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Possible power sources for the Jovian polar infrared hot spots

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POSSIBLE POWER SOURCES FOR THE JOVIAN POLAR INFRARED HOT SPOTS

BY

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ABSTRACT

Strong 8-μm infrared hot spots in the polar regions of Jupiter exhibit different behaviors: the northern polar hot spot (hereafter, NPHS) tends to remain fixed in System III longitude while the southern polar hot spot (SPHS) drifts. Joule heating associated with Pedersen currents that are generated by the spinning magnetized ionosphere (the Faraday disc dynamo) is proposed as a possible power source for the hot spots. A quantitative perturbation model is used to show that the NPHS is confined by a steep longitudinal magnetic-field gradient to a System III longitude of approximately 175°, in agreement with observations. The model also shows that a Joule heating power of about 10^{14}–10^{15} Watts can be dissipated in the hydrocarbon layer, significantly larger than particle-precipitation power and the radiated power of the hot spots. The drift of the SPHS is hypothesized as being caused by gravity waves. The total energy provided by Joule heating and by the dissipation of the waves constitutes the power for the hot spot; propagation of the waves causes the location of the total energy deposition to move, thus causing the drift of the SPHS. Because of the asymmetry in the polar magnetic field configurations between the two hemispheres, these gravity waves are more likely to deposit energy comparable to the Joule heating energy in the south to heat up the hydrocarbon layer where IR emission originates. The ranges of wavelength and frequency are investigated
for waves that propagate mainly in north-south direction. These waves can cause the SPHS to drift at the observed speed of \( \sim 5 \text{km/s} \) and dissipate heat that is comparable to Joule heating in the south but less important than the Joule heating in the north. The current-driven joule-heating model, with the presence of wave modulation, can thus account for the primary features of the Jovian polar hot spots: their power output, the fixed location of the NPHS, and the drift motion of the SPHS.
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1. INTRODUCTION

The Jovian IR hot spots have been observed at 8μm with the NASA Infrared Telescope Facility for more than a decade [Caldwell et al., 1980, 1983, 1988; Kim et al., 1985]. The northern polar hot spot (hereafter NPHS) and the southern polar hot spot (hereafter SPHS) reveal different characteristics. The NPHS is always observed to be at a System III (1965) longitude near 180°, latitude 60°N, including the first Voyager encounter in 1979 [Caldwell et al., 1988; Clarke et al., 1989]. In contrast, the southern polar hot spot (hereafter SPHS), although fixed in latitude, moves with respect to System III; it is fixed with respect to neither the subsolar point nor Io's position. It drifts persistently at a speed of up to several km/sec for a time period of an hour or so. This infrared activity is the most energetic phenomenon (>10 ergs/sec-cm²) compared with other auroral activities [Kim, 1988]: extreme ultraviolet aurora (~10 ergs/sec-cm²) [Broadfoot et al., 1979; Yung et al., 1982], and X-ray aurora (~ 10⁻⁴ ergs/sec-cm²) [Metzger et al., 1983].

Stratospheric methane (CH₄) is thought to be the source of emission through its ν₄ fundamental vibration [Gillett and Westphal, 1973; Orton, 1977; Kim et al., 1985]. Because there is no known process on Jupiter that would lead to a local anomalous increase in methane abundance in the stratosphere at the north polar hot spot, one is led to the conclusion that the enhanced emission is due to increased excitation of methane by either internal or external sources. Several possible internal sources have been rejected by Halthore et al. [1988]: (1) Existence of a "convective" type internal source could be verified by anomalous velocities in the tropospheric cloud patterns near the hot spots, but no such patterns have been reported in the literature; other internal sources of energy (for example, gravity waves) could cause the inferred heating of the stratosphere but are unlikely to cause the observed alterations in hydrocarbon abundances in the northern hemisphere reported by Halthore et al. [1988] and Kostiuk et al. [1987]. (2) Of the external sources, solar
ultraviolet radiation cannot excite a region that is confined to a specific longitude. One of the remaining possibilities, an external source of energy of magnetospheric origin for the NPHS, has been proposed by Halthore et al. [1988]. They specifically propose that the NPHS is caused by the precipitation of magnetospheric charged particles guided by the magnetic field into the polar atmosphere. The hypothesis is founded on the observation that the effect is confined to fixed longitude ranges in System III coordinates. "The infrared and the UV brightness enhancements may thus be two aspects of the same phenomenon—that of energetic particle interaction with atmospheric gases" [Halthore et al., 1988].

Of the precipitating charged particles, heavy ions such as oxygen and sulfur have been widely thought to be responsible for the energy source of UV aurora [see, for example, Thorne, 1981, 1983; Gehrels and Stone, 1983]. The total power delivered by these heavy ions may exceed $10^{12}$ W in the Jovian atmosphere [Gehrels and Stone, 1983; Herbert et al., 1987] which is the lower limit for the IR hot spots. However, most of the power is deposited at an average depth of $10^{19}$ cm$^{-2}$ of H$_2$ (about 500 km above the ammonia cloud top), which is above the homopause [Gehrels and Stone, 1983], while the IR hot spot is located between 1 mb and 1 mb pressure level (390 km and 150 km above the cloud top, respectively), which is below the homopause [Caldwell et al., 1988]. We reproduce in Figure 1 the altitude distribution of energy deposition into the Jovian atmosphere for two different energy lower limits to the flux spectra (the two limits were chosen to illustrate the deposition of $10^{13}$ W to $10^{14}$ W), shown in Figure 14 of Gehrels and Stone [1983]. By virtue of the distribution, we conclude in Chapter 2 that the energy deposition below the homopause and within the hydrocarbon layer accounts for only a small fraction of the total deposition energy. As a matter of fact, the power is about $2.4 \times 10^{10}$ W which is smaller than the radiation power of $2 \times 10^{12}$ W for the infrared hot spot by a factor of 80. This lack of sufficient precipitating power to excite the methane below the homopause makes it difficult to relate the IR hot spot to heavy-ion precipitation.
Fig. 1. Illustrative calculation of the altitude dependence of the oxygen and sulfur energy deposition in the Jovian atmosphere for two lower limits of the extrapolated oxygen and sulfur energy spectra (from Gehrels and Stone [1983]).
Electron precipitation is another external source of power that may be related to the hot spots. Waite et al. [1988] suggest a scenario where both energetic heavy ions (>300 kev/nucleon) and energetic electrons (10 to 30 kev) play roles in Jovian auroral processes. They conclude that the heavy-ion auroral energy deposition is concentrated at altitude below the homopause and energetic electrons are responsible for the bulk of the observed UV and EUV emissions because they deposit their energy above the methane-absorbing layer. The scenario is inspired by the difficulties with the hypothesis of a predominantly heavy-ion aurora, such as the low emission value of IUE observations for 1304 Å OI emission which is at least 50 times lower than that by model calculation of Horanyi et al. [1988] for heavy-ion precipitation. However, this mixed-precipitation scenario does not provide plausible power for the IR hot spots in that most of the electron deposition occurs above the homopause whereas the hot spots are believed to be located below the homopause.

