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MECHANICAL BEHAVIOR OF TWO-DIMENSIONAL
DRILL STRINGS IN INCLINED HOLES

by

SERGIO E. FIRPO

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

MASTER OF SCIENCE

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March, 1986
MECHANICAL BEHAVIOR OF TWO DIMENSIONAL
DRILL STRINGS IN INCLINED HOLES

SERGIO E. FIRPO

ABSTRACT

A two dimensional model of the deflected configuration of the bottom hole assembly of a drill string is developed in this thesis. The analytical solution of the governing equation of the beam-column problem is treated in matrix form using the transfer matrix method which permits the solution of boundary value problems.

A simple, easy to use computer method is developed to solve problems including gravity effects in inclined holes, and multiple stabilizers in straight and curved holes, and the effect of approximating the actual variable compressive axial force with a constant force is studied. Results of the program describe the mechanical behavior of the drill string by solving for the drill string deflection, slope, moment and shear force at any point from the drill bit to the tangency point. From these results, stress and strain at any point, and the resultant force at the bit can easily be computed.

Accuracy of the results, for a multiple stabilizers case, is verified by two alternative methods. Simple-beam theory is used to verify the results for the limiting case of a horizontal hole, and the results of the beam-column problem are verified using a computer library, Continuous System Modeling Program (CSMP), which utilizes a Runge-Kutta integration method.
ACKNOWLEDGEMENT

The author wishes to express his sincere appreciation for the advice and assistance of Dr. John B. Cheatham, Jr. throughout this research. The author also wishes to thank the members of the Drilling Engineering Association, Dr. D. Hearn, ARCO Resource Technology, Mr. R. W. Pittman and Mr. R. W. Hall, Jr., Texaco Inc., Dr. C. E. Murphey and Mr. R. Millhone, Chevron, for financing this research. Finally, he is extremely grateful to Linda Anderson for her careful and accurate typing (and retyping!) of this thesis.
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Nomenclature

\( \alpha \) = acute angle between hole axis and the vertical
\( \beta \) = positive constant
\( \gamma \) = weight per unit length
\( \lambda \) = positive constant
\( \epsilon \) = constant
\( \phi \) = angle between resultant force at the bit and the vertical
\( \Theta \) = beam or rod slope
\( EI \) = bending stiffness
\( L \) = drill string length from the bit to tangency point
\( L_i \) = location of the i-th stabilizer
\( M \) = bending moment
\( P \) = compressive axial force
\( r \) = effective radius of the hole
\( R \) = reaction force
\( S \) = state vector = \([y \ \Theta \ M \ V \ 1]\)^T
\( S_i \) = state vector at \( x = x_i \)
\( U \) = transfer matrix
\( U_{ij} \) = i-th row and j-th column matrix element
\( V \) = shear force
\( x \) = distance along hole axis
\( y \) = beam deflection
GLOSSARY

Bottom-Hole Assembly: A particular combination of drill bit, drill collars and stabilizers at the lower section of a drill string.

Drill collars: Heavy drill pipe used in the bottom-hole assembly to provide the weight on bit.

Drill string: Combination of Bottom-Hole Assembly and drill pipe used in drilling operations.

Dog-Leg: A sharp change in the direction of the borehole.

Key seats: A groove of appreciable length worn into the wall of the hole by the tool joints of the drill string.

Stabilizer: A tool of diameter larger than the drill-collars used to centralize the drill collars in the borehole.
1.0 INTRODUCTION

Many authors [1-4] studying the problem of predicting the deflection of a drill string have stated the importance of controlling hole deviation during drilling operations. The most important aspects are hole trajectory prediction, the prevention of severe dog legs which may cause the formation of key-seats and excessive wear on drill collars, and the prevention of crooked holes and twist-off in the drill pipe. A primary objective of this work is to provide an improved theoretical basis for engineering calculation of the mechanical behavior of the bottom-hole assembly of a drill string. This study can assist in providing better definitions of the operational limits for drill strings by relating how the stresses relate to the hole and drill string configurations.

The complete study will include analysis of the following:
Straight holes, inclined from the vertical
Hole curvature, increasing and decreasing
Gravity effects
Constant and variable axial force
Different drill collar sizes
Multiple stabilizers in straight and curved holes.
Torque effects on helical buckling

In most of this work the drill string deflections and the borehole geometry are considered to be contained in a single vertical plane. The procedure developed here yields the values of the deflection, slope, moment and shear force at every point of the drill string.
2.0 REVIEW OF PREVIOUS WORK

Lubinski and Woods [1] considered the problem of the deflection of the drill string in an inclined, straight hole. Their results, showing the relationship between hole diameter and drill-collar length, are used in the present work to verify preliminary results. The effect of stabilizers was also analysed by Lubinski and Woods [2] in straight, inclined holes. It was established that a straight, vertical hole is very expensive and difficult to drill.

B. A. Walker and M. B. Friedman [3] developed a three dimensional mathematical model of a drill string. Deflections were constrained within a cylinder with straight axis. In their work the drill string was divided into small elements, for which the solution of the linear differential equation was written. The solution for each element is "shot" from the bit to the top of the bottom-hole assembly using a procedure based on combinations of independent solutions of ordinary differential equations. In this method eight solutions and their first, second and third derivatives are saved for every element.

A Finite Element Method was used by Millheim [10] to analyse the behavior of bottom-hole assemblies. His method of solution uses a commercial, general-purpose, finite element program. Effects of stabilizers were considered in straight holes only. In analysing curved holes without stabilizers some discrepancies with actual field data were found. This was resolved using a curved-beam, finite element representation. It was suggested that "the curved-beam formulation using the nonlinear foundation option should serve as the point of departure if curved borehole assemblies are analyzed".
The most complete study of drill strings in two dimension was presented by J. Fisher [4] using a finite difference method. Deflections, moments and forces were computed for complex bottom-hole assemblies in straight and curved holes. His method involved the iterative solution of pentadiagonal matrices.

All of the numerical procedures described above are based on dividing the total length of the drill string into small elements. Accuracy of these methods depends on the size of the mesh chosen, which also determines the number of computations and storage required.

W. Bradley [12] analyzed the use of heavy, stiff bottom collars in controlling hole deviation and increasing penetration rate. He used a transfer matrix approach to study the section of drill collars immediately above the bit. This analysis was restricted to straight holes, first without stabilizers and then with a single stabilizer.

In the present work the emphasis is on developing a simple procedure, requiring few computations, for the solution of more general two-dimensional drill string problems. Like Bradley a transfer matrix approach is used, but here the analysis is extended to consider curved holes and multiple stabilizers.

3.0 ANALYSIS OF A MULTIPLE STABILIZER DRILL STRING

The bottom-hole assembly of a drill string consists of a drill bit, a combination of different drill collars which provide the required weight on bit and a variable number of stabilizers (Fig. 1). Stabilizers of various lengths and diameters are used, to restrict the deflection and inclination of the drill string.
Schematic of a drill string

Figure 1
A two-dimensional model of the deflected configuration is developed in this thesis. The model is based on a number of assumptions which are described in the next two sections.

3.1 Idealization of the drill string

In this study the drill string is assumed to be inextensible, elastic and piecewise constant diameter, and the bit is centralized in the borehole and exerts no bending moment on the formation. Stabilizers only restrict the deflection of the drill string. The clearance of a stabilizer is the difference between the stabilizer and hole radii. For simplicity, stabilizers with zero clearance are considered.

The maximum drill string deflection is limited to the difference between the bit and drill collar radii, which is termed the effective radius of the hole. Since the effective radius of the hole is of the order of a few inches, a very good approximation is to consider the length of the drill string the same as the length of the borehole axis. Above the point of contact between the drill collars and the borehole wall, the drill collars are assumed to remain in contact with the hole. The borehole wall is parallel to the borehole axis, which lies entirely within a single vertical plane. Constant hole curvature is considered. Since the maximum drill string deflection is very small compared with its length, slender beam theory applies.
Two dimensional drill string with two stabilizers
3.2 Idealization of Loading

The loads considered are a constant compressive axial force, a uniformly distributed load acting in the vertical direction due to the drill collars' weight per unit length, and lateral concentrated forces acting at the bit and stabilizers. The effect of torque is studied in Appendix IX and it is found to be negligible compared with the effect of the forces described above. Influences of rotation and fluid dynamics are not considered and the contact between the drill string and the borehole wall is considered frictionless.

3.3 Beam Column Governing Equation and General Solution

The bottom section of a drill string under the effect of a compressive axial load constitutes a beam-column problem. For a practical situation, the drill string deflection is of the order of a few inches, in a beam length of several hundred inches, therefore, the curvature of the beam can be written as

\[
\frac{1}{p} = \frac{\frac{d^2y}{dx^2}}{[1 + (\frac{dy}{dx})^2]^{3/2}} \approx \frac{d^2y}{dx^2}
\]

since

\[\frac{dy}{dx} \ll 1.\]

The governing equation is the nonhomogeneous differential equations:

\[
\frac{d^4y}{dx^4} + \beta^2 \frac{d^2y}{dx^2} = \lambda
\]

[1]
where:

\[ \beta^2 = \frac{P}{EI}; \quad \text{and} \quad \lambda = \lambda_0 \frac{\sin \alpha}{EI} \]

are constant and

- \( P \) is the compressive axial force
- \( EI \) is the bending stiffness
- \( \lambda_0 \) is the drill collars' weight per unit length
- \( \alpha \) is the angle of inclination with respect to the vertical
- \( y \) is the lateral or "radial" deflection
- \( x \) is the distance along the hole axis.

The general solution of equation \( 1 \) is

\[ y = C_1 + C_2x + C_3 \sin \alpha + C_4 \cos \alpha + \frac{\lambda}{\beta^2} \frac{x^2}{2} \quad [2] \]

which combines the solution for the homogeneous equation with a particular solution for the nonhomogeneous equation. The constants \( C_1 \), \( C_2 \), \( C_3 \) and \( C_4 \) are determined using the boundary conditions of the problem.

3.4 **Boundary Conditions**

A Cartesian coordinate system is used with the origin at the bit. The \( x \)-axis is taken along the axis of the hole and positive \( y \) deflection is downward as shown in Figure [2]. At the drill bit we have a pin-end condition, centralized in the borehole. The boundary conditions are

\[ M(0) = -EI \frac{d^2y}{dx^2} = 0; \quad \text{and} \quad y(0) = 0 \]
Above the top stabilizer, the drill collars contact the borehole wall at a point called the tangency point. Boundary conditions for the most general case are:

\[ M(L) = -EI \frac{d^2 y}{dx^2} = -\frac{EI}{\rho}, \]

\[ y(L) = \rho(1 - \cos \frac{L}{\rho}) \pm r, \]

and

\[ \theta(L) = \frac{L}{\rho} \]

where

- \( r \) is the effective radius of the hole.
- \( L \) is the distance from the bit.
- \( \rho \) is the radius of curvature of the hole.
- \( \theta \) is the slope of the hole.

In the particular case of a straight hole, \( \rho = \infty \) and the boundary conditions reduce to:

\[ y(L) = r \]
\[ \theta(L) = 0 \]
\[ M(L) = 0 \]

3.5 Solution Procedure

The boundary conditions at the bit arranged in vector form are considered as the initial parameters of the problem. This vector \( S \), comprised of state variables (displacement \( y \), slope \( \theta \), moment \( M \) and shear force \( V \)) is called the state vector.
A transfer matrix is used to transfer the state variables, \( S_0 \), at \( x = 0 \) to any axial location \( x = x_i \) according to the following equation:

\[
S_i = U S_0 \tag{3}
\]

In a drill string with two stabilizers, as in Figure 2, \( U_1 \) is a field matrix, presented in Appendix I, which transfers the state vector from the bit to the left of the first stabilizer:

\[
S_1 = U_1 S_0 \tag{4}
\]

\( U_2 \) is point matrix, presented in Appendix II, that considers the effect of the concentrated force at the first stabilizer. The state vector to the right of the first stabilizer is:

\[
S_2 = U_2 S_1
\]

and replacing \( S_1 \) gives

\[
S_2 = U_2 U_1 S_0
\]

Considering the complete drill string we have

\[
S = U_5 U_4 U_3 U_2 U_1 S_0 \tag{5}
\]

where \( U_5, U_3, U_1 \) are field matrices for the drill collars segments, and \( U_4, U_2 \) are point matrices at the stabilizers. The distance between the top stabilizer and the tangency point is unknown. An initial guess is used to generate the first approximation for the field matrix \( U_5 \). Performing the matrix multiplication, equation [5], can be written as:
\[ S = U S_0 \]  

where \( U \) is the overall transfer matrix. From the right side of equation [6], we can write an expression for the moment at the tangency point where \( y_0 \) and \( M_0 \) are known and \( \theta_0 \) and \( V_0 \) the unknown. Thus

\[ S(3) = M(L) = y_0 U(3,1) + \theta_0 U(3,2) + M_0 U(3,3) + V_0 U(3,4) + U(3,5) \]  

\[ = - \frac{EI}{\rho} \]

From equation [3], the expression for the displacement at the first stabilizer is:

\[ y_1 = y_0 U(1,1) + \theta_0 U(1,2) + M_0 U(1,3) + V_0 U(1,4) + U(1,5) = 0 \]

Equations 7 and 8 can be solved for the unknowns, \( \theta_0 \) and \( V_0 \).

With the initial state vector known, it is possible now to calculate the state vector at any point of the drill string. At the tangency point, the tangency condition is used to iterate on the length of the drill collars in the top section.

The inclination of the hole at the tangency point is

\[ \theta_h = \frac{L}{\rho} \]

if \( \theta \) is the slope of the drill collars at the same point then

\[ \theta = \theta_h \]

An iteration procedure is considered with the condition
\[ |\theta - \theta_n | < \varepsilon \]

for \( \varepsilon \) sufficiently small as the convergegence criterium. The BASIC PROGRAM presented in Appendix III can be used to apply this transfer matrix method to the analysis of different drill string configurations in inclined straight and curved holes.

4.0 RESULTS

The transfer matrix method is used as the basis for the computer programs STAB, TANGENCY and TANG-VAR, presented in Appendices III, VII and VIII respectively. Program STAB analysis considers the effects of multiple stabilizers, in straight and curves holes with a constant axial force. The input data required defines the drill string configuration, the type of hole and the operating conditions. The drill string is defined by:

\( r \): the effective radius of the hole resulting of the combination of drill bit and drill collars used.

\( n \): the number of stabilizers

\( L_1, \ldots, L_n \): the location of the stabilizers

\( I_1, \ldots, I_n \): the moment of inertia for each section of drill collars

The type of hole is defined by:

\( \alpha \): the hole inclination with respect of the vertical

\( c \): the deviation rate for the cases of curved holes.

The operating conditions are defined by:

\( P \): the weight on bit desired
\( \lambda_0 \): the weight per unit length of drill collars considering the mud weight. In the numerical examples this value is taken to be equal to the weight in air, which would be the case when drilling with air or gas as the drilling fluid. The numerical examples assume an 8-3/4 in. bit and 6-1/4 in. drill collars with full gauge stabilizers.

A first case of a drill string with only 1 lb axial force can be useful for verifying the accuracy of the results by comparing with a simple beam solution. The statically indeterminate beam problem of a drill string with two stabilizers is solved in Appendix V using the principle of superposition. Its results agree with those given by the transfer matrix approach. Results are tabulated in Table I.

The accuracy of the results for a beam-column problem is verified using a computer library called "Continuous System Modeling Program", CSMP, in which a variable step Runge-Kutta integration method is utilized. Since Runge-Kutta is an initial value solution the results of the transfer matrix approach at the bit are used to start the integration. A drill string with two stabilizers located at 500" and 1000" above the bit, in a straight hole, is used as a numerical example. Results are tabulated in Table II. These results show excellent agreement. A detailed computer output is presented in Appendix VI.

Variation of the compressive axial force is analyzed in Appendix VIII, where the transfer matrix approach is used to obtain a numerical approximation to the exact solution of the non-linear differential equation defining the problem of variable axial force. Results of this
Table I

COMPARISON OF RESULTS USING TRANSFER-MATRIX AND SIMPLE-BEAM THEORY

Horizontal Hole, 8 3/4 IN. BIT, 6 1/4 in. Drill Collars

<table>
<thead>
<tr>
<th></th>
<th>Transfer Matrix</th>
<th>Beam Theory</th>
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<tbody>
<tr>
<td>weight-on-bit (lb)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>hole curvature (deg/100ft)</td>
<td>+3(INC)</td>
<td>+3(INC)</td>
</tr>
<tr>
<td>number of stabilizers</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>L(1):in</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>L(2):in</td>
<td>382.5</td>
<td>382.4</td>
</tr>
<tr>
<td>slope at the tangency point (rad)</td>
<td>0.05159</td>
<td>0.05158</td>
</tr>
</tbody>
</table>
Table II

COMPARISON OF RESULTS USING TRANSFER MATRIX AND CSMP

8 3/4 in. Bit and 5/3/4 in. Drill Collars

\( \text{Alpha} = 90 \text{ Deg; } R = 1.5 \text{ in.} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Transfer Matrix</th>
<th>CSMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>weight-on-bit (lb)</td>
<td>25,000</td>
<td>25,000</td>
</tr>
<tr>
<td>hole curvature (Deg/100')</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>number of stabilizers</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>( L(1) ) (in)</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>( L(2) ) (in)</td>
<td>500</td>
<td>500</td>
</tr>
<tr>
<td>( L(3) ) (in)</td>
<td>402.29</td>
<td>402.5</td>
</tr>
<tr>
<td>deflection (in)</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>slope at the tangency point (Rad)</td>
<td>5.73 E-6</td>
<td>5.91 E-6</td>
</tr>
<tr>
<td>shear force at tangency point (lbs)</td>
<td>-810</td>
<td>-801.0</td>
</tr>
</tbody>
</table>
Table III

TRANSFER MATRIX RESULTS USING A VARIABLE
AND A CONSTANT AXIAL FORCE

8 3/4 in. bit and 6 1/4" Drill Collars

$\alpha = 30\ \text{Deg};\ R = 1.5\ \text{in.};\ P = 15,000\ \text{lb}$

VARIABLE AXIAL FORCE

<table>
<thead>
<tr>
<th>Hole Curvature</th>
<th>Length of Tangency Section (in)</th>
<th>Slope at the Bit (Rad)</th>
<th>Shear Force at the Bit (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>392.2</td>
<td>0.00764</td>
<td>-439</td>
</tr>
<tr>
<td>$+3$ (INC)</td>
<td>447.6</td>
<td>0.002286</td>
<td>-562</td>
</tr>
<tr>
<td>$-3$ (DEC)</td>
<td>335.5</td>
<td>-0.00322</td>
<td>-219</td>
</tr>
</tbody>
</table>

CONSTANT AXIAL FORCE

<table>
<thead>
<tr>
<th>Hole Curvature</th>
<th>Length of Tangency Section (in)</th>
<th>Slope at the Bit (Rad)</th>
<th>Shear Force at the Bit (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>390.9</td>
<td>0.00767</td>
<td>-433</td>
</tr>
<tr>
<td>$+3$ (INC)</td>
<td>442.8</td>
<td>0.00228</td>
<td>-543</td>
</tr>
<tr>
<td>$-3$ (DEC)</td>
<td>335.9</td>
<td>-0.00322</td>
<td>-223</td>
</tr>
</tbody>
</table>
approach are then compared with a similar case using a constant axial force. The axial force of 15,000 lb is at the lower range of normally used values of weight on bit. Results are tabulated in Table III.

