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RICE UNIVERSITY

Design of a Harmonic Drive Test Apparatus for Data Acquisition and Control

by

Scott W. Hejny

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree

Master of Science

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Houston, Texas
April, 1997
Design of a Harmonic Drive Test Apparatus for Data Acquisition and Control

Scott W. Hejny

Abstract

Harmonic drive gear reducers were developed during the mid-1950’s and are used in many industrial and military applications. The harmonic drive is a compact, “in-line” gear reducer capable of producing reduction ratios of up to 320:1. The devices are known for their efficiency and precision, but they also possess undesirable qualities characterized by nonlinear behavior. These undesirable aspects include the presence of both static and dynamic friction, flexibility (compliance), and kinematic, or positional, error.

In the thesis, the mechanical design of a test platform for the study of the system nonlinearities was developed and fabricated. Experimental testing through computer controlled data acquisition validated the apparatus as a system for the examination and control of kinematic error. A new model for kinematic error was developed for the test system. Observations were made concerning the way in which flexibility effects the magnitude of the kinematic error. PD control schemes were implemented in both motor state and load state feedback control efforts. The load state feedback effort successfully compensated for the effects of kinematic error during regulation of
the load position. The experimental results were compared to a model and discussed in detail.
To my wife Julie,

my parents

Bill and Marie Hejny,

and

James, Brodwynne, and Brad Patak

for

their constant support and love.
Acknowledgments

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Chapter 1

Introduction

The theory underlying the operation of the harmonic drive gear reducer was developed during the mid-1950s [13, 14]. The technology has advanced since that point, but research into the theoretical aspects of the transmissions and their inherent inaccuracies has not been extensive. Most of the research on this topic has been performed by engineers in the former Soviet Union [3, 12, 25]. The rest of the research has been performed by a scattering of engineers in Canada [23, 24, 11], Japan [9, 21, 7], and the United States [26, 15, 10, 20, 28]. The following review will consist of a brief description of the concepts behind harmonic drive applications and operation.

1.1 Applications of Harmonic Drive Technology

The harmonic drive has been used for many years in robotic and military applications. A typical military application would be in the actuation of missile fins. Small motors and harmonic drives can be fit inside the cramped body of the missile and still generate enough torque to guide the missile fin during high speed maneuvering where the aerodynamic loads are at a peak. Harmonic drives are especially well suited for precision applications such as in robots that manufacture precision components (such as printed circuit boards, etc.) and in precise measuring devices. The small size of the harmonic drive makes them an excellent choice for use in revolute robotic joints. Smaller motors may be used since the harmonic drive will magnify the output torque. The harmonic drive is making headway in commercial and industrial applications as well. There are several manufacturers of harmonic drives in the United States. These
companies offer a wide variety of harmonic drive configurations and sizes.

Harmonic drives have both advantages and disadvantages. The advantages include a high torque capacity due to the multiple tooth contact and near zero backlash. The mechanical efficiencies (expressed as the percentage of the rated torque output achieved for a given velocity) of harmonic drives approach 90 percent and harmonic drives have a concentric geometry and a compact design [8].

Harmonic drives have disadvantages as well. For example, the flexible spline experiences high torsional loading due to the highly effective torque transmission capabilities. The flexspline flexes due to these loads and produces a nonlinear response. This nonlinearity is difficult to model theoretically due to the presence of hysteresis and makes prediction of the behavior of the harmonic drive under heavy loading conditions difficult. This type of error is known as flexibility or "compliance" error. Another disadvantage is the tendency of the harmonic drive to vibrate at resonant frequencies. These vibrations serve as an energy sink and have been known to produce dramatic torque losses and velocity fluctuations [26, 23]. Prediction of the natural frequencies is a difficult proposal due to the nonlinear dynamic characteristics of the system. Furthermore, friction plays an inevitable role in the harmonic drive system. There is a static friction (soft-windup) that must be overcome before motion is initiated. There is also an additional velocity-dependent dynamic friction that is closely related to the system damping parameters. This nonlinear frictional component is a challenge to predict and model because of the idiosyncrasies of each specific harmonic drive.
1.2 Principles of Operation

The harmonic drive consists of three basic components: The wave-generator, the flexible spline (or flexspline), and the circular spline. Figure 1.1 identifies these structures. The wave-generator is composed of a rigid steel ellipse surrounded by a flexible bearing. The shaft (which may serve as either the input or the output) is connected to the wave-generator. The connection may be made through the use of an Oldham coupling, a pin-type configuration, or a keyed slot in the wave-generator. The connection at this point seeks to eliminate any backlash that may produce undesirable lags, or “dead-zones”, during operation. The joint must have a high torsional stiffness.

The flexspline is a flexible, thin-walled cup that has teeth around the outer circumference of one end. The circular spline is a rigid steel ring that has teeth machined into the inner circumference. The teeth of the circular spline have a slightly larger pitch diameter than those of the flexspline. A properly assembled harmonic drive is shown
in Figure 1.2. As can be seen, the wave-generator is inserted into the tooth-bearing end of the flex spline. The flex spline deforms and assumes the elliptical shape of the wave-generator by following the contours of the race bearing. The circular spline fits over the deformed flex spline such that the teeth of the circular spline and the teeth of the flex spline mesh at the major axes of the wave-generator ellipse. These two contact points consist of several fully meshed teeth and a number of teeth in varying states of engagement. The number of teeth in contact depends on the gear reduction ratio of the harmonic drive. The larger the gear reduction ratio, the larger the number of teeth present on the circular and flexible splines. The larger the number of teeth, the larger the number of engaged teeth at any one time.

The circular spline is manufactured with two more teeth than the flex spline. Thus, for a full 360° revolution of the wave-generator, the circular spline rotates by only two teeth. This manner of tooth engagement and relative rotation enables there to be very large gear reduction ratios in very compact drives. The fact that many teeth

**Figure 1.2:** Fully Assembled Harmonic Drive Gear Transmission[6]
are meshed together during operation helps ensure efficient torque transmission and eliminates the risk of tooth slippage. Recent advances in tooth design have brought about the "S" tooth [17]. This tooth design helps ensure better contact between the teeth of the flexspline and the circular spline and has reduced the kinematic error and torque loss (or compliance error) that will be discussed later. Figure 1.3 describes the deflection seen in a 180° rotation of the wave-generator [26]. The gear reduction

![Figure 1.3: 180° Revolution of the Wave-Generator](image)

ratio for a harmonic drive is determined by the following equation.

\[ N = \frac{FST}{FST - CST} \]  

(1.1)

The nomenclature for Equation (1.1) and for the other equations in the chapter is presented in Table 1.2. Gear reduction ratios for harmonic drives vary from 50:1 to as high as 320:1 [1, 6, 17].

The high gear reductions available for harmonic drives are indicators of the corre-
<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>FST</td>
<td>Number of Flexible Spline Teeth</td>
</tr>
<tr>
<td>CST</td>
<td>Number of Circular Spline Teeth</td>
</tr>
<tr>
<td>N</td>
<td>Gear Reduction Ratio</td>
</tr>
<tr>
<td>$T_{wg}$</td>
<td>Wave-Generator Torque</td>
</tr>
<tr>
<td>$T_{cs}$</td>
<td>Circular Spline Torque</td>
</tr>
<tr>
<td>$T_{fs}$</td>
<td>Flexible Spline Torque</td>
</tr>
<tr>
<td>$\theta_{wg}$</td>
<td>Wave-Generator Rotation</td>
</tr>
<tr>
<td>$\theta_{cs}$</td>
<td>Circular Spline Rotation</td>
</tr>
<tr>
<td>$\theta_{fs}$</td>
<td>Flexible Spline Rotation</td>
</tr>
</tbody>
</table>

Table 1.1: Nomenclature for Chapter 1 Equations

Corresponding torque magnifications that may be seen. The reduction in rotational velocity is converted to increased torque and is described by the following equation [5]:

$$ T_{wg} = \frac{1}{N+1} T_{cs} = -\frac{1}{N} T_{fs} \tag{1.2} $$

As can be seen, the torque produced by the circular spline is $N+1$ times greater than the input torque (when considering the wave-generator to be the input and the flexspline fixed to ground). Similarly, if the wave-generator is again the input while the circular spline is fixed to ground, the flexspline torque is $N$ times greater than the input. This is a very large increase in torque for such a compact device. This type of torque increase may enable small, compact motors to overcome very large loads [1]. Thus, smaller motors may be used with harmonic drives to perform functions previously relegated to larger and more bulky actuators with gear boxes.

A similar relationship may be established between the input and output rotational positions. This relationship follows [26]:

$$ \theta_{cs} = \frac{\theta_{wg} + N\theta_{fs}}{N+1} \tag{1.3} $$
The exact relationship may be determined by deciding which of the two previously mentioned configurations is being used (i.e., whether the flexspline or the circular spline is grounded). Thus, either very low speeds with very high torques or very high speeds with very low torques may be achieved through the use of a harmonic drive. The most common configuration is the use as a speed reduction and torque magnification device. Note from Equation (1.2) and Equation (1.3) that if the circular spline is grounded, the flexspline rotates in the direction opposite that of the wave-generator. Also note that the equations shown above ignore system nonlinearities [1].

Finally, there is a type of error known as kinematic error. This error is the deviation between the expected load state position and the actual load state position. This error has been improved with new tooth geometries. However, since the kinematic error has been linked to the meshing of teeth necessary for the operation of the harmonic drive, it is a type of error that will be ever-present since machine tolerances are finite. Thus, some method must be found to compensate for this inherent error before near-ideal operation is achieved.

1.3 Outline of the Thesis

This thesis will describe the construction of a harmonic drive test apparatus that will be used in an attempt to characterize the various types of error inherent in the operation of harmonic drives. Chapter 2 will discuss the design and fabrication of the mechanical system. The various design issues will be highlighted and the design strategy will be revealed. Chapter 3 will describe the design of the electromechanical and data acquisition interfaces. The sensors chosen for the experiment and their relevance will be outlined and the method for achieving computer based control of the system will be described. Chapter 4 contains a discussion of the kinematic error
present in the harmonic drive system. An effort was made to identify the error and to evaluate the frequency content of the waveform. Chapter 5 discusses the validity and practical application of PD (proportional-derivative) control in the harmonic drive system. The control will be implemented using both motor state and load state feedback, and the results and implication of each case will be examined. The control effort will seek to drive the load to a desired position by compensating for kinematic error in the system while ignoring dynamic friction and compliance. Chapter 5 also contains a brief presentation of the static friction present in the system and how it affects the operation and control schemes. Chapter 6 provides a comparison between the experimental results and a simulation developed in [28]. The similarities and differences are discussed in detail and a new phenomenon, the influence of flexibility on the magnitude of the kinematic error, is described. The results of the thesis research and future recommendations are then identified in the Conclusions.
Chapter 2

Mechanical Design of the Harmonic Drive Test Assembly

The mechanical design for the harmonic drive test apparatus proceeded through a series of design stages. The process and the results are presented in the following discussion which describes the various design features and issues that had to be addressed. The development of the design is critical because we must establish a platform for the study of the nonlinearities present in the harmonic drive. This requires a stable experimental device that will provide the means for measuring the nonlinear errors while preventing any external error component from being imposed. Thus, a great deal of effort must be placed upon the design in order to achieve these goals.