Recently, Barbosa [1990] carried out a calculation of the X-ray spectrum according to an auroral electron beam model. The electrons are assumed to be accelerated by a field-aligned potential drop and penetrate into the atmosphere as a Maxwellian beam of primaries that are scattered, degraded in energy, and merged with a population of ionization secondaries having a power-law energy distribution. The scenario suggests a continuum X-ray emission that fits the observations of Metzger et al. [1983] fairly well. This is a strong evidence against the hypothesis of Metzger et al. [1983] who interpret the X-rays as being generated by K-shell emission from precipitating energetic O+ and S+ ions. However, we are skeptical of Barbosa's model mainly because: (1) one of the important parameters, the flux of primaries, is loosely constrained by UV auroral energy requirements. Thus, it has a large uncertainty [Barbosa, 1990]; (2) the strong coupling between the primary electron beam parameters and secondary beam parameters is ignored in the model [Waite, private communication, 1990]; (3) neither the flux of the primary electrons or the flux of the secondary electrons is altitude-dependent, an assumption that
inevitably brings unrealistic excess power deposited in the atmosphere. We postulate that this may be the major reason for the large power output of his model. We thus suspect that the electron precipitation may not contain sufficient power to be responsible for the observed X-ray emission. Whether the Jovian auroral processes are dominated by ion precipitation or electron precipitation remains an open question, but neither contains enough power for the hot-spot emission below the homopause.

An alternative hypothesis for the source of the hot spots is Joule heating resulting from the dissipation of Pedersen current flowing in the polar ionospheres. This hypothesis is motivated by an important fact: According to Caldwell et al. [1980] and Halthore et al. [1988], the (peak of the) NPHS is always situated on or poleward of the theoretical last-closed-field-line boundary [Acuña et al., 1983] within which all field lines are open and extend to the magnetotail, as shown in Figure 2. This is different from the widely acknowledged UV auroral geometry, which is somehow related to Io's torus [Prange, 1990]. (A quick glance at the figure may give rise to some doubts about the accuracy of the statement: the auroral locus derived from the Voyager UVS experiments lies at about the same mean latitude but differs in detail from the Io torus footprint [Herbert et al., 1987]. This discrepancy may be attributed to the difference between the O₄ magnetic field model predictions and the real surface magnetic field [Prange, 1990]. On the other hand, the latitude of the auroral peak at 210° System III longitude is about the same as that of the torus footprint, which supports the idea that Io plasma is involved in the generation of the UV aurora. Because of the relatively well-defined longitudinal dependency of the UV aurora, the dotted points in Figure 2 near 210° should be given more weight when one considers the latitudinal dependency of the aurora.) Thus there is a growing conviction that the magnetospheric sources for the emission in various spectral regions may be different, in particular, UV and IR emissions arise from different causes: The UV would be excited by particles precipitating on the Io drift-shell and/or even larger L shells, whereas the IR would
Fig. 2. Comparison of the computed Io torus northern footprint [Roederer et al., 1977] (dashed line) and the auroral locus (dotted points) by direct observation [Broadfoot et al., 1981] (dotted line) as well as the theoretical last-closed-field-line [Roederer et al., 1977; Connerney et al., 1981] (solid line). The peanut shaped region represents the NPHS with its peak located poleward of the last-closed-field-line boundary (180° longitude, 60° latitude) as indicted.
be the result of precipitation on open field lines in a localized longitude range [Caldwell et al., 1988; Clarke et al., 1989], or the result of thermal excitation by Joule heating. Recently, people have argued about the positions and hence the excitation mechanisms of the UV and IR emissions [Dessler, 1990; Prange, 1990] through direct comparison between the UV aurora and the IR hot spot shown in Figure 2: the NPHS "is exactly nested along the UVS experimental equatorward boundary of the UV auroral oval in the region of the UV intensity peak (same shape, same orientation coincidentally at constant latitude, same latitude and longitude). This strongly suggests that the ultimate magnetospheric process generating the UV and IR emissions must be the same, at least in the longitudinal range of the north maximum." [Prange, 1990]. Whether the (northern) UV aurora and the IR NPHS are located in the same region depends strongly on the spatial resolutions of the IR hot-spot contour and the UV aurora locus (including their shapes and positions, especially their peak longitudes/latitudes). We feel that it is too early to determine their spatial separation; on the other hand, there is, at this time, insufficient basis to claim that the northern UV aurora and the IR NPHS occur at the same geographic position just by viewing and comparing the two maps that lack adequate spatial resolution. Judging from the early study of Isbell et al. [1984], we point out that the total energy dissipation from the Joule heating in the polar region is large enough ($\sim 10^{15}$ W) to generate the hot spots. We thus propose the following scenario:

The UV aurora is produced in the closed-field-line region by particle precipitation originating from the Io torus and larger L shells, whereas the IR hot spot is produced by Joule heating in the open-field-line region mapping to the magnetopause boundary layer. The peak of the UV aurora is mainly determined by the 'windshield wiper effect' of maximum $|dB/d\lambda_{\text{III}}|$ [Dessler and Hill, 1979], whereas the peak of the IR hot spot is controlled not only by the local maximum magnetic field but also by the ion/electron
concentration distribution produced nearby in the UV auroral precipitation. (The UV aura and IR hot spot are close to each other in latitude so that the nonuniform ion/electron concentration distribution due to the precipitation can affect the concentration where the hot spot is nested through diffusion, an assumption whose validity can only be justified \textit{a posteriori}).

The apparent alignment of the bulk shape of the IR hot spot with the equatorward boundary of the UV aura (rather than with the last-closed-field-line) shown in Figure 2, which is obviously related with the longitudinal extent of the hot spot, may be attributed to the effect of stratospheric zonal winds [Caldwell et al., 1988; Halthore et al., 1988].

Our hypothesis of Joule heating for the hot spots is also motivated by the example of the Earth, with which we are quite familiar: more than half of the total power dissipated into the Earth's upper atmosphere appears in the form of thermal energy created by dissipation of electrical currents, i.e., Joule heating [Wolf, 1988]. This suggests that Joule heating may also play a fundamental role in producing the Jovian hot spots. Although the power for Jupiter's aura is drawn from the kinetic energy of Jupiter's spin (instead of the solar wind as for the Earth) [Hill et al., 1983a], many Jovian auroral process are similar to those in the terrestrial case. Among these is the expectation that powerful Birkeland currents flow into and out of the auroral ionosphere. As illustrated in Figure 3, inside the auroral disc (Faraday disc) which is defined here by magnetic mapping of the magnetopause boundary layer onto the atmosphere, the inward and outward Birkeland currents close the Pedersen current that is responsible for the Joule heating power. In the boundary layer, the Birkeland currents sustain the twisted magnetic field, which is produced by a quasi-viscous interaction (e.g., magnetic merging or solar wind entry) that tends to couple the solar wind and the magnetic field of Jupiter. The ultimate power source of these currents is the spinning ionosphere of the planet acting as a Faraday disc dynamo with the solar wind constituting the load. If the polar magnetic field is independent of System III
Fig. 3. Magnetospheric and ionospheric currents that must flow to produce the helical magnetic field in the magnetotail (after Isbell et al. [1984]). The case illustrated is for a magnetic moment $M$ oriented antiparallel to the planetary spin vector $\Omega$. Currents flow in the tail to produce the toroidal component of magnetic field $B_\psi$ that is associated with the helical twist. The currents close the corresponding Birkeland currents that flow into and away from the auroral disc, as described in the text. For cases with $M$ and $\Omega$ parallel to each other like Jupiter, all the arrows in the picture showing the directions of the B-field and the currents should be reversed.
longitude, the Faraday Disc Dynamo produces constant potential drop across the auroral disc. (The outer edge of the disc, according to the definition, is the last-closed-field-line and is magnetically mapped to the cylindrical surface separating the magnetopause boundary layer from the solar wind. The inner edge of the disc is the boundary magnetically mapped to the cylindrical surface separating the magnetopause boundary layer from the vacuum-like tail lobe. The magnetic mapping relation is shown in Figures 3 and 4). In real situations, however, the surface magnetic field is not uniform with respect to longitude (see Figure 5). One consequence of this nonuniform field configuration is that ion concentration becomes a function of System III longitude because of the longitudinally dependent particle precipitation induced by the nonuniform field (the "windshield wiper effect"), which causes the preferred "active sector" where the UV auroral intensity is a maximum (at \(\sim 210^\circ\)) and presumably the ion/electron concentration is a maximum. We take a basic assumption that the nonuniform ion/electron concentration in the closed-field-line region affects the concentration in the adjacent open-field-line region through a process such as diffusion which will not be discussed in this work. Thus, the Pedersen conductivity in the open-field-line region is generally nonuniform too, owing to the asymmetric surface magnetic field as well as the induced local nonuniform ion/electron concentration. Another consequence of the nonuniform field configuration is that the Birkeland current density becomes a function of longitude because it is in general dependent on both magnetic field and conductivity. Naturally, these nonuniform effects cause the power of Joule heating in the ionosphere to vary with longitude, which can make the hot spots occur at certain preferred longitudes.

