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DEVELOPMENT OF A 500 MW,
ONE-MICROSECOND, MULTI-KILOAMPERE
RELATIVISTIC KLYSTRON AMPLIFIER

by

WILLIAM BRIAN HAYNES

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE
DOCTOR OF PHILOSOPHY

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Houston, Texas

May, 1996
Abstract

Development of a 500 MW, One-Microsecond, Multi-KiloAmpere Relativistic Klystron Amplifier

by

William Brian Haynes

This work presents research on the one-microsecond, L-band relativistic klystron amplifier (RKA) project conducted at Los Alamos National Laboratory. A collection of theoretical analyses is presented for rf cavities, intense electron beams, beam-cavity interactions, and small-signal klystron bunching. Electromagnetic field simulations were done for three dimensional cavity structures using HFSS with very accurate results. Particle-in-cell simulations of the complete RKA were done using the two dimensional code ISIS. Extraction efficiency for intense modulated beams is discussed and verified in simulations. Designs for input and idler cavities are reviewed. Extremely low-Q, single-gap, output cavities are investigated for coupling rf power from very low-impedance, modulated, electron beams.

Output cavities with a $Q$ less than 4 have been designed, measured, and tested. Methods were implemented for designing 2D equivalent output cavity structures to model 3D structures in 2D codes. A technique for ex-situ rf conditioning of the output cavity gap pieces is presented. A beam-pipe center conductor, intended to reduce the space-charge potential depression of the beam, is discussed. Diagnostics for intense-beam and high-power rf measurements are presented.
A coaxial directional coupler and load, capable of handling more than 500 MW at 1300 MHz, were designed. Mode conversion from coax to waveguide is discussed for >100 MW power levels. Methods for determining the gap voltage in an operating cavity are presented. Pulse-shortening of the rf in the RKA is also discussed.

A 650 kV, 5 kA, one-microsecond, annular beam has been produced from a stainless-steel, explosive-field-emission cathode. The beam current was modulated up to 70% \((I/I_0 = 70\%)\) using a two-cavity bunching section operating at 1300 MHz. RKA structures simulated in ISIS have extracted up to 250 MW. This number was consistent with the extracted power actually measured in the equivalent experiment. Overall energy extraction was as high as 160 J per pulse. The average rf output power coupled into the 6-inch-diameter coax transmission line was approximately: 300 MW for 300 ns, 250 MW for 500 ns, and 100 MW for 1 \(\mu s\). Peak power levels as high as 475 MW have also been produced.
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When you pass through, no one can pin you down, no one can call you back.

Ying-an

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Never ask what to make of life. Life is what you make of it. My path is always under my feet.

WBH

Even though you know a thousand things, ask the man who knows one.

Turkish Proverb

My Thesis Committee:

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*An ancient buddha said, "Mountains are mountains; waters are waters." These words do not mean mountains are mountains; they mean mountains are mountains.*

Dogen

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_Egil Skallagrimsson (Egil’s Saga)_

My blade, blue Dragvendil, Was a blunt shield-biter, Short-Atl’s arts Enfueled its edges. On the prating sword-pusher I used my true power, My teeth solved my troubles And tore out his throat.

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_For the wisdom of this world is foolishness with God._

I Corinthians 3:19

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Chapter I

Introduction

1.1 Historical Perspective

Every year since the first microwave tube was invented, physicists and engineers have been pushing the limits of microwave tubes in the areas of continuous and peak power, upper frequency, efficiency, and reliability. A lot of the microwave tube research and development was done in the early stages of the field, back in the days when science could be done in the form of basic research. Those days are gone now, but the legacy that the early pioneers of microwave research left to the world will never be forgotten. The purpose of this dissertation is to expand the development of one of the latest generations of high power microwave tubes, the Relativistic Klystron Amplifier.

The first microwave source was a gridded tube, the triode, based on the invention by Lee De Forest in 1906 [1]. As the frequency of the gridded tubes was increased, the inductance of the connection leads and the capacitance of the grid structures made life difficult for the early tube designers. The biggest problem, however, was the transit time of the electrons [2-9]. The radio frequency (rf) fields could go through one or more cycles before the electrons from the cathode even made it past the grid. The grid spacings were decreased to a few thousandths of an inch, but the limit for the low-power triode is still near 6 GHz even today. The magnetron also came along early in the game. The earliest reports on the development of the magnetron date back to 1921 [10, 11]. A magnetron is somewhat limited in that it is
only an oscillator and not an amplifier. Nevertheless, the development of the magnetron into a viable radar source is often credited with giving the Allies victory in Europe during the second World War. Today, the extensive array of commercial and experimental microwave tubes is impressive. Typical types of tubes include triodes, tetrodes, klystrodes, klystrons, magnetrons, traveling wave tubes, backward wave oscillators, twystrons, ubitrons, peniotrons, and gyrotrons. In general, each tube has its advantages in one or two areas. Once the tube is classified as either an oscillator or an amplifier, the important parameters are usually operating frequency, output power, efficiency, and bandwidth. More detailed information on different types of tubes can be found in books like Gilmour's [12-17].

The first klystron was made by Russell H. Varian and Sigurd F. Varian in 1939 [18]. By utilizing the inductance and capacitance of the cavity structures, and the transit time of the electrons, they were able to make a microwave tube that could be scaled to higher frequencies in the microwave regime. Their klystron was a two-cavity gridded tube like the one shown in Figure 1.1

![Diagram of a klystron](image)

Figure 1.1: The first klystron, as depicted by the Varian brothers
An electron beam is produced by the cathode at the bottom towards the collector at the top. A radio frequency voltage is built up across the gap of the first cavity by an external source. The gap voltage modifies the velocity of the electrons in the beam as they pass through. At a given phase, the electrons are slowed, and as the voltage reverses, the electrons are accelerated. These changes in velocity are relatively small compared to the initial beam velocity, but as the beam continues to drift downstream, the faster electrons begin to catch up with the slower ones ahead in the beam. The velocity modulation produces a density modulation after the electrons are allowed to drift. Soon afterwards, the beam consists of regions of higher density separated by regions of lower density. The regions of higher density are referred to as electron bunches. The bunches then pass through the gap of another cavity, in this case similar to the first cavity but without any external drive. If the second cavity resonates such that the electron bunches are retarded by the induced gap voltage, the cavity will absorb the kinetic energy from the bunched beam and convert the energy into the cavity fields. These fields have been increased substantially from the level seen in the first cavity because of the large kinetic energy present in the bunches of the beam due to the initial DC acceleration from the cathode to the anode. The power in the second cavity can be coupled out, in a similar fashion to the way it was coupled into the first, and these cavities comprise an amplifying tube. The Varian brothers showed good insight into the operation of their tube by naming it a klystron, after the Greek verb "klyzo," which describes the action of waves breaking on the beach. In this case, the bunches comprise a space-charge wave, and the output cavity gap is the beach where the bunches deposit their energy. An Appelgate diagram can depict the bunching more graphically as shown in Figure 1.2. The lines linearly represent electron trajectories. Where the lines come together, bunches have formed. The lines
Figure 1.2: Appelgate Diagram for a cavity klystron [14]

do not cross because this particular diagram is taking space charge effects into account. The operation frequency of the tube in their report was 2.3 GHz, but they did not mention the output power. They also realized that if they fed back a portion of the output signal into the input cavity, the klystron would become an oscillator.

Today the klystron is still the best source for efficient, high peak output power, phase stability, and frequency stability, which makes it the source of choice for accelerators and modern radars. The modern cavity klystron can have up to eight or even more cavities. There are no grids in these tubes because of the high beam power levels. Grids would produce too much plasma from beam impingement and would erode quickly, severely limiting the tube's lifetime. The intermediate cavities are usually referred to as idler cavities since they are only driven by the current modulation on the beam. The idler cavities add gain by increasing the modulation on the beam. These cavities can either be tuned synchronously at the same frequency for high gain, or they can be stagger tuned to increase the bandwidth at a reduced gain.
The only exception to the typical tuning in a high power tube is the last cavity before the output cavity, often termed the penultimate cavity. The penultimate is not loaded like the output cavity. Therefore, the penultimate's shunt impedance is relatively high. If the penultimate is tuned on resonance, the gap voltage induced by the beam would be high enough to break down. Instead, the penultimate is tuned slightly above the operating frequency by approximately 1.5%. Because the penultimate is being driven below the resonance peak, the gap appears inductive which tends to tighten the bunches more for better extraction at the output cavity. Typical commercial high-power klystrons can output 20-30 MW for 10 μs at 100 Hz repetition rate in L-band (1300 MHz). Special high-power tubes, made by Darryl Sprehn and others at SLAC specifically for their accelerator, have been known to put out 150 MW for 3 μs in S-band (2.998 GHz) [19].

For a moderate attempt at completeness, the reflex klystron should also be mentioned. The reflex klystron contains a single cavity for both bunching and output. Like the cavity klystron, the electrons are velocity modulated as they leave the cavity gap. Instead of allowing the bunches to form downstream, the electrons are turned around by the reflector, which is at a large negative potential. If the bunches arrive back at the gap with the correct phase to reinforce the currents in the cavity, the tube will oscillate from the feedback of the beam. A diagram of the reflex klystron is shown in Figure 1.3.
Reflex klystrons are usually low-power sources (< 1 W), but they can be made to have low noise and are phase-lockable.

1.2 Relativistic Electron Beams and Klystrons

The name relativistic klystron is something of a misnomer in that the beams are mildly relativistic, but in many cases not much more so than the beam energies in typical high-power klystrons. The beam voltage in a standard high power klystron is on the order of 200-300 kV, and relativistic klystrons have beam voltages of 400 kV or more. The SLAC/LLNL developmental relativistic klystron operates at 1.3 MV and 600 A [20, 21]. The physical mechanism for bunching in conventional klystrons, as described above in Section 1.1, and their relativistic brothers is mostly equivalent. Where relativistic klystrons are very different is in the intensity and geometry of the beam. Typical beam currents in a high power tube are on the order of a couple of hundred amps, whereas the beam currents in a relativistic klystron usually range from two to tens of kiloamps. The cathode loading of standard klystrons is on the order of
10 A/cm², but the relativistic klystron can have cathode current densities of up to 1000 A/cm². Because of the large amount of space charge in the beam, the geometry is changed to an annulus to increase the space charge limiting current and decrease the potential energy residing in the Coulomb fields. If the beam is propagated at a significant fraction of the space charge limiting current, perturbations in the beam tube, such as the gap of a cavity, cause nonlinear bunching effects in the beam of the RKA due to changes in the energy partitioning. Another consequence of the large beam currents in RKA’s is that the cavities have to be designed for much more beam loading. This implies that the input and output cavities in an RKA must have more external coupling. As the external coupling is increased, it becomes more difficult to design the cavity for the correct resonant frequency. Typical output power levels of commercial high power klystrons are on the order of 10-50 MW, not including recent advances of the SLAC tubes. Relativistic klystrons are being developed for the 200 MW to 10 GW regime and possibly even higher. The reason the RKA is capable of more output power is simply that there is much more power in the beam to modulate. The difficulty lies in trying to extract the large amounts of modulated power present on the intense beam.

Moshe Friedman originated the concept of the relativistic klystron amplifier (RKA) at the Naval Research Laboratory. He started work in the Seventies on intense beams and their interaction with cavities. His first published work was on an autoacceleration scheme for the intense beam [22]. Soon after, Friedman started looking at automodulating the beam current with four coaxial cavities along the beam line[23]. One year later, Friedman had put an output cavity on a twenty-cavity buncher section and made the first relativistic klystron oscillator, or RKO [24]. He achieved about 600 MW output for 50 ns at 2.9 GHz. Much later, Friedman started to
develop a theory for the operation of his RKO [25]. This was also his first report with some numerical simulations using the MASK 2D code. He contended that the oscillation frequency was set not by the cavity resonance frequency, but by the spacing between the cavity gaps. The feedback mechanism was provided by the beam much like in the reflex klystron except that another cavity gap has taken the place of the reflector. The next work Friedman did was to study the effects of a bunched beam on a cavity gap experimentally, theoretically, and numerically [26-29]. In 1985, Friedman finally used an external source to drive a buncher section [30]. He modulated a 5 kA intense relativistic beam to 80% above the DC current value at 1328 MHz. Along with J. Krall and Y. Y. Lau, Friedman studied the external modulation in great detail both theoretically and experimentally [31-33]. The first true RKA was reported by Friedman and analyzed by Lau in 1989 and 1990 [34-36]. This RKA is shown below in Figure 1.4.

![Figure 1.4: Moshe Friedman's 100 ns RKA](image)

Friedman showed an output pulse averaging near 2.5 GW for 50 ns at 1.3 GHz. The next year, Friedman claimed to have over 10 GW in a 100 ns pulse, but did not show an actual waveform [37]. The large amount of output power possible from RKA's sparked a lot of interest in the high power microwave community, and soon after, a
number of RKA projects sprang up across the nation including the one at Los Alamos National Laboratory.

1.3 Applications of the Relativistic Klystron Amplifier

As alluded to earlier in this chapter, most research these days needs a near-term application if any funding is to be expected, and the RKA is no exception. One major application would be as a microwave source for high-gradient accelerators. An RKA could provide up to ten times the amount of power that is currently being generated by the most state-of-the-art standard klystron [38]. The needs of the Next Linear Collider (NLC), an electron-positron collider with a 1 TeV center of mass collision energy, are estimated to be on the order of 4000 to 5000 tubes. Cutting this number by a factor of ten would make the NLC considerably more feasible.

Another logical application of the RKA is in the field of electronic warfare for damaging or upsetting electronics. The high field intensities could be enough to affect electronics at significant distances from the source.

Burning coal is still one of the major energy sources for electrical generating stations. Coal has a lot of impurities that make it even more dirty to burn than if it were pure. It has been suggested that microwaves could desorb some of these impurities before the coal is burned and decrease the cost of scrubbing the exhaust. The practicality of this method needs much study, but it is a possibility.

Recently, there has also been a lot of interest in microwave catalysis of chemical reactions. Microwaves have the ability to deposit energy directly on the catalyst surface as opposed to having a large reaction chamber at the appropriate temperature. With microwaves to drive them, catalytic reactions can take place at near atmospheric pressure and with the bulk of the reaction temperature near ambient
instead of using several atmospheres of pressure and temperatures up to 1000 °C. Since the catalyst surface is where the reaction takes place, microwave heating can produce more efficient reactions or even different types of reactions based on modifying the electric field at the surface.

1.4 Overview of the RKA Project at Los Alamos and the Author's Contribution

The development of the RKA at Los Alamos started late in 1990. The difficult task of extending Friedman's work at 100 ns to the one-microsecond pulse length regime was chosen. The ambitious goal of the program was to produce 1 GW for 1 μs at 1.3 GHz, or 1 kJ of microwave energy per pulse. Eventually, the RKA was to take full advantage of the BANSHEE modulator and be pulsed at 5 Hz or more. The first design of the RKA used a loop-driven input cavity, a fixed-frequency idler cavity, and a dual waveguide loaded output cavity [39]. A diagram of this RKA is shown in Figure 1.5. The beam energy at this time was only 1 GW, with a modulated beam power near 350 MW. Even this quickly-made, proof-of-principle RKA coupled 50 MW into the output waveguide. These early RKA results brought in funding for the project to take root. The input cavity was redesigned and built with iris coupling between the waveguide and the cavity, because the drive level was to be increased to near 500 kW from the previous level of 5 kW. A tuning slug was added to the idler cavity, and a TM010 pillbox
output cavity with coaxial coupling was being designed, based on early simulations with the particle-in-cell code ISIS. The hardware for an improved electron beam diode, to be capable of 2.5 GW of beam power, was also in the works.

It was at this point, in late June of 1991, that the author became involved with the RKA project. The author's most important responsibilities included, but were not limited to, the following: assisting in the mechanical design details of the first pillbox output cavity and the rest of the output hardware; testing the viability of the 3D electromagnetic code HFSS, to see if it could be used to design a new output cavity; modification and redesign of the output cavity; design, assembly, and testing of the high-power vacuum output coaxial directional coupler and load; design of a 2D equivalent output cavity for ISIS simulations and comparison to the experimental data; design and implementation of a high-Q conditioning cavity for the output cavity gap surfaces, and a center conductor for the beam pipe at the output cavity gap.
Other responsibilities were: the operation and repair of BANSHEE, the high-voltage, pulsed-power modulator; optimization of the diode’s surface finish and the A-K gap along with periodic resurfacing; modifications to the drive magnetron’s waveguide attenuator for high power operation; characterization of the input cavity; characterization, modification, and redesign of the idler cavity; modification of the B-dot array beam pipe used for measuring beam modulation as a function of distance for high power operation; calibration of all B-dots and determining the cavity gap voltage relational factor; calibration of all of the signal cables, waveguide, attenuators, filters, detectors, directional couplers, and beam current Rogowski coils used in the experiment; determining the necessary specifications for a 10 kiloGauss DC magnet for the RKA and bidding it out; obtaining power supplies to provide the magnet with the required 650 kW of continuous power.

1.5 Overall System Description

Before going into a detailed description of each section of the experiment, the overall system will be reviewed here. A diagram of the main sections of the experiment is shown below in Figure 1.6.

![Diagram of the RKA experiment](image-url)
Every microwave tube needs a high voltage (HV) supply to provide the kinetic energy necessary for the electrons in the beam. For the RKA, BANSHEE is the HV modulator. BANSHEE is a thyatron-switched, line-type modulator. The pulse is produced by four, lumped-element Blumleins connected in parallel to a 10:1 step-up transformer. BANSHEE can provide up to 1 MV and 10 kA of pulsed power for 1 \( \mu \)s. BANSHEE is currently operating in single shot mode, using a single spark gap in place of the thyatrons. When using the thyratrons, BANSHEE can be configured for repetitious firing at a rate of up to 5 Hz.

The heart of any tube is the electron beam. Most tubes use thermionic cathodes to provide the electrons for the beam, but the high current density needed for the RKA, and a limited budget, made an explosive field emission cathode the only feasible choice. The process for explosive field emission is somewhat up for debate, but present hypotheses are typically much like that found in Miller's book [40]. The high negative potential provided by BANSHEE starts to strip electrons from the microwhiskers on the stainless steel surface. The high current density in the whiskers heats them rapidly, and they explode, producing a plasma. The plasma is then the primary electron source from the cathode. As the plasma expands across the anode-cathode gap, the impedance drops, as does the impedance of the beam. Emission typically starts when the cathode voltage becomes more negative than about -400 kV.

The cathode tip is cylindrical to produce the annular beam used in the RKA. The electrons follow the lines of a solenoidal magnetic field which converge slightly as the field enters the beam pipe. The solenoid is a pulsed magnet composed of 10 to 11 pancake segments spaced 5 cm apart. The overall length is 1.22 m, and the bore is 25.4 cm in diameter. The approximate field at the center of the bore is 0.5 T. The magnet is energized by a 20 kV, 40 kJ capacitor bank. The magnet current rise time
is about 3.5 ms to a peak of 1100 A with an L/R decay time of 11 ms. With pulsed magnetic field systems, the diffusion of magnetic flux into the RKA cavities, flanges, and drift pipes is a major concern, because distortions in the axial guide field can be introduced. To mitigate this effect, all of the components were made from 304 non-magnetic stainless steel, and excess material was removed from the flanges without sacrificing mechanical integrity or incurring unreasonable machining expenses.

The RKA is a three cavity klystron. The input cavity takes up to 400 kW of 1.3 GHz rf from a magnetron drive source to form 15-20 kV of rf potential across its gap for the initial beam modulation. The current modulation impressed on the beam can be described in terms of its Fourier components and is of the form

\[ I(t, z) = I_0 + I_1(z)\cos(\omega t + \phi_1) + I_2(z)\cos(2\omega t + \phi_2) + \ldots, \quad (1.1) \]

where \( I_0 \) is the DC beam current and \( I_1, I_2, \) etc. are the harmonic components of the beam current. At the idler cavity gap, the fundamental current modulation \( I_1 \) is about 10% of the dc beam current \( (I_1/I_0 = 10\%) \). The idler cavity has no external drive, but is driven by the modulation on the beam. The idler cavity's resonant frequency is tuned slightly above the drive frequency \( \omega \), so that it can increase the modulation on the beam without breaking down. A typical idler cavity gap voltage is about 200 kV, and the current modulation is around 66% at the location of the output cavity gap. The output cavity is a very heavily rf-loaded, low-Q cavity. The low \( Q \) is necessary to keep the output cavity gap fields from overcoming the kinetic energy in the electrons of the beam. As the electron bunches are slowed in the output cavity gap, the rf energy in the output cavity is coupled out into a low-impedance coaxial line. The coax then tapers out into standard 6 in., 50 \( \Omega \) coax where the power is
measured with a directional coupler and then dumped into a coaxial load at the end of
the line. Once the beam is past the output cavity gap, the electrons follow the
diverging field lines of the solenoidal magnetic field and impact into the wall of the
beam dump. A somewhat more detailed summary of this experiment than the above
paragraph is available in a special issue of the IEEE Transactions on Plasma Science
[41].

1.6 Organization of this Work

Descriptions of each aspect of the experiment are to be found in later chapters.
Chapter 2 covers much of the theory of the RKA, including basic microwave cavity
analyses and terminology, intense relativistic electron beam physics, beam-cavity
interactions, classical space charge bunching, relativistic bunching, and computer
modeling. The beam and related items are covered in Chapter 3. Power measurement
and high power mode conversion from TEM coax to TE$_{10}$ waveguide are discussed in
Chapter 4. Chapters 5, 6, and 7 cover the input, idler, and output cavities,
respectively. Chapter 8 presents conclusions from the experiment and many possible
areas for future work. In the Appendices, there is information on useful mathematical
relationships for relativistic beams, some experimental details for different parts of
the tube, ISIS input files, and mechanical drawings for much of the RKA.
Chapter II
Theory

There are a number of areas to analyze in the RKA. One is the fundamental rf response of the cavities and coupling structures alone. The next is an introduction to the physics of intense relativistic electron beams. Then comes the analysis of the cavities when beam is present in the drift tube. Small-signal bunching is reviewed for both classical and relativistic klystrons. The last area that must be examined is the modeling of the electromagnetic fields in the cavities and the entire tube, including the beam, in as similar a fashion to the real device as is feasible. Because of the nonlinear action in the RKA, the last area of analysis is necessarily dominated by particle-in-cell (PIC) simulation codes.

2.1 Cavity Analysis

Resonant circuits and structures have two important parameters in common across all scientific boundaries; the resonant frequency and the quality factor. The quality factor is designated $Q$ and relates the energy stored to the energy lost as is shown in Eqn. (2.1).

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy lost per cycle}}$$  (2.1)
In practice, a more convenient way to measure the $Q$ is to find the resonant peak's location ($f_o$), and then find the frequencies ($f_i$ & $f_f$) above and below $f_o$ where the response has decreased by 3 dB (0.707). The $Q$ can then be calculated using Eqn. (2.2).

$$Q = \frac{f_o}{\Delta f}, \quad \text{where} \quad \Delta f = f_f - f_i,$$

(2.2)

To simplify the analysis, the particular mode of interest can be represented by a series or parallel lumped-element circuit. In order to effectively use the lumped circuit models, the concept of detuned reference planes must be introduced. When the cavity is being excited well off of resonance, there will be a standing wave in the coupling waveguide. The location of the maxima and minima are the locations of the planes of the detuned open and detuned short, respectively, as shown in Figure 2.1.

![Figure 2.1: Locations of the planes of the detuned short and open](image)
At resonance, the impedance of the circuit model will be real, as will the impedance of the actual cavity if viewed at the detuned plane positions. The deciding factor in choosing one position or the other depends on the desired circuit model. A series resonant circuit will have a high impedance off resonance which corresponds the detuned open position. The parallel resonant circuit has a low impedance off resonance and is the model of choice when at the plane of the detuned short. It is most convenient to model the beam as a current source into a parallel load; therefore, the parallel resonant circuit model will be used here, and it will be implicitly assumed that we are using the reference plane of the detuned short. The parallel circuit model is shown in Figure 2.2.

![Parallel circuit model for a microwave cavity](image)

**Figure 2.2: Parallel circuit model for a microwave cavity**

The components $R_p$, $C_p$, and $L_p$ make up the overall impedance of the cavity, $R_p$ is the shunt impedance, $M$ is the mutual inductance of the coupling transformer, $Z_0$ is the characteristic line impedance, and $V_s$ is the gap voltage. In most instances, these definitions are made along the centerline of the cavity. This is because the standard klystron still uses TM_{010} cavities and a pencil beam on axis. Later on, to make these definitions more useful to the RKA, the shunt impedance and gap voltage will be defined relative to our beam's location. In this way, the gap voltage can be immediately calculated from the induced current and the shunt impedance.
Additional concepts that must be introduced are those of cavity coupling and loaded $Q$. To be useful, a cavity must be connected in some fashion to the external world. The degree of coupling is indicated by the parameter $\beta$. The introduction of the coupling parameter is usually brought about from a circuit analysis of the cavity. The analysis that follows is one from Ginzton [42], who modeled the cavity as a series circuit as shown in Figure 2.3.

![Diagram](image)

**Figure 2.3**: Series model of a resonant cavity

Losses in the coupling circuit are neglected in this simplified model. The circuit in Figure 2.3d is simplified to represent the primary of the coupling transformer as a
coupled impedance in series with the cavity parameters. The impedance coupled in series with the cavity parameters due to a matched generator is given by

\[
Z = \frac{(\omega M)^2}{Z_0 + j\omega L}, \quad (2.3)
\]

or

\[
Z = \frac{(\omega M)^2}{Z_0 \left[1 + (\omega L / Z_0)^2\right]} \left(1 - j \frac{\omega L}{Z_0}\right). \quad (2.4)
\]

Now define \( \beta \) as

\[
\beta = \frac{(\omega M)^2}{Z_0 R_s} \frac{1}{1 + \left(X_s / Z_0\right)^2}, \quad (2.5)
\]

or

\[
\beta = \beta_s \frac{1}{1 + \left(X_s / Z_0\right)^2}, \quad (2.6)
\]

where

\[
\beta_s = \frac{(\omega M)^2}{Z_0 R_s}. \quad (2.7)
\]

The factor \( \beta_s \) is the ratio of coupled resistance to the cavity resistance \( R_s \). Letting \( X_s = \omega L_s \), Eqn. (2.4) becomes

\[
Z = \beta R_s \left(1 - j \frac{\omega L_s}{Z_0}\right). \quad (2.8)
\]

The loaded \( Q \) value of the system (\( Q_L \)) is defined as the ratio of total reactance to total series loss. It is given by

\[
Q_L = \frac{\omega L - \beta R_s X_s / Z_0}{R_s (1 + \beta)}, \quad (2.9)
\]

or

\[
Q_L = \frac{\omega L / R_s - (\beta R_s / Z_0) (X_s / \omega L)}{R_s (1 + \beta)}. \quad (2.10)
\]
The second term in the numerator of Eqn. (2.10), representing the ratio of coupled reactance to the cavity reactance, is usually small compared to unity and can be neglected. Equation (2.10) then simplifies to

$$Q_i = \frac{Q_0}{1 + \beta}, \text{ where}$$

$$Q_o = \frac{\omega L}{R_1}. \quad (2.12)$$

When the coupled resistance and the cavity resistance are equal, $\beta_i = 1$, and the cavity is said to be critically coupled. When $\beta_i < 1$, the cavity is undercoupled; when $\beta_i > 1$, the cavity is overcoupled. Under most circumstances, the second term in Eqn. (2.6) is nearly equal to unity and $\beta_i \sim \beta$. From Eqn. (2.11), it is apparent that at critical coupling, $Q_i = Q_o/2$ [43].

Equation (2.11) can be written as

$$\frac{1}{Q_i} = \frac{1}{Q_o} + \frac{\beta}{Q_o}, \text{ or}$$

$$\frac{1}{Q_i} = \frac{1}{Q_o} + \frac{1}{Q_{re}}. \quad (2.14)$$

The measured $Q$ is the loaded $Q$ ($Q_i$) and actually represents a combination of the unloaded cavity $Q$ ($Q_o$) and the the external $Q$ ($Q_{re}$) of the external circuitry that connects to the cavity. It should be noted that any loading on the cavity, either by external circuitry or resistive wall losses, will lower the resonant frequency compared to the unloaded case [12, 44].

The coupling parameter can be easily determined by measuring the cavity response. Looking at $S_1$ on a Smith chart, the cavity will trace out a response similar
to one of the examples in Figure 2.4. (The reference plane is at the detuned short position)

![Diagram showing undercoupled, unity coupled, and overcoupled states](image)

Figure 2.4: Measurement of $S_n$ for different degrees of cavity coupling

The voltage standing-wave-ratio, or VSWR, in the waveguide is equal to

$$VSWR = \frac{1 + |S_n|}{1 - |S_n|} .$$

(2.15)

Once the type of coupling and the VSWR are determined, $\beta$ is given as follows:

- Undercoupled: $\beta = 1/VSWR$  
  \hspace{1cm} (2.16)
- Unity coupled: $\beta = 1$  
  \hspace{1cm} (2.17)
- Overcoupled: $\beta = VSWR$  
  \hspace{1cm} (2.18)

The degree of coupling desired in any given cavity depends on its function, as will be shown in great detail in the following chapters. The input cavity needs to be heavily overcoupled with no beam in the gap. Once beam is present, the losses in the cavity increase drastically, and the cavity becomes unity coupled, which allows all of the injected rf power to go into modulating the beam. The output cavity of the RKA needs to be even more overcoupled than the input. The low $Q$ keeps the gap fields from becoming so high as to reflect electrons in the beam back upstream. The rf
conditioning cavity for the output cavity gap pieces needs to have a high $Q$ so that large fields can be generated for relatively low input power, but the conditioning cavity also needs to be unity coupled to minimize the reflected power returning to the klystron and to maximize the internal fields of the cavity.

Another important parameter, that is often referred to in cavity analysis, is $R/Q$. The definition of $R/Q$ is given in Eqn. (2.19) below.

$$\frac{R}{Q} = \frac{\left( \int \mathbf{E} \cdot d\mathbf{l} \right)^2}{\omega \epsilon \int |\mathbf{E}|^2 dV} \tag{2.19}$$

The line integral in the numerator is taken along a path defining the gap voltage. The denominator is twice the power lost by the cavity multiplied by the $Q$. This expression is governed by geometrical factors and is valid regardless of the amount of external loading and, to a good approximation, independent of the beam loading [42, 45]. In practice, it is often easier to experimentally determine the shunt impedance $R_s$ from a measurement of $R/Q$ and $Q$ than to measure the shunt impedance directly. Ginzton covers an experimental method for determining $R/Q$ using a small perturbation to the cavity in the region where high electric fields are present.

With accurate simulations (see Section 2.6.1), however, the shunt impedance can be determined from an integral of the peak electric field along the beam trajectory which gives the gap voltage $V_s$. The power lost by the cavity every second is

$$P_{\text{lost}} = \frac{V_s^2}{2R_s} \tag{2.20}$$

In the case of the RKA output cavity, the input power to the drive probe is known and the power transmission is known from the square of $S_{21}$. Since almost all of the power is coupled out into the coax, the power transmitted through the structure
essentially equals the power lost from the cavity. Once the gap voltage for a given power loss is determined, the shunt impedance is found by solving Eqn. (2.20) for $R_p$.

$$R_p = \frac{V^2}{2P_{ms}}$$

(2.21)

Now, $R/Q$ can be found by dividing the shunt impedance $R_p$ by the cavity $Q$ as determined from a frequency sweep.

2.2 Intense Relativistic Beams

Intense relativistic electron beams have a number of problem areas that need attention when designing high-power-electron devices. High-current beams have large self-magnetic and electric fields which can lead to beam instabilities. The large amount of space charge stores significant energy in the beam's Coulomb fields which can slow and even stop portions of the beam, if the current approaches or exceeds the space-charge-limiting current. The beam heavily loads any cavity it passes through, making cavity design difficult. Any electron beam will have some non-ideal distribution function through the cross-section. The portion of the beam outside the most dense section is called the beam's halo. In an intense beam, hundreds of amps can be in the halo, giving a higher probability for electrons to be in the wrong area. For instance, the nose of a cavity gap where the electric field is greatest. The topics of beam instabilities and the space-charge-limiting current will be touched upon in this section, and the beam's effect on cavites will be covered in Section 2.3.
2.2.1 Beam Instabilities

The diocotron, or slipping stream, instability has been known for some time [46]. The diocotron instability arises from azimuthal rotational shear between the outer and inner portions of the beam. It has been extensively studied in crossed-field and parallel-field devices [40, 47-54].

