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RICE UNIVERSITY

Dynamic Response of Concrete-Faced Rockfill Dams in Rectangular Canyons

by

Yi Guo

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

Master of Science

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ABSTRACT

Dynamic Response of Concrete-Faced Rockfill Dams
in Rectangular Canyons

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An analytical closed-form solution has been developed for steady-state lateral response of concrete-face rockfill dams built in rectangular canyons. The canyon is assumed to be rigid, while the dam is idealized as a three-dimensional linearly-hysteretic elastic body deforming in shear and bending. Both free and base induced oscillations are studied for various canyon geometries. A parametric study of the effects of the canyon narrowness and the dam slope on the response is undertaken. Finally, a more rigorous numerical formulation is used for verification of the closed-form solution.
ACKNOWLEDGEMENT

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NOTATION

\( \alpha \) slope of the dam \( \alpha = \frac{B}{H} \)

\( \alpha_0 \) dimensionless frequency \( \alpha_0 = \frac{\omega_{nm} H}{V_s} = \sqrt{r_{nm} + \left(\frac{H}{L}\right)^2 (2m - 1)^2 \pi^2} \)

\( \beta_d \) material hysteretic damping ratio of the dam rockfill

\( \beta_{slab} \) material hysteretic damping ratio of the slab

\( \gamma_{yx} \) shear strain along horizontal direction

\( \gamma_{yz} \) shear strain along vertical direction

\( \Delta_{nm} \) normalized displacement of the nth mode along the height and mth mode along the length

\( \varepsilon_{vv, nm} \) axial strain of the nth mode along the height and mth mode along the length

\( \zeta \) dimensionless variable \( \zeta = z/H \)

\( \eta \) dimensionless variable \( \eta = x \pi / L \)

\( \Theta \) \( \Theta = -\sin \theta / H \)

\( \phi \) slope of the dam after deformation

\( \xi_{nm} \) modal damping factor

\( \rho_d \) mass density of the dam rockfill

\( \rho_{slab} \) mass density of the slab

\( \gamma \) Poisson’s ratio of the dam rockfill

\( \Phi \) part of the displacement related to dimensionless variable \( \zeta \)

\( \varphi \) cross-sectional rotation
\( \omega_{nm} \)  
- frequency of harmonic steady-state vibration of the dam

\( \omega_{*nm} \)  
- damped frequency

\( b_j \)  
- coefficients of the power series

\( g \)  
- gravity acceleration

\( k \)  
- simplifying factor  
\[
    k = \frac{C + F}{A} = \frac{12G_d}{\alpha^2E} \left( \frac{\rho_d \omega^2 H^2}{G_d} \right) \left( \frac{2m - 1}{L} \right)^2 H^2
\]

\( m \)  
- modes in x direction

\( n \)  
- modes in z direction

\( q \)  
- simplifying factor  
\[
    q = \frac{k}{r} = \frac{6}{\alpha^2(1 + \nu)}
\]

\( r \)  
- simplifying factor  
\[
    r = \frac{R + D}{A} = \frac{1}{G_d} \left( \frac{\pi}{L} \right)^2 \left( 2m - 1 \right)^2 H^2
\]

\( r_{nm} \)  
- roots of the frequency equation

\( t \)  
- time

\( u \)  
- lateral displacement of the dam due to bending and shear force relative to the boundary \( u = u(x, z, t) \)

\( u_a \)  
- absolute displacement of the dam

\( u_b \)  
- lateral displacement of the rock base

\( v \)  
- volume of the infinitesimal element of the dam

\( A \)  
- simplifying factor  
\[
    A = \frac{E}{12H^2}
\]

\( \text{AF} \)  
- amplification function

\( R \)  
- simplifying factor  
\[
    R = \frac{E \rho_d \omega^2}{12G_d}
\]

\( C \)  
- simplifying factor  
\[
    C = \frac{\rho_d \omega^2}{\alpha^2}
\]
D simplifying factor \( D = \frac{1}{12} E \left( \frac{\pi}{L} \right)^2 (2m - 1)^2 \)

F simplifying factor \( F = \frac{1}{\alpha^2} G_d \left( \frac{\pi}{L} \right)^2 (2m - 1)^2 \)

B maximum width of the dam (at the base)

E young's modulus of the dam rockfill

\( G_d \) constant shear modulus of the dam rockfill

\( G_d^* \) complex valued shear modulus \( G_d^* = G_d (1 + 2i\beta_d) \)

\( G_{ini} \) initially assumed dynamic shear modulus

\( G_{obt} \) shear modulus obtained from iterations

\( G_{slab} \) shear modulus of the concrete slab

H height of the dam

I Geometrical moment of inertia

\( K_{nm} \) simplifying factor \( K_{nm} = E_{slab} A_{slab} \alpha(P_{nm} V_{nm}(t)) \Theta \)

L length of the dam

\( L_{nm} \) simplifying factor \( L_{nm} = \alpha(P_{nm} V_{nm}(t)) \)

M moment acting on the infinitesimal element of the dam

\( N_{vv,nm} \) axial force of the slab

\( P_{nm} \) modal participation factor

\( Q_{xy} \) average shear force acting on vertical face of the infinitesimal element of the dam

\( Q_{yz} \) average shear force acting on horizontal face of the infinitesimal element of the dam
$S_d$  response spectrum of displacement  

$S_v$  response spectrum of velocity  

$S_a$  response spectrum of acceleration  

$T_{nm}$ period of the rocking vibration  

$U$ scalar amplitude of the displacement  

$V_s$ shear wave velocity  

$V_{nm}$ duhamel integral
## ABBREVIATION

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<td>AF</td>
<td>Amplification Function</td>
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<td>CFRD</td>
<td>Concrete-Face Rockfill Dam</td>
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<tr>
<td>MDOF</td>
<td>Multi-Degree-Of-Freedom</td>
</tr>
<tr>
<td>SDOF</td>
<td>Single-Degree-Of-Freedom</td>
</tr>
<tr>
<td>1-D</td>
<td>One Dimensional</td>
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<tr>
<td>2-D</td>
<td>Two Dimensional</td>
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<tr>
<td>SH</td>
<td>Shear Wave</td>
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<td>FEA</td>
<td>Finite Element Analysis</td>
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CHAPTER 1
INTRODUCTION

1.1 General Objective

Due to recent advances in soil and rock engineering, significant progress has been made in the design and construction of rockfill dams. For dams in seismic regions, strong earthquakes may potentially cause substantial damage or even failure to such structure, endangering lives and causing vast property damage and serious environmental problems. Reliable assessment and better understanding of the dynamic behavior and seismic response characteristics of dams can prevent such catastrophic failures by allowing remedial measures in unsafe existing dams and by improving the design of future dams.

During the past decade, remarkable progress has been made in understanding the dynamic characteristics and seismic behavior of earth and rockfill dams during strong earthquakes. A series of studies have focused mainly on three dimensional effects of the canyon geometry, the nonlinear and inelastic behavior of the materials, the inhomogeneity of the materials, the relative flexibility between the dam and the canyon or foundation soil, and the composition and spatial variability of excitation. Related developments have been reviewed in two state-of-the art papers by Gazetas (1987) and Gazetas and Dakoulas (1991) and also in other references (Dakoulas and Hashmi 1991; Dakoulas and Hsu 1995). Despite this significant progress, the effects of
some factors influencing the behavior of rockfill dams are yet to be fully understood.

The main objective of this research is to study the seismic behavior and design of CFR dams built in a rectangular canyon subjected to strong seismic shaking. Precisely, the present work is to develop a new analytical model that will allow a comprehensive study of the individual and combined effects of factors on the seismic response of such dams.
1.2 Review Of Literature On Concrete-Face Rockfill Dam


A limited number of studies has been devoted to the seismic behavior of the concrete face rockfill dam. Among these studies are those by Seed et al (1985), Bureau et al (1985), Gazetas and Dakoulas (1992), Uddin (1992), and Uddin and Gazetas (1995). Hatanaka (1955) reported a fundamental consideration on the earthquake resistant properties of the earth dam and concluded that the vibration of the dam having gentle grade of surfaces of slope can be considered as the shear vibration.

Dakoulas and Hashmi (1991) focused on the wave passage effects on the response of earth dams. They developed a new mathematical solution for evaluating the lateral response of earthfill and rockfill dams in flexible canyons subjected to asynchronous excitation consisting of SH waves incident at an arbitrary angle. The effects of several factors such as flexibility of canyon rock, angle of incidence of waves, the canyon narrowness and frequency of SH excitation influencing the dam response were clarified.

Dakoulas and Hsu (1993) studied the response of earth and rockfill dams in semi-elliptical canyons to oblique waves. The results indicate consistently the effect of the
impedance ratio on the dam response is dramatic for the entire frequency spectrum, and especially more dramatic for high-frequency excitation. This is because the impedance ratio affects both the amount of energy radiated back to the canyon and the spatial variability of the ground motion along the dam canyon interface. The effect of the canyon geometry on the response amplification seems to be almost as important as that of the impedance ratio. The combined effects of the radiation damping and the spatial variability of the ground motion are more dramatic for dams in narrow canyons subjected to high-frequency excitation. When obliquely incident waves impinge on the dam-canyon interface, they induce both symmetric and antisymmetric response components. Even at small incidence angles, the antisymmetric components may lead to substantially higher response in the dam than the response caused by vertically propagating waves.

Uddin and Gazetas (1995) performed a dynamic plane-strain finite-element study of the response of a typical 100-m-tall concrete-faced rockfill dam to strong seismic shaking. In their study, an equivalent-linear analyses have been performed with Coulomb's friction law governing the behavior of the face-slab-rockfill interface. The results show that the effects of the concrete slab on the vibration characteristics of the CFR dam are neglectable in that there is only 2-4% increase of the natural frequencies of the dam due to the presence of the slab and the transverse modal shapes of the dam sections with and without slab are practically identical. Thus estimation of the vibrational characteristics of the CFR dam can be performed by conventional method. The consequences of the high acceleration near the midcrest of the dam are likely to
be serious for the face slab and crest wall but minor for the overall safety of the dam. The face slab experiences very strong axial forces, but insignificant bending moment and shear force for the plane-strain model. They also presented a closed-form solution for the plane-strain problem based on the shear beam model and dynamic properties of the dam material including both the rockfill and concrete were studied (Uddin 1992).
1.3 Scope Of Work

The purpose of work is to perform systematic study on the dynamic characteristics of Concrete-face rockfill (CFR) dam subjected to strong base excitation. In this study the effects of canyon geometry, slope of the dam and properties of the dam materials on the response of rockfill dams to steady-state harmonic are investigated.