The magnetic field of Jupiter can, for our purpose, be considered as time independent. The mathematical characteristics of the field are described by many models, such as \(O_4\) [Acuña and Ness, 1976] and \(P11(3,2)A\) [Davis et al., 1975]. If the assumption of Joule heating is valid, it requires that the Pedersen conductivity configuration in System III
Fig. 4. Simplified drawing of the geometry of the auroral disc and the current system. The inner and outer edges of the disc are represented by colatitudes $\theta_1$ and $\theta_2$, or equivalently by the polar radii $\rho_1$ and $\rho_2$. $\phi$ stands for the conventional longitude and $h$ is the thickness of the disc. $\pi(\theta_1)$ and $\pi(\theta_2)$ are sheet currents at the two edges that flow into and away from the disc. Within $\theta_1$ and $\theta_2$, Birkeland current $j_R$ flows into the disc and closes with the current in the tail that causes magnetopause-boundary-layer field lines to twist. Current continuity requires that the azimuthal current $j_\phi$ and the Pedersen current $j_\theta$ be closed by currents flowing in and out of the disc.
Fig. 5. Diagram indicating the magnetic field intensities at the northern foot of the last-closed-field-line (thick line) and at the southern one (thin line), from the O4 model. Also shown in the figure are the sinusoidal curves that simulate the above two curves in the north (thick broken line) and in the south (thin broken line), respectively.
coordinates be approximately time-independent (except for day–night variations) in order that the NPHS be fixed with respect to System III coordinates. One might be concerned that the SPHS should also be fixed in longitude as is the NPHS if the SPHS is produced by the same mechanism as the NPHS, whereas the observations have shown persistent drift motion of the SPHS in System III. We propose that the difference in localization of the hot spots between the two polar regions is the result of the distinct feature of surface field gradients in the southern polar region compared with that of the northern polar region. As can be seen from the O$_4$ model (see Figure 5), the magnetic field varies sharply with respect to System III longitude in the northern polar region (it changes from a minimum of $\sim 8.5$ Gauss to a maximum of $\sim 14$ Gauss), while in the southern polar region the maximum magnetic field is about 10 Gauss and the minimum magnetic field is about 8 Gauss. Also, the field maximum in the south covers a wide range of longitude from 130° to 320°. This flatness of the field in the southern polar region causes less prominent longitudinal variation of particle precipitation than its northern counterpart and correspondingly decreases the conductivity variation in System III longitude. Therefore, in the southern polar region, the effect of the conductivity and/or the surface magnetic field on confining the hot spot at a certain longitude is weak compared with that in the northern polar region. This weak confining tendency makes it possible for other small atmospheric and/or magnetic perturbations, along with day-night effect, to control the position of the SPHS. These perturbation can, for example, lead to an increase in heating rate and/or in methane abundance and thereby to an enhancement of IR emission. If the perturbation is not localized, the increases in the heating and/or in the methane abundance may not be localized either, which can lead to the motion of the SPHS.

In this thesis, I present a theoretical analysis of the Joule heat dissipated in the polar ionosphere. The aim is to examine the Joule heating hypothesis for the hot spots, by evaluating the total Joule heating power and its longitudinal distribution in the framework
of the Faraday disc dynamo theory. The asymmetric behaviors of surface magnetic field, Birkeland current, and conductivity as well as modulation of the Birkeland current are taken into account. I find that the Joule heating hypothesis can plausibly account for the longitudinal confinement of the NPHS as well as the power output. I also present a "scissors" model to interpret the motion of the SPHS by introducing the modulation of the longitudinal position of the SPHS due to gravity wave packets. The resulting estimate of the speed of the SPHS caused by the modulation is consistent with the observations, although, at this point, we do not attempt to exclude other possible mechanisms that can act in a similar way to cause the motion of SPHS.
2. OVERVIEW

The IR hot spots are thought to be located between the 1-μb and 1-mb pressure levels [Halthore et al., 1988]. The upper end of this altitude range is set at the homopause above which the methane concentration becomes so low that it is unlikely to produce strong infrared emissions. The lower end of this altitude range is determined by comparing the P-T profiles inside and outside the hot spots. The temperature of the hot spots is found unaltered at the altitude level where pressure is higher than 1 mbar. The 1-mbar level is thus interpreted as the lower limit for the altitude of the hot spots.

2.1 Energy Deposition by Energetic Ion Precipitation Below the Homopause

UV auroral activity on Jupiter is one of the most energetic phenomena associated with the magnetosphere, requiring a continuous power input of $10^{13} - 10^{14}$ W into the auroral zone [Broadfoot et al., 1981; Yung et al., 1982]. This corresponds to an energy flux of 10 ergs/cm^2-s (or $1\times10^{-6}$ W/cm^2) if one identifies the precipitating particles as electrons [Waite et al., 1983; Atreya et al., 1981; Yung et al., 1982; Broadfoot et al., 1981; Gérard and Singh, 1982]. Precipitating ions require a lower energy flux to produce the observed emissions.

We infer from both the latitude of the observed UV aurora and the Voyager in situ particle measurements that the Io plasma is involved in the generation of energetic particles responsible for the auroral emissions [Broadfoot et al., 1981; Bridge et al., 1979; Krimigis et al., 1979; Vogt et al., 1979; Gehrels et al., 1981]. Experiments on the Voyagers have detected intense fluxes of charged particles in the inner magnetosphere of Jupiter. Both energetic and thermal ion populations in this region are largely composed of relatively
heavy ions such as oxygen and sulfur.

Most earlier magnetospheric studies of wave-particle interactions have suggested that electrons were more likely than ions to precipitate into the Jovian atmosphere (cf., Waite et al., 1983). The identified sources of energetic electron scattering, however, appear to be somewhat inadequate to explain the inferred auroral power budget [Waite et al., 1988]. Goertz [1980] suggested proton precipitation and Thorne [1981, 1983] considered heavy-ion precipitation owing to the measured heavy-ion composition of the Io torus region. Early evidence in favor of ion precipitation includes X-ray observations of Jupiter made by the Einstein X-ray observatory [Metzger et al., 1983] showed that a total auroral power of $10^{15} - 10^{16}$ W would be required to explain the observed X-ray emission if Bremsstrahlung radiation from electron scattering was assumed. They suggested K-shell excitation of energetic precipitating S and O to explain the X-ray observation with a reasonable total power in line with that inferred by the UV observations. Gehrels and Stone [1983] reported observations of 1 to 20 Mev/nuc oxygen, sodium and sulfur ions in the Jovian magnetosphere by the Cosmic Ray Subsystem (CRS) onboard Voyagers 1 and 2. They calculated the inward diffusion rate of these heavy energetic ions and assumed that they are scattered into the loss cone, resulting in auroral power input into the Jovian atmosphere. They also extrapolated the energy spectra to energies below those actually observed by the CRS in order to obtain the $10^{13} - 10^{14}$ W total auroral power required to explain the observed UV observations.