Early in the characterization of the RKA beam, fast framing photographs were taken at the end of the drift tube. During the beginning of the pulse, there was evidence of some turbulent instability in distinct spots around the circumference. A fast-framing picture is shown below in Figure 2.5. The beginning of the pulse starts at the lower left hand side of the picture, and the frames are 167 ns apart.

![Fast-framing picture](image)

Figure 2.5: End view of RKA annular beam taken by a fast framing camera

(Shot 414)

This instability was thought to be due to the diocotron instability since it was similar in form to pictures in the literature [47, 51].

Davidson analyzes the diocotron instability for an annular relativistic beam, as does Han Uhm [52, 54]. Using Davidson's formulae,
(Dimensionless beam geometric factors)

\[ \Delta_i = \frac{x_b^*}{d} \quad , \quad \text{(2.22)} \]

\[ \Delta_b = \frac{x_b^* - x_i^*}{d} \quad , \quad \text{(2.23)} \]

\[ \Delta_o = \frac{d - x_b^*}{d} \quad , \quad \text{(2.24)} \]

where \( d \) is the beam pipe radius, \( x_b^* \) is the outer beam radius, and \( x_i^* \) is the inner beam radius. The relativistic flow parameter (in cgs) is

\[ \theta = \kappa(x_b^* - x_i^*) = \frac{4\pi \hat{n}_i e(x_b^* - x_i^*)}{B_o} \quad . \quad \text{(2.25)} \]

In the long wavelength regime \( (k^2 d^2 << 1) \), the necessary and sufficient condition for instability is

\[ g \equiv \frac{2(\Delta_o \Delta_i)^{\frac{1}{2}}}{\Delta_b} > \frac{\sinh \theta}{\theta} \quad . \quad \text{(2.26)} \]

Using typical parameters for the RKA at midpulse,

\[ x_b^* = 2.7 \text{ cm}, \ x_i^* = 3.1 \text{ cm}, \ d = 3.65 \text{ cm}, \]

\[ \hat{n}_i = 1.27 \times 10^{11} \text{ cm}^{-3} \quad (I = 4 \text{ kA}, \beta = 0.9), \ B_o = 5000 \text{ G}, \]

\[ \text{and} \quad e = 4.803 \times 10^{-10} \text{ statC}. \]

This makes \( g = 6.1 \) and \( \theta = 0.06 \). The condition for instability, Eqn. (2.26), is satisfied for these beam parameters.

Uhm incorporated a factor for charge neutralization in his analysis. Assuming a low fraction of space charge neutralization from a background of ions (i.e. a good
vacuum), there is a possibility for several different instability modes using typical RKA beam parameters. Even though the diocotron instability cannot be completely discounted by these analyses, there is little possibility that the beam is undergoing any such breakup. The reasons for this are twofold. Firstly, the beam should become more distorted as it travels further down the beam pipe. Witness plate shots near the end of the RKA still show a smooth annular profile. Secondly, the instability seems to be most prevalent in the early portion of the pulse, when the current is relatively low. As the current increases, the instability should grow instead of essentially disappearing as it does in Figure 2.5. The instabilities shown in the early part of the pulse are most likely due to some filamentation process as the cathode starts to turn on, since it is unlikely that the field-emission breakdown should occur in a consistent and smooth fashion around the circumference of the cathode.

Although not an instability per se, another problem area for electron beam driven tubes is the electron cyclotron resonance. In general, this is more of a problem for BWO's and TWT's since the rf structure is distributed all along the beam, but the cyclotron resonance can, in principle, adversely affect any tube with a magnetic field. The relativistic cyclotron frequency is given by [52]

$$\omega_{cb} = \frac{eB_0}{\gamma_v mc} \quad \text{(cgs)}$$

(2.27)

Using the parameters given above for the RKA ($\gamma_v = 2.2$),

$$\omega_{cb} = 4.0 \times 10^{10} \text{ s}^{-1},$$

$$f_{cb} = 6.4 \text{ GHz}.$$  

Therefore, the cyclotron resonance is well above the operation frequency of 1.3 GHz.
2.2.2 The Space-Charge-Limiting Current

One of the most important concepts to cover in relation to high power microwave tubes is that of the space-charge-limiting current. The analysis for relativistic beams, similar to those used in relativistic klystrons, was first done by Bogdankevich and Rukhadze in 1971 [55]. For a given beam energy, beam pipe, and beam geometry, there is a current value which cannot be exceeded. This is the space-charge-limiting current. If this value is exceeded, the excess space charge piles up into a plasma which oscillates at the plasma frequency. The oscillating plasma is a form of virtual cathode and can radiate microwave fields. In fact, there are high-power microwave sources based on this fundamental principle called virtual cathode oscillators [56].

The concept of the space-charge-limiting current is also intimately related to the partitioning of the beam energy into kinetic and potential portions. In general, only the beam's kinetic energy can be extracted, so it is necessary to track the partitioning closely as the beam is injected and subsequently modulated. The analysis that follows is equivalent to that done by Carlsten and Miller [40, 57], and also to that done by Nation and Humphries [58, 59]. For any given injection voltage in a diode, the actual beam kinetic energy will be lower since some potential energy is required to create the Coulomb fields inside the grounded beam pipe. The potential of an infinitely thin annular electron beam with radius \( r_s \) and initial gamma \( \gamma_{m0} \), in the diode, must satisfy Laplaces' equation,

\[
\nabla^2 \phi = 0 ,
\]

in the regions inside and outside the beam up to the wall radius \( r_w \), or in cylindrical coordinates,
\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \phi}{\partial r} \right) = 0.
\]  
(2.29)

The solution is subject to the boundary conditions

\[
\phi(r_w) = 0
\]  
(2.30)

and

\[
\left. \frac{\partial \phi}{\partial r} \right|_{r_w} = \frac{I}{2\pi \varepsilon_0 r_w v_0},
\]  
(2.31)

where \( I \) is the total beam current, and \( v_0 \) is the longitudinal electron velocity which is uniform across the beam. The solution is [60]

\[
\phi(r) = \frac{I}{2\pi \varepsilon_0 v_0} \ln \frac{r}{r_w}, \quad r_w < r < r_w,
\]  
(2.32)

and

\[
\phi(r) = -\frac{I}{2\pi \varepsilon_0 v_0} \ln \frac{r_w}{r_b}, \quad r \leq r_b,
\]  
(2.33)

where the beam velocity is given by

\[
v_0 = c \sqrt{1 - \left( \gamma_{sw} - \frac{e\phi(r_w)}{m_0 c^2} \right)^2}.
\]  
(2.34)

The velocity of light is \( c \), and the electronic charge \( e \) is positive for these calculations. At \( r = r_b \), the beam potential must satisfy \( \phi_b = \phi(r_b) \)

\[
\frac{e\phi_b}{m_0 c^2} \sqrt{1 - \left( \gamma_{sw} - \frac{e\phi_b}{m_0 c^2} \right)^2} = \frac{e}{2\pi \varepsilon_0 m_0 c^3} I \ln \frac{r_w}{r_b} = \frac{I}{8.5 \text{kA}} \ln \frac{r_w}{r_b}.
\]  
(2.35)

Miller designates the left hand side of the above equation as \( F(\phi_b) \), and this is shown in Figure 2.6.
The function $F(\phi_b)$ is largest for a beam potential energy increase and a corresponding kinetic energy decrease of

$$e\phi_b = (\gamma_{\text{ej}} - \gamma_{\text{ej}}' \gamma_{\text{ej}}^{-1})m_pc^2,$$  \hspace{1cm} (2.36)

which gives the largest possible current to be

$$I_{\text{max}} = \frac{2\pi e_0 m_pc^3}{e \ln \frac{r_x}{r_b}} \left( \gamma_{\text{ej}}^{2/3} - 1 \right)^{1/2}.$$  \hspace{1cm} (2.37)

It is convenient to define a normalized threshold current $I_a$ to be

$$I_a = \frac{2\pi e_0 m_pc^3}{e \ln \frac{r_x}{r_b}} = \frac{8.5\text{kA}}{\ln \frac{r_x}{r_b}}.$$  \hspace{1cm} (2.38)

This recasts Eqn. (2.37) as

$$I_{\text{max}} = I_a \left( \gamma_{\text{ej}}^{2/3} - 1 \right)^{1/2}.$$  \hspace{1cm} (2.39)

Conservation of energy dictates that
\[ \gamma_{\text{eq}} = \gamma_0 + \frac{I}{I_0 \beta_0} \]  

(2.40)

where the zero in \( \gamma_0 \) and \( \beta_0 \) denotes the DC beam values in the beam pipe. It is useful to see the partitioning of energy in the beam as the space-charge-limiting current is reached. The beam's kinetic energy at this point is

\[ E_k = (\gamma_{\text{eq}}^{10} - 1)m_c c^2 \]  

(2.41)

and the potential energy is

\[ E_p = (\gamma_{\text{eq}} - \gamma_{\text{eq}}^{10})m_c c^2 \]  

(2.42)

For practical values of injection voltage, where \( \gamma_{\text{eq}} \) is about 2, only 26% of the energy is kinetic and the other 74% of the energy is potential. In fact, for all \( \gamma_{\text{eq}} > 1 \), the potential energy of the beam will always exceed the kinetic energy. Only the kinetic energy of the beam is extractable into microwaves. As the beam is bunched, more space charge is being grouped together and thus the potential energy of the bunches is increased and the bunches slow down. Therefore, for any given beam geometry, there is a point where increasing the bunching does not increase the amount of extractable power, since the additional energy goes into the potential fields of the beam. Another way to see this is to transform Eqn. (2.35) into an expression for the minimum energy needed to transport beam. Using energy conservation principles once again,

\[ \gamma_{\text{eq}}^{20} = \left( \frac{E_k + E_p}{m_c c^2 + 1} \right)^{20} = \left( \frac{I_{\text{peak}}}{8.5 kA} \ln \frac{r_b}{r_k} \right)^{20} + 1 \]  

(2.43)

For a given peak current, \( I_{\text{peak}} \), the difference between \( m_c c^2 \gamma_{\text{eq}} \) and \( m_c c^2 \gamma_{\text{min}} \) is the kinetic energy available for conversion to microwaves. The potential energy in the
Coulomb fields is lost. Thus, the maximum power that can be extracted as microwaves is given by
\[ P_{\text{max}} = \frac{1}{2} I_i (511 kV) (\gamma_{\text{inj}} - \gamma_{\text{max}}), \]  
(2.44)
where \( I_i \) is the peak modulated current in the fundamental component. Some forms of RKA output structures convert the potential energy of the beam back into kinetic energy by allowing the beam to intercept a ground plane at the end of the extraction gap such as was done by Friedman [35]. Friedman could do this because his pulse was only 100 ns long, which is shorter than the time it would take the plasma produced by the impact to close and short the extraction gap. For our 1 \( \mu s \) RKA, we had to insure that the beam dumped a reasonable distance (> 20 cm) away from the extraction gap. This fact made it impossible to reconvert any of the potential energy back to kinetic in the same way in our output cavity.

2.3 Beam-Cavity Interactions

The presence of beam in a microwave cavity can have a major effect on the cavity’s response. The beam loads the input cavity and affects the resonant frequency, but the most drastic change is in the coupling. As was mentioned before, the input cavity needed to be matched with the beam present. Both of these topics will be covered in the next two sections. An introduction to the cavity transit-time factor can be found in Section 2.3.3.

2.3.1 Cavity Resonant Frequency Shift with Beam Loading

Rob Ryne, at Los Alamos, analyzed the effect of beam loading on the resonant frequency of a cavity [61]. Most of his analysis is reproduced below.
Let \( \tilde{A}_\alpha \) and \( \omega_\alpha \) represent the cavity eigenmodes and eigenfrequencies, respectively, of a klystron input cavity that is initially excited by a DC beam. For a given mode \( \alpha \), the cavity will also be driven close to resonance, \( \omega \sim \omega_\alpha \). The fields in the cavity will predominantly be in the driven mode, and the vector potential inside the cavity can be written approximately as

\[
\tilde{A}(\vec{x},t) = a(t)\tilde{A}_\alpha(\vec{x}) ,
\]  

where

\[
a(t) = \text{Re}\{b(t)e^{-i\omega t}\} ,
\]  

and \( b(t) \) is a slowly varying function of time. The function \( b(t) \) describes the envelope of the transient ring up of the signal in the cavity as a function of time. If you take the dot product of the eigenmodes of the cavity, which satisfy the wave equation, with the driven electric fields of the cavity, which also satisfy the wave equation with a drive term, and then make certain approximations for a finite cavity \( Q \), it can be shown that \( b(t) \) satisfies the following differential equation:

\[
\dot{b} + \left( \frac{\omega_\alpha}{Q_\alpha} - 2i\omega \right)b + \left( \omega_\alpha^2 - \omega^2 - i \frac{\omega_\alpha \omega}{Q_\alpha} \right)b = \frac{1}{\varepsilon_0} \int \vec{J}_{\text{rf}} \cdot \vec{A}_\alpha \, d^3x ,
\]  

where

\[
\vec{J}_{\text{rf}}(\vec{x}) = \frac{1}{\pi} \int_{-\pi}^{\pi} \vec{J}(\vec{x},t)e^{i\omega t} \, d(\omega t) .
\]

The right hand side of Eqn. (2.47) is the drive term, and \( \vec{J}_{\text{rf}} \) represents the rf beam current density.

Now consider an infinitely thin annular beam of radius \( r = r_s \). For an initially DC beam, it is possible to calculate the right hand side of Eqn. (2.47) in the small-signal approximation. The result is given by
\[ \frac{1}{\varepsilon_0} \int_i A_i \cdot A_i d^3x = G \left[ \frac{I_{dc}}{\gamma_0 (\gamma_0^2 - 1)} \right] \frac{\omega q/mc^2}{\varepsilon_0} b, \]  

(2.49)

where \( I_{dc} \) is the DC beam current, \( \gamma_0 \) is the beam relativistic factor, \( q \) is the electronic charge, and \( G \) is a geometrical factor given by

\[ G = \frac{\omega}{v_0} \int_0^r A_a(z) e^{i \omega z/v_0} \int_0^r A_a(z') e^{-i \omega z'/v_0} dz'' dz' dz, \]  

(2.50)

where \( A_a(z) = A_a(r = r_s, z) \) is the vector potential at \( r = r_s \) and \( v_0 \) is the DC beam velocity. Integrating by parts, it follows that

\[ G = \frac{\omega}{v_0} \int_0^r A_a(z) e^{i \omega z/v_0} \left[ \int_{z_0}^{z_1} A_a(z') e^{-i \omega z'/v_0} dz' - \int_{z_1}^{z_2} A_a(z') e^{-i \omega z'/v_0} dz' \right] dz \]  

(2.51)

Now let the geometrical factor be broken into real and imaginary parts, or

\[ G = G' + iG''. \]  

(2.52)

Integrating by parts again, we obtain the following expression for \( G' \):

\[ G' = \left( \int_0^r \frac{\omega z}{v_0} A_a \sin \frac{\omega z}{v_0} \right) \left( \int_0^r \frac{\omega z}{v_0} A_a \cos \frac{\omega z}{v_0} \right) - \left( \int_0^r \frac{\omega z}{v_0} A_a \cos \frac{\omega z}{v_0} \right) \left( \int_0^r \frac{\omega z}{v_0} A_a \sin \frac{\omega z}{v_0} \right). \]  

(2.53)

(The beam loaded \( Q \) is actually related to \( G' \).) Rewriting the circuit equation, Eqn. (2.47), we obtain

\[ \hat{b} + \left( \frac{\omega_a}{Q_a} - 2i \omega \right) \hat{b} + \left( \omega_a^2 - \omega^2 - i \frac{\omega \omega_a}{Q_a} \right) \hat{b} = G \left[ \frac{I_{dc}}{\gamma_0 (\gamma_0^2 - 1)} \right] \frac{\omega q/mc^2}{\varepsilon_0} b. \]  

(2.54)

Now let \( \Delta \omega \) equal the difference between the cavity frequency and the drive frequency

\[ \Delta \omega = \omega_a - \omega. \]  

(2.55)

Then

\[ \omega_a^2 = \omega^2 + 2 \omega \Delta \omega + \Delta \omega^2. \]  

(2.56)
Neglecting terms of order $\Delta \omega^2$, we obtain

$$
\ddot{b} + \left( \frac{\omega_a}{Q_a} - 2i\omega \right) \dot{b} + \left( 2\omega \Delta \omega - i \frac{\omega \omega_a}{Q_a} \right) b = G \frac{I_{dc}}{\gamma(\gamma^2 - 1)} \frac{\omega q/mc^2}{\epsilon_0} b.
$$

(2.57)

Now it is convenient to define the beam loaded $Q$, $Q_a$, and the effective $Q$, $Q_{\sigma}$, much like was done in Eqn. (2.14).

$$
\frac{1}{Q_{\sigma}} = \frac{1}{Q_a} + \frac{1}{Q_{bi}},
$$

(2.58)

where

$$
Q_{bi} = \left[ \frac{\gamma (\gamma^2 - 1)}{I_{dc} G_i} \right] \frac{\omega_a \epsilon_0}{q/mc^2}.
$$

(2.59)

Similarly, there is also a frequency shift due to the beam loading, $\Delta \omega_a$, and an effective frequency shift, $\Delta \omega_{\sigma}$, according to

$$
\Delta \omega_{\sigma} = \Delta \omega + \Delta \omega_{bi},
$$

(2.60)

where

$$
\Delta \omega_{bi} = - \frac{q/mc^2}{\epsilon_0} \left[ \frac{I_{dc} G^i / 2}{\gamma (\gamma^2 - 1)} \right].
$$

(2.61)

The effective resonant frequency is obtained by setting $\Delta \omega_{\sigma} = 0$:

$$
\omega_{\sigma} = \omega_a - \frac{q/mc^2}{\epsilon_0} \left[ \frac{I_{dc} G^i / 2}{\gamma (\gamma^2 - 1)} \right].
$$

(2.62)

Ryne goes on to analytically determine the geometrical factors for a simple pillbox cavity, and thus the frequency shift. But for cavities of arbitrary shape, analytic expressions for $A_e$ cannot be easily found, and numerical methods must be used.

Ryne wrote a computer code, BLQ, that calculates $Q_e$ and $\omega_{\sigma}$ from numerical values of the cavity eigenmode obtained from the rf cavity code SUPERFISH. BLQ uses these values to numerically evaluate Eqn. (2.53) for $G^i$ and Eqn. (2.52) in
conjunction with Eqn. (2.51) for $G'$. At first, the code was verified using the analytical solution for the pillbox cavity. The numerical results agreed well with the analytical model.

Next the code was used to evaluate the input cavity of the RKA. Ryne used beam parameters of 500 keV, 5 kA, and a nominal beam radius of 3.3 cm. SUPERFISH calculated the resonant frequency of the unloaded structure to be 1310 MHz. Running BLQ on resonance gave the following results:

$$Q_0 = 88;$$

$$\omega_0 = 2\pi \times 1303 \text{ MHz}.$$ 

The beam-loaded $Q$ is a sensitive function of the annular beam radius. In Figure 2.7, the variation of $Q_0$ as a function of $r_e$ can be readily seen.

![Figure 2.7: Beam-loaded $Q$ as a function of beam radius](image)

The beam-loaded $Q$ changes rapidly as the beam radius approaches the beam pipe radius (3.65 cm). The shift in cavity frequency, however, was less than one might expect. For a beam radius range of 0 to 3.5 cm, the effective frequency shift was only
5 MHz. The unloaded $Q$ of the input cavity was about 27, which implies the 3 dB bandwidth is about 48 MHz, which is much larger than the frequency shift.

2.3.2 Input Cavity Drive Reflection with Beam Loading

Now that we know the effect of the beam on the cavity frequency and $Q$, we can look at the effect of the beam on the input coupling match. The equivalent circuit of a cavity with beam is shown in Figure 2.8.

![Figure 2.8: Equivalent cavity circuit for beam-cavity interactions.](image)

Note the beam acts as a current generator, $i_b$, in parallel with some stray beam capacitance, the beam impedance, and the cavity. For the input cavity, the beam rf current is zero; therefore, $i_b$ is also zero in the model. In the cavity model, $\beta$ is defined as the coupling coefficient similarly to Eqn. (2.7).

$$\beta = \frac{\omega^2 M^2 Q_0^2}{Z_0 Z_{cav}}$$  \hspace{1cm} (2.63)

The input waveguide has a characteristic impedance $Z_0$, $Q_0$ is the unloaded cavity $Q$, $Z_{cav}$ is the cavity impedance, and $M$ is the mutual inductance of the coupling. In terms of the parallel impedance with the cavity driven on resonance,

$$\beta = \frac{\omega^2 M^2 Q_0^2}{Z_0 R_p}.$$  \hspace{1cm} (2.64)
From a SUPERFISH run, the cavity $R_p/Q = 76$, and a measurement of the unloaded cavity $Q_o$ (with the coupling slot taped over) was 2060. This implies that

$$R_p = \left( \frac{R_p}{Q_o} \right) Q_o = 1.56 \times 10^4 \Omega \quad .$$

(2.65)

The cavity input reflection was measured with a network analyzer which gave a value for $S_{11}$. Using Eqns. (2.15) and (2.18), the unloaded beam coupling parameter, $\beta_{beam}$, is calculated to be 38.2. Now, the last missing section of Equation (2.64) can be found.

$$\frac{\omega M}{Z_o} = 1.40 \quad .$$

(2.66)

Since Eqn. (2.64) can be recast as

$$\beta = \frac{\omega^2 M^2}{Z_o} \left( \frac{Q_o}{R_p} \right)^2 R_p \quad ,$$

(2.67)

or with measured values,

$$\beta = 1.40 \left( \frac{1}{76} \right)^2 R_p = 2.42 \times 10^{-4} R_p \quad ,$$

(2.68)

the coupling can be calculated once the beam resistance is known.

From STORK [45], the parallel resistance with the beam is

$$R_p = 1.70 \times 10^{-5} \left[ \frac{\gamma(y^2 - 1) m_0 c^2}{I_0} \right] R_b(r) \quad .$$

(2.69)

The beam impedance as a function of radius $R_b(r)$ is shown below in Figure 2.9. Note the similarity between the beam impedance variation with beam radius and the beam-loaded $Q$ as shown back in Figure 2.7.
Using Equations (2.68) and (2.69), the reflected power in the cavity as seen from the waveguide port can now be calculated from

\[ P_{\text{ref}} = \left( \frac{1-\beta}{1+\beta} \right)^2. \] (2.70)

As the beam impedance drops, the loaded \( Q \) of the cavity also drops as would be expected. Since the beam detuned the cavity frequency by only a small amount, the beam capacitance can be neglected.

The reflected power was calculated for the changing beam parameters and overlaid with an actual reflected power measurement from the RKA. A couple of points are calculated as an example in Table 2.1.

Table 2.1: Calculated beam parameters at two times for shot 422

<table>
<thead>
<tr>
<th>( t (\mu s) )</th>
<th>( V (kV) )</th>
<th>( I (kA) )</th>
<th>( \gamma )</th>
<th>( R_s (k\Omega) )</th>
<th>( \beta )</th>
<th>( P_{\text{ref}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4</td>
<td>420</td>
<td>0.5</td>
<td>1.82</td>
<td>14.6</td>
<td>3.53</td>
<td>0.312</td>
</tr>
<tr>
<td>3.0</td>
<td>625</td>
<td>3.9</td>
<td>2.223</td>
<td>3.89</td>
<td>0.941</td>
<td>0.00092</td>
</tr>
</tbody>
</table>
The results are shown below in Figure 2.10.

![Graph showing reflected power vs time for measured and theoretical data.](image)

Figure 2.10: Experimental and theoretical input reflection coefficients during a pulse

Note the good agreement between the two curves, even though the voltage and current waveforms from the beam were not very smooth.

### 2.3.3 The Cavity Transit Time Factor

As electrons move through the field of an accelerating (or decelerating) cavity, the phase of the rf signal is also changing. The net result of this is that any given electron experiences a lower average accelerating field than the peak value. There is a correction factor which relates the peak cavity gap field to an equivalent average value that the electron sees as it passes through the gap. This factor is called the transit-time factor. A condensed analysis from Wangler is given here as an introduction [62]. We start with a gap geometry like that shown in Figure 2.11. The origin is placed at the center of the gap so that the electric field is an even function in the analysis. The longitudinal electric field as seen by a particle on axis, with velocity \( v \), passing through the accelerating gap in the cavity shown in Figure 2.11 can be
Figure 2.11: Geometry and field distribution across an accelerating gap

described by

\[ E_z(r = 0, z, t) = E(0, z) \cos[\omega t(z) + \phi] \]  \hspace{1cm} (2.71)

where

\[ t(z) = \int_0^z \frac{dz}{u(z)} \]  \hspace{1cm} (2.72)

is the time the particle is at position \( z \). At time \( t = 0 \), the particle is at the origin where \( z = 0 \), and the phase of the electric field relative to the crest is \( \phi \).

The energy gain of a particle with charge \( q \) traveling over the distance \( L \) is

\[ \Delta W = q \int_{-L/2}^{L/2} E(0, z) \cos[\omega t(z) + \phi] dz . \]  \hspace{1cm} (2.73)

The length \( L \) is chosen long enough to include all of the field in the gap region. Using a trigonometric identity, the energy gain can be written as

\[ \Delta W = q \int_{-L/2}^{L/2} E(0, z) \left[ \cos \omega t \cos \phi - \sin \omega t \sin \phi \right] dz . \]  \hspace{1cm} (2.74)
We want to express this result in terms of the spatial average of the accelerating field. Therefore,

\[ \Delta W = qE_0 TL \cos \phi \quad , \quad (2.75) \]

where the spatial average of the accelerating field is given by

\[ E_0 \equiv \frac{1}{L} \int_{-L/2}^{L/2} E(0,z)dz \quad , \quad (2.76) \]

and a quantity we call the transit-time factor is defined by

\[ T \equiv \frac{\int_{-L/2}^{L/2} E(0,z) \cos \omega t(z)dz}{\int_{-L/2}^{L/2} E(0,z)dz} - \tan \phi \frac{\int_{-L/2}^{L/2} E(0,z) \sin \omega t(z)dz}{\int_{-L/2}^{L/2} E(0,z)dz} \quad . \quad (2.77) \]

The phase \( \phi \) is 0 if the particle arrives at the cavity origin when the field is at the crest, negative if the particle arrives earlier than the crest, and positive if the particle arrives after the field crests. Maximum acceleration occurs when \( \phi = 0 \), which is often the choice for relativistic electrons.

Since the origin is at the electrical center of the gap, and \( E(z) \) is at least approximately an even function,

\[ 0 = \int_{-L/2}^{L/2} E(0,z) \sin \omega t(z)dz \quad . \quad (2.78) \]

The transit-time factor now becomes

\[ T = \frac{\int_{-L/2}^{L/2} E(0,z) \cos \omega t(z)dz}{\int_{-L/2}^{L/2} E(0,z)dz} \quad . \quad (2.79) \]

The transit-time factor becomes the average of the cosine factor experienced by the particle, weighted by the field. The transit-time factor increases when the field is more concentrated longitudinally near the origin, where the cosine factor is the
largest, which is why accelerating cavities often have noses to decrease the gap length.

In many cases the change in a particle’s velocity through the gap is small compared to the initial velocity. If we ignore the change in velocity, we can write

$$\omega t = \frac{\omega z}{v} = \frac{2\pi z}{\beta \lambda},$$  \hspace{1cm} (2.80)

where $\beta = v/c$ and $\beta \lambda$ is recognized as the distance the particle travels in an rf period. The transit-time factor now becomes

$$T = \frac{\int_{-\lambda/2}^{\lambda/2} E(0,z) \cos(2\pi z/\beta \lambda) dz}{\int_{-\lambda/2}^{\lambda/2} E(0,z) dz}. $$ \hspace{1cm} (2.81)

If the electric field across the gap is constant and falls to zero just to either side of the gap, the transit-time factor simplifies to

$$T = \left[ \frac{\sin(\pi g/\beta \lambda)}{\pi g/\beta \lambda} \right]. $$ \hspace{1cm} (2.82)

This is the common sync function that is usually used as the transit-time factor. Obviously, for an exact calculation, one would have to go back to the definition in Eqn. (2.77) or more simply, Eqn (2.81). Once again, since the trajectory of the electrons in the RKA beam are off-axis, the integrals would also have to be taken at the same radius. However, Eqn. (2.82) is usually good enough to give a reasonable approximate value, especially for cavities like the input and idler in the RKA. Typical numbers for the RKA bunching cavities are

$$g = 2cm, \quad \beta = 0.9, \text{ and } \lambda = 23.06cm,$$

which give a transit-time factor of

$$T = 0.985.$$
Transit-time factors for the output cavity can be extremely difficult to calculate, because if the output cavity is extracting all of the electron energy in the bunch, the electron velocity goes to zero just as the electron is at the trailing edge of the gap fields. In practice, Eqn. (2.82) only can give a rough estimate of the transit-time factor for the output cavity.

2.4 Small Signal Theory of Classical Klystrons and the RKA

The theory developed for RKA analysis is, for the most part, small-signal. This is because the non-linear nature of the bunching process tends to preclude nice, linear, closed-form solutions. Instead, as the bunching increases past the 10% level, PIC codes must be used to obtain any meaningful results. At least two people have done some work towards analytically modeling the nonlinear aspects of the modulated beam and the klystron, Han Uhm and Y. Y. Lau [32, 63-65]. Their work is mentioned for reference purposes, but will not be reviewed here. The next two sections provide an overview of space charge wave bunching for classical and relativistic electron beams.

2.4.1 Classical Space Charge Wave Bunching

Some insight into the operation of the RKA can be gained by analyzing the bunching process in a conventional klystron. Early klystrons used nonrelativistic beams, and so the analyses used the notation where \( \beta_o = \omega / v_o \), where \( v_o \) is the velocity of the electrons in the beam. Analyses on the nonrelativistic klystrons can be found in books such as those by Slater, Chodorow, and Collin [12, 13, 17]. Since this analysis must take relativity into account, we will let \( \beta_o = v_o / c \), the electron velocity normalized to the speed of light, and define \( k_o = \omega / v_o \), the DC beam
propagation constant. Except for the Bessel functions introduced later, the DC components are denoted by zeros in the subscripts whereas the fundamental rf quantities are denoted by a one in the subscript. Bessel functions can be recognized by the two variable arguments in parentheses following them. The Bessel functions should not be confused with the current $I$ and current density $J$ which do not have arguments.

An axial magnetic field is used to confine the radial motion of the electron beam. The rf signal will have small signal components for the beam velocity, charge density, and the axial electric field. The desired relation is an equation relating $k$ to the drive frequency $\omega$, called the dispersion equation. It will be shown that, in general, there are two values of $k$ supported for any given frequency. These values of $k$ are termed the fast and slow space charge waves, because one will have a velocity faster, and one slower, than light. The initial conditions determine the relative sizes of these two waves. We start with an initial rf current of zero on a DC beam. When the DC beam passes an rf cavity with a gap voltage $V_s$, the sinusoidal gap voltage results in a beam voltage modulation $T_{cov} V_s e^{i\omega t}$, where $T_{cov}$ is the cavity transit-time factor. The transit-time factor relates the change in the gap voltage as the electron passes through the gap to an equivalent constant average value as was discussed above in Section 2.3.3.

The standard method followed is to use the Lorentz force equation in conjunction with conservation of charge to find the rf velocity $v_i$ and the rf charge density $\rho_i$ as functions of the rf axial electric field $E_a$ [57]. Then, Maxwell's equations provide the solution of $E_a$ in terms of $v_i$ and $\rho_i$, and a transcendental equation in which both $k$ and $\omega$ must be satisfied. In the klystron geometry, the
transcendental equation can be reduced to a quadratic, which yields two solutions for $k$.

