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Model of superthermal ions in the dayside Venus ionosphere

Kramer, Leonard, Ph.D.

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MODEL OF SUPERTHERMAL IONS IN THE DAYSIDE VENUS IONOSPHERE

by

LEONARD KRAMER

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APPROVED, THESIS COMMITTEE

Paul A. Cloutier
Professor of Space Physics and Astronomy
Chairman

Richard A. Wolf
Professor of Space Physics and Astronomy

Wendell Horton
Adjunct Professor of Space Physics and Astronomy

Balasubramaniam Ramaswamy
Assistant Professor of Mechanical Engineering and Material Science

Houston, Texas
February, 1993
ABSTRACT

MODEL OF SUPERHEATED IONS IN THE
DAYSIDE VENUS IONOSPHERE

by

Leonard Kramer

A model is presented which simulates the behavior of superthermal ions previously reported in the dayside ionosphere of Venus. The model considers effects of $\mathbf{E} \times \mathbf{B}$ and gradient drifts, charge exchange and collisions with the ambient neutral atmosphere and the possible effects of a wave-particle (anomalous) scattering process. Results indicate that scattering processes are required if superthermal ions are the explanation for the observed "missing pressure" component in the dayside Venus ionosphere. The scattering scale length required to match the "missing pressure" distribution is similar to the scale length previously predicted for growth of a lower hybrid beam instability.
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I. INTRODUCTION

The solar wind interaction with terrestrial planets can be divided roughly between two types. One of these is characterized by planets such as the Earth which possess a strong intrinsic magnetic field which deflects the solar wind and couples the momentum of the diverted flow directly to the planet's core. The other type is characterized by planets such as Venus or Mars which apparently have no strong intrinsic field. For these planets, the interaction is a continuous external fluid flow in which the vertical flow momentum directly loads the ionosphere and ultimately the lower atmosphere.

Figure 1 reproduced from Brace et al. [1983], illustrates most of the features of the solar wind interaction with the Venus environment which are now well understood. The vertical (or radial) scale of the Ionosphere in this figure is exaggerated by approximately a factor of 5. We can obtain from this figure the often noted observation that the solar wind interaction with Venus is rather comet-like exhibiting a wake region similar to a cometary tail. The major difference dynamically from the comet-like interaction is that the atmospheric composition is, of course, different and the presence of gravity produces a different physical process than would be active in the cometary prototype. Specifically, due to the lack of gravity, the comet exhibits a high speed outgassing of volatile neutral constituents which does not prevail in a planetary scale interaction.

Although Venus apparently has no intrinsic magnetic field, the solar wind flowing past the planet is, however, a magnetized plasma. Since the neutral atmosphere is subject to solar ultraviolet radiation, ions of planetary origin form from the neutral atmosphere and interact with the flowing plasma. The overlying external flow creates electric currents which react in such a way so as to produce a draping of the embedded solar wind magnetic field over the planet. This draping and the associated convection effects on the
Figure 1. Reproduced from Brace et al. [1983]. A convenient illustration of the Venus-Solar Wind interaction and other well understood features of the Venus aeronomical environment.
embedded field is not shown in Figure 1. In addition any stream lines that flow through
the ionopause (the boundary between the ionosphere and the ionosheath region) have
been de-emphasized. The author of Figure 1 has taken a conservative position on these
points because, as we will discuss, the nature of the magnetic field, and the steady state
flow field through the ionosphere has been contested by different research groups.

The Mach number of the solar wind flowing past planetary scale objects in the solar
wind is typically quite high; on the order of 8 to 10. A simple one dimensional analysis
[Cloutier 1983] and more complex aerodynamic simulations [Spreiter and Stahara, 1966,
1980, 1992] have been successful in explaining or describing the shock and other gross
features of the flow field. Although the mean free path for ion collisions in the solar
wind governing conventional fluid effects is comparable to the distance between the
planets, the particles in the plasma are, nevertheless, coupled by long range collective
forces. Evidently, these collective interactions provide the mechanism for “effective”
collisions which randomize particle trajectories and transmit the information about the
presence of the planet to the supersonic flow on a shorter distance scale than Coulomb
ion-ion collisions. The gyro-radius has sometimes been invoked as this mean free path
but a randomizing effect such as afforded by non-linear wave-particle interaction appears
to be needed as well. Spreiter et al. [1966] make the point that the shock envelope shape
is rather insensitive to the thermodynamic ratio of specific heats, γ, or to the Mach
number which suggests that there is considerable “lee-way” in any effective collision
mean free path.

Although both Venus and Mars are believed to be prototypes for the non-magnetic
solar wind interaction, Venus has been studied to a much greater extent. This is due to
the long duration and high quality of measurements from the Pioneer Venus spacecraft
which operated for 14 years at Venus. The recent U.S. Magellan space craft was
dedicated to radar imaging of Venus and sent back detailed topographic images of the
planet. In contrast, our understanding of the solar wind interaction with a Martian ionosphere has not benefited from comprehensive examination and there is no such analog to Figure 1 available for Mars. The United States Viking spacecrafts which went to Mars in 1976 were focused primarily on biology and planetary surface science. These spacecraft observations did not emphasize aeronomical observations and lacked most significantly any magnetic field measurements. The Soviet Phobos spacecraft which did carry a full complement of such instrumentation, unfortunately stopped working after a short time. This situation will be remedied to some extent by the Mars Observer mission which, at this writing, is on its way to Mars and will arrive at that planet in August, 1993.

The availability of extensive Venus data therefore tends to orient us toward study of that planet although attention to Mars has increased within the past few years with the Phobos and Mars Observer missions. The question of how the momentum of the external solar wind flow is transferred to an underlying planet that is un-magnetized has occupied the attention of several research groups for the past few years. Most particularly, the group headed by C.T. Russell at UCLA and by P.A. Cloutier at Rice University have had differing points of view on the subject. An appreciation for the subtleties involved in the physical interaction might be gained by considering that, as we have mentioned, the flow is a High Mach number regime in which compressible fluid effects are present and the fluid is a magnetized plasma with an inhomogenous composition. The constituents are chemically reactive and flow past an obstacle that presents a significant gravitational potential. In addition the plasma exhibits non-equilibrium velocity space and spatial density distributions giving rise to possible plasma instabilities and further, effective mass addition occurs from solar ionization meaning that the continuity equation has a particle source term. These are all factors that place the problem outside the usual area of expertise of aerodynamists but also make it a challenging and interesting problem in its own right. Although the Spreiter et al. [1966,
1980 and 1992] works mentioned earlier provide an excellent picture of the flow field streamline structure in the ionosheath, to date, no self consistent, three dimensional simulation of the prototype exists for the entire flow field and due to the complexities we have noted, such efforts are likely to be rather unproductive for a very long time into the future. Rather, we believe, that careful examination of individual components of the system invoking reasonable approximations are much more fruitful in producing insights into the nature of the physical system. The main thrust of this present work is to address that part of the problem related to how the vertical component of the solar wind momentum gets transferred to the planet.

Initially, pressure balance was invoked between the ionospheric thermal pressure in the ionosphere and the magnetic pressure in the shocked solar wind. Central to that idea was the postulated existence of an impermeable boundary or "ionopause" between the ionosphere and the shocked, magnetized solar wind in the ionosheath [Brace et al., 1980; Elphic et al., 1980; Russell and Vaisberg, 1983; Luhmann et al., 1984; Phillips et al., 1984]. An alternate explanation is described by Cloutier et al. [1987] in a simulation of the steady state flow through the ionopause in which the vertical component of solar wind ram pressure $\rho V^2$ directly loads the underlying atmosphere and is balanced by the sum of magnetic and thermal pressure, plus any left-over ram pressure. Evidently, from these results, we conclude that the ionopause is indeed a hard obstacle to the solar wind but the small amount of plasma that does leak through the boundary is, nevertheless, essential to its dynamics. It is also important to recognize that this requires a continuous flow in the ionosphere similar to that seen in Figure 2. The detailed streamline structure within the ionosphere is rather speculative and this uncertainty is reflected by the dotted stream lines in the ionosphere seen in that figure.

The horizontal flow in the ionosphere has been modeled by Whitten et al. [1984]. The two panels in Figure 3 reproduced from that work show the altitude and solar zenith angle
Figure 2. Illustration of dayside flow field at Venus. Radial scale of the ionosphere is exaggerated for clarity and the bundle of streamlines in the nose of the interaction is larger than the true interaction. Exact details of the stream lines within the ionosphere are deliberately vague because the precise configuration is unknown.
Figure 3. Reproduced from Whitten et al. [1984]. Presents what we believe to be the best quantitative details of flow in ionosphere. Panel (a) shows horizontal velocity as a function of solar zenith angle. Panel (b) provides altitude structure at several solar zenith angles. Altitude in panel (b) is distance above planet’s surface.
dependence of the horizontal flow velocity in the ionosphere. Whitten et al. have used an implicit in time finite difference scheme to solve the continuity and momentum equations assuming a source and loss term related to known chemistry and ionization processes. The upshot of that work is that horizontal velocities are of kilometer per second scale and can explain the required convection to supply a night-side ionosphere. It is important to note however that their model assumes a closed ionosphere in conformity with, what we regard as an incorrect interpretation by the Russell et al. group about the nature of the ionopause. The effects of solar wind absorption by the ionosphere have not been treated in the boundary conditions of such models and probably represent a future research opportunity. The Orbiter Retarding Potential Analyzer (ORPA) is said to be able to resolve ionospheric vector velocities and some provocative results suggesting rapid super-rotation in the ionosphere have been presented [Knudsen, 1991]. Velocity measurements from the ORPA are, in fact, plotted with error bars in the Whitten et al. figure reproduced here in Figure 3a. revealing basic agreement with the Whitten model.

The one dimensional aspects of the vertical flow have been studied to a considerable extent. These aspects of the ionosphere been treated in the Cloutier et al. [1987] model which successfully replicates the steady-state vertical flow field. We think that the fundamental insights of the Cloutier et al. [1987] model are 1) that the flow field is a steady state (time independent) process that adjusts rapidly to changes in the overlying external solar wind flow and 2) that mass loading from the production of ions within the ionosphere absorbs the flow momentum to the maximum extent possible. In the picture developed from the model, excess flow is diverted around the planet and the lower Venus ionosphere acts as a nozzle with a constriction caused by the increasing interference of neutral planetary atmospheric species as the ion flow progresses downward. The way in which the momentum is partitioned among the magnetic, thermal, gravitational and neutral drag forces is predicted at each altitude by numerically integrating the
momentum, energy and continuity equations downward in altitude using initial conditions estimated from data at high altitude. Essentially, however, the flow is driven by a vertical pressure gradient associated with a finite pressure at high altitude contrasted with an effectively zero pressure at low altitude associated with complete absorption of ions at the exobase. This leads to a choked flow condition in a region referred to as the sonic layer attained in the vicinity of the exobase. The choked flow implies a fundamental limit on the capacity of the "nozzle" to accept flow regardless of how large the driving pressure gradient may be. The complexity inherent in just the one-dimensional aspects, as described in the Cloutier model, foreshadows the difficulty in modeling the two dimensional field.

Work by Cravens et al. [1980] is similar to the Cloutier work except lower boundary conditions are imposed on the magnetic field and the solution is time dependent. The continuous flow described by the Cloutier and Cravens groups illustrated in Figure 2 and 3 is now accepted. The acceptance of a continuous flow paradigm removes an obstacle to understanding details of the dayside solar wind interaction.

As previously mentioned this thesis concerns the details involving missing components in the momentum and energy budgets in the ionosphere. In particular the presence of a non-thermal distribution of ions in the post-shock flow field at Venus is now firmly established experimentally. Although quantitative measures of number density distribution of this high energy population are lacking, and the details of identification in the spacecraft data is rather sketchy, the combination of a missing momentum component in the flow field seen in observations [Cloutier et al. 1992] and a missing heat source needed to enforce balance in the energy and momentum budgets [Cravens et al., 1980, Cloutier et al., 1987] suggest the importance of these ions to understanding the ionospheric flow and represents the motivation for the present work.
In the master's thesis by Kramer [1991], a straightforward effort was initiated to model the behavior of these superthermals. In that work, a two-dimensional computer model was constructed using a test particle approach. The high energy test particles were dropped into the flow field to simulate a production of superthermal ions and the dynamic effects from the draped magnetic field and frame effect motional electric field was simulated by integrating the equation of motion. In this way, the ion's history was used to examine the contribution of the small unmeasured superthermal population to the missing pressure. That work established most of the mathematical and numerical tools necessary to investigate how a population of particles is represented by the trajectory of a single test particle. It also demonstrated the possibility that the ions could populate the low altitude region and provide a potential explanation for the missing pressure.