A simple drill string without stabilizers is used to compare the results with results published previously by Lubinsky [1]. Dimensionless units were used in finding the drill string length corresponding to a given hole size. In Figure VII-4, results for a constant and a variable axial force are shown. The marks (+) correspond to data published by Lubinski [1], and the curves correspond to the analyses presented in Appendices VII and VIII. It can be seen that the variable axial force results agree with the data published previously.

Different drill strings with multiple stabilizers have been analyzed in straight and curved holes with different weights on bit. The results for the length of the tangency section are shown in Table IV. In the first case for a drill string with a single stabilizer located 400 in. above the bit, the tangency point is found to be 394.25 in above the stabilizer.

A drill string with two stabilizers, located at 400" and 800" above the bit is used as an example of a multiple stabilizer problem. Results for the length of the tangency section are presented in Table IV. Once the tangency point is located it is possible to compute the displacement, slope and shear force at every point along the drill string from the bit to the tangency point. Detailed computer outputs are presented in Appendix IV, and the deflection of the drill string is plotted in the following figures. Figures 3, 4, and 5 correspond to an 8 3/4" bit, 6 1/4" drill collars with one stabilizer placed 500 in.
Table IV
LENGTH OF TANGENCY SECTION OF
A 6 1/4 IN. DRILL COLLAR STRING
IN HORIZONTAL HOLES

Alpha = 90 Deg, R = 1.25 in.

<table>
<thead>
<tr>
<th>weight-on-bit (lb)</th>
<th>hole curvature (Deg/100')</th>
<th>number of stabilizers</th>
<th>L(1) (in)</th>
<th>L(2) (in)</th>
<th>L(3) (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>400</td>
<td>394.25</td>
<td></td>
</tr>
<tr>
<td>25,000</td>
<td>0</td>
<td>1</td>
<td>400</td>
<td>389.08</td>
<td></td>
</tr>
<tr>
<td>50,000</td>
<td>0</td>
<td>1</td>
<td>400</td>
<td>384.92</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>2</td>
<td>400</td>
<td>400</td>
<td>378.30</td>
</tr>
<tr>
<td>1</td>
<td>+3(INC)</td>
<td>2</td>
<td>400</td>
<td>400</td>
<td>382.40</td>
</tr>
<tr>
<td>1</td>
<td>-3(INC)</td>
<td>2</td>
<td>400</td>
<td>400</td>
<td>374.01</td>
</tr>
<tr>
<td>25,000</td>
<td>0</td>
<td>2</td>
<td>400</td>
<td>400</td>
<td>368.76</td>
</tr>
<tr>
<td>25,000</td>
<td>+3(INC)</td>
<td>2</td>
<td>400</td>
<td>400</td>
<td>361.95</td>
</tr>
<tr>
<td>25,000</td>
<td>-3(DEC)</td>
<td>2</td>
<td>400</td>
<td>400</td>
<td>377.16</td>
</tr>
</tbody>
</table>
Dimensionless Drill String Length for Different Hole Sizes

Const.-Var. axial force.

Straight hole
Dimensionless axial force \( p = 2 \)

+ Data published by Lubinski and Wood [1]

Figure VII-4
Figure 3
Figure 4

- $P = 2.0 \times 10^4$ lbs

Deflection (in)
Figure 7

DRILL STRING DEFLECTION

DEFLECTION (in)

DISTANCE FROM THE BIT (in)

$P = 2.0 \times 10^4 \text{ lbs}$
DRILL STRING DEFLECTION

DISTANCE FROM THE BIT (in)

DEFLECTION (in)

--- P = 4.0E4 lbs,

Figure 8
above the bit, with one, 20,000 and 40,000 lbs of weight on bit, respectively. In Figures 6, 7, and 8 the same drill string is considered in a curved hole. The curvature is +3° (INC) Deg/100", where INC denotes increasing hole angle from tangency point to the bit.

5.0 DISCUSSION

Accuracy of the results of the procedure presented here is verified for the problem of a simple drill string without stabilizers by comparison with the results published by Lubinski [1]. In a more complex problem of a drill string with stabilizers simple-beam theory and CSMP verification showed no significant discrepancies.

Determination of the proper boundary conditions is very important in the correct definition of the problem. A pin-ended condition at the bit is considered by many authors [1-4] based on the assumption that the bit exerts no bending moment on the formation at the bit face. At a certain distance above the bit the drill collars contact the borehole wall and the drill string remains in contact with the wall above this point. This condition provides the remaining boundary conditions. In curved holes the value of the moment is determined by the hole curvature. The unknown location of the tangency point makes solving the problem more difficult. Three boundary conditions are known at the tangency point; two are used in the solution for the initial parameters and the remaining one is used in determining the location of the tangency point by iteration. Choosing the slope of the drill string as the boundary condition to be satisfied in the iterative process proved to be the most effective way to achieve fast convergence. The number of
iterations required for convergence is printed at the end of the results. Fewer than 10 iterations were required in all the cases analyzed. In each iteration the complete procedure required the multiplication of small matrices, \([5 \times 5]\), and the solution of a system of two equations with two unknowns. The total number of matrix multiplications is equal to the number of different elements considered in the drill string configuration. Once the correct location of the tangency point is determined, the state vector can be computed at any point along the length of the drill string. These results permit analysis of the stresses and strains along the entire length of the drill string. The value and location of the maximum stress is important in considering different drill string configurations for the same operating conditions. In Appendix IV, results for a drill string with a single stabilizer 400" above the bit, in a straight-hole with 25,000 lbs weight on bit shows the maximum bending moment located at the stabilizer equal to 161,600 lb-in. The corresponding stress is:

\[
\sigma = \frac{Mc}{I} + \frac{P}{A}
\]

\[M = 161,600 \text{ lb-in}, \text{ bending moment}\]
\[C = 3.125 \text{ in}, \text{ radius of the drill collars}\]
\[I = 70.9 \text{ in}^4, \text{ moment of inertia}\]
\[P = 25,000 \text{ lbs}, \text{ axial force}\]
\[A = 7.52 \text{ in}^2, \text{ drill collar cross section}\]
\[\sigma = 10,447 \text{ lb/in}^2\]
For the two-stabilizer drill string with the same weight on bit the maximum bending moment is 125,400 lb-in and the stress is

$$\sigma = 8,850 \text{ lb/in.}$$

Corresponding strains are

$$\varepsilon_{1st} = \frac{\sigma}{E} = 3.48 \times 10^{-4} \text{ in/in}$$

$$\varepsilon_{2nd} = \frac{\sigma}{E} = 2.95 \times 10^{-4} \text{ in/in}$$

where $E = 3 \times 10^7 \text{ lb/in}^2$, Young's modulus.

Past theoretical studies have assumed that the bit drills in the direction of the resultant force on the bit in uniform or isotropic formations [13]. The lateral force acting at the bit, $V$ is given by the state vector. The inclination of the resultant force is calculated with the following expression

$$\tan(\phi - \alpha) = \frac{V}{P}$$

where:  
$\phi$ is the angle between the resultant force and the vertical  
$\alpha$ is the hole inclination  
$P$ is the weight on bit.

The post buckled configuration of a rod constrained within a circular cylinder, subjected to a compressive axial load and external torque was studied. In this post buckled configuration the rod takes a helical shape. The contribution of torque to that configuration is found to be negligible compared to the contribution of the axial force.
6.0 SUMMARY AND CONCLUSIONS

In the present work a simple, easy to use computer method was developed for solving general two dimensional drill string problems. The effect of gravity on the drill string configuration in inclined holes was included by considering the weight per unit length of drill collars in air. Mud weight can be included easily by considering the buoyancy forces.

Simple drill strings without stabilizers were used in straight inclined and curved holes. Results were presented in dimensional and dimensionless form. The effects of multiple stabilizers in both straight inclined and curved holes were considered. This study permitted the evaluation of the ability of the procedure to be used to analyze more complex configurations without any increase in the difficulties in the solution procedure.

In this work the general approach requires multiplication of 5 x 5 field and point matrices where the number of matrices is determined by the number of different elements of the drill string. In determining the initial unknown parameters a small system of only two equations in two unknowns has to be solved. Development of the field matrix was based on the solution of the differential equation for this particular beam-column problem, and the point matrix was developed based on the deflection of the next support. This feature added an extra term to the point matrices presented in reference [5].

Results describe the drill string configuration from the bit to the tangency point giving at any point along the drill string the deflection, slope, bending moment and shear force, from which one can
compute the maximum stress for the deflected drill string and the resultant force at the bit. From the direction of this resultant force the direction in which the bit will drill can be determined in isotropic formations.

Since including the variation of the axial force in the calculations produces results that are only slightly different from those obtained by assuming a constant axial force, this latter assumption can usually give satisfactory results. Calculations made for the constant axial force case are considerably easier than those involving varying axial force.

Torque was found to have a small effect on the post buckled helical configuration of a rod constrained within a cylinder subjected to an axial force and external torque. This result can be used for the following three dimensional study. This present study is intended to serve as a starting point for a comprehensive study of the three-dimensional drill string problem. From the future research limits of applicability of the present results can be established.
REFERENCES


FIELD MATRIX ANALYSIS

The general solution of the beam-column problem can be written in matrix form to facilitate the treatment of complex nonuniform beams. The nonhomogeneous governing differential equation is

$$\frac{d^4y}{dx^4} + \lambda^2 \frac{d^2y}{dx^2} = \gamma$$

where $x$ is the distance along the beam axis and $y$ is the beam deflection. In the above equation $\lambda$ is defined as:

$$\lambda^2 = \frac{p}{EI}$$

and $\gamma$ is defined as

$$\gamma = \gamma_0 \frac{\sin \alpha}{EI}$$

where $P$ is the compressive axial force and $EI$ the beam bending stiffness, $\gamma_0$ is the weight per unit length of the beam and $\gamma$ is the component normal to the beam axis, and $\alpha$ is the angle between the beam axis and the vertical. Both $\lambda$ and $\gamma$ are constant over the length of the segment considered. Since the differential equation is linear, the solution is found by adding a particular solution to the solution of the homogeneous differential equation. The solution, with $\gamma = 0$ is:
\[ y_c = c_1 + c_2 x + c_3 \sin \lambda x + c_4 \cos \lambda x . \]

A particular solution of the complete equation is
\[ y_p = \frac{\gamma}{\lambda^2} \frac{x^2}{2} . \]

Thus, the general solution of nonhomogeneous equation
\[ y = c_1 + c_2 x + c_3 \sin \lambda x + c_4 \cos \lambda x + \frac{\gamma}{\lambda^2} \frac{x^2}{2} . \]

The slope, moment and shear force can be put as:
\[
\begin{align*}
\theta &= -\frac{dy}{dx} = -c_2 - \lambda c_3 \cos \lambda x + \lambda c_4 \sin \lambda x - \frac{\gamma x}{\lambda^2} \\
M &= -EI \frac{d^2y}{dx^2} = Pc_3 \sin \lambda x + Pc_4 \cos \lambda x - \frac{EI}{\lambda^2} \gamma \\
V &= -EI \frac{d^3y}{dx^3} - P \frac{dy}{dx} = Pc_2 - \frac{\gamma}{\lambda^2} x .
\end{align*}
\]

This is transformed into initial parameter form by solving for the initial values \( y_0, \theta_0, M_0 \) and \( V_0 \)
\[
\begin{align*}
y_{x=0} &= y_0 = c_1 + c_4 \\
\theta_{x=0} &= \theta_0 = -c_2 - \lambda c_3 \\
M_{x=0} &= M_0 = Pc_4 - \frac{\gamma}{\lambda^2} \\
V_{x=0} &= V_0 = -Pc_2 .
\end{align*}
\]
Solving for the constants c's and substituting in the equations above yields in matrix form:

\[
\begin{bmatrix}
  y \\
  \theta \\
  M \\
  V \\
  1
\end{bmatrix} =
\begin{bmatrix}
  1 & \frac{s_\lambda x}{\lambda} & \frac{c_\lambda x - 1}{P} & \frac{1}{P}\left(\frac{s_\lambda x}{\lambda} - x\right) & \frac{y}{4EI\lambda}\left(1 - c_\lambda x - \frac{x^2}{2}\right) \\
  0 & c_\lambda x & \frac{1}{P}(1 - c_\lambda x) & \frac{y}{EI\lambda}\left(x - s_\lambda x\right) \\
  0 & \frac{PS_\lambda x}{\lambda} & c_\lambda x & \frac{s_\lambda x}{\lambda} & \frac{y}{\lambda^2}\left(1 - c_\lambda x\right) \\
  0 & 0 & 0 & 1 & -\gamma x \\
  0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
y_0 \\
\theta_0 \\
M_0 \\
V_0 \\
1
\end{bmatrix}
\]

where \( s = \sin _\lambda x \) and \( c = \cos _\lambda x \). Denoting by \( S \) a state vector consisting of the deflection, slope, moment and shear force we can write

\[
S_1 = US_0
\]

where \( S_0 \) is the state vector at \( x = 0 \), and \( U \) the field matrix that transfers the state vector to \( x = x_1 \).
APPENDIX II

POINT MATRIX ANALYSIS

Point Matrices consider the effect of concentrated occurrences, such as supports or concentrated forces on a continuous beam. Stabilizers of a drill string are considered as simple supports. For the case of full gauge stabilizers the centerline of the drill string coincides with the hole centerline.

At the stabilizer, where the deflection is $Y_s$, a Point Matrix is developed to solve for the unknown reaction $R$. For the drill string analysed only one type of point matrix is required. Figure II-1 shows a full gauge stabilizer and the coordinate system. The equation for the deflection at $x = L$, written in terms of the components of the field matrix to the right of the support and the initial parameters at the support is:

$$Y_L = Y_s U_{11} + \theta_s U_{12} + M_s U_{13} + (V_s + R)U_{14} + U_{15}$$

where $Y_s$, $\theta_s$, $M_s$ and $V_s$ are the deflection, slope, moment and shear force at the support. $R$ is the unknown reaction at the support.

Solving for $R$

$$R = -Y_s \frac{U_{11}}{U_{14}} - \theta_s \frac{U_{12}}{U_{14}} - M_s \frac{U_{13}}{U_{14}} - V_s \frac{U_{15}}{U_{14}}.$$

Two dimensional drill string model

Figure II-1
This can be expressed in matrix form as

\[
\begin{bmatrix}
Y_s^+ \\
\theta_s^+ \\
M_s^+ \\
V_s^+ \\
1
\end{bmatrix}
= 
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
\frac{U_{11}}{U_{14}} & \frac{U_{12}}{U_{14}} & \frac{U_{13}}{U_{14}} & 0 & \frac{Y_L - U_{15}}{U_{14}} \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
Y_s \\
\theta_s \\
M_s \\
V_s \\
1
\end{bmatrix}
\]

The left-hand side expresses the state vector immediately to the right of the support. One row is added to agree with the field matrix presented in Appendix I.

Notation: \( U_{ij} \) is the element of the i-th row and j-th column of the transfer matrix to the right of the support.
APPENDIX III
MULTIPLE-STABILIZER COMPUTER LISTING

A BASIC computer program given in this Appendix uses the Transfer Matrix procedure for solving the two-dimensional model presented in the main text. This program solves for the state vector of any drill string consisting of one to four stabilizers, with constant stiffness for each segment of drill pipe between stabilizers. The following cases were treated:

i) Straight hole, ii) Constantly increasing curvature hole and iii) Constantly decreasing curvature hole.

The program was implemented on an HP9816 computer using Advanced BASIC with its matrix handling capabilities. Variable definitions given within the program correspond to the definitions used in the main text. The output file of the program contains the input data used together with the results of the cases named earlier.
PROGRAM STAB

10 !
20 !
30 !
40 !
50 !
60 !
70 !
80 !
90 !
100 !
110 !
120 !
130 !
140 !
150 !
160 !
170 !
180 !
190 !
200 !
210 !
220 !
230 !
240 !
250 !
260 !
270 !
280 !
290 !
300 !
310 !
320 !
330 !
340 !
350 !
360 !
370 !
380 !
390 !
400 !
410 !
420 !
430 !
440 !
450 !
460 !
470 !
480 !
490 !
500 !
510 !
520 !

