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A zonally and annually averaged study of potential early Martian atmospheres

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A ZONALLY AND ANNUALLY AVERAGED STUDY
OF POTENTIAL EARLY MARTIAN ATMOSPHERES

by

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ABSTRACT

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Observations of the surface of Mars suggest a high probability of surface water activity in that planet's past. Consequently, many studies of Mars' early atmosphere have attempted to estimate the carbon dioxide level by requiring that surface temperatures be high enough to support surface liquid water. In the main, these studies have employed one-dimensional, radiative-convective climate models capable of considering only a single solar zenith angle, typically chosen to represent a global and annual average. Such models are hence not well suited for considering meridional variations in the temperature profile, which are affected by variations in the orbital obliquity and the meridional redistribution of heat by dynamic processes.

I describe modifications to a more complex model, the multi-level energy balance model designed at NASA's Goddard Laboratory for Atmospheric Sciences, which make it suitable for study of an atmosphere with varying carbon dioxide levels. Vertically and meridionally defined, the model includes heating and cooling by radiation, mean meridional circulation, large-scale (baroclinic) and small-scale (convective) eddies, and surface turbulent flux. I present annually-averaged results for an examination of potential atmospheres of early Mars, given that its carbon dioxide level may range from 0 to 500 Pa and the orbital obliquity from 0° to 50°. These results are compared with those obtained from a radiative-convective model.
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I. INTRODUCTION

Mars is today a barren planet: chill, dry and dusty. Due to its low atmospheric pressure, surface water would rapidly boil away, but a great deal of geologic evidence suggests that in the past, this condition was far from the case. As summarized in Clifford et al. (1988), McEwen (1991) and Baker et al. (1991), the face of Mars is marked by numerous features indicative of water activity. The cratered highlands contain networks of small valleys, and emerging from the highlands north of Valles Marineris are great outflow channels containing what appear to be fluvial patterns in their scoured beds. Other such features include lobate ejecta patterns from craters, wet landslides and layered deposits in Valles Marineris, degraded impact craters and possible glacial formations. It has even been argued that geological formations at the termination of the large outflow channels are evidence of sediment deposition as might result in a large lake or possibly a sea.

Estimates of the volume of water and the methods by which they were determined which might have been present in the past are varied. Carr (1987) estimated that the volume of water necessary to erode the channels was equivalent to a global ocean 500 m deep, assuming that the region of the channels was representative of the rest of the planet. Scambos and Jakosky (1990) more recently employed an argument involving the early outgassing history and composition of Mars' atmosphere and the composition of a small group of achondritic meteorites known as SNC meteorites, which may been ejected from Mars during its cratering history (e.g., Vickery and Melosh 1987),
to derive a water inventory of depth 90-3000 m. (In comparison, Earth’s oceans presently contain enough water to cover the planet to a depth of about 2700 m if spread evenly over the surface.) Although the summary presented in McKay and Stoker (1989) notes that lesser water amounts have also been determined, the more recent studies listed have found amounts equivalent to global oceans of depth 80 m or greater.

The current location of this water is open to question. Owen et al. (1988) have found that the deuterium to hydrogen (D/H) ratio of Mars is presently enriched over the terrestrial ratio by a factor of about 6±3. If one assumes that Mars and Earth began with similar D/H ratios, this enrichment suggests that Mars has lost at least 83% of its water through photodissociation and thermal escape. Extrapolating the present water loss rate backwards, Hunten et al. (1989) estimate that 2.5 to 20 meters of water may have escaped Mars’ atmosphere, depending on how the early solar EUV flux is considered in the analysis. In combination, these arguments imply an initial water abundance for Mars of no more than 25 m, significantly less than the estimates cited above. To account for the difference, Owen et al. (1988) theorized that the water loss rate might have been much higher in Mars’ early history. However, other researchers have argued that the missing water remains on Mars, and since the amount of water in the present atmosphere and polar caps is much lower than the difference, that it is probably geologically confined, perhaps in the form of subsurface ice. Until further evidence is available, the implications of the D/H ratio remain open for interpretation (McEwen 1991).

There is, moreover, debate over the state of the water that created the fluvial features on Mars, as many suggest that liquid water at the surface is
not necessary to explain how many of these features might form. For instance, subsidence and slumping due to subsurface water movement and the melting of ground ice by subsurface heating have been suggested as the source of the valley networks in the highlands, and perhaps the actual cause is a combination of both. Nevertheless, the preponderance of the evidence seems to suggest that surface temperatures for early Mars were higher than they are today.

A. The faint young Sun paradox

With the evidence leaning toward past surface water activity and the concomitant warmer surface temperatures, it is apparent that the climate of early Mars must have been much different than it is now. A complicating factor is that models of stellar evolution suggest that the luminosity of the Sun would have been 30% lower 4 Gyr ago than it is at present (Newman and Rood 1977) and that less radiation was therefore available to warm the early planets. A solution to this contradictory relationship, since dubbed the “faint young Sun paradox”, was first discussed by Sagan and Mullen (1972) and recently reviewed by Kasting and Grinspoon (1991). Sagan and Mullen (1972) postulated that an ammonia greenhouse in the atmosphere would explain the geologic evidence indicating that Earth also had above-freezing temperatures 3.8 Gyr ago. The NH₃ greenhouse theory fell by the wayside when it was realized that ammonia would photochemically dissociate on a short time scale (Kuhn and Atreya 1979), but it should be noted that an attempt to resurrect this theory has recently been made (Brown and Kasting 1992). A similar argument has been used against methane, and instead, a carbon dioxide greenhouse has apparently become the favored solution. However, since CO₂
is a less efficient absorber than ammonia or methane, the existence of a CO₂ greenhouse also requires a much denser atmosphere than had been anticipated. Various studies, beginning with Pollack (1979) and Cess et al. (1980), have explored the strength of the CO₂ greenhouse for early Mars, and a review of these studies may be found in Kasting and Toon (1989). The most notable of these studies is perhaps that of Pollack et al. (1987), who estimated that a carbon dioxide partial pressure between 75 and 500 kPa (0.75 and 5 bars; note that the present-day total surface pressure is about 0.7 kPa, or 7 mbar) would be required to keep the surface temperature of Mars above freezing during its early history, the lower estimate corresponding to optimal heating conditions of low surface albedo, perihelion and equatorial latitudes.

One is left to wonder what happened to all this carbon dioxide. Processes which may have removed the carbon dioxide permanently from the planet are thermal escape (Hunten et al. 1989) and impact erosion (Melosh and Vickery 1989). Additionally, Carr (1989) and Pollack (1991) discuss the rate at which, in a warm and moist atmosphere of its own creation, the CO₂ would have been converted to carbonate rocks, a rate so fast that there would also have to be some process which would return it to the atmosphere if warm conditions were to have prevailed for any length of time. Possible solutions suggested for this recycling method are volcanic outgassing and impact erosion. Attempts to estimate the amount of CO₂ stored within the regolith have been made (Kahn and Appleby 1991), but a definitive answer must presumably wait for in situ evidence gathering.
B. Astronomical theory of climate change

An additional element that adds to the attraction of studying the potential atmospheres of early Mars is the astronomical (or Milankovitch) theory of climate change, which depends on periodic variations in the orbital variables: obliquity, eccentricity and argument of perihelion. This theory has been advanced as an explanation for the advance and retreat of Earth's glaciers (e.g., Hays et al. 1976), and its implications for Mars may potentially be greater (Toon et al. 1980). For Earth, the oscillation in obliquity is about one degree about the present value of 23.5° and produces only relatively small variations in climate, though many would not call the creation of continental ice sheets a small variation. For Mars, however, the obliquity currently oscillates 12° about the present 25.1° over a period of 120 kyr (Ward 1979) and has great potential for radically different climate states. Moreover, Ward et al. (1979) have suggested that prior to major geologic events — in particular the creation of the Tharsis uplift — Mars may have had an average obliquity of 32° with extrema near 25° and 46°, and Bills (1990) has postulated a maximum obliquity of 51.4° when orbital resonance conditions occur. Such high obliquities result in redistribution of insolation toward the poles, and the annual insolation at the poles may vary by 100% between the extremes of the possible obliquities (Ward 1974). Toon et al. (1980) have noted that for obliquities near 50°, the poles would receive as much solar energy on an annual basis as does the equator. In addition to oscillations in its orbital obliquity, Mars' eccentricity may reach values as high as 0.141 (Ward 1974), offering up the possibility of major climate variations depending on the planet's position in its orbit. For maximum obliquity and eccentricity, the possibility of intermittent liquid
water activity during the year is quite possible and may be of catastrophic nature.

C. Selecting an atmospheric model

The varying meridional profile of the solar input resulting from the applied orbital elements suggests that it would be beneficial when constructing a climate model of early Mars to include a meridional dimension. As noted above, Pollack et al. (1987) found a range of carbon dioxide levels which support a greenhouse which would allow above-freezing temperatures, the lower end of which corresponded to a solar zenith angle appropriate to equatorial latitudes. However, their model did not include the meridional dimension and thus omitted any possible impact of the redistribution of heat between the latitudes by dynamic processes.

In dealing with surface water at or near the freezing point, one would expect improvement in results if the albedo feedback of surface ice were included. (One study which attempted to portray the possible effect of clouds on the climate of early Earth [Rossow et al. 1982] was criticized for neglecting variations in surface albedo [Kasting et al. 1984b].) Any reasonable treatment of variable surface albedo must, however, consider its latitudinal dependence, which cannot be adequately simulated with a one-dimensional radiative-convective model (RCM) such as has been used in many of the past studies (e.g., Pollack et al. [1987] and Durham and Chamberlain [1989]).

With these factors in mind, I began searching for a climate model which would permit processes varying with and/or dependent upon latitude. Perhaps the easiest step to have taken would have been to employ a so-called
one-and-a-half dimensional RCM such as is detailed by Wang and Stone (1980). In their model, the mean annual surface temperature profile at a given latitude is assumed to fit a series expansion of the first two Legendre polynomials. By specifying the surface temperature at the boundary of the polar ice-cap, Wang and Stone (1980) are able to extract the latitude of that boundary as a function of the global mean surface temperature and thence calculate a global average surface albedo. The only true consideration of meridional variations which occurs in this model is in the calculation at three different latitudes of the vertical profile of the radiative flux, but they are then averaged together to obtain the global mean flux profile. Horizontal heat transport is included in the model only so far as the temperature profile equation accurately represents the meridional temperature profile. In addition to omitting such meridional heat transport, this type of model is not well-suited for the study of seasonal effects, and although the research described in this thesis does not include the study of seasonal variation, it was my intent to employ a model which would permit continued study beyond the present research project. Consequently, I felt it best to go a step further in selecting a new model.

A second alternative would have been the simple, or single-layer, energy-balance model (SEBM), which is primarily a one-dimensional (meridional) scheme, although two-dimensional (full surface) versions exist (e.g., North et al. 1983). Popularized by Budyko and Sellers, these models have been used extensively to examine time-stepping climate problems (North et al. 1981), especially the Milankovitch theory of Earth's ice ages. However, in suppressing the vertical dimension, most of these models have come to rely
on empirical approximations of radiative effects which are reliant on limited
temperature and pressure ranges. These may be well-suited for examinations
of Earth's (relatively) recent climate history, but would require a great deal of
modification for atmospheres much different in composition or thickness,
such as occur on the early terrestrial planets. Additionally, the stability of such
models against sudden climatic transitions has been questioned (Warren and
Schneider 1979).

A third possibility would have been a full-scale general circulation
model (GCM), but such is far more costly in time and money than, for
instance, an SEBM. Furthermore, Stone and Risbey (1990) have noted that
"meridional transports of heat simulated by GCMs used in climate change
experiments differ from observational analyses and from other GCMs by as
much as a factor of two." They also question whether current GCMs are in fact
superior to simpler models for simulating temperature changes associated
with global-scale climate change.