This Ph. D. thesis advances the understanding of superthermal behavior in several ways. In the master's a straightforward effort was invoked to model the superthermal behavior by integrating the equation of motion from a mono-energetic population. That is, a test ion particle with a singular energy was followed and the mathematical and numerical tools were developed to represent the contribution of the single ion to the density and current structure. We demonstrated the feasibility of supplying a missing component in the ion population from the superthermal source. At that time, however, no effort was made to model the vertical component of the pressure tensor resulting from the superthermal population. In this Ph. D. Thesis, our mathematical tool inventory has been augmented by developing a method to calculate the pressure contribution from the test particle.

Second, as mentioned, the density and currents presented in the master's were derived from a mono-energetic population. In this work, the contribution from arbitrary energy distributions of superthermals will be shown. Although, as we will discuss, the quantitative details of the superthermal distribution is poorly known, arbitrary
distributions are assumed based on logical assumptions. The mono-energetic contribution from each class of test particle represents a sample from that distribution. We have therefore used the mono-energetic model to predict dynamic parameters at many energies spanning the range of energies covered by the assumed distributions and then produced weighted integrals of dynamic parameters at all altitudes.

Third, we have now found a plausible mechanism for energization of superthermals at the ionopause and we will discuss this. It involves a beam-particle instability associated with the flow of shocked solar wind at 100 km/sec scale velocities through ionospheric ions of planetary origin at and just above the ionopause. Several other candidates for energization are also discussed.

Fourth, and perhaps most significantly we will describe a surprising result involving the requirement for an anomalous scattering mechanism. The draped magnetic field in the ionosphere configures itself in such a way so as to produce a significant vertical magnetic field gradient. When the otherwise downward flowing superthermal ions encounter this gradient they exhibit such strong horizontal drift they are inhibited from populating the low altitude region. In addition, the ions generate large, coherent currents which have been suggested to be unstable. To remedy this problem we have invoked an ad-hoc scattering mechanism which appears to be similar to a procedure followed by Holland and Chen [1991] in their simulation of nonlinear wave-particle interactions. This procedure allows the ions to cross the field gradients and produce low altitude pressure profiles similar to observed profiles of missing pressure.

Finally, we will flesh out the details of the work regarding planetary atmosphere loss mechanisms which were touched on in the master's. We have considered that the superthermals have energies which are much larger than needed to escape the planet. They nevertheless do not escape because they are tied to the overlying draped magnetic field. When they undergo charge exchange, a process treated in the model, they are
transformed into high energy neutral atoms which are no longer subject to the dynamic effects of electric fields or the Lorentz force from the magnetic field. Since they still have essentially all the energy of the parent ion they can now escape the planet. We will show that the calculated escape rates are reasonable from comparison with other published estimates. Although our atmospheric loss mechanism is speculative and is really a rather tangential by product of the work, it has broad significance to the origins of planetary atmospheres and comparative planetology in general since there is substantial interest in the overt differences between planets, especially Venus and Earth, which are otherwise similar in size and distance from the sun.
II. OBSERVATIONAL EVIDENCE FOR SUPERThERMAl IONS

A superthermal layer in the Venus ionosphere was initially reported from careful examination of Pioneer Venus Orbiter ion mass spectrometer (OIMS) data [Taylor et al. 1980]. The Ion Mass spectrometer is a Bennett type which is a radio frequency resonance tube admitting positive ions with discrete charge to mass ratios. Historically, the Bennett type spectrometer employed a continuous sweep of stopping voltage at the back end of the instrument with a voltage-current efficiency curve determining the associated resonant ion density at approximately discrete points along the voltage sweep. The Pioneer Venus ion spectrometer was an innovative design in that it increased the time resolution of measurements by directly "stepping" to the approximate voltage location of expected ions and so eliminated significant lost measurement time during the continuous sweep. Onboard computer logic "dithers" fine tuning servo voltages between prescribed limits to automatically find the peak in the current output. This information is then encoded in a data word and inserted into the telemetry stream sent to Earth. The axis of the instrument is oriented parallel to the spacecraft spin axis. This means that the apparent orifice angle to the spacecraft velocity vector must be accounted for to eliminate the effects of ram velocity which the instrument reports as an upward bias in the density measurements. The removal of this bias is done in post processing on the ground. We should point out that, although the spacecraft spin is coaxial with the ion spectrometer, the data is usually spin modulated by about a factor of 2. This has been attributed to the ambient flow of ions in the ionosphere and the effect of shadowing from the antenna mast.

The direct evidence for superthermal ions consists of random, impulsive signals in the mass channels that are not otherwise expected to exhibit activity. The Neutral Mass Spectrometer has been referred to as a "quadrapole" spectrometer. It operates by
subjecting neutral constituents in the ambient medium to an ionizing electron beam. The ions generated from the neutrals are then accelerated by focusing grids toward a deflector target. Secondary electrons emitted by the deflector are then detected. To eliminate ambient ions from contaminating the intended neutral constituent measurements, the instrument employs a repeller grid covering the entrance aperture to produce a positive potential of about 40 Volts which has no effect on neutral species but excludes the thermal ions. Early in the mission it was noticed that impulsive telemetry counts were appearing in the ONMS similar in shape and time occurrence to the superthermal signatures seen by OIMS. These occurred at inappropriate altitudes for neutrals to be detected and exhibited a non-physical morphology best described as chaotic or impulsive with a wide variation over several orders of magnitude in density.

The results are reported in a paper by Kasprzak et al. [1982]. Figure 4, reproduced from that work, shows a cross periapsis pass illustrating the superthermal signatures. General agreement in time and location of impulsive signals is noted between OIMS and ONMS. The electron density measurement from the Orbiter Electron Temperature Probe (OETP), at the same time registers a disturbed condition suggesting the presence of large amplitude waves possibly resulting from plasma instabilities, and finally, the electric field detector (OEFD) shows some increase in activity at the lower frequencies. None of these instruments are designed to measure the quantitative distribution of superthermals and so the ambient background population usually masks their presence and variations in the apparent aperture area of instruments relative to the directed flow of high energy components might produce intermittent signatures.

Another view of the superthermals can be seen in altitude profiles for an inbound portion of orbit 413 in figure 5. In this figure, ion concentration of several species are shown along with magnetic field, electron density and electric field activity. We identify the "ionopause" as the point where the ions and electrons exhibit a sharp reduction in
Figure 4. Reproduced from Kasprzak et al. [1982]. Inter comparison of telemetry from 4 instruments on Pioneer Venus for a dayside periapsis pass containing superthermal activity. The hump in the middle of the ONMS and OETP measurements reflects the normal influence of the lower atmosphere. Superthermals are indicated by the impulsive signals between about 25700 and 25800 seconds.
density. As noted earlier, the expectation of thermal and magnetic pressure balance caused some to identify the ionopause as the point where this balance is achieved. Since we believe this theoretical expectation to be incorrect we identify the ionopause from observations as the point where the ions exhibit a sharp density gradient. Or, as several others have put it, the "ionopause is the place where the ions pause."

Figure 5 was selected to reflect many of the essential characteristics seen in a survey of about 80 dayside periapsis passes distributed among the first 3 years of Pioneer Venus telemetry. The electric field observations are from a 4 channel detector which was intended to span a wide range of frequency. The channels are 30% half maximum bandpass filters centered at 100 Hz, 730 Hz, 5.4 kHz and 30 kHz. The data is severely compromised by a low frequency ripple clearly seen in the 730 Hz, 5.4 kHz and 30 kHz channels. The modulation occurs at multiples of the spacecraft spin rate, occurring at all times when the orbiter is in sunlight. It has been suggested that the problem has to do with "...sun-oriented anisotropy of the plasma sheath surrounding the spacecraft", but apparently no comprehensive physical explanation for the effect has been exposed [Scarf 1980] although it is known the electric field detectors on other spacecraft also exhibit the problem. Despite this difficulty, it is evident that the instrument does respond to variations in the plasma environment as can be seen in a comparison with the ion data of Figure 5e and 5f. We note that the 5.4 and 30 kHz channels almost always show a increase in background baseline at the ionopause. This correlation is so strong that in most cases we can identify the ionopause in the high frequency OEFD without looking at the plasma densities. This effect is due to a change in dielectric properties of the plasma as the dipole "rabbit ears" of the detector crosses the ionopause.

The antenna array forms a capacitor and when the spacecraft enters the ionosphere the permitivity of the dielectric between its "plates" increases permitting a higher energy density of the noise. This is detected as a higher ac current which is in turn
Figure 5. Altitude profiles of data from 4 instruments for dayside Pioneer Venus pass (orbit 413 inbound). Panels (a) through (d) are 4 channels from the Orbiter electric field detector. Panel (e) contains the major ion constituents from the ion mass spectrometer including O$^+$ (mass 16), O$_2^+$ (mass 32) and CO$_2^+$ (mass 44). Panel (f) illustrates 4 minor ions channels: (masses 1, 12, 14 and 28). Ions are displayed as individual plot symbols and the electron density from the electron temperature probe is over-plotted as a solid line in panel (4). These data are selected as representative of impulsive signals characteristic of the superthermal ion signature.
reported as a higher electric baseline field magnitude. It is difficult to test the effect quantitatively since the energy density spectrums and attendant response to the bandpass characteristics of the detector are unknown, however the sense of the effect as the spacecraft enters the ionosphere is in the right direction. The effect does not apply at the lower frequencies closer to the electron and ion gyro-frequencies. This is because the plasma permittivity and therefore the electric field energy density cannot be expressed as a simple expression owing to resonances and cutoffs in the dispersion relation for the plasma at these lower frequencies. This is consistent with the much richer set of characteristics seen in the 100 Hz and 730 Hz channels as the spacecraft transits the ionopause.

In the altitude range of about 650 to 825 kilometers there is a disturbed condition active across 4 channels (the identification in the lowest channel at 100 Hz we concede to be problematical due to all the other structure seen in that channel) suggesting a broadband characteristic or at least a high frequency component in the electric field activity. As altitude decreases this abruptly (within 10 to 20 kilometer) ceases and is replaced by disturbed activity seen in the ion detector data; particularly Figure 5e. This behavior is seen on several inbound orbit segments but apparently has not been reported before. It may be evidence of a superthermal energization process propagating downward or of superthermal activity in different latitude regions since in addition to altitude, the spacecraft's roughly north to south orbit traverses a substantial latitude extent. In any event the presence of a sharp boundary between the disturbed electric field and the ion data is compelling and is likely associated with directed horizontal flow that we mentioned earlier since it is not seen on outbound segments of the periapsis pass. We do not think it represents the bow shock signature since the altitude is rather low and the solar zenith angle in this region is between 40° and 45° which is not consistent with the
bowshock signature. It is also not characteristically repeatable on adjacent orbits as would be expected for the bowshock which is a relatively stationary feature.

In Figures 5e and f, we see 7 of the 15 possible ions detected by the OIMS. In addition to altitude profiles of the data we also see plotted to the extreme right side, so-called data flags, which the ion telemetry processing system identifies from several criterion among which is the "pegging" of the servo voltages mentioned earlier on one side or the other of the allowed range of energies associated with the resonant ion in the OIMS. In reference to the OIMS data flagging we should emphasize that no comprehensive documentation exists for the flagging criterion. The program module CLENUP8.FOR contains the essential code related to data flagging and it is seen that some ions are always flagged and others are flagged if some other ion rises above a particular threshold, for example. From our experience, the telemetry processing, as implemented, is as complex as any electrical or mechanical system on the spacecraft. We believe that the failure to document these important aspects to at least the same level as the hardware components, is an unrecognized flaw.

The electron density determined from the OETP is over-plotted as a solid line in 5e. The flagged ion data are most prominent on this particular orbit, between 350 and 700 kilometers. The ionopause, we would identify near the bottom of this interval; about 350 to 375 kilometers. We note that there is strong peaking up of flagged telemetry in mass bins 12, 14, and 28 seen in Figure 5(f) at and just above the ionopause. We believe this feature, shaped like the profile of a human nose, is highly characteristic of the superthermal ion signature on the dayside and is seen in many examples. The signature also appears to be much more prevalent on the inbound pass as opposed to outbound.

The most plausible physical explanation for this is that the superthermals are aligned with the flow direction. When the spacecraft enters the ionospheric environment, it moves against the ambient flow. On the outbound segment, the flow is from behind.
This means that the instrument aperture has more direct exposure to the plasma on the inbound leg and the spacecraft velocity "adds" to the apparent energy of the plasma in a way that is not accommodated by the modeled efficiency curves of the telemetry processing systems.

