.2 Analysis of Dipole Results

A phase portrait was created for the test particle in Section 3.3. The proton has an energy of 107 eV and an initial equatorial pitch angle of $40^\circ$. This energy corresponds to the bounce resonant energy of this particle with a 2 mHz Pc 5 wave. Figure 3.4 shows the phase portrait for the proton. In the phase plot, the ordinate axis represents
\( \rho_\parallel \) and the abscissa is measured in normalized units of \( \chi \). Corresponding values of latitude, \( \theta_i \), in degrees, can be obtained using Figure 2.1. Recall that \( \chi = 0 \) designates the equatorial plane. The line \( \rho_\parallel = 0 \) represents where the particles have zero kinetic energy parallel to the field line; in other words, mirror points of the particles lie on this axis.

This curve is consistent with the expected bounce motion. The closed curve in the figure indicates the conservation of the second adiabatic invariant. In Figure 3.4, the particle is started in the equatorial plane with velocity along the field line, which corresponds to Point A. The particle then fills out a quasiperiodic curve on the \( L = 3 \) surface. From Point A in Figure 3.4, the proton fills in the curve counterclockwise as the particle moves below the equatorial plane and eventually bounces at Point B, where \( \rho_\parallel = 0 \). The proton's velocity then increases until it reaches the equatorial plane (Point C). After this, the particle starts slowing until it reaches its mirror point.
above the equatorial plane, Point D, where it again bounces and continues to complete the curve.

The phase curves of eight particles were computed next; these particles have equatorial pitch angles which range from 10° to 80°. The equatorial pitch angles, energies, and mirror points of each particle are shown in Table 3.1, and Figure 3.5 shows the phase portrait for all eight particles. Each particle has the same magnetic moment and was started in the equatorial plane with positive velocity.

The innermost curve in Figure 3.5 corresponds to the particle with an initial pitch angle of 80°. This curve appears fairly ellipsoidal; the magnetic field with which this particle interacts is nearly quadratic since the particle remains near the equatorial plane. Particles with smaller equatorial pitch angles travel further along the field line before bouncing. The outermost curve corresponds to the α = 10° particle; this curve has the most exaggerated shape because this particle travels the furthest along the field line, mirroring near the loss cone.

In Section 3.2, the loss cone at L = 3 was found to contain all particles that mirror beyond χ = ±0.82; thus, the loss cone appears as two separate areas on these plots. A particle entering the area which corresponds to χ ≤ −0.82 represents loss in the Southern Hemisphere, and likewise a particle entering the area where χ ≥ 0.82 represents precipitation in the Northern Hemisphere.

In spite of the change in shape of the phase curves for the particles, the same interpretation applies for each: the particles start in the equatorial plane with positive velocity, then proceed to exhibit normal bounce motion. Note that none of the curves cross since each particle has the same magnetic moment, and the first invariant is conserved in these simulations.

In the next chapter, Figure 3.5 and this analysis are used as a basis for understanding and analyzing the effects of various perturbations to the dipole system.
<table>
<thead>
<tr>
<th>$\alpha_i$</th>
<th>Energy (eV)</th>
<th>Mirror Point ($x_{\text{max}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^\circ$</td>
<td>174.5</td>
<td>0.64</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td>148.4</td>
<td>0.23</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>125.8</td>
<td>0.12</td>
</tr>
<tr>
<td>$40^\circ$</td>
<td>107.0</td>
<td>0.076</td>
</tr>
<tr>
<td>$50^\circ$</td>
<td>91.97</td>
<td>0.049</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>80.61</td>
<td>0.032</td>
</tr>
<tr>
<td>$70^\circ$</td>
<td>72.70</td>
<td>0.019</td>
</tr>
<tr>
<td>$80^\circ$</td>
<td>68.06</td>
<td>0.0092</td>
</tr>
</tbody>
</table>