In light of the drawbacks connected with these three types of climate
models, I decided that the optimum approach to include latitudinal vari-
tions would be a fourth method, a two-dimensional model which retains the
vertical dimension for calculation of radiation transport, along with a merid-
ional dimension for the inclusion of varying solar zenith angle, surface
albedo, and meridional heat transport. A multi-layer energy balance model
(MLEBM) combines these features, and I selected the model designed at the
Goddard Laboratory for Atmospheric Sciences (GLAS) — and described by
The GLAS MLEBM evaluates a heat balance equation in each cell of the two-dimensional grid. In addition to radiative heat flux, it includes dynamic heat transport, which is divided into primary regimes, separated by the poleward boundary of the Hadley circulation. This boundary is located on the basis of conservation of momentum within a statically-stable, highly inviscous, Boussinesq fluid. Within the Hadley cells, the temperature profile is forced to match the moist adiabatic lapse rate. Outside the tropics, the major dynamic heat transport is that of longitudinal baroclinic eddies, approximated as a two-dimensional diffusion process. This MLEBM also includes sensible and latent heat exchange at the surface, small-scale eddies, and growth of ice/snow fields and their associated albedos. And like other energy balance models, it may consider annually-averaged or seasonal climate studies by specification of constant or varying solar insolation.

The GLAS MLEBM was, however, designed to examine the modern Earth atmosphere, and the next chapter of this thesis describes a number of changes which I have made in the model so that it may be applied to the various potential atmospheres of early Mars.

D. Similar studies in the past

Before proceeding with a description of the model used in this study, I must note that there have been a few prior applications of energy-balance models to the study of ancient climates of Mars. For example, Hoffert et al. (1981) employed an SEBM to examine an early Mars with a CO$_2$ partial pressure of 100 kPa (1.0 bar), finding a surface temperature distribution that
allowed for liquid water activity over 95% of the planet. But their model did not include consideration of orbital obliquity variations.

Postawko and Kuhn (1986) found it doubtful that a surface temperature on Mars of 273 K could be maintained by CO₂ partial pressures less than 300 kPa (this value apparently chosen since Pollack and Black [1979] estimated that that much CO₂ was outgassed on Mars), suggesting that fluvial features were instead the result of subsurface water activity. Such a contention, however, is at variance with many of the analyses of the origin of Martian surface features (e.g., Carr 1987). Although their study employed what was primarily a one-dimensional, meridional model, Postawko and Kuhn (1986) did include vertical definition of the atmosphere for radiation calculations. However, they employed the dry adiabatic lapse rate to determine the temperature of the vertical layers rather than employ any dynamic transport. The dry lapse rate is not well suited to study of Earth's present atmosphere (Hummel and Kuhn 1981, Stone and Carlson 1979), in which observed lapse rates lie closer to the moist adiabat. Its application to the study of the atmosphere of early Mars is similarly questionable when one examines CO₂ levels in excess of several hundred kPa, since the consequent higher temperatures permit increased atmospheric water vapor contents.

Most recently, an SEBM was used by François et al. (1990) to examine the conditions under which permanent CO₂ polar caps could exist, and they included in their model consideration of reduced solar luminosity, high orbital obliquity and increased CO₂ levels. However, the CO₂ levels considered only reached as high as 0.5 bars, and consequently this study did not venture into temperature domains where liquid water activity can be examined.
These previous studies have tended to be limited in the cases which they examined. This study will examine a wider range of possible climates resulting from the various carbon dioxide levels and orbital elements which may have existed for early Mars.
II. THE MODEL

This chapter describes a multi-layer energy balance model (MLEBM) which has been adapted for study of potential atmospheres of early Mars. The model is based on the model described by Peng et al. (1982, 1987), which was originally designed for study of the atmosphere of modern Earth. Additional details about features not fully described in this chapter may be found in the appendices or in the cited papers.

The model treats the atmosphere as a two-dimensional plane stretching from pole to pole and from surface to top. This plane is divided into a network of cells, in each of which a heat balance equation is evaluated. The terms in this equation include the significant heating processes, including radiative transfer and various dynamic transports. The relative importance of the latter terms depends on location, as the poleward boundary of the tropical circulation effectively divides the atmosphere into two regimes. The model also includes heat exchange at the planetary surface and allows for variation in the surface albedo result from temperature change.

Although the purpose of this study is to determine mean annual temperature profiles, the heat balance equation is a function of time. The averages are determined by annually averaging the solar input and letting the model run until the heating rate in each cell is approximately zero. This equilibrium state is considered to have occurred when the global average of the net incoming solar flux and outgoing thermal flux are in balance and the rate
of change in the surface temperature profile has fallen to a small value (less than 0.1 K yr\(^{-1}\)). In a sense, the temperatures obtained are not truly the annual mean in that they do not average together temperatures determined for each day and season of the year. One might instead consider the application of mean solar input as representing a “perpetual spring” or “perpetual autumn.”

A. Grid structure

The two-dimensional atmospheric grid employed in this study is shown in Fig. 1. The planet is evenly divided along a north-south meridian into latitude zones of width \(\Delta \phi = 10^\circ\), for a total of 18 zones. In each of these zones, the model may include land, open ocean and/or ice-covered ocean at the surface. Vertically, the atmosphere is divided into five layers when

![Diagram of the two-dimensional atmospheric grid. The pressure at the surface and at the radiative-convective boundary must be input. The thicknesses of layers two, three, four and five are equal in \(\Delta P\).](image)
evaluating heat transport. Radiative transfer is the most important process in determining the temperature profile in the uppermost layer and dynamic processes dominate in the four lower layers. The boundary between these realms is roughly coincident with the tropopause, and hence the uppermost layer may be considered equivalent to the stratosphere and the lower four equivalent to the troposphere. The four lower layers are also assumed to be equal in pressure differential, \( \Delta P \). The vertical layers are sub-divided during radiative transfer calculations, as described below in §2.C.1. The independent vertical coordinate is atmospheric pressure, and the pressure at the upper limit of the model is 100 Pa (1.0 mbar).

One drawback to the atmospheric model described in this thesis is that the altitude of the level lying between the radiative and convective (dynamic) regions, a point lying slightly below the tropopause, is not interactively predicted but must be specified by the user. In the bulk of the cases presented in this thesis, I have specified that the pressure at this boundary be equal to one-fifth the surface pressure. This approximation is close to the actual value for lower surface pressures but is too high by about a factor of two for the higher surface pressures studied here. The effect of this is a slight heightening in surface temperatures, as will be discussed below.

B. Parameters

1. Surface albedos

As mentioned in the introduction, an important reason for constructing a latitude-dependent atmospheric model is to allow for albedo variations along a meridian, whether as a result of variations in surface temperature or
surface type. As is described in the next chapter, I have specified the land albedo in the present study by two different methods. In the first, I specify a single value for all zones; in the second a zonal distribution is specified. As a result, the varying size of possible polar caps and of their effect on the climate has been omitted from this study.

The albedos for the oceanic surface types are calculated according to the parameterization described by Peng et al. (1987). For open ocean, the albedo is dependent upon atmospheric conditions and the angle of the sun. For clear conditions, this albedo is taken to be

$$ \alpha_{\text{sea}} = \frac{0.055}{\cos \phi} , \quad (2.01) $$

with a maximum value of 0.54. The albedo of sea ice is a function of surface temperature and ranges from 0.45 to 0.70 according to

$$ \alpha_{\text{ice}} = \begin{cases} 
0.70 & T_I \leq 263 \, \text{K} \\
0.70 + 0.025 \, \text{K}^{-1} \, (T_I - 273 \, \text{K}) & 263 \, \text{K} < T_I < 273 \, \text{K} \\
0.45 & 273 \, \text{K} \leq T_I 
\end{cases} , \quad (2.02) $$

where $T_I$ is the zonal sea ice surface temperature.

2. Water vapor profile

The prescribed tropospheric vertical relative humidity profile for a latitude zone is determined according to the parameterization of Manabe and Wetherald (1967), which was derived to fit mean annual, hemispherically averaged data for Earth and which has since been used in numerous radia-
tive-convective climate models. Thus, the relative humidity (percent of saturation) is specified by

\[ r_h \equiv \frac{q}{q_{sat}} = r_{\text{hs}} \left[ \frac{(P/P_s) - 0.02}{1 - 0.02} \right]^\Omega, \]  

(2.03)

in which \( q \) is the specific humidity (the water vapor mass mixing ratio), \( q_{sat} \) is the specific humidity for saturation, \( r_{\text{hs}} \) is the surface relative humidity, \( P \) is pressure and \( P_s \) is surface pressure. The form of the exponential parameter \( \Omega \) has varied depending on the modeler and has been set equal to unity (Manabe and Wetherald 1967), made a function of surface temperature (Cess 1976) and made a function of pressure (Kasting and Ackerman 1986). I have taken the simplest approach, setting \( \Omega = 1 \), since the difference between mean surface temperatures obtained using this choice and those obtained using the more complex parameterizations is small.

Typically set at values near 0.77 in one-dimensional climate models, the surface relative humidity in this study is determined by averaging the relative humidities over land, sea ice and open ocean in each latitude zone; these values are specified to be 0.68, 0.78 and 0.84, respectively. The specific humidity is then readily determined, as the saturation value is

\[ q_{sat} = \frac{\nu \ P_{v, sat}}{\mu_{\text{ave}} \ P} = \frac{\nu \ P_{v, sat}}{\mu_{\text{dry}} \ P}, \]  

(2.04)

where \( \nu, \mu_{\text{ave}} \) and \( \mu_{\text{dry}} \) are the molecular masses of water, the average atmospheric gas and the dry component of the atmosphere, respectively, and \( P_{v, sat} \) is the water vapor partial pressure for saturation. The latter value is calculated from the Clausius-Clapeyron equation, which relates the slope of the saturation pressure curve to the temperature:
$$\frac{dP_{v, sat}}{P_{v, sat}} = \frac{L}{R_v} \frac{dT}{T^2}.$$  \hspace{1cm} (2.05)

Here \( L \) is the latent heat of either vaporization or sublimation, and \( R_v \) is the gas constant for water vapor.

To determine the water vapor content above the troposphere, I have adopted a procedure described by Visconti (1982). Here, the water vapor mixing ratio is set equal to either the saturation mixing ratio or to the mixing ratio at the radiative-convective transition level, whichever value is smallest.

3. Clouds

Because the treatment of clouds in the various hierarchies of climate models still provokes much debate in studies of the modern Earth atmosphere and because their presence would have added an extra degree of complexity (with its accompanying increase in computation time) to this model, I have adopted the approach of several previous climate modelers (e.g., Lindzen et al. 1982, Kasting et al. 1984a) by omitting clouds entirely. One would expect that as heightened CO\(_2\) levels provoke surface temperature increases and expansion of the troposphere that the overall cloudiness would also increase. However, deciding the altitude at which the clouds would occur, their extent and their optical properties is a complex matter which I felt was better left outside the scope of this study. The implications of this decision are discussed below.
C. Heating mechanisms

As was mentioned previously, each latitude zone of the planetary surface is sub-divided into three regions: land, open sea and sea ice. A surface temperature is calculated for each of these regions, and the zonal surface temperature is determined by averaging the results. The formulae for determining heating rates at the surfaces are

\[
C_L D_L \frac{\partial T_L}{\partial t} = F_{S,L} + F_{T,L} + F_{SH,L} + F_{LH,L} \tag{2.06}
\]

\[
C_W D_W \frac{\partial T_W}{\partial t} = F_{S,W} + F_{T,W} + F_{SH,W} + F_{LH,W} + COF - F_{WI} \tag{2.07}
\]

\[
\frac{1}{2} C_I D_I \frac{\partial T_I}{\partial t} = F_{S,I} + F_{T,I} + F_{SH,I} + F_{LH,I} + F_{IC} + F_{WI} \left( \frac{1 - A_I}{A_I} \right) \tag{2.08}
\]

where the subscripts \( L, W \) and \( I \) indicate land, open ocean and sea ice, respectively. The heat capacity of a surface type is \( C \), and its effective depth is \( D \). Net downward heat fluxes (per unit area) at the surface are those of solar radiation, \( F_S \); thermal radiation, \( F_T \); sensible heat, \( F_{SH} \); and latent heat, \( F_{LH} \). Other flux terms are the upward heat conduction into the bottom of the sea ice layer from the water below, \( F_{IC} \); the heat loss (per unit area) of open sea from lateral heat transport into the sea ice region, \( F_{WI} \); and the horizontal convergence of heat flux in the open ocean by current systems, \( COF \). In a latitude zone, the fraction of ocean covered by sea ice is \( A_I \). The factor of one-half on the left-hand-side of equation (2.08) is included because the bottom of the ice layer is held fixed at the temperature of the water below it, which can not fall below the freezing point. In Earth's oceans, upwelling from the deeps also contributes to the heating of the surface mixed layer, but I have omitted this
potential heating source in this study due to the uncertainties with the possible depth and cover of any potential ocean on early Mars.