For earth dams, the slope is usually moderate, the effect of the bending moment induced by the inertia force is generally small and negligible. But for rockfill dams with steep slope, the effect of the moment might be significant and it has to be considered. The study utilizes a simplified analytical model for the dam, which is idealized as a linearly hysteretic elastic body deforming due to both shear force and bending moment. This generalized shear beam model extends in the vertical and longitudinal directions and assumes uniform response values for the upstream-downstream direction. The canyon has a rectangular shape.

In the first part, the study investigates the effect of canyon geometry for a dam on a rigid rock base subjected to lateral synchronous identical motion along the dam-canyon interface. The natural frequencies and modes of vibration of dams with different geometry have been presented. Note in this part of the work, the base is assumed to be rigid and no dam canyon interaction and radiation damping are considered. Rigorous analytical closed-form solutions are present for the steady state and harmonic response based on the generalized shear beam model. Furthermore a parametric investigation elucidates the effects of the slope of the dam, the ratio of length to width and the frequencies of excitation on the response along the crest of the
dam. The second part of the study utilizes a numerical formulation for the evaluation of the steady-state response of a CFR dam. Results in terms of the natural frequencies, displacement mode shape and amplification factors, lateral deflection and shear stress of the concrete slab are presented.
CHAPTER 2

LATERAL RESPONSE OF DAM IN RECTANGULAR RIGID CANYON

2.1 Introduction

In recent years, the Concrete-Faced Rockfill Dam (CFRD) has been used increasingly. primarily because using compacted rockfill can greatly eliminate the settlement problems caused by soil and earthfill. Current design trends, method of construction and performance of many recent dams can be found in the proceedings of a 1985 ASCE symposium and ASCE publication "Concrete Face Rockfill Dams-Design, Construction and Performance" edited by J. Barry Cooke and James L. Sherard (1985) and "Concrete-Faced Rockfill Dam: I. Assement, and II. Design" (Sherard and Cooke, 1985; Cooke and Sherard, 1987).

For such dams built in narrow canyons, the natural periods decrease with the presence of relatively rigid abutments which creates a three-dimensional (3-D) stiffening effect, and in the meantime, the accelerations near the dam crest increase significantly as the canyon becomes narrower. Several studies concerning various canyon geometries have demonstrated that the 3-D geometry effects on the seismic response characteristics of dams are very important.

It is usually assumed for simplicity due to the complexity of 3-D modeling and analysis of the dam-canyon system that the supporting canyon vibrates as a rigid body
resulting to synchronous identical excitation along the dam-canyon interface. Based on this assumption, numerical results have been achieved for several idealized canyon shapes, such as semi-cylindrical, rectangular, trapezoidal, as well as some actual canyon geometries. A few of these studies give consideration to three-dimensional inelastic conditions, while most of them are linear analyses. A review of the most representative studies and the methods is given.

Hanataka (1952) and Ambraseys (1960) reported closed-form solutions for the lateral response of dams in rectangular canyons. Dakoulas and Gazetas (1986) presented a simple closed-form solution for the lateral response of a homogeneous earth dam in a semi-cylindrical canyon, while Dakoulas and Hsu (1992) derived a complete closed-form solution of earth dam in both semi-elliptical rigid and semi-elliptical flexible canyon. These solutions are obtained with the shear beam concept which assumes only lateral displacement and shear deformation distributed uniformly along the lateral (upstream-downstream) direction of the dam. Based upon these assumptions, the solutions obtained are exact and no other approximation is introduced. The results are in the form of simple algebraic expressions for vibration displacement, natural periods, modal shapes, steady-state amplitude functions, and modal participation factors for seismic excitation.

Martinez & Bielak (1980) have developed an efficient numerical procedure for dams having a plane of symmetry perpendicular to the longitudinal axis, by neglecting the longitudinal deformation, discretizing in finite elements only the dam midsection, and using Fourier series to compute the displacement distribution along the longitudinal
direction. Results on natural frequencies and modal shapes for rectangular, trapezoidal and triangular canyons indicate significant differences due to canyon geometry.

By dividing the dam body through vertical closely spaced transverse planes into superelements behaving as shear-beams, Ohmachi et al. (1982) have developed an approximate efficient formulation in which no symmetry is required. The solution is obtained by using a linear interpolation function for the displacement shape in the longitudinal direction and by enforcing compatibility of deformation between super-elements. Results on natural frequencies and modal shapes for rectangular, trapezoidal and triangular canyons confirm the conclusions drawn by Martinez and Bielak (1980). Using the Rayleigh-Ritz method, Abdel-Ghaffar and Koh (1982) have presented a semi-analytical solution for dams built in canyons having a plane of symmetry. This solution utilizes the shear-beam modal shapes or sinusoids as basis functions, and involves a transformation of the dam geometry into a cuboid. The method is versatile and could be used for an approximate solution of the nonlinear problem.

Makdisi et al. (1982) have developed a special 3-D dynamic finite-element formulation by replacing the 2-D plane-strain isopartametric elements of the computer code LUSH with prismatic longitudinal elements having six faces and eight modal points. To reduce computer storage and time requirements, the longitudinal displacements are ignored, and only shear waves propagating vertically and horizontally in the embankment are considered. Results have been presented for steady-state and transient response of homogeneous dams in triangular canyons. In similar studies, Mejia et al. (1983) used a 3-D finite-element formulation but without restricting the longitudinal
deformations.

Finally Prevost et al. (1985) used a kinematic multi-surface plasticity constitutive modal for soil in 3-D finite-element modeling of dams in arbitrarily-shaped canyons. The dam body is discretized in eight-node isoparametric ‘brick’ elements. However, the finite-element mesh used in such 3-D analyses seems to be coarse, due to the very substantial components are artificially filtered out or at least reduced as they propagate through a coarse mesh, affecting adversely the computation of accelerations near the crest zone.

More details about the above developments may be found in the given references and in two state-of-the-art papers by Gazetas (1987) and Gazetas and Dakoulas (1991). The above studies have provided valuable insight and help improve our understanding of the effects of the canyon geometry on the response of dams. Despite the substantial progress, much still has to be learned regarding the behavior of such structures built in narrow canyons. As stated above, analytical closed-form solutions are particularly valuable as they lead to results in the form of especially simple algebraic expressions for natural periods, modal shapes, and response to steady-state harmonic and transient seismic excitation. Such solutions allow extensive parametric studies offering considerable insight on the effects of the various key parameters influencing the response and provide a means of checking more complex numerical solutions, as well as a valuable tool for preliminary design computations.

However, the closed-form solutions obtained so far for the three dimensional problem considering only shear action and ignore the effect of bending. It is of
interest to study the seismic behavior of the concrete-face rockfill dams in three dimensions subjected to both shear and bending action.

Based on the work of Hanataka (1955) and subsequent work by Uddin (1992) and Uddin and Gazetas (1995) of plane-strain CFR dams, a study is perform to extend the strain-plane model to a three-dimensional model which approximates better many actual dams. To this end, the objective of this study is to develop a closed-formed analytical solution for the dynamic lateral response of three-dimensional concrete-face rockfill dams and investigate the response in terms of accelerations, displacements, shear stress and strains. The results will be used to gain more insight on the effects of canyon narrowness on the seismic response characteristics, by using canyons with length to height ratios ranging from 2 to infinity (plane strain conditions) and comparing with results from other canyon shapes. The solution will be used to evaluate the response characteristics of the concrete face slab.
2.2 Simplifying Assumptions and Mathematical Model

The typical cross section of actual rockfill dam is trapezoidal, but for simplicity, in the following discussion the dam will be considered as a triangular cross-section, because the effect of the upper-cut triangle on the form of dam vibration is small. Figure 2.1 portrays a 3-D perspective view of the dam in a rectangular canyon. The dam has a triangular cross-section, consisting of homogeneous and linearly hysteretic rockfill with a constant mass density $\rho_d$, a constant shear modulus $G_d$, and a material hysteretic damping ratio $\beta_d$. The canyon consists of rigid rock and vibrates exclusively in the lateral (upstream-downstream) direction. Therefore, all points along the dam-canyon interface experience identical and synchronous (in-phase) oscillations. No slippage is allowed at the dam base.

The response of the dam to the seismic excitation, applied along the rectangular boundary with the canyon, is assumed to be only in horizontal direction consisting of deformation induced by lateral shear and deformation due to bending moment with the upstream-downstream displacements, $u$, uniformly distributed across the width of the dam. In other words, the dam is idealized as a generalized “shear beam”, which extends in the vertical and longitudinal directions. The response values are assumed either uniform or average for the upstream-downstream direction. The uniformity of displacements, $u$, across the width of the dam has been confirmed as a reasonable approximation by a series of seismic analyses of earth dams with finite-element and shear-beam models (Dakoulas and Gazetas 1985, 1986, Gazetas 1987). Indeed, comparisons of acceleration, displacement and shear stress time histories between
shear-beam and numerical analyses at various points within the dam showed excellent agreement. Figures 2.3 (a) and (b) show the longitudinal section and the maximum cross-section and the coordinate with the origin resting on the middle point of the top of the dam at the left vertical edge. Figure 2.3 shows an infinitesimal element $b\Delta x\Delta z$ and the corresponding shear forces (using an average across the width $b$) $Q_{yx}$ and $Q_{yz}$ applied on the horizontal and vertical sides, respectively. In reality, the distributions of the $Q_{yx}$ and $Q_{yz}$ are almost uniform for most of the width $b$, except near the two edge of the dam where they vanish due to the decrease of the confining pressure. Nevertheless, by considering a total shear force and an average shear modulus across the width, no assumption regarding their exact distribution is required. To improve the model, a bending moment $M$ is introduced acting on the horizontal face of the dam.
Figure 2.1 Three dimensional view of a dam in a rectangular canyon
Figure 2.2 Dam in rectangular canyon: (a) longitudinal section; (b) maximum cross-section
Figure 2.3 Shear forces and bending moment acting on infinitesimal element
2.3 Free Vibration: Analysis And Results

Consider the dynamic average shear forces $Q_{yx}$ and $Q_{yz}$ and bending moment $M$ acting on an infinitesimal horizontal element of the dam with volume $v = b\,dx\,dz = (B/H)\,z\,dx\,dz$. The net shear forces induced by earthquake shaking on the horizontal and vertical faces of the element are respectively (with unit width and height):

$$\frac{\partial Q_{yz}}{\partial z} \quad \text{and} \quad \frac{\partial Q_{yx}}{\partial x}$$