Waite et al. [1988] suggested that electron as well as ion precipitation plays a role in Jovian auroral processes. They proposed that heavy-ion auroral energy deposition is concentrated at altitudes below the homopause (i.e., 100-300 kev/nuc) and is responsible for the X-ray emissions while electrons with energies 10 to 30 kev deposit their energy above the homopause and are responsible for the bulk of the observable UV and EUV emissions. Barbosa [1990] used a theoretical model, the features of which are well grounded in
observations of the terrestrial aurora, to explain the Jovian X-ray emission in terms of bremsstrahlung from auroral electrons. The model seems to be compatible with the X-ray data and other quantitative conclusions drawn from the model are in perfect agreement with the UV observations of both Voyager and the IUE satellite with regard to electron energy flux and penetration depth. Yet, as mentioned in the Introduction, the flux of the primary electrons and the flux of the secondary electrons are assumed independent of each other and of the altitude, which is physically implausible.

Whether the UV aurora is produced through direct ion precipitation or through electron precipitation is an open question that will not (and need not) be addressed in this work. However, we have noticed that if the auroral interpretation of the hot spots is correct (i.e., the hot spots are generated by direct particle impact), then the extra infrared emission undoubtedly comes from the levels at which CH₄ exists on Jupiter, a level that only energetic particles can reach. Caldwell et al. [1980] stated there were some questions whether in fact the electrons could reach the hydrocarbon level. Thus, energetic precipitating ions are more likely to be responsible for the hot spots. We adopt the ion precipitation scenario here and compare hot-spot energy budget with that of ions in the following.

Figure 1 is a reproduction from Gehrels and Stone [1983] that shows the altitude distribution of energy deposition in the Jovian atmosphere for two different energy lower limits to the flux spectra which were chosen to illustrate the deposition of $10^{13}$ W to $10^{14}$ W. For the assumed spectrum extrapolation, most of the energy deposition occurs between $2\times10^{18}$ and $5\times10^{19}$ cm$^{-2}$ of H$_2$, corresponding to an altitude range of approximately 450 to 500 km (the reference altitude is taken as the ammonia cloud top). The location of the peak depends on the minimum energy of the spectrum but is above the homopause for both of the representative lower limits shown in the figure [Gehrels and Stone, 1983]. Yung et al. [1982] find, in their study of the observed wavelength dependence of the UV auroral emission, that the energy deposition must occur between $5\times10^{17}$ and $2\times10^{20}$ cm$^{-2}$,
which is consistent with Gehrels and Stone's result. This is also the result found for electron precipitation by Waite et al. [1983] who obtained the peak volume emission rate above the hydrocarbon layer, which is around 350 km to 400 km in the equatorial region [Atreya et al., 1981; Festou et al., 1981]. Therefore, the energy deposition below the homopause and within the hydrocarbon layer accounts for only a small fraction of the total energy deposition. This can be estimated easily by the following empirical deposition rates derived from Figure 1 assuming the rates can be extrapolated linearly to lower altitude:

\[ F_s = (1.3 \times 10^{-7}) \times 10^{0.32h} \quad \text{(W/m)} \]  

(2.1.1)

and

\[ F_o = (7.6 \times 10^{-8}) \times 10^{0.32h} \quad \text{(W/m)} \]  

(2.1.2)

where \( h \) represents the altitude (in unit of 10\(^6\) km) and the two subscripts "s" and "o" stand for sulfur and oxygen, respectively. We integrate these rates with respect to the altitude to obtain the power deposited below the homopause which is assumed at the altitude of 400 km. The power is about \( 2.4 \times 10^{10} \) W. Using their data of early 1980, Caldwell et al. [1980] estimated the energy involved in the infrared brightenings and indicated that the energy required to produce the observed brightenings is \( \sim 9 \) ergs cm\(^{-2}\) sec\(^{-1}\), or a total power of \( 2 \times 10^{12} \) W for a hot-spot area of \( 9100 \times 25000 \) (km)\(^2\) [Caldwell et al., 1988]. From the data they provide, we suspect that this is actually the IR radiation power [Chamberlain, private communication, 1990; Kim, private communication, 1990] and the real input power is even larger. Therefore, the input energy flux of \( 2.4 \times 10^{10} \) W from ion precipitation is too small (by a factor of at least 80) to produce the observed IR emissions. Furthermore, the power flux from ion precipitation calculated above is actually an upper limit for the energy input for the IR hot spot in that the IR hot spot layer (1\(\mu\)b-1mb) is only part of the region below the homopause while the power of \( 2.4 \times 10^{10} \) W represents the
total energy deposited below the homopause.

The problems associated with particle penetration are orders of magnitude more severe against \textit{NH}_3 than they are against \textit{CH}_4 [Caldwell et al., 1980]. Thus, the UV aurora and the IR emissions are unlikely to be directly caused by the same energizing mechanism.

2.2 Joule Heating as an Alternative Power Source

There is insufficient flux of energetic particles to relate the hot spots directly to energetic ion precipitation. Furthermore, the observed NPHS peak near 180° is neither at the position of minimum B (where enhanced precipitation would occur if the pitch-angle-scattering is very weak [Herbert, private communication, 1990]) nor at the position of maximum dB/d\lambda_m (i.e., the "windshield wiper effect", if the pitch-angle-scattering is stronger. [in cases when the scattering is very strong, however, the precipitation distribution becomes fairly uniform]). These problems have led people to turn their attention to other mechanisms such as Joule heating caused by dissipation of large Pedersen currents in the polar ionosphere. The mechanism is driven by the polar ionosphere acting as a Faraday disc dynamo to convert kinetic energy of planetary spin into power P_I dissipated as Joule heating in the ionosphere and power P_{SW} delivered to the magnetotail and solar wind [Hill et al., 1983b; Isbell et al., 1984].

A list of these powers for various planets is provided by Isbell et al. [1984], who assumed that quasi-viscous interactions such as magnetic merging or solar wind entry cause the field lines in the magnetotail to spiral. The Birkeland currents that sustain the spiraling field were assumed to be present throughout the tail lobe and the magnetopause boundary layer. The currents map along the magnetic field lines to the ionospheric polar cap. Based on this work, Zhan [1989] considered helical hydromagnetic waves in the tail
magnetopause by restricting the Birkeland current within the magnetopause boundary layer so that the tail lobe is vacuum-like. Therefore, the polar ionosphere was assumed to contain a region where no current is injected into the ionosphere and thus no Joule heating is produced there. The region, with the radius of \( \rho_1 = R_1 \theta_1 \) (\( \rho \): polar distance, and \( \theta \): colatitude), is a disc centered at the north pole in the simplified case when the magnetic dipole moment and the spin axis are aligned together, as shown in Figures 3 and 4. Only the region between \( \theta_1 \) and \( \theta_2 \) in the polar ionosphere, which is referred to as auroral disc here, can generate Joule heat and produce hot spots under the above assumption.

