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Rice University

The Wage Gradient and the Interaction Between Employment and Residential Decentralization in Urban Areas

By

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A Thesis Submitted
In Partial Fulfillment of the Requirements for the Degree

Doctor of Philosophy

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ABSTRACT

The Wage Gradient and the Interaction Between Employment and Residential Decentralization in Urban Areas

Debashis Ghosh

Mathematical models of urban spatial structure in cities with decentralized employment involve the interaction between a minimum of three markets: land, labor and capital. I study the interaction between the land and the labor markets carefully. Standard theory assumes that when firms decentralize, firms located away from the Central Business District (CBD) offer lower wages to their workers. The continuous downward adjustment in wages as firms move further from the CBD is characterized as the wage gradient. By this approach, following firm decentralization, the adjustment in wages is sufficient to leave everyone indifferent between the monocentric city and the latter case. Consequently, no one relocates and the city is left essentially unchanged.

I refute this argument on the grounds that it does not hold in a model with a full labor market. First, I consider the wage gradient approach, where labor demand is infinitely elastic. Next, I look at the case where labor demand becomes vertical. I consider several special cases, and find that when proportionally more employment moves further out, everyone in the city is better off. Finally, I include a labor market where firm movements take place due to movement of capital, a complimentary factor of production. I show that the wage gradient posited by standard urban theory breaks down. Instead, as firms decentralize in a city with fixed population, an upward sloping utility gradient appears as firms move further out.
Finally, I consider a case where there are a fixed number of high skilled and low skilled workers in the city with a full labor market. I study where they live, given the proximity and strength of the employment subcenter. When the CBD and the subcenter locate close to each other, the two groups stay segregated, all low skilled workers living close to the center and all high skilled workers occupying a low density suburban area. As more jobs move further out, this pattern changes, and low skilled residential areas and high skilled living areas intermingle. In all, I find five possible patterns of residential location.
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Table of Contents

ABSTRACT ........................................ iii

Acknowledgements .............................. v

Table of Contents .............................. vi

List of Tables ................................ xi

List of Figures ................................. xv

Preface
I. A Study of Decentralization in Urban Areas ........ xxiii
II. The Hamilton Mills Conjecture: Unaltered Patterns of Household Rents after Employment Decentralization? .... xxxii
III. Partial and General Equilibrium Models of Urban Areas .... xxxiii

Chapter 1. A Study of Partial Equilibrium Models of City Structures under Various Modes of Decentralization

1.1. The Monocentric City ...................... 1
   1.1a. The Model ................................ 3
   1.1b. Simulation Techniques, Results and Analysis .... 6
   1.1c. The 'Open' versus the 'Closed' City .......... 12

1.2. The City with Suburban Employment .......... 14
   1.2a. The Model ................................ 13
Chapter 2. Special Cases of Employment Decentralization in Partial Equilibrium Models of City Structures

2.1. Decentralization Outside the Initial City Limits ....73
2.2. When a Fixed Number of Jobs and Workers move in from outside the Urban Area ....76
   2.2a. Simulation Techniques, Results and Analysis ....76
2.3. Employment Decentralization in Two Subcenters ....107
   2.3a. The Model ....107
   2.3b. Simulation Techniques and Results ....108

Chapter 3. A Partial General Equilibrium Model of a City with a Full Labor Market and Decentralized Employment

3.1. The Monocentric City with Homogeneous Labor ....134
   3.1a. The Model ....134
   3.1b. Simulation Techniques and Results ....138
3.2. The City with Homogeneous Labor and Decentralized Employment ....142
  3.2a. The Model ....142
  3.2b. Simulation Techniques, Results and Analysis ....147
3.3. Sensitivity Analysis: Changes in Gwage Cities with Changes in Labor Productivity ....173
3.4. Comparing the Partial and General Equilibrium Approaches ....184

Chapter 4. A Partial General Equilibrium Model of a City with Decentralized Employment, a Full Labor Market and a Heterogeneous Workforce

4.1. The Monocentric City with Heterogeneous Labor ....189
  4.1a. The Model ....189
  4.1b. Simulation Techniques and Results ....195
4.2. The City with Heterogeneous Labor and Decentralized Employment ....201
  4.2a. The Model ....202
  4.2b. Simulation Techniques, Results and Analysis ....208
  4.2c. Sensitivity Tests: Results and Analysis ....246

Bibliography ....261
List of Tables

Table 1.1a: Parameter Values for Monocentric City ....8
Table 1.1b: Population Distribution in Monocentric City ....9
Table 1.2: Parameter Values for Decentralized City offering HM wages at SBD ....21
Table 1.3a: Equilibrium Values for Closed HM City with Decentralized Employment ....23
Table 1.3b: Equilibrium Values for Open HM City with Decentralized Employment ....27
Table 1.4a: Equilibrium Values for Closed City with 40% of the Jobs Decentralized ....42
Table 1.4b: Comparing Monocentric, HM and Ewage Equilibria when Employment Decentralizes Close to the CBD ....47
Table 1.4c: Comparing Monocentric, HM and Ewage Equilibria when Employment Decentralizes Far from the CBD ....52
Table 1.4d: Equilibrium Values for Open Ewage City with Decentralized Employment ....54
Table 2.1: The Effects of 5000 New Jobs moving into the City with Two Employment Centers and 40% Jobs Initially Decentralized ....77
Table 2.2: The Effects of 10000 New Jobs moving into the City with Two Employment Centers and 40% Jobs Initially Decentralized ....79
Table 2.3: Comparing Structures of Semi-Closed Cities when CBD and SBD are close to each other ....95
Table 2.4: Comparing Structures of Semi-Closed Cities when CBD and SBD are far from each other ....100
Table 2.5: Equilibrium Values for Closed Wage (60000/20000/20000) Cities .... 109
Table 2.6: Comparing Two and Three Centered Wage Cities (Base: SBD1 10 miles out) .... 114
Table 2.7: Comparing Two and Three Centered Wage Cities (Base: SBD1 16 miles out) .... 116
Table 2.8: Equilibrium Values for Wage (40/60) Cities .... 118
Table 2.9: Equilibrium Values for Closed Wage (40/30/30) Cities .... 118
Table 3.1: Parameter Values for Monocentric City with a Full Labor Market .... 139
Table 3.2: Monocentric City Equilibria: Different Factor Productivities .... 140
Table 3.3: Equilibria in Gwage City (θ = .72; ϕ = .28) with 75% of Total Capital Stock concentrated at the CBD .... 150
Table 3.4: Equilibria in Gwage City (θ = .72; ϕ = .28) with 50% of Total Capital Stock concentrated at the CBD .... 151
Table 3.5: Equilibria in Gwage City (θ = .72; ϕ = .28) with 25% of Total Capital Stock concentrated at the CBD .... 152
Table 3.6: Equilibria in HM City (θ = .72; ϕ = .28) .... 153
Table 3.7: Equilibria in HM City (θ = .9; ϕ = .1) .... 173
Table 3.8: Equilibria in Gwage City (θ = .9; ϕ = .1) with 75% of Total Capital Stock concentrated at the CBD .... 174
Table 3.9: Equilibria in Gwage City (θ = .9; ϕ = .1) with 50% of Total Capital Stock concentrated at the CBD .... 175
Table 3.10: Equilibria in Gwage City (θ = .9; ϕ = .1) with 25% of Total Capital Stock concentrated at the CBD .... 176
Table 4.1: Parameter Values for Monocentric City with Heterogeneous Labor .... 197
<table>
<thead>
<tr>
<th>Table 4.2:</th>
<th>Equilibria for Monocentric Cities with Heterogeneous Labor</th>
<th>198</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 4.3:</td>
<td>Equilibria for HwageI City with 75% of Total Capital Stock at CBD</td>
<td>212</td>
</tr>
<tr>
<td>Table 4.4:</td>
<td>Equilibria for HwageI City with 50% of Total Capital Stock at CBD</td>
<td>213</td>
</tr>
<tr>
<td>Table 4.5:</td>
<td>Equilibria for HwageII City with 50% of Total Capital Stock at CBD</td>
<td>247</td>
</tr>
<tr>
<td>Table 4.6:</td>
<td>Equilibria for HwageIII City with 75% of Total Capital Stock at CBD</td>
<td>252</td>
</tr>
<tr>
<td>Table 4.7:</td>
<td>Equilibria for HwageIII City with 50% of Total Capital Stock at CBD</td>
<td>253</td>
</tr>
</tbody>
</table>
List of Figures

Figure 1: Depicting the Wage Gradient Argument (in Preface) ....xxxii

Figure 1.1: Rents in Monocentric City ....11

Figure 1.2a: Population Density in Monocentric City ....11

Figure 1.2b: Population Density Changes in Open and Closed Cities. ....13

Figure 1.3a: Comparison of Rents in Monocentric City to closed HM City with SBD 6 and 11 miles out. ....25

Figure 1.3b: Close Views of Figure 1.3a ....25

Figure 1.4a: Comparison of Rents in Monocentric City to closed HM City with SBD 11, 16 and 20 miles out. ....26

Figure 1.4b: Close Views of Figure 1.4a ....26

Figure 1.5: Comparison of Rents in Monocentric City to open HM City with SBD 6, 11, 16 and 20 miles out.(close views) ....28

Figure 1.6: Sketch of a Circular City Area with Decentralized Employment ....31

Figure 1.7: Sketch accompanying Proof of Proposition 1 (see Page 32). ....33

Figure 1.8: Changes in Land Market with Decentralization ....34

Figure 1.9a: Comparison of Rents in Monocentric City to closed HM and Ewage ....44

Figure 1.9b: Close Views of Figure 1.9a ....44

Figure 1.10a: Comparison of Rents in Monocentric City to closed HM and Ewage ....45

Figure 1.10b: Close View of Figure 1.10a ....45

Figure 1.11: Changes in Land Market in Ewage City with Employment Decentralizing Close to the CBD ....46
| Figure 1.12a: | Comparison of Rents in Monocentric City to closed HM and Ewage | 49 |
| Figure 1.12b: | Close Views of Figure 1.12a | |
| Figure 1.13a: | Comparison of Rents in Monocentric City to closed HM and Ewage | 50 |
| Figure 1.13b: | Close Views of Figure 1.13a | |
| Figure 1.14: | Changes in Land Market in Ewage City with Employment Decentralizing Far from the CBD | 51 |
| Figure 1.15: | Comparing Rents in Monocentric City and Open Ewage City | 55 |
| Figure 1.16: | Comparing Rents in Monocentric City, Open HM City and Open Ewage City | 56 |
| Figure 1.17a: | Comparing Rents in Monocentric City, Open HM City and Open Ewage City with SBD 6 and 11 miles out (Close Views near Boundaries) | 57 |
| Figure 1.17b: | Comparing Rents in Monocentric City, Open HM City and Open Ewage City with SBD 16 and 20 miles out (Close Views near Boundaries) | 57 |
| Figure 1.18: | Comparing Normalized SBD Wages in Closed HM and Ewage Cities | 61 |
| Figure 1.19: | Comparing Normalized Values of Interface Between CBD and SBD Workers in Closed HM and Ewage Cities | 62 |
| Figure 1.20: | Comparing Normalized Utility Levels in Closed HM and Ewage Cities | 64 |
| Figure 1.21: | Comparing Boundaries in Closed HM and Ewage Cities | 66 |
| Figure 1.22: | Comparing Commuting Patterns in Closed HM and Ewage Cities | 67 |
| Figure 1.23: | Comparing Normalized SBD Wages in Closed and Open HM and Ewage Cities | 68 |
Figure 1.24: Comparing Normalized Populations in Closed and Open HM and Wage Cities ....69

Figure 1.25: Comparing Commuting Patterns in Closed and Open HM and Wage Cities ....70

Figure 2.1: Comparing Rents when Employment locates outside City Boundary in Closed and Open Wage City ....74

Figure 2.2: Close Views near boundary of Figure 2.1 ....75

Figure 2.3: Comparing Rents (SBD 6 miles out) when 5000 and 10000 New Jobs Locate at CBD ....81

Figure 2.4: Comparing Rents (SBD 6 miles out) when 5000 and 10000 New Jobs are spread out equally over CBD and SBD ....82

Figure 2.5: Comparing Rents (SBD 6 miles out) when 5000 and 10000 New Jobs Locate at SBD ....83

Figure 2.6: Comparing Rents (SBD 6 miles out) under Different Modes of Decentralization when 5000 New Jobs come into the City ....84

Figure 2.7: Comparing Rents (SBD 6 miles out) under Different Modes of Decentralization when 10000 New Jobs come into the City ....85

Figure 2.8: Comparing Rents (SBD 18 miles out) when 5000 and 10000 New Jobs Locate at CBD ....86

Figure 2.9: Comparing Rents (SBD 18 miles out) when 5000 and 10000 New Jobs are spread out equally over CBD and SBD ....87

Figure 2.10: Comparing Rents (SBD 18 miles out) when 5000 and 10000 New Jobs Locate at SBD ....88

Figure 2.11: Comparing Rents (SBD 18 miles out) under Different Modes of Decentralization when 5000 New Jobs come into the City ....89
Figure 2.28: Comparing Rents in Ewage City with SBDs 16 and 20 miles out when more Jobs Decentralize ....121

Figure 2.29: Comparing Rents in Ewage City with SBDs 5 and 22 miles out when more Jobs Decentralize ....122

Figure 2.30: Comparing Rents in Ewage City with SBDs 16 and 22 miles out when more Jobs Decentralize ....123

Figure 2.31: Effects on Land Market in the Interior of Three Centered City ....126

Figure 2.32: Normalized Utility Levels in Three Centered Cities (SBDs 10 and 16) ....129

Figure 2.33: Comapring Rents in Ewage 22 Cities with Different Modes of Decentralization ....131

Figure 3.1: Comparing Rents in Monocentric Cities with Different Factor Productivities ....141

Figure 3.2: Comparing Population Densities in Monocentric Cities with Different Factor Productivities ....141

Figure 3.3: Comparing Rents and Population Densities for Gwage Cities (SBD 8 miles out) with Different Divisions of Capital ....154

Figure 3.4a: Close Views of Rent Profiles in Figure 3.3 ....155

Figure 3.4b: Close Views of Population Density Profiles in Figure 3.3 ....156

Figure 3.5: Comparing Rents and Population Densities for Gwage Cities (SBD 16 miles out) with Different Divisions of Capital ....157

Figure 3.6a: Close Views of Rent Profiles in Figure 3.5 ....158

Figure 3.6b: Close Views of Population Density Profiles in Figure 3.5 ....159
Figure 3.7: Commuting Patterns in HM and Gwage Cities (Different capital Allocations) ....160
Figure 3.8: Changes in Labor Markets when SBD forms close to CBD ....162
Figure 3.9: Relative SBD Wages for HM and Gwage Cities ....163
Figure 3.10: Changes in Land Market when SBD forms close to CBD ....165
Figure 3.11: Normalized Utility Levels for HM and Ewage Cities ....167
Figure 3.12: Changes in Labor Markets when SBD forms far from CBD ....169
Figure 3.13a: Changes in Rents in Gwage City with SBD 20 miles out ....171
Figure 3.13b: Changes in Rents in Gwage City with SBD 22 miles out ....171
Figure 3.14a: Comparing Rents in Gwage (θ = .9; φ = .1; SBD 8 miles out) Cities ....177
Figure 3.14b: Comparing Population Densities in Gwage (θ = .9; φ = .1; SBD 8 miles out) Cities ....177
Figure 3.15: Close Views of Rent Profiles in Figure 3.14 ....178
Figure 3.16: Comparing Rents in Gwage (SBD 8 miles out) Cities with Different Factor Productivities ....179
Figure 3.17a: Comparing Rents in Gwage (θ = .9; φ = .1; SBD 16 miles out) Cities ....180
Figure 3.17b: Comparing Population Densities in Gwage (θ = .9; φ = .1; SBD 16 miles out) Cities ....180
Figure 3.18: Comparing Rents in Gwage (SBD 16 miles out) Cities with Different Factor Productivities ....181
Figure 3.19: Relative SBD Wages in Gwage Cities ....182
Figure 3.20: Normalized Utility Levels in Gwage Cities ....183
Figure 3.21: SBD Labor Market in Partial and General Equilibrium Models when SBD forms close to CBD ....185
Figure 3.22: SBD Labor Market in Partial and General Equilibrium Models when SBD forms far from CBD ....186
Figure 4.1: Comparing Rents in Monocentric Hwage I, II and III Cities ....199
Figure 4.2: Comparing Population Densities in Monocentric Hwage I, II and III Cities ....199
Figure 4.3: Comparing Rents in Hwage City with Decentralized Employment (sub 6, 7, 11, 18 and 20 miles out; 50% capital at CBD) ..214
Figure 4.4: Comparing Rents in Hwage City with Decentralized Employment (sub 7 miles out) and Different Capital Allocations ..215
Figure 4.5: Comparing Rents in Hwage City with Decentralized Employment (sub 11 miles out) and Different Capital Allocations ..215
Figure 4.6: Comparing Rents in Hwage City with Decentralized Employment (sub 18 miles out) and Different Capital Allocations ..216
Figure 4.7: Comparing Rents in Hwage City with Decentralized Employment (sub 20 miles out) and Different Capital Allocations ..216
Figure 4.8a: Comparing Population Densities in Hwage City with Decentralized Employment (sub 7 miles out) and 75% of Capital at CBD ..217
Figure 4.8b: Comparing Population Densities in Hwage City with Decentralized Employment (sub 7 miles out) and 50% of Capital at CBD ..217
Figure 4.9a: Comparing Population Densities in Hwage City with Decentralized Employment (sub 11 miles out) and 75% of Capital at CBD ..218
Figure 4.9b: Comparing Population Densities in Hwage City with Decentralized Employment (sub 11 miles out) and 50% of Capital at CBD ..218
Figure 4.10a: Comparing Population Densities in Hwagel City with Decentralized Employment (sub 18 miles out) and 75% of Capital at CBD ...219

Figure 4.10b: Comparing Population Densities in Hwagel City with Decentralized Employment (sub 18 miles out) and 50% of Capital at CBD ...219

Figure 4.11a: Comparing Population Densities in Hwagel City with Decentralized Employment (sub 20 miles out) and 75% of Capital at CBD ...220

Figure 4.11b: Comparing Population Densities in Hwagel City with Decentralized Employment (sub 20 miles out) and 50% of Capital at CBD ...220

Figure 4.12a: Pattern I of Residential Location ...223

Figure 4.12b: Pattern II of Residential Location ...224

Figure 4.12c: Pattern III of Residential Location ...225

Figure 4.12d: Pattern IVa of Residential Location ...226

Figure 4.12e: Pattern IVb of Residential Location ...227

Figure 4.13: Comparing Population Densities in Hwagel City with Decentralized Employment (sub 6 miles out) and 50% of Capital at CBD ...224

Figure 4.14a: Relative SBD Wages for High Skilled Workers in Hwagel city ...229

Figure 4.14b: Relative SBD Wages for Low Skilled Workers in Hwagel city ...229

Figure 4.15a: Comparing Ratio of Wages at CBD and SBD ...230

Figure 4.15b: Comparing Ratio of Utility Levels at CBD and SBD ...230

Figure 4.16a: Normalized Utility Levels for High Skilled Workers in Hwagel city ...231

Figure 4.16b: Normalized Utility Levels for Low Skilled Workers in Hwagel city ...231

Figure 4.17: Changes in Labor Markets when SBD forms close to CBD ...233

Figure 4.18: Changes in Labor Markets when SBD forms far from CBD ...237
Figure 4.19a: Changes in Land Market when SBD forms close to CBD (Pattern I) ... 239

Figure 4.19b: Changes in Land Market when SBD forms close to CBD (Pattern III) ... 240

Figure 4.19c: Changes in Land Market when SBD forms far from CBD ... 241

Figure 4.20a: Comparing Rents in Hwagel and II Cities with Decentralized Employment (sub 7 miles out) and 50% of Capital at CBD ... 248

Figure 4.20b: Comparing Population Densities in Hwagel and II Cities with Decentralized Employment (sub 7 miles out) and 50% of Capital at CBD ... 248

Figure 4.21a: Comparing Rents in Hwagel and II Cities with Decentralized Employment (sub 12 miles out) and 50% of Capital at CBD ... 249

Figure 4.21b: Comparing Population Densities in Hwagel and II Cities with Decentralized Employment (sub 12 miles out) and 50% of Capital at CBD ... 249

Figure 4.22a: Comparing Rents in Hwagel and II Cities with Decentralized Employment (sub 18 miles out) and 50% of Capital at CBD ... 250

Figure 4.22b: Comparing Population Densities in Hwagel and II Cities with Decentralized Employment (sub 18 miles out) and 50% of Capital at CBD ... 250

Figure 4.23a: Comparing Rents in Hwagel and III Cities with Decentralized Employment (sub 7 miles out) and 50% of Capital at CBD ... 254

Figure 4.23b: Comparing Population Densities in Hwagel and III Cities with Decentralized Employment (sub 7 miles out) and 50% of Capital at CBD ... 254

Figure 4.24a: Comparing Rents in Hwagel and II Cities with Decentralized Employment (sub 12 miles out) and 50% of Capital at CBD ... 255
Figure 4.24b: Comparing Population Densities in HwageI and II Cities with Decentralized Employment (sub 12 miles out) and 50% of Capital at CBD

...255

Figure 4.25a: Comparing Rents in HwageI and III Cities with Decentralized Employment (sub 18 miles out) and 50% of Capital at CBD

...256

Figure 4.25b: Comparing Population Densities in HwageI and III Cities with Decentralized Employment (sub 18 miles out) and 50% of Capital at CBD

...256

Figure 4.26a: Relative SBD Wages for High Skilled Workers in HwagIII City

...257

Figure 4.26b: Relative SBD Wages for Low Skilled Workers in HwagIII City

...257

Figure 4.27a: Comparing Ratio of Wages at CBD and SBD in HwagII City

...258

Figure 4.27b: Comparing Ratio of Utility Levels at CBD and SBD in HwagIII City

...258

Figure 4.28a: Normalized Utility Levels for High Skilled Workers in HwagIII city

...259

Figure 4.28b: Normalized Utility Levels for Low Skilled Workers in HwagIII city

...259
Preface

I. A STUDY OF DECENTRALIZATION IN URBAN AREAS

Urban theory views the city as a marketplace where people engage in the production and export of one or more goods. Early theory attempted to explain the clustering of employment at certain points in the city. One reason advanced was the presence of export nodes through which export goods are shipped. Firms locate around these nodes to minimize freight transportation costs. By assumption, only one such node exists, at the exact center of the city, in the first instance. This prompts all firms to cluster there, forming the Central Business District (henceforth CBD). Workers at the CBD then choose where to live. More recently, the tone of research has changed. Over the last half 50 years or more, a large number of US cities have experienced outward movements by both firms and households, not necessarily in that order. The forces of decentralization at work in the American urban landscape has prompted the emergence of a new genre of urban research, which attempts to answer two basic questions: (1) why do firms decentralize? (2) why do households decentralize? The two questions are sometimes posed in conjunction. When firms decentralize, do households follow and vice versa?

Mathematical models of urban spatial structure are general equilibrium models of the land, housing, capital and labor markets. Earlier models study mainly the land market, using only a rudimentary labor market with the wage rate held exogenous and all employment located at the center. In addition, either the total demand for labor is assumed to be completely elastic (the open model) or the total labor supply to the urban area is assumed to be fixed (a closed model). Solow (1973) first introduces a model with a full labor market. One of the insights gained after Muth (1967) incorporated the housing market is that building structures on plots of land does not alter the general pattern of rents: both a land rent gradient and a housing rent gradient would decrease with distance from the center. In recent years, a number of researchers have used the simplifying assumption of having the households consume land directly. I use the latter approach.

In the monocentric model, where all employment is located at the CBD, workers have the option of living close to the CBD, commuting small distances but paying higher
rents; or living in the far suburbs -- incurring higher commuting costs but enjoying cheaper land. The interaction of these two forces causes the monocentric city to feature a downward sloping rent gradient and population density gradient as one moves away from the CBD. This is a direct consequence of the fact that to reach equilibrium, people must enjoy the same level of utility, regardless of where they live in the city. These models are analytically tractable and comparative static exercises on both the closed and open city frameworks can be easily carried out.

In this setup, unless something changes, firms have no incentive to locate away from the center. Which brings us back to the first question: why do firms decentralize? Earlier theory attributes this to the formation of a suburban export node, which gives some firms the opportunity to locate in the suburbs, in search of more land etc. In modern cities, a number of incentives may occur. Firms may possess different technologies, formation of faster freight transportation routes may link the suburbs to export outlets, fiscal incentives geared towards 'developing' the suburbs may emerge, so on and so forth. Some theorists have suggested a link between the first and second questions -- namely, that household decentralization occurs due to 'flight from blight' in inner cities, and that employment decentralization follows -- service sector employment decentralizes when the demand for them grows in the suburbs. This last effect may reinforce an already ongoing process of decentralization, but research has not proved it to be an important underlying factor. The effects of employment decentralization on households is likely to be an important question to probe, in and of itself. In my research, I concentrate on analyzing this latter question, for a number of reasons.

First, most treatments of why firms move out from the CBD suffer from the drawback that there is not much meaningful discussion of why employment clusters form at the CBD, let alone at the Suburban Business District (henceforth SBD), in a modern metropolitan area. While urban theory tends to dwell on manufacturing industries and export nodes, the modern city with its office sector, shopping facilities, freeway systems and automobile transportation does not always fall within the purview of such a scenario. Subcenters often form at the juncture of major freeways or around beltways and rarely next to the airport at the outskirts of the city, making accessibility more of a consideration than closeness to the suburban export node.
Second, even acknowledging the importance of freight transportation and export nodes, standard theories do not add much to a general equilibrium setting. Consider the commonest explanation for suburbanization of employment, the construction of a suburban export node. The location of such a node is exogenous to the models. Since profit maximizing suburban firms will want to locate at the suburban node under most circumstances, the location of the SBD itself becomes mostly exogenous to the analysis. This produces an outcome where firms are clustered next to the export nodes at the center and the suburb. Then the question of what happens to firm rent bids is not as interesting as what happens to rent patterns for residents, since the overall rent profile of the city is determined principally by the (changing) rent profile of the residential sector. This is certainly the case when firms are more centralized than residences and we eschew the improbable solution where firms and households coexist everywhere in the city (the so-called 'integrated' solution). Hence it is permissible to assume that firms occupy a point at the center and a narrow belt at the suburb.

Third, the explanation that subcenters form due to the clustering of service sector industries and shopping facilities is similarly not adequate. This is a step in the natural evolution theory of suburbanization: when households move to the suburbs, some of the industries which provide services to them follow. Similar rationale (service oriented industry following already suburbanized communities) is used for explaining the formation of SBDs under the "flight form blight" class of theories. Besides the fact that this is not an extremely attractive motivation for employment suburbanization, it is also different from the reality of modern cities, where suburbs are no longer automatically stratified by income levels in the absence of fiscal constraints like zoning. Also, employment subcenters do not consist primarily of shopping malls; indeed, the modern perception is that offices have tended to stay in the center and manufacturing has tended to move to the suburbs (principally due to the increased ease of freight transportation). Movements by service firms may bolster the SBD cluster, but they are not the principal cause of the formation of the SBD. While homogeneously rich suburbs may have been a feature of the monocentric cities, for reasons like the two mentioned above, there has been increased outward movement by the 'blue-collars' in recent years. All in all, it is much more interesting, and realistic, to first consider the effect suburbanized employment has on household
(re)location. This would produce more economically integrated\textsuperscript{1} suburbs in the absence of fiscal restrictions.

Finally, most general equilibrium models of urban areas are difficult to solve, which is why numerical analysis has become a powerful vehicle of study in this field. The major problems arise in finding the boundaries between different kinds of firms and households. Under these circumstances, allowing that firm rent bids affect the overall rent profile of the city in very limited locales, it is useful to ignore the inconvenience and analyze household rent bids in a more tractable framework.

From the points discussed above, I wish to draw a couple of conclusions. First, the issue of why subcenters form is still decidedly murky. The explanation I offer in Chapters 3 and 4 of my thesis, that of capital movement towards the suburbs, is a realistic possibility, as is the possibility of the formation of a suburban export node (e.g. an airport). Second, I do not dwell on the issue of why or where capital moves or where the suburban node forms, instead treating it is as an exogenous impetus, since it is not important to sacrifice tractability (especially in the more complex version of the model with heterogeneous households) for uncertain gains. Once again, it is more fruitful to study how households would react if a certain amount of capital were to shift a certain exogenously determined distance away (this is also realistic since some of the capital moving to the suburbs might be social capital, or may move out there as a direct result of fiscal incentives from city councils and the like). For subsequent research, the models I use can be modified to include firm bid rent functions.

In an attempt to investigate the effects of employment decentralization on households in a partial equilibrium setting (without a full labor market), early urban theorists have used the so-called 'wage gradient' approach. When firms move out of the CBD, wages and rents in the urban area could be expected to adjust. In the wage-gradient approach, all the adjustment is expected to be in wages, leaving all workers indifferent between the first and the second equilibria, and the rents in the decentralized city identical to those in the monocentric city! White (1976) categorized the saving in wage bills to

\textsuperscript{1} Unlike the standard definition of 'integration' in urban economics, viz. the co-existence of firms and residences everywhere, I use the expression in the more commonly used sense - the co-existence of different economic classes and/or ethnic groups of households in the same region of the city.
subcenter firms as an additional incentive to move out of the CBD. I found this argument to be very counterintuitive. Cities with decentralized employment and a reasonable transportation system, Houston being a prime example, often feature rent gradients which are very flat compared to the predictions of the monocentric city model.

One easy criticism of the wage gradient model is that it lacks a full labor market, and hence is incapable of investigating how the land market and labor market interact with one another. Apart from White (1976), no one has really attempted to deconstruct the wage gradient argument and come up with an alternate theory of interaction between the two markets. Since Solow (1972), urban theorists have been concerned with the solution and analysis of general equilibrium models which incorporated at least the land, labor and capital markets. In the process, it has been discovered that due to the complexities involved, obtaining closed form solutions were not viable for much beyond the early monocentric structure. Consequently, most recent research on this topic has used numerical analysis. Some researchers continue attempts to find closed form solutions to general equilibrium systems, but such equilibria are established only after defining boundary conditions rather rigidly. Even under these conditions, not much beyond the signs of results from basic comparative static experiments like increasing the speed of commute, increasing income etc. can be established.

In my research, I use numerical analysis for three reasons. First, the experiment I concentrate on, finding the effects on rents in the city as suburban employment locates further and further out, has not been attempted before. The changing boundaries between groups who work at the center and those who work at the subcenter make it very difficult to identify changes, especially in a closed city -- where with a fixed population, utility level changes when employment decentralization shifts the distribution of population. Second, to check on the validity of the wage gradient conjecture, I need an approach which quantifies results if and when any changes occur. Third, in the last chapter of my thesis, I introduce heterogeneity in the workforce, necessitating more boundaries between resident groups in the city and increasing the complexity. On all counts, numerical analysis serves the purpose admirably. I build on a basic structure introduced by White and Hotchkiss (1993).
I am primarily interested in investigating the effect of employment decentralization on household location by studying the interactions between the labor and land markets in isolation. Not only is this question important independent of the issue of why firms decentralize, but the theory I wanted to investigate (the wage gradient) is aligned directly to it. It is clear to me that regardless of everything else going on in the city, the interactions between the labor and land markets is not well understood. The wage gradient idea is still in vogue. Currently, general equilibrium models, which allow CBD wages to vary, nevertheless allow SBD wages to differ from them by the amount of the round trip commuting cost. In fact, as I mentioned before, the saving in wage bills are counted as an added inducement for firms to decentralize. To me, this proves that either the wage gradient was correct -- which seems strange -- or that researchers are concentrating so hard on complete general equilibrium structures which yield simultaneous answers to the two basic questions that they somehow assume the veracity of the wage gradient approach.

In my thesis, I look at both closed and open cities. In the closed city, the total population is held fixed, so any changes in the parameters are reflected by changes in the equilibrium level of utility. In the open city, both in and out migrations are allowed costlessly, so that any changes in parameters end up changing the city's population but leaving the utility level unchanged. The difference is philosophical. The closed city approach assumes that any changes taking place in a particular city are being replicated by similar cities in the region. The open city assumes that either the city has a large hinterland, or that only one city among many experiences changes.

In the Chapter 1 of my thesis, I first set up a partial equilibrium model along the lines of Hamilton and Mills, two prominent theoreticians whose mention of the wage gradient theory started me thinking. Wages at the center are fixed, the location of the subcenter is exogenous and wages there determined accordingly. I find a mistake in the structure of the wage gradient, so that subcenter workers are in fact not indifferent but slightly overcompensated when paid according to the wage gradient. This causes a flattening of the rent gradient in the closed city. The mistake is a quantitative effect which increases initially with the subcenter locating further out, peaks and then starts going down. However, this does not disprove the wage gradient. The city structure is altered little. All people inside the SBD loop still commute to the center, and all people outside commute to the SBD. The spirit of the wage gradient is intact.
In the second part of Chapter 1, I note that with the HM approach, the labor demand curve is perfectly elastic everywhere in the city. This raises the troubling notion that SBD firms cannot control much. Their wage offer is determined exogenously. If they need to hire a certain number of workers, they must pick the one spot in the city where -- after decentralization -- exactly the number of workers needed live outside the SBD loop. But since the location of the subcenter is exogenous (close to the location of an airport, say) even this is not a choice variable. I go to the other extreme and assume a perfectly vertical labor demand curve in the SBD market. No matter where they locate, the SBD firms demand a fixed portion of the labor, and adjust prices accordingly. Now I find significant changes in structure when employment decentralization occurs far from the CBD. The relative SBD wages do not fall as much as the wage gradient predicts. As employment locates further away, the relative SBD wage curve stays very flat. Utility levels, on the other hand, rise prodigiously as employment located far from the CBD. In addition, rents by suburban households peak around the subcenter, signifying reverse commuting -- a complete break from the monocentric city structure. However, when employment locates close to the CBD (not too close, either), the changes in the city are in fact smaller than in the previous case and the wage gradient approach seems to work well, since the utility changes were truly small.

In Chapter 2, I probe a number of special cases of decentralization. First, I look at employment decentralizing to a point outside the initial city boundary. Under these circumstances, when the SBD labor demand curve is vertical, a "hole" is created in the city as the SBD workers break off from the main city. Next, I move away from the closed and open city approaches momentarily, assuming instead that a fixed number of specialized jobs move into the city. The previous employees move in with the job. They can live where they want, but must work at the site where the new jobs locate. This is a 'semi-closed' city. Lastly, I consider the effects of employment moving out to two subcenters. In general, the two later models show that when proportionately more employment decentralizes, every resident is unambiguously better off in the closed city.

In my third chapter, I present a partial general equilibrium model with a full labor market, with the express intention of examining the relations between the land and the labor markets. Here I introduce capital as a complimentary factor of production. The amount of capital in the urban area is given exogenously. All capital is concentrated at the CBD in the
monocentric city. The experiment I carry out here is an extension of the previous one. Employment subcenters form due to the movement of some capital out of the center and to some other point in the city. I find the effects on the land market when different amounts of capital move to different locations in the city. There is a downward sloping labor demand curve. With wages flexible everywhere, the equilibrium wages are determined by the labor market. Now the wage gradient approach really breaks down. With the subcenter close to the CBD, relative SBD wages adjust downwards as employment moved out, more so than the wage gradient suggests. When employment locates in the far suburbs and a sufficient amount of capital moves there, the SBD wages hold at the same level as the suburb moves out. Instead of the wage gradient, an upward sloping utility gradient appears everywhere in the city as the subcenter moves out. This is the final refutation of the wage gradient approach.

The general equilibrium model proves categorically what the previous incomplete structure of the labor model had only hinted at, and what is completely the opposite of the wage gradient psychology. As more jobs move further out, everyone is better off in the closed city. Rents and population density flatten out, no part of the city is particularly congested, and everyone is better off. If the city were open, people would migrate in, eating away at the utility. Either way, the CBD loses its prime importance when sufficient employment moves far enough out. Rents peak at the subcenter and the monocentric city commuting patterns are reversed.

There is a catch. The idea of capital movements is new, but a full capital market is absent. There are obviously bounds to more capital moving further away from the center. This, however, is not difficult to add. Capital tends to relocate when there are different rates of return at different locations -- the movements cease when the returns were equalized. There are a myriad of reasons why such differentials could exist, differences in production technology, fiscally imposed differences, so on and so forth. Regardless of where and when the out movement ceases, the effect would be a less dense, better off populace in the closed city.

In Chapter 4, I introduce a heterogeneous labor force where people possess high and low levels of skill. The general trends of this exercise are in line with Chapter 3, i.e. upward sloping utility gradients exist for both skill levels when employment moves further
out. The most interesting issue here is to see whether the location of employment subcenters has any effect on the creation of segregation in residential patterns. I assume that the city is segregated when large blocks of people of the same skill level living without much contact with the other kind. And I do find such a connection. The low skilled workers are tied to their workplace, be it the center or the subcenter. Therefore, when employment suburbanizes close to the CBD, the low skilled groups at the center and the subcenter are located almost where they were in the monocentric city, packed in high density neighborhoods close to the CBD. The high skilled workers are all located in low density suburbs. This very segregated profile changes only when enough employment moves far enough out, drawing with it the low skilled workers at the subcenter, creating gaps between the two low skilled groups which are then filled in with high skilled workers. All in all, five patterns of residential location emerge.

I have a significant number of future research agendas which can be pursued within this framework. I obtain the following original insights out of this work:

(a) The innovation of using capital movements as a cause for employment decentralization provides a setup which can be easily completed and used for further research. This leads to the next two items.

(b) I discover true problems with the wage gradient analysis, and why it finally fails.

(c) By setting up the experiment with employment moving further out, I can isolate and observe the presence of the utility gradient, which underlines the common sense proposition that in a closed city, more decentralization is good for everybody.

(d) I find out the effects of employment decentralization on residential segregation, or lack thereof.

(e) I also discover a horde of other insights, e.g. the effects of employment decentralizing to more than one employment center, the effect of new jobs and workers moving into the city, why 'hole' may exist in a city with decentralized employment etc.

(f) The computational techniques I use can be easily adapted to tackle further extensions of the model.
II. THE HAMILTON MILLS CONJECTURE: UNALTERED PATTERNS OF HOUSEHOLD RENTS AFTER EMPLOYMENT DECENTRALIZATION?

The analytical argument presented by Hamilton and Mills (hereafter HM) is reproduced here in its essentials. This is a well-stated case for the standard assumptions under which employment decentralization in an urban area leaves the rent structure and commuting pattern of households completely undisturbed. HM consider a uniformly circular city with its Central Business District (henceforth CBD) at the center of the circle. A firm which locates away from the CBD will offer its workers a lower wage than it would if the firm were at the center. Workers do not have to commute as far to get to a suburban office as they would have to get to the CBD job site; so they accept a lower wage. HM assume that commuting costs include both the time (opportunity) cost of commuting, as well as the material cost (per mile) of operating an automobile. In Figure I below, if a firm is to locate at S, 5 miles from the CBD, consider the wage it offers with respect to the CBD wage. If the city boundary is at B, anyone living on the segment SB would have to commute 10 fewer miles each day (considering round-trip commutes) if they work at S rather than at the CBD. If the CBD wage is w, the time spent in commuting one mile is t and the fixed material cost per mile is m, then a firm locating at S can offer a wage up to $10(wt + m) less than the CBD wage. This essentially signifies that in addition to the rent gradient, the city with decentralized employment possesses a wage gradient, whereby wages decrease by $2(wt + m) for every mile a business moves away from the center.

![Figure I](image-url)
Consider the choices available to workers who live along the ray from the CBD to B. Anyone living closer than 5 miles to the CBD will not want to work at S, since the saving in commuting cost is less than $10(wt + m), which makes them unwilling to accept the lower sub center wage. Therefore, the rent decrease gradient is unchanged over the segment from the CBD to S. Workers living along the segment SB will be indifferent between working at the CBD and working at S. Their lower wages are exactly offset by the saving in commuting costs. As a result, the rent gradient remains unaltered even here after employment decentralizes. Workers still commute inwards every morning, some to the CBD and some only as far as S. To quote HM: "The fact that the same location equilibrium condition holds in a world with decentralized employment leads to a somewhat counterintuitive result: the shape of the rent function is completely unaffected by the decentralization of employment." It might seem that the city's rent function would get progressively flatter as employment becomes more decentralized; that is, as the dominance of the CBD was eroded. However, that is wrong......

There are several features in the above set up which warrant attention. First, the HM scenario is a partial equilibrium analysis of employment decentralization. Wage offers at the center never change in response to decentralization of employment. The question of why employment decentralizes and how the suburban business location is chosen is ignored. At the same time, lower wage bills provide an extra incentive for firms to move out. Second, HM make the implicit assumption that the demand for labor is perfectly elastic throughout the urban area. This eschews the question of circumstances where labor scarcity may cause a suburban firm to re-negotiate with its employees and offer them higher wages than dictated by the wage gradient, which is the cornerstone of the HM analysis.

III. PARTIAL AND GENERAL EQUILIBRIUM MODELS OF URBAN SPATIAL STRUCTURE

Early comparative static analysis of urban areas have usually analyzed the land market, without the addition of a full labor market. Wheaton (1974) performs comparative statics on both closed and open partial equilibrium models, using a general utility function with two arguments, a composite good and land. Brueckner (1987) extends this analysis

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2 HM's italics
to consider housing produced by land and capital. Sasaki (1987) extends it to consider the redistribution of rents, which makes income endogenous, even though wages are still exogenous.

Solow (1973) first introduces a model with a full labor market. The model maintains the standard assumption of employment clusters being driven by export nodes. Solow solves the model for the "Santa Claus" case in which land and labor are used in equal proportions. Models with more realistic production functions are outlined, but not explicitly solved.

Moving from the monocentric city to the next stage, several authors analyze urban models with suburban employment, without a full labor market -- White (1976, 1978), Wicand (1987) and Yinger (1992, 1993) among them. These models are analytically tractable. White (1976) provides the most complete partial equilibrium analysis of the wage gradient approach to date. The analysis has two separate parts. In the first part, a general equilibrium structure is used to analyze why firms decentralize. In addition to the standard explanation of a suburban export node used by Solow, White mentions possible wage savings as an additional incentive for firms to decentralize. Other means by which employment may decentralize -- e.g. differences in technology -- are also discussed.

In the second part, focusing on how employment decentralization affects households, White switches to a partial equilibrium analysis, with wages fixed at the center. One of the first persons to simplify the land market by assuming that firms locate at a point in the center and are spread evenly around a beltway in the suburb, White also assumes a vertical labor supply curve for SBD firms. White does not identify the mistake in the definition of the wage gradient. Consequently, when employment decentralizes close to the center (as long as the number of people living outside the loop are greater than or equal to the number of workers demanded by SBD firms) in this model, there are no changes in the city structure, as SBD workers are indifferent between working at either employment location. When employment decentralizes to the far suburbs, however, White finds results similar to my findings in the latter part of Chapter 1. The SBD firms face labor scarcity, and must raise their wage offers. This pulls people outwards, dropping rents near the center, raising rents in the suburbs and raising the overall level of utility in the closed city. This analytical framework is not conducive to carrying on the experiment
of tracking the changes in the city as employment decentralizes further and further out -- hence White does not find the utility gradient over some part of the urban area. Also, without a full labor market, the wage gradient cannot be dispelled.

White (1978) analyzes a model with racial discrimination, one of the few analytical attempts to study residential segregation. The approach is still partial equilibrium and wage differentials between white and black workers are arbitrarily established. In Chapter 4 of my thesis, I investigate a similar issue in a partial general equilibrium setup, with workers differing in productivity and all changes in the labor and land market endogenous.

Hotchkiss and White (1992) present a simulation partial equilibrium model with two discrete employment subcenters, two member families and the possibility of one or both of them working. Wages at all employment locations for both genders are posited exogenously. This model introduces a technique for solving for endogenous utility level in a closed city, which I modify and use in my thesis.

Since urban models following the Solow outline become analytically intractable, simulation has become the preferred tool for urban theoreticians analyzing models with a full labor market. Straszheim (1984) presents an open city model with a Cobb-Douglas production function featuring a Hicks neutral shift factor. Sullivan (1983) develops a more elaborate simulation model with a full labor market, suburban employment, and a variety of other features. Sullivan (1986) extends the analysis to model a small, open city with endogenous CBD wages. The production functions feature shift factors -- agglomerative economies which influence firm rent offers. Differences in agglomeration economies influences outward movement by firms. The primary goal of Sullivan is to study the effects of zoning regulations. Though these models use a full labor market and analyze the location of the subcenter endogenously, they incorporate the wage gradient when analyzing how the labor and land markets are linked together. Also, the increases in utility as employment decentralizes cannot be seen without analyzing closed city models.

A few recent papers have attempted to devise strategies for obtaining closed form solutions to urban models with full labor markets. Sasaki and Katayama (1990) analyze a closed model with a linearly homogeneous production function with land and a fixed labor
supply as inputs. They perform comparative static analysis of changes in the urban transportation system.

Except the Sasaki model, all the other models use absentee landlords. In other words, the land rents disappear from the system. Fujita (1989) develops an open model with city land ownership, where rents are distributed equally among city residents. Fujita shows that no differences in land use patterns emerge as a result of this extension. Models of individual land ownership have not been attempted. In the circumstances, I use the absentee landlord approach in my thesis.

Ross and Yinger (1994) attempt the most comprehensive analytical framework of a city with decentralized employment and a full labor market. The model of land, labor, housing and capital markets analyzes an open city with absentee landlords and both discrete and beltway locations of the SBD. However, this model continues to assume the presence of a wage gradient linking the CBD and SBD wages. Ross and Yinger find a subpeak in household rent offers at the SBD in spite of the wage gradient. This results from an assumption made in the paper to the effect that both CBD and SBD workers must live in the space between the two employment centers. This places an additional constraint on the land market, with fixed land supplies being allotted to the two groups of workers. This creates unwanted complications, e.g. it does not allow for cases where the CBD and SBD are close together, and only the CBD workers live between the two centers.

All things considered, the fully analytical general equilibrium models have not been able to go beyond the framework of the changes in the usual parameters. For the experiments I undertake, these tools are inadequate, unless stringent assumptions along the lines of Yinger are made. I choose to use simulation models instead. Also, the inclusion of the wage gradient in all these models underlines the need to explore the interaction between the land and the labor markets.
CHAPTER 1: A STUDY OF PARTIAL EQUILIBRIUM MODELS OF CITY STRUCTURES UNDER VARIOUS MODES OF EMPLOYMENT DECENTRALIZATION

In this chapter, I analyze the validity of the Hamilton-Mills (henceforth HM) conjecture by developing a model similar to the one hinted at by them and partially analyzed by White (1976). To make the setup comparable to them, the model lacks a full labor market (I incorporate this in the next chapter). I point out various points of departure from the HM analysis. Most of the material was independently developed by me when analyzing the effects of decentralization on city structure. I provide a complete analysis of the issues raised by HM and general urban theory vis a vis the importance of the wage gradient and its role in maintaining the residential rent profile of the monocentric city even when a Suburban Business District (henceforth SBD) is formed.

Initially, I am interested in seeing how the city develops around downtown. As people are settling down, I want to get a sense of what drives the rent patterns over the metropolitan area. To this end, I solve the consumers' utility maximization problem. In the HM case, the wage offer at the Central Business District (henceforth CBD) is given exogenously. The labor demand curve is assumed to be infinitely elastic throughout the urban area, so at any wage offer at the center and the suburb, the labor supply forthcoming can be absorbed in production. Hence the HM model looks only at the labor supply (i.e. the land consumption equilibrium) in the urban area, given exogenous wage offers at the center and the suburb(s).

1.1. THE MONOCENTRIC CITY

As I mentioned in the literature review section, the formulation of urban model(s) in general terms is not conducive to advanced comparative static analysis. Straszheim (1984) writes: "The complexity.....is such that a closed form solution is not possible. Conducting the usual comparative statics is generally uninstructive since the resultant expressions in the differentials of interest are themselves complex nonlinear differential equations which are not readily solved. (Instead) the solution is examined..by numerical methods, using representative values of the parameters...." I follow the approach used by Hotchkiss and
White (1993) among others and assume specific forms of utility (and later production) functions, commuting cost functions, so on and so forth. At a relatively moderate computer cost, these models permit numerical analysis of a large variety of circumstances dealing with the specifics of production, preferences, in and out migration, forms of decentralization, local government controls etc. I share the opinion of the pioneers in this field regarding the utility of such techniques in analyzing an otherwise intractable system, and feel that there is a net gain imparted by the added flexibility, without which very limited studies of equilibria would be possible.

Assumptions

(i) The urban area has no special topographical features at the outset, save some feature that encourages the formulation of a CBD. This could be any of a number of things: an export node, a high concentration of capital at the center of the city, so on and so forth. I look at a circular shaped possible metropolitan area. Hence the CBD is assumed to be in place at (0,0) initially. As discussed before, the CBD is assumed to occupy a point or a tight cluster and not cause significant distortions in the rent profile of the city. All firms in the urban area are assumed to be homogeneous in all respects.

(ii) All distances are measured radially in miles. To simulate the city structure looked at by HM, commuters are assumed to travel along ubiquitous radial roads.

(iii) There is only one type of household occupying the urban area. Each household consists of one person and she works at the one CBD, earning a common wage. Following White's (1992) formulation, households are given to possess Cobb-Douglas utility functions and assumed to spend 8 hours per day in work and commute. Leisure is not explicitly included in the utility function.\(^1\) In the final equilibrium, all households must have the same utility, regardless of their location in the city.\(^2\)

(iv) Land is taken as a proxy for housing. Households are assumed to demand land rather than housing services.\(^3\) Rent bids are in dollars/unit of land (e.g. $/square mile). Rents on land are collected by absentee landlords, and each plot of land is sold to the highest bidder.

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\(^{1}\) The inclusion of leisure does not reverse the general trend of any of the results in the analyses.

\(^{2}\) The people who commute more have lower net incomes, but they face lower rents in the suburbs. This relationship between higher commuting costs (lower net income) and lower rents as one moves away from the center creates the rent gradient.