**Table 3.1** These statistics correspond to eight protons whose phase space curves are shown in Figure 3.5.

**Figure 3.5** The $\rho_\parallel - \chi$ phase space curves for eight protons in the dipole field.
Chapter 4

Results for the Perturbed Dipole

4.1 The Hydromagnetic Wave Perturbations at $L = 3$

The perturbations in this study correspond to compressional hydromagnetic waves which are antisymmetric about the equatorial plane. Figure 4.1 illustrates the magnetic perturbation. The effect of wave perturbations are minimal at the loss cone; this

![Graph showing wave perturbations](image)

**Figure 4.1** The basic structure of compressional waves added to the dipole magnetic field. The frequency and amplitude were treated as parameters in this study.

is typical of waves generated near the equatorial plane due to plasma disturbances [Chan 1991].

In the next section, energies and characteristics of ions in resonance with hydromagnetic waves are presented. Adiabatic invariants in the perturbed dipole system are discussed in Section 4.3. Section 4.4 reports and analyzes the results for adding
a single hydromagnetic wave perturbation to the dipole system, and Section 4.5 discusses the results of adding several wave modes to the dipole system.

4.2 Resonant Interaction with Magnetospheric Waves

4.2.1 Characteristics of Resonance Condition Solutions

As described in Chapter Two, bounce resonant and drift-bounce resonant solutions are obtained from the resonance condition given in Equation (2.17):

$$\omega - M\omega_d = \mathcal{K}_l\omega_b$$

The effects of $M$, the azimuthal wave mode number (if $\lambda$ is the azimuthal wavelength on a given L-shell, $M = 2\pi LR_E/\lambda$), and L-shell on resonant energies are considered. The small parameter $\epsilon$ is evaluated, and relativistic effects are also considered.

For bounce-resonant particles,

$$\omega \approx \omega_b \quad (4.1)$$

Therefore, by comparison with Equation (2.17), the term $M\omega_d$ is very small for bounce-resonant solutions, and the mode number does not strongly affect bounce resonant energies. For the remainder of this study, a wave mode of $M = 100$ is used for wave perturbations unless otherwise noted. This value is consistent with measured values [Takahashi 1988].

The resonant energies are dependent on L-shell. Figure 4.2 shows the dependence of $H^+$ and $O^+$ bounce-resonant solutions on L-shell. Each ion's resonant energy is shown for 2 mHz and 20 mHz waves. Resonant energies scale with particle mass; more massive particles need higher energies to be in resonance with the wave. Wave frequency also scales resonant energy. The 2 mHz Pc 5 wave is used in this study to model bounce-resonant energies that may correspond to the precipitation peaks found by Kozyra, et al and described in Section 1.1.

Bounce-resonant energies for electrons, protons and singly-ionized oxygen interacting with a 2 mHz wave are in Table 4.1. Particles with small equatorial pitch
Figure 4.2  \( H^+ \) and \( O^+ \) bounce-resonant energies as functions of L-shell for 2 mHz and 20 mHz waves.

<table>
<thead>
<tr>
<th>( \alpha_i )</th>
<th>Electron</th>
<th>Proton</th>
<th>( O^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>0.09507</td>
<td>174.5</td>
<td>2301</td>
</tr>
<tr>
<td>20°</td>
<td>0.08138</td>
<td>148.4</td>
<td>1988</td>
</tr>
<tr>
<td>30°</td>
<td>0.06889</td>
<td>125.8</td>
<td>1713</td>
</tr>
<tr>
<td>40°</td>
<td>0.05919</td>
<td>107.0</td>
<td>1480</td>
</tr>
<tr>
<td>50°</td>
<td>0.05167</td>
<td>91.97</td>
<td>1290</td>
</tr>
<tr>
<td>60°</td>
<td>0.04453</td>
<td>80.61</td>
<td>1143</td>
</tr>
<tr>
<td>70°</td>
<td>0.04012</td>
<td>72.70</td>
<td>1040</td>
</tr>
<tr>
<td>80°</td>
<td>0.03788</td>
<td>68.06</td>
<td>978.7</td>
</tr>
</tbody>
</table>

Table 4.1  Bounce-resonant energies, in eV, for particles interacting with a 2 mHz Pc 5 wave.
angles have the highest resonant energies since bounce and drift-bounce frequencies are inversely proportional to pitch angle.