The heating rate in the tropospheric cells of the atmospheric grid is calculated from

\[ C_p \frac{\partial T}{\partial t} = H_S + H_T + H_{RLH} + H_{LSE} + H_{SSE} + H_{MMC}, \] (2.09)

where \( C_p \) is the atmospheric heat capacity and \( H_x \) is the heat convergence resulting from any process \( x \) heating or cooling the cell. The subscripts \( S, T, RLH, LSE, SSE \) and \( MMC \) indicating the various processes respectively represent solar radiation, thermal radiation, latent heat release, large-scale eddies, small-scale eddies and mean meridional circulation. It should be noted that the heating rates resulting from large- and small-scale eddies are only applied outside the tropics and the mean meridional circulation (Hadley transport) only inside the tropics.

Equation (2.09) is solved only for zonally-averaged air temperatures, rather than for air temperatures over each of the three surface types. Peng et al. (1987) note that this is equivalent to assuming an infinite coupling coefficient between the tropospheric air masses over the three surfaces but that the effect is not serious. Additionally, the possible error, particularly that in determining surface fluxes, is reduced by interpolating air temperatures over the surface type when determining surface turbulent fluxes (see §II.C.5).

Heating rates are also calculated in the stratosphere, that part of the atmosphere which lies above the troposphere and which is approximately in radiative equilibrium. Dynamic heating processes do occur in this region, and
Peng et al. (1987) described a scheme for approximating them which is dependent on observed temperature gradients for modern Earth. Desiring not to employ such a planet-dependent scheme and noting that changes in the stratospheric temperature profile have little effect on surface temperatures, I have omitted large-scale stratospheric heat transport in the present study. Consequently, radiative flux is the only heating source considered in the stratosphere.

Instead of direct formulations for the heating rates $H_x$ included in equation (2.09), several of the following sections present methods for determining meridional and vertical heat fluxes, $[Tv]$ and $[T\omega]$ respectively. (Here $v$ is the meridional velocity and $\omega$ the vertical. The latter is expressed in isobaric coordinates). For a process $x$, the local heating rate may be determined from these fluxes according to

$$H_x = -\frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left\{ [Tv]_x \cos \phi \right\} - \frac{\partial}{\partial P} [T\omega]_x \, ,$$

(2.10)

in which $a$ is the planetary radius and $\phi$ is the latitude.

1. Radiative fluxes

To allow for more accurate flux determination, the number of layers in the vertical grid structure of the model is increased during the radiative transfer calculations, as is shown in Fig. 2. The greatest amount of subdivision occurs in the stratosphere, where radiative heating is the most important process and where the rate of change in the atmospheric pressure is greatest. For thermal (or IR) flux calculations, the layers of the primary grid are subdivided into 20 layers: twelve in the stratosphere and eight in the troposphere. The
thermal spectrum from 2.23 μm longwards is divided into 38 bands, in which absorption by H₂O, CO₂ and O₃ may be calculated (Kasting et al. 1984a,b), although the latter gas is only included when the model is calibrated against modern Earth conditions. The thermal flux at each vertical level is determined by numerically evaluating the integral equation of transfer (Kasting et al. 1984a, Schmunk 1990).

For solar flux calculations, the secondary grid has ten layers: six in the stratosphere and four in the troposphere. Solar flux wavelengths from 0.25 μm to 4.0 μm are represented by 26 spectral bands. Absorption by O₂, H₂O, CO₂ and O₃ are included (Haberle et al. 1985), although O₂ and O₃ are omitted when the model is applied to early Mars. Rayleigh scattering is also calculated (Liou 1980) and allowance is made for the fact that pure CO₂ scatters more efficiently than the dry gas of the modern Earth atmosphere. The delta-two-stream method is used to solve the differential form of the equation of trans-

![Diagram of vertical layers for radiative flux calculations]

**FIG. 2.** Subdivision of the vertical layers for radiative flux calculations.
fer (Toon et al. 1989, Schmunk 1990). The incident solar flux is specified, and the meridional profile of the solar zenith angle may be determined from the elements of the planetary orbit, as described in Appendix A.

2. Large-scale eddies

In grid cells outside the tropics, there are two dynamic heat fluxes to be considered, those of large- and small-scale eddy fluxes. It is the former of these which is responsible for meridional heat transport in the extratropics and it results from longitudinal baroclinic eddies. A baroclinic atmosphere is one in which horizontal temperature gradients cause the pressure to vary along a surface of constant density, and vice-versa. Evaluating the circulation in such a region may be performed using a quasi-geostrophic two-parameter, or two-layer, baroclinic model (e.g., Holton 1979). However, Stone (1974) demonstrated that the fluxes resulting from these eddies may be approximated with two-dimensional diffusion-like formulae. According to his approach, the horizontal and vertical transport of heat and specific humidity by baroclinic eddies may be written

\[ [\theta v]_{LSE} = -K_{LSE} \left( \frac{\partial \theta}{\partial y} + \gamma \frac{\partial P}{\partial P} \right), \]

(2.11a)

\[ [\theta \omega]_{LSE} = -K_{LSE} \left( \gamma \frac{\partial \theta}{\partial y} + \gamma^2 \frac{\partial \theta}{\partial P} \right) = \gamma [\theta v]_{LSE} , \]

(2.11b)

\[ [q v]_{LSE} = -K_{LSE} \left( \frac{\partial q}{\partial y} + \gamma \frac{\partial q}{\partial P} \right), \]

(2.12a)

\[ [q \omega]_{LSE} = -K_{LSE} \left( \gamma \frac{\partial q}{\partial y} + \gamma^2 \frac{\partial q}{\partial P} \right) = \gamma [q v]_{LSE} , \]

(2.12b)
where \( y = a \phi \) and \( \theta \) is the potential temperature (see Appendix D for a discussion of the relationship between \( \theta \) and \( T \)). \( K_{LSE} \) is an eddy diffusion coefficient and \( \gamma \) is the mean slope of the eddy mixing surface in the \( y-P \) plane. Values for the eddy diffusivity and slope are determined from the properties of the atmosphere. In describing this method, Stone (1974) assumed a constant Coriolis parameter \( f \) and then made a correction to \( \gamma \) for the effect of non-zero \( \partial f / \partial \phi \). Peng et al. (1982) have modified the correction term to obtain for the eddy diffusivity and slope:

\[
K_{LSE} = 0.14368 \frac{R^{3/2}}{f_0^2} \left( \frac{-P}{\partial P} \right)^{1/2} \left| \frac{\partial \theta}{\partial y} \right| [1 - 2.5 h(P)]^{-1},
\]

(2.13)

\[
\gamma = -8.526 \left( \frac{k^2}{\sqrt{2} + k^2} \right) \frac{\partial \theta}{\partial y} \left( \frac{\partial \theta}{\partial P} \right)^{-1} h(P),
\]

(2.14)

in which

\[
k^2 = \sqrt{2} \left( \sqrt{2 + 4.743 \alpha^2 + 1.257 \alpha^4} - 1 - 1.121 \alpha^2 \right),
\]

(2.15)

\[
\alpha \equiv \cot \phi \left( -\frac{P}{\partial P} \right) \left( \frac{\partial \theta}{\partial \phi} \right)^{-1},
\]

(2.16)

\[
h(P) \equiv \frac{z(P)}{z_t} \left\{ 1 - \frac{z(P)}{z_t} \right\},
\]

(2.17)

where \( z_t \) is the tropopause altitude.

3. Small-scale eddies

The other dynamic flux outside the tropics is that of small-scale eddies, which are approximated as a vertical diffusion process according to the method presented in Peng et al. (1987). The heat flux is
\[ [T \omega]_{SSE} = - \rho g K_{SSE} \{ \rho g \frac{\partial T}{\partial P} - (\Gamma_d - \Gamma_p) \} , \]  

(2.18)

where \( K_{SSE} \) is an eddy diffusion coefficient, here set equal to a constant value of 1.0 m\(^2\) s\(^{-1}\). The dry adiabatic lapse rate, denoted \( \Gamma_d \), is equal to \( g / C_p \). The counter-gradient factor, \( \Gamma_c \), is a correction term inserted so that a small-scale process may be evaluated on a large grid and is here defined as

\[ \Gamma_c = r_h (\Gamma_d - \Gamma_w) , \]  

(2.19)

in which \( r_h \) is the relative humidity and \( \Gamma_w \) is the moist adiabatic lapse rate. This latter term may be determined by noting that

\[ \Gamma_w = - \frac{dT}{dz} \bigg|_{moist} = \rho g \frac{dT}{dP} \bigg|_{moist} , \]  

(2.20)

and by applying an equation given by Kasting (1988) for the variation of pressure with temperature under saturated conditions:

\[ \frac{T \, dP}{P \, dT} \bigg|_{moist} = \frac{L}{R_v T} - \frac{\varepsilon L}{T} \frac{C_p d - q_{sat} \left( C_e - \frac{L}{T} + \frac{dL}{dT} \right)}{1 + \frac{q_{sat}}{\varepsilon}} \left( L \frac{q_{sat}}{T} + R_d \right) . \]  

(2.21)

in which \( C_c \) is the specific heat of water condensate, \( R_d \) is the gas constant for dry air and \( \varepsilon = \mu_v / \mu_{dry} \). In presenting the moist adiabatic lapse rate in this form, I have assumed that there is no condensed water present and that the approximation of equation (2.04) is still valid, or \( \mu_{ave} = \mu_{dry} \). (For temperatures below 300 K, the error in the latter assumption is less than 4%.)

Moisture flux also results from the small-scale eddies and is calculated from
\[ (q_w)_{SSE} = \rho^2 g^2 K_{SSE} \frac{\partial q}{\partial P} \]  

(2.22)

4. Mean meridional (Hadley) transport

Within the tropic atmosphere, the only dynamic transport mechanism to be considered is that of the mean meridional circulation, which occurs in the two Hadley cells lying in the low latitudes on each side of the equator. I follow the approach of Peng et al. (1987), which avoids a complex parameterization of the Hadley transport by assuming that vertical temperature profile in the tropics will fit a pre-determined moist adiabatic lapse rate, which Stone and Carlson (1979) have noted is very close to observed annually-averaged lapse rates in the tropics on modern Earth. With this information in hand, the actual temperature profile may be determined once the horizontal heat transport in the tropical zones is known. However, Peng et al. (1987) provide a parameterization for mean meridional transport reliant upon a number of derived constants. As these values are appropriate only for modern Earth, I have returned to a scheme described by Held and Hou (1980) and Hou (1984) in order to simulate this flux for a different set of planetary characteristics.