\(^{3}\) I am ignoring the distinction between floor space and yard space, i.e. how much housing capital is invested on a plot of land, and how the rent payment in one's budget is split between land rents and cost of
(v) Each person's cost of commuting to work includes:

(a) a time cost of commuting: \(2. w. t(i, j)\), where commute time is valued at the lost wage;

(b) a monetary cost of commuting: \(2. m. d(i, j)\), where \(m\) is the per mile material cost of commuting. Distances of commute (measured in miles along the radial roads) and the time of commute from a point \((i, j)\) to the center \(^4\) are given by \(d(i, j) = \sqrt{i^2 + j^2}\) and \(t(i, j) = \frac{1}{f} d(i, j)\) respectively, \(^5\) where \(w\) is the (fixed) wage offer at the center and \(f\) is the average speed of commute in miles/hour.

(vi) The HM conjecture does not really specify whether the urban area has a fixed population before and after suburbanization of employment. White uses a city with a fixed population (i.e. a 'closed' city) for her analysis. I analyze my results for the closed city as well as the city with costless in and out-migration (an 'open' city). The definition of closed and open is presented later. In Chapter 2, I consider a case where there is limited migration, with a fixed number of people entering or leaving the city.

1.1a. THE MODEL

Each household's problem is to maximize

\[ U = g(d)^\alpha. L(d)^\beta \]

with respect to \(g(i, j)\) and \(L(i, j)\), subject to the budget constraint

\[ 8w = g(i, j) + R(i, j). L(i, j) + 2w(i, j) + 2md(i, j) \]

\[ \ldots (1.1) \]

where \(g(i, j)\) and \(L(i, j)\) are respectively the consumption of the numeraire production good and the amount of land demanded at location \((i, j)\), \(R(i, j)\) is the rent per unit of land at \((i, j)\) \(^6\) and \(d(i, j)\) and \(t(i, j)\) are defined as before.

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\(^4\) I lay out the initial model of the city in terms of discrete distances in co-ordinate space, finding this to be a convenient format in which to lay out a city, especially for later inclusion of taxi cab modes of commuting. As I show later, my simulation models use continuous radial distances and density functions.

\(^5\) \(w\) is taken to be the opportunity cost of any time not spent in working, i.e. time of commute.

\(^6\) The unit in which rents are measured in this setup is a little complicated. If the rent offer at distance \(d\) form the center is \(S_y\), each person who lives at a distance \(d\) is not obligated to pay \(S_y\). The total land available (around a thin circumferential strip) at a distance \(d\) from the center is \(2\pi d\). If a person were to buy the whole land, she would have to pay \(S_y\). As it stands, the land demand of one individual is only a
From the first order conditions of utility maximization, the demands for $g$ and $L$ are

\[ g(i,j) = \alpha \{8w - 2wt(i,j) - 2md(i,j)\} \]  

\[ L(i,j) = \left( \frac{\beta}{R(i,j)} \right) \{8w - 2wt(i,j) - 2md(i,j)\} \]  

\[ \ldots (1.2a) \]

\[ \ldots (1.2b) \]

I use (1.2a) and (1.2b) to find the indirect utility function

\[ V = \left( \frac{\alpha^\alpha \beta^\beta}{R(i,j)^\beta} \right) \{8w - 2wt(i,j) - 2md(i,j)\} \]  

\[ \ldots (1.3) \]

For maintaining equilibrium, the households in the city must get the same amount of utility from locating anywhere in the city, i.e. $V$ must be the same at all $(i,j)$. I solve the general equilibrium system simultaneously and find $V^*$, the equilibrium level of utility. Given $V^*$, I find $R(i,j)$ by the following modification of (1.3):

\[ R(i,j) = \left( \frac{\alpha^\alpha \beta^\beta}{V^*} \right)^\frac{1}{\beta} \{8w - 2wt(i,j) - 2md(i,j)\}^\frac{1}{\beta} \]  

\[ \ldots (1.4a) \]

Using the time of commute function, I rewrite the rent gradient as

\[ R(i,j) = \left( \frac{\alpha^\alpha \beta^\beta}{V^*} \right)^\frac{1}{\beta} \{8w - 2w\left(\frac{1}{f}\right) d(i,j) - 2md(i,j)\}^\frac{\alpha}{\beta} \]  

\[ \ldots (1.4b) \]

From (1.4b), it is evident that rent offers decline with increasing distance from center. The city boundary is reached when the rent at a particular distance equals $R_a$, the agricultural rent. If $b$ be the distance of the city boundary from the center, the boundary can be found where rents equal $R_a$:

\[ R_a = \left( \frac{\alpha^\alpha \beta^\beta}{V^*} \right)^\frac{1}{\beta} \{8w - 2w\left(\frac{1}{f}\right) b - 2mb\}^\frac{1}{\beta} \]  

\[ \ldots (1.5) \]

small fraction of the total land. If $n$ people live at distance $d$, each of them pay $(y/n)$ in rent and consume $(2.pl.d/n)$ units of land.
Households bid for land at every location in the city. Once the rent offers are determined, I find the population density per square mile at each location \( n(i,j) = 1/L(i,j) \).\(^7\) This is the residential density per unit amount of land (e.g. per acre or per square mile). Using (1.2b) and the definition of \( L(i,j) \),

\[
\frac{R(i,j)}{\beta} \left[ 8w - 2w \left( \frac{1}{f} \right) d(i,j) - 2md(i,j) \right]^{\frac{1}{\beta}}
\]

and I can substitute the value of \( R(i,j) \) for every location to get

\[
\frac{\alpha}{V^*} \left[ 8w - 2w \left( \frac{1}{f} \right) d(i,j) - 2md(i,j) \right]^{\frac{\alpha}{\beta}}
\]

.....(1.6b)

I find the total population of the urban area (where the urban area is again defined for all \( R(i,j) \geq R_a \)) by summing the population at each point inside city limits:

\[
N = \sum_i \sum_j n(i,j) = \sum_i \sum_j \left[ \frac{\alpha}{V^*} \left[ 8w - 2w \left( \frac{1}{f} \right) d(i,j) - 2md(i,j) \right]^{\frac{\alpha}{\beta}} \right]
\]

or

\[
N = \frac{\alpha}{V^*} \left[ 8w - 2w \left( \frac{1}{f} \right) d(i,j) - 2md(i,j) \right]^{\frac{\alpha}{\beta}} \quad \text{for all } R(i,j) \geq R_a \cdots (1.7)
\]

From equation (1.7), I can derive the equilibrium utility level for the city as a function of the total population of the city and the other parameters. If \( N = \bar{N} \) be a fixed population for a city, I can find the equilibrium utility level \( V^* \) as

\[
V^* = \frac{\alpha}{N^\beta} \sum_i \sum_j \left( \frac{1}{8w - 2w \left( \frac{1}{f} \right) d(i,j) - 2md(i,j)} \right)^\alpha
\]

.....(1.8)

The value of \( V^* \) is then used in equation (1.4) to determine equilibrium rents simultaneously with the other variables.

---

\(^7\) Here land supply at a co-ordinate point is fixed at 1 unit (1 square mile), so \( n(i,j)L(i,j) = 1 \) at every \( (i,j) \). In the later version of the model, where distances are measured continuously along radii, the total area available at a distance \( d_c \) is \( 2\pi d_c \), so the population density is \( 2\pi d_c / L(i,j) \).
1.1b. SIMULATION TECHNIQUES, RESULTS AND ANALYSIS

To compute the equilibrium for the monocentric city, I solve for the equilibrium level of utility $V^*$. When equations (1.4) and (1.7) are solved successively, I get the following equation in one variable by setting $N = \bar{N}$, where $\bar{N}$ is the exogenously given total population of the urban area:

$$\bar{N} = \left(\frac{\alpha}{N^*}\right)^{\beta} \sum_i \sum_j \left(8w - 2w\left(\frac{1}{\bar{f}}\right)d(i,j) - 2md(i,j)\right)^{\alpha}\beta \quad \text{for all } R(i,j) = R_a. \quad \text{(1.9a)}$$

Adapting (1.8) then yields $V^*$

$$V^* = \left(\frac{\alpha}{N^*}\right)^{\beta} \sum_i \sum_j \left(\frac{1}{8w - 2w\left(\frac{1}{\bar{f}}\right)d(i,j) - 2md(i,j)}\right)^\alpha \quad \text{.....(1.9b)}$$

I utilize two matlab packages to evaluate $V^*$. First, I use FSOLVE to solve the system simultaneously. I also use continuous radial distance functions. If $d_c$ is the distance from center, and the city has a circular area, then the population density $n'$ in any thin strip of infinitesimal width at distance $d_c$ is found by equating the total land demand at a distance $d_c$ to the total land supply at the same distance (i.e. $n'(d_c).L(d_c) = 2.pi.d_c$)

$$n' = 2.pi.d_c.n$$

where $n = 1/L(d_c)$ is the population density per square mile:

$$n = \left[\left(\frac{\alpha}{V^*}\right)\left(\frac{1}{8w - 2w\left(\frac{1}{\bar{f}}\right)d_c - 2md_c}\right)\right]^{\frac{1}{\beta}} \quad \text{.....(1.9c)}$$

To find $N$, the total population in the urban area, I solve the differential equation

$$N = 2.pi.d_c.n \quad \text{.....(1.9d)}$$

for initial values $d_c = 0, N = 0$, $V^* =$ guess value of $V^*$ and final value $d_c = 30$ using an ODE45 solver, which uses standard Range-Kutta methods of solving 4th and 5th order differential equations. The matlab programs are used in the following manner.
First, (1.9a) is written in implicit form:

\[ \bar{N} - N = 0 \quad \text{for all} \quad R(d_c) \geq R_a \quad \text{.....(1.9e)} \]

Next, the population (labor supply) of the urban area \( N \) is found by initially guessing a value for \( V^* \). I use the following loop:

\[ \begin{align*}
\text{Guess} & \quad V^* \quad \text{------------------------------}
\downarrow & \quad \uparrow
\downarrow & \quad \uparrow
\text{Given} & \quad V^* \text{ and all other parameters, find all} \quad R(i,j) \geq R_a \quad \uparrow
\text{Hence find population density} \quad N \text{ at all locations} & \quad \uparrow
\downarrow & \quad \uparrow
\downarrow & \quad \uparrow
\text{Check if} \quad N = \bar{N}. \text{ If not} & \quad \text{-------------------------------} \quad \uparrow
\downarrow & \quad \\
\text{Once} & \quad V^* \text{ has been determined and the city laid out,} & \\
\text{use equation (1.5) to determine} & \quad b, \text{ the city boundary.} & 
\end{align*} \]

Given the parameter values, this appears to be a fairly simple system of one equation in one unknown (\( V^* \)), but complications arise in the way FSOLVE algorithms work. FSOLVE uses IMSL techniques of minimization to find the new guess values for \( V^* \) and move the system towards equilibrium, but the guesswork involves evaluating the gradients of the functions (the population density function in this case) for the guess values, checking to see how far they are off and making fresh guesses. At the interface where \( R = R_a \), the population density drops off to zero, as the subsequent points are outside the boundary of the city. This creates a kink in the density function at the boundary point \( b \) and throws the guesswork off, since the function becomes non-differentiable at that point. To transcend this problem, I introduce a smoothed \( N \) function which approximates the actual \( N \) function, but is smoothed out at the kink. I use the function

\[ N = 2.\pi.c.d_n \cdot \gamma e^{(R_{\text{diff1} - 1})} \text{ where } R_{\text{diff1}} = \frac{R(d_c) - R_a}{|R(d_c) - R_d|} \quad \text{.....(1.9f)} \]
When $R(d_c) \geq R_a$, $e^{(R_{diff}-1)} = e^0 = 1$, so (1.9e) reverts to (1.9d). However, when $R(d_c) < R_a$, Riftone becomes negative and $\gamma e^{(R_{diff}-1)}$, and hence $N$, approaches 0. The higher the value of $\gamma$, of course, the faster $N$ approaches 0. At the limit, when $\gamma$ approaches $\infty$, $N$ is closest to 0. However, to smooth the $N$ function, I use a value of $N$ which allow FSOLVE to find a solution without running into non-differentiability. There is thus an element of error, since the solution value of $V^*$ is such that when the rent and population density profiles are evaluated later, there will be a slight adjustment. After the iteration results emerge, I check the solutions using (1.9d). Still, due to the non-linearity of the functions, there are some rounding errors. I consider error levels within ± 1% to be non-significant, and all results conform to that. Table 1.1a below lists the parameter values used to numerically simulate the monocentric city.

<table>
<thead>
<tr>
<th>PARAMETERS</th>
<th>VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.8</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.2</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>10</td>
</tr>
<tr>
<td>$d_c$</td>
<td>0 : 30 miles</td>
</tr>
<tr>
<td>$w$</td>
<td>$$20 / hour</td>
</tr>
<tr>
<td>$f$</td>
<td>30 mi / hour</td>
</tr>
<tr>
<td>$m$</td>
<td>$$0.40 / mile</td>
</tr>
<tr>
<td>$R_a$</td>
<td>$$1000</td>
</tr>
<tr>
<td>$N$</td>
<td>100000</td>
</tr>
</tbody>
</table>

**TABLE 1.1a: PARAMETER VALUES FOR MONOCENTRIC CITY**

Using the above values, I find the equilibrium utility level $V^*$ to be 17.5938, which generates a population $N = 100000$ exactly. Table 1.1b below lists the cumulative population density (total population living within a certain distance from the center) for the city. Of special interest are the value of $b$, the city boundary, where rent offers in the urban area fall to $R_a$, the agricultural rent. Also of importance for later comparison is the point beyond which 40% of the population live in the initial equilibrium, $d_L = 14.2159$ miles in the table.
<table>
<thead>
<tr>
<th>Distance from Center ($d_c$)</th>
<th>Population living between 0 - $d_c$</th>
<th>Population living between $d_c$ - b</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>15480</td>
<td>84520</td>
</tr>
<tr>
<td>10</td>
<td>35446</td>
<td>64554</td>
</tr>
<tr>
<td>$d_1 = 14.2159$</td>
<td>60000</td>
<td>40000</td>
</tr>
<tr>
<td>$d^* = 14.572$</td>
<td>62210</td>
<td>37790</td>
</tr>
<tr>
<td>16</td>
<td>70830</td>
<td>29170</td>
</tr>
<tr>
<td>18</td>
<td>82897</td>
<td>17103</td>
</tr>
<tr>
<td>20</td>
<td>94730</td>
<td>5270</td>
</tr>
<tr>
<td>b = 20.8469</td>
<td>100000</td>
<td>0</td>
</tr>
</tbody>
</table>

**TABLE 1.1b: POPULATION DISTRIBUTION IN MONOCENTRIC CITY**

There is a small discrepancy between the distances found using the two types of programs I use: the ODE45 solver using smoothened population density functions and the program using the original versions of (1.3)-(1.9a), where rents and population densities fall to zero as soon as rent offers fall below $R_d$. I use the former to help in finding the solutions using FSOLVE and use the latter to generate the rent profiles etc. of the city. For example, using the first program, the boundary of the monocentric city is found to be 21.4653 miles out, whereas the second (original) method begets 20.8469 miles. The discrepancy is due to the fact that smoothening the population density functions in the ODE45 programs causes the population to not drop abruptly to zero at the interface between city and agricultural land. On the other hand, the graphs for rents will do exactly that. Though the second method would find the boundaries more exactly, it does not work for all my purposes. When I need to compare areas occupied by specific numbers and groups of people before and after an experiment, the ODE45 programs are ideal since they generate cumulative density functions which can easily be analyzed to yield information along the lines of Table 1.1b. The "exact" programs only generate a profile, i.e. the rent offers and population figures along a ray from the origin to the boundary of the city. The mitigating factor in all of this is that while a particular distance figure would be different depending on which program was used, the comparisons between different boundaries (as parameters change) are unaffected by which standard is used. It does not make a difference whether 20.8469 miles or 21.4653 miles is used as the basic monocentric city boundary.
I use the truer measures of distance (the rent profiles) whenever I can. This is possible when there are distinct interfaces between groups offering different rents at the same point, e.g. at the city boundary. When I use the ODE45 measures, I note that specifically and limit myself to comparing areas from the origin to the distance measured, to avoid any inconsistencies. Besides convenience, the error margins are low (between 1 - 4% of the rent function estimates). Due to the prevalence of smoothing and rounding errors, I consider discrepancies of less than 0.045 miles to be insignificant.

Figures 1.1a and 1.1b below depict the equilibrium rent and population density profiles of the monocentric city for the given parameter values. Figure 1.1b illustrates the problem that arises due to the smoothed population density functions. The population does not abruptly fall to zero at the boundary. There will also be the aforementioned slight discrepancy in the value of b depending on whether I use the rent profile (Figure 1.1a) or the population density profile (Figure 1.1b). In this part of the analysis, I use the rent profile to derive the boundary of the city. For the information in Table 1.2b (except b), I use the cumulative density functions from the ODE45 program.

In subsequent analyses, I have used either the rent gradient, or the population density gradient, sometimes both, to depict changes in city structure in response to an outside stimulus. As I mentioned, the two gradients will behave similarly under some stimuli and (sometimes) differently under others. When the two gradients show equivalent changes, I have only used the rent gradients for the most part. When the changes are not accurately depicted by the rent gradient (one relevant case being for wages in high ranges), I have used the population density gradients, since they always show a true picture of the changes in the patterns of habitation in the city.

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8 In the graphs, I only look at the positive quadrant in coordinate space. This is permissible since the city area is symmetric.
**FIGURE 1.1a: RENTS IN THE MONOCENTRIC CITY**

**LEGEND:** rlc -- monocentric city rents

**FIGURE 1.1b: POPULATION DENSITY IN THE MONOCENTRIC CITY**

**LEGEND:** nlc -- monocentric city population
1.1c. THE 'OPEN' VERSUS THE 'CLOSED' CITY

One of two basic assumptions can be made when looking at the changes in rent profiles in the city - the so called "open city" and "closed city" cases. Let us suppose that some exogenous variable changes, e.g. CBD wages. The open city allows the population N to change in response to this stimulus, whereas the closed city would hold the population fixed at the initial level N.

The difference between the two approaches is not only one of philosophy, but is a product of the assumptions made about the urbanization process in a region. The closed city is a case where people do not migrate in or out. As a result, the initial population is given exogenously (N = N) and the equilibrium utility level V could rise or fall everywhere in the city in response to an outside stimulus. Consider an increase in wage income. With increased income, people demand more housing. Since housing is relatively cheaper in the suburbs, people tend to move in that direction, causing rents to increase towards the city boundary. Rents near the center, on the other hand, fall (or do not increase as much) as people move away. Less people live near the center and more toward the suburbs than before. The end result is a more spread out city with people enjoying a higher utility level. When people earn more, utility should rise everywhere if no more people came into the city. This could happen when the region has a number of cities and all of them are experiencing the same kind of improvements in standards of living, so people have no urge to migrate out of the city they originally live in.

In the open city, people are able to migrate in and out of the city costlessly. When income increases in a city, more people come in (in response to the lure of higher utility), and this increases land demand and drives rents up, till the point is reached where the initial utility gain is completely wiped out. Obviously, there is no in-migration past this point. The assumptions here make sense if the city being considered has a large hinterland and people migrate to the 'big city' whenever there is a tangible improvement in living

---

9 The validity of these results depends on the form of the utility function. The important consideration is whether the income elasticity of housing demand is at least 1, or below 1. For a Cobb-Douglas function exhibiting constant returns to scale, the income elasticity of land demand (in this case) is 1, so the results are always true. If the income elasticity of land demand is below 1, then the result holds as long as the income elasticity of commuting costs is lower than the former. As people move away, they save in rents but incur higher costs of commute. When the people gain by this, an outward movement occurs.
conditions, since the expected benefit of such a relocation becomes more lucrative. Alternatively, and in the same vein, there could be many cities, but only one experiences a boost in living conditions, due to some characteristic peculiar to that city. Under the exact same increase in wage income, then, rents would definitely rise everywhere in the city. The end result is a city more crowded everywhere, with a higher income level but the same utility level as before. Figure 1.2 demonstrates the effects of a wage increase (from $20 to $21 per hour) on the population density gradient in the open versus the closed city cases. In the closed city (new \( V^* = 18.4317, N = \bar{N} = 100000 \)), less people live near the center and more live near the edge of the city as people move out in response to higher housing demand. In the open city (\( V = V^* = 17.5938, \text{new} \ N = 262820 \)), population density rises everywhere in the city till all initial utility gains are wiped out.\(^{10}\)

![Graph showing population density changes in open and closed cities.]

**FIGURE 1.2: POPULATION DENSITY CHANGES IN OPEN AND CLOSED CITIES**

LEGEND: \( n1c(w=20) \), \( n1c(w=21; \text{closed}) \), \( n1c(w=21; \text{open}) \) - as before, \( w = \text{wages} \).

\(^{10}\) In the closed city case in this example, rents are higher everywhere, but they rise more in the suburbs than in the interior of the city. Mathematically speaking, due to the characteristics of the Cobb-Douglas utility function and the starting wage level, the rent gradients for the monocentric closed cities intersect in the second quadrant. The population gradients tell the true story of relocation here.
1.2. THE CITY WITH SUBURBAN EMPLOYMENT

Assumptions

(i) The CBD wage is always fixed at $w = w_c$.
(ii) The SBD forms as a belt at $d_{sub}$, which is at a distance $d_s$ from the CBD, and the firms are spread evenly around the loop.\(^{11}\) Some firms stay at the CBD and hire $N_c$ workers, while the ones that go out to the suburbs hire $N_s$ workers. These two groups of firms could be identical (except it's unclear why some move out) or part of a separate industry. As I mentioned before, my chief concern here is to investigate the effects of employment decentralization on household location choices.
(iii) Firms are assumed to occupy a negligible amount of land relative to residential use.
(iv) SBD firms will attempt to pay the minimum wage possible to attract the number of people they wish to hire. When the demand for labor is completely elastic, urban theory claims this to be the HM wage. I intend to verify this conjecture.

1.2a. THE MODEL

There are now two groups of people living in the urban area, workers who work at the CBD at a wage of $w_c$ and workers who work at the SBD at a wage rate $w_s$. CBD workers commute $2d_c$ miles to and from work every day, while SBD workers commute $d_s = 2(d_{sub} - d_f)$ miles to and from work every day. At every location, they compete with one another for land. For all points in the urban area ($R(i,j) \geq R_0$) the

$$R_c(i,j) = \left\{ \frac{\alpha \beta}{\nu^*} \right\}^{1/\gamma} \left[ 8w_c - 2w_s \left( \frac{1}{1-\gamma} \right) d_c(i,j) - 2md_c(i,j) \right]^{1/\gamma} \quad \text{.....(1.10a)}$$

$$R_s(i,j) = \left\{ \frac{\alpha \beta}{\nu^*} \right\}^{1/\gamma} \left[ 8w_s - 2w_s \left( \frac{1}{1-\gamma} \right) d_s(i,j) - 2md_s(i,j) \right]^{1/\gamma} \quad \text{.....(1.10b)}$$

\(^{11}\) This reinforces the point made earlier about one possible distinction between CBD and SBD firms. If there are agglomerative economies, obviously the firms which can take advantage of it stay in a cluster at the CBD, while the firms dispersed around the circumference of a SBD loop are not equipped to taking advantage of similar economies. The strength of this argument depends on factors such as the cost of transportation around the SBD loop, whether agglomerative economies require more than low cost transportation of goods and personnel, so on and so forth.
where $t_s = t(i,j) - t_{sub}$ and $d_s = d(i,j) - d_{sub}$.

I use the following rule for land allocation:

If $R_c > R_s$, the CBD workers get the land
If $R_s \geq R_c$, the SBD workers get the land

Rules (1) and (2) above hold iff $\max \{R_c(i,j), R_s(i,j)\} \geq R_a$. ........(1.11)

(1.11) is chosen as the allocation in order to be consistent with the implicit assumption in the HM setup that workers who are indifferent between working at the center and the suburb will work at the suburb.\footnote{Notice that making the same rent offer implicitly translates to obtaining the same level of utility, since rents are the residual payments from individuals' budgets in this framework.}

Given that there are two kinds of people occupying the urban area, there will now be a partitioning of the area occupied by CBD workers (population density $n_c$), and the area occupied by the SBD workers (population density $n_s$). $N_c$ and $N_s$, the total populations of CBD and SBD workers in the city, can then be found by adapting (1.7):

$$N_c = \left(\frac{\alpha \alpha}{V^*}\right)^1 \frac{1}{\beta} \sum_i \sum_j \left\{8w_c - 2w_c \left(\frac{1}{f}\right) d_c(i,j) - 2md_c(i,j)\right\}^\alpha$$

iff $R_c > R_s$ and $R_c \geq R_a$ ........(1.12a)

$$N_s = \left(\frac{\alpha \alpha}{V^*}\right)^1 \frac{1}{\beta} \sum_i \sum_j \left\{8w_s - 2w_s \left(\frac{1}{f}\right) d_s(i,j) - 2md_s(i,j)\right\}^\alpha$$

iff $R_s \geq R_c$ and $R_s \geq R_a$ ........(1.12b)

When the rent offer from a particular group of workers is not dominant at a certain location, their population falls to zero. The total population of the urban area is given by

$$N = N_c + N_s$$

...............(1.13a)

with the provision mentioned above. Equation (1.13a) can be rewritten as
\[ N = \begin{bmatrix} \frac{\alpha \beta}{\beta V^*} \end{bmatrix} \sum_i \sum_j \left[ 8w_c - 2w_d(\frac{1}{r}) d_c(i,j) - 2md_c(i,j) \right]^{\alpha \beta} + \sum_i \sum_j \left[ 8w_s - 2w_s(\frac{1}{r}) d_s(i,j) - 2md_s(i,j) \right]^{\alpha \beta} \] 

\[ \text{......(1.13b)} \]

From (1.8) and (1.13), given the urban population \( \bar{N} \) for the closed city and the other parameter values, I find the equilibrium utility level in the same way as in the previous section. \( V^* \) is determined when the system is solved simultaneously. For the open city, utility is held fixed at the initial \( V^* \) and population is allowed to vary.

1.2b. EMPLOYMENT DECENTRALIZATION AND THE STRUCTURE OF THE CITY

HM make two problematic assumptions regarding the wage offers put forth by suburban firms. First, their characterization of the proper wage offer at the sub center is not quite correct. Second, they implicitly make the assumption that firms moving out choose either their location or the number of workers they hire endogenously (i.e. suburban firms never choose where they will locate and how many people they wish to hire independently of the other parameters in the model). In fact, given that the location of the subcenter is often motivated by an external stimulus, the firms which locate there cannot choose anything.

I analyze the effect of employment decentralization on the structure of the city in two phases. In the first part, I follow the wage gradient approach advocated by HM and standard urban theory to determine wages at the subcenter. In this part, I analyze the effects on both a closed and an open city, pointing out a fundamental mistake in the standard wage gradient and discussing methods of correcting for it. In the second part of the analysis, I consider a labor demand function that is completely inelastic. White introduced one end of the problem, that of possible labor scarcity for firms located far out in the suburbs. I complete the discussion by considering the problem of excess supply of labor for a subcenter located close to the CBD.

I extend the analysis in a number of ways. One of them is to look at the effects of moving the employment subcenter further out, which has not been done before. As I
mentioned before, the normal approach has been to hold the distance of decentralization fixed and concentrate on the effects of changes in parameters like wages, agricultural rents, cost of transportation etc. My mode of analysis helps in two ways. First, it helps to flush out the effects on the two endogenous variables \( V^* \) and \( w_a \) as the city decentralizes further and further out. Second, I divide the initial monocentric city into two zones: the Excess Demand Zone and the Excess Supply Zone, which help in understanding the effects of the two kinds of wages structures imposed on the decentralized city. I look at all these results in the context of the open city as well. It would be easy to carry out the standard exercises with this methodology, but there is not much additional gain. Different city structures, which are difficult to analyze theoretically due to the non-linearity of the standard functional forms, can thus be analyzed numerically.

1.2b.1. EMPLOYMENT DECENTRALIZATION WITH FULLY ELASTIC LABOR DEMAND (THE HM CASE)

HM describe a circular city springing up around one CBD at the center of the urban area, with the city structure being determined along the lines described in the previous section. Next, firms choose to decentralize along a circumferential 'band', at some distance from the center. The suburban firm, according to HM, will offer its employees a lower wage compared to the CBD firm, the difference being reflected in the saving in commuting cost for the worker who lives in the suburb.\(^{13}\) At every point in the urban area, there will now be two kinds of people bidding for land, the CBD employee and the (SBD) employee. The worker(s) with the higher rent offer wins the land after the auction. To make the setup comparable to the HM scenario, if SBD workers' rent bid is equal to that of the CBD workers, they win the land. This is akin to the HM assumption that workers indifferent between the CBD and the SBD will work at the latter employment center.

1.2b.1a. DETERMINING THE WAGE OFFER AT THE SUBCENTER

Following HM's conjecture, the SBD firm must offer the workers who live at the SBD a wage rate which would make them indifferent between working at the CBD for a higher wage but commuting daily, and working at the SBD itself and not commuting at all. In the context of our framework, the rent offer forthcoming from the person who lives at

\(^{13}\) HM's definition of this wage compensation is subject to scrutiny, as I discuss later.
the SBD and works there must be equal to the offer from the person who lives at the SBD who works at the CBD. If \( w_c \) and \( w_s \) are the wage rates at the CBD and SBD respectively and \( d_{sub} \) is the distance from the center at which the suburban firm locates, then

\[
R_c (w_c, d_{sub}) = R_s (w_s, 0) \quad \quad \quad \quad \quad \text{.....(1.14)}
\]

At this point, I want to reemphasize that when HM refer to commuting costs, they consider both time costs of commuting (with time being measured in wages lost per hour) and material costs of commuting. The importance of this will be evident shortly. From (1.8), I find the rent offers from the CBD and the SBD employees.

\[
R_c (w_c, d_s) = \left( \frac{\alpha^\alpha \beta^\beta}{V^*} \right) \left[ 8w_c - 2w_c \left( \frac{1}{f} \right) d_{sub}(i,j) - 2md_{sub}(i,j) \right]^{\frac{1}{\beta}} \quad \quad \quad \quad \quad \text{.....(1.15a)}
\]

\[
R_s (w_s, 0) = \left( \frac{\alpha^\alpha \beta^\beta}{V^*} \right) \left[ 8w_s \right]^{\frac{1}{\beta}} \quad \quad \quad \quad \quad \text{.....(1.15b)}
\]

Equating (1.15a) and (1.15b), I get \( w_s \) in terms of the other parameters:

\[
w_s = w_c - 0.25w_c \left( \frac{1}{f} \right) d_{sub} - 0.25md_{sub} \quad \quad \quad \quad \quad \text{.....(1.16)}
\]

The value of \( w_s \) from equation (1.16) is the SBD wage function that HM allude to as the wage gradient -- the SBD wage offer which leaves all people residing outside the SBD loop indifferent between working at the CBD and the SBD. The spirit of the offer is that SBD firms make the minimum possible wage offer which would get them some workers. Especially when the SBD forms far out in the suburbs, the HM wage is just enough to get the few workers who live outside the belt, but no more.

1.2b. SIMULATION TECHNIQUES, RESULTS AND ANALYSIS

Since \( w_s \) is determined exogenously (given \( d_{sub} \)), there is still only one endogenous variable in the HM framework. In the closed city, it is the equilibrium level of utility \( V^* \). The values of \( N_s \) and \( N_c \) are determined when the equilibrium is established, but their values are not fixed, only their sum is fixed at \( N \) by equation (1.13). In the open city, \( V^* \)
is fixed at the monocentric city level as a result of the migration which occurs following any shock. Given $V^*$, $w_c$, $w_s$ and $d_s$, the total population $N$ can be determined endogenously.

**THE CLOSED CITY CASE**

To start with, I analyze the closed city under employment decentralization. All common parameters from Table 1.1 have the same values. The location of the subcenter, $d_{sub}$, is given exogenously. I test the solutions for several values of $d_{sub}$. Radial distances from the center, $d_c$, are allowed to vary from 0 to 30 miles as before. For a given subcenter location, $w_s$ is determined according to the assumptions made about the behavior of the SBD firms, especially regarding whether they obey the HM wage gradient or not. I discuss the possible wage functions in Section 1.2b.1a. Given $w_c$, $w_s$ and all other parameters, the FSOLVE program starts out by guessing an initial value of $V^*$. Equations (1.10a), (1.10b) and the land allocation rule (1.11) determine rents all over the city and also who lives where. Once the two populations are settled, equations (1.12a) and (1.12b) are adapted to measure distances continuously:

$$n_c = 2\pi d_c n_c \quad \text{where} \quad n_c = \left[\frac{\alpha^\alpha}{V^*}\right]^{\frac{1}{\beta}} \left[8w_c - 2w_c \left(\frac{1}{f}\right)d_c - 2md_c\right]^{\frac{1}{\beta}} \quad \ldots(1.17a)$$

and

$$n_s = 2\pi d_c n_s \quad \text{where} \quad n_s = \left[\frac{\alpha^\alpha}{V^*}\right]^{\frac{1}{\beta}} \left[8w_s - 2w_s \left(\frac{1}{f}\right)d_s - 2md_s\right]^{\frac{1}{\beta}} \quad \ldots(1.17b)$$

The population densities $n_c$ and $n_s$ fall to zero when the group in question cannot outbid the other and offer higher than agricultural rent. I then solve the differential equations (1.17a) and (1.17b) to find $N_c$ and $N_s$, the total populations of CBD and SBD workers respectively, using the initial values $d_c = 0$, $N_c = 0$, $N_s = 0$, $V^* = $ initial guess. Once again, I use ODE45 for solving the differential equations. Using the total population values $N_c$ and $N_s$, the FSOLVE program then checks for closure -- $\bar{N} = N_c + N_s$.

For each possible location of the subcenter, $d_{sub}$, the following loop iterates to the solution for the closed city:
Guess $V^*$

Given $d_{\text{sub}}$ and $w_{\text{c}}$, determine $w_\text{s}$ from (1.14), (1.15a) and (1.15b)

Given all parameter values, determine rents at all $d_c$ for CBD and SBD workers

Use rule (1.11) to determine who lives where by outbidding the other group.

Check that $R_c \geq R_a$ and $R_s \geq R_a$

Once the groups are settled, use the ODE45 solver to

determine $N_c$ and $N_s$ by solving the differential equations (1.17a) and (1.17b)

Use equation (1.13) to check if the closure property is satisfied.

If not

Once all other characteristics of the city have been determined,

adapt equation (1.5) to find the new values of $d'_l$, $d'_l'$, $b'$\textsuperscript{14}

Table 1.2 lists the parameter values used for the HM city with decentralized employment. Once again, though the iteration process looks fairly simple, it is complicated by the algorithms used in FSOLVE to guess new values of $V^*$. There are now two kinks in the rent and population density functions: one at the interface where one of the two groups of workers stop living and the other group take over and another at the interface of the city's rent gradient with agricultural rent at the boundary at the city. To maintain differentiability, I introduce "smooth" population density functions similar to the one used for the monocentric city solution. The mathematical explanations are exactly the same as before. The smooth functions are

\textsuperscript{14} $d'_l$ is the interface between the residential areas of the CBD and SBD workers, $d'_l'$ is the distance beyond which 40000 workers live and $b'$ is the new boundary when the decentralized city offers HM wages. I find $d'_l$ and $b'$ using the rent profiles, while the ODE45 program yields $d'_l'$. The latter allows consistent comparisons, since the distance between which and the center 60000 people lived in the monocentric city was found similarly.
\[ N_c = 2.\pi.d_c.n_c.\gamma. e^{(R_{diff1} + R_{diff2} - 2)} \]

\[ N_s = 2.\pi.d_c.n_s.\gamma. e^{(-R_{diff1} + R_{diff3} - 2)} \]

where \( R_{diff1} = \frac{R_c - R_s}{|R_c - R_a|} \), \( R_{diff2} = \frac{R_c - R_s}{|R_c - R_d|} \) and \( R_{diff3} = \frac{R_s - R_a}{|R_s - R_d|} \).

\[ \cdots \cdots \text{(1.18)} \]

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<tr>
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<td>( \overline{N} ) (for closed city)</td>
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**TABLE 1.2: PARAMETER VALUES FOR DECENTRALIZED CITY OFFERING HM WAGES AT THE SBD**

Again, the value of \( \gamma \) is of paramount importance in maintaining the tractability of the system vis-a-vis the FSOLVE algorithm. This system is more sensitive to the value of \( \gamma \), due to the greater number of kinks which the function exhibits. After the iteration process is complete, however, the model is allowed to run with the solution value of \( V \) and deviations due to low values of \( \gamma \) are corrected by trial and error. When I analyze the closed city, \( \overline{N} \) must remain constant as I change \( d_{sub} \) or other parameters.

\( \text{15} \) \( w_s \) is determined in accordance with whether one follows the HM wage gradient or allows for a downward sloping labor demand schedule (as in Section 1.B.2.b). I use equation (38) for simulating the HM city.
Standard urban theory predicts that when I use the wage gradient, there should be no change at all in the household rent profile, and hence by extension in the utility level, size of the metropolitan area, land values etc. The first part of my exercise is therefore to check the validity of this prediction. Referring back to Table 1.1b, I have the cumulative populations as I move further and further away from the center. Then, according to standard theory, if firms decentralize to any distance within city limits, I should find the same number of people living inside and outside the SBD loop as before. For example, if employment decentralizes to a loop 10 miles from the CBD, and SBD firms follow the wage gradient in setting wages, 35446 people should continue to live inside the loop and commute to the center, while another 64554 would live outside the SBD loop and be indifferent between commuting all the way to the center or only up to the loop. In addition, the utility level should stay at 17.5938, the city boundary should still be 20.8469 miles out; in short, nothing should change. I list the equilibrium values for the HM cities with different distances of decentralization in Table 1.3a below for comparison with Table 1.1b.

The results displayed in Table 1.3a are somewhat puzzling at first sight. It appears that there are small but significant changes from the monocentric city in most cases, well beyond the scope of the error factor. This directly contradicts HM, White and other standard theorists. There is a clear movement towards the suburbs, raising the possibility that the HM compensation in fact makes the people outside the SBD loop better off if they work at the subcenter as opposed to the center. The increase in income will have predictable effects on the closed city, as I analyze presently.

Due to the non-linearity of the system of equations, I do not consider fractional values of $d_{sub}$ (except for $d^* = 14.572$) which is an important point of reference for subsequent discussion. Though the results are not difficult to iterate, they will be somewhat tainted by rounding errors. Also, I present values of the endogenous variables up to four significant digits. I check for the closest values of all the variables by using a program which does not use the smoothed functions but the original allocation mechanism. There may still be very small distortions. I do take those into account when analyzing. For example, values of $b'$ are a little distorted by the fact that the iterated total populations are not always 100000 exactly. For this reason, I consider deviations within 0.045 miles to be non significant. I make similar corrections for all the variables analyzed.

16 Firms decentralizing outside the monocentric city boundary are discussed as a special case later.
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<th>$d_{sub}$</th>
<th>$w_s$</th>
<th>$V^*$</th>
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<th>$d_l''$</th>
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**TABLE 1.3a: EQUILIBRIUM VALUES FOR THE CLOSED HM CITY WITH DECENTRALIZED EMPLOYMENT (BASE POPULATION = 100000)**

Comparing Table 1.1b and Table 1.3a, the first thing to note is that all changes will be reflected by the changes in the overall utility level, since this is a closed city experiment. And I find that apart from the case where employment suburbanizes to the boundary of the monocentric city, the utility level always exceeds 17.5938 -- which was the utility level in the monocentric city. White finds such a benefit from suburbanization only when labor scarcity in the suburbs forces a higher SBD wage offer than the HM case. That is not a viable explanation here.

Some other characteristics of the succession of static equilibria are noticeable. As the subcenter moves further and further away from the CBD, the utility level for the city as a whole starts rising at first. This carries on till the SBD is 11 miles out. After that, the

---

17 For comparison purposes, remember $d_{sub}$ here is comparable to $d_e$ in column 1 of Table 1.1b. Columns 4 and 5 here are thus comparable to columns 2 and 3, respectively, in Table 1.1b.
18 The reason $N$ is mentioned here is because of the possible error factor mentioned earlier. The values given should be compared with 100000, the starting population for the closed city, to check that error limits do not exceed ±1.5%.
19 Due to the small errors mentioned before, all distance measures are rounded up to the third decimal point from now on.
utility level starts declining, and when the SBD forms at or beyond the original boundary of
the city and everything reverts to the monocentric case. I discuss the last case (firm
decentralization at or beyond the original city boundary) later.

The effects on the city boundary follow the effects on the utility level. Once again,
for \(0 \leq d_{sub} \leq 11\), the city stretches out further (grows in land area) as employment locates
further away from the CBD. For the region \(11 < d_{sub} \leq 21\), however, the city starts to
shrink in size as employment decentralizes further out. Some deviations, as in going from
d_{sub} = 14\) to \(d_{sub} = 14.572\), are within the range of anomalies arising from rounding errors
etc. In the case mentioned, the increase in city boundary can be attributed to the extra 321
people staying in the latter city.

Comparing with the population densities, I find that compared to the monocentric
city, there is a small but definite movement towards the suburbs. When the SBD moves
out and offers HM wage, between 2.5-5% of the population initially living inside the loop
relocate to the suburbs. All the population figures from Table 1.3a are well within the
prescribed error margin of ±1.5%, so the movement towards the suburbs cannot be
attributed to non linearity problems. It is interesting to note, however, that \(d_1'\) is exactly
equal to \(d_{sub}\). I discuss the significance of this later.

Figures 1.3a and 1.4a depict the comparative static results discussed above,
comparing the rent profiles for cities with SBDs 6 and 11 miles out (Figure 1.3a) and 11,
16 and 20 miles out (Figure 1.4a). Some of the results are difficult to see due to scaling
problems. To alleviate this problem and establish the relations between the rent profiles in
the various cases under consideration, I present "close up" views of Figures 1.3a and 1.4a
(at distances near the CBD and the boundaries of the cities) in Figures 1.3b and 1.4b.
FIGURE 1.3a: COMPARISON OF RENTS IN THE MONOCENTRIC CITY TO THE HM CITY WITH THE SBD 6 AND 11 MILES OUT (LONG VIEW)

LEGEND: r1c -- monocentric city rents; r2cHM/6, r2cHM/11 -- decentralized city rents

FIGURE 1.3b: CLOSE VIEWS OF FIGURE 1.3a

LEGEND: r1c, r2cHM/6, r2cHM/11 -- same as above
FIGURE 1.4a: COMPARISON OF RENTS IN THE MONOCENTRIC CITY TO THE HM CITY WITH THE SBD 11, 16 AND 20 MILES OUT (LONG VIEW)

LEGEND: as before

FIGURE 1.4b: CLOSE VIEWS OF FIGURE 1.4a

LEGEND: as before
THE OPEN CITY CASE

Iterations for the open city are carried out are carried out under the same assumptions as the closed city case, with one important exception. Equation (1.13) is no longer valid. The endogenous variable is the total population N. This system is much easier to solve. For all values of $d_{sub}$, the utility level in the city is held constant at the level of the monocentric city -- 17.5938, by the assumption that people will migrate into the city till all possible utility gains have been wiped away. I do not need to use the FSOLVE mechanism, nor the 'smooth' population density functions. Since $w_s$ is exogenous, the city is allowed to form freely to determine all endogenous variables. Table 1.3b below lists the results from the iterations. I only list the values of $d_{sub}$ which are necessary for comparing the open city results with the closed city graphs.

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**TABLE 1.3b: EQUILIBRIUM VALUES FOR THE OPEN HM CITY WITH DECENTRALIZED EMPLOYMENT (UTILITY LEVEL = 17.5938)**

Figure 1.5 below depicts the succession of equilibria for the open city, comparing the monocentric city to the decentralized HM city with subcenters at a distance of 6, 11, 16 and 20 miles from the CBD. Since the results are difficult to see on one graph, I present a succession of close views for the same graph, looking at equilibria over the distances 0-6 miles from the center, 6-12 miles from the center, 12-17 miles from the center and 17-21.4 miles from the center successively. Seeing the precise location of the different rent profiles helps in analyzing the results.

The open city table and figures shed further light on the issues highlighted from analyzing the closed city case. First, the effects on the open city can also be seen to depend on whether employment decentralizes between 0-11 miles out, or between 11-21 miles out,
as is evident from the fact that population peaks when $d_{sub}$ is 11 miles out. Similarly, the city boundary is furthest out when the SBD is 11 miles from the center. This bolsters the results from the closed city, as I expect. However, there are some other interesting features. From the open city figures, it can be seen that between 0-6 miles from the CBD, there are almost no discernible effects on the city under any mode of decentralization. Between 6-12 miles, an increase in rents is noticed for the city with its SBD 6 miles out but not for any other. Similarly, in the next graph, there is an increase in rents for the cities with SBD 6 and 11 miles out. This trend continues, prompting the conclusion that for an open city with a subcenter at $d_{sub}$, there is no change in the rent profile (and population) in the interior of the city. All the activity is at or beyond the SBD. People from outside the city come and live near (at and beyond the SBD) demanding more land and driving rents up. This is an important discovery, since it pinpoints the possible source of the deviation from the HM conjecture. Whatever the cause, it only affects the people who work at the SBD. Any effects on rents and utility (in the closed city) or population (in the open city) of
the CBD workers is a fallout from the desire of more people to work at the SBD and live in the suburbs.

GENERAL ANALYSIS

I now analyze the results from the closed and open city cases together. In the region 0-11 miles from the CBD, the effects of the formation of the SBD become more prominent as the subcenter moves further away. This takes the form of rents falling close to the center and rising in the suburbs, accompanied by an overall increase in utility in the closed city. The phenomenon can be studied better in the open city, where rents rise only in the suburbs. In either case, proportionally less people live near the center (diminishing land demand) and more move to the suburbs (increasing land demand there). In the region 11-22 miles out, these effects of firm suburbanization then begin to diminish as the SBD moves away, till the SBD reaches the edge of the city. Close to the boundary (e.g. SBD 20 miles out), the effects of decentralization are barely discernible and the rent profile is back to that of the monocentric city.

From the information contained in Tables 1.3a-b and Figures 1.3-1.5, it is evident that there is some aspect of the standard wage gradient that has not been fully analyzed. The effects on the urban area reaches a maximum when the utility gain (or the population increase) is also the maximum (SBD 11 miles out). This suggests that the two might emerge from the same source. Something causes the workers outside the subcenter loop to prefer working at the SBD, which is wholly contradictory to the spirit of the wage gradient, viz. suburban firms pay just enough to make workers living outside indifferent between working for them or for the CBD and not a cent more. All indications point to the fact that subcenter workers get a boost in income, which causes predictable results in the city case. Since wages at the CBD are fixed at $20 an hour, the workers at the SBD must be getting overcompensated.

To resolve this issue, I examine the commuting cost function carefully. The standard formulation of the commuting cost function includes both time and material costs of commuting. The source of the anomaly is the first of these. Time costs of commute are found by evaluating the time lost in commute in terms of lost wages, the opportunity cost. When equation (1.16) is used to calculate the HM wage offer at the suburb, the formulation
succeeds in making the person who lives right at the SBD belt indifferent between working at the center for \( w_c \) but commuting \( d_{sub} \) miles to and from work each day, and working at the suburban job site for \( w_s \) and not commuting at all. But consider the options for anyone who lives outside the SBD loop, at a distance \( d_1 \), say. She has the choice of working at the CBD at \( w_c \) and commuting \( 2d_1 \) miles every day or working at the subcenter at \( w_s \) and commuting only \( 2(d_1 - d_{sub}) \) miles every day. By (1.16),

\[
    w_s = w_c - 0.25w_c \left( \frac{1}{f} \right) d_{sub} - 0.25md_{sub}
\]

If she works at the CBD, her daily wage net of commuting costs, \( n w_c \), is

\[
    n w_c = 8w_c - 2w_c \left( \frac{1}{f} \right) d_1 - 2md_1
    = 8w_c - 2w_c \left( \frac{1}{f} \right) d_{sub} - 2md_{sub} - 2w_c \left( \frac{1}{f} \right) \Delta d - 2m\Delta d \quad \ldots \ldots (1.18a)
\]

where \( \Delta d = d_1 - d_{sub} \), and if she works at the SBD, her daily net wage \( n w_s \) would be

\[
    n w_s = 8(w_c - 0.25w_c \left( \frac{1}{f} \right) d_{sub} - 0.25md_{sub}) - 2w_s \left( \frac{1}{f} \right) \Delta d - 2m\Delta d \quad \ldots \ldots (1.18b)
\]

From (1.18a) and (1.18b), I find the overcompensation per SBD worker to be

\[
    n w_s - n w_c = 2(w_c - w_s) \left( \frac{1}{f} \right) d_{sub} \quad \ldots \ldots (1.18c)
\]

Equation (1.18c) identifies the anomaly. Under the HM wage offer, once the SBD worker living beyond the loop decides to work at the subcenter, she values the saving in commute time from the SBD to the CBD in terms of \( w_s \). However, the HM wage compensates her in terms of \( w_c \). According to the structure of her utility function, this leaves her better off. To make the person at \( d_1 \) perfectly indifferent between working at the CBD and the SBD, the compensation should instead be in terms of \( w_s \), since the SBD worker places a lower value on the commute from \( d_{sub} \) to the CBD.
Consider a numerical example from Figure 1.6 above. The wage at the center is $12.50 / hour. The round trip commute from $d_{sub}$ to the CBD costs $1.50 / hour when evaluated at the central wages. Now consider two individuals, one living at $d_{sub}$ exactly, and the other living at $d'$, which is further out than $d_{sub}$. Assume the round-trip commuting cost from $d'$ to $d_{sub}$ (evaluated at CBD wages) to be $1/hour. According to the HM wage gradient, the wage offer at the subcenter should be $(12.50 - 1.50) = 11$ /hour.

Consider the individual at $d_{sub}$. She can commute to the center, earn $12.50 in gross wages, spend $1.50 in commuting and end up with a net wage of $11/hour. Alternatively, she can work where she lives, at $d_{sub}$, earn $11/hour and spend no time and cost in commuting, for a net wage of $11/hour. She is thus truly indifferent between working at the center and the suburb.

But now consider the individual living at $d'$. He can work at the center, earning $12.50 but spending $2.50 an hour in commuting, giving him a net wage of $10/hour. If he chooses to commute to the SBD instead, he earns a gross wage of $11/hour but incurs a commuting cost of $t/hour. If $t = 1$, then the individual at $d'$ would be perfectly indifferent between the CBD and the SBD. But he values his commute time at the SBD wage of $11/hour, not at the CBD wage of $12.50 an hour. Hence $t < 1$, and he is better off working at the SBD rather than at the CBD. I present a formal proof of this proposition.
**Proposition 1:** The HM wage offer overcompensates people who live beyond the SBD belt and work at the subcenter.