Figure 4.3 shows a graph of the dimensionless parameter, $\epsilon$, for $L = 2$ protons and electrons as a function of energy. Equation (4.2) shows the relativistic relation used to calculate the Larmor radius over a range of $L$-values.

$$a_c = \frac{\gamma m v_\perp}{q B_0} = \frac{m v_\perp}{q B_0 \sqrt{1 - \frac{v^2}{c^2}}}$$

This result is compared directly to the nonrelativistic case. The points at which the curves in this figure diverge indicate the limit of nonrelativistic theory. Figure 4.3 shows the energies used in this study are well within the nonrelativistic scheme, as expected.

Using curves of constant energy, Figure 4.4 illustrates the functional relationship between the small parameter $\epsilon$ and $L$-shell value. For protons at $L = 3$ that are in
bounce-resonance with a 2 mHz wave, ε remains a small parameter on the order of $10^{-4}$.

To conclude this section, Table 4.2 shows drift-bounce resonant energies for the electrons, protons and $O^+$ ions; these particles have initial pitch angles of 40° at $L = 3$. These energies are much larger, as expected, than the bounce-resonant energies given in Table 4.1. The electron is relativistic, though the proton and oxygen ion are not. We will not consider the drift-bounce resonance any further in this thesis.

<table>
<thead>
<tr>
<th>Drift-Bounce Resonant Energy (KeV)</th>
<th>Electrons</th>
<th>Protons</th>
<th>$O^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$6.528 \times 10^6$</td>
<td>3594</td>
<td>259.9</td>
</tr>
</tbody>
</table>

**Table 4.2** Drift-bounce resonant energies of particles and a 2 mHz wave.
4.2.2 Resonant Particles and Energies in this Study

In the remainder of this thesis, we consider the motion of protons in the perturbed dipole system. The two main cases investigated in this chapter are ("Case One") resonant protons with initial pitch angles of 40° and ("Case Two") resonant protons with initial pitch angles of 10°. The first case corresponds to the predictions of Kozyra, et al, and the second case represents the most likely to result in precipitation: the particle already mirroring near the loss cone is in bounce resonance with the perturbation.

All particles in the phase portraits have the same magnetic moment; thus, only one particle may be in bounce-resonance with the wave in each plot. Tables 4.3 and 4.4 list energies of the particles in each Case, as well as equatorial gyration and bounce frequencies of each particle. These frequencies are important to characterizing the motion of test particles and are used in the next section to assess the adiabaticity of particle motion in the perturbed dipole.

4.3 Adiabatic Invariants in the Perturbed Dipole

According to adiabatic theory, the three invariants discussed in the previous chapter may remain constants of motion even as the dipole magnetic field changes. In this

<table>
<thead>
<tr>
<th>$\alpha_i$ ($\alpha_{res} = 40^\circ$)</th>
<th>Energy (eV)</th>
<th>Bounce Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10°</td>
<td>1466</td>
<td>7.378</td>
</tr>
<tr>
<td>20°</td>
<td>378.0</td>
<td>3.747</td>
</tr>
<tr>
<td>30°</td>
<td>176.8</td>
<td>2.562</td>
</tr>
<tr>
<td>40°</td>
<td>107.0</td>
<td>1.993</td>
</tr>
<tr>
<td>50°</td>
<td>75.34</td>
<td>1.673</td>
</tr>
<tr>
<td>60°</td>
<td>58.95</td>
<td>1.450</td>
</tr>
<tr>
<td>70°</td>
<td>50.07</td>
<td>1.364</td>
</tr>
<tr>
<td>80°</td>
<td>45.59</td>
<td>1.301</td>
</tr>
</tbody>
</table>