The relativistic Lorentz force equation is the starting point:

$$\gamma v m_0 \frac{dv_i}{dt} = \gamma v m_0 \left( \frac{\partial}{\partial t} + v \frac{\partial}{\partial z} \right) v_i = e E_{cl} ,$$  \hspace{1cm} (2.83)

or

$$j v^3 m_0 (\omega - vk) v_i = e E_{cl} ,$$  \hspace{1cm} (2.84)

where the total beam velocity is the sum of the DC and rf portions

$$v \equiv v_0 + v_i ,$$  \hspace{1cm} (2.85)

Charge conservation gives

$$\frac{\partial I_i}{\partial z} = \frac{\partial \rho_i}{\partial t} ,$$  \hspace{1cm} (2.86)

where, to first order, the rf current density $J_i$ is related to the rf velocity $v_i$ and charge density $\rho_i$ by

$$J_i = \rho_i v_0 + v_i \rho_o ,$$  \hspace{1cm} (2.87)

and gives

$$k (\rho_i v_0 + v_i \rho_o) = \omega \rho_i ,$$  \hspace{1cm} (2.88)

which results in

$$v_i \rho_o k = \rho_i (\omega - kv_o) .$$  \hspace{1cm} (2.89)

The rf current is given by

$$J_i = v_i \rho_o \left( \frac{\omega}{\omega - kv_o} \right) .$$  \hspace{1cm} (2.90)

Now we know $J_i$ as a function of $E_{cl}$. A typical technique used by Collin is to solve the wave equation for the axial vector potential [17]

$$\nabla^2 A_{cl} + \frac{\omega^2}{c^2} A_{cl} = -\mu_o J_i ,$$  \hspace{1cm} (2.91)
and \( E_\alpha \) is given by

\[
E_\alpha = -j \frac{\omega^2 - k^2 c^2}{\omega} A_\alpha .
\]  

(2.92)

The solutions for \( A_\alpha \) and \( E_\alpha \) are zero order Bessel functions. In each region, either \( A_\alpha \) or \( E_\alpha \) can be solved by matching the axial fields between the regions, using Gauss' law for the radial fields between regions, and the boundary condition that the axial field must vanish at \( r = r_* \). Collin gives a typical nonrelativistic solution for an axial uniform beam of radius \( r_\star \),

\[
p \frac{J'_\alpha(pr_\star)}{J_\alpha(pr_\star)} = h \frac{K_\alpha(hr_\star)I'_\alpha(hr_\star) - K'_\alpha(hr_\star)I_\alpha(hr_\star)}{K_\alpha(hr_\star)I_\alpha(hr_\star) - K'_\alpha(hr_\star)I'_\alpha(hr_\star)} .
\]  

(2.93)

where

\[
h^2 = k^2 - \frac{\omega^2}{c^2} ,
\]  

(2.94)

and

\[
p^2 = \left( k^2 - \frac{\omega^2}{c^2} \right) \left[ 1 - \left( \frac{\omega}{\omega_p} \right)^2 \left( \frac{\omega}{\omega_p} + \frac{\omega}{\omega_p - k} \right) \right] .
\]  

(2.95)

The \( J_\alpha(pr) \) terms are Bessel functions of the first kind, and the \( K_\alpha(hr) \) and \( I_\alpha(hr) \) are modified Bessel functions. The apostrophe designates the first derivative of the function. The plasma frequency \( \omega_p \) is given by

\[
\omega_p^2 = \frac{\rho \epsilon}{m_0 \epsilon_0} .
\]  

(2.96)

The solution for \( k \) in terms of \( \omega \) is in general transcendental, which usually leads to an infinite set of solutions. Each solution represents a different space charge wave. In the limit of large \( r_\star \) and \( r_* \), Equation (2.93) can only be satisfied by four values of \( k \):

\[
k = \pm \frac{\omega}{c} .
\]  

(2.97)
\[ k = \frac{\omega}{v_0} \pm \frac{\omega_p}{v_0}. \quad (2.98) \]

The first two solutions are trivial, implying that all the rf quantities are zero, i.e. no bunching has occurred. The other two solutions are the desired result and are called the fast \( k_f \) and slow \( k_s \) space charge waves, respectively.

For most typical klystron geometries, the solution to the transcendental equation in some reasonable limit can be written

\[ k_f, s = \frac{\omega}{v_0} \pm k_q, \quad (2.99) \]

and

\[ k_q = \frac{\omega_p R}{v_0}, \quad (2.100) \]

where \( R \) is the geometrical plasma reduction factor, and \( k_q \) is the reduced propagation constant.

Since the space charge wave is propagating, the rf current can be written as

\[ J_i = J_f e^{i(\omega-\Delta \omega z)} + J_s e^{i(\alpha-\Delta \omega z)} . \quad (2.101) \]

From Eqn. (2.90), the rf velocity is

\[ v_i = \frac{J_f}{\rho} \left( \frac{\omega - k_f v_0}{\omega} \right) e^{i(\omega-\Delta \omega z)} + \frac{J_s}{\rho} \left( \frac{\omega - k_s v_0}{\omega} \right) e^{i(\alpha-\Delta \omega z)}. \quad (2.102) \]

We define the rf gap to be at \( z = 0 \), and at the gap \( J_i = 0 \) so

\[ J_f = -J_s = \frac{J_i}{2}. \quad (2.103) \]

The total rf current density \( J_d \) is the combination of the fast and slow space charge waves. The current modulation is now

\[ J_i = j J_d e^{i \left( \frac{\omega - \Delta \omega}{\omega_0} \right) \sin(k_q z)}. \quad (2.104) \]
Since
\[ \frac{\omega - k_\perp v_\perp}{\omega} = \pm k_\perp \frac{v_\perp}{\omega}, \tag{2.105} \]
the amplitudes of the fast and slow velocity waves are equal, and they will beat resulting in
\[ v_i = \frac{J_0}{\rho} k_\perp \frac{v_\perp}{\omega} e^{i\left(\omega - \frac{\omega}{\omega_{\perp}}\right)} \cos(k_\perp z). \tag{2.106} \]
If the average of \( k_\perp \) and \( k_\parallel \) is not \( \omega/v_\perp \), the amplitudes of the fast and slow velocity waves are not equal and an additional rf velocity term with a \( \sin(k_\perp z) \) dependence comes in.

It is instructive to calculate the beam's kinetic energy as a function of axial position. If there is an increase in the beam's potential energy, in accordance with energy conservation, the kinetic energy must decrease. If we use
\[ V_i = \frac{m_0 v_\perp^2}{e} v_i \tag{2.107} \]
for the rf beam voltage (this is derived in the next section), the beam power is
\[ \text{Re}\left\{ (I_0 + I_1)(V_\perp + V_i) \right\} = \text{Re}\left\{ I_0 V_\perp + I_1 V_\perp + I_0 V_i + I_1 V_i \right\}. \tag{2.108} \]
Since
\[ \text{Re}\left\{ I_1 V_\perp \right\} = 0, \tag{2.109} \]
the time averaged beam power at any point is \( I_0 V_\perp \), and no power is converted from DC kinetic to rf potential.

The ratio of the magnitudes of the rf kinetic power at the cavity and at the maximum current modulation is (nonrelativistic)
\[ \frac{|I_1 V_\perp|}{|I_0 V_i|} = \frac{\omega}{2\omega_\perp}, \tag{2.110} \]
therefore, \( \omega_e = \omega/2 \) to get any gain. In a classical klystron, the idler cavity at the maximum current modulation acts like a transformer, and the new voltage modulation on the beam is \( I_e Z_{\text{car}} = V_{\text{nec}} \). The increase in voltage modulation is

\[
\frac{V_{\text{nec}}}{V_1} = \frac{\omega}{2\omega_e} \frac{Z_{\text{car}}}{Z_{\text{beam}}}. 
\]

(2.111)

Since the impedance of the beam \( (Z_{\text{beam}}) \) is around 1 kΩ, and \( Z_{\text{car}} \sim 10^5 \Omega \), there is a large amount of gain from cavity to cavity.

### 2.4.2 Relativistic Intense Annular Beam Klystrons

Now that the classical picture of bunching has been reviewed, a comparison can be made to the derivation for the intense relativistic annular beam. R. J. Briggs is usually credited with giving the first dispersion analysis for space-charge waves on a relativistic annular beam near the space-charge limit [66], but the derivations that follow are based on an interpretation by Carlsten of the small signal theory of Friedman and Lau [33, 57, 67]. For the sake of simplicity, an infinitely thin annular beam will be assumed. Only axisymmetric TM modes with a phase dependence of \( e^{j(\omega t-kz)} \) will be considered. The Lorentz force equation is the same as given above in Equation (2.83), and the continuity equation from Equation (2.89) are combined to give

\[
 j \frac{\gamma_e m_e}{\rho_e k} (\omega - k v_e) \rho_i = e E_a. 
\]

(2.112)

The longitudinal electric field \( E_a \) must be solved for in each region. The infinitesimally thin beam is located at radius \( r_b \) in a conductive pipe of radius \( r_c \). The solutions for \( E_a \) will be matched in the two regions, region I \((r < r_c)\) and region II \((r_b < r < r_c)\).
Since \( E_a \) also satisfies the wave equation of \( A_a \), we can write solutions in terms of the modified Bessel functions

\[
E_a = AI_o(hr) \quad (2.113)
\]

and

\[
E_n = -A \frac{jk}{h} I_i(hr) \quad , \quad (2.114)
\]

for \( r < r_* \), where \( h \) is given by

\[
h^2 = k^2 - \frac{\omega^2}{c^2} . \quad (2.115)
\]

The velocity of the space charge wave is less than \( c \); therefore, \( h \) is real. The \( K_n \) modified Bessel function is not a valid solution in this region, since it is infinite at \( r = 0 \). The solution for \( E_a \) outside of the annulus \( (r_0 < r < r_*) \) is

\[
E_a = BI_o(hr) + CK_o(hr) \quad , \quad (2.116)
\]

and

\[
E_n = -\frac{jk}{h} [BI_i(hr) + CK_i(hr)] . \quad (2.117)
\]

Continuity of \( E_a \) at \( r = r_* \) gives

\[
A = B + C \frac{K_n(hr_0)}{I_n(hr_0)} . \quad (2.118)
\]

The axial boundary condition at the conducting pipe gives

\[
0 = BI_o(hr_0) + CK_o(hr_0) . \quad (2.119)
\]

The change in the rf radial electric field \( E_a \) across the beam is given by

\[
\vec{\nabla} \cdot \vec{E}_a = \frac{\rho_1}{\epsilon_0} , \quad (2.120)
\]

where \( \rho_1 \) is the infinitesimally thin surface charge and comes from

\[
-\frac{jk}{h} [BI_i(hr_0) - CK_i(hr_0) - AI_i(hr_0)] = \frac{\rho_1}{\epsilon_0} . \quad (2.121)
\]
Solving for the surface charge in Equation (2.112) yields

\[
\rho_s = \frac{jep_v k}{\gamma_s m_0 (\omega - kv_0)^2} E_o = \frac{jep_v k}{\gamma_s m_0 (\omega - kv_0)^2} AI_0(h r_s). \tag{2.122}
\]

Equations (2.118, 119, 121, and 122) have four relations between the four unknowns \(r, A, B\) and \(C\), which can now be solved. Combining Equations (2.118) and (2.119) gives

\[
B = -C \frac{K_0(h r_s)}{I_0(h r_s)}. \tag{2.123}
\]

and

\[
A = C \left[ \frac{K_0(h r_s)}{I_0(h r_s)} - \frac{K_0(h r_s)}{I_0(h r_s)} \right]. \tag{2.124}
\]

Now, Equation (2.121) becomes

\[
\frac{jk}{\varepsilon_o h} A \left[ \frac{K_0(h r_s)}{I_0(h r_s)} I_1(h r_s) + K_1(h r_s) \right] = \frac{K_0(h r_s)}{I_0(h r_s)} \frac{K_0(h r_s)}{I_0(h r_s)} + I_1(h r_s). \tag{2.125}
\]

Using Equations (2.122 and 2.125), the transcendental dispersion relation is now apparent.

\[
\frac{ep_v}{\gamma_s m_0 (\omega - kv_0)^2} E_o = \frac{1}{h} \frac{I_1(h r_s)}{I_0(h r_s)} \left[ \frac{K_0(h r_s)}{I_0(h r_s)} + \frac{K_0(h r_s)}{I_0(h r_s)} \right] \tag{2.126}
\]

This equation is rather complicated, but is not beyond simplification. The right-hand-side is the same as

\[
\frac{1}{h} \frac{I_1(h r_s)}{I_0(h r_s)} \frac{K_0(h r_s)}{I_0(h r_s)} I_0(h r_s) + \frac{K_0(h r_s)}{I_0(h r_s)} I_1(h r_s). \tag{2.127}
\]

The numerator is the Wronskian of \(K_s\) and \(I_s\), so [68]
\[ K_i(x)I_o(x) + I_i(x)K_o(x) = \frac{1}{x}, \quad (2.128) \]

and Equation (2.126) can be rewritten as

\[
(\omega - kv_o)^2 = \left( k^2 - \frac{\omega^2}{c^2} \right) \frac{e \rho_o}{\gamma_o^3 m_o e_o} r_b \frac{I_0(hr_o)}{I_0(hr_o)} \left[ K_o(hr_o)I_o(hr_o) - K_o(hr_o)I_o(hr_o) \right], \quad (2.129)
\]

which is the same form that Lau uses. Lau defines a space charge reduction factor \( R \), so that Eqn. (2.129) can be written

\[
(\omega - kv_o)^2 = \left( k^2 - \frac{\omega^2}{c^2} \right) \alpha_o R, \quad (2.130)
\]

where

\[
\alpha_o = \frac{e \rho_o}{\gamma_o^3 m_o e_o} \frac{r_b}{c^2} \ln \frac{r_o}{r_b} = \frac{l}{I_o \gamma_o^3 \beta_o}. \quad (2.131)
\]

The threshold current, \( I_o \), is the same as used in Eqn. (2.32), and

\[
R = \frac{1}{\ln \frac{r_o}{r_b} \frac{I_0(hr_o)}{I_0(hr_o)}} \left[ K_o(hr_o)I_o(hr_o) - K_o(hr_o)I_o(hr_o) \right] \quad (2.132)
\]

is the space charge reduction factor. The main problem is that \( R \) is a function of the space charge wave propagation number \( k \), and Eqn. (2.129) is still difficult to solve. Lau assumes that \( R \) is not a strong function of \( k \) (which is reasonable) and is most often close to 1. Lau then rewrites the dispersion relation as

\[
(\omega - kv_o)^2 = \alpha(k^2 c^2 - \omega^2) \quad (2.133)
\]

and assumes \( \alpha = \alpha_o R \) is a constant. Now, Equation (2.133) is a trivial quadratic, and \( k \) can be found using the quadratic formula,

\[
k = \frac{2 \omega v_o \pm \sqrt{4 \omega^2 v_o^2 - 4(1 + \alpha)(v_o^2 - \alpha^2) \omega^2}}{2(v_o^2 - \alpha^2)}, \quad (2.134)
\]
which reduces to

\[ k = \frac{\omega}{v_0} \left[ 1 \pm \left( \frac{\alpha^2 + \alpha/\gamma_0^2}{\beta_0^{\prime2}} \right)^{1/2} \right] \frac{\beta_0^2}{\beta_0^{\prime2} - \alpha} \quad . \]  \hspace{1cm} (2.135)

At this point, Friedman defines

\[ \alpha \mu = \left( \alpha^2 + \frac{\alpha}{\gamma_0^2} \right)^{1/2} \beta_0 \quad \]  \hspace{1cm} (2.136)

and

\[ \delta = \frac{\beta_0^2}{\beta_0^{\prime2} - \alpha} \quad , \]  \hspace{1cm} (2.137)

so that

\[ k = \frac{\omega}{v_0} (1 \pm \alpha \mu) \delta \quad , \]  \hspace{1cm} (2.138)

or

\[ k_j = \frac{\omega}{v_0} (1 - \alpha \mu) \delta \quad , \]  \hspace{1cm} (2.139)

and

\[ k_i = \frac{\omega}{v_0} (1 + \alpha \mu) \delta \quad . \]  \hspace{1cm} (2.140)

The rf current is written as above

\[ J_i = J \ e^{i(\omega-z)}} + J \ e^{i(\omega-z)}} . \]  \hspace{1cm} (2.141)

The initial current modulation at the cavity is zero at \( z = 0 \), which implies that

\[ J_j = -J_i \]  \hspace{1cm} (2.142)

and results in a current modulation of the form

\[ J_i = jJ \ e^{\left(\frac{\omega - k}{v_0}\right)} \sin \left( \frac{\omega - k}{v_0} \delta \alpha \mu z \right) , \]  \hspace{1cm} (2.143)

where \( J_{\sigma} = 2J_j \) as before. The corresponding velocity modulation is

\[ u_i = \frac{J_j}{\rho_0} \left( \frac{\omega - k_j v_0}{\omega} \right) e^{i(\omega-z)}} + \frac{J_{\sigma}}{\rho_0} \left( \frac{\omega - k_i v_0}{\omega} \right) e^{i(\omega-z)}} . \]  \hspace{1cm} (2.144)
This time, however,

\[
\left( \frac{\omega - k_z v_b}{\omega} \right) \neq \left( \frac{\omega - k_z u_b}{\omega} \right)
\]

(2.145)

and the velocity waves do not simply beat. The rf velocity is now

\[
v_i = \frac{I_{\sigma}}{4 \rho \sigma r_b \pi} \left[ 1 - \delta(1 - \alpha \mu) \right] e^{\left( \frac{\omega - \omega_u}{v_b} \right)} e^{\frac{\omega \delta \alpha \mu z}{v_b}} -
\]

\[
\frac{I_{\sigma}}{4 \rho \omega r_b \pi} \left[ 1 - \delta(1 + \alpha \mu) \right] e^{\left( \frac{\omega - \omega_u}{v_b} \right)} e^{-\frac{\omega \delta \alpha \mu z}{v_b}}
\]

(2.146)

which is equivalent to

\[
v_i = \frac{I_{\sigma}}{2 \pi \rho \sigma r_b} \delta \alpha \mu e^{\left( \frac{\omega - \omega_u}{v_b} \right)} \cos \left( \frac{\omega}{v_b} \delta \alpha \mu z \right) -
\]

\[
\frac{I_{\sigma}}{2 \pi \rho \sigma r_b} (\delta - 1) e^{\left( \frac{\omega - \omega_u}{v_b} \right)} \sin \left( \frac{\omega}{v_b} \delta \alpha \mu z \right)
\]

(2.147)

and is the same result that Friedman gives.

One more manipulation will allow us to express the rf voltage in terms of the rf velocity. The energy of an individual particle is

\[
E = m_c c^2 \left( \gamma_{in} - 1 \right)
\]

(2.148)

so the voltage change is related to the velocity change by

\[
eV_i = m_c c^2 \left( \frac{\partial \gamma_n}{\partial \gamma_0} \frac{\partial \gamma_0}{\partial \beta_0} \frac{v_i}{c} \right)
\]

(2.149)

Now

\[
\frac{\partial \gamma_0}{\partial \beta_0} = \beta_0 \gamma_0
\]

(2.150)

and we can recall from before that

\[
\gamma_{in} = \gamma_0 + \frac{I_0 + I_1}{I_0 \beta_0}
\]

(2.151)

If we assume \( I_1 \ll I_0 \) and changes in it can be neglected, then
\[
\frac{\partial \gamma_{\omega}}{\partial \gamma_0} = 1 - \frac{I_0}{I_0 \beta_0^2} \frac{\partial \beta_0}{\partial \gamma_0} = \frac{1}{\delta} .
\] (2.152)

Therefore
\[
e V_1 = \frac{m_0 v_0 \gamma_0}{\delta} v_1 .
\] (2.154)

Recall from above that the beam's rf power was a constant of axial position after the cavity. The DC power is still
\[
\text{Re}\{I_0^* + I_1^* (V_0^* + V_1^*)\} = \text{Re}\{I_0 V_0^* + I_1 V_0^* + I_0 V_1^* + I_1 V_1^*\} .
\] (2.155)

Time averaging the first term still gives the DC beam current, and the middle terms average to zero, but the time average of the last term is not zero. It is
\[
\text{Re}\{I_1 V_1^*\} = -\frac{I_0^2}{2 \pi \sigma_0 \rho_0} (\delta - 1) \frac{e\delta}{m_0 v_0 \gamma_0} \sin^2 \left( \frac{\omega}{v_0} \delta \alpha \mu z \right) ,
\] (2.156)

which is negative (because \(\delta > 0\)). The decrease in DC power results from an increase in the rf potential power. As the beam bunches and increases the rf current \(I_\sigma\), DC power is being converted into rf power, stored as potential energy.

The current modulation can also be related to an initial gap voltage, \(V_{\text{gap}}\). At \(z = 0\),
\[
v_1 = \frac{I_{\sigma}}{2 \pi \sigma_0 \rho_0} \delta \alpha \mu e^{i \omega z}
\] (2.157)

\[
= e V_{\text{gap}} \frac{\delta}{m_0 v_0 \gamma_0} e^{i \omega z}
\] (2.158)

so
\[
I_\sigma = \frac{2 \pi \sigma_0 \rho_0 e}{\alpha \mu m_0 v_0 \gamma_0} V_{\text{gap}} .
\] (2.159)
The DC current is related to the charge density by

$$I_0 = 2\pi r_s v_s \rho_0 .$$  \hspace{1cm} (2.160)

Some examples of calculations may give a better understanding of the physics involved. Using equations (2.159) and (2.160), the current modulation is

$$\frac{I_L}{I_0} \propto \frac{1}{\gamma_0^3} V_{sep} .$$  \hspace{1cm} (2.161)

The more relativistic the device, the larger the gap voltage must be. For low currents, $\alpha$ is small, and

$$\delta = 1 + \frac{\alpha}{\beta_0^2} .$$  \hspace{1cm} (2.162)

The term $\alpha \mu$ is also small,

$$\alpha \mu \sim \frac{\alpha}{\gamma_0 \beta_0} ,$$  \hspace{1cm} (2.163)

which is the relativistic plasma frequency times the square root of the beam radius divided by the speed of light.

As $\alpha$ increases, the bunching distance $z_s$ decreases, and $\delta$ and $\alpha \mu$ both increase. In addition, the ratio of the phase velocity depression to the additional potential energy of bunching increases. Equation (2.147) says that the voltage depression is comparable to the gap voltage when

$$\delta \alpha \mu = \delta - 1$$  \hspace{1cm} (2.164)

or

$$\alpha = 1 - \frac{1}{\gamma_0^2 \beta_0^2} .$$  \hspace{1cm} (2.165)
The space charge reduction factor \(\ln(r_*/r_0)\) is usually quite small. For example, it is only 0.13 for a 3.2 cm radius beam in a 3.65 cm radius beam pipe.

Next, it is very instructive to calculate the bunching distance \(z_b\) and the current modulation \(I_d/I_0\) for various beam geometries. The beam injection voltage will be 450 kV (\(\gamma_n = 1.881\)) and the current will be set at 3 kA. Various different beam radii will be used. In Table 2.2, the space charge limited current \(I_{\infty}\) for different beam radii are listed.

<table>
<thead>
<tr>
<th>(r_0) (cm)</th>
<th>(I_{\infty}) (kA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>6.6</td>
</tr>
<tr>
<td>2.0</td>
<td>14.1</td>
</tr>
<tr>
<td>2.6</td>
<td>25.1</td>
</tr>
<tr>
<td>3.2</td>
<td>64.6</td>
</tr>
</tbody>
</table>

From Eqn. (2.134), we can use

\[
\frac{I_d}{I_0} = \frac{eV_{\text{gap}}}{m_0c^3} \frac{1}{\beta_0^2 \gamma_0^3 \alpha \mu}
\]

(2.166)

to calculate the current modulation and

\[
z_b = \frac{\pi V_0}{2\omega \delta \alpha \mu}
\]

(2.167)

to calculate the distance to the maximum current modulation. The ratio of the voltage depression to the gap voltage is also calculated with

\[
\zeta = \frac{\delta - 1}{\alpha \mu \delta}
\]

(2.168)

The comparison for different beam radii is given below in Table 2.3.
Table 2.3: Beam parameters for a 450 kV injection voltage at different beam radii

<table>
<thead>
<tr>
<th>( r_o (cm) )</th>
<th>2.0</th>
<th>2.6</th>
<th>3.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_o (kA) )</td>
<td>3.0</td>
<td>3.0</td>
<td>3.0</td>
</tr>
<tr>
<td>( I_{\max} (kA) )</td>
<td>5.38</td>
<td>9.55</td>
<td>24.63</td>
</tr>
<tr>
<td>( \gamma_o )</td>
<td>1.611</td>
<td>1.735</td>
<td>1.826</td>
</tr>
<tr>
<td>( \nu_o (\alpha) )</td>
<td>(2.35 \times 10^{10})</td>
<td>(2.45 \times 10^{10})</td>
<td>(2.51 \times 10^{10})</td>
</tr>
<tr>
<td>( V_o (kV) )</td>
<td>312</td>
<td>375.6</td>
<td>422</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.0644</td>
<td>0.0279</td>
<td>0.00906</td>
</tr>
<tr>
<td>( \alpha \mu )</td>
<td>0.217</td>
<td>0.1226</td>
<td>0.0632</td>
</tr>
<tr>
<td>( \delta )</td>
<td>1.117</td>
<td>1.0436</td>
<td>1.0131</td>
</tr>
<tr>
<td>( z_b (cm) )</td>
<td>18.6</td>
<td>36.8</td>
<td>75</td>
</tr>
<tr>
<td>( I_o \sigma / I_o (%) )</td>
<td>1.75</td>
<td>2.29</td>
<td>3.63</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>0.483</td>
<td>0.341</td>
<td>0.205</td>
</tr>
</tbody>
</table>

The fact that the bunching distance is less than a meter makes the tube more feasible to build. It is now time to check on Lau's assumption about the space charge reduction factor \( R \) for the 3.2 cm case. The wave number \( k \) is close to \( \omega / v_o = 31.9 \) m\(^{-1}\). Therefore,

\[
h = \sqrt{k^2 - \frac{\omega^2}{c^2}} = 16.6 \text{ m}^{-1}
\]

\[
hr_o = 0.531
\]

\[
hr_c = 0.606
\]

\[
I_o (.531) = 1.0742
\]

\[
I_o (.606) = 1.0935
\]

\[
K_o (.531) = 0.87504
\]

\[
K_o (.606) = 0.76979
\]

and

\[
R = 7.6(1.018)(0.1300) = 1.0053
\]

which is very close to 1, and Lau's assumption seems to be valid for this case.
Some ISIS/MERLIN simulations were done by Anthony Peratt and Tom Kwan at LANL for a 450 kV, 3 kA beam, which were comparable to the numbers in the table above [69]. The beam radius was 3.2 cm, but the beam pipe wall in the simulation was at 3.9 cm, which was slightly larger than in the RKA. The small signal theory predicted a current modulation of 2.97% at 57 cm, which was close to the values found from the simulation.

2.5 Ramo's Theorem

No discussion of microwave tube theory would be complete without introducing Ramo's Theorem [70]. The microwave power extracted from the beam is given by Ramo's Theorem as

$$P(t) = \int_{\gamma} \vec{J}(r,t) \cdot \vec{E}(r,t) dV,$$  \hspace{1cm} (2.169)

where $\vec{J}$ is the beam current density, and $\vec{E}$ is the electric field in the region of the cavity gap. It is apparent from Eqn. (2.144) that only the beam's kinetic energy can be extracted as was mentioned before. Since $E \sim e^{i\omega t}$, power can only be extracted at the fundamental frequency of, or the harmonics of, the beam modulation. To have the highest output power, both the kinetic energy of the beam and $I$, must be simultaneously maximized. Ramo's Theorem has been derived for a variety of cases, including nonrelativistic beams in high-Q cavities [71], sinusoidal relativistic current distributions [72], and most recently for relativistic beams with arbitrary periodic current distributions in low-Q cavities [45]. For the interested reader, the derivations begin with a normal mode analysis much like that done by Slater, Collin, and Van Bladel, but will not be reproduced here [12, 17, 73]. As was proved in the low-Q cavity case, a certain amount of $Q$ is needed to maintain mode purity of the
extraction. In other words, if the cavity doesn't resonate at all, it will not be selective in pulling the desired frequency off of the beam, and essentially noise will be extracted.

2.6 Computer Simulation Codes

The role of computers in the design of rf structures has been increasing as workstations become more powerful, and large computer codes become more accurate. Two codes in particular were used in the design of the RKA: Hewlett-Packard's (HP) High Frequency Structure Simulator (HFSS) and Los Alamos' ISIS particle-in-cell (PIC) code. HFSS is a 3D electromagnetic field solver that runs on a HP or Sun workstation, so it was used for designing cavities of a certain Q and resonant frequency. ISIS, on the other hand, is a fully self consistent particle-pushing electromagnetic code which can model not only the rf structure (in 2D), but also the action of macro particles in the beam. Because of the complexity of ISIS, it is only run on the mainframe CRAY's or on Thinking Machines' massively parallel Connection Machine CM-5.

2.6.1 HFSS

HFSS was actually developed by Ansoft Corporation, but was modified and distributed by HP. HFSS computes the scattering parameter (S-parameter) response and electromagnetic field distributions for passive three-dimensional structures. HFSS is a finite element code, and can model objects with up to 3 orders of magnitude difference between the largest dimension and the smallest segment. It can also include losses due to resistive boundaries and lossy dielectrics. The beauty of HFSS is that it is fully 3D, and allows for traveling-wave boundary conditions.
Essentially, port surfaces are defined much like they would be in a real structure. The volume of the object is divided up into small tetrahedra. A traveling wave is assumed at the port boundaries, and the solution for the rest of the structure is calculated at each tetrahedron from those boundaries. If the solution has not sufficiently converged, the tetrahedral mesh is refined in the areas of greatest error, and the solution process is repeated. The ability to do realistic 3-D electromagnetic modeling is proving to be critical for developing new, realistic output cavity designs. An output cavity is a true three dimensional problem with traveling wave boundary conditions. To our knowledge there is no computational code, other than HFSS, available that can solve 3-D problems with traveling wave boundary conditions in the frequency domain. A different electromagnetic code (which can push particles), MAFIA, can address similar problems in the time domain, but the calculation is not as straightforward [74].

HFSS has been tested many times here at LANL, and has been able to predict the behavior of a number of microwave structures with astounding accuracy. The comparisons between the simulations and the experiment are reserved for the sections in the following chapters where the structures are actually described. To give an idea of the way HFSS is used, some examples of the output cavity calculations described earlier in this chapter are given here.

First the cavity structure is drawn in a CAD-like interface that is built into the program. Objects with curves are approximated with straight-line segments. Simple shapes can be united together or subtracted to produce more complex geometries. Another method is to draw the cross-section in 2D and revolve it around its axis to create a 3D object. An example of the final output cavity geometry is shown below in Figure 2.12.
Once the object has been drawn, the material parameters must be specified. The object can be covered with a perfect conductor or with a surface with finite resistivity. HFSS treats any resistive impedance boundary as only a resistance, that is, it only calculates loss in the resistance, not the fields penetrating in the skin depth. Lossy dielectrics and magnetic materials can also be specified as needed. Next, port surfaces are defined in regions of constant cross-section at the end of a transmission line. Boundaries where the model has been split for modeling purposes can also be defined. Lines across the port surface can be defined for setting the polarity of the field or calculating the impedance at the port. The number of modes that the user is interested in at each port is also entered at this point. Then the solution parameters are set. These include the operation frequency, the accuracy of the port fields, the number of adaptive passes allowed, the option to frequency sweep, and the option to seed the starting mesh. There is a fast frequency sweep option, but for the best accuracy, the solution must be recalculated using the adapted mesh at each point. This can take many hours for complicated problems. Once everything is set the adaptive process is started. HFSS creates an initial mesh (if it was not seeded) and calculates a solution in the interior volume based on the fields of a propagating wave,

Figure 2.12: The RKA final output cavity (RKA5) model in HFSS
at the ports. It goes back and takes the curl of the original magnetic field to compare to the solved electric field, and then it takes the curl of the original electric field and compares it to the solved magnetic field. If the reciprocal comparison falls within an acceptable tolerance, the solution has sufficiently converged. Otherwise, the mesh is refined in the areas of greatest error, and the solution process is repeated.

Once the solution has converged, as indicated by a small change in the S-parameters over the last two solutions, then the fields can be viewed or choices from a multitude of different calculator operations can be used to compute the desired field value. The field plots can be shown on any defined plane. Animated versions of the magnitude of the electric or magnetic fields can be viewed to verify that the model is behaving as expected. The software claims that you do not even need to know Maxwell's equations to get results, but this is only true in a limited sense. It is always best to check the field solutions to be sure there are not any regions of peak electric fields and so forth in odd locations. An example of the cross-section of the RKA conditioning cavity is shown with the magnitude of the electric field plotted inside is shown in Figure 2.13.

![Image of RKA5 cavity with electric field magnitude](image)

Figure 2.13: Magnitude of the electric field in the plane of symmetry of RKA5
To get the gap voltage calibration factor, the phase of the animated plot was stepped until the peak in the electric field was across the gap. The gap voltage at this phase was then integrated across the gap pieces. Then the magnitude of the magnetic field was stepped until it was a maximum in the region of the B-dot loop. Since the effective area of the B-dot probe was known from microwave measurements (described in chapter 5), the voltage induced in the coax of the B-dot could be related to the peak gap voltage. Therefore, the power in the cavity measured from the cavity B-dot could be related directly to the cavity gap voltage. This method proved to be reasonably accurate as can be seen in Table 7.2, where the energies of electrons accelerated in the RKA conditioning cavity gap were compared to the gap voltage measured in the cavity.