The first step is to determine the poleward boundary of the Hadley cell in each hemisphere. Held and Hou (1980) locate this boundary by evaluating the basic atmospheric equations for a statically-stable Boussinesq fluid (a fluid in which density is treated as a constant except in the buoyancy term of the vertical momentum equation) confined between the surface and the tropopause. Flow is forced by radiative heating, and linear diffusion of heat and momentum is the only small-scale mixing. Stress is assumed to be zero at the top and proportional to wind-speed at the surface. In the limit that the
viscosity goes to zero, such that viscous stresses are so small that angular momentum is conserved within poleward flow, the Hadley transport boundary is identified as the latitude at which the meridional velocity goes to zero. This latitude, denoted $\phi_H$, was found by Held and Hou (1980) to solve the relation

$$\frac{1}{3} (4R - 1) \psi_H^3 - \frac{\psi_H^5}{1 - \psi_H^2} - \psi_H + \frac{1}{2} \ln \left[ \frac{1 + \psi_H}{1 - \psi_H} \right] = 0 \quad , \quad (2.23)$$

in which $\psi \equiv \sin \phi$ and $R$ is a "thermal Rossby number":

$$R = \frac{g z_t \Delta H}{\Omega^2 a^2} . \quad (2.24)$$

Here, $\Delta H$ is the fractional change from equator to pole in the vertically integrated (surface to tropopause) potential temperature and $\Omega$ is the planetary angular velocity. Applying equations (2.23) and (2.24) as they stand predicts a Hadley boundary for modern Earth which is several degrees too low. I compensate for this by doubling $\Delta H$ in (2.24), noting that Held and Hou (1980) applied a constant value for $\Delta H$ which is about twice the actual value. I also calculate $z_t$ rather than apply a constant value and consequently obtain $\phi_H \approx 35^\circ$, close to the observed Earth annual average.

Within the boundaries imposed by this approximation, Hou (1984) also demonstrates that the vertically-averaged meridional heat flux may be determined from

$$[\bar{\theta} \bar{v}]_{MMC} = \bar{\theta}_E \left( \frac{a \Delta H}{\tau} \right) (1 - \psi^2)^{1/2} F(\psi) \quad , \quad (2.25)$$
where $\bar{\theta}_E$ is a horizontally and vertically-averaged potential "radiative-equilibrium" temperature, $\tau$ is a radiative damping time and the function of latitude is defined

$$F(\psi) = \frac{1}{3} \left( 1 + \frac{1}{2R} \right) \frac{\psi}{\psi_H} \left[ 1 - \left( \frac{\psi}{\psi_H} \right)^2 \right] \psi_H^3 + \frac{1}{4R} \left[ \ln \left( \frac{1+y}{1-y} \right) - \frac{\psi}{\psi_H} \ln \left( \frac{1+\psi_H}{1-\psi_H} \right) \right].$$

(2.26)

In order to reduce the number of necessary calculations, I assume in evaluating equation (2.25) that $\theta_E = \theta$, as did Peng et al. (1987). According to Houghton (1986), the damping time for a slab of thickness $z_l$ and density $\rho$ is

$$\tau = \frac{C_p \rho z_l}{8 \sigma T^3},$$

(2.27)

in which $\sigma$ is the Stefan-Boltzmann constant.

In their seasonally-varying model, Peng et al. (1987) altered the formulation for $[\bar{\theta} \bar{v}]_{MMC}$ of Held and Hou (1980) to allow for movement of the intertropical convergence zone (ITCZ), the location where the north and south Hadley cells meet, as a result of variation in the subsolar latitude. Since all cases presented in this study are for annual averages, I effectively set the ITCZ at the equator by leaving the latitude dependence described by Hou (1984) unmodified.

Similar to the heat flux equation (2.25), the vertically-averaged meridional water vapor flux is calculated from

$$[\bar{q} \bar{v}]_{MMC} = \bar{q} \left( \frac{a \Delta_H}{\tau \Delta_V} \right) (1 - \psi^2)^{1/2} F(\psi).$$

(2.28)
where \( \bar{q} \) is the vertically-averaged value of the specific humidity and \( \Delta \nu \) is the fractional change in \( \theta \) from surface to tropopause.

Once the meridional transports have been determined, the vertically-integrated potential temperature in a tropical zone is calculated from

\[
\Sigma_\nu(\theta(t+\Delta t)) = \Sigma_\nu(\theta^*) - \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \left\{ \cos \phi \left( \Sigma_\nu(\theta \nu)_{MMC} \right) \right\} \Delta t 
\]

\[
= \Sigma_\nu(\theta^*) - \frac{4}{a \cos \phi} \frac{\partial}{\partial \phi} \left\{ \cos \phi \left[ \theta \nu \right]_{MMC} \right\} \Delta t ,
\]  
(2.29)

in which \( \theta^* \) is the grid cell's potential temperature from the previous time-step, plus the effect of all applicable heating other than the Hadley transport. The function \( \Sigma_\nu \) is shorthand for the vertical sum within the tropospheric layers; i.e.,

\[
\Sigma_\nu(x) \equiv \sum_{j=2}^{5} x_j .
\]  
(2.30)

As mentioned above, the vertical potential temperature profile may be determined from (2.29) if a lapse rate profile is already known. If \( \theta_j \) is the potential temperature at the middle of layer \( j \) (see Fig. 2 for a description of layer geometry), the relationships between the lapse rates and the layer potential temperatures are

\[
\gamma_{23} = \frac{\theta_3 - \theta_2}{P_3 - P_2} ,
\]  
(2.31a)

\[
\gamma_{34} = \frac{\theta_4 - \theta_3}{P_4 - P_3} ,
\]  
(2.31b)

\[
\gamma_{45} = \frac{\theta_5 - \theta_4}{P_5 - P_4} .
\]  
(2.31c)
The four unknowns \( \theta_2, \theta_3, \theta_4, \) and \( \theta_5 \) are thus easily calculated from the equations (2.29) and (2.31) once the lapse rates are supplied. In their presentation of equations (2.31), Peng et al. (1987) applied the parameterization suggested by Rennick (1977), in which the lapse rates are expressed as polynomial functions of surface temperature and are fitted to observed tropical temperatures. Because the surface temperatures and pressures considered in this study generally differ by large amounts from those considered by Peng et al. (1987), I have instead used the moist adiabatic lapse rate formula as presented in equations (2.20) and (2.21) to calculate the lapse rates \( \gamma_{23}, \gamma_{34} \) and \( \gamma_{45} \) in the tropics.

5. Surface turbulent fluxes

Heat transport resulting from surface turbulence is calculated in each latitude zone using the formulae provided by Peng et al. (1987). The surface sensible heat flux is determined from

\[
F_{SH,x} = C_p \rho K_{turb,x} (\Gamma_x - \Gamma_d + \Gamma_c)
\]

where the subscript \( x \) indicates the surface type (land, sea ice or open ocean), \( K_{turb,x} \) is an eddy diffusion coefficient, \( \rho \) is the density of air, \( \Gamma_x \) is the lapse rate near the surface, \( \Gamma_d \) is the dry adiabatic lapse rate and \( \Gamma_c \) is a counter-gradient factor, again set equal to the relative humidity times the difference between the dry and moist adiabatic lapse rates. Determining the various values for \( \Gamma_x \) requires knowledge of the air temperatures above the land surface, but as was noted above, the model does not calculate air temperatures over each surface type. Consequently a parameterization is made in which it is assumed the difference between the temperatures over a surface type and the mean temperature profile decreases at a rate such that at the middle of the bottom-most
layer of the atmosphere, the difference is $e^{-1/3}$ times the difference at the surface, or

$$T_{x,5} = T_5 + (T_{x,s} - T_5) e^{-1/3} ,$$

(2.33)

in which the subscript 5 indicates the middle of layer five, the bottom layer (see Fig. 2). Given $T_{x,s}$, $T_{x,5}$ and the altitude at the middle of the bottom layer the lapse rate is easily calculated.

The diffusion coefficient required by equation (2.32) is determined from

$$K_{turb,x} = K_{turb,0x} \frac{v(\phi)}{v(0)} + \max(\Gamma_x,0) \times 800 \text{ m}^3 \text{ s}^{-1} \text{ K}^{-1} ,$$

(2.34)

in which $v$ is a zonal mean surface wind speed, for which I have applied observed modern Earth values extracted by Peng et al. (1987). Values specified for the coefficient $K_{turb,0x}$ are 4.2 m$^2$ s$^{-1}$ for land, 2.0 m$^2$ s$^{-1}$ for open ocean, and 1.4 m$^2$ s$^{-1}$ for sea ice.

The surface latent heat flux is determined from

$$F_{LH,x} = L \ W_x \ p \ K_{turb,x} \frac{\partial q}{\partial z} ,$$

(2.35)

where $W_x$ is a surface "water availability" parameter and is set equal to 0.6 for land, 0.2 for sea ice and 1.0 for open ocean.

6. Oceanic heat transport

Due to the many uncertainties about the potential amount of liquid water on the surface of early Mars, an attempt to realistically determine the rate of heat transport by oceanic current systems would be an unnecessary
complication. In the cases where open ocean is present, I have thus employed a simple, conduction-like formula similar to that described by Temkin and Snell (1976) and Lee and Snell (1977). The total northward heat flow across a latitude circle is given by

\[ J = -k_{ocn} \frac{\pi \cos \phi}{\Omega} \int_{\sin \phi} f_W (1 - f_W) (1 - A_I) D_W \frac{\partial T_W}{\partial \phi}, \]  

(2.36)

where \( k_{ocn} \) is a constant coefficient (set equal to \( 1.88 \times 10^7 \) J K\(^{-1}\) m\(^{-1}\) s\(^{-2}\)), \( \Omega \) is the planetary angular velocity, \( f_W \) is the portion of the latitude circle \( \phi \) which is ocean, \( A_I \) is the fraction of the ocean amount which is ice-covered and \( D_W \) is the "effective" depth of the oceanic mixing layer, specified as 100 m throughout this study. Because equation (2.36) renders an infinite value at the equator \((\phi = 0)\), oceanic heat transport at that latitude is instead taken as the average of the values calculated for the two latitudes immediately north and south of the equator \((\phi = \pm 10^\circ)\). Additional information about the derivation of equation (2.36) may be found in Appendix B.

Once oceanic heat transport values have been determined, their convergence in a unit area of the oceanic mixing layer may be calculated from

\[ COF = \left\{ 2 \pi a^2 f_W (1 - A_I) \cos \phi \right\}^{-1} \frac{\partial J}{\partial \phi}. \]  

(2.37)

7. Sea ice effects

The scheme by which sea ice thickness and coverage are calculated is taken directly from Peng et al. (1987) and is described in detail in Appendix C. This approach is entirely thermodynamic and ignores dynamic processes such as drifting and precipitation onto the ice. The thickness of the ice layer is
affected by surface heating and by conduction between its top and bottom, the latter being held fixed at the freezing point of sea water, taken in this model as 271.2 K. Net warming at the top leads to ice-top melting, while cooling lowers the surface temperature. Net downwards conduction through the ice causes melting on the bottom face while net upwards conduction causes accretion. Zonal ice cover decreases when the adjacent open water is warmed and heat flows from the water to the ice; it increases when the open water cools below the freezing point. When the ice cover changes, the open water and sea ice surface temperatures calculated from (2.07) and (2.08) are altered to account for averaging the temperatures of the new water and new ice with those of the old.

8. Latent heat release

The release of latent heat created by condensation of water vapor in the troposphere is determined by assuming that all excess water in a zone precipitates. This excess is defined as the zonal convergence of water vapor due to all dynamic transports less the change in the water vapor content as prescribed according to the procedure in §II.B.2. The basic precipitation rate in a zone is thus

\[ Pr_\alpha = \int_{P_t}^{P_s} Dq \, dP - \int_{P_t}^{P_s} \Delta q \, dP, \quad (2.38) \]

where

\[ Dq = \frac{1}{a \cos \phi} \frac{\partial}{\partial \phi} \{ [q\nu] \cos \phi \} + \frac{\partial}{\partial P} [q\omega], \quad (2.39) \]
and \( \Delta q \) is the change in the prescribed specific humidity. The \([qv]\) and \([qw]\) terms are determined by summing all moisture fluxes appropriate to the zone and layer, as given by equations (2.12), (2.22), (2.28) and (2.35). Since there may be zones in which the basic precipitation rate \( Pr_\alpha \) is less than zero, moisture is drawn from other zones to account for the loss and precipitation rates in the other zones correspondingly adjusted. Consequently, in each hemisphere, I determine adjusted zonal precipitation rates from

\[
Pr_{\beta,\pm i} = \max(0, Adj_{\pm i} Pr_{\alpha,\pm i}) ,
\]

(2.40)

where

\[
Adj_{\pm i} = \left( \sum_{i=1}^{9} Pr_{\alpha,\pm i} \cos \phi_i \right) \left( \sum_{i=1}^{9} \max(0, Pr_{\alpha,\pm i}) \cos \phi_i \right)^{-1} .
\]

(2.41)

Heating in layer \( j \) due to latent heat released by precipitation is then found from

\[
H_{RLH,j} = \eta_j \frac{L}{C_p} Pr_\beta ,
\]

(2.42)

where for simplicity's sake a relative distribution, \( \eta \), of latent heat release is specified. The values assigned to the four tropospheric layers are \( \eta_2=0, \eta_3=0.5, \eta_4=0.45 \) and \( \eta_5=0.05 \).
III. RESULTS

To apply the multi-layer energy balance model to the possible atmospheres of early Mars, I made a number of assumptions. The first was that the atmosphere may be treated solely as a mixture of carbon dioxide, nitrogen and water vapor. The amount of CO₂ is specified for the individual case being examined, and the water vapor profile is determined according to equation (2.07). Following Durham and Chamberlain (1989), I have fixed the amount of N₂ throughout this study at a partial pressure of 14.4 kPa (0.144 bars). Since nitrogen is not a radiatively active gas, the accuracy of the latter estimate is of little consequence.