In order to associate hot spots with Joule heating, it is important to find out the total Joule dissipation in the auroral disc. We follow the analysis of Isbell et al. [1984] to obtain the electric field measured in the reference frame of the corotating auroral disc (see Appendix B):

\[
E_{\phi} = \frac{BR_1 \Omega}{\sigma_0 + 1} \theta \left( 1 - \frac{\theta_1^2}{\theta^2} \right) \tag{2.2.1}
\]

The power \( P_1 \) dissipated in the auroral disc as Joule heating and the power \( P_{SW} \) delivered to the magnetotail and solar wind:

\[
P_1 = 2 \pi \Sigma \left[ \frac{R_1^2 B \Omega}{\sigma_0 + 1} \right]^2 \left[ \frac{1}{4} (\theta_2^4 - \theta_1^4) - \theta_1^2 (\theta_2^2 - \theta_1^2) + \theta_1^4 \ln(\frac{\theta_2}{\theta_1}) \right] \tag{2.2.2}
\]

\[
P_{SW} = -P_1 + \frac{2 \pi \Sigma (R_1^2 B \Omega)^2}{(\sigma_0 + 1)^2} \left[ \frac{1}{4} (\theta_2^4 - \theta_1^4) - \frac{1}{2} \theta_1^2 (\theta_2^2 - \theta_1^2) \right]
\]

\[
= \frac{2 \pi \Sigma (R_1^2 B \Omega)^2}{(\sigma_0 + 1)^2} \left[ \frac{1}{4} \sigma_o (\theta_2^4 - \theta_1^4) - \frac{1}{2} \sigma_o \theta_1^2 (\theta_2^2 - \theta_1^2) - \theta_1^2 \ln(\theta_2 / \theta_1) \right] \tag{2.2.3}
\]

Numerical values of these powers along with some other interesting physical quantities are listed in Table 1, for several planetary systems. These results are consistent with those of Isbell et al. [1984]. In fact, their results are the \( \theta_1 = 0 \) limit of the present results.
<table>
<thead>
<tr>
<th>Planet</th>
<th>$B_p$ (Tesla)</th>
<th>$B_T$ (Tesla)</th>
<th>$R_p$ (km)</th>
<th>$R_T$ (km)</th>
<th>$\theta_1$ (deg)</th>
<th>$\theta_2$ (deg)</th>
<th>$R_I$ (km)</th>
<th>$R_2$ (km)</th>
<th>$\Sigma$ (mho)</th>
<th>$\Omega$ (s^-1)</th>
<th>$M$ (Tesla-m)</th>
<th>$\Delta\Omega/\Omega$</th>
<th>$\max \alpha_p$ (deg)</th>
<th>$P_t$ (w)</th>
<th>$P_{sw}$ (w)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>$3 \times 10^{-7}$</td>
<td>$5.1 \times 10^{-8}$</td>
<td>2,439</td>
<td>-</td>
<td>57</td>
<td>-</td>
<td>$4.9 \times 10^5$</td>
<td>$2.2 \times 10^{12}$</td>
<td>1</td>
<td>$0.2 \times 10^{10}$</td>
<td>$0^0$</td>
<td>$0^0$</td>
<td>$0^0$</td>
<td>$0^0$</td>
<td></td>
</tr>
<tr>
<td>Earth</td>
<td>$6 \times 10^{-5}$</td>
<td>$2.5 \times 10^{-4}$</td>
<td>6,378</td>
<td>10</td>
<td>16</td>
<td>$5.6 \times 10^4$</td>
<td>$9.0 \times 10^4$</td>
<td>20</td>
<td>$10^0$</td>
<td>$7.27 \times 10^{-5}$</td>
<td>$7.8 \times 10^{15}$</td>
<td>$\leq 0.055$</td>
<td>$0.89$</td>
<td>$9.1 \times 10^6$</td>
<td>$1.9 \times 10^8$</td>
</tr>
<tr>
<td>Jupiter</td>
<td>$5 \times 10^{-4}$</td>
<td>$5.5 \times 10^{-9}$</td>
<td>69,000</td>
<td>8</td>
<td>15</td>
<td>$4.6 \times 10^6$</td>
<td>$6.9 \times 10^6$</td>
<td>5</td>
<td>$10^0$</td>
<td>$1.76 \times 10^{-4}$</td>
<td>$1.3 \times 10^{20}$</td>
<td>$\leq 0.20$</td>
<td>$68$</td>
<td>$4.2 \times 10^{14}$</td>
<td>$2.0 \times 10^{15}$</td>
</tr>
<tr>
<td>Saturn</td>
<td>$7 \times 10^{-4}$</td>
<td>$3.0 \times 10^{-4}$</td>
<td>60,330</td>
<td>6</td>
<td>11</td>
<td>$9.6 \times 10^5$</td>
<td>$1.7 \times 10^5$</td>
<td>10</td>
<td>$5^0$</td>
<td>$1.64 \times 10^{-4}$</td>
<td>$7.6 \times 10^{18}$</td>
<td>$\leq 0.12$</td>
<td>$32$</td>
<td>$3.0 \times 10^{11}$</td>
<td>$2.8 \times 10^{12}$</td>
</tr>
<tr>
<td>Neptune</td>
<td>$3 \times 10^{-5}$</td>
<td>$7.1 \times 10^{-10}$</td>
<td>24,765</td>
<td>7</td>
<td>11</td>
<td>$5.9 \times 10^5$</td>
<td>$9.3 \times 10^5$</td>
<td>&lt;2</td>
<td>$&lt;1^0$</td>
<td>$1.08 \times 10^{-4}$</td>
<td>$2.0 \times 10^{17}$</td>
<td>$\leq 0.29$</td>
<td>10</td>
<td>$7.1 \times 10^8$</td>
<td>$2.3 \times 10^9$</td>
</tr>
</tbody>
</table>

(*) Values taken from arbitrary assumption.
(a) Values taken from Isbell et al. [1984].
(b) Values taken from Lyons and Williams [1984].
(c) Values taken from Lui [1987].
(d) Values taken from Ness [1977].
(e) Values taken from Connerney et al. [1981].
(f) Values taken from Stone and Owen [1984].
(g) Values taken from Ness et al. [1986].
(h) Values taken from Hill et al. [1983].
(i) Values taken from Ness et al. [1989].
(j) Values taken from Stone and Miner [1989].
The power dissipation in terms of Joule heating in the Jovian ionosphere is of the order of \(10^{14} - 10^{15}\) W, which is significantly larger than available from energetic particle precipitation, and it is larger than required for the observed power output of the IR hot spots.

2.3 Hot-Spot Altitude

We have hypothesized that the hot spot is caused by Joule heating associated with Pedersen currents in the lower ionosphere. We are here interested in determining if the maximum in the Joule heating power distribution occurs at or near the estimated hot-spot altitude range (1μb-1mb). If the maximum-power altitude is too far away from the hot-spot range (say, tens of scale heights above or below), the net power dissipated in the hot-spot range may not be sufficient to generate the IR emissions even though there is enough total power. The power distribution in altitude can be obtained by examining the altitude profile of the Pedersen conductivity.

The Pedersen conductivity in the ionosphere of Jupiter is a complicated matter especially at low altitude (near the homopause), because of the multilayer structure of electron concentration measured by Pioneer 10 and 11 [Fjeldbo et al., 1975]. Voyager 1 and 2 measurements of the electron-number-density profiles do not provide much help because these profiles give no useful information about the concentration below the 1000 km level (see Figure 2.4 of Strobel and Atreya [1983]). Despite the fact that the electron concentration calculated using the photochemical scheme described by Strobel and Atreya [1983] and for the geometry of the observation corresponding to the Voyage 1 entry occultation looks rather smooth, some bottomside layers are considered real structure [Strobel and Atreya, 1983]. For mathematical convenience, Huang and Hill [1989] smoothed the layer structure so as to derive the Pedersen conductivity. We adopt their smoothed electron/ion
concentration profile in this work. The uncertainty and error incurred from this approximation depend strongly on the confidence of the electron concentration measurements made by Pioneer and by Voyager as well.