2.1 Initial Design Proposals

The first design proposal was to have the major axis of motion of the apparatus located in the horizontal plane. A drawing of the initial design is presented below in Figure 2.1. As can be seen, gravity could play a significant role in the vertical deflection of the system. This is highly undesirable since concentricity and perpendicularity between the motor and load sides of the harmonic drive is critical for proper operation. If we wish to isolate specific behavioral characteristics of the harmonic drive, we cannot afford to introduce additional misalignment errors due to bending. The misalignment is due to the inability of the wave-generator and flexspline to handle radial loading. In fact, this radial loading can lead to an asymmetric preloading condition known as dedoidal that has been identified by previous researchers as being
Figure 2.1: Initial Design in the Horizontal Plane

a serious source of output error [15]. Furthermore, more supports would be necessary to prevent the load and torque sensor from deflecting the shaft. The flexion of the output shaft would introduce many new undesirable errors in the system analysis.

It is possible to operate in the horizontal position. We could have designed bearing supports that are located at the input and output sides of the harmonic drive in order to remove any radial loading. These supports could have been integral components to a housing that completely enclosed the harmonic drive. However, complicated and expensive design issues led to the proposal of an alternative solution.

The second design configuration, Figure 2.2, operates in the vertical plane. In this configuration, the radial loading problem may be avoided if proper alignment issues are addressed during the design and machining of the various components of the system. The plates may be machined and set in their proper configuration and alignment grooves may be used to help maintain the integrity of the axial alignment. This design configuration was determined to be less than optimal. The rectangular struc-
tural configuration would most likely require that the orientation of the plates be alternated on every level to maintain torsional integrity of the system. Furthermore, the design and machining of the plates would be labor intensive.

2.2 Final Mechanical Design Configuration

Therefore, a third and final design was fully developed. The design is shown in Figure 2.3. The third design replaces many of the vertical support plates with cir-
Figure 2.3: Final Design Configuration
cular steel pipe sections. The three sections of pipe have a sufficient diameter and wall thickness to provide support on all sides of the apparatus. The strength of the circular cross-section will reduce the required thickness of the other support plates and enable these plates to be made from aluminum rather than steel. The aluminum will be much easier to machine and will reduce the cost. The overall weight of the structure will be reduced as well.

From the drawing, the horizontal plates are now flange-type structures that align with the inner surface of the steel pipe. The flanges are bolted to the steel pipe around their circumference. This eases the complexity of the design and reduces machining. Alignment in an axial sense is still the most critical issue. Each of the flanges and support plates have alignment circles that maintain the critical alignment of the system. Note that the inner surface of the pipe has been bored to insure that a concentric and perpendicular mating surface is provided for the flanges. Also note that the portion of the structure surrounding the load remains similar to the second design configuration. The box-like support in this region permits the use of a larger radius load disk and provides access for future modifications.

The vertical load generated by the various components of the system is supported by a single thrust bearing located adjacent to the support table. The load is held in place with collar pins and set screws. Further axial support is provided by a journal bearing that fits over the upper shaft of the rotating torque sensor. This support eliminates any radial motion induced by rotating elements of the experiment. The output encoder is located underneath the support table. This makes accessing the encoder more difficult, but it prevents us from having to use an output gear train that connects the encoder to the rotating output shaft. Gear trains such as these
have been shown to have definitive signals that appear in the frequency spectrum of experimental systems. We wish to avoid as much of this type of clutter as possible. The location of the output encoder also prevents any further increase in the height of the system. We wish to keep the load as close to the primary support (the table) as possible in order to maintain a high level of stability.

The linkage that joins the motor, harmonic drive, and the torque sensor is very rigid. This aspect is important since we wish to avoid any torsion in the system produced by elements other than the flexspline and the harmonic drive as a whole. The connection between the motor and the wave-generator is made with a keyed shaft. A keyed coupling is also used to connect the flexspline to the torque sensor. Both of the shafts have been designed to high tolerance specifications (± 0.001 diametrically) so that all backlash in the system is eliminated. Care was taken to insure that the torsional stiffness of the couplings was greater than that of the flexible spline. Since we are interested in the flexspline deflection, we must isolate the phenomenon.

The remainder of the connections are made with flexible couplings. These couplings will compensate for any small axial misalignments present that could impair the operation of the torque sensor and the output encoder. Again, these couplings were chosen specifically for their high torsional stiffness. The fully assembled system is mounted on a steel table whose surface is 1.062 inches thick and weighs over 700 pounds. This surface provides a firm and stable base for the experiment, and the large mass of the base should help prevent vibrations from affecting the system. The table has been carefully leveled to insure vertical operating conditions. The portion of the experiments adjacent to the table have alignment rings that are fitted to a cir-
cular hole drilled through the table. This is the means chosen to maintain alignment between elements above and below the table.

2.3 Summary of Design Achievements and Challenges

The design of the mechanical system was one of the most important processes in the research project. Care had to be taken to insure that the system would be a valid testbed for the identification and control of the nonlinear kinematic error, friction, and flexibility in the system. The design was a new approach because, as far as literature indicates, a dedicated system for research into harmonic drives has not been previously constructed. Most of the existing research has been done on existing robotic platforms that use harmonic drives in the link joints. Therefore, an effort was made to study the existing literature and identifying what configuration would best suit the research apparatus. A highly stable platform was desirable because literature indicates the sensitivity of the harmonic drive to vibrations. The system also had to maintain the critical axial alignment discussed thoroughly in the previous sections. These alignment issues demanded high quality designs that included very high tolerance interfaces so that tolerance buildup would not adversely effect the system. The drawings were all done on AutoCAD and were made to scale so that we knew exactly how the system would fit together upon assembly. Effort was also put into making the load removable so that different loads could be used in the system and so that other experiments that focus on topics such as shaft flexibility could be performed.

The design and fabrication of the system was a precision effort. As mentioned previously, the drawings were exact, so the sensors and other components had to be selected before the design could be frozen. This alone produced many iterations as the optimal combination of sensors needed for the experiment was determined. Finally, actual
assembly of the manufactured product is a challenging task in itself. Small design errors had to be compensated for and in some instances corrected. Fortunately, the amount of time spent on the design itself paid off because all changes made after the machining of the parts were very minor and typically consisted of modifying parts to match the purchased components (such as the motor and sensors) whose dimensions varied slightly from the drawings obtained from manufacturers. Again, these changes were small but necessary due to the fact that almost all tolerances in the system are ±0.001 inches.
Chapter 3

Electromechanical System Specifications and Hardware Interfacing for Data Acquisition and Processing

The following chapter will describe the various characteristics of the harmonic drive test apparatus. The system specifications and the characteristics of the sensors employed in the research project will be fully detailed. The reasoning behind the sensor selection will be defined as well. First, however, the operational mode will be described.

The basis for the harmonic drive test apparatus is the study and control of non-linear characteristics of harmonic drives. The motor and the sensors, along with their various interfaces, will assist in providing the control and feedback necessary for the identification, study, and control of the kinematic error, dynamic friction, and flexibility found in the harmonic drive system. The system will be run such that the sensors in the system will record both input (motor side) and output (load side) characteristics of the system. A view of the complete system setup is shown in Figure 3.1 at the end of the chapter. The primary measurement concerns are the input and output torque and displacements. The input and output positions will provide the means necessary for the measurement of the kinematic error. The torque measurements will be useful in characterizing the compliance and frictional losses in the harmonic drive test system. All of these measurements will enable the development of an accurate
and precise dynamic model for a harmonic drive gear reducer in the presence of the
kinematic, frictional, and flexibility error components.

3.1 HDC-040-50 Harmonic Drive Gear Reducer

The focus of the system is the harmonic drive itself. The harmonic drive chosen for
this experimental project is the Harmonic Drive Technologies HDC-40-050. The gear
reduction ratio \( (N) \) of the drive is 50 (i.e., the drive produces one output revolution
for every 50 input revolutions). The circular spline of the drive is grounded to an
aluminum plate and the flexspline serves as the output port. The motor shaft drives
the wave-generator, which serves as the input port. The primary characteristics of
the HDC-40-050 are presented in Table 3.1. The characteristics listed in the table

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>HDC-40-050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Torque Output</td>
<td>226 Nm</td>
</tr>
<tr>
<td>Maximum Output Torque</td>
<td>312 Nm</td>
</tr>
<tr>
<td>Static Torque Limit</td>
<td>828 Nm</td>
</tr>
<tr>
<td>Ratchet Torque Limit</td>
<td>746 Nm</td>
</tr>
<tr>
<td>Maximum Input Speed (Grease Lubrication)</td>
<td>2800 RPM</td>
</tr>
<tr>
<td>Standard Wave-Generator Inertia</td>
<td>4.36 kg-cm(^2)</td>
</tr>
<tr>
<td>No-Load Starting Torque</td>
<td>14.1 Ncm</td>
</tr>
</tbody>
</table>

Table 3.1: Characteristics of the HDC-40-050 Harmonic Drive Gear Set

are defined as follows. The *Rated Torque Output* is the "operating" torque, or the
average torque used to make predictions about the operating life of the harmonic
drive. This is the recommended operating torque [6]. The *Maximum Output Torque*
is a limit that must be maintained during operation. The torque produced with a
dynamic input torque should not exceed this limit whether they be momentary or
continuous. The *Maximum Input Speed* is limited by the size of the gear set and
the type of lubrication. The smaller drives can handle a larger input speed than the
larger sets. Oil lubricated drives can operate at higher speeds than grease lubricated drives [6]. The Static Torque Limit is the maximum torque that may be exerted to the output when the input is locked. The Ratchet Torque Limit is the limit which should be avoided such that ratcheting will not occur. “Ratcheting” occurs under severe dynamic overload conditions in which separating forces are generated such that the tooth mesh is deflected radially. The deflection prevents correct operation and causes the flexspline to move off center by one tooth. Overloads of this type frequently occur during emergency stop conditions [6].

The HDC-40-050 was selected due to torque, output velocity, and lubrication considerations. The HDC-40-050 is a large harmonic drive and, as listed above, can withstand a rated torque of 226 N-m. This is important since the motor used in the system has a rated torque of 3.8 N-m. When this input torque is magnified by the harmonic drive, the rated output torque of the system is 190 N-m. We do not plan to operate in this torque range, but the system is designed to withstand the torque should the need arise. Furthermore, the operating speed of the motor will be limited to 3000 rpm. The reduction in speed produced by the harmonic drive will result in a maximum output rotational velocity of 60 rpm. A grease lubricated drive was selected because it does not have to be isolated in a sealed housing. A grease cartridge is used instead and is held in place against the wave-generator bearing by a support located in the interior of the flexible spline. A view of the configuration of the harmonic drive in the existing system is shown in Figure 3.2 at the end of the chapter.
3.2 Motor, Power Supply, and Controller

The system that will be used to initiate motion in the harmonic drive gear set is based on an AC servo motor that is driven by a power supply and controller purchased from Atlas-Copco Controls, Inc. [16] and shown in Figure 3.3. The motor has the following characteristics. The motor has a resolver in the housing whose signal is returned to

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>AC Servo Motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rated Torque Output</td>
<td>3.8 Nm</td>
</tr>
<tr>
<td>Stall Torque</td>
<td>4.6 Nm</td>
</tr>
<tr>
<td>Maximum Torque Output</td>
<td>18 Nm</td>
</tr>
<tr>
<td>Rated Current</td>
<td>8.80 A</td>
</tr>
<tr>
<td>Maximum Current</td>
<td>26 A</td>
</tr>
<tr>
<td>Rated Speed</td>
<td>4000 rpm</td>
</tr>
<tr>
<td>Maximum Speed</td>
<td>4000 rpm</td>
</tr>
<tr>
<td>Rated Power</td>
<td>1.592 W</td>
</tr>
<tr>
<td>Armature Inertia</td>
<td>0.29 kgm²x10⁻³</td>
</tr>
<tr>
<td>Voltage Constant</td>
<td>44 V/krpm</td>
</tr>
<tr>
<td>Torque Constant</td>
<td>0.60 Nm/A</td>
</tr>
</tbody>
</table>

Table 3.2: Characteristics of the AC Servo Motor

the motor controller. Effectively the motor is treated as being DC because control is performed upon a rectified current in the motor controller. The power supply is an Atlas-Copco BA4/30-50-310. The power supply is isolated from the 220 VAC three-phase building power supply by a three-phase isolation transformer. The power supply transmits a rectified voltage to the Atlas-Copco ST1/25-3:0 controller. The controller then inverts the modified voltage signal back into an AC waveform for the motor. The controller contains a microprocessor and is used to set up motor operation and perform control functions. In the test system, the controller is used to set the motor into a “torque-control” mode and to process the resolver feedback signal coming from the motor. The controller also contains an analog board and an
expansion test card that will be discussed in the electrical specifications section. The complete signal flow schematic is shown in Figure 3.4. Note that the electrical box contains the power supply for the torque sensor and the encoders. It is a junction box where all of the acquired signals are routed to the dSPACE board.