THIS PROGRAM SOLVES FOR THE HOLE-RADIUS AND THE LENGTH OF THE
SECTION IN THE DRILLING STRING FROM THE LAST STABILIZER TO THE
POINT OF CONTACT (TANGENCY) BETWEEN THE HOLE AND THE DRILL PI-
TE

OPTION BASE 1
DIM U5(5,5),U1(5,5),U2(5,5),U3(5,5),U4(5,5),U5(5,5),U6(5,5)
DIM U7(5,5),U8(5,5),U9(5,5),U10(5,5),U11(5,5),U12(5,5),U13(5,5),U14(5,5)
DIM S5(5),S1(5),S2(5),S3(5),S4(5),S5(5),S6(5),S7(5),S8(5),S9(5),S10(5),S11(5)
PRINT IS 701

PROGRAM "STAB"
MULTIPLE-STABILIZERS SOLUTION
PRINT
PRINT "ENTER DATE",Dat$;
PRINT "DATE",Dat$
PRINT "1."
PRINT "ENTER WEIGHT ON BIT (lb), (Def.=1)",P
PRINT "WEIGHT ON BIT",TAB(40),"P = ";P;TAB(54),"lb"
PRINT "ENTER RATE OF INCLINATION (Deg./100)"; Def.=5",D
PRINT "RATE OF INCLINATION",TAB(40),"C = ";D;TAB(54),"Deg./100"
Rho=68755/D
PRINT USING 260;Rho
PRINT "HOLE-CURVATURE RADIUS: RHO = ";D.DDE," in"
PRINT "ENTER EFFECTIVE RADIUS OF THE HOLE (in), (Def.=1.25)",R
PRINT "EFFECTIVE RADIUS OF THE HOLE",TAB(40),"R = ";R;TAB(54);"in"
PRINT "ENTER NUMBER OF STABILIZERS (Def.=1)"; N
PRINT "NUMBER OF STABILIZERS",TAB(40),"N = ";N
PRINT "ENTER E (1b/in^2) (Def.=3.0E7)",E
PRINT "YOUNG'S MODULUS",TAB(40),"E = ";E;TAB(54);"1b/in^2"
PRINT "ENTER HOLE INCLINATION (Deg) (Def.=90)",Alpha
PRINT "HOLE INCLINATION",TAB(36),"ALPHA = ";Alpha;TAB(54);"Deg"
PRINT "1."
FOR K=1 TO N
    DISP "DIST. FROM BIT TO";K;"ST STABILIZER (in)=";
ELSE
    DISP "DIST. FROM STABILIZER";K-1;"TO STABILIZER";K;" (in)=";
END IF
PRINT L(K)
DISP "I(";K;") <in^4>=";
PRINT In(K)
DISP "W(";K;") <lb/in>=";
PRINT W(K)
W(K)=W(K)*SIN(Alpha*PI/180)
ON K GOTO 490,530,590,630,620
PRINT "LENGTH BIT TO 1ST STABILIZER L(";K;") =";L(K);TAB(54);"in"
PRINT "MOMENT OF INERTIA 1ST SECTION I(";K;") =";In(K);TAB(54);"in^4"
PRINT "WEIGHT PER UNIT LENGTH W(";K;") =";W(K);TAB(54);"lb/in"
GOTO 640
530  PRINT ""
540  PRINT "LENGTH 1ST TO 2ND STABILIZER" L(";K;") =";L(K);TAB(S4),"in"
550  PRINT "MOMENT OF INERTIA 2ND SECTION" I(";K;") =";I(N);TAB(S4),"in^4"
560  PRINT "WEIGHT PER UNIT LENGTH" W(";K;") =";W(K);TAB(S4),"lb/in"
570  GOTO 640
580  PRINT ""
590  PRINT "LENGTH 2ND TO 3RD STABILIZER" L(";K;") =";L(K);TAB(S4),"in"
600  PRINT "MOMENT OF INERTIA 3RD SECTION" I(";K;") =";I(N);TAB(S4),"in^4"
610  PRINT "WEIGHT PER UNIT LENGTH" W(";K;") =";W(K);TAB(S4),"lb/in"
620  GOTO 640
630  PRINT "LENGTH 3RD TO 4TH STABILIZER" L(";K;") =";TAB(S4),L(K);"in"
640  NEXT K
650  PRINT ""
660  L(N+1) = L(N)
670  DISP "INPUT MOMENT OF INERTIA FOR TANGENCY SECTION (in^4)"
680  INPUT IN(N+1)
690  PRINT "MOMENT OF INERTIA TAN. SEGMENT"
700  PRINT "I(";N+1;") =";IN(N+1);TAB(S4),"in^4"
710  INPUT "ENTER DISTRIBUTED WEIGHT (lb/in)" W(N+1)
720  PRINT "DISTR. WEIGHT TANGENCY SEGMENT"
730  PRINT "W(";N+1;") = ";W(N+1);TAB(S4),"lb/in"
740  W(N+1) = W(N+1) * SIN(Alph*S.141592/180)
750  DISP "S>lected or D>eleted output results?"
760  ON KBD GOTO 770
770  A$ = UPC$ (KBD$)
780  IF A$ = "S" AND A$ = "D" THEN 780
790  OFF KBD
800  IF UPC$ (A$) = "S" THEN
810  Outp=1
820  END IF
830  DISP ""
840  PRINT ""
850  PRINT ""
860  PRINT "***************************************************************************"
870  PRINT ""
880  FOR G=1 TO 3
890  ON G GOTO 910,950,1040
900  Km=0
910  PRINT ""
920  PRINT "RESULTS FOR A STRAIGHT HOLE"
930  GOTO 1110
940  Km=1
950  Ky=1
970  END IF
980  PRINT ""
990  PRINT "RESULTS FOR CURVED HOLE"
1000 END
1010 PRINT USING 1020;D
1020 IMAGE "INCREASING CURVATURE",2D.D," DEGREES/100 FEET"
1030 GOTO 1110
Km=1
Ky=1
PRINT ""
PRINT USING 1080;D
IMAGE "DECREASING CURVATURE.",2D,D," DEGREES/100 FEET"
Km=1
Ky=1
PRINT ""
IMAGE " Ltotal L("D,") THETA PHI EPS "
Count=1
Eps1=0
Eps2=0
Lt=0
FOR K=1 TO N+1
Lt=Lt+L(K)
Zt=1-COS(Lt/Rho)
IF ABS(Zt)<1.E-10 THEN
Zt=0
END IF
IF K>N+1 THEN
Y(K)=Ky*Rho*Zt+C1(K) ! C1(K) is the clearance of the stabs.
ELSE
Y(K)=Ky*Rho*Rz+R
END IF
B(K)=SQRT(P/E*In(K))
M(K)=B(K)*L(K)
FIELD MATRIX
U(K,1,1)=1
U(K,1,2)=-SIN(M(K))/B(K)
U(K,1,3)=(COS(M(K))-1)/P
U(K,1,4)=(SIN(M(K))-M(K))/B(K)*P
U(K,1,5)=W(K)*(1-COS(M(K))-M(K)*2/2)/(E*In(K)*B(K)^4)
U(K,2,2)=-COS(M(K))
U(K,2,3)=B(K)*SIN(M(K))/P
U(K,2,4)=-U(K,1,3)
U(K,2,5)=W(K)*SIN(M(K))/(E*In(K)*B(K)^3)
U(K,3,2)=P*SIN(M(K))/B(K)
U(K,3,3)=U(K,2,2)
U(K,3,4)=-U(K,1,2)
U(K,3,5)=-W(K)*(1-COS(M(K)))/B(K)^2
U(K,4,4)=1
U(K,4,5)=-W(K)*L(K)
U(K,5,5)=1
POINT MATRIX
Up(K,1,1)=1
Up(K,2,2)=1
Up(K,3,3)=1
Up(K,4,1)=-Up(K,1,1)/Up(K,1,4)
Up(K,4,2)=-Up(K,1,2)/Up(K,1,4)
Up(K,4,3)=-Up(K,1,3)/Up(K,1,4)
Up(K,4,5)=(Y(K)-Up(K,1,5))/Up(K,1,4)
Up(K,5,5)=1
FOR J=1 TO 5
FOR I=1 TO 5
1590  ON K GOTO 1600,1620,1650,1680,1710
1600  UI(I,J)=U(K,I,J)
1610  GOTO 1730
1620  UI(I,J)=U(K,I,J)
1630  UI(I,J)=Up (K,I,J)
1640  GOTO 1730
1650  UI(I,J)=U(K,I,J)
1660  UI(I,J)=Up (K,I,J)
1670  GOTO 1730
1680  UI(I,J)=U(K,I,J)
1690  UI(I,J)=Up(K,I,J)
1700  GOTO 1730
1710  UI(I,J)=U(K,I,J)
1720  UI(I,J)=Up(K,I,J)
1730  NEXT J
1740  NEXT K
1750  NEXT K
1770  !
1780  S(1)=0
1790  S(3)=0 ! > BOUNDARY CONDITIONS AT THE BIT
1800  S(5)=1
1810  S(1)=Y(N+1)
1820  S(3)=Km*(E*In(N+1)/Rho) ! > BOUNDARY CONDITIONS AT THE TANG. POINT
1830  S(5)=1
1840  FOR K=1 TO N
1850  ON K GOTO 1870,1900,1940,1980
1870  MAT Ui= UI#UI
1880  MAT Ur= US#UI
1890  GOTO 2000
1900  MAT Ui= U4#Ur
1910  MAT Ur= US#UI
1920  MAT Ui= Ur
1930  GOTO 2000
1940  MAT Ui= U6#Ur
1950  MAT Ur= U7#Ui
1960  MAT Uiii= Ur
1970  GOTO 2000
1980  MAT Ui= Ub#Ur
1990  MAT Ur= U9#Ui
2000  NEXT K
2010  !
2020  A(1,1)=UI(I,1,2)
2030  A(1,2)=UI(I,1,4)
2040  A(2,1)=Ur(3,2)
2050  A(2,2)=Ur(3,4)
2060  !
2070  Q(1)=UI(I,1,5)+Y(1)
2080  Q(2)=Ur(3,5)+S(3)
2090  !
2100  MAT A_inv= INV(A)
2110  MAT X= A_inv*Q
2120  S0(2)=X(1)
2130  S0(4)=X(2)
MAT S1= U1*S0
IF N=2 THEN
MAT S= U2*S1
MAT S2= U3*S
END IF
IF N=3 THEN
MAT S= U4*S2
MAT S3= U5*S
END IF
MAT S= U*S0
Lamda=.873
Eps=S(2)-(Km*(Lt/Rho))
IF ABS(Eps)>1.E-5 THEN
\ CONVERGECY
L(N+1)=L(N+1)-Lamda*Eps*Rho
END IF
Eps=Eps
\ CRITERIUM
Count=Count+1
IF Count>100 THEN
PRINT "At Count 100 Eps=";Eps;" and Length=";L(N+1)
PRINT "MODIFY Count OR EXIT"
PAUSE
END IF
GOTO 1160
END IF
IF Outp=1 THEN
PRINT USING 2390;L(N+1)
IMAGE "LENGTH OF TANGENCY SECTION"
L="",M4D.2D,", in"
PRINT ""
PRINT "STATE VECTORS"
PRINT ""
PRINT "DEFLECT. SLOPE MOMENT SHEAR FORCE"
PRINT ""
PRINT " in  rad  lb-in  lb"
PRINT ""
PRINT USING 2480;S0(1),S0(2),S0(3),S0(4)
IMAGE "AT THE BIT"
PRINT ""
FOR I=1 TO N
ON I GOTO 2520,2540,2560,2580
PRINT USING 2590;S1(1),S1(2),S1(3),S1(4)
GOTO 2630
PRINT USING 2600;S2(1),S2(2),S2(3),S2(4)
GOTO 2630
PRINT USING 2610;S3(1),S3(2),S3(3),S3(4)
GOTO 2630
PRINT USING 2620;S4(1),S4(2),S4(3),S4(4)
PRINT "AT THE 1ST STAB.",M2D.3D,2X,MD.5D,2X,M6D.D,4X,M5D.D
PRINT "AT THE 2ND STAB.",M2D.3D,2X,MD.5D,2X,M6D.D,4X,M5D.D
PRINT "AT THE 3RD STAB.",M2D.3D,2X,MD.5D,2X,M6D.D,4X,M5D.D
PRINT "AT THE 4TH STAB.",M2D.3D,2X,MD.5D,2X,M6D.D,4X,M5D.D
NEXT I
PRINT USING 2660;S(1),S(2),S(3),S(4)
IMAGE "AT THE TAN. POINT",
PRINT ""
PRINT ""
2690 PRINT USING 2700: Count, Eps
2700 IMAGE "Count="; "2D," ERROR IN THE SLOPE AT TAN. POINT EPS="; MD.3DE
2710 PRINT ""
2720 PRINT ""
2730 ELSE
2740 Ltot=0
2750 FOR K=1 TO N+1
2760 Ymax=0
2770 Xmax=0
2780 PRINT "X Y(hc) Y THETA M V"
2790 FOR I=1 TO 30
2800 D1=I#(K)/30
2810 Yc=Ky*Rho*#(1-COS(D1/Rho))
2820 B(K)=SQR(P/(E*In(K)))
2830 M(K)=B(K)*D1
2840! 
2850 UC(1,1)=1
2860 UC(1,2)=-SIN(M(K))/B(K)
2870 UC(1,3)=(COS(M(K))-1)/P
2880 UC(1,4)=-SIN(M(K))-M(K))/B(K)*P
2890 UC(1,5)=-W(K)*(1-COS(M(K))-M(K)^2)/{E*In(K)*B(K)^4)
2900 UC(2,2)=COS(M(K))
2910 UC(2,3)=B(K)*SIN(M(K))/P
2920 UC(2,4)=UC(1,3)
2930 UC(2,5)=W(K)*M(K)-SIN(M(K))/E*In(K)*B(K)^3
2940 UC(3,2)=-P*SIN(M(K))/B(K)
2950 UC(3,3)=UC(2,2)
2960 UC(3,4)=-UC(1,2)
2970 UC(3,5)=W(K)*(1-COS(M(K)))/B(K)^2
2980 UC(4,4)=1
2990 UC(4,5)=-W(K)*D1
3000 UC(5,5)=1
3010! 
3020 ON K GOTO 3030, 3050, 3080
3030 MAT Si= UC*S0
3040 GOTO 3100
3050 MAT SO= U2*S1
3060 MAT Si= UC*S0
3070 GOTO 3100
3080 MAT SO= U4*S2
3090 MAT Si= UC*S0
3100! 
3110 PRINT USING 3120; D1+Ltot; Yc; Si(1); Si(2); Si(3); Si(4)
3120 IMAGE M42.2D, 2X, M2D.3D, 2X, M2D.3D, 2X, M6D.3D, 2X, M6D.3D, 2X, M6D.D, 2X, M6D.D
3130 IF Iprint=1 THEN GOTO 3220
3140 IF ABS(ABS(Si(1))-ABS(Yc))>R THEN
3150 Iprint=1
3160 PRINT "" 
3170 PRINT "AT X="; D1+Ltot; " DEFORMATION Y="; Si(1) - Yc; " >= R"
3180 PRINT ""
3190 ON K GOTO 3200, 3220, 3250
3200 PRINT "STABILIZER LOCATED TOO FAR"
3210 END IF
3220 IF ABS(Ymax)<ABS(Si(1)) THEN
3230 Ymax=Si(1)
Xmax=Di+Lt

END IF

NEXT I

Iprint=0

Lt=Lt=+L(K)

PRINT ""

PRINT "FOR L(";K-1;") < X < L(";K+1") , Y(X) < R "

PRINT ""

PRINT "MAX DEFLECTION Y=";Ymax;" AT X=";Xmax

PRINT ""

NEXT K

END IF

NEXT G

PRINT IS 1

END
APPENDIX IV

DETAILED RESULTS FOR MULTIPLE STABILIZERS CASE

Presented here is the sample output of the computer program of Appendix III. The values of the deflection, slope, moment and shear force in the drill string are given for the entire length analyzed. This example corresponds to a bottom-hole assembly composed of an 8 3/4 in. drill bit, 6 1/4 in. drill collars and two full gauge stabilizers. A more complete description of the drill string is shown at the beginning of the computer printout together with the values used for the weight on bit, hole inclination, location of the stabilizers and hole deviation rate.

Output variable definitions and dimensions are as follows:

x: independent variable, in.
y(hc): distance of hole centerline to the x-axis, in.
y: distance of drill string centerline to x-axis, in.
θ: slope of drill string centerline to x-axis, in.
M: moment, lb-in.
V: shear force, lb.
**PROGRAM "STAB"**
**MULTIPLE-STABILIZERS SOLUTION**

**DATE**

<table>
<thead>
<tr>
<th>WEIGHT ON BIT</th>
<th>P = 1</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>RATE OF INCLINATION</td>
<td>C = 3</td>
<td>Deg./100'</td>
</tr>
<tr>
<td>HOLE-CURVATURE RADIUS</td>
<td>RH = 2.29E+04 in</td>
<td></td>
</tr>
<tr>
<td>EFFECTIVE RADIUS OF THE HOLE</td>
<td>R = 1.25 in</td>
<td></td>
</tr>
<tr>
<td>NUMBER OF STABILIZERS</td>
<td>N = 1</td>
<td></td>
</tr>
<tr>
<td>YOUNG'S MODULUS</td>
<td>E = 3.0E+7 lb/in^2</td>
<td></td>
</tr>
<tr>
<td>HOLE INCLINATION</td>
<td>ALPHA = 90 Deg</td>
<td></td>
</tr>
<tr>
<td>LENGTH BIT TO 1ST STABILIZER</td>
<td>L(1) = 400 in</td>
<td></td>
</tr>
<tr>
<td>MOMENT OF INERTIA 1ST SECTION</td>
<td>I(1) = 70.925 in^4</td>
<td></td>
</tr>
<tr>
<td>WEIGHT PER UNIT LENGTH</td>
<td>W(1) = 6.692 lb/in</td>
<td></td>
</tr>
<tr>
<td>MOMENT OF INERTIA TAN. SEGMENT</td>
<td>I(2) = 70.925 in^4</td>
<td></td>
</tr>
<tr>
<td>DIST. WEIGHT TANGENCY SEGMENT</td>
<td>W(2) = 6.692 lb/in</td>
<td></td>
</tr>
</tbody>
</table>

********************************************************************************

**RESULTS FOR A STRAIGHT HOLE**

| LENGTH OF TANGENCY SECTION | L = 394.25 in |

**STATE VECTORS**

<table>
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<tr>
<th>DEFLECT.</th>
<th>SLOPE</th>
<th>MOMENT</th>
<th>SHEAR FORCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>in</td>
<td>rad</td>
<td>lb-in</td>
<td>lb</td>
</tr>
<tr>
<td>AT THE BIT</td>
<td>0.000</td>
<td>-.00345</td>
<td>0.0</td>
</tr>
<tr>
<td>AT THE 1ST STAB.</td>
<td>-0.000</td>
<td>-.00148</td>
<td>-157426.0</td>
</tr>
<tr>
<td>AT THE TAN. POINT</td>
<td>1.250</td>
<td>-0.000000</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Count= 2 ERROR IN THE SLOPE AT TAN. POINT EPS=-1.516E-06
PROGRAM "STAB"
MULTIPLE-STABILIZERS SOLUTION