Table 4.3 Particles and energies used in Case One simulations.
<table>
<thead>
<tr>
<th>$\alpha_i$ ($\alpha_{res} = 10^\circ$)</th>
<th>Energy (eV)</th>
<th>Bounce Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^\circ$</td>
<td>174.5</td>
<td>1.990</td>
</tr>
<tr>
<td>$20^\circ$</td>
<td>44.97</td>
<td>1.010</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>21.04</td>
<td>0.6909</td>
</tr>
<tr>
<td>$40^\circ$</td>
<td>12.73</td>
<td>0.5374</td>
</tr>
<tr>
<td>$50^\circ$</td>
<td>8.964</td>
<td>0.4509</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>7.014</td>
<td>0.3989</td>
</tr>
<tr>
<td>$70^\circ$</td>
<td>5.957</td>
<td>0.3676</td>
</tr>
<tr>
<td>$80^\circ$</td>
<td>5.424</td>
<td>0.3508</td>
</tr>
</tbody>
</table>

**Table 4.4** Particles and energies used in Case Two simulations.

Section, characteristic timescales for each adiabatic quantity are compared with the hydromagnetic wave frequency to assess the adiabaticity of particles in the perturbed dipole.

The first adiabatic invariant, $\mu$, is conserved when the gyrofrequency is much shorter than the perturbative wave frequency, $\omega$; also, $\epsilon$ must be a small parameter for the first adiabatic invariant to be conserved. For the second adiabatic quantity, the bounce frequency, $\omega_b$, must be much longer than $\omega$; likewise, the bounce-averaged drift frequency, $\bar{\omega}_d$, must be much longer than $\omega$ for the third invariant to be conserved.

The first adiabatic invariant is conserved in all test particle simulations. In all simulations, particles have equatorial gyrofrequencies equal to 109.3 Hz, which is much greater than the 2 mHz wave frequencies. The small parameter $\epsilon$ was computed to be $O(10^{-3})$. Thus, the particles' cyclotron motion is not affected by the perturbations; the first adiabatic invariant is conserved.

All of the bounce frequencies for particles in each Case studied are of the same order, though only one particle in each case is in resonance with the wave. Thus, the second adiabatic invariant is not necessarily conserved for particles used in simulations; all particles have $\omega_b = O(\omega)$. Note that particles with small initial pitch angles
have the longest bounce frequencies; for some particles, $I_2$ may only be broken for particles with large initial pitch angles.

From Section 3.1.3, the drift period of the particles is about 40 hours, which is much longer than the bounce period. Therefore, $I_3$ is not strictly conserved as an adiabatic quantity.

4.4 Analysis of Single-Frequency Hydromagnetic Waves

4.4.1 Phase Portraits in a Nonautonomous System

The addition of a time-dependent magnetic field to the system complicates the phase-space analysis of guiding center motion. The addition of hydromagnetic waves expands phase space from two dimensions $(\chi, \rho_{||})$ to three dimensions $(\chi, \rho_{||}, t)$, where time is no longer an independent variable.

In general, time-dependent perturbations in phase portraits allow phase curves to cross, due to the fact that the phase portrait is now the projection of three-dimensional motion onto two dimensions. In phase space, a particle orbit winds around the surface of a torus. Figure 4.5 illustrates the relation between $\chi$, $\rho_{||}$, the phase of the wave, $\xi$,

![Diagram](image)

**Figure 4.5** Motion on the surface of a torus in phase space.
and the dimensions of the torus.

In this study, the perturbations are oscillatory, and the complication of three-dimensional phase space can be eliminated. The hydromagnetic waves are azimuthally asymmetric, though they are periodic. The mode number, \( M \), defines the number of periods in \( 2\pi \) radians. By plotting points only when

\[ M\zeta - \omega t = 2n\pi \]

(4.3)

the azimuthal dependence of the wave is eliminated from the phase portraits, and the phase space is essentially reduced back to the two dimensions \( \chi \) and \( \rho_\parallel \). When phase-space points lie on curves, the curves will not cross, again, due to uniqueness of solutions for guiding centers with different total energies.

Note that if \( M \to 0 \), the azimuthal dependence of the perturbation is removed from the system, and the phase space collapses to the original two dimensions of the previous chapter.