The Pedersen conductivity is given by

\[ \sigma = \frac{n_e e}{B} \frac{v_{in} \omega_{ci}}{v_{in}^2 + \omega_{ci}^2} \]  \hspace{1cm} (2.3.1)

where B and \( n_e \) are the magnetic field strength and electron number density, \( v_{in} \) and \( \omega_{ci} \) are the ion-neutral collision frequency and the ion gyrofrequency. The contribution of the electrons to the conductivity is unimportant and is neglected here. The electron number density is smoothed by Huang and Hill [1989] in such a way that it takes a form in the altitude range 300–700 km:

\[ n_e = 2.6 \times 10^3 \ e^{(0.67 \times 10^{-7} h - 2.2 \times 10^{-4} h^2)} / \text{cm}^3 \]  \hspace{1cm} (2.3.2)

where \( h \) is in unit of km. The empirical approximation of \( v_{in} \) was taken by them as

\[ v_{in} = 2.9 \times 10^{-9} n \ \text{cm}^3/\text{sec} \]  \hspace{1cm} (2.3.3)

where \( n \) is the number density of neutral particles and is given by:

\[ n = 4.3 \times 10^{19} \ e^{-4.6 \times 10^{-7} h + 2.3 \times 10^{-5} h^2} / \text{cm}^3 \]  \hspace{1cm} (2.3.4)

For these neutral and ion number density distributions, the Pedersen conductivity as a function of altitude is shown in their Figure 2 and is reproduced here as Figure 6. The peak of the curve appears at 400 km (corresponding to \( \omega_{ci} = v_{in} \)), which is within the estimated main contribution region of 300-700 km to the conductivity [Strobel and Atreya,
Fig. 6. Altitude distribution of Pedersen conductivity. 
(After Huang and Hill, 1989)
1983] and near the upper end of the estimated hot-spot altitude range (1 μb pressure level, or 390 km above the ammonia cloud top). The Joule heating power is therefore maximum near the estimated altitude of the hot spot. The hot spot altitude is somewhere between maximum Joule heating power (~400 km, which is near the homopause) and maximum methane number density (below the homopause). The optimum altitude for hot-spot formation is therefore near but slightly below the 400 km level.

The idealized single-peak conductivity profile would be replaced by a multi-peak profile if the multilayer structure of the electron concentration were taken into account. The assumption that the IR hot spots are generated by enhanced Joule heating near the altitude where the Pedersen conductivity is enhanced implies that the IR hot spot may have a similar layered structure; no observation of such structure is yet possible. In the final chapter of this paper, we will discuss a problem related with this multi-peak conductivity scenario, i.e., the difficulty that the parallel current would have in reaching low altitude because of conductivity gaps between adjacent layers.

Note that the profile of Pedersen conductivity is based on the "unconverged" atmosphere model, a terminology we adopt from Horanyi et al. [1988], which means the atmosphere is first obtained by running the model with only solar radiation as an energy input (i.e., no auroral precipitation or Joule heating). As discussed by Horanyi et al. [1988], the most noticeable difference between the "converged" and the "unconverged" calculation are the large H and H+ (and thus n_e) abundances for the former, which is a consequence of the large auroral dissociation and ionization rates. Because of the fact that for an "unconverged" atmosphere, [H_2] >> [H] near or below the homopause, it is difficult for the precipitation process to produce enough H to overwhelm the number density of H_2 and become the leading neutral. In fact, Horanyi et al. [1988] have shown in their Figure 19 that even for a "converged" atmosphere, [H_2] is still much greater than [H] below 500 km. Thus, the neutral particle number density can be considered unaltered by precipitation
or heating. The electron number density, on the other hand, increases significantly; but as long as the change of \( n_e \) under the effect of precipitation or heating maintains a smooth appearance, the maximum \( \sigma \) always occurs at the altitude where \( v_m = \omega_d \) (this condition depends approximately on only the neutral particle density and the local B field). Thus, particle precipitation or Joule heating can increase the magnitude of the conductivity, but it will not significantly change the altitude of peak conductivity.

2.4 Zenographic Location of the NPHS

The north polar hot spot was reportedly near the polar cap (Caldwell et al., 1988) which includes all open field lines that extend into the magnetotail based on the \( O_4 \) model. In fact, one can see in Figures 3 and 6 of Caldwell et al. [1988] that the main part and the peak of the NPHS are inside the polar cap. It appears, therefore, that the NPHS is located poleward of the last-closed-field-line. The IR hot spot is then separate from the UV aurora [Clarke et al., 1989]. As discussed in the Introduction and in Section 2.2, the Pedersen current that causes the Joule heating and thus the hot spot, maps magnetically to the current flowing in the magnetotail, which causes the field lines in the tail to twist into a helix. The quantitative mapping relations between the ionosphere and the tail and the related power dissipated in the ionosphere and power delivered to the magnetotail and solar wind have been discussed in detail by Hill et al.[1983b] for the pole-on geometry then expected for Uranus and by Isbell et al. [1984] for Jupiter. We have, in Section 2.2, redone the calculation for the case of a thin magnetopause boundary layer. We have, however, assumed that the surface magnetic field strength and the Pedersen conductivity are uniform with respect to planetary longitude. As a consequence, the power from Joule heating by the Pedersen current is uniform and independent of longitude. This would lead to a cloud of moderately
"warm" spots evenly distributed throughout the auroral disc, a situation that may correspond to the observed non-zero background emission (see Figure 3 of Caldwell et al., 1988), but we would be unable to explain the enhancement at a certain longitude. A localized hot spot must be explained in terms of something nonuniform or asymmetric. An important factor that can influence the location of the hot spot and that is generally nonuniform is the surface magnetic field.

The magnetic field of Jupiter is not a spin-axis-oriented dipole. It is tilted and offset with respect to the spin axis, and higher-order moments cause it to vary irregularly with longitude. A detailed surface magnetic field map may be obtained from either the O₄ model [Acuña and Ness, 1976] or the P11(3,2)A model [Davis et al., 1975]. Connerney [1981], using a generalized inverse approach, derived a model that includes an azimuthal sheet current and is therefore more realistic. However, we will use the O₄ model here because the majority of the references we cite for this work used the O₄ model and because the error caused by using the O₄ model is probably negligible compared with other possible errors. The isocontour map of surface field intensity [Acuña et al., 1983] derived using the O₄ model shows that in a wide range of latitude, including 60° lat where the NPHS peak is reported to occur, the field varies nearly as a sine function with respect to longitude and its maximum value of about 14 Gauss occurs near λ₃ =150° (Figure 5). In the following analysis, we will, for mathematical convenience, approximate this magnetic field variation by a sinusoidal form (as depicted in Figure 5):

\[ B = B_0 \left( 1 + \beta \sin \phi \right) \]  \hspace{1cm} (2.4.1)

where \( B_0 = 10 \) Gauss and \( \beta = 0.4 \). The east azimuthal angle \( \phi \) is defined to be 90° at \( \lambda_{III} =150° \) where the surface magnetic field reaches its maximum.

Such a variation of the surface magnetic field in System III longitude makes the
Birkeland current that flows into the auroral disc nonuniform in longitude. If, for simplicity, the Jovian magnetospheric tail is assumed to have a figure-8 cross-section, instead of the more realistic theta (θ) cross-section (see Dessler and Juday [1965], Isbell et al. [1984]), as used in Section 2.2 and in Appendix A, j/B in the tail can be assumed constant. Correspondingly, j/B above the auroral disc is constant too according to (2.2.1), which leads to j ∝ B above the disc, viz, the Birkeland current is enhanced near the longitude of maximum surface magnetic field (~150°). This enhancement of Birkeland current would tend to make the Joule heating power maximize at 150° if the Birkeland current were the only thing altered by the nonuniform surface magnetic field configuration. Other factors that can be altered by the field and that play important roles in controlling the power distribution are the electron/ion concentration and/or the Pedersen conductivity.