Consider an individual with a strictly quasi-concave utility function of the form $U(g, L)$ which she maximizes subject to the budget constraint $8w - g - rL - 2tw - 2md = 0$, where the variables and parameters are all as discussed before.

I maximize the Lagrangian

$$V(d) = \max \{ U(g, L) + \lambda [8w - g - R L - 2tw - 2md] \}$$

The equilibrium condition requires $\frac{\partial V}{\partial d} = 0$, so from the envelope theorem

$$\frac{\partial V}{\partial d} = -\lambda [2w \left( \frac{1}{t} \right) + 2m] + R'L = 0$$

Now consider an employer at a sub center location $d_{sub}$ who pays a wage $w_S$ such that an individual at $d_{sub}$ is indifferent between commuting to the center or working at $d_{sub}$. If $U(g, L)$ is strictly quasi-concave it must be that

$$w_S \leq w_C - [2w_C \left( \frac{1}{t} \right) + 2m] d$$

since strict equality would imply that her previous consumption bundle was feasible.

Figure 1.7 below demonstrates this proposition. If the HM conjecture is correct then rents and land demands (hence population densities) should be invariant. However, since $w_C > w_S$ for some $d > d_{sub}$, this implies that for $d > d_{sub}$

$$\frac{\partial V}{\partial d} = -\lambda [2w_C \left( \frac{1}{t} \right) + 2m] + R'L > 0 \quad \text{.....(1.19)}$$

Condition (1.19) proves that for the HM conjecture to be correct under the circumstances described by them, the net wage for all people who commute to the SBD must be lower than $w_C - [2w_C \left( \frac{1}{t} \right) + 2m] d$. 
Now that the source of the anomaly is established, it becomes easy to understand the results from Tables 1.3a-b and Figures 1.3-1.4. For example, given an increase in real income of the SBD workers in the closed city, we can postulate that the rent gradient will flatten over the urban area, with rents near the center falling as people gravitate towards the suburbs. At the same time, with the total urban population fixed, there will be an increase in the overall utility level of the city. The reason that the increase in income does not continue as the SBD locates further and further out is also clear. The amount of real income increase, hence the change in utility in the closed city, will depend on two things:

(a) the increase in income for each SBD worker;
(b) the total number of SBD workers.

As firms decentralize further and further away from the CBD, (a) rises but (b) falls, since the number of workers depend on the number of people living outside the loop after decentralization. Since the change in utility depends on the product of (a) and (b), there will be a maximum utility for the closed city when decentralized firms offer HM wage. From Table 1.3a, the optimum distance of decentralization is 11 miles, when the utility level for the city rises to 17.7454 and 61.22% of the population is decentralized. Also, in all cases, as long as some people are working at the SBD (i.e. whenever employment

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20 Remember that some people move to the suburb after employment decentralization.
decentralizes within the initial commuting zone), people living outside the loop are not merely indifferent between working at the CBD and the SBD; they prefer working at the SBD. Now I analyze the changes in different endogenous variables.

**Effect on Rents, Utility and Boundary in the closed city**

I depict the changes in the structure of the city in Figure 1.8 below. The graph shows the household rent profiles before and after employment decentralizes.

![Figure 1.8](image)

Notice from the tables that the boundary between the CBD and the SBD workers $d'_l$ is always at $d_{sub}$. $d_l$ shows where the same number of people as the CBD workers were living in the monocentric city. For example, using Tables 1.1b and 1.3a, when the SBD locates 10 miles out, 33309 people are living inside the loop (i.e. within 10 miles of the center). In the monocentric city, 35446 people were living within a radius of 10 miles from the center, while 33309 people used to live within a radius 9.57 miles from the CBD. Since everyone must have the same utility level in the final equilibrium, I can analyze the effects on overall utility in the city by looking at the plight of the 33309 people. They earn the same wage as before ($20), but they now occupy $pi.(0.43)^2$ more land. Since the price elasticity of demand for the Cobb-Douglas utility function is -1, the increase in land supply causes an equal percentage decrease in rent for central city residents, since land demand increases and rents are inversely related to land demand.
From the indirect utility function, I compare the utility levels of a person living at the center before and after suburbanization. Since wages are fixed for CBD workers, the relative utility level is \( \frac{V^*_2}{V^*_1} = \frac{w^*/R_2(0)}{w^*/R_1(0)^\alpha} = \left( \frac{d^*_2}{d^*_1} \right)^{2\alpha} > 1 \), where \( V^*_2 \) and \( V^*_1 \) are the equilibrium levels of utility in the decentralized and monocentric cities respectively. At equilibrium, all households in the city must have the same level of utility. Hence utility rises everywhere.

From Tables 1.1b and 1.3a, this is true of all values of \( d_{\text{sub}} \) (\( d_i < d_{\text{sub}} = d_i' \)), unless they are too close to the edge of the initial city. The overcompensation of SBD employees makes them want to move outwards to consume more land, leaving the remaining CBD workers with more land, decreasing rents in the interior of the city and in the process increasing everyone's utility. The more the movement towards the suburbs, the further the city boundary stretches, alleviating the crowding in the suburbs and maintaining a higher utility level. The thin dashed line in Figure 1.8 (R2cHMc) shows the possible equilibrium. Notice that the decentralized city is always compartmentalized, no one inside the loop works at the SBD and no one outside works at the CBD. More about this later.

**Effects on Rents, Population and Boundary in the open city**

The effects on rents and population in the open city is pretty similar to the depiction in Figure 1.8, with one salient exception. As Figure 1.5 shows, effects on rents are only felt near the subcenter and beyond. There are very insignificant changes close to the CBD. The overcompensation of SBD workers causes more people to want to work in the suburbs, prompting a movement towards the suburbs among the old city residents and new laborers from outside. However, the new laborers prefer to work at the SBD and cease to seek jobs there only when overcrowding of residential land near the SBD erodes the utility they get from the relatively higher wages. Either way, there is an increase in land demand and rents near the SBD and in the suburbs, and the inflow of new labor continues until the original utility gains have been wiped out. Thus in the context of Figure 1.8, there will be very insignificant changes in rents near the center, but a larger increase in rents close to the SBD and beyond. The bold dashed line in Figure 1.8 (R2cHMo) shows the possible equilibrium rent profile.
Conclusions

The convenient part about the anomaly in the wage gradient is that it is relatively small, even though it is significant. In fact, using the wage rate as the opportunity cost of time lost commuting is an economic construct that is subject to alternate viewpoints. For example, when leisure is not specifically in the utility function (like in my model), it is not always clear that opportunity costs should perforce depend on wages. However, within the confines of this model, there is a problem.

There are two ways of eliminating the problem:
(a) If commuting costs are fixed per mile, there are no problems.
(b) If (a) does not hold, all SBD commuters must be forced to give up some portion of their income that will take away their gain in income and leave them exactly as well off as in the monocentric city. They will each be charged the amount of the overcompensation

\[ 2(w_c - w_s) \left( \frac{1}{f} \right) (d_{sub}) \]

collecting a total of

\[ \int_{d_{ll}}^{b^*} 2.\pi.d_c.n_s [ 2.(w_c - w_s) \left( \frac{1}{f} \right) d_{sub} ] \]

for the entire city. This approach has some practical problems. While it is not impossible to levy some tax on a particular group of workers, workers must be taxed differently, depending on how far they commute. This is difficult to accomplish in a community where people drive to work. If they rode a shuttle service instead, ticket prices could be adjusted to include the amount of the overcompensation.

Incidentally, the seemingly common sense approach of finding a SBD wage which would leave all SBD workers indifferent between the center and the subcenter and not change the profile of the city will not work here. If a wage lower than HM wage is offered, the people living at and very close to the SBD will no longer be able to work at the SBD -- violating another basic assumption of the HM framework.
Getting beyond the subject of the anomaly, there are other problems with the HM approach. The wage gradient, when used by the SBD firms, does maintain the inward commuting pattern of the monocentric city, but at a price. The city with decentralized employment is completely segmented. All people who live inside the SBD loop work at and commute to the center, while everyone who lives outside will commute to the SBD loop. This complete compartmentalization of the city is not consistent with reality. Usually, there is some cross commuting between such zones. In this set up, it would be normal to investigate when there could be people who live beyond the loop but commute to the center and when there could be people who live near the center but commute outward to suburban job sites. The HM city maintains the dominance of the CBD. With fixed wages at the center and the wage gradient approach to setting SBD wages, the inward pattern of commuting shows the control the CBD exerts over the city. By contrast, the SBD -- even when it locates close to the center and attracts a significant workforce -- is truly a secondary center. Not even one worker alters her direction of commute.

In the same vein, the SBD firm has no control over the number of people it wants to hire. If it offers the HM 'minimum wage', it will get the people who live further than its location. If the SBD location is near the center, then there is a large supply of labor. If the SBD location is far out, there is a very limited amount of labor available. In reality, production circumstances and profit considerations determine labor demand. Under the HM wage scenario, firms are assumed to have an infinitely elastic labor demand function, so they are satisfied with whatever labor supply they get. When I consider the same firm choosing between alternate SBD locations, or a social planner debating which SBD location to subsidize, the HM approach leaves a firm with a certain fixed labor demand one and only one feasible location in the city. For example, in Table 1.3a, if the SBD demands 40% of the labor force, the only feasible location would be approximately 14.572 miles out. What if the local government provided incentives for the SBD to locate 5 miles out instead? Or 18 miles out? These questions must be dealt with separately, for the former location will have an excess supply of labor, while the latter will experience labor scarcity. The best way to analyze this is to include a full blown labor market in the model, which I do in Chapter 3. An intermediate step would be to hold the amount of decentralization fixed at a certain fixed amount. I do this next in Section 1.2b.2. At the end of that section, I compare the values of the endogenous variables from the two kinds of decentralization.
Finally, as the last two rows of Table 1.3a shows, the HM wage does not work when the SBD location occurs at or beyond the boundary of the initial monocentric city, at a point like S' in Figure 1.1. I treat this as a special case in Chapter 2.

1.2b.2. THE DECENTRALIZED CITY WITH FIXED LABOR DEMAND

Assumptions

(i) SBD firms demand 40% of the total labor force (40000 laborers), wherever they locate. This is completely different from the HM approach. However, the labor demand curve becomes vertical instead of horizontal. So this is not a complete story either.
(ii) Wages at the center are still fixed. This is not a general equilibrium setting.

1.2b.2a. THE MODEL

From Table 1.3a, if the SBD firms offer HM wage, there is only one feasible location $d^*$ for them, approximately 14.572 miles from the CBD. As I mentioned before, the location of the SBD is often dictated by other considerations, like the presence of an export node at a specific location (which may be different than $d^*$). It is highly unlikely that firms faced with varying labor supply will either choose to locate 14.572 miles out or take whatever labor supply they can find at the optimal location. In other words, since the labor demand function is not flat but vertical, how can SBD firms be in a position to control the size of their work force, regardless of where they locate?

Assuming the SBD firms want to hire 40% of the workforce, the possible suburban locations can be divided into two categories for the purpose of analysis. I analyze the effects of employment decentralization under the following heads:

(a) when employment decentralizes up to $d^*$ (14.572 miles) from the CBD;
(b) when employment decentralizes further out than $d^*$ but within the initial city boundary;

From Table 1.3a, SBD firms offering HM wages will find themselves faced with a shortage of labor in the zone $d^*$-$b$, since less than 40% of the city's population live outside
the subcenter belt when the SBD locates there. Since there is an excess demand for labor in the zone $d^*-b$, I call it the Excess Demand Zone (henceforth EDZ). White (1976) analyzes the labor demand function in this zone in terms of the area from which labor needs to be attracted to the suburban job site. If the SBD locates at $d_{sub}$ ($d_{sub} > d^*$ here) less than 40% of the workforce live outside the loop, so the firms will have to make a wage offer of $w_s = w^* - 2td_L + td_{sub}$ ($w^*$ is the fixed CBD wage) to attract all workers living at or beyond the point $d_L$ in the city, where $d_L < d_{sub}$. As a result, households relocate towards the suburbs, there is outward commuting, the utility level of the urban area increases, so on and so forth. I use numerical simulation and analysis to look at the city as employment decentralization occurs further and further beyond $d^*$.

The other side of the problem is to examine what happens to the city when employment decentralization takes place inside 14.572 miles from the CBD. In the presence of a downward sloping labor demand function, there would appear to be an excess supply of labor in this region. For example, from Table 1.3a, when employment decentralizes 5 miles out and the SBD firms offer HM wage, almost 90% of the labor force wants to work at the SBD. However, White's analysis suggests that when firms maximize their profit function, it is permissible to allow an excess supply of labor to persist.

Why is equilibrium in the labor market over the zone 0-$d^*$ not a requirement for White's model? The answer lies with the anomaly from the previous section. In the standard approach, the HM wage offer is supposed to leave the people living outside the subcenter loop completely indifferent between working at the CBD and the SBD. In that case, consider a scenario when the SBD locates 6 miles out. There are now 13972 workers living inside the loop who will prefer to work at the center. In addition, there are 90000 workers living outside the beltway who will be indifferent between working for the CBD or the SBD. If the SBD offers HM wage, it can hire the first 40000 workers who show up. The next 50000 can continue to work at the CBD with no loss of efficiency, since they are not any worse off by having to do. When the anomaly exists, the situation changes. As proved before, the workers outside the loop will actually be overcompensated by HM wage. Hence all workers living outside the loop will prefer working at the SBD to working at the CBD. In this scenario, an excess supply of labor will cause inefficiencies in the market, since some people will be worse off by being forced to work at the CBD.
1.2b.2b. SIMULATION TECHNIQUES, RESULTS AND ANALYSIS

Now there is a fundamental change in the scenario through the introduction of an additional constraint. $N^D$ is fixed, and for the subcenter locating anywhere in the city, we must have $N^D = N^S$. It is therefore necessary for SBD firms to stop offering HM wages. Instead, they must offer a wage higher than HM wage in the excess demand zone (EDZ) inducing some people living in the central city to commute outwards to the SBD. In addition, for the reasons explained above, labor market equilibrium must also be satisfied in the ESZ. Here the inherent anomaly in the construction of the HM wage gradient breaks down the very spirit of the wage gradient. The HM wage is meant to represent a minimum wage which the SBD firms could offer and get people to work for them. Now the standard theory fails. If the HM wage overcompensates suburban workers and the firm faces an excess supply of labor, it is clearly not the minimum wage offer. For example, if the SBD locates 6 miles out, where the HM wage offer would generate 86027 workers, while the firms need only 40000, they can offer a lower wage, one that will get them exactly 40000 workers and no more. Once again, this anomaly would go away under one of the assumptions or taxes mentioned before, but the corrections are difficult to implement. Instead, I present an equilibrium where both $V^*$ (for the closed city), $N$ (for the open city) and $w_s$ are determined simultaneously.

THE CLOSED CITY CASE

I use the model with suburban employment presented in Section 1.3a, with one change. Equation (1.13) is now altered to

$$N_C = \bar{N}_C$$  \hspace{1cm} \text{.....(1.20a)}

and

$$N_S = \bar{N}_S$$ \hspace{1cm} \text{where} \hspace{0.5cm} \bar{N}_S = (0.667)\bar{N}_C$$  \hspace{1cm} \text{.....(1.20b)}

Unlike in the HM wage case, $w_s$ is now no longer determined by equations (1.14) and (1.15). Instead the model is solved numerically on MATLAB, using FSOLVE and ODE45 as before. The following loop now iterates to the solution, given $d_{sub}$:
Guess $V^*$ and $w_s$

\[ \downarrow \]

Given all parameter values, determine rents at all $d_e$ for CBD and SBD workers

Use rule (1.11) to determine who lives where by outbidding the other group.

Check that $R_C \geq R_a$ and $R_S \geq R_a$

\[ \downarrow \]

Once the groups are settled, use the ODE45 solver to

determine $N_C$ and $N_S$ by solving the differential equations (1.17a) and (1.17b)

\[ \downarrow \]

Use equations (1.20a) and (1.20b) to check if the closure properties are satisfied.

\[ \downarrow \]

If not

\[ \downarrow \]

Once all other characteristics of the city have been determined,

adapt equation (1.5) to find the new city boundary $b''$ and $d_L''$ \(^{21}\)

There are now two equations, (1.20a) and (1.20b), where the population densities are found by solving the differential equations (1.17a) and (1.17b) as before, with the same initial and final conditions. Also as before, I use the smoothened versions of the population density functions to help the system iterate towards a solution without crashing.

I use the parameter values from Table 1.2, with the exception of $w_s$, as mentioned, and $\gamma = 2.5$. The value of $\gamma$ is still of paramount importance. Since the guesswork gets more complicated here, $\gamma$ needs to be smaller, i.e. the population density functions need to be smoother at the kinks. The simultaneous solution of equations (1.20a) and (1.20b) then yields $V^*$ and $w_s$. I correct for the deviations due to the value of $\gamma$ chosen by using the solutions from FSOLVE and checking through trial and error. Due to the non linearity of the system, there are some cases where exact solutions cannot be reached. I get the closest possible solutions and accept errors in the ±2% range.

\(^{21}\) $b''$ is the new boundary of the city. $d_L''$ now refers both to the interface between CBD and SBD workers and the distance beyond which 400000 people live in the city. Notice that these are the same.
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**TABLE 1.4a: EQUILIBRIUM VALUES FOR CLOSED CITY WITH 40% OF THE JOBS DECENTRALIZING**

Table 1.4a above lists the equilibrium values I find for the closed city equilibrium wage offers for all values of $d_{sub}$. $d_L''$, the point at which a worker is indifferent between working at the CBD and the SBD (i.e. the inner boundary of the area from which the subcenter must draw its workers), is found using the ODE45 programs. Though the values can be found more exactly from the rent profile, I use the ODE45 values to maintain consistency with the previous measures of $d_L$ and $d_L'$. I calculate the new city boundary $b''$ from the rent profiles. This is both exact and consistent with the previous values $b$ and $b'$.

As mentioned before, I divide the urban area into the ESZ (0-$d^*$ miles) and the EDZ ($d^*$-$b$ miles). First, consider the exact borderline $d^*$. From Table 1.3a and 1.3a, the equilibrium values for $d_{sub} = d^* = 14.572$ miles are identical. This is not surprising, indeed it is the definition of $d^*$. When SBD firms locating at $d^*$, they find exactly 40000 workers when they offer HM wage. Hence, they will not offer any other wage. However, this is not the case when firms locate either inside or outside $d^*$. 
(A) **Formation of the SBD in the Excess Supply Zone (0-\(d^*\) miles)**

In the ESZ, offering HM wages will create an excess supply of labor. This is inefficient, since the people unemployed at the suburb will have to work at the center and be worse off, and their loss does not transcend into someone else's gain. From the viewpoint of the firms, HM wage is no longer the minimum wage offer they can make. Since they do not want to hire more than 40000 workers, they will be better off trying to capture the population living beyond \(d_L\) (where \(d_L\) is the distance beyond which 40000 people live in the monocentric city) by offering a wage lower than HM wage. The firms look at the population distribution in the monocentric city (Table 1.1b) and find \(d_L\) to be 14.2159 miles. However, when they offer wages \(w_s\), aimed at capturing the people beyond \(d_L\), there is a relocation towards the suburbs due to the overcompensation factor, as I explain later. Eventually, 40000 people end up living beyond a distance \(d_L''\) from the center. From Table 1.4a, the value of \(d_L''\) varies from 14.345 miles (\(d_{sub} = 6\)) to 14.572 miles (\(d_{sub} = 14.572\)). The subcenter locates at \(d_{sub}\), so \(d_L > d_{sub}\) (except for the short span \(14.2159 < d_{sub} < 14.572\)) and \(d_L'' > d_{sub}\). I will refer to the new wage offer as equilibrium wage (Ewage for short) from now. I will refer to this type of city as the Ewage city.

Figures 1.9a and 1.10a below depict the changing structure of the city when the SBD locates 6 and 11 miles out in the HM and Ewage cities. Figures 1.9b and 1.10b show close ups of the corresponding graphs near the center and the boundary of the city for visual comparisons.

Figures 1.9-1.10 show that when employment decentralizes in the ESZ, the closed Ewage city is less spread out than the HM city, but more so than the monocentric city. Comparing these graphs to the information in Table 1.4a, I find them to be completely consistent. Note that over the ESZ, wage offers are consistently lower in the Ewage city compared to the HM city. This happens since with a vertical labor demand curve, firms must make lower wage offers in order to capture only a portion (40000 to be precise) of the people living outside the SBD loop. \(d_L''\) shows the distance beyond which the required number of people live after relocation.
FIGURE 1.9a: COMPARISON OF RENT PROFILES WITH SBD 6 MILES OUT IN CLOSED CITY, UNDER HM WAGE AND EQUILIBRIUM WAGE OFFERS AT THE SBD (LONG VIEW)

**LEGEND:** r1c -- monocentric city rents; r2cHM/6/cl, r2cW/6/cl -- HM and E wage city rents

FIGURE 1.9b: CLOSE VIEWS OF FIGURE 1.9a

**LEGEND:** r1c, r2cHM/6/cl, r2cW/6/cl - as before
FIGURE 1.10a: COMPARISON OF RENT PROFILES WITH SBD 11 MILES OUT IN CLOSED CITY, UNDER HM WAGE AND EQUILIBRIUM WAGE OFFERS AT THE SBD (LONG VIEW)

**LEGEND:** r1c, r2cHM/11/cl, r2cW/11/cl -- as before

FIGURE 1.10b: CLOSE VIEWS OF FIGURE 1.10a

**LEGEND:** r1c, r2cHM/11/cl, r2cW/11/cl -- as before
Figure 1.11 above analyzes what happens to rents in the Ewage city when employment locates in the ESZ. Consider the rent profile of the city as it goes from being monocentric to having a subcenter at $d_{sub}$. The SBD firms first choose their wage offer on the assumption that they can offer HM wage, which would leave all people outside $d_{sub}$ indifferent between working for them and the CBD. This wage offer ($w_s = w_c - 2t_c d_{sub}$) \(^{22}\) is aimed at capturing all people living between $d_L$ (= 14.2159 miles) and $b$ in the monocentric city. However, due to the overcompensation effect, $w_s$ ends up making the people outside the loop actually prefer to work at the SBD. The rent gradient flattens and people move out towards the suburbs. Now in the HM city, 40000 people live beyond $d_L'$. This relocation to the suburbs alters the map of the city and causes the SBD firms to adjust their wages downward. Their new wage offer $w_s' (< w_s)$ is $w_s' = w_c - 2t_c d_L'$.

In the next stage of the problem, the SBD firms want to capture precisely 40% of the population, they start off by offering $w_s'$. The lower suburban wage now causes the central city residents to be relatively better off compared to the HM city. This causes a steepening of the rent gradient and a movement of households towards the center of the city. Finally, the 40000 suburban workers end up living between $d_L''$ and $b''$, the new city boundary. The final wage offer is $w_s'' = w_c - 2t_c d_L''$ ($w_s' < w_s'' < w_c$).

---

\(^{22}\) $t_c$ refers to the commuting cost evaluated in terms of $w_c$. 
Compared to the HM city, the Ewage city will offer lower wages. I characterize the change in wages as the wage effects (WE for short). The changes in the overcompensation effect (OE for short) compared to the HM city is ambiguous. When \( w_s \) decreases, the overcompensation per worker increases. But the number of SBD workers falls, decreasing the total amount of the OE. The WE will create a Ewage city which is more spread out and suburbanized than the monocentric city, but less so than with the HM city. Figure 1.11 depicts the changes from the monocentric city (rent profile \( R_m \)) to the Ewage city (rent profile \( R_{2e} \)) and the relative positions of the various distance measures.

Table 1.3a lists the HM wage \( w_s \) and \( d_{L'} \), while Table 1.4a lists values for \( w_s" \) and \( d_{L}" \) for the city offering Ewage at the SBD. The intermediate wage offer, \( w_s' \), is not tabulated. Table 1.4b below shows how the information from Tables 1.1b, 1.3a and 1.4a can be put together to analyze the series of events discussed and depicted in Figure 1.11.

<table>
<thead>
<tr>
<th>( d_{sub} )</th>
<th>MONO</th>
<th>HM city</th>
<th>Ewage city</th>
</tr>
</thead>
</table>

**TABLE 1.4b**

Having analyzed the process of household relocation, comparing the monocentric case with the two kinds of decentralization becomes fairly easy. When the SBD firms locate in the ESZ and end up offering Ewage, the wage effect is to decrease the incomes of the subcenter workers relative to that of SBD workers in the HM city. The overcompensation effect is ambiguous since there are offsetting changes. Since the total increase in income of the SBD workers is lower than in the HM city, the SBD workers do not move towards the suburbs quite as much in the Ewage city. Consequently, the increase in land supply in the interior of the city is smaller and rents higher than in the HM city. The household utility level is lower and the city boundary, though further out than 20.8469 miles -- the boundary of the monocentric city -- is closer than those in the HM city. As SBD locations move further and further out, the household utility levels in the Ewage city approaches those in the HM city. At the boundary of the ESZ (i.e. at \( d^* \)), the two utility levels converge, as do the values of \( b^* \) and \( d_{L}" \).
While the Ewage city is not the most efficient characterization of the equilibrium, it is a step forward in the analysis -- since the compartmentalization of the HM city is broken down. No longer do all workers living inside the loop commute to the center and all workers living outside the loop commute to the subcenter. Since $d_L'' > d_{sub}$, all workers living between the center and $d_L''$ now commute to the CBD, while everyone living past $d_L''$ commute to the subcenter. The pattern of inward commuting still persists, but there will be cross commuting as some people will live outside the loop and commute all the way to the center.

(B) Formation of the SBD in the Excess Demand Zone ($d^*-b$ miles)

When the subcenter forms in the EDZ, SBD firms offering HM wages will now face labor scarcity, since less than 40000 people initially live outside the loop. In this case, the SBD firms will need to increase the wage offer to attract people living inside the loop. Now $d_{sub} > d_L'$ (note that $d_L' < d^* < d_{sub}$), and SBD firms must start with a wage offer $w_s' = w_c - 2td_{L'}' + td_{sub}$. In addition to the overcompensation, whose amount decreases as firms move further and further out in this zone, the offer of $w_s'$ ($> w_s$) now causes a relocation towards the suburbs.

In Figures 1.12a and 1.13a overleaf, I compare the rent profiles of the HM city and the Ewage city as the subcenter locates 16 and 20 miles out respectively with that of the monocentric city. Figures 1.12b and 1.13b show close ups of the corresponding graphs near the center and the city boundary, to allow a more detailed analysis of the effects on the city as the SBD forms in the EDZ. Note that unlike some of the earlier cases, the close ups in Figures 1.12b and 1.13b can be used to compare the relative effects on the rent profile of the city as the subcenter forms 16 and 20 miles out, since they use the same scaling.

I use the information from Figures 1.12 and 1.13 and from Table 1.4a to analyze the broad trends noticed in the closed city when Ewage offers are made in the EDZ. I find the results to be consistent. In this zone, there are significant changes being wrought on the rent profile of the city. Due to the higher wage offers, more people are being enticed to work at the subcenter and live in the suburbs. The wage gradient breaks down, with SBD wages falling very slightly as one moves closer to the original boundary of the city. On the other hand, $V^*$, $d_L''$ and $b^*$, increase very significantly compared to when the SBD forms
FIGURE 1.12a: COMPARISON OF RENT PROFILES WITH SBD 16 MILES OUT IN CLOSSED CITY, UNDER HM WAGE AND EQUILIBRIUM WAGE OFFERS AT THE SBD (LONG VIEW)

**LEGEND:**
- r1c -- monocentric city rents;
- r2eHM/16/cl, r2eW/16/cl -- HM and Ewage city rents respectively

FIGURE 1.12b: CLOSE VIEWS OF FIGURE 1.12a

**LEGEND:** r1c, r2eHM/16/cl, r2eW/16/cl - same as above
FIGURE 1.13a: COMPARISON OF RENT PROFILES WITH SBD 20 MILES OUT IN CLOSED CITY, UNDER HM WAGE AND EQUILIBRIUM WAGE OFFERS AT THE SBD (LONG VIEW)

**LEGEND:** r1c, r2cHM/20/cl, r2cW/20/cl -- as before

FIGURE 1.13b: CLOSE VIEW OF FIGURE 1.13a

**LEGEND:** r1c, r2cHM/20/cl, r2cW/20/cl -- as before
in the ESZ. This makes perfect sense. With SBD wage offers significantly higher than the HM level when the SBD locates further out, people living close to $d_{sub}$ get a big boost in their income compared to the monocentric and HM city level. Now a number of people living inside the SBD loop prefer to work at the subcenter, causing a relocation of housing towards the suburbs, producing a local rent peak at the SBD. Even these higher rents do not offset the increase in utility from the income boost, causing a more spread out city which is also less crowded in the interior.

Figure 1.14 above is a generic graph depicting the effects on the city when SBD firms locate in the EDZ and offer Ewage. The rent profile of the monocentric city is not shown on the graph. Initially, 40000 people live beyond $d_L$ (14.2159 miles). When the subcenter forms beyond $d^*$ and the SBD firm offers HM wage $w_s = w_c - 2t_c d_{sub}$, the city is rearranged so that 40000 people now lives beyond $d_L'$ ($d_L' < d^* = 14.572$ miles). In Figure 1.14, the thick line $R_{HM'}$ represents the HM city, with $d_L' < d^*$. SBD firms wanting precisely 40000 workers will then offer the higher wage $w_s' = w_c - 2t_c d_L' + t_c d_{sub}$, aimed at capturing the population beyond $d_L'$. This will produce the rent profile $R_{W'}$, shown as the dashed line in Figure 1.14, with the high wage offers in the suburb drawing out people towards the SBD loop. As Figures 1.12 and 1.13 show, this effect is more prominent the further out the SBD locates.
However, this is not the end of the relocation process. Due to the outward relocation by households, more than 40000 people now live between $d_L''$ and $b''$, the area from which the SBD gleans its labor supply. As a result, firms will now lower their wage offer to $w_s'' = w_c - 2t_c d_L'' + t_c d_{sub}$, where $d_L'' > d_L''$. The city area now shrinks inwards a bit, with $b''$ being the new boundary. Even at this point, there is a problem. The SBD workers are now being overcompensated 23 due to the anomaly. This intermediate rent profile is not depicted in the figure. Following a further movement towards the suburbs, so the SBD wage must be lower still. The end result is still a more spread out city than the corresponding HM one with a local rent peak at the subcenter and lower rents near the center, as shown by the rent gradient $R_W''$ in Figure 1.14.

The information in Table 1.4b does not show the values for the intermediate rent gradient $R_W''$. The bottom half of the table, for values of $d_{sub}$ greater than $d^*$, shows the values for the final equilibrium of the city, viz. the values associated with $R_W''$. The rent profiles shown in Figures 1.12 and 1.13 represent the final equilibria in the city, equivalent to the final rent gradient $R_W''$ in Figure 1.14. The values of $w_s''$, $d_L''$ and $b''$ can be found from Table 1.4a, while the values for $w_s$, $d_L$ and $b'$ can be found from 1.3a. Table 1.4c above shows how data from Tables 1.1b, 1.3a and 1.4a can be tabulated to find wages and distance measures used in Figure 1.14. Notice that the regions $d_L$-$b$, $d_L$-$b'$ and $d_L''$-$b''$ are not comparable due to differences in computational methods, along the lines described in Section 1.1b.

<table>
<thead>
<tr>
<th>$d_{sub}$</th>
<th>$d_L$</th>
<th>$w_s$</th>
<th>$b'$</th>
<th>$d_L'$</th>
<th>$w_s''$</th>
<th>$b''$</th>
<th>$d_L''$</th>
</tr>
</thead>
</table>

**TABLE 1.4c**

Referring back to Table 1.4a, one difference between the ESZ and the EDZ is immediately apparent. In the case of the closed $E_w$ city, there is always an increase in utility as $d_{sub}$ moves further and further out. However, the effects on the city, on all

---

23 The amount of overcompensation is $2(w_c - w_s)(1/f)d_L''$. Since $w_s$ and the number of SBD workers are both higher, it is uncertain whether the OE is positive or negative.
counts, are much larger when employment decentralizes in the EDZ as compared to when it locates in the ESZ. This is not surprising.

When employment locates in the ESZ, the upper limit for changes in the Ewage city is set by the HM city level. Since the latter is derived from the overcompensation effect, the changes are not too big to start with. The wage effects in the Ewage city push SBD wages down and cause the changes to be even smaller. There are some discernible effects compared to the monocentric city, since the SBD wage offers even in the Ewage city do not leave people indifferent between working at the center and the subcenter.

On the other hand, the wage effect when employment locates in the EDZ in the Ewage city is by definition big, especially when the suburb moves close to the boundary of the original city. For example, HM wage would generate less than 6% of the labor force 20 miles out. To induce 40% of the workforce to travel to the SBD instead, the Ewage offer must perform more be significant. As the SBD moves towards the original boundary, the overcompensation effect is ambiguous, since \( w_s \) and \( N_s \) both increase from the HM city levels. However, the gain in strength of the Ewage effect is quite significant. This can be seen from the graphs and the table, the average effect is about 10 times stronger (per unit mile move outwards by \( d_{sub} \)) in the EDZ compared to the ESZ. All information in Table 1.4a and Figures 1.12-1.14 is completely consistent with the analysis at the beginning of this section.

THE OPEN CITY CASE

Deriving the equilibrium values for the open Ewage city is more complicated than it was doing the same for the open HM city. The utility level is held fixed at 17.5938 and the total population is allowed to vary, but there is one constraint. The subcenter must employ 40% of the total population. I again use the model with suburban employment presented in Section 1.3b, but the following modification of equation (1.13) is now used (instead of equations (1.20a - b)) to determine the only endogenous variable \( w_s \):

\[
N_c = (0.667).N_s
\]  

(1.21)

Given \( d_{sub} \), the model is then solved numerically using FSOLVE and ODE45, via the following loop:
Guess \( w_s \)

\[
\begin{align*}
&\downarrow \\
&\downarrow \\
&\text{Given all parameter values, determine rents at all } d_c \text{ for CBD and SBD workers} \\
&\text{Use rule (1.11) to determine who lives where by outbidding the other group.} \\
&\text{Check that } R_c \geq R_s \text{ and } R_s \geq R_a. \\
&\downarrow \\
&\downarrow \\
&\text{Once the groups are settled, use the ODE45 solver to} \\
&\text{determine } N_c \text{ and } N_s \text{ by solving the differential equations (1.17a) and (1.17b).} \\
&\downarrow \\
&\downarrow \\
&\text{Check if equation (1.21) is satisfied.} \\
&\text{If not} \\
&\downarrow \\
&\downarrow \\
&\text{Once all other characteristics of the city have been determined,} \\
&\text{adapt equation (1.5) to find the new city boundary } b'' \text{ and } d_L''.
\end{align*}
\]

<table>
<thead>
<tr>
<th>( d_{sub} )</th>
<th>( w_s )</th>
<th>( N_c )</th>
<th>( N_s )</th>
<th>( N_c / N_s )</th>
<th>( d_L'' )</th>
<th>( b'' )</th>
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<td>1.4961</td>
<td>19.4</td>
<td>27.043</td>
</tr>
</tbody>
</table>

**TABLE 1.4d: EQUILIBRIUM VALUES FOR THE OPEN EWAGE CITY WITH DECENTRALIZED EMPLOYMENT (UTILITY LEVEL = 17.5938)**

\[ ^{24} d_L'' \text{ is still the interface between the CBD and SBD workers, beyond which 40\% of the total population of the city reside.} \]
Table 1.4d lists the equilibrium values for selected distances of decentralization. Comparing the equilibria in Figure 1.15 below, points of similarity with and difference from the open HM city equilibria are immediately noticeable. Like the open HM city, there are no perceptible changes in rents in the innermost part of the city. However, when employment decentralized in the open HM city, there was a rise in rents at the subcenter and beyond. For example, when the SBD formed 6 miles out, rents would rise from 6 miles out and beyond. In the open Ewage city, however, there are no changes in rents up to a distance of approximately 15 miles out.

**FIGURE 1.15: COMPARISON OF RENTS: MONOCENTRIC CITY AND OPEN EWAGE CITY WITH THE SBD 6, 11, 16 AND 20 MILES OUT (CLOSE VIEWS)**

**LEGEND:** r1c -- monocentric city rents; r2cW/6/11/16/20 -- Ewage city rents
Figure 1.16 above compares the HM and Ewage cities. From Figures 1.5 and 1.15, I know that when employment decentralizes beyond 15 miles out, there are no effects in the region 0-15 miles out from the CBD. Furthermore, even for decentralization 6 and 11 miles out, there are no effects on rents close to the center. These conclusions allow me to choose the scaling of Figure 1.16 above in a way as to best facilitate the presentation of the available information. In other words, even though the rent profiles for the cities with SBDs at 16 and 20 miles are omitted and I exclude the first 10 miles of the city, all the pertinent information is included in Figure 1.16. As mentioned before, the difference between the open HM city and the open Ewage city is that the former city shows rising rents from the subcenter and beyond, whereas the latter do not show any discernible effects in the Excess Supply Zone.

Next, I compare the effects on the city when employment decentralization occurs under the HM and the Ewage frameworks in the open city. I analyze the changes with close-ups near the boundary in Figure 1.17a and b below.
FIGURE 1.17a: COMPARISON OF RENTS: MONOCENTRIC CITY, OPEN HM CITY AND OPEN EWAGE CITY WITH SBD 6 AND 11 MILES OUT (CLOSE VIEWS)

FIGURE 1.17b: COMPARISON OF RENTS: MONOCENTRIC CITY, OPEN HM CITY AND OPEN EWAGE CITY WITH SBD 16 AND 20 MILES OUT (CLOSE VIEWS)

LEGEND: as before
The thing to notice from the close views near the boundaries of the monocentric, open HM and open Ewage cities in Figures 1.17a-b is the difference between when employment decentralizes in the ESZ (Figure 1.17a) and when employment decentralizes in the EDZ (Figure 1.17b). In every case, the city stretches out. In the case of employment decentralizing in the ESZ, the effects on the HM city are greater than the corresponding Ewage city equilibria. In the EDZ, the effects on the Ewage city are huge, compared to the corresponding HM city results. This distinction is most blatant when employment decentralizes very close to the original city boundary. For decentralization 20 miles out in the open HM city, there is a significant rise in rents at and beyond the SBD, but all the action occurs in the outskirts of the city, and there are no changes in commuting patterns. The CBD is still dominant. In the Ewage case, however, there is a huge change - to the extent that there is a local rent peak at the SBD, and some people living between the CBD and the subcenter start commuting outwards. The same is true for decentralization 16 miles out, but the difference is smaller.

The effects on rents (and population) in the Ewage city, as seen from Table 1.4d and Figures 1.15-1.17, are consistent with the analysis of the closed city cases. First, the effects of decentralization and offering Ewage are felt only by the subcenter workers. Second, there is again a distinction between decentralizing in the ESZ (0-\(d^*\) miles out) and in the EDZ (\(d^*-b\) miles out). In the ESZ, less than 40% of the original population would be living outside the SBD loop originally and the SBD has to attract people from outside the loop. Comparing the values of \(d_L^*\) from Table 1.4d and \(d_L'\) from Table 1.3b, the major difference between the HM city and Ewage city is again apparent. The SBD in the former city gets all the workers from outside the loop and none of the workers from inside the loop. Since the CBD wage remains at $20, the benefits of suburbanization accrue only to the SBD workers and any effects on rents are felt where this group lives. In the HM city, the SBD workers live outside the loop and all effects are felt there. In the Ewage city, there are two possibilities. In the ESZ, only a portion of the workers outside the loop work at the subcenter (from 14.43 miles for \(d_{\text{sub}} = 6\) and 14.57 miles for \(d_{\text{sub}} = 11\)). Hence effects on rent are only felt beyond that point. In the EDZ, some workers from inside the loop commute in the reverse direction (from 15.5 miles for \(d_{\text{sub}} = 16\) and from 17.9 miles for \(d_{\text{sub}} = 20\)) and since benefits accrue to those working at the subcenter, the effects on rent are felt from the interface inside the loop.
The effects on the city when employment decentralizes in the ESZ cannot be larger than in the HM city, since the wage effects dampen the changes. When employment decentralizes in the EDZ in the Ewage city, the large boost in wages compared to the HM case produces large differences. In both cases, the overcompensation effects are uncertain, since there are contradictory effects.

GENERAL CONCLUSIONS

A number of facts can be observed when decentralization occurs under the Ewage regime. First, decentralization in the Ewage city initially benefits only the SBD workers. This benefit ends up making people want to work at the SBD and move towards the suburbs to consume more land. In the closed city, the outward movement ends up improving the lot of the CBD workers with fixed wages as well, since they have more land to themselves. Rents fall in the interior of the city and rise near the boundary, though the amount of flattening depends on the zone of decentralization. In the open city, in migration occurs till everyone is back to the initial level of utility. Population increases everywhere, but rent increases only occur close to and beyond the interface where the SBD workers start to outbid the CBD workers for the land.

Second, the importance of $d^*$ is pivotal. The value of $d^*$ depends on the amount of labor the SBD firms demand in the Ewage city. It divides the urban area into the Excess Supply Zone and the Excess Demand Zone, and the characteristics of decentralization in the Ewage city are very dependent on this division. There are two things which happen to change the face of the Ewage city -- the overcompensation effect and the wage effect. In the ESZ, the wage effect is small -- it is a toned down version of the overcompensation effect from the HM city. In the EDZ, both effects are present (though the OE is may be higher or lower) but it is not possible to tell them apart as I solve for equilibrium values.

Third, the Ewage city breaks down the compartmentalized nature of the HM city. When the subcenter locates in the ESZ, Ewage offers generate cross commuting from the suburbs to the CBD. However, the pattern of inward commuting continues, akin to the monocentric and the HM city. Offering Ewage to overcome labor scarcity in the suburbs is more of a deviation off the familiar path. There is cross commuting in the reverse direction; some people who live in the central city now commute to the suburbs every day. This is
more than just completing the other side of the puzzle from the case of decentralization within the ESZ. The whole basis of the monocentric city is now broken down with the pattern of reverse commuting. No longer are jobs essentially more centralized than households, especially when a fixed amount of employment moves out close to the original boundary.

Finally, the Ewage framework is not the most efficient framework, since there is not a downward sloping labor demand curve. When employment locates in the ESZ, the CBD is still dominant. When employment locates in the EDZ, however, the dominance of the CBD is eroded, the more so as the SBD moves closer to the original boundary. This is a strong refutation of the HM conjecture and the validity of the wage gradient approach.

1.2b.3. COMPARING THE HM AND EWAGE CASES FOR CLOSED CITIES

In this section, I consolidate the findings from the previous two sections by comparing and contrasting the nature of equilibria in the HM city and the Ewage city as employment decentralizes further and further from the CBD. I look at five major indicators of the structure of the city as $d_{sub}$ varies from 6 miles to 20 miles out from the center: SBD wages (CBD wages are fixed), utility levels, boundaries, the location of $d_{L'}$ (in the HM city) and $d_{L''}$ (in the Ewage city), and commuting patterns. Except in the last case, I look at normalized (with respect to the monocentric city levels) values of the indicators. Where relevant, the monocentric city level is marked at 1 on the graph as a point of comparison.

SBD Wages

Figure 1.18 below depicts the standard SBD wage gradient when decentralization occurs in the HM city, shown against the wage function as employment decentralizes in the Ewage city. The point where the two curves intersect is of course $d^*$ (14.572 miles). The HM wage gradient has a constant downward slope throughout the city area, as expected from the equation form $w_{shm} = w_c - tz_{sub}$. The Ewage offers are lower than the HM wage offers in the excess supply zone (0-14.572 miles from the center), as SBD firms offer equilibrium wages to attract only 40000 workers, as opposed to all that they would get with HM wages. In the EDZ (14.572-21 miles out from the center), however, the Ewage
gradient flattens -- almost to the point of being horizontal. This is due to the SBDs desire to maintain the same number of workers (40000) even as $d_{sub}$ locates further out and the labor scarcity problem worsens. The salient feature of the comparison is the breakdown of the wage gradient when employment decentralizes in the EDZ in the Ewage city. The significantly higher relative wages for this zone in the Ewage city usher in the results that follow.

**FIGURE 1.18: NORMALIZED SBD WAGES IN CLOSED HM AND EWAGE CITIES**

**LEGEND:** wHM -- wages in HM city; wEwage -- wages in Ewage city

How much land area do the 60000 people closest to the CBD occupy?

Figure 1.19 below shows the changes in the land area occupied by the 60000 workers who live closest to the CBD as $d_{sub}$ increases. I use $d_L$ (14.2159 miles), the distance beyond which 40000 people live in the monocentric city as the numeraire. It is instructive to compare $d_L'$, the distance beyond which 40000 people live in the HM city, and $d_L''$, the distance beyond which 40000 people live in the Ewage city as $d_{sub}$ increases. The comparison brings to light the basic differences between the modes and zones of decentralization. The curves are aligned around $d^*$ (14.572 miles), the point where the two curves interact. From previous analysis, I know that with CBD wages fixed at $20, an
increase in land area per inhabitant will make these people, and by the equal utility condition everyone else in the city, better off. The changes in \( d_L' \) are thus directly reflected in changes in the utility level. Also, the movement towards the suburbs will cause the city to stretch its boundaries. The changes in the middle of the city arise from the same source as, and thus are mirrored in, the changes in \( V^* \) and \( b \).

![Equilibrium levels of \( d_L \) and \( d_L'' \)](image)

**Figure 1.19: Normalized \( d_L' \) in Closed HM City and \( d_L'' \) in Ewage City**

**Legend:**
- `mono`:
- `dLHIM`:
- `dLEwage`:

Distance of SBD from Center (\( d_{sub} \))

Consider the HM case. If the HM wage offer left subcenter firms completely indifferent between working at the CBD and the SBD, the rent and population density gradients should remain unaltered, i.e. everyone in the city should remain where they were before. Figure 1.19 confirms what I determined before, viz. the existence of the overcompensation effect. The overcompensation of suburban employees causes them to be better off. Higher land demand from them causes movement out towards the suburbs. The amount of movement is reflected in the upward shift in \( d_L' \), signifying more land for the 60000 people who live between 0-\( d_L' \). As \( d_{sub} \) increases, \( d_L' \) starts to rise, signifying increasing land area for the 60000 people and a corresponding increase in overall utility. The overcompensation effect is highest, and \( d_L' \) peaks, when \( d_{sub} \) is 11 miles out. As \( d_{sub} \)
increases further, the overcompensation effect is on the wane, and the shift in $d_L'$ grows smaller. A corresponding decrease in utility level would be expected.

In the Ewage city, when $d_{sub}$ increases in the ESZ, more than 40000 workers are available to the SBD when they offer HM wages. The SBD firms will thus offer an equilibrium wage lower than HM wage, decreasing the overcompensation to the SBD workers as a whole and reducing the amount of movement towards the suburbs. $d_L''$, the corresponding distance to $d_L'$, will thus be between $d_L$ and $d_L'$ and stable, especially between $11-d^*$ miles, where the overcompensation effect has begun to wear off and there is not a big excess supply. In the EDZ, it is a completely different story. Once $d_{sub}$ moves beyond 14.572 miles, the SBD offering HM wage cannot find 40000 workers. When it increases its job offer to attract all people living beyond $d_L'$ (the corresponding HM city distance) inside the loop, the offer proves too high. First, the higher wages prompt a move towards the suburbs to buy cheaper land and live near the workplace. Second, the wage offer will still overcompensate people as long as time costs are evaluated at wages. Due to the combination of these factors, the effects on $d_L''$ as employment decentralizes further out in the EDZ is much more significant. As Figure 1.18 shows, wages fall little as the SBD moves closer to the boundary. This translates into a progressively bigger movement outward as $d_{sub}$ increases. Now a much larger effect on the city - both on the size and the utility level - may be expected as the CBD workers (60000 of them) have more and more area to live in. The effects certainly far outweigh the effects in the HM city, where the overcompensation effect steadily dwindles as the SBD moves closer to the boundary.

Utility Levels

Next, I look at the effects on the utility levels in the closed HM city and Ewage city as $d_{sub}$ moves further and further out. Figure 1.20 depicts the changes in utility levels, which are compared against the numeraire utility level in the monocentric city (17.5938). Once again, the utility curves for the HM city and the Ewage city intersect at $d^*$. In the previous analysis, I predicted that the changes in the utility level will mirror the changes in $d_L'$ and $d_L''$. This is borne out by the graph.

The HM city experiences an increase in utility in the region 0-11 miles out from the center, and utility levels decline thereafter till they reach the monocentric city level when the
SBD forms at the initial boundary of the city (20.8469 miles out). The effect of the overcompensation are reflected in the outward movement of population (reflected by changes in $d_L$' in Figure 1.19). These are smallest in the zone 10-14 miles out, similarly for utility levels.

Utility gains in the Ewage city are lower than in the HM city when employment decentralizes in the ESZ, due to smaller overcompensation (to only 40000 workers) and consequently, the comparatively smaller space that 60000 people have to live in (shown by the smaller movements in $d_L$' in Figure 1.19). After $d_{sub}$ crosses $d^*$ and moves into the EDZ, however, the utility gains not only exceed those in the HM city over the same region, they are far larger than the gains in the ESZ.

This demonstrates that when employment decentralizes in the EDZ, the gains in utility from the SBD switching from HM wage to Ewage in a closed city are far greater than only the gains from the overcompensation of suburban workers through the HM wage offers. Moreover, the effects on utility in the EDZ can be said to represent a "utility
gradient", which presence of which is the antithesis of the wage gradient approach to employment decentralization. Reviewing the nature of the problem, it is not difficult to see why. The Cobb-Douglas functions make it simpler mathematically, but the result is easy to intuit.

Consider the labor supply functions in equation (1.13). With \( \bar{N} \) remaining fixed in the closed city, the effects of suburbanization will be felt on one of the endogenous variables on the right hand side of the equation. When employment decentralizes in the ESZ in the HM city, with the SBD wages declining exogenously, utility levels must rise by only small amounts to counter the overcompensation effects. For the Ewage city, the effects are even smaller.

When employment decentralizes in the EDZ in the Ewage city, \( w_e \) falls by very small amounts. To compensate the effects of the equilibrium wage offer, \( V^* \) must rise significantly. The downward sloping wage gradient in the HM city thus gets replaced by the upward sloping utility gradient when employment decentralizes in the EDZ in the Ewage city. In the HM city, the significant and continuous drop in wages confine the effects on utility to small levels.