4.4.2 Case One: \( \alpha_{res} = 40^\circ \)

In this Section, the results of adding single-frequency hydromagnetic wave perturbations of various amplitudes to the dipole system are presented. The amplitude of the first perturbation modeled is 5.0% of the background dipole field; in other words, \( \frac{\delta B_\parallel}{B} = 5.0\% \). The eight test particles of Table 4.3 were simulated in this system, and Figure 4.6 shows the phase space curves for each guiding center. The innermost curve corresponds to the proton with equatorial pitch angle of 80°, and the outermost curve corresponds to the proton with \( \alpha_t = 10^\circ \).

Each particle represented in Figure 4.6 is associated with its own torus in phase space. Figure 4.7 illustrates how nested phase curves may result from nested tori. The phase curves for protons with initial pitch angles of 10°, 20°, 30°, 60°, 70° and 80° are all the result of nested tori. The shapes of the tori are distorted for smaller initial
Figure 4.6  A $\rho_\parallel - \chi$ phase portrait when a 5%, 2 mHz wave is added to the Case One system.

Figure 4.7  Nested tori result in nested phase curves in the phase space.
pitch angles due to the nonlinearity of the dipole field lines on which the particles are trapped.

The two phase curves for the $\alpha_i = 40^\circ$ and $50^\circ$ protons significantly differ from the other curves in this phase portrait. In phase space, these protons wind around tori like the others. However, these tori are not nested as shown in Figure 4.7. Instead, they are wrapped around the torus of the $\alpha_i = 60^\circ$ proton.

These phase curves are called island structures, or islands. The $\alpha_i = 40^\circ$ and $50^\circ$ protons form islands because each of the particles has a bounce frequency near the wave frequency. These particles are most strongly affected by the wave. They exhibit bounce motion which is modulated by the amplitude of the perturbation. Each particle bounces when the superposition of the background dipole field and the perturbation equal the mirror force, and both particles can travel farther up the field line (in the direction of $+\chi$) when the wave perturbation is turned on.

The largest initial pitch angle particles, with $\alpha_i = 60^\circ$, $70^\circ$, and $80^\circ$, show distinct asymmetries to their curves, but are otherwise unaffected by the wave; this is due to the natural bounce periods of the particles and their relation to the wave frequency, $\omega$. These particles have bounce periods faster than that of the wave. From the point of view of the large pitch angle particles, the magnetic field is essentially a dipole that is not symmetric through the equatorial plane; the field below the equatorial plane appears weaker than the magnitude above this plane.

Particles with initial pitch angles of $10^\circ$ and $20^\circ$ have the largest energies. These particles have the fastest bounce frequencies and are hardly affected by the wave perturbations. These particles hardly “see” any perturbation to the dipole system. Particularly near their bounce points, where these particles spend most of their time, no change is seen because the perturbation amplitude approaches zero near the loss cone. These particles bounce when they reach their normal bounce points.

The overall asymmetry of the phase portraits in this Chapter is due to the asymmetry of the perturbations. By changing the phase of the initial wave by $180^\circ$, mirror
images (flipped about $\chi = 0$) of the plots are obtained. No particles are lost, or precipitated, from the magnetic field for this first low-amplitude perturbation.

Next, the hydromagnetic perturbation was enhanced by increasing the amplitude to 10% of the background field. The effects of this increase are shown in Figure 4.8. The enhanced perturbation creates more island structures; particles with initial pitch angles of 60° and 70° form islands similar to the 40° and 50° pitch angle particles. These particles have energies near the resonant particle. Antisymmetric effects are evident for the largest initial pitch angle particle, $\alpha_i = 80°$. No significant change is noted on the $\alpha_i = 10°$ or 20° curves, and no particles are precipitated.

![Figure 4.8](image.png)

**Figure 4.8** A $\rho_\parallel - \chi$ phase portrait when a 10%, 2 mHz wave is added to Case One; the resonant particle has $\alpha_i = 40°$.