The total inertia force on the element equals:

$$\rho_d \ddot{u}(x, z, t) \frac{B}{Hz}$$

where $u(x, z, t)$ is the lateral displacement due to shear force and bending moment relative to the boundary and $\ddot{u} = \frac{\partial^2 u}{\partial t^2}$. Consider the dynamic equilibrium of the above three forces acting on the infinitesimal element:

$$\rho_d \ddot{u} - \frac{B}{Hz} = \frac{\partial Q_{yz}}{\partial z} + \frac{\partial Q_{yx}}{\partial x}$$

Letting $\alpha = \frac{B}{Hz}$ leads:

$$\rho_d \ddot{u} = \frac{1}{az\left(\frac{\partial Q_{yz}}{\partial z} + \frac{\partial Q_{yx}}{\partial x}\right)} \quad (2.1)$$

Then consider the equilibrium due to bending neglecting the moment caused by shear force on the vertical face of the element which is small:

$$\frac{\partial M}{\partial z} = \rho_d \ddot{\Phi} \frac{\partial^2 \Phi}{\partial t^2} + Q_{yz} \quad (2.2)$$
Where $\varphi$ is the cross-sectional rotation due to bending given by:

$$\varphi = \frac{\partial u}{\partial z}$$

and $I$ is the geometrical moment of inertia given by:

$$I = \frac{\alpha^3 z^3}{12}$$

Substituting $I$ and $\varphi$ into equation (2.2) leads to:

$$\frac{\partial M}{\partial z} = \frac{1}{12} \rho_d \alpha^3 z^3 \frac{\partial^3 u}{\partial z \partial t^2} + Q_{yz}$$

(2.3)

Taking into account the stress-strain relation for shear:

$$\frac{Q_{yx}}{az} = G_d \frac{\partial u}{\partial x}$$

(2.4)

and the moment-curvature relationship for bending

$$-\frac{M}{EI} + \frac{1}{G_d \partial z (\alpha z)} \frac{\partial Q_{yz}}{\partial z} = \frac{\partial^2 u}{\partial z^2}$$

(2.5)

From equation (2.5):

$$M = E \frac{\alpha^3 z^3}{12} \left[ \frac{1}{G_d \partial z (\alpha z)} \left( \frac{\partial^2 u}{\partial z^2} \right) \right]$$

(2.6)

Differentiating equation (2.6) with respect to $z$:

$$\frac{\partial M}{\partial z} = \frac{E}{G_d} \left( \frac{\alpha^2 z^2 \partial Q_{yz}}{12} + \frac{\alpha^2 z^2 \partial Q_{yz}}{12} - \frac{\alpha^2}{12} Q_{yz} \right) - \frac{E \alpha^3 z^3 \partial^2 u}{12 \partial z^2} - \frac{E \alpha^3 z^3 \partial^3 u}{12 \partial z^3}$$

(2.7)
Eliminating $\frac{\partial M}{\partial z}$ from equation (2.3) and equation (2.7) and differentiating it with respect to $z$:

$$\frac{E}{G_d}\left(\frac{\alpha^2 z^2 \frac{\partial^2 Q_{yz}}{\partial z^2}}{4} + \frac{\alpha^2 z^2 \frac{\partial^3 Q_{yz}}{\partial z^3}}{12}\right) - \frac{E}{G_d}\left(\frac{\alpha^3 z \frac{\partial^2 u}{\partial z^2}}{2} + \frac{\alpha^3 z \frac{\partial^3 u}{\partial z^3}}{2} + \frac{\alpha^3 z \frac{\partial^4 u}{\partial z^4}}{12}\right) = \frac{\partial Q_{yz}}{\partial z} + \frac{1}{4}\rho_d \alpha^2 z^2 \frac{\partial^3 u}{\partial z \partial t^2} + \frac{1}{12}\rho_d \alpha^3 z^3 \frac{\partial^4 u}{\partial z^2 \partial t^2}$$

(2.8)

From equation (2.1) and Equation (2.4), solving for $\frac{\partial Q_{yz}}{\partial z}$:

$$\frac{\partial Q_{yz}}{\partial z} = \rho_d \alpha z \frac{\partial^2 u}{\partial t^2} - G_d \alpha z \frac{\partial^2 u}{\partial x^2}$$

(2.9)

Differentiating equation (2.9) with respect to $z$:

$$\frac{\partial^2 Q_{yz}}{\partial z^2} = \rho_d \alpha \frac{\partial^2 u}{\partial t^2} + \rho_d \alpha z \frac{\partial^3 u}{\partial z \partial t^2} - G_d \alpha \frac{\partial^2 u}{\partial x^2} - G_d \alpha z \frac{\partial^3 u}{\partial z \partial x^2}$$

(2.10)

$$\frac{\partial^3 Q_{yz}}{\partial z^3} = 2 \rho_d \alpha \frac{\partial^3 u}{\partial z \partial t^2} + \rho_d \alpha z \frac{\partial^4 u}{\partial z^2 \partial t^2} - 2 G_d \alpha \frac{\partial^3 u}{\partial z \partial x^2} - G_d \alpha z \frac{\partial^4 u}{\partial z^2 \partial x^2}$$

(2.11)

Substituting equation (2.9), (2.10) and (2.11) into equation (2.8), we have the following equation corresponding to the shear-bending vibration:
\[
E \left( \frac{1}{4} \rho_d \frac{\partial^2 u}{\partial t^2} + \frac{5}{12} \rho_d \frac{\partial^3 u}{\partial z \partial t^2} - \frac{5}{12} G_d z \frac{\partial^3 u}{\partial z^2 \partial x^2} - \frac{1}{4} G_d \frac{\partial^2 u}{\partial x^2} + \frac{1}{12} \rho_d \frac{\partial^2 u}{\partial z^2 \partial t^2} \right)
\]
\[
- \frac{1}{12} G_d \frac{\partial^2 u}{\partial z^2} \frac{\partial^4 u}{\partial x^4} - \frac{1}{12} G_d \frac{\partial^2 u}{\partial x^2} \frac{\partial^4 u}{\partial z^4} = \frac{1}{4} \rho_d \frac{\partial^2 u}{\partial z \partial t^2}
\]
\[
+ \frac{1}{12} \rho_d \frac{\partial^2 u}{\partial z^2} \frac{\partial^4 u}{\partial x^4} + \frac{1}{\alpha^2} \rho_d \frac{\partial^2 u}{\partial t^2} - \frac{1}{\alpha^2} G_d \frac{\partial^2 u}{\partial x^2}
\]

(2.12)

To take into account the dissipation of energy due to inelastic soil behavior, a linear hysteretic damping is introduced in equation (2.12) by replacing \( G_d \) with the complex valued shear modulus \( G_d^* = G_d(1 + 2i\beta_d) \), where \( \beta_d \) is the damping ratio and \( i = \sqrt{-1} \).

For harmonic steady-state vibration of frequency \( \omega \), the total displacement of the dam, \( u_d \), may be written as

\[
u = U(x, z) \exp(i\omega t)
\]

(2.13)

Substituting equation (2.13) into equation (2.12) leads to

\[
E \left( \frac{1}{4} \rho_d \omega^2 U + \frac{5}{12} \rho_d \omega^2 z \frac{\partial U}{\partial z} + \frac{5}{12} G_d z \frac{\partial^3 U}{\partial z^2 \partial x^2} - \frac{1}{4} G_d \frac{\partial^2 U}{\partial x^2} + \frac{1}{12} \rho_d \omega^2 z \frac{\partial^2 U}{\partial z^2} \right)
\]
\[
+ \frac{1}{12} G_d \frac{\partial^2 U}{\partial z^2} \frac{\partial^4 U}{\partial x^4} + E \left( \frac{1}{2} \frac{\partial^2 U}{\partial z^2} + \frac{1}{2} \frac{\partial^3 U}{\partial z^3} + \frac{1}{12} \frac{\partial^4 U}{\partial z^4} \right)
\]
\[
= \frac{1}{4} \rho_d \omega^2 z \frac{\partial U}{\partial z} + \frac{1}{12} \rho_d \omega^2 z \frac{\partial^2 U}{\partial z^2} + \frac{1}{\alpha^2} \rho_d \omega^2 U - \frac{1}{\alpha^2} G_d \frac{\partial^2 U}{\partial x^2}
\]

(2.14)

Due to the geometry of the canyon, it is convenient to introduce dimensionless variables
\[ \eta = \frac{z}{H} \quad 0 \leq \eta \leq 1 \]
\[ \zeta = \frac{x\pi}{L} \quad 0 \leq \zeta \leq \pi \]

Equation (2.14) may be rewritten as:

\[
\frac{E}{G_d} \left( \frac{1}{4} \rho_d \omega^2 U + \frac{5}{12} \rho_d \omega^2 \eta \frac{\partial U}{\partial \eta} + \frac{5}{12} G_d \eta \left( \frac{\pi}{L} \right)^2 \frac{\partial^2 U}{\partial \eta^2} + \frac{1}{4} G_d \left( \frac{\pi}{L} \right)^2 \frac{\partial^2 U}{\partial \zeta^2} \right) \\
+ \frac{1}{12} \rho_d \omega^2 \eta^2 + \frac{1}{12} G_d \eta^2 \left( \frac{\pi}{L} \right)^2 \frac{\partial^4 U}{\partial \eta^2 \partial \zeta^2} \right) + \frac{E}{H^2} \left( \frac{1}{2} \frac{\partial^2 U}{\partial \eta^2} + \frac{1}{2} \frac{\partial^3 U}{\partial \eta^3} + \frac{1}{12} \frac{\partial^4 U}{\partial \eta^4} \right) \\
= \frac{1}{4} \rho_d \omega^2 \eta \frac{\partial U}{\partial \eta} + \frac{1}{12} \rho_d \omega^2 \eta^2 \frac{\partial^2 U}{\partial \eta^2} + \frac{1}{\alpha^2} \rho_d \omega^2 U - \frac{1}{\alpha^2} G_d \left( \frac{\pi}{L} \right)^2 \frac{\partial^2 U}{\partial \zeta^2} 
\]

For this differential equation, the solution may be found using variable separation. It can be assumed that \( U \) is in the form:

\[ U(\zeta, \eta) = \Phi(\eta) \sin(2m - 1)\zeta \]  

(2.16)