**City Size**

Figure 1.21 below illustrates the effects of employment decentralization on the size of the city. These can be judged by the effects on the boundary. The shifts in the boundary are direct consequences of what happens to the three previous variables. When SBD workers earn more, they will demand more land. To avail of cheaper land, they move to the suburbs, stretching the boundary and making for a less dense city. The only thing of note is the common sense but politically incorrect observation that wastage of space actually increases the well-being of all city residents. This result is obtains when I consider a city in isolation. For an entire region, it might not be more efficient to have less crowded cities with high utility levels and more crowded cities with low utility levels. In the absence of barriers (e.g. international borders and immigration laws), such a situation cannot persist, since people will migrate and equalize utilities.
Commuting Patterns

Figure 1.22 shows the changes in commuting patterns as the decentralized city moves from a HM regime to a Ewage regime. The graph plots the equilibrium values for the interface between the CBD and SBD workers as $d_{sub}$ increases - $d_1'$ for the HM city and $d_1'' (= d_2''$) for the Ewage city. As shown by the 45° curve for the HM city, the city is fully compartmentalized with no cross commuting between the areas inside and outside the SBD loop. The Ewage city curve, which deviates from the former, shows the pattern of cross commuting between the regions.

In the ESZ, $d_1'' > d_1'$, implying some of the people living outside the SBD loop commute to the CBD workplace. In spite of the cross commuting, the inward pattern of commuting implies that rents still fall continuously as one moves away from the center, maintaining a basic characteristic of the monocentric city structure. In the EDZ, however, $d_1'' < d_1'$, implying that some people living inside the loop now commute outward towards
the SBD job site. This reverse commuting represents the final breakdown of the monocentric city residential structure.

![Graph showing commuting patterns](image)

**FIGURE 1.22: COMMUTING PATTERNS IN CLOSED HM CITY AND EWAGE CITY**

**LEGEND:** dlHM -- $d_i$ in HM city; dlEwage -- $d_i''$ in Ewage city

### I.2b.4. COMPARING THE HM AND EWAGE CASES FOR OPEN CITIES

To wrap up the analysis of the effects of employment decentralization in the homogeneous city under the two basic regimes, I compare the changes in some of the major variables in the closed and the open cities.

**SBD Wages**

Figure 1.23 compares the relative SBD wage offers in the closed and open cities. Both the closed and open HM cities show the same wage gradient. The difference occurs in the open city cases. However, even the SBD wage offers in the open HM and Ewage

---

25 Note that $d_i'' = d_L''$ in the Ewage city.
cities do not differ by much in the ESZ. They are both lower than the HM wages, but not significantly different from each other. In the EDZ, Ewage offers in the open city are lower than the corresponding offers in the closed city, and both of them are higher than the wage offers in the HM cities. The thing to note is that in the EDZ, the downward slope of the wage offer curve in the open Ewage city still deviates from the HM wage gradient by a significant degree. Compared to the very flat wage offer curve for the closed Ewage city, the wage offers in the open Ewage city are also fairly flat, though they are lower everywhere.

**Total Population**

Figure 1.24 analyzes the effects of employment decentralization on the total population of the city. Population levels are normalized to 100000, the population in the monocentric city, the closed HM city and the closed Ewage city. Since $V^*$ is fixed at
17.5938 in the open city, the influences of employment suburbanization will be felt on the population level -- similar to the changes in utility levels in the closed city.

![Graph](image)

**FIGURE 1.24: NORMALIZED POPULATIONS IN CLOSED AND OPEN HM AND EWAGE CITIES**

**LEGEND:** mono -- population in monocentric, closed HM and closed Ewage cities; NHM (open), NEwage (open) -- population in open HM and Ewage cities

The curves for the open HM city and the open Ewage city appear to be somewhat similar in nature to the corresponding utility gradients from the closed cities. This is to be expected, of course. When employment decentralizes in the HM city, the amount of the overcompensation and the constant decrease in wages lead to comparatively small increases in population, which are strongest over the zone 10-14 miles and dwindle near the CBD and the city boundary. In the Ewage city, population rises but to a smaller extent than in the HM city, since the overcompensation now accrues to only 40% of the population. In the EDZ, however, there is a upward "population gradient". The effects on population will be smaller, since the wage offers in the open Ewage city are lower than those in the closed Ewage city.
Commuting Patterns

Figure 1.25 below compares the commuting patterns in the closed and open cities, based on the shifting values of the interface between CBD and SBD workers. The $45^\circ$ curve represents the interface in both the open and closed HM cities, representing the compartmentalization of the city even after employment decentralizes. In both the open and closed Ewage cities, this no longer holds. When employment decentralizes in the ESZ, people living outside the loop commute in to work at the CBD; and when employment decentralizes in the EDZ, people living inside the loop commute the opposite way every morning to work at the SBD. The latter case breaks down the last link to the monocentric city -- the pattern of commuting inward towards the CBD. What can be observed from this graph is that the interface values are higher for the open Ewage city everywhere, but the difference is much more prominent for decentralization in the EDZ. What this signifies is that in the ESZ, the distance of commute will be slightly greater in the open Ewage city compared to the closed city case, whereas in the EDZ, the distance of commute will be
significantly lower in the open city than the closed city. This should make sense. When employment decentralizes in the EDZ and major benefits accrue to SBD workers, the amelioration is somewhat tainted, and the desire to move outward somewhat dampened, by the people coming in from outside the city. The in migrants will also want to work at the SBD and live close to the workplace, increasing the supply of labor closer to the SBD. Hence SBD firms will draw workers from a smaller, but more crowded, area. In the ESZ, on the other hand, the in migration will tend to make less land available for everybody, packing the people closer together close to the SBD.

GENERAL ANALYSIS AND CONCLUSIONS

When analyzing and contrasting the effects on the open and the closed Ewage cities, the first thing to recognize is that the effects are originating from the two sources mentioned before, viz. the (comparatively small) overcompensation effect by itself in the ESZ and the overcompensation reinforced by the offering of Ewage in the EDZ.

(A) Decentralization in the ESZ

The HM wage gradient comes close to making all people perfectly indifferent between working at the center and the suburb, hence the small overcompensation effect. When population is fixed and wages adjust downward rapidly in the closed HM city, there will be a relatively small rise in utility level, just enough to compensate for the overcompensation of suburban employees. Since the overcompensation accrues only to SBD employees, the benefits also accrue to them in the first instance. But as they demand more land with their higher income, there is a population shift towards the suburbs, increasing the land area available to the CBD workers, who get a utility boost even with fixed incomes. A similar pattern is seen in the open HM city. With utilities fixed, the wage overcompensation translates into a rise in population, but there are no effects on city structure close to the CBD. For the Ewage cities, the upper limits are set by the HM city. All Ewage adjustments are downward. In the closed Ewage city, less people have to be enticed into working at the SBD, hence the lower wage offers. The offers are even lower in the open city, since some of the SBD labor comes from outside the city, and a smaller income boost to SBD workers is sufficient to generate this inflow.
(B) Decentralization in the EDZ

In the EDZ, however, firms must really break away from the HM mentality if they are to follow an Ewage structure. The effects on the closed city when employment decentralizes beyond $d^*$ are much more pronounced than in the HM case, as firms must make wage offers high enough to lure people living inside the SBD belt to commute in the reverse direction every morning. Thus the SBD workers are given a substantial income boost compared to the monocentric and HM city. Their increased demand for land, the outward shift in population and the increased land area for the CBD workers are all proportionately higher.

Finally, the movement from the HM regime to the Ewage regime generates endogenous wages at the subcenter. When employment decentralizes far enough from the center, the dominance of the CBD is also broken down. In the process, an upward sloping utility gradient replaces the standard downward sloping wage gradient for far away locations of the SBD. However, this does not complete the story, since there is no full labor market yet. Switching form a perfectly horizontal to a perfectly vertical labor demand curve affords some insights into the process of employment suburbanization, but only a downward sloping labor demand curve will tell the full story. I do this in Chapter 3, with a partial general equilibrium model which incorporates a full labor market.
CHAPTER 2: SPECIAL CASES OF EMPLOYMENT DECENTRALIZATION
IN PARTIAL EQUILIBRIUM MODELS OF URBAN AREAS

2.1. DECENTRALIZATION OUTSIDE THE INITIAL CITY LIMITS

Referring back to Figure 1.1, all discussion till now has centered around cases where firms decentralize within the initial commuting zone, i.e. somewhere on the ray running from the CBD to B (the city boundary). If decentralization occurs at a point like S, the HM wage gradient is an option. Consider, however, a suburban firm which locates at S', outside the original commuting zone.1 Would HM wages still be viable?

Consider someone living at S. The HM wage offer should leave her indifferent between working at the CBD and the SBD. Hence a CBD worker and a suburban worker should be able to offer the exact same rent if they were to vie for land at S. There are no conceptual problems with this approach so long as S is within the city limits.

If firms locating at S' were to offer HM wages, then a worker living at S' must be indifferent between working at the CBD or at S' itself. This would mean that this person would only be able to offer rents lower than that made by someone living at B and working at the CBD. But B is defined as the distance beyond which the land rents fall below the agricultural rent $R_a$ for the monocentric city. Hence, if the firms were to offer HM wages, no one working at the CBD would gain access to land at S'. Tables 1.3a and 1.3b show that the rent profile of the city remains completely unaltered when suburban firms locate beyond the initial boundary in the closed and open HM city. Not even one person is leaving her CBD job.

The minimum viable wage offer from SBD firms at S' would thus have to be higher than the HM wage offer at B. In real life, they can afford this for any number of reasons -- an excess demand for labor and cheaper land, to name two.

At the moment, I do not have a full-fledged labor market in the model. When I do so in the next section, I can find the labor demand (and the wage offer necessary to obtain

1 As I mentioned before, the location of the SBD often reflects the opportunities made available to the firms, whether as a function of their own requirements, as a result of local planning, fiscal concessions etc. Hence it is possible for the SBD to form anywhere in the city, and also outside its boundaries.
them) at points like B and S' endogenously. For the moment, as HM wage is not a plausible wage offer, I consider the case where 40% of the jobs move out to the SBD. Figure 2.1 below depicts the effect of firms decentralizing outside the initial urban area (22 miles out, well outside the original city boundary) in the closed and the open city. The Ewage offers and other important variables for these cities can be found in Tables 1.4a-b.

![Graph showing rent profiles](image)

**Figure 2.1: Comparison of Rent Profiles When SBD Forms 22 Miles**

**Legend:**
- r1c - rents in monocentric city
- r2cEwage22 (closed) - rents in closed city
- r2cEwage22 (open) - rents in open city
- Ra - agricultural rents

For the closed city in Figure 2.1, the pattern of residential location is clearly affected both inside and outside the initial commuting zone. The rent peak around the subcenter shows the existence of reverse commuting. Most importantly, as households move to stay near their new suburban jobs and reduce commuting, the city actually tends to break in two -- with a satellite city forming around the suburban employment center. In the graph, there is a 'hole' in the city in the region 17.611-18.774 miles from the center. The subcenter city exists between 18.774-25.226 miles. This scenario seems to parallel the experience of cities like Houston (with large areas relative to population and decentralized employment at considerable distance from downtown), which tend to exhibit large tracts of undeveloped land, creating 'holes' in the city.
When employment decentralizes 22 miles out in the open city, a rent peak forms around the sub center and the outer boundary of the city is stretched out. This is to be expected, given that SBD firms do not decrease wages incrementally as they move away from the center. There is an increase in population all over the city. However, note that the *city rent structure is not affected very much in the interior of the city*. Another interesting observation is that the 'hole' in the city no longer exists. Clearly, in the open city, people move in from outside the city to claim the land which would otherwise fall vacant.

Figure 2.2 below reemphasizes the analysis by presenting close views (near the CBD and near the boundary) of the open and closed Ewage cities in Figure 2.1.

![Diagram](https://example.com/diagram.png)

**FIGURE 2.2: CLOSE VIEWS OF FIGURE 2.1 NEAR THE CBD AND THE BOUNDARY**

**LEGEND:** as in Figure 2.1

The case where employment decentralizes outside the original city limits in the closed city is interesting, since it offers an explanation for the existence of vacant lots in the interior of cities with dispersed employment. However, it is not interesting in another sense -- once the city splits up, there are essentially two cities whose utility level is being maximized, subject to the constraint that their equilibrium levels of utility must be equalized at the end. As such, studying the effects of displacing more employment further out falls outside the purview of the analysis. The assumption that a given group of people can spread out over larger and larger areas without encroaching on anyone from outside the city is rather unrealistic. It is important, perhaps, to know the precise moment -- in terms of the location and amount of decentralized employment -- when the city breaks up in two, but beyond that, there is not much to be gained by forcing the issue.
2.2. WHEN A FIXED NUMBER OF JOBS AND WORKERS MOVE IN FROM OUTSIDE THE URBAN AREA

The analyses in the preceding sections have differentiated between the closed city and the open city cases. The assumptions made to preserve either of these frameworks are somewhat rigid. In the real world, while it is true that in heavily urbanized countries any changes in structure are likely to affect all urban areas simultaneously (the closed city case), the creation and location of subcenters and the general movement of jobs can easily happen under circumstances that are unique to a particular city. On the other hand, completely 'free' in-migration is not always possible, except in the case of 'mega cities' with huge hinterlands. It is difficult to find cities which are completely open or closed. More plausibly, I assume that migration occurs due to the movement of job specific human capital, i.e. people who are specialized in a particular job and are forced to relocate when their jobs move. I consider the case of an otherwise closed city which attracts a firm (or an industry) from outside. When the firm(s) move to the urban area, they bring in a fixed number of job openings. But they are not interested in hiring any available labor. Their workers are specialized in their particular line of work, and they (the workers) choose to relocate with the firm. Hence the effects on the city are twofold: N' new jobs move to the city and the same number of workers move in. The firm has a choice of locations. It decides where to locate by maximizing its profit function. When the firm locates at a particular spot, its workers are free to live where they want, but must work at the location chosen by the new firm. The effects depend on where the new firm locates.

2.2a. SIMULATION TECHNIQUES, RESULTS AND ANALYSIS

I consider a firm (or industry) entering the city with 40% of its employment originally decentralized, as described in the Sections 1.2b.2. The new firm brings in N' laborers, and has the option of operating at the CBD, at the SBD or spreading out and splitting its operations between the CBD and SBD. For numerical simulations, I use the same techniques I used for the Ewage city in Section 1.2b.2b, with the exception that the values for the number of jobs at the center and the suburb are now given by \( N'_c \) and \( N'_s \) (which change according to the mode of location selected by the incoming firm), instead of \( N_c \) and \( N_s \), the values used previously. I will call this the "semi-closed" city.
The following modifications are made to the constraints on population:

\[ N_C = N_c' \]
\[ N_S = N_s' \]

where \( N_c' = N_c' + \overline{N}_c \) \( ..... (2.1) \)

where \( N_s' = N_s' + \overline{N}_s \) \( ..... (2.2) \)

and \( N' = N_c' + N_s' \); \( \overline{N}_c' + \overline{N}_s' = \overline{N}' \).

\( N_c' \) and \( N_s' \) are the number of new jobs at the CBD and SBD, respectively, after the new firm has moved into the urban area. The iterations are carried out using the same loop as in 1.2b.2b, where equations (1.20a) and (1.20b) are used to calculate the populations at the center and the subcenter given the parameters and the guess values of the dependent variables, then the results are checked against equations (2.1) and (2.2). I analyze three modes of decentralization: (i) all new employment locates at the CBD, (ii) all new employment locates at the SBD and (iii) half the employment is located at the center and half is located in the suburb. Table 2.1a-c below list the comparative static results for \( N' = 5000 \) and SBD's at distances of 6-22 miles from the center.

### TABLE 2.1

**THE EFFECTS OF 5000 NEW JOBS MOVING INTO THE CITY WITH TWO CENTERS (\( N = 100000 \), \( w_c = 20 \) AND 40% INITIAL DECENTRALIZATION)**

<table>
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<th>( d_{sub} )</th>
<th>( w_g )</th>
<th>( V^* )</th>
<th>( N_c )</th>
<th>( N_s )</th>
<th>( d_L'' )</th>
<th>( b'' )</th>
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**TABLE 2.1a: THE DECENTRALIZED CITY WITH ALL NEW JOBS LOCATED AT THE CBD (\( \overline{N}_c = 65000 \); \( \overline{N}_s = 40000 \), \( d_L = 15.04 \))**
<table>
<thead>
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<th>(w_s)</th>
<th>(V^*)</th>
<th>(N_e)</th>
<th>(N_s)</th>
<th>(d_L'')</th>
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TABLE 2.1b: THE DECENTRALIZED CITY WITH NEW JOBS SPREAD BETWEEN THE CBD & THE SBD \(\overline{N_e} = 62500; \overline{N_e} = 42500, d_L = 14.6276\)  

<table>
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<th>(w_s)</th>
<th>(V^*)</th>
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<td>60132</td>
<td>44869</td>
<td>17.9951</td>
<td>25.7987</td>
</tr>
</tbody>
</table>

TABLE 2.1c: THE DECENTRALIZED CITY WITH ALL NEW JOBS LOCATED AT THE SBD \(\overline{N_e} = 60000; \overline{N_e} = 45000, d_L = 14.2159\)  

In the tables above, \(d_L''\) is both the boundary between the residential area occupied by the central workers and that occupied by the subcenter workers (equivalent to \(d_f''\)) and the distance inside which \(\overline{N_e}\) workers used to live in the monocentric and initial Ewage city. These values are to be compared against the monocentric city and the initial Ewage city. Notice that the yardstick for comparison \(d_L\) changes with each mode of decentralization.\(^2\) The relevant values of \(d_L\) are listed with each table. Finally, \(b''\) is the boundary of the city in each case. The values of \(b''\) must be compared to the values from Table 1.4a.

\(^2\) Since each mode of decentralization involves a different number of CBD workers - 60000, 65000 and 70000 - the value of \(d_L\) must be the one that accommodated the corresponding number of people in the monocentric city.
Next, I look at the effects of 10000 new jobs moving into the urban area, using the same three modes of decentralization. Tables 2.2a-c list the results. As before, the changing yardstick $d_L$ is provided for every table.

### Table 2.2
**The Effects of 10000 New Jobs Moving into the City with Two Centers ($\overline{N_c} = 100000$, $w_c=20$) and 40% Initial Decentralization**

<table>
<thead>
<tr>
<th>$d_{sub}$</th>
<th>$w_s$</th>
<th>$V^*$</th>
<th>$N_c'$</th>
<th>$N_s'$</th>
<th>$d_L''$</th>
<th>$b''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>18.2648</td>
<td>17.4304</td>
<td>69951</td>
<td>40022</td>
<td>15.6516</td>
<td>22.0455</td>
</tr>
<tr>
<td>8</td>
<td>17.7258</td>
<td>17.44</td>
<td>69763</td>
<td>40086</td>
<td>15.6527</td>
<td>22.0514</td>
</tr>
<tr>
<td>10</td>
<td>17.2068</td>
<td>17.459</td>
<td>69996</td>
<td>40037</td>
<td>15.7457</td>
<td>22.1539</td>
</tr>
<tr>
<td>11</td>
<td>16.954</td>
<td>17.4708</td>
<td>70019</td>
<td>39980</td>
<td>15.7921</td>
<td>22.2038</td>
</tr>
<tr>
<td>12</td>
<td>16.705</td>
<td>17.4718</td>
<td>70000</td>
<td>40000</td>
<td>15.7922</td>
<td>22.1975</td>
</tr>
<tr>
<td>14</td>
<td>16.2183</td>
<td>17.4858</td>
<td>70066</td>
<td>39945</td>
<td>15.848</td>
<td>22.2534</td>
</tr>
<tr>
<td>18</td>
<td>15.824</td>
<td>17.854</td>
<td>70073</td>
<td>39804</td>
<td>17.0891</td>
<td>23.3192</td>
</tr>
<tr>
<td>22</td>
<td>15.7865</td>
<td>18.377</td>
<td>69974</td>
<td>39911</td>
<td>18.6962</td>
<td>18.845</td>
</tr>
</tbody>
</table>

### Table 2.2a: The Decentralized City with All New Jobs Located at the CBD ($\overline{N_c} = 70000$; $\overline{N_s} = 40000$, $d_L = 15.5$)

<table>
<thead>
<tr>
<th>$d_{sub}$</th>
<th>$w_s$</th>
<th>$V^*$</th>
<th>$N_c'$</th>
<th>$N_s'$</th>
<th>$d_L''$</th>
<th>$b''$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>18.2772</td>
<td>17.4345</td>
<td>64987</td>
<td>45259</td>
<td>14.877</td>
<td>22.0751</td>
</tr>
<tr>
<td>8</td>
<td>17.7409</td>
<td>17.4505</td>
<td>64767</td>
<td>45234</td>
<td>14.8893</td>
<td>22.0893</td>
</tr>
<tr>
<td>10</td>
<td>17.2246</td>
<td>17.464</td>
<td>65325</td>
<td>45200</td>
<td>15.0187</td>
<td>22.22</td>
</tr>
<tr>
<td>11</td>
<td>16.973</td>
<td>17.4819</td>
<td>65001</td>
<td>45047</td>
<td>15.0203</td>
<td>22.2193</td>
</tr>
<tr>
<td>12</td>
<td>16.7255</td>
<td>17.49</td>
<td>64983</td>
<td>45011</td>
<td>15.044</td>
<td>22.2427</td>
</tr>
<tr>
<td>14</td>
<td>16.241</td>
<td>17.502</td>
<td>65216</td>
<td>45007</td>
<td>15.1153</td>
<td>22.3161</td>
</tr>
<tr>
<td>18</td>
<td>16.0431</td>
<td>17.996</td>
<td>65130</td>
<td>45003</td>
<td>16.6739</td>
<td>23.669</td>
</tr>
<tr>
<td>22</td>
<td>15.995</td>
<td>18.48</td>
<td>65171</td>
<td>44928</td>
<td>18.2797</td>
<td>18.6139</td>
</tr>
</tbody>
</table>

### Table 2.2b: The Decentralized City with New Jobs Spread between the CBD & the SBD ($\overline{N_c} = 65000$; $\overline{N_s} = 45000$, $d_L = 15.04$)
\[\begin{array}{|c|c|c|c|c|c|c|}
\hline
d_{\text{sub}} & w_s & v^* & N_e' & N_e'' & d_L'' & b'' \\
\hline
8 & 17.7568 & 17.4635 & 60142 & 49999 & 14.1863 & 22.1339 \\
10 & 17.244 & 17.4762 & 59982 & 50037 & 14.1874 & 22.1333 \\
22 & 16.2349 & 18.6513 & 59874 & 50134 & 17.9099 & 18.1914 \\
\hline
\end{array}\]

**TABLE 2.2c: THE DECENTRALIZED CITY WITH ALL NEW JOBS LOCATED AT THE SBD \(N_e = 60000; \bar{N}_c = 50000, d_L = 14.2159\)**

From Tables 2.1 and 2.2, the range of values of \(d^*\) are still between 14 and 15. Thus the locations of the ESZ and EDZ are the same as in the Ewage city with 100000 people. I choose cities with \(d_{\text{sub}} 6\) and 18 miles out to demonstrate the effects on the semi-closed city in the two zones. In light of the range of values of \(d^*\), these two distances represent the heart of the ESZ and the EDZ respectively. Figures 2.3-2.12 depict the effects on the cities under different modes of decentralization. The graphs use the monocentric city with a population of 100000 as the base case.

Figures 2.3-2.5 compare the cities with different populations under the same mode of decentralization, when the SBD loop forms 6 miles out, comparing each mode (e.g. the cities with 5000 and 10000 new jobs when all of them locate at the CBD) with the monocentric city, the closed Ewage city and the open Ewage city. The graphs use close-ups of the rent profiles near the CBD and the boundary to illustrate the different effects under the various modes and population. Figures 2.6-2.7 then compare the cities with the same population but different modes of decentralization (e.g. cities with 5000 new jobs under all three modes of decentralization), completing the picture and allowing comparisons across modes of decentralization and population levels. Figures 2.8-2.12 then repeat the process for the cities with the SBD 18 miles out.
FIGURE 2.3: COMPARISON OF RENT PROFILES (SBD 6 MILES OUT) WHEN
5000 AND 10000 NEW JOBS LOCATE AT THE CBD (CLOSE UPS NEAR CBD, b)

**LEGEND:**
- r1c -- monocentric city
- rEwage6 (c), rEwage6 (o) -- initial closed and open Ewage city
- rEwage6 (sc65/40), rEwage6 (sc70/40) -- semi closed Ewage city with 5000 and 10000 new jobs,
  respectively, moving to the CBD
FIGURE 2.4: COMPARISON OF RENT PROFILES (SBD 6) WHEN 2500 AND 5000 NEW JOBS LOCATE AT THE CBD AND THE SBD (CLOSE UPS NEAR CBD, b)

**LEGEND:**
r1c, rEwage6 (c), rEwage6 (c) -- as before; rEwage6 (sc 62.5/42.5), rEwage6 (sc 65/45) -- semi closed Ewage city with new jobs split between 2 centers
FIGURE 2.5: COMPARISON OF RENT PROFILES (SBD 6) WHEN 2500 AND 5000 NEW JOBS LOCATE AT THE CBD AND THE SBD (CLOSE UPS NEAR CBD, b)

**LEGEND:**
- r1c
- rEwage6 (c)
- rEwage6 (o)
- rEwage6 (sc60/45)
- rEwage6 (sc60/50)

rEwage6 (sc 60/50) -- semi closed Ewage city with new jobs split between 2 centers
FIGURE 2.6: COMPARISON OF RENT PROFILES (SBD 6) UNDER DIFFERENT MODES OF DECENTRALIZATION WHEN 5000 NEW JOBS COME INTO THE CITY (CLOSE UPS NEAR CBD, b)  
LEGENDS: as in Figures 2.3-2.5
FIGURE 2.7: COMPARISON OF RENT PROFILES (SBD 6) UNDER DIFFERENT MODES OF DECENTRALIZATION WHEN 10000 NEW JOBS COME INTO THE CITY (CLOSE UPS NEAR CBD, b)

ALL LEGENDS: as in Figures 2.3-2.5
FIGURE 2.8: COMPARISON OF RENT PROFILES (SBD 18) WHEN 5000 & 10000 NEW JOBS LOCATE AT THE CBD (CLOSE UPS NEAR CBD, b)

**LEGEND:** as in Figure 2.3
FIGURE 2.9: COMPARISON OF RENT PROFILES (SBD 18) WHEN 2500 & 5000 NEW JOBS LOCATE AT THE CBD AND THE SBD (CLOSE UPS NEAR CBD, b)

LEGEND: as in Figure 2.4
FIGURE 2.10: COMPARISON OF RENT PROFILES (SBD 18) WHEN 5000 & 10000
NEW JOBS LOCATE AT THE SBD (CLOSE UPS NEAR CBD, b)

LEGEND: as in Figure 2.5
FIGURE 2.11: COMPARISON OF RENT PROFILES (SBD 18) UNDER DIFFERENT MODES OF DECENTRALIZATION WHEN 5000 NEW JOBS COME INTO THE CITY (CLOSE UPS NEAR CBD, b)

LEGEND: as in Figures 2.8-2.10
FIGURE 2.12: COMPARISON OF RENT PROFILES (SBD 18) UNDER DIFFERENT MODES OF DECENTRALIZATION WHEN 10000 NEW JOBS COME INTO THE CITY (CLOSE UPS NEAR CBD, b)

LEGEND: as in Figures 2.8-2.10
GENERAL ANALYSIS AND CONCLUSIONS

To analyze the changes in the structure of the city when a fixed number of jobs come into the urban area, information from all preceding sections must be brought together. Given the nature of the population density functions and the forces at work in shaping the city, it is evident that the effects originating from the two sources mentioned in Section 1.2b.2 are still present. For the purposes of this analysis, it is convenient to look at the effects of the different modes of decentralization on the CBD workers, since there are no absolute changes in \( w_c \). In equilibrium, everyone in the city must receive equal utility. Hence any changes in the utility level of the CBD workers at the final equilibrium must reflect the changes for others as well. All references to land demand and land supply changes in the following analysis refers to the relevant levels for CBD workers.

The general effects on the area occupied by the CBD workers can be categorized under the following heads:

(1) Changes in population

(a) Overall Population Effect (OPE): Increase (decrease) in total city population will increase (decrease) demand for land, increasing (decreasing) rents and decreasing (increasing) utility levels everywhere.

(b) Population Effect at Employment Center (PEEC): Increase (decrease) in population around a particular employment center will increase (decrease) land demand near that center, affecting rent offers of that particular group of workers disproportionately more than the rest of the city. When changes occur near the CBD, the effects on land demand are maximum there. When changes occur near the SBD, the effects on land demand close to the CBD are small. Also, when more (less) jobs locate in the SBD, there are more (less) workers benefiting from the overcompensation effects. This will affect the supply of land to CBD workers.

(2) Changes in SBD wages

(a) Overcompensation Effect (OE): The total overcompensation of SBD workers for fixed values of \( w_c \) and \( f \) is given by
OE (d_{sub}, w_s, d_L'', b'') = \int_{d_L''}^{b''} 2 \pi \cdot d_c \cdot n_s \cdot \left( \frac{1}{r} \right) d_{sub} \left( w_c - w_s \right) \quad \ldots (2.3) \\

where \( n_s \) is the population density of the people in the band of land occupied by the SBD workers (between \( d_L'' \) and \( b'' \)) and \( 2(w_c - w_s) \left( \frac{1}{r} \right) d_{sub} \) is the amount of the overcompensation per SBD worker. The following changes are possible.

(i) When more (less) jobs locate in the SBD, \( n_s \) rises (falls), increasing (decreasing) the total overcompensation available to SBD workers.

(ii) When more (less) jobs locate in the SBD, \( w_s \) rises (falls), decreasing (increasing) the total amount of the OE.

Due to the combination of (i) and (ii), the OE peaks at some value of \( d_{sub} \) in the interior of the city (11 miles out in Section 1.2b.2). The more (less) the OE, the more (less) the SBD workers will attempt to get to the suburbs, causing the land supply for CBD workers to increase (decrease), decreasing (increasing) rents and increasing (decreasing) utility.

(b) Wage Effect (WE): To illustrate the WE, consider what happens as SBD firms switch from the HM to the E wage regime. When SBD firms face an excess supply of labor in the ESZ, they decrease wages (from the HM city) to attract only the number of workers they need. Since the number of SBD workers decreases and the amount of OE per worker increases, the total change in the OE is small. The WE dominates, causing less outward movement by SBD laborers, hence lower supply of land and higher rents to CBD workers, lowering the utility level. When SBD firms are faced with an excess demand for labor in the EDZ, they raise wage offers enough to entice people living inside the loop to commute outwards to the loop. This extra fillip in wages causes SBD workers to move towards the suburbs, increasing the supply of land to the CBD workers, decreasing rents and increasing utility. Also, the number of workers increase, raising the OE. But SBD wages increase, depressing the OE. Thus ultimately, the wage effects dominate in the E wage city. The further the SBD is from \( d^* \), the bigger the wage effects.

Overall, the population effects tend to overshadow the weak effects from SBD wages when employment decentralizes in the ESZ. Hence outward shifts in land demand dominate supply changes and rents tend to rise everywhere in the city. When employment
decentralizes in the EDZ, the strong effects on SBD wages are harder to swamp. Land supply to the CBD workers tends to increase more, dropping rents close to the CBD.

The HM wage gradient comes close to making all people perfectly indifferent between working at the center and the suburb, so the overcompensation effect is comparatively small. In the HM city and the open and closed (60/40) Ewage cities discussed in Chapter 1, the analysis is simpler since either the utility level or the population is fixed. The main lesson to be learnt from there is the difference in the approach of SBD employers depending on whether or not the city allows in migration. Wage offers in the open cities will be lower than in the corresponding closed cities, since the labor supply available to the employers is expanded. The complication that arises in the case of the semi-closed city is that both population and utility levels are changing. For the purposes of comparing the various semi-closed cities, Figures 2.13-2.15 look at the wages and utility levels in these and the earlier cities.

**FIGURE 2.13: RELATIVE SBD WAGES IN SEMI-CLOSED CITIES**

**LEGEND:** as in Figures 2.3-2.12

---
3 I use the same terminology as in Figures 2.3-2.12 to denote the different modes of decentralization.
FIGURE 2.14: CLOSE VIEW \( (d_{\text{sub}} = 14 - 18) \) OF FIGURE 2.13
LEGEND: as in Figures 2.3-2.12

FIGURE 2.15: COMPARISON OF UTILITY LEVELS IN SEMI-CLOSED CITIES
LEGEND: as in Figures 2.3-2.12
(A) Employment Decentralization in the ESZ (0-d* miles)

Populations are increasing in the semi-closed city, and this would make the effects on city structure automatically smaller, unless there is a compensating increase in income from some source. However, given that all parameters except wages for SBD workers are held constant, the effects cannot exceed the maximum overcompensation effects, from the closed HM city. To illustrate the point, Table 2.3 below looks at the city structures under different modes of employment location for a case of decentralization in the ESZ (d_sub 6 miles out). As before, I concentrate on the CBD workers for gauging and analyzing the effects of employment decentralization under the various modes under consideration.

<table>
<thead>
<tr>
<th>MODE</th>
<th>w_s</th>
<th>v*</th>
<th>N_C</th>
<th>N_s</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>mono</td>
<td>N.A.</td>
<td>17.5938</td>
<td>N.A.</td>
<td>N.A.</td>
<td>100000</td>
</tr>
<tr>
<td>HM cl</td>
<td>18.4</td>
<td>17.7214</td>
<td>13972</td>
<td>86027</td>
<td>99999</td>
</tr>
<tr>
<td>HM op</td>
<td>18.4</td>
<td>17.5938</td>
<td>14485</td>
<td>91449</td>
<td>105934</td>
</tr>
<tr>
<td>Ewage cl</td>
<td>18.2804</td>
<td>17.6447</td>
<td>59902</td>
<td>40172</td>
<td>100074</td>
</tr>
<tr>
<td>(60/40)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ewage open</td>
<td>18.2785</td>
<td>17.5938</td>
<td>61367</td>
<td>40861</td>
<td>102228</td>
</tr>
<tr>
<td>Ewage sc</td>
<td>18.273</td>
<td>17.522</td>
<td>65132</td>
<td>39860</td>
<td>104992</td>
</tr>
<tr>
<td>(65/40)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ewage sc</td>
<td>18.2648</td>
<td>17.4304</td>
<td>69951</td>
<td>40022</td>
<td>109973</td>
</tr>
<tr>
<td>(70/40)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ewage sc</td>
<td>18.278</td>
<td>17.5361</td>
<td>62659</td>
<td>42529</td>
<td>105188</td>
</tr>
<tr>
<td>(62.5/42.5)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ewage sc</td>
<td>18.2772</td>
<td>17.4345</td>
<td>64987</td>
<td>45259</td>
<td>110244</td>
</tr>
<tr>
<td>(65/45)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ewage sc</td>
<td>18.2846</td>
<td>17.536</td>
<td>60138</td>
<td>44853</td>
<td>104991</td>
</tr>
<tr>
<td>(60/45)</td>
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<td></td>
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<tr>
<td>Ewage sc</td>
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<td>17.434</td>
<td>60135</td>
<td>49789</td>
<td>109924</td>
</tr>
<tr>
<td>(60/50)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 2.3: COMPARISON OF CITY STRUCTURES FOR VARIOUS MODES OF EMPLOYMENT LOCATION WHEN EMPLOYMENT DECENTRALIZES IN THE EXCESS SUPPLY ZONE (6 MILES OUT)**

Except for the closed HM and Ewage (60/40) cities, all other modes increase the total population of the city. The TPE would thus tend to depress the utility level of the city by raising land demand and rents everywhere, lowering the amount of land consumed by each person in equilibrium. However, some of the modes also boost the income of the
people who live in the suburbs and go to work at the SBD. The effects from this source
tend to increase the utility level of all residents of the city, along the lines mentioned
in Chapter 1. The net effect on the city is determined by the relative strengths of all effects.
The maximum utility gain occurs in the closed HM city, since population is held constant at
100000 and $w_s$ is the highest among all modes of decentralization.

Compare the effects when all new jobs are located at the CBD. When 5000 new
jobs enter the city, the TPE is lower than in the open HM city and higher than in the open
Ewage city. The PEEC, however, is significantly higher than both the HM and Ewage
cities. The total population effect is strongly in favor of increasing land demands and
raising rents everywhere. Since all new jobs locate at the CBD, relative wages fall at the
SBD (and the pure WE thus decreases), compared to the HM and Ewage (60/40) cases,
causing less outward movement by SBD employees, tightening the supply of land for the
CBD employees, increasing rents and depressing utility level. The OE effect is about the
same as in the HM case (lower number of SBD employees, lower wages) but higher than
the Ewage case (same number of SBD employees, lower $w_s$), which tends to compensate
the pure WE somewhat. All the effects of the wage change are relatively weak in the ESZ,
therefore the population effects dominate and rents rise and utility falls everywhere.

When 2500 of the new jobs move to the SBD, the TPE is unchanged. The PEEC is
lower since not all the increase in land demand occurs near the CBD. The land demand
schedule for the CBD land area shifts out less than in the Ewage (65/40) city. On the other
hand, the pure wage effect is higher, since $w_s$ must be higher to attract more people to the
SBD. The OE is unambiguously higher than the HM case (more SBD employees, lower
$w_s$) but about the same as the (65/40) mode (more SBD employees, higher $w_s$). The net
effect is that the land supply curve for CBD employees shifts out further. The population
effects still dominate so rents are higher (and utility lower) than in the monocentric city, but
rents fall (and utility increases) a little compared to the (65/40) mode. The net effect is a
city which is packed tighter together everywhere, the boundary $b^*$ shifts in.

Finally, all new jobs locate at the SBD. Though $w_s$ adjusts upwards, there is no
effect on the rents and utility levels, as evident from Figure 2.6 and Table 2.3. The city
gets more spread out, with $b^*$ increasing. There is no effect on the amount of land
consumed in the interior of the city. Outside $d_{1L}$, there is an increase in population density
which does not change much with distance, implying a significant rise in population in the suburbs. The lack of movement in land demand and supply near the CBD (as evidenced by the lack of change in both land consumption and rent levels) as the city moves from the (62.5/42.5) mode to the (60/45) mode must be attributed to the movement of the last 2500 workers who switch workplaces from the CBD to the SBD. The increase in the SBD wage offer is just enough to induce these marginal workers to switch. If they were living at the edge of the CBD land area (between 14.388 - 14.628 miles, the values of \( d_L \)" in the (60/45) and (62.5/42.5) cities) their switching will have minimal effects on the interior of the city. The effect on the SBD workers is much more definitive, since they get a boost in income which causes them to move out, spreading the suburb (notice that \( d_L \)" decreases and \( b^n \) increases as the last 2500 workers switch from the CBD to the SBD). Figure 2.16 illustrates this by comparing close views of population density profiles around \( d_L \)" showing that profiles of the (62.5/42.5) and (60/45) cities diverge only in the suburbs.

When 10000 new jobs enter the urban area, the population effects are even stronger, even the TPE exceeding both the HM and Ewage cases. The wage effects are lower for the first two modes of decentralization, exacerbating the effects on rents and utility. As Figures 2.3-2.7 and Table 2.3 show, the comparative effects on the city are the same as jobs locate according to the three modes, but the level of rents are higher (and utility levels lower) in all cases due to the stronger population effects. The switch from the (65/45) to the (60/50) city still causes minimal change in the interior of the city, however, and the utility levels change very little in the process.

Ultimately, the direction of all changes when employment locates in the EDZ are driven by the stronger population effects, with the wage effects influencing the magnitude of the changes. Among the values of \( d_{sub} \) that I look at, the wage effects are strongest when the SBD is 6 miles out, and the weakest when the SBD is 14 miles out. From Tables 2.1 and 2.2, while the pattern of effects from one mode of decentralization to the other holds for all locations of the subcenter in the ESZ (6-14 miles out), with the utility levels increasing but remaining below monocentric city levels; the levels get closer to 17.5938 as \( d_{sub} \) moves closer to the boundary between the ESZ and the EDZ. When the SBD locates 14 miles out and 5000 new jobs locate in the city, the wage effects are the smallest (i.e. the downward adjustments in \( w_s \) compared to the HM and Ewage (60/40 city are the smallest)
FIGURE 2.16: POPULATION DENSITY CLOSE TO d_L IN SEMI-CLOSED CITIES

**LEGEND:**
- nlc -- population density in monocentric city;
- n2cEwage60/40, n2cEwage(65/40), n2cEwage(62.5/42.5), n2cEwage(60/45) -- population densities in cities with different modes of decentralization
and $V^*$ approaches the monocentric city level of utility. When 10000 new jobs move in, however, the population effects dominate and utility levels are significantly below the monocentric level.

(B) Employment Decentralization in the EDZ ($d^*-b^+ \text{ miles}$)

When employment decentralizes in the EDZ in the closed (60/40) Ewage city, the wage effects are pronounced (and much higher than those in the ESZ) due to the significant upward deviation in SBD income levels. Due to the rise in $w_s$, the OE decreases. But due to the increase in $N_s$, the OE increases. The net effect on land supply for CBD workers is dominated by the pure WE. In the semi-closed cities, the SBD wages are lower than the Ewage (60/40) city, but significantly higher than the HM wages. As Figure 2.13 shows, even most of the cities with at least some of the new jobs locating at the CBD (the 65/40, 62.5/42.5 and 65/45 cases) have a higher $w_s$ than the open Ewage city. This is due to the significantly higher population of CBD workers in the open Ewage city.

The total population of the semi-closed cities is also lower than the open Ewage city. Near the CBD, land demands and rent levels are comparatively low, while the opposite is true near the boundary. Utility levels in all semi-closed cities are thus higher than the open Ewage city. The wage effects tend to dominate the population effects, causing rents to fall near the center and rise near the suburbs. The population effects set the magnitudes of the changes in rents, utilities etc. I look at decentralization 18 miles out as an example of the EDZ. Table 2.4 below looks at the effects on the city under various modes of decentralization.

The general trend of results is evident from Figures 2.13-2.15. Studying the effects on SBD wages and overall utility levels, the first thing that can be established is the much stronger deviations in the EDZ, as opposed to the ESZ. Also, the population effect, the strength of which causes cities with the same population to bunch together when employment decentralizes in the ESZ (notice how the wages and utility levels for the semi-closed cities with 105000 and 110000 people are close together when employment locates upto 14 miles from the CBD) is dominated by the strong wage effects when the SBD locates in the EDZ.
<table>
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<tr>
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<th>$N_C$</th>
<th>$N_s$</th>
<th>$N$</th>
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<td>110008</td>
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**TABLE 2.4: COMPARISON OF CITY STRUCTURES FOR VARIOUS MODES OF EMPLOYMENT LOCATION WHEN EMPLOYMENT DECENTRALIZES IN THE EXCESS DEMAND ZONE (18 MILES OUT)**

Hence $w_s (65/40) >> w_s (70/40)$, $w_s (62.5/42.5) > w_s (65/45)$ and $w_s (60/45) << w_s (60/50)$. The utility levels follow the wage effect. Hence $V^* (65/40) >>> V^* (70/40)$, $V^* (62.5/42.5) >> V^* (65/45)$ and $V^* (60/45) > V^* (60/50)$. The trend clearly is that the cities with more suburban employment in the EDZ overcome the population effects better, to the extent where the Ewage (60/50) city actually is better off than the smaller but more centralized (65/40) and (62.5/42.5) cities. This is completely contrary to the experience with employment locating in the ESZ. The Ewage (65/45) and (70/40) cities do not enjoy this high utility level, implying that though the wage effect dominates the population effect, the latter is still strong enough to subdue utilities below the levels of the smaller cities.

Given this overview, it is relatively simple to see what happens in the EDZ and correlate it to the depiction in Figures 2.8-2.12. The effects of the population changes can be traced through Figures 2.8-2.10. As population increases for the same mode of decentralization, rents go up everywhere in the city. When more of the jobs locate at the CDB (Figure 2.8), the difference in rents is more prominent in the interior, while when
more of the jobs locate at the SBD (Figure 2.10), the difference in rents is more prominent in the suburbs. What is interesting to note is that in the former case (all new jobs at the CBD), there are still distinguishable differences in rents at the suburbs; while in the latter case (all new jobs at the SBD) rents near the CBD are almost the same for the two populations, and very close to the Ewage (60/40) city. This illustrates that most changes occur near the employment center where the new jobs locate and that for all employment at the CBD, the population effects on the entire city are much more significant than for the case where all new employment locates in the suburbs, when the wage effects strongly pull people around the SBD. The bottom left corners of Figures 2.11-2.12 also show the shift in the interface between the areas occupied by the CBD and SBD workers.

Figures 2.17-2.21 below depict the equilibria in the semi-closed city when the SBD locates 22 miles out. This is an extreme example of the EDZ, being outside the boundary of the original city, and it is interesting to note that the 'holes' in the city reappear. The exogenously determined increases in population in the semi-closed cities are not sufficient to close the holes. The open Ewage city represents a population level at which the wage effect is fully compensated by the population effect and hence leave rents in the interior of the city unchanged. As before, Figures 2.17-2.19 depict the population effects, through the rent profiles for the same mode of decentralization but different populations. Figures 2.20-2.21 compare the modes for the same populations. In Figure 2.19, the (60/40), (60/45) and (60/50) cities show no difference in rents close to the CBD. This is natural. With the city essentially split up in two, the changes in the CBD occur near the boundary of the area occupied by the CBD workers. As \( w_s \) increases, people switch to SBD jobs. At the same time, there is enough in-migration to generate the required final equilibrium. The end result is a bigger SBD and a smaller 'hole' as population increases.
FIGURE 2.17: COMPARISON OF RENT PROFILES (SBD 22) WHEN 5000 & 10000 NEW JOBS LOCATE AT THE CBD (CLOSE UPS NEAR CBD, b)

LEGEND: as in Figure 2.3
FIGURE 2.18: COMPARISON OF RENT PROFILES (SBD 22) WHEN 2500 & 5000 NEW JOBS LOCATE AT THE CBD AND THE SBD (CLOSE UPS NEAR CBD, b)

LEGEND: as in Figure 2.4
FIGURE 2.19: COMPARISON OF RENT PROFILES (SBD 22) WHEN 5000 & 10000 NEW JOBS LOCATE AT THE SBD (CLOSE UPS NEAR CBD, b)

LEGEND: as in Figure 2.5
FIGURE 2.20: COMPARISON OF RENT PROFILES (SBD 22) UNDER DIFFERENT MODES OF DECENTRALIZATION WHEN 5000 NEW JOBS COME INTO THE CITY (CLOSE UPS NEAR CBD, b)

**LEGEND:** as in Figure 2.3-2.7
FIGURE 2.21: COMPARISON OF RENT PROFILES (SBD 22) UNDER DIFFERENT MODES OF DECENTRALIZATION WHEN 10000 NEW JOBS COME INTO THE CITY (CLOSE UPS NEAR CBD, b)

LEGEND: as in Figure 2.3-2.7
2.3. EMPLOYMENT DECENTRALIZATION IN TWO SUBCENTERS

Finally, I want to study the characteristics of the city when more than one subcenter loop forms. For comparison, I have looked at cities with subcenters 10, 16, 20 and 22 miles out, and looked at the effects of a second subcenter forming 5, 10, 16, 20 or 22 miles out. I have picked some of the possible permutations to present an idea of the changes in the rent profiles of the city. All the cities considered are closed, for obvious reasons.

2.3a. THE MODEL

To simulate the city with three employment centers, some adjustments to the population constraints are necessary. For the closed city cases, I use the following rent functions for the rent offers from workers at the CBD, SBD₁ and SBD₂, assuming that distances are measured discretely to start with:

\[ R_c(i,j) = \left( \frac{\alpha}{V^*} \right)^{\frac{1}{\beta}} \left[ 8w_c - 2w_c \left( \frac{1}{f} \right) d_c(i,j) - 2md_c(i,j) \right]^{\frac{1}{\beta}} \]  
\[ R_{s1}(i,j) = \left( \frac{\alpha}{V^*} \right)^{\frac{1}{\beta}} \left[ 8w_{s1} - 2w_{s1} \left( \frac{1}{f} \right) d_{s1}(i,j) - 2md_{s1}(i,j) \right]^{\frac{1}{\beta}} \]  
\[ R_{s2}(i,j) = \left( \frac{\alpha}{V^*} \right)^{\frac{1}{\beta}} \left[ 8w_{s2} - 2w_{s2} \left( \frac{1}{f} \right) d_{s2}(i,j) - 2md_{s2}(i,j) \right]^{\frac{1}{\beta}} \]

where \( d_{s1} = d_c - d_{sub1} \) and \( d_{s2} = d_c - d_{sub2} \). I designate the subcenter furthest away from the CBD as SBD₂.

Given the rent bids, I use the following rules for land allocation:

1. If \( R_c(i,j) > \max [R_{s1}(i,j), R_{s2}(i,j)] \), CBD workers get the land.
2. If \( R_{s1}(i,j) > R_{s2}(i,j) \) and \( R_{s1}(i,j) \geq R_c(i,j) \), SBD₁ workers get the land.
3. If \( R_{s2}(i,j) \geq \max [R_c(i,j), R_{s1}(i,j)] \), SBD₂ workers get the land.

Rules (1) - (3) above hold iff max \( [R_c(i,j), R_{s1}(i,j), R_{s2}(i,j)] \) \( \geq R_0 \).
The rules above maintain the assumption that in order to get workers, the firms at the furthest subcenter need only to make workers indifferent between working for them and their competitors closer to the center of the city. The population densities for each group of workers is then given by

\[
    n_c(i,j) = \left( \frac{\alpha}{\sqrt{\pi}} \right)^\frac{1}{\beta} \left( 8w_c - 2w_d \left( \frac{d_c(i,j)}{d} \right) - 2md_c(i,j) \right)^\frac{\alpha}{\beta}
\]  \hspace{1cm} \text{(2.8)}

\[
    n_{s1}(i,j) = \left( \frac{\alpha}{\sqrt{\pi}} \right)^\frac{1}{\beta} \left( 8w_{s1} - 2w_{s1} \left( \frac{d_{s1}(i,j)}{d} \right) - 2md_{s1}(i,j) \right)^\frac{\alpha}{\beta}
\]  \hspace{1cm} \text{(2.9)}

\[
    n_{s2}(i,j) = \left( \frac{\alpha}{\sqrt{\pi}} \right)^\frac{1}{\beta} \left( 8w_{s2} - 2w_{s2} \left( \frac{d_{s2}(i,j)}{d} \right) - 2md_{s2}(i,j) \right)^\frac{\alpha}{\beta}
\]  \hspace{1cm} \text{(2.10)}

The population densities are non-zero only when the relevant group of people win the land by the allocation mechanism (2.7). If they lose, the population falls to zero.