In the third scenario for the Case One particles, the hydromagnetic wave has an amplitude such that $\frac{\sigma_B}{B} = 20\%$, which corresponds to the measured wave amplitudes for the February, 1986 event. These results are shown in Figure 4.9. Similar island structures are formed for the six particles with initial pitch angles of 30°, 40°, 50°, 60°, 70° and 80°. These particles' phase-space curves are forced very close together,
leaving two "empty" areas in phase space: one is inside the island structures, and the other is below the equatorial plane along the line \( \rho_\parallel = 0 \). All of these particles follow similar orbits. Like the previous two cases, the addition of the wave to the dipole system causes no precipitation of the test particles. Particles with \( \alpha_i = 10^\circ \) and \( 20^\circ \) follow orbits which create phase curves similar to the unperturbed case.

The effect of congregating particles in a narrow region of phase space is exemplified when a very large amplitude wave is simulated. The results of adding a perturbation amplitude equal to 40\% of the background magnetic field is shown in Figure 4.10; this amplitude is consistent with the largest observed Pc 5 waves. This figure shows that the particles with \( \alpha_i \geq 30^\circ \) follow similar phase-space curves when subjected to a high-amplitude Pc 5 waves.

In all of these simulations for Case One with a single-frequency wave perturbation, none of the particles reach the loss cone; no precipitation occurs.

**Figure 4.9** A \( \rho_\parallel - \chi \) phase portrait when a 20\%, 2 nHz wave is added to the Case One system.
4.4.3 Case Two: $\alpha_{res} = 10^\circ$

The set of phase portraits presented in this section are for the eight particles defined as Case Two in Table 4.4. These particles all have the same magnetic moment, and the $\alpha_i = 10^\circ$ particle is resonant with the perturbations. The $\alpha_i = 10^\circ$ proton mirrors very near the loss cone in an unperturbed dipole field, so the particle should not require much of a "push" along the field line to cause precipitation.

Figure 4.11 shows the results for a small-amplitude perturbation equal to 5% of the background field. The particle with an initial pitch angle of $10^\circ$ forms a single, elongated island structure in phase space. No other particles appear to be affected by the perturbation; no particles are precipitated.

The results of increasing the amplitude of the perturbation to 10% of the background field are shown in Figure 4.12. The $\alpha_i = 10^\circ$ particle exhibits a phase curve similar to Figure 4.11. The phase curves for the $\alpha_i = 40^\circ$ and $50^\circ$ particles
Figure 4.11  A $\rho_\parallel - \chi$ phase portrait when a 5\%, 2 mHz wave is added to Case Two; the resonant particle has $\alpha_i = 10^\circ$.

Figure 4.12  The $\rho_\parallel - \chi$ phase portrait for a 10\% perturbation on the Case Two system.
show asymmetric effects from the waves, but the other particles do not appear to be affected by the perturbations.

Increasing the amplitude of the perturbation to 20% of the background field, as is shown in Figure 4.13, affected the $\alpha_i = 40^\circ$ and $50^\circ$ particles. These particles mirror at latitudes near the maxima of the perturbation wave; thus, these particles are most affected by the waves. These particles also have bounce frequencies near one-half of the wave frequency.

The $\frac{B'}{B} = 20\%$ did not produce precipitation; nor did the largest amplitude perturbation, 40% of the background magnetic field. The mirror point of the $\alpha_i = 10^\circ$ particle is moved further along the field line, toward the critical latitude, but not far enough cause the particle to be lost. Figure 4.14 illustrates the effects of the very large amplitude (40% of the background field) perturbation for Case Two.

![Figure 4.13](image)

**Figure 4.13** The effect of a 20% perturbation on the $\rho - \chi$ phase portrait for the Case Two system.
4.5 Analysis of Multiple-Mode Hydromagnetic Waves

4.5.1 Case One Revisited: $\alpha_{res} = 40^\circ$ for Two Wave Modes

In the previous section, a single hydromagnetic perturbation with $M = 100$ was applied to the dipole system. In reality, the observed Pc 5 waves show a finite, albeit very narrow, frequency spectrum. The wave mode number was shown in Section 4.2.1 to hardly affect the energy of the resonant particle; however, as a demonstration, Case One and Case Two are briefly revisited with two wave modes.