Another observation that we note is that the solid line in Figure 5 (e) representing the electron density rises above the dominant O\(^+\) ion background at the ionopause. This is an almost ubiquitous feature in the all the clean examples that we were able to look at. The difference in density amounts to on the order of 100 to 1000 cm\(^{-3}\). The documentation coming with the OETP electron density tapes includes no particular caveats or qualification about accuracy which would be relevant to the comparison we are making here and so we tentatively identify the difference to be associated with the indirectly sampled superthermal population. We checked with L. Brace, who is the principal investigator for the OETP. He is conservative in his opinion that neither the OIMS or the OETP are capable of resolving abundances in the region of the ionopause but that the deficit between OIMS and OETP cannot be positively ruled out as being a crude measure of the superthermal population. We have also checked with H. A. Taylor, the original PI for the OIMS, who agrees that the difference between electron density and dominant ion background is a crude measure to perhaps an order of magnitude of the superthermal population.
III. MOMENTUM AND ENERGY BUDGET

The vertical momentum budget on the dayside can be directly investigated using the PVO measurements and several workers have done this. Elphic et al. [1980] compare thermal pressure ($nk(T_e+T_i)$) to magnetic pressure ($B^2/2\mu_0$). Figure 6 incorporates two figures from that work which show interesting discontinuities at times between magnetic and thermal pressure. The Elphic et al. work makes the essential point that magnetic and thermal pressures are comparable. We observe in Figure 6(a) that the downward excursion in plasma pressure between about 2032 and 2033 UT is not continuous with the magnetic pressure. Likewise, panel (b) shows a sharp jump at about 2102 UT. Elphic et al. at the time these figures were produced did not have access to the ion temperature directly and so modeled ion temperature by assuming it was 1/2 the electron temperature. We cannot rule out the possibility that the pressure differences we point to here are not due to deviations from this modeled behavior but they do stimulate speculation in the direction of a missing component.

Brace et al. [1980] also investigated the vertical momentum budget by examining the relationship between maximum magnetic field pressure in the vicinity of the ionopause to the ram pressure observed outside the bowshock. Their results are illustrated in figure 7a. The maximum magnetic field pressure should estimate the total pressure to a reasonable level of confidence. A 45°, straight line relationship between maximum magnetic pressure and external solar wind momentum reflects exact balance. Brace et al. comment on the systematic deficit seen in figure 7a, and note that it neglects the fact that only the vertical (or radial) component of momentum actually loads the top of the atmosphere. By incorporating this radial component of velocity outside the shock boundary they more closely fit the straight line and speculate that the non-thermal component in the plasma detected by Taylor et al. [1980] is a possible source for the
Figure 6. Reproduced from Elphic et al. [1980]. Possible evidence of missing component in the pressure balance on the dayside. Magnetic field and thermal pressure resolved into equivalent pressure units to illustrate transfer of momentum to planet. Panel (a) shows a downward excursion in plasma pressure between about 2032 and 2033 UT not continuous with the magnetic pressure. Panel (b) shows a sharp jump at about 2102 UT.
Figure 7. From Brace et al. [1980]. Further evidence of missing pressure in the ionospheric flow field. Both panels show maximum magnetic pressure, assumed to be a measure of total pressure near the ionopause, plotted versus solar wind ram pressure in the unshocked solar wind. Panel (b) uses ram pressure weighted by $\cos^2(\chi)$ term to account for vertical component of momentum. Closer agreement is noted in panel (b). Residual discrepancy may be due to missing, unmeasured superthermal component.
remaining discrepancy.

Supporting work for a missing component in the momentum and energy budget comes from Cravens et al. [1980] who suggest that the ion and electron temperatures are substantially higher than can be accounted for by known energetic inputs and Cloutier et al. [1987] who show that an anomalous heat source is required to satisfy the energy budget in their model.

Quantitative measure of the missing pressure term is provided by Cloutier et al. [1992] based on results from a Ph. D. thesis by B. K. Stewart [1991]. Solar wind ram pressure, \( (\rho v^2) \) obtained outside the bowshock from the Pioneer Venus plasma analyzer (OPA) experiment was compared to the sum of kinetic particle pressure \( (nk(T_e+T_i)) \) and magnetic pressure \( (B^2/2\mu_o) \) in the post shock flow field. Data were obtained from the publicly available Unified Abstract Data Set (UADS) containing OPA, OETP, the retarding potential analyzer (ORPA) and the magnetometer (OMAG). Figure 8, reproduced from Cloutier et al. [1992], illustrates the missing pressure for low, medium and high solar wind pressures. In this figure the quantity:

\[
\text{"MISSING PRESSURE" } = 1 - \frac{B^2 + nk(T_i+T_e)}{2\mu_o K \rho (V \cos \chi)^2}
\]  

was calculated where \( \chi \) is the solar zenith angle required by a Newtonian pressure balance criterion. This equation estimates the amount of disagreement between the observations and the momentum conservation requirement.

The Cloutier [1992] paper discusses Equation 1 and other issues related to the missing pressure. Several points should be emphasized. First, a rather minor point, is that residual ram (or dynamic, \( \rho v^2 \)) pressure contributes, in principle, to total pressure. However, these are easily shown to be negligible in the ionosphere. Considering any residual ram pressure we note that the flow field model predicts a downward flow velocity of order 30 m/s [Cloutier et al., 1992] and the density is of order \( 10^5 \text{ cm}^{-3} \) from
Figure 8. Evaluation of missing pressure derived by comparing available solar wind ram pressure (ρV²) from the PV OPA, magnetic pressure (B²/2μ₀) from OMAG and kinetic thermal pressure (nk(Tₑ + Tᵢ)) derived from OPA and ORPA experiments [from Cloutier et al. 1992].
Figure 5 with an O\(^+\) dominant constituent. A calculation of \(\rho V^2\) from these values gives a residual ram pressure of \(2.4 \times 10^{-3}\) nano-\(\text{Pa}\) which is negligible compared to values reported for missing pressure. It has also been determined that, at the lowest levels in the ionosphere, a small contribution from the gravitational potential causes Equation 1 to understate the missing pressure at the lowest altitudes. In addition, a small error occurs in the vicinity of the exobase at 170 km because isentropic flow along streamlines begins to break down due to collisional processes with the neutral atmosphere.

There appears to be significant confusion about the role of \(K\) in the momentum budget relationship noted in Equation 1. Some of the early literature suggests the possibility that the solar wind flow might be reflected at the shock in a kind of specular reflection. Thus a distinction can be made between the completely inelastic flow with a \(K\) factor of unity and a specular interaction in which the vertical flow component is completely reversed thus doubling the momentum flux. The shock interaction is now assumed to be completely inelastic and so there is no doubling of the momentum flux.

There is nevertheless a hydrodynamic effect which governs the ratio of stagnation pressure at the ionopause to the free stream ram pressure. Spreiter and Stahara [1992] refer to this hydrodynamic \(K\) factor as the "Newtonian" approximation and point out that for non-zero solar zenith angles the relation cannot be derived analytically but has been found to be "empirically" true for the dayside interaction. They use a hydrodynamic "\(K\)" factor in the range of .881 to .844. In justifying \(K\) less than unity, Spreiter and Stahara [1992] cite Landau and Lifshitz's fluid mechanics text which uses the well known shock jump conditions followed by adiabatic transfer along streamlines starting at the post-shock point and stagnating in the ionosphere. Incidentally, the derivation of the \(K\) factor using the shock jump conditions and adiabatic transfer along the streamline stagnating at the nose of a blunt body has become a classic plasma physics homework problem. From
examining their [Spreiter and Stahara, 1992] work we believe that their description of the relationship as "empirical" is not sufficiently articulate to describe what they mean. We think it is better to say that the relation accurately describes the pressure balance relation at the ionopause as determined from the self-consistent solution of the shock and ionosheath interaction. The word "empirical" suggests that it is determined from measurements which is not correct. It is, however, a very accurate relationship for solar zenith angles less than about 50° as indicated by the figures and descriptions in Spreiter and Stahara [1992].

To see the \( K \) factor effect in another way consider the integration of force per unit volume along a radial line extending from the unshocked solar wind to the surface of the planet in Spreiter and Stahara's flow field. If we did this we would find that in addition to the static pressure gradient and gravity we would have a component of momentum flux along the radial line and a component of centrifugal force due to the curving of the streamlines around the planet supporting some of the load of the overlying flow. This is the origin of the hydrodynamic \( K \) factor.

An important complication was noted recently. The solar wind has typically been assumed to be composed of protons. However, the 3 to 5\% Helium (alpha particle) abundance tends to increase the effective momentum flux. It has been proposed that this effect cancels the less than unity \( K \) factor cited above leaving an effective \( K \) of approximately 1.0 [see Cloutier et al. 1992 and works cited therein]. The bottom line with respect to the \( K \) factor in Equation 1 is that \( K \) is of order 1.0 to 1.05.

We might object that the missing pressure seen in Figure 8 might reflect an inaccuracy in the \( K \) factor discussed above. A little thought, however, causes us to realize that any effect resulting from \( K \) factor inaccuracy manifests itself as a stepwise offset rather than a monotonic decline with altitude as seen in Figure 8. We concede that there remains the possibility that some form of sampling bias from the spacecraft coverage might result in
the structure seen in figure 8. Nevertheless it is difficult to see how this might happen unless there was a residual solar zenith angle dependence on the K factor not modeled by the $\cos^2(\chi)$ term in equation 1.

Equation 1, then, provides a quantitative estimate of the amount of disagreement between the observations and the momentum conservation requirement. In principle, the ability to resolve any missing component in the momentum budget could be affected by transient variability in the solar wind or the transient decay of the previously existing conditions. The works by Cravens et al. [1984] and by Cloutier [1984] establish, however, that the flow-field adjusts to transient variability in the solar wind on time scales that are short compared to the convective time scale of the flow in the ionospheric plasma. This implies that we should not expect the flowing medium to exhibit any significant "memory" of its previous condition. Neither should we expect that Pioneer Venus observations would reflect the decay of "remnant" features of that condition. This further implies that time dependent variations in the solar wind will be transmitted within minutes along fluid flow lines extending from the solar wind through the ionosheath and into the ionospheric envelope where some flow lines will stagnate at the lowest parts of the ionosphere. Thus, it is sufficient to examine the physics of a steady state response embodied in Equation 1. This should not be understood to mean that transients in the solar wind are unimportant. On the contrary; because the effects of the transient variability are transmitted so quickly through the flowing medium, we must account for the possibility of confusing spatial features of the steady state with temporal variations in the solar wind as the spacecraft moves along its orbital track. Brace et al. [1980] speculate that this could account for some of the scatter seen in Figure 7 but that does not explain the systematic deficit seen. In the Cloutier et al. [1992] missing pressure paper, solar wind variability was minimized by examining the solar wind velocity from the Plasma Analyzer (OPA) and carefully eliminating orbits from consideration which
exhibited large variations in solar wind velocity outside the shock. This procedure reduces the chance that sudden increases in the solar wind, possibly associated with dynamic changes in the underlying ionosphere, would be misidentified in the statistical result as a missing pressure signature. We should add however that this viewpoint is in strong contrast to opinions stated by, for example Luhman et al. [1984], who, perhaps until recently promoted the idea that the transient variability will only manifest itself in the upper most layers of the ionosphere because of a proposed shielding effect of the ionosphere.
IV. ION ACCELERATION MECHANISMS

From the preceding observational scenario it is possible that the missing pressure seen in Figure 8 may be explained by a downward momentum flux of a poorly measured superthermal population. Assuming the existence of such a population at and just above the ionopause, we will examine various mechanisms affecting the density and associated pressure of these ions in the ionosphere. We also examine the proposed acceleration mechanisms and attempt to come to some logical conclusions about the superthermal source.

The source of the superthermal ions is a compelling issue that has been addressed by several workers in a rather peripheral way. A newly created ion in the fore-shock region will be subject to solar wind pickup and acceleration to energies of up to several keV. The ions are observed to be quickly transported downstream to populate the wake region [Intrilligator 1989, Slavin et al. 1989]. Transferring them to low altitude on the dayside with an intervening draped B field present is difficult to understand. We speculate that if there were extremely high energy, kilo-volt class ions produced outside the shock they might be able to penetrate but this scenario would be inconsistent with some of the observations which we will discuss shortly. Another mechanism involving plasma wave interaction through the Landau damping of whistler mode waves has been proposed [Brace et al. 1980] and some evidence of this is reported by Taylor et al. [1981] and Kasprzak et al. [1982] in the form of the apparent decrease in amplitude of low frequency electric field waves seen in Figure 1 as the satellite crosses periapsis. The theoretical justification is mostly speculative but the observation appears to be sound. Some theoretical discussion and empirical analysis of the coupling of low frequency wave activity to the plasma environment at Mars, is provided by Nagy et al. [1990] and by
Sagdeev et al. [1990] and we expect the analogy between flows at Mars and Venus to make these studies relevant eventually.