Substituting equation (2.16) into equation (2.15)

\[
\{ E \eta \frac{\partial^4 \Phi}{\partial \eta^4} + 6E \eta \frac{\partial^3 \Phi}{\partial \eta^3} + \left[ 6E - E \left( \frac{\pi}{L} \right)^2 (2m - 1)^2 H^2 \eta^2 + \frac{E}{G_d} \rho_d \omega^2 H^2 \eta^2 \right] \frac{\partial^2 \Phi}{\partial \eta^2} \\
+ \left[ 5 \frac{E}{G_d} \rho_d \omega^2 H^2 \eta - 5E \left( \frac{\pi}{L} \right)^2 (2m - 1)^2 H^2 \eta \right] \frac{\partial \Phi}{\partial \eta} + \left[ 3 \frac{E}{G_d} \rho_d \omega^2 H^2 \\
- 3E \left( \frac{\pi}{L} \right)^2 (2m - 1)^2 H^2 - 12 \frac{\rho_d \omega^2}{\alpha^2} H^2 + 12 \frac{G_d \left( \frac{\pi}{L} \right)^2 (2m - 1)^2 H^2}{\alpha^2} \right] \Phi \} \sin(2m - 1)\zeta = 0 
\]

(2.17)

In order that equation (2.17) is satisfied for any values of \( \zeta \) and \( \eta \), the following equation can be obtained
\[
\begin{align*}
E\eta \frac{\partial^4 \Phi}{\partial \eta^4} + 6E\eta \frac{\partial^3 \Phi}{\partial \eta^3} + \left[ 6E - E\left(\frac{\pi}{L}\right)^2 (2m - 1)^2 H^2 \eta^2 + \frac{E}{G_d} \rho_d \omega^2 H^2 \eta \right] \frac{\partial^2 \Phi}{\partial \eta^2} \\
+ \left[ 5 \frac{E}{G_d} \rho_d \omega^2 H^2 \eta - 5E \left(\frac{\pi}{L}\right)^2 (2m - 1)^2 H^2 \eta \right] \frac{\partial \Phi}{\partial \eta} + \left[ 3 \frac{E}{G_d} \rho_d \omega^2 H^2 \\
- 3E \left(\frac{\pi}{L}\right)^2 (2m - 1)^2 H^2 \eta - 12 \frac{\rho_d \omega^2}{\alpha^2} H^2 \eta + 12 \frac{G_d \left(\frac{\pi}{L}\right)^2 (2m - 1)^2 H^2 }{\alpha^2} \right] \Phi = 0
\end{align*}
\] (2.18)

For free vibrations, the solution must satisfy the boundary condition of zero displacement along the rock base and the right and left boundaries (no slippage). The canyon is assumed to be rigid, at the dam base \((\eta = 1)\),

\[
U(\zeta, 1) = 0 \quad 0 \leq \zeta \leq \pi
\] (2.19)

at the left vertical boundary \((\zeta = 0)\),

\[
U(0, \eta) = 0 \quad 0 \leq \eta \leq 1
\] (2.20)

and at the right vertical boundary \((\zeta = \pi)\),

\[
U(\pi, \eta) = 0 \quad 0 \leq \eta \leq 1
\] (2.21)

Moreover, it should yield zero shear stress at the crest

\[
G_d^* \frac{\partial U}{\partial \eta}(\zeta, 0) = 0 \quad \text{and} \quad G_d^* \frac{\partial U}{\partial \zeta}(\zeta, 0) = 0 \quad 0 \leq \zeta \leq \pi
\] (2.22)

We apply the boundary condition that at right and left vertical boundary the total displacements of the dam relative to the rock base are zero i.e:

at the left vertical boundary \((\zeta = 0)\),

\[
U(0, \eta) = 0 \quad 0 \leq \eta \leq 1
\] (2.23)
and at the right vertical boundary ($\zeta = \pi$),

$$U(\pi, \eta) = 0 \quad 0 \leq \eta \leq 1$$  (2.24)

In order that equation (2.16) satisfies equation (2.23) and (2.24) for any value of $\eta$, $\sin m \alpha = 0$ and $\sin m \pi = 0$ are obtained, so $m$ must take values $m = 1, 2, 3, \ldots$.

Letting

$$A = \frac{E}{12H^2}, \quad R = \frac{E\rho_d \omega^2}{12G_d}, \quad C = \frac{\rho_d \omega^2}{\alpha^2}$$

$$D = \frac{1}{12} E \left(\frac{\pi}{L}\right)^2 (2m - 1)^2$$

$$F = \frac{1}{\alpha^2} C \left(\frac{\pi}{L}\right)^2 (2m - 1)^2$$

Equation (2.17) can be written in the simple form

$$A\eta^2 \frac{\partial \Phi^4}{\partial \eta^4} + 6A\eta \frac{\partial \Phi^3}{\partial \eta^3} + (6A + R\eta^2 + D\eta^2) \frac{\partial \Phi^2}{\partial \eta^2} + (5R\eta + 5D\eta) \frac{\partial \Phi}{\partial \eta} + (3R + 3D + C + F) \Phi = 0$$  (2.25)

Note that if the dam is infinitely long, i.e. $L \to \infty$, the problem becomes a plain problem, and equation (2.26) becomes

$$A\eta^2 \frac{\partial \Phi^4}{\partial \eta^4} + 6A\eta \frac{\partial \Phi^3}{\partial \eta^3} + (6A + R\eta^2) \frac{\partial \Phi^2}{\partial \eta^2} + 5R\eta \frac{\partial \Phi}{\partial \eta} + (3R + C) \Phi = 0$$  (2.26)

Equation (2.27) quite agrees with the analysis of Hatanaka (1955) for plane strain conditions.

For the ordinary differential equation representing rocking oscillations of the homogeneous rockfill dam, i.e. equation (2.25), the solution can be found using the method of Frobenius. Note that the point $\eta = 0$ is a regular singular point, i.e the
general solution of a linear combination of convergent series exists.

We assume the following power series satisfies equation (2.25)

$$\Phi(\eta) = \sum_{0}^{\infty} b_j \eta^j$$  \hspace{1cm}  \text{(2.27)}

where $b_j$ are coefficients to be determined, from equation (2.28) we have

$$\eta \frac{\partial \Phi}{\partial \eta} = \sum_{0}^{\infty} (j + 2)(j + 1)j(j - 1) b_{j + 2} \eta^j$$

$$\eta \frac{\partial^3 \Phi}{\partial \eta^3} = \sum_{0}^{\infty} (j + 2)(j + 1) j b_{j + 2} \eta^j$$  \hspace{1cm}  \text{(2.28)}

$$\frac{\partial^2 \Phi}{\partial \eta^2} = \sum_{0}^{\infty} (j + 2)(j + 1) b_{j + 2} \eta^j$$

Furthermore, letting

$$r = \frac{R + D}{A} = \frac{\rho_d \omega^2 H^2}{G_d} - \left(\frac{\pi^2}{L}\right)(2m - 1)^2 H^2$$  \hspace{1cm}  \text{(2.29)}

$$k = \frac{C + F}{A} = -\frac{12 G_d}{\alpha^2 E} \left(\frac{\rho_d \omega^2 H^2}{G_d} - \left(\frac{\pi^2}{L}\right)(2m - 1)^2 H^2\right)$$  \hspace{1cm}  \text{(2.30)}

Substituting equation (2.29), (2.30) and (2.31) into equation (2.26) leads

$$\left[\sum_{0}^{\infty} (j + 1)(j + 2)^2(j + 3) b_{j + 2} + \sum_{0}^{\infty} [r j(j - 1) + 5 r j + 3 r + k] b_j\right] \eta^j = 0$$  \hspace{1cm}  \text{(2.31)}

The left-hand side of equation (2.31) is the Taylor series expansion of a function
whose expansion coefficients are contained in the braces. In order for this equation to be satisfied over an interval about the origin, the expression in the brackets must be identically zero. Thus, for every value of \( j \)

\[
(j + 1)(j + 2)^2(j + 3)b_{j+2} + [rj(j - 1) + 5rj + 3\alpha + k]b_j = 0
\]  

(2.32)

From equation (2.32), the recurrence relations which determine successively the coefficients \( b_1, b_2, b_3 \) \ldots \ldots in terms of \( b_0 \) are given by

\[
b_{j+2} = \frac{[rj(j - 1) + 5rj + 3\alpha + k]}{(j + 1)(j + 2)^2(j + 3)}b_j
\]  

(2.33)

Further, we let

\[
\frac{k}{r} = \frac{12G_d}{\alpha^2 E} = \frac{6}{\alpha^2(1 + v)} = q
\]

, then equation (2.33) can be written as

\[
b_{j+2} = \frac{r[(j + 1)(j + 3) + q]}{(j + 1)(j + 2)^2(j + 3)}b_j
\]  

(2.34)

The coefficients \( b_j \) in equation (2.27) can be determined from equation (2.34) and the boundary conditions (2.19) and (2.11). The solution of free rocking vibration may be written in the form

\[
\Phi_m(\eta) = b_0(1 + \Gamma_2 r \eta^2 + \Gamma_4 r^2 \eta^4 + \Gamma_6 r^3 \eta^6 + \Gamma_8 r^4 \eta^8)
\]

\[
+ \left( \frac{\chi_1}{\chi_2} \right)(\eta + \Gamma_3 r \eta^3 + \Gamma_5 r^2 \eta^5 + \Gamma_7 r^3 \eta^7 + \Gamma_9 r^4 \eta^9)
\]  

(2.35)

where

\[
\chi_1 = 1 + \Gamma_2 r + \Gamma_4 r^2 + \Gamma_6 r^3 + \Gamma_8 r^4
\]
and \[ \chi_2 = 1 + \Gamma_3 r + \Gamma_5 r^2 + \Gamma_7 r^3 + \Gamma_9 r^4 \] (2.36)

where \( \Gamma_2, \Gamma_3 \ldots \ldots \Gamma_9 \) are functions of \( b_2, b_3 \ldots \ldots b_9 \) respectively and their expression are given in the Appendix A. In this case we use the vibration of the fundamental and second rocking mode only, because of their predominance, and neglect the terms with order higher than \( r^5 \) as they are very small.

The frequency equation can be obtained by forcing equation (2.35) to satisfy the boundary conditions equation (2.19), and the natural frequencies are recovered from the roots of

\[ F(r, \alpha) = 0 \] (2.37)

where \( r \) is a function of length to height ratio of the dam \( L/H \), \( m \) is the mode number for the longitudinal direction, \( \omega_{nm} \) is the natural frequency of the shear rocking vibration and \( \alpha \) is the slope of the upstream and downstream faces of the dam.