### 2.3b. SIMULATION TECHNIQUES AND RESULTS

For the purposes of simulating the city with three employment centers, I derive continuous population density functions in a circular city using radial distances \( d_c, d_{s1} \) and \( d_{s2} \) in place of the corresponding \((i,j)\) values in the rent and population functions from (2.4-2.10). Then the following total population functions emerge, along the lines of (1.18):

\[
    N_c = 2 \pi d_c n_c
\]  \hspace{1cm} \text{(2.11)}

\[
    N_{s1} = 2 \pi d_{s1} n_{s1}
\]  \hspace{1cm} \text{(2.12)}

\[
    N_{s2} = 2 \pi d_{s2} n_{s2}
\]  \hspace{1cm} \text{(2.13)}

where \( N_c, N_{s1} \) and \( N_{s2} \) are the total populations of CBD, SBD\(_1\) and SBD\(_2\) workers respectively. I can also get 'smoothened' functions by adding on the exponential parts at the end of equations (2.11)-(2.13), as in equation (1.18).

I postulate an Ewage framework on the cities with three centers, and study the effects when a fixed number of jobs move out to the two subcenters. In the first instance, I study the city where 40000 jobs move out of the center, only now they split up equally
between SBD1 and SBD2. Next, I repeat the process for 60000 jobs moving out of the center and locating at the two subcenters.

Equations (2.11)-(2.13) can be used to generate the population constraints:

\[ N_c = \overline{N_c} \]

(2.14)

\[ N_{s1} = \overline{N_{s1}} \]

(2.15)

\[ N_{s2} = \overline{N_{s2}} \]

(2.16)

where \( \overline{N_c} + \overline{N_{s1}} + \overline{N_{s2}} = \overline{N} \).

\( \overline{N} \) is the fixed total population. The iterations are carried out along the same lines as in Section 1.2b.2. There are now three endogenous variables, \( w_{s1} \), \( w_{s2} \) and \( V^* \) and the three constraints from equations (2.14)-(2.16). I start FOLVE with guess values of the three endogenous variable, go through the loop using ODE45 to solve for the total populations, check them against the constraints, so on and so forth. The results are listed in Table 2.5. Figures 2.22-2.25 show the results. I use close-ups where necessary.

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<th>( V^* )</th>
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**TABLE 2.5: EQUILIBRIUM VALUES FOR CLOSED EWAGE (60/20/20) CITIES**
FIGURE 2.22: CHANGES IN RENT PROFILES IN EWAGE (60/20/20) CITY WITH SBD_1 10 MILES OUT WHEN A SECOND SBD FORMS (CLOSE UPS NEAR CBD, b)

LEGEND: r1c, r2cHM, r2cEwage (60/40), Ra -- as before; r3cEwage5/10(60/20/20), r3cEwage10/20(60/20/20), r3cEwage10/22(60/20/20) -- cities with 2 SBD's with 60000 jobs at the CBD, 20000 each in the 2 SBD's
FIGURE 2.23: CHANGES IN RENT PROFILES IN EWAGE (60/20/20) CITY WITH SBD_1 16 MILES OUT WHEN A SECOND SBD FORMS (CLOSE UPS NEAR CBD, b)

LEGEND: as before
FIGURE 2.24: CHANGES IN RENT PROFILES IN EWAGE (60/20/20) CITY WITH SBD₁ 22 MILES OUT WHEN A SECOND SBD FORMS (CLOSE UPS NEAR CBD, b)

LEGEND: as before
Figures 2.22-2.24 compare the rent profiles in the Ewage (60/40) city to the three-centered cities with the same number of jobs at the center, but the other 40000 jobs split up between the original SBD (10, 16 and 22 miles out in Figures 2.22, 2.23, 2.24 respectively) and an additional subcenter which forms elsewhere in the city. The original SBD's are respectively in the ESZ, in the EDZ and outside the original boundary of the city. For comparison, I look at the formation of a second subcenter, either closer to the center or further out in the suburbs.

When comparing the effects, it is useful to remember that the total population of the city does not change, so there are no changes in land demand near the CBD from that source. There will be changes in population near the subcenter, but these changes do not affect the land demand near the CBD significantly, since the number of jobs there are fixed. So if rents in the interior of the city change, they do so due to changes in land supply from the wage effect and overcompensation effect. These originate from fluctuating SBD wages as the SBD workforce shifts employment venues.

(A) First Subcenter in the ESZ

Figure 2.22 depicts changes in the Ewage10 (60/40) city when a second subcenter locates 5, 20 and 22 miles out. For comparison, the figure depicts the rent gradients in the monocentric, closed HM10 and closed Ewage10 (6/4) cities. In addition, Table 2.6 below lists more information to help in the analysis, including the equilibrium values of wages and utilities from the closed HM and Ewage (6/4) cities with SBD's at distances of 5, 20 and 22 miles from the CBD. Looking at the graph and the table, it is possible to detect and justify the changes that occur due to the formation of a second subcenter. As can be seen, there is a significant difference depending on whether the second SBD locates closer to the CBD, or further out, than the first one. Consider the Ewage5/10 (6/2/2) city with comparable ones. From the graph, the rent gradient for the Ewage5/10 (6/2/2) city is steeper (rents higher near the center, lower near the boundary) than the one for the Ewage10 (6/4) city, which in turn is steeper than the rent gradient for the HM10 city. This is natural, since both subcenter are in the ESZ. The wages to the workers at SBD10 decrease form the HM to the Ewage (6/4) to the Ewage (6/2/2) city, hence the wage effect.

---

4 For this section, population numbers in brackets are in '0000s. For example, (6/4) refers to (60000/40000), (6/2/2) refers to (60000/20000/20000), so on and so forth.
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<th>(w_{s2})</th>
<th>(V^*)</th>
<th>(N_e)</th>
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<td>59728</td>
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<td>19976</td>
</tr>
<tr>
<td>Ewage10/22(6/2/2)</td>
<td>22</td>
<td>17.198</td>
<td>15.239</td>
<td>18.114</td>
<td>59909</td>
<td>20123</td>
<td>19936</td>
</tr>
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</table>

**TABLE 2.6: COMPARISON OF TWO AND THREE CENTERED EWAGE CITIES**
(BASE CASE: SBD₁ 10 MILES OUT; \(w_{s1}\) refers to wages at SBD₁, except for 5/10)

will be smaller, decreasing the incomes of the SBD10 workers, and providing them with less of an inducement to move to the suburbs in search of more land. The OE will be ambiguous, as \(w_{s}\) and \(N_{s}\) both decrease. The land supply near the CBD decreases as modes of decentralization change, raising rents in the interior and lowering utility everywhere in the city.

Comparing the Ewage5/10 city with the HM5 and Ewage5 cities, however, the former has a lower utility than the HM5 city, but a higher utility than the Ewage5 city. This suggests two things. First, the upper limit for changes when employment decentralizes in the ESZ are still set by the HM city (with the lowest \(w_{s}\) and highest \(N_{s}\), hence the highest OE). Compared to the HM city, all Ewage modes will have negative WE and small OE. When both SBD's locate in the ESZ, the total effects on the city will be small. Second, when the second subcenter locates further away than the first one, the city as a whole is better off under all circumstances. As evident from Table 2.6, the SBD5 wage offer changes little when the mode of decentralization switches from Ewage5 (6/4) to Ewage5/10 (6/2/2). This suggests minimal disturbance of the population inside \(d_{l,2}\). It must be the case that very few people get displaced by the switch, i.e. there must have been close to 20000 workers living between \(d_{l,1}\) and \(d_{l,2}\) in the Ewage5(6/4) city, and very little
inducement is needed to get the people outside $d_{L2}$ to work at SBD10. The wage offer at SBD10 is understandably lower, and the effects on rents in the interior smaller, but very close to the Ewage5 (6/4) city, since only the SBD10 workers are affected significantly.

When the second subcenter locates in the EDZ, with SBD1 still 10 miles out, the effects on the city are predictably large. As people at the second SBD will get a much stronger wage boost when they are induced to work close to the boundary (essentially being forced to reverse commute) there is a significant movement towards the suburbs, increasing land supply near the center, dropping rents and increasing utility. The results are evident from comparing the Ewage10/20 (6/2/2) and Ewage 10/22 (6/2/2) cities in Figures 2.22 and Table 2.6. The wage offer for the SBD10 workers fall with a lower demand for labor there, but the dampening effect on utility is more than compensated for by the land vacated by people who want to be near the second subcenter now. As Table 2.5 shows, the land occupied by SBD workers (given by $2.\pi.(d_{L2} - d_{L1})$) increases significantly as the second SBD forms further and further out. Similarly, $d_{L1}$ increases, showing how CBD workers now have access to more land and are thus better off. Compared to the Ewage10 (6/4) city, the land area available to CBD workers is almost unchanged in the Ewage 5/10(6/2/2) city (after compensating for numerical errors), hence utility changes should be small, as is in fact the case. For the second SBD further out, there are significant increases in CBD land area, which show up in the lower rents near the center and higher utilities everywhere. When SBD2 locates 22 miles out, the effects on the city are strong enough to cause a 'hole' to appear in the city.

(B) First Subcenter in the EDZ

When the first subcenter locates in the EDZ, the Ewage (6/4) base case is of course significantly altered from the monocentric and HM cities. As Figure 2.23 shows, the Ewage16 (6/4) city already has some SBD workers commuting outwards in the morning. Table 2.7 below helps in viewing the relevant comparisons.

The comparisons are interesting. Since SBD1 is not far out of the ESZ, the wage effects are not huge enough to overcome the inward pull of labor when the mode of decentralization switches from Ewage16 (6/4) to Ewage5/16 (6/2/2). There are quite significant changes near the CBD and in the middle region, but almost none near the
<table>
<thead>
<tr>
<th>MODE</th>
<th>SBD₂</th>
<th>wₘ₁</th>
<th>wₘ₂</th>
<th>V*</th>
<th>Nₑ</th>
<th>Nₘ₁</th>
<th>Nₘ₂</th>
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<td>Monocentric</td>
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<td>17.5938</td>
<td>100000</td>
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<td>N.A.</td>
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<td>17.718</td>
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<td>89966</td>
<td>N.A.</td>
</tr>
<tr>
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<td>17.625</td>
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<td>40310</td>
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<td>HM16</td>
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<td>15.7333</td>
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<td>31202</td>
<td>N.A.</td>
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<td>Ewage16(6/4)</td>
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<td>HM20</td>
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<td>5925</td>
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<td>Ewage20(6/4)</td>
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<td>100000</td>
<td>0</td>
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<tr>
<td>Ewage22(6/4)</td>
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<td>N.A.</td>
<td>18.6619</td>
<td>60019</td>
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<td>18.55</td>
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<td>61522</td>
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<td>Ewage16/20(6/2/2)</td>
<td>20</td>
<td>15.92</td>
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<td>59970</td>
<td>20311</td>
<td>20207</td>
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<td>Ewage16/22(6/2/2)</td>
<td>22</td>
<td>15.776</td>
<td>15.29</td>
<td>18.17</td>
<td>59853</td>
<td>20000</td>
<td>19926</td>
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</table>

TABLE 2.7: COMPARISON OF TWO AND THREE CENTERED EWAGE CITIES
(BASE CASE: SBD₁ 16 MILES OUT; wₘ₁ refers to wages at SBD₁, except for 5/16)

boundary. This suggests, again along the lines of the Ewage5/10 case, that a significant portion of the SBD16 workers (those occupying the middle of the city) needed little inducement to switch allegiance to SBD5 in the three-centered city. The differences in the city arise from the loss in wages suffered by people around SBD16 in the Ewage16 city, and land supply in the interior falls as people move in, driving up rents and lowering utility. In the final equilibrium, the rent peak around SBD16 disappears. The people near the boundary are not affected much, they keep on working at SBD16 as before. Hence the rent profiles for Ewage16 and Ewage5/16 nearly converge after they rise above the monocentric city rents.⁵

When the second subcenter locates further out in the suburbs, the flattening effects on rents in the interior get exacerbated. When SBD₂ forms 20 miles out, 20000 people get an even larger fillip in wages, prompting them to move outwards, diminishing rents in the

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⁵ There seems to be an anomaly in moving from the Ewage5/10 to the Ewage5/16 city, as the utility level seems to decrease, in contradiction with the hypothesis that for a fixed location of SBD₁, utility increases as SBD₂ moves further from the center. The results are compromised by the rounding errors in the Ewage5/16 equilibrium, the nearly 900 extra people and too many people at the CBD and SBD₁ dilute the effects on land supply in the interior of the city, forcing rents up and lowering utility.
interior (including depressing the rent peak at SBD16) and causing a second rent peak to form at SBD20. The city as a whole is much better off, as the outward movement by the 20000 SBD20 workers increases land supply both near the CBD and in the middle, lowering rents. When SBD2 forms 22 miles out, the outward movement by the 20000 workers who now work there is significant enough to cause major changes in the city. First, enough land is freed up in the interior and middle of the city to erase the rent peak at SBD16. Secondly, a 'hole' appears in the city and a second rent peak springs up around SBD22.

An extreme case of the first subcenter being in the EDZ occurs when the Ewage22 city is taken as a base case. Since all the second subcenters are inside SBD22, the effects on the city are noticeably smaller than in the Ewage22 (6/4) city. This is natural as 20000 jobs now revert back towards the center of the city. Several points of comparison are of interest, though. First, compared to the Ewage22 (6/4) city, the other three modes of decentralization leave rents near the center significantly higher and with relatively small variations in steepness to separate them. The same is true of the profiles near the boundary. Since all the three - centered cities have the same number of people at SBD22, there are no differences in the rent peaks around SBD22. The set up essentially functions as two different cities, with the smaller one around SBD22 having almost identical characteristics while the bigger interior city with 80000 people is affected as if it were a smaller version of the Ewage city with one SBD, respectively 5, 10 and 16 miles out. Not surprisingly, the effects on the interior of the 5/22 and 10/22 cities are close together, both SBDs being located in the ESZ. The interior city in the Ewage16/22 case does not show any rent peaks around SBD16, since even this subcenter is now essentially in the ESZ for a Ewage16 (60000/20000) case.

The direction of the results remain the same, the further out a city suburbanizes, the better off everyone is. It is interesting to analyze the city with three centers since it may provide some answers regarding which effect is stronger: the effect of the same amount of employment decentralizing to one or more subcenters which locate further and further away from the center or the effect of more employment moving out to subcenter(s) that are already existing. To complete the second part of the analysis, Table 2.9 below looks at the equilibria when the subcenters each employ 30000 workers, leaving only 40000 at the center. While this is somewhat out of the purview of analysis along HM lines, since even
jobs close to the CBD need no longer necessarily be more centralized than households, analyzing the results from Tables 2.5 and 2.9 together point out some of the differences between more 'traditional' centralized cities and cities where employment is more spread out. As base cases, Table 2.8 lists the equilibria for some Ewage (40/60) cities.

<table>
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<tr>
<th>SBD</th>
<th>$w_s$</th>
<th>$V^*$</th>
<th>$N_e$</th>
<th>$N_s$</th>
<th>$d_{L_1}^*$</th>
<th>$b^*$</th>
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<td>10</td>
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<td>13.5403</td>
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<tr>
<td>20</td>
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<td>22</td>
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**TABLE 2.8: EQUILIBRIUM VALUES FOR EWAGE (40/60) CITIES**

<table>
<thead>
<tr>
<th>$s_1$</th>
<th>$s_2$</th>
<th>$w_{s_1}$</th>
<th>$w_{s_2}$</th>
<th>$V^*$</th>
<th>$N_e$</th>
<th>$N_{s_1}$</th>
<th>$N_{s_2}$</th>
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<td>18.5946</td>
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<td>30022</td>
<td>15.5097</td>
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<td>26.0149</td>
</tr>
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</table>

**TABLE 2.9: EQUILIBRIUM VALUES FOR CLOSED EWAGE (40/30/30) CITIES**

Figures 2.25-2.30 depict the changes in some of the three-centered cities as more jobs move to the exogenously determined SBD worksites. I use close-ups for maximum effect.
FIGURE 2.25: CHANGES IN RENT PROFILES IN EWAGE CITY WITH SBD’S 5 & 10 MILES OUT WHEN MORE JOBS MOVE OUT (CLOSE UPS NEAR CBD, b)

LEGEND: as before
FIGURE 2.26: CHANGES IN RENT PROFILES IN EWAGE CITY WITH SBD'S 10 & 20 MILES OUT WHEN MORE JOBS MOVE OUT (CLOSE UPS NEAR CBD, b)

**LEGEND:** as before
FIGURE 2.27: CHANGES IN RENT PROFILES IN EWAGE CITY WITH SBD’S 5 & 16 MILES OUT WHEN MORE JOBS MOVE OUT (CLOSE UPS NEAR CBD, b)

**LEGEND:** as before
FIGURE 2.28: CHANGES IN RENT PROFILES IN EWAGE CITY WITH SBD'S 16 & 20 MILES OUT WHEN MORE JOBS MOVE OUT (CLOSE UPS NEAR CBD, b)

**LEGEND:** as before
FIGURE 2.29: CHANGES IN RENT PROFILES IN EWAGE CITY WITH SBD'S 5 & 22 MILES OUT WHEN MORE JOBS MOVE OUT (CLOSE UPS NEAR CBD, b)

**LEGEND:** as before
FIGURE 2.30: CHANGES IN RENT PROFILES IN EWAGE CITY WITH SBD'S 16 & 22 MILES OUT WHEN MORE JOBS MOVE OUT (CLOSE UPS NEAR CBD, b)

LEGEND: as before
When analyzing this part of the three-centered city, one important perspective to take notice of is whether or not jobs stay more centralized than households as more jobs move out of the center. Wherever it is useful and non-obvious, I include graphs for the Ewage (4/6) city as another basis of reference. Also note from Table 2.6 that the definition of the ESZ changes when 60000 jobs move out of the center. SBD10 is still in the ESZ, but 16, 20 and 22 are well out of it. Finally, though the total population in the city is unchanged, land demand in the interior of the city may be changing as the number of workers at the employment centers change. Figures 2.25-2.30 and Tables 2.6, 2.8 and 2.9 are used together in the analysis.

(A) First Subcenter in the ESZ

From Figures 2.25 and 2.26, consider the changes in the city with its first subcenter 10 miles out when a second one forms 5 and 20 miles out, respectively, and each subcenter employs 30000 workers. For comparison, the monocentric, HM, Ewage (6/4) and Ewage (4/6) cities are provided. For the Ewage 5/10 city, the upper limit on changes from the monocentric model is still set by the HM10 case, since both SBD5 and SBD10 are in the ESZ. Close to the CBD, the rent offer curves for the HM city and the Ewage5/10 cities are related as follows: (6/2/2) > (6/4) > (4/3/3) > (4/6) > HM. This preserves the earlier hypothesis about the pattern of effects. In all cases, having the second subcenter form closer to the CBD than the first one will cut land supply and raise rents near the center and lower utility levels everywhere. However, moving from the cities with 60000 jobs at the CBD to the cities with 40000 jobs at the CBD, there is a negative effect on land demand near the center. As the graph shows, the supply effects dominate. In the case of the (4/6) city, the increase in land supply is bigger than the (4/3/3) city (they have the same land demand close to the CBD), so rents are lower near the center in the former city. Finally, even with 60000 jobs decentralizing, SBD10 is in the ESZ, implying that jobs remain more centralized than residences in all these cities.

The rent gradients of the HM and Ewage5/10 (4/6) city intersect between 10 and 11 miles from the CBD, and HM rents are higher near the boundary. In figure 2.26, when SBD2 locates 20 miles out, the effects on the Ewage (4/3/3) city are much more pronounced compared to the Ewage (6/2/2) city. As the tables show, the wage offer at SBD10, which was lower than the HM and Ewage (6/4) levels for the Ewage (6/2/2) case,
will now be substantially higher. Consequently, the strong wage effects for the SBD20 workers will now be supplemented by the wage effects for the SBD10 workers. Also, land demand falls. This, along with the increase in land supply, decreases rents substantially in the interior of the city, causing everyone to be much better off.

When the second subcenter locates 20 miles out, the cities with three subcenters show much lower rents near the CBD, since the supply and demand effects now move in the same direction. As might be expected, the CBD land supply increases from the (6/2/2) to the (4/3/3) city as more jobs move out to SBD20, and the effects can be gauged from the substantial decrease in CBD rents. The effects near the boundary are predictable.

(B) First Subcenter in the EDZ

![Figure 2.31](image)

When 60000 jobs move out of the center, SBD16 is in the middle of the EDZ, rather than at the edge. The effects on the Ewage (4/6) city are therefore much more pronounced. When the second SBD locates 5 miles out, the wage effect diminishes for the SBD16 workers as their income falls significantly, even compared to the Ewage16 (6/4) city. However, the wage at SBD16 increases when, starting from the (6/2/2) city, 10000 more jobs move into each of the two subcenters. Figure 2.31 above provides a simple
sketch of changes in land demand, supply and rents near the CBD to illustrate the effects of decentralization in the interior of the city.

In the Ewage5/16 (6/2/2) city, the inward movement of households (compared to the HM16 city) puts severe enough pressure on land supply as to actually raise rents in the interior. In the Ewage5/16 (4/3/3), however, less jobs are leaving SBD16 (the land supply to the interior is higher than the Ewage5/16 (6/2/2) city, though still lower than the HM16 city) while a vastly lower number of people are vying for land close to the CBD. The decline in land demand negates the tighter supply and pushes rents below the HM16 level. In the case of the Ewage16 (6/4) and (4/6) cities, the increases in land supply near the center are much higher, and land rents much lower, as all decentralized jobs locate 16 miles out. The Ewage16 (4/6) city also has lower land demand close to the CBD, compounding the downward trend on rents near the CBD.

Viewing the suburbs of the 5/16 cities, in addition to the HM and Ewage5/16 cities almost converging close to the boundary, the rent peak at SBD16 is almost gone for the Ewage5/16 (4/3/3) city. The rents in the suburb are lower than in the Ewage16 (60/40) city, demonstrating the strength of the inward pull exerted on SBD16 workers when 30000 jobs move to SBD5, which counteracts the higher land demand (compared to Ewage16 (6/4)) near the subcenter. The Ewage16 (4/6) city has a higher effect on land demand and a lower increase in land supply at SBD16, hence there are large effects on the SBD rent peak.

Looking at the Ewage 16/20 cities from Figure 2.28, many of the results are expected. When more jobs decentralize and the second subcenter locates close to the boundary, the decrease in land demand and increase in land supply produce a huge negative effect on rents close to the center. The separate effects are clearly seen in Figure 2.28. The relationships between the rent profiles close to the CBD for the various modes of decentralization are as follows: mono > HM16 >> Ewage16 (6/4) > Ewage16/20 (6/2/2) >>> Ewage16 (4/6) > Ewage16/20 (4/3/3). This clearly shows the supply induced effect (the difference between HM16 and Ewage16 (6/4)) and also the demand effect compounding the supply effect (the difference between Ewage16/20 (6/2/2) and Ewage16 (4/6)). In the suburbs, the effects are predictable, the only thing of interest being the proximity between rents from the Ewage16 (6/4) and Ewage16/20 (4/3/3) cities over the region 16-19 miles from the CBD, indicating that much of the movement when switching
from the former to the latter mode is taking place through CBD workers switching allegiance to SBD20.

Figures 2.29-2.30 look at SBD22 as the first subcenter, again an extreme case of employment decentralizing in the EDZ. I do not show the rents for the Ewage22 (4/6) city on the graphs, since the effects are obvious from comparing the utility level from Table 2.8 to the other cases under consideration. Also, including the Ewage22 (4/6) city introduces scaling problems which inhibit the clarity of the effects near the boundary for the other cities. When a second subcenter forms 5 miles out, decentralizing 20000 extra jobs causes the rents near the center to fall from the Ewage5/22 (6/2/2) case, but they are still higher than the Ewage22 (6/4) case. However, the gap between these two modes is slightly less prominent than with similar modes in the SBD16 cities. This shows the strength of the positive effect on land supply when the first subcenter forms 20 (as opposed to 16) miles out. Near the boundary, the peak at SBD22 is most prominent for Ewage22 (6/4), followed by Ewage5/22 (4/3/3), followed by Ewage5/22 (6/2/2), accompanied by a corresponding outward shift, and decline in size, of the 'holes'. This is natural, since the interior of the city always behaves as a separate entity, causing changes in rents to occur at the boundaries of the holes.

Similar logic applies to the changes near the center in the Ewage16/22 cities. The second subcenter locating in the EDZ will cause workers to move in less than in the corresponding Ewage5/22 cities, hence land rents should be lower. The interesting effect occurs near the boundary. In the Ewage16/22 (4/3/3) city, there is no 'hole'. It also shows a rent peak at the subcenter, and the significant increase in suburban land demand is the underlying reason. This is the first time that a decentralized closed city with a subcenter outside the boundary of the original monocentric city has showed no signs of vacant land in the interior. I discuss this further in the next segment. Figure 2.32 below depicts the changes in utility for the cities with the first subcenter at 10 and 16 miles from the CBD under the various modes of decentralization.
(C) Number of jobs Decentralizing vs. Distance of SBD

To compare and contrast the relative effects on the city as both the number of decentralized jobs and the distance of decentralization changes, I will look at some selected examples from the last segment. One broad conclusion that can be drawn from studying the three-centered modes of decentralization is that for a fixed location of the first SBD, the city has a higher utility the further out the second SBD locates. What is left open for debate is whether the city gets more of a boost in utility when a greater number of jobs locate to the same subcenter location or when some of the employment decentralizes further out.

Both changes tend to raise the utility level in the city. An increase in the number of decentralized jobs will decrease land demand near the center, lowering rents there, at the
same time putting upward pressure on rents in the suburbs. When subcenters locate further out, the positive wage effects for SBD2 workers will induce them to demand more land and move out to the suburbs where rents rise, simultaneously increasing the supply of land close to the center and lowering rents there again. Consequently, rents will fall significantly near the center and rise near the boundary. The wage gains at the suburb are somewhat dampened by the rise in rents there, but the overall effect is higher utility. Question is: which of the effects is stronger?

The answer may be found by inspecting different circumstances of decentralization. When the location of the first subcenter is given, if the second SBD locates close to the center, land supply near the CBD falls as people move closer in. At the same time, if more jobs move out of the subcenter, land demand will also fall, tending to diminish rents near the CBD. It is easier to find which effect dominates in these and such other circumstances.

Figure 2.31 provides one such example in moving from the Ewage5/16 (6/2/2) to the Ewage5/16 (4/3/3) city. Comparing rents near the center, the first city shows rents higher than the HM16 city, while the second city shows rents lower than the HM city. Considering the (4/3/3) city, CBD land supply is still tighter than in the HM city, but the decrease in land demand more than compensates for that. However, the supply shift is likely to be small here, while the demand shift is bigger. The fact that rents at the CBD do not fall by much suggests that the effects of the demand decrease affect the equilibrium less than the supply decrease.

Consider another scenario. Moving from the Ewage5/16 (4/3/3) city to the Ewage16 (6/4) city, rents decline substantially near the center, suggesting that the effect of 40000 jobs moving to SBD16 with no change in land demand is much greater than the effect of 30000 jobs each moving to SBD5 and SBD16, even when the latter is accompanied by a big decrease in land demand. The same lesson can be learnt from similar circumstances, e.g. comparing the Ewage 5/10 (4/3/3) with the Ewage 6/4 city, the Ewage10/20 (4/3/3) with the Ewage 20 (6/4) city, so on and so forth. All of this supports the hypothesis that a fixed amount of employment locating close to the boundary will make everyone better off than having more employment move out of the center, but with only part of the decentralized employment locating closer to the boundary.
Some corroboration regarding the strength of the supply effect is found in considering the Ewage16/22 (4/3/3) city. As I mentioned in the last segment, it is the only city which comes close to the monocentric city rent levels in the suburbs (16+ miles out), besides the Ewage open city, which has a lot more people living in it. This indicates that when a substantial number of people locate in several subcenters close to the boundary of the original city, it may be possible to eliminate vacant land ('holes') within city limits, while at the same time providing people with much higher utility levels than the open city. Figure 2.33 revisits some Ewage cities with SBD1 22 miles out from the CBD.

![Graph showing rents comparison](image)

**FIGURE 2.33: COMPARISON OF RENTS IN EWAGE22 CITIES WITH DIFFERENT MODES OF DECENTRALIZATION**

However, the logic may be carried too far. As Figure 2.33 shows, the Ewage20/22 cities do not come close to emulating the Ewage16/22 (4/3/3) city in terms of rents approaching monocentric city levels in the suburbs. While the 'hole' does disappear in the Ewage (6/2/2) city, rent levels in the suburbs are low with not enough people congregated
around the employment subcenters. Going back to the Ewage16/22 (4/3/3) city, and comparing it with the Ewage open city, the latter has a much more pronounced rent peak at SBD22, but the former city -- through the sheer effect of enough jobs decentralizing far enough out -- comes close to matching it over 16-19 miles. This demonstrates the power of the combined effects of decreased land supply and increased land demand at SBD22, almost matching the substantial Total Population Effect in the open city.

In conclusion, the land supply effects seem to have more strength than the land demand effects, especially when the subcenters form far from the center. This is not surprising, being a logical extension of a conclusion that I drew at the end of the last section, viz. the wage effects (the driving force behind changes in land supply) outweigh the population effects (the driving force behind the changes in land demand) when employment decentralizes in the Excess Demand Zone.

(D) Some Thoughts on the Open City model

In the above analysis, I have investigated the effects of the formation of additional subcenters on a closed city. Though I do not explicitly investigate an open city, it is easy to posit what would happen there. With all changes affecting population density, when employment decentralizes to the first CBD, there is little change in the interior of the city. With the location of the first subcenter given, the formation of a second subcenter closer to the CBD will diminish the increase in population near SBD₁ and cause population density to rise in the area between SBD₂ and SBD₁. When the second subcenter forms further away than SBD₁, density of population rises more in the far suburbs. The area around SBD₁ becomes less congested. In both cases, the most interior portion of the city is left undisturbed, as before.
VOLUME II

THE WAGE GRADIENT AND THE INTERACTION BETWEEN
EMPLOYMENT AND RESIDENTIAL DECENTRALIZATION IN URBAN AREAS

BY

DEBASHIS GHOSH
CHAPTER 3: A PARTIAL GENERAL EQUILIBRIUM MODEL OF A CITY WITH DECENTRALIZED EMPLOYMENT AND A FULL LABOR MARKET

In this chapter, I extend the model of the city with decentralized employment to include a full labor market. Wages at the CBD are no longer fixed. Wages at the SBD are not determined by the Hamilton-Mills (HM) wage gradient, nor are they determined for an exogenous number of workers demanded at the subcenter, as in the Equilibrium wage (Ewage) city. Instead, the urban area as a whole is the repository of an exogenous amount of capital inputs for production. When production at CBD and SBD sites are carried on using identical methods, the level of labor demand is set by the division of capital between the employment sites. The CBD is always at (0,0). The location of the SBD(s) and the wage offers at the employment sites determine the pattern of household location. Equilibrium on the labor supply side is achieved when everyone in the urban area (all workers are homogeneous in this section) has the same level of utility. For closure, the amount of labor demanded must be equal to the amount supplied at the equilibrium wage rate in each employment center. Finally, the total number of workers (the population) must be invariant under all circumstances in the closed city.

As in the first two chapters, I discount the effect firm bid rents have on the profile of the city. The reasons are unchanged, though somewhat more specific. The location of the SBD is contingent on an exogenous occurrence -- in this case capital moving out of the center of the city. A lot of production is tied to durable assets like buildings, storage facilities, production areas, office space and such. Not all of this is convertible over the short or medium run, and there is often a substantial fixed cost of doing so even in the long run. This would tend to explain why capital outflows do not often result in a total vacuum at the center. I do probe the case of what happens when more than 50% of the capital moves out to the suburb, which may be an intermediate step between the cities where employment is always more centralized than households and the city where employment and residences are 'integrated'. Also, I do not dwell on the issue of why or where capital movements occur, instead treating it is as an exogenous impetus, since it is not important to sacrifice tractability (especially in the more complex version of the model with heterogeneous households) for uncertain gains. It is more fruitful to study how households would react if a certain amount of capital were to shift a certain exogenously
determined distance away (this is also realistic since some of the capital moving to the suburbs might be social capital, or may move out there as a direct result of fiscal incentives from city councils and the like). This issue is important in itself, regardless of an answer to the question of why firms decentralize.

If the location and amount of the decentralized capital is determined exogenously, the location of the SBD becomes exogenous too. As White (1976), Sullivan (1986), Yinger (1992) and others find, the CBD and SBD firms' bid rents for land would peak at the precise locations of the capital, with the amplitude increasing with increases in amounts of capital. This adds nothing to the pattern of changes in rents in the city, which occur due to the movements of the labor force as they switch allegiance from one employment center to another and thus shift places of residence, demand more or less land, so on and so forth. A complete general equilibrium framework may include firms' bid rents for land, but I choose to have the firms locate at a point in the center and evenly around a belt of infinitesimal width in the suburbs.

3.1. THE MONOCENTRIC CITY WITH HOMOGENEOUS LABOR

3.1a. THE MODEL

Assumptions

(i) The urban area has no special topographical features at the outset, save a high concentration of capital at the center of the city which encourages the formation of a Central Business District (henceforth CBD) there at the outset. I look at a circular shaped possible metropolitan area. Hence the CBD is assumed to be in place at (0,0) initially.

(ii) All distances are measured radially in miles. Commuters are assumed to travel along ubiquitous radial roads.

(iii) There is only one type of household occupying the urban area. All of them work at the one CBD, earning identical wages. Following White's (1992) formulation, households are given to possess Cobb-Douglas utility functions and assumed to spend 8 hours per day in work and commute. Leisure is not explicitly included in the utility function.
(iv) Land is taken as a proxy for housing. Households are assumed to demand land rather than housing services. Rent bids are in dollars/unit of land (e.g. $/sq mile). Rents on land are collected by absentee landlords, and each plot of land is sold to the highest bidder.

(v) Each person's cost of commuting to work includes:

(a) a time cost of commuting: \(2.\text{w}.t(i,j)\), where commute time is valued at the lost wage;

(b) a monetary cost of commuting: \(2.\text{m}.d(i,j)\), where \(m\) is the per mile material cost of commuting. Distances of commute are measured in miles along the radial roads, and the time of commute from a point \((i,j)\) to the center, for example, is given by \(d(i,j) = \sqrt{i^2 + j^2}\) and \(t(i,j) = \left(\frac{1}{f}\right)d(i,j)\), where \(w\) is the wage offer at the center and \(f\) is the average speed of commute in miles/hour. Later, I use radial distances in continuous population density functions for numerical iterations.

(vi) Labor and capital are compliments in production. Initially, all production in the urban area is carried on using the same technology. The firms are assumed to be homogeneous in all other respects as well.

(vii) The production function has a Hicks neutral shift factor \(A\), which could be taken to represent agglomerative effects along the lines of Straszheim (1983) and Sullivan (1986). Unlike those models, however, I do not use the shift factor to introduce a difference in the production technologies of CBD and SBD firms.

(viii) The amount of capital in the urban area is fixed. Initially, all the capital stock is concentrated at the center.

(ix) Firms do not demand land in this model. The CBD is assumed to occupy a point or a tight cluster and not cause significant distortions in the rent profile of the city.

(x) I present the closed city case initially. The open city and semi-closed city cases are also investigated in some instances.

**Labor Demand**

From the first order conditions of profit maximization, the labor demand functions in the urban area can be determined. By assumption, firms do not demand land for production purposes. Hence the production function combines labor and capital:

\[ Q = A.N^\theta.K^\varphi \]
where $Q$ is the numeraire good produced and $A$ is the Hicks neutral shift factor. If $w$ represents wages and $r$ the rental rate on capital, the firm maximizes the profit function

$$\pi = AN^\theta K^\varphi - wN - rK$$

(...) (3.1)

where the numeraire good has a price of 1. The amount of capital in the urban area is held fixed at $K$, and $\theta + \varphi = 1$, so the first order condition for profit maximization yields

$$A\theta N^{\theta - 1} K^\varphi - w = 0$$

(...) (3.2)

from which I get the labor demand function $N^D$ in terms of $w$

$$N^D = \left(\frac{A\theta K^\varphi}{w}\right)^{\frac{1}{\varphi}}$$

(...) (3.3)

**Labor Supply**

On the labor supply side, households work at the CBD for the wage offer $w$. They maximize utility functions with respect to demand for land $L$ and the numeraire good $Q$, subject to their income net of commuting costs. The equilibrium is found exactly along the lines of the monocentric city models examined previously. Hence they maximize the Lagrangian

$$V = Q^\alpha L^\beta - \lambda \left[8w - 2t_c d(i,j) - Q - R(i,j) L(i,j)\right]$$

(...) (3.4)

with respect to $Q$ and $L(i,j)$, generating the demand functions for $Q$ and $L$ as functions of all parameters and distance from center, where the distance function $d(i,j)$ is evaluated as before.

From the first order conditions of maximization, the demands for $Q$ and $L$ are

$$Q(i,j) = \alpha \left\{8w - 2wt(i,j) - 2md(i,j)\right\}$$

(...) (3.5)

$$L(i,j) = \left(\frac{\beta}{R(i,j)}\right) \left\{8w - 2wt(i,j) - 2md(i,j)\right\}$$

(...) (3.6)

Equations (3.5) and (3.6) can be used to find the indirect utility function
\[ V^* = \left( \frac{\alpha^\beta}{R(i,j)^\beta} \right) \left\{ 8w - 2w(t(i,j)) - 2md(i,j) \right\} \]  \quad \text{.....(3.7)}

Equation (3.7) in turn yields the rent function

\[ R(i,j) = \left( \frac{\alpha^\beta}{V^*} \right)^{\frac{1}{\beta}} \left\{ 8w - 2w\left(\frac{1}{f}\right) d(i,j) - 2md(i,j) \right\}^{\frac{1}{\beta}} \]  \quad \text{.....(3.8)}

using the time of commuting function, and where \( V^* \) is the equilibrium utility level, determined when the system of equations is solved for a given population \( N \). For maintaining equilibrium, each household must get the same utility, regardless of where they live. The rent gradient in the monocentric city has a negative slope. Rents decline till the boundary of the city is reached \( b \) miles from the center, where the rent offer by city residents falls below the agricultural rent offer \( R_a \)

\[ R_a = \left( \frac{\alpha^\beta}{V^*} \right)^{\frac{1}{\beta}} \left\{ 8w - 2w\left(\frac{1}{f}\right) b - 2mb \right\}^{\frac{1}{\beta}} \]  \quad \text{.....(3.9)}

For all \( R(i,j) \geq R_a \), then, the population density at each location can be found by setting the demand for land equal to the supply. Assuming that the land supply at each coordinate point is 1, I then find the population density at each location as

\[ N(i,j) = \left( \frac{\alpha^\beta}{V^*} \right)^{\frac{1}{\beta}} \left\{ 8w - 2w\left(\frac{1}{f}\right) d(i,j) - 2md(i,j) \right\}^{\frac{\alpha}{\beta}} \]  \quad \text{.....(3.10)}

The total population of the urban area is then given by summing up the populations at each point. The total labor supply in the city is

\[ N^S = \sum_i \sum_j \left( \frac{\alpha^\beta}{V^*} \right)^{\frac{1}{\beta}} \left\{ 8w - 2w\left(\frac{1}{f}\right) d(i,j) - 2md(i,j) \right\}^{\frac{\alpha}{\beta}} \text{ for all } R(i,j) \geq R_a \]

\[ = \left( \frac{\alpha^\beta}{V^*} \right)^{\frac{1}{\beta}} \sum_i \sum_j \left\{ 8w - 2w\left(\frac{1}{f}\right) d(i,j) - 2md(i,j) \right\}^{\frac{\alpha}{\beta}} \text{ for all } R(i,j) \geq R_a \]  \quad \text{.....(3.11)}
For a given population in the closed city case, I can find \( V^* \) in terms of all the other parameters as follows

\[
V^* = \left( \frac{\alpha^\alpha}{N^\beta} \right) \sum_i \sum_j \left( \frac{1}{8w - 2w(\frac{1}{f}) d(i,j) - 2md(i,j)} \right) \alpha
\]  
\text{.....(3.12)}

The value of \( V^* \) from (3.12) is used in equation (3.8) to determine rents.

Closure

To close out the general equilibrium model, the total demand for labor (from equation (3.3)) must be equal to the total supply of labor (from equation (3.11b)):  

\[
N^D = N^S \quad \text{.....(3.13a)}
\]

In the closed city, if the population is fixed at \( \bar{N} \), (3.13a) becomes

\[
N^D = N^S = \bar{N} \quad \text{.....(3.13b)}
\]

3.1b. SIMULATION TECHNIQUES AND RESULTS

There are two variables to be determined -- \( w \) and \( V^* \). Their equilibrium values are found from the simultaneous solution to the two equations:

\[
\bar{N} - N^D = 0 \quad \text{.....(3.14a)}
\]

\[
\bar{N} - N^S = 0 \quad \text{.....(3.14b)}
\]

Equations (3.14a) and (3.14b) are derived from the closure condition (3.13b). Substituting for labor demand and supply from equations (3.3) and (3.11b) respectively, the two equation system is

\[
\bar{N} - \left( \frac{A_0 K^q}{w} \right)^{\frac{1}{q}} = 0 \quad \text{.....(3.15)}
\]
\[
\bar{N} - \left[ \frac{\alpha}{V^*} \right]^\frac{1}{\beta} \sum_i \sum_j \left( 8w - 2w \left( \frac{1}{f} \right) d(i,j) - 2m d(i,j) \right)^\frac{\alpha}{\beta} = 0
\]  

(3.16)

Given the values of all other parameters, equations (3.15) and (3.16) are simultaneously solved to yield the equilibrium values of \( w \) and \( V^* \). (3.16) is solved using the boundary condition \( R(i,j) \geq R_0 \). I use MATLAB programs to iterate the system to a solution.

As before, I use FSOLVE to solve the two equation system simultaneously. To find \( N^S \), I utilize the ODE45 solver. The system is more complicated than in the case with no labor demand, since the labor demand and labor supply systems are sensitive to changes in guess values for \( w \) in different degrees. Since kinks in the functions throw FSOLVE out of kilter, I use the continuous population density function

\[
N = \left[ \frac{\alpha}{V^*} \right]^\frac{1}{\beta} \left( 8w - 2w \left( \frac{1}{f} \right) d_c - 2m d_c \right)^\frac{\alpha}{\beta}
\]  

(3.17a)

where \( d_c \) is the distance measured along a line from the center. The following 'smooth' differential equation is used to find \( N^S \)

\[
N^S = 2.\pi. d_c. N. \gamma. e^{(Rd(\text{diff}) - l)} \text{ where } Rd(\text{diff}) = \frac{R(d_c) - R_0}{R(d_0) - R_d}
\]  

(3.17b)

(3.17b) is then solved for initial values \( d_c = 0 \), \( N^S = 0 \) and final value \( d_c = 30 \). Table 3.1 lists the parameter values used to set up the first equilibrium.

<table>
<thead>
<tr>
<th>LABOR DEMAND</th>
<th>LABOR SUPPLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARAMETERS</td>
<td>VALUES</td>
</tr>
<tr>
<td>( A )</td>
<td>16.531</td>
</tr>
<tr>
<td>( R )</td>
<td>1000000</td>
</tr>
<tr>
<td>( \bar{N} )</td>
<td>1000000</td>
</tr>
<tr>
<td>( f )</td>
<td>30 miles / hour</td>
</tr>
<tr>
<td>( m )</td>
<td>$0.40 / mile</td>
</tr>
<tr>
<td>( d_c )</td>
<td>0 - 30 miles</td>
</tr>
</tbody>
</table>

| TABLE 3.1: PARAMETER VALUES FOR MONOCENTRIC CITY WITH A FULL LABOR MARKET |
In Table 3.1, the value of $\overline{N}$ is of course common to both the labor demand and the supply sides. The solutions are obtained via the following loop:

\[
\text{Guess } V^* \text{ and } w \quad \downarrow \quad \uparrow \\
(Labor \ Demand) \\
\text{Given } w \text{ and all other parameters, find } N^D. \\
\uparrow \quad \uparrow \\
(Labor \ Supply) \\
\text{Given } V^*, w \text{ and all other parameters, find all } R \geq R_a. \\
\uparrow \quad \uparrow \quad \uparrow \\
\text{Hence find population density } \Pi \text{ at all locations.} \\
\uparrow \\
\text{Sum up } \Pi \text{ over the urban area to find } N^S. \\
\uparrow \\
\downarrow \\
\text{Check if } N^D = \overline{N} \text{ and } N^S = \overline{N}. \text{ If not} \\
\downarrow \\
\text{Once } V^* \text{ has been determined and the city laid out,} \\
\text{use equation (3.25) to determine } b, \text{ the city boundary.}
\]

After the FSOLVE solutions emerge, I use the original (kinked) population density function from equation (3.17a) to check the results by trial and error. The coefficients of the production function, $\theta$ and $\varphi$, are not included in Table 3.1 since I use two different sets of values for sensitivity analysis. Table 3.2 below presents the two monocentric city equilibria that emerge from using the two sets of coefficients. All other parameters are at the level designated in Table 3.1. Due to the increased complexity of the equilibrium, I accept rounding errors in the ±5% limit. Figures 3.1 and 3.2 depict the rent and population density gradients in the monocentric cities.

<table>
<thead>
<tr>
<th>$\vartheta$</th>
<th>$\varphi$</th>
<th>$w_s$</th>
<th>$V^*$</th>
<th>$N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.72</td>
<td>.28</td>
<td>22.6794</td>
<td>19.8633</td>
<td>d=100000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>s=100000</td>
</tr>
<tr>
<td>.9</td>
<td>.1</td>
<td>18.7302</td>
<td>16.5055</td>
<td>d=100000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>s=100160</td>
</tr>
</tbody>
</table>

**TABLE 3.2: MONOCENTRIC CITY EQUILIBRIA FOR DIFFERENT COEFFICIENTS FOR LABOR AND CAPITAL IN PRODUCTION FUNCTION**
FIGURE 3.1: COMPARISON OF RENT PROFILES IN MONOCENTRIC CITIES WITH DIFFERENT LABOR PRODUCTIVITY

**LEGEND:** \( r1c(x/y) \) -- \( x, y \) coefficients of labor and capital respectively in production

FIGURE 3.2: COMPARISON OF POPULATION DENSITIES IN MONOCENTRIC CITIES WITH DIFFERENT LABOR PRODUCTIVITY; **LEGEND:** same as above
3.2. THE CITY WITH HOMOGENEOUS LABOR AND DECENTRALIZED EMPLOYMENT

3.2a. THE MODEL

Assumptions

In addition to the assumptions in the monocentric city model, I also assume:

(i) The Suburban Business District (henceforth SBD) forms as a belt of infinitesimal width at \( d_{sub} \), which is at a distance \( d_s \) from the CBD, and the firms are spread evenly around the loop. Some firms stay at the CBD and hire \( N_c \) workers, while the ones that go out to the suburbs hire \( N_s \) workers. This movement occurs as a result of a movement of capital from the center to the suburbs. As I mentioned before, my chief concern here is to investigate the effects of employment decentralization on household location choices.

(ii) Both CBD and SBD firms will attempt to pay the minimum wage possible to attract the number of people they wish to hire, i.e. the labor market clears at all times.

(iii) In the general equilibrium setup, the wage at the suburb is no longer calculated in terms of a fixed wage at the center. Both the CBD wage \( w_c \) and the SBD wage \( w_s \) are determined simultaneously in equilibrium.

(iv) The total amount of capital in the city is still fixed at \( \bar{K} \). However, this will now be divided into the capital at the center \( K_c \), and the capital at the subcenter \( K_s \). The division of capital between the two employment locations is determined exogenously. To preserve durability of capital, I do not allow all capital to move out to the suburbs.

Labor Demand

The labor demand functions in the urban area can be determined from the first order conditions of profit maximization. Now there are two sites, where production is carried on by identical firms with identical technologies but different endowments of capital. Production uses labor and capital, as follows

\[
Q_c = A.\{N_c\}^\theta.\{K_c\}^\phi
\]

\[
Q_s = A.\{N_s\}^\theta.\{K_s\}^\phi
\]
where $Q_c$ and $Q_s$ are the quantities produced of the numeraire good (price = 1) and $A$ is the Hicks neutral shift factor. The amount of capital in the urban area is fixed:

$$K_c + K_s = \bar{K} \quad \text{.....(3.18)}$$

The division of capital between the center and the subcenter are exogenously determined, i.e. $K_c = \bar{K_c}$ and $K_s = \bar{K_s}$. If $w_c$ and $w_s$ represent wages at the CBD and the SBD respectively, and $r$ is the rental rate on capital, the firms maximize the profit functions

$$\pi_c = A_c (N_d^\theta K_c^\psi - w_c N_c - r_c \bar{K_c}) \quad \text{.....(3.18a)}$$

$$\pi_s = A_s (N_d^\theta K_s^\psi - w_s N_s - r_s \bar{K_s}) \quad \text{.....(3.18b)}$$

The rental rate on capital is probably not equalized between the center and the suburb. However, since I am not interested in differences in rental rates at different locations as possible inducements for capital to move out; instead focusing on the effects of capital movement on labor demand, supply and ultimately rent profiles, the determination of rental rates is not important to my model. Given that $\theta + \varphi = 1$, the first order conditions for profit maximization yield

$$A_c \theta (N_d^\theta K_c^\varphi - w_c = 0 \quad \text{.....(3.19a)}$$

$$A_s \theta (N_d^\theta K_s^\varphi - w_s = 0 \quad \text{.....(3.19b)}$$

from which I get the labor demand functions $N_c^D$ and $N_s^D$ in terms of $w_c$ and $w_s$ respectively

$$N_c^D = \left(\frac{A_c \theta (K_c^\varphi)}{w_c} \right)^\frac{1}{\varphi} \quad \text{.....(3.20a)}$$

$$N_s^D = \left(\frac{A_s \theta (K_s^\varphi)}{w_s} \right)^\frac{1}{\varphi} \quad \text{.....(3.20b)}$$

**Labor Supply**

On the labor supply side, households work either at the CBD for the wage offer $w_c$ or at the SBD for the wage offer $w_s$. They maximize utility functions with respect to
demand for land $L$ and the numeraire good $Q$, with respect to their income net of commuting costs. The equilibrium is found along the lines of the monocentric city. Hence the CBD and SBD workers respectively maximize the Lagrangians

$$V_c = \left[ Q_d \right]^{\alpha} \left[ L_d \right]^{\beta} - \lambda_1 \left[ 8w_c - (2w_c + m)t_c d_c(i,j) - Q_c - R_c(i,j).L_c(i,j) \right] \quad (3.21a)$$

$$V_s = \left[ Q_s \right]^{\alpha} \left[ L_s \right]^{\beta} - \lambda_2 \left[ 8w_s - (2w_s + m)t_s d_s(i,j) - Q_s - R_s(i,j).L_s(i,j) \right] \quad (3.21b)$$

with respect to $Q$'s and $L$'s, generating the demand functions for $Q$ and $L$ as functions of all parameters and distance from center $d_c$ and distance from the SBD $d_s$, where

$$d_c(i,j) = \sqrt{i^2 + j^2}, \quad t_c(i,j) = \left( \frac{1}{f} \right) d_c(i,j), \quad d_s(i,j) = l d_c - d_{subl} \text{ and } t_s(i,j) = \left( \frac{1}{f} \right) d_s(i,j)$$

where $f$ is the speed of commute and $d_{sub}$ the location of the subcenter.