DATE

WEIGHT ON BIT
RATE OF INCLINATION
HOLE-CURVATURE RADIUS
EFFECTIVE RADIUS OF THE HOLE
NUMBER OF STABILIZERS
YOUNG’S MODULUS
HOLE INCLINATION

\[
P = 1
\]
\[
C = 3
\]
\[
\rho = 2.29E+04
\text{ in}
\]
\[
R = 1.25
\text{ in}
\]
\[
N = 1
\]
\[
E = 3.5E+7
\text{ lb/in}^2
\]
\[
\alpha = 90
\text{ Deg}
\]
\[
L(1) = 400
\text{ in}
\]
\[
i(1) = 70.925
\text{ in}^4
\]
\[
W(1) = 6.692
\text{ lb/in}
\]
\[
i(2) = 70.925
\text{ in}^4
\]
\[
W(2) = 6.692
\text{ lb/in}
\]

*********************
RESULTS FOR A STRAIGHT HOLE

\[
\begin{array}{cccccc}
X & Y(hc) & Y & \theta & M & V \\
13.33 & 0.000 & 0.046 & -0.00342 & 12003.0 & 855.6 \\
26.67 & 0.000 & 0.091 & -0.00331 & 22816.3 & 766.4 \\
40.00 & 0.000 & 0.134 & -0.00313 & 32439.9 & 677.2 \\
53.33 & 0.000 & 0.174 & -0.00290 & 40873.9 & 587.9 \\
66.67 & 0.000 & 0.211 & -0.00262 & 48118.1 & 498.7 \\
80.00 & 0.000 & 0.244 & -0.00230 & 54172.6 & 409.5 \\
93.33 & 0.000 & 0.272 & -0.00195 & 59037.5 & 320.2 \\
106.67 & 0.000 & 0.296 & -0.00156 & 62712.7 & 231.0 \\
120.00 & 0.000 & 0.314 & -0.00116 & 65198.1 & 141.8 \\
133.33 & 0.000 & 0.327 & -0.00075 & 66493.9 & 52.6 \\
146.67 & 0.000 & 0.334 & -0.00033 & 66600.0 & -36.7 \\
160.00 & 0.000 & 0.335 & 0.00008 & 65516.3 & -125.9 \\
173.33 & 0.000 & 0.332 & 0.00049 & 63243.0 & -215.1 \\
186.67 & 0.000 & 0.323 & 0.00087 & 59780.0 & -304.3 \\
200.00 & 0.000 & 0.308 & 0.00123 & 55127.3 & -393.6 \\
213.33 & 0.000 & 0.290 & 0.00156 & 49284.9 & -482.8 \\
226.67 & 0.000 & 0.267 & 0.00185 & 42232.8 & -573.0 \\
240.00 & 0.000 & 0.241 & 0.00209 & 34031.0 & -661.2 \\
253.33 & 0.000 & 0.212 & 0.00227 & 24619.6 & -750.5 \\
266.67 & 0.000 & 0.180 & 0.00239 & 14018.4 & -839.7 \\
280.00 & 0.000 & 0.148 & 0.00245 & 2227.6 & -928.9 \\
293.33 & 0.000 & 0.116 & 0.00242 & -10755.0 & -1018.2 \\
306.67 & 0.000 & 0.084 & 0.00231 & -24925.2 & -1107.4 \\
320.00 & 0.000 & 0.054 & 0.00210 & -40265.1 & -1196.6 \\
333.33 & 0.000 & 0.028 & 0.00180 & -56832.7 & -1285.8 \\
346.67 & 0.000 & 0.007 & 0.00139 & -74572.0 & -1375.1 \\
360.00 & 0.000 & -0.008 & 0.00086 & -93501.0 & -1464.3 \\
\end{array}
\]

************
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<tr>
<th>L (0) &lt; X &lt; L(1), Y(X) &lt; R</th>
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<td>MAX DEFLECTION Y = 0.335459904887 AT X = 160</td>
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<table>
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<tr>
<th>L</th>
<th>X</th>
<th>Y(hc)</th>
<th>Y</th>
<th>THETA</th>
<th>M</th>
<th>V</th>
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<td>413.14</td>
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FOR L(1) < X < L(2), Y(X) < R

<table>
<thead>
<tr>
<th>MAX DEFLECTION Y = 1.25 AT X = 794.25</th>
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<tbody>
<tr>
<td>794.250</td>
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FOR L(2) < X < L(3), Y(X) < R

<table>
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<th>MAX DEFLECTION Y = 1.25 AT X = 794.25</th>
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<tbody>
<tr>
<td>794.250</td>
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</table>
PROGRAM "STAB"
MULTIPLE-STABILIZERS SOLUTION

DATE

| WEIGHT ON BIT | P = 25000 lb |
| RATE OF INCLINATION | C = 3 Deg./100' |
| HOLE-CURVATURE RADIUS | RHO = 2.29E+04 in |
| EFFECTIVE RADIUS OF THE HOLE | R = 1.25 in |
| NUMBER OF STABILIZERS | N = 2 |
| YOUNG'S MODULUS | E = 3.0E+7 lb/in^2 |
| HOLE INCLINATION | ALPHA = 90 Deg |

| LENGTH BIT TO 1ST STABILIZER | L(1) = 400 in |
| MOMENT OF INERTIA 1ST SECTION | I(1) = 70.925 in^4 |
| WEIGHT PER UNIT LENGTH | W(1) = 6.692 lb/in |

| LENGTH 1ST TO 2ND STABILIZER | L(2) = 400 in |
| MOMENT OF INERTIA 2ND SECTION | I(2) = 70.925 in^4 |
| WEIGHT PER UNIT LENGTH | W(2) = 6.692 lb/in |

| MOMENT OF INERTIA TAN. SEGMENT | I(3) = 70.925 in^4 |
| DIST. WEIGHT TANGENCY SEGMENT | W(3) = 6.692 lb/in |

***************************************************************

RESULTS FOR A STRAIGHT HOLE

LENGTH OF TANGENCY SECTION | L = 368.76 in |

STATE VECTORS

| DEFLECT.  | SLOPE  | MOMENT  | SHEAR FORCE |
| in        | rad    | lb-in   | lb          |
| AT THE BIT | 0.000  | -0.00601 | 0.0         | 1067.4     |
| AT THE 1ST STAB. | -0.000 | 0.00250  | -108400.3   | -1609.4    |
| AT THE 2ND STAB. | 0.000  | -0.00305 | -125454.7   | -1381.0    |
| AT THE TAN. POINT | 1.250  | -0.00000 | 0.0         | -978.4     |

Count= 4 ERROR IN THE SLOPE AT TAN. POINT EPS=-1.448E-06

RESULTS FOR CURVED HOLE
INCREASING CURVATURE. 3.0 DEGREES/100 FEET

LENGTH OF TANGENCY SECTION  
\[ L = 361.95 \, \text{in} \]

STATE VECTORS

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Count= 4  ERROR IN THE SLOPE AT TAN. POINT EPS=-2.286E-06

DECREASING CURVATURE. 3.0 DEGREES/100 FEET

LENGTH OF TANGENCY SECTION  
\[ L = 377.16 \, \text{in} \]

STATE VECTORS

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Count= 3  ERROR IN THE SLOPE AT TAN. POINT EPS=-7.761E-07
PROGRAM "STAB"
MULTIPLE-STABILIZERS SOLUTION

DATE

WEIGHT ON BIT \( P = 25000 \) lb
RATE OF INCLINATION \( C = 3 \) Deg./100'
HOLE-CURVATURE RADIUS \( RHO = 2.29E+04 \) in
EFFECTIVE RADIUS OF THE HOLE \( R = 1.25 \) in
NUMBER OF STABILIZERS \( N = 1 \)
YOUNG'S MODULUS \( E = 3.5E+7 \) lb/in^2
HOLE INCLINATION \( \text{ALPHA} = 90 \) Deg

LENGTH BIT TO 1ST STABILIZER \( L(1) = 400 \) in
MOMENT OF INERTIA 1ST SECTION \( I(1) = 70.925 \) in^4
WEIGHT PER UNIT LENGTH \( W(1) = 6.692 \) lb/in

MOMENT OF INERTIA TAN. SEGMENT \( I(2) = 70.925 \) in^4
DIST. WEIGHT TANGENCY SEGMENT \( W(2) = 6.692 \) lb/in

******************************************************************************

RESULTS FOR A STRAIGHT HOLE

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<th>Y</th>
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For \( L(0) < X < L(1) \), \( Y(X) < R \)

Max Deflection \( Y = 0.380847194453 \) at \( X = 160 \)

For \( L(1) < X < L(2) \), \( Y(X) < R \)

Max Deflection \( Y = 1.25 \) at \( X = 789.082307968 \)
PROGRAM "STAB"
MULTIPLE-STABILIZERS SOLUTION

DATE

WEIGHT ON BIT P = 50000 lb
RATE OF INCLINATION C = 3 Deg./100'
HOLE-CURVATURE RADIUS RHO = 2.29E+04 in
EFFECTIVE RADIUS OF THE HOLE R = 1.25 in
NUMBER OF STABILIZERS N = 1
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HOLE INCLINATION ALPHA = 90 Deg

LENGTH BIT TO 1ST STABILIZER L(1) = 400 in
MOMENT OF INERTIA 1ST SECTION I(1) = 70.925 in^4
WEIGHT PER UNIT LENGTH W(1) = 6.692 lb/in
MOMENT OF INERTIA TAN. SEGMENT I(2) = 70.925 in^4
DIST. WEIGHT TANGENCY SEGMENT W(2) = 6.692 lb/in

*****************************************************************************

RESULTS FOR A STRAIGHT HOLE

<table>
<thead>
<tr>
<th>X</th>
<th>Y (hc)</th>
<th>Y</th>
<th>THETA</th>
<th>M</th>
<th>V</th>
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<td>M</td>
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For \( L(0) < X < L(1) \), \( Y(X) < R \)

Max deflection \( Y = 0.442784625711 \) at \( X = 160 \)

Max deflection \( Y = 1.25 \) at \( X = 784.916329532 \)

Stabilizer located too far

For \( L(1) < X < L(2) \), \( Y(X) < R \)
### PROGRAM "STAB"
#### MULTIPLE-STABILIZERS SOLUTION

<table>
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<tr>
<th>Variable</th>
<th>Value</th>
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<td>C, RATE OF INCLINATION</td>
<td>3</td>
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<td>RH0, HOE-CURVATURE RADIUS</td>
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<td>R, EFFECTIVE RADIUS OF THE HOLE</td>
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<tr>
<td>W(1), WEIGHT PER UNIT LENGTH</td>
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**RESULTS FOR A STRAIGHT HOLE**

**THE RESULTS ARE**

**LENGTH OF TANGENCY SECTION**

\[ L = 368.76 \text{ in} \]

**STATE VECTORS**

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<th>SHEAR FORCE</th>
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<td>in rad</td>
<td>lb-in</td>
<td>lb</td>
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</tbody>
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- **AT THE BIT**
  - 0.000  
  - -0.00601
  - 0.0
  - 1067.4

- **AT THE 1ST STAB.**
  - 0.000
  - 0.00250
  - -108400.2
  - -1609.4

- **AT THE 2ND STAB.**
  - 0.000
  - -0.00305
  - -125454.8
  - -1381.0

- **AT THE TAN. POINT**
  - 1.250
  - -0.00000
  - 0.0
  - -978.4

**Count = 4**  
**ERROR IN THE SLOPE AT TAN. POINT EPS = 1.429E-06**
RESULTS FOR CURVED HOLES

INCREASING CURVATURE, 3 DEGREES/100 FEET

THE RESULTS ARE

LENGTH OF TANGENCY SECTION $L = 361.95$ in

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<td>rad</td>
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Count= 4 ERROR IN THE SLOPE AT TAN. POINT EPS=-2.268E-06

DECREASING CURVATURE, 3 DEGREES/100 FEET

THE RESULTS ARE

LENGTH OF TANGENCY SECTION $L = 377.16$ in

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<th>SHEAR FORCE</th>
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Count= 3 ERROR IN THE SLOPE AT TAN. POINT EPS=-7.961E-07
PROGRAM "STAB"
MULTIPLE-STABILIZERS SOLUTION

DATE

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<td>C = 3 Deg./100°</td>
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<tr>
<td>HOLE-CURVATURE RADIUS</td>
<td>RHO = 2.29E+04 in</td>
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<tr>
<td>EFFECTIVE RADIUS OF THE HOLE</td>
<td>R = 1.25 in</td>
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<td>YOUNG'S MODULUS</td>
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<td>HOLE INCLINATION</td>
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| LENGTH BIT TO 1ST STABILIZER    | L(1) = 400 in |
| MOMENT OF INERTIA 1ST SECTION   | I(1) = 70.925 in^4 |
| WEIGHT PER UNIT LENGTH          | W(1) = 6.692 lb/in |

| LENGTH 1ST TO 2ND STABILIZER    | L(2) = 400 in |
| MOMENT OF INERTIA 2ND SECTION   | I(2) = 70.925 in^4 |
| WEIGHT PER UNIT LENGTH          | W(2) = 6.692 lb/in |

| MOMENT OF INERTIA TAN. SEGMENT  | I(3) = 70.925 in^4 |
| DIST. WEIGHT TANGENCY SEGMENT   | W(3) = 6.692 lb/in |

*******************************************************************************

RESULTS FOR A STRAIGHT HOLE

<table>
<thead>
<tr>
<th>X</th>
<th>Y(hc)</th>
<th>Y</th>
<th>THETA</th>
<th>M</th>
<th>V</th>
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For $L(0) < X < L(1)$, $Y(X) < R$

Max Deflection $Y = 0.671027900436$ at $X = 186.66666667$

<table>
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<th>Y</th>
<th>THETA</th>
<th>M</th>
<th>V</th>
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For $L(1) < X < L(2)$, $Y(X) < R$

Max Deflection $Y = -0.103603793445$ at $X = 720$

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FOR L(2) \times X \times L(3)

MAX DEFLECTION Y = 1.25 AT X = 1168.75559683

RESULTS FOR CURVED HOLE

INCREASING CURVATURE. 3.0 DEGREES/100 FEET
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**AT X = 432.33333333333333 DEFLECTION Y=-3.73380977564 >= R**

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FOR L(1) < X < L(2), Y(X) < R
MAX DEFLECTION $Y = -13.961203208$ AT $X = 800$

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AT $X = 812.065149392$ DEFORMATION $Y = -14.3457465782$ $\geq R$

FOR $L(0) < X < L(1)$, $Y(X) < R$

DECREASING CURVATURE. 3.0 DEGREES/100 FEET

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MAX DEFLECTION $Y = 3.49056661661$ AT $X = 400$

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AT $X = 413.33333333$ DEFORMATION $Y = 3.67581081799$ $\geq R$

STABILIZER LOCATED TOO FAR

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<td>-977.2</td>
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<td>-1066.4</td>
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<td>-1155.6</td>
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<td>1.877</td>
<td>10.309</td>
<td>-0.03077</td>
<td>-116469.2</td>
<td>-1244.9</td>
</tr>
<tr>
<td>706.67</td>
<td>2.052</td>
<td>10.724</td>
<td>-0.03152</td>
<td>-123281.4</td>
<td>-1334.1</td>
</tr>
<tr>
<td>720.00</td>
<td>2.234</td>
<td>11.150</td>
<td>-0.03222</td>
<td>-131025.6</td>
<td>-1423.3</td>
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<td>-139685.7</td>
<td>-1512.5</td>
</tr>
<tr>
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<td>-0.03407</td>
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<td>-1601.8</td>
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<td>2.827</td>
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<td>-0.03504</td>
<td>-159679.2</td>
<td>-1691.0</td>
</tr>
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<td>773.33</td>
<td>3.041</td>
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<td>-170970.8</td>
<td>-1780.2</td>
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<td>-0.03718</td>
<td>-183094.9</td>
<td>-1869.5</td>
</tr>
<tr>
<td>800.00</td>
<td>3.491</td>
<td>13.961</td>
<td>-0.03837</td>
<td>-196026.0</td>
<td>-1958.7</td>
</tr>
</tbody>
</table>

FOR L(1) < X < L(2), Y(X) < R

MAX DEFLECTION Y = 13.961203208 AT X = 800

X Y'(mc) Y THETA M V
S12.57 0.003 14.451 -0.03948 -179607.6 290.6

AT X = 812.572165909 DEFLECTION Y = 14.4472187916 >= R
APPENDIX V

SIMPLE BEAM VERIFICATION

A. SIMPLE BEAM THEORY.

Simple Beam Theory is used to verify the solutions given by the Transfer Matrix approach for the limiting case of a beam-column with an axial force sufficiently small that its effect is negligible in comparison to the effect of the remaining forces.

A drill string with stabilizers and no axial force acting constitutes a statically indeterminate beam problem that can be solved using the principle of superposition. Fig. 1 shows the different types of loads and their associated beam deflections. Boundary conditions and assumptions are the same as in the beam column case.

The unknown concentrated forces at the stabilizers are found using the condition of prescribed displacements at each of those points. The proper length of the segment of the drill string between the last stabilizer and the point of contact with the bore-hole wall is found by iteration using the condition that the drill string remains in contact with the wall of the hole above the contact point.

Equations for the deflection and slope for each type of load were obtained from Spotts (7); they are reproduced in Fig. 2. A short computer program -BEAM_SOL_2-, presented at the end of this appendix, carries out the calculations and its results compare with the results obtained by the transfer matrix method, using $P = 1$ lb, as follows:
<table>
<thead>
<tr>
<th></th>
<th>Beam Solution</th>
<th>Transfer Matrix</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of last segment</td>
<td>382.5 in.</td>
<td>382.4 in.</td>
</tr>
<tr>
<td>Slope of tangency point</td>
<td>0.0519 rad</td>
<td>0.0518 rad</td>
</tr>
</tbody>
</table>
Simple Beam Principle of Superposition Loads and Corresponding Deflection

Figure 1
Beam deflection and slope for different types of loads

\[ y = \frac{wx}{24EI} \left( L^3 - 2Lx^2 + x^3 \right); \quad \theta = \frac{wL^3}{24EI} \]

\[ y = \frac{Mx}{6EI} \left( Lx - x^3 \right); \quad \theta = \frac{ML}{3EI} \]

\[ y = \frac{Fx}{6EI} \left( x^2 - b^2 - x^2 \right); \quad \theta = \frac{FaL}{6EIL} \left( L^2 - a^2 \right) \]

Figure 2
B. SIMPLE BEAM COMPUTER LISTING.