Little progress appears to have been made in understanding the details of the flow at and just below the ionosphere and this is likely due to deviations from MHD behavior associated with the superthermal activity or collisional processes. The Spreiter and Stahara work employs an a priori ionopause boundary and so their work does not help in understanding its formation. Also, their's is not a full MHD simulation but rather a hydrodynamic solution with the magnetic field configuration determined by the frozen in flux condition.

At and just above the Venus ionosphere, the shocked solar wind flows through stationary ions of planetary origin. This could stimulate stream instabilities and some progress has been made in this area which will be discussed in a later section. In general an instability must be present in order to produce a high energy population of ions and collisions are necessary to increase the entropy of the plasma as it relaxes to a characteristic Maxwellian. The Boltzman equation is:

\[ \frac{\partial f}{\partial t} + v \cdot \nabla f + (E + v \times B) \cdot \nabla f = C(f) \]  \hspace{1cm} (2)

where the \( C(f) \) term represents collisions or wave-particle interactions associated with coordinate or velocity space encounters between plasma particles. In the Vlasov equation, which is usually invoked, the \( C \) is set equal to 0 and the equation is usually linearized:

\[ -i(\omega - k \cdot v) f_{ij} + \frac{q_i}{m_j} (E + v \times B) \cdot \nabla f_{oj} = 0 \]  \hspace{1cm} (3)

Equation (3) along with the Maxwell equations which are already linear in \( f \), \( E \) and \( B \) constitute a self consistent system which can be used to examine instability and wave phenomenon. The principal conclusion from the study of the self consistent plasma oscillations is that when there is a population inversion of high velocity particles, such as
streaming ions, then the dispersion relation exhibits complex characteristic frequencies \( \omega \) with the a positive imaginary part \( \gamma = \text{Im}(\omega) > 0 \). These represent growth rates of unstable oscillations.

Several authors have made the important point, however, that with \( C(f) = 0 \), the linearized system is reversible and so no increase in entropy is possible [Swanson, 1989; Shapiro and Shevchenko, 1988]. Perhaps a simple way to see this is to consider the analysis of a cold beam component. The Vlasov equations (defined here as the linearized Boltzman equation with the 4 Maxwell equations plus the continuity equation) can be used to show that the beam leads to exponential growth in the amplitude of longitudinal waves. However, efforts to step the equations forward in time do not evolve toward an equilibrium because there is no term tending to remove particles from the beam and redistribute them to lower velocities. It is also apparent that due to the deterministic nature of the Vlasov equations, evolving the phase space trajectory of a system with the time reversed, will regenerate the original conditions. So, in the system without a collision term, no information is lost and no evolution toward an equilibrium occurs. Collisions, in an abstract sense, represent the discontinuous "destruction" of a particle in a specific state \( (r, v) \) and its simultaneous "creation" at some other point in phase space. This is another way of saying that entropy is increased by the collisions.

The collision term, however, is not simply confined to particle-particle interaction. Non-linear interaction of the particles with waves can redistribute particles in velocity space and thus lead to effective relaxation to equilibrium. This point is made by considering Figure 9 which is a cartoon representing the two dimensional potential seen by a near resonance particle. Assuming the energy density of the waves is large so that it is not much disturbed by the particle motion, we see how the ion effectively "falls" into a hole and then emerges with essentially the same energy but rotated into a new direction. This picture has relevance to a wave-particle scattering mechanism which we believe to
Figure 9. Cartoon illustrating how a resonant ion can interact with the disturbed electric field component to effectively "fall" into a potential hole and emerge with a rotated velocity vector.
be operative on the superthermal ions in the atmosphere and this will be discussed in a following section.

Recently, work by Szego et al. [1991] has suggested a stream instability, associated with the shocked solar wind flowing in the vicinity of the ionopause might be responsible for superthermal production. Some aspects of the beam instability may be understood by considering the dispersion relation in the unmagnetized case:

\[ 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pb}^2}{(\omega - k \cdot v)^2} = 0 \]  \hspace{1cm} (4)

where the "pe" and "pb" subscripts refer to the plasma frequency associated with, respectively, the background plasma and the beam component electrons. The beam effectively "sees" a Doppler shifted frequency which explains the second term involving \( k \cdot v \) [Landau and Pitaevskii, 1981]. The expression is derived by substituting the linearized forms of the continuity equation and the momentum equation into the Poisson equation for self consistent electric fields. Equation 4 follows from seeking roots giving a homogeneous solution (setting determinant equal to zero).

If we seek near resonance solutions to Equation 4, \( \omega = k \cdot v + \delta \) where \( \delta \ll \omega \), we recognize that any complex component in \( \delta \) represents a growth rate which makes the mode unstable to perturbations. Substituting \( \omega = k \cdot v + \delta \) into Equation 4 gives:

\[ 1 - \frac{\omega_{pe}^2}{(k \cdot v)^2} - \frac{\omega_{pb}^2}{(\delta)^2} = 0 \]  \hspace{1cm} (5)

which can be rearranged to yield:

\[ \delta = \frac{\pm \omega_{pb}}{\sqrt{1-(\omega_{pe}/k \cdot v)^2}} \]  \hspace{1cm} (6)

from which we realize that an imaginary root arises when:

\[ (k \cdot v)^2 < (\omega_{pe})^2 \]  \hspace{1cm} (7)
At and just above the ionopause $v \sim 100$ km/sec and so we can determine the scale length of excited waves assuming an unmagnetized limiting case or that the flow is aligned with the magnetic field. This results are summarized in Table 1.

<table>
<thead>
<tr>
<th>ionosphere $N_e$ [cm$^{-3}$]</th>
<th>$f_{pe} = (\omega_{pe}/2\pi)$</th>
<th>$2\pi/k$ (max wavelength) assuming $v=100$ km/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>63.7 kHz</td>
<td>1.6 meters</td>
</tr>
<tr>
<td>1000</td>
<td>285 kHz</td>
<td>.35 meters</td>
</tr>
</tbody>
</table>

Table 1. Wavelength of unmagnetized longitudinal waves is too short and frequency is too high to explain electric field measurements from PVO.

We see that the frequencies and required wavelength in the simple unmagnetized case do not correspond to the measured quantities.

In the Szego et al. work, the above simple analysis has been extended to a magnetized plasma with two preferred directions. One, in the direction of the magnetic field and the other in the direction of wave propagation. When this is done, they find growth rates comparable to the lower hybrid frequency, with propagation direction nearly perpendicular to the ambient magnetic field.

This mode, in which $k \cdot B = 0$, is sometimes referred to as a flute mode because it resembles fluted columns in laboratory plasmas with cylindrical symmetry. Szego et al. suggest that the post-shock and ionospheric plasma may be treated as a cold magnetized plasma with an unmagnetized beam component resulting from the shocked solar wind flow passing through cold plasma of planetary origin. The cold plasma dispersion relation for a plasma consisting of electrons and a streaming ion component is:

$$D(\omega,k) = K \frac{\omega_{pe}^2}{\omega_e^2} - \frac{k^2 \omega_{pe}^2}{k^2 \omega_e^2} - \frac{\omega_{pb}^2}{(\omega - k \cdot v)^2} = 0$$

(8)

where the pe, ce and pb subscripts again refer to respectively, electron plasma, electron cyclotron and ion beam plasma frequencies and $K$ is an electromagnetic correction...
deriving from the ratio of parallel and perpendicular components of the Electric field [by personal correspondence K. Szego 1992]:

\[ K = 1 + \frac{\alpha_{pe}^2}{k^2c^2} \]  

(9)

In their solution they also make the assumption that \( \textbf{k} \) and \( \textbf{v} \) are parallel and oppositely directed.

Solving this dispersion relationship, they show that waves might be generated in the high altitude post-shock mantle and propagate downward providing a mechanism for superthermal ion production near the ionopause. They make the important point that the distance scale for \( \textbf{E} \times \textbf{B} \) pickup in this region is too long to be an acceptable mechanism near the ionopause. They show, however, that the energy flux from the observed wave activity is more than sufficient to energize superthermal ions.

On the other hand, the cold plasma relation which they use may not be appropriate since the thermal electron velocity is comparable to the superthermal ion velocity. The thermal background ionosphere exhibits temperatures of a few thousand Kelvin [Knudsen 1979]. From the temperature, a thermal velocity may be expressed:

\[ V_t = \sqrt{\frac{2kT_j}{m_j}} \]  

(10)

A characteristic superthermal velocity is

\[ V_{sj} = \sqrt{\frac{2E}{m_j}} \]  

(11)

where \( E \) is the superthermal ion energy and \( \omega_{cj0} = qB_0/m_j \) is a gyrofrequency determined from a characteristic magnetic field magnitude of \( B_0 = 100 \) nT.

The parameter \( \rho_j \) in Table 2 is the distance that any of the charged species travel over its nominal gyro-period based on a 100 nT field. This provides a scale length measuring the degree to which the various plasma components are affected by the overlying magnetic field. The larger this parameter the closer the species is to effecting a
straight arc rather than a gyro-motion around the magnetic field. Comparison of the numbers in Table 2 reveals that the superthermals are unmagnetized relative to both the

<table>
<thead>
<tr>
<th>Thermal component</th>
<th>Velocity</th>
<th>Magnetization scale length</th>
</tr>
</thead>
<tbody>
<tr>
<td>thermal e⁻ (2000 K)</td>
<td>V₁ₑ = 250 km/sec</td>
<td>0.014 km</td>
</tr>
<tr>
<td>thermal O⁺ (1000 K)</td>
<td>V₁ᵢ = 1 km/sec</td>
<td>1.7 km</td>
</tr>
<tr>
<td>10 eV O⁺ (superthermal)</td>
<td>Vₛᵢ = 11 km/sec</td>
<td>18.4 km</td>
</tr>
<tr>
<td>100 eV O⁺ (superthermal)</td>
<td>Vₓₛᵢ = 140 km/sec</td>
<td>230 km</td>
</tr>
<tr>
<td>Shocked solar wind, H⁺</td>
<td>Vₛₜₜ = 100 km/sec</td>
<td>10 km</td>
</tr>
</tbody>
</table>

| Table 2. Comparison of plasma component velocities and magnetization scale lengths. The parameter ωₑ is the cyclotron frequency: qB₀/mₑ. |

electrons and the ions in the plasma. The parallel component of the electron motion does appear to be dominantly thermal and so we can see that parallel wave motion will be strongly damped. For comparison, the inverse wavenumber of the 100 Hz band activity, (assuming slow phase speed and a spacecraft speed of order 10 km/sec) is dimensionally the same as the magnetization scale length. That is, \( \frac{1}{k} = \frac{Vₛₑ}{ω₁₀₀Hz} \) and is equal to of order 10 to 20 meters.

A generalized solution to the hot, magnetized dispersion relation, however, appears to be extremely difficult and very likely intractable if for no other reason than that the velocity distribution functions are not well known. Swanson [1989] presents a fully general relation reflecting the complexity of such an undertaking. He does discuss so-called Bernstein modes which propagate perpendicular to the magnetic field and reduce to lower hybrid waves in the cold plasma limit. The dispersion relation for the Bernstein modes is:

\[
D(k₂,k₁,ω) = k₂(1 - \frac{ωₑ²}{ω²}) + k₁(1 + \frac{ωₑ²}{ωₑ}) - \frac{ωₚᵢ²}{Vᵢ²}Z'(-\frac{ω}{kVᵢ}) = 0
\]  

(12)

Where the \( Z(ζ) \) and \( Z'(ζ) \) are the so called plasma dispersion function and its derivative which are defined in appendix B.
Before describing the simulation of superthermal ions we note that any instability analysis, only predicts the onset of growth whereas, the actual Venus environment reflects a fully developed flow. We know that considerable horizontal velocity shear exists at and above the ionopause. Kelvin-Helmholtz instabilities have been invoked to describe observed "flux-rope" type magnetic field observations [Cloutier 1983]. Another suggested possibility for an instability source are ion-cyclotron waves which are associated with anisotropic temperature distributions in which the perpendicular temperature ($T_\perp$) is greater than the parallel temperature ($T_\parallel$). These kind of waves have been associated with the Pc 1 and Pc 2 pulsations in the Earth's magnetosphere [Gary, 1992, Fraser et al., 1992]. Like the lower hybrid beam instability, they have growth rates which are maximum in the direction perpendicular to the overlying magnetic field but their frequency, on the order of .1 to 10 Hz, appears to be much too low to account for the 100 Hz energy observed in the Pioneer Venus measurements.
V. ION TRAJECTORY MODEL

The model integrates the equation of motion for a population of ions using virtual test particles selected from a distributed population and using time history of the test particle as a tracer for subsets of the distributed population. The dynamic effects from many such subsets are then added up and presented as a normalized missing density and pressure.