From the first order conditions, the demands for $Q$ and $L$ at the CBD are

$$Q_c(i,j) = \alpha \left\{ 8w_c - 2w_c t_c(i,j) - 2md_c(i,j) \right\} \quad (3.22)$$

$$L_c(i,j) = \left( \frac{\beta}{R_c(i,j)} \right) \left\{ 8w_c - 2w_c t_c(i,j) - 2md_c(i,j) \right\} \quad (3.23)$$

Corresponding equations for $Q_s$ and $L_s$ can be found. Equations (3.22), (3.23) and the corresponding ones for the subcenter can be used to find the indirect utility functions

$$V_c^* = \left( \frac{\alpha^\alpha \beta^\beta}{R_c(i,j)} \right) \left\{ 8w_c - 2w_c t_c(i,j) - 2md_c(i,j) \right\} \quad (3.24a)$$

$$V_s^* = \left( \frac{\alpha^\alpha \beta^\beta}{R_s(i,j)} \right) \left\{ 8w_s - 2w_s t_s(i,j) - 2md_s(i,j) \right\} \quad (3.24b)$$

For equilibrium, all homogeneous individuals in the city should have the same level of utility, regardless of where they locate. Therefore

$$V_c^* = V_s^* = V^* \quad (3.25)$$

Equations (3.24a), (3.24b) and (3.25) in turn yield the rent functions
\[ R_c(i,j) = \left( \frac{\alpha^\alpha \beta^\beta}{V^*} \right)^{\frac{1}{\beta}} \left[ 8w_c - 2w_f \left( \frac{1}{f} \right) d_c(i,j) - 2md_c(i,j) \right]^{\frac{1}{\beta}} \]  

\[ R_s(i,j) = \left( \frac{\alpha^\alpha \beta^\beta}{V^*} \right)^{\frac{1}{\beta}} \left[ 8w_s - 2w_s \left( \frac{1}{f} \right) d_s(i,j) - 2md_s(i,j) \right]^{\frac{1}{\beta}} \]  

Equations (3.26a) and (3.26b) use the time of commuting functions. The equilibrium utility level \( V^* \) is determined when the system of equations is solved for given populations at the center and the suburbs. The rent gradients for each groups of people is negatively sloped.

The two groups of people compete with one another for land at every location in the city. I use the following rule for allocating land and deciding who lives where:

If \( R_c > R_s \), the CBD workers get the land

If \( R_s = R_c \), the SBD workers get the land

Rules (1) and (2) above hold iff max \([R_c(i,j), R_s(i,j)]\) \( \geq R_a \). 

The allocation is carried out along the lines hinted at by Hamilton and Mills (HM), where I assume that if workers are indifferent between working at the center and the subcenter, they will work at the SBD.1

One of the two groups 'wins' the land at each location in the interior of the city by allocation rule (3.27). The process continues till the boundary of the city is reached b miles from the center, where the rent offer by the dominant group of city residents falls below the agricultural rent offer \( R_a \), i.e.

\[ R_a < \left( \frac{\alpha^\alpha \beta^\beta}{V^*} \right)^{\frac{1}{\beta}} \left[ 8w - 2w \left( \frac{1}{f} \right) b - 2mb \right]^{\frac{1}{\beta}} \]  

where \( w \) is the wage offer made to the dominant group at \( b \).

---

1 As mentioned before, making the same rent offer implicitly signifies having the same level of utility.
For all $R(i,j) = R_a$, then, the population density at each location can be found by setting the demand for land equal to the supply. Assuming that the land supply at each coordinate point is $1$, I find the population densities for each group of workers at each location along the lines of equation (3.10). The total population of each group in the urban area is then found by summing up the populations at each point:

$$N_c^s = \sum_i \sum_j \left( \frac{\alpha \gamma}{\gamma^*} \right)^\beta \left[ 8w_c - 2w_d \left( \frac{1}{f} \right) d_c(i,j) - 2md_c(i,j) \right]^{\frac{\alpha \gamma}{\beta}}$$

iff $R_c > R_s$ and $R_c(b) = R_a$ ....(3.29a)

and

$$N_s^s = \sum_i \sum_j \left( \frac{\alpha \gamma}{\gamma^*} \right)^\beta \left[ 8w_s - 2w_d \left( \frac{1}{f} \right) d_s(i,j) - 2md_s(i,j) \right]^{\frac{\alpha \gamma}{\beta}}$$

iff $R_s > R_c$ and $R_s(b) = R_a$ ....(3.29b)

where $N_c^s$ and $N_s^s$ represent the supplies of CBD labor and SBD labor respectively.

Equations (3.29a) and (3.29b) can be rewritten as

$$N_c^s = \left( \frac{\alpha \gamma}{\gamma^*} \right)^\beta \sum_i \sum_j \left[ 8w_c - 2w_d \left( \frac{1}{f} \right) d_c(i,j) - 2md_c(i,j) \right]^{\frac{\alpha \gamma}{\beta}}$$

......(3.30a)

$$N_s^s = \left( \frac{\alpha \gamma}{\gamma^*} \right)^\beta \sum_i \sum_j \left[ 8w_s - 2w_d \left( \frac{1}{f} \right) d_s(i,j) - 2md_s(i,j) \right]^{\frac{\alpha \gamma}{\beta}}$$

......(3.30b)

where $N_c^s$ and $N_s^s$ satisfy the allocation rule and the boundary condition, as in (3.29). The total population of the city is given by

$$N^s = N_c^s + N_s^s$$

......(3.31)

The equilibrium level of utility $V^*$ is found in a closed city when the total population $N^s$ is held constant and the equation system is solved simultaneously.
Closure

To close out the general equilibrium model, the total demand for CBD labor (from equation (21a)) must be equal to the total supply of CBD labor (from equation (3.30a)). Similarly, the total demand for SBD labor (from equation (3.20b)) must equal the total supply of SBD labor (from equation (3.30b)):

\[ N^D_c = N^s_c \quad \ldots (3.32a) \]

\[ N^D_s = N^s_s \quad \ldots (3.32b) \]

In addition, in the closed city, if the total city population is fixed at \( \bar{N} \), the sum of the two populations cannot exceed \( \bar{N} \). Hence

\[ N^s = \bar{N} \quad \ldots (3.32c) \]

Note that the satisfaction of (3.32a) through (3.32c) maintains both the solution of the general equilibrium system and the closed city assumption.

3.2b. SIMULATION TECHNIQUES, RESULTS AND ANALYSIS

There are now three endogenous variables: \( w_c \), \( w_s \) and \( V^* \). Their equilibrium values are found from the simultaneous solution to the three equations:

\[ \bar{N} - N^D_c - N^D_s = 0 \quad \ldots (3.33a) \]

\[ N^D_c - N^s_c = 0 \quad \ldots (3.33b) \]

\[ N^D_s - N^s_s = 0 \quad \ldots (3.33c) \]

Equations (3.33a), (3.33b) and (3.33c) are derived from the closure conditions (3.32a)-(c). Substituting for labor demand and supply from equations (3.20) and (3.30) respectively, the three equation system is

\[ \bar{N} - \left( \frac{A \cdot \theta_c (K_c)^p}{w_c} \right)^\frac{1}{\phi} - \left( \frac{A \cdot \theta_s (K_s)^p}{w_s} \right)^\frac{1}{\phi} = 0 \quad \ldots (3.34) \]
\[
\left(\frac{A_0(K_c)}{w_c}\right)^{1/\varphi} - \left(\frac{\alpha\alpha_1}{\varphi^{1/\varphi}}\right) \sum_i \sum_j \left[ 8w_c - 2w_c \left(\frac{1}{f}\right) d_c(i,j) - 2md_c(i,j) \right]^{\alpha_1}_{\beta_i} = 0 \quad \ldots \quad (3.35)
\]

\[
\left(\frac{A_0(K_s)}{w_s}\right)^{1/\varphi} - \left(\frac{\alpha\alpha_1}{\varphi^{1/\varphi}}\right) \sum_i \sum_j \left[ 8w_s - 2w_s \left(\frac{1}{f}\right) d_s(i,j) - 2md_s(i,j) \right]^{\alpha_1}_{\beta_i} = 0 \quad \ldots \quad (3.36)
\]

Given the values of all other parameters, equations (3.34)-(3.36) are simultaneously solved to yield the equilibrium values of \(w_c, w_s\) and \(V^*\). Also, (3.35) and (3.36) are solved using the boundary condition \(\max[R_c(i,j), R_s(i,j)] \geq R_s\).

As before, I use FSOLVE to solve the three equation system simultaneously. To find \(N_c^g\) and \(N_s^g\), I utilize the ODE45 solver. The system is complicated since the labor demand and supply systems are sensitive in different degrees to changes in guess values for \(w_c\) and \(w_s\). To avoid kinks, I use the continuous population density function

\[
n_c^g = \left(\frac{\alpha\alpha_1}{\varphi^{1/\varphi}}\right) \left[ 8w_c - 2w_c \left(\frac{1}{f}\right) d_c - 2md_c \right]^{\alpha_1}_{\beta_i} \quad \ldots \quad (3.37a)
\]

\[
n_s^g = \left(\frac{\alpha\alpha_1}{\varphi^{1/\varphi}}\right) \left[ 8w_s - 2w_s \left(\frac{1}{f}\right) d_s - 2md_s \right]^{\alpha_1}_{\beta_i} \quad \ldots \quad (3.38a)
\]

where \(d_s = d_c - d_{sub}\), and \(d_c, d_s\) and \(d_{sub}\) are measured along a line from the center. I use the following 'smooth' differential equations to find \(N^g\)

\[
N_c^g = 2.\pi.d_c.N_c^g.y.e^{(Rdiff1-2)} \text{ where } Rdiff1 = \frac{R_d(d_c) - R_s}{|R_d(d_c) - R_s|} \quad \ldots \quad (3.37b)
\]

\[
N_s^g = 2.\pi.d_c.N_s^g.y.e^{(Rdiff2-2)} \text{ where } Rdiff2 = \frac{R_d(d_s) - R_s}{|R_d(d_s) - R_s|} \quad \ldots \quad (3.38b)
\]

I then use ODE45 to solve (3.37b) and (3.38b) solve for initial values \(d_c = 0, N_c^g = 0, N_s^g = 0\) and final value \(d_c = 30\). After the solutions are plugged back in FSOLVE and the final solutions emerge, I use the kinked (but continuous) functions (3.37a) and (3.38a) to check the results via trial and error. The solutions are obtained via the following loop:
Guess \( V^* \), \( w_c \) and \( w_s \)

\[ \downarrow \]

(Labor Demand)

Given \( w_c, w_s \) and all other parameters, find \( N_c^P \) and \( N_s^P \).

\[ \uparrow \]

(Labor Supply)

Given \( V^* \), \( w_c, w_s \) and all other parameters, find all \( R_c(i,j), R_s(i,j) \)

such that \( \max[R_c(i,j), R_s(i,j)] \geq R_a \).

Hence find population densities \( \Pi_c^S \) and \( \Pi_s^S \) at all locations.

Solve the differential equations (3.37b) and (3.38b) over the urban area
to find \( N_c^S \) and \( N_s^S \).

\[ \uparrow \]

Check if conditions (3.33a)-(3.33c) hold. If not

\[ \rightarrow \rightarrow \rightarrow \rightarrow \]

Once \( V^* \) has been determined and the city laid out,
determine \( d_L \), the interface between CBD and SBD workers
and \( b \), the city boundary.

All parameters that are common to this model and the monocentric model have the
same values as in Table 3.1. For the coefficients \( \theta \) and \( \psi \), I use both sets of values. First,
I study the city with more productive labor (\( \theta = .72; \psi = .28 \)). Tables 3.3-3.5 below
present the equilibria for the two centered General Equilibrium city (henceforth Gwage)
with flexible wages at both centers as the exogenous distribution of the (fixed) total capital
stock in the city changes from all capital at the CBD (monocentric city) to 75% of the capital
at the CBD, 50% of the capital at the CBD and 25% of the capital at the CBD. Due to the
complexity of the solution process and the different degrees of non-linearity on the demand
and supply side, I accept errors of \( \pm .5\% \) in the solutions.
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**TABLE 3.3:** EQUILIBRIA IN THE GWAGE CITY ($\theta = .72; \ \Phi = .28$) WITH 75% OF THE TOTAL CAPITAL STOCK OF THE CITY CONCENTRATED AT THE CBD
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**TABLE 3.4: EQUILIBRIA IN THE GWAGE CITY ($\theta = .72; \varphi = .28$) WITH 50% OF THE TOTAL CAPITAL STOCK OF THE CITY CONCENTRATED AT THE CBD**
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*TABLE 3.5: EQUILIBRIA IN THE GWARE CITY ($\theta = .72; \varphi = .28$) WITH 25% OF THE TOTAL CAPITAL STOCK OF THE CITY CONCENTRATED AT THE CBD*
I compare the succession of equilibria shown in Tables 3.3-3.5 to the monocentric city with the same values of \( \theta \) and \( \Psi \). Also, to better contrast the changes as the city moves from a fixed central wage to a Gwage framework, I look at the decentralized city under the HM regime for the values of \( d_{sub} \) that I choose to depict. As in the previous chapters, I look at a city with its SBD fairly close to the center (in the ESZ), one with its SBD far out in the suburbs (in the EDZ) and one with its SBD very close to, or beyond, the boundary of the original monocentric city. I choose SBDs at distances of 8, 16 and 20 miles respectively for the three cases. Table 3.6 below lists the HM equilibria for SBD 8 and 16 miles out. I exclude the equilibrium for an SBD 20 miles out, since I know from the exercises in Chapter 1 that when employment locates very close to the boundary, the rent profile will be almost identical to that of the monocentric city. For iterating the HM city, the same techniques as in Section 9 and the same parameter values as in Table 9 are used, with the one exception that the CBD wage is assumed fixed at $22.6794, the equilibrium wage in the monocentric city for \( \theta = .72, \Psi = .28 \). Without a full labor market, SBD wages are determined by the HM wage gradient and the result in a segmented city with all workers outside the SBD loop working at the subcenter.

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**Table 3.6: Equilibria in HM City \((w_c = 22.6794)\)**

Figure 3.3 below depicts the rent and population density profiles when the SBD locates 8 miles out in the three Gwage (.72/.28) cities. Figures 3.4a and b then provide 'close ups' of the rent and population densities near the CBD and the boundary of the Gwage cities. Figure 3.5, 3.6a and b repeat the procedure for the three Gwage16 (.72/.28) cities. In all cases, I compare the Gwage cities to the HM cities from Table 3.6. The slopes of the profiles should not be compared across graphs, since I use different scales to highlight different aspects of the equilibria.
FIGURE 3.3: COMPARISON OF RENT AND POPULATION DENSITY PROFILES IN GWAGE CITIES (SBD 8 MILES) WITH DIFFERENT DIVISIONS OF CAPITAL

LEGEND: r1c(x/y), r2cHM8 -- as before; r2cGwage8 (capc=?) -- showing concentration of capital at CBD
FIGURE 3.4a: CLOSE VIEWS (NEAR CBD AND BOUNDARY) OF RENT PROFILES IN GWAGE CITIES (SBD 8 MILES) WITH DIFFERENT DIVISIONS OF CAPITAL

LEGEND: r1c(x/y), r2cHM8, r2cGwage8 (capc=?), as before
FIGURE 3.4b: CLOSE VIEWS (NEAR CBD AND BOUNDARY) OF POPULATION DENSITY PROFILES IN GWAGE CITIES (SBD 8 MILES) WITH DIFFERENT DIVISIONS OF CAPITAL

**LEGEND:** r1c(x/y), r2cHM8, r2cGwage8 (capc=?) -- as before
FIGURE 3.5: COMPARISON OF RENT AND POPULATION DENSITY PROFILES IN GWAGE CITIES (SBG 16 MILES) WITH DIFFERENT DIVISIONS OF CAPITAL

LEGEND: r1c(x/y), r2cHM16, r2cG wage16 (capc=?) -- as before
FIGURE 3.6a: CLOSE VIEWS (NEAR CBD AND BOUNDARY) OF RENT PROFILES IN GWAGE CITIES (SBD 16 MILES) WITH DIFFERENT DIVISIONS OF CAPITAL

LEGEND: r1c(x/y), r2cHM16, r2cGwage16 (capc=?) -- as before
FIGURE 3.6b: CLOSE VIEWS (NEAR CBD AND BOUNDARY) OF POPULATION DENSITY PROFILES IN GWAGE CITIES (SBD 16 MILES) WITH DIFFERENT DIVISIONS OF CAPITAL

LEGEND: r1c(x/y), r2cHM16, r2cGwage16 (capc=?) -- as before
GENERAL ANALYSIS AND CONCLUSIONS

From Figures 3.3-3.6, and reviewing the numbers from Tables 3.5-3.7, the effects of capital movements in the city can be seen when all wages are flexible. It is important to look at commuting patterns in the Gwage city, since the area from which the CBD draws its labor is an important piece of the puzzle in this case. Figure 3.7 below depicts the commuting patterns in the Gwage cities with different concentrations of capital at the two employment centers, through studying the interfaces between the living area of CBD and SBD workers. The interfaces of these two groups in the corresponding HM cities (which always occurs at the SBD location, and is hence a 45° line on the graph) are presented as a point of reference.

![Graph showing commuting patterns in HM and Gwage cities with different concentrations of capital at the CBD](image)

**FIGURE 3.7: COMMUTING PATTERNS IN HM AND GWAGE CITIES WITH DIFFERENT CONCENTRATIONS OF CAPITAL AT THE CBD**

**LEGEND:**
- $dL2eHM (.72/.28)$ -- figures for HM cities compatible with Gwage (.72/.28) cities;
- $dL2eGwage (capc = ?)$ -- as before

Looking at the tables, it is evident that as more and more capital shifts out to the suburbs, the labor employed at the CBD decreases and that employed in the SBD increases. However, the rate of increase is not constant. One way to pinpoint the reasons behind these and other effects of decentralization on the residents of the city is through reviewing
the information in Figure 3.7. As is clearly seen from the graph, each of the Gwage cities has one and exactly one location of $d_{sub}$ at which the city is completely segmented (i.e. there is no cross commuting among the zones inside and outside the SBD loop). These points, at which the graphs for the various Gwage cities intersect with the HM city line, yield the respective values of $d^*$ -- the dividing line between the Excess Supply Zone (ESZ) and the Excess Demand Zone (EDZ). The ESZ and EDZ are now defined as follows for a given location of the SBD: Assume the CBD wage is fixed at $w^*$, the monocentric city equilibrium level. Assume the SBD offers a reservation wage $w_{rs}$ sufficient to make the people outside the loop indifferent between working at either the center or the subcenter (by allocation rule (3.27), they will then work at the SBD). If the number of people located inside the SBD loop are sufficient to meet the demand for labor at the CBD when $w^*$ is offered there, and the number of people living outside the loop are sufficient to meet the labor demand at the SBD when $w_{rs}$ is offered there, there is instant equilibrium in the labor market. If there are too many people willing to work at the SBD, the subcenter is assumed to be in the ESZ. If there are too few people willing to work at the SBD, I assume it is located in the EDZ.

Combining the information from Figure 3.7 and Tables 3.4-3.6, it can be seen that for any given division of capital, when the SBD forms in the ESZ, the SBD employment rises comparatively fast with increases in $d_{sub}$. When the SBD forms in the EDZ, the rise in SBD employment is comparatively much smaller. Also, the employment at the SBD (and the CBD) increases with the allocation of more capital to the suburb, shifting $d^*$ out. From Figure 3.7, the ESZ stretches over 0-16 miles when 25% of the capital decentralizes, over 0-12 miles when 50% of the capital decentralizes, and over 0-8 miles when 75% of the capital decentralizes. The higher the amount of capital at the center, the higher is the labor demand and $d^*$ is the furthest out.

I look at the changes in city structure when employment decentralizes in the ESZ and in the EDZ. For each of these, I analyze the effects on wages, land rents and utility. The major difference in the Gwage structure is, of course, that wages are flexible at both employment centers.

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2 The small discrepancies at the points of intersection between the Gwage and HM curves are due to rounding errors from the ODE45 programs.
(A) Employment Decentralization in the ESZ

Changes in Wages and Employment

In the monocentric city, all capital is concentrated at the CBD and all people work there. With no capital outside the CBD, labor demand is zero everywhere else. In this scenario, consider an SBD loop forming at $d_{sub}$, where $d_{sub} < d^*$, and a certain amount of the capital stock moving there. When the capital stock at the CBD falls, the labor demand falls. Similarly, labor demand rises at the SBD.

Assume the SBD offers a reservation wage $w_{rs}$, while the CBD wage stays fixed at $w^*$ in the first instance. For convenience, start by assuming that commuting costs are not dictated by time lost in commute. The reservation wage is then exactly the same as the HM wage. Figure 3.8 below depicts the pressures on wages at the employment centers. I use employment figures from Tables 3.4 and 3.6 (SBD 8) as an example.

![Diagram](image)

**FIGURE 3.8**

When capital moves out to the SBD, labor demand falls to $N^D_c(K_2)$ at the CBD and rise to $N^S_c(K - K_2)$ at the SBD. When the SBD offers HM wages, the labor supply to the SBD (which was zero in the monocentric city) rises to 79125 along the given labor supply
curve $N^e_w(w^*)$. Simultaneously, the labor supply to the CBD shrinks to $N^e_w(w^*)$.\(^3\) At a wage $w^*$, there will now be an excess demand for labor ($N^e_w - 20891$) at the center. So CBD wages must adjust upwards. There are two effects on $w_s$. As CBD wages rise, the reservation wage for the SBD also increases. But, at the reservation wage, there is an excess supply of labor (on the graph, I show the excess supply at the initial reservation wage $w_{rs}$: $(79125 - N^e_r)$) at the subcenter. This will tend to depress SBD wages. Hence the end result will be increases in both CBD and SBD wages, but SBD wages will always be below reservation wages, which in turn are lower than CBD wages. Hence relative SBD wages ($w_s/w_c$) in the Gwage city will be below relative SBD wages in the HM city, compensating for the excess supply. Figure 3.9 below depicts the movement of relative SBD wages in the Gwage city, comparing them to the HM wage level.

\(^3\) Two things to note here: first, I use the HM city employment figures for convenience, though they are obviously tainted by the distortions from the overcompensation effects. Second, the labor supply curve for both centers will have the same shapes, since the two groups of workers have identical preferences. However, when the employment at one center increases, the labor supply for the other center will shift inwards, to maintain the closed city assumptions.
Two other trends can also be noticed from Figure 3.9. First, for a given location of the subcenter, the higher the amount of capital at the SBD, the higher relative SBD wages are. This makes perfect sense. With more capital at the SBD, the labor demand there increases (shifting $N^*_3(K - K_2)$ out further in Figure 3), causing less of a downward adjustment in $w_s$ due to excess supply. Simultaneously, labor demand falls more at the CBD, causing wages there to increase less. When enough capital moves to the SBD, it may even escape being in the ESZ altogether, instead shifting over into the EDZ. This can be observed for SBDs 8 miles out and beyond as more and more capital moves out of the center.

Second, for a given division of capital, the closer $d_{sub}$ is to $d^*$, the higher the relative SBD wages are. Again, this makes sense. The closer the SBD is to the boundary of the ESZ, the lower the excess supply when the SBD offers reservation wages. Hence the lower the downward adjustment in $w_s$. When capital moves out to the ESZ, outward shifts in the SBD location will cause relative SBD wages to start below HM wage rates, but the Gwage city relative wages will fall at a slower rate than the HM city rates, eventually catching up to the latter when employment decentralization occurs near $d^*$.

The other thing to observe from the labor market is that for a given distribution of capital, the amount of employment at the SBD decreases as employment decentralization occurs further out. In the ESZ, there are significant changes in employment with changes in $d_{sub}$. This happens due to another feature of the labor supply function -- the importance of commuting costs. When the SBD locates close to the center, people near and just beyond $d_L$ do not benefit by a large amount through the save in commute by switching from the CBD to the SBD. As a result, comparatively larger changes in SBD wages are required to win people over. This can be identified as a situation where the CBD is dominant over the SBD. Since CBD wages rise more than SBD wages, CBD labor demand and hence employment is higher when the two centers are close. As the SBD moves further away, the dominance of the CBD is eroded as commuting costs become more important. SBD labor demand and employment rises significantly. When the SBD reaches $d^*$, the level of employment at both centers reach an 'equilibrium level'. Thereafter, as employment decentralizes in the EDZ, the rise in SBD employment is much smaller.
In my numerical analysis, the results will be somewhat affected by the presence of the overcompensation effects, since I include time costs in my calculations. However, the overcompensation effects would be small, since wages and number of workers at the SBD tend to move in opposite directions during the adjustment process.

Changes in Land Rents

Given $d^*$, Figure 3.10 below depicts what happens when employment decentralization occurs in the ESZ ($d_{sub}$ less than $d^*$). Two Gwage cities for different values of the capital stock at the CBD are shown. Comparing to Figure 3.3, $capc_1$ corresponds to $.75*cap$ and $capc_2$ corresponds to $.25*cap$. Comparing Gwage ($capc=.75*cap$) to the HM city when $d_{sub}$ locates in the ESZ, when the SBD offers reservation wages, there will be a significant excess supply of labor at the SBD and an excess demand for labor at the CBD. Hence under the Gwage ($capc=.75*cap$) regime, the CBD firms must increase their wage offers to induce people living outside the loop to commute all the way in to the center. SBD wages will also rise relative to the HM city level, for the reasons mentioned before. With the higher SBD wages, the amount of overcompensation increases per worker, though the number of SBD workers decreases.

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4 Basically, $capc_2 < capc_1$ always.
There are some ambiguous effects on the land market in the Gwage city. First, consider the effects on land rents in the interior of the city. With lower CBD employment, land demand falls in the interior. The existing CBD workers demand more land, offsetting the first effect. On the land supply side, there are several changes. Since both CBD and SBD workers get higher wages, they want to move out to satisfy their quest for more land. The secondary effect (from the outward movement of CBD and SBD workers) would thus be to increase land supply and lower rents in the interior. The net effect on rents in the interior depends on which of these effects are stronger. In the suburbs, higher SBD wages increase the demand for land, while higher CBD and SBD wages constrict land supply. Rents thus rise unambiguously, even above the HM city level.

As depicted in Figure 3.10, the overall rent gradient is flatter in the Gwage (capc=.75*cap) city. The opposing effects on land demand and supply close to the center cancel each other out, as rents are almost identical to the monocentric city levels. In the suburbs, rents are unambiguously higher, even more so than the HM city levels. Figure 3.4a matches this analysis. As for population density, the city gets less dense everywhere, with a significant number of people now living beyond the boundary of the monocentric cities. The population in the suburbs for the Gwage (capc=.75*cap) city will be lower than for the HM city, since the HM city is less crowded in the interior (Figure 3.4b).

When more capital moves out to the subcenter, the excess supply of labor to the SBD at reservation wages is smaller, since the labor demand is higher at the SBD. Consequently, land supply is higher near the center and tighter in the suburbs. The effects on land demand are smaller near the center and larger in the suburbs. The net effect is to drop rents in the interior, and raise them even higher than in the previous case near the boundary. This analysis pertains to Gwage(capc2) in Figure 3.10, which in turn corresponds to the r2c Gwage8 (capc=.25*cap) curves in Figures 3.3 and 3.4a.

**Changes in Utility Levels**

Figure 3.11 below maps the changes in utility levels with employment decentralization. Concentrating on decentralization in the ESZ, there are some conflicting pressures on utility. Consider the changes close to the CBD in a Gwage city with a fixed
distribution of capital. I have already looked at the effects on wages, patterns of commute and land rents in the interior of the city. Combining all the information, the CBD workers get higher wages and face equal or lower rents close to the center. Thus utility should unambiguously increase for them. Though the picture is less clear for SBD workers, whose wage increases are somewhat offset by higher rents in the suburbs, it is clear from the information in Tables 3.3-3.5 (and due to the equal utility condition) that the positive wage effects sweep the negative effects of the increases in rents.

General Comments

The above analysis matches the information in Tables 3.3-3.5 and Figures 3.8-3.11. From Figure 3.9, when employment decentralizes in the ESZ, the relative level of SBD wages falls at about the same rate as the wage gradient in the HM city. The relative SBD wages are lower than HM wages; however, the drop in SBD employment is huge compared to the small fall in relative SBD wages. Thus the CBD wage is just high enough
(and the relative SBD wages just low enough) to overcome the excess demand in the HM city by weaning away workers away from the subcenter (e.g. in going from the HM8 city to the Gwage8 (capc = .75*cap) city, the CBD induces approximately 46000 SBD workers to switch. The increase in CBD wages is proportional). The employment at the subcenter increases as the SBD locates further out, moving the relative SBD wages closer to the HM levels, since more people live inside the loop and less people living outside have to be lured to the CBD.

Finally, as employment decentralizes further out, total commuting decreases (the interface between CBD and SBD workers moves closer to the subcenter) and overall utility increases. For a fixed location of the SBD, greater decentralization of capital will have the same effect -- more the employment at the subcenter, lower the land demand and greater the land supply near the CBD, and higher the utility level. What is interesting to note is that in all Gwage cities, utility will rise continually as employment decentralizes, be it in the ESZ or the EDZ. This reintroduces the idea of the utility gradient, which first surfaced in the context of the Ewage city, but only in the EDZ. A quantitative analysis of changes in major variables with increasing capital decentralization is presented at the end of this section.

(B) Employment Decentralization in the EDZ

When employment decentralization takes place in the EDZ, the effects on residential location will naturally be dictated by the amount of the excess demand at the SBD (and by extension the amount of excess supply at the CBD). Once again, I look at the effects on wages, land rents and utility.

Changes in Wages and Employment

When the subcenter locates beyond d*, the effects on the CBD and SBD labor markets will be exactly the opposite of the situation depicted in Figure 3.8. Figure 3.12 below sketches what happens. When capital decentralizes far away from the CBD, a comparatively large number of workers live inside the SBD loop. Labor demand falls at the CBD and rises at the SBD. If the CBD continues to offer w* and the SBD offers the reservation wage wr (aimed at capturing all the people outside the SBD loop), there is an
excess supply of labor at the center and an excess demand for labor at the subcenter. I use numbers from HM16 and Gwage16 (capc = .75 * cap) cities as an example.

There is upward pressure on SBD wages. When \( w_s \) becomes high enough so that too many people want to work at the subcenter and an excess demand is created at the CBD, \( w_c \) must also rise. However, the initial excess supply at the CBD exerts a downward pressure on \( w_c \). Also, people move outwards with higher incomes, changing labor supply. Relative SBD wages will stay far above HM levels, but will change little with employment locating further out.

When more capital moves out to the same subcenter location, the CBD labor demand curve in Figure 3.12 shifts further in, increasing the excess supply at the CBD. Simultaneously, the SBD labor demand curve shifts out, increasing the excess demand there. Thus in the Gwage cities with 50% and 75% of the capital at the subcenter, relative SBD wages are even higher. This is where the SBD becomes increasingly more dominant, especially when employment decentralizes close to the original boundary. This trend is evident from tables 3.3-3.5.

As employment decentralizes further and further out in the EDZ, the CBD labor supply curve shifts in, widening the excess supply there. SBD labor supply shifts out, raising the excess demand. There is more upward pressure on SBD wages, as well as
more downward pressure on CBD wages form this source. However, when SBD wages rise, CBD wages will have to increase to maintain employment levels, especially in view of the fact that with commuting costs between the CBD and the SBD increasing, the SBD now gains workers with comparatively smaller inducements. In addition, when higher wages are offered, the recipients tend to demand more land and move towards the suburbs, increasing labor supply there but decreasing it close to the center. Overall, relative SBD wages will be almost constant. Similarly, increases in employment as decentralization occurs further out will be small. From Tables 3.3-3.5 and Figure 3.9, the labor demand and supply effects seem to cancel each other out; thus relative SBD wages and employment levels change little when employment locates beyond a certain distance in the EDZ.

Changes in Land Rents

The effects on land rents are fairly unambiguous when employment locates in the EDZ. Compared to the HM city, more people work in the suburbs and less work at the center. Also, more land is available in the interior as people move out in search for cheaper suburban land. Considering the effects on the people who live in the interior, land demand falls and land supply rises, causing rents to decrease. Rents increase significantly in the suburbs, since the positive effects on SBD wages are strong enough to cause a rent subpeak in the suburbs and induce some people to commute outwards every morning.

When more capital moves into the suburbs, the effects on rents get more pronounced. As Figures 3.5 and 3.6a show, when 25% of the capital shift out to the SBD, the effects on rents are small -- in fact, practically nil near the center. There is just the hint of a peak at the subcenter. The drop in rents at the CBD and the rise near the SBD become more pronounced with more capital moving out.

As the SBD locates further out, the dominance of the center gets eroded and the effects on rents are bigger. Figures 3.13a and b below depict the situations when employment locates 20 and 22 miles out respectively. Notice that a 'hole' appears when 75% of the capital locates 22 miles out. This is when the SBD becomes 'independent'. 
FIGURE 3.13a: RENTS IN THE GWAGE CITY WITH SBD 20 MILES OUT

FIGURE 3.13b: RENTS IN THE GWAGE CITY WITH SBD 22 MILES OUT

LEGEND: as before
Changes in Utility

From Figure 3.11, utilities in the Gwage cities keep on increasing as employment locates in the EDZ. When CBD and SBD wages become constant, lower land rents due to expansions in land supply raise the consumption of land and hence increase utility everywhere in the city. Also, as more capital shifts to the suburbs, land supply increases more, increasing utility. What is interesting to note is that the slope of the utility curve does not change much as the SBD switches from the ESZ to the EDZ. This is a marked change from the situation in the Ewage cities where utility levels changed little when employment decentralized in the ESZ, then rose sharply when employment decentralized in the EDZ. More about this at the end of the section.

The Effects of more Capital shifting to the SBD

As more and more capital shifts to the SBD, the effects are obvious. With increased labor demand there, SBD wages rise. When employment locates in the EDZ and 75% of the capital is in the subcenter, relative SBD wages are no longer merely constant, but are actually rising (Figure 3.9). The interface between CBD and SBD workers move further in, as the SBD captures an increasingly greater portion of the city's workforce. Rents are increasingly lower in the interior of the city and increasingly higher in the suburbs. The overall utility of the city increases as with more jobs in the suburbs, the city spreads out, people switch to more lucrative SBD jobs and enjoy more open spaces. Also, as more and more capital shifts out to the SBD, the dominance of the CBD gets eroded. Similarly, when the SBD moves further out, saving the commute from the SBD loop to the CBD gains importance, decreasing the dominance of the CBD. When the above two circumstances combine, i.e. a large amount of capital moves very far out in the suburbs, the dominance of the CBD can disappear. This prompts the case in Figure 3.13b where the city actually splits up in two -- as evidenced by the 'hole' in the middle. Even in the Gwage (capc = .75*cap) city, if the SBD moves far enough out (e.g. 24 miles out -- Table 3.3), the SBD will eventually break the dominance of the CBD. However, the combination of increasing distance of decentralization and amount of decentralized capital accomplishes this much faster.
Another point that emerges from viewing the utility 'gradient's in Figure 3.11 is that with a fixed location of the SBD loop, increasing the amount of capital at the subcenter from 25% to 50% of the total capital stock raises utilities more than the process of increasing the amount of capital at the SBD from 50% to 75% of the total stock. This is especially true when employment locates in the EDZ. The reasons behind this can be found via the effects on labor supply. When employment locates close to the center, the CBD is very dominant. The effect of increasing the SBD capital from 25% to 50% of the total stock may or may not involve breaking the dominance of the center, i.e. inducing some workers to commute outwards. When the SBD locates in the EDZ, reverse commuting is introduced and a significant rent peak crops up at the SBD. This causes a significant amount of benefit to people who lived close to the suburbs and switched. When more capital switches to the suburb, the effect is only to raise the SBD rent peak further -- the additional benefits to SBD workers is not as pronounced as in the previous case.

3.3. SENSITIVITY ANALYSIS: CHANGES IN GWAGE CITIES WITH CHANGES IN LABOR PRODUCTIVITY

For a sensitivity analysis, I next consider the Gwage city where labor is less productive -- \( \theta = 0.9; \ \psi = 0.1 \) -- than in the previous case. Table 3.2 lists the equilibrium wage and utility level in the monocentric city with lower productivity. Notice that when labor is less productive, the equilibrium wages and utility levels decline; since less output being produced with the same amount of labor and capital. Figures 3.1a and 3.1b depict the rent and population density gradients in the cities with different labor productivity. For comparison, I use the HM cities with subcenter 8 and 16 miles out, same as before. Table 3.7 below lists the equilibria (with \( w_c \) now fixed at $18.7302). Tables 3.8, 3.9 and 3.10 then list the equilibria for the Gwage cities. Figures 3.14-3.18 then depict the rent and population density gradients (using close-ups where necessary) and compare them with the previous Gwage cities.

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**TABLE 3.7: EQUILIBRIA IN HM CITY (w_c = 18.7302)**
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**TABLE 3.8: EQUILIBRIA IN THE GWAGE CITY ($\theta = .9$; $\Psi = .1$) WITH 75% OF THE TOTAL CAPITAL STOCK OF THE CITY CONCENTRATED AT THE CBD**
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TABLE 3.9: EQUILIBRIA IN THE GWAGE CITY ($\theta = .9; \varphi = .1$) WITH 50% OF THE TOTAL CAPITAL STOCK OF THE CITY CONCENTRATED AT THE CBD
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**TABLE 3.10: EQUILIBRIA IN THE GWAGE CITY ($\theta = .9; \varphi = .1$) WITH 25% OF THE TOTAL CAPITAL STOCK OF THE CITY CONCENTRATED AT THE CBD**
FIGURE 3.14a: RENTS IN GWAGE ($\theta = .9$, $\phi = .1$) CITY WITH SBD 8 MILES OUT

FIGURE 3.14b: POPULATION DENSITIES IN GWAGE8 ($\theta = .9$, $\phi = .1$) CITY

LEGEND: as before
FIGURE 3.15: RENTS IN GWAGE8 (θ = .9; ϕ = .1) CITIES
(CLOSE UPS NEAR CBD AND BOUNDARY)

LEGEND: as before
FIGURE 3.16: COMPARISON OF RENTS IN GWAGE8 CITIES WITH DIFFERENT PRODUCTIVITY (CLOSE UPS NEAR CBD AND BOUNDARY)

LEGEND: as before
FIGURE 3.17a: RENTS IN GWAGE16 ($\theta = .9$, $\varphi = .1$) CITIES

FIGURE 3.17b: POPULATION DENSITIES IN GWAGE16 ($\theta = .9$, $\varphi = .1$) CITIES

LEGEND: as before
FIGURE 3.18: COMPARISON OF RENTS IN GWAGE16 CITIES WITH DIFFERENT PRODUCTIVITY (CLOSE UPS NEAR CBD AND BOUNDARY)

LEGEND: as before
From Figures 3.14-3.18 and Tables 3.7-3.10, I find the expected results. With the more labor intensive methods of production, the total output diminishes since in this particular city, the total capital stock is bigger than the total population. With the income effect, all wages are smaller in absolute value. As Figure 3.19 shows, however, relative SBD wages are higher in the Gwage (.9/.1) cities, especially so when employment decentralizes in the EDZ.

As a result of the higher relative SBD wages, normalized utility levels are higher for all SBD locations in the Gwage (.9/.1) city. However, absolute levels of utility are smaller. This implies that starting though workers in the Gwage (.9/.1) city start from a lower salary and utility level, decentralization of capital to a certain distance from the center benefits them relatively more than a similar change in the Gwage (.72/.28) city with higher
initial wages and utility levels. Figure 3.20 shows the changes in normalized values of equilibrium levels of utility in the Gwage cities with different labor productivity.

![Figure 3.20: Comparison of normalized utility levels in Gwage cities](image)

The important thing, however, is that the pattern of changes wrought on the city are the same in the Gwage (.9/.1) cities. With higher SBD wages, the employment at the subcenter are higher than in the corresponding Gwage (.72/.28) cities. Consequently, the interface between the CBD and the SBD move closer to the center. The values of $d^*$ are now around 12, 9 and 6 miles for the Gwage (.9/.1) cities with 75%, 50% and 25% of the total capital stock at the center, as compared to 16, 12 and 8 miles out for the Gwage (.72/.28) cities.

Since the changes in the city accrue to a population with low starting income and utility, the positive effects are heightened in the Gwage (.9/.1) city. This is borne out by
Figures 3.14-3.18. For example, in Figure 3.14, the Gwage8 (capc = .25*cap) city shows reverse commuting and significant rent peak near the SBD. From Table 3.10, this makes sense since for this city, d* is 6 miles out, so a subcenter 8 miles out is in the EDZ. From Figure 3.17, even with only 25% of the capital at the subcenter in the Gwage (.9/.1) city, there is a significant peak at the subcenter. Compare this to the corresponding Gwage (.72/.28) city, where there was a relatively small perturbation in the rent gradient, since in that city, the SBD at 16 was barely out of the ESZ and in the EDZ.

Finally, as Figure 3.16 and 3.18 show, the rents in the Gwage (.9/.1) cities are lower everywhere than in the corresponding Gwage (.72/.28) cities. With capital being less productive, more labor must be employed.

3.4. COMPARING THE PARTIAL AND GENERAL EQUILIBRIUM APPROACHES

The equilibria in the Gwage city differ significantly from those obtained following the HM and Ewage approaches. To start with, there is a fundamental difference in allowing all wages to be endogenous and tied to movements in the complimentary factor, viz. capital. In the partial equilibrium approaches -- both for the HM and Ewage cities -- improvements for the inhabitants of the city were constrained significantly by the demand conditions in the city.

Figure 3.21 depicts the SBD labor demand curves in the HM, Ewage and Gwage cities when the subcenter forms in the ESZ. Since the SBD is in the ESZ, there is an excess supply of labor at the reservation wage. Several assumptions are made for convenience. First, the SBD labor demand for the Gwage city (FBG in the graph) is assumed to be at the position shown on the graph after a certain amount of capital has shifted out to the suburb. The labor demand curve is then assumed to hold this position throughout this portion of the analysis. Second, the reservation wage shown on the graph is assumed to be the HM wage, since it sets the level for the HM city demand curve ABC. This is true only if (a) there are no time costs of commuting; (b) if the reservation wage is assumed to hold constant. Even if (a) is true, (b) is not likely to hold. As I discussed earlier, simultaneous with excess supply in the SBD market, there is excess demand in the
CBD market. As CBD wages adjust, so will SBD wages. Third, with the shifts in population densities, the labor supply curve for the SBD will shift. I assume this does not happen. Even given all these perturbations, though, SBD wages must fall against CBD wages, which allows the simplified diagram below. The demand curve for the Ewage city is held fixed at DBE.

![Diagram](image)

**FIGURE 3.21**

From the figure, the inefficiencies involved with the HM and Ewage cities can be seen. Point j is the only efficient point on the graph. Neither the HM nor the Ewage equilibria are at j. The HM city reaches an equilibrium at h. The Ewage city reaches an equilibrium at k. In general, when the SBD locates close to the center, equilibrium SBD wages (and by extension relative SBD wages) in the Gwage city will be above Ewage levels but below HM levels. Figure 3.9 bears this out.

Figure 3.22 below shows the SBD labor market equilibria in the HM, Ewage and Gwage cities when the subcenter forms in the EDZ. Only the Gwage equilibrium at j is represents equilibrium in the SBD market. The HM SBD wage at h and the Ewage solution at k are not true equilibria. In general, when the SBD locates far to the center, equilibrium SBD wages (and by extension relative SBD wages) in the Gwage city will be below Ewage levels but above HM levels. Figure 3.9 bears this out too.
In the above analysis, I have not formally characterized the inefficiencies that arise with the HM and Ewage framework (the 'dead weight loss' areas can be seen in Figures 3.21 and 3.22). The reason for this is that as the analysis in this chapter reveals, the 'inefficiencies' that arise out of the HM and Ewage structures are not the result of market failures, but rather due to arbitrarily placed constraints on the nature of the labor market. The HM city represents one extreme -- a completely flat labor demand curve -- while the Ewage framework goes to the other extreme of a perfectly vertical labor demand curve. Neither is a realistic representation of the labor market.

The urban area has four principal factor markets -- the land market, the housing market, the labor market and the capital market. I assume that people consume land directly, leaving only the land, labor and capital markets. The models described in Chapters 1 and 2, which are widely used in urban theory, characterize only the land market fully. I include the labor market in this chapter, with capital as the complimentary factor of production. As I mentioned before, analytical tractability is my main reason for not including the capital market, added to the fact that an additional firm rent peak around the subcenter does not add anything substantial to the analysis.

This analysis extends urban theory in several ways. First, almost all of standard urban theory has continued to use the wage gradient approach to solving for equilibria in a
city with decentralized employment, even in a general equilibrium setting (see Ross and Yinger ('94) etc.). As the relative SBD wage curves in Figure 3.19 show, this approach is completely wrong. When employment locates far away from the center and enough capital moves to the subcenter, final SBD wages do not fall by the amount of the commuting costs between the CBD and the SBD in the general equilibrium model. Instead, a utility gradient appears as employment moves further and further from the center. Since the Gwage city reaches the most efficient solution, the equilibrium utility level must also be the best adjusted in this framework. As can be seen from the equilibrium utility levels in Figure 3.11, the utility gradient -- first observed for decentralization in the EDZ in the Ewage cities -- now exists for decentralization anywhere in the Gwage city. With wages flexible at all places, the Gwage city thus comes close to being the perfect "minimum wage" city with a full labor market. The important difference comes from the flexibility of CBD wages, the 'dominance' of the CBD is broken, whether decentralization of employment occurs close to the CBD or far away from it. Whether the CBD remains dominant and dictates the pattern of commuting, say, is governed by the location of the CBD and the amount of the complimentary factor of production -- capital -- which moves out of the center, and not on arbitrarily fixed central wages and constraints on labor demand conditions. Ironically, the concept of the 'minimum wage' city, which was the basis for the HM conjecture and the wage gradient, is best accomplished when the wage gradient disintegrates and a positively sloped utility gradient emerges.

This reinforces my earlier objection to sacrificing tractability. In most versions of general equilibrium models of the city, finding closed form solutions has proved a substantial barrier to analyzing anything but monocentric city structures. In the few later models that have attempted to capture decentralized cities (e.g. Ross and Yinger ('94)), strict assumptions regarding boundaries between resident groups have marred the flavor. This is partly the reason behind the lack of any earlier attempts to investigate the effects of the first exogenous shock that I consider -- shifting the SBD loop further and further away

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5 Ross and Yinger posit that both CBD and SBD workers must live between the two employment centers. At the same time, they maintain that the wage gradient holds. There are two major problems here. First, the authors are only analyzing the case where the SBD locates far from the CBD, since they eliminate the case where only CBD workers live between the two employment centers. Second, in spite of having SBD wages determined by the wage gradient, they find a household rent subpeak at the SBD. The first assumption produces the second problem. Since the interface between CBD and SBD workers is predetermined, the land supply to the workers is restricted. This pushes rents up at the SBD.
from the subcenter. As Straszheim mentions, the expressions that show the directions of change are simply too cumbersome (and indeterminate, in the absence of exogenously imposed boundary conditions) to compare.

In the case of models that have used numerical simulation models (Hotchkiss and White ('93), Straszheim ('84), Sullivan ('83,'86) etc.), they have mostly looked at the changes in standard parameters -- transportation, income etc. -- or at special problems like changes in fiscal structure or the introduction of two income families. Somehow, this basic feature of urban decentralization has been ignored. The myth about the wage gradient has persisted, perhaps due to the assumption that enough was known about the interaction between the land and the labor market. White ('76) studied a model similar to my Ewage framework; however, as this chapter shows, that is not the full story. No one has disturbed the myth since.

My approach is also incomplete. Moving more capital further out (or having a more decentralized city) is good, but if the problem is not constrained, there is no unique solution. Decentralization would continue till the city breaks up, or all the capital moves out of the center. The constraint would exist in the capital market. Now that the interaction between the land and labor markets is understood, it is easy to extend this analysis and probe the reason for the employment moving outwards. In this framework, there are likely unequal returns to capital at the center and the suburbs. As capital moves to the suburbs, however, rental rates fall there and rise in the center. The movement stops when returns are equalized. The important fact is: as capital moves out, the accompanying outward movement of jobs creates a more decentralized city, where everyone is unambiguously better off. This is the major contribution of my analysis.

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6 See Chapter 1, Section 1.1.
CHAPTER 4: A PARTIAL GENERAL EQUILIBRIUM MODEL OF A CITY WITH DECENTRALIZED EMPLOYMENT, A FULL LABOR MARKET AND A HETEROGENEOUS WORKFORCE

In this chapter, I extend the partial general equilibrium framework introduced in Chapter 3. The model includes a full land and labor market, as before. In addition, I introduce a complexity in the structure of the workforce. Workers are no longer homogeneous in production, there are high skilled workers and low skilled workers. I want to explore the possibility of integration in the urban area in the absence of fiscal constraints. In this context, a segregated city is one where high skilled and low skilled workers live isolated from one another. An integrated solution is where there are a significant number of interfaces between the areas occupied by the two groups of workers. Remember that in this framework, only one group of people can win a land auction at a particular location. Hence co-existence at the same location is not possible, except through rounding errors. However, the difference lies in whether there are zones of skilled workers punctuated by zones of unskilled workers (the integrated solution with many interfaces between the rich and poor) or if all skilled laborers live in one uninterrupted zone and all unskilled laborers live in another (the segregated solution).