The shunt impedance was also calculated for the model of RKA5 using the method discussed at the end of Section 2.1. A line was defined in HFSS at the midpoint of the beam (x = 2.95 cm) from the start to the end of the beam tube (z = -8 to +10 cm, where the upstream wall of the output cavity was at z = 0 cm). The peak electric field was plotted along this line. The line points were entered into the line calculator where the values were assigned. The z-component of the E-field was chosen and the magnitude was taken. Then the integral was performed along the line. The peak value found was \( V_x = 4.447 \text{ V} \). Next, Equation (2.21) was used to find the shunt impedance and \( R/Q \) was found by dividing \( R_x \) by the \( Q \). The results are given below.

\[
R_x = \frac{(4.447 \text{ V})^2}{2(0.04928 \text{ W})}
\]

\[
R_x = 201 \Omega
\]
\[ Q = 4.14 \]

\[ \frac{R}{Q} = 48.5 \]

If the modulated current on the beam is about 2.5 kA, the shunt impedance would induce a gap voltage of 503 kV. If the output cavity was 100% efficient, and the bunched-beam kinetic energy was matched at 500 keV, the power extraction would be about 625 MW (\( P = VI/2 \)). In practice, nothing is 100% efficient, but these numbers verified that the RKA5 output cavity design was close to what was needed.

Since HFSS calculates the frequency sweep at discrete points much like a network analyzer, low-Q cavities are more suited to modeling in HFSS than are high-Q cavities. High-Q cavities are not impossible to model, just more difficult. If HFSS was adapting close to the resonant frequency of a high-Q cavity, the resonant frequency shifts slightly as the mesh is refined. The best procedure is to let HFSS adapt close to the resonant frequency. Then once the solution has converged, do a frequency sweep to determine the resonance peak. Lastly, have HFSS resolve the model at the resonance peak, but without any additional adaptation of the mesh. A solution arrived at in this manner is usually quite good.

The limitations of HFSS are being eliminated, even at this writing. It still cannot push particles, which is the final requirement for any serious tube designer, but can give accurate solutions to fully 3D problems on a desktop workstation. In combination with a particle-pushing code such as ISIS, a reasonable and realizable tube design can be arrived at. ISIS is used to determine the gap geometry and \( Q \) of the output cavity, and then the required design parameters are implemented in HFSS, after which they can be translated to the actual cavity geometry.
2.6.2 ISIS

ISIS is the PIC code which was used to simulate the RKA. ISIS was developed at Los Alamos for simulating plasma physics [75], and is a self-consistent, fully electromagnetic, relativistic Maxwell solver. In addition, ISIS allows for traveling wave boundary conditions, so power flow from the output cavity and down the transmission line can be modeled.

2.6.2.1 Introductory PIC Code Description

A short introduction to the methodology of PIC simulation is given below. This summary is a reproduction of that done by Rickey Faehl and others, with some additional observations by the author [76]. A more comprehensive description of PIC codes in general can be found in Birdsall and Langdon [77] and similar works.

"Particles" are moved by solving equations of motion with the Lorentz force

$$\frac{d\vec{p}}{dt} = \frac{q}{m}(\vec{E} + \vec{v} \times \vec{B}), \tag{2.170}$$

where $\vec{E}$ is the electric field, $\vec{B}$ is the magnetic field, $q/m$ is the charge-to-mass ratio for a given species, and the momentum $\vec{p}$ is related to the velocity $\vec{v}$ by

$$\vec{p} = \frac{\vec{v}}{\sqrt{1 - \frac{\vec{v} \cdot \vec{v}}{c^2}}}, \tag{2.171}$$

where $c$ is the velocity of light. Once $\vec{p}$ is found, the particle is moved by solving

$$\frac{d\vec{r}}{dt} = \vec{v}. \tag{2.172}$$
It should be emphasized that the particles are often macroparticles representing many
times the charge and mass of a single particle, such as an electron, since attempting to
track the huge numbers of electrons involved in most experiments would overwhelm
even the most powerful of supercomputers.

For the following ISIS simulations, all three components of momentum were
calculated, and two spatial coordinates were retained. This type of simulation is often
termed 2.5D since it does track the momentum in 3D. In these calculations, the \( r-z \)
cylindrical geometry was used, so the \( \theta \) spatial component was ignored. However,
field quantities and momenta in the \( \theta \) direction were retained.

Each particle also has a charge \( Q \) attached to it. These charges are chosen so
that the initial density distribution will be properly computed when the charge from
each particle is interpolated onto an Eulerian mesh. Current densities are constructed
in a similar fashion, by weighting the quantities \( Q_j v_j \) onto a mesh. The procedure
then is to start each time step with an arbitrary distribution of particles at positions
\( \bar{x}_j \). Charge and charge current density distributions at mesh locations \((I,J)\), that is
\( \rho(I,J) \) and \( \bar{J}(I,J) \), are constructed by interpolating the particle quantities \( Q \) and
\( Q_j v_j \) onto the mesh. The charge weighting scheme used in ISIS is known as a "charge
conserving algorithm" [78].

The fields needed to solve Eqn. (2.170) are obtained from the solution of
Maxwell's equations

\[
\frac{\partial \vec{D}}{\partial t} = -\bar{J}_r + \nabla \times \vec{H} ,
\]

\[
\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E} ,
\]

where \( \vec{D} = \varepsilon \vec{E} \), \( \varepsilon \) is the dielectric coefficient, \( \vec{B} = \mu \vec{H} \), \( \mu \) is the permeability
coefficient, and \( \bar{J}_r \) is the sum of particle currents \( \bar{J} \) and conduction currents \( \bar{J}_r = \sigma \bar{E} \),
and $\sigma$ is the conductivity. Since the solutions to Eqns. (2.173) and (2.174) can be obtained once $\mathbf{j}$ is known and the boundary conditions are specified, it is clear that Eqns. (2.170), (2.173), and (2.174) comprise a self-consistent model for the spatial and time evolution of both fields and particles. This is the essence of PIC algorithms.

Conductors, semiconductors, and insulators are defined on the numerical mesh by defining regions with appropriate values of $\sigma$ and $\varepsilon$. All materials in these calculations were chosen with $\mu = \mu_{0}$, the vacuum value. Vacuum and plasma-filled regions are assigned vacuum values $\varepsilon = \varepsilon_{0}$, as well as $\mu = \mu_{0}$. The only dispersion for electromagnetic fields in nonmaterial regions is due to plasma effects, plus a certain intrinsic factor which arises from the finite differencing of Eqns. (2.173) and (2.174). Stability of the field solving occurs if the numerical time step $\Delta t$ is chosen small enough to satisfy a Courant condition related to the cell size, $\Delta x$ and $\Delta y$.

An advantage of using an electromagnetic field solver, aside from its simplicity, vectorizability, and robustness, is that fields propagate at the speed of light and in a causal manner. Voltage is not specified directly, but the appropriate voltages are established by imposing field conditions on the boundaries. Since potential gradients cannot be maintained along the surface of good conductors, the boundary-imposed fields propagate and reflect in such a fashion that the proper field and potential distributions are established within the configuration after the transients die away.

### 2.6.2.2 ISIS Simulations of the Complete RKA

The only readily available version of ISIS was the 2.5D program. A 3D version is being developed at the present, but it was not available for these simulations. The 2.5D version was fine for modeling the buncher section, but could
not simulate the complexity of the actual output cavity. In order to run PIC code simulations of the entire structure, a 2D equivalent of the RKA output cavity was designed in HFSS using dielectric plates in place of the vanes. The dielectric plates act as discontinuities in the transmission line much like the vanes in the output cavity. The reflection from such discontinuities can be calculated by methods such as those used by Schwinger and Collin [79, 80]. For this analysis, it is only necessary to know that the discontinuities cause reflections so as to define a cavity boundary. It was an absolute necessity to keep the geometry of the nose and upstream wall the same as in the actual cavity. As long as the gap geometry, mode, resonant frequency, and $Q$ of the cavity were the same as in the actual cavity, the cavities should be equivalent for the simulation when operating at resonance. The diagram of the 2D cavity is shown in Figure 2.14.

![Diagram of 2D cavity](image)

**Figure 2.14:** HFSS 2D equivalent of the RKA output cavity for ISIS simulations

The frequency response for this cavity is shown overlaid with the RKA cavity model's response in Figure 2.15.
Figure 2.15: Response of the 2D and 3D cavity models

Although the shapes of the resonant curves are slightly different, at resonance the cavities are essentially equivalent.

The cross-sections of the 2D equivalent output cavity and the idler cavity were entered into the ISIS input files. The details of these files can be found in Appendix C. The actual geometry of the input cavity was not included in these simulations; instead, the electric-field distribution at the center of the beam pipe produced by the input cavity was found using the code SUPERFISH (a 2D electromagnetic eigenmode solver). The distribution throughout the beam pipe was then approximated in the ISIS geometry based on the axial values entered from the SUPERFISH simulation. The geometry for the ISIS model is shown in Figure 2.16. The radial dimension (y) is expanded compared to the z dimension for clarity. The beam was located at 2.7 to 3.2 cm, the beam voltage was 650 kV, and the beam current was set to 4.5 kA. The idler
cavity in the model was tuned to give the approximate current modulation found in the experiment (see Chapters 5 and 6). The amplitude of the input cavity fields were adjusted until the current modulation just before the idler cavity gap was around 12%. Then the back wall of the idler cavity was adjusted back and forth until the current modulation just before the output cavity gap was about 70%.

Once the idler had been tuned up, a number of simulations were done with different input cavity amplitudes. A graph of the modulation near the output cavity gap versus output power is shown in Figure 2.17. Initial simulations done by Bruce Carlsten showed that maximum power extraction took place at about 75% beam modulation [81]. Part of the difference was probably due to these simulations being done at a somewhat higher beam voltage (650 kV) and slightly reduced beam current (4.5 kA). The important confirmation here was that more modulation does not always increase the output power. The most power extracted from this geometry, without any tweaking on the output cavity, was about 250 MW. Since these particular simulations were done after all of the experimental work, the retrospective conclusion for output cavity RKA5 may be that it might have needed a slightly higher $Q$ for
Figure 2.17: Output power versus beam modulation after the idler cavity

these simulated beam parameters. However, as discussed in the conclusions, the RKA's beam was inconsistent enough to make any small changes in the output cavity's performance unrecognizable. To demonstrate the large amount of power in the harmonics of the beam modulation, a fast-Fourier transform (FFT) was performed on the signal from a current probe is shown in Figure 2.18.

Figure 2.18: FFT of the current modulation just before the output cavity gap
The scatter plots have been reproduced below in Figures 2.18 and 2.19. The beam modulation near the output cavity gap was about 78%. The extracted power was 250 MW. The first three plots show the geometry of the RKA with the beam at three different time snapshots covering approximately one rf cycle of operation. Note the behavior of the beam as the bunches approach and pass through the output cavity gap. This behavior is even more graphically illustrated in the last three plots. The y-axis shows the momentum of the particles all along the z-axis. The more positive particles are traveling faster and become the rear portion of a bunch, whereas the less positive particles represent the slower portion at the front of the bunch. As the bunch moves through the output gap at 59 cm to 61 cm, kinetic energy is extracted in the form of microwaves, and all of the particles in the bunch all slow down. Sometimes the bunching becomes strong enough that some particles are left behind as the space-charge-limiting current is approached. Other particles are actually accelerated in the negative direction and continue back to the origin unless reaccelerated in the gap of the idler cavity. The time in ISIS is given in cm, and the three times listed correspond to the three geometrical figures on the previous page. If the output cavity happened to be perfectly efficient, the momentum of the bunch would be dropped to nearly zero while passing through the cavity gap, leaving only enough velocity to transport the particles away from the gap. For this particular design, there is still a significant amount of kinetic energy in the bunch even after passing through the gap.
Figure 2.19: ISIS RKA geometry and beam for three different times
Figure 2.20: ISIS beam momentum plots along the length of the RKA
Chapter III
The Modulator, Diode, and Beam Diagnostics

3.1 The Modulator

The modulator for the RKA was BANSHEE. BANSHEE was designed to be a 1 MV, 10 kA, 1 μs pulse for driving a load impedance of about 100 Ω. With thyratron switching, BANSHEE is capable of a 5 Hz pulse repetition frequency (PRF), which makes it state of the art in the world of pulsed-power. Performance to date has achieved 600 kV at 6 kA for 1 μs at a 1 Hz PRF [82]. A PRF of 5 Hz has also been achieved.

A block diagram of BANSHEE is shown in Figure 3.1.

![Block diagram of BANSHEE](image)

Figure 3.1: Block diagram of BANSHEE

The prime power source is a 90 kW, variable-voltage dc power supply. An intermediate capacitive energy storage bank of 37.5 μF is used with a small thyratron
to command resonantly charge (CRC) the pulse forming Blumlein system. A schematic of BANSHEE is shown in Figure 3.2.

![Figure 3.2: Schematic diagram of BANSHEE](image)

The Blumlein system consists of four lumped element Blumleins. The four Blumleins each consist of two six-stage pulse forming networks (PFN's). Each set of two Blumleins is switched by a newly-developed 120 kV, hollow anode thyatron (EEV CX-1812). The CX-1812 is designed for 100 kA peak current, 10 μs pulse width, and a 500 Hz PRF. The Blumleins are discharged through a 10:1 step-up transformer to achieve 1 MV and 10 kA at the 100 Ω load. This modulator design is possible because of the ability of the CX-1812 thyatron to hold off 100 kV and conduct a peak current of 100 kA with a di/dt of over $10^{11}$ A/s. The design of the CX-1812 will allow operation of the BANSHEE modulator at 1 MV and 10 kA for a 1 μs pulse length at an ultimate PRF of 500 Hz. It must be emphasized that the CX-1812 is still an experimental tube under development, and it has not been tested to its full specifications. During the last portion of the rep-rate testing on BANSHEE, it
was found that the tube in the CRC circuit was dissipating about 20% of the energy it was switching due to a spurious oscillation. At BANSHEE's power levels, the CRC tube could not survive very long. EEV has since developed a better tube for the CRC, but there has not been any funding to test this new tube.

For the RKA, only single shot mode was needed from BANSHEE. To accomplish this, without the added complication of the thyratrons and CRC, the four Blumleins were connected to one large spark gap which was triggered by two HV pulse generators connected in series. Since no main energy storage was needed, the HV power supply was connected through some limiting resistors directly to the PFN's. In addition to charging the BANSHEE Blumleins for each shot, the pulsed magnet capacitor bank had to be charged and the data acquisition system had to be readied. The complete process took about 4 minutes per shot under the best of circumstances.

3.2 The Diode

The diode for the electron beam is the heart of any vacuum tube. Although the most desirable type of cathode is thermionic, this experiment used a stainless steel, explosive field-emission, cold cathode because of experimental and fiscal constraints. The advantages of a thermionic cathode are numerous, the most important being the constant impedance and the level of maturity of the technology. When correctly designed, a thermionic cathode gives a beam with very little voltage and current fluctuation, which are important for being able to extract the maximum power from the klystron. The main disadvantages of the thermionic cathode are: (1) the maximum current emission density is low (< 50 A/cm²); (2) it needs a very clean vacuum so as not to be spoiled; (3) eventually the metals that give the cathode a low
work function are depleted from the surface; (4) for the current required in the RKA, a thermionic gun would be very expensive. The advantages of the stainless steel cathode are: (1) very high current densities are possible (> 1 kA/cm²); (2) inexpensive; (3) can work in a relatively poor vacuum; (4) needs no heating and peripheral cooling. The main disadvantage is the variation of impedance during the pulse caused by plasma closure of the A-K gap. The cross-section of the diode is shown in Figure 3.3.

![Diagram of the RKA anode and cathode geometry](image)

Figure 3.3: Side view of the RKA anode and cathode geometry

The lines of the solenoidal B-field are also shown to emphasize the necessity for the complex shape. As the voltage on the cathode rises, electrons are not only emitted from the front corner edge as desired, but also from the shank. These electrons follow
the magnetic field lines both in the forward and in the backward directions. Because of the taper on the cathode, the electrons emitted from the shank in the backward direction tend to intercept the cathode surface. This decreases the back wall current and increases the efficiency and impedance of the diode. There is still some shank current that is emitted from the rear curved surface, but the amount of loss is fairly low. The final setting of the A-K gap spacing was 13.5 cm from the front edge of the cathode to the downstream end of the taper of the anode. At this spacing, the diode was about 75% efficient. In this case, the efficiency of the diode is derived by taking the current in the beam pipe divided by the current at the output of BANSHEE. After approximately every 100 shots, the outside cathode surface needed to be polished to remove surface damage caused by light arcing in spots. The procedure for polishing the cathode is detailed in Appendix B. Typical performance traces of the beam voltage and current are given in Figure 3.4.

![Figure 3.4: Beam voltage and current](image)
The diode power for this shot peaked at about 4 GW and the actual beam power is about 3 GW. If the RKA could be made to be 33% efficient, this would imply an output power of nearly 1 GW.

3.3 Beam Diagnostics

Several different types of beam diagnostics were used in BANSHEE and on the RKA: resistive voltage dividers, Pearson probes, Rogowski Probes, B-dots, and witness plates. The Pearson probe is a multiturn toroid similar in construction to the Rogowski described below, except that the Pearson has an iron core torus and consequently, a slower response time. A diagram of the RKA as set up for beam modulation experiments is shown in Figure 3.5, which shows the different locations of the diagnostics.

![Diagram of the RKA with beam diagnostics]

Figure 3.5: Diagram of the RKA with beam diagnostics (The RKA is configured for beam modulation measurements after the idler)
The beam voltage was measured at the output of the pulse transformer in BANSHEE using a high voltage copper sulfate resistor divider. A long plastic cylinder filled with copper sulfate solution has an electrode on each end with an additional electrode to tap off a small portion of the signal. This signal is forced through another set of parallel resistors and then to the oscilloscope. The resistive dividers had good frequency response and were consistently free from noise. The current was measured with a calibrated Pearson monitor in BANSHEE, and with Rogowski coils at a number of places on the beamline; the Rogowskis were used to track current loss and determine the final current in the beam. A witness plate could be placed towards the end of the beam to check the beam alignment and cross-section. The B-dots are a single-turn loop used for measuring the current modulation on the beam.

3.3.1 The Rogowski Loop

A Rogowski loop is a multturn toroidal inductor that encircles the current to be measured [83]. A typical geometry for a Rogowski used in the RKA is shown in Figure 3.6 below.

Figure 3.6: A Rogowski coil
If we model the above geometry as a number of very closely spaced coils that are perpendicular to the large loop at radius \( r \), we can approximate the output analytically. The first step is to find the magnetic field at radius \( r \) around a line current. Starting with Maxwell's equation for the magnetic field due to a current [84, 85],

\[
\oint \vec{H} \cdot d\vec{l} = \int \left( \vec{J} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{s}.
\]  

(3.1)

The left hand side is the component of the magnetic field perpendicular to the small loops integrated around the circumference of the large loop at radius \( r \). The right hand side is just the current enclosed in \( r \). Therefore,

\[
\int_{0}^{2\pi} H_{\theta} r d\theta = I,
\]  

(3.2)

and

\[
H_{\theta} = \frac{I}{2\pi r}.
\]  

(3.3)

Since the current is actually a function of time, the magnetic field follows as

\[
\frac{\partial H_{\theta}}{\partial t} = \frac{1}{2\pi r} \frac{\partial I}{\partial t}.
\]  

(3.4)

Using the constitutive relation, \( \vec{B} = \mu \vec{H} \), the above equation becomes

\[
\frac{\partial B_{\theta}}{\partial t} = \frac{\mu}{2\pi r} \frac{\partial I}{\partial t}.
\]  

(3.5)

Now it is time to find the voltage induced on the small loops due to the changing magnetic field. We start with

\[
\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}.
\]  

(3.6)
The left hand side is the voltage on the loop, and the right hand side is the component of the magnetic field penetrating the loop. For this geometry, the only component is $B_\theta$. Therefore,

$$V = -A \frac{\partial B_\theta}{\partial t} .$$

(3.7)

There are actually a number of small loops in series, so the voltage is multiplied by the number of loops $N$, or

$$V = -NA \frac{\partial B_\theta}{\partial t} .$$

(3.8)

Now substituting Eqn. (3.5) into Eqn. (3.8) above gives

$$V = -\frac{\mu NA \partial I}{2\pi r} \frac{\partial I}{\partial t} .$$

(3.9)

Now solving for the current in the beam in terms of the measured voltage gives

$$I = -\frac{2\pi r}{\mu NA} \int V dt .$$

(3.10)

The main simplification in this analysis is that the inductance of the loops was neglected. This is a good assumption for the Rogowski coils used in the RKA. The Rogowski coils were made from semi-rigid coax with a spiral etched into the outer conductor and had low inductance for the 1 $\mu$s pulse length. These are the non self-integrating type of Rogowski. It is readily apparent from Eqn. (3.10) that to determine the beam current, the voltage signal must be integrated. The integrators were simply a single-order low-pass filter, but they needed to have good frequency response. For this purpose, T&M Research Products 30 $\mu$s integrators were used. The Rogowski-integrator combinations were calibrated by running a sequence of shorted shots. A shorting rod was connected to the end of the tapered shank in place of the cathode. The shorting rod extended along the beamline through all of the Rogowski coils to be calibrated. A number of shots were fired and then the
calibration factor for each Rogowski was calculated so as to give the same current value as the internal current monitor. A typical waveform from an integrated Rogowski is shown below in Figure 3.7.

![Figure 3.7: Beam current in the RKA measured with a Rogowski](image)

The Rogowski coil was placed in a groove in a flange so that the coil would not be in the actual beam pipe diameter. There was a small gap between the flanges to allow the magnetic field lines of the beam current to couple to the coil.

### 3.3.2 Witness Plate Diagnostics

Two types of witness plates were used: a Pyrex disc and a stainless disc. The disc was placed in the location along the beam pipe where the beam profile or alignment was in question. The RKA would be returned to vacuum and one to five shots would be fired. The stainless type disc was only useful for one shot since the beam would melt the surface in the areas of highest current density. If more than one shot was fired into the stainless, the beam would actually start to erode the surface,
which sprayed small particles of molten stainless in the general area and all of the way up the beam pipe to the cathode. The cathode would then start to grow small whiskers oriented along the magnetic field lines, and the diode impedance would drop to unusable levels. The Pyrex discs, however, could take several shots before the surface started to fracture. After the shot(s), either type of disc would be removed, and the damaged area could be measured. The advantage of the Pyrex discs was that the beam could also be profiled over the pulse length.

The beam profile was examined over time by using a Pyrex witness plate at the end of the beam line. A vacuum window was placed just after the Pyrex disc to allow the emitted light to propagate to the camera. The electrons, which are traveling at about 90% of the speed of light, enter the Pyrex and are now traveling faster than light in the material. This creates a shock wave which emits photons and is known as Čerenkov radiation [86, 87]. The light emitted was captured using a fast-framing camera. The photo in Figure 3.8 (similar to Figure 2.5) shows the progression of the beam in time. The time per frame is about 167 ns starting at the initiation of current flow at the lower left of the picture.

![Image of beam profile](image)

Figure 3.8: Fast-framing photograph of the RKA beam profile (Shot 344)
The size of the beam in this picture is about 2.95 cm radius with a nominal thickness of 5 mm. These pictures can be deceiving in that there may also be some beam halo that is not visible. The brightness of the main portion of the beam forces a certain exposure with the camera which is then not sensitive enough to capture light at a much lower level. The significance of the photographs is that the beam is well-behaved throughout the relevant portion of the pulse.

3.4 Beam Current Modulation Measurements

Before the idler or output cavities were added to the RKA, the current modulation had to be measured. B-dot probes were used for this measurement because of their fast response. B-dots are so named because their output is proportional to the time derivative of the magnetic field coupling to the loop.

3.4.1 The B-dot Pipe

The beam current modulation was measured using a linear array of 11 B-dot probes located along the length of a section of beam drift pipe. The B-dots for this diagnostic were made by soldering a loop of 0.026 in. wire to an SMA bulkhead feedthrough. A 25 in. section of thick-wall beam pipe with a 3.1 in. ID was modified to include B-dots every 2 in. Each B-dot resided in a small cylindrical cavity with a coupling hole at the bottom looking towards the beamline. The B-dots were calibrated by placing the center conductor from a 3 1/8 in. diameter coax down the middle and clamping 3 1/8 in.-to-N type adapters to the ends. A TEM wave was launched from one end using the network analyzer and the coupling coefficient for each B-dot was measured. The other transition was terminated into 50 Ω. A diagram
of the B-dot pipe and calibration set-up is shown in Figure 3.9. Only three of the 11 B-dots are shown for clarity.

![Figure 3.9: B-dot pipe and calibration fixture](image)

The calibration factor for determining the current modulation can be calculated using the following method. Let port #1 be the input to the transition on the left, and port #2 be connected to the SMA feedthrough to the B-dot loop. The network analyzer measures \( S_{21} \) which is defined as

\[
S_{21} = \frac{V_2}{V_1},
\]

or in decibels,

\[
S_{21}(dB) = 20 \log \left( \frac{V_2}{V_1} \right) .
\]

Solving for the ratio of the B-dot voltage \( V_2 \) to the voltage in the pipe with the center conductor \( V_1 \) yields

\[
\frac{V_2}{V_1} = 10^{S_{21}/20} .
\]

The impedance in the coax beam pipe is \( V_1 = I_c Z_i \). Substituting this into Eqn. (3.13) gives

\[
\frac{V_2}{I_c Z_i} = 10^{S_{21}/20} .
\]

The characteristic impedance of this particular geometry is \( Z_i = 50 \Omega \). Now we have
\[
\frac{V_2}{I_1} = 50\Omega \left(10^{5/20}\right) .
\] (3.15)

The power measured at the SMA feedthrough is the power in the coaxial line from port #2 or
\[
P_2 = \frac{V_2^2}{2Z_2} .
\] (3.16)

Solving for the voltage \(V_2 = \sqrt{2P_2Z_2}\), and substituting into Eqn. (3.15) gives
\[
\frac{\sqrt{2P_2Z_2}}{I_1} = 50\Omega \left(10^{5/20}\right) .
\] (3.17)

The impedance of the coax connecting to the SMA feedthrough is also \(Z_2 = 50\Omega\), so
\[
I_1 = \frac{\sqrt{2Z_2\sqrt{P_2}}}{50\Omega \left(10^{5/20}\right)} ,
\] (3.18)

and therefore
\[
I_1 = \frac{\sqrt{P_2}}{5\Omega \left(10^{5/20}\right)} .
\] (3.19)

where \(I_1\) is the rf current in the pipe we want to calculate. The power \(P_2\) is the power measured by the detector after all of the interconnecting attenuation factors between the diode and the SMA connection have been taken into account. With no interconnecting attenuation, this implies
\[
P_{\text{dod}} = P_2 ,
\] (3.20)

or, in conclusion,
\[
I_1(A) = \frac{\sqrt{P_{\text{dod}}(W)}}{5\Omega \left(10^{5/20}\right)} .
\] (3.21)

The modulated current in the beam pipe is now related to the power detected from the B-dot.
There is some question as to whether the space charge wave couples in the same fashion as the TEM wave and thus Eqn. (3.21) would give an erroneous indication of the current modulation [33-35, 88]. This is because the space charge wave is propagating slower than the TEM calibration wave, which travels at the speed of light. In general, Friedman's group thought the modulated current might be underestimated by a factor of up to 40%, but Bekefi's simulations and analysis indicated that the factor is only about 10% off for typical RKA beam geometries and currents. Another complication may come from the change in the electric field of relativistic charged particles. As a charged particle approaches the speed of light, its electric field starts to compact into a plane perpendicular to the direction of travel [44, 86, 89, 90]. One fact is certain, however, and that is the closer the beam gets to the space charge limiting current, the slower the beam will travel, and the correction factor will become larger. Since it is impossible to determine the exact modulation factor without the exact beam geometry and parameters in the simulations, all modulation data reported in this work is given without any correction. If there is an error, these numbers represent modulations smaller than may actually be present on the beam. We did not feel the need to correct for this since the extracted power never exceeded the modulated power on the beam before the output cavity.

3.4.2 B-dot Pipe Modification

As the input drive power was increased to approach the level of modulation necessary for full output, a pulse-shortening phenomenon was noticeable in the beam pipe B-dot diagnostics located downstream of the idler cavity. A number of possible explanations for this problem were investigated such as crystal diode breakdown,
attenuator breakdown, B-dot loop breakdown, and even cable breakdown. An example of the B-dot signal is shown in Figure 3.10.

![B-dot waveform](image)

**Figure 3.10: Representative B-dot waveform before modification**

There was not a breakdown in modulation because the B-dot in the idler cavity showed a reasonable cavity signal for most of the beam pulse. After eliminating the cable, attenuator, and detector breakdown possibilities, the next step was to check for stray beam electrons causing loop breakdown. To do this, a small disc of Teflon was placed over the coupling hole for the loop that would be able to absorb any stray electrons. This modification did not give any improvement to the problem. The only remaining possibility was field induced breakdown of the loop near the connection point on the SMA bulkhead feedthrough, possibly stimulated by x-rays from the beam collector located nearby. This was actually the most likely possibility from the beginning, but also the most difficult to remedy. To correct this, the area of the loops needed to be decreased, and then they needed to be recalibrated as described earlier. The original loop areas were about 1 cm² and were coupling nearly 2 kW from a 60% modulated beam. The loops were decreased to 0.27 cm², which dropped the coupled power out by about an order of magnitude, and the pulse length indicated at the B-
dots now corresponded to the signals from the input and idler cavities as is shown in Figure 3.11. Note that the level of modulation measured is also an order of magnitude greater than in the waveform of Figure 3.10.

![Figure 3.11: B-dot signal after loop modification](image)

From this experience, it is apparent that SMA bulkhead feedthroughs should not be used for coupling over 1 kW of power in the microsecond regime, especially in the presence of x-rays. An additional benefit from the modified B-dots was that they became more directional. That is, the isolation between a signal traveling parallel to the plane of the loop and one traveling perpendicular increased from 5 dB to 13 dB. This implied that the loops were behaving correctly and coupling more strongly to the magnetic field than to the electric field.
Chapter IV
Measurement, Dissipation, and
Conversion of Microwave Power

4.1 Calibration Methods

A number of researchers, in the area of high power microwaves, typically say that HPM is a 3 dB science. This is usually because almost all of the high power rf levels are measured using crystal diode detectors. To discriminate between the fundamental output, other harmonics, and noise, the signal paths are often filtered with appropriate bandpass filters. The methods used to calibrate the cables, B-dots, and diodes can be varied, and this adds to the uncertainty in the measurements. The crystal detectors used in this experiment were made by Hewlett-Packard (HP). There were two types, the HP423B and the HP8470A. The two detectors were very similar except that the HP8470A had a higher frequency response. Calibrating the diodes consisted of measuring the incident power and noting the output voltage over a range from -5 to +23 dBm. The power meter and sensor head were the HP436A and the HP8481A, respectively. There are two common ways to measure the power during calibration. A 6 dB power splitter can be used with the diode on one end and the power meter on the other. This method is not very accurate because the diode and the sensor do not have an extremely low VSWR, and the power level range needed for the diode puts the typical sensor head for the meter in a non-linear regime. The HP8481A sensor head was most accurate in the lower portion of its range (< +10
dBm). A better way to calibrate the diodes was to use a directional coupler instead of the power splitter. A diagram for this calibration method is shown in Figure 4.1.

![Diagram of equipment used for diode calibration](image)

**Figure 4.1:** Schematic diagram of equipment used for diode calibration

The output voltage was measured for 15 points in the above range. A typical curve for the HP423B is given in Figure 4.2.

![Graph showing typical response of the HP423B detector diode](image)

**Figure 4.2:** Typical response of the HP423B detector diode
These points were entered into the acquisition program data base. The program generated a fifth-order polynomial fit to the data points (as was done in the graph above) so that incoming voltage readings taken off the scopes would be instantly converted to measured power. The diodes were usually run near the upper part of their range for the best accuracy and signal-to-noise ratio.