In calculating the solar zenith angle, there are three orbital parameters which must be considered: obliquity, eccentricity, and argument of perihelion. The argument of perihelion may, of course, take any value between \( \omega = \pm 180^\circ \), and for early Mars, the obliquity may have been as large as \( \Theta = 51.4^\circ \) (Bills 1990) and the eccentricity as high as \( e = 0.141 \) (Ward 1974), resulting in large variations of the zenith angle along a meridian. However, the most interesting studies of this effect are those of the seasonal cycle of the climate. In this annually averaged study, this effect is limited to a hemispheric asymmetry in the zenith angles for a particular latitude. For example, the combination of \( e = 0.141, \Theta = 50^\circ \) and \( \omega = \pm 45^\circ \) results in a 9.4% difference between the cosines of the zenith angle at \( \pm 85^\circ \). Since simulations with hemispherical symmetry require less computation time and in order to avoid examining the multiplicity of possible combinations of the three orbital parameters, I have
limited this study to cases in which the zenith angles for a particular latitude are the same in both hemispheres, which occur when \( \omega = 0^\circ \) or \( 180^\circ \). There remains a slight variation in the cosine of the zenith angle as a result of the eccentricity, but this effect is small, less than a 0.5% difference at the poles when \( e = 0 \) and \( e = 0.141 \) are compared. Consequently, I have also specified that \( e = 0 \) throughout this study. Cosines of the zenith angles which result from these two simplifications are presented in Table I.

The final assumption which I made was that the solar luminosity was about 25% to 30% lower 4 Gyr ago than it is today (e.g., Newman and Rood 1977). For Mars, the annual mean of the solar "constant" would thus have been about 460 W m\(^{-2} \) (\( \approx 78\% \times 586 \) W m\(^{-2} \)). When Mars is at perihelion during periods of maximum eccentricity, the incoming solar flux attains values 38% greater than this annual mean. (In combination, these values

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</tr>
<tr>
<td>85° N/S</td>
<td>0.0870</td>
</tr>
</tbody>
</table>

**Table I.** Zonal values for the cosine of the solar zenith angle for \( \omega = 0^\circ \) and \( e = 0 \).
result in an early solar flux at perihelion about 8% greater than the annual mean of the modern solar flux.) In order to facilitate comparison between the results of this study and those of studies employing radiative-convective models (RCMs), I include here a number of simulations in which this higher flux value is prescribed. However, the meridional surface temperature profiles obtained for these cases should be viewed with caution since zonal mean annual zenith angles are still applied. (This is not important with RCMs since only a single zenith angle, as opposed to a meridional profile, must be applied in those models.) These cases are labeled with the term “perihelion”.

Several different series of simulations are now presented, the difference between them being the manner in which the surface is described.

A. Case A: no body of water, land albedo = 0.24

The first series of simulations which I performed included greatly simplified surface characteristics. First, no body of water was present, thus negating the need to calculate the various terms in the open sea and sea ice heat balance equations (2.07) and (2.08). Additionally, the albedo of the land surface was set equal to 0.24 in all latitude zones. This last specification of course omits the direct effect of any polar ice caps which might exist, but it indirectly includes them by averaging their high albedos with those of the lower latitudes, which are typically less than 0.24. This particular choice of albedo was chosen since it is extremely close to the value used in the RCM employed by Durham and Chamberlain (1989). It is also within a few percent of the observed planetary mean value for the present-day Mars. Since these
simplifications, together with the solar zenith angles described previously, result in hemispherical symmetry of the various inputs to the model, calculations were made for only one hemisphere and their results assumed to be true for the other.

Fig. 3 presents the response of the MLEBM to variations in the partial pressure of carbon dioxide, and for a specified obliquity of $\Theta = 25^\circ$. Also shown are the results of Durham and Chamberlain (1989). Two pairs of curves have been drawn. The lower pair denotes the response of the two models for annual averages of the solar input: zenith angles and incident flux. The upper pair shows the response for averaged zenith angles but for the “perihelion” flux. Besides the MLEBM’s use of mean annual zenith angles, the “perihelion” set should be viewed with a bit of skepticism since both MLEBM and RCM have been run until equilibrium conditions were achieved, whereas perihelion is only a transitory condition.

Nevertheless, the results of the two models are in fairly close agreement. Both models achieve temperatures in excess of 273 K at about 400 kPa for the mean case and near 200 kPa for the “perihelion” case. This agreement is expected since the same radiative transfer code is used by both this model and Durham and Chamberlain (1989), and the planetary albedos (the total planetary reflectivity) which they calculate differ by no more than 2% for the pressures examined, rising for the mean flux case from about 0.37 for $P(\text{CO}_2) = 100$ kPa to 0.48 for $P(\text{CO}_2) = 500$ kPa. Discrepancies between them are the result of a number of small differences between the models, including the spacing of their vertical pressure grids, the parameterization of the vertical water vapor profile, temperature lapse rates outside the tropical
FIG. 3. Response of mean surface temperature to variations in the CO$_2$ partial pressure for case A (no large body of water and a land albedo set to 0.24 in all zones) and for an obliquity of 25°. The curves labeled MLEBM are the results of this study; those labeled RCM are from Durham and Chamberlain (1989).
region (the lapse rate of the convective region in the RCM is constrained to exactly match the moist adiabatic lapse rate), and the altitude of the boundary between the radiative and convective heating regimes. As noted previously, the pressure of the boundary altitude, $P_{RC}$, has been set at one-fifth the surface pressure in the present model, whereas the RCM of Durham and Chamberlain (1989) interactively calculates it. At lower CO$_2$ partial pressures, the difference is quite small but becomes larger at higher surface pressures. Since $P_{RC}$ in the present model is at a lower altitude, the effective radiating altitude is also lower and surface warming results. For example, for $P$(CO$_2$) = 500 kPa, the difference in $P_{RC}$ is about 40 kPa and the present model is warmed by about 5 K. This difference also explains the slightly different shape of the MLEBM and RCM curves in Fig. 3, resulting in the MLEBM curves crossing over to warmer temperatures as $P$(CO$_2$) increases.

Incident solar flux for the "perihelion" case is about 8% more than the solar flux which reaches Mars at the mean radius of its present day orbit, and hence one wonders what the results of the MLEBM would be if applied to that case. For a 1 kPa (slightly more than Mars' actual pressure near 0.7 kPa) atmosphere composed of 95% carbon dioxide and 5% nitrogen and receiving a flux of 586 W m$^{-2}$, the model finds a global mean surface temperature of 216 K, slightly below the observed Martian mean of about 220 K. This agreement seems good, though again a number of variables such as the specified values $P_s$ and $P_{RC}$, the water vapor profile and the omission of warming by the dust in Mars' atmosphere all have some impact on this result.
Of course, the main objective of this study is to obtain horizontal temperature profiles. Such are presented for selected CO\textsubscript{2} levels in Figs. 4 and 5, the former displaying profiles for averaged solar input and the latter for the "perihelion" case. The most notable feature of these two figures is the flattening of the temperature profile as higher CO\textsubscript{2} levels are examined. This is a result of the increasing amounts of atmosphere being more capable of transporting heat horizontally, and may be traced back to the pressure dependence of the diffusion coefficient $K_{LE}$ for large-scale eddies in equation (2.13), which attains values of about $10^5$ m\textsuperscript{2} s\textsuperscript{-1} at the higher pressures. This feature is also observed in the temperature profiles presented by Postawenko and Kuhn (1986) for diffusion coefficients between $10^4$ and $10^5$ m\textsuperscript{2} s\textsuperscript{-1}. One important effect of such shallow profiles is that if the atmosphere of early Mars did indeed contain CO\textsubscript{2} partial pressures above 200 kPa, Fig. 5 indicates that temperatures in excess of the freezing point of water could possibly be obtained over large portions of the planet during perihelion.

It should also be noted that for the lower CO\textsubscript{2} levels presented that surface temperatures in the polar regions may fall below the freezing point of carbon dioxide (approximately 200±10 K for the pressures studied here), and the atmosphere would begin to freeze out, a process which is observed during southern hemispherical winter for modern Mars. Even at temperatures above this point, carbon dioxide condensation (the formation of CO\textsubscript{2} rain, perhaps) would occur. The results of such activity are not well understood, and it has been suggested (Kasting 1991) that this may negate the theory of a carbon dioxide greenhouse providing a solution to the faint young Sun paradox. Alternatively, the presence of carbon dioxide clouds might serve to
FIG. 4. Horizontal temperature profiles for case A with the annually averaged solar flux. Note that the $P(\text{CO}_2) = 500 \text{ kPa}$ curve is above the freezing point of water except poleward of $60^\circ$, but the $10 \text{ kPa}$ curve is below the freezing point of carbon dioxide poleward of $60^\circ$. 
FIG. 5. Horizontal temperature profiles for case A with "perihelion" flux. Note that the $P(\text{CO}_2) = 300$, 400 and 500 kPa curves are above the freezing point of water at all latitudes and that the 200 kPa curve exceeds freezing equatorward of 25°.
warm the atmosphere, but the characteristics of such clouds are also poorly understood.

Also of some interest are vertical profiles of the temperature, several curves for which are presented in Fig. 6. Two sets of curves are presented, again a set for the mean solar input and a set for "perihelion", and in each profiles for latitudes 15°, 45° and 75° are shown. Note that in all of the profiles that a tropopause has formed at an altitude near \( P_t = 0.03\ P_s \) (or \( \log[P/P_s] = -1.5 \) using the units of Fig. 6) above the radiative-convective boundary \( P_{RC} \). This relationship also occurs in models in which \( P_{RC} \) is interactively determined. The region between these two altitudes is opaque enough to be heated by absorption of radiation but not to such an extent that convection begins to overturn the air.

Heretofar, all results presented have been for cases of an obliquity of 25°, the mid-point of the obliquities which Mars would have experienced during its early history. One would expect that the surface temperature profile would vary for other applied values, and such is demonstrated for the case of \( P(\text{CO}_2) = 200\ \text{kPa} \) in Fig. 7. The difference between the zenith angles near the equator and near the poles is greatest for the smaller obliquities, and consequently the temperature profile for the \( \Theta = 5° \) case shows the most variation and the \( \Theta = 45° \) curve shows the least. However, it must be noted that this result for an obliquity of 45°, and to a lesser extent for 35°, is seriously endangered by the application of annually averaged solar zenith angles and fluxes. When Mars is at its maximum obliquity (about 51°), the annual average of the solar flux at the poles will equal the average of the flux at the equator, but unlike the lower latitudes, the poles receive this radiation during
FIG. 6. Vertical temperature profiles for selected latitudes for case A and a CO$_2$ partial pressure of 300 kPa. The three curves to the left are for the average solar flux, the three to the right for the flux of perihelion.
Fig. 7. Variation in surface temperature profile for case A due to obliquity variations for a 200 kPa atmosphere with mean solar flux. The differential between equator and pole is greatest for the smallest obliquity, reflecting that that case experiences the smallest zenith angles in the equatorial region and the largest zenith angles in the polar regions. See also Table III.
a limited time interval, followed by a period in which they receive little or no solar radiation. The results presented here do not take into account that extreme seasonality, and the obliquity cases of 35° and particularly 45° should not be paid a great deal of attention.