### 3.3 System Characteristics and Sensor Specifications

The motor is used to displace the inertial load and the various inertias of the system components. Table 3.3 identifies the system inertias. The sensors chosen for the har-

<table>
<thead>
<tr>
<th>Mechanical Component</th>
<th>Inertia</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave-Generator</td>
<td>4.360x10^{-4} kgm^2</td>
</tr>
<tr>
<td>Motor Shaft</td>
<td>0.29x10^{-3} kgm^2</td>
</tr>
<tr>
<td>Motor/Harmonic Drive Connector</td>
<td>2.0729x10^{-5} kgm^2</td>
</tr>
<tr>
<td>Flexspline/Torque Sensor Connector</td>
<td>2.38x10^{-4} kgm^2</td>
</tr>
<tr>
<td>Torque Sensor</td>
<td>1.1299x10^{-3} kgm^2</td>
</tr>
<tr>
<td>BK2-80 Flexible Coupling</td>
<td>0.65x10^{-3} kgm^2</td>
</tr>
<tr>
<td>MK2-100 Flexible Coupling</td>
<td>1.60x10^{-5} kgm^2</td>
</tr>
<tr>
<td>Inertial Load</td>
<td>4.7969x10^{-2} kgm^2</td>
</tr>
<tr>
<td>Output Shaft</td>
<td>1.4163x10^{-4} kgm^2</td>
</tr>
<tr>
<td>Canon Encoder</td>
<td>≈ 1.0x10^{-5} kgm^2</td>
</tr>
<tr>
<td>Sumtak Encoder</td>
<td>≈ 1.0x10^{-5} kgm^2</td>
</tr>
</tbody>
</table>

**Table 3.3:** Inertial Components of the Experimental System

monic drive test apparatus include a laser rotary encoder to measure load position, a rotary encoder to measure the motor position, and a rotating torque sensor to measure the load torque. The integration of these sensors is described in Figure 3.4. The laser rotary encoder is manufactured by Canon and has a resolution of 50,000 pulses per revolution (0.007°). This rate of acquisition will assist in the measurement of the output position such that the kinematic error signature should be easily identifiable. The Canon M-1 encoder is connected to the dSPACE digital signal processing (DSP)
board present in the host PC. The encoder is a necessity in the system since we must have a means of measuring the output (load-side) state of the system.

The motor side position may be obtained from either a built-in resolver located in the motor housing or from a Sumtak encoder attached to the motor shaft. The Sumtak encoder was purchased after it was recognized that the resolver was imposing inaccuracies and noise in the measurement system. The identification and characterization of the resolver error will be discussed in Chapter 4. This addition produced the need for additional mechanical design as the encoder had to be mounted to the motor casing in a structure that would enable the encoder to couple to the motor shaft. A view of the Sumtak encoder mounting configuration is shown in Figure 3.5 at the end of the chapter. This is accomplished through the use of a flexible coupling that compensates for any misalignment between the motor and encoder shafts. Both sensor outputs will be shown in the discussion since they are relevant to the system identification. but the encoder, with a resolution of 2000 pulses per revolution (0.180°), produces much more accurate positional information. The resolver does not have the resolution of the encoder and the signal contains much more noise. The resolver signal will have to be filtered in some manner before it may be evaluated.

The torque sensor used in the experimental setup is a DC operated non-contact torque sensor. The sensor is characterized by the specifications in Table 3.3. The

<table>
<thead>
<tr>
<th>Torque Sensor Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Torque Range</td>
<td>226 Nm</td>
</tr>
<tr>
<td>Torque Overload</td>
<td>904 Nm</td>
</tr>
<tr>
<td>Rotating Inertia</td>
<td>1.1299x10^{-3} kgm²</td>
</tr>
</tbody>
</table>

**Table 3.4:** Torque Sensor Specifications
torque sensor has a very large capacity and matches that of the harmonic drive. The range is large considering the small torque that will be used in the experiment, but this sensor was selected to provide flexibility in the design of future applications and experiments. The torque sensor is a non-contact device, and this is ideal for the system where unnecessary frictional components and excessive noise in the collected data could corrupt the analysis. The torque sensor is important to the overall experiment. In order to characterize the friction and compliance errors, the output torque experienced by the flexspline must be a measurable quantity. The availability of the output torque will enable the computation of the torque lost across the flexible spline. The motor torque will be acquired by monitoring the motor current through the controller as described below.

3.4 Data Acquisition and Processing

The final signals necessary for the system identification and control are acquired from the ST1 controller. Again, the flow of signals may be observed in Figure 3.4. Two circuit boards are connected to the ST1. The first is the Atlas-Copco LA3 Analog Input Board. This board is connected to the ST1 microprocessor and is used for the input of the torque setpoint. The setpoint is input from an outside source in the form of an analog voltage. The LA3 board then uses analog-to-digital conversion (ADC) technology to transform the voltage into the appropriate motor current. The second board is the Atlas-Copco "test card." This card is normally used as an interface with the host PC so that the motor parameters may be entered into and stored in the ST1 Controller EEPROM. The test card also has pins that may be used as analog outputs for the motor position (it provides access to the resolver signal) and the motor torque. These are the final two output signals that are necessary for the system identification.
So, in summary, the signals that are to be used for the system identification and control are shown in Table 3.4. Recall that the torque setpoint will be sent to the

<table>
<thead>
<tr>
<th>Signal</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input (Motor) Torque</td>
<td>Test Card (Measure of Motor Current)</td>
</tr>
<tr>
<td>Input (Motor) Position</td>
<td>Sumtak Encoder or Test Card (Resolver)</td>
</tr>
<tr>
<td>Output (Load) Position</td>
<td>Himmelstein Rotating Torque Sensor</td>
</tr>
<tr>
<td>Output (Load) Torque</td>
<td>Canon Laser Rotary Encoder</td>
</tr>
</tbody>
</table>

Table 3.5: Torque and Position Signal Origin

LA3 board from the host PC.

The signals and feedback will be processed by a dSPACE digital signal processing (DSP) board located in the host PC (a Compaq 486DX2/66). The dSPACE board has the ability to read the signals through an interrupt routine running in the background. dSPACE then performs the necessary control functions based on the signals themselves and inputs from a Windows based program that will be used to set the torque and start the system. The Windows program and the background dSPACE routine share memory addresses such that data may be passed between the two applications. The dSPACE capability to perform the dual role of acquisition and processing is due to the fact that the dSPACE board contains two microprocessors. One of the processors performs the data acquisition interfacing while the other performs slave application such as the computation of control algorithms. The real-time data acquisition capabilities available from dSPACE will be valuable in the implementation of PD control on the system. The data acquired may also be displayed at the terminal in real-time through the dSPACE TRACE application [2]. An example of this capability is shown at the end of the chapter in Figure 3.6. This program displays the
acquired data for a set number of passes through the interrupt routine.

The dSPACE system also has the ability to store acquired data for later processing through the TRACE application. This data will be loaded into Matlab and evaluated. These evaluations will serve as the basis for the development of models for kinematic error, friction, and compliance.

3.5 Summary of the Electromechanical Design

The electrical interfacing and design of the system was a valuable experience. A knowledge of electrical engineering was required and was implemented during the course of the development. The development of the system required both the wiring and the design of sensor/hardware interfaces. A knowledge of the operation of and safety issues surrounding high voltage systems was required to complete the wiring for the power supply and controller. Important issues arose after powering up the system because the power supply was short-circuiting through the rectifier. A thorough examination of the electrical wiring diagram indicated that the absence of a floating ground was the source of the problem. The solution to this issue required the purchase and installation of a 220 VAC 3-phase transformer that isolates the power supply from the building ground. Once again, mechanical design was required in order to construct an enclosure for the high voltage transformer, power supply, and controller. The transformer and power supply are completely enclosed in a metal computer console. The controller sits atop the console and is covered with a plexiglas shield that prevents contact with the 310 VDC terminal while providing a measure of ventilation.

The computer interface the control system had to be carefully designed. All of the sensors were interfaced with the appropriate dSPACE board address and then calibrated.
The programs necessary for the operation and control of the system were developed so that the user could communicate with the system through a Windows interface and dSPACE. The programming was performed in C and C++. The programming itself was a valuable project as experience was gained in adapting the programs for data acquisition and processing through the sensors and dSPACE board. This experience insured that the correct signals were being monitored and controlled while a measure of safety was maintained in the operating system.
Figure 3.1: Complete View of the Test System Indicating the Harmonic Drive Test Apparatus and the PC Control Station
Figure 3.2: View of the Harmonic Drive Test Apparatus
Figure 3.3: Power Supply, Controller, and Electrical Junction Box in the Harmonic Drive Test Apparatus System
Figure 3.4: Data Acquisition and Signal Flow Diagram
Figure 3.5: View of the Sumtak Encoder Mounted on the Motor Housing
Figure 3.6: View of the PC Monitor as Data is Displayed in Real-Time Using TRACE[2]
Chapter 4

Kinematic Error

Chapter 3 will discuss the test procedure and results for the examination of kinematic error in the harmonic drive test apparatus. First, the kinematic error as described in the literature will be introduced. This discussion will provide the basis for the identification and study of the error in the experimental system. Kinematic error is a common problem with harmonic drive gear reducers and has been identified as being a function of $\theta_m$. Assuming that there is no flexibility between the motor and load side, the equation that describes the error is:

$$\tilde{\theta} = \frac{1}{N} \theta_m - \theta_L$$  \hspace{1cm} (4.1)

where $N$ is the gear reduction ratio, $\theta_m$ is the input position defined by the rotation of the motor shaft, $\theta_L$ is the position measured at the load end of the harmonic drive system, and $\tilde{\theta}$ is the kinematic error. A typical kinematic error waveform available in the literature is described below in Figure 4.1. The model is characterized by a low frequency error that carries higher frequency components. Other models have been developed by researchers. Hsia [10] proposed a model that varies sinusoidally as a function of the elliptical wave-generator geometry. A similar approximation was developed by Ramson [20]. Other researchers have tried methods centered on vector analysis [3, 12, 18].