The BASIC computer program that follows employs the procedure described in section A to solve the simple beam problem analyzed. It was implemented on an HP9816 computer. Variable definitions are within the program and it presents the results with the input data used.
: PROGRAM BEAM_SOL_2

! THIS PROGRAM USES BEAM THEORY TO SOLVE THE CASE OF A
! DRILL STRING WITH TWO STABILIZERS IN A CURVED HOLE

! INCREASING CURVATURE, WITH P -> 0

! COMPRESSION ON THE LENGTH OF THE SECTION FROM THE
! LAST STABILIZER TO THE TANGENCY POINT.

100 PRINT " PROGRAM BEAM_SOL_2"
110 PRINT ""
120 PRINT " BEAM SOLUTION FOR CURVED HOLE "
130 PRINT " INCREASING CURVATURE"
140 PRINT " TWO STABILIZERS"
150 PRINT ""
160 INPUT "ENTER DATE",Dat$,
170 PRINT " DATE",Dat$
180 PRINT ""
190 E=3.0E+7
200 R=1.25
210 A1=90
220 W0=6.692
230 I=70.925
240 L1=300
250 L2=600
260 INPUT "ENTER DEVIATION RATE Def. 3 Deg/100"",Deg.
270 INPUT "ENTER HOLE INCLINATION Def. 90 Deg. ",A1
280 INPUT "ENTER DIST. WEIGHT Def. 6.692 lb/in",W0
290 INPUT "ENTER MOMENT OF INERTIA Def. 70.925 in^4",I
300 EI=E*I
310 INPUT "ENTER HOLE CURVATURE ",Rho=68755/Deg
320 INPUT "ENTER DISTANCE FROM BIT TO 1ST STAB. (in) ",L1
330 INPUT "ENTER DISTANCE FROM BIT TO 2ND STAB. (in) ",L2
340 INPUT "ENTER EFFECTIVE RADIUS (in) ",R
350 A1=A1*PI/180
360 W=W0*SIN(A1)
370 PRINT "INPUT DATA"
380 PRINT ""
390 PRINT USING 885;R
400 PRINT USING 881;Deg
410 PRINT USING 884;Rho
420 PRINT USING 887;A1
430 PRINT USING 882;L1
440 PRINT USING 883;L2
450 PRINT USING 886;I
460 PRINT USING 887;E
470 PRINT ""
480 OPTION BASE 1
490 DIM A(2,2),A_inv(2,2),B(2),F(2)
500 Y1=Rho*(1-COS(L1/Rho))
510 Y2=Rho*(1-COS((L2/Rho))
520 L1d=L1*L1
530 L2d=L2*L2
540 L1d=L1d*L1d
550 L2d=L2d*L2d
560 A(1,1)=Y1+L1d
570 A(1,2)=Y1
580 A(2,1)=Y1
590 A(2,2)=Y1+L2d
600 A_inv(1,1)=(1/A(1,1))
610 A_inv(1,2)=-A_inv(1,1)*A(1,2)
620 A_inv(2,1)=-A_inv(1,1)*A(2,1)
630 A_inv(2,2)=(1/A(2,2))
640 B(1)=A(1,1)+A_inv(1,2)*A(1,2)
650 B(2)=A(2,1)+A_inv(2,1)*A(2,1)
660 F(1)=B(1)
670 F(2)=B(2)
680 A_inv(1,2)=A_inv(2,1)
690 A_inv(2,1)=A_inv(1,2)
700 A_inv(1,1)=A_inv(2,2)
710 A_inv(2,2)=A_inv(1,1)
720 PRINT "1ST STABILIZER DEFL.
730 PRINT "2ND STABILIZER DEFL."
L2sq=L2*L2
Ltan=(L1+(L2-L1))/2 ! LENGTH OF TAN. SECTION
D1=0
FOR I=1 TO IS
Ltan=Ltan+D1
Lt=L2+Ltan
Phi=Lt/Rho
Y=Rho*(1-COS(Phi))
H=R*COS(Phi)
Beta=(Y-H)/Lt
H1=L1*Beta
H2=L2*Beta

SLOPE AT THE TAN. PT. DUE TO THE DIST. LOAD
Alpha1=WL*Lt^3/24/Ei
Delw1=WL*L1*(Lt^3-2Lt*L1sq*L1^3)/24/Ei
Delw2=WL*L2*(Lt^3-2Lt*L2sq*L2^3)/24/Ei
Alpha2=Lt/3/Rho
DELTA AT 1ST STAB. DUE TO THE MOMENT
Del1=Lt*LI*(1-(L1/Lt)^2)/6/Rho
DELTA AT 2ST STAB. DUE TO THE MOMENT
Del2=Lt*L2*(1-(L2/Lt)^2)/6/Rho
B(1)=Delw1+Del1-h1+y1
B(2)=Delw2+Delm2-h2+y2
Den=3*Ei*Lt
A(1,1)=Lisq*(Lt-L1)^2/Den
A(1,2)=L1*Ltan*(Lt-Lt-Ltan*Lisq)/(2*Den)
A(2,1)=A(1,2)
A(2,2)=L2sq*(Lt-L2)^2/Den
MAT A_inv= INV(A)
MAT F= A_inv*B
SLOPE DUE TO FORCE AT 1ST STA.
Alpha3=F(1)*L1*(Lt-L1sq)/(2*Den)
SLOPE DUE TO FORCE AT 2ND STA.
Alpha4=F(2)*L2*(Lt-L2sq)/(2*Den)
The=Alpha1+Alpha2-Alpha3+Alpha4+Beta
Eps=The-Phi
IF (ABS(Eps)<=1.E-8) THEN 811
DI=Eps*21500
NEXT I
811 PRINT "RESULTS"
812 PRINT ""
820 PRINT USING 890;Ltan
840 PRINT USING 900;The
850 PRINT ""
860 PRINT USING 910;Eps
880 PRINT USING 920;I
881 IMAGE "HOLE DEVIATION (Deg/100)"
882 IMAGE "DIST. FROM BIT TO 1ST STAB.
883 IMAGE "DIST. FROM BIT TO 2ND STAB.
884 IMAGE "HOLE CURVATURE"
885 IMAGE "EFFECTIVE RADIUS OF THE HOLE"
886 IMAGE "MOMENT OF INERTIA"
887 IMAGE "YOUNG'S MODULUS"
890 IMAGE "LENGTH OF TAN. SECTION"
890
Ltan="",M40.D
900  IMAGE "SLOPE AT THE TAN. POINT"  THETA= ",MD.5D"
910  IMAGE "ERROR IN THE SLOPE"  EPS= ",MD.2DE"
920  IMAGE "NUMBER OF ITERATIONS"  COUNT= ",3D"
930  END
C. SIMPLE BEAM COMPUTER RESULTS.

Output of the computer program of section B is presented here. These results correspond to the case of a drill string with two full gauge stabilizers. The drill bit is 8 3/4 in. and drill collars are 6 1/4 in. O.D., 2 in. I.D. The hole has a 3°/100 feet constant curvature, and the inclination is 90° from the vertical. Following these results are the results given by the transfer matrix method for the same case.
PROGRAM BEAM_SOL_2

BEAM SOLUTION FOR CURVED HOLE
INCREASING CURVATURE
TWO STABILIZERS

INPUT DATA

EFFECTIVE RADIUS OF THE HOLE \( R = 1.250 \)
HOLE DEVIATION (Deg/100') \( C = 3.0 \)
HOLE CURVATURE \( RHO = 2.29E+04 \)
HOLE INCLINATION FROM VERT. \( \alpha = 90.0 \)
DIST. FROM BIT TO 1ST STAB. \( L1 = 400.0 \)
DIST. FROM BIT TO 2ND STAB. \( L2 = 800.0 \)
MOMENT OF INERTIA \( I = 70.925 \)
YOUNG'S MODULUS \( E = 3.00E+07 \)

RESULTS

LENGTH OF TANGENCY SECTION \( L_{tan} = 382.5 \)
SLOPE AT THE TAN. POINT \( \theta = 0.05159 \)
ERROR IN THE SLOPE \( \epsilon = 2.35E-09 \)
NUMBER OF ITERATIONS \( \text{COUNT} = 5 \)
APPENDIX VI

BEAM COLUMN VERIFICATION

A. Description of the procedure.

The Continuous System Modeling Program (CSMP), a computer library developed by IBM, is utilized in this appendix to verify the results obtained with the transfer matrix method for the case of a drill string with two stabilizers. Results of the CSMP method agree with those of the transfer matrix method presented in Appendix IV.

A detailed description of CSMP can be found in Speckhart and Green [6]. It is utilized here to solve the ordinary differential equation that describes the moment-curvature relationship, for small deflections, for a beam column

\[ \frac{d^2y}{dx^2} = \frac{M}{EI} . \]

\[ M = Py + V_0x - w \frac{x^2}{2} , \quad 0 < x < L_1 \]
\[ M = Py + V_0x - w \frac{x^2}{2} + F_1(x - L_1), \quad L_1 < x < L_2 \]
\[ M = Py + V_0x - w \frac{x^2}{2} + F_1(x - L_1) + F_2(x - L_2), \quad L_2 < x < L \]

with:

- \( P \) weight on bit, lb.
- \( V_0 \) lateral force at the bit, lb.
- \( w \) vertical component of drill collar weight per unit length, lb/inch.
- \( F_1 \) concentrated force at lower stabilizer, lb.
- \( F_2 \) concentrated force at upper stabilizer, lb.
\( l_1 \) distance bit-lower stabilizer, in.
\( l_2 \) distance bit-upper stabilizer, in.
L distance bit-contact point, in.
y drill string deflection, in.

A variable step Runge Kutta algorithm of CMSP is used to solve the differential equation. Integration begins at the bit and continues up to the tangency point. Initial values for this algorithm are obtained from The state vector of the transfer matrix at the bit. The program stops when the condition:

\[
M(x) = \frac{EI}{\rho}
\]

is satisfied. \( \rho \) is the hole radius of curvature.

A comparison of the results at the tangency point given by both methods is presented in the following example:

Drill string with two stabilizers.
P = 25,000 lb weight on bit.
Constant increasing curvature hole.

<table>
<thead>
<tr>
<th></th>
<th>Transfer Matrix</th>
<th>CSMP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total length, in.</td>
<td>1402.29</td>
<td>1402.5</td>
</tr>
<tr>
<td>Deflection, in.</td>
<td>1.5</td>
<td>1.499</td>
</tr>
<tr>
<td>Slope, rod.</td>
<td>(-57 \times 10^{-6})</td>
<td>(-59 \times 10^{-6})</td>
</tr>
<tr>
<td>Moment, lb-in.</td>
<td>0</td>
<td>163</td>
</tr>
<tr>
<td>Shear force, lb.</td>
<td>(-810)</td>
<td>(-805.9)</td>
</tr>
</tbody>
</table>
Small differences between the results originate in the numerical approximation to the exact solution in the CSMP procedure.

B. COMPUTER LISTING.

The CSMP program that follows implements the procedure described in part A of this appendix. Variable definitions within the program are:

\[
\begin{align*}
YDD &= \frac{d^2y}{dx^2} \\
M &= -EI \frac{d^2y}{dx^2} \text{ Moment} \\
YD &= \frac{dy}{dx} \text{ Shear force} \\
YDO &= \frac{dy(0)}{dx} \text{ Initial Conditions.} \\

V &= \frac{dM}{dx} \\
VO &= \frac{dM(0)}{dx}
\end{align*}
\]

Functions are defined as:

\[
\begin{align*}
\text{INTGRL (IC,Fx)} &= \int Fx \, dx, \quad \text{IC initial condition} \\
\text{DERIV (IC, Fx)} &= \frac{dFx}{dx}, \quad \text{IC initial condition} \\
\text{STEP(x)} &= \begin{cases} 
0 & x < x_0 \\
1 & x > x_0 
\end{cases}
\end{align*}
\]

Execution is controlled by the following parameters:

FINTIM Final integration time.

PRDEL Output interval.

DELT Size of integration step.

FINISH Conditional to stop integration.
C. **CSMP Computer Results**

Numerical results of the problem of a drill string with two stabilizers in a curved hole is presented in this section. The drill string is composed of

- 8 3/4 in. drill bit.
- 6 1/4 in. O.D., 3 in. I.D. drill collars.
- Two full gauge stabilizers located at 400 and 800 in. from the bit.

In this program the hole has a constant increasing curvature of 3°/100 feet. Two different output intervals are used. The first prints the results at 50 in. intervals, and the second one prints the results every 1/4 in. Output variables and units are

- y beam deflection, in.
- θ beam slope, rad.
- M moment, lb-in.
- V shear force, lb.
****CONTINUOUS SYSTEM MODELING PROGRAM****

***PROBLEM INPUT STATEMENTS***

* PROGRAM CB.SCND.STA.CSMP
* *
* SOLVES THE 2ND ORDER D.E FOR A BEAM COLUMN SIMULATION
* INTEGRATION STARTS AT THE BIT.
* USES AS INITIAL CONDITIONS VALUES OBTAINED IN THE
* TRANSFER MATRIX APPROACH.
* 
* LABEL COLUMN BEAM SOLUTION (TWO-STABILIZER, FROM THE BIT)
* LABEL STRAIGHT HOLE  \( P = 25000.0 \) (LB)
* RENAME TIME=X
* CONSTANT \( I = 66.378, \ E = 2.8E7, \ P = 25000.0, \ W = 5.021, \ V_0 = 970.511167, \)
* \( Y_0 = 1.1003844E-2, \ F_1 = 2844.661254, \ F_2 = 2415.049608, \)
* \( L_1 = 500, L_2 = 1000, R = 1.5 \)
* \( YDD = W * X ** 2 / 2 - P * Y - V_0 * X * F_1 * (X - L_1) \times \text{STEP}(L_1) - F_2 * (X - L_2) \times \text{STEP}(L_2)) \). \( / (E * I) \)
* \( YD = \text{INTGRL}(YD0, YDD) \)
* \( Y = \text{INTGRL}(0.0, YD) \)
* \( M = -E*I*YDD \)
* \( V = \text{DERIV}(V0, M) \)
* \( \text{THETA} = YD \)
* \( 
\begin{align*}
\text{TIMER} & \ FINTIM = 1500.0, \ 
\text{PRDEL} = 50 \\
\text{FINISH} & \ X = 1400.0 \\
\text{PRINT} & \ Y, \text{THETA}, M, V \\
\text{CONTINUE} & \\
\text{TIMER} & \ PRDEL = 0.25 \\
\text{FINISH} & \ M = 1.0E-15. \\
\text{PRINT} & \ Y, \text{THETA}, M, V
\end{align*}
\) 

END
STOP

OUTPUT VARIABLE SEQUENCE
YDD  YD  Y  M  V  THETA

OUTPUTS  INPUTS  PARAMS  INTEGS  + MEM BLKS  FORTRAN  DATA CDs
10(500)  41(1400)  13(400)  2*  0= 2(300)  8(600)  13

ENDJOB
*** CSMP/360 SIMULATION DATA ***

LABEL COLUMN BEAM SOLUTION (TWO-STABILIZER, FROM THE BIT)

LABEL STRAIGHT HOLE  P=25000.0 (LB)

CONSTANT  I=66.378, E=2.8E7, P=25000.0, W=5.021, V0=970.51167,...

YD0=1.1003844E-2, F1=2844.661254, F2=2415.049608,...

L1=500, L2=1000, R=1.5

TIMER FINTIM=1500.0, PRDEL=50

FINISH X=1400.0

PRINT Y, THETA, M, V

CONTINUE

TIMER VARIABLES
DELT = 3.1250E+00
DELMIN= 1.5000E-04
FINTIM= 1.5000E+03
PRDEL = 5.0000E+01
OUTDEL= 0.0
<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>THETA</th>
<th>M</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>0.0</td>
<td>1.1004E-02</td>
<td>0.0</td>
<td>9.7051E+02</td>
</tr>
<tr>
<td>5.0000E+01</td>
<td>5.3695E-01</td>
<td>1.0225E-02</td>
<td>5.5673E+04</td>
<td>9.7782E+02</td>
</tr>
<tr>
<td>1.0000E+02</td>
<td>1.0006E+00</td>
<td>8.1375E-03</td>
<td>9.6962E+04</td>
<td>6.7944E+02</td>
</tr>
<tr>
<td>1.5000E+02</td>
<td>1.3357E+00</td>
<td>5.1492E-03</td>
<td>1.2248E+05</td>
<td>3.5271E+02</td>
</tr>
<tr>
<td>2.0000E+02</td>
<td>1.5078E+00</td>
<td>1.6966E-03</td>
<td>1.3138E+05</td>
<td>1.1662E+03</td>
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<td>2.5000E+02</td>
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<td>1.2235E+05</td>
<td>-3.2978E+02</td>
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<td>-6.6016E+02</td>
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<tr>
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<td>1.0407E+00</td>
<td>-6.9344E-03</td>
<td>5.8161E+04</td>
<td>-9.6841E+02</td>
</tr>
<tr>
<td>4.0000E+02</td>
<td>6.6645E-01</td>
<td>-7.7901E-03</td>
<td>3.1857E+03</td>
<td>-1.2442E+03</td>
</tr>
<tr>
<td>4.5000E+02</td>
<td>2.8931E-01</td>
<td>-6.9925E-03</td>
<td>-6.4414E+04</td>
<td>-1.4782E+03</td>
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<tr>
<td>5.0000E+02</td>
<td>-3.1710E-05</td>
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<td>-1.4237E+05</td>
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<tr>
<td>5.5000E+02</td>
<td>-1.2863E-01</td>
<td>-1.1706E-03</td>
<td>-6.6633E+04</td>
<td>2.4517E+03</td>
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<tr>
<td>6.0000E+02</td>
<td>-1.4002E-01</td>
<td>5.1616E-04</td>
<td>-4.0507E+04</td>
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<td>-4.2078E-02</td>
<td>9.2500E-04</td>
<td>1.7095E+04</td>
<td>3.2940E+02</td>
</tr>
<tr>
<td>7.5000E+02</td>
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<td>9.0000E+02</td>
<td>-8.2033E-02</td>
<td>-5.7638E-04</td>
<td>-2.4230E+04</td>
<td>-7.2413E+02</td>
</tr>
<tr>
<td>9.5000E+02</td>
<td>-8.5831E-02</td>
<td>6.0965E-04</td>
<td>-6.5788E+04</td>
<td>9.4820E+02</td>
</tr>
<tr>
<td>1.0000E+03</td>
<td>-8.7060E-06</td>
<td>3.0559E-03</td>
<td>-1.1766E+05</td>
<td>-1.1405E+03</td>
</tr>
<tr>
<td>1.0500E+03</td>
<td>2.1801E-05</td>
<td>5.3974E-03</td>
<td>-5.8023E+04</td>
<td>2.3078E+03</td>
</tr>
<tr>
<td>1.1000E+03</td>
<td>5.1527E-01</td>
<td>6.2727E-03</td>
<td>-8.9591E+03</td>
<td>8.7555E+02</td>
</tr>
<tr>
<td>1.1500E+03</td>
<td>8.2594E-01</td>
<td>5.9891E-03</td>
<td>2.7886E+04</td>
<td>6.1473E+02</td>
</tr>
<tr>
<td>1.2000E+03</td>
<td>1.1006E+00</td>
<td>4.8930E-03</td>
<td>5.1282E+04</td>
<td>3.3341E+02</td>
</tr>
<tr>
<td>1.2500E+03</td>
<td>1.3079E+00</td>
<td>3.5777E-03</td>
<td>6.0437E+04</td>
<td>4.0711E+01</td>
</tr>
<tr>
<td>1.3000E+03</td>
<td>1.4355E+00</td>
<td>1.7716E-03</td>
<td>5.5052E+04</td>
<td>-2.5312E+02</td>
</tr>
<tr>
<td>1.3500E+03</td>
<td>1.4907E+00</td>
<td>5.2460E-04</td>
<td>3.5300E+04</td>
<td>-5.3869E+02</td>
</tr>
</tbody>
</table>