Applying (2.36) to boundary condition equation (2.18), the periods of free vibration can be solved from the roots of the following equation

\[
\left(9\Gamma_9 - 7\Gamma_8 - \frac{7}{18}\Gamma_7 + \frac{1}{3}\Gamma_6 + 5\Gamma_7\Gamma_2 - 3\Gamma_6\Gamma_3 + \Gamma_5\Gamma_4 \right)^4 \\
+ \left(7\Gamma_7 - 5\Gamma_6 - \frac{5}{14}\Gamma_5 + \frac{1}{6}\Gamma_4 + 3\Gamma_5\Gamma_2 - \Gamma_4\Gamma_3 - \frac{1}{20}\Gamma_3\Gamma_2 \right)^3 \\
+ \left(5\Gamma_5 - 3\Gamma_4 - \frac{3}{10}\Gamma_3 + \frac{1}{12}\Gamma_2 + \Gamma_3\Gamma_2 \right)^2 + \left(3\Gamma_3 - \Gamma_2 - \frac{1}{6} \right)^2 + 1 = 0
\] (2.38)

By calling \( r_{11} \) the minimum positive root of equation (2.38), from equation (2.29), the fundamental frequency of the dam rocking vibration can be written in the form
\[ \omega_{11} = \sqrt{\gamma_{11} + \left(\frac{H}{L}\right)^2 \pi^2 \frac{V_s}{H}} \]  

(2.39)

where \( V_s \) is the shear wave velocity expressed as

\[ V_s = \sqrt{\frac{G_d}{\rho_d}} \]  

(2.40)

The natural frequencies can be expressed in a dimensionless form

\[ \alpha_{nm} = \frac{\omega_{nm} H}{V_s} = \sqrt{r_{nm} + \left(\frac{H}{L}\right)^2 \left(2m - 1\right)^2 \pi^2} \]  

(2.41)

corresponding to the vibrational mode \((n, m)\).

Tables 2.1 to 2.3 give the values of the dimensionless frequencies for \( n=1, 2, 3 \) and \( m=1, 2, 3 \) and for various aspect ratios \( L/H \), with the slope of the dam \( \alpha = 3.6 \) (1:1.8). The corresponding roots for equation (2.38) are \( r_1 = 5.6709 \), \( r_2 = 29.8357 \) and \( r_3 = 70.9243 \). Tables 2.4 to 2.6 give these values with the slope of the dam \( \alpha = 2.6 \) (1:1.3). The corresponding roots for equation (2.38) are \( r_1 = 5.8387 \), \( r_2 = 31.8353 \) and \( r_3 = 75.4832 \). The value of Poisson's ratio is taken as 0.3 for dry and nearly dry rockfill.

Notice in these tables that for \( L/H \rightarrow \infty \), the results approach those of the independent plane-strain shear beam solution. The results confirm that the natural frequencies of the dam increase as canyon narrowness increases, indicating a stiffening effect. Practically for aspect ratios \( L/H > 6 \), the natural frequencies of the dam approach those for the dams under plane strain conditions. These tables also
show that the natural frequency of the dam increases slightly when the slope decreases.

It is of interest to investigate the natural periods of the rocking vibration of the dam. The period $T_{nm}$ corresponding to the mode $(n, m)$ can be given as

$$T_{n1} = \frac{2\pi}{\omega_{n1}} = \frac{2\pi}{\sqrt{r_{n1} + \left(\frac{H}{L}\right)^2 \pi^2}} \frac{H}{V_s} \quad (2.42)$$

When the aspect ratio $L/H \rightarrow \infty$, equation (2.42) becomes

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{r_n}} \frac{H}{V_s} \quad (2.43)$$

Figure 2.11 shows a comparison between the period of free vibration obtained in case of a dam in rectangular canyon and that corresponding to plane strain conditions. The normalized period $\frac{T_{n1}}{T_n}$, given by

$$\frac{T_{n1}}{T_n} = \frac{\sqrt{r_n}}{\sqrt{r_{n1} + \left(\frac{H}{L}\right)^2 \pi^2}} \quad (2.44)$$

is plotted versus the aspect ratio $L/H$. It can be seen that when the length is more than 4 times of the height, the difference between the periods of the three dimensional dam and the plain strain dam is less than 5%. When $L/H$ is greater than 3, the difference is less than 10%.

Finally, the displacement mode shape is given by
\[ \Phi_{nm}(\eta) = b_0 (1 + \Gamma_2 r_{nm} \eta^2 + \Gamma_4 r_{nm} \eta^4 + \Gamma_6 r_{nm} \eta^6 + \Gamma_8 r_{nm} \eta^8) \\
+ \left( \frac{\chi_1}{\chi_2} \right) (\eta + \Gamma_3 r_{nm} \eta^3 + \Gamma_5 r_{nm} \eta^5 + \Gamma_7 r_{nm} \eta^7 + \Gamma_9 r_{nm} \eta^9) \]  \hspace{1cm} (2.45)

The values of \( \chi_1 \) and \( \chi_2 \) can be obtained from the expression (2.36) by substituting \( r = r_{nm} \).

The modal displacement shapes along the height, normalized to the unit value at the midcrest are given by \( \Delta_{nm} = \frac{\Phi_{nm}(\eta)}{\Phi_{nm}(0)} \). Figures 2.4 to 2.7 plot the normalized displacement for vibrational modes for \( n=1, 2, 3 \) and 4. The displacement shapes are plotted along the height of the dam at mid-section. Figures 2.8 to 2.10 plot the normalized displacement for vibrational modes for \( m=1, 2, 3 \). The displacement shapes are plotted along the dam crest. Note that the displacement shapes along the height of the dam are practically independent of the aspect ratio \( L/H \).
<table>
<thead>
<tr>
<th>Aspect Ratio</th>
<th>$\omega_{nm}H/V_d$ for $n=1$ and $\alpha = 3.6$ (1:1.8)</th>
</tr>
</thead>
<tbody>
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Table 2.1 Dimensionless frequency for $n=1$ and $m=1, 2, 3$ and various dam length to height ratio
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Table 2.2 Dimensionless frequency for n=2 and m=1, 2, 3 and various dam length to height ratio
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<th>Aspect Ratio</th>
<th>$\omega_{nmH/V_d}$ for $n=3$ and $\alpha = 3.6$ (1:1.8)</th>
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Table 2.3 Dimensionless frequency for $n=3$ and $m=1, 2, 3$ and various dam length to height ratio
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<th>$\omega_{nm}H/V_s$ for $n=1$ and $\alpha = 2.6$ (1:1.3)</th>
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<td>2.4163</td>
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</table>

Table 2.4 Dimensionless frequency $n=1$ and $m=1, 2, 3$ and various dam length to height ratio
| Aspect Ratio (L/H) | $\omega_{2mH}/V_s$ for n=2 and $\alpha = 2.6$ (1:1.3) |
|-------------------|-----------------|-----------------|-----------------|
|                   | m=1             | m=2             | m=3             |
| 2                 | 5.8569          | 6.4579          | 7.3513          |
| 2.5               | 5.7805          | 6.1767          | 6.7858          |
| 3                 | 5.7386          | 6.0185          | 6.4579          |
| 3.5               | 5.7132          | 5.9210          | 6.2519          |
| 4                 | 5.6967          | 5.8569          | 6.1145          |
| 4.5               | 5.6853          | 5.8125          | 6.0185          |
| 5                 | 5.6772          | 5.7805          | 5.9488          |
| 6                 | 5.6665          | 5.7386          | 5.8569          |
| 7                 | 5.6601          | 5.7132          | 5.8007          |
| 8                 | 5.6559          | 5.6967          | 5.7640          |
| 9                 | 5.6531          | 5.6853          | 5.7386          |
| 10                | 5.6510          | 5.6772          | 5.7205          |
| $\infty$          | 5.6423          | 5.6423          | 5.6423          |

Table 2.5 Dimensionless frequency for n=2 and m=1, 2, 3 and various dam length to height ratio.
<table>
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<tr>
<th>Aspect Ratio</th>
<th>$\omega_{3mH/V_s}$ for n=3 and $\alpha = 2.6$ (1:1.3)</th>
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Table 2.6 Dimensionless frequency for n=3 and m=1, 2, 3 and various dam length to height ratio
Figure 2.4 Displacement Mode Shape of the First Mode along the Height (L/H=3)
Figure 2.5 Displacement Mode Shape of the Second Mode along the Height (L/H=3)
Figure 2.6 Displacement Mode Shape of the Third Mode along the Height (L/H=3)
Figure 2.7 Displacement Mode Shape of the Fourth Mode along the Height (L/H=3)
Figure 2.8 Displacement Mode Shape of the First Mode along the Length (L/H=3)
Figure 2.9 Displacement Mode Shape of the Third Mode along the Length (L/H=3)
Figure 2.10 Displacement Mode Shape of the Third Mode along the Length (L/H=3)
Figure 2.11 Natural Period of a 3-D Dam Normalized by the Period of a Plain-Strain Dam
2.4 Earthquake Response Analysis

In the previous section we derived the natural frequencies and modes of free vibration of concrete-faced rockfill dam built in rectangular canyon. In this section we will derive the earthquake response of the same dam subjected to uniform horizontal transverse ground acceleration $\ddot{u}_h(t)$.

The equation of motion of the dam relative to the rock base excited by the ground motion can be written as

$$\frac{E}{G_d}\left(\frac{1}{4}\rho_d \frac{\partial^2 u}{\partial t^2} + \frac{5\rho_d z}{12} \frac{\partial^3 u}{\partial z \partial t^2} - \frac{5G_d z}{12} \frac{\partial^3 u}{\partial z \partial x^2} = \frac{G_d \rho_d^2}{4} \frac{\partial^2 u}{\partial x^2} + \frac{\rho_d z^2}{12} \frac{\partial^4 u}{\partial z^2 \partial t^2} - \frac{G_d z^2}{12} \frac{\partial^4 u}{\partial z^2 \partial x^2}\right)$$

$$-E\left(\frac{1}{2} \frac{\partial^2 u}{\partial z^2} + \frac{1}{2} \frac{\partial^3 u}{\partial z^3} + \frac{1}{12} \frac{\partial^4 u}{\partial z^4}\right) - \frac{1}{4}\rho_d \frac{\partial^2 u}{\partial z \partial t^2} - \frac{1}{12}\rho_d z^2 \frac{\partial^2 u}{\partial z^2 \partial t^2} - \frac{1}{\alpha^2} \rho_d \frac{\partial^2 u}{\partial t^2} + \frac{1}{\alpha^2} G_d^2 \frac{\partial^2 u}{\partial x^2}$$

$$-\frac{1}{\alpha^2} \rho_d \frac{\partial^3 u}{\partial z \partial t^2} + \frac{1}{\alpha^2} G_d z^2 \frac{\partial^3 u}{\partial z \partial x^2} = \rho_d \ddot{u}_h(t)$$

(2.46)

It can be shown that the orthogonality condition is satisfied. Hence, we can use modal superposition in which $U(x, z, t)$ is obtained as a summation of a discretely infinite number of displacement histories, corresponding to the each natural frequencies of the dam. The earthquake response of the dam can be written as the following equation, using the generalized coordinates

$$u(x, z, t) = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{nm}(x, z) P_{nm} V_{nm}(t)$$