Another researcher used experimental results to generate a kinematic error model [26, 27]. This model examines kinematic error as a function of the frequencies present in
the output position signal of a harmonic drive. The frequency spectrum of the experimental data was examined and a model was developed which incorporated the harmonic frequencies present in the output position signal and their relative magnitudes. The model is shown below

\[
\dot{\theta} = A_1 \sin(2\theta_m + \phi_1) + A_2 \sin(4\theta_m + \phi_2) + \ldots
\]  \hspace{1cm} (4.2)

where \(\theta_m\) is the motor position, \(A_1\) and \(A_2\) are magnitudes obtained from experimental data, and \(\phi_1\) and \(\phi_2\) are phase shifts (also found in the experimental data). The magnitudes and phase shifts may be computed from the real and imaginary components of the FFT of the error signal. More harmonics may be examined, but the first two harmonics have the greatest magnitude and have been shown by previous research to be the predominant instigators of the kinematic error [15, 26]. This kinematic error model was recently used to develop a simulation for a harmonic drive
test system composed of a motor, a harmonic drive, and an inertial load [28]. The results of the simulation were used to develop a PD control algorithm that regulated the kinematic error in point-to-point movements in the simulated environment.

Regulation as well as tracking capabilities are desirable for the control of the harmonic drive test apparatus. In order to accomplish this task, the kinematic error that is present in the experimental system must be identified and analyzed. This will be accomplished through a detailed examination of the difference between the motor position measured by both the resolver and the Sumtak encoder (divided by the gear reduction ratio, $N$) and the load position measured by the Canon encoder.

4.1 Experimental Procedure for Kinematic Error Recording

The harmonic drive test apparatus is operated in a manner such that data can be taken at different rotational velocities. A typical data acquisition run for the examination of the kinematic error waveform is described below.

4.1. The power stage of the motor controller is enabled with Atlas-Copco Socasin Expert software [16]. The software is then exited since the motor is now in the "torque-control" mode and is waiting for an input.

4.2. The dSPACE software and the Windows program are executed so that a torque input can be sent to the motor controller. The dSPACE program is started first so that it may run in the background and exchange information with the Windows program. The TRACE application is started as well so that the real-time data, such as motor position and load position, may be displayed on the
PC monitor.

4.3. The torque input is adjusted from the keyboard so that a slow, steady rate of rotation is established after the static friction in the system has been overcome. The slow rotational velocity is maintained throughout the course of the experiment.

4.4. Data acquisition is automatically initiated through the interrupt routine in the dSPACE software, which then communicates with the Windows application. Data is stored in ASCII files so that it may be loaded into Matlab for later analysis.

4.5. The positional error is calculated using the formulation in Equation (4.1) and is displayed along with the other signals. This provides an immediate measure of the desired signal.

4.6. The shape and frequency content of the kinematic error signature or the results of the PD control may then be examined using post-processing files in Matlab.

### 4.2 Experimental Results for the Kinematic Error Analysis

The experimental analysis of the kinematic error present in the harmonic drive test apparatus was extensive and covered a wide range of rotational velocities. It is critical to identify the kinematic error and to make sure that the data is consistent in defining the error waveform. The actual acquisition and identification of the error serves to
validate the test platform and provides a clear view of how well the system has been designed. Therefore, the determination of the presence of the kinematic error is a very important step in the research. Furthermore, the kinematic error analysis was performed with both the resolver and the Sumtak encoder as the measure of the input position. As mentioned previously, the encoder is a much more accurate means for position measurement, but it is interesting and valuable to study the results obtained from both sensors in order to realize the importance of sensor selection and to examine the types of noise and interference that sensors can impose on the system. Both sets of results are presented in the following sections.

The first sets of data taken in the experiment used the resolver as the motor position measurement device, and the results were examined in both the dSPACE TRACE environment and during post-processing in Matlab. The first idea of the kinematic error, therefore, came in the real-time dSPACE acquisition and is shown in Figure 4.2. The input torque for this error trace was 0.20 N-m. As can be seen, the signal is very noisy and it is difficult to interpret the trends present in the waveform. An effort was made to identify the source of the high-frequency noise present. The results indicate that the noise is present in the input position measurement from the resolver. A plot of the resolver signal is shown in Figure 4.3. The noise present in the resolver signal tends to pulse once per input revolution. The noisy signal is then transferred to the kinematic error when the motor and load positions are compared. The resolver signal also tends to migrate during the course of the test. This indicates that either the resolver calibration is incorrect or that the calibration is changing with time. Either case is unacceptable since the kinematic error component that we are measuring has a relatively small magnitude (compared to the magnitude of the noise) and a low frequency. Therefore, since the noise is present on the input or motor side rather than
Figure 4.2: dSPACE Plot of the Kinematic Error Waveform (Resolver Position Measurement)

on the load side, it may be stated that the high frequency signal is not caused by the harmonic drive and is not, then, a valid component of the kinematic error signature.

After determining that the previous analysis was correct, the kinematic error signal was then filtered in Matlab. The filter design in Matlab was of some concern since it is possible that a portion of the kinematic error itself could be filtered out if the cutoff frequency of the filter was too low. Thus, caution was used in developing a low-pass filter from the options available in the Matlab Signal Processing Toolbox such that the filter removed the high-frequency resolver noise without extracting frequency content from the kinematic error signal. This process was begun by testing several filters on the input position signal. We were confident that if the filter did not adversely alter the noisy input signal, the filter would not adversely effect the kinematic
Figure 4.3: Enlarged View of the Resolver Input Position Signal

error.

After the completion of the filter design, the kinematic error signal was filtered and examined as a function of time and as a function of input revolutions (wave-generator revolutions). The results are shown in Figure 4.4. As can be seen, the kinematic error is a summation of several sinusoids that oscillate about zero. The primary sinusoidal component appears to occur approximately once per input revolution. This signal serves as a carrier for several higher frequencies as well. The next step in the procedure is to analyze the error with the Sumtak encoder as the motor position measurement device.

Figure 4.5 is an example of the kinematic error seen in the TRACE display during the experiment. Figure 4.6 provides a comparison between the kinematic error signal seen from the resolver input and that from the encoder input. It is very inter-
Figure 4.4: Filtered Kinematic Error Waveform (Resolver Input)

Interesting to compare the two signals in Figure 4.6. The resolution and lack of noise in the encoder position measurement were amazing. No filtering of the data was necessary due to the high resolution of both the input and output position signals. It is obvious that the sensors used for research applications (especially application where very high resolution is required) must be selected with care. Sensors that are noisy and unreliable can produce unsatisfactory results such as those shown in the figure. The encoder results are much more pleasing. The exact kinematic error waveform may be
seen in the dSPACE TRACE display. The results are satisfying and are completely repeatable. Tests run at different velocities exhibited very little variation in the waveform. Small differences were due only to very high frequency fluctuations along the lower frequency kinematic error waveform. These results correspond closely to results published in literature [26]. Some small changes were noted in the amplitude of the kinematic error as well. These variations will be discussed in a later chapter. The
Figure 4.6: Kinematic Error Waveforms with Resolver Input (Top) and Encoder Input (Bottom)

results provide a high level of confidence and validate the system as a test apparatus for the study of kinematic error in harmonic drives. The test procedure was repeated for several different input orientations to further confirm the consistency of the error signal. The addition of the input encoder to the system made a huge difference in the quality and repeatability of the results obtained in the kinematic error testing.
4.3 Discussion of Results

A frequency analysis of the kinematic error signature will provide an idea of the frequency content of the signal. The frequency analysis was performed by taking an FFT of the kinematic error and examining plots such as the one shown below in Figure 4.7. The first examination will be of the frequency spectrum when the resolver is used as the motor position signal input. The FFT was produced after the data was filtered. This FFT yields troubling results. The primary component of the kinematic error appears at a frequency of approximately one cycle per input revolution. This result is disturbing since literature supports the peak at two cycles per input revolution as
being that characteristic of the kinematic error in harmonic drives. However, as mentioned previously, the noise in the resolver pulses at one cycle per input revolution. The pulsing shows up in the frequency spectrum as the dominant peak but should be ignored since the input position signal cannot bear a portion of the kinematic error signature.

Figure 4.8 shows an FFT of the kinematic error signal obtained through use of the Sumtak encoder as the input position measurement device. The results in this case are much more satisfying. The large peak at one cycle per input revolution is gone.
since the resolver has been removed from the data analysis. This tends to confirm the previous diagnosis as to the source of the cyclic behavior occurring once per input revolution. The second peak appears at two cycles per input revolution. This dominant peak corresponds to the kinematic error signature in the harmonic drive. The frequency corresponds to the meshing of gear teeth as described in literature [15, 26]. The appearance of the error at this frequency is due to the fact that, for every complete rotation of the wave-generator, the flexspline rotates two teeth relative to the grounded circular spline. The appearance of harmonics varies from test to test. Typically, harmonics are apparent at even multiples of the input revolution frequency. The even components correspond to the harmonic nature of the kinematic error signal and are the second, third, fifth, etc., harmonics. In Figure 4.8, the next harmonic clearly visible is that at six cycles per input revolution. Other peaks are present at eight and ten cycles per input revolution. There is a peak at four cycles per input revolution, but the scale of the figure makes it appear very small. These are the harmonics of the kinematic error. Also note that there are other peaks at one, three, and five cycles per input revolution. These are most likely due to friction in the system that is encountered once per input revolution. The amplitude of the frequency components remained relatively constant throughout the testing procedure. Velocity also had little effect on the magnitude of the kinematic error waveform.

The validation of the apparatus was determined through repeated tests. These tests indicate that the kinematic error is repeatable for several tests at the same rotational velocity (input torque of 0.20 N-m) and for tests at different velocities as well (at an input torque ranging from 0.20 → 0.50 N-m). The results were also compared to previous results published in [26], and while there are small differences, the results of the experimental apparatus agree with the published kinematic error waveforms.
It should be noted that small differences were present in different experiments. The waveform changed slightly from test to test in all cases. These variations may be attributed to the dynamics of the experimental system. Starting position had no effect on the kinematic error since tests were started both from defined and random positions.

Another source of variation is due to the disassembly and subsequent reassembly of the drive. If alignment is not perfect, binding may occur and the harmonic drive will require a greater torque in order to initiate motion. This phenomenon was experienced during the testing procedure. The harmonic drive test apparatus was disassembled in order to tighten a loose component. When reassembled, the motor torque required to start motion was significantly higher than previously experienced (0.50 N-m as compared to 0.20 N-m). After disassembling the drive, it was observed that the flexible spline had shifted during final assembly and that the wave-generator had been pushed an excessive distance into the flex spline. This misalignment caused the teeth of the circular spline and flexible spline to lock together, preventing movement. This condition has been described in literature and has been identified as "high pre-loading" [26]. Realignment and reassembly produced the results previously observed for satisfactory alignment. Thus, the alignment of the various components of the harmonic drive must be meticulously maintained. The slightest variation could induce incorrect or destructive results. Furthermore, imprecise assembly could add to the friction already present in the system. As will be discussed later, the friction in the system must be kept to a minimum in order for control of kinematic error to be achieved.
Chapter 5

Control of Kinematic Error

To the best of our knowledge, control of the kinematic error present in harmonic drives has not been addressed in literature except in [15]. However, model based control of kinematic error has been addressed only in [28], where the regulation of kinematic error in a harmonic drive system consisting of a motor, a harmonic drive, and an inertial load has been discussed. In this research, it has been shown that for regulation (i.e., set-point control) PD (proportional-derivative) compensation with both motor state and load state feedback will produce asymptotic stability of the equilibrium. Detailed proofs of these statements may be found in [28] as well. Furthermore, the equations of motion for the harmonic drive system in terms of motor side and load side parameters were developed [28]. The following chapter will discuss the theory and implementation behind the control effort. The experimental results for the PD control of the system will then be presented and discussed in detail. The chapter will begin with a brief introduction to static friction in the system.