The geometry for this model is shown in Figure 10. The magnetic field is dependent on the Z coordinate with a single component in the X direction and the electric field is constant over the altitude range and oriented in the Y direction. As touched upon previously, Cloutier et al. [1987] predict that the overlying downward bulk flow of plasma associated with the solar wind absorption must impress a nearly constant horizontal "frame effect" electric field along streamlines extending from the high altitude region in the post shock mantle all the way down through the ionosphere to a stagnation point at the exobase. Due to the approximate requirement that $\mathbf{E} = -\mathbf{V} \times \mathbf{B}$ and that $\nabla \times \mathbf{E} = \mathbf{0}$ in steady state, the electric field must be orthogonal to the velocity streamline and equipotential along it with magnitude given by $\mathbf{E} = \mathbf{V}_H \mathbf{B}_H$ where the H subscript indicates high altitude (i.e. just below the ionopause). Their model result shows that the expected downward velocity is of order $\mathbf{V}_H = 30 \text{ m/s}$ which is the value we have adopted for the present work. We emphasize that we are only seeing the vertical component. We assume that the ion in the flow field also has a field aligned, horizontal component (the horizontal component of Figure 2) parallel to the X direction in the coordinate system.

In the regions where superthermal signatures are seen, $\text{O}^+$ dominates and we assume that the test particles employed in the model are composed of that species. Discounting for the moment the effects of collisions, the time evolution of the particle's position is:

$$\frac{d^2 \mathbf{R}}{dt^2} = \frac{q}{m_0}(\mathbf{E} + \mathbf{V} \times \mathbf{B})$$  \hspace{1cm} (2)
Figure 10. Coordinate system used in model. Z coordinate is up. "Frame effect" electric field lies entirely in Y direction and is constant down to exobase with magnitude of $V_H B_H$ where $V_H$ is the downward bulk plasma flow velocity and $B_H$ is the magnetic field in the region just above the ionopause. Magnetic field is horizontal and is taken to be in the X direction. Particle trajectories are modeled in the Y-Z plane. Test particles with a superthermal energy are dropped into the flow and dynamic effects of the impressed electric and magnetic field are integrated. Various collision processes are also simulated. Note that there is field aligned hydrodynamic flow in the X direction assumed as indicated in Figure 2 but not evident in this figure.
where $q$ is ion charge and $m_0$ is its mass.

The prototype magnetic field profile used in the model is shown in Figure 11a which was chosen to reflect essential characteristics of the observed dayside magnetic field. A 4th order Runge-Kutta method was used to integrate Equation 2 in the Y-Z plane and the result for a 10 eV ion starting at 540 km is shown in Figure 11b. The most significant advantage of the Runge-Kutta method is that it is an explicit method which only requires information about the initial state of the system and the derivatives at that point. Studies indicate that the 4th order Runge-Kutta integrator used in this work delivers the most economical tradeoff between accuracy and execution time [Gerald 1978]. The test shown in Figure 11 neglects the effects of the neutral atmosphere and wave particle interactions to illustrate some salient features. First, the dark vertical line to the right hand side of the figure between about 300 and 520 km represents the superposition of many nearly overlapping circles as the ion cyclotron motion $\mathbf{E} \times \mathbf{B}$ drifts downward. The axes have been expanded to show the entire trajectory causing us to lose resolution of the cyclotron orbit in the strong field region. Most startling is the large horizontal drift associated with the ion encountering the field depletion at about 300 km. The oscillating orbit in Figure 11b clearly follows the direction of magnetic gradient drift upon encountering the region of magnetic field depletion, reversing direction at the maximum and minimum of magnetic field. We have referred to this as a "pseudo-gradient" drift to distinguish it from the MHD gradient drift in which the gyro-radius is small compared to the scale of the gradient. The semantic distinction is necessary because an analytical expression exists for the MHD case. For thermal velocities, gradient drift ($V_{\text{gradient}} = m_0 V^2/2qB^2$) is only a small fraction of the drift motion seen in Figure 11. We did determine that the model approaches the ideal gradient drift for initial ion velocities which are close to thermal velocities.
Figure 11. Charge exchange and wave-particle effects are turned off to illustrate startling horizontal drift and other interesting features. Trajectory of 10 eV O⁺. Without some scattering mechanism, ion makes many brief excursions to the exobase where it accumulates significant probability of charge exchange without penetrating very deep into the ionosphere on average.
Figure 12. Illustration relating how the single ion trace acts as a proxy to the trajectory of an infinite horizontal plane of particles all having the same initial altitude and orientation.
The single ion trace acts as a proxy to the path of a population of identical particles having initial energy and initial release angle confined to narrow ranges. This subset of the superthermal population is characterized by a production rate per unit horizontal area which can be thought of as a downward flux rate. If we could somehow remove all the other particles in the population except this particular component we should witness the concerted motion of an infinite, horizontal plane of particles as illustrated in Figure 12. These move downward along the trajectory with a new plane of particles created continuously in an average period ($\tau$) related to a production frequency.

It is shown in the Appendix B that by employing the Dirac delta function descriptions of this vertical coordinate of the population, and then time averaging over a production period, the contribution to number density and horizontal current is given by

$$\langle n(Z) \rangle = \frac{\lambda}{\tau \Delta Z} \sum_i \Delta t_i$$

(3)

$$\langle j(Z) \rangle = \frac{q\lambda}{\tau \Delta Z} \sum_i \Delta Y_i$$

(4)

and that downward momentum flux or pressure is:

$$P(Z) = \left[ m_0 n V_Z^2 \right] = \frac{m_0 \lambda}{\tau} \sum_i |V_Z(t_i)|$$

(5)

where the additional parameters are $\lambda$, which is initial density per unit horizontal area; $\tau$, the production period; $\Delta Z$, a small audit zone with a center at altitude $Z$; $\Delta t_i$ and $\Delta Y_i$ which are time spent and aggregate horizontal distance covered by the particle during the $i$'th traversal of $\Delta Z$. $V_Z(t_i)$ is vertical velocity component at the $i$'th point of intersection of the trajectory with a referenced altitude $Z$. The summations are taken, in the case of equation (3) and (4) to be over the total number of traversals that the particle makes in the altitude zone $\Delta Z$ and in the case of equation (5), over the trajectory intersections with the referenced altitude $Z$. Equation (3), (4) and (5) represent the contributions from the subset of ions all having the same initial energy and initial starting orientation. We will
see in a later section that appropriate weighed averages across a spectrum of initial energies and averages over many trajectories using a Monte-Carlo simulation of charge exchange are used to develop total macroscopic density, current and pressure.

The current in equation (4) is assumed to be due to drift of ions only. Since the electron gyro-radius is smaller than the O$^+$ gyro-radius by a factor of approximately 29000 ($= m_o/m_e$), the electrons are tightly coupled to the ambient magnetic field over vertical gradient scale lengths and are well within the MHD regime. Coulomb interactions with ions is the dominant form of interaction experienced by electrons in the altitude range of interest. Due to the ion's large mass, Coulomb collisions have no effect on ion momentum. These same collisions are, in strong contrast, highly effective in randomizing any non-thermal electron motion. From this physical argument we realize that the electrons are thermalized on a time scale which is much shorter than a comparable thermalization time scale for ions. Thus we assume that the electrons are 1) dominated by processes described by ideal MHD theory governing a thermal, magnetized plasma and 2) any electron velocity space anisotropies are relaxed to Maxwellian distributions on time scales that are negligible compared to the ions. These assumptions imply that the electron's response to the magnetic gradient will not contribute to the horizontal current and is the justification for our neglecting them in this work.

Figure 13 is the numerical evaluation of Equations 3, 4 and 5 for the trajectory in Figure 11 using an algorithm developed for the master's thesis [Kramer, 1991]. The modeled ion density (normalized to unit production rate) in Figure 13a exhibits a startup transient between about 540 and 470 km where the density is increasing to a uniform normalized value determined to be 33.33 s/km. The transient region is 2 gyro-radii wide. Since the altitude range does not sample all parts of the circular arcs until the ion has drifted down this far, the density gradually increases to the uniform value in the region
Figure 13. Numerical evaluation in accord with equations 3, 4 and 5 of (a) density, (b) pressure and (c) horizontal currents for 10 eV ion shown in Figures 11. Units may be rationalized by multiplying each profile by the downward flux of superthermal ions ($\lambda/\tau$). Flat density profile of 33.33 sec/km in (a) between 350 and 560 km corresponds to the 30 m/s bulk flow resulting from $E \times B$ drift in this region. Large horizontal currents in (c) may result in beam or other instability.
from about 350 to 470 km and is consistent with an average flow of 30 m/s as required by the imposed conditions (ie: 1/30 m/s = 33.33 s/km). Thus we confirm the ability of the code to reproduce $E \times B$ drift which is an essential feature of the flow mechanism. As distinct from magnetic field gradient drift discussed earlier, the $E \times B$ drift mechanism does not have a requirement that the gyro-radius be small. Kinetic energy calculated from modeled total velocity and potential energy determined from horizontal displacement against the imposed electric field were compared to test consistency. It was found from these independent model outputs, that the model conserves energy to a level of resolution necessary for this investigation.

The large horizontal current seen in Figure 13 at about 300 km results from the ion encountering the field depletion at about 300 km. This may not be realistic since it is believed that very large coherent currents will be unstable to beam instabilities. As mentioned briefly in our discussion of ion acceleration mechanisms, the beam instability, arises from small charge density fluctuations which lead to opposing fluctuations in the beam velocity due to the requirement of charge flux conservation. The associated feedback effect on density causes exponential growth in the original fluctuations. It is possible that this instability represents a scattering mechanism for the superthermals and we will address that in a following section.

In addition, the ac electric field activity in the moving spacecraft frame is quite large and we might anticipate momentum coupling between electrostatic waves and any superthermal ions. The Pioneer spacecraft provides measurements of electric field activity in the form of a four channel electric field detector. Although the instrument has the sun oriented spin modulation defect noted earlier the 12 second averaged UADS database removes this modulation and may provide a diagnostic of the region associated with superthermal energization. An example height profile of the OEFD data from UADS is shown in Figure 14. We can see that the low frequency channel peaks several
Figure 14. Altitude profile of "de-spun" electric field and electron density data produced from the UADS data set. Peaking up of electric field as noted in figure 5 is also present here. Electric field activity might be used as a diagnostic of the superthermal energization region. In this thesis, the ion production region was varied over a large range of altitudes. The uniqueness of a particular shape of the production region cannot be determined. It was found however, that a normally distributed production region peaking in the same altitude region as the electric field 100 Hz signal was found to satisfactorily populate the lower altitude ionosphere and match missing pressure.
tens of kilometers above the ionopause designated by the point where the electron density makes its sharp turn near 330 kilometers. This electric field enhancement appears to be consistently associated with the ionopause on many orbits. The actual mechanisms determining the location of ionopause are still not well understood but its association with superthermals has been postulated [Matney 1991]. These observations are also consistent with the lower hybrid beam instability reported by Szego et al. [1991].

As the ions drift downward, they eventually reach a point where the neutral atmosphere becomes important to their dynamics. As it moves along its path we see that the ion samples the low altitude region near the exobase many times and so accumulates significant probability of charge exchange with the neutral atmosphere. Collision probability is treated as a modified Poisson process with the distribution resulting from the fact that \( v \), the collision frequency, is not constant but varies due to the changing neutral density with altitude as the particle's altitude changes. From Figure 15, showing the neutral atmosphere density [Kasprzak 1990] we see that the altitude variation of the dominant atomic oxygen is several orders of magnitude.

In a Poisson process the time between collisions is an exponential random variable with a cumulative distribution

\[
F(t) = 1 - \exp\left(-\int_0^t v dt\right)
\]  

(6)

which follows from accumulated collision probability over many small time segments.