4.1. THE MONOCENTRIC CITY WITH HETEROGENEOUS LABOR

4.1a. THE MODEL

Assumptions

(i) The urban area has no special topographical features at the outset, save a high concentration of capital at the center of the city which encourages the formation of a Central Business District (henceforth CBD) there at the outset. I look at a circular shaped possible metropolitan area. Hence the CBD is assumed to be in place at (0, 0) initially.

(ii) All distances are measured radially in miles. Commuters are assumed to travel along ubiquitous radial roads.

(iii) There is only one type of household occupying the urban area. All of them work at the one CBD, earning identical wages. Following White's (1992) formulation, households
are given to possess Cobb-Douglas utility functions and assumed to spend 8 hours per day in work and commute. Leisure is not explicitly included in the utility function.

(iv) Land is taken as a proxy for housing. Households are assumed to demand land rather than housing services. Rent bids are in dollars/unit of land (e.g. $/sq mile). Rents on land are collected by absentee landlords, and each plot of land is sold to the highest bidder.

(v) Each person's cost of commuting to work includes:
   (a) a time cost of commuting: \(2w.t(i,j)\), where commute time is valued at the lost wage;
   (b) a monetary cost of commuting: \(2m.d(i,j)\), where \(m\) is the per mile material cost of commuting. Distances of commute are measured in miles along the radial roads, and the time of commute for a typical worker from a point \((i,j)\) to the center, for example, is given by \(d(i,j) = \sqrt{i^2 + j^2}\) and \(t(i,j) = \left(\frac{1}{f}\right) d(i,j)\), where \(w\) is the wage offer at the center and \(f\) is the average speed of commute in miles/hour. Later, I use radial distances in continuous population density functions for numerical iterations.

(vi) Labor and capital are compliments in production. Initially, all production in the urban area is carried on using the same technology. The firms are assumed to be homogeneous in all other respects as well.

(vii) There are two kinds of labor available for hire in the city: skilled laborers with higher marginal productivity and unskilled laborers with lower marginal productivity.

(viii) Once hired, the workers have no choice but to work for their designated employers, they cannot move to a different employment location unless demand conditions dictate such a move.

(ix) The production function has a Hicks neutral shift factor \(A\), which could be taken to represent agglomerative effects along the lines of Straszheim (1983) and Sullivan (1986). Unlike those models, however, I do not use the shift factor to introduce a difference in the production technologies of CBD and SBD firms.

(x) The good produced is a numeraire, with a price of 1.

(xi) The amount of capital in the urban area is fixed. Initially, all the capital stock is concentrated at the center.

(xii) Firms do not demand land in this model. The CBD is assumed to occupy a point or a tight cluster and not cause significant distortions in the rent profile of the city.

(xiii) The city is assumed to be closed.
Labor Demand

From the first order conditions of profit maximization, the labor demand functions for high skilled and low skilled laborers can be determined. By assumption, firms do not demand land for production purposes. Hence the production function combines high skilled labor $N^h$, low skilled labor $N^l$ and capital $K$:

$$Q = A(N^h)^{\theta_1} (N^l)^{\theta_2} K^{\varphi}$$

where $Q$ is the numeraire good produced and $A$ is the Hicks neutral shift factor. If $w^h$ and $w^l$ represent high and low skilled wages respectively, and $r$ the rental rate on capital, the firm maximizes the profit function

$$\pi = A(N^h)^{\theta_1} (N^l)^{\theta_2} K^{\varphi} - w^h N^h - w^l N^l - rK$$  \hspace{1cm} ......(4.1)

The amount of capital in the urban area is held fixed at $\bar{K}$, and $\theta_1 + \theta_2 + \varphi = 1$, so the first order condition for profit maximization yields

$$A\theta_1 (N^h)^{\theta_1 - 1} (N^l)^{\theta_2} \bar{K}^{\varphi} - w^h = 0$$  \hspace{1cm} ......(4.2a)

$$A\theta_2 (N^h)^{\theta_1} (N^l)^{\theta_2 - 1} \bar{K}^{\varphi} - w^l = 0$$  \hspace{1cm} ......(4.2b)

from which I get the labor demand functions $N^{hD}$ and $N^{lD}$ in terms of $w^h$ and $w^l$

$$N^{hD} = \left(\frac{A(\theta_1)^{\theta_1 - \theta_2} (\theta_2)^{\theta_2} \bar{K}^{\varphi}}{(w^h)^{\theta_1} (w^l)^{\theta_2}}\right)^{\frac{1}{\varphi}}$$  \hspace{1cm} ......(4.3)

$$N^{lD} = \left(\frac{A(\theta_1)^{\theta_1} (\theta_2)^{\theta_2 - 1} \bar{K}^{\varphi}}{(w^h)^{\theta_1} (w^l)^{\theta_2 - \theta_1}}\right)^{\frac{1}{\varphi}}$$  \hspace{1cm} ......(4.4)

Labor Supply

On the labor supply side, there are now two groups of workers vying for land in the city. Both the high skilled and low skilled households work at the CBD for wage offers
wh and wL. They maximize utility functions with respect to demand for land L and the numeraire good Q, subject to their income net of commuting costs. The equilibrium is found exactly along the lines of the monocentric city models examined previously, since the two groups have identical preferences. Hence they maximize the Lagrangians

\[ V^h = \left( Q^h \right)^\alpha \left( L^h \right)^\beta \beta \left[ 8w^h - 2\left( t^h \right)^\kappa d^h(i,j) - Q^h - R^h(i,j).L^h(i,j) \right] \quad \text{...(4.5a)} \]

\[ V^l = \left( Q^l \right)^\alpha \left( L^l \right)^\beta \beta \left[ 8w^l - 2\left( t^l \right)^\kappa d^l(i,j) - Q^l - R^l(i,j).L^l(i,j) \right] \quad \text{...(4.5b)} \]

with respect to Q's and L(i,j)'s, generating the land demand functions for Q and L as functions of all parameters and distance from center, where the distance functions d(i,j)'s are evaluated as before.

From the first order conditions of maximization, the demands for Q and L respectively are

\[ Q^h(i,j) = \alpha \left\{ 8w^h - 2w^h \left( \frac{1}{f} \right) d^h(i,j) - 2md^h(i,j) \right\} \quad \text{...(4.6)} \]

\[ L^h(i,j) = \left( \frac{\beta}{R^h(i,j)} \right) \left\{ 8w^h - 2w^h d^h(i,j) - 2md^h(i,j) \right\} \quad \text{...(4.7)} \]

\[ Q^l(i,j) = \alpha \left\{ 8w^l - 2w^l \left( \frac{1}{f} \right) d^l(i,j) - 2md^l(i,j) \right\} \quad \text{...(4.8)} \]

\[ L^l(i,j) = \left( \frac{\beta}{R^l(i,j)} \right) \left\{ 8w^l - 2w^l \left( \frac{1}{f} \right) d^l(i,j) - 2md^l(i,j) \right\} \quad \text{...(4.9)} \]

Equations (4.6)-(4.9) can be used to find the indirect utility functions

\[ V^{h*} = \left( \frac{\alpha \beta^h}{R^h(i,j)} \right) \left\{ 8w^h - 2w^h \left( \frac{1}{f} \right) d^h(i,j) - 2md^h(i,j) \right\} \quad \text{...(4.10)} \]

\[ V^{l*} = \left( \frac{\alpha \beta^l}{R^l(i,j)} \right) \left\{ 8w^l - 2w^l \left( \frac{1}{f} \right) d^l(i,j) - 2md^l(i,j) \right\} \quad \text{...(4.11)} \]
There are now two levels of utility associated with the two skill levels. Equations (4.10) and (4.11) in turn yield the rent offers for the two skill groups

\[ R^h(i,j) = \left( \frac{\alpha}{V^h} \right)^{1/\beta} \left( 8w^h - 2w^h \frac{1}{f} d^h(i,j) - 2md^h(i,j) \right)^{1/\beta} \quad .... (4.12) \]

\[ R^l(i,j) = \left( \frac{\alpha}{V^l} \right)^{1/\beta} \left( 8w^l - 2w^l \frac{1}{f} d^l(i,j) - 2md^l(i,j) \right)^{1/\beta} \quad .... (4.13) \]

using the time of commuting function, and where \( V^h \) and \( V^l \) are the equilibrium utility levels for the high skill and low skill groups respectively, determined when the system of equations is solved for given populations of the two groups. To determine who lives where, I follow the same allocation rules as before, viz. whoever makes the highest rent bid wins the land (if the bids are the same, the high skilled workers get the land). Thus

1. If \( R^h \geq R^l \), the high skilled workers get the land
2. If \( R^l > R^h \), the low skilled workers get the land

Rules (1) and (2) above hold iff \( \max [R^h, R^l] \geq R_a \). \quad .... (4.14)

For maintaining equilibrium, each household of the same skill class must have the same utility, regardless of where they live. The rent gradients for both groups have negative slopes in the monocentric city. Rents decline till the boundary of the city is reached b miles from the center, where the rent offer by city residents falls below the agricultural rent offer \( R_a \).

For all \( R(i,j) \geq R_a \), then, the population density at each location can be found by setting the demand for land equal to the supply. Assuming that the land supply at each coordinate point is 1, I then find the population density at each location as

\[ n^h(i,j) = \left( \frac{\alpha}{V^h} \right)^{1/\beta} \left( 8w^h - 2w^h \frac{1}{f} d^h(i,j) - 2md^h(i,j) \right)^{\alpha/\beta} \quad .... (4.15) \]

\[ n^l(i,j) = \left( \frac{\alpha}{V^l} \right)^{1/\beta} \left( 8w^l - 2w^l \frac{1}{f} d^l(i,j) - 2md^l(i,j) \right)^{\alpha/\beta} \quad .... (4.16) \]
The total population of the urban area is then given by summing up the populations at each point. The total labor supply in the city is

$$N^{hS} = \sum_i \sum_j \left( \frac{\alpha}{V^{h*}} \right)^{1/\beta} \left( 8w^h - 2w^h \left( \frac{1}{f} \right) d^h(i,j) - 2md^h(i,j) \right)^{\alpha/\beta}$$

$$= \left( \frac{\alpha}{V^{h*}} \right)^{1/\beta} \sum_i \sum_j \left( 8w^h - 2w^h \left( \frac{1}{f} \right) d^h(i,j) - 2md^h(i,j) \right)^{\alpha/\beta} \quad \ldots(4.17)$$

and

$$N^{lS} = \sum_i \sum_j \left( \frac{\alpha}{V^{l*}} \right)^{1/\beta} \left( 8w^l - 2w^l \left( \frac{1}{f} \right) d^l(i,j) - 2md^l(i,j) \right)^{\alpha/\beta}$$

$$= \left( \frac{\alpha}{V^{l*}} \right)^{1/\beta} \sum_i \sum_j \left( 8w^l - 2w^l \left( \frac{1}{f} \right) d^l(i,j) - 2md^l(i,j) \right)^{\alpha/\beta} \quad \ldots(4.18)$$

Equations (4.17) and (4.18) above hold only when the particular group of worker referred to win the land auction with the other skill group and the agricultural rent offer. For a given population in the closed city case, I can find $V^{h*}$ and $V^{l*}$ in terms of all the other parameters by simultaneously solving the system of equations. The values of $V^{*}$'s from (4.17)-(4.18) are then used in (4.12)-(4.13) to determine rents.

Closure

To close out the general equilibrium model, the total demand for each kind of labor from equations (4.3)-(4.4) must be equal to the total supply of the corresponding kind of labor from equations (4.17) and (4.18). Also, in the closed city, the total amounts of high skilled and low skilled labor are fixed

$$N^{hD} = N^{hS} = \bar{N}^h \quad \ldots(4.19)$$

$$N^{lD} = N^{lS} = \bar{N}^l \quad \ldots(4.20)$$
4.1b. SIMULATION TECHNIQUES AND RESULTS

There are four variables to be determined -- $w^h$, $w^l$, $V^{h*}$ and $V^{l*}$. Their equilibrium values are found from the simultaneous solution to the four equations:

\[
\overline{N^h} - N^{hD} = 0 \quad \quad \quad \quad (4.21)
\]
\[
\overline{N^h} - N^{hS} = 0 \quad \quad \quad \quad (4.22)
\]
\[
\overline{N^l} - N^{lD} = 0 \quad \quad \quad \quad (4.23)
\]
\[
\overline{N^l} - N^{lS} = 0 \quad \quad \quad \quad (4.24)
\]

Equations (4.21)-(4.24) are derived from the closure conditions (4.19)-(4.20). Substituting for labor demand and supply from equations (4.3)-(4.4) and (4.17)-(4.18) respectively, the four equation system is

\[
\overline{N^h} - \left( \frac{A.(\theta_1)^{\theta_1}.(\theta_2)^{\theta_2}.(K)^{\theta_0}}{(w^h)^{\theta_1}.(w^l)^{\theta_2}} \right) \frac{1}{\varphi} = 0 \quad \quad \quad \quad (4.25)
\]
\[
\overline{N^h} - \left( \frac{\alpha^\alpha}{V^{h*}} \sum_i \sum_j \left[ 8w^h - 2w^h(\frac{1}{f}) d^h(i,j) - 2md^h(i,j) \right]^\alpha_{\beta} \right) = 0 \quad \quad \quad \quad (4.26)
\]
\[
\overline{N^l} - \left( \frac{A.(\theta_1)^{\theta_1}.(\theta_2)^{\theta_2}.(K)^{\theta_0}}{(w^h)^{\theta_1}.(w^l)^{\theta_2}} \right) \frac{1}{\varphi} = 0 \quad \quad \quad \quad (4.27)
\]
\[
\overline{N^l} - \left( \frac{\alpha^\alpha}{V^{l*}} \sum_i \sum_j \left[ 8w^l - 2w^l(\frac{1}{f}) d^l(i,j) - 2md^l(i,j) \right]^\alpha_{\beta} \right) = 0 \quad \quad \quad \quad (4.28)
\]

Given the values of all other parameters, equations (4.25)-(4.28) are simultaneously solved to yield the equilibrium values of $w^h$, $w^l$, $V^{h*}$ and $V^{l*}$. Of course, (4.26) and (4.28) are solved contingent on the satisfaction of (4.14). I use MATLAB programs to iterate the system to a solution.
I use FSOLVE to solve the four equation system simultaneously. To find \( N^{hs} \) and \( N^{ls} \), I utilize the ODE45 solver. The system is more complicated than in the case with no labor demand, since the labor demand and labor supply systems are sensitive to changes in guess values for wages in different degrees. Since kinks in the functions throw FSOLVE out of kilter, I use continuous population density functions

\[
n^h = \left( \frac{\alpha^h}{v^{h^*}} \right)^\frac{\alpha}{\beta} \left[ 8w^h - 2w^h \left( \frac{1}{f} \right) d_c - 2md_c \right]^\frac{\alpha}{\beta} \tag{4.29}
\]

\[
n^l = \left( \frac{\alpha^l}{v^{l^*}} \right)^\frac{\alpha}{\beta} \left[ 8w^l - 2w^l \left( \frac{1}{f} \right) d_c - 2md_c \right]^\frac{\alpha}{\beta} \tag{4.30}
\]

where \( d_c \) is the distance measured along a line from the center. The following 'smooth' differential equations are used to find \( N^{hs} \) and \( N^{ls} \)

\[
N^{hs} = 2.\pi d_c n^h \gamma e^{(R_{diff1} + R_{diff2} - 2)} \tag{4.31}
\]

\[
N^{ls} = 2.\pi d_c n^l \gamma e^{(-R_{diff1} + R_{diff3} - 2)} \tag{4.32}
\]

where

\[
R_{diff2} = \frac{R^h(d_c) - R^l(d_c)}{R^h(d_c) - R^l(d_c)}, \quad R_{diff1} = \frac{R^h(d_c) - R_a}{R^h(d_c) - R_d}, \quad R_{diff3} = \frac{R^l(d_c) - R_a}{R^l(d_c) - R_d}
\]

Equations (4.31) and (4.32) are then solved for initial values \( d_c = 0, N^{hs} = 0, N^{ls} = 0 \) and final value \( d_c = 30 \). After the FSOLVE solutions emerge, I use differential equations in the original kinked population density functions (equations 4.29-4.30) to check the results by trial and error. The solutions are obtained via the following loop
Guess \( w^h, w^l, V^h^* \) and \( V^l^* \).

\[ \downarrow \]

(Labor Demand)

Given \( w^h, w^l \) and all other parameters, find \( N^{hD} \) and \( N^{lD} \).

\[ \uparrow \]

(Labor Supply)

Given \( w^h, w^l, V^h^*, V^l^* \) and all other parameters, find all \( R^h, R^l \) that satisfy (4.14).

Hence find population densities \( \Pi^h \) and \( \Pi^l \) at all locations.

Sum up \( \Pi^h \) and \( \Pi^l \) over the urban area to find \( N^{hD} \) and \( N^{lD} \).

\[ \uparrow \]

Check if the closure conditions (4.25)-(4.28) are satisfied. If not → → → →

↓

Once \( w^h, w^l, V^h^*, V^l^* \) have been determined and the city laid out, the system can be solved for interfaces between high and low skilled workers etc.

Table 4.1 lists the parameter values used to set up the first equilibrium. The value of \( N^h \) and \( N^l \) are of course common to both the labor demand and the supply sides.

<table>
<thead>
<tr>
<th>LABOR DEMAND</th>
<th>LABOR SUPPLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>PARAMETERS</td>
<td>VALUES</td>
</tr>
<tr>
<td>( A )</td>
<td>16.531</td>
</tr>
<tr>
<td>( N^h )</td>
<td>50000</td>
</tr>
<tr>
<td>( N^l )</td>
<td>50000</td>
</tr>
<tr>
<td>( f )</td>
<td>30 miles / hour</td>
</tr>
<tr>
<td>( m )</td>
<td>$0.40/mile</td>
</tr>
<tr>
<td>( d_c )</td>
<td>0 - 30 miles</td>
</tr>
</tbody>
</table>

**TABLE 4.1: PARAMETER VALUES FOR MONOCENTRIC CITY WITH A FULL LABOR MARKET AND HETEROGENEOUS LABOR INPUTS**

The three other parameters on the labor demand side, \( \theta_1, \theta_2 \) and \( \Phi \), are not included in Table 4.1 since I use two different sets of values for the purposes of analyzing the sensitivity of the system to changes in the coefficients of the production function. In addition, I analyze the effects of introducing more capital in the urban area. I compare the
city when 1000000 and 100000000 units of capital exist at the center of the urban area. Table 4.2 below presents the three sets of values which I will consider in this chapter, and the corresponding monocentric city equilibria that emerge from using the three sets of coefficients. All other parameters are at the level designated in Table 4.1. The labor demand values are the indicators of employment level. Due to the extreme complexity of the process of iterating to a solution, I accept labor supply figures within ±5% of demand.

<table>
<thead>
<tr>
<th>$\bar{K}$</th>
<th>$\theta_1$</th>
<th>$\theta_2$</th>
<th>$\varphi$</th>
<th>$w^h$</th>
<th>$w^l$</th>
<th>$V^h*$</th>
<th>$V^l*$</th>
<th>$N^h$</th>
<th>$N^l$</th>
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<td></td>
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<td></td>
<td></td>
<td></td>
<td>s=50483</td>
<td>s=49981</td>
</tr>
<tr>
<td>$10^6$</td>
<td>.6</td>
<td>.3</td>
<td>.1</td>
<td>13.383</td>
<td>6.6915</td>
<td>12.3352</td>
<td>5.8043</td>
<td>d=49999</td>
<td>d=49999</td>
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<td></td>
<td>s=50009</td>
<td>s=50001</td>
</tr>
<tr>
<td>$10^8$</td>
<td>.6</td>
<td>.3</td>
<td>.1</td>
<td>21.211</td>
<td>10.6055</td>
<td>19.3504</td>
<td>9.2239</td>
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<td></td>
<td></td>
<td>s=50183</td>
<td>s=49949</td>
</tr>
</tbody>
</table>

**TABLE 4.2: HETEROGENEOUS MONOCENTRIC CITY EQUILIBRIA**
(DIFFERENT VALUES OF $\bar{K}$, $\theta_1$, $\theta_2$ AND $\varphi$ IN PRODUCTION FUNCTION)

There is more capital than there is labor in the urban area at all times. Comparing with Table 3.2 in Chapter 3, notice that with everything else being the same, when the total labor share in production is split up between high skilled and low skilled workers, there is a marked negative income effect. Total production goes down since half of the scarcer factor is now only half as productive, while the more abundant factor (capital) is unchanged. Hence the same amount of capital now produces less. Wages are lower, so are equilibrium levels of utility. I refer to the three cities in Table 4.2 as Hwage (short for Heterogeneous Gwage cities) I, II and III for the rest of this analysis.

For the equilibrium values mentioned in Table 4.2, Figures 4.1a and b depict the rent and population density gradients in the three monocentric Hwage cities. In the population density graph, I have split up the two skill groups. I continue this throughout the analysis, as it gives us a good idea of the differences in changes experienced by the rich and the poor groups when employment decentralization occurs in the city. Due to the 'smoothened' density functions used while iterating, there is some overlapping of skill groups at the interface of the zones they occupy in the city. This does not show integration.
FIGURE 4.1 (top) and 4.2 (bottom): RENT AND POPULATION DENSITY GRADIENTS IN MONOCENTRIC HWAGE I, II AND III CITIES

LEGEND: r1c HwageI / II / III -- rents in Hwage I, II and III respectively; nh1c Hwage I / II / III, nl1c HwageI / II / III: high and low skilled populations in Hwage I, II and III cities;
(see Table 4.2 for definitions of Hwage I/II / III)
In Figures 4.1a and b, the effects of differing labor productivity and capital stocks on the monocentric city follow expected patterns. For a fixed amount of capital (HwageI and II cities), decreasing $\varphi$ -- the coefficient of capital in the production function -- will cause an income effect by decreasing the productivity of the variable factors. From Table 4.2, compared to the HwageI city, the HwageII city has lower wages and utility level for both groups of workers. In Figure 4.1, the loss in income causes the monocentric city to be more crowded in the interior. Since land demands are lower, people move inwards, raising rents close to the center, lowering them in the suburbs and pulling the city boundary inwards. In Figure 4.1a, the HwageI city has the steepest rent gradient. In Figure 4.1b, lower productivity increases population density everywhere and both low skilled and high skilled workers are packed more tightly in.

For fixed coefficients of production, the effects of increasing the capital stock can be seen by comparing the HwageI and III cities in Figures 4.1a and b. For given amounts of labor, increasing the amount of the fixed factor increases productivity of both classes of labor, increasing their incomes. With extra income, people demand more land and move outwards, lowering rents close to the center and raising them in the suburbs. The city is less crowded everywhere, with both low and high skilled laborers enjoying more space.

The HwageI and III cities cannot be compared in general terms. In the specific example set up here, the effect of a higher capital stock overshadows the lower productivity effects, and HwageIII has higher wages and utility and is more spread out.\footnote{Comparing the HwageI city to the HwageIII city in Figure 4.1b, the boundary of the zone occupied by low skilled workers does not shift out. From Table 4.2, there are more low skilled workers living in HwageI, hence the apparent anomaly.}

Finally, notice that the monocentric city is completely 'segregated'. I define a 'segregated' city as one in which the number of interfaces between high skilled and low skilled workers is the minimum -- namely, one. This is the case in all the monocentric Hwage cities. Due to the labor productivity coefficients (high skilled laborers are twice as productive as their brethren, meaning that their wage income will be close to twice as much) and the utility functions (income elasticity of demand for land is not lower than the income elasticity of commuting costs, signifying that people with higher incomes will prefer to consume more land in the suburbs and commute more) and an equal number of
laborers of each group in the city, the high skilled 'rich' group lives in the suburbs, while the low skilled 'poor' workers inhabit the inner city. I call this the $N^h / N^l$ city, with every '/' denoting an interface between a skilled and an unskilled group. Note that the segregation does not arise from the usual reasons (ethnic bias, flight from blight, property values etc.), but from the nature of labor productivity differentials (which drive income levels) and preferences in consumption (which drive land demand).

In the next section, I look at the employment decentralized version of the model. Without introducing any additional shocks except decentralization of employment through movements of capital in the city, I examine the possibility of finding a more 'integrated' city, i.e. a city with more interfaces between rich and poor workers.\(^2\)

### 4.2. THE CITY WITH HETEROGENEOUS LABOR AND DECENTRALIZED EMPLOYMENT

There are two categories of people -- divided into four wage classes -- living in the urban area. They are:
- skilled laborers working at the center earning $w_{\parallel}$ per hour;
- unskilled laborers working at the center earning $w_\perp$ per hour;
- skilled laborers working at the suburb earning $w_{\parallel}$ per hour;
- unskilled laborers working at the suburb earning $w_\perp$ per hour.

The wage offers from the demand side of the model have to provide each of these classes with enough money to live in the urban area with positive utility. In equilibrium, each worker of the same skill level must get the same utility regardless of where they live in the city. The equilibrium utility level will be higher for the high skilled group and lower for the low skilled group, as in the previous section.

\(^2\) Remember, though, that the movement of capital out of the CBD is not as innocent as it might seem at face value. Since I do not include a full capital market in this model, what is missing from this set up is the reason behind capital movements. If capital moves out to the suburbs in response to specific fiscal circumstances, for example, institutional factors will be at least partly responsible for creating a more (or less) segregated city. Even so, a large number of factors which are normally cited as the base cause of segregation along income lines -- e.g. zoning, property base considerations, bias against the poor -- are absent from this framework, making it an idealized framework for studying the effects of employment decentralization on residential segregation.
4.2a. THEMODEL

Assumptions

In addition to the assumptions in the monocentric city model, I also assume:
(i) The Suburban Business District (henceforth SBD) forms as a belt of infinitesimal width at \( d_{sub} \), which is at a distance \( d_s \) from the CBD, and the firms are spread evenly around the loop. Some firms stay at the CBD and hire \( N^h_s \) high skilled workers and \( N^l_s \) low skilled workers, while the ones that go out to the suburbs hire \( N^h_s \) high skilled workers \( N^l_s \) low skilled workers. This movement occurs as a result of exogenous movements of capital from the center to the suburbs.
(ii) Both CBD and SBD firms will attempt to pay the minimum wage possible to attract the number of people they wish to hire, i.e. the labor market clears at all times.
(iii) In the general equilibrium setup, wages are determined simultaneously in equilibrium.
(iv) The total amount of capital in the city is still fixed at \( K \). However, this will now be divided into the capital at the center \( K_c \), and the capital at the subcenter \( K_s \). The division of capital between the two employment locations is determined exogenously, i.e. \( K = K_c + K_s \). To preserve durability of capital, I do not allow all capital to move out to the suburbs.

Labor Demand

The labor demand functions in the urban area can be determined from the first order conditions of profit maximization. Now there are two sites, where production is carried on by identical firms with identical technologies but possibly different endowments of capital. Production at both sites use the two kinds of labor available and capital, as follows

\[
Q_c = A_s \left( N^h_c \right)^{\theta_1} \left( N^l_c \right)^{\theta_2} \left( K_c \right)^{\varphi}
\]

\[
Q_s = A_s \left( N^h_s \right)^{\theta_1} \left( N^l_s \right)^{\theta_2} \left( K_s \right)^{\varphi}
\]

where \( Q_c \) and \( Q_s \) are the quantities produced of the numeraire good (price = 1) and \( A \) is the Hicks neutral shift factor. The amount of capital in the urban area is fixed:

\[
K_c + K_s = \bar{K}
\]

\[(4.33)\]
The division of capital between the center and the subcenter are exogenously determined, i.e. $K_c = \overline{K}_c$ and $K_s = \overline{K}_s$. If $r_c$ and $r_s$ are the rental rates on capital at the CBD and SBD respectively, the firms maximize the profit functions

$$
\pi_c = A_i \left( N_c^h \right)^{\theta_1} \left( N_c^l \right)^{\theta_2} \left( \overline{K}_c \right)^{\varphi} - w_c^h N_c^h - w_c^l N_c^l - r_c \overline{K}_c \quad \ldots(4.34)
$$

$$
\pi_s = A_i \left( N_s^h \right)^{\theta_1} \left( N_s^l \right)^{\theta_2} \left( \overline{K}_s \right)^{\varphi} - w_s^h N_s^h - w_s^l N_s^l - r_s \overline{K}_s \quad \ldots(4.35)
$$

I do not consider endogenous perturbations in the capital market due to differences in rental rates at different locations; instead making capital movements exogenous and focusing on the effects of capital movement on labor demand, supply and ultimately rent profiles. Given that $\theta_1 + \theta_2 + \varphi = 1$, the first order conditions for profit maximization yield the following labor demand functions for the four groups of workers

$$
N_{c}^{ID} = \left( \frac{A_i (\theta_1)^{1 - \theta_2} (\theta_2)^{\theta_2} (\overline{K}_c)^{\varphi}}{w_c^h (\theta_1)^{1 - \theta_2} (w_c^l)^{\theta_2}} \right)^{\frac{1}{\varphi}} \quad \ldots(4.36)
$$

$$
N_{c}^{ID} = \left( \frac{A_i (\theta_1)^{1 - \theta_2} (\theta_2)^{\theta_2} (\overline{K}_c)^{\varphi}}{w_c^h (\theta_1)^{1 - \theta_2} (w_c^l)^{\theta_2}} \right)^{\frac{1}{\varphi}} \quad \ldots(4.37)
$$

$$
N_{s}^{ID} = \left( \frac{A_i (\theta_1)^{1 - \theta_2} (\theta_2)^{\theta_2} (\overline{K}_s)^{\varphi}}{w_s^h (\theta_1)^{1 - \theta_2} (w_s^l)^{\theta_2}} \right)^{\frac{1}{\varphi}} \quad \ldots(4.38)
$$

$$
N_{s}^{ID} = \left( \frac{A_i (\theta_1)^{1 - \theta_2} (\theta_2)^{\theta_2} (\overline{K}_s)^{\varphi}}{w_s^h (\theta_1)^{1 - \theta_2} (w_s^l)^{\theta_2}} \right)^{\frac{1}{\varphi}} \quad \ldots(4.39)
$$

**Labor Supply**

On the labor supply side, households work either at the CBD for the wage offers $w_c^h$ or $w_c^l$, depending on skill level, or at the SBD for the wage offers $w_s^h$ or $w_s^l$. They maximize utility functions with respect to demand for land $L$ and the numeraire good $Q$, with respect to their income net of commuting costs. The equilibrium is found along the
lines of the monocentric city model. Hence the high and low skilled CBD and SBD workers respectively maximize the Lagrangians

\[ V^h_c = (Q^h_c)\alpha(L^h_c)^\beta - \lambda_1 [8w^h_c - (2w^h_c + m)t_c d_c(i,j) - Q^h_c - R^h_c(i,j)\cdot L^h_c(i,j)] \] . \hspace{1cm} (4.40a)

\[ V^l_c = (Q^l_c)\alpha(L^l_c)^\beta - \lambda_2 [8w^l_c - (2w^l_c + m)t_c d_c(i,j) - Q^l_c - R^l_c(i,j)\cdot L^l_c(i,j)] \] . \hspace{1cm} (4.40b)

\[ V^h_s = (Q^h_s)\alpha(L^h_s)^\beta - \lambda_3 [8w^h_s - (2w^h_s + m)t_s d_s(i,j) - Q^h_s - R^h_s(i,j)\cdot L^h_s(i,j)] \] . \hspace{1cm} (4.40c)

\[ V^l_s = (Q^l_s)\alpha(L^l_s)^\beta - \lambda_4 [8w^l_s - (2w^l_s + m)t_s d_s(i,j) - Q^l_s - R^l_s(i,j)\cdot L^l_s(i,j)] \] . \hspace{1cm} (4.40d)

with respect to Q's and L's, generating the demand functions for Q and L as functions of all parameters and distance from center \( d_c \) and distance from the SBD \( d_s \), where

\[ d_c(i,j) = \sqrt{r^2 + j^2}, \quad t_c(i,j) = \left( \frac{1}{l} \right) d_c(i,j), \quad d_s(i,j) = kd_c - d_{sub} \text{ and } t_s(i,j) = \left( \frac{1}{l} \right) d_s(i,j) \] where \( l \) is the speed of commute and \( d_{sub} \) the location of the subcenter.

From the first order conditions, the demands for Q and L are

\[ Q^h_c = \alpha\{8w^h_c - 2w^h_c t_c(i,j) - 2md_c(i,j)\} \] ..... (4.41a)

\[ L^h_c = \left( \frac{\beta}{R^h_c(i,j)} \right) \{8w^h_c - 2w^h_c t_c(i,j) - 2md_c(i,j)\} \] ..... (4.41b)

\[ Q^l_c = \alpha\{8w^l_c - 2w^l_c t_c(i,j) - 2md_c(i,j)\} \] ..... (4.42a)

\[ L^l_c = \left( \frac{\beta}{R^l_c(i,j)} \right) \{8w^l_c - 2w^l_c t_c(i,j) - 2md_c(i,j)\} \] ..... (4.42b)

\[ Q^h_s = \alpha\{8w^h_s - 2w^h_s t_c(i,j) - 2md_c(i,j)\} \] ..... (4.43a)

\[ L^h_s = \left( \frac{\beta}{R^h_s(i,j)} \right) \{8w^h_s - 2w^h_s t_c(i,j) - 2md_c(i,j)\} \] ..... (4.43b)

\[ Q^l_s = \alpha\{8w^l_s - 2w^l_s t_c(i,j) - 2md_c(i,j)\} \] ..... (4.44a)
\[ L_s^1 = \left( \frac{\beta}{R_s^1(i,j)} \right) \{8w_s^1 - 2w_s^1t_c(i,j) - 2md_c(i,j)\} \]  

\((4.44b)\)

I then use equations (4.41)-(4.44) to find the indirect utility functions

\[ V_c^{i*} = \left( \frac{\alpha^\beta}{R_c^{i*}(i,j)} \right) \{8w_c^i - 2w_c^it_c(i,j) - 2md_c(i,j)\} \]  

\((4.45)\)

\[ V_c^{h*} = \left( \frac{\alpha^\beta}{R_c^{h*}(i,j)} \right) \{8w_c^h - 2w_c^ht_c(i,j) - 2md_c(i,j)\} \]  

\((4.46)\)

\[ V_s^{h*} = \left( \frac{\alpha^\beta}{R_s^{h*}(i,j)} \right) \{8w_s^h - 2w_s^ht_c(i,j) - 2md_c(i,j)\} \]  

\((4.47)\)

\[ V_s^{l*} = \left( \frac{\alpha^\beta}{R_s^{l*}(i,j)} \right) \{8w_s^l - 2w_s^lt_c(i,j) - 2md_c(i,j)\} \]  

\((4.48)\)

For equilibrium, all similar individuals in the city should have the same level of utility, regardless of where they locate. Therefore

\[ V_c^{h*} = V_s^{h*} = V_c^{*} \]  

\((4.49)\)

\[ V_c^{l*} = V_s^{l*} = V_s^{*} \]  

\((4.50)\)

Equations (4.45)-(4.50) in turn yield the rent functions

\[ R_c^{h(i,j)} = \left( \frac{\alpha^\beta}{V_c^{h*}} \right)^{\frac{1}{\beta}} \left\{8w_c^h - 2w_c^ht_c(i,j) - 2md_c(i,j)\right\}^{\frac{1}{\beta}} \]  

\((4.51)\)

\[ R_c^{l(i,j)} = \left( \frac{\alpha^\beta}{V_c^{l*}} \right)^{\frac{1}{\beta}} \left\{8w_c^l - 2w_c^lt_c(i,j) - 2md_c(i,j)\right\}^{\frac{1}{\beta}} \]  

\((4.52)\)

\[ R_s^{h(i,j)} = \left( \frac{\alpha^\beta}{V_s^{h*}} \right)^{\frac{1}{\beta}} \left\{8w_s^h - 2w_s^ht_c(i,j) - 2md_s(i,j)\right\}^{\frac{1}{\beta}} \]  

\((4.53)\)
\[ R_s^l(i,j) = \left( \frac{\alpha \beta}{V_s^*} \right)^{\frac{1}{\beta}} \left[ 8w_s^l - 2w_s^l \left( \frac{1}{f} \right) d_s(i,j) - 2md_s(i,j) \right]^{\frac{1}{\beta}} \]  \quad \ldots (4.54)

The equilibrium utility levels \( V_c^* \) and \( V_s^* \) are determined when the system of equations is solved for given populations at the center and the suburbs. The rent gradient for each group of people have negative slopes.

The four groups of people compete with one another for land at every location in the city. I use the following rule for allocating land and deciding who lives where:

1. If \( R_c^h \geq \max [R_c^h, R_s^h, R_s^l, R_a] \), they get the land.
2. If \( R_c^l > \max [R_c^h, R_s^h, R_s^l] \) and \( R_c^l \geq R_a \), they get the land.
3. If \( R_s^h > \max [R_c^h, R_c^l, R_s^l] \) and \( R_s^h \geq R_a \), they get the land.
4. If \( R_s^l < R_s^h \) and \( R_s^l \geq \max [R_c^l, R_s^h, R_a] \), they get the land.

In (1)-(4) above, 'they' refers to the group designated.  \quad \ldots (4.55)

One of the four groups 'wins' the land at each location in the interior of the city by allocation rule (4.55). The process continues till the boundary of the city is reached b miles from the center, where the rent offer by the dominant group of city residents falls below the agricultural rent offer \( R_a \), i.e.

\[ R_a > \left( \frac{\alpha \beta}{V^*} \right)^{\frac{1}{\beta}} \left[ 8w - 2w \left( \frac{1}{f} \right) b - 2mb \right]^{\frac{1}{\beta}} \]  \quad \ldots (4.56)

where \( w \) and \( V^* \) are the values associated with the dominant group at \( b \).

For all \( R(i,j) \geq R_a \), then, the population density at each location can be found by setting the demand for land equal to the supply. Assuming that the land supply at each coordinate point is 1 in the discrete version of the model, I find the population densities for each group of workers at each location along the lines of equations (4.15) and (4.16). I find the total population of each group in the urban area by summing up the populations at each point and rearranging terms:
\[ N^h_c = \left( \frac{\alpha^{\alpha}}{V_c^*} \right)^{\alpha} \beta \sum_i \sum_j \left[ 8w^h_i - 2w^h_i \left( \frac{1}{f} \right) d_c(i,j) - 2md_c(i,j) \right]^{\alpha} \]  
\[ N^l_c = \left( \frac{\alpha^{\alpha}}{V_s^*} \right)^{\alpha} \beta \sum_i \sum_j \left[ 8w^l_i - 2w^l_i \left( \frac{1}{f} \right) d_c(i,j) - 2md_c(i,j) \right]^{\alpha} \]  
\[ N^h_s = \left( \frac{\alpha^{\alpha}}{V_c^*} \right)^{\alpha} \beta \sum_i \sum_j \left[ 8w^h_s - 2w^h_s \left( \frac{1}{f} \right) d_s(i,j) - 2md_s(i,j) \right]^{\alpha} \]  
\[ N^l_s = \left( \frac{\alpha^{\alpha}}{V_s^*} \right)^{\alpha} \beta \sum_i \sum_j \left[ 8w^l_s - 2w^l_s \left( \frac{1}{f} \right) d_s(i,j) - 2md_s(i,j) \right]^{\alpha} \]  

where \( N^h_c \), \( N^l_c \), \( N^h_s \) and \( N^l_s \) satisfy allocation rule (4.55). The total supplies of high and low skilled workers in the city are given by

\[ N^h = N^h_c + N^h_s \]  
\[ N^l = N^l_c + N^l_s \]

The equilibrium level of utility \( V^* \) is found in a closed city when the total populations \( N^h \) and \( N^l \) are held constant and the system of equations is solved simultaneously.

Closure

To close out the general equilibrium model, the total demand for labor (from equations (4.36)-(4.39)) must be equal to the corresponding total supply of labor (from equations (4.57)-(4.60)).

\[ N^h_c = N^h_s \]  
\[ N^l_c = N^l_s \]  
\[ N^h_s = N^h_s \]  
\[ N^l_s = N^l_s \]
In addition, in the closed city, the total supply of high skilled workers in the city is fixed at $N^h$, while the total supply of low skilled workers is fixed $N^l$.

\[ N^{hS} = N^h \]  \hspace{1cm}  \text{.....(4.67)}

\[ N^{lS} = N^l \]  \hspace{1cm}  \text{.....(4.68)}

Note that the satisfaction of (4.63)-(4.68) maintains both the solution of the general equilibrium system and the closed city assumption.

4.2b. SIMULATION TECHNIQUES, RESULTS AND ANALYSIS

There are now six primary endogenous variables: $w^h$, $w^l$, $w^h$, $w^l$, $v_c^*$ and $v_s^*$. Their equilibrium values are found from the simultaneous solution to the six equations:

\[ N^{hd}_c - N^{hS}_c = 0 \]  \hspace{1cm}  \text{.....(4.69)}

\[ N^{ld}_c - N^{lS}_c = 0 \]  \hspace{1cm}  \text{.....(4.70)}

\[ N^{hd}_s - N^{hS}_s = 0 \]  \hspace{1cm}  \text{.....(4.71)}

\[ N^{ld}_s - N^{lS}_s = 0 \]  \hspace{1cm}  \text{.....(4.72)}

\[ N^h - N^{hS}_c - N^{hS}_s = 0 \]  \hspace{1cm}  \text{.....(4.73)}

\[ N^l - N^{lS}_c - N^{lS}_s = 0 \]  \hspace{1cm}  \text{.....(4.74)}

Equations (4.69)-(4.74) are derived from expressing the closure conditions (4.63)-(4.68) in implicit form. The precise functions for labor demand and supply are given by equations (4.36)-(4.39) and (4.57)-(4.60) respectively. Given the values of all other parameters, I solve equations (4.69)-(4.74) simultaneously to get the equilibrium values of $w^h$, $w^l$, $w^h$, $w^l$, $v_c^*$ and $v_s^*$. Once they are determined, I find the equilibrium values of the secondary endogenous variables $N^{hd}_c$, $N^{ld}_c$, $N^{hd}_s$, $N^{ld}_s$, $N^{lS}_c$, $N^{lS}_s$, $N^h$ and $N^l$. The labor supply equations must satisfy the allocation rule (4.55).

As before, I use FSOLVE to solve the six equation system simultaneously. To find $N^{hS}_c$, $N^{lS}_c$, $N^{lS}_s$ and $N^{lS}_s$, I utilize the ODE45 solver. The system is complicated since the
labor demand and supply systems are sensitive in different degrees to changes in guess values for \( w^h, w^l, w^b, w^l \). To avoid kinks, I use the 'smooth' and continuous population density functions

\[
N^h_c = 2. \pi. d_c. N^h_c.r. e^{(R_{diff1} + R_{diff2} + R_{diff3} + R_{diff4} - 4)} \quad \cdots (4.75)
\]

\[
N^l_c = 2. \pi. d_c. N^l_c.r. e^{(R_{diff1} + R_{diff5} + R_{diff6} + R_{diff7} - 4)} \quad \cdots (4.76)
\]

\[
N^h_s = 2. \pi. d_c. N^h_s.r. e^{(R_{diff2} - R_{diff5} + R_{diff8} + R_{diff9} - 4)} \quad \cdots (4.77)
\]

\[
N^l_s = 2. \pi. d_c. N^l_s.r. e^{(R_{diff3} - R_{diff6} - R_{diff8} + R_{diff10} - 4)} \quad \cdots (4.78)
\]

where \( d_s = d_c - d_{sub} \), and \( d_c, d_s \) and \( d_{sub} \) are measured along a line from the center. The continuous density functions are

\[
r^h_c = \left( \frac{\alpha^h_c}{V^{h^h}*} \right)^{\frac{1}{\beta}} \left[ 8w^h - 2w^h \left( \frac{1}{f} \right) d_c - 2md_c \right]^{\frac{\alpha}{\beta}} \quad \cdots (4.75a)
\]

\[
r^l_c = \left( \frac{\alpha^l_c}{V^{l^l}*} \right)^{\frac{1}{\beta}} \left[ 8w^l - 2w^l \left( \frac{1}{f} \right) d_c - 2md_c \right]^{\frac{\alpha}{\beta}} \quad \cdots (4.76a)
\]

\[
r^{h_s} = \left( \frac{\alpha^{h_s}}{V^{h^s}*} \right)^{\frac{1}{\beta}} \left[ 8w^h - 2w^h \left( \frac{1}{f} \right) d_s - 2md_s \right]^{\frac{\alpha}{\beta}} \quad \cdots (4.77a)
\]

\[
r^{l_s} = \left( \frac{\alpha^{l_s}}{V^{l^s}*} \right)^{\frac{1}{\beta}} \left[ 8w^l - 2w^l \left( \frac{1}{f} \right) d_s - 2md_s \right]^{\frac{\alpha}{\beta}} \quad \cdots (4.78a)
\]

and

\[
R_{diff1} = \frac{R^h_c(d_c) - R^l_c(d_c)}{R^h_c(d_c) - R^l_c(d_c)} \quad R_{diff2} = \frac{R^h_s(d_c) - R^l_s(d_c)}{R^h_s(d_c) - R^l_s(d_c)}
\]

\[
R_{diff3} = \frac{R^h_c(d_c) - R^l_s(d_s)}{R^h_c(d_c) - R^l_s(d_s)} \quad R_{diff4} = \frac{R^h_c(d_c) - R_c}{R^h_c(d_c) - R_c}
\]
\[ \text{Rdiff5} = \frac{R^l(d_c) - R^h_s(d_s)}{R^l(d_c) - R^h_c(d_s)}, \quad \text{Rdiff6} = \frac{R^l(d_c) - R^l_s(d_s)}{R^l(d_c) - R^l_c(d_s)}, \]

\[ \text{Rdiff7} = \frac{R^l(d_c) - R_a}{R^l(d_c) - R_d}, \quad \text{Rdiff8} = \frac{R^h_s(d_s) - R^h_c(d_s)}{R^h_s(d_s) - R^h_c(d_s)}, \]

\[ \text{Rdiff9} = \frac{R^h_s(d_s) - R_a}{R^h_s(d_s) - R_d}, \quad \text{Rdiff10} = \frac{R^l(d_c) - R_a}{R^l(d_c) - R_d}. \]

where \( d_s = d_c - d_{\text{sub}}, \) and \( d_c, \, d_s \) and \( d_{\text{sub}} \) are measured along a line from the center. For finding the labor supplies, I use ODE45 to solve the differential equations (4.75)-(4.78) for initial values \( d_c = 0, \, N_c^h = 0, \, N_c^S = 0, \, N_s^h = 0, \, N_s^S = 0 \) and final value \( d_c = 30. \) I then use the labor supply values in the main FSOLVE program to solve equations (4.69)-(4.74) simultaneously. The solutions are obtained via the following loop:

\[ \text{Guess } w^*_h, \, w^*_c, \, w^*_l, \, w^*_{l}, \, V^*_c \text{ and } V^*_s \quad \uparrow \]

\[ \text{(Labor Demand)} \quad \uparrow \]

Given \( w^*_h, \, w^*_c, \, w^*_l, \, w^*_l, \, V^*_c \text{ and } V^*_s \) and all other parameters, find \( N_c^h, \, N_c^c, \, N_c^S, \, N_s^h, \, N_s^S \).

\[ \uparrow \]

\[ \text{(Labor Supply)} \quad \uparrow \]

Given \( w^*_h, \, w^*_c, \, w^*_l, \, w^*_l, \, V^*_c \text{ and } V^*_s \) and all other parameters,

\[ \text{find all } R_c(i,j), \, R_s(i,j) \text{ such that } \max [R_c(i,j), \, R_s(i,j)] \geq R_a. \quad \uparrow \]

Hence find population densities \( n_c^h, \, n_c^c, \, n_s^h \text{ and } n_s^S \) at all locations.

\[ \uparrow \]

Solve the differential equations (4.75)-(4.78) over the urban area to find \( n_c^h, \, n_c^c, \, n_s^h, \text{ and } n_s^S \).

\[ \uparrow \]

Check if conditions (4.69)-(4.74) hold. If not

\[ \text{Once } V^*_c \text{ and } V^*_s \text{ have been determined and the city laid out, determine the interfaces between different groups of workers and } b^*, \text{ the city boundary.} \quad \downarrow \]
All parameters that are common to this model and the monocentric model have the same values as in Table 4.1. For the coefficients $\theta_1$, $\theta_2$, $\varphi$ and $\kappa$, I use all three sets of values. After the FSOLVE solutions emerge, I use the original kinked (but continuous) functions to check the results via trial and error. The solution process is extremely complicated due to the different degrees of non-linearity on the labor demand and supply side and the large number of groups vying for land in the city. Again, the values of labor demand are the true indicators of employment. I accept labor supply figures within $\pm$5%.

First, I study what happens to the city with more productive labor ($\theta = .72$; $\varphi = .28$) per unit of capital when part of the capital stock moves into the suburbs, creating a second employment center. For analyzing all the Hwage setups, I consider 50% capital decentralization as the primary case. I also consider the city with 75% of the capital stock at the CBD to study the effects of continuing capital decentralization on the city. Tables 4.2-4.4 present equilibrium values for the Hwagel cities with the exogenous distribution of the (fixed) total capital stock in the city changing from all capital at the CBD (monocentric city) to 75% of the capital at the CBD to 50% of the capital at the CBD.

To depict the equilibrium profiles with various distances of decentralization and divisions of capital at the employment centers, it is now important to study both the rent and the population density profiles. Figure 4.3 shows the rent profiles in the Hwagel city with 50% of the capital at the CBD when employment decentralizes 6, 7, 11, 18 and 20 miles out. As I analyze presently, these five cases represent five possible distributions of population in the urban area. To bring this out, I then present the population density gradients for decentralization 7, 11, 18 and 20 miles out in Figures 4.8-4.11 (I show the population density in the city with SBD 6 miles out during the course of later analysis).