***SIMULATION HALTED***

X = 1.4000E+03

1.4000E+03 1.5000E+00 -4.7698E-06 1.8508E+03 -8.0597E+02
*** CSMP/360 SIMULATION DATA ***

TIMER PRDEL=0.25
FINISH M=1.0E-15
PRINT Y,THETA,M,V

END

TIMER VARIABLES
DELT = 1.5625E-02
DELMIN= 1.5000E-04
FININT= 1.5000E+03
PRDEL = 2.5000E-01
OUTDEL= 0.0
<table>
<thead>
<tr>
<th>X</th>
<th>Y</th>
<th>THETA</th>
<th>M</th>
<th>V</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4000E+03</td>
<td>1.5000E+00</td>
<td>-4.7698E-06</td>
<td>1.8508E+03</td>
<td>-8.0597E+02</td>
</tr>
<tr>
<td>1.4002E+03</td>
<td>1.5000E+00</td>
<td>-5.0080E-06</td>
<td>1.6522E+03</td>
<td>-7.4652E+02</td>
</tr>
<tr>
<td>1.4005E+03</td>
<td>1.5000E+00</td>
<td>-5.2167E-06</td>
<td>1.4507E+03</td>
<td>-8.1618E+02</td>
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<tr>
<td>1.4007E+03</td>
<td>1.5000E+00</td>
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<td>1.2505E+03</td>
<td>-7.9980E+02</td>
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<tr>
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<td>-5.5830E-06</td>
<td>1.0472E+03</td>
<td>-8.1900E+02</td>
</tr>
<tr>
<td>1.4012E+03</td>
<td>1.5000E+00</td>
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<td>-7.8300E+02</td>
</tr>
<tr>
<td>1.4015E+03</td>
<td>1.5000E+00</td>
<td>-5.7809E-06</td>
<td>6.4975E+02</td>
<td>-8.2300E+02</td>
</tr>
<tr>
<td>1.4017E+03</td>
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<td>-5.8541E-06</td>
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<td>-8.0662E+02</td>
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<tr>
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<td>1.5000E+00</td>
<td>-5.9003E-06</td>
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<td>-8.0912E+02</td>
</tr>
<tr>
<td>1.4022E+03</td>
<td>1.5000E+00</td>
<td>-5.9192E-06</td>
<td>3.8062E+01</td>
<td>-8.1500E+02</td>
</tr>
</tbody>
</table>

***SIMULATION HALTED*** \[ M = -1.6319E+02 \]

1.4025E+03 1.4999E+00 -5.9109E-06 -1.6319E+02 -8.0100E+02
** PROGRAM "STAB" **
** MULTIPLE-STABILIZERS SOLUTION **

<table>
<thead>
<tr>
<th>Weight on Bit</th>
<th>P = 25000 (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rate of Inclination</td>
<td>C = 3 (Deg./100 Feet)</td>
</tr>
<tr>
<td>Hole-Curvature Radius</td>
<td>RHo = 2.29E+04 (in)</td>
</tr>
<tr>
<td>Effective Radius of the Hole</td>
<td>R = 1.5 (in)</td>
</tr>
<tr>
<td>Number of Stabilizers</td>
<td>N = 2</td>
</tr>
<tr>
<td>Young's Modulus</td>
<td>E = 2.8E+7 (lb/in^2)</td>
</tr>
<tr>
<td>Hole Inclination</td>
<td>Alpha = 90 (Deg)</td>
</tr>
<tr>
<td>Distance from Bit to 1st Stabilizer</td>
<td>L(1) = 500 (in)</td>
</tr>
<tr>
<td>Moment of Inertia 1st Section</td>
<td>I(1) = 66.378 (in^4)</td>
</tr>
<tr>
<td>Weight Per Unit Length</td>
<td>W(1) = 5.021 (lb/in)</td>
</tr>
<tr>
<td>Distance from 1st to 2nd Stabilizer</td>
<td>L(2) = 500 (in)</td>
</tr>
<tr>
<td>Moment of Inertia 2nd Section</td>
<td>I(2) = 66.378 (in^4)</td>
</tr>
<tr>
<td>Weight Per Unit Length</td>
<td>W(2) = 5.021 (lb/in)</td>
</tr>
<tr>
<td>Moment of Inertia for Tangency Section</td>
<td>I(3) = 66.378 (in^4)</td>
</tr>
<tr>
<td>Distributed Weight Tangency Section</td>
<td>W(3) = 5.021 (lb/in)</td>
</tr>
</tbody>
</table>

************************************************************

STRAIGHT HOLE

THE RESULTS ARE

Length of Tangency Section
L = 402.296759654 (in)

At the Bit
Deflection | Y = 0 (in) |
Slope | Theta = -0.011003843924 (Rad) |
Moment | M = 0 (LB.in) |
Shear Force | V = 970.51116752 (Lb) |

At the 1st Stabilizer
Deflection | Y = 1.59872115546E-14 (in) |
Slope | Theta = -.004231383291993 (Rad) |
Moment | M = -142369.41624 (LB.in) |
Shear Force | V = -1539.9893248 (Lb) |

At the 2st Stabilizer
Deflection | Y = 1.7763568394E-15 (in) |
Slope | Theta = -.00305587966116 (Rad) |
Moment | M = -117658.205258 (LB.in) |
Shear Force | V = -1205.82757604 (Lb) |

At the Tangency Point
Deflection | Y = 1.5 (in) |
Slope | Theta = 5.73098587997E-6 (Rad) |
Moment | M = 0 (LB.in) |
Shear Force | V = -810.71458359 (Lb) |

Count = 5
Eps = 5.73098587997E-6
APPENDIX VII

SIMPLE DRILL STRING ANALYSIS

In this appendix the Transfer Matrix procedure is applied to solve the beam-column problem in a simple drill string. Results obtained agree with those found by Lubinski and Wood (1) and show the effect of the weight on bit and hole curvature on buckling of the bottom hole assembly.

For this problem the drill string consists of a drill bit and drill collars, and the loads considered are (a compressive axial force, due to) the weight on the bit, and (a uniformly distributed force normal to the drill string due to) the drill-collars' weight per unit length. The origin of the coordinate system is taken at the tangency point with the x-axis tangent to the hole, and the hole inclination is the angle the x-axis makes with the vertical. Figure 1 shows the drill string and the coordinate system.

Boundary conditions are established by the following assumptions:

i) The bit is centralized in the borehole, and there is no bending moment between the bit and the formation.

ii) The drill pipe remains in contact with the borehole wall above the contact point.

All other assumptions considered in the main text also apply here. The boundary conditions are:

\[ y(0) = 0 \quad y(L) = R(1 - \cos \lambda / \rho) \pm r \]
TWO DIMENSIONAL DRILL STRING WITH NO STABILIZERS

Figure VII-1
\[
\frac{dy(0)}{dx} = 0 \quad \frac{d^2y(L)}{dx^2} = 0
\]

(1)

\[
\frac{d^2y(0)}{dx^2} = \frac{1}{\rho}
\]

\( y \) is the deflection of the drill string
\( \rho \) is the hole radius of curvature
\( r \) is the effective radius of the hole
\( L \) is the length from the bit to the tangency point

The governing differential equation is

\[
\frac{d^4y}{dx^4} + \beta^2 \frac{d^2y}{dx^2} = \frac{w}{EI}
\]

(2)

with \( \beta^2 = \frac{\rho}{EI} \) constant.

Writing the solution in matrix form, as in Appendix I, we have:

\[
S = U S_0
\]

(3)

\( S_0 \) is the state vector at the tangency point
\( U \) is the field matrix of Appendix I
\( S \) is the state vector at the bit

Solution results are presented in two forms: For the first one the condition

\[
\frac{d^2y(L)}{dx^2} = 0 \quad \text{or} \quad M(L) = 0
\]
is used to solve for the shear force at the tangency point for a given length of the drill string.

The equation for the moment at $L$ is:

$$M(L) = -\frac{P}{\beta} \sin \beta L \theta_0 + \cos \beta L M_0 + \frac{\sin \beta L}{\beta} V_0 - \frac{w}{\beta^2} (1 - \cos \beta L) \quad (4)$$

Solving for $V_0$ and writing in terms of the transfer matrix components

$$V_0 = (U_{3,2} \theta_0 - U_{3,3} M_0 + U_{3,5})/U_{3,4} \quad (5)$$

With all the initial parameters known, the transfer matrix $U$ of equation (3) transfers the state vector at the tangency point to the bit. Since the length of this portion of drill pipe has been chosen arbitrarily the procedure yields a hole size which is a function of that length.

Results are presented in dimensionless units in Figures 2, 3, and 4, where curves show the location of the tangency point and the corresponding hole size. Dimensionless parameters are defined as follows:

$$m = \frac{3}{2} \sqrt{\frac{EI}{w}} ; \quad p = \frac{P}{mw} \quad (6)$$

$$w = w_0 \sin \alpha \quad (7)$$

$w_0$ weight per unit length of drill string

$P$ = weight on bit

$\alpha$ = hole inclination at the tangency point.
A second solution procedure iterates on the distance from the bit to the tangency point until the specified displacement at the bit is satisfied. Results are in dimensional form and consist of the state vector at the bit, length of the drill string and state vector at the tangency point.

A sample drill string consisting of an 8 3/4 in. drill bit and 6 1/4 in. O.D. - 3 in. I.D. drill collars is used in the analysis of the results.

Curves in Figure 2 show the location of the tangency point versus hole size in dimensionless units. Different curves are obtained with 18,260 lb., 9,130 lb. and 4,565 lb. of weight on bit. These values correspond to \( p = 4 \), \( p = 2 \), and \( p = 1 \), respectively, in dimensionless units.

Results from Lubinski and Woods (1) for \( p = 2 \) are plotted for comparison. It can be seen that both solution methods yield the same results for values of \( m \sin \alpha/r \) above 10; the solutions diverge at lower values. The Lubinski and Woods method considers the variation of the compressive axial force, while the transfer-matrix results in this Appendix correspond to a constant axial force, equal to the weight on bit, over the entire length studied. The transfer matrix method yields results identical to Lubinski's when a variable axial force - as in Appendix X - is considered. In Figure 4 the dashed curve corresponds to the variable-force solution.
Location of the tangency point for drill string with no stabilizers.

Const. axial force. Straight hole.

Figure VII-2

Data from Lubinski and Woods [1] for \( p = 0.2 \)
Location of the tangency point for drill string with no stabilizers.

**Straight and curved holes.**

Effect of hole curvature

- **STR = 0 Deg./100 feet**
- **INC = +3 Deg./100 feet**
- **DEC = -3 Deg./100 feet**

Figure VII-3
Dimensionless Drill String Length for Different Hole Sizes

Const.-Var. axial force.

Straight hole
Dimensionless axial force \( p = 2 \)

Data published by Lubinski and Wood [1]

Figure VII-4
Figure 3 shows the effect of hole curvature on the location of the tangency point. Curves correspond to constant hole curvatures of $3^\circ$, $0^\circ$, and $-3^\circ$ per hundred feet. The value of the dimensionless weight on bit is $p = 2$.

Current values of $m \sin \alpha / r$ are presented in the following table, where four bit-collar combinations are considered at three different hole inclinations.

<table>
<thead>
<tr>
<th>Drill bit (in)</th>
<th>Drill-collars O.D. (in)</th>
<th>$m$</th>
<th>$\alpha = 30^\circ$</th>
<th>$\alpha = 50^\circ$</th>
<th>$\alpha = 10^\circ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 1/4</td>
<td>8 1/2</td>
<td>812</td>
<td>375</td>
<td>75</td>
<td>7.5</td>
</tr>
<tr>
<td>9 7/8</td>
<td>8 1/2</td>
<td>812</td>
<td>590</td>
<td>102</td>
<td>20</td>
</tr>
<tr>
<td>9 7/8</td>
<td>6 1/4</td>
<td>682</td>
<td>188</td>
<td>32</td>
<td>6.5</td>
</tr>
<tr>
<td>8 3/4</td>
<td>6 1/4</td>
<td>682</td>
<td>272</td>
<td>47</td>
<td>9.5</td>
</tr>
</tbody>
</table>

Lubinski and Woods (1), investigating straight inclined holes, concluded that perfectly vertical holes cannot be drilled even in isotropic formations. Thus, in practice, $m \sin \alpha / r = 10$ is a lower bound.

In the example which follows, the iterative solution procedure is used for the solution of the state vector at the bit and at the tangency point. Although only weight on bit was varied the effect of changing any of the other parameters could be studied.
The solution is given for the cases of straight and curved holes inclined 10° from the vertical. Results along with a detailed description of the drill string used, are presented in the output file of the computer program "Tangency" at the end of this Appendix.

The BASIC computer program that follows applies the transfer matrix method in the solution of the state vectors for the drill string described earlier in this Appendix. This computer program was implemented on a Hewlett Packard 9816 computer. Variable definitions within the program are as follows.

State vector: at the bit; at the tangency point

\[
\begin{align*}
S(1) &= y \\
S(2) &= \text{Theta} \\
S(3) &= M \\
S(4) &= V \\
S_0(1) &= y_0 \\
S_0(2) &= \text{Theta}_0 \\
S_0(3) &= M_0 \\
S_0(4) &= V_0
\end{align*}
\]

\[L = \text{length of drill string from bit to the tangency point.}\]

\[B = \text{Beta.}\]
PROGRAM TANGENCY

PROGRAM TO SOLVE THE EFFECTIVE RADIUS FOR GIVEN TANGENCY LENGTH

FOR NO-STABILIZERS CASES, GIVEN A HOLE-DEVIATION RATE, IT
SOLVES FOR THE CASES OF STRAIGHT, INCREASING AND DECREASING-
CURVATURE HOLE.
THE PROGRAM ITERATES UNTIL THE CALCULATED HOLE SIZE EQUALS
THE EFFECTIVE RADIUS OF THE HOLE

110!
120 DIM U(S,S), S(S), S0(S)
130 PRINTER IS 701
140 PRINT " PROGRAM TANGENCY"
150 PRINT " FOR NO-STABILIZER CASES"
160 PRINT "
170 PRINT " Enter effective hole radius /in"", Ra
180 PRINT " Enter deviation rate (Deg./100 Feet) ", D
190 PRINT " Enter hole inclination from vert. (DEG) ", A1
200 PRINT " Enter effective hole radius /in"", E
210 PRINT " Enter weight on bit (lb/in^2) ", W
220 PRINT " Enter distributed weight (lb/in) ", W0
230 PRINT USING 340; D
240 IMAGE " Effective deviation rate "
250 C = ", M2D, " (Deg./100'")
260 PRINT USING 340; Rad
270 PRINT USING 355; A1
280 IMAGE " Effective hole inclination from vert. alpha = ", M2D, " (Deg)"
290 PRINT USING 340; Rad
300 IMAGE " Effective hole curvature radius "
310 RHO = ", M2D, " (in)"
320 PRINT USING 371; Ra
330 IMAGE " Effective hole radius "
340 R = ", M2D, " (in)"
350 PRINT USING 380; E
360 IMAGE " Elastc constant "
370 E = ", M2D, " (lb/in^2)"
380 PRINT USING 390; I
390 IMAGE " Moment of inertia "
400 I = ", M2D, " (in^4)"
410 PRINT USING 400; P
420 IMAGE " Effective weight on bit "
430 P = ", M2D, " (lb)"
440 PRINT USING 410; W0
450 IMAGE " Effective weight per unit length "
460 W = ", M2D, " (lb/in)"
470 PRINT " 
480 IF J>2 THEN 570
460 IF J=2 THEN 520
470 Km=0
480 Ky=0
490 PRINT "******************************************************************"
491 PRINT ""
500 PRINT "STRAIGHT HOLE"
510 GOTO 610
520 Km=1
530 Ky=1
540 PRINT "******************************************************************"
541 PRINT ""
550 PRINT "INCREASING CURVATURE HOLE ",D," DEGREES/100 FEET"
560 GOTO 610
570 Km=1
580 Ky=1
590 PRINT "******************************************************************"
591 PRINT ""
600 PRINT "DECREASING CURVATURE HOLE ",D," DEGREES/100 FEET"
610 PRINT ""
630 FOR I=0 TO 50
640 B=SQRT(P/Ei)
650 K=B*L1
660 U(1,1)=1
680 U(1,2)=-SIN(K)/B
690 U(1,3)=(COS(K)-1)/P
700 U(1,4)=(SIN(K)-K)/B*P
710 U(1,5)=-W*(1-COS(K)-K^2/2)/(Ei*B^4)
720 U(2,2)=COS(K)
730 U(2,3)=B*SIN(K)/P
740 U(2,4)=(1-COS(K))/P
750 U(2,5)=-W*(K-SIN(K))/((Ei*B^3)
760 U(3,2)=P*SIN(K)/B
770 U(3,3)=U(2,2)
780 U(3,4)=-U(1,2)
790 U(3,5)=-W*(1-COS(K))/B^2
800 U(4,4)=1
810 U(4,5)=-W*L1
820 U(5,5)=1
830 S0(1)=0
840 S0(3)=Km*(Ei/Rad)
850 S0(4)=-(U(3,1)*S0(1)+U(3,2)*S0(2)+U(3,3)*S0(3)+U(3,5))/U(3,4)
860 S0(5)=1
870 MAT S= U*S0
890 !
900 !
910 Eps=S(1)-Ky*Rad*(1-COS(L1/Rad))
911 Eps1=ABS(Eps)-Ra
920 IF ABS(Eps1)>1.E-5 THEN
930 L1=L1-Eps1*25
940 ELSE
950 GOTO 980
960 END IF
970 NEXT I
980 H=ABS(S(1)-Ky*Rad*(1-COS(L1/Rad)))
981 PRINT USING 990;L1
990 IMAGE "LENGTH
L="M3D.2D," in."
991 PRINT USING 1000;H
1000 IMAGE "CALCULATED HOLE SIZE R="M2D.3D," in."
1010 PRINT ""
1011 PRINT "STATE VECTOR ";
1013 PRINT " DEFLECT. SLOPE MOMENT SHEAR FORCE"
1014 PRINT " ";
1015 PRINT " in rad lb-in lb"
1016 PRINT ""
1017 PRINT USING 1018;S(1),S(2),S(3),S(4)
1018 IMAGE "AT THE BIT " M2D.3D,JX,MD.5D,2X,MSD.D,3X,MD.D
1019 PRINT USING 1020;S0(1),S0(2),S0(3),S0(4)
1020 IMAGE "AT THE TAN. POINT",M2D.3D,JX,MD.5D,2X,MSD.D,3X,MD.D
1030 PRINT ""
1150 NEXT J
1160 PRINTER IS 1
1170 END
PROGRAM TANGENCY
FOR NO-STABILIZER CASES