The collision frequency of a projectile such as the superthermal ion moving through a distributed population of targets like the neutral atmosphere may be expressed:

\[
v = n \sigma V
\]  

(7)

where \( n \) is neutral number density of targets (in our case neutral density which is a function of altitude), \( \sigma \) is the collision cross section and \( V \) is the speed of the projectile (superthermal ion). The charge exchange cross section for the reaction:
Figure 15. Neutral density from PV orbiter [Kasprzak 1990]. Atomic oxygen dominates on dayside. Suprathermal ions are assumed to be $O^+$ primarily because it is the dominant species in the proposed energization region. There is a remote possibility that they could be composed of some other species. This atomic Oxygen curve was fitted to a simple exponential and used in the determination of charge exchange probability for Monte-Carlo simulation.
O^{+}\text{HOT} + O \rightarrow O^{+} + O\text{HOT} \quad (8)

was studied by Rutherford and Vroom [1974] and we determined that the equation:

$$\sigma_o = (5.59 - 0.474 \log_{10}(K[\text{eV}])) \times 10^{-16} \text{ cm}^2 \quad (9)$$

where $K$ is the lab frame kinetic energy of the particle, fits their charge exchange data and is applicable in the range of 60 to 500 eV. A recent study of momentum transfer in charge exchange reactions by Hodges and Breig [1991] suggests that deviation from pure forward scattering in H-H\text{+} reactions does not become important until the projectile energy is substantially less than about 10 eV. Since the quantum mechanical spin symmetries involved in the interaction potential of the O-O\text{+} reaction of Equation 8 is much more complicated than the H-H\text{+} reaction, we must acknowledge some uncertainty in extending Equation 9 as low as 10 eV, but we have invoked this procedure as a best effort until better values $\sigma_0(K)$ become available.

Likewise, from the neutral density profile shown in Figure 15 reproduced from Kasprzak [1990], the equation

$$n_0(Z) = 3.9 \exp \left[ - \frac{Z[\text{km}]-140}{16.927} \right] \times 10^{-9} \text{ cm}^{-3} \quad (10)$$

was fitted to the noon time (12h LST) atomic oxygen curve.

Given these empirical expressions for cross sections and neutral density, collision frequency may be evaluated at any point in the trajectory and so Equation 6 becomes:

$$\int_0^S n_0 \sigma dS = - \ln(F(S)-1) \quad (11)$$

where $S$ is the distance traveled or path length along the curving trajectory ($dS = Vdt$). A similar equation has been derived by Hodges and Tinsley [1981] in their Monte-Carlo simulation of hot exospheric hydrogen at Venus and by Ip [1988] in a study of the atomic oxygen corona at Mars. Equation 11 may be used by substituting for $F(S)$, a uniform random deviate between 0 and 1 (a random number generator), and then numerically
accumulating the integral on the left hand side until it exceeds the value chosen on the right. This event then signifies a charge exchange and the end of the integration of the trajectory.

The question of ion-ion Coulomb collisions in this region has not been studied quantitatively. Although the abundance of ions is 3 to 4 orders of magnitude smaller than that of the neutrals, the screened Coulomb interaction is a long range force law with collision cross sections correspondingly large compared to the charge exchange cross section. According to Banks and Kockarts [1973], the total ion-ion collision cross section is, after some rearrangement, given by:

\[
\sigma_T = 2\pi\frac{k}{E}\left(\frac{\lambda_D}{E}\right) - \frac{k}{E}
\]

where \(k\) is the Coulomb constant (\(k = 1.439 \times 10^{-7} \text{ eV cm}\)), \(E\) is the energy (eV) of the projectile ion, and

\[
\lambda_D = 6.9\left(\frac{T_i[\text{eV}]\cdot L}{n[\text{cm}^{-3}]^{1/2}}\right) \text{ cm}
\]

is the Debye radius. The term in Equation 12, \(k/E\) is negligible compared to \(\lambda_D\) for characteristic ion temperature and density and so 12 can be rewritten:

\[
\sigma_T = 2\pi\frac{k\lambda_D}{E}
\]

\[
= 9.04 \times 10^{-7} \frac{\lambda_D[\text{cm}]}{E[\text{eV}]} \text{ cm}^{-2}
\]

These collisions represent the short range limit of particle interaction in a plasma and are one of the mechanisms for relaxation of a non-equilibrium velocity distribution (such as that associated with the superthermals) to a Maxwellian. Such terms are conventionally neglected because they are slow, but are important since Coulomb scattering and non-linear wave-particle interactions are the only way to increase entropy and thus relax to the equilibrium state. For conventional choices of ion temperature, density and particle energy, this Coulomb collision cross section in Equation 14 is roughly 6 orders of magnitude larger than the corresponding ion-neutral charge exchange cross section. That
having been noted, we investigate effectiveness of Coulomb collisions in transferring momentum. This is done by considering the cumulative cross section as a function of scattering angle:

$$\sigma(\theta) = 2\pi \left( \frac{k}{E} \right)^2 \frac{1}{\Lambda} \left( \frac{1}{1 - \cos \theta} \right)$$

(15)

where $\theta$ is the center of mass scattering angle, and $\Lambda = k/(E\lambda_D)$ is related to a minimum scattering angle [after Banks & Kockerts 1973]. A random scattering angle can be determined from the cumulative collision cross section given in Equation 15 according to:

$$\theta_{\text{RANDOM}} = \cos^{-1} \left( 1 - \frac{\Lambda}{1 - r^2(\Lambda - 1)} \right)$$

(16)

where $r$ is a uniform random deviate between 0 and 1. When the Coulomb collision is simulated, the energy and momentum of the projectile is decremented according to the geometry dictated by $\theta_{\text{RANDOM}}$. When this algorithm is put in the code, it is found that, although there are many ion-ion collisions, they are statistically ineffective in transferring significant amounts of momentum or in changing the energy of the superthermal ions. Small momentum transfer can be readily demonstrated analytically for this class of collisions by integrating the angle weighted change in momentum of a particle (a term proportional to $1 - \cos(\theta)$) but the associated small drag from many collisions under the conditions in the Venus ionosphere cannot be modeled in any obvious way except in the Monte-Carlo simulation. We nevertheless conclude that ion-ion interactions are ineffective in scattering or cooling the downward flowing superthermals.

In support of this we examine the work by Luhmann [1984] showing that in the ionosphere, ion-electron collision frequency dominates. However, these are only scattering collisions which alter the momentum of a target electron having no effect on the ion's momentum. Ion-ion collisions should have a similar frequency but, by the
theoretical discussion above we expect no effect from these interactions. Also, because O$^+$ has no molecular vibrational degree of freedom the way, for example O$_2^+$ has, electron recombination is insignificant for O$^+$ since it requires a three body collision to carry away the ion's kinetic energy. We then expect the height variation of neutral density to be a governing agent in determining how the superthermals are distributed. In particular, we might expect a monotonic drop in superthermal density as altitude decreases.

Equations 3, 4 and 5 are the tools necessary to determine the dynamic effect of a mono-energetic population. As it stands, the single trajectory is not useful to understanding the effects of a population of such particles. As noted earlier in our discussion referring to Figure 12, the single test particle trajectory does, however, act as a tracer for the subset of ions with a singular energy and initial release direction selected from a high altitude source region having an extended range of energies and initial orientations. By attributing a particular energy distribution to the population at high altitude, discrete subsets of the population may be followed downward. The total density and pressure from the initial superthermal population is then just the sum of the contributions from each discrete subset weighted according to the distribution of the initial population determined artificially at high altitude. In effect, to evaluate superthermal pressure we integrate:

$$P_T(Z) = \int_0^\infty w(E_H)P(E_H, Z)dE_H$$

where the term $P(E_H, Z)$ is the pressure for each energy ($E_H$) component determined from Equation 5 and averaged over multiple trajectories subject to Monte-Carlo charge exchange simulation. The function $w(E_H)$ is the initial energy distribution of the superthermal ions at high altitude and has the property:
Figure 16. Three postulated initial energy distributions for the source region spanning low, medium and high temperature distributions in the range of 10 to 200 eV. These were used to weight integrals of momentum flux for Monte-Carlo runs at several energies in this range. The point here is, not to quantitatively match the exact superthermal distribution, but to examine the gross effects of 1) a high energy tail. 2) an "bump on the tail" centered on an intermediate energy. 3) a possible high energy bump on the tail characterized by a increasing energy distribution. Should emphasize that the bulk thermal ion component, with energy below about 5 eV is not shown but would appear as a sharp spike with mean value barely distinguishable from zero on this scale.
\[ 1 = \int_{0}^{\infty} w(E_h) dE_h \]  

(18)

To examine effects of varying superthermal distribution we postulate the existence of hot, intermediate and low temperature distributions as shown in Figure 16. In practice, the integration of Equation 17 is done numerically at each altitude from model output at intervals distributed between initial energies ranging from 10 to 200 eV. We should emphasize that the distributions shown in Figure 16 do not include the bulk thermal background plasma. The thermal particles, having energies less than about 2 to 5 eV, if plotted, would appear on this figure as a sharply peaked Maxwellian energy distribution with a mean energy that might be barely distinguished from zero on this scale.

We had originally speculated that as the ions encounter the magnetic field depletion region at about 300 kilometers altitude, they would "break" loose from the magnetic field and quickly traverse the gap. When we carefully examine individual particle trajectories as in Figure 11, however, we observe that the superthermal ion exhibits a very shallow angle of attack when it encounters the low altitude magnetic field drop off near 300 km. We tried various magnetic field configurations including step-wise field depletions between 200 and 300 kilometers but were never able to obtain a configuration that lead to rapid transit across the gap. Thus, we conclude that nothing in the model can scatter the projectile ions directly to the nominal exobase. Rather, the ions exhibits many successive excursions to the region near the exobase where they accumulate significant probability of charge exchange without penetrating, on average, very deep into the ionosphere.

Ions also exhibit the large horizontal drifts associated with vertical gradients in the magnetic field. These drifts can drive large unstable currents in the plasma leading to a drag force or angle scattering proportional to the energy density in the local wave
Figure 17. Anomalous, cumulative scattering cross section used to choose angle in Monte-Carlo scattering events. This function was chosen in lieu of any measured or theoretically available information on how test particles scatter in the ionospheric region of interest. Increasing $\alpha$ has the effect of tending to more narrowly forward scatter the ions.
turbulence. Estimate of the energy density of local wave turbulence is beyond the scope of the present work. In any event the large modeled drift amounts to distances which, in some cases, is larger than the radius of the planet. These should be unphysical since the dayside is actually a magnetic "pinch" region. Ions would be expected to escape the ionosphere as they spiral around the planet along the field lines (along the "z" direction in this model) before they encountered the lower ionosphere.

After noting the failure of the model to populate the low altitude region, it was decided to incorporate an anomalous scattering of the ions. The scattering is simulated by assuming that ions recoil from scattering centers which are distributed along the trajectory. As mentioned earlier, such a process results in an exponentially distributed interval between collisions and when such an event is signaled in the simulation a scattering angle is selected from an artificial differential scattering cross section permitting a forward scattering or focusing of the particles. The cumulative cross section is shown in Figure 17 and is given by the simple relation:

$$\int_{0}^{\theta} \frac{1}{\sigma_{A}} \frac{d\sigma_{A}}{d\theta'} \, d\theta' = 1 \cdot \left( \frac{1 - \theta/\pi}{1 + \alpha \theta/\pi} \right)$$

(19)

The parameter $\alpha$ is a focusing parameter which when increased, results in a greater degree of forward scattering of the ions in collisions. We emphasize that this differential scattering cross section is chosen gratuitously based upon the physical expectation that ions will be primarily forward scattered. Since scattering from electrostatic waves is equally likely to cause either gain or loss of energy by the superthermals, no effort was made to adjust the post collision energy of the ions.

Although no particular cause for the scattering is assumed we postulate that such a mechanism is physically reasonable given the possibility of resonant interaction with the plasma turbulence associated with the disturbed ac component in the electric field. A similar collision operator has been invoked in a paper describing non-linear effects in the
Figure 18  Same as figure 11 except that now, charge exchange and wave-particle effects are turned on. Basic idea is that some process like this is necessary if the superthermals are the source of the missing pressure. In addition to momentum, these ions can also contribute to downward flux of energy.
Figure 19. Density, momentum flux (pressure) and horizontal current associated with individual trajectory of Figure 18. Note that the large horizontal currents of Figure 13c are small and appear to vary randomly around zero.
Earth magnetotail [Holland and Chen, 1991]. Although they model particles in phase space, the "ad hoc" nature of the collision process is similar to ours (indeed we have adopted the term "ad hoc" from that work). An alternate justification for the scattering is that a beam instability associated with the pervasive tendency of the high energy superthermal components to exhibit very large modeled horizontal drifts could manifest itself as a scattering. Without this ad-hoc scattering mechanism these currents can be strong even in association with rather weak gradients or random fluctuations in the magnetic field. To our knowledge, such an instability mechanism has not been described in the literature.

When this wave-scattering algorithm is incorporated in the model, the resulting trajectories are as shown in Figure 18 for a 10 eV ion. This trajectory was chosen as representative and it is seen that there is considerable stochastic variation in the path as expected. Figure 19, which is an evaluation of number density, momentum flux and horizontal current plotted to the same scale as in Figure 13 makes the point that the lower altitude region is populated to a much higher degree than in the result without wave-scattering. We also report that the overall residence time in audit zones is greatly reduced, and the large horizontal current disappears. Examination of number density, momentum flux and horizontal current reveal that the lower altitude region is populated to a much higher degree than in the result without wave-scattering.