The electron/ion concentration at IR hot-spot altitude could be uniform with respect to System III longitude (except for day-night variations) if no energetic particles precipitated into the atmosphere or if the precipitation were uniform in longitude. It has already been shown that the precipitating particles do not have enough power to produce the observed IR hot spot. On the other hand, particle precipitation does alter ion and electron concentrations at the precipitating point and nearby regions owing, partly, to some highly energetic ions (~Mev) or electrons (~ 30 kev) that are able to precipitate down to the hot spot altitude and partly (perhaps) to diffusion that further alters the electron/ion concentration of nearby longitudes and latitudes. Obviously, the maximum enhancement of ionospheric concentration occurs in the longitudinal region where maximum particle precipitation occurs. It was pointed out, in connection with the Earth's radiation belt, that nondipolar anomalies (i.e., higher-order moments) in the Earth's magnetic field create preferred geographical areas where energetic particles will be lost into the atmosphere [Dessler, 1959; Cladis and Dessler, 1961]. The same effect must occur at Jupiter, where the nondipole components of the magnetic field are relatively larger than at Earth [Acuña
and Ness, 1976]. A simple model depending on particle drifts and slow-pitch angle diffusion [Dessler and Hill, 1979] suggests that the maximum precipitation should occur at the maximum of negative dB/dλ_{III} in the case of subcorotating electrons and at the maximum dB/dλ_{III} for supercorotating positive ions. Observations of the Jovian UV aurora by Voyager found the maximum signal in the northern hemisphere occurred at System III longitude λ_{III} =210° ± 10° [Herbert et al., 1987], corresponding to the Dessler and Hill description for the case of electron-generated aurora. This contradicts the argument that it is ions (rather than electrons) that excite the observed UV aurora. This difficulty can be overcome by invoking an alternative drift mechanism, related with the observed lag of the magnetospheric plasma population near the torus below corotational speed, which would allow an ion-generated aurora to be consistent with the Voyager observations (see, for example, Herbert et al. [1987]). The electric field (in the corotating frame) associated with subcorotation causes charged particles to drift westward at the subcorotation rate in addition to whatever gradient and curvature drift they undergo. Thus ions with sufficiently slow gradient/curvature drift experience a net subcorotational drift. At L=6, for example, a 1-Mev ion executes subcorotational drift if the local plasma population lags behind corotation by 5%. With increasing L, larger corotation lag is expected [Hill, 1979] and thus particles with higher energy will be able to drift subcorotationally and thereby precipitate into the region near λ_{III} =210° ± 10°. Ions with energy in the Mev range are energetic enough to precipitate down to the altitude near the homopause. These particles ionize the atmospheric molecular hydrogen so as to increase the local electron and ion concentrations, as do the high-energy electrons (∼30 kev, although the number of these electrons may be relatively small). In this study, we phenomenologically prescribe the electron/ion concentration at the auroral disc using another sinusoidal form:

\[ N = N_0 \left[ 1 + \alpha \sin(\phi + \phi_0) \right] \]  
(2.4.2)
where the constant $N_o$ is the average electron column number density (height-integrated number density) and $\phi_o (= 210^\circ - 150^\circ = 60^\circ)$ is the phase difference between the maximum $B$ and maximum $N$. The parameter $\alpha$ is a measure of the influence of particle-precipitation enhancement on the electron/ion concentration at the hot-spot latitude/altitude. We are not aware of any observational result for this parameter, and we will assume that $\alpha$ is a constant less than unity for mathematical simplicity. Although the electron/ion concentration may be enhanced by factors of 10 to 100 [Caldwell et al., 1988] at the UV aurora peak, the influence of the enhancement should decrease at the IR hot-spot latitude/altitude. Thus our assumption of $\alpha < 1$ is plausible.

Notice that the azimuthal angle is defined in the opposite direction of the System III longitude, i.e., $\phi$ is increasing in the corotating direction while $\lambda_{III}$ is decreasing. The following mapping relations:

For max $B$: \[ \phi = 90^\circ \iff \lambda_{III} = 150^\circ \]

For max $N_i$: \[ \phi + \phi_o = 90^\circ \iff \lambda_{III} = 210^\circ \]

give $\phi_o = 60^\circ$ and $\lambda_{III} = 240^\circ + 360^\circ \times H(\phi - 240^\circ) - \phi$, where $H(x)$ is the step function: $H(x) = 1$ if $x > 1$, and $H(x) = 0$ otherwise.

Because the height-integrated Pedersen conductivity ($\Sigma_p$) is proportional to electron/ion concentration and inversely proportional to the local magnetic field strength, the conductivity should be nonuniform in System III longitude and is enhanced at a certain longitude. A mathematical approximation of the variation of $\Sigma_p$ in System III can be written as:

\[
\Sigma_p \sim \frac{N}{B} = \frac{\Sigma_0}{1 + \frac{\alpha \sin(\phi + \phi_o)}{\beta \sin \phi}}
\]

Such a longitude-dependent conductivity affects the Joule heating process in many
ways, among which the following two are obvious and important: (1) It causes the Joule heating to distribute in such a way that maximum power dissipation in the auroral disc occurs at the minimum $\Sigma_p$, and minimum power dissipation at the maximum $\Sigma_p$, if $\Sigma_p$ were to act alone in determining the Joule heating distribution (note the auroral disc is a source rather than a load); (2) The maximum $\Sigma_p$, and maximum Birkeland current, which is proportional to surface magnetic field, occur at different longitudes [Compare Equations (2.4.1) and (2.4.3)]. Mother Nature probably does not like this to happen and would presumably modulate the Birkeland current and tend to increase the current at or near the maximum conductivity [Wolf, private communication, 1990]. If the current modulation did not exist, the Joule heating power would maximize at the longitude between the maximum B and minimum $\Sigma_p$. For example, if we take $\alpha=0.5$, the minimum $\Sigma_p$ would occur near 120° System III, and the maximum Joule heating power would hence occur between 120° and 150°. If we include the current modulation, the maximum Birkeland current will move to higher longitude and thus the maximum Joule heating power will also move to higher longitude. More detailed quantitative analysis leading to the Joule heating distribution will be conducted in Chapter 3.

2.5 Motion of the SPHS

The drift motion of the SPHS is mysterious. According to Clarke et al. [1989], the SPHS may be found anywhere from System III longitude 300° to 90°, and when tracked for a period of an hour or so, is observed to drift toward higher longitudes. No satisfactory explanation has been given to account for the motion. However, it serves as additional evidence that the polar IR hot spots are not generated by direct particle precipitation. No systematic drift motion has been reported for the UV aurora, which is thought to be
produced by particle precipitation. If the hot spots and the UV aurora were indeed two aspects of the same phenomenon—that of energetic particle interaction with atmospheric gases, as proposed by Halthore et al. [1988], drift motion of UV the aurora, at least in the southern polar region, should also have been observed by Voyager and/or IUE.