(2.47)

where $U_{nm}(\zeta, \eta) = \Phi_{nm}(\eta) \sin(2m - 1)\zeta$ for the mode $(n,m)$. 
\( P_{nm} \) is the modal participation factor given by

\[
P_{nm} = \frac{\int_0^\pi \int_0^\pi \xi \eta U_{nm}(\xi, \eta) d\xi d\eta}{\int_0^\pi \int_0^\pi \xi \eta U_{nm}^2(\xi, \eta) d\xi d\eta}
\]

(2.48)

and \( V_{mn}(t) \) is the Duhamel integral given by

\[
V_{nm}(t) = \frac{1}{\omega_{nm}^*} \int_0^t \dot{u}_b(\tau) e^{-\xi_{nm} \omega_{nm}(t-\tau)} \sin \omega_{nm} \sqrt{(1 - \xi_{nm}^2)(t-\tau)} d\tau
\]

(2.49)

where \( \xi_{nm} \) is the modal damping factor, \( \omega_{nm} \) is the natural frequency and \( \omega_{nm}^* \) is the damped frequency given by

\[
\omega_{nm}^* = \omega_{nm} \sqrt{(1 - \xi_{nm}^2)}
\]

(2.50)

Following Uddin’s approach for the plane strain dam (Uddin 1992), the model participation factors can be expressed as follows:

\[
P_{nm} = \frac{\Omega_1 + \Omega_2 + \Omega_3 + \Omega_4 + \Omega_5 + \Omega_6 + \Omega_7 + \Omega_8 + \Omega_9 + \Omega_{10}}{\Psi_1 + \Psi_2 + \Psi_3 + \Psi_4 + \Psi_5 + \Psi_6 + \Psi_7 + \Psi_8 + \Psi_9 + \Psi_{10}}
\]

(2.51)

The values of \( \Omega_1, \Omega_2, \ldots, \Omega_{10} \) and \( \Psi_1, \Psi_2, \ldots, \Psi_{10} \) are given in Appendix B.

For \( n=1, 2, 3 \) and \( m=1, 2, 3 \) and for various aspect ratios \( L/H \), the values of the model participation factors are given in Table 2.7 to Table 2.9.

From equation (2.46), we see that due to the orthogonality property of the mode shapes, we can simplify the problem to that of the earthquake response of a single-degree-of-freedom (SDF) system. If the mode shapes and the factors \( P_{nm} \) are known,
the axial stress and axial strain can be computed.

From equation (2.47), the absolute acceleration $\ddot{u}_a$ is directly recovered

$$\ddot{u}_a(x, z, t) = \ddot{u}_b(t) + \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} U_{nm}(x, z) P_{nm} \dot{V}_{nm}(t) \quad (2.52)$$

Evaluation of displacements requires only the first few terms of the series. For the absolute accelerations, however, more terms are required due to slow series convergence.

The expression for the shear strains $\gamma_{yx}$ and $\gamma_{yz}$ are given by

$$\gamma_{yx} = \frac{1}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\partial U_{nm}(x, z)}{\partial x} P_{nm} V_{nm}(t) \quad (2.53)$$

$$\gamma_{yz} = \frac{1}{2} \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} \frac{\partial U_{nm}(x, z)}{\partial z} P_{nm} V_{nm}(t) \quad (2.54)$$

In addition, the response spectrum can be used to evaluate maximum displacement, stress and strain.
<table>
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<tr>
<th>Aspect Ratio</th>
<th>Participation Factors $P_{nm}$ for n=1</th>
</tr>
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Table 2.7 Participation factors for n=1 and various dam length to height ratios
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Table 2.8 Participation factors for $n=2$ and various dam length to height ratios
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Table 2.9 Participation factors for n=3 and various dam length to height ratios
2.5 Steady-State Harmonic Response

In the previous section, the expression for earthquake response of the dam has been obtained. For an arbitrary earthquake excitation, the solution itself can not give a clear understanding of various factors that influence the response of the dam, such as the canyon geometry and the slope of the dam. So it is of great interest to obtain the steady state response to a harmonic base excitation, as it allows numerical study of the effects of these factors. For a harmonic base excitation \( \ddot{u}_b(t) = \dot{U}_b e^{i\omega t} \), the total motion inside the dam \( U_d \) is expressed as

\[
 u(\eta, \zeta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} U_{mn}(\eta, \zeta) P_{mn} V_{mn}(t) 
\]

(2.55)

which has to satisfy the shear-bending vibration equation (2.12) and the boundary condition equation (2.22) of zero stress at the dam crest.

where \( V_{mn}(t) \) is the Duhamel integral given by

\[
 V_{mn}(t) = \frac{1}{\omega_{mn}} \int_0^t \dot{u}_b(\tau) e^{-\xi_{mn} \omega_{mn}(t-\tau)} \sin(\omega_{mn} \sqrt{1 - \xi_{mn}^2}(t-\tau)) d\tau 
\]

(2.56)

and \( U_{mn}(\eta, \zeta) = \Phi_n(\eta) \sin(2m - 1) \zeta \)

(2.57)

For the simple harmonic base excitation \( \ddot{u}_b(t) = \dot{U}_b e^{i\omega t} \), equation (2.56) can be simplified as

\[
 V_{mn}(t) = \frac{\dot{U}_b}{\omega_{mn}} \int_0^t e^{i\omega_{mn}(t-\tau)} e^{-\xi_{mn} \omega_{mn}(t-\tau)} \sin(\omega_{mn} \sqrt{1 - \xi_{mn}^2}(t-\tau)) d\tau 
\]

(2.58)
Using the result of equation (2.50) and (2.54) and further defining time independent function $\ddot{U}_{d_{\text{max}}} (\zeta, \eta)$, the absolute acceleration of the dam may be written as

$$\ddot{U}_d (\zeta, \eta) = \ddot{U}_b + \ddot{U}_{d_{\text{max}}} (\zeta, \eta)$$  \hspace{1cm} (2.59)

The results of the steady-state response are presented in the form of an amplification function, $AF = AF (\zeta, \eta, \omega)$, defined as the ratio of the total acceleration of the dam over its base acceleration, i.e.

$$AF = \frac{\ddot{U}_b + \ddot{U}_{d_{\text{max}}} (\zeta, \eta)}{\ddot{U}_b}$$  \hspace{1cm} (2.60)

A parameter study is undertaken to investigate the influence of the canyon narrowness and the contribution of bending on the steady-state response of a dam subjected to steady train of propagating SH waves.

For a given canyon shape, the canyon narrowness may be expressed by using the ratio of the length over the height of the dam, $L/H$. To investigate the effect of the canyon narrowness on the response, four dams are considered, all having the same $S$ wave velocity of $V_s$ and material damping $\beta_d = 0.10$ (for the rockfill), and corresponding to four different values of $L/H = 2, 3, 5$ and $\infty$.

Figure 2.12 plots the amplification factor AF at the midcrest ($\eta = 0, \zeta = \frac{\pi}{2}$) of three dams built in rectangular rigid canyon with length to height ratio $L/H = 2, 3, 5$ versus a dimensionless frequency $\alpha_0$ defined as
\[ \alpha_0 = \frac{\omega H}{V_3} \] (2.61)

The amplification factor of a dam under plane-strain condition \( L/H \to \infty \) is also plotted for comparison in Figure 2.13.

The results in Figure 2.12 show that for narrow canyons with \( L/H < 5 \) the midcrest amplification at the first resonance is a little above 10, whereas as \( L/H \to \infty \), as in Figure 2.13, it reduces to about 6. This is in agreement with earlier results for rectangular canyon (Dakoulas and Hashmi 1991) and semi-elliptical canyon (Dakoulas and Hsu 1993). The amplification at higher frequencies is much larger for dams built in narrow canyons than that for dams under plane-strain conditions. For example, the amplification factor \(|AF|\) at the third resonance peak value is about 6.0 for \( L/H = 2 \), and this value reduces to about 2.8 for \( L/H = 3 \) and to about 2.2 for \( L/H = 5 \). This substantially higher amplification for dams in canyons with \( L/H = 2 \) is due to two factors: (a) the expected stiffening effect of the canyon narrowness and (b) possible wave focusing phenomena at midcrest. As expected, increasing canyon narrowness results in higher amplification for the high frequency part of spectra in Figures 2.12 to 2.14. This stiffening effect of the canyon narrowness is due to the increasing proximity of the rigid boundaries of the rock canyon. As \( L/H \to \infty \), the effect of these factors diminishes and the high midcrest amplifications drop dramatically. Figures 2.12 and 2.13 also show that the amplification function approach to 1 when the dimensionless frequency \( \alpha_0 = \omega H/V_d \) becomes larger than 10.

Figure 2.14 plots the contribution of the first term \( (m = 1) \) to the midcrest
amplification for the same dams shown in Figure 2.12. Comparison between the results of Figure 2.12 and Figure 2.14 demonstrates that the first term in equation (2.53) is very important.

Figures 2.12 to 2.14 demonstrate the effects of only the length to width ratio $L/H$ on the steady-state midcrest amplification of dams built in canyons with rectangular shapes. In addition to the effect of length to width ratio, it is also of interest to examine the effect of the canyon shapes on the response, by considering the amplifications of dams built in canyons with rectangular and semi-elliptical shapes with the same length to width ratio. The amplification $AF$ of a dam in a semi-elliptical rigid canyon is given by (Dakoulas and Hsu, 1995).

$$AF = 2 \sum_{n=0, 2, \ldots}^{\infty} \frac{i^n R_{on}^{(1)}(c_d, l) \, S_{on}^{(1)}(c_d, 0)}{R_{on}^{(1)}(c_d, \zeta_b) \, N_{on}} \, R_{on}^{(1)}(c_d, \zeta) \, S_{on}^{(1)}(c_d, \eta) \quad (2.62)$$

where $R_{on}^{(1)}$ is the radial prolate spheroidal wave function of the first kind with order 0 and degree $n$ and $R_{on}^{(1)}$ is the angular prolate spheroidal wave function of the first kind with order 0 and degree $n$.

Figures 2.15 to 2.17 plot the amplifications for the two canyon shapes with $L/H$ ratios equal to 2, 3, and 5, respectively, using equations (2.59) and equation (2.61). It can be noticed in Figure 2.15 that the amplification values of the dam in the semi-elliptical canyon ($L/H=2$) are substantially higher than those of the dam in rectangular canyon. It is due to the higher boundary proximity as well as the wave focusing phenomena that is less remarkable for rectangular canyons. For $L/H=3$, the significant differences in the response shown in Figure 2.16, for dimensionless frequencies $\alpha_0$ ranging from
about 3 to 8, are due to the difference between the two canyon shapes, with the semi-elliptical canyon being “narrower” than the rectangular one. In Figure 2.17, for L/H=5 or higher, the effect of the canyon shape is less significant especially for higher frequencies, and the midcrest amplifications for both dams approach that of a dam under plane strain conditions.