Figure 4.15 shows the result of adding a 2 mHz Pc 5 wave with two modes, $M = 50$ and $M = 51$ to the dipole field. The amplitude of the perturbation is $\frac{\delta B_H}{B} = 10\%$. The effects of this perturbation on the phase portraits are similar to the results in the previous section where $\frac{\delta B_H}{B} = 10\%$. Nearly identical island structures are formed for particles with $\alpha_i$ between $30^\circ$ and $60^\circ$, and the waves minimally affect the particles with initial pitch angles of $10^\circ$ and $20^\circ$. Thus, two wave modes of amplitude
Figure 4.15  Two wave modes acting on Case One particles; a $\rho_\parallel - \chi$ phase portrait for a 10% perturbation.

Figure 4.16  The $\rho_\parallel - \chi$ phase portrait for two waves modes with amplitude 20% of the background field.
$\frac{\delta B}{B} = 10\%$ cause these particles to trace phase-space curves that are very similar to the set of curves for one wave mode. In other words, as expected, two wave modes do not significantly change the results of the previous section.

Increasing the perturbation amplitude to 20% further illustrates this point. Figure 4.16 shows the effects of two wave modes at this amplitude. Nearly identical results were observed for one mode perturbation for the same amplitude perturbation. As before, the result of strong amplitude waves is to cause a large range of pitch angle particles to trace similar phase space curves. The particles, therefore, follow similar orbits. No precipitation is directly caused by wave-particle interactions.

### 4.5.2 Case Two Revisited: $\alpha_{res} = 10^\circ$ for Two Wave Modes

Two 2 mHz wave modes, $M = 100$ and $M = 101$ are present in Figure 4.17. The amplitude of both waves is 10% of the background magnetic field; the addition of the second wave mode does not qualitatively change the results from Section 4.4.3.

![Figure 4.17](image.png)

**Figure 4.17** A $\rho_l - \chi$ phase portrait for two waves modes with amplitude 10% of the background field; $\alpha_i = 10^\circ$. 
Increasing the amplitude of these two modes to 20% of the background field yields the same results as one mode, as expected.

**Figure 4.18** The $\rho_l - \chi$ phase portrait for two waves modes with amplitude 20% of the background field.
Chapter 5

Summary, Conclusions and Future Work

5.1 Summary of Main Results

Dynamics of ring current ions in bounce-resonance with magnetohydrodynamic (MHD) waves in the Earth’s magnetosphere are modeled. These wave-particle interactions are investigated as possible precipitation mechanisms. At L=3, the background magnetic field of the Earth is approximated by a dipole, and Hamiltonian guiding center equations are integrated to obtain the trajectory of the particles’ guiding center.

Phase portraits of particles in a magnetic dipole are constructed in the $\rho_\parallel - \chi$ plane to facilitate the analysis and interpretation of results. Nested, smooth curves represent the guiding center bounce motion in a dipole field. Protons with different equatorial pitch angles and energies trace out each phase curve; the phase portraits are symmetric with respect to $\chi$ and $\rho_\parallel$.

In Case One, a $\alpha_i = 40^\circ$ proton is in bounce resonance with a single antisymmetric hydromagnetic perturbation. The amplitude of the perturbation is a parameter in this study; it is varied from 5% of the background magnetic field to the maximum observed by spacecraft, about 20% of the background field.

The addition of the wave perturbation changes the $\chi$-symmetry of the phase portrait. This broken symmetry is due to the asymmetry of the wave. In Case One, the hydromagnetic wave most strongly affects the resonant particle. As the perturbation amplitude is increased, particles with pitch angles near the resonant particle are affected by the wave; they form island structures. Particles which initially mirror near the equatorial plane (i.e. $\alpha_i = 80^\circ$ and particles which mirror near the loss cone (i.e. $\alpha_i = 10^\circ$) are least affected by the perturbations.
The effects of the Pc 5 waves diminish at the precipitation boundary. Even at the largest modeled perturbation amplitude, the particles which mirror near the loss cone are the least affected. However, particles with $\alpha_i \geq 30^\circ$ trace out very similar phase curves; these particles follow similar orbits in the system. No precipitation is observed.