DEVIATION RATE C = 3 (Deg./100')
HOLE INCLINAT. FROM VERT. ALPHA = 90 (Deg)
HOLE-CURVATURE RADIUS RHO = 2.29E+04 (in)
EFFECTIVE HOLE RADIUS R = 1.250 (in)
ELASTIC CONSTANT E = 3.00E+07 (lb/in^2)
MOMENT OF INERTIA I = 70.9 (in^4)
WEIGHT ON BIT P = 1 (lb)
WEIGHT PER UNIT LENGTH W = 6.692 (lb/in)

*****************************************************************************

STRAIGHT HOLE

LENGTH L = 312.52 in.
CALCULATED HOLE SIZE R = 1.250 in.

STATE VECTOR DEFLECT. SLOPE MOMENT SHEAR FORCE
       in    rad    lb-in    lb
AT THE BIT -1.250 0.00800 0.0  -1045.7
AT THE TAN. POINT 0.000 0.00000 0.0  1045.7

*****************************************************************************

INCREASING CURVATURE HOLE 3 DEGREES/100 FEET

LENGTH L = 359.55 in.
CALCULATED HOLE SIZE R = 1.250 in.

STATE VECTOR DEFLECT. SLOPE MOMENT SHEAR FORCE
       in    rad    lb-in    lb
AT THE BIT -4.070 0.02003 0.0  -1461.2
AT THE TAN. POINT 0.000 0.00000 92840.5 944.8

*****************************************************************************

DECREASING CURVATURE HOLE 3 DEGREES/100 FEET

LENGTH L = 271.63 in.
CALCULATED HOLE SIZE R = 1.250 in.

STATE VECTOR DEFLECT. SLOPE MOMENT SHEAR FORCE
       in    rad    lb-in    lb
AT THE BIT 0.360 -0.00067 -0.0  -567.1
AT THE TAN. POINT 0.000 0.00000 -92840.5 1250.7
PROGRAM TANGENCY

FOR NO-STABILIZER CASES

DEVIATION RATE $C = 3$ (Deg./100')
HOLE INCLINATION FROM VERT. ALPHÁ = 10 (Deg)
HOLE-CURVATURE RADIUS $RHO = 2.29E+04$ (in)
EFFECTIVE HOLE RADIUS $R = 1.250$ (in)
ELASTIC CONSTANT $E = 3.00E+07$ (lb/in^2)
MOMENT OF INERTIA $I = 70.9$ (in^4)
WEIGHT ON BIT $P = 25000$ (lb)
WEIGHT PER UNIT LENGTH $W = 6.692$ (lb/in)

*****************************************************************************

STRAIGHT HOLE

LENGTH $L = 451.92$ in.
CALCULATED HOLE SIZE $R = 1.250$ in.

STATE VECTOR DEFLECT. SLOPE MOMENT SHEAR FORCE

<table>
<thead>
<tr>
<th>in</th>
<th>rad</th>
<th>lb-in</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT THE BIT</td>
<td>-1.250</td>
<td>.00553</td>
<td>0.0</td>
</tr>
<tr>
<td>AT THE TAN. POINT</td>
<td>0.000</td>
<td>0.00000</td>
<td>0.0</td>
</tr>
</tbody>
</table>

*****************************************************************************

INCREASING CURVATURE HOLE 3 DEGREES/100 FEET

LENGTH $L = 502.56$ in.
CALCULATED HOLE SIZE $R = 1.250$ in.

STATE VECTOR DEFLECT. SLOPE MOMENT SHEAR FORCE

<table>
<thead>
<tr>
<th>in</th>
<th>rad</th>
<th>lb-in</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT THE BIT</td>
<td>-6.760</td>
<td>.02305</td>
<td>0.0</td>
</tr>
<tr>
<td>AT THE TAN. POINT</td>
<td>0.000</td>
<td>0.00000</td>
<td>92840.5</td>
</tr>
</tbody>
</table>

*****************************************************************************

DECREASING CURVATURE HOLE 3 DEGREES/100 FEET

LENGTH $L = 369.85$ in.
CALCULATED HOLE SIZE $R = 1.250$ in.

STATE VECTOR DEFLECT. SLOPE MOMENT SHEAR FORCE

<table>
<thead>
<tr>
<th>in</th>
<th>rad</th>
<th>lb-in</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>AT THE BIT</td>
<td>1.734</td>
<td>-.00661</td>
<td>0.0</td>
</tr>
<tr>
<td>AT THE TAN. POINT</td>
<td>0.000</td>
<td>0.00000</td>
<td>-92840.5</td>
</tr>
</tbody>
</table>
PROGRAM TANGENCY FOR NO-STABILIZER CASES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEVIATION RATE</td>
<td>C = 3 (Deg./100°)</td>
</tr>
<tr>
<td>HOLE INCLINAT. FROM VERT. ALPHA</td>
<td>10 (Deg)</td>
</tr>
<tr>
<td>HOLE-CURVATURE RADIUS</td>
<td>RHO = 2.29E+04 (in)</td>
</tr>
<tr>
<td>EFFECTIVE HOLE RADIUS</td>
<td>R = 1.250 (in)</td>
</tr>
<tr>
<td>ELASTIC CONSTANT</td>
<td>E = 3.00E+07 (lb/in^2)</td>
</tr>
<tr>
<td>MOMENT OF INERTIA</td>
<td>I = 70.9 (in^4)</td>
</tr>
<tr>
<td>WEIGHT ON BIT</td>
<td>P = 50000 (lb)</td>
</tr>
<tr>
<td>WEIGHT PER UNIT LENGTH</td>
<td>W = 6.692 (lb/in)</td>
</tr>
</tbody>
</table>

**STRAIGHT HOLE**

<table>
<thead>
<tr>
<th>State Vector</th>
<th>Deflect.</th>
<th>Slope</th>
<th>Moment</th>
<th>Shear Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>In</td>
<td>rad</td>
<td>lb-in</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>AT THE BIT</td>
<td>-1.250</td>
<td>0.00592</td>
<td>0.0</td>
<td>-97.3</td>
</tr>
<tr>
<td>AT THE TAN. POINT</td>
<td>0.000</td>
<td>0.00000</td>
<td>0.0</td>
<td>393.4</td>
</tr>
</tbody>
</table>

**INCREASING CURVATURE HOLE**

<table>
<thead>
<tr>
<th>State Vector</th>
<th>Deflect.</th>
<th>Slope</th>
<th>Moment</th>
<th>Shear Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>In</td>
<td>rad</td>
<td>lb-in</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>AT THE BIT</td>
<td>-5.191</td>
<td>0.02109</td>
<td>0.0</td>
<td>145.3</td>
</tr>
<tr>
<td>AT THE TAN. POINT</td>
<td>0.000</td>
<td>0.00000</td>
<td>92840.5</td>
<td>639.2</td>
</tr>
</tbody>
</table>

**DECREASING CURVATURE HOLE**

<table>
<thead>
<tr>
<th>State Vector</th>
<th>Deflect.</th>
<th>Slope</th>
<th>Moment</th>
<th>Shear Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>In</td>
<td>rad</td>
<td>lb-in</td>
<td>lb</td>
<td></td>
</tr>
<tr>
<td>AT THE BIT</td>
<td>2.436</td>
<td>-0.00365</td>
<td>0.0</td>
<td>-309.3</td>
</tr>
<tr>
<td>AT THE TAN. POINT</td>
<td>0.000</td>
<td>0.00000</td>
<td>-92840.5</td>
<td>168.4</td>
</tr>
</tbody>
</table>
PROGRAM TANGENCY

FOR NO-STABILIZER CASES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviation Rate C</td>
<td>3 Deg./100'</td>
</tr>
<tr>
<td>Hole Inclination From Vert. Alpha</td>
<td>90 Deg</td>
</tr>
<tr>
<td>Hole-Curvature Radius Rhq</td>
<td>2.29E+04 in</td>
</tr>
<tr>
<td>Effective Hole Radius R</td>
<td>1.250 in</td>
</tr>
<tr>
<td>Elastic Constant E</td>
<td>3.00E+07 lb/in</td>
</tr>
<tr>
<td>Moment of Inertia I</td>
<td>70.9 in^4</td>
</tr>
<tr>
<td>Weight on Bit P</td>
<td>25000 lb</td>
</tr>
<tr>
<td>Weight per Unit Length W</td>
<td>6.692 lb/in</td>
</tr>
</tbody>
</table>

****************************

Straight Hole.

| Length L                          | 303.67 in |
| Calculated Hole Size R            | 1.250 in  |

State Vector: Deflect, Slope, Moment, Shear Force

<table>
<thead>
<tr>
<th></th>
<th>in</th>
<th>rad</th>
<th>lb-in</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>At the Bit</td>
<td>-1.250</td>
<td>0.0823</td>
<td>0.0</td>
<td>-913.2</td>
</tr>
<tr>
<td>At the Tan. Point</td>
<td>0.000</td>
<td>0.0000</td>
<td>0.0</td>
<td>1119.0</td>
</tr>
</tbody>
</table>

****************************

Increasing Curvature Hole 3 Degrees/100 Feet

| Length L                          | 335.01 in |
| Calculated Hole Size R            | 1.250 in  |

State Vector: Deflect, Slope, Moment, Shear Force

<table>
<thead>
<tr>
<th></th>
<th>in</th>
<th>rad</th>
<th>lb-in</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>At the Bit</td>
<td>-3.698</td>
<td>0.1959</td>
<td>0.0</td>
<td>-1122.1</td>
</tr>
<tr>
<td>At the Tan. Point</td>
<td>0.000</td>
<td>0.0000</td>
<td>92840.5</td>
<td>1119.8</td>
</tr>
</tbody>
</table>

****************************

Decreasing Curvature Hole 3 Degrees/100 Feet

| Length L                          | 271.62 in |
| Calculated Hole Size R            | 1.250 in  |

State Vector: Deflect, Slope, Moment, Shear Force

<table>
<thead>
<tr>
<th></th>
<th>in</th>
<th>rad</th>
<th>lb-in</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>At the Bit</td>
<td>0.360</td>
<td>-0.0064</td>
<td>-0.0</td>
<td>-600.1</td>
</tr>
<tr>
<td>At the Tan. Point</td>
<td>0.000</td>
<td>0.0000</td>
<td>92840.5</td>
<td>1217.6</td>
</tr>
</tbody>
</table>
PROGRAM TANGENCY
FOR NO-STABILIZER CASES

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deviation Rate</td>
<td>C = 3 (Deg./100')</td>
</tr>
<tr>
<td>Hole Inclination From Vert. Alpha</td>
<td>( \alpha = 90 ) (Deg)</td>
</tr>
<tr>
<td>Hole-Curvature Radius</td>
<td>( R_H = 2.29 \times 10^4 ) (in)</td>
</tr>
<tr>
<td>Effective Hole Radius</td>
<td>( R = 1.250 ) (in)</td>
</tr>
<tr>
<td>Elastic Constant</td>
<td>( E = 3.00 \times 10^7 ) (lb/in^2)</td>
</tr>
<tr>
<td>Moment of Inertia</td>
<td>( I = 70.9 ) (in^4)</td>
</tr>
<tr>
<td>Weight on Bit</td>
<td>( P = 50000 ) (lb)</td>
</tr>
<tr>
<td>Weight per Unit Length</td>
<td>( W = 6.692 ) (lb/in)</td>
</tr>
</tbody>
</table>

**************************************************************

STRAIGHT HOLE

<table>
<thead>
<tr>
<th>Length</th>
<th>Calculated Hole Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = 295.08 ) in.</td>
<td>( R = 1.250 ) in.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State Vector</th>
<th>Deflect.</th>
<th>Slope</th>
<th>Moment</th>
<th>Shear Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>at the bit</td>
<td>-1.250</td>
<td>.00847</td>
<td>0.0</td>
<td>-775.5</td>
</tr>
<tr>
<td>at the tan. point</td>
<td>0.000</td>
<td>0.00000</td>
<td>0.0</td>
<td>1199.2</td>
</tr>
</tbody>
</table>

**************************************************************

INCREASING CURVATURE HOLE 3 DEGREES/100 FEET

<table>
<thead>
<tr>
<th>Length</th>
<th>Calculated Hole Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = 314.95 ) in.</td>
<td>( R = 1.250 ) in.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State Vector</th>
<th>Deflect.</th>
<th>Slope</th>
<th>Moment</th>
<th>Shear Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>at the bit</td>
<td>-3.414</td>
<td>.01929</td>
<td>0.0</td>
<td>-866.6</td>
</tr>
<tr>
<td>at the tan. point</td>
<td>0.000</td>
<td>0.00000</td>
<td>92840.5</td>
<td>1300.9</td>
</tr>
</tbody>
</table>

**************************************************************

DECREASING CURVATURE HOLE 3 DEGREES/100 FEET

<table>
<thead>
<tr>
<th>Length</th>
<th>Calculated Hole Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L = 271.56 ) in.</td>
<td>( R = 1.250 ) in.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>State Vector</th>
<th>Deflect.</th>
<th>Slope</th>
<th>Moment</th>
<th>Shear Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>at the bit</td>
<td>.359</td>
<td>-.00061</td>
<td>0.0</td>
<td>-632.8</td>
</tr>
<tr>
<td>at the tan. point</td>
<td>0.000</td>
<td>0.00000</td>
<td>-92840.5</td>
<td>1184.4</td>
</tr>
</tbody>
</table>
APPENDIX VIII

VARIABLE AXIAL LOAD ANALYSIS

In this appendix the effects of considering a variable axial load acting on the drill collars at the bottom of a drill string is studied and the results show that for practical values of weight on bit the assumption of a constant axial force yields less than 2% error for a limiting case.

In other sections of this work the axial force acting on the drill collars has been assumed constant along all the length analyzed. Since the weight on bit is made up of the weight of several drill collars this compressive axial force decreases upwards.

An exact solution of the problem considering a variable axial force is complex; however, a numerical approximation is easily obtained using the transfer matrix method. By dividing the drill string into a finite number of segments, the continuously variable axial force can be assumed constant over each segment of sufficiently small length, letting the force vary from one segment to another. As the length of the segments becomes smaller this approximation approaches the exact solution.

This case is solved using the Transfer Matrix method. Basically the field matrix is the same for all the segments with different numerical values for the coefficient $\beta$. Since $\beta^2 = \frac{P}{EI}$, where $P$ is the axial force and $EI$ the bending stiffness, it allows one to consider different types of drill collars from one segment to another.
Variable Axial Load Drillstring Model

Figure VIII-1
Let the length of the drill collar from the bit to the contact point be divided as:

$$0 < x_1 < x_2 < \ldots < x_i \ldots < x_{n-1} < 1$$

and $U_i$ be the field matrix of one of the segments, by progressive multiplication of the matrices from left to right we have the overall matrix:

$$U = U_nU_{n-1} \ldots U_i \ldots U_1$$

then the state vector comprised of the deflection, slope, moment and shear force at $x = 1$ is:

$$S_{x=L} = US_0$$

with $S_0$ the state vector at the bit.

Assuming constant bending stiffness, we have

$$\beta_x^2 = (P_b - wx \cos \alpha)/EI$$

with

- $P_b$ weight on bit
- $w$ drill collar weight per unit length
- $\alpha$ hole-axis angle respective to the vertical
- $x$ distance from the bit
Defining $h = x_i - x_{i-1}$ as the segment length and considering equal length segments, for the $i$-th segment we have:

$$\beta_i^2 = (P_d - ihw \cos \alpha)/EI.$$ 

The case of a drill string composed of 8 3/4 inches bit and 6 1/4 inches drill collars without stabilizers has been solved with a computer program implemented on an HP9816 computer. Differences found in various parameters for the cases of constant and variable axial force are as follows:

Data used:

- $P = 15,000 \text{ lb}$
- $\alpha = 30^\circ$
- $w = 5.021 \text{ lb/in.}$
- $EI = 1.85 \times 10^9 \text{ lb-in.}^2$

Results:

- Axial force variation $\Delta P = 1704 \text{ lb}$.
- Tangency length variation (%): $\frac{\Delta L}{L} \times 100 = 0.5\%$
- Slope variation at the bit (%): $\frac{\Delta \theta}{\theta} \times 100 = 0.4\%$
- Shear force variation at the bit (%): $\frac{\Delta V}{V} \times 100 = 1.4\%$

This shows that the constant axial force assumption is valid with less than 2% error. The weight on bit used is at the lower limit of practical values. B. Walker (3) shows for a similar bit-drill collar configuration a range of 20,000 to 60,000 lb of weight on bit, and as the weight on bit increases the error of using a constant force becomes much smaller.
Another way of comparing the influence a constant axial force has in the results is shown in Figure 1, where values for such cases are plotted with the curves considering a variable axial load presented by Lubinski and Woods (1).