It is of interest to study the contribution of bending by comparing the results derived from the present study to those for a dam in a rectangular canyon responding only in shear vibration. (Dakoulos and Hashim 1992). Figures 2.18 to 2.20 plot the midcrest amplification for two dams built in rectangular canyon for shear only and shear-bending deformation, with the aspect ratios L/H=2, 3, 5, respectively, also versus the dimensionless frequencies $\alpha_0$. Figure 2.18 shows that for the first two resonances the amplification values of the dam considering the influence of the moment are a little higher than that of dams without moment. For higher frequencies, the response of the two dams tends to have little difference. From Figures 2.19 and 2.20, for both L/H=3 and L/H=5, the difference is more substantial and lies on the entire frequencies spectra. This is due to the stiffening effect for the much narrower canyons which causes substantially higher amplification for higher frequencies. Consequently, the difference between the shear-bending response and pure shear response is more significant at the higher frequencies.

Figures 2.21 to 2.23 plot the distribution of the amplification along the crest for dams built in canyons with rectangular shapes of L/H ratios equal to 2, 3, and 5, respectively, evaluated at the first resonance of the dam. The maximum response
occurs at the central region along the entire length, while at the two ends of the dams the amplification values of the response are about unit.

It is also of interest to examine the distribution of the shear strain developed within the dam body, as the level of shear strains controls the amount of stiffness degradation and material hysteretic damping, associated with the level of the nonlinearity of the soil response induced during a seismic shaking. Figure 2.24 plots the distribution of the normalized shear strain \( \frac{\gamma_{yz} H}{U_b} \) developed during the shaking evaluated at the mid-section versus the normalized depth \( z/H \) for four dams built in rectangular canyons with aspect ratios \( L/H = 2, 3, 5 \) and \( \infty \) (dam under plane strain conditions). The shear strain is computed for 10% material hysteretic damping for all dams. The results suggest that the canyon geometry has little effect of the magnitude of the shear strain \( \gamma_{yz} \) developed at the mid-section as the same phenomena noticed in Chapter 2.3 for mode shapes of the response. Indeed, from the figure it can be seen that the shear strain curves show very consistent trends and similar values for all aspect ratios \( L/H \). As \( L/H \) increases, the shear strain decreases slightly. It should be noted that the shear strain \( \gamma_{yx} \) at the dam midsection is zero due to symmetry.
Figure 2.12 Midcrest Amplification for dams in rectangular canyons with L/H=2, 3, 5 versus dimensionless frequency.
Figure 2.13 Midcrest Amplification for dams in rectangular canyons versus dimensionless frequency
Figure 2.14 First term (m=1) of Midcrest Amplification for dams in rectangular canyons with L/H=2, 3, 5 versus dimensionless frequency
Figure 2.15 Midcrest Amplification for two dams built in rectangular and semi-elliptical with L/H=2
Figure 2.16 Midcrest Amplification for two dams built in rectangular and semi-elliptical with L/H=3
Figure 2.17 Midcrest Amplification for two dams built in rectangular and semi-elliptical with $L/H=5$
Figure 2.18 Midcrest Amplification for two dams built in rectangular canyon shear and shear-bending, L/H=2
Figure 2.19 Midcrest Amplification for two dams built in rectangular canyon for shear and shear bending, L/H=3
Figure 2.20 Midcrest Amplification for two dams built in rectangular canyon for shear and shear-bending, $L/H=5$
Figure 2.21 Midcrest Amplification along the crest for $L/H=2$

at the first resonance
Figure 2.22 Midcrest Amplification along the crest for $L/H=3$

at the first resonance
Figure 2.23 Midcrest Amplification along the crest for $L/H=5$

at the first resonance
Figure 2.24 Normalized shear strain at first resonance at midsection versus depth for dams in rectangular canyons with \( L/H = 2, 3, 5 \) and \( \infty \).
2.6 Summary And Conclusions

The dynamic response of dams supported by canyons of different shape have been investigated. A closed-form analytical solution has been developed for the dynamic response of concrete-face rockfill dams resting on rigid rectangular canyons assumed to respond in lateral shear and bending. The results are used to obtain more insight on the effect of different factors such as canyon narrowness, slope of dam face, shapes of the canyon on the seismic response characteristics of the dams. For this purpose, the rectangular canyon shape is particularly useful as it allows significant flexibility in modeling various degrees of canyon narrowness. The solution for the dam response is given in terms of the product of the summation of a power series and a sine function. Results have been presented for natural frequencies, modal displacement shapes, participation factors, and response to transient and steady-state harmonic base excitation for various dam length to height ratios. Parametric studies have been performed to examine the effect on the response of the length to height ratio; the slope of the dam face and effect of the canyon shape. The response for dams built in semi-elliptical and rectangular canyons has been compared.

The conclusions drawn from this study are summarized as follows:

1. The developed model for the lateral response of dams in rectangular canyons is a generalization of the shear bending beam model. For the special case of length to height ratio $L/H \rightarrow \infty$, the solution becomes that of dams under plane-strain conditions.

2. The results confirm the stiffening effect of the canyon: the natural frequencies of the dam increase as canyon narrowness increases. For aspect ratios $L/H > 5$, the natural
frequencies of the dam can be practically seen as identical to those for dams under plane-strain conditions. For dams with the same aspect ratio L/H, the natural frequencies of the dam increases as the slope of the dam becomes sharper.

3. The displacement shapes along the height of the dam in the first mode are practically independent of the aspect ratio L/H. Bigger differences are seen in higher modes. The modal shapes along the length of the dam are expressed as sine functions.

4. Results on steady-state response to harmonic excitation suggest that the high midcrest amplifications observed for high frequency motion in dams with rectangular canyon shape are the combined result of canyon narrowness and wave focusing phenomena. These high-frequency amplifications are reduced as L/H increases. As a special case, for the aspect ratio L/H \(\rightarrow \infty\), the amplification of the first resonance reduces to about 6 compared to that of L/H=2 which has a value of a little more than 10.

5. For L/H<5, the midcrest amplification is affected significantly by both the L/H ratio and the particular canyon shape. For L/H>5 the high-frequency midcrest amplification approaches that of a dam under plane-strain conditions in which the amplifying effect basically concentrate within the first three resonances.

6. The results on shear strain amplification evaluated at mid-section indicate no significant differences for L/H=2, 3 and 5. However, the shear strains corresponding to the plain strain dam are about 50% smaller.

7. The presented closed-form solution for the lateral response of dams in rigid rectangular canyons can be a valuable tool for extensive parametric studies and
preliminary design computations.
CHAPTER 3

NUMERICAL SOLUTIONS OF LATERAL RESPONSE
OF DAM IN RECTANGULAR RIGID CANYON

3.1 Introduction

In the previous chapter, the dynamic behavior of the three dimensional Concrete-Faced Rockfill Dam (CFRD) in a rectangular rigid canyon has been studied. The theoretical results in terms of the natural frequencies and steady-state response to harmonic shear wave excitation, including amplification factors, strains and stresses have been obtained based on the shear bending beam model. It is of interest to compare these results to solutions obtained using a numerical method that can model the dam body in a more rigorous way so that the closed-form solutions can be verified.

In this chapter, a numerical study of the dynamic response of the Concrete Faced Rockfill Dam in a rectangular rigid canyon is presented. The study is performed using the Finite Element Method which is widely used in the analysis and design of earth dams. This study is performed by using the general-purpose commercial finite element code ABAQUS.
(a) Perspective View of the Dam

(b) Typical Cross Section of the Dam

Figure 3.1 Finite Element Discretization of the Dam Body
3.2 Finite Element Model of the Concrete Face Rockfill Dam

Similar to the model in the theoretical study, the concrete-faced rockfill dam in the numerical study consists of a rockfill embankment covered with an upstream concrete face slab. The dam is built in a rectangular rigid canyon. The basic features of CFR dam are outlined in Figure 3.1, which shows a perspective view of the dam and a typical cross-section. The main body of the CFR dam consists exclusively of rockfill which has a material density of 2000 kg/m$^2$ and Young's Modulus of $1.109 \times 10^9$ Pa. The concrete face slab covers the entire upstream face and extends out 4.0 m horizontally at the foot and 2.5 m vertically at the top of dam. The face slab is 0.8 m thick and made of 20 MPa strength concrete. The concrete material density is 2460 kg/m$^2$ and the Young's Modulus is $1.50 \times 10^9$ Pa. The slab thickness is uniform along the height of the dam. The characteristics of the dam are summarized in Table 3.1.

There are two types of elements in the model, as shown in Figure 3.2. The main body of the dam is discretized with eight-node 3D solid elements, while the face slab and the adjacent rockfill are discretized with six-node 3D solid elements.

![Figure 3.2 Finite Elements Types Used in the Numerical Model](image)
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Table 3.1 Summary of the Geometric Characteristics and Material Properties of the Dam
3.3 Numerical Study of the CFRD

The numerical analysis consists of two steps. The first step involves the extraction of the natural modes and frequencies of the dam. Based on the extracted natural frequencies, the second step performs steady state response analysis using modal superposition, assuming as excitation harmonic shear waves traveling in the vertical direction.

3.3.1 Natural Frequencies and Modal Shapes

In order to get a clear understanding of the free vibration characteristics of the dam body, the first 40 modes are calculated for this inhomogeneous CFRD dam. Table 3.2 shows the first forty natural frequencies in rad/s and cycle/s respectively.

The results show that for the first mode, the theoretical study and numerical study give almost the same results. The numerical study gives the first natural frequency $f_{11}=10.357$ rad/s for the dam with a slope of $\alpha=1:1.8$. For a dam height $H=100$ m and shear wave velocity $V_s=400$ m/s, the dimensionless frequency $\alpha_0 = \frac{\omega_{nm}H}{V_s}$ is 2.5893. The theoretical model gives a fundamental dimensionless frequency $\alpha_0 = 2.6014$ as shown in Table 2.1. There is only a 0.4% difference between the theoretical and numerical predictions. Figure 3.3 and Figure 3.4 show the first vibrational mode in both the longitudinal and transverse directions. Note that the first mode is the most important mode representing the most significant characteristics of the dam response. Therefore, good agreement between the theoretical and numerical predictions mean that the total response will be also in agreement.