The modern network analyzer is really a miracle of today's technology. It can take measurements of phase and amplitude at over a thousand points in seconds and correct for the interconnecting cables. It then can present the data in a number of useful formats. One can truly get an appreciation for the efforts early researchers put into making microwave measurements with only detectors and slotted lines. The network analyzer works by measuring the forward and reflected voltage amplitude and phase at each port. Because even modern electronic circuits are difficult to implement at microwave frequencies, the forward and reflected signals are mixed down to a lower frequency with a local oscillator. Then the voltage and phase can be determined using conventional electronics. By having known standards such as a load, an open, and a short, the effects of the connection cables and adapters can be calibrated out. All passive devices used for the RKA were characterized using the HP8753B network analyzer with the HP85047A S-parameter test set. This included signal cables, waveguide lines, directional couplers, attenuators, filters, B-dots, and cavities. Because of the S-parameter test set, the connections for characterizing a microwave device were trivial. A typical connection to a cavity is shown in Figure 4.3. The actual output power from the analyzer's internal source was not very accurate, so it could not be used to calibrate the diodes. The network analyzer is designed to make relative measurements. All calibrations for the analyzer were of the full two-port type and done at the ends of the male N-type coaxial cable connectors.
This method necessitated the use of a short female barrel for the "thru" calibration. A slight error is introduced in the correction from the non-zero electrical length of the barrel [91]. Later, calibrations were done using cables with one female and one male end to avoid any additional errors.

4.2 The High-Power Directional Coupler

At NRL, Friedman measured the output power of his RKA by radiating from an open horn out the door of his lab, and sampling the output using a small receiving antenna [35]. Since Friedman claims to have the world's record for RKA output in the 50 ns regime [37], it is important to examine the viability of his measurement scheme. Firstly, the only way to know the total power radiated from an antenna is to measure the entire radiation pattern to determine the mode in the antenna. This requires either a lot of sensors covering the radiated area, or a lot of shots with a sampling antenna at a different location for each shot, assuming that the same mode is
present every time. The sampling method requires many shots and is not very accurate because of shot-to-shot variability. To make matters worse, the large size of components used in RKA's usually means the components might be running overmoded, which gives them a good probability for transmitting in several modes. These modes might be consistent or they might vary from any tuning obstacles placed in the line or the output power level itself. An additional problem is that the output power from an unconditioned tube is very inconsistent from shot to shot, and it is unlikely that a consistent radiation pattern could be determined or even trusted.

To avoid these obvious problems, the LANL RKA was designed with a coax directional coupler in the output line. High power, vacuum couplers were not available commercially, so this coupler had to be made in-house. The standard 50 Ω, six inch line dimensions were chosen for this section in order to use commercial coax-to-N type transitions and a network analyzer for calibration. Other complications to the design were the requirements for slots in vacuum pumping ports and a place for an ionization gauge tube. The slots in the outer conductor were needed to keep the electromagnetic fields from coupling into the six inch vacuum ports. The slots in the inner conductor helped the vacuum pumping inside of the conductor. The macroscopic geometry of the coupler is shown in Figure 4.4. In general, the coupler is a reciprocal device, but it was better to have the perturbations caused by the vacuum slots after the power measurement. The coupler made for this experiment was based on the modified Bethe design [92]. The trick to making a coax-to-coax or waveguide-to-coax coupler is to balance the voltage induced in the coupling loop by the electric field with the voltage induced on the loop from the magnetic field such that the waves on the loop cancel for one direction and add for the
opposite. The other fact which one soon learns by trying to make a coupler, by intuition or after the fact, is that the only way to get good directivity is to have the

![Diagram of a coaxial directional coupler]

Figure 4.4: Mechanical geometry of the coaxial directional coupler

reflection coefficients at the coupling ports very low. A more elegant way to say this is that any four port reciprocal device with matched ports is a directional coupler. This was proven by Montgomery in the 1940's [93]. To make the design conservative, it was assumed that the main coax line might contain up to 1 GW. The
coupling ports used SMA bulkhead vacuum feedthroughs, and it was desired to keep the peak power coupled out of them to be less than 1 kW. Therefore, the approximate coupling factor was targeted at -65 dB. The electric field coupling has the dominant effect on the coupling factor and this is set by three main parameters: the coupling hole dimensions (aperture and thickness), the spacing from the wire to the hole, and the width of the wire. The magnetic field coupling gives the directivity and is controlled by the following parameters: the coupling hole aperture, the loop area, and

Figure 4.5: Side view of the directional coupler loop and aperture
the orientation of the loop relative to the power flow. A side view of the actual geometry is shown in Figure 4.5. The procedure for adjusting the directional coupler is reviewed in Appendix B. The final performance of the directional coupler is recorded in Figure 4.6.

![Graphs showing forward and reverse coupling](image)

Figure 4.6: Coupling and directivity of the RKA's custom coaxial coupler

At 1300 MHz, the forward coupling was -64.58 dB and the directivity was better than 25 dB over a fairly broad range. Directivity this high was due to an artifact of the measurement process. The coax load described below was the load used in the calibration process out of necessity. The reflection off of this load was -20 dB, so the best directivity possible at the coupler could not have been better than that. However, since we were mainly concerned with the directivity for the forward direction, adjusting the value to be unusually low was still valid because the coupler was
connected to the load just as it was to be in the experiment. This coupler never broke
down, even when measuring peak power levels as high as 475 MW. Coupler
breakdown would have been obvious if the coupler signal did not track with the
output cavity B-dots.

4.3 The Coaxial Vacuum Load

Very high power vacuum loads are essentially as difficult to come by as
directional couplers, so this was another piece that had to be made in-house. In many
commercial high power loads, ferrites are used for the absorbing medium for their
high attenuation factors per unit volume at lower microwave frequencies. For the
RKA, the peak power could be in a range where typical ferrite materials saturate. To
avoid saturation problems, a porous, soft, ceramic-like silicon carbide material was
used. The material was obtained from Resin Systems Inc. in New Hampshire under
the model number RS-4200CHP. It attenuates well at 10 GHz, but is not as effective
at 1.3 GHz. The advantages of the silicon carbide are threefold. It has no saturation
problem like ferrites do since it absorbs the electric field as opposed to the magnetic
field. It can handle very high temperatures (>1000 °C) and is compatible with
vacuum systems into the 10^-7 Torr range. The down side of the material is that it
came in 1.5 x 8 x 12 in. blocks. These blocks could be cut and machined by Resin
Systems, but not in any complicated way. It was desirable to keep the outer radius of
the coax load such that the vacuum conductance was high and electric field
enhancement points were no closer than the original 3 in. radius. For this reason, the
outer radius was increased by 0.5 in. to hold a half-inch thick layer of silicon carbide.
It was relatively easy for Resin-Systems to machine smaller rectangular blocks from
the larger ones, so it was decided to fill the outer circumference with small-cross-
section, long bars of silicon carbide. A formula for calculating the number of rectangles that can be placed around the inner circumference of a circle is given in Appendix B.

After all of the exciting geometry analysis in Appendix B, it was decided that there should be some gap between the sides of the pieces for better vacuum, so the number of segments was dropped to 18. Originally, the bars were going to be epoxied to the wall with Torr-Seal™. The main drawback to this method was that if the bars were ever accidentally squirted with ethanol or another volatile liquid, they would probably never be suitable for vacuum without a lot of baking under vacuum. A method for mechanically holding the bars in position was devised as an alternative. Three support rings were made from soft, 0.125 in., 5052-H32 aluminum alloy sheet. The material needed to be soft and malleable so that the split rings could be bent and fit through the flanges on the side. Small 0.063 in. dowel pins were pressed into holes in the rings for spring supports. Views of the support rings are shown in Figure 4.7. Only three spring clips are shown for clarity. The width of the rings was only 0.5 in. so that they would not protrude above the level of the silicon carbide. Thin spring steel strips were bent into the clip shape seen above to give a friction grip to the sides of adjacent bars. The rings were placed in the load, one on each end and one in the middle, and then the clips were placed on the dowel pins as the bars were forced in between the dowel pins. Only two of the blocks are shown below for clarity. A simplified view of the load is shown in Figure 4.8. The inside and outside conductors were made out of stainless because any extra attenuation was desirable. Slots in the inner conductor gave an outlet to the main tube for better vacuum conductance. The center conductor was supported by a four post spider with Multilam™ in the center support ring to make a good electrical connection. The connection at this end was
Figure 4.7: Support rings for the silicon carbide bars in the coaxial load

Figure 4.8: Geometry of the coax load

mostly for any residual beam current that did not return through the output cavity vanes. The entire center conductor for the load and the coupler could rotate and translate to adjust the output cavity gap.
The performance of the load was actually slightly better than expected. The desired VSWR was less than 1.2. The impedance at the outer conductor discontinuity was about 59.5 Ω. The VSWR expected from a transition to the higher impedance line is 1.19. The measured VSWR was 1.20, as can be seen in Figure 4.9.

![Graph](image_url)

**Figure 4.9: Reflection performance of the RKA coaxial load (VSWR)**

A Fourier transform was performed in the network analyzer which shows the relative position of the reflections reference to the calibrated N-port. This result is shown in Figure 4.10. Plots of this nature can give the approximate location of the reflections in a transmission line. In this case, the coax load started about 1.9 m from the measuring port’s reference plane. From the above plot, most of the reflection comes from the discontinuity in the line at the entrance to the load. A well matched load could be made if the outer conductor and absorbing medium were adiabatically tapered out to the new dimension. Single-pass loss for the load was estimated to be at least 20 dB, as calculated from the reflection at the end of the load (Fig. 4.10) and
from transmission measurements using an open N-to-waveguide transition placed near the open portion at the end of the load.

4.4 Design of a Very High Power Coax-to-Waveguide Mode Converter

Having a lot of output power from a tube is great, but if you cannot transport that power to the required destination, you have not completed the task. Waveguide is usually the preferred choice because of its low attenuation and high power handling capability. Accelerators typically need rf power in the TE_{10} mode. There have been a number of designs for coax-to-waveguide converters. Most of these feature variations on a common theme: adiabatically changing the cross-section from coax into waveguide so as not to cause any acute disturbances in the impedance. Friedman proposed one such converter in his paper on the RKA [35]. His converter also
employed magnetic insulation to further increase the breakdown limit. A diagram of his concept is shown in Figure 4.11.

![Diagram of External Magnetic Field Coil, Coaxial, E-Field Lines, Rectangular](image)

Figure 4.11: Friedman's mode converter concept

Friedman demonstrates the converter with four waveguide outputs, but notes that this number could be increased as needed. A more generalized version of Friedman’s multi-fin converter was analyzed in great detail by Shen and Wu [94]. They concluded that such a converter could have a very low reflection coefficient at discrete frequencies, but good broadband response would only come from a long structure. The cases they examined were $N = 12, 14, 16, \text{ and } 18$, where $N$ is the number of fins. The $N = 12$ case actually gave the lowest reflection coefficient overall.

In microwave engineering, it is almost always possible to make a near perfect device at a single frequency. The difficulty usually arises in making devices that are broadband. Narrowband devices have their place, but broadband devices are much more adaptable to different needs. With this in mind, a wideband mode converter from $TM_{01}$ to $TE_{11}$ circular waveguide was designed by Bob Eisenhart while working with Hughes Missile Systems Group. This converter was made for use with
the PASOTRON™ high power microwave source [95]. Eisenhart demonstrated over 30% bandwidth with less than 0.5 dB of transmission loss using this converter. A diagram and the performance of the converter are detailed in Figure 4.12.

![Diagram](image)

Figure 3. Measured Insertion Loss for the Fullsize Mode Converter.

Figure 4.12: Eisenhart's TM₀₁ to TE₁₁ mode converter for circular waveguide

One of the most impressive aspects of the design was that at each step of the conversion, the geometry was designed to suppress any multimoding that would usually occur in the more drastic line changes. The relevant aspect of this design for the RKA was that the intermediate conversion modes included going from TEM coaxial to TE₁₀ rectangular waveguide. He rated the power handling capability of the converter portion to be about 130 MW-GHz, which also makes it a suitable contender for the RKA. With some careful modifications, the power rating could probably be
substantially increased. The details of the geometry were held as proprietary information by Hughes, so it was necessary to create a new geometry in HFSS to test any modifications.

The most probable place to improve the convertor was in the region of the transition from the coax to the waveguide. One way to decrease the surface field on the center conductor in this area was to increase the area of the center conductor and to change its shape. The shape shown in Figure 4.13 should accomplish this goal.

![Waveguide and Center Conductor Cross-Section](image)

Figure 4.13: High power modification to the convertor center conductor

Now the peak surface field is spread over two regions instead of just one. Funding for the RKA project ran out before the converter could be designed so the output power numbers quoted later are all measured in the six-inch coax.
Chapter V

The Input Drive Source and Input Cavity

5.1 The Input Source

The input drive power for the RKA comes from an L-band magnetron radar source obtained from Radio Research Inc. (RRI). The unit is capable of 500 kW output for 5 µs at up to 200 Hz repetition rate. Since we were only going to use the radar in single-shot mode, two resistors (R17 and R27, in reference to the supplied schematic) that bled charge off the PFN were removed. In single shot mode, the magnetron cathode never comes up to full operating temperature, which makes it less efficient than normal. By raising the voltage to the tube, some of this loss could be regained, but the typical output power was about 300-350 kW average during the RKA beam pulse. The coaxial output of the JAN5J26 tube was converted to WR-650 waveguide with a transition, and then fed through a circulator, variable attenuator, and a dual directional coupler respectively before being transported to the input cavity. A diagram of the input drive system is shown in Figure 5.1.

![Diagram of the RKA drive system](image_url)

Figure 5.1: Diagram of the RKA drive system
The isolator and directional coupler were also obtained from RRI. The isolator had 0.2 dB insertion loss and 20 dB of isolation. The directional coupler had approximately 15 dB directivity and -34 dB coupling at 1300 MHz. The loss through the connecting waveguide, the window, and the stainless taper amounted to 0.23 dB. This loss was taken into account when determining the actual input power present at the input cavity coupling iris. No commercial high-power (low duty factor) variable attenuators were available so an FXR L160 attenuator was modified for this experiment. The attenuator originally used nichrome-coated glass vanes, 3 in. tall and 9 in. long, which could be adjusted in towards the center to increase the absorption. The vanes were not able to withstand the high electric field from the magnetron pulse. Therefore, the vanes were replaced by a single rectangular (0.5 in. thick x 1.7 in. x 12 in. long) bar of ferrite loaded epoxy, more commonly known as Eccosorb™. The ferrite was still very lossy even when fully retracted, so when the highest power was needed, the bar was completely removed. Several complex vane shapes were tried, but the simple bar gave the best combination of input VSWR with attenuation variability. The performance of the attenuator is shown in Figure 5.2.
Figure 5.2: Attenuation and input VSWR for the modified FXR L160 attenuator

The precision and accuracy of the attenuator was not important since the power was measured after attenuation.

5.2 The Input Cavity

The input cavity in the initial RKA was a quarter-lambda, coaxial, reentrant-nose type with loop-drive coupling. The maximum input power to this cavity was about 5 kW [39]. Since the new RKA design was going to need up to 500 kW of input drive, the loop coupling had to be replaced by an input waveguide with iris coupling. A picture of the new cavity is shown in Figure 5.3. Past the outside magnet diameter, the drive waveguide tapers down from full height WR-650 waveguide to an inner height of 0.75 in. The reduced height waveguide was necessary to be able to pass radially through adjacent magnet coils of the segmented solenoid. At the end of the reduced height waveguide, an iris connects the cavity to
the waveguide. The coupling iris subtends 90 degrees of arc and has radiused edges to prevent arcing. A vacuum window was placed between the waveguide and tapered section to seal the input of the tube. The vacuum window was manufactured by Microwave Techniques Inc. (MTI) for use in pressurized waveguide, but was not originally suited for vacuum use. The rexolite used for the vacuum barrier had to be polished to a smooth (900 grit) finish and sealed with lightly greased vacuum o-rings.

The input cavity and waveguide taper were constructed with non-magnetic 304 stainless steel. This cavity was not copper plated like the other cavities because the rf loaded Q was only about 32, and the beam loading was even more drastic. The measured response ($S_{21}$) of the cavity is shown in Figure 5.4.
Figure 5.4: Measured response curve for the input cavity

The cavity response was measured using two B-dots in the side wall of the cavity spaced 90° apart and opposite to the coupling iris. Using the typical parallel circuit model, the unloaded cavity shunt impedance (i.e. without the beam) is on the order of 100 kΩ, whereas the beam impedance is around 2 to 4 kΩ. Therefore, the overall circuit impedance is dominated by the beam and not by the losses in the stainless steel walls. In order to use the magnetron drive power efficiently, the input cavity must be matched (unity coupled, $\beta = 1$) to the transmission line when the beam is present in the gap. This required that the coupling slot be made large. With no beam, the cavity is strongly overcoupled. As was reviewed in Chapter 2, the coupling coefficient $\beta$ is determined from the VSWR by

$$\beta = 1/VSWR, \quad (\beta < 1) \quad \text{undercoupled}$$

$$\beta = VSWR, \quad (\beta > 1) \quad \text{overcoupled}.$$
Whether the cavity is overcoupled or undercoupled can be determined in a number of ways, one of them being to watch the reflection coefficient on a Smith chart while sweeping in frequency around resonance as depicted back in Figure 2.4 of Chapter 2. An undercoupled cavity will trace a small circle at the edge. A unity coupled cavity will trace a circle that passes through the center at resonance, i.e. the cavity looks like a matched load on resonance. An overcoupled cavity will trace a larger circle which can approach the outer boundary for large $\beta$. In the case of the RKA cavity, the VSWR was 38 (heavily overcoupled) without the beam; this implies that $\beta = 38$. Once the beam loading is present in the gap, the cavity becomes near unity coupled and reflects very little of the incident power. This is portrayed in Figure 5.5 where the reflected power signal from the cavity is overlaid with the beam power trace.

![Diagram](image)

**Figure 5.5: Magnetron reflected power and beam power**

As the beam power increases, the input reflection drops to nearly zero, and then as the beam power starts to drop off the reflection coefficient returns to its previous high level. The degree of matching can be estimated using the method outlined in Chapter
2. The theoretical reflected power is compared to typical experimental data in Figure 5.6 below (repeated from Figure 2.10 for convenience).

![Graph showing comparison between theoretical and experimental reflected power at the input cavity.](image)

**Figure 5.6:** Comparison between theoretical and experimental reflected power at the input cavity

Efficiently coupling all the power into the input cavity would not normally be absolutely necessary since the gain of the tube can be increased by adding more idler cavities, but in this development effort it was desired to keep the gain as low as possible to deter oscillation problems, to prevent possible breakdown in the additional idler cavities, and for simplicity.

The beam modulation after the input cavity necessary for full output power was determined by using ISIS to be around 10%. As is apparent from Figure 5.7, the input cavity is capable of over 10% modulation with 200kW of input drive power. Since current modulation is proportional to the square root of the drive power, a second-order polynomial fit was applied to the above data. The row of B-dots allowed experimental determination of the area where peak modulation occurred. This would be the location for the gap of the next cavity. This distance was also determined with ISIS. The location for peak modulation after the input cavity was
calculated to be 30 cm which was in good agreement with the experimentally
determined distance of 31 cm from the center of the input gap.

5.3 Determination of the Gap Voltage

It was necessary to determine the gap voltage of the input cavity in order to
simulate the cavity correctly in the PIC code. An actual input cavity model was not
used in the PIC code since the chance for oscillations and other noise amplification
problems might mask important aspects of the klystron's operation. Another issue is
that numerical instabilities can result if the simulation time is allowed to run too long.
Thus, cavity ring-up time may take long enough that there isn't any run time left to
see the equilibrium operation. By definition, long computer simulations are also
expensive. The actual details of the RKA ISIS model are discussed in Chapter 2.
The main issue here is to match the simulated gap voltage to the level present in the
experiment. To do this, the input cavity was modeled in HFSS. From the computed
solution, the peak gap voltage was related to the peak magnetic field at the location of
the cavity B-dots. The effective area of the B-dots was found by placing them in the midpoint of the narrow wall of standard WR-650 waveguide. Since the fields in waveguide are well known, the coupling constant given by the network analyzer could be used to find the effective area of the B-dot. Using Maxwell's equation for the induced voltage, the effective area is determined as follows:

\[ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \]  

(5.1)

or in integral form,

\[ \oint \vec{E} \cdot d\vec{l} = -\frac{\partial}{\partial t} \oint \vec{B} \cdot d\vec{S}, \]  

(5.2)

where the left hand side is the voltage induced on the B-dot loop. Since the plane of the loop is oriented at 90 degrees to the magnetic field, the inner product simplifies to a direct multiplication. The loop is small relative to the cavity volume so the magnetic field inside the loop is relatively constant; therefore, the integral over the surface gives the effective loop area \( A_{\sigma} \) at the average magnetic field.

\[ V = -A_{\sigma} \frac{\partial B}{\partial t} \]  

(5.3)

The magnetic field is of the form \( e^{i\omega t} \), and we need the constitutive relation \( \vec{B} = \mu \vec{H} \) to put the equation into the necessary form.

\[ V = -A_{\sigma} \omega \mu H \]  

(5.4)

We will use peak field values for magnetic field and induced voltage for convenience. In waveguide, the peak magnetic field in the z-direction is given by

\[ H_z = \frac{\pi}{\omega \mu a} \sqrt{\frac{4P_{ext} \eta_{iv}}{ab}}, \]  

(5.5)
where $\eta_w$ is the waveguide characteristic impedance as determined from the power and voltage (\( \eta_w = V^2/P \)). The variables $a$ and $b$ correspond to the width and height of the waveguide respectively. The power induced in the B-dot coax is equal to

\[
P_{cx} = \frac{V_{cx}^2}{2R_{cx}}. \tag{5.6}
\]

Solving this for coax voltage, substituting for the LHS of equation 5.4, and placing the equivalent form of $H_z$ from Eqn. (5.5) in the RHS gives

\[
\sqrt{2R_{cx}P_{cx}} = \frac{A_{eff} \pi}{a} \sqrt{\frac{4P_w \eta_w}{ab}}. \tag{5.7}
\]

Solving for the effective area leaves

\[
A_{eff} = \sqrt{\frac{a'bR_{cx}}{2\pi^2 \eta_w}} \left( \frac{P_{cx}}{P_{wx}} \right). \tag{5.8}
\]

The ratio of the coax power to the power in the waveguide is exactly what the network analyzer gives in the form of $S_{21}$.

\[
\frac{P_{cx}}{P_{wx}} = 10^{-S_{21}/10} \tag{5.9}
\]

This works out well because we do not need to be concerned with the absolute amount of power flowing in the waveguide as long as there is very little power reflected from the load end. Now the equation can be cast in its final form.

\[
A_{eff} = \sqrt{\frac{a'bR_{cx}}{2\pi^2 \eta_w} \left( 10^{-S_{21}/10} \right)} \tag{5.10}
\]

The B-dots used in the cavities in this experiment were made from SMA vacuum bulkhead feedthroughs purchased from M/A-COM (pt.# 2052-312X-00, X=1,2,3). These feedthroughs used a hermetic fused glass seal as the vacuum barrier. The
center lead was bent over at a right angle and soldered to the outer grounded conductor. Typical effective areas for this type of B-dot at 1300 MHz were approximately 1 mm². Once the effective area for a B-dot was determined, the power measured from the B-dot could be related to the cavity gap voltage through the cavity field simulation, Eqn (5.6), and Eqn (5.4). The gap voltage to B-dot voltage factor for the input cavity was 1869 V/V. Putting this technique into practice yields the results in Figure 5.8.

![Graph showing gap voltage over time](image)

**Figure 5.8:** Input cavity gap voltage during the magnetron pulse

This data was taken with approximately 300 kW of input drive power. The spike at 2.3 μs is due to an oscillation in the RKA at the end of the output pulse. Before the beam turns on, the cavity voltage is around 40 kV. Once the beam starts to load the cavity, the gap voltage drops to around 20 kV even though the cavity is reflecting very little of the input power. This was the value of gap voltage entered into ISIS for simulating the RKA.
Chapter VI

The Idler Cavity

6.1 Description

The idler cavity for the initial RKA was a simple quarter-wavelength reentrant-nose coaxial cavity. This cavity was modified to include a tuning ring so that the idler could be tuned from about 1300 to 1400 MHz. The cavity is depicted in Figure 6.1. It is important to be able to tune the idler cavity above the operation frequency.

![Diagram of the idler cavity](image)

Figure 6.1: Cross-sectional view of the RKA idler cavity
If the idler is tuned to the drive frequency, the electrons will be decelerated in the gap, thereby extracting power from the beam into the high shunt-impedance idler cavity. This action will induce a very large voltage across the gap causing it to break down. If the idler is tuned below the operating frequency it will appear as a capacitive impedance, resulting in the exact opposite phase actually needed for bunching. As bunches pass the gap, the field would tend to accelerate the electrons at the front of the bunch and decelerate the ones at the tail end of the bunch. This would smear out the bunches and decrease the modulation on the beam. However, if the cavity is tuned to resonate above the operating frequency, it will appear as an inductive impedance which will decelerate electrons at the front of the bunch and accelerate the ones at the tail. This velocity modulation will compress the bunches further, as is desired, and will increase the modulation on the beam. The usual procedure is to start with the idler tuned at least 5% (65 MHz) above the operating frequency with the input drive at maximum power. Then the tuning is slowly brought closer to the drive frequency until maximum modulation is measured without having much idler cavity breakdown. In practice, the idler could never be tuned closer than 1% (13 MHz) above the operation frequency without breaking down.

Even though the beam loading in the idler was very heavy, it was important to keep the unloaded $Q$ as high as possible so that a minimum amount of energy would be lost to resistance in the cavity. To keep the $Q$ high, the cavity was copper plated to a depth of about 0.003 in. The skin depth of copper is about 2 $\mu$m at 1300 MHz, which is less than .0001 in. The aluminum o-ring for the two cavity halves was sanded with 600 grit paper and cleaned with ethanol immediately before assembly. In addition, the tuning slug often needed a light tap on the adjusting rods after changing
the position to make sure the contacts seated well. The unloaded $Q$ of the cavity was usually around 2000.

6.2 Idler Modulation Performance

One might be tempted to think that more modulation is always better since power cannot be extracted from an unmodulated beam, but this does not tell the whole story. With a large amount of space charge in the beam, it is possible to modulate the beam enough to convert all of the kinetic energy into the potential energy fields. That is, the charge of the beam cannot propagate and simply stops. This is commonly known as reaching the space charge limiting current as described in Chapter 2. Only the kinetic energy of the beam is extractable as microwaves, so it doesn't make sense to overmodulate the beam. Using ISIS, the amount of extractable energy peaked at about 66% modulation of the DC beam current ($I_n$) into the fundamental $I_i$ ($I_i/I_n = 0.66$). In the process of finding the optimal tuning for the idler cavity, the output cavity became available so the complete range of idler tuning and drive power was not explored. It was found that any tuning closer than about 1325 MHz resulted in too much breakdown and pulse shortening, even at medium drive levels. Therefore, the idler was tuned to 1332 MHz and characterized at high power to make sure the 66% modulation level could be reached with the available input power. Consequently, the data shown below in Figure 6.2 are actually for two different tuning settings. The low and medium power data were taken for a 1344 MHz tuning, and the high power data were taken with the idler tuned to 1332 MHz. The power curve is representative, since the low and medium levels did not differ much with a small change in tuning frequency, and since the idler was already a couple of percent away from resonance.
As done in Chapter 5, a fit was applied to the above data: in this case a second-order polynomial. From Figure 6.2, it is apparent that the buncher section could provide the necessary modulation to drive the output cavity near 500 MW. The bunching did not go to zero as the drive was turned off because there was always enough noise in the beam to induce some bunching after the input cavity. The best example showing the modulation came from shot 842. In Figure 6.3, the modulation current can be seen on the left and expressed as a percent modulation of the DC beam current on the right. The modulation is over 65% for most of the 1 μs beam voltage pulse. The early ISIS simulations indicated that the modulation should have peaked at about 14 cm, but measurements showed that the peak modulation was near 22 cm after the idler gap. The difference was probably due to the set up in ISIS. If the idler frequency was tuned closer to resonance in the model, the modulation would be higher and peak in a shorter distance from the idler cavity gap.
Early in the characterization of the output cavity, the idler seemed more susceptible to breakdown from perturbations in the beam caused by the extraction gap. To decrease some of this susceptibility, the idler was tuned back to 1344 MHz.

6.3.1 Idler Cavity Modifications

In later stages of the output cavity development, the idler cavity underwent field/beam induced breakdown more often than was acceptable, and seemed to be
getting worse. After disassembly, the damage was seen to be concentrated on the edge of the reentrant nose and the opposite cavity wall. The design of this cavity was originally based on Friedman's RKA which operated at a pulse length of 100 ns. Friedman argued that there is a certain amount of insulation provided by the space charge of the beam. He termed this property electrostatic insulation [33, 35, 96]. We have seen little evidence of electrostatic insulation, most likely because our beam radius was 2.95 and the wall radius was 3.65 cm, whereas Friedman's beam radius was 13.2 cm and the wall was at 14 cm [35]. The other reason our RKA was much more prone to breakdown is the long pulse length. The probability of breakdown goes up by orders of magnitude as the pulse length is increased [19]. This is because there is more time for any stray electrons that are emitted or are in the general area to be caught up in the fields of the cavity and accelerated into the wall. If enough electrons are created from the plasma of the initial arc, the cavity breaks down and that is the end of the pulse. On the other hand, if it takes a number of rf cycles for the electrons to multiply to breakdown levels from secondary impacts, this is called multipactor [83]. When a cavity is multipactoring, it may not necessarily breakdown. Sometimes the fields are clamped at the multipactor level. Multipactor is a resonance phenomenon that often happens in high-gradient rf resonant structures.

To help reduce the probability for breakdown, it was decided to put a larger radius on the reentrant nose so that the peak surface field would be reduced. A ring was machined from 304 stainless and friction fit on the original nose. The new nosepiece was not copper plated since it would not drop the $Q$ very much. The new radius was 0.100 in. and the actual geometry is shown below in Figure 6.4. The much larger radius of curvature considerably lowered the peak surface field.
It was known that the increase in surface area of the nose would increase the gap capacitance and lower the cavity's resonant frequency, so the gap was increased about 0.10 in to compensate for the change. Both the upstream and downstream sides of the gap were polished to a near mirror-like finish with the 0.05 µm alumina polish compound to reduce the chance for field emission induced breakdown.

6.3.2 Final Idler Performance

After the modification, the idler's tuning range was slightly higher, which implies that the gap should have been increased a little less than had been done. In any case, the tuning range was about 1310-1410 MHz which was still sufficient for the RKA. The portion of the transit time factor, associated with the gap length increase, only dropped by half of a percent, so the cavity was expected to perform
essentially as well as it had previously, except with less vulnerability to breakdown. This was not the case. In general, the modified cavity had to be tuned closer to the operation frequency to get the same level of beam modulation. The difference must have been due to the gap fields being concentrated farther away from the beam. This effect would also decrease the overall transit time factor. The cavity was still capable of modulation near the 60% level, as is shown in Figure 6.5 below.

![Image of graph showing beam current and modulation over time]

Figure 6.5: Modulation after the modified idler cavity on shot 1516

However, shots with this modulation level tended to be more infrequent than those with the unmodified cavity. An advantage of the new geometry was that the breakdown problem was not as prevalent as with the original idler geometry.
6.4 Redesign of the Idler Cavity

The best solution for the idler cavity was to eliminate the nose altogether. A new cavity was designed which was of the noseless pillbox type. Noseless designs are less prone to breakdown and multipactoring [97]. The 2 cm gap spacing was chosen for this idler, as the first idler seemed to produce more consistent modulation near the 67% level. The tuning slug was eliminated since the approximate resonant frequency needed was already known, and it would have disturbed the simple geometry of the pillbox, which might make breakdown more likely. The geometry of this cavity design is shown in Figure 6.6.

Figure 6.6: Geometry of a TM_{010} pillbox idler cavity
Unfortunately, the funding for the RKA ran out before this new geometry could be built and tested.
Chapter VII
The Output Cavity

7.1 Goals

The output cavity, in any klystron, performs the toughest task. It must slow the electron bunches sufficiently to extract the maximum amount of power while leaving enough energy in the beam to transport the used electrons away from the cavity gap and to the beam dump. The output cavity has to do all of this without breaking down or allowing undesirable beam instabilities such as virtual cathodes to form. If the fields in the cavity are too high, yet under the breakdown limit, the electrons in the beam will actually turn around and return to the input cavity gap where they can induce oscillations or, at the very least, cause a change in the gain of the system. A more elegant way to state the major goal in output cavity design is that the shunt impedance of the output cavity must be a good match to the modulated beam impedance.