5.1 Static Friction in the Harmonic Drive Test Apparatus

Friction is a difficult parameter to identify in any system. And, no matter how well a system is designed, friction will almost always play a role. This is the case with the harmonic drive test system. Typically there are two types of friction that must be considered. The first is the static friction which must be overcome when initiating motion. The second is known as dynamic friction. This type of friction is a function of the velocity of the system. Both of these types of friction have been identified as
playing a significant role in harmonic drive systems [26]. As indicated in the experimental results of the previous chapter, friction is a force that must be compensated for in order to produce acceptable results. The experiment made use of a simple model for static friction in the system. The source of the static friction is discussed below.

In the control implementations, a simple model was used to calculate the static friction in the system. The static friction in the system is due to the contact between the many rotating components. As an example, the bearings in the motor, the wave-generator, and in the drive train require some force to initiate motion. Once they start rotating, the required force decreases. Note, however, that this friction force may vary from experiment to experiment as both position and lubrication states change. Therefore, due to the many aberrations that may cause the friction to change, a simple approximation was made to model the friction. The torque input to the system was slowly increased until motion was initiated. This value for the torque was then added to the PD regulated torque input equation. This method is approximate and, as seen in the experimental phase of the project, is not always accurate. An example is the fact that the friction present in the harmonic drive test apparatus is not constant within the 360° rotational domain. This variation in the static friction makes the motor state feedback effort even more difficult. The difficulty arises because the torque required to escape the static friction then produces high velocities on the motor side and induces overshoot. The effect of the static friction was not noticed as much in the load state feedback. Any effects were compensated for by the gear reduction (and the subsequent reduction in velocity), the feedback of the load position and velocity, and the fact that the system dynamics were associated to the control effort. The friction model described above was adopted primarily because we
are only rotating in one direction and are not concerned with the behavior of the static friction as we approach or pass through zero velocity. There are sources in literature which address this issue and compensate for the friction without imposing discontinuity at zero velocity.

5.2 Motor State Feedback

The equation of motion in terms of the motor variables is

\[
\begin{bmatrix}
J_m + Y^2(\dot{\theta}_m)J_L \\
B_m + Y^2(\dot{\theta}_m)B_L
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_m \\
\dot{\theta}_m
\end{bmatrix}
+ \begin{bmatrix}
J_LY(\dot{\theta}_m) & J_LY(\dot{\theta}_m)
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_m \\
\dot{\theta}_m
\end{bmatrix}
= \tau_m.
\] (5.1)

The above equation is defined by the terminology presented in Table 5.1. The application of motor state feedback to the system is graphically illustrated in Figure 5.1. In this type of feedback control, kinematic error is ignored and the desired position for the motor, \(\dot{\theta}_m^d\), is defined as a function of \(\theta_L\) such that \(\dot{\theta}_m^d = N\dot{\theta}_L^d\). In this manner, the system is considered to be ideal and the desired motor position is a constant (the gear reduction ratio, \(N\)) multiplied by the desired load position. From this point of view, the following theorem is defined.

<table>
<thead>
<tr>
<th>Term</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(J_m, J_L)</td>
<td>Motor and Load Side Inertias</td>
</tr>
<tr>
<td>(B_m, B_L)</td>
<td>Motor and Load Side Damping Terms</td>
</tr>
<tr>
<td>(\theta_m, \theta_L)</td>
<td>Motor and Load Side Positions</td>
</tr>
<tr>
<td>(\dot{\theta}_m, \dot{\theta}_L)</td>
<td>Motor and Load Side Velocities</td>
</tr>
<tr>
<td>(\ddot{\theta}_m, \ddot{\theta}_L)</td>
<td>Motor and Load Side Accelerations</td>
</tr>
<tr>
<td>(\tau_m)</td>
<td>Motor Torque</td>
</tr>
<tr>
<td>(\dot{\theta})</td>
<td>Kinematic Error (= \frac{\theta_m}{N} - \theta_L)</td>
</tr>
<tr>
<td>(\ddot{\theta}_m)</td>
<td>(\frac{d}{d\theta_m}(\dot{\theta}_m))</td>
</tr>
<tr>
<td>(\dddot{\theta}_m)</td>
<td>(\frac{d}{dt}(\ddot{\theta}_m))</td>
</tr>
<tr>
<td>(Y(\ddot{\theta}_m))</td>
<td>(\frac{1}{N} + \ddot{\theta}_m)</td>
</tr>
</tbody>
</table>

Table 5.1: Nomenclature for the System Equations of Motion
Figure 5.1: Motor State Feedback PD Control Schematic

Theorem 5.1  Given the equation of motion in terms of motor variables presented in Equation (5.1), let the control input to the dynamic system be given by the following PD controller

\[ \tau_m = K_p \theta_m^e - K_D \dot{\theta}_m \]  \hspace{1cm} (5.2)

where \( \theta_m^e = \theta_m^d - \theta_m \) is the error between the constant desired motor position \( \theta_m^d \) and the actual motor displacement \( \theta_m \). If PD control is implemented in this fashion, then the motor will be driven to the desired position such that \( (\theta_m^d - \theta_m) \rightarrow 0 \) and \( \dot{\theta}_m \rightarrow 0 \) as \( t \rightarrow \infty \).

The type of motor state feedback described above is typically adopted in industrial drive systems because of the ease of implementation. Typically, motors are purchased with built-in position sensors such that this type of feedback requires little effort, expense, or modification. However, the load will not necessarily achieve the desired position because the kinematic error that exists between the motor and load states will not be compensated. If we only feed back the motor variables, we ignore the kinematic error that is present between the motor and load in the harmonic drive such that this scheme only controls the motor state. The proof of this theorem is presented in [28].
5.2.1 Implementation Issues

The implementation of the control scheme described in Equation (5.2) requires no model calculations since the feedback will be based upon the motor position $\theta_m$ and motor velocity $\dot{\theta}_m$ alone. The experiment will incorporate a simple compensation for the static friction present in the system. The experiment has been designed so that no reversal of direction will occur such that friction will not change signs and complicate the simple model. Thus, the motion in the experiment will be limited to

$$0^\circ \leq N\theta^d_m \leq 90^\circ .$$

The friction compensation consists of the addition of a constant torque, $F_s$, determined by experiment to equal the static friction in the system. The control input becomes

$$\tau_m = K_p\theta^e_m - K_D\dot{\theta}_m + F_s .$$  \hspace{1cm} (5.3)

An effort will be made to avoid the excitation of flexibility in the system since this nonlinear effect has not been included in the model. Furthermore, the torque input into the system will be held within the range of

$$-1 \ \text{Nm} \leq \tau_m \leq 1 \ \text{Nm} .$$

Torques in this range will not violate the parameters defined in the ST1 Controller or in the software. Thus the control law is defined and may be implemented in the dSPACE software interrupt routine. The gains $K_p$ and $K_D$ may be tuned to meet the needs of the system dynamics and the theorem is valid as long as the gains remain positive.

Experimental Control Results

The equation necessary for the application of PD control with motor state feedback, Equation (5.2), was programmed into a dSPACE software routine. For the motor
state feedback experiment, \( N\theta_m^d \) is defined to be 67.5°. The results are shown below in Figure 5.2. The results seem to indicate that both variables are driven to the

![Graph showing motor position and load position over time](image)

**Figure 5.2:** Motor State Feedback PD Control with Static Friction Compensation

desired position. However, a closer examination disproves this statement. Figure 5.3 describes the steady state error in both the motor and load positions after the motor position has been driven towards the desired position and Figure 5.3(a) indicates that the motor position goes to zero for the given torque input. The actual value for the motor position error is \(-0.165^\circ\). The negative sign in the value indicates that there was a small overshoot rather than an undershoot since \( \theta_m^e = \theta_m^d - \theta_m \) and \( \theta_m > \theta_m^d \) in this case. This value is zero to within the accuracy of the Sumtak encoder that mea-
Figure 5.3: Expanded View of the Steady State Error in the Motor and Load Positions after Motor State Feedback

sures motor position and whose resolution is 0.180°. It can be seen in Figure 5.3(b) that the load position error is $-0.0223^\circ$. This value cannot be accepted as zero, however, since the Canon encoder that measures load position has a higher resolution of 0.007°. So, the use of motor state feedback will not drive the load to the desired angular position, in this case 67.5°. This result was predicted by the theorem, shown in the simulations developed in [28], and now proven experimentally.

Figure 5.4(a) shows the kinematic error present in the system during the step while Figure 5.4(b) shows the torque required to complete the movement. It is evident
that when the desired position is attained, the torque is maintained at a constant level due to the effects of static friction. Note that the torque plot indicates that a required torque of over 1.0 N-m was computed by the control law. This is only the computed torque for achieving the desired position, but the motor torque, as mentioned previously, has been limited in the software to ±1.0 N-m. The kinematic error also displays predictable trends as it pulses rapidly at first due to the high velocity. The error then settles to a constant value of −0.0257° as the velocities of the motor and load sides go to zero. An interesting check may be made with the kinematic error. In theory, the difference between the motor position and load position is the
kinematic error. Furthermore, the difference between the load position error and the motor position error is also the kinematic error. The relationship between the load position error, $\theta^e_L$, and the motor position error, $\theta^e_m$, is

$$\frac{\theta^e_m}{N} - \theta^e_L = \frac{0.165}{50} - (-0.0223) = -0.0257$$

which is exactly the observed kinematic error as measured experimentally. The results for the motor state feedback control of the harmonic drive system are not satisfactory. The poor results are predictable since the PD control only takes into account the motor position in the feedback loop. The kinematic error is between the motor side and the load, and no compensation is made for this error component. Furthermore, friction plays a significant role in the motor state feedback control effort.

5.3 Load State Feedback

The equation of motion in terms of the load variables is

$$\begin{bmatrix} \frac{J_m}{Y^2(\dot{\theta}_m)} + J_L \end{bmatrix} \ddot{\theta}_L + \begin{bmatrix} \frac{B_m}{Y^2(\dot{\theta}_m)} + B_L \end{bmatrix} \dot{\theta}_L - \frac{J_m}{Y^3(\dot{\theta}_m)} \ddot{\theta}_m \dot{\theta}_L = \frac{\tau_m}{Y(\dot{\theta}_m)}$$ (5.4)

The nomenclature for Equation (5.4) is shown in Table 5.1 of the previous section. A schematic diagram for the load state feedback experiment is shown in Figure 5.5.

![Figure 5.5: Load State Feedback PD Control Schematic](image-url)
Theorem 5.2  Given the equation of motion in terms of load variables presented in Equation (5.4), let the control input to the dynamic system be given by the following augmented PD controller

\[
\tau_m = Y(\dot{\hat{\theta}}_m) \left( K_p \theta^e_L - K_D \dot{\theta}_L \right) = \left( \frac{1}{N} + \hat{\theta}_m \right) \left( K_p \theta^e_L - K_D \dot{\theta}_L \right) \tag{5.5}
\]

where \( \theta^e_L \) is defined as \( \theta^d_L - \theta_L \). If the PD control is implemented in this fashion, then the load will be driven to the desired position such that \( (\theta^d_L - \theta_L) \to 0 \) and \( \dot{\theta}_L \to 0 \) as \( t \to \infty \).