B. Case B: no body of water, land albedo of Postawko and Kuhn (1986)

In the second series of simulation runs, I have again specified a planetary surface which is entirely land. This time, however, I have employed a distribution of the surface albedo as is described by Postawko and Kuhn (1986), the values for which are presented in Table II. (Note: Postawko and Kuhn [1986] list a value of 0.28 for the albedo at 35°. However, the global mean

<table>
<thead>
<tr>
<th>Zone</th>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5° N/S</td>
<td>0.240</td>
<td>0.180</td>
<td>0.180</td>
</tr>
<tr>
<td>15° N/S</td>
<td>0.240</td>
<td>0.190</td>
<td>0.190</td>
</tr>
<tr>
<td>25° N/S</td>
<td>0.240</td>
<td>0.180</td>
<td>0.180</td>
</tr>
<tr>
<td>35° N/S</td>
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<td>0.180</td>
<td>0.180</td>
</tr>
<tr>
<td>45° N/S</td>
<td>0.240</td>
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<td>0.230</td>
</tr>
<tr>
<td>55° N/S</td>
<td>0.240</td>
<td>0.250</td>
<td>0.250</td>
</tr>
<tr>
<td>65° N/S</td>
<td>0.240</td>
<td>0.450</td>
<td>0.450</td>
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<tr>
<td>75° N/S</td>
<td>0.240</td>
<td>0.650</td>
<td>0.650</td>
</tr>
<tr>
<td>85° N/S</td>
<td>0.240</td>
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<td>0.650</td>
</tr>
<tr>
<td>Mean</td>
<td>0.240</td>
<td>0.247</td>
<td>—</td>
</tr>
</tbody>
</table>

Table II. Specified land albedos. Setting $\alpha_L = 0.180$ at 25° for cases B and C is made on the assumption that the 0.28 listed for that latitude by Postawko and Kuhn (1986) is a misprint. No global mean is presented for case C since for that case the land fraction in a zone may be less than one.
albedo would consequently be about 0.26 instead of the 0.25 which they claim. I have assumed that the albedo in this zone should be 0.18.) The global mean value of the albedo for this particular albedo profile is within one percent of the mean applied in case A, and so the response of the model to increases in the carbon dioxide level is very close to the response shown in case A.

This variation in this specified albedo profile serves to explicitly describe an ice cap in the polar regions. But since the albedo is specified, it also means that any possible growth or decay of the ice cap is not allowed to occur. I have adopted this simplistic ice cap description since a more complex, and potentially more accurate, treatment requires either a more fully developed hydrologic cycle than is employed in this model or a parameterization of a process for which no observations are available.

Meridional surface temperature profiles for this surface albedo distribution are shown in Figs. 8 and 9. Again, the flattening of the temperature curve is seen as the pressure increases. In these cases, however, a greater steepening is seen for the lower CO$_2$ levels, this being a response to the increased albedo in the polar regions as compared to the simulations in case A. Again, one must note that in the $P$(CO$_2$) = 10 kPa case, the temperature falls below the freezing point of CO$_2$ for latitudes poleward of 60°.

Fig. 10 further demonstrates the difference between the profiles of case A and case B. At the lower CO$_2$ pressures, there is a crossover of the curves due to the difference in albedos. Case B has the lower albedo, and hence warmer temperatures, near the equator, while case A has the lower albedo and warmer temperatures near the poles. At higher pressures, the increased
FIG. 8. Horizontal temperature profiles for case B (no body of water, with land albedo as specified by Postawko and Kuhn [1986]) with the annually averaged solar flux. The $P(\text{CO}_2) = 500$ kPa curve is above the freezing point of water except poleward of $60^\circ$; the 10 kPa curve is below the freezing point of carbon dioxide poleward of $60^\circ$. The effect of the albedo variation is damped out as denser atmospheres are examined.
FIG. 9. Horizontal temperature profiles for case B with “perihelion” flux. The 300, 400 and 500 kPa curves are above the freezing point of water at all latitudes and that the 200 kPa curve exceeds freezing equatorward of 25°.
FIG. 10. Comparison of surface temperature profiles for cases A and B with average solar flux. Note the crossover at lower pressures due to the varying albedo of case B. For the 500 kPa curves, this effect is obscured by the greater opacity of the atmosphere so that case A is warmer at all latitudes due to its slightly lower mean albedo.
opacity of the atmosphere washes out this effect, and case A is warmer at all latitudes.

The effect of obliquity variations for case B is presented in Fig. 11, and again it is seen that shallower temperature profiles result from increasing obliquities. A comparison of the effect of obliquity variations between cases A and B is shown in Table III. Due to its varied surface albedo profile, the temperature difference between the equatorial and polar zones is seen to be greater for Case B.

C. Case C: boreal ocean of Baker et al. (1991)

The final series of simulations which I conducted includes a number of features of the MLEBM which were omitted in cases A and B. In this series I have included a body of water on the planetary surface, using zonal ocean fractions adapted from Baker et al. (1991), who describe a possible fluvial history of Mars which includes brief, episodic inundations of much of the northern hemisphere between extended periods of minimal surface activity. These values were extracted from sketch maps presented in Baker et al. (1991).

<table>
<thead>
<tr>
<th>Obliquity</th>
<th>Case A</th>
<th>Case B</th>
</tr>
</thead>
<tbody>
<tr>
<td>5°</td>
<td>23.0 K</td>
<td>25.0 K</td>
</tr>
<tr>
<td>15°</td>
<td>22.7 K</td>
<td>23.6 K</td>
</tr>
<tr>
<td>25°</td>
<td>18.8 K</td>
<td>22.9 K</td>
</tr>
<tr>
<td>35°</td>
<td>15.2 K</td>
<td>19.4 K</td>
</tr>
<tr>
<td>45°</td>
<td>10.1 K</td>
<td>17.1 K</td>
</tr>
</tbody>
</table>

TABLE III. Equator (5°) to pole (85°) temperature differential for a 200 kPa atmosphere at selected obliquities.
FIG. 11. Variation in surface temperature profile for case B due to obliquity variations for a 200 kPa atmosphere with mean solar flux. See also Table III.
shown here as Fig. 12, and are listed below in Table IV. These values are plainly not hemispherically symmetric, and consequently this series of simulations calculates heating in both hemispheres and provides pole-to-pole temperature profiles. Land albedos for this case are again those specified by Postawko and Kuhn (1986). Noting from the results of cases A and B that surface temperature profiles are below the freezing point for almost all simulations in which the mean annual value of the reduced solar flux was applied, I restrict the study of the oceanic case to locating the CO₂ levels at which the ocean would be ice covered and ice free for the "perihelion" flux.

The response for this case of the model to variations in the carbon dioxide level is shown in Fig. 13. Also shown is the response of case A to the same input. The curves are vastly different due to the variation of the surface albedo for the watery Mars. For CO₂ levels below about 160 kPa, the boreal ocean freezes over entirely, rendering a global mean surface albedo of 0.359,

<table>
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</tr>
</thead>
<tbody>
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<td>5° S</td>
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<td>25° S</td>
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<td>35° S</td>
<td>0.00</td>
</tr>
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<td>45° S</td>
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<td>55° S</td>
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</tr>
<tr>
<td>65° S</td>
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</tr>
<tr>
<td>75° S</td>
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<td>85° S</td>
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</table>

<table>
<thead>
<tr>
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<th>( f_W )</th>
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</thead>
<tbody>
<tr>
<td>5° N</td>
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<tr>
<td>15° N</td>
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<td>25° N</td>
<td>0.40</td>
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<tr>
<td>35° N</td>
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<tr>
<td>45° N</td>
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<td>55° N</td>
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<td>1.00</td>
</tr>
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<td>85° N</td>
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</tr>
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</table>

**TABLE IV.** Specified ocean fraction for case C, estimated from the boreal ocean study of Baker *et al.* (1991).
FIG. 12. Sketch maps of the Baker et al. (1991) boreal ocean (taken from Nature 352, 589-594). See also Table IV.
FIG. 13. Response of mean surface temperature to variations in the CO₂ partial pressure for case C, including the Baker et al. (1991) ocean fractions with the Postawko and Kuhn (1986) land albedos. “Perihelion” flux is applied, as is an obliquity of 25°. Also shown is the response of case A. Below 160 kPa, the ocean of case C is entirely frozen over and mean surface albedo is 0.359. Above 290 kPa, the ocean is entirely ice free and the mean albedo falls to 0.197.
and thus case C achieves global mean surface temperatures about 11 K less than those of case A. Above this pressure, the ice begins to melt, and for pressures in excess of 290 kPa, the ocean is ice free. At 300 kPa, the mean surface albedo has fallen to 0.197, and the surface temperature is now about 4 K higher than that of case A. As with both preceding cases, one should treat this "perihelion" study with a certain degree of wariness. Additionally, one would also expect contraction of any ocean which might be present during the icier conditions, thus reducing the territory which would be covered by ice and increasing the exposed land area. However, these results do demonstrate the possibility of open-water conditions existing for CO₂ levels in excess of 200 K, much the same result as was found in case A or in RCM studies (e.g., Durham and Chamberlain 1989).

Also presented for the simulations of a boreal ocean are surface temperature profiles (Fig. 14) for selected values of the CO₂ partial pressure. In the 100 kPa case, the ocean is entirely iced over and the maximum surface temperature occurs in the 5°S zone, which is almost entirely land and where the surface albedo is 0.26, much less than the 0.34 in the 5°N zone immediately adjacent to it. The 200 kPa curve does not show quite so noticeable a maximum temperature. Here, though, the northern hemisphere has become the warmer as the water between the equator and about 50° is ice free, reducing the average albedo in these zones to about 0.14. The 300 kPa curve shows an even flatter temperature profile, but with a greater disparity between the polar temperatures since the northern polar region is now ice free.
FIG. 14. Horizontal temperature profiles for case C for selected CO$_2$ partial pressures with "perihelion" flux. Note that for 100 kPa, the boreal ocean is completely frozen over, while for the 300 kPa case, it is ice-free.
D. Comparison to previous studies

It was previously noted that this study realized surface temperature profiles which exhibited similar behavior to the results of Postawko and Kuhn (1986) for values of the large-scale eddy diffusion coefficient between $10^4$ and $10^5$ m$^2$ s$^{-1}$. (Unfortunately, they did not consider variation about the present obliquity and so no comparison could be made of its effect on the temperature profile.) It must be noted that the results of this study found mean surface temperatures in excess of those of Postawko and Kuhn (1986). While we agree that a CO$_2$ partial pressure of 300 kPa is insufficient to push the surface temperature above the freezing point for water for a reduced early solar flux, the mean surface temperature found in this study for case A or B exceeds that of Postawko and Kuhn (1986) by about 18 K. However, the fashion in which they tabulate their results confuses the matter. Despite their claim of, for example, a surface temperature of 241 K for a $P$(CO$_2$) $= 300$ kPa atmosphere when a reduced solar flux and water vapor saturation are considered, they then present zonal surface temperature values for selected diffusion coefficients in two cases of which the equatorial temperature is below the claimed planetary mean.

In another single-layer energy balance model study, Hoffert et al. (1981) examined the response of the early Mars atmosphere to carbon dioxide levels up to about 200 kPa, applying the modern value of the solar luminosity. They found that CO$_2$ levels in excess of 100 kPa would warm the planet enough that the surface temperature distribution allowed for liquid water activity over 95% of the planet. However, this result is at variance with the results of this study and several recent RCM studies, such as those of Pollack et al. (1987)
and Durham and Chamberlain (1989). I found that CO\textsubscript{2} pressures in excess of 200 kPa are necessary to support liquid water activity for such a luminosity. The cause of the large difference is probably, as noted by Pollack \textit{et al.} (1987), the omission of upward convection in the Hoffert \textit{et al.} (1981) model.
IV. DISCUSSION

Before summarizing the results of the studies which I have presented in this thesis, a few comments are in order about ways in which the model which I have employed could be improved and also about possible directions for future studies.