Dimensionless quantities of the figure are defined as follows:

- \( m \) = length of one dimensionless unit defined by \( m = \frac{3\sqrt{EI}}{W} \)
- \( w \) = weight in mud per unit length of drill pipe
- \( r \) = effective radius of the hole
- \( \alpha \) = hole inclination from the vertical
- \( \xi \) = distance from bit to tangency point
- \( p \) = dimensionless axial force, defined as \( p = \frac{P}{mW} \); \( P \) is the weight on bit.

The computer program which follows solves for the state vector for the case of variable axial load. Variable definitions are the same as in Appendix VIII. Two numerical examples are presented at the end of this section. A weight on bit of 25,000 lb and 15,000 lb respectively is used in each example. The drill string is composed of a 8 3/4 in. drill bit and 6 1/4 in. drill collars. The computer output contains the results of the problem for the cases of straight and curved holes.
PROGRAM TANG-VAR

PROGRAM TO SOLVE THE EFFECTIVE RADIUS FOR GIVEN TANGENCY LENGTH

FOR NO-STABILIZER$ CASES, GIVEN A HOLE-DEVIATION RATE, IT
SOLVES FOR THE CASES OF STRAIGHT, INCREASING AND DECREASING-
CURVATURE HOLE, WITH VARYING AXIAL FORCE.

THE PROGRAM ITERATES UNTIL THE CALCULATED HOLE SIZE EQUALS
THE EFFECTIVE RADIUS OF THE HOLE

DIM U(5,5), S(5,5), S0(5,5), Sx(5,5)
PRINT "PROGRAM TANG-VAR"
PRINT "FOR NO-STABILIZER CASES"
PRINT "VARYING AXIAL FORCE P"
PRINT ""
Ra=1.5
D=5
E=2.8E+7
I=66.378
PO=1
WO=5.021
L1=400
INPUT "ENTER EFFECTIVE HOLE RADIUS (in)", Ra
INPUT "ENTER RATE OF INCLINATION (Deg./100 Feet)", D
INPUT "HOLE DEVIATION ALPHA(Deg)", A1
INPUT "ENTER E (lb/in^2)", E
INPUT "ENTER I (in^4)", I
INPUT "ENTER WEIGHT ON BIT (lb)", PO
INPUT "ENTER DISTRIBUTED WEIGHT (lb/in)", WO
INPUT "GUESS INITIAL LENGTH "
C= "D;" (Deg./100 Feet)
Alpha= "A1;" (Deg)
Rad=6755/D
R= "Rad;" (in)
E= "E;" (lb/in^2)
I= "I;" (in^4)
P= "PO;" (lb)
W= "WO;" (lb/in)
E1=EI
A1=A1*PI/180
W=WO*SIN(A1)
FOR J=1 TO 3
IF J>2 THEN 570
IF J=2 THEN 520
Km=0
Ky=0
PRINT "******************************************************************"
PRINT "STRAIGHT HOLE"
GOTO 610
Km=1
KY=-1
PRINT "***********************************************************************
PRINT "INCREASING CURVATURE HOLE ",D," DEGREES/100 FEET"
GOTO 610
KM=-1
KY=1
PRINT "***********************************************************************
PRINT "DECREASING CURVATURE HOLE ",D," DEGREES/100 FEET"
PRINT ";
PRINT ";
FOR I=0 TO 50
DIM Uold(5,5),Ui(5,5)
MAT Uold= IDN
X0=L1/30
K=B*X0
Pt=P0-L1*K*X0*COS(A1)
FOR G=1 TO 30
X=G*X0
F=Pt-X*K*X0*COS(A1)
B=SQRT(F/Ei)
K=B*X0
END
GOSUB Fm
FOR G=1 TO 30
MAT Ui= Uold*Ui
MAT Uold= Ui
NEXT G
S0(1)=0
S0(3)=KM*(Ei/Rad)
S0(4)=-(U(3,1)*S0(1)+U(3,2)*S0(2)+U(3,3)*S0(3)+U(3,5))/U(3,4)
S0(5)=1
MAT S= U*S0
Eps=S(1)-KY*Rad*(1-COS(L1/Rad))
Eps=ABS(Eps)-Ra
IF ABS(Eps1)>1.E-4 THEN
L1=L1-Eps1*S0
ELSE
GOTO 980
END IF
NEXT I
H=ABS(S(1)-KY*Rad*(1-COS(L1/Rad)))
PRINT "LENGTH",TAB(30),"L=";L1
PRINT "CALCULATED HOLE SIZE",TAB(30),"R=";H
PRINT ";
PRINT ";
PRINT "STATE VECTOR AT THE BIT"
PRINT ";
PRINT "ROD DEFLECTION",TAB(30),"Y=";S(1)
PRINT "SLOPE",TAB(26),"THETA=";S(2)
1080 PRINT ""
1090 PRINT "STATE VECTOR AT THE TANGENCY POINT"
1100 PRINT"
1110 PRINT "ROD DEFLECTION",TAB(30),"Y",S0(1)
1120 PRINT "SLOPE",TAB(26),"THETA",S0(2)
1130 PRINT "MOMENT",TAB(30),"M",S0(3)
1140 PRINT "SHEAR FORCE",TAB(30),"V",S0(4)
1151 ! COMPUTATION ALONG THE ROD
1152 GOTO 1174
1153 Alpha=SGR(P/Ei)
1155 PRINT"
1156 PRINT "DIMENSIONLESS POSITION,MOMENT AND SHEAR FORCE"
1157 PRINT"
1158 PRINT " X    Y    THETA    M    V"
1159 PRINT"
1160 PRINT USING 1172;Lx;Y,S0(2),S0(3),S0(4)
1161 Lx=L1
1162 DI=L1/20
1163 FOR I=1 TO 20
1164 L1=I*D1
1165 K=B*L1
1166 GOSUB Fm
1167 MAT Sx=U*S0
1168 Y=-Sx(1)/Ra
1169 M=Sx(3)/(P*Ra)
1170 Vi=Sx(4)/(Alpha*P*Ra)
1171 PRINT USING 1172;L1/Lt,Sx(1),Sx(2),Sx(3),Sx(4)
1172 IMAGE 4D,3D,2X,MD.SDE,2X,MD.SDE,2X,MD.SDE,2X,MD.SDE,XX,MD.SDE
1173 NEXT I
1174 NEXT J
1175 PRINTER IS 1
1176 STOP
1177 Fm: !
1178 U(1,1)=1
1179 U(1,2)=-SIN(K)/B
1180 U(1,3)=(COS(K)-1)/P
1181 U(1,4)=(SIN(K)-K)/(B*P)
1182 U(1,5)=-W*(1-COS(K)-K^2/2)/(Ei*B^4)
1183 U(2,2)=COS(K)
1184 U(2,3)=B*SIN(K)/P
1185 U(2,4)=(1-COS(K))/P
1186 U(2,5)=-W*(K-SIN(K))/(Ei*B^3)
1187 U(3,2)=-P*SIN(K)/B
1188 U(3,3)=U(2,2)
1189 U(3,4)=-U(1,2)
1190 U(3,5)=-W*(1-COS(K))/B^2
1191 U(4,4)=1
1192 U(4,5)=-W*X0
1193 U(5,5)=1
1198 RETURN
1200 END
PROGRAM TANG-VAR
FOR NO-STABILIZER CASES
VARYING AXIAL FORCE P

RATE OF INCLINATION C = 3 (Deg./100 Feet)
HOLE DEVIATION FROM VERTICAL Alpha = 30 (Deg)
HOLE-CURVATURE RADIUS Rho = 22918.333333 (in)
EFFECTIVE HOLE RADIUS R = 1.5 (in)
ELASTIC CONSTANT E = 2.8547 (lb/in^2)
MOMENT OF INERTIA I = 66.378 (in^4)
WEIGHT ON BIT P = 15000 (lb)
DISTRIBUTED WEIGHT W = 5.021 (lb/in)

***********************************************************************
STRAIGHT HOLE

LENGTH L = 392.185124386
CALCULATED HOLE SIZE R = 1.50002548185

STATE VECTOR AT THE BIT

ROD DEFLECTION Y = -1.50002548185
SLOPE THETA = 0.00764298545554
MOMENT M = 0
SHEAR FORCE V = -439.375145573

STATE VECTOR AT THE TANGENCY POINT

ROD DEFLECTION Y = 0
SLOPE THETA = 0
MOMENT M = 0
SHEAR FORCE V = 545.205609197

***********************************************************************
INCREASING CURVATURE HOLE 3 DEGREES/100 FEET

LENGTH L = 447.594250383
CALCULATED HOLE SIZE R = 1.50008802551

STATE VECTOR AT THE BIT

ROD DEFLECTION Y = -5.87069914989
SLOPE THETA = 0.0228632254049
MOMENT M = 2.91038304567E-11
SHEAR FORCE V = -562.754473505

STATE VECTOR AT THE TANGENCY POINT

ROD DEFLECTION Y = 0
SLOPE THETA = 0
MOMENT M = 81095.9493855
SHEAR FORCE V = 560.930692081

***********************************************************************
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DECREASING CURVATURE HOLE</td>
<td>3</td>
</tr>
<tr>
<td>DEGREES/100 FEET</td>
<td>3</td>
</tr>
<tr>
<td>LENGTH</td>
<td>L= 335.558844904</td>
</tr>
<tr>
<td>CALCULATED HOLE SIZE</td>
<td>R= 1.49996573499</td>
</tr>
<tr>
<td>STATE VECTOR AT THE BIT</td>
<td></td>
</tr>
<tr>
<td>ROD DEFLECTION</td>
<td>Y= -0.956533287769</td>
</tr>
<tr>
<td>SLOPE</td>
<td>THETA= -0.00322345698369</td>
</tr>
<tr>
<td>MOMENT</td>
<td>M= 2.91658304567*10^-11</td>
</tr>
<tr>
<td>SHEAR FORCE</td>
<td>V= -219.99293306</td>
</tr>
<tr>
<td>STATE VECTOR AT THE TANGENCY POINT</td>
<td></td>
</tr>
<tr>
<td>ROD DEFLECTION</td>
<td>Y= 0</td>
</tr>
<tr>
<td>SLOPE</td>
<td>THETA= 0</td>
</tr>
<tr>
<td>MOMENT</td>
<td>M= -81095.9493855</td>
</tr>
<tr>
<td>SHEAR FORCE</td>
<td>V= 622.427547071</td>
</tr>
</tbody>
</table>
PROGRAM TANGENCY_1
FOR NO-STABILIZER CASES
CONSTANT AXIAL FORCE P

RATE OF INCLINATION  C = 3 (Deg./100 Feet)
HOLE DEVIATION FROM VERT.  ALPHA = 30 (Deg)
HOLE-CURVATURE RADIUS  RHO = 22918.333333 (in)
EFFECTIVE HOLE RADIUS  R = 1.5 (in)
ELASTIC CONSTANT  E = 2.8E+7 (lb/in^2)
MOMENT OF INERTIA  I = 66.378 (in^4)
WEIGHT ON BIT  P = 15000 (lb)
DISTRIBUTED WEIGHT  W = 5.021 (lb/in)

************************************************************
STRAIGHT HOLE
************************************************************

LENGTH  L = 390.944341257
CALCULATED HOLE SIZE  R = 1.49999992685

STATE VECTOR AT THE BIT

ROD DEFLECTION  Y = -1.49999992685
SLOPE  THETA = 0.00767372624978
MOMENT  M = -2.91038304567E-11
SHEAR FORCE  V = -435.17993489

STATE VECTOR AT THE TANGENCY POINT

ROD DEFLECTION  Y = 0
SLOPE  THETA = 0
MOMENT  M = 0
SHEAR FORCE  V = 548.285831236

************************************************************
INCREASING CURVATURE HOLE  5 DEGREES/100 FEET
************************************************************

LENGTH  L = 442.783526398
CALCULATED HOLE SIZE  R = 1.50000004775

STATE VECTOR AT THE BIT

ROD DEFLECTION  Y = -5.77716868033
SLOPE  THETA = 0.0227865186581
MOMENT  M = 0
SHEAR FORCE  V = -543.243525269

STATE VECTOR AT THE TANGENCY POINT

ROD DEFLECTION  Y = 0
SLOPE  THETA = 0
MOMENT  M = 81095.9493855
SHEAR FORCE \( V = 568.364517754 \)

DECREASING CURVATURE HOLE \( \frac{3}{100} \) DEGREES

LENGTH \( L = 335.922983011 \)
CALCULATED HOLE SIZE \( R = 1.49999995034 \)

STATE VECTOR AT THE BIT

ROD DEFLECTION \( Y = 0.961833303588 \)
SLOPE \( \theta = -0.00324570923748 \)
MOMENT \( M = 0 \)
SHEAR FORCE \( V = -223.20382772 \)

STATE VECTOR AT THE TANGENCY POINT

ROD DEFLECTION \( Y = 0 \)
SLOPE \( \theta = 0 \)
MOMENT \( M = -81095.9493855 \)
SHEAR FORCE \( V = 620.130821129 \)
PROGRAM TANGENCY_1

FOR NO-STABILIZER CASES
CONSTANT AXIAL FORCE P

RATE OF INCLINATION  C = 3 (Deg./100 Feet)
HOLE DEVIATION FROM VERT.  ALPHA = 30 (Deg)
HOLE-CURVATURE RADIUS  RHO = 22918.333333 (in)
EFFECTIVE HOLE RADIUS  R = 1.5 (in)
ELASTIC CONSTANT  E = 2.8E+7 (lb/in^2)
MOMENT OF INERTIA  I = 66.373 (in^4)
WEIGHT ON BIT  P = 25000 (lb)
DISTRIBUTED WEIGHT  W = 5.921 (lb/in)

******************************************************
STRAIGHT HOLE

LENGTH  L = 382.454624186
CALCULATED HOLE SIZE  R = 1.49999997967

STATE VECTOR AT THE BIT

ROD DEFLECTION  Y = -1.49999997967
SLOPE  THETA = .00784406768704
MOMENT  M = 0
SHEAR FORCE  V = -382.025520922

STATE VECTOR AT THE TANGENCY POINT

ROD DEFLECTION  Y = 0
SLOPE  THETA = 0
MOMENT  M = 0
SHEAR FORCE  V = 578.127013098

******************************************************
INCREASING CURVATURE HOLE  3 DEGREES/100 FEET

LENGTH  L = 418.314924959
CALCULATED HOLE SIZE  R = 1.49999991865

STATE VECTOR AT THE BIT

ROD DEFLECTION  Y = -5.31752268211
SLOPE  THETA = .0222550233768
MOMENT  M = 0
SHEAR FORCE  V = -401.158975215

STATE VECTOR AT THE TANGENCY POINT

ROD DEFLECTION  Y = 0
SLOPE  THETA = 0
MOMENT  M = 81095.9493855
SHEAR FORCE  V = 649.020645896
**DECREASING CURVATURE HOLE**

<table>
<thead>
<tr>
<th>LENGTH</th>
<th>L</th>
<th>338.049441392</th>
</tr>
</thead>
<tbody>
<tr>
<td>RADIUS</td>
<td>R</td>
<td>1.50000007394</td>
</tr>
</tbody>
</table>

**STATE VECTOR AT THE BIT**

| ROD DEFLECTION | Y       | -0.99309903705 |
| SLOPE          | THETA   | -0.0025517249099 |
| MOMENT         | M       | -1.45519152284E-11 |
| SHEAR FORCE    | V       | -257.886135513 |

**STATE VECTOR AT THE TANGENCY POINT**

| ROD DEFLECTION | Y       | 0 |
| SLOPE          | THETA   | 0 |
| MOMENT         | M       | -81095.9493855 |
| SHEAR FORCE    | V       | 590.786987102 |
APPENDIX IX

TORQUE ANALYSIS

In this appendix the post buckled configuration of a rod constrained within a circular cylinder, subjected to a compressive axial load and external torque is studied. In the post buckled configuration the rod takes a helical shape, Figure 1. Analysis described below shows that the contribution of torque to that configuration is negligible compared to the contribution of the axial force. A solution from Love's A Treatise on the Mathematical Theory of Elasticity [8] is used. Variable definitions are:

\( p \) = pitch of rod helical configuration

\( \gamma \) = helix angle

\( r \) = radius of the cylinder - radius of the rod

\( P \) = compressive axial load

\( T \) = externally applied torque (parallel to the axis of the cylinder)

\( EI \) = rod bending stiffness

\( C \) = rod curvature in the helical configuration

\[ EI C = Pr \cos \gamma + T \sin \gamma. \]

Replacing in terms of \( r \) and \( p \):

\[ EI 4\pi^2 r/(p^2 + (2\pi r)^2) = Prp/(p^2 + (2\pi r)^2)^{1/2} + T 2\pi r/(p^2 + (2\pi r)^2)^{1/2}. \]

This equation gives the relation between the external loads \( P \) and \( T \) to maintain the helical equilibrium configuration, and can be written as:
A drill string is used as an example to show the relation of the pitch with the applied loads. The drill string consists of steel drill collars of 6 1/4 in. O.D. and 3 in. I.D. Results are given in Figure 2 and 3. In Figure 2 the helix pitch corresponding to an axial load equal to 5,000 lb and varying torque is shown. Helix pitch changes from 4,100 in. to 4,075 in. by increasing torque from zero to 50,000 lb-in. Figure 3 shows the helix pitch variation for different axial forces.
Helical configuration of a rod under an axial compressive force $P$ and external torque $T$. 

Figure IX-1
Effect of Torque on Helix Pitch.

Figure IX-2

Torque (lb-in * 1000)

Pitch (in * 1000)
P = 5000
Effect of Torque on Helix Pitch.

Figure IX-3