The system input in Equation (5.5) is a function of the kinematic error since \( Y(\dot{\hat{\theta}}_m) \) is defined in as:

\[
Y(\dot{\hat{\theta}}_m) = \left( \frac{1}{N} + \hat{\theta}_m \right) \tag{5.6}
\]

and \( \hat{\theta}_m = \frac{d}{d\hat{\theta}_m}(\ddot{\theta}) \). In the feedback design, the motor will not achieve a desired position. This is not crucial since we wish to control the load position. We are not interested in this case in the final position of the motor side as long as the load state objective is attained. Control in this manner can successfully compensate for the kinematic error since the load state is observed. The PD control with respect to load state feedback is now a closed loop control effort, unlike the motor state feedback case in which the dynamics relating \( \theta_m \) to \( \theta_L \) are open loop.

5.3.1 Implementation Issues

The implementation of the control scheme described in Equation (5.5) requires a knowledge of the kinematic error since the control input is multiplied by \( Y(\dot{\hat{\theta}}_m) = \frac{1}{N} + \hat{\theta}_m \). Therefore, in order for the control expression to be nonsingular, the value of \( Y(\dot{\hat{\theta}}_m) \) must never be zero. The following argument will show that \( Y(\dot{\hat{\theta}}_m) \) is never zero.
Fact 5.3.1 For all harmonic drives,

\[
\frac{1}{N} > \bar{\theta}_m \tag{5.7}
\]

so that \( Y(\bar{\theta}_m) \neq 0 \).

The argument for Equation (5.7) comes from existing harmonic drive literature. From literature, the kinematic error may be modeled without loss of generality and for the sake of this discussion with the following equation. Since the predominant frequency driving the kinematic error occurs at twice the input rotational frequency, \( \bar{\theta} = \bar{\theta}(2\theta_m) \):

\[
\bar{\theta} = A \sin(2\theta_m) \tag{5.8}
\]

Now, the harmonic drive catalog [6] lists the maximum positional error for different harmonic drive sizes. The maximum error listed is for the smallest harmonic drive and is \( \pm 150 \) arc-seconds. We will consider this "worst-case" to show that \( Y(\bar{\theta}_m) \) is always positive.

Recall from Table 5.1 that \( \bar{\theta}_m = \frac{d\bar{\theta}}{d\theta_m} \), so it follows from Equation (5.8) that

\[
\bar{\theta}_m = 2A \cos(2\theta_m) \tag{5.9}
\]

The maximum of this function occurs when \( 2\theta_m = \pm n\pi \) degrees such that

\[
|\bar{\theta}_m|_{\text{max}} = 2A \tag{5.10}
\]

From Equation (5.8), we see that this value corresponds to \( 2|\bar{\theta}|_{\text{max}} \) such that

\[
|\bar{\theta}|_{\text{max}} = A = \frac{|\bar{\theta}_m|_{\text{max}}}{2} \tag{5.11}
\]

Therefore, we may assume that \( |\bar{\theta}_m|_{\text{max}} = 2|\bar{\theta}|_{\text{max}} = \pm 300 \) arc-seconds. This quantity corresponds to \( \pm 0.001454 \) radians. Using this value in the equation for \( Y(\bar{\theta}_m) \), we see
that

\[ Y(\hat{\theta}_m) = \left( \frac{1}{N} - \hat{\theta}_m \right) \]
\[ = \left( \frac{1}{N} - 0.001454 \right). \]

(5.12)

In the worst-case for Equation (5.12), the gear reduction ratio will be the maximum available for harmonic drive gear reducers. At present, the maximum reduction ratio is 320:1 [6]. Using this value in Equation (5.12) yields

\[ Y(\hat{\theta}_m) = \frac{1}{320} - 0.001454 = 0.003125 - 0.001454 = 0.001671 > 0. \]

(5.13)

Thus, for the maximum value of \( \hat{\theta}_m \), \( Y(\hat{\theta}_m) \) is always positive.

Another estimation for the maximum kinematic error is presented in [15]. Here,

\[ \hat{\theta}_{max} = \frac{8}{d_m}. \]

(5.14)

where the maximum error in arc-minutes, \( \hat{\theta}_{max} \), is defined as a function of the pitch diameter of the circular spline, \( d_m \), in inches. Again, the maximum error occurs in the smallest harmonic drive. From the catalog, the pitch diameter of the smallest harmonic drive is 1.03 inches. Therefore,

\[ \hat{\theta}_{max} = \frac{8}{d_m} = \frac{8}{1.03} = 7.767. \]

(5.15)

This value is in arc-minutes and corresponds to 0.0011297 radians. If we use this value in Equation (5.12), and select the 320:1 reduction ratio again,

\[ Y(\hat{\theta}_m) = 0.003125 - 0.0011297 = 0.001995 > 0. \]

(5.16)

These results from practical data available in existing literature indicate that Fact 1 is valid. So, \( Y(\hat{\theta}_m) \) is never zero and may be used in the PD control scheme without compromising the validity of the control law. However, a problem arises in the practical implementation of Equation (5.5). A valid means for calculating \( \hat{\theta}_m \) at each sampling period must be derived.
Computation of $\ddot{\theta}_m$

$\ddot{\theta}_m$ may be computed at each sampling period as

$$\frac{\Delta \theta_L}{\Delta \theta_m}$$

where $\Delta \theta_L = \theta_L(t) - \theta_L(t - T)$ and $\Delta \theta_m$ is similar with $T$ being equal to the sampling period. Recall that

$$\ddot{\theta} = \frac{1}{N} \theta_m - \theta_L$$

where $\ddot{\theta}$ is the kinematic error as a function of the input (motor) position, $\theta_m$, the output (load) position, $\theta_L$, and the gear reduction ratio, $N$. Taking the derivative of $\ddot{\theta}$ with respect to $\theta_m$ produces the following relationship:

$$\frac{d\ddot{\theta}}{d\theta_m} = \frac{1}{N} - \frac{d\theta_L}{d\theta_m}$$

such that

$$\ddot{\theta}_m = \frac{1}{N} - \frac{d\theta_L}{d\theta_m}.$$  \hspace{1cm} (5.19)

This, in experimental terms, may be approximated as

$$\ddot{\theta}_m \approx \frac{1}{N} - \frac{\Delta \theta_L}{\Delta \theta_m}$$

and the quantity

$$\frac{\Delta \theta_L}{\Delta \theta_m}$$

may be measured experimentally by comparing the variation in the motor and load positions between sampling periods.

Singularity issues arise when $\Delta \theta_L$ and $\Delta \theta_m$ approach zero. However, when velocities go to zero, $\ddot{\theta}_m$ is zero because the kinematic error, $\ddot{\theta}$, is a constant. Because of this, when the velocity goes to zero, Equation (5.20) implies that

$$\frac{\Delta \theta_L}{\Delta \theta_m} = \frac{1}{N}$$

(5.22)
as $\Delta \theta_L$ and $\Delta \theta_m \to 0$. This is valid through both examination of the equations and from experimental results.

The results, therefore, permit the practical application of the PD control load state feedback algorithm to the experimental apparatus. The experiment will incorporate the same simple compensation for the static friction that was used in the motor state feedback case. Again, the experiment has been designed so that no reversal of direction will occur such that friction will not change signs and produce complications in the simple model. Thus, the motion in the experiment will be limited to

$$0^\circ \leq \theta_L^d \leq 90^\circ.$$

The friction compensation consists of the addition of a constant torque, $F_s$, determined by experiment to equal the static friction in the system. The control input becomes

$$\tau_m = Y(\hat{\theta}_m) \left( K_p \theta_L^d - K_D \hat{\theta}_L \right) + F_s.$$  \hspace{1cm} (5.23)

An effort will be made to avoid the excitation of flexibility in the system since this nonlinear effect has not been included in the model. Furthermore, the torque input into the system will again be held within the range of

$$-1 \text{ Nm} \leq \tau_m \leq 1 \text{ Nm}.$$

Thus the control law is defined and may be implemented in the dSPACE software interrupt routine. The gains $K_p$ and $K_D$ are approximate and may be tuned to meet the needs of the system dynamics.

**Experimental Control Results**

The load state feedback issue is much more complicated than the motor state feedback. The computation of $\hat{\theta}_m$ is a difficult problem to overcome since the numerical
differentiation required is inherently discontinuous. These discontinuities lead to singularities in the control algorithm since, as shown previously, the control input is a function related to the kinematic error. However, the value for \( \dot{\theta}_m \) must be calculated since it is the term in the load state feedback equation which will compensate for the kinematic error. Otherwise, the PD control implementation would ignore kinematic error in the same way that the motor state feedback case did and there would be no way to eliminate the steady state error in the load position. Therefore, the software was tailored to compute \( \dot{\theta}_m \) and to smooth any singularities that would induce sharp peaks in the torque input. The results for the load state feedback are shown in Figure 5.6. Furthermore, the motor and load positional errors are shown in Figure 5.7. As can be seen in Figure 5.7(a), the motor position error for the load state feedback case is 0.736084\(^\circ\). This is insignificant, however, since we are interested in driving the load position to the \( \theta^d_L \), not the motor position to \( \theta^d_m = N\theta^d_L \). The load position error, as shown in Figure 5.7(b), is −0.002731\(^\circ\). This is zero since the resolution of the Canon encoder is 0.007\(^\circ\). Furthermore, it may be seen in Figure 5.8(a) that the steady state value for the kinematic error is −0.0175\(^\circ\). In this case, the relationship between the load position error, \( \theta^e_L \), and the motor position error, \( \theta^e_m \), is

\[
\frac{\theta^e_m}{N} - \theta^e_L = \frac{0.736084}{50} - (-0.002731) = -0.0175
\]

which is exactly equal to the computed kinematic error shown in Figure 5.8. Note that the load state position error is much less than the kinematic error because the \( \dot{\theta}_m \) term in the load state feedback equation successfully compensates for the nonlinear oscillation. The experimental plot of \( \dot{\theta}_m \) is shown in Figure 5.8(c) and is expanded in Figure 5.9. It is obvious from Figures 5.8(c) and 5.9 that rapid fluctuations occur because of the approaching zero velocity condition and the subsequent singularity condition described in Fact 2. The function used in the software to represent \( \dot{\theta}_m \) is not continuous, as mentioned previously, and these oscillations have been common to
all of the load state feedback experiments. It has been mentioned previously that this condition could bring a chattering phenomenon into the torque equation if proper properties of the value for $\hat{\theta}_m$ present in the previous arguments are not exploited. Also note that the sudden fluctuations in $\hat{\theta}_m$ produce a small oscillation in the required torque, but the magnitude of the fluctuations in $\hat{\theta}_m$ are so small that the torque fluctuation is negligible. The results would have been much more noticeable if the numerical singularities in $\hat{\theta}_m$ had not been smoothed in the software application. An initial concern was that too much filtering of $\hat{\theta}_m$ would prevent the load state feedback control from properly compensating for the kinematic error. However, the
results have shown that the methods used in the software are valid.