A. Directions for improvement

Despite the complexity of the model, there were several features which were simplified and which it would be worthwhile to improve in future studies. First, the altitude of the boundary between the radiative and convective regimes was specified to lie at $P_{RC} = 0.2 P_s$, which despite a relatively small effect on the surface temperature (for example, the 5 K decline in the mean surface temperature found for a 500 kPa atmosphere when $P_{RC}$ is raised from 100 kPa to 60 kPa, the latter value being obtained from the RCM of Durham and Chamberlain [1989]), is an extremely crude approximation. In RCMs, this boundary is located by comparing the radiative-equilibrium lapse rate to a critical lapse rate, typically either a specified average value or the lapse rate for moist adiabatic processes. Where the two lapse rates match is where the boundary occurs. The interactive approach could be extended to the present model by varying the upward level to which the dynamic heat transport processes are allowed to extend. However, doing so would require increasing the resolution of the vertical grid, and the modeler must also
allow for the fact that the boundary will lie at different levels for different latitude zones, such as occurs for modern Earth.

More important, however, would be improvement of the method by which moisture transport is treated in this model. Although dynamic transport is calculated, the actual water vapor profile is determined by the parameterization of Manabe and Wetherald (1967). Instead, an equation for moisture conservation could be applied, and where the humidity exceeds specified values, condensation would occur, with possible precipitation. A possibility for such an approach is described by Liou et al. (1985), but again, this is a improvement which would require a finer vertical grid resolution.

Improvement in determining the water vapor profile could also provide a suitable approach for cloud prognostication, which indeed is the intent of the scheme of Liou et al. (1985), as it could allow for the possibility of varied cloud cover, extent and droplet size distribution, an important factor in describing a cloud in terms suitable for inclusion in radiative transfer calculations. However, this could also be a dangerous path to venture, since the nature of cloud feedback in climate modeling is still a hotly debated issue. In a study such as has been described here, one would expect that increasing CO₂ levels, with the consequent surface temperature increases and expansion of convective region of the atmosphere, would result in increasing cloudiness. One is left to question, though, whether the increased reflection of solar radiation by the cloud top (a negative, moderating feedback) would prove stronger than the more efficient absorption of thermal radiation within the cloud (a positive feedback). In this matter, one is tempted to avoid the issue until
better data for a relatively well-known climate, that of modern Earth, are available before essaying a greatly different atmosphere.

A final reason to improve the moisture transport is to improve the treatment of the surface albedo. I have specified the land albedo directly in this model, rather than let it be interactively determined, and consequently there is no growth/decay in the size of the polar ice caps as a result of changes in the surface temperature, an important seasonal effect for modern Mars. If moisture transport were improved, the size of the ice caps could be determined through examination of precipitation and surface temperatures.

B. Directions for future study

In addition to improving the manner in which the model presently works, there are a few new directions in which studies might be pursued. The most obvious of these would be its modification to the study of seasonal variation in Mars' early climate, particularly in regards to the effect of maximum orbital eccentricity. Only two processes in this model would require much modification before such studies could commence. The first of these is the solar input, which would be most readily treated by using daily averaging to the zenith angle and incident flux rather than the annual averaging applied here. Additionally, one must also allow for daily variation in the period in which the Sun is above the horizon. However, procedures for such calculations are well-established and have been used by Peng et al. (1987) in their study of modern Earth using the original version of the model described in this thesis.
The other necessary modification for seasonal variation is to the configuration of the Hadley cells. In this study I have assumed that the latitude of peak heating lies at the equator, so that the boundary between the north and south Hadley cells (the ITCZ) also occurs at that latitude. This latitude would vary in a seasonal model and could be accounted for by applying the method of Lindzen and Hou (1988), which itself is a modification of the scheme of Hou (1984) used in this study. This modification would be especially valuable in that Lindzen and Hou (1988) have noted that placing the ITCZ at the equator results in a weaker annual mean circulation for modern Earth than is actually observed. If this latitude is displaced by even a few degrees they found that the strength of the winter-side cell was greatly amplified and the summer-side cell suppressed.

Another less obvious direction for future consideration is carbon dioxide condensation. In a recent study employing an RCM, Kasting (1991) examined the possible effect of this process on the atmosphere of Mars. For the current solar luminosity, he found that condensation would begin to occur at high altitudes for surface pressures in excess of 35 kPa (0.35 bar). The heat released by this condensation raises the effective radiating altitude, so that less radiation is required from the surface to maintain radiative equilibrium and the surface temperature is decreased. As an example, he notes for a surface pressure of 200 kPa (2 bar), the surface temperature is about 9 K below the results of RCM studies in which CO$_2$ condensation was not considered. For reduced solar luminosities, the reduction becomes more significant.

Despite some uncertainties in his model, including the omission of any possible effect of CO$_2$ condensate clouds and possible error in his treat-
ment of the CO$_2$ equation of state, Kasting (1991) concludes that a CO$_2$ greenhouse solution does not provide an adequate solution to the faint young Sun paradox, suggesting that alternative solutions should be investigated. These include possible warming by CO$_2$ clouds, about which little is currently known; the presence of additional greenhouse substances such as CH$_4$, NH$_3$ and dust; and alternative luminosity paths for the early Sun. The second course has been suggested in the past (e.g., Sagan and Mullen 1972) and since been disregarded, but examinations of NH$_3$ (Brown and Kasting 1992) and volcanic gases (Kasting 1992) have resumed. In a study of the rate of solar mass loss, Graedel et al. (1991) have proposed the latter solution, suggesting a much smaller variation in the solar luminosity over time.

C. Summary

In this thesis I have described a two-dimensional, vertically and meridionally defined model of the atmosphere of Mars and presented results of the application of that model to possible early states of that atmosphere. The response of the model to variations in the carbon dioxide partial pressure were found to be very similar to those of studies employing radiative-convective models when similar surface albedos were applied. In particular, this study found that surface temperatures greater than the freezing point of water could be obtained for CO$_2$ levels exceeding 400 kPa for the mean solar flux when a reduced early solar luminosity is assumed. At this pressure, increased horizontal heat transport causes a relatively shallow meridional surface temperature profile such that these temperatures could also be achieved outside the equatorial latitudes, even when one allows for the increased surface albedo of a polar ice cap. Above freezing temperatures could be obtained for
lower CO₂ levels (200 kPa and higher) when the specified solar flux is raised to a value appropriate to the perihelion distance during periods of maximum eccentricity. The effect of varying obliquities was also examined, and as expected, it was found that the lowest obliquities provided the greatest difference between the surface temperatures at the equator and the poles. Finally, the model was extended to examine a possible intermittent ocean on Mars as has been described by Baker et al. (1991). For the "perihelion" flux, this ocean made the transit from totally ice-covered to totally ice free between 150 and 300 kPa.
APPENDIX A. CALCULATION OF SOLAR ZENITH ANGLES

The solar flux striking a point at the top of a planet's atmosphere at a given time is calculated from

\[ S = S_0 \left( \frac{d_0}{d} \right)^2 \cos Z(\phi, \delta, h) \]  
(A.01)

in which \(S_0\) is the appropriate solar "constant" (=1360 W m\(^{-2}\) for modern Earth), \(d_0\) is the average distance between sun and planet, \(d\) is the current distance and \(Z\) is the solar zenith angle for the point in question. The latter is a function of latitude \(\phi\), solar declination \(\delta\), and hour angle \(h\) (measured from local noon) and may be written

\[ \cos Z(\phi, \delta, h) = \sin \phi \sin \delta + \cos \phi \cos \delta \cos h \]  
(A.02)

The solar declination varies according to

\[ \sin \delta = \sin \theta \sin \Theta \]  
(A.03)

where \(\theta\) is the angular distance from northern hemispheric spring equinox (a function of eccentricity \(e\) and the argument of perihelion \(L\)) and \(\Theta\) is the obliquity of the ecliptic. Since variation in \(\delta\) is relatively slow, it may be considered constant during the course of one day, and the average solar zenith angle for that day can be determined from

\[ \cos Z(\phi, \delta) = \frac{1}{2H} \int_{-H}^{H} (\sin \phi \sin \delta + \cos \phi \cos \delta \cos h) \, dh \]

\[ = \sin \phi \sin \delta + H^{-1} \cos \phi \cos \delta \sin H \]  
(A.03)
where $\pm H$ is the sunrise/sunset angle, determined by setting $\cos Z = 0$ in equation (A.02) and solving for $h$. When $\tan \phi \tan \delta \geq 1$, the sun is above the horizon all day and $H$ is set to $\pi$; for $\tan \phi \tan \delta \leq -1$, the sun never ascends above the horizon and $\cos Z(\phi, \delta) = 0$. The annual average of $\cos Z(\phi)$ is found by the taking the time-weighted average of the daily zenith angles $\cos Z(\phi, \delta)$.
APPENDIX B. DETAILS ON OCEANIC HEAT TRANSPORT

Oceanic heat transport by current systems is approximated by means of a conduction-like formula, as described in Temkin and Snell (1976). If one considers a latitude $\phi$, heat is transported by the ocean across this latitude through a total cross-sectional area

$$A = (2 \pi a \cos \phi) f_W (1 - A_I) D_W,$$  \hspace{1cm} (B.01)

where $f_W$ is the fraction of latitude $\phi$ which is oceanic (either open water or sea ice), $A_I$ is the fraction of ocean which is ice-covered and $D_W$ is an "effective" depth for the oceanic mixed surface layer. The rate of heat transport northward is then

$$J = -k(\phi) A \frac{\partial T_W}{\partial y} = -k(\phi) (2 \pi a \cos \phi) f_W (1 - A_I) D_W \frac{\partial T_W}{\partial \phi},$$

$$= -k(\phi) (2 \pi \cos \phi) f_W (1 - A_I) D_W \frac{\partial T_W}{\partial \phi},$$  \hspace{1cm} (B.02)

where $T_W$ is the open water temperature and $k(\phi)$ is a latitudinally-dependent transport coefficient. Since ocean currents are deflected by the Coriolis force, Temkin and Snell (1976) wrote this coefficient as

$$k(\phi) = k_{ocn} (2 \Omega a \| \sin \phi \|)^{-1},$$  \hspace{1cm} (B.03)

where $k_{ocn}$ is a zonally-independent coefficient and $\Omega$ is planetary angular velocity. Thence, the heat flow is

$$J = -k_{ocn} \frac{\pi \cos \phi}{\Omega a \| \sin \phi \|} f_W (1 - A_I) D_W \frac{\partial T_W}{\partial \phi}.$$  \hspace{1cm} (B.04)

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Lee and Snell (1977) modified this equation by multiplying the right-hand-side by a factor of \((1 - f_W)\) in order to achieve results closer to observed values for modern Earth, explaining this "as introducing non-continental meridional channelling of ocean currents." Including this term gives

\[
J = -k_{ocn} \frac{\pi \cos \phi}{\Omega a} \left| \frac{f_W}{\sin \phi} \right| f_W (1 - f_W) (1 - A_I) D_W \frac{\partial T_W}{\partial \phi}. \tag{B.05}
\]

If the coefficient \(k_0\) is taken to equal \(1.88 \times 10^7 \text{ J K}^{-1} \text{ m}^{-1} \text{ s}^{-2}\), as in Temkin and Snell (1976), a unit analysis of equation (B.04) or (B.05) reveals that \(J\) is returned in terms of power per length \((W \text{ m}^{-1})\) when it should be simply power \((W)\). Additionally, when this scheme is included in the model during simulations run for modern Earth, the calculated oceanic heat transports are too small by a factor of \(10^6\) to \(10^7\). This suggests that one too many planetary radii have been included, and consequently I alter equation (B.03) to read

\[
k(\phi) = k_{ocn} (2 \Omega | \sin \phi |)^{-1}, \tag{B.06}
\]

and thus obtain for the oceanic heat transport

\[
J = -k_{ocn} \frac{\pi \cos \phi}{\Omega | \sin \phi |} f_W (1 - f_W) (1 - A_I) D_W \frac{\partial T_W}{\partial \phi}. \tag{B.07}
\]

Since equation (B.06) renders an infinite value at the equator \((\phi = 0)\), the heat transport at that latitude is taken as the average of the values calculated at the two grid points immediately north and south of the equator \((\phi = \pm 10^\circ)\).
APPENDIX C. DETAILS ON THE SEA ICE MODEL

The scheme by which water temperatures and sea ice coverage are calculated is that of Peng et al. (1987). This approach is entirely thermodynamic and does not include such dynamic processes as wind-forced ice drifting and ice accretion due to snowfall.