7.2 The Initial Output Cavity Design

The first design for the output cavity was based on approximations using the 2.5D ISIS code. Since the code assumed an axisymmetric geometry, the actual features of the output coupling slot could not be included. A picture of the initial design is shown below in Figure 7.1. This cavity incorporated adjustments for the coupling aperture, gap length, and resonant frequency. It was thought that by making
the total area of the slot, in its full open position, equivalent to the area in a full circumference slot, the coupling would be nearly the same. This, however, was not a satisfactory approximation. The behavior of the cavity is completely dependent on the geometry and interconnections between the two end walls. As a result, the loaded cavity $Q$ was much higher than expected. The desired loaded $Q$ was estimated to be near 5 but the measured $Q$ ranged from 100 to 200 depending on the adjustable settings. A sample of a frequency sweep on the cavity is shown in Figure 7.2.

Figure 7.1: The initial design for the output cavity

Figure 7.2: Measured response of the initial output cavity
This value was much higher than expected, and the cavity could not be tuned on resonance without producing electric fields that exceeded the level for RF breakdown. Such fields would also be high enough to reflect electrons in the beam. After a number of shots, no consistent RF output pulses were produced, even at very low input drive levels. The output cavity was disassembled at this point and it was apparent that the cavity was arcing mostly near the adjustable output coupling vanes. The first corrective step was to machine away all of the stationary vane portion of the coupling iris. Not surprisingly, the measured $Q$ was still high, ranging from 230 to 540 with a resonant frequency around $1318 \pm 6$ MHz. The $Q$ may have increased because the rotor vanes made better contact with the wall without the stator vanes. The next step was to machine away most of the vanes on the rotor also. This left about a 7.2 mm coupling slot around the circumference with four 1.43 cm wide vanes spaced at almost 90 degrees. This modification dropped the $Q$ to about 84 at the lowest point, but not as far as was needed.

At this point, the coupling from two B-dots in the cavity was low enough that the network analyzer was making measurements very near the noise floor and, as a result, could not be trusted for anything more than a rough estimate. To alleviate this problem, an E-field probe was made from 0.144 in semirigid coax. The probe tip was made by stripping 0.75 in of the outer shield and 0.5 in of the insulation from the end. The coax was supported in the center of the beam pipe by a short cylinder of aluminum about 3.5 in back from the probe tip. This probe worked better than the B-dot, but the frequency response was not as level as it could be. Later, a better E-field probe was made which is shown in detail in the next section of this chapter.

The tuning slug had to be fully extended into the cavity volume to raise the resonant frequency as much as possible. The new $Q$ range was about 84 to 270
depending on the gap spacing. The gap spacing had almost no effect on the 1255 MHz resonant frequency for this configuration. Even though the $Q$ was still too high, a number of shots were fired to test the output cavity. The high $Q$ was not seen as too much of a problem since the cavity was so far off resonance. For 5 kW of input RF power, the output power was about 6 MW. With 20 kW of input drive, the output was near 25 MW average for some shots. Examples of these shots are shown in Figure 7.3.

Figure 7.3: Sample shots from the early output cavity configuration
In addition, the output pulse length had increased to the microsecond regime. As the input drive was increased further, the pulse shortening reappeared, but this sequence of shots proved that the output cavity modifications were headed in the right direction.

7.3 Major Modifications to the Initial Design

The quickest way to increase the output coupling was to increase the size of the coupling slot. Because of the modular design of the output cavity, it was relatively easy to change the downstream nose and end-wall of the cavity, which defined the coupling slot. The first step was to make a test nose out of aluminum with no vanes. This piece had three nylon screws to keep it centered in the outer conductor. Starting with the coupling slot entirely covered with 1 in. wide strips of copper tape, the cavity response was measured as the slot was gradually and symmetrically opened up. It was found that the best combination for a cavity $Q$ around 10 and a reasonably close resonant frequency used three to four strips symmetrically placed around the circumference.

A new output cavity downstream end wall (the one containing the coupling irises) was built, based on the cold tests, with a loaded $Q$ of ~10. The cavity gap had to be opened up to 2.75 cm to bring the resonant frequency close to the desired value of 1300 MHz. As can be seen in Figure 7.6 below, the resonant frequency was about 1280 MHz. It was fortunate that the coupling could be changed so drastically while still keeping the frequency and gap within reasonable bounds. In fact, the very low $Q$ of the cavity made the shift in frequency almost unimportant. The much lower loaded $Q$ resulted in much lower electric fields across the cavity gap, thereby reducing the rf breakdown problem. This change created a better match between the modulated
beam impedance and the cavity shunt impedance for better conversion efficiency from beam power to microwave power. A comparison between the original and modified cavity designs is shown in Fig. 7.4. The original geometry is shown above the centerline, and the modification is indicated by the heavy lines below the centerline.

Figure 7.4: Modification to the cavity end wall

By this time, it was necessary to make an E-field probe with a broader frequency response to more accurately measure low-Q cavities. The probe was fashioned after the design used in the HFSS calculations. In this way, the measurements would also
correspond more closely with the calculations. The details of this probe are shown in Figure 7.5 below.

![Diagram of the E-field probe](image)

**Figure 7.5:** Construction details of the new E-field probe

Measurements of the modified cavity are shown in Figure 7.6.

![Graph of measured response](image)

**Figure 7.6:** Measured response of the modified output cavity
The modified cavity almost instantly showed vastly improved output. For the first 70 shots at low drive power (20 kW), the output was typically 25 MW average for 1 µs or 50 MW average for 0.5 µs. On the third shot the input drive was increased to 90 kW, the output was near 100 J. This shot is shown in Figure 7.7 below.

![Graph](image)

**Figure 7.7:** Early output from the modified output cavity

The highest energy obtained with the modified output cavity, in a single pulse, was

![Graph](image)

**Figure 7.8:** The highest energy pulse, 375 MW peak, 160 J (Shot 1308)
160 J. Data are shown in Fig. 7.8 where rf output power, beam voltage, and beam current are all overlaid on the same time scale. The peak power is 375 MW. Note that the rf power goes away before the beam voltage reaches its maximum value. This will be discussed later. The frequency down-converted mixer IF signal from this shot, shown in Fig. 7.9, is relatively clean. The FFT of this mixer signal is shown in Fig. 7.10. The 3 dB bandwidth is 0.9 MHz.

![Figure 7.9: Mixed IF from shot 1308](image)

![Figure 7.10: FFT of the mixer output for shot 1308](image)
Other shots have recorded higher peak powers, although at a slightly lower energy per pulse. Fig. 7.11 shows the pulse from a different shot with a peak power of 475 MW and an energy of 140 J.

![Output Power vs Time Graph](image)

**Figure 7.11:** The highest peak power shot

The signals from the two B-dot loops located 90 degrees apart on the upstream

![Beam Voltage vs Time Graph](image)

**Figure 7.12:** Output and idler cavity gap voltages for shot 1308
wall of the output cavity are shown in Fig. 7.12. They track together, indicating that the output cavity is operating in the proper mode. The B-dot signals are plotted in Fig. 7.12 in terms of the output cavity gap voltage they represent as calculated using the techniques described in Chapter 5, Section 5.3. The actual voltage across the 2.76 cm output gap at 375 MW reaches 370 kV, producing an average electric field of about 134 kV/cm.

During this stage of output cavity testing, a complete redesign of the cavity in HFSS was in progress. It was desired to test the validity of the computer model before actually building the new RKA output cavity. A simpler cavity, without the impedance taper and radiused corners, was built into standard 6-in-diam. coax for cold-test measurements. Figure 7.13 shows the simulated structure.

![Diagram](image)

**Figure 7.13:** RKA output cavity mock-up for testing the validity of HFSS

Once the actual test cavity was built and measured, the same geometry was entered into the HFSS modeling program. The results of the actual measurements and the computer model are shown below in Fig. 7.14. The experimentally measured curve used for the comparison has been heavily smoothed to remove spikes and fluctuations (artifacts of the measurement process) due to reflections and possible multimoding in the coax line caused by the 6-in.-coax to N-type transition at the load end.
Figure 7.14: Measured and calculated response for the RKA model

The actual numbers are given in Table 7.1 for direct comparison.

Table 7.1: Comparison of RKA output cavity model and HFSS simulation

<table>
<thead>
<tr>
<th></th>
<th>$f_0$ (MHz)</th>
<th>$Q_L$</th>
<th>$S_{21}$</th>
<th>$S_{21}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Linear</td>
<td>(dB)</td>
</tr>
<tr>
<td>HFSS Model</td>
<td>1462</td>
<td>23</td>
<td>0.425</td>
<td>-7.43</td>
</tr>
<tr>
<td>Measured (smoothed)</td>
<td>1463</td>
<td>25.3</td>
<td>0.388</td>
<td>-6.93</td>
</tr>
<tr>
<td>Measured (unsmoothed)</td>
<td>1461</td>
<td>27.6</td>
<td>0.450</td>
<td>-8.21</td>
</tr>
</tbody>
</table>

Note that the resonant frequencies in the cold test experiment and the computer model are practically identical, and also that the $Q$ values are close. The magnitude of $S_{21}$ (transmission coefficient between the drive E-probe and the output coaxial transmission line) lies between the smoothed and unsmoothed values. This comparison gave considerable credibility to basing an entirely new design on the simulated model.
To see what the modified output cavity was capable of, the geometry was entered into HFSS. The cavity was modeled with a lot of detail, and it was uncertain whether HFSS could generate the mesh for the geometry. The cross section of the model is shown in Figure 7.15:

![Figure 7.15: Cross sectional view of the modified output cavity model](image)

The results of the simulation are overlaid with the measurement and are shown in Figure 7.16 below.

![Figure 7.16: Comparison between the measured and modeled modified output cavity](image)
Once again, HFSS accurately predicted the resonant frequency and loaded $Q$ of the structure. The $Q$ was about 2% off, and the resonant frequency was less than 0.02% off. From the solution, the magnitude of the peak gap voltage across the geometrical cavity gap was found to be as high as 538 kV, with a peak surface electric field of 225 kV/cm when 500 MW was being coupled into the output coax. This gap voltage is high enough to turn electrons around in the beam. Therefore, it was obvious that the modified output cavity would never be able to reach the desired output power level of 1 GW or even 500 MW.

Figure 7.17: Overlay of magnetron power, input and idler cavities
At this point in the experiment it was not certain why the rf pulse was terminating prematurely. The most reasonable explanation then was that the output cavity was producing fields high enough to reflect electrons back up the beam-line and/or start some beam oscillation phenomenon such as a virtual cathode. Fig. 7.17 shows traces representing magnetron forward and reflected power, the B-dot loop signal in the idler cavity, and the B-dot signal in the input cavity. All of these signals have a spike at the time the rf output power goes away. These signals are bandpass filtered with a bandwidth of several hundred MHz around 1.3 GHz, so the spike appears to be the result of an rf modulation on the beam (space charge wave) traveling back up the beam toward the cathode. The interpretation of the observation is complicated by the fact that the output gap appears to be breaking down at this time as well.

7.4 Output Cavity Redesign

The adjustable tuning, output loading, and gap length features in the original output cavity were not useful because the output loading could not be made strong enough, even by machining most of the vanes away to open the irises almost fully. It was decided that a new optimized cavity design, which eliminated most of the adjustable features of the original design, would be best for extracting the high power present on the very low impedance modulated beam.

The new output cavity was designed using HFSS. The HFSS modeling indicated that the loaded $Q$ of the cavity was very sensitive to the size of the small vanes across the output iris used to support the inner conductor. A loaded $Q$ of 4 was difficult to achieve in our geometry, which is basically a TM$_{010}$ pillbox cavity with iris coupling into coaxial transmission line. The cavity was so heavily loaded by
the output iris that the resonant frequency drops considerably. Consequently, the cavity diameter had to be significantly decreased to raise the resonant frequency back to the desired value. The coupling iris needed to be as large as possible while still maintaining enough support vanes to keep the cavity operating in the desired TM_{010} mode. As designed, the new output cavity had 4 support vanes between the downstream cavity end wall and the inside of the outer conductor where the output cavity irises feed the coaxial transmission line. Fewer vanes would lower the loaded Q even more, but increase the likelihood of having a higher order mode excited in the coax. Four vanes represented a reasonable design compromise. Also, the size of the coupling iris cannot result in too much of the downstream end wall area being removed, or else the edge of the iris adjacent to the beam drift pipe would end up too close to the cavity gap. If the iris edge came too close to the beam pipe edge at the output gap, there would have been a rise in the peak surface electric field at the nose of the gap, increasing the probability of rf breakdown. Many of the machined radii necessary for corners that see high fields were significant compared to the other cavity dimensions. For example, the radius at the output cavity gap nose was 0.5 cm, and the beam pipe radius was only 3.65 cm; therefore, all radii had to be included in the model accurately enough to simulate their effect, but not so accurately as to require an impossibly complex mesh. Because the support vanes help carry the beam return current in addition to the cavity rf currents, their contact footprint typically had to be much larger than their cross-section.

Views of the new design are shown in Fig. 7.18. The end view only shows one half of the structure because, due to symmetry, it was not necessary to model the complete structure. An electric-field probe was added on the left side to supply an input signal for modeling purposes.
The beam passes from left to right. The output cavity section was coupled to the low impedance coaxial line through 4 irises defined by the 4 vanes. The output coax then tapered to the standard 6-in-diam, 50 Ω line dimensions. According to HFSS, this cavity had a resonant frequency of 1300 MHz and a loaded $Q$ of about 4.1.

The coupling slots were increased in both height and angular width. The variable coupling adjustment was removed, and the tuning slug was also eliminated. A separate tuning control was desirable, but the only available location would have put the tuning slug in the position where the magnetic field was strongest (near the cavity's outer wall), and this was the location desired for the output coupling irises. The one adjustment that was retained was the gap length, which could tune the frequency to some extent. The new output cavity had a lower output gap voltage, due to the decreased shunt impedance, thereby reducing the possibility for electron reflection. For an output power of 500 MW, the geometrical gap voltage was calculated to be 365 kV, and the peak surface field was 246 kV/cm. The peak surface field has not increased substantially over the value in the modified original cavity, but the gap voltage has been drastically decreased. The peak surface field is what leads to
rf breakdown, while electron reflection is determined by the line integral of the electric field along the electron trajectory across the geometrical output gap and the fringing fields just inside the drift pipe flanking the gap. The lower gap voltage should have allowed the new cavity to consistently extract 0.5 to 1 GW for the microsecond-long beam pulse.

7.4.1 Mechanical Construction Details of the New Output Cavity

The main constraint on any computer model for this project was that it must be able to be implemented into actual hardware. The areas of concern were electrical connections, vacuum issues, magnetic field diffusion, machineability, and ease of assembly. These issues were all dealt with in the mechanical design of the new output cavity.

The most important aspect of the design was the electrical connections. Without good contacts, the cavity would not resonate correctly, or might arc in crucial areas and produce a plasma which might cause the main gap to breakdown also. The connection from the upstream half of the cavity to the coax not only carried RF currents, but also formed the vacuum seal at this point. A soft aluminum 0.063 in. cross section O-ring was used here. Copper is usually not used as a vacuum o-ring material because, once deformed, it tends not to press back on the contact surfaces as much as aluminum does. The disadvantage of aluminum is it has a very thick oxide layer on it in the natural state that needs to be removed in order to make a good electrical contact. The aluminum was sanded with 600 grit sandpaper along the circumference and cleaned with ethanol immediately before the two cavity halves were bolted together. This step is important to follow since aluminum forms the oxide very quickly in air. The inner coax also needed a good connection to the outer
surface of the downstream nose. This was accomplished by using a 0.030 in. diameter pure silver wire. Indium was tried at this location but quickly proved to be the wrong choice since the nose threaded down on the contact surface. The indium galled and balled up in the O-ring groove making disassembly very difficult. Gold wire of the correct diameter would have also worked well here. The support vanes fit into notches in the nose with great precision, and were tightened in with stainless flat-head socket screws. The contact feet of the vanes needed special attention since they had to maintain contact with the outer coax when the gap position was changed. The rf current and the beam current had to conduct through these small surfaces. The connection was made using Multi-lam™, an almost flat contact spring material. The Multi-lam was friction fit into a dovetail groove in the top of the vane. The Multi-lam was only 0.027 in. thick and could not take up much slack; therefore, the nose and vane assembly had to fit in the coax with only about 0.020 in. of clearance. The Multi-lam worked well for motions in the same direction as the spring material, but did not function as well for perpendicular motions, such as rotation. The spring sections tended to dig into the soft copper plating. Using the Multi-lam in the original design where the pieces had to rotate was not a good idea, but it worked well for the modifications and the redesign.

For the most part, designing for vacuum system entails making sure there are no places where air could be trapped (which would make a virtual leak) and having enough conductance to the pumps. The additional difficulty with vacuum microwave cavities is the need for vacuum seals that also make good electrical connections. This was the purpose of the soft aluminum O-rings mentioned in the previous section. For ease of assembly, the O-ring was set into a groove so it would hold itself in position while the upstream cavity half was being fitted. To keep any air from being trapped
around the beam dump and behind the nose, ports were milled in the side and notches were cut in the end that the nose butted against. It was important to put the side ports in a location where the beam did not dump. The high current in the beam would have melted any surface that it struck perpendicularly. On a side note, the beam dump was made of titanium to decrease the x-ray yield when the energetic beam electrons impacted. Not only are x-rays hazardous, but they can also induce breakdown by providing free electrons in cavities.

Since the 0.5 T magnetic field for the RKA was pulsed, it was necessary to keep the material in the beamline and cavities as thin as possible. As the magnetic field tried to penetrate the tube, circulating eddy currents were set up in the circumferences. The RKA was fired as the magnet current crested, which gave time for the smaller eddy currents to dissipate. The construction material had to be non-magnetic 304 stainless, as were all of the bolts and nuts. The poor conductivity of stainless was helpful for the field diffusion, but any cavity where the $Q$, or the power, was high needed to be copper plated. The plating was only on the order of a few thousandths of an inch so most of the eddy currents were carried in the stainless. Anywhere there was a flange of significant thickness, it was scalloped between bolt holes to remove excess material. Many of the beamline flanges were the conflat type, but the copper gaskets could not be used under any circumstances. The copper gaskets acted like a single-turn transformer to the pulsed magnetic field and kept the magnetic field from penetrating to the center. Instead, the square Viton type gasket was used early on. Later, flanges were designed with self-holding round Viton o-rings that made assembly much easier. The aluminum o-rings mentioned above were all thin enough not to cause problems.
Making the cavity machineable meant trying to keep as many of the operations on either the lathe or the mill. There was some hand finishing required on the vanes, but this was unavoidable. Most of the details for making the cavity easy to assemble have already been covered previously. In retrospect, the thread used to hold the nose to the center conductor should have been less fine. The combination of a fine thread, and only having four small threaded feet, made the nose easy to crossthread.

7.4.2 Performance of the New Output Cavity

When the new output cavity was built, the response was measured and compared to the simulation. The results are shown in Figure 7.19.

![Graph showing simulated and measured response of the new output cavity (RKA5)]

Figure 7.19 Simulated and measured response of the new output cavity (RKA5)

Once again, the simulated and measured resonant frequency and $Q$ of the cavity correlated almost perfectly. With great anticipation, the new output cavity was bolted up to the rest of the RKA.
The initial performance of the new output cavity was encouraging. At low and medium input drive levels, the output pulses tended to be longer. However, the abrupt pulse termination was still there. Typical pulses at medium drive are shown in Figure 7.20 below.

Figure 7.20: Output from the new output cavity at medium drive levels (90 kW)

It was hoped that the cavity might condition up if the drive power was slowly raised after many shots. However, this was not possible, as it takes hundreds of thousands of shots to actually condition a tube, and these experiments were only single shot due to the pulsed nature of the magnetic solenoid and the limits of the HV power supply. High power shots were tried, but produced less peak power than the modified cavity
had. Even though the peak power was lower, in general, the pulse length was closer to a microsecond than ever before at the 100-200 MW power level. Some of the better shots are shown in Figure 7.21. Shot 2339 averages about 100 MW for 850 ns. The spike at the end is not being included since the RKA had shifted in output frequency at that point.

![Graph of Shot 2364 and Shot 2360 Power vs Time](image1)

![Graph of Shot 2339 Power vs Time](image2)

**Figure 7.21: Output at high power drive levels (300 kW)**

Disassembly of the output cavity showed evidence of arcing across the gap. The worst damage was on the upstream wall where the plating had actually been eroded away in spots. Any arc would almost surely initiate on the downstream nose, where
the field was much greater, but it seemed the greatest damage came from the electron impact into the other side.

Although it was unlikely that the fields generated in the output cavity were high enough to cause electron reflection, there was still evidence of feedback on the beam. Ever since the RKA started to produce 100 MW pulses, the tube has always had essentially two modes of operation. These two modes are evident in Figure 7.20 above. One mode was where the RKA behaved essentially as an amplifier and coupled power out for nearly the entire beam pulse, but at a relatively low level (Shots 2339 and 2360). The other mode was what could best be described as a locked oscillator. The output power grew quickly to a very high level, early in the pulse, and then terminated prematurely (Shot 2364). There was a need to try and separate out the differences between straight breakdown in the cavity, and oscillation from feedback in the beam and consequent breakdown. Some experimentation was done by varying the time the drive magnetron initiated bunching in the input cavity. It was thought that there might be more probability for reflecting electrons when the diode first turned on since the electron energy was relatively low at that point in the pulse. Electrons reflected early in the pulse could have started building an oscillation as the output power increased. As the beginning of the input drive was adjusted towards the middle of the beam voltage, the RKA typically only produced output pulses at the lower level, with fewer early terminations. This was not an acceptable way to operate the RKA, however, because half of the beam pulse was being sacrificed.

Another possible driving factor for oscillations was the third harmonic from the beam current modulation. The beam tube was not cut off to the third harmonic, and if any beam energy was converted to rf, that rf could propagate all through the tube. To check for the third harmonic, a B-dot loop was placed in the region of the
diode. A number of shots were performed with different filters on the B-dot signal. If there was a spike on the output pulse, there was almost always a concurrent spike in the diode B-dot. The spike was most pronounced when the signal was being filtered at the third harmonic (3.9 GHz), but the spike was also present at the fundamental and second harmonic. An additional confirmation that the third harmonic could be present actually came from the ISIS simulations. In Appendix C, the plots for the current modulation after the input cavity (probe 10) clearly show a third harmonic component building about one-third of the way into the run. A good experiment to eliminate the third harmonic would have been to put an rf sever between the idler and output cavities. The sever would absorb any rf fields in the beam pipe while allowing the space charge wave on the beam to pass through. Severs are commonly used in traveling-wave tubes between gain sections to prevent oscillations. In any event, no matter what initiated the breakdown, it seemed the RKA might benefit from some sort of conditioning.

7.5 The RKA Conditioning Cavity Experiment

The problem of conditioning for electron tubes has been around for many years. Cold emission from tube electrode surfaces was studied by Bennett as early as 1931 [98]. All high power microwave tubes that come from industry have been conditioned in order to reach their full power potential. This is also true for accelerator structures. At SLAC, an S-band accelerator structure was used for an rf breakdown study [99]. They shot millions of pulses into their structure to condition it. A graph of their conditioning timeline is shown in Figure 7.22. It does not seem to matter how well the cavity surfaces are polished and prepared, there in no substitution for the process of lightly arcing the cavity under vacuum to remove microscopic field-
enhancement spots, absorbed gases, and impurities [100, 101]. This process is used in many other places where high-gradient rf surface fields are encountered [102]. Kilpatrick was one of the first researchers to try to quantify the relation between peak surface field breakdown and frequency [103]. He based his research on relatively low frequency data and with relatively poor vacuum compared to today's standards. His formula for the breakdown surface field, $E_b$ (in MV/m), is given below.

$$f \ (MHz) = 1.64 E_b^{3/5} e^{45 E_b}$$  \hspace{1cm} (7.1)$$

Since the actual parameter to solve for is $E_b$, a solution can be obtained by iteration. The Kilpatrick field is always a good reference point, but in today's accelerators and rf structures, the gradients often reach two to three times $E_b$ [62, 104]. In the RKA, the Kilpatrick field is near 32 MV/m. The highest field anticipated in the output cavity was about 25 MV/m, assuming 500 MW output, so this did not appear to be
any major problem. The RKA could not be rep-rate conditioned since this would require a major upgrade in BANSHEE, a DC magnet, a thermionic cathode, and special cooling for the beam dump. With this in mind, an alternate method for conditioning the cavity pieces was devised.

A high $Q$ cavity that could hold the RKA output gap pieces approximately in the standard RKA orientation, under vacuum, and with waveguide input coupling was designed. The geometry was determined using HFSS. A diagram of the design is shown in Figure 7.23. The dark hashed areas represent OFHC copper and the lighter hashed regions are stainless steel. The vacuum pump used was a Balzers 450 l/s turbopump.

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**Figure 7.23:** View of the RKA output gap conditioning cavity
The cavity had to be unity coupled at the resonant frequency, so that all of the incident power would go into the cavity fields. The loaded $Q$ of this cavity was near 6000 according to HFSS, and could recreate the high surface gradients seen on the gap pieces in the RKA with only 1 MW of input power. The cavity was transported to a high-power commercial klystron capable of 10 MW, 5 $\mu$s pulses at up to 100 Hz. Because each pulse could contain up to 50 J, the exterior of the cavity had to be water cooled. The cavity body was made with 1 in. thick OFHC copper, and channels for water flow were milled in the sides. Copper plates were then brazed over the channels to keep the water flowing in the correct direction.

To prepare the RKA cavity pieces for assembly, the surfaces that were damaged by previous arcing were sanded down to the bare stainless steel surface under the copper plating. The stainless was then polished as described in Appendix B similar to the cathode surface.

Once the cavity had been assembled, the $Q$ and the coupling were found to be drastically affected by the connection the nosepiece made with the end wall. The rf joint wire o-ring had to be made of silver or gold to get a reliable contact. In any case, the best measured $Q$ obtained was near 3500 once the ceramic waveguide window was in place. The $Q$ was low not only because of the rf joint at the nose, but also because the surfaces of the RKA cavity pieces were mostly stainless steel. The steady-state response of the cavity can be seen in Figure 7.24 below. As can be seen below, the cavity is slightly undercoupled. If the $Q$ would have been able to be brought closer to the ideal value by improving the nosepiece's connection, the cavity would have been nearly unity coupled as designed. It was also crucial for the cavity's resonant frequency to be at 1300 MHz +/- 5 MHz, since the drive amplifier for the klystron had a very narrow bandwidth.
Figure 7.24: Measured response of the RKA conditioning cavity

The actual resonant frequency was 1301.4 MHz, which was perfect.

The cavity was connected up to the high power klystron as shown in Figure 7.25.

Figure 7.25: Schematic of the conditioning cavity experiment
Forward and reflected power were measured using an in-house built waveguide directional coupler, and the power in the cavity was determined from one of the B-dots already on the RKA cavity upstream wall. The gap voltage was determined from the power at the cavity B-dot as done with the other cavities. An example of the waveforms is shown in Figure 7.26.

![Waveform Diagram](image)

Figure 7.26: Forward, reflected, and cavity B-dot signals

The $Q$ of the conditioning cavity can also be estimated from the fall time of the cavity power. The fill time constant $\tau$ is related to the cavity's $Q$ by

$$\tau = \frac{Q}{\omega},$$

(7.1)

where

$$\omega = 2\pi f_0,$$

(7.2)
and \( f_r \) is the cavity's resonant frequency. In more general terms, Lance gives the following formula [105]:

\[
Q = 4.343 \frac{\omega \Delta t}{A},
\]

(7.3)

where \( A \) is the decibel ratio of the cavity power levels at \( t_2 \) and \( t_r \). Using the cavity waveform from Figure 7.25 and converting the diode signal to measured power, the cavity \( Q \) turns out to be

\[
Q = 4.343 \frac{1301 \text{MHz} (1 \mu s)}{81 \text{mW}} \times \frac{81 \text{mW}}{6.2 \text{mW}}
\]

or

\[
Q = 3200
\]

This value is sufficiently close the value measured with the network analyzer of 3500.

Note that there is also an x-ray detector in Figure 7.24. This was an intrinsic high-purity germanium crystal for determining the energy spectrum of the x-rays emitted from the cavity. The counts from the detector were taken by a multichannel analyzer which was calibrated by a \(^{133}\text{Ba}\) source for the low energy regime and a \(^{137}\text{Cs}\) source for the high energy regime. The x-ray detector was useful for correlating the gap voltage in the cavity with the energy of the electrons accelerated in the gap. Several x-ray spectra were run early on in the experiment and these results are correlated with the calculated cavity gap voltage in Table 7.2 below.

<table>
<thead>
<tr>
<th>X-Ray Spectrum #</th>
<th>Drive Power (MW)</th>
<th>X-Ray Energy (keV)</th>
<th>Gap Voltage * (.87) (kV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.295</td>
<td>400</td>
<td>404</td>
</tr>
<tr>
<td>2</td>
<td>1.4</td>
<td>764</td>
<td>902</td>
</tr>
<tr>
<td>3</td>
<td>1.8</td>
<td>910</td>
<td>1020</td>
</tr>
</tbody>
</table>
The multiplier to the gap voltage is the approximate transit time factor across the gap of the cavity. The agreement between the energy and the gap voltage is reasonable, especially when the error of the x-ray energy (+/- 50 keV) and the cavity voltage (+/- 0.3 dB) are taken into consideration.

The conditioning cavity was run for many hours with a typical input of 1.5 MW to 2.3 MW for 4.5 µs at 40 Hz from the klystron. The vacuum was usually about 6 x 10^-7 Torr before a run and near 1 x 10^-6 Torr when near the maximum surface field. At the beginning of a run, the drive power was started at a low level and then brought up after 30 to 60 minutes, if the cavity was not arcing more than a few times over a 5 minute period. Cavity arcs were evident from decreased cavity signal pulse lengths and an increase in the reflected power. As the upper limit of the surface field was approached, the cavity had to be left at that power level longer before attempting an increase, otherwise the cavity arced too much and was lightly damaged. Occasionally, the cavity did arc heavily and the power had to be dropped to a lower level. Then the conditioning process had to start over at that point. The damage was always able to be conditioned back out with about 500,000 shots. A plot of the surface field on the gap nose piece is shown in Figure 7.27. Note that for the runs from 2 - 4.2 million pulses that the maximum surface field was slowly increasing, but that the safe starting field had increased significantly. The peak at 3.7 million pulses is due to a short run at a 2.5 µs pulsewidth. It is apparent that even a 50% drop in pulsewidth instantly allows a significantly higher surface field to be held. The damage near the end happened right at the end of the available klystron time and could not be conditioned out in the standard fashion. However, the pulse repetition rate was dropped to 10 Hz, and the surface field was able to be drastically increased even with the damage. This may be due to the vacuum system being overloaded with
Figure 7.27: Cavity gap surface field during the latter part of the conditioning process desorbed molecules from the walls at the 40 Hz rep rate. With this knowledge the experiment probably should have been run at the 10 Hz rate from the beginning, but this would have increased the conditioning time needed.

An important experiment was conducted before the run starting at 5.3 million pulses. The cavity was vented to atmospheric pressure using UHP nitrogen and left there for 5 min. The cavity was then returned to vacuum, and the run was started. The cavity essentially followed the same conditioning track as if it had been under vacuum all along. This served to confirm that most all of the positive effects of conditioning could be retained after exposure to air.

An enigma of this experiment was the location of the actual arcs in the cavity. The viewport on the vacuum cross was there to view a portion of the cavity gap during operation. When the cavity was arcing, it was obvious from the light generated inside, but no arcs ever seemed to originate in the gap. When the cavity was disassembled later, the only evidence of arcing was near the outer rf joint of the
nosepiece. The crucial surface of the nosepiece did not have any arc damage, but it did look exceptionally clean. In fact, the surface looked like the stainless on the cathode sidewall where very small scratches were accentuated by high voltage stress. One might be tempted to believe that the gap was never actually stressed, but that is not correct. The x-ray spectra were taken at drive levels where the cavity was not arcing. This is because the cavity arcs produced too many x-rays for the detector, and because arcs would give erroneous results. The energy of the x-rays correlated well with the calculated gap voltage, which implies that the gap was producing the electrons that were accelerating into the gap surfaces.

Once the conditioning experiments were ended, the cavity was let up to UHP nitrogen and only briefly exposed to atmosphere as the vacuum ports were sealed off. The cavity was transported over to the RKA experiment where the gap pieces were removed and then reassembled in the RKA while taking great care to never touch the conditioned surfaces.

7.6 RF Glow Discharge Cleaning

It has been widely seen, in the accelerator community, that structures conditioned under vacuum and then exposed to air return to their conditioned state quickly, once back in operation [106]. This was a necessary fact for the RKA conditioning to be effective, and seemed valid for the RKA conditioning cavity as was described in the section above. What was not known was whether the cavity pieces could be handled carefully enough while moving them from the conditioning cavity back to the output cavity so as to retain their surface quality.