Thus, the load state feedback approach based on Equation (5.23) successfully compensates for the kinematic error present in harmonic drive gear reducers. The arguments presented previously also indicate that the PD control of the system achieves asymptotic stability of the equilibrium. The importance of the kinematic error to the success of the load state feedback algorithm cannot be emphasized enough. In an isolated test, the load state feedback equation was modified such that $Y(\dot{\theta}_m)$ was approximated as $\frac{1}{N}$, which is equivalent to ignoring kinematic error. The steady state
Figure 5.8: Kinematic Error, Torque, and $\tilde{\theta}_m$ Plots for Load State Feedback

errors in the load state and motor state positions for this experiment are shown in Figure 5.10. The results were poor because the removal of $\tilde{\theta}_m$ from the control equation effectively removed all compensation for the kinematic error. The steady state error on the load side is $-0.3280^\circ$. The steady state error on the motor side and the kinematic error are $-15.3250^\circ$ and $-0.0215^\circ$, respectively. It is obvious that the desired condition of $\theta^d_L - \theta_L = 0$ has not been achieved.
5.4 Conclusions

The experimental data collected for the PD compensation with motor state feedback confirms the theory that compensation for kinematic error is necessary for the load to reach the desired position, $\theta^d_L$. The motor state feedback does not compensate for the kinematic error, and, as $t \to \infty$, $\theta^d_L - \theta_L \not= 0$. This fact is demonstrated with the application of the PD regulation with load state feedback to the system. The results for this case indicate that the compensation for the kinematic error through the inclusion of the $Y(\dot{\theta}_m)$ term in the PD control expression succeeds in achieving the desired response, namely that $\theta^d_L - \theta_L \to 0$ and $\dot{\theta}_L \to 0$ as $t \to \infty$. The impor-
Figure 5.10: Expanded View of the Steady State Error in the Motor and Load Positions after Load State Feedback with $Y(\hat{\theta}_m)$ Approximated as $\frac{1}{N}$


tance of the presence of kinematic error in the load state case was shown when $Y(\hat{\theta}_m)$ was approximated as $\frac{1}{N}$. In this case, the kinematic error was again ignored and the results were not the desired.

A final observation of interest comes when we examine the value $Y(\hat{\theta}_m)$. The control law for load state feedback is given in Equation (5.5). If we substitute for $\hat{\theta}_m$, the
equation becomes

\[ \tau_m = \left( \frac{2}{N} - \frac{\Delta \theta_L}{\Delta \theta_m} \right) \left( K_p \theta_L^0 + K_D \theta_L \right) \]  

(5.24)

Since \( \theta_L \) and \( \theta_m \) are related by the gear reduction ratio, \( Y(\tilde{\theta}_m) \approx \frac{1}{N} \) because

\[ \frac{\Delta \theta_L}{\Delta \theta_m} \approx \frac{1}{N} \]  

(5.25)

This result implies that \( \frac{\Delta \theta_L}{\Delta \theta_m} \) is similar to a gear reduction ratio that fluctuates about the constant value \( \frac{1}{N} \). This has been observed in literature where the kinematic error has been attributed to fluctuations in the gear reduction ratio due to the elliptical geometry of the wave-generator [10]. The previous results have shown that this dynamic modification of the gear reduction ratio is necessary if desired load state position is to be attained.
Chapter 6

Comparison of Experimental and Simulated Results for the PD Control of Kinematic Error

As mentioned previously in the text, a simulation for a harmonic drive system has been developed in [28]. The simulation has been based on the experimental system consisting of the motor, the harmonic drive, the sensors and couplings, and the inertial load and makes use of all of the experimental parameters. The following discussion will compare the experimental data to the simulation in order to evaluate similarities and differences. The simulation has been modified from [28] so that a simple static friction model may be included. First, however, the simulation equations and the modeling of kinematic error will be addressed. The equation used in the simulation for motor state feedback is given by Equation (6.1) below

\[
\left[ J_m + Y^2(\dot{\theta}_m)J_L \right] \ddot{\theta}_m + \left[ B_m + Y^2(\dot{\theta}_m)B_L \right] \dot{\theta}_m + J_L Y(\dot{\theta}_m) \ddot{\theta}_m \dot{\theta}_m = \tau_m \quad (6.1)
\]

and the computed torque is

\[
\tau_m = K_p \theta_m^e - K_D \dot{\theta}_m + F_s. \quad (6.2)
\]

The load state feedback equation is Equation (6.3)

\[
\left[ \frac{J_m}{Y^2(\dot{\theta}_m)} + J_L \right] \ddot{\theta}_L + \left[ \frac{B_m}{Y^2(\dot{\theta}_m)} + B_L \right] \dot{\theta}_L - J_{in} \frac{\dot{\theta}_m \ddot{\theta}_L}{Y(\dot{\theta}_m)} = \frac{\tau_m}{Y(\dot{\theta}_m)} \quad (6.3)
\]

and the computed torque in this case is

\[
\tau_m = Y(\dot{\theta}_m) \left( K_p \theta_L^e - K_D \dot{\theta}_L \right) = \left( \frac{1}{N} + \ddot{\theta}_m \right) \left( K_p \theta_L^e - K_D \dot{\theta}_L \right) + F_s. \quad (6.4)
\]
The kinematic error in the simulation is computed as

\[
\hat{\theta} = A_1 \sin(\theta_m) + A_2 \sin(2\theta_m) + A_3 \sin(3\theta_m) + A_4 \sin(4\theta_m) \\
+ A_5 \sin(6\theta_m) + A_6 \sin(8\theta_m)
\]

which makes use of the two dominant frequencies identified to be the prime contributors to the kinematic error in harmonic drive systems (those components at a frequency of 2\(\theta_m\) and 4\(\theta_m\)) as well as four additional terms. This model is a reasonable approximation of the kinematic error seen in harmonic drives. The model is essentially a trigonometric approximation composed of only six terms. The formal definition of the trigonometric approximation will be discussed later in the section.

The magnitude gains were computed in the following manner. An FFT of the experimental data was performed using Matlab. The result produced a complex vector containing the information relating to the magnitude of the error. This vector was multiplied by the conjugate such that a real valued magnitude vector was generated. These magnitudes were modified because multiplication by the complex conjugate produces a vector whose values are the sum of the squares of the real and imaginary components. So, the actual magnitudes should be equal to the square root of the values. The results are the peak-to-peak amplitudes of the kinematic error waveform. The values were divided by two to produce the half-amplitudes that are to be multiplied by the \textit{sine} functions. The results of this process produced the gains that are used in the simulation and are shown in Table 6. The model computed in this form produces the results seen in Figure 6.1. As may be seen, the model follows the general trends in the error waveform. Of particular importance is the approximation of the trends occurring at twice and four times the motor rotation frequency. The magnitude of the component at twice the input rotational frequency, (2\(\theta_m\)), in particular, comprises the majority of the kinematic error signature as the magnitude of this component is an order of magnitude larger than any other contributor. This was shown
<table>
<thead>
<tr>
<th>Amplitude</th>
<th>Gain Value (deg)</th>
</tr>
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<tbody>
<tr>
<td>$A_1$</td>
<td>0.0015</td>
</tr>
<tr>
<td>$A_2$</td>
<td>0.0180</td>
</tr>
<tr>
<td>$A_3$</td>
<td>0.0015</td>
</tr>
<tr>
<td>$A_4$</td>
<td>0.0050</td>
</tr>
<tr>
<td>$A_5$</td>
<td>0.0040</td>
</tr>
<tr>
<td>$A_6$</td>
<td>0.0020</td>
</tr>
</tbody>
</table>

Table 6.1: Amplitude Gains Used in the Kinematic Error Simulation

previously in Figure 4.8. Obviously, the higher frequency terms are ignored, but the model is reasonable since it reproduces the most important trends and permits the comparison between simulation and experiment.

6.1 Development of a Complete Model for Kinematic Error

The kinematic error as described in Equation (6.5) is in the form used in literature to model the kinematic error. It is interesting to determine if a model for the kinematic error observed in this specific system can be developed. This curve fit would preferably be made with a harmonic series as used in previous literature. Research indicates that there are several ways to do this. One approach makes use of the describing function method [4]. This is approach is similar to the approach used in [26]. Here, the basis function, $N(\theta_m, \phi)$, is defined as

$$N(2\theta_m, \phi_1) = A_1 \sin(2\theta_m + \phi_1)$$

(6.6)

where the primary frequency component is the dominant kinematic error contributor and the $\phi_1$ accounts for the phase of the waveform. The rest of the components may be summed into a residual term that will contain the $N(4\theta_m, \phi_2), N(6\theta_m, \phi_3)$, etc. components (i.e., the higher frequency terms). This is actually a hybrid form of the describing function method since, typically, the input to the describing function is to
be sinusoidal. In this case, our input is a linearly increasing function of the motor position, $\theta_m$. Other options were explored as well.

A second way to model the kinematic error is through the use of a trigonometric approximation, typically referred to as a discrete Fourier expansion [19, 22]. This type of model is well suited for a periodic function such as the kinematic error. In this method, the kinematic error waveform data, $y(x)$, is approximated by a series of sines and cosines as

$$y(x) = \frac{a_0}{2} + \sum_{k=1}^{L} \left( a_k \cos \frac{2\pi}{2L+1} kx + b_k \sin \frac{2\pi}{2L+1} kx \right)$$  \hspace{1cm} (6.7)
where

\[ a_k = \frac{2}{2L + 1} \sum_{x=0}^{2L} y(x) \cos \frac{2\pi}{2L + 1} kx, \quad k = 0, 1, \ldots, L \quad (6.8) \]

\[ b_k = \frac{2}{2L + 1} \sum_{x=0}^{2L} y(x) \sin \frac{2\pi}{2L + 1} kx, \quad k = 1, 2, \ldots, L \quad (6.9) \]

such that \( L \) is the number of data points in one period \( P \) of the signal to be modeled. A simplification may be made because the kinematic error appears to be an odd function such that \( y(-x) = -y(x) \) as in \( \sin(x) \). The simplification exploits symmetry and the fact that, for an odd function, the cosine terms in Equation (6.7) will vanish. The equations in this form are given as

\[ y(x) = \sum_{k=1}^{L-1} b_k \sin \frac{\pi}{L} kx \quad (6.10) \]

where \( b_k \) is now defined as

\[ b_k = \sum_{x=1}^{L-1} y(x) \sin \frac{\pi}{L} kx, \quad k = 0, 1, \ldots, L - 1. \quad (6.11) \]

The period of the kinematic error has been defined in literature and in this research as one complete input (motor shaft) revolution \((\theta_m = 0 \rightarrow 2\pi)\). In the experimental data, one input revolution is composed of 788 data points. The method employed will fit one period of the data such that this one period may be reproduced to fit the entire curve. Thus, we have \( L - 1 \) coefficients \((b_k)\) and \( L - 1 \) terms to approximate the odd, periodic function. The results for the 788 point model are shown in Figure 6.2.

In the figures that follow, the top figure provides a view of the fit of the model to the sampled period while the bottom figure is a view as the modeled period is repeated over the entire waveform. The modeled curve is a perfect fit for the sample period and a close fit for the remainder of the kinematic error waveform. The variation is to be expected unless we were to construct a complete 5001 point model for the entire waveform. This is impractical since slight discrepancies are expected due to
Figure 6.2: Comparison Between Experimental Data (Solid) and the 788 Point Model (Dashed) for the Kinematic Error Waveform

slight system variations and sampling properties. The model is accurate but large. Figure 6.3 and Figure 6.4 represent models consisting of 23 and 5 terms, respectively.