In Case Two, the bounce-resonant particle has an initial pitch angle of $10^\circ$. The results for this Case are qualitatively similar to the above scenario. The resonant particle forms and island structure in all of the phase portraits. These islands are narrow, but they do not intercept the loss cone.

Particles with $\alpha_i = 40^\circ$ and $50^\circ$ are the only other particles to form islands. These particles have bounce frequencies nearly equal to half of the wave frequency. Other particles in the phase portraits are minimally affected by the presence of the wave; their phase curves show slight asymmetries for the largest amplitude perturbation.

When two wave modes are added to Case One and Case Two scenarios, essentially the same results for one mode are obtained. Low-amplitude perturbations for two modes, however, more strongly affect the particles than the same amplitude single mode perturbations.

### 5.2 Relevancy to Subauroral Precipitation

These simulations show that bounce-resonant wave-particle interactions are incapable of causing significant particle precipitation from the ring current. Though changes in the dynamics of resonant, and near-resonant, particles are caused by the waves, no precipitation is observed for any of the Cases studied.

Case Two was expected to be the most likely to cause precipitation since the resonant interaction involved a particle initially very near the loss cone. However, since the amplitude of the perturbation wave decreases near the precipitation boundary, the background magnetic field proves to be a strong enough mirror force to prevent precipitation.
Particles with initial pitch angles of 40° and 50° display the most dynamical effects due to interaction with the waves. In both Case One and Case Two, islands were formed by these particles. These particles interacted strongly with the waves for two reasons: (1) the particles' bounce frequencies are approximately one-half of the perturbation frequency, and (2) the particles spend most of their time near the peak of the perturbations.

While the results in this study show that wave-particle interactions alone do not cause notable precipitation of ring current ions, the results in this thesis suggest that their effect on particles may be part of a multi-stage process. We have shown that the waves can move the mirror points of bounce-resonant particles to higher latitudes. Perhaps a second process, such as a cyclotron-resonant pitch-angle scattering process could then precipitate these particles. This multi-stage process may work to precipitate particles during the recovery phases of magnetic storms. The similar phase curves shown in Figure 4.9 show that all of the particles with initial pitch angle greater than 30° occupy nearby areas in phase space. This accumulation, or increased density, of particles is unstable. By forming an unstable state of the system, wave-particle interactions may "set the stage" for another process to precipitate these protons, such as Coulomb collisions or plasma instabilities.

As long as particles conserve the first adiabatic invariant, this model will not precipitate particles; in any realistic scheme the mirror force of the background field is never diminished to let trapped particles precipitate.

5.3 Suggestions for Future Work

The phase portraits in this study may be studied in more detail to better understand the dynamics of particles interacting with hydromagnetic waves. Regions near the precipitation boundary, as well as inside island structures, were not investigated here.

The addition of more waves, of various frequencies, to the model might cause pitch angle scattering and precipitation.
The eight test particles for Case One are chosen to represent a range of pitch angles, from a particle that mirrors very near the loss cone initially, to a particle that initially mirrors near the equatorial plane. These eight particles are not meant to represent a realistic distribution of protons at $L = 3$; however, a distribution function for ring current particles could be defined, and the Dipole Orbit Code could be used to further investigate the role of wave-particle interactions in the ring current.

According to Liouville's theorem, the distribution function for a particular particle remains the same along its entire orbit. This theorem is extremely helpful in assessing the effects of magnetic fields on a distribution of particles (see e.g. [Martin 1994]). Liouville's Theorem may applied as follows: (1) an initial phase-space distribution of particles is determined; (2) test particles are simulated in the perturbed system; (3) the distribution function for the final state of the system at any time is calculated using the Theorem. The results may improve the understanding of wave-particle interactions in the ring current, and their possible role in a precipitation mechanism.
Bibliography