To clean any residual contamination caused by transferring the pieces once they were back in the RKA, an argon rf glow discharge was used. Plasma cleaning
techniques have also shown utility in decreasing the required conditioning time for high gradient structures [107]. A manifold was made to mount where the beam pipe connected to the RKA output cavity. The manifold is shown in Figure 7.28. Only the turbomolecular pump was used to bring the output section of the RKA to vacuum.

![Diagram](image-url)

Figure 7.28: Argon glow discharge manifold for the RKA output cavity

The manifold had an rf feedthrough on the end, an argon gas fitting, a thermocouple gauge, and a viewport window on the opposite sides. The vacuum coax feedthrough had to be extended to the point where the cavity gap was located, otherwise the rf glow would start near the feedthrough and use all the rf power before the glow could spread to the cavity. The small wire on the end was also necessary to improve rf
coupling into the cavity. The shape of the wire was very crucial to getting a decent match at the input. To start the glow discharge, the vacuum pump was first switched to 2/3 speed, and the UHP argon flow was brought up until the pressure in that arm of the vacuum cross was about 60 microns. The 50 W rf source was turned on at full power, and a Tesla coil was brought near the viewport to initiate the glow. Once the glow started, the argon flow was reduced until the pressure dropped to about 10 microns. This seemed to produce the best glow in the cavity gap. The glow discharge was continued for several hours. At the end of the treatment, the glow was extinguished and the system was returned to vacuum overnight. Before disconnecting the manifold, the RKA output section was brought up to 1 psig pressure with UHP nitrogen. The manifold was removed and the cavity was immediately reconnected to the RKA beam line.

The first shots on the cavity were done at low power to avoid any immediate arcing damage and to make sure the beam was behaving correctly. After a number of shots the power was increased to a medium drive level for 30 shots and the cavity was still operating well. Then the drive power was raised to the maximum (300 kW). The RKA was still operating as it did before with two modes of amplification, one as an amplifier and the other as a driven oscillator. Unfortunately, any time the driven oscillator mode was excited, the power in the output cavity grew rapidly and the cavity arced. Any arcing at high power damaged the surface of the output cavity to the point where the effect of any surface treatment would be masked. The bottom line was that no obvious improved performance was apparent. Another data point here is that cavities without electron beams in them can sustain extremely large gradients, but the same cavities with beam in them breakdown at much lower levels [108].
7.7 The RKA Beam Pipe Center Conductor Experiment

A concept that had been entertained in thought only, early in the RKA's development, was adding a center conductor to the beam pipe in order to lower potential depression of the beam. A center conductor could not be added without some thought, because it would short out the cavity mode. One way to possibly avoid shorting the cavity mode would be to place a gap in the center conductor. The gap could be connected to a resonant cavity of its own in the center conductor. The tuning of the center conductor cavity could then be adjusted to produce the appropriate impedance at the gap so the overall cavity mode would not be drastically affected. This is precisely the experiment the Air Force Phillips Lab RKA team, led by Kyle Hendricks, performed [109]. With their coaxial extractor geometry, they were able to extract 1 GW for about 60 ns. This power level was five times higher than the amount of extracted power before adding the center conductor. Their original output cavity was based on a design made by the author. The major difference between the Hendricks' cavity and the LANL cavity was that theirs was based on a TEM resonance rather than a $\text{TM}_{010}$ mode because of the large geometry of their cavity and beam. It was likely that a center conductor resonant gap could match the requirements of a TEM output cavity fairly easily, but that the match for a $\text{TM}_{010}$ cavity might be very difficult if not impossible.

Realizing that time and available funds were running short, we designed and built a center conductor out of 304 stainless steel based on the geometry used by Hendricks. The geometry of this center conductor is shown in Figure 7.29. The cavity in the center conductor was a quarter-wavelength TEM resonator. This should have placed a high impedance at the gap of the center conductor and ideally supported the output cavity gap voltage and not destroyed the cavity mode. Once the center
Figure 7.29: The LANL RKA output cavity center conductor

Conductor was installed in the output cavity, the cavity's response was measured and is shown below in Figure 7.30.

Figure 7.30: Measured response of the RKA output cavity with the center conductor

It was completely obvious that the center conductor had destroyed the mode of the output cavity, but some shots were taken to determine how the operation of the RKA might be affected. The output pulses all had a common theme: a short spike of about
35 MW at the pulse beginning and about 5 MW to 10 MW of power for the rest of the pulse.

In order to understand the differences between the results of the Hendricks' cavity and the LANL cavity, some simulations were done in HFSS. The results for the original design for Hendricks' cavity alone and with a center conductor are overlaid in Figure 7.31.

![Graph showing S\(_21\) (dB) vs. Frequency (MHz) for cavities with and without center conductor.](image)

Figure 7.31: Hendricks' output cavity simulated with and without the center conductor

Note that the center conductor cavity's resonance merely adds a small perturbation to the original response of the cavity, ignoring the increase in coupling caused by the center conductor. With a small amount of tuning, the center conductor cavity could be adjusted so that the resonance would overlap the output cavity resonance, and the output cavity would resonate as it did before. Now, the same simulations were applied to the LANL RKA output cavity and the results are shown in Figure 7.32.
The center conductor did not just add a small resonance as it had with the TEM output cavity, it completely changed the cavity response. This is why the LANL RKA output cavity's extraction did not improve with the center conductor. If the funding ever returns, the LANL RKA's output cavity could be redesigned for a TEM resonance, and then the center conductor could be tried again.
Chapter VIII
Conclusions

8.1 The RKA

Pioneering research on the RKA single gap extractor has optimized extremely low output cavity $Q$'s. However, for resonant extraction, the cavity $Q$ should not be dropped below about 4 in order to maintain good mode purity and thus efficiency. The performance of the RKA was somewhat less than expected, due to several elusive problems. The most prominent of these, the lack of a consistent beam, made the experiment extraordinarily difficult. However, the RKA performed a number of noteworthy feats. The extraction of 160 J in a single pulse with an average power near 250 MW for 0.5 $\mu$s is one of them. The only other high peak power, microsecond-pulse-length microwave tube that has extracted more energy is Bruce Miller's Super-Reltron (250J), but it has two output cavities [110]. Another feat of the RKA is the extraction of 475 MW peak power. It is tempting to discount the final version of the output cavity as being inferior to the previous version, but this would be a mistake. The main goal of this experiment was to extend high power microwave generation in RKA's to the microsecond regime. The final output cavity for the RKA showed this was possible. It extracted an average of about 100 MW for 850 ns. This is a much longer pulse than the previous design extracted at any significant power level. It is also much longer than the typical 100ns output pulses for any high-power klystron with a field emission cathode. The RKA produced this pulse even though the output cavity gap was heavily pitted and the plating was eroding in some areas.
Never before has any single gap extraction cavity shown as much promise for relativistic klystrons with field emission cathodes.

8.2 Utility of the Codes

Computer codes for modeling electromagnetic fields, and the interaction of particles with those fields have made a great impact on microwave tube design. The two codes primarily used here were HFSS and ISIS.

8.2.1 HFSS

The author's success with HFSS, for the cases presented in this thesis, has been nothing less than phenomenal. The performance of three cavities was duplicated, and two other cavities were built based solely on the simulated 3D structure. In each case, the response curve of the simulated structure showed all the detail of the measured device, especially when the device was measured in the same fashion as in the simulator. For low $Q$ cavities ($Q < 500$), HFSS is one of the best codes to use. In each case here, the measured resonance frequency matched the simulated result by better than 0.5%, and the measured cavity $Q$'s were within 20% of the simulated values. It should also be noted that it is actually very difficult to measure very low $Q$ values like those in the RKA output cavity. Typical measurement probes and techniques usually do not have a broad range of flat response, although the electric-field probe designed in Chapter 7 was very good for measuring broad frequency sweeps. High $Q$ cavities, such as the RKA conditioning cavity, are more difficult to solve in HFSS, and high $Q$ cavities are usually better left to eigenmode solvers such as SUPERFISH or MAFIA. In our case, HFSS was still the only code useful for the conditioning cavity. The complex geometry and the fact that the input port had to be
a very good match for the drive klystron made HFSS the clear choice. Even so, the resonant frequency of the conditioning cavity was accurate to better than 0.5%. The $Q$ of the conditioning cavity was far off, but this was due to the rf connection between the nose piece and end wall, and between the exposed stainless steel surfaces. More importantly, the reflection coefficient for the drive port was very low, as designed. The question that is begged is, where is the limit of HFSS? HFSS is nearly impossible to use when the structure is many wavelengths long and/or the structure is highly overmoded. However, programmers continue to churn out improved versions of HFSS, and as workstations become more powerful there may be little to limit the program's use in the future.

8.2.2 ISIS

As good as HFSS is, the ultimate computer code for any kind of tube design has to be able to push particles. ISIS is one of the best PIC codes in the world. ISIS has body-fitted coordinates and open boundary conditions for traveling waves. The downside of ISIS is that it is incredibly user unfriendly, but its utility of ISIS is that it can often get results where other codes fail.

For the first time, a 2D equivalent of a 3D output cavity structure was designed so that believable results could be obtained from the ISIS code. If the output cavity gap geometry, resonant frequency, and $Q$ cannot be accurately described in ISIS, the results obtained for power extraction and beam behavior are practically useless.

The maximum power extracted from the complete RKA ISIS model never exceeded about 250 MW. It is interesting to note that the peak power extracted using RKA5 never was above 300 MW. Most of the time, the output power was in the 100
to 200 MW range. These numbers give considerable credibility to the ISIS simulations. The ultimate test for ISIS would be in the area of extracted power versus beam current modulation. Unfortunately, the real RKA would have to be working consistently in order to make any such comparison. As was mentioned back in Section 2.6.2.2, the ISIS simulations tend to recommend an output cavity with a slightly higher $Q$, at least for the given beam parameters. Another possibility would be to run a beam with more modulated current, which would induce higher fields in the output cavity, and extract more of the bunch's kinetic energy.

An interesting result that came in the early ISIS simulations was that once the output cavity was included in the model, the input cavity had to be excluded. The input cavity was replaced with an equivalent electric field value in the correct location. The reason for the exclusion was that the small number of particles reflected back from the output cavity would return to the input cavity and cause the tube to oscillate. It was felt that this oscillation was a numerical artifact and so the input cavity was removed so that useful results could be seen [111]. More investigation into this problem is needed because the RKA seemed to also suffer from oscillation problems which were possibly driven by reflected electrons from the output cavity.

Since we are presently limited to a 2D version of ISIS, the best way to make use of each code is to separate a tube design problem into several parts. The first part involves using ISIS to determine the maximum beam modulation for maximum extraction, given the previously determined beam geometry, impedance, and energy. The next step is to design an output cavity in HFSS with the right shunt impedance to match the beam impedance at maximum modulation. Then an equivalent 2D model of the output cavity is also designed in HFSS. The 2D output cavity geometry can
then be translated back into ISIS, where benchmarks for output power and efficiency can be obtained. Information gleaned at this juncture might also point towards slight improvements in the output cavity shunt impedance for more efficient extraction.

8.3 Future Work

There are a number of areas to improve or extend the performance of the RKA. The funding for doing any such work at Los Alamos has disappeared for now, but there is always a possibility that interest in RKA’s might return. When that day comes, the topics listed below will be excellent areas for development.

8.3.1 Constant Impedance Electron Beam Diode

Without a doubt, the single biggest impediment to the performance of any long-pulse RKA is the lack of a good electron source for the beam. This is not to say that an explosive field emission diode cannot be made to perform correctly for long pulses. It only says that none has been developed to date. The only way an output cavity can be correctly designed is for it to be matched to the beam impedance. If the beam impedance is not constant, there is no way to match the beam with any one geometry.

The shot-to-shot variation of the field-emission diode was not the problem. Over the short term (10-20 shots), the peak voltage might vary as little as 5%. After about 100 shots, the voltage would have dropped about 20% to 30%, and the cathode would have to be repolished. The variation over the pulse was the crucial factor. The beam voltage would usually vary about 20% from the point where the voltage crossed above 600 kV to the point where it dropped below 600 kV. Not only that, the beam current would always rise from about 4 to 5.5 kA during the same period due to the
plasma closure of the A-K gap. The combination of the rising current as the beam voltage turned over the crest made good conditions for virtual cathodes to form. Whether or not they actually formed is up for debate, but a virtual cathode in an output gap would instantly short the gap.

Another possible explanation for the early-pulse-termination problem might have been that there was a significant number of lower-energy electrons towards the center of the annulus, which would be reflected from any typical gap voltage. These reflected electrons could build up an oscillation in the tube and cause the tube to break down.

Yet another interesting problem for ISIS would be to model the existing beam voltage and current in time for the field emission cathode. This might show if there were any problems induced in the beam as the voltage crested.

The standard type of diode used in microwave tubes is the thermionic type. As detailed in Chapter 3, the main limitation with a thermionic cathode is the current density. The RKA needs a current density of approximately 0.5 kA/cm², which is almost two orders of magnitude greater than the typical rating of a thermionic source. With that in mind, one might be tempted to not even give the idea a second thought, but the problem is not as far fetched as it sounds. Most thermionic cathodes for klystrons and other linear-beam tubes use some form of magnetic compression to create the necessary current densities in the device while keeping the current density at the cathode surface low (< 10 A/cm²). The compression factor can range as high as ten in some designs. The compression factor cannot be made arbitrarily high, because the electrons tend to gain too much transverse momentum as they enter the beam pipe which creates transverse oscillations on the beam. The other way to increase the current density is to change the cathode surface. Some thermionic
cathodes have been made with current densities as high as 100 A/cm\(^2\). These cathodes do not have as long a lifetime as the standard type, but they can be produced. Using both maximum compression and an increased current density, a thermionic cathode for the RKA suddenly becomes more feasible. Using a slightly less extreme type of cathode with 50 A/cm\(^2\) capability and a compression factor of 10:1, the diode would be capable of 500 A/cm\(^2\), which is sufficient for the RKA. This simple analysis does not take into account the difficulties with compressing an annular beam, but it does show that the idea is plausible.

### 8.3.2 Repetitive Pulse Operation

It has been mentioned earlier that BANSHEE was originally designed to be pulsed at up to 5 Hz. One large impediment to rep-rating the RKA was the lack of a DC magnet. A DC magnet was finally procured in 1994, and power supplies that could source the necessary 650 kW were located. This magnet was designed to produce a field of up to 1 T in a 28 cm diameter bore. Another benefit of a DC magnetic field greater than 0.5 T is that it might make the RKA more consistent or even increase the extracted power. Unfortunately, the cost of connecting the input power supplies is more than could be afforded at this point.

The RKA might never put out the required power without the in-situ rf conditioning that other high power tubes receive. Conditioning can only be feasible with repetitive pulsing. But for the sake of argument, assume the RKA did put out the desired amount from any given single shot, and that the DC magnet was in operation. We would probably have operated the RKA in rep-rate mode until it melted. Once that had been tried, there would be a few places that would definitely need to be redesigned. The most damage would be located in the beam dump. For
long term repetitive pulsing, the beam dump would need to be water cooled since almost 4 kJ of energy is deposited in it with each pulse. Whatever diode, thermionic or other, was used, it and the load would probably need some sort of cooling as well.

8.3.3 Rf Windows

If all of the extracted rf power was converted to a TE_{10} waveguide mode, a window to separate the vacuum in the tube from the pressurization in the waveguide would have to be designed. This would be a non-trivial task. Even these days, it is often window technology that limits high power tube development. Windows are susceptible to damage by high peak power from surface flashover. For high continuous power operation, windows are mostly limited by heat dissipation. Needless to say, window design for high rf power levels is a complete study in itself, and beyond the scope of this work.

8.3.4 Post Acceleration

The most obvious method of increasing the efficiency and output of the RKA is to increase the kinetic energy in the beam relative to the potential energy. One way to accomplish this is to post accelerate the beam. After the beam has been sufficiently bunched, it is passed through an acceleration gap with a large DC or rf potential that is driven by an external source. The beam picks up additional energy from the gap, but all of the energy adds directly to the kinetic energy portion of the beam. In fact, since the bunches have a higher velocity after the acceleration, the potential energy requirements decrease as is discussed in Chapter 2. As stated before, it is only the kinetic that can be extracted, so post acceleration is a very attractive method for increasing the output power. In theory, the beam could be bunched, post accelerated,
have some power extracted, reaccelerated, extract power again, and so forth. In reality, the beam bunches would probably have to be reshaped after each extraction cavity, but the possibility for many stages of post acceleration exists. Some ISIS simulations were done by Bruce Carlsten on an RKA geometry with some post acceleration [112]. He found that the extraction efficiency, limited to one-half the harmonic current multiplied by the post acceleration voltage, can exceed 50%. Another interesting result was that the energy spread of the accelerated bunch had decreased, which can also improve the extraction efficiency.

8.3.5 Harmonic Extraction

From the spectral analysis of the ISIS runs in Chapter 2, it is apparent that a large portion of beam energy resides in higher harmonics, especially the second and third. It is highly feasible that an output cavity could be designed to extract at one of these harmonic frequencies. In our RKA, the beam tube is cut off to the fundamental and the second harmonic (2.6 GHz). This is important because the cavities need to be isolated from one another in order to keep feedback from the high power levels in the output cavity from coupling back to the previous cavities and inducing oscillations. To extract at a harmonic, the output cavity would be designed to resonate at that frequency in a TEM or TM_{0n0} mode. The most difficult aspect of this type of design would be that the size of the beam pipe sets the minimum dimensions for the cavity. In general, the higher the resonant frequency, the smaller the cavity. A TEM cavity is not quite as limited as the TM in this respect, but there would a number of possible resonating modes competing with the desired mode even with the best of designs. An advantage of harmonic extraction is that the same buncher section could be used for a number of different output cavities. The main disadvantage is that the size of the
output cavity and output ports would be the same as in a tube that was designed to operate at the harmonic output frequency. This means that the output section would be limited by the same breakdown problems, which results in a method no more practical than building a standard tube with fundamental extraction. Extraction of this type has been accomplished in a tube called the Super-Reltron [110].

8.4 Other Types of Output Cavities

To decrease the possibility of breakdown in RKA's, a number of efforts to distribute power over a larger volume in the output section have resulted in different possibilities for output cavity designs. Some of the following designs have been tried with limited success in other high power tubes.

8.4.1 Multigap Extraction

Multigap extraction is the simplest form of distributing the output power. As shown in Figure 8.1, additional single-gap cavities can be added to the beamline. Each cavity is tuned to extract only a fraction of the total output power, keeping the load per cavity down. This type of extraction scheme has been used by SLAC on an 11.5 GHz X-band tube [113], and by Miller in the Super-Reltron tube [110]. Problems with this type of design are that the output is separated in many waveguides, which might need to be recombined into one, and it may be difficult to obtain a good phase and pulse shape match between the individual outputs. The reason for the pulse shape change is that there is an optimal position to extract maximum modulation from the beam. As the beam is allowed to propagate past this point, the bunch tends to spread out again due to space charge forces.
8.4.2 Disc-Loaded Gap

Friedman's group at NRL has developed an interesting type of high power cavity for beam modulation and rf extraction. The biggest problem with small extraction gaps is that they are prone to electric field breakdown. One difficulty with increasing the gap length is that the possibility of forming virtual cathodes by exceeding the space charge limiting current goes up dramatically. By placing washer-like discs in the gap, the space charge limiting current can be kept larger than the modulated beam current. A picture of this geometry is shown in Figure 8.2. The capacitance of the washers divides the voltage across the gap and lowers the potential across each individual gap. The washers are supported by thin metal rods across the gap at the radius of the beam pipe. The size of the support rods is such that their inductance is low enough to allow compensating charge to follow the beam, but high enough that the rf does not short out across the gap. The increased gap length
Figure 8.2: NRL's Disc-loaded gap cavity

decreases the transit-time factor, but an analysis done by Lampe shows that the effect is not too detrimental for a gap length exceeding half a wavelength [114]. NRL has also done PIC code simulations using a disc-loaded gap for a buncher [115]. Up to 40% current modulation was seen from the simulations. Friedman has tested this geometry in a monotron oscillator and achieved 70% current modulation from a 7 kA beam [116].

8.4.3 Traveling Wave Output Cavities

The ultimate structure for an output cavity is the traveling wave (TW) type. The TW structure is similar to having multiple extraction cavities, except the cavities are connected closely to one another, forming a number of cells in a larger structure. Instead of having a standing wave structure and coupling rf energy from the standing wave fields, a TW structure extracts a fraction of the total energy in each cell. An example of one type of TW output cavity is shown in Figure 8.3. The power from the first cell flows into the second cell as more beam energy is added and so forth. The difficulty with making a TW structure is to design it such that the power only flows in the direction of the beam's propagation and the rf is phased correctly to extract power
at each cell. Traveling wave output cavities for highly relativistic klystrons have been analyzed by Ryne [117], and have been used experimentally for a number of high power microwave tubes [118, 119]. These tubes were designed for short pulse lengths (< 200 ns), but the TW structure was more successful than previous designs. The most prohibitive aspect of the TW design is the expense as the structure is costly to design and to machine. A TW output cavity for the RKA would probably have the best chance for extending the output power to the GW regime, since the $Q$ for the single gap cavity was already at the lowest possible value where efficient extraction could still be expected.
Appendices

Appendix A

Useful Relationships

A.1 Relativistic Mechanics [62]:

\( v = \text{particle velocity}, \ c = 2.997924 \times 10^8 \text{ m/s}, \ \text{the speed of light in vacuo} \).

Rest mass of the electron \( m = 510.999 \text{ keV/c}^2 \).

(Sometimes given as \( m_0 \) and \( m_c \) in the text)

Charge of the electron \( e = 1.6022177 \times 10^{-19} \text{ C} \).

The normalized velocity factor is
\[
\beta = \frac{v}{c}.
\]

Velocity to kinetic energy:
\[
\gamma = \sqrt{1 - \beta^2} ;
\]
\[
E_k = (\gamma - 1)mc^2 .
\]

Kinetic energy to velocity:
\[
\gamma = \frac{E_k + mc^2}{mc^2} ;
\]
\[
\beta = \sqrt{1 - \frac{1}{\gamma^2}} .
\]

Total energy:
\[
U = E_k + mc^2 = \gamma mc^2 .
\]
Energy to momentum:

\[ \beta \gamma = \sqrt{\gamma^2 - 1} . \]

For small differences:

\[ \delta \gamma = \gamma \beta \delta \beta, \delta \gamma = \beta \delta (\beta \gamma); \delta E_z = mc^2 \delta \gamma, \delta p = mc \delta (\beta \gamma) . \]

Dynamics:

\[ \ddot{r} = \frac{d \dot{p}}{dt} = m \frac{d (\gamma \dot{v})}{dt} . \]

A.2 Intense Annular Beams

The space-charge-limiting current:

\[ I_{\text{max}} = \frac{2 \pi e_0 m_0 c^3 \left( \gamma_{\text{ej}}^2 - 1 \right)^{3/2}}{e \ln \frac{r_e}{r_b}} = \frac{8.525 kA}{\ln \frac{r_e}{r_b}} \left( \gamma_{\text{ej}}^{2/3} - 1 \right)^{3/2} . \] (2.37)

The beam velocity in a drift pipe:

\[ v_0 = c \sqrt{1 - \frac{e \phi(r_b)}{m_0 c^2}} . \] (2.34)

The minimum \( \gamma \) needed to propagate the beam:

\[ \gamma_{\text{min}}^{2/3} = \left( \frac{E_p + E_z}{m_0 c^2} + 1 \right)^{2/3} = \left( \frac{I_{\text{peak}} \ln \frac{r_e}{r_b}}{8.5 kA} \right)^{2/3} + 1 . \] (2.43)

The maximum amount of extractable power from a modulated beam:

\[ P_{\text{max}} = \frac{1}{2} I_i (511 kV) (\gamma_{\text{ej}} - \gamma_{\text{min}}) . \] (2.44)

A.3 Microwave Formulae

Although some confusion may be caused, the coupling parameter in microwave work has traditionally been defined as \( \beta \).
The guide wavelength [85]:

\[
\lambda_g = \frac{\lambda}{\sqrt{1 - (\lambda / \lambda_c)^2}}.
\]

The group and phase velocities:

\[
v_g = \frac{1}{\sqrt{\mu \varepsilon}} \sqrt{1 - \left(\frac{\lambda}{\lambda_c}\right)^2},
\]

\[
v_p = \frac{1}{\sqrt{\mu \varepsilon \sqrt{1 - (\lambda / \lambda_c)^2}}}.
\]

The cutoff wavelength for a TE\textsubscript{10} mode in rectangular waveguide with width \(a\):

\[
\lambda_c = 2a.
\]

The reflection coefficient:

\[
\Gamma = S_{11} = \frac{Z_{\text{load}} - Z_{\text{lim}}}{Z_{\text{load}} + Z_{\text{lim}}} = \frac{1 - \text{VSWR}}{1 + \text{VSWR}} = \frac{1 - \beta}{1 + \beta}.
\]

Reflected or transmitted power:

\[
P_r = |S_{11}|^2.
\]

The voltage standing-wave-ratio, or VSWR, in waveguide:

\[
\text{VSWR} = \frac{1 + |S_{11}|}{1 - |S_{11}|}. \tag{2.15}
\]

Once the type of cavity coupling and the VSWR are determined, \(\beta\) is given as follows:

- Undercoupled: \(\beta = 1 / \text{VSWR}\) \quad (2.16)
- Unity coupled: \(\beta = 1\) \quad (2.17)
- Overcoupled: \(\beta = \text{VSWR}\) \quad (2.18)
Appendix B

Experimental Details

Some of the more critical details of the RKA experiment have been included here for reference purposes. The cathode polishing technique was also used for any of the high-electric-field gradient surfaces in the cavities.

B.1 Cathode Polishing

The polishing procedure started with 600 grit wet sanding on a lathe, and continued with the same piece of paper until the surface had no more scratches resembling 600 grit and the sandpaper looked closer to 1000 grit. Then the surface was cleaned with water and a clean dry cloth. The next step was to apply some 0.05 mm alumina polish compound to a damp towelette and continue polishing until a mirror-like finish was obtained. If the damage was not too severe, sometimes the sanding step could be skipped in favor of using only the alumina polish. Since the front tip of the cathode was where electron emission needed to be encouraged, it had to be prepared in a different manner. While the cathode was still on the lathe, the front edge was filed flat with a fine single-cut file. Afterwards, approximately 2 mm of the side edge near the tip was abraded with fresh 600 grit sandpaper. After installation, it would take about 5-10 shots to clean off residual contamination on the surface and bring the diode up to full operating potential.
B.2 Directional Coupler Calibration

The process would typically follow this procedure (also see Figure 4.4): A conflat copper gasket had to be crushed between the flanges to the same degree as it would in the final assembly. Then a loop was fashioned out of a light gauge wire (0.026 in. diam.) and shaped in such a way so that it ran close to the walls of the coupling chamber and close to the coupling hole. The assembly was bolted up and the coupling factor checked. If the coupling factor was within 5 dB, this could be tweaked by adjusting the proximity of the loop to the hole. If the factor was further off, the hole size or the gauge of the loop had to be changed. If standard gauge wire proved to be too thin, sections could be cut from copper sheet material. Once the coupling factor was close, it was time to adjust the loop in the sides and near the top flange to get a good match between the coupling ports. Next, the directivity was measured as the loop's rotation was changed. If high directivity could not be obtained, either the size of the loop usually needed to be increased, or the electric field coupling needed to be decreased. Then the process would start over. If the directivity was good, a new copper gasket replaced the development one, the parts were thoroughly washed with ethanol and the pieces reassembled. As the conflat flanges were being torqued, the directivity was monitored. As long as the directivity was increasing, all was well. If the directivity started to drop, the flanges could be loosened and the loop rotation adjusted appropriately. This assumed that the conflat knife edges were in very good condition and there was still enough crush in the gasket to get a vacuum seal.
B.3 Fitting Rectangles Around the Circumference of a Circle

An interesting geometry problem had to be solved to figure out what size of SiC blocks to order for the coax load in Chapter 4. The basic geometry is shown in Figure B.1.

![Diagram of fitting rectangular boxes around the circumference of a circle](image)

Figure B.1: Fitting rectangular boxes around the circumference of a circle

Solving this problem produces the following equation for the half-width \( w \) of the rectangles.

\[
w = \sin \theta_1 \cos \theta_1 \left[ -h \pm \sqrt{h^2 - \sec^2 \theta_1 (h^2 - r^2)} \right]
\]  \hspace{1cm} (B.1)

The height of the rectangles was going to be 0.5 in., and there needed to be enough to approximate the circle well. Nineteen blocks would give a width of about 1 in. For 19 segments, \( \theta_1 = 9.474^\circ \), and \( h = 0.5 \) in. Putting these numbers into equation B.1 gives \( w = 0.495 \) in. The final bar dimensions as ordered were 0.5 x 0.990 x 12 in.
Appendix C

ISIS Input Files and Examples

C.1 Introduction

Being that ISIS is an extraordinarily complicated computer code, the input files are included here in the hope of helping anybody so unfortunate as to have to actually try to run the code. An example run and post-processing section are also included at the end. Multitudinous thanks go to Dr. Bruce "Sensei" Carlsten for putting these together into a working form for me.

Most of the units used in ISIS are a bit different than one might be used to, so they will be reviewed here. Energy is given in terms of the electron’s rest mass energy, 511.1 keV. A related value is the voltage which would accelerate an electron to the point where the kinetic energy equals the rest mass energy, so the units of voltage are the same.

\[ V_{\text{sis}} = 511.1 \text{keV} \]  
(C.1)

Current is measured in terms of the Alfven Current, defined by

\[ I_{A} = \frac{4\pi e m_e c^2}{e} = 17.05 \text{ kA} \]  
(C.2)

Of course the units are not exactly the Alfven current: they are divided by \( 4\pi \), which gives

\[ I_{\text{sis}} = \frac{17.05 \text{ kA}}{4\pi} = 1.357 \text{ kA} \]  
(C.3)

Power is voltage multiplied by the current,

\[ P_{\text{sis}} = V_{\text{sis}} I_{\text{sis}} = 693.4 \text{ MW} \]  
(C.4)
The momentum of particles in the scatter plots is given in terms of $\gamma \beta_z$. The relativistic factor $\gamma$ can be found from
\[
\gamma = \sqrt{(\gamma \beta_z)^2 + 1}.
\]  \hfill (C.5)

C.2 The Files

To help explain some of the operations in the codes, comments have been added into the files by the author and are denoted by curly brackets {}.