The 23 point model is still very close to both the sampled period and the entire waveform. This was expected because the amplitudes of the first 23 terms were the largest components of the approximation. However, a 23 point model is still large. Therefore, the 5 point model was developed to gain an understanding of how typical kinematic error models follow the data. As can be seen, the curve follows the trends of the error while not accounting for all of the higher frequency components. This is still a reasonable model, however, because the dominant frequency of two cycles per input revolution is clearly seen. However, if a more complete model is available for use
Figure 6.3: Comparison Between Experimental Data (Solid) and the 23 Point Model (Dashed) for the Kinematic Error Waveform

in the simulation, the confidence in the results generated would be higher. Because of this, a new model for the kinematic error is being developed for implementation in the simulation. The new model is based upon the 788 point model described above and will be integrated into the simulation program. The integration is an issue as the coefficients generated for the 788 point model are most readily acquired from a table accessed at each step of the numerical integration. This may require the production of a new simulation program in C since array access is much easier in the format than in integration schemes available in programs such as Matlab. This simulation is a definite course for future research. Therefore, the simulated results that are
compared to the experiment in the following section are based upon Equation (6.5). With this established, the results of the comparison may now be observed.

6.2 Comparison of the Results for Motor State Feedback

The comparison between experiment and simulation for the PD control with motor state feedback is shown in Figure 6.5. The motor and load position curves show similar trends. This is a good result since the model does not take into account dynamic friction or compliance. The difference between the two curves is most likely due to these factors. The kinematic error traces are similar in trend as well. The experiments have shown, however, that the amplitude of the frequency components
Figure 6.5: Comparison Between Experiment (Solid) and Simulation (Dashed) for the Motor State Feedback Experiment

of the kinematic error seem to show the effects of flexibility in the system. As the velocity decreases, the relative magnitude of the kinematic error decreases as well. This is evident from the plot of the experimental data in Figure 6.5 and accounts for the differences between curves as time increases. The cause of this phenomenon is most likely due to the increased torsion seen in the flexible spline at higher velocities. This trend will be discussed in a later section.

The comparison of the torque curves for the motor state feedback experiment is presented in Figure 6.6. The curves are similar in trend. However, definite differences
**Figure 6.6:** Comparison Between Experimental (Solid) and Simulated (Dashed) Torques for the Motor State Feedback Experiment

...do exist. The simulated torque decreases at a more gradual rate than that in the experiment and exceeds the level required in experiment. These results are partially due to the fact that different gains were used in the simulation and in the experiment. The simulation was tuned to match the position response curves as closely as possible. This discrepancy is directly related to the fact the simulation does not compensate for dynamic friction or flexibility. Both of the curves settle to a steady state value equal to the static friction present in the system. The system slows and stops as it passes into the region dominated by static friction as no external braking is applied.

Figure 6.7 describes the behavior of the motor and load position errors with increasing time. Figure 6.8 provides an expanded view of the region near zero as the curves approach. As seen in Figure 6.8, the motor position error goes to zero for both...
Figure 6.7: Comparison Between Experimental (Solid) and Simulated (Dashed) Motor and Load Position Errors for the Motor State Feedback Experiment

the simulation and the experiment while the load position errors are finite. Recall that a "zero" motor position for the experiment is a position less than 0.20° (the limit of resolution for the motor side encoder). The load position for the experiment is nonzero since the resolution of the output encoder is 0.007°. Obviously, the simulation does not have such limitations and it is therefore possible to observe a zero motor position error for the motor state feedback case. These results re-emphasize the fact that motor state feedback of the harmonic drive system is not successful since the kinematic error in the transmission cannot be compensated in the motor variables.
Figure 6.8: Expanded View of the Region Near Zero for the Comparison Between Experimental (Solid) and Simulated (Dashed) Motor and Load Position Errors for the Motor State Feedback Experiment

6.3 Comparison of the Results for Load State Feedback

Figure 6.9 describes the comparison between experiment and simulation in the PD control with load state feedback experiment. In the case of load state feedback, the experiment and simulation are again similar in trend. The difference between the two curves is relatively small and, again, is most likely due to the exclusion of the other nonlinear flexibility and dynamic friction from the model. Note that there is a slight overshoot in the simulation. This is due to the absence of dynamic friction and compliance from the simulation. The kinematic error plot exhibits the same tendency mentioned previously. The magnitude of the kinematic error again seems to
Figure 6.9: Comparison Between Experiment (Solid) and Simulation (Dashed) for the Load State Feedback Experiment

be effected by compliance in the system. As in the motor state case, the kinematic error curves converge to the same value as the steady-state error is maintained.

Figure 6.10 provides a comparison between experimental and simulated torques. The comparison between the two curves indicate that a smaller torque was required by the simulation to produce the desired results. This is again partially due to the tuning of the gains to match the position response. Once again, however, the simulation fails to account for dynamic friction and flexibility. This is most likely the reason for the differences. The curves display the same trends discussed previously as the
desired position is attained. Figure 6.11 describes the general trends in the motor and load position errors with increasing time and Figure 6.12 provides an expanded view of these results. Figure 6.12 indicates that the load position errors for both the experiment and the simulation go to zero while the motor position errors are finite. These are the desired results for the PD control experiment. They show that PD control with load state feedback is the only way to fully compensate for the system dynamics introduced by the kinematic error is to feed back the load state that bears the kinematic error signature.

6.4 Flexibility Effects in the Kinematic Error Waveform

The kinematic error model used in the simulation is an approximate fit of the experimental kinematic error waveform. The most noticeable discrepancies, however, are
most likely due to the effects of flexibility in the system. These effects will be isolated and described in future research.

In order to better exemplify the flexibility effects present, let us first discuss the nature of the measurement of the kinematic error. The kinematic error is determined in the software at each sampling period from the following equation

$$\bar{\theta} = \frac{\theta_m}{N} - \theta_L .$$

(6.12)

Equation (6.12) assumes that the only error present is the kinematic error. This assumption indicates the presence of a rigid link between the motor and the load. This is not the case in the harmonic drive. The flexible spline may exhibit a torsional
deflection if the torque applied to the system is sufficiently large. Thus, the error measured in the previous equation is actually a sum of the effects of kinematic error and flexibility as shown in

$$\tilde{\theta} = \tilde{\theta}_{ke} + \tilde{\theta}_f$$  \hspace{1cm} (6.13)

where the $\tilde{\theta}_{ke}$ is the kinematic error and $\tilde{\theta}_f$ is the torsional displacement. To get a better understanding of the effect of flexibility on the kinematic error magnitudes, several experiments were run using the load state feedback PD control routine. In the experiments, the gain $K_p$ was varied in order to produce different responses. The gain was adjusted so that the rise times (and, thus, the velocities) varied from trial
to trial. The results of the experiment are shown in Figure 6.13. As can be seen, the

figure shows the kinematic error plots obtained from three different runs at increasing values for $K_p$. In the top plot ($K_p = 15$), the magnitudes of the peaks are almost constant because the relatively low gain produced a slow response from the system. Still, it is apparent that the magnitude of the first several peaks is slightly larger than the magnitudes of the peaks occurring after $t > 1.0$ second. This tends to reaffirm the observation that, at a low torque level and near constant velocity, the magnitude of the kinematic error peaks will be the same. The second plot ($K_p = 30$) shows a different response. The initial peaks are relatively small as the motor torque is small.
The magnitudes then increase noticeably as the maximum torque is attained before decreasing to a much smaller value as the steady-state kinematic error is approached. The third plot \((K_p = 50)\) shows an even larger increase in the magnitude of the peaks. This effect is seen as well in Figure 6.14. This figure compares the last two cases where \(K_p = 30\) and \(K_p = 50\). The increase in magnitude seen at the increased gain value is obvious.

The results seen in Figures 6.13 and 6.14 support the finding that the equation
we are using to compute kinematic error, namely Equation (6.12), is dependent upon flexibility in the system. This result did not effect the goals in the PD control experiments because the effects of flexibility diminish as the velocity approaches zero. When the effects of flexibility disappear, the only error component remaining to be compensated is the kinematic error. Thus, the control approach is successful even in the presence of flexibility.

6.5 Conclusions

The comparison between the experimental and simulated results generally indicate that while the simulation follows the trends in the experimental data, there are noticeable differences that must be accounted for. The most obvious of these are the presence of flexibility and dynamic friction. These are the factors that produced most of the differences in the kinematic error and torque comparisons, respectively. The kinematic error model used in the simulation, as mentioned previously, is only approximate. However, the results are similar even though flexibility is ignored. This result indicates that while the model is not complete, it is a reasonable approximation because it allowed us to validate the theoretical PD control results by comparing the theory based simulation to the experimental data. A detailed analysis of the effects of flexibility and dynamic friction upon the system and the kinematic error waveform is for future research.
Chapter 7

Conclusions and Future Recommendations

Harmonic drive gear reducers are attractive for use in fine pointing applications because of their compact nature and their high precision. However, the harmonic drive possesses unwanted characteristics as well. Typically, three types of nonlinear behavior that have been associated with harmonic drives are kinematic error, friction, and compliance. During the course of the research experiment, a thorough literature review indicated that while some effort had been put forth to identify the errors present in harmonic drives, very few researchers have attempted to compensate for the nonlinearities with active control.

The main contributions from the thesis consist of the following.

• The mechanical design and fabrication of the test apparatus. This process consisted of the selection and development of a detailed plan incorporating all necessary system components into a stable, high-tolerance experimental platform.

• The development of the hardware and software interfaces necessary for the operation of the test system. All sensors, including the Canon laser rotary encoder, the Sumtak encoder, the Himmelstein torque sensor, and the resolver in the motor housing were interfaced with the dSPACE environment. This enabled
the data acquisition and control of the test apparatus to take place through the host computer.

- The identification and characterization of the kinematic error in the experimental system. The acquired kinematic error signature was used to produce a detailed model that may be adapted for use in the simulation.

- The application of the PD control methodology to the motor and load state feedback control of the kinematic error in the harmonic drive system. The results from both cases indicate that the theories defined in the thesis are valid and that the desired load position may be attained in the presence of kinematic error. Furthermore, the load state feedback is successful while the motor state feedback fails because the load state control takes into account the kinematic error present between the motor and load sides by closing the loop. The motor state feedback is an open loop scheme which simply ignores kinematic error and is, for this reason, inadequate.

The construction of the test system was a challenging process that began with a literature review and a knowledge of the error components that we wished to identify. From this point the basic structure and sensor requirements were identified and tuned to form a precision system. The design and fabrication of the harmonic drive test apparatus provides a valuable system that may be used for research into the nonlinearities seen in the harmonic drive. Furthermore, the apparatus may be used as a tool for future experiments focused upon issues such as the modeling and control of shaft flexibility and friction. Sensor selection and software development was another
challenging issue to address. This process required an identification of the sensors necessary to examine the nonlinearities integration of the sensors into a high resolution system capable of being operated through the coordination of dSPACE and Windows programs.

The most important conclusion that may be drawn from this research is that PD control with load state feedback can effectively compensate for the kinematic error present in harmonic drives when driving the load to a desired position. This conclusion confirms the previous proofs derived from Lyapunov stability theory that insure the asymptotic stability of the equilibrium when the equations of motion are subject to PD regulation [28]. This result is very important because, even though kinematic error has a relatively small magnitude, it may cause significant error in fine pointing applications. Thus, the ability to drive the load position error to zero is a critical factor in the operation of systems which employ harmonic drives.

Future research performed with the system should seek to implement trajectory tracking control as well as other control methods such as adaptive and robust control. Future research should also focus upon the identification of the friction and compliance present in the system. Once these two additional nonlinearities have been identified and characterized, nonlinear control laws need to be devised to provide compensation.
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