I also consider the effects of changing capital distribution by showing the changes in city structure when 25% and then 50% capital moves out to the suburbs. I compare the effects on the rent gradients (for SBD 7, 11, 18 and 20 miles out) in Figures 4.4-4.7, and later compare the effects on population density for cities with the same subcenters in parts a and b of Figures 4.8-4.11. The population graphs get extremely messy if I attempt to show more than one decentralized city on the same graph. So for comparing the effects of changes in the distribution of capital in this set, I use two graphs top and bottom, maintaining the same scale on the axes.
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Table 4.3: Equilibria for Wage 1 City with 75% of the Total Capital Stock at the CBD

($\theta_1 = .48, \theta_2 = .24; \varphi = .28; K = 1000000$)

212
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<td>17.8194</td>
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<td>29808</td>
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<td>8</td>
<td>19.6045</td>
<td>9.8553</td>
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<td>19575</td>
<td>30319</td>
<td>30245</td>
</tr>
</tbody>
</table>

**Table 4.4: Equilibria for HWAGE I City with 50% of the Total Capital Stock at the CBD**

$(\theta_1 = .48; \theta_2 = .24; \varphi = .28; \bar{K} = 1000000)$
FIGURE 4.3: RENT GRADIENTS FOR HWAGEI CITY \((\theta_1 = .48, \theta_2 = .24, \varphi = .28, K = 1000000)\) WITH ONE AND TWO EMPLOYMENT CENTERS (SUB = 6, 7, 11, 18, 20; 50% CAPITAL AT CBD)

**LEGEND:**
- r1c Hwagei: highest rent offers in monocentric city;
- r2c Hwagei (sub=7/11/18): highest rent offers in decentralized city with subcenters 7, 11 and 18 miles out respectively.
FIGURE 4.4 (TOP)-4.5 (BOTTOM): RENT GRADIENTS FOR MONOCENTRIC AND DECENTRALIZED (SUB = 7 AND 11; 75% AND 50% CAPITAL AT CBD) HWAGEI CITIES ($\theta_1 = .48, \theta_2 = .24, \phi = .28, \bar{K} = 1000000$)

LEGEND: as before
FIGURE 4.6 (TOP)–4.7 (BOTTOM): RENT GRADIENTS FOR MONOCENTRIC AND DECENTRALIZED (SUB = 18 AND 20; 75% AND 50% CAPITAL AT CBD) HWAGEI CITIES ($\theta_1 = .48, \theta_2 = .24, \phi = .28, \overline{K} = 1000000$)

LEGEND: as before
FIGURE 4.8a (TOP)-b (BOTTOM): COMPARISON OF POPULATION DENSITIES IN DECENTRALIZED (SUB = 7) HWAREGI CITIES WITH 75% (TOP) AND 50% (BOTTOM) OF CAPITAL STOCK AT THE CBD

LEGEND: nhc1c, nlc1c.: population of skilled and unskilled labor in monocentric city; nhc2c., nlc2c.: population of skilled and unskilled labor working at the CBD in decentralized city; nhs2c., nls2c.: population of skilled and unskilled labor working at the SBD in decentralized city.
FIGURE 4.9a (TOP)-b (BOTTOM): COMPARISON OF POPULATION DENSITIES IN DECENTRALIZED (SUB = 11) HWAGEI CITIES WITH 75% (TOP) AND 50% (BOTTOM) OF CAPITAL STOCK AT THE CBD

LEGEND: as before
FIGURE 4.10a (TOP)-b (BOTTOM): COMPARISON OF POPULATION DENSITIES IN DECENTRALIZED (SUB = 18) HWAGEI CITIES WITH 75% (TOP) AND 50% (BOTTOM) OF CAPITAL STOCK AT THE CBD

LEGEND: as before
FIGURE 4.11a (TOP)-b (BOTTOM): COMPARISON OF POPULATION DENSITIES IN DECENTRALIZED (SUB = 20) HWAGEL CITIES WITH 75% (TOP) AND 50% (BOTTOM) OF CAPITAL STOCK AT THE CBD

LEGEND: as before
Tables 4.3-4.4 and Figures 4.3-4.11 bring out several trends and characteristics of the equilibria in the city with full land and labor markets when two groups of laborers with high and low skill levels compete for jobs and residential land at multiple employment centers. As I show shortly, the basic trends displayed by wages, equilibrium level of utility etc. are along the lines of the results found in Chapter 3 for the homogeneous version of this model. What is of interest in the heterogeneous case is the issue of who stays where -- i.e. the possibility of cohabitation among the rich and the poor. As I mentioned at the outset, the nature of the land allocation process (see the allocation rule in equation (55)) precludes the possibility of rich and poor people cohabiting exactly the same plot of land, except through rounding errors. Following the previous definitions of 'segregated' and 'integrated' solutions (from Section 4.1b), I look at the number of interfaces between zones inhabited by the high skilled 'rich' workers and the low skilled 'poor' workers as an indicator of the degree of residential integration -- the higher the number, the more integrated the city is. The monocentric city shows the highest possible degree of segregation, with all the skilled workers living in one mass in the suburbs, all the unskilled workers living in one mass close to the center and only one interface between the two groups. I study the population density graphs (Figures 4.8-4.11) to explore the impact of employment decentralization on residential integration.

Finally, the results in the model depend on whether the subcenter forms very close to the CBD or far away from it, and also on the amount of capital that moves out to the subcenter. The combination of these two factors will determine whether the SBD is located in the Excess Supply Zone (henceforth ESZ) or in the Excess Demand Zone (henceforth EDZ). Consider a subcenter loop forming outside the CBD. If the wage offer at the CBD is held fixed at \( w^* \), the monocentric city level, the SBD reservation wage \( w_{rs} \) is defined as the wage offer at which all people living outside the loop will be willing to work for the SBD. If the SBD locates close to the center and not enough capital moves there, the SBD has an excess supply of labor (and the CBD an excess demand for labor). The SBD is then said to be in the ESZ. Similarly, the SBD is said to be in the EDZ when the subcenter forms far away from the center and enough capital moves there to create a demand for labor greater than the number of people living outside the loop. Note that the definitions of ESZ and EDZ are geared towards the situation in the SBD labor market. Figures 4.12-4.14 below show various aspects of the movements of high and low skilled wages and utility levels when employment decentralization occurs anywhere in the city.
As I begin to analyze the nature of the equilibria in the Hwage city, I first consider the effect of employment decentralization on segregation (or integration) in residential patterns. The thing to remember here is that the amount of 'mixing' -- or lack thereof -- that I find here depends strictly on the distance of the subcenter and the amount of capital that moves to the SBD, and not on the usual causes (social, fiscal etc.) that are cited in such analyses. This makes for the possibility of using these patterns as base cases to look for real life deviations from the norm.

**Effects Of Employment Decentralization On Residential Integration**

For a given distribution of capital in the urban area, there are five general types of residential location patterns that could emerge in the decentralized Hwage cities. I create five categories: I, II, III, IVa and IVb. In the following analyses, I identify the categories, using examples from the population density diagrams to illustrate them. I then evaluate the patterns in the context of analyzing which of them show a greater degree of segregation among the high skilled and low skilled inhabitants of the city. In the graphs sketching the patterns of residential location, I depict the first quadrant only (the city is symmetrical around (0, 0)). Also, the interfaces between 'rich' and 'poor' groups are marked in very thick boundary lines, while the interfaces between groups of the same income levels are marked in light boundary lines. The coordinate axes are marked in dashed lines, and the city boundary is marked with a line of medium thickness. I mark each location band with the population who reside there.

The monocentric city has the most segregated solution with all poor, low skilled laborers hugging the CBD and all rich, high skilled workers inhabiting the suburbs. There is only one interface between the two income groups. In the city with decentralized employment, there exists the possibility of more interfaces between high and low income workers. There also exists the possibility of groups with the same skill levels employed at different centers sharing interfaces with one another. For any two groups X and Y, I use the notation X/Y to imply an interface between two groups with different skill level, and the notation X--Y to imply an interface between the groups when X and Y have the same skill level. More interfaces of the former kind (between different skill levels) imply more integration, since zones with homogeneous populace are smaller. More interfaces of the latter kind (between same skill levels) imply higher segregation.
Pattern I \( N^c_s \rightarrow N^l_s \rightarrow N^h_s \)
(e.g. Decentralization upto 6 miles out in the HwageI (capc = .5*cap) city)

\[ \text{FIGURE 4.12a: PATTERN I} \]

This pattern of residential location is visible when the SBD locates very close to the center. The nature of the production function (high skilled workers have twice the marginal productivity) and the labor resources (equal number of high and low skilled workers in the city) ensure that high skilled workers earn about twice the income level of the low skilled group. This, combined with homogeneous preferences where the income elasticity of land demand is higher than the income elasticity of commuting costs, ensures that high income groups will move out towards the suburbs (flatter rent gradient) while the low income groups will hug the employment centers (steeper rent gradient). In this context, it is easy to see that when the two employment center are located very close together, the two low skilled groups will also be very close together, hugging the employment locations, and both high skilled groups will occupy the suburbs. The inner city features a high population density, while the suburbs are very low density. Figure 4.8a depicts such a city, when employment locates 7 miles out in the HwageI (capc = .75*cap) case. Another case, when employment decentralizes 6 miles out in the HwageI (capc = .5*cap) city, is depicted in Figure 4.13 below.

What is remarkable in the two cases (Figure 4.8a and 4.13) is that the shape of the overall areas occupied by the high skilled and low skilled workers in the monocentric city is hardly disturbed. The boundaries, especially, are quite intact. This is quite plausible. In this equilibrium, there is no residential relocation across the boundaries in the monocentric city. For example, low skilled laborers in the decentralized city occupy the same zone they
did before; but now, some of them commute to the center, while others commute only to the SBD. Similar effects occur in the area occupied by the high skilled laborers. This pattern of decentralization, then, preserves the kind of segregation between high and low income groups as is seen in the monocentric city.

Pattern II ($N^h_c / N^h_s / N^h_c / N^h_s \ldots N^h_i$)

(e.g. Decentralization 7-8 miles out in the Hwagei (capc = .5*cap) city)
When the subcenter forms a bit further out, the low skilled workers at the SBD move outwards along with it. This vacates some land close to the center, between the two low skilled groups. High skilled workers who want to work at the center and have a steeper rent offer curve than their counterparts who work at the SBD will squeeze into this gap. However, not all of the high skilled workers at the center will fit in here, and some of them must live beyond the area occupied by the low skilled workers at the SBD, creating the second pattern seen in Figure 4.12b above. This is a more integrated pattern than in the first case; but the presence of a large low density, high income suburb mars the effect.

In Figure 4.8, the HwageI city with subcenter 7 miles out shows the first pattern when only 25% of the capital is in the SBD. When 50% of the capital moves to the subcenter, however, the effect is big enough to cause substantial residential relocation -- evolving into a city with the second pattern. What can be observed from two cities exhibiting this pattern (in Figures 4.8b and 4.9a) is that in moving from the first to the second pattern, very significant changes are wrought in the city. Comparing Figures 4.13 and 4.8b, the CBD loses (and the SBD gains) a substantial portion of its unskilled labor when the subcenter moves from 6 to 7 miles out. This occurs when the SBD reaches the critical distance at which a bunch of people switch allegiance from the CBD to the SBD, since they are better off by doing so. This produces perturbations in the labor market, as I analyze later.

Pattern III ($N^h_c / N^h_s / N^l_s / N^l_c$)
(e.g. Decentralization 10-12 miles out in the HwageI ($capc = .5*cap$) city)

FIGURE 4.12c: PATTERN III
This, in some sense, is the most 'normal' pattern that is expected to emerge in these kinds of cities. When the subcenter locates a medium distance away from the CBD, and enough capital moves to the SBD, the most integrated pattern of residential location (each of the four groups staying together, with no overlapping of territory) is seen, with even and alternate bands of low and high income workers. Figure 4.9b depicts such a city.

In this scenario, the subcenter is formed at a distance which is just right to accommodate each of the four groups of people in their 'proper' sequence. The reason for this statement can be seen from Table 4.4. When the subcenter forms between 10-12 miles out, the CBD can afford to stop paying rising wages (as $d_{sub}$ increases) to attract high skilled workers from the far suburbs. As the low skilled workers at the SBD vacate land close to the center, high skilled workers can be induced to turn them into low density suburbs close to the inner city, and CBD wages then start to decline (as $d_{sub}$ increases) as the high skilled stop demanding high compensation for long commutes. The emergence of the third pattern thus signifies a turning point for the labor market. I fully analyze this later.

**Pattern IVa** ($N_c^l / N_c^h -- N_s^l / N_s^h$)

(e.g. Decentralization 14-16 miles out in the HwageI (eapc = .5*cap) city)

![Pattern IVa Diagram](image)

When the subcenter moves further out than in the previous case, the evenness of the land distribution is disturbed. As the low income people working at the SBD move further out, following their job site, a vacancy is created in the middle of the city which cannot be filled only by the high skilled workers at the center. After the latter group have their fill of land, the remaining land is used by the fourth group -- high skilled workers at the SBD.
There is not enough, however, to accommodate all of them, so the remainder of the group continue to occupy the furthest suburbia -- as they do in all patterns.

Figures 4.10a and b depict cities of this pattern. Note that while the proximity of the high skilled and low skilled CBD workers to their job site continue to lower their wages, the high skilled SBD group is about equidistant from their employment site on either side of the SBD. This will slow the increase in SBD wages somewhat, but not as much as in the case of when the high skilled CBD workers moved from the middle suburbs (in the first pattern) to just outside the inner city (by the third pattern). A further advantage to SBD firms is that when they locate in the far suburbs and offer reasonably high wages, workers living close to them and working there get a tremendous boost in income, much more so than when the SBD locates close to the CBD. This means that as $d_{sub}$ increases, firms do not have to offer very much higher wages. This happens in the case of low skilled SBD wages in Table 4.4 -- the level remains about the same as the SBD moves out.

Pattern IVb ($N^h_c / N^b_c$ - 'hole' - $N^h_s / N^b_s$)  
(e.g. decentralization 18+ miles out in the HwageI (cape = .5*cap) city)

As seen from the definition, pattern IVb is a variation of the previous pattern. The difference is that the subcenter has moved far enough out to split the city up in two parts. A 'hole' appears in the middle of the city as the land falls vacant (it is denoted by an empty band in Figure 4.12e). What is noteworthy is that since even high skilled workers at the subcenter must be attached in some fashion to their place of employment, there is never a solution where all the land between the zones occupied by the low skilled workers at the two centers is taken up by a low density suburb. Pattern IVa comes closest to that
solution, remembering that in the symmetrical city, even the CBD has a suburb of high skilled workers across the vertical axis.

Once the city splits up in two, the effects on the labor market are quite markedly different from those in the previous equilibria. With effectively two cities, as the subcenter moves further away, the labor supply at the CBD shortens (and the labor supply at the SBD increases) as whole chunks of people from the boundary of the first (inner) city switch to the SBD. As expected, these effects are felt much more in the high skilled labor markets. I discuss these changes later, in my analysis of the labor markets.

Figures 4.11a and b depict such cities. From the figures, notice that when more capital moves into the SBD in the far suburbs, the second (outer) city simply gets bulkier at the expense of the first one. The 'hole' moves inwards.

For the next part of the analysis, I look at the effects on the land and labor markets in the urban area. For analyzing the two markets, whether the subcenter locates close to the center (in the ESZ) or far away (in the EDZ) becomes relevant. Also, the case where the subcenter forms so far away from the center that the city splits up in two must be analyzed as a separate case, even though it falls under the head of employment decentralization in the EDZ. For each of the markets, then, I separate the cases while analyzing.

Figures 4.14-4.16 overleaf show some of the general trends followed by relative wages and utility levels as employment decentralizes further and further out in the suburbs. They also provide graphs for changes in the variables when more capital moves to the suburb. Figure 4.14a tracks the movement of relative SBD wages ($w_s/w_o$) for high skilled workers for two given levels of capital at the suburb. Figure 4.14b does the same for low skilled workers. Figure 4.15a then depicts the movement in relative high skilled wages ($w^h/w^l$) at the two centers as $d_{sub}$ increases. As expected, high skilled wages are about twice the level of low skilled wages, but there are some small deviations. Figure 4.15b finds a similar pattern for changes in relative high skilled utility levels ($V^h/V^l$) -- as expected, the value is always close to 2, but there are small deviations. Finally, Figures 4.16a and b track the changes in normalized levels of utility for high skilled and low skilled workers as $d_{sub}$ increases.
FIGURE 4.14a (TOP) AND b (BOTTOM) - CHANGES IN RELATIVE SBD WAGES FOR HIGH AND LOW SKILLED WORKERS AS THE SBD FORMS FURTHER OUT (COMPARISONS FOR DIFFERENT AMOUNTS OF CAPITAL AT THE SBD)

LEGEND: whs/whc Hwagel (capc=?) - relative high skilled SBD wages for different capital distributions; wls/wlc Hwagel (capc=?) - relative low skilled SBD wages for different capital distributions.
FIGURE 4.15a (TOP) AND b (BOTTOM) - COMPARISONS OF WAGE AND UTILITY RATIOS (HIGH/LOW SKILLED) AT THE TWO CENTERS AS THE SBD FORMS FURTHER OUT (COMPARISONS FOR DIFFERENT DISTRIBUTIONS OF CAPITAL)

**LEGEND:**
- whc/wlc Hwagel (capc=?)
- whs/wls Hwagel (capc=?)
- whc/wlc Hwagel (capc=.5*cap)
- whs/wls Hwagel (capc=.5*cap)
- Vh/VI Hwagel (capc=?)
- Vh/VI Hwagel (capc=.5*cap)

whec/wlc: wage ratios at CBD and SBD; whs/wls: wage ratios at CBD and SBD; Vh/VI: ratio of utility levels at CBD and SBD.
FIGURE 4.16a (TOP) AND b (BOTTOM) - CHANGES IN NORMALIZED UTILITY LEVELS FOR HIGH AND LOW SKILLED WORKERS AS THE SBD FORMS FURTHER OUT (COMPARISONS FOR DIFFERENT DISTRIBUTIONS OF CAPITAL)

LEGEND: $V^h_{WageL}$ (capc=?): utility levels for high skilled workers for different capital distributions; $V^l_{WageL}$ (capc=?): utility levels for low skilled workers for different capital distributions.
Effects On The Labor Market

The first thing to note from Tables 4.3-4.4 is that the effects of employment decentralization on wages and employment do not change much when differences in labor productivity are introduced. By the assumptions that the total amount of capital is fixed, that there are an equal number of workers of each kind and that one group is twice as productive as the other, a high skilled worker always earns close to double the salary of a low skilled worker who works at the same employment location. Apart from this variation between the high skilled and low skilled workers at each employment location (and the fact that the numbers of the two kinds of workers hired at each location are not exactly equal -- which I explain shortly), the total employment at the center and the subcenter can be analyzed along very similar lines to that employed in Chapter 3 to analyze the homogeneous version of the general equilibrium model of land and labor markets. I start by dividing the analysis in two parts.

(A) When the SBD forms in the Excess Supply Zone

In this part, I consider the case where the SBD loop forms close to the center of the city, and a significant but not huge amount of capital moves there. To begin with, I consider the situation faced by CBD and SBD firms when they bid for any one kind of labor. Take the hypothetical situation where the SBD loop forms close to the CBD and 50% of the capital in the city (say) moves there. If the CBD were to keep on offering \( w^* \), the monocentric level of wages, and the SBD were to offer a reservation wage \( w_{rs} \) -- a wage just high enough to lure to the SBD all the workers living outside the loop -- what happens in the two labor markets?

In Figure 4.17 below, \( K_1 = \bar{K} \), the total amount of capital in the urban area. In the monocentric city, there is no capital and hence no production at the SBD location. The CBD market is at equilibrium at \( w^* \) and \( N^* \). When some capital moves to the suburb, the CBD labor demand curve moves in to \( N_e^0(K_2) \), where \( K_2 \) is the reduced amount of capital at the center. Simultaneously, the labor demand curve at the SBD shifts out to \( N_s^0(K_1 - K_2) \). For the rest of the analysis, the labor demand curves do not move.
FIGURE 4.17

Due to the proximity of the SBD loop to the CBD, a large number of people live outside the location of the SBD loop in the monocentric city. Unless the amount of capital moving to the suburb is sufficient to accommodate all these people, the SBD market will face a glut of labor (shown as $(N_2' - N_1')$ in the graph), while the CBD market will face a shortage (shown as $(N_1 - N_2)$ in the graph). This puts upward pressure on CBD wages and downward pressure on SBD wages. This is the main feature of wage adjustment in all four labor markets when the SBD forms close to the center.

As the SBD moves away from the center, however, the equilibrium does not stay unchanged. The location of the SBD dictates some changes in labor supply which affect the final equilibrium. There are two effects on labor supply, one of which is unambiguous. Following the formation of the SBD, some people gain by working there and saving on commuting costs. This translates into a number of CBD workers -- who live at the outer edge of the zone occupied by others in their group -- switching allegiance from the CBD to the SBD. This causes the CBD to lose part of its workforce (inward shift in CBD labor supply) and the SBD to gain workers (outward shift in SBD labor supply), putting further

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3 In real life terms, the following example can be constructed. Consider a city with an SBD 5 miles from the CBD. In this equilibrium, the CBD draws workers from as far as 10 miles out (a likely solution for suburban employment locating in the ESZ). Now consider the SBD moving 10 miles from the CBD. It is very possible (due to the overcompensation of SBD workers) that the save in 20 miles worth of round trip commuting costs will be sufficient to lure some of the CBD workers living close to 10 miles out to switch allegiance to the SBD.
upward pressure on CBD wages and downward pressure on SBD wages. When the CBD forms very close to the center, this effect is small. But as the SBD moves further out, this effect becomes more and more important. I have referred to this earlier as breaking down the dominance of the CBD. The effects are also different for high and low skilled workers, which produces some differences in the two markets, which are evident from Figures 4.14 and 4.15a. I discuss these presently.

The second set of effects on labor supply arise from the direct changes in income from the initial forces. When the CBD wage offer rises, its workers want more land and move out to the suburbs in search of it, possibly causing a further depletion of the labor supply close to the center, exerting an upward pressure on wages. However, when the SBD wage offer falls, the loss of income causes its workers to demand less land and move inwards, easing the pressures on labor supply close to the CBD and increasing them in the suburbs. This tends to increase the upward pressure on CBD wages and ease the downward pressure on SBD wages. This second effect on labor supply is rather difficult to isolate unambiguously. Whether or not the effects are enough to make people switch from one center to the other is difficult to see. For example, switching from the CBD to the SBD may result in a loss of income for the individual (when the CBD and SBD are very close to each other) and take away the incentive for consuming more land.

Figure 4.17 shows the primary effects on the generic (without separating the high skilled or low skilled laborers) CBD and SBD markets, without introducing the changes in labor supply. For a given location of the SBD close to the center, CBD wages will rise above \( w^* \) and SBD wages will fall from the reservation wage level. As the SBD locates further and further away from the CBD, the labor demand curves for each type of labor at each center will hold at the levels shown in Figure 4.17. However, as the SBD moves towards the edge of the ESZ, the initial position of the CBD labor supply curve will shift inwards (more people inside the SBD loop) and that of the SBD labor supply curve will shift outwards (less people outside the SBD loop). Thus the excess demand in the CBD labor market (the upward pressure on CBD wages) and the excess supply in the SBD labor market (the downward pressure on SBD wages) will be lower. At the same time, labor supply decreases at the CBD (more upward pressure on CBD wages) and increases at the SBD (more downward pressure on SBD wages) due to people switching from CBD to SBD jobs. Tables 4.3 and 4.4 show that the latter effects dominate, regardless of the actual
amount of capital in the SBD. The SBD moving out does break the dominance of the CBD, raising wages and lowering employment in the CBD market, and doing exactly the reverse in the SBD market. This story is consistent with the changing pattern of residential location as the subcenter moves out. As the city population profile changes from Pattern I to Pattern II, for example, the CBD loses a chunk of people.

The previous analysis (backed up by the numerical results) suggest that relative SBD wages decrease for both high and low skilled workers as $d_{sub}$ rises. Figures 4.14a and b show this clearly. Thus, there is a downward sloping wage gradient when SBD employment locates close to the center. This is not, however, the wage gradient employed by standard urban theory. This has been shown in Chapter 3, and can be easily seen by considering that standard theory would estimate the relative SBD wage at $w_{rel}/w^*$, while relative SBD wages in this framework are unambiguously lower.

Figure 4.14 and Tables 4.3-4.4 also show that the upturn of relative SBD wages (signifying the end of the zone in which the SBD faces an excess supply of labor in the first instance) occurs at different distances for high skilled and low skilled workers. The SBD seems to face a (small) excess supply of low skilled workers for a little further distance of decentralization. For example, in Table 4.4, wages for low skilled workers at the center continue to rise till the SBD locates beyond 11 miles out, while the wages for high skilled workers at the center continue to fall only till the SBD locates beyond 10 miles out. This can be explained by observing an obvious feature of the residential location equilibria. As I mentioned earlier, the richer workers tend to live further away from the employment centers, while the poorer workers hug the employment center in all circumstances. Figures 4.7-4.11 bear this out. From the previous analysis of patterns of residential locations, recall that when the city reaches the boundary of the ESZ, the pattern changes from II to III. For the first time, the high skilled workers at the center move in between the two low skilled groups. As $d_{sub}$ increases, all of the high skilled workers decrease commuting costs significantly, and thus are willing to work for lower wages. The downturn in wages is a little later in coming for the low skilled workers, since they are not located right at the interface where the changes first start to occur. Even when the subcenter locates sufficiently far away to just create an excess supply of high skilled workers close to the CBD, not enough low skilled people are willing to switch to the CBD, thus maintaining a small excess demand for them. The boundary between the ESZ and the EDZ is somewhat
fuzzy in this setup, since there is a middle zone of excess supply of high skilled CBD workers and an excess demand of low skilled CBD workers. This band occurs between 10-11 miles out in the Hwagel (capc = .5*cap) city.

Comparing the changes in relative SBD wages in the two Hwagel cities (with 75% and 50% of the capital at the SBD, respectively) in Figure 4.14, I notice a difference in the way capital distribution affects them, depending on whether the market is for high skilled or for low skilled workers. The outward movement of capital creates a significant (downward) change in relative SBD wages for low skilled workers, but the relative wages for low skilled workers is not affected by as much.

For an explanation, it is necessary to look at the effects of capital redistribution on the population density profile of the city. As can be seen in Figure 4.8, for example, when the SBD goes from having 25% of the total capital to having 50% of the total capital, the pattern of residential location shifts from Pattern I (almost no disturbances in the low skilled workforce) to Pattern II (where a substantial chunk of the low skilled workforce relocate). This shift forces the SBD firms, especially, to have a pay proportionately higher wages to compensate a large number of people for extra commuting costs. Throughout the ESZ, this feature holds. Most of the supply of high skilled labor come from the far suburbs, hence the changes in both CBD and SBD high skilled wage offers do not vary much. There are, however, significant changes in the location etc. of the low skilled (especially SBD) laborers. To induce them to relocate outwards, the SBD firms have to increase their wage offer more than proportionately (compared to the low skilled CBD wages), hence the relative SBD wages drop.

(B) When the SBD forms in the Excess Demand Zone

In this part, I analyze the effects on the CBD and SBD labor markets when the SBD loop forms far from the center of the city, and a significant amount of capital moves outside the CBD. There are two subparts, depending on whether or not the SBD locates in a zone where the city 'holds together' (Pattern IVa), or whether the SBD moves far enough out that a 'hole' is created in the middle of the city -- in which case there are effectively two cities with employment centers in the middle of each (Pattern IVb).
As before, Tables 4.3-4.4 and Figures 4.14 and 4.15a show the trends of changes in the labor market when 25% and 50% of the capital moves away from the CBD. There are significant differences in the effects on the labor market depending on exactly how far out the SBD locates in the EDZ. Figure 4.17 below depicts at the generic effect on the CBD and SBD labor markets (not considering differences in skill level) when the SBD locates far in the suburbs.

![Figure 4.18](image)

**FIGURE 4.18**

When employment decentralizes far in the suburbs, and I set up the same initial conditions as in the previous analysis, with the CBD offering monocentric city wages and the SBD offering a reservation wage which captures all people living outside the SBD loop, it is evident that there will be an excess demand for labor in the SBD labor market (too few people live outside the loop) and an excess supply of labor in the CBD labor market. Figure 4.18 shows this, and the resulting downward pressure on CBD wages and upward pressure on SBD wages. The labor demand curve stays fixed for the rest of the analysis.

There are, however, effects on labor supply that change the equilibrium as the SBD locates further out. When the SBD forms in the EDZ, people near it get a large boost in income by switching allegiance from the CBD to the SBD. The CBD labor supply shrinks (putting downward pressure on CBD wages), while the SBD labor supply expands (putting upward pressure on SBD wages). There are contradictory effects on wages at the two centers. The first effects dominate -- relative SBD wages do not fall after the SBD moves into the EDZ. Employment rises in the CBD and falls in the SBD.
As $d_{sub}$ increases, though, the effects on relative wages and employment are not as significant as they were in the case where employment located close to the center. This is understandable. First, when employment locates in the EDZ, the dominance of the center has already been broken down. Any further movements of the SBD cause more perturbations in the immediate locality, rather than the deep interior of the city. Second, since the suburbs are significantly less dense than the interior, the changes in labor supply as the SBD loop moves out (depending on the number of people who live inside and outside the loop initially) are smaller. Thirdly, the primary movements in labor demand and labor supply tend to keep wages constant with distance of decentralization.

There is, then, no wage gradient as employment decentralizes further and further out in the EDZ. In the first part of the EDZ (as long as the city holds together, i.e. Pattern IVa), CBD wages continue to fall for both high and low skilled workers while SBD wages continue to rise for both groups. Relative SBD wages are constant. Employment continues to rise at the CBD and fall at the SBD for both high and low skilled workers, which is consistent with fixed labor demand and the labor supply shifts discussed above.

When the SBD locates far enough to cause a 'hole' in the middle of the city (Pattern IVb), the trends change slightly. With two cities, any outward movement in the SBD causes perturbations to the other market only in so far that a few extra people are weaned away from the CBD. To prevent too many people from wanting to go over to the other city, the CBD has to increase its wage offers to both high and low skilled labor. The SBD can afford to even drop wages slightly and still remain an attractive proposition just by putting more distance between itself and the CBD. In Figure 4.14, slight downturns in relative SBD wages can be seen at the tail end of the graphs.

When more capital shifts into the subcenter in the Hwage city with employment suburbanized in the EDZ, there are comparatively huge effects on both high and low skilled subcenter wages. With the dominance of the CBD ahving disappeared, an SBD with a higher labor demand causes wages for both skill levels to shoot up. The CBD loses workers fast. Overall, there is a big increase in relative SBD wages. The gap between the two groups is apparent in Figure 4.14. Both relative high and low skilled SBD wages take a downturn when the city splits up in two, but as the figure shows, the gap between them is maintained.
Effects On The Land Market

When the effects on wages and employment and wages arising from the labor market are clearly understood, the effects on the land market -- manifested in the changes in land rents and overall levels of utility -- become easier to follow. I will split up the analysis in two parts, as before, depending on whether employment locates in the ESZ or the EDZ.

(A) When the SBD forms in the Excess Supply Zone

From Tables 4.3-4.4 and the previous analysis, as employment decentralizes further and further out in the ESZ, $w^l$ and $w^h$ rise, while $N^h_c$ and $N^l_c$ tend to fall in the CBD labor market. Simultaneously, $w^l_s$ and $w^h_s$ tend to fall, while $N^h_s$ and $N^l_s$ tend to rise in the SBD labor market. Again, the first thing to remember is that low skilled workers are tied to whichever center they work at, while high skilled workers commute more and live in the suburbs. Rents offers from the former group are always steeper than from their high skilled counterparts working at the same center. When suburban employment locates very close to the center, the rent offer curves are as follows:

![Diagram](image)

**FIGURE 4.19a**

In Figure 4.19a, the upper envelope of the four rent offer curves is the rent gradient which emerges in Figures 4.3-4.5, for example. This envelope is marked with a bold line. To avoid overcrowding, I have not sketched the monocentric city rents in Figure 4.19a, but
Figures 4.3-4.5 show clearly what happens. In the above pattern (corresponding to Pattern I of residential location), the boundaries of the zones occupied by high and low skilled workers in the monocentric city are almost invariant. So land demand does not change much from monocentric city levels in the area closest to the CBD. However, since wages for both CBD groups are higher than in the monocentric city, there is some relocation inside the zone occupied by low skilled workers. The pressure on land supply lessens very close to the center. Rents will fall slightly in the most interior part of the city, but rise at the subcenter and beyond, reflecting the small outward shift in population. Overall, the utility level increases as the SBD moves out.

As the subcenter moves further out, the low skilled workers there move out along with their place of employment. Rents in the interior of the city fall much more as Pattern I is replaced by Pattern II and then finally Pattern III, and high density low skilled residential zones are repalced by an inner city suburb, made up of high skilled CBD workers. At the very edge of the ESZ, the land rents are as follows:

![Diagram](image)

**FIGURE 4.19b**

The rent profile shown in Figure 4.19b corresponds to residential location Pattern III. Figures 4.3-4.5 show Hwagel cities with subcenter 11 miles out and varying capital distributions. The Hwagel (capc = .5*cap) city corresponds to Figure 4.19b. As the subcenter pulls away from the center, pressures on land supply close to the center ease off significantly, compared to the monocentric city. Coupled with the steady decrease in CBD employment, rents near the the center fall and those near the suburbs rise, much more so
than when the SBD forms very close to the CBD. The utility levels for both high and low skilled workers continue to rise with distance of decentralization, as Figure 4.16 showed.

(B) When the SBD forms in the Excess Demand Zone

When the SBD forms far enough from the center to create residential location Pattern IVa, the effects on the labor market have slowed down. As distance of employment decentralization increases, \( w^h \) and \( w^l \) fall, while \( N^h \) and \( N^l \) now begin to rise in the CBD labor market. Simultaneously, \( w^h \) and \( w^l \) tend to rise, while \( N^h \) and \( N^l \) tend to fall in the SBD labor market. The changes are smaller than when employment decentralizes in the ESZ. The rent profile in the city looks as follows:

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Figure 4.19c
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Figure 4.19c shows how the pattern changes as the SBD low skilled workforce vacate more space in the middle of the city, following the outward movement of their employment location. Some high skilled suburban workers now live between the CBD and the SBD. With small decreases in employment at the CBD, land demand drops close to the center. Even though CBD workers earn less, SBD workers earn significantly more with increased distance of decentralization. The latter group moves outward, spreading the city out as land supply increases (land rent decreases) in the interior of the city and land supply tightens (land rent increases) in the suburbs. The overall utility level continues to rise for both groups of workers as \( d_{sub} \) increases.
As the SBD moves further and further out, the second rent peak at the subcenter -- signifying the degree of reverse commuting prevalent in the city -- grows in stature. In the extreme case, the subcenter workforce break off from the initial city and form their own (Pattern IVb). Once this stage is reached, the utility level stops following the steady upward rise that it displayed till now.

When for a given distance of decentralization, more capital moves into the suburbs, the utility level for both groups of workers increases unambiguously, as Figure 4.16 shows. This is logical, since more capital and employment moving to the subcenter will hasten the outward movement of population, easing pressures on land and raising everyone's land consumption and utility level.

Figure 4.15b shows another trend. Though the high skilled workers are expected to be roughly twice as well off as their low skilled counterparts due to the assumptions of the model, the actual trend shows that the former group are in fact more than twice as well off for all distances of employment decentralization. This makes sense, too. Since the low skilled laborers hug the employment centers, any improvements in terms of easing pressure off land is likely to benefit the higher income suburbanites. As the SBD moves away from the center, the disparity lessens somewhat, as low skilled suburban workers can avail of relatively cheaper land, away from the competition of the low skilled CBD workers. The former group also gains in income as the SBD breaks away from the dominance of the CBD. Simultaneously, the high skilled workers lose some utility when they forsake the far suburbs and come to inhabit part of the interior of the city.

Beyond a certain distance, however, the ratio of high to low utility levels flattens out. Both high and low skilled workers are benefitting equally from suburbanization. The zones of each group is well defined, and no one group gains in relative terms.

Figure 4.15b also shows that when more capital moves to the suburbs, low skilled workers benefit more than high skilled ones. This is understandable; while the low skilled workers continue to live in congested conditions near the employment center, the suburban workers get a significant boost in income, raising the relative low skilled utility level.
General Analysis And Conclusions

As in the general equilibrium model with homogeneous labor, the most noteworthy aspect of this framework is the complete dissolution of the standard wage gradient approach used by urban theorists. The results from Chapter 3 conclusively show that when a full labor market is introduced, in this case through an exogenously distributed complimentary factor of production (capital), the standard wage gradient is non-existent regardless of the distance to which employment decentralizes. In this chapter, I find that the introduction of a population which is heterogeneous in productivity does not change the general conclusion. There are differences in the wages and utility levels for the high and low skilled workers, as expected, but for both groups, relative SBD wages do not decrease at the rate proposed by the standard wage gradient as the SBD forms further and further away from the center. For both groups, the relative SBD wages decrease more than with the wage gradient when employment decentralizes close to the center. In this zone, relative SBD wages fall as suburban employment moves further away, but the slope of the wage adjustment schedule is steeper than the wage gradient. When suburban employment locates far away from the CBD, the relative SBD wages stop falling. Instead, they hold steady or even increase slightly. Finally, when the employment decentralization occurs so far from the center as to cause the city to break up into two, the relative SBD wages show a very slight downward turn.

Instead of the wage gradient, then, the process of employment and residential decentralization produces a utility gradient in the urban area. As employment decentralizes further and further out, utility levels rise steadily. As I mentioned in Chapter 3, this is natural in a model with a full labor market. Since all wages adjust to the process of employment decentralization (labor demand curves are downward sloping), residential utility levels are bound to rise as land rent offers change. Again, high skilled workers are better off than their low skilled counterparts, however, both groups experience the steady increase in utility as employment decentralizes. For a given distance of decentralization, shifting more capital to the subcenter also increases employment and wages at the SBD and increases utility. When the city breaks up in two, the utility increases slow down slightly.

The concern that remains to be resolved in this framework is the seemingly unconstrained gains in utility for all workers when more employment locates further out.
This suggests that the full story is yet to be told. To complete the scenario, there are a number of possible approaches. First, the capital market story needs to be chronicled in greater detail. In the approach I have used so far, capital movements to the suburbs are wholly exogenous. One obvious constraining factor that would exist in this setup are falling returns to capital in the suburbs. Presumably, there are unequal returns to capital at the center and in the suburbs when the out movement of capital occurs. Such inequalities may be introduce by various means, e.g. fiscal incentives set up with the express purpose of moving capital out of the center. When capital moves out to the suburbs, labor demand increases there, wages and employment rise. This lowers the return to capital. Simultaneously, the return to capital rises in the center as labor demand shifts inward, decreasing wages and employment. Presumably, the capital movement stops when returns are equalized everywhere in the city. Without an explicit capital market, what I present in this model is a series of short run production equilibria, to find the final equilibrium, the minimum cost point for all parties involved must be discovered.

Other factors relating to the capital market may also be important. The foremost among them is the durability of capital. Even when strong incentives to move capital to the suburb are present, it may not always be possible to move more than a certain amount of capital out of the center. There are certain durable forms of capital which do not yield to such pressures easily. Also, there are durable forms of housing capital which may cause a problem in real life. If old housing stocks exist close to the center of the city, the fixed costs of demolishing them and building new housing in the suburbs may not be achieved instantaneously. This is likely to be a further constraining factor in real life. Models of filtering (Steen etc.) analyze the impact of existing durable housing stock on the urban area.

Second, I assume that production in the center and the suburbs is carried on using the same technology. This is unlikely to be true in a modern city, which features both manufacturing and office activities. If differences in production technologies are introduced in a full capital market, firms will sort themselves out in the same manner that households do, and this further friction between components of the final equilibrium will constrain the outcome. Normally, if transportation technologies are heterogeneous -- humans and freight cars have different costs of commuting -- firms of one kind will inhabit the center and firms of the other kind will inhabit the suburbs. Their workers will then align themselves accordingly.
Third, a main reason for the lack of constraints is the closed city format. When a city gains in utility from employment decentralization, people from outside the city are likely to migrate in. Even if similar changes are taking place in all cities, it is not reasonable to assume that after the city breaks up in two halves, the second half can continue to move further and further away from the first city without encountering anything except agricultural land. In real life, improvements in living conditions are usually followed by some in migration, which causes congestion, extra pressure on land etc. and lower utility. In the scenario where there are no changes in population, having more land available for consumption leads to everyone being better off.

Finally, the importance of fiscal constraints cannot be forgotten. Local government regulations can affect the equilibria on both the production and the consumption side. On the production side, governments may provide fiscal incentives for capital to move out of the center. They may also undertake investments in infrastructure (e.g. a radial road for faster intracity freight transportation, a suburban export node) which facilitate the location of certain kinds of production at certain points in the city. On the residential side, zoning and property tax laws will affect the land allocation process significantly, constraining the possible patterns of residential location.

After mentioning the drawbacks and the possible redemptive approaches, however, let me comment on the positive aspects of the framework that I have used so far. As I mentioned in Chapter 3, before this model, standard urban theory has not examined the interactions between the land and the labor market carefully enough, as is evidenced by the free use of the wage gradient approach, even in current works like Yinger (1994). Part of the reason is the intractability of the general equilibrium approach when closed form solutions have been attempted. White (1976) has been the only person who has attempted to investigate the wage gradient, albeit in a partial equilibrium framework with CBD wages fixed. Later researchers, even when they have switched to numerical analysis (Straszheim (84), Sullivan (83, 86)) have not examined the wage gradient issue carefully. This has partly been due to the problems of finding solutions to the basic experiments I have attempted, viz. the impact of employment decentralizing further and further away from the CBD. Also, few people (White (78), Straszheim (83)) have attempted to examine the effects of employment decentralization in a city with a heterogeneous labor force, certainly not in the context of a general equilibrium model.
Simplifying the model, and other iterating techniques I use, pays dividends in so far that it provides a better understanding of the relationships and friction between the labor and land markets in the urban area. In addition, it lays a solid foundation into which other aspects of decentralization, including a full capital market, can easily be incorporated.

A very important insight that arises from the analysis with heterogeneous labor are the different patterns of residential location. These are determined based solely on the location and strength (in terms of attracting labor) of the SBD. The underlying feature is that low skilled workers do not move out of the inner city unless their jobs leave. High skilled workers will take up residence in the interior of the city only if the land is vacated by suburban low skilled workers. This can be set up as a basis for comparing real life situations, to see if the patterns are disturbed by social, fiscal or other institutional factors.

4.2c. SENSITIVITY TESTS: RESULTS AND ANALYSIS

Comparing The Hwagel And II Cities

To analyze the effects of changing factor productivity, I first compare the Hwagel and Hwagel II cities, with 50% of the capital moving out to the suburbs in each case. The two cities are then similar in every respect except for the coefficients of production. In the Hwagel city, \( \theta_1 = .48, \theta_2 = .24, \varphi = .1 \); in the Hwagel II city, \( \theta_1 = .6, \theta_2 = .3, \varphi = .1 \). I leave the relative productivity of high and low skilled workers unchanged, but the capital in second city is less productive than in the first. The monocentric cities are compared in Table 4.2 and Figure 4.1-4.2. As mentioned before, Hwagel II is a city with lower levels of income for everyone, since the productivity of the more abundant factor (capital) drops.

The iterations are carried out using exactly the same parameters and techniques as in Section 4.2b. Table 4.5 lists the equilibrium value for the Hwagel II cities with 50% of the capital moving out to different distances. The values in Table 4.5 are to be compared with those in Table 4.4. Figures 4.20a, 4.21a and 4.22a compare the equilibrium rent gradients in the Hwagel (capc = .5*cap) city to those in the Hwagel II (capc = .5*cap) city when employment decentralization occurs 7, 12 and 18 miles from the CBD. Figures 4.20b, 4.21b and 4.22b then compare the population density profiles for the same cases.
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TABLE 4.5: EQUILIBRIA FOR HWAGE II CITY WITH 50% OF THE TOTAL CAPITAL STOCK AT THE CBD
$(\theta_1 = .6; \theta_2 = .3; \rho = .1; K = 1000000)$
FIGURE 4.20a (TOP) - b (BOTTOM): COMPARISON OF RENTS (TOP) AND POPULATION DENSITIES (BOTTOM) IN DECENTRALIZED (SUB = 7) HWAGEI AND HWAGEII CITIES WITH 50% OF CAPITAL STOCK AT THE CBD

**Legend:** as before
FIGURE 4.21a (TOP) - b (BOTTOM): COMPARISON OF RENTS (TOP) AND POPULATION DENSITIES (BOTTOM) IN DECENTRALIZED (SUB = 12) HWAGEI AND HWAGEII CITIES WITH 50% OF CAPITAL STOCK AT THE CBD

LEGEND: as before
FIGURE 4.22a (TOP) - b (BOTTOM): COMPARISON OF RENTS (TOP) AND POPULATION DENSITIES (BOTTOM) IN DECENTRALIZED (SUB = 18) HWAGEI AND HWAGEII CITIES WITH 50% OF CAPITAL STOCK AT THE CBD

LEGEND: as before
The effects of lowering the productivity of capital are easily understood by comparing the HwageI and HwageII monocentric cities. Everyone earns less in the HwageII city -- hence, it is more dense and compact than the HwageI city. Employment decentralization creates the same pattern of effects as in the HwageI city. Table 4.5 shows that the Excess Supply Zone ends only 7 miles out for the HwageII city, and when employment decentralizes 16 miles from the CBD and beyond, the city splits up. Besides the scale effects, everything else follows the previous analysis, including the trends of high and low skilled wages and utilities. The wage gradient does not exist, a utility gradient is noticed instead. Figures 4.20-4.22 show the extent to which loss in productivity affects the city. The residential location patterns for employment decentralization 7 miles out, for example, resemble Pattern II in the HwageI city but most definitively Pattern III in the HwageII city. Similarly for employment decentralization 12 miles out. For the last case, there is a huge hole created in the HwageII city, as compared to nary a hint of one in the HwageI city.

Comparing The HwageI And HwageII Cities

Finally, I compare the effects of introducing more capital in the urban area, while maintaining the same relative factor productivity. Again, all parameters and techniques are the same as in Section 4.2b. Comparing the monocentric cities in Table 4.2 and Figures 4.1-4.2, the results can be expected. Increasing the endowment of the complementary factor of production increases the income level of everyone in the city, creating a larger, less dense city. The results should follow. For example, the ESZ and the EDZ should shift out when employment decentralizes, the boundary between them will be further out. Tables 4.6-4.7 below list the equilibria for the HwageI and HwageII cities with 75% and 50% of the total capital in the center. Figures 4.23, 4.24 and 4.25 then depict the rent gradients and population density profiles for decentralized HwageI (capc = .5*cap) cities with subcenters 7, 12 and 18 miles out respectively, comparing them with the corresponding HwageI cities.

Figures 4.26 and 4.27 then look at the movements in relative SBD wages and utility levels for the HwageII cities with different capital distributions. I use these graphs to compare the trends between the HwageI and HwageII cities, not only with regard to the shifting out of the SBD location, but also to investigate the effects of shifting more capital out to a given employment subcenter.
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TABLE 4.6: EQUILIBRIA FOR HWAGE III CITY WITH 75% OF THE TOTAL CAPITAL STOCK AT THE CBD
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**TABLE 4.7: EQUILIBRIA FOR HWAGE III CITY WITH 50% OF THE TOTAL CAPITAL STOCK AT THE CBD**

$(\theta_1 = .6; \theta_2 = .3; \varphi = .1; \bar{K} = 100000000)$
FIGURE 4.23a (TOP) - b (BOTTOM): COMPARISON OF RENTS (TOP) AND POPULATION DENSITIES (BOTTOM) IN DECENTRALIZED (SUB = 7) HWAGEL AND HWAGEIII CITIES WITH 50% OF CAPITAL STOCK AT THE CBD

**Legend:** as before
FIGURE 4.24a (TOP) - b (BOTTOM): COMPARISON OF RENTS (TOP) AND POPULATION DENSITIES (BOTTOM) IN DECENTRALIZED (SUB = 12) HWAGEI AND HWAGEIII CITIES WITH 50% OF CAPITAL STOCK AT THE CBD

**LEGEND:** as before
Figure 4.25a (top) - b (bottom): Comparison of rents (top) and population densities (bottom) in decentralized (sub = 18) HWAGEI and HWAGEIII cities with 50% of capital stock at the CBD. Legend: as before.
FIGURE 4.26a (TOP) AND b (BOTTOM) - CHANGES IN RELATIVE SBD WAGES FOR HIGH AND LOW SKILLED WORKERS AS THE SBD FORMS FURTHER OUT (COMPARISONS FOR DIFFERENT AMOUNTS OF CAPITAL AT THE SBD)

LEGEND:  
whs/whc HwageIII (capc=75*cap): relative high skilled SBD wages for different capital distributions;  
wls/wlc HwageIII (capc=5*cap): relative low skilled SBD wages for different capital distributions.
FIGURE 4.27a (TOP) AND b (BOTTOM) - COMPARISONS OF WAGE AND UTILITY RATIOS (HIGH/LOW SKILLED) AT THE TWO CENTERS AS THE SBD FORMS FURTHER OUT (COMPARISONS FOR DIFFERENT DISTRIBUTIONS OF CAPITAL)

LEGEND: whc/wlc HwageIII (capc=.75*cap), whs/wls HwageIII (capc=.75*cap), whc/wlc HwageIII (capc=.5*cap), whs/wls HwageIII (capc=.5*cap).

V*h/V*l HwageIII (capc=.75*cap), V*h/V*l HwageIII (capc=.5*cap).

Vh/Vl HwageIII (capc=?): ratio of utility levels at CBD and SBD.
FIGURE 4.28a (TOP) AND b (BOTTOM) - CHANGES IN NORMALIZED UTILITY LEVELS FOR HIGH AND LOW SKILLED WORKERS AS THE SBD FORMS FURTHER OUT (COMPARISONS FOR DIFFERENT DISTRIBUTIONS OF CAPITAL)

LEGEND: \( V^h \) HwageIII (capc=?) utility levels for high skilled workers for different capital distributions; \( V^l \) HwageIII (capc=?) utility levels for low skilled workers for different capital distributions.
The patterns of decentralization change as the production coefficients change. The changes are consistent with the increase in income from the Hwägel to the HwägelIII framework. The most noticeable difference from the Hwägel to HwägelIII regime is the shift of population towards the suburb. This is natural, due to the nature of the utility function. As Figures 4.27-4.28 show, the relative SBD wages and overall utility level behave similarly to the previous cases. The overall conclusions are not changed with changes in labor productivity and income.
References