C.2.1 The Cosmos Deck

136 /usr/tmp/bhaynes fred cosrka
cosrka 286 lines
*1*
1 # c shell
2 # c shell script to run isis
3 set dirname = "/bhaynes/rka"
4 set ptag = "af"
5 set savepath = "/usr/tmp/mejl."
6 set pathname = $dirname"/$ptag
7 set ifrun = "yes"
8 #
9 # get the change deck for historian
10 #
11 cat > changes << endhere
12 *ident,changes
13 *d,smoothj.154
14 *d,smoothj.310
15 *i,smoothj.196
16 nxx1m=680 {smoothing, very important}
17 nxx1b=681 {beams stop is at mesh point z = 681}
18 nxx1=679
19 *d,parm21.8
20 parameter (nsrf=10)
21 *d,smoothj.200
22 aj1(1,nxx1b)=2.*aj1(1,nxx1m)-aj1(1,nxx1)
23 *d,smoothj.203
24 590 aj1(m,nxx1b)=aj1(m,nxx1m)
25 *d,smoothj.206
26 aj1(m,nxx1b)=aj1(m,1)
27 *d,sMOOTHJ.027
28 610 ajsv(m)=aj1(m,nxx1m)+2.*aj1(m,1)+aj1(m,2)
29 *d,sMOOTHJ.209
30   do 630 i=1,nxx1m
31 *d,sMOOTHJ.210
32 630 aj1(m,nxx1b+1-i)=aj1(m,nxx1b+1-i)+aj1(m,nxx1b-i)
33 *d,sMOOTHJ.212
34   do 640 i=2,nxx1m
35 *d,param.03.6
36   parameter (idn=10, ispmax=1)
37 *d,param.03.8
38   parameter(nx1=699, nx2=79)  {these numbers = largest mesh points-2}
39 *d,param.21.12
40   parameter (ntcp=3*(nx1b+nx2b+4))
41 *d,param.31.6
42   parameter (maxcoils=100, maxsols=10)
43 *i, wpar.255
44   write(93) t, np
45 *i, wpar.377
46   write(93) (tvec(ipq), ipq=1,7)
47 *i, whist.119
48   write(92) thst
49   do 200 i=1,iprbmx
50 200 write(92) iprtpb(1,i),iprtpb(2,i),iprtpb(3,i),prbhst(i)
51 *i, endmp.58
52   close(92)
53   close(93)
54 *i, strtdmp.24
55 *CA parm21
56 *CA emisco
57   dimension ntcp(nsrf)
58 *i, strtdmp.29
59 c open file for history, particle ascii dumps
60 c write ics array into mypar dump
61   open(92, file='myhis', form='unformatted')
62   open(93, file='mypar', form='unformatted')
63   write(93) nsrf
64   do 20 m=1,nsrf
65   ntcp(m)=0
66   if(ncath(m).ne.8h default) ntcp(m)=ncath(m)
67 20 continue
68   write(93) ((ntcp(m)), m=1,nsrf)
69   write(93)(((xivcs(i,1,m), xivcs(i,2,m)),
70   li=1, ntcp(m)), m=1,nsrf)
71   write(92) iprbmx
72 *define, zrtheta, fext, emiss, twolhd, beam
73 endhere
74 #
75 # get the historian program library and make changes
76 #
77 if (-e ispl010 ) goto hist
78 cfs get dir=/088688/isis/ver10 ispl010
79 hist:
80 (historn i=changes,p=ispl010,c=ischm.f,l,f > outhist)
81 set iflag = `awk '/ERROR/ {print $0}' < outhist`
82 if ( $iflag != """) goto quit
83 if (-e netcdf.inc ) goto compile
84 cfs get dir=/088688/isis/ver10 netcdf.inc
85 compile:
86 #
87 # compile changes
88 #
89 cft77 -esz -o agress -l isis iscom.f >& outerr
90 #
91 if (-e libnetcdf.a ) goto load
92 cfs get dir=/088688/isis/ver10 libnetcdf.a
93 load:
94 set lib = ",-lcfi1ib,clams,ipc,rpc,io libnetcdf.a"
95 set exeq = "isis" sptag
96 cat > ldrinp<< endhere
97 COMMONS=AAAASCMBASIC,CAVIT,DIPS,DMPCOM,BEAMCOM,
98 Flag,FLDBND,GPTEM,HISTO,HYBRD,INJBND,KWANCOM,MCCOM,
99 MOMENT,PARBND,QUADS,
100 PROBE,RANDCOM,SEEDCOM,SMUTCOM,SOLEN,STATCOM,TCOEF,
101 TEMCOM,TIM,ZZZZSCM,
102 AAAALCM,ABC0M,AEC0M,COLFLDS,FSOLV,GRADTE,HISTORY,
103 METCOM,TMPFLD,ZZZZLCM
104 endhere
105 ssegdr -o $exeq -H 800000+0 -Ddynamic=BUF -l ldrinp -O mc Slib iscom.o
106 #
107 # set up the
108 cat > inamSptag<< endhere
109 $list
110 label=ae,2.5Cav RKA 3Deq input4.5kA 
111 x1min=0.0,x1max=70.0,
112 x2min=0.0,x2max=8.0,
113 i per1l=1,ip er1u=2,iper2l=3,iper2u=1,
114 vp1u=1.0,
115 gac1l=1.0,gac1u=1.0,gac2l=0.0,gac2u=1.0,
116 denx1=405,405,0.0,0.0,
117 posx1=0.0,0.1,0.101,26.28,
118 denx2=0.0,0.0,1.0,1.0,0.0,0.0,
119 posx2=0.0,27.2,701,3.2,0.3,201,3.000,
121 ifnij=1, vfl=2.040,
122 rtime=5.0,
123 nspx1=2,nspx2=4,
124 nsm=2,
125
126 ics(1,1,1)=701,81, 701,74, 592,74, 592,41, 591,41, 591,40,
127 590,40, 590,39, 589,39, 589,38, 386,38, 386,39, 385,39,
128 385,40, 384,40, 384,76, 343,76, 343,39, 355,39, 355,43,
129 359,43, 359,42, 360,42, 360,41, 361,41, 361,40, 360,40
130 360,39, 359,39, 359,38, 1,38, 1,81, 701,81,
131 ics(1,1,2)=701,51, 701,1, 681,1, 681,38, 615,38, 615,39,
132 614,39, 614,40, 613,40, 613,41, 612,41, 612,48, 613,48,
133 613,49, 614,49, 614,50, 615,50, 615,51, 701,51,
134 ics(1,1,3)=645,74, 645,51, 630,51, 630,74, 645,74,
135 ics(1,1,4)=629,74, 629,51, 617,51, 617,74, 629,74,
136 mtyp(3)=3,4,
137 epsil(3)=11.2,16.8,
138 far2max=3.0,
139 iw=85,
140
141 ndump=0160, ndxyy=0160,
142 np1d(1)=1, np2d(1)=1,
143
144 ndhis=3,
145
146 tlim=1000., dt=0.060,
147 timtotal=10000.,
148 b0=5.8,
149
150 zcavmin=0.0,200.,
151 itypcav=1,
152 phase=0.0,0.0,
153 ampcav=.30,0.0,
154
155 wca=0.2725,
156 mtop=4,
157 mbos=5,
158
159 iprbpt(1,1)=11,2,41, 11,40,41, 11,80,41, 11,120,41, 11,160,41,
160 11,200,41, 11,240,41, 11,280,41, 11,320,41, 11,360,41,
161 11,400,41, 11,440,41, 11,480,41, 11,520,41, 11,580,41,
162 11,600,41, 11,610,41, 11,620,41, 11,630,41, 11,640,41,
163 11,650,41, 11,660,41, 11,670,41, 11,680,41, 11,690,41,
164 50,15,20, 50,45,20,
165 22,685,1,
166 6,685,62,
167 iphot(1,1,1)=359,38, 386,38,
168 iphot(1,1,2)=589,38, 615,38,
170
171  
172 endhere
173 #
174 # get the restart dump file if it exits
175 #
176 if ($ifrun != yes ) goto quit
177 #
178 # set timtotal for production jobs to allow for clean up
179 #
180 # get job name
181 #
182 pwd | awk '{print substr($1,15)}' > tmp
183 set job = `cat tmp`
184 rm tmp
185 prod j | grep $job | awk '{print $4}'> tmp
186 @ tmax = `cat tmp`
187 rm tmp
188 #
189 # save 3 minutes for storing files
190 #
191 @ tmax -= 3
192 if($tmax == -3) goto dump
193 @ tmax *= 60
194 @ lines = `awk 'END { print NR }' < inam$ptag`
195 @ lines -= 1
196 head -$lines inam$ptag > o$$.tmp
197 rm inam$ptag
198 mv o$$.tmp inam$ptag
199 cat >> inam$ptag << endhere
200 timtotal = $tmax,
201 \$
202 endhere
203 dump:
204 #
205 # get the restart dump file if it exits
206 #
207 cfs list $pathname"/i0"" awk 'i0/ {print $1}'
208 | sort -r | awk 'NR==1 {print $1}' > o$$.tmp
209 awk 'length($0) < 7' < o$$.tmp > o$1.tmp
210 set fname = `cat o$1.tmp`
211 rm o$$.tmp
212 rm o$1.tmp
213 if($fname == "") then
214 @ ifdump = 0
215 # dont do anything
216 else
217 @ ifdump = 1
218 set pn = $pathname"/$fname # retrieve desired file
cfs get $pn
@ lines = `awk 'END { print NR }' < inam$ptag`
@ lines -= 1
head -$lines inam$ptag > o$$.tmp
rm inam$ptag
mv o$$.tmp inam$ptag
cat >> inam$ptag << endhere
dmpname = '$fname',
$ endhere
def
osis$ptag
#
if error occur during the run save everything
#
set chck = `ls core`
if ( $chck == core ) then
mv * $savepath # save checkpoint file
endif
set filel = `ls i0*` # store restart dumps
set chkcfs='network'
while ($chkcfs != "")
set chkcfs = `cfs store dir=$pathname $filel | grep network`
if ( $chkcfs == ") break
# sleep
sleep 60
end
set filel = `ls id*`
set chkcfs='network'
while ($chkcfs != "")
set chkcfs = `cfs store dir=$pathname $filel | grep network`
if ( $chkcfs == ") break
# sleep
sleep 60
end
set filel = `ls ip*`
set chkcfs='network'
while ($chkcfs != "")
set chkcfs = `cfs store dir=$pathname $filel | grep network`
if ( $chkcfs == "") break
# sleep
sleep 60
end
set filel = `ls if*`
set chkcfs='network'
while ($chkcfs != "")
set chkcfs = `cfs store dir=$pathname $filel | grep network`
if ( $chkcfs ==") break
# sleep
sleep 60
end
268 sleep 60
269 end
270 set filel = `ls ih*`
271 set chkcf5=`network`
272 while ($chkcf5 != "")
273 set chkcf5 = `cfs store dir=$pathname $file1 | grep network`
274 if ($chkcf5 == "") break
275 # sleep
276 sleep 60
277 end
278 set filel = `ls ix*`
279 set chkcf5=`network`
280 while ($chkcf5 != "")
281 set chkcf5 = `cfs store dir=$pathname $file1 | grep network`
282 if ($chkcf5 == "") break
283 # sleep
284 sleep 60
285 end
286 quit:
*end

C.2.2 The Inam File

137 /usr/tmp/bhaynes fred inamaf
inamaf  66 lines

*!
  1 $list
  2 label='ae,2.5Cav RKA 3Deq input4.5kA ',
  3
  4 x1min=0.0,x1max=70.0,     {Min and Max dimensions of the model, in cm}
  5 x2min=0.0,x2max=8.0,     {x1 = z, x2 = r}
  6
  7 iper1l=1,iper1u=2,iper2l=3,iper2u=1,
  8 vp1u=1.0,
  9
 10 gac1l=1.0,gac1u=1.0,gac2l=0.0,gac2u=1.0,
 11
 12 denx1=.405,.405,0.0,0.0, \{affects the current in the beam and the \}
 13 posx1=0.0,0.1,0.101,26.28, \{beam position. There are 4 streams \}
 14 denx2=0.0,0.1,0.1,0.0,0.0, \{spaced at 0.125 cm increments from \}
 15 posx2=0.0,2.7,2.701,3.20,3.201,3.000, \{2.7 to 3.2 cm. Current = 4.5 kA \}
 16 ifinj=t., vfl1=2.040, \{Beam voltage in γBz. Voltage = 650 kV \}
 17 rtime=5.0, \{internal connected surfaces entered in z,r \}
 18 nspx1=2,nspx2=4, \{these are mesh units starting at 1,1. The mesh spacing for this model is 0.1cm \}
 19 nsml=2,
ics(1,1,1)=701,81, 701,74, 592,74, 592,41, 591,41, 591,40,  
590,40, 590,39, 589,39, 589,38, 386,38, 386,39, 385,39,  
385,40, 384,40, 384,76, 343,76, 343,39, 355,39, 355,43,  
359,43, 359,42, 360,42, 360,41, 361,41, 361,40, 360,40,  
360,39, 359,39, 359,38, 1,38, 1,81, 701,81,  
ics(1,1,2)=701,51, 701,1, 681,1, 681,38, 615,38, 615,39,  
614,39, 614,40, 613,40, 613,41, 612,41, 612,48, 613,48,  
613,49, 614,49, 614,50, 615,50, 615,51, 701,51,  
ics(1,1,3)=645,74, 645,51, 630,51, 630,74, 645,74,  
dielectric disc 1  
ics(1,1,4)=629,74, 629,51, 617,51, 617,74, 629,74,  
dielectric disc 2  
mtyp(3)=3.4,  
define the ics's 3 &4 as dielectrics  
epsil(3)=11.2,16.8,  
assign relative permittivity  
far2max=3.0,  
whatever this is, it needs to be bigger than x2max  

ndump=0160,ndxy=0160,  
nplt1d(1)=1,nplt2d(1)=1,  
ndhis=3,  
tlim=1000.,dt=0.060,  
timtotal=10000.,  
magnetic field on axis, B = 1 T  
zcavmin=0.0,200.,  
fitting the input cavity fields  
itypcav=1,  
phase=0.0,0.0,  
ampcav=30.0,0.0,  

drive amplitude = 0.30}  

wcav=0.2725,  
operation frequency in cm^{-1}  
mtop=4,  
mbot=5,  

{Probes follow: 11=current, 22=power, 6=radial E-field. Referenced Conseq.}  
iprbtp(1,1)=11.2,41, 11.40,41, 11.80,41, 11.120,41, 11.160,41,  
11.200,41, 11.240,41, 11.280,41, 11.320,41, 11.360,41,  
{Probes 1-29}  
11.400,41, 11.440,41, 11.480,41, 11.520,41, 11.580,41,  
50,15,20, 50,45,20,  
{Probe descriptions: 10 = I before idler, 15 = I}  
22,685,1,  
{before output gap, 16 = I in middle of oc gap}  
6,685,62,  
{28 = Power in the output coax, 29 = E_r in out. coax}  
ijpot(1,1,1)=359,38, 386,38,  
ijpot(1,1,2)=589,38, 615,38,  

*end
138 /usr/tmp/bhaynes
C.2.3 The Input Cavity File

4 /usr/tmp/bhaynes fred icavaf
icavaf 61 lines
*t*

1  60  {This is the E-field description on axis for the input cavity}
2  0.,  2.3989156751879e-2  {This came from a file generated in SUPERFISH}
3  0.3389830508474,  2.4225510132237e-2  {ISIS imports this while running}
4  0.6779661016949,  2.5963228808778e-2
5  1.016949152542,  2.8442944249901e-2
6  1.35593220339,  3.2108034021687e-2
7  1.694915254237,  3.7041243790469e-2
8  2.033898305085,  4.3429167464609e-2
9  2.372881355932,  5.1634806161267e-2
10  2.71186440678,  6.1729247577308e-2
11  3.050847457627,  7.4559564561706e-2
12  3.389830508475,  9.0012417198392e-2
13  3.728813559322,  1.094520272773
14  4.067796610169,  0.132860048145
15  4.406779661017,  0.1617645376997
16  4.745762711864,  0.1968602839118
17  5.084745762712,  0.2394612881304
18  5.423728813559,  0.2916310120189
19  5.762711864407,  0.3538349032859
20  6.101694915254,  0.4305379484098
21  6.440677966102,  0.5208173888171
22  6.779661016949,  0.6309791784804
23  7.118644067797,  0.7599940137567
24  7.457627118644,  0.9128121361577
25  7.796610169492,  1.090352830513
26  8.135593220339,  1.292274998606
27  8.474576271186,  1.521724913555
28  8.813559322034,  1.768081073427
29  9.152542372881,  2.030266267297
30  9.491525423729,  2.308259481712
31  9.830508474576,  2.576674124116
32  10.16949152542,  2.821160597902
33  10.50847457627,  3.025305081388
34  10.84745762712,  3.17260926925
35  11.18644067797,  3.2500224595
36  11.52542372881,  3.250822423463
37  11.86440677966,  3.173671070231
38  12.20338983051,  3.025198948341
39  12.54237288136,  2.818735169999
40  12.8813559322,  2.570758666455
41  13.22033898305,  2.299737664876
42  13.5593220339,  2.022602544592
43  13.89830508475,  1.753479833338
44  14.23728813559,  1.502615608262
C.3 Running the Code and Post-Processing

5 /usr/tmp/bhaynes nice -20 csh cosaf -xv &  {1st step: Compile the Cosmos Deck}
[1] 30484
6 /usr/tmp/bhaynes ldr-334 segldr: CAUTION  {Ignore these}
                        File 'librcc.a' cannot be found in any of the search directories.
                        CAUTION
                        File 'libiob.o' cannot be found in any of the search directories.
                        CAUTION
                        Common block 'DIPS' was specified by directive but was not referenced. It
                        has been discarded.
                        CAUTION
                        Common block 'HYBRD' was specified by directive but was not referenced.
                        It has been discarded.
                        CAUTION
                        Common block 'MCCOM' was specified by directive but was not referenced.
                        It has been discarded.
                        CAUTION
                        Common block 'MOMENT' was specified by directive but was not referenced.
                        It has been discarded.
                        CAUTION
                        Common block 'QUADS' was specified by directive but was not referenced.
                        It has been discarded.
                        CAUTION
                        Common block 'RANDCOM' was specified by directive but was not referenced.
                        It has been discarded.
ldr-187 segldr: CAUTION
   Common block 'TCOEF' was specified by directive but was not referenced.
   It has been discarded.
ldr-187 segldr: CAUTION
   Common block 'COLFLDS' was specified by directive but was not referenced.
   It has been discarded.
ldr-187 segldr: CAUTION
   Common block 'GRADTE' was specified by directive but was not referenced.
   It has been discarded.
ldr-167 segldr: CAUTION
   The length of common block 'OLAY2' has been redefined as larger by module
   'BNDP' from file 'iscom.o'.
ldr-167 segldr: CAUTION
   The length of common block 'OLAY2' has been redefined as larger by module
   'DIVE' from file 'iscom.o'.
ldr-167 segldr: CAUTION
   The length of common block 'OLAY2' has been redefined as larger by module
   'STRTDMP' from file 'iscom.o'.
ldr-133 segldr: CAUTION
   Unsatisfied external references have been encountered.

Unsatisfied external references
Entry name        Modules referencing entry

RUNF    ERRRUN

**CFS0417: list /u0/bhaynes/rka/af/i0* failed
*     No paths match the wild card expression. 04/29 00:38
warning   smoothing turned on
   {Here's where ISIS starts}
2-1/2 d cylindrical (z-r-theta) coordinates initialized
fitting rf cavities
fitting routine terminated with info= 1 residual= 5.735E-05
isisaf  initialized on 04/29/96 00:39:08
   isis version 10.5 of nov 13 93
   ae,2.5Cav RKA 3Deq input4.5kA
initialization complete from dump 0 n= -1
   {The cosmos file then initiates an ISIS run, but this run was terminated}

6 /usr/tmp/bhaynes ps
   PID TTY  TIME COMMAND
14681 p012  0:00 csh
30484 p012  0:00 csh
30573 p012  0:32 isisaf
30593 p012  0:00 ps
7 /usr/tmp/bhaynes kill -9 30573
8 /usr/tmp/bhaynes Killed
core: No such file or directory
cfs: send file: Broken pipe
**CFS0001: save /094928/fazio/rka/af/i001af failed
   * /094928/fazio/rka/af does not exist. 04/29 00:39
    cfs: send file: Broken pipe
**CFS0001: save /094928/fazio/rka/af/id00af failed
   * /094928/fazio/rka/af does not exist. 04/29 00:39
    cfs: send file: Broken pipe
**CFS0001: save /094928/fazio/rka/af/ip00af failed
   * /094928/fazio/rka/af does not exist. 04/29 00:39
**CFS0371: save /094928/fazio/rka/af/rp00af01 failed
   * IP00AF01 local file is empty. Cannot store. 04/29 00:39
    cfs: send file: Broken pipe
**CFS0001: save /094928/fazio/rka/af/if00af failed
   * /094928/fazio/rka/af does not exist. 04/29 00:39
**CFS0001: save /094928/fazio/rka/af/ih00af failed
   * /094928/fazio/rka/af does not exist. 04/29 00:39
**CFS0371: save /094928/fazio/rka/af/ix00af failed
   * IX00AF local file is empty. Cannot store. 04/29 00:39

[1] Done csh cosaf -xv

8 /usr/tmp/bhaynes

{ Once the cosmos file was run, it created the inam file which contained all of the
experimental parameters for the run. From then on, only the inam file needed to be
edited, and then the isisaf program could be run directly. }

17 /usr/tmp/bhaynes fred inamaf
    inamaf  66 lines
    *t48
    48 ampcav=.22,0.0,
    *rp 48
    p:.22
    r:.20
    48 ampcav=.20,0.0,
    *end
    inamaf  66 lines
18 /usr/tmp/bhaynes nice -20 isisaf &
   {drop the input drive to 0.20}
[1] 10580
19 /usr/tmp/bhaynes warning smoothing turned on
   2-1/2 d cylindrical (z-r-theta) coordinates initialized
   fitting rf cavities
   ffiting routine terminated with info= 1 residual= 5.735E-05
   isisaf initialized on 04/28/96 14:41:47
   isis version 10.5 of nov 13 93
   ae,2.5Cav RKA 3Deq input4.5kA
   initialization complete from dump 0 n= -1
   a isavaf 11261 blocks
   a it01af 992 blocks
Bruce wrote a postprocessing program for plotting the probe data and computing the fourier transform of the data. The program was called peh.x

```sh
19 /usr/tmp/bhaynes peh.x
0 for history, 1 for scat, 2 for exit
0 {Plot the probe waveforms}
you have 50 probes at 5556 time steps; t=999.8999997777
enter fundamental frequency (cm-1)
.2725
iprobe, tstart, tend
10 0 1000 {look at probe 10 from t=0 to 1000}
average current=-3.104860405962
harmonic currents: I1= 0.414453493714I2= 0.1105565613559I3= 0.1155788322087
{there were often points that needed elimination}
```

```
>>> POINTS OUT OF RANGE. THEY WILL BE IGNORED.
>>> 1  X(I)   Y(I)
>>>  1  0.0000E+00   6.2097E+00
>>>  2  5.0005E-04   6.1509E+00
>>>  3  1.0001E-03   5.9764E+00
{etc., etc}
>>>  47  2.3002E-02   5.6158E-01
>>>  48  2.3502E-02   5.3918E-01
>>>  49  2.4002E-02   4.8752E-01
iprobe, tstart, tend
10 500 1000 {look at probe 10 once the signal has stabilized, relatively}
average current=-3.25273603046
harmonic currents: I1= 0.4634879770328I2= 0.1456921022866I3= 0.1842002286408
{calculating I1/Iave here gives the current modulation, I1/I0}
{here, I1/I0 = 14.25%. the modulation from the input cavity}
```

```
>>> POINTS OUT OF RANGE. THEY WILL BE IGNORED.
>>> 1  X(I)   Y(I)
>>>  1  0.0000E+00   6.5055E+00
>>>  2  9.9992E-04   6.4378E+00
>>>  3  1.9998E-03   6.2374E+00
>>>  4  2.9998E-03   5.9118E+00
```
>>> 5  3.9997E-03  5.4729E+00
>>> 6  4.9996E-03  4.9369E+00
>>> 7  5.9995E-03  4.3234E+00
>>> 8  6.9994E-03  3.6545E+00
>>> 9  7.9993E-03  2.9536E+00
>>> 10 8.9992E-03  2.2449E+00
>>> 11 9.9992E-03  1.5520E+00
>>> 12 1.0999E-02  8.9720E-01
>>> 16 1.4999E-02  9.9824E-01
>>> 17 1.5999E-02  1.2371E+00
>>> 18 1.6999E-02  1.3757E+00
>>> 19 1.7999E-02  1.4182E+00
>>> 20 1.8998E-02  1.3729E+00
>>> 21 1.9998E-02  1.2511E+00
>>> 22 2.0998E-02  1.0670E+00
>>> 23 2.1998E-02  8.3627E-01
>>> 30 2.8998E-02  7.4093E-01
>>> 31 2.9998E-02  8.1615E-01
>>> 32 3.0998E-02  8.3747E-01
>>> 33 3.1997E-02  8.0708E-01
>>> 34 3.2997E-02  7.2997E-01

iprobe, tstart, tend
14 500 1000
average current=-3.225894169766
harmonic currents:I1= 2.004879670419I2= 1.289349887793I3= 0.848546556386

>>> POINTS OUT OF RANGE. THEY WILL BE IGNORED.
>>>  I  X(I)  Y(I)
>>>  1  0.0000E+00  6.4518E+00
>>>  2  9.9992E-04  6.3856E+00
>>>  3  1.9998E-03  6.1895E+00
>>>  4  2.9998E-03  5.8708E+00
>>>  5  3.9997E-03  5.4410E+00

iprobe, tstart, tend
15 500 1000  {Probe 15 is the current modulation just before the output cavity gap}
average current=-3.274487148166
harmonic currents:I1= 2.546602908382I2= 1.445495009072I3= 0.8664368532889

{I1/I0 = 77.77% from the idler cavity}

iprobe, tstart, tend
16 500 1000  {this current probe is right in the output gap}
average current=-3.273912136416
harmonic currents:I1= 2.487762384906I2= 1.570015335631I3= 0.8641360362172

>>> POINTS OUT OF RANGE. THEY WILL BE IGNORED.
>>> I        X(I)        Y(I)
>>> 1  0.0000E+00  6.5478E+00
>>> 2  9.9992E-04  6.4787E+00
>>> 3  1.9998E-03  6.2738E+00
>>> 4  2.9998E-03  5.9410E+00
>>> 5  3.9997E-03  5.4928E+00
>>> 6  4.9996E-03  4.9458E+00
>>> 7  5.9995E-03  4.3204E+00
>>> 8  6.9994E-03  3.6392E+00
>>> 9  7.9994E-03  2.9266E+00
>>> 271 2.6998E-01  2.3669E+00
>>> 272 2.7098E-01  2.4543E+00
>>> 273 2.7198E-01  2.4878E+00
>>> 274 2.7298E-01  2.4663E+00
>>> 275 2.7398E-01  2.3913E+00
>>> 276 2.7498E-01  2.2661E+00

iprobe, tstart, tend
28 500 1000  {Probe 28 is the output power in the output coax}
average current=1.058014050603
harmonic currents:i1= 1.44906106161612= 0.419921437463813= 0.118581456255
iprobe, tstart, tend
29 0 1000  {Probe 29 is the radial electric field in the output coax}
average current=-7.3855520497078E-2
harmonic currents:i1= 7.9096615952623E-2i2= 5.5325088689446E-3i3= 4.06703825013
95E-3

>>> POINTS OUT OF RANGE. THEY WILL BE IGNORED.
>>> I        X(I)        Y(I)
>>> 1  0.0000E+00  1.4771E-01
>>> 2  5.0005E-04  1.4651E-01
>>> 3  1.0001E-03  1.4295E-01
>>> 1663 8.3108E-01  4.0099E-03
>>> 1664 8.3158E-01  3.8916E-03
>>> 1665 8.3208E-01  3.7175E-03
>>> 1666 8.3258E-01  3.4948E-03
>>> 1667 8.3308E-01  3.2317E-03
>>> 1668 8.3358E-01  2.9379E-03
>>> 1669 8.3408E-01  2.6238E-03

iprobe, tstart, tend
29 500 1000
average current=-8.3728832237466E-2
harmonic currents:i1= 9.5186884944894E-2 i2= 8.1063488904611E-3 i3= 4.1078721184321E-3
(In the steady state, I₁ from probe 29 is the radial electric field (near steady state) used to calculate the output power once the calibration factor has been determined using the plots)

\( \text{iprobe, tstart, tend } \)
0'0'0
0 for history, 1 for scat, 2 for exit
1
number of conductors=10
1607, 279, 77, 71, 6*0
which time (cm)?
800

\{End the history analysis\}

\{Make the particle scatter plots\}

dump time=0. number of parts=8
dump time=9.599999999996 number of parts=511
dump time=9.599999999996 number of parts=696
dump time=19.19999999999 number of parts=511
\{etc., etc.\}
dump time=796.7999999856 number of parts=3572
dump time=796.7999999856 number of parts=4083
dump time=796.7999999856 number of parts=4582
dump time=796.7999999856 number of parts=5093
dump time=796.7999999856 number of parts=5604
dump time=796.7999999856 number of parts=5974
number of particles=5974
which time (cm)?
810
dump time=806.3999999852 number of parts=511
dump time=806.3999999852 number of parts=1022
dump time=806.3999999852 number of parts=1533
dump time=806.3999999852 number of parts=2044
dump time=806.3999999852 number of parts=2555
dump time=806.3999999852 number of parts=3066
dump time=806.3999999852 number of parts=3577
dump time=806.3999999852 number of parts=4088
dump time=806.3999999852 number of parts=4599
dump time=806.3999999852 number of parts=5110
dump time=806.3999999852 number of parts=5621
dump time=806.3999999852 number of parts=6095
number of particles=6095
which time (cm)?
820
dump time=815.9999999848 number of parts=511
dump time=815.9999999848 number of parts=1022
dump time=815.9999999848 number of parts=1533
dump time=815.9999999848 number of parts=2044
dump time=815.9999999848 number of parts=2553
dump time=815.9999999848 number of parts=3063
dump time=815.9999999848 number of parts=3572
dump time=815.9999999848 number of parts=4082
dump time=815.99999999848 number of parts=4592
dump time=815.99999999848 number of parts=5103
dump time=815.99999999848 number of parts=5614
dump time=815.99999999848 number of parts=6125
dump time=815.99999999848 number of parts=6402
number of particles=6402
which time (cm)?

plot done. pages = 23. words = 106097
graphics cl = u
END OF DISSPLA 11.0 9212, DRIVERS 9111 -- 211422 VECTORS IN 22
PLOTS.
RUN ON 4/28/96 USING SERIAL NUMBER 2545 AT LOS ALAMOS
NATIONAL
LABORATORY
PROPRIETARY SOFTWARE PRODUCT OF COMPUTER ASSOCIATES, INC.
89000 VIRTUAL STORAGE REFERENCES; 8 READS; 29 WRITES.
STOP (called by EXIT )
CP: 62.312s, Wallclock: 446.763s, 1.7% of 8-CPU Machine
HWM mem: 1430345, HWM stack: 2048, Stack overflows: 0
20 /usr/tmp/bhaynes pscan

{A program called pscan displayed the plots in Versaterm-Pro, Tek 4014 emulation}

? d1  {d1 displays the first plot, d+2 would display the plot after the next, n for next}

Figure C.1: Current Probe # 10, the modulation just before the idler cavity

Note that some third harmonic noise starts to build later on in the signal.
Figure C.2: A closer look at the quasi-steady state portion of the current modulation

Figure C.3: An FFT of Figure C.2; the third harmonic is actually bigger than the 2nd.  
(The fundamental is at 0.2725 cm\(^{-1}\))

Figure C.4: Probe 14, current modulation between the input and idler cavities
Figure C.5: Probe 15, current modulation just before the output cavity gap

Figure C.6: An FFT of the current measured by probe 15.

Note the large amounts of power in the second and higher harmonics.
Figure C.7: Probe 16, current modulation measured in the middle of the output gap

Figure C.8: Probe 28, power output in the coaxial transmission line

Figure C.9: Probe 29, the radial electric field in the coaxial transmission line
Figure C.10: The last portion of the waveform from probe 29

Figure C.11: FFT of the electric field in Figure C.10

Note the selectivity of the output cavity by comparing Figure C.11 to the current modulation FFT in Figure C.6. The scatter plots have not been reproduced here since they are shown at the end of Chapter 2.

The procedure for calculating the output power is as follows: First the peak excursions, in the regions of steady-state operation, of probes 28 and 29 are measured from Figures C.8 and C.10, respectively. In this case, probe 28 = 3.2 x 693.4 MW =
2.219 GW. The peak excursions from probe 29 are about -0.18 - +0.02 = -0.20. The calibration factor is \( \alpha \), which is given by
\[
\alpha = \frac{\text{Power}}{(E_{r1(peak)})^2} .
\]  
(C.6)

Thus,
\[
\alpha = \frac{2.219 \text{ GW}}{(-0.20)^3} ,
\]  
(C.7)

so
\[
\alpha = 55.48 \text{ GW} .
\]  
(C.8)

Now the output power can be calculated using
\[
P_{\text{out}} = \frac{1}{2} \alpha E_{r1}^2 ,
\]  
(C.9)

where \( E_{r1} \) is equal to the value of \( I_1 \) from the FFT analysis on probe 29 in the pest.x analysis above. Therefore,
\[
P_{\text{out}} = \frac{1}{2} (55.48 \text{ GW})(.09519)^2 ,
\]  
(C.10)

so the output power is
\[
P_{\text{out}} = 251.4 \text{ MW} .
\]  
(C.11)

The output power measured in the experiment was comparable to the above value.
Appendix D
RKA Mechanical Drawings

The mechanical drawings for the RKA were created by a number of draftspersons. Early on, Tony Cimabue and Helena Korhoner from AT-4 made the drawings for the initial hardware. They took care of a multitude of details in order to bring the first version of the RKA output cavity and output coax together. Lorraine Dominguez made the reductions from the full size drawings. The modified RKA cavity and beam pipe center conductor were drawn by Rick Lovato in AT-9. The conditioning cavity was drawn by Paul Haddock. Many thanks go to Tony, Helena, Lorraine, and especially Rick and Paul for all of their efforts.

The first output cavity and coax            pp. 214-226
The final output cavity (RKA5)           pp. 227-234
The beam pipe center conductor          pp. 235-238
The RKA conditioning cavity             pp. 239-245

The Author looking over BANSHEE and the RKA  p. 246
(Photograph courtesy of Dr. Michael Fazio on top of a 30 ft pneumatic platform)
{Dr. Fazio uses only Nikon cameras and lenses}
References