To simplify calculations, lateral heat transport from open water to the adjacent ice pack is assumed only to melt the edge of the ice, reducing its fractional area, and not to heat the ice surface. Consequently, a new ice top temperature may be calculated by modifying (2.08) to read

$$\frac{1}{2} C_I D_I \frac{\partial T_I}{\partial t} = F_{S,I} + F_{T,I} + F_{SH,I} + F_{LH,I} + F_{IC} \quad (C.01)$$

The volumetric specific for water, $C_I$, is set equal to $1.8424 \times 10^6$ J K$^{-1}$ m$^{-3}$. The final term in (C.01) is that of conduction through the ice. In order to maintain a continuous temperature profile, the bottom of the ice sheet and the water below are held at the freezing for sea water, $T_{frz} = 271.2$ K, a value representative of water with salinity equal to that of modern Earth oceans. Consequently, the conduction term may be written

$$F_{IC} = \frac{k_{ice} (T_{frz} - T_I)}{D_I} \quad (C.02)$$

in which $k_{ice}$ is the thermal conduction coefficient for sea ice, $2.04$ W K$^{-1}$ m$^{-1}$. This conduction also causes ablation or accretion along the lower face of the
ice, and with no other heating processes affecting the ice bottom, the resulting change in the ice thickness is found from

\[
\frac{\partial D_I}{\partial t} = \frac{F_{IC}}{Q_I} = \frac{k_{ice} (T_{frz} - T_I)}{D_I Q_I}.
\]  

(C.03)

where \( Q_I \) is the volumetric heat of fusion for ice, \( 3.02 \times 10^8 \) J m\(^{-3}\).

If the value calculated for the ice top temperature by equation (C.01) is greater than the melting point of water ice, \( T_0 = 273.15 \) K, then the surface temperature is set at the melting point and the excess heat is used to melt the top of the ice. To determine the amount of melting, one must consider the effect of melting a thin layer of thickness \(-\Delta D_I\). The heat necessary to melt this thickness is \(-Q_I \Delta D_I\), and the top of the underlying ice experiences a temperature decrease of

\[
-\Delta T_I = -\frac{Q_I \Delta D_I}{\frac{1}{2} C_I (D_I + \Delta D_I)} = -\frac{2 Q_I \Delta D_I}{C_I (D_I + \Delta D_I)}.
\]  

(C.04)

With the requirement that the ice top temperature become \( T_0 \), or \( \Delta T_I = T_0 - T_I \), a little algebra shows that the necessary melting reduces the thickness to

\[
\Delta D_I = \frac{(T_0 - T_I') C_I D_I}{2 Q_I - (T_0 - T_I') C_I}.
\]  

(C.05)

where \( T_I' \) is the ice temperature which was calculated from equation (C.01).

Lateral heat transport from open water to existing sea ice is assumed to occur only when the sum of all other open water heating sources is positive. Since the temperature of the open water must be equal to or greater than the freezing point, this requirement means that heat flux from open water to ice,
\( F_{WI} \) is constrained to be either positive or zero. When this transport does occur, a portion of this sum proportional to the ice area is lost. Thus,

\[
F_{WI} = \begin{cases} 
0 & F_{\text{net}} < 0 \\
A_I F_{\text{net}} & F_{\text{net}} > 0 
\end{cases} ,
\]  

(C.06)

where \( A_I \) is the fractional ice cover and \( F_{\text{net}} \) is defined by

\[
F_{\text{net}} = F_{S,W} + F_{T,W} + F_{SH,W} + F_{LH,W} + COF .
\]  

(C.07)

Since there are two possible values for \( F_{WI} \), the remaining discussion of water temperatures and the sea ice model considers the possibilities separately.

**Lateral heat transport occurs.** When \( F_{\text{net}} > 0 \), there is heat transport from the open water to the sea ice. The temperature of the open water is consequently calculated by combining equations (2.07), (C.06) and (C.07), or

\[
C_W D_W \frac{\partial T_W}{\partial t} = F_{\text{net}} - F_{WI} = (1 - A_I) F_{\text{net}} .
\]  

(C.08)

The fraction of the sea ice melted is

\[
\Delta A_I = - \Delta A_W = - \frac{F_{WI} (1 - A_I)}{D_I \{ Q_I + C_I (T_0 - T_{ave}) \}} ,
\]  

(C.09)

where \( T_{ave} \) is the average temperature of the ice layer, \( 0.5 \times (T_I + T_{frz}) \). In the area where the ice melted, the water has a temperature which is the average of the ice just melted and the mixed oceanic layer which was below it. Noting that the ratio of the density of ice to the density of sea water is 0.88, the resulting temperature in the area of melting is

\[
T_{\text{melt}} = \frac{T_{frz} (D_W - 0.88 D_I) + T_0 (0.88 D_I)}{D_W} .
\]  

(C.10)
At the end of the time step, the average temperature of all open water is the weighted average of $T_W$ calculated from (C.08) and $T_{melt}$ calculated from (C.10). The new average open water temperature is then

$$T_{W,n} = \frac{T_{W,n-1} A_{W,n-1} + T_{melt} (A_{W,n} - A_{W,n-1})}{A_{W,n-1} + \Delta A_W}$$

$$= \frac{T_{W,n-1} A_{W,n-1} + T_{melt} (A_{W,n} - A_{W,n-1})}{A_{W,n}} .$$  \hspace{1cm} (C.11)

(The subscript $n$ used here is to differentiate between the changing values of a variable recalculated more than once during a time step.) The open water area determined from equation (C.09) may exceed 100% (all ice has melted), in which situation $A_{W,n}$ is reduced to that value in (C.11). The heat which would have melted the excess ice is applied to further heat the open water;

$$T_{W,n} = T_{W,n-1} + \frac{(A' - 1) Q_I D_I}{D_W C_W} ,$$  \hspace{1cm} (C.12)

where $A'$ is the open water area originally calculated from equation (C.09).

_Lateral heat transport does not occur._ When $F_{net} < 0$, there is no lateral heat transport from the open water to the sea ice, or $F_{WI} = 0$, and the open water temperature is calculated from

$$C_W D_W \frac{\partial T_W}{\partial t} = F_{net} .$$  \hspace{1cm} (C.13)

As long as the new open water temperature does not fall below $T_{frz}$, no change is made occurred in the ice area or thickness. If it does, the water temperature is raised back to $T_{frz}$, and the necessary heat is extracted by increasing the ice cover. This increase is determined by assuming that the
open water temperature within a zone bounded by latitudes $\phi_1$ and $\phi_2$ varies with $\cos \phi$, or linearly with the variable $\Psi$, defined as

$$\Psi = \frac{\cos \phi - \cos \phi_1}{\cos \phi_2 - \cos \phi_1}.$$  

(C.14)

This relationship is illustrated in Fig. 15, in which the two curves show the open water temperature before and after cooling. (Note: ice formation in water which is initially ice-free is only considered if the adjacent zone is iced over, in which case $\Psi_1(t) = \Psi_2$.) Since the water temperature is restricted from falling below the freezing point, an area $\Delta A_I$ must ice over in response to cooling. This area is determined by noting that the slopes of the two curves.

---

**FIG. 15.** Relationship between cooling of open water and change in ice cover. The open water temperature is assumed to vary linearly with $\Psi$, defined in equation (C.14). The solid curve is the initial open water temperature; the dashed curve is the temperature after cooling. The water temperature may not fall below the freezing point, and so the fractional area $\Delta A_I$ must ice over. (From Fig. A1 of Peng et al. [1987].)
are equal, or

\[ 2 \frac{T_{frz} - T_W(t)}{\Psi_I(t) - \Psi_1} = \frac{\Delta T_W}{\Psi_I(t) - \Psi_I(t+\Delta t)} \]  \hspace{1cm} (C.15)\]

where \( \Psi_I \) represents the latitude at which the ice line occurs. Since initial \( \Psi_I(t) = 1 - A_I(t) \), equation (C.15) may be rewritten

\[ 2 \frac{T_{frz} - T_W(t)}{1 - A_I} = \frac{\Delta T_W}{\Delta A_I} \]  \hspace{1cm} (C.16)\]

which may be manipulated to obtain

\[ \Delta A_I = \frac{1}{2} \frac{\Delta T_W}{T_{frz} - T_W(t)} \]  \hspace{1cm} (C.17)\]

If ice formation did not occur, the water in the area \( \Delta A_I \) would be at an average temperature \( T_{frz} + 0.5 \Delta T_W \). The thickness of new ice can thus be determined by the amount of energy required to raise this water back to the freezing point:

\[ D_{I,new} = -\frac{\Delta T_W C_W D_W}{2 Q_I} \]  \hspace{1cm} (C.18)\]

A new zonal ice surface temperature and thickness are then found by averaging \( T_{frz} \) (the temperature of the new ice) and \( D_{I,new} \) with the surface temperature and thickness of the older ice. The average open water temperature is also adjusted to reflect the change in its area and is found from

\[ T_W(t+\Delta T) = \frac{1}{2} \left( T_{frz} + T_W(\Psi_1, t+\Delta t) \right) = \frac{1}{2} \left( T_{frz} + T_W(\Psi_1, t) + \Delta T_W \right) \]

\[ = T_W(t) + \frac{\Delta T_W}{2} \]  \hspace{1cm} (C.19)\]
APPENDIX D. POTENTIAL TEMPERATURE

This study uses potential temperature, $\theta$, in place of temperature, $T$, in many of its calculations. This value is the temperature a parcel of ideal gas would come to if it was adiabatically moved from some pressure $P$ to a standard pressure $P_0$. If the initial temperature is $T$, then the potential temperature is

$$\theta = T \left( \frac{P_0}{P} \right)^\kappa$$  \hspace{1cm} (D.01)

$$\kappa = \frac{R_{dry}}{C_{P,dry}} ,$$  \hspace{1cm} (D.02)

where $R_{dry}$ is the gas constant (J kg$^{-1}$ K$^{-1}$) of the dry atmosphere and $C_{P,dry}$ is its specific heat (J kg$^{-1}$ K$^{-1}$) at constant pressure. For modern Earth, the standard pressure is typically set at $P_0 = 1 \times 10^5$ Pa. When $R_{dry} = 287.05$ J kg$^{-1}$ K$^{-1}$ and $C_{P,dry} = 1005$ J kg$^{-1}$ K$^{-1}$, the exponent is $\kappa = 0.2856$.

In this study, however, I consider atmospheres much different from that of modern Earth and so the values commonly applied in (D.01) and (D.02) must be altered. In all cases, I have set the standard pressure equal to the surface pressure, or $P_0 = P_s$. The atmospheric gas constant and the specific heat are calculated by taking the mass-weighted average of the individual values for the component gases

$$X_{dry} = \left( \sum_{i=1}^{n} c_i \mu_i X_i \right) \left( \sum_{i=1}^{n} c_i \mu_i \right)^{-1} ,$$  \hspace{1cm} (D.03)
where $c_i$ is the volumetric mixing ratio of gas species $i$, $\mu_i$ is its molecular weight and $X$ is either gas constant or specific heat. Gas constants for the various species may be found in various texts; specific heats are determined from formulae presented in Toulokian and Makita (1970).

Since the value of $C_{P,dry}$ for a particular grid point is constantly changing, the value of $\kappa$ to be applied in equation (D.01) is subject to variation in time and space. In order to reduce the number of calculations which must be made, I have instead specified a single planetary value for $\kappa$, which is periodically calculated. In all other equations, values for $C_{P,dry}$ are otherwise calculated as required.
REFERENCES