Figures 20 and 21 show the evaluation for a similar trajectory at an energy of 40eV to illustrate the consistency of the result. These tests were also performed for higher energy ions with the same result.

We should also report a final observation about the integration time step. The accumulated charge-exchange and anomalous collision probability is proportional to the path length traversed during each integration step. The model places an implicit lower limit or bias to the distribution function since there is no way for the model to select path
Figure 20. Same as Figure 18 except for a higher energy ion (40 eV).
Figure 21. Same as Figure 19 except for higher energy ion (40 eV).
lengths smaller than the that afforded by the distance traveled by the particle in one integration time step. In other words, the numerical model cannot resolve distance between collisions smaller than the distance traveled in a integration time step. In our initial efforts using the lower energy ions, this presented no problem since the distance traveled by the ions was short compared to 99% of the random trajectory segments. However, a subtle effect was noticed when we investigated expected collision frequency versus that calculated in the model. Significant divergence was observed for the 150 and 200 eV particles. In thinking about this we realized that the quadratic dependence of energy on velocity and hence path length per time step (ie: Energy = mV^2/2) caused us to neglect the lower limit bias in the distributions. The problem was fixed simply by making sure the time step was small enough so that any trajectory segment was smaller than 95% of the random segments selected from the population. We also monitored the collision frequency to assure that it was within tolerances of that imposed by the theoretical mean collision frequency.
VI. RESULTS AND CONCLUSIONS

Integration of Equation 17 with the charge exchange logic turned on but the wave scattering process turned off for 70000 Monte-Carlo trajectories with initial energies distributed evenly among 10, 20, 40, 80, 100, 150 and 200 eV is shown in Figure 22. The source region has an isotropically distributed initial release angle and is normally distributed in altitude at 350 km with a standard width of 5 km. The prototype B field profile of Figure 11a is used. The abscissa scale is in units normalized to unit superthermal production rate and particular emphasis is placed on the comparison with the overplotted, observed missing pressure from Cloutier et al. [1992] in the region between 350 km and the exobase at approximately 170 km.

Although the magnitude of the maximum pressure does increase with the location of the peak in the weighting function as might be expected, the overall shape of the pressure profile is rather independent of the initial energy distribution. With the wave-particle interaction process absent, we have difficulty matching the missing pressure in the region below about 300 km. We attempted to estimate various parametric dependencies to see if we could match the lower altitude region to missing pressure. Sensitivity to charge exchange cross section was tested by reducing and then increasing it by an order of magnitude. These changes had no essential effect other than to produce a relatively minor altitude shift in the entire curve. Failure to populate the low altitude region by this test excludes the possibility that we are encountering some exotic problem with the cross section or that we are using an inappropriate neutral atmosphere model. We also varied the source region width and altitude and found that only by distributing the source across low altitude could we populate it with modeled pressure. This is unsatisfactory since the ions are not believed to be produced inside the ionopause and in any event the resulting fit to the observed missing pressure is poor.
Figure 22. Without any mechanism to scatter ions directly across magnetic field depletion region, model fails to satisfactorily populate region below about 300 km. Figure shows modeled superthermal pressure for the several temperature profiles shown in Figure 16 with the effect of charge exchange using equation 9 and neutral density from Figure 15 (Equation 10). The unsatisfactory comparison is indicated particularly by the lack of correspondence between the model (smooth curves) and the measured missing pressure (big dotted curves).
The wave scattering algorithm has essentially two parameters, 1) a mean free path, 1, and 2) the focusing parameter $\alpha$ in Figure 17 (and Equation 19). These were systematically varied in decades to find a result which populated the low altitude region with a pressure profile similar to the "observed" missing pressure. Best fits were found for $\alpha = 10$ and $l=10$ km and that result is shown in Figure 23. These are also the parameters selected post-hoc in our presentation of the stochastic trajectory examples shown in Figure 18 and 20. A value of $l = 1$ kilometer was also found to satisfactorily populate the low altitude region. This suggests that the scale length for scattering is of order 10 km or less. Although Cloutier et al.’s [1992] average missing pressure only spans altitudes below 360 km, we examined the individual pressure profiles and found that the missing pressure does indeed level off with altitude above the ionopause in the same way as is seen in the modeled result. For altitudes much higher than the ionopause we might expect horizontal convection to remove the superthermals. Alternatively, we speculate that the wave scattering mechanism is not operative for regions more than a few gyro-radii above the ionopause which is in accord with the theoretical predictions of Szego et al. [1991]. In that case, any superthermals scattered upward would be reflected back into the ionosphere.

As previously discussed, we have shown that the identification of the "missing pressure" in the dayside Venus ionosphere as superthermal ions is possible if the superthermal ions are scattered by a mechanism involving a scale length of order 10-20 km. The correspondence of this scale length with the wavelength calculated for the growth of a lower hybrid beam instability suggests that the scattering mechanism is associated with the solar wind interaction at the ionopause, which may be responsible both for the production of the superthermal ions and their subsequent distribution through the lower ionosphere.
Figure 23. Same as Figure 22 except that now, anomalous wave-particle scattering is included in the model. Mean free path for wave-particle interaction is 10 km and the angle distribution function parameter, α=10, from Figure 17. These parameters were selected empirically by varying them in decades until the lower altitude region below 300 km could be populated with a pressure profile similar to the observed missing pressure. This makes the point that the simple wave/particle scattering mechanism can produce pressures in the low altitude region.
This suggestion is reinforced by examination of Figure 24, in which we have plotted "missing pressure" as a function of distance below the ionopause, rather than as a function of absolute altitude for different solar wind conditions as shown in Figure 8, taken from Cloutier et al. (1992). Figure 24 clearly indicates that "missing pressure" exhibits an orderly variation with distance below the ionopause independent of solar wind pressure or ionopause altitude. This in turn implies that the missing pressure distribution is independent of any planet-fixed feature such as neutral density. Thus we conclude that the missing pressure is controlled by mechanisms associated with the ionopause consistent with wave-particle scattering due to the presence of a lower hybrid type electrostatic fluctuation. This interpretation is further reinforced by the fact that the location of our model's source region, just above the magnetic field depletion region, coincides with observed low frequency electric field activity seen in the OEFD data and noted by Szego et al. [1991] as nearly the same frequency and location as that associated with their lower hybrid instability.

A further conclusion of the model is that superthermals apparently undergo a stochastic "random walk" path rather than the ordered guiding center \( \mathbf{E} \times \mathbf{B} \) downward drift postulated as the dominant mechanism in ionospheric models of thermal ions [Cloutier et al., 1987; Cravens et al., 1980]. The gyro-radii of these high energy ions are quite large: 10 to 100 km or more. When compared to the gyro-radii of the thermal ions and electrons as was done in our discussion relating to Table 2, or to the vertical gradients in density and magnetic field, superthermals execute broad, nearly straight arcs rather than tight circles and thus should rightfully be considered unmagnetized components of the plasma. Physical agents governing the motion of the superthermals should, therefore be different from that of the background ionosphere. If, for example, the scattering is a strongly resonant mechanism with the unmagnetized superthermals, but relatively ineffective for thermal ions tied to the magnetic field, then we can see how the
Figure 24. Missing pressure profiles for all solar wind conditions and ionopause altitude plotted versus distance below ionopause. Nearly monotonic variation and small standard error lend credibility to the suggestion that the missing pressure is due to superthermal ions energized and scattered by waves associated with the ionopause.
random walk effect would dominate only the motion of the superthermals, whereas conventional $E \times B$ drift would govern the background plasma.

Our presentation of modeled superthermal density and pressure have, to this point, been normalized to unit production rate. This is necessary since the rates are unknown. However, the ability to resolve the missing pressure from the spacecraft measurements allows us to infer a rate and therefore estimate sensible concentration as a function of altitude. This is shown in Figure 25 for the three initial energy distributions. We see in this Figure that maximum concentrations never exceed a few hundred cm$^{-3}$. This corresponds to a ratio of superthermal to total ion concentration of 1 to 2% in the altitude region of interest and also corresponds to the instrument threshold below which OIMS and OETP begin to see their respective superthermal responses (as in Figure 4 and in Taylor et al. [1980]). It also coincides with the observed difference between OIMS dominant ion density and the OETP electron density as indicated in Figure 5e.

Finally, we would like to speculate on the post-charge exchange fate of the superthermals. Upon charge exchange, the ion turns into a high velocity neutral atom possessing essentially all the energy of the parent superthermal but decoupled from the plasma and not subject to any dynamic effects from the magnetic field. Re-ionization through charge exchange with the thermal ionosphere is effectively closed as a loss mechanism since the ions are 3 to 6 orders of magnitude smaller in number than the neutral atmosphere. The same is true of photoionization.

So-called "knock-on" or neutral-on-neutral collisions do open up as an interaction channel. Cross sections for atomic oxygen on oxygen collisions appear to be unavailable due to apparent difficulty in handling atomic oxygen. It is unlikely, however, that such cross sections are more than a few $10^{-16}$ cm$^2$ since other species, notably Helium or Argon are governed by similar quantum-mechanical interaction processes in which the neutral projectile "sees" a neutral target consisting of a central positive charge
Figure 25. Modeled superthermal density inferred from flux rate determined by comparison of measured to modeled missing pressure at 350 km. Due to inexact nature of calculations, values should be regarded as a rough estimate. Result indicates, however, that superthermals make up on the order of 1 to 2% of total ion population and the density of several hundred ions per cm$^3$ seen in this figure corresponds to the threshold levels for detection of superthermals by the OETP measurements in Figure 1 and in the OIMS as shown by Taylor et al. [1980].
surrounded by an electron cloud. In the master's thesis [Kramer, 1991] some interesting results are reported regarding the so-called knock-on collisions and that work includes quantum mechanical calculations using code supplied by Ru-Shan Gao and Ken Smith from Rice University. However, the bottom line of these calculations is that the atomic oxygen resulting from the charge exchange of superthermals basically execute 10 to 100 eV ballistic trajectories. Since the planetary escape energy is in the range of about 8.6 eV most of these trajectories are hyperbolic with half of the projectiles intercepting and thus being lost to the lower atmosphere and the other half escaping the planet.

Neglecting the complicated effects of knock-on collisions processes, we can assume that to first order half of the downward flux of superthermals are essentially lost from the planet. From dimensional considerations the loss rate may be expressed:

$$\Phi = \frac{1}{2} \frac{f \rho V^2_{s.w.}}{m_o \Sigma |V_Z|}$$  \hspace{1cm} (20)

where $f$ is the fraction of the solar wind pressure representative of the missing pressure seen in Figure 2, $\rho V^2_{s.w.}$ is the solar wind pressure, the characteristic value of which we take from Cloutier [1992] at $1.0 \times 10^{-9}$ kg/(m$\cdot$sec$^2$), $m_o = 25.6 \times 10^{-27}$ kg is the mass of O$^+$, and $\Sigma |V_Z|$ is the modeled superthermal pressure normalized to unit production rate.

Taking a conservative characteristic value for the modeled pressure at 10 km/s from Figure 14 and a value for the percent missing pressure from Figure 2 of 10% giving $f = 0.1$ we obtain a loss rate in Equation 20 of:

$$\Phi = 1.9 \times 10^7 \text{ cm}^{-2} \text{ sec}^{-1}$$ \hspace{1cm} (21)

which is larger but within an order of magnitude of values published by McElroy [1982] for loss of atomic oxygen although the mechanism cited in that work is entirely different.

Although the model described here has been framed in terms of the pressure contribution of superthermal ions to total ionospheric pressure, it is equally important to consider the constraints this model places on the production rates of superthermal ions.
and downward energy flux contributed by the superthermal population since any proposed physical mechanism must be able to accommodate these constraints.