The numerical solution gives many more modes than the theoretical closed-form
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Table 3.2 First Forty Natural Frequencies of the Dam
solution. By considering the modal participation, the most important modes can be identified. For example, the 33rd mode in the numerical study represents the second mode from the closed-form solution, corresponding to dimensionless frequencies of 5.7965 and 5.5617 respectively. Figures 3.5 and 3.6 show the displacement mode shapes \( f = 3.69 \) in longitudinal and transverse directions. At higher modes the difference between the theoretical solution and numerical result increases. Figure 3.7 and Figure 3.8 show the displacement mode shapes at frequency \( f = 10.83 \) in the longitudinal and transverse directions, respectively.
Figure 3.3 First Displacement Mode Shape along the Dam Height

Figure 3.4 First Displacement Mode Shape along the Dam Length
Figure 3.5 Displacement Mode Shape at Frequency f=3.69 cycle/s along the Dam Height

Figure 3.6 Displacement Mode Shape at Frequency f=3.69 cycle/s along the Dam Length
Figure 3.7 Displacement Mode Shape at Frequency $f=10.83$ cycle/s along the Dam Height

Figure 3.8 Displacement Mode Shape at Frequency $f=10.83$ cycle/s along the Dam Length
3.3.2 Steady State Response of CFRD

In this section, a study of the steady state response of the CFRD is performed using modal superposition. The ground excitation is synchronous harmonic lateral displacement of unit amplitude. Figure 3.9 portrays the displacement time history of the input ground motion.

![Figure 3.9 Ground Motion](image)

Figure 3.9 shows the mid-section of the dam with the points at which the amplification factors are computed. Figures 3.11 to 3.15 show the amplification factors versus frequency at different heights in the mid-section. Figure 3.15 shows that the
Figure 3.10 Mid-Cross Section of the Dam

crest amplification at the first natural frequency is slightly over 10, while at the second natural frequency, it drops to about 6. These results are in agreement with the theoretical solutions shown in Figure 2.15. Figure 3.16 shows the variation of the amplification factor along the height of the dam.
Figure 3.11 Amplification at Point A
Figure 3.12 Amplification at Point B
Figure 3.13 Amplification at Point C
Figure 3.14 Amplification at Point D
Figure 3.15 Amplification at Point E
Figure 3.16 Variation of the Amplification Factor along the Height at the First Natural Frequency
3.3.3 Response of Face Slab

There is no established method to determine the effect of the deformation of the rockfill embankment on the behavior of the concrete slab. It is important to understand the dynamic behavior of the concrete slab utilizing the results of the numerical analysis.

Figure 3.17 shows a perspective view of the concrete slab with the points at which the shear stresses are computed. Figures 3.18 to 3.20 portray the variation of the shear stress versus the frequency of the base excitation at points FF, GG and HH respectively. The maximum shear stress has a sharp peak, however, at larger frequencies its value decreases significantly. For concrete slab at the top the dam in the mid-section, the shear stress has the maximum value of $2.78 \times 10^5$ Pa at $f=1.6484$ Hz, it drops to $4.9 \times 10^4$ Pa at $f=3.69$ Hz. With the frequency increases, the shear stress approaches a constant value of $2.7 \times 10^4$ Pa. Since the first mode is the most important mode, by studying the parameters of the slab at the first natural frequency, predictions of the dynamic behavior of the concrete slab can be made. It can also be noticed that the shear stress of the concrete slab increases with the height of the dam. The shear stress is 4120 Pa at height of 4.8 m, it increases to $9.2 \times 10^4$ Pa at height of 48.5 m and reaches the maximum value of $2.78 \times 10^5$ Pa at the top of the dam of the mid-section.

It is also of interest to study the lateral deformation of the slab, so the bending behavior of the concrete slab can be predicted. Figure 3.21 shows the normalized relative lateral displacement of the concrete slab with respect to the base displacement.
versus the height of the dam at the first resonance, it can be seen that the lateral displacement of the concrete slab increases with the height of the dam and reaches the maximum value of 9.63 at the top of the dam. By studying the variation of the lateral displacement and the bending-moment behavior of the concrete slab, the theoretical shear beam model can be verified which is beyond the scope of this study and
demands further investigation.

Figure 3.18 Shear Stress of the Slab at Point HH
Figure 3.19 Shear Stress of the Slab at Point GG
Figure 3.20 Shear Stress of the Slab at Point FF
Figure 3.21 Normalized Relative Lateral Displacement of the Concrete Slab with Respect to the Base Displacement vs. Height of the Dam at the First Resonance
3.4 Summary And Conclusions

A numerical study has been undertaken to investigate the dynamic response of dams supported by rigid rectangular canyons.

Results have been presented for natural frequencies, modal displacement shapes, amplification factors, displacements and shear stresses. Comparison of the results from the numerical analysis to those derived from theoretical closed-form solutions shows that:

1. The first natural frequency of the concrete faced rockfill dam computed by the theoretical closed-form solution and the more rigorous numerical method are practically identical, the error being less than 1%.

2. The amplification of the steady-state response at the mid-crest computed by the theoretical closed-form solution AF=10.1 and the numerical solution AF=10.5 are practically identical.

3. The maximum normalized deflection of the concrete slab with respect to the base displacement evaluated by the theoretical solution and the numerical solution are 9.2 and 9.65, respectively.

The results from the numerical study consistently agree with the closed-form solution.
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Santiago, Chile.


36. M. Hatanaka (1955), "Fundamental Considerations on the Earthquake Resistant Properties of the Earth Dam", Disaster Prevention Research Institute, Kyoto University Bulletins No. 11, pp1-36.


APPENDIX A

\[ \Gamma_2 = -\frac{3 + q}{12} \]

\[ \Gamma_3 = -\frac{8 + q}{72} \]

\[ \Gamma_4 = -\frac{15 + q}{240} \Gamma_2 \]

\[ \Gamma_5 = -\frac{24 + q}{600} \Gamma_3 \]

\[ \Gamma_6 = -\frac{35 + q}{1260} \Gamma_4 \]

\[ \Gamma_7 = -\frac{48 + q}{2352} \Gamma_5 \]

\[ \Gamma_8 = -\frac{63 + q}{4032} \Gamma_6 \]

\[ \Gamma_9 = -\frac{80 + q}{6480} \Gamma_7 \]
APPENDIX B

\[ \Omega_1 = \frac{1}{2} \left( \frac{\pi \cos(m\pi)}{m} \right) \]

\[ \Omega_2 = \frac{1}{3} \left( \frac{\pi \cos(m\pi)}{m} \right) \]

\[ \Omega_3 = \frac{\Gamma r_{nm}^2}{4} \left( \frac{\pi \cos(m\pi)}{m} \right) \]

\[ \Omega_4 = \frac{\Gamma r_{nm}^2}{5} \left( \frac{\pi \cos(m\pi)}{m} \right) \]

\[ \Omega_5 = \frac{\Gamma r_{nm}^2}{6} \left( \frac{\pi \cos(m\pi)}{m} \right) \]

\[ \Omega_6 = \frac{\Gamma r_{nm}^2}{7} \left( \frac{\pi \cos(m\pi)}{m} \right) \]

\[ \Omega_7 = \frac{\Gamma r_{nm}^2}{8} \left( \frac{\pi \cos(m\pi)}{m} \right) \]

\[ \Omega_8 = \frac{\Gamma r_{nm}^2}{9} \left( \frac{\pi \cos(m\pi)}{m} \right) \]

\[ \Omega_9 = \frac{\Gamma r_{nm}^2}{10} \left( \frac{\pi \cos(m\pi)}{m} \right) \]
\[ \Omega_{10} = \frac{\Gamma_{9 r_{nm}}}{11} \left( -\frac{\pi \cos(m \pi)}{m} \right) \]

\[ \Psi_1 = \frac{1}{2} \left( \frac{1}{8m^2} + \frac{2\pi m^2 - \cos(2m \pi)}{8m^2} \right) \]

\[ \Psi_2 = \frac{2q}{3} \left( \frac{1}{8m^2} + \frac{2\pi m^2 - \cos(2m \pi)}{8m^2} \right) \]

\[ \Psi_3 = \frac{q^2 + 2\Gamma_{2 r_{nm}}}{4} \left( \frac{1}{8m^2} + \frac{2\pi m^2 - \cos(2m \pi)}{8m^2} \right) \]

\[ \Psi_4 = \frac{2q(\Gamma_2 + \Gamma_3) r_{nm}}{5} \left( \frac{1}{8m^2} + \frac{2\pi m^2 - \cos(2m \pi)}{8m^2} \right) \]

\[ \Psi_5 = \frac{(\Gamma_{2 r_{nm}}^2 + 2q\Gamma_3 + 2\Gamma_4 r_{nm}) r_{nm}}{6} \left( \frac{1}{8m^2} + \frac{2\pi m^2 - \cos(2m \pi)}{8m^2} \right) \]

\[ \Psi_6 = \frac{2q r_{nm}^2 (\Gamma_2 \Gamma_3 + \Gamma_4 + \Gamma_5)}{7} \left( \frac{1}{8m^2} + \frac{2\pi m^2 - \cos(2m \pi)}{8m^2} \right) \]

\[ \Psi_7 = \frac{q^2 r_{nm}^2 (\Gamma_{3 r_{nm}}^2 + 2\Gamma_5 + 2\Gamma_2 \Gamma_4 r_{nm} + 2\Gamma_6 r_{nm}) r_{nm}}{8} \left( \frac{1}{8m^2} + \frac{2\pi m^2 - \cos(2m \pi)}{8m^2} \right) \]

\[ \Psi_8 = \frac{2q r_{nm}^3 (\Gamma_2 \Gamma_5 + \Gamma_3 \Gamma_4 + \Gamma_6 + \Gamma_7)}{9} \left( \frac{1}{8m^2} + \frac{2\pi m^2 - \cos(2m \pi)}{8m^2} \right) \]
\[ \Psi_9 = \frac{r_{nm}^3 (2q^2(\Gamma_3 \Gamma_5 + \Gamma_7) + \Gamma_4^2 r_{nm} + 2\Gamma_2 \Gamma_6 r_{nm} + 2\Gamma_8 r_{nm})}{10} \left( \frac{1}{8m^2} + \frac{2\pi m^2 - \cos(2m\pi)}{8m^2} \right) \]

\[ \Psi_{10} = \frac{2qr_{nm}^4 (\Gamma_4 \Gamma_5 + \Gamma_3 \Gamma_6 + \Gamma_2 \Gamma_7 + \Gamma_8 + \Gamma_9)}{11} \left( \frac{1}{8m^2} + \frac{2\pi m^2 - \cos(2m\pi)}{8m^2} \right) \]