To this end we note that equation 21 is an estimate of half the superthermal production rate integrated over the altitude extent of the normally distributed production region. We really have no a priori information about the detailed shape or distribution of the real production region except as indicated by low frequency electric field variations appearing in the OEFD data on nearly every day-side periapsis pass. The Gaussian distribution that we selected for the production region in the model has a standard width of 5 km and is centered at 350 km. It was selected to roughly match the altitude distribution of the electric field perturbations and by extension, the theoretical expectations in the Szego et al. [1991] work. We also wanted to select a distribution which was simple to implement and did not exhibit any significant production of superthermals over the low altitude region below the magnetic field depletion. This was desired since we wish to investigate the connection between the superthermals and missing pressure. As we have noted in the model description section, we also tried various other configurations and altitudes for the source region to develop confidence in the profile that we selected. We found that the detailed shape is not particularly important although a location just above the model magnetic field gradient appears to be critical. If the source region is too high, the combination of a relatively long residence time at high altitude and accumulated charge exchange probability below about 300 kilometers adversely distorts the result enough to produce an unfavorable comparison with the observed missing pressure. If it is too low, the ions execute short trajectories that terminate in the region of the exobase at about 170 km and again, produce an poor match to missing pressure. The assumption of Gaussian altitude distribution for superthermal production implies that the production of superthermals per unit volume is:
\[ p(Z) = \frac{2\Phi}{\sigma \sqrt{2\pi}} \exp\left(-\frac{(Z-\mu)^2}{2\sigma^2}\right) \]  

(22)

where \( \Phi \) is as in Equation 21, \( \sigma \) is the standard width of 5 km and \( \mu \) is the center altitude of 350 km. The maximum value of Equation 22 is an upper limit to the production rate per unit volume of superthermals with numerical value:

\[ \frac{2\Phi}{\sigma \sqrt{2\pi}} = 30 \text{ cm}^{-3} \text{sec}^{-1} \]  

(23)

Although there are many crude assumptions inherent in the calculation of Equation 23 any mechanism for superthermal production can be compared to this upper limit perhaps to within an order of magnitude.

We are also able to calculate an order of magnitude estimate of the superthermal energy flux. Using the value presented in Equation 21 and postulating an average energy for the superthermals of \( \langle E \rangle = 100 \text{ eV} \) (from figure 7), we can obtain a crude estimate of superthermal energy flux of order of:

\[ 2\Phi \langle E \rangle = 3.8 \times 10^9 \text{ eV cm}^{-2} \text{ s}^{-1} \]  

(24)

which compares favorably to the energy flux of \( 4 \times 10^9 \text{ eV cm}^{-2} \text{ s}^{-1} \) for the low frequency waves reported by Szego et al. [1991]. The wave-particle coupling efficiency is not likely to be as near to unity as suggested by the fortuitous close correspondence between these two numbers. This could indicate that average energy of 100 eV selected is too high. We emphasize though, that the modeled missing pressure is not very sensitive to the postulated superthermal energy distribution as indicated by Figure 14. We can also estimate the energy flux required for the missing "Q" term needed to solve the Cloutier flow field. In Figure 8 of the Cloutier et al. [1987] work, the heating term is has a characteristic value on the order of 1000 eV cm\(^{-3}\) sec\(^{-1}\). If we estimate the thickness of that region to be of order 10 km we obtain an estimate of the height integrated heat flux of on the order of \( 10^9 \text{ eV cm}^{-2} \text{ s}^{-1} \), which, although an extremely crude estimate is again easily within expected tolerances of the value reported in equation...
24. The order of magnitude agreement between the Szego et al. electric field energy flux, the Cloutier model and that calculated here for the modeled superthermals increases confidence in the energization mechanism suggested by that paper. For reference, we report the pre-shock solar wind energy flux ($\rho V^3/2$) is of order $10^{10}$ or $10^{11}$ eV cm$^{-2}$ s$^{-1}$ indicating that there is ample solar wind flow energy to supply this mechanism.

Thus we can see then, that several results from the model have converged to establish credibility for the physical features simulated. We think that foremost of these is the ability of the model to populate the altitude region below about 300 km with superthermal ions and match the observed missing pressure. Second, the model source region located just above the ionopause agrees with the theoretically predicted acceleration region. Third, estimates of sensible density fit nicely into the range of densities believed to be attributable to the superthermal ions and a crude estimate of atmospheric loss resulting from the superthermals is similar to other published estimates. Although the wave-scattering mechanism is empirically modeled in the present work, its scale length is similar to the theoretically predicted growth distance for instability and energy flux calculations from the present model correspond to that theoretically predicted.

The model and the principal results and conclusions described here have been accepted for publication [Kramer et al., 1993]. There are, however, several questions that are still pending future work. Data analysis related to solar wind magnetic field control of superthermal distributions and effects of the horizontal ionospheric flow on their dayside location have not been done. The Venus ionosphere is clearly, an MHD turbulent region and the required wave-particle interaction is not well understood. Although we favor the analysis of the cold plasma dispersion relation of Szego et al. [1991], that work only estimates the onset of instabilities which are expected to cascade via nonlinear couplings
to the higher frequencies in the fully developed regime. Our extension of Szego et al.'s theoretical analysis to the wave-particle scattering is rather speculative and a more comprehensive analysis in the future could fill in these gaps. In particular, the superthermal scattering mechanism needed to match observed missing pressure should logically also scatter the dominant thermal background ions. As we have pointed out, this might be resolved if the scattering is a resonant mechanism affecting only the high-energy tail of the ion distribution while allowing the bulk thermal ion distribution to behave as an MHD fluid.
REFERENCES


APPENDIX A

Using the Vlasov equations to solve for first order perturbations in the particle distributions we encounter integrals such as:

\[ \sigma_{et} = \frac{q^2}{im_e} \int_{-\infty}^{\infty} v_x \left( \frac{\partial f_{oe}}{\partial v_x} \right) d^3v \]  

(B.1)

which, incidentally, happens to be the electron component of the conductivity tensor transverse to the magnetic field due to motion of electrons.

With the assumption of a Maxwellian electron distribution, this becomes:

\[ \sigma_{et} = - \frac{n_0 q^2}{im_e k V_e} \left[ \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-\xi^2}}{(\xi - \zeta)} d\xi \right] \]  

(B.2)

where \( \zeta = \omega/k V_e \). The term in brackets:

\[ Z(\zeta) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{e^{-\xi^2}}{(\xi - \zeta)} d\xi \]  

(B.3)

is the Plasma dispersion function for \( \text{Im}(\zeta) > 0 \) and is tabulated in references [Swanson, 1989]. It has the property:

\[ Z'(\zeta) = \frac{dZ(\zeta)}{d\zeta} = -2(1 + \zeta Z(\zeta)) \]  

(B.4)

We emphasize that equation A.1 and similar formulations are fully general but the expressions involving the \( Z \) function have the explicit assumption of Maxwellian.
APPENDIX B

Given a population of ions distributed evenly in a horizontally oriented plane and all moving according to the same equation of motion, the density of such a population as a function of time, t, and altitude, Z is:

\[ n(Z,t) = \lambda \delta(Z-Z_0(t)) \]  \hspace{1cm} (A.1)

where \( \lambda \) is initial density per unit horizontal area, \( Z_0(t) \) is the vertical coordinate of the particle as a function of time and \( \delta \) is the Dirac delta function operator. If instead of a single plane of particles, a production region is simulated by creating such a plane every \( \tau \) seconds, then the sensible density is the time average of \( n(Z,t) \) given in equation A.1 over a production period given by:

\[
n(Z) = \frac{\int_0^\tau n(Z,t)dt}{\int_0^\tau dt} = \frac{\lambda}{\tau} \int_0^\tau \delta(Z-Z_0(t))dt \hspace{1cm} (A.2)
\]

The delta function, \( \delta(f(t)) \), has the property:

\[
\delta(f(t)) = \sum_i \frac{\delta(t-t_i)}{|df(t_i)| dt} \hspace{1cm} (A.3)
\]

where the \( t_i \) are the roots of \( f(t) \) (ie: solutions to \( f(t_i) = 0 \)). Using this property in (A.2) we obtain:
\[ n(Z) = \frac{\lambda}{\tau} \int_0^{\tau} \sum_i \frac{\delta(t-t_i)}{|V_Z(t_i)|} dt \quad (A.4) \]

or

\[ n(Z) = \frac{\lambda}{\tau} \sum_i \frac{1}{|V_Z(t_i)|} \quad (A.5) \]

where \( V_Z(t_i) \) is the vertical velocity at points where \( Z_0(t) \) intersects \( Z \) over the interval of time \( \tau \) (i.e., the roots of \( Z-Z_0(t_i) = 0 \)). Less rigorously, Equation A.5 may be interpreted as the amount of time spent by the particles in a prescribed altitude range. Since these particles propagate downward, it is known a priori that, ignoring production and loss, downward flux is constant and Equation A.5 embodies this concept.

Following the reasoning leading to Equation A.1, horizontal current density may be expressed,

\[ j(Z, t) = q n V_Y \quad (A.6) \]

where \( q \) is ion charge, \( V_Y \) is horizontal velocity. Substituting A.1 for \( n(Z, t) \) and time averaging over a production period provides:

\[ j(Z) = \frac{q\lambda}{\tau} \sum_i \frac{\int_0^{\tau} V_Y(t) \delta(t-t_i) dt}{|V_Z(t_i)|} \]

\[ = \frac{q\lambda}{\tau} \sum_i \frac{V_Y(t_i)}{|V_Z(t_i)|} \quad (A.7) \]

The actual algorithm for estimating \( n(Z) \) and \( j(Z) \) is derived by considering an average of Equation A.5 and A.7 over a small altitude range. This is done to avoid sampling the occasional near zero value of \( V_Z \) at discrete altitudes causing round-off problems in accumulation the sums numerically. Physically, these singularities are a manifestation of the infinite horizontal extent of the production region assumed in the model. The
limitation is acknowledged but definitely not a source of error in the calculation since the
total vertical scale of the modeled region is a small fraction of the horizontal dimension
of the day-side ionosphere. Performing the average of Equation A.5 over the audit zone,
\( \Delta Z \), we obtain:

\[
\langle n(Z) \rangle = \frac{\int_{Z}^{Z+\Delta Z} n(Z) dZ}{\Delta Z} = \frac{\lambda}{\tau \Delta Z} \int_{Z}^{Z+\Delta Z} \sum_{i} \frac{dZ}{|V_{Z}(t_i)|} \\
= \frac{\lambda}{\tau \Delta Z} \int_{Z}^{Z+\Delta Z} \sum_{i} \frac{dt}{|dZ|} dZ
\]  

(A.8)

Which leads immediately to Equation 3 in the text where the angle brackets "\(<>" indicate
average over an altitude range and \( \Delta t_{i} \) is amount of time spent by the particle as it
traverses \( \Delta Z \) with \( i \) now denoting which traversal the particle makes through the audit
zone. For trajectories such as that shown in Figure 4, the particle doubles back, making
multiple transits through the audit zone thus requiring the summation in Equation A.8.

A very similar analysis can be applied to equation A.7:

\[
\langle j(Z) \rangle = \frac{\int_{Z}^{Z+\Delta Z} j(Z) dZ}{\Delta Z} = \frac{q \lambda}{\tau \Delta Z} \int_{Z}^{Z+\Delta Z} \sum_{i} \frac{V_{Y} dZ}{|V_{Z}|}
\]  

(A.9)

which reduces to Equation 4 in the text.

Equation 4 indicates that current density is proportional to the aggregate horizontal
distance traveled by the particle while it is confined to the altitude range \( \Delta Z \). It is entirely
analogous to Equation 3 except that instead of time, current is proportional to, in some
sense, the total "amount of Y" spent in the interval.
Equations 3 and 4 are adequate to evaluate the number and current density as they stand. However, the work by Kramer [1991] may be helpful for readers wishing to replicate this work as it describes a trick that improves efficiency of the numerical calculation.

The vertical momentum flux is given by:

$$P(Z,t) = m_0 n V_Z^2$$  \hspace{1cm} (A.10)

where \(m_0\) is the ion mass, \(n\), its number density and \(V_Z\) its vertical velocity component. Substituting \(n = \lambda \delta(Z-Z_0(t))\) we obtain an expression for the momentum flux or pressure associated with an initial population starting at some original altitude:

$$P(Z,t) = m_0 \lambda \delta(Z-Z_0(t))V_Z^2$$  \hspace{1cm} (A.11)

In (A.2) the additional parameters are \(\lambda\), which is a density per unit horizontal area, \(\delta\) is the Dirac delta function operator, \(Z_0(t)\) is the altitude of the ion as its trajectory evolves. Time averaging over a production period for a continuously produced population of such ions per unit area obtains:

$$P(Z) = \frac{m_0 \lambda}{\tau_0} \int_0^{\tau_0} \delta(Z-Z_0(t))V_Z^2 dt$$

$$= \frac{m_0 \lambda}{\tau_0} \int_0^{\tau_0} \sum_i \left( \frac{\delta(t-t_i)}{|V(t_i)Z|} \right) V_Z^2 dt$$  \hspace{1cm} (A.12)

where the summation is taken over all intersections with the independent altitude \(Z\) and \(V_Z(t_i)\) is the vertical velocity component at that point. Equation A.12 leads directly to Equation 5. There is no need to average over an audit zone, \(\Delta Z\), since the sum accumulated in Equation 5 is stable with respect to singularities.