The longitudinal limitation of the drift motion of the SPHS (to the range of 300°–90°) is not understood, either. In certain cases, the drift seems to be sun-related, although statistically the SPHS has no fixed position with respect to the subsolar point [Caldwell et al., 1988]. For example, the SPHS drifted roughly at the subsolar rate when it was observed during May 5, 1984 (see Figure 7). On the other hand, it was observed drifting at a slower speed on April 18, 1983 and July 26, 1984. However, the evidence of the slower speeds may not be sufficient to rule out the possibility that the drift motion may be related to the sun which can, for example, increase the photoionization rate near the subsolar point. As a matter of fact, the correlation between the sun and the SPHS on May 5, 1984 seems to be pretty convincing, because of the nearly constant longitude offset of the SPHS from the central meridian longitude during the 90-minute observation period (as of –13° from the beginning and of –19° at the end of the observation). The offset may be interpreted as the time lapse between the peak sunshine and the peak emission due to the delayed maximum photoionization at the hydrocarbon layer, perhaps. It may also be interpreted as the longitudinal offset of maximum Joule heating power with respect to maximum conductivity. On the other hand, if the motion is indeed generated by the Sun, the (motion of the) SPHS should have been seen outside the limited longitude range. Caldwell et al. [1980, 1983, and 1988] have shown negative results outside that range. The disappearance of the hot spot remains unexplained and will not be discussed in detail in this work. However, we suspect that, if the disappearance is real and the sun is indeed responsible for the drift, the disappearance could be at least partially due to time variations of solar wind.
Fig. 7. A summary of the SPHS's motion plotted as change in longitude with time (from Caldwell et al., 1988). Each symbol represents one image, with a different type of symbol for each day. The motion of the SPHS for each day is shown by a least-square fit as a solid line connected to the symbols it represents. Broken lines are drawn to show the motion of hypothetical features moving at the rate of Io, the Sun, and Jupiter.
The assumption that the sun induces the drift motion of the SPHS can not be used to explain those observations which (1) show the SPHS drifts at slower speed and (2) indicate the SPHS has no fixed position with respect to the subsolar point because of its variable longitudinal offset [Caldwell et al., 1988]. For the time being, however, we do not attempt to rule out the possibility of a solar-induced-drift. Rather, we want to point out that the Sun as well as other effects (atmospheric waves, for example) may take part in controlling the drift motion of the SPHS in addition to the magnetic field that has a weak longitudinal variability. In this study, we discuss gravity waves as a possible mechanism that controls the drift of the hot spot.

The slow variation of the surface magnetic field in the southern polar region with respect to the System III longitude, as shown in Figure 5, may also be modeled in terms of a simple sinusoidal function like Equation (2.4.1), with smaller \( B_o \) and \( \beta \). Within \( 180^\circ-300^\circ \), the field strength reaches its maximum value of about 10 Gauss. This longitudinal region approximately coincides with the region where the SPHS was not observed. The magnetic field at other longitudes, i.e., where the SPHS has been found, has a relatively steep gradient. If the disappearance of the SPHS within \( 90^\circ-300^\circ \) is real, particle precipitation and/or Birkeland current must play roles in order to prevent the SPHS from being observed in the longitude range where it was most likely to occur otherwise. (Maximum Joule heating power would have occurred near the longitude of maximum B, i.e., minimum conductivity if electron/ion concentration and Birkeland current were uniform in longitude.) In case the disappearance of the SPHS is not real and the failure to observe the SPHS was solely because of the observers' bad luck, the hot spot may be equally possible to occur at any longitude. This can happen if the electron/ion concentration and the Birkeland current as well as the conductivity are more or less uniform in longitude. Whether or not the disappearance is real, the Joule heating rate, like the magnetic field strength, should have less variability in longitude in the southern hemisphere as
compared to the north, and in any case should be stable in time. To cause the SPHS to move, additional physical process(es) must be present.

The presence of additional process(es) should not contradict the observed localization of the NPHS, which can be described fairly well by the Joule heating hypothesis (see Chapter 3). The key is the B-field configuration of the planet. From the discussion in the previous section, we see that the Joule heating power depends rather strongly on the local B field, which presents different features in each hemisphere. In the northern polar region, the B field varies quite rapidly with changing longitude. Thus the first-order perturbation term in (2.4.1) is not negligible (0.4 compared to 1). Because of this rapid variation of B field with respect to longitude, the "windshield wiper effect" is large and therefore strong particle precipitation can be produced in the active sector. The resultant ion concentration is enhanced near the active sector and at higher latitude (i.e., the hot-spot latitude) compared to its background. Mathematically, it is represented by a non-negligible $\alpha$ in (2.4.2). The combined effect of these relatively large parameters causes the Joule heating power to be a sensitive function of the longitude, equivalent to a strong confining tendency that restricts the NPHS to a certain longitude. The southern polar region is different. The lesser longitudinal variability of the surface magnetic field in this region causes $\alpha$ and $\beta$ to be small compared to unity so that the contrast between the maximum power and the minimum is small. This weak confining tendency of the magnetic field makes it possible for other physical processes to participate in controlling the longitudinal position of the hot spot by affecting the conductivity and/or the local excitation of methane through dissipating heat, for example. These processes may possess asymmetric north-south features (partially) because of the B-field asymmetry between the two hemispheres. For example, the additional heat deposition from these processes may be less than Joule heating in the north but comparable to Joule heating in the south; correspondingly, the maximum IR emission longitude is determined mainly by Joule heating in the north while determined by the
combined effect of Joule heating and the additional process(es) in the south. If this happens, the SPHS could "slide" along the auroral disc, depending on the total effect of the combined processes. If the total effect is variable, a drift motion of the hot spot can be expected.

Possible additional processes could be either external or internal. Of the external sources other than solar radiation, Io modulation has been excluded by Caldwell et. [1988] because there is a lack of obvious correlation between Io and the motion of the SPHS. One of the remaining possibilities for external sources, particle precipitation has also been excluded. Of the internal sources, the cloud motion observed by Pioneer [Smith and Hunt, 1976] and Voyager [Smith et al., 1979] could be a candidate if the speeds were fast enough. The zonal velocity profiles of different cloud features [Smith et al., 1979] shows that the velocity of the high-speed north temperate jet is only 150 m/s, at least 20 times smaller than that of the SPHS. The cloud features in the southern polar region move even slower than the jet. We propose that the drift of the SPHS may be interpreted as the result of atmospheric wave effects such as a solitary wave or a normal plane wave. Furthermore, we propose that a wave packet is the proper candidate—a wave packet is more realistic than a monochromatic plane wave because in actual circumstances any observable wave consists of a finite width (Δk) of wave number (k). The packet represents a wave chain consisting of several wavelengths and moving at a specific group velocity. (The group velocity is the velocity at which the energy of the wave travels). Therefore, as the wave packet travels, the hot spot moves.

The SPHS has been observed to move at a speed of several km/sec. This is greater than the sound speed (~1.2 km/sec) at the hot-spot altitude. Because the SPHS is proposed to be moved forward directly by the wave (or wave packet), we introduce a new hypothesis in which the motion of the SPHS is analogous to that of the intersection point of the two blades of a pair of scissors. As the wave front (equivalent to one blade) approaches the
auroral disc (equivalent to the other blade), the intersection point of the two blades (which is where the hot spot should be) moves at a different speed and in a different direction from the relative speed of the two blades (as illustrated in Figure 8). The fast drift speed of the SPHS is caused by the small angle between the wave front and the auroral disc. We redraw here as Figure 9 the L=constant drift shell intersections with the Jovian southern ionosphere produced by Roederer et al. [1977]. We see that, within 300°–90° longitude where the SPHS has been observed, the last-closed-field line boundary is almost parallel to the circle of constant latitude of 70°S, with a small angle between them. In order to make the intersection point between the wave front and the arc move fast, the normal to the wave front (i.e., the wave vector) must have a large component in the south-north direction. In other words, the wave packet moves mainly south-north, while the SPHS moves mainly east-west.

One important test of the hypothesis is to see whether the wave packet characterized above can quantitatively produce the observed speed for the SPHS. Such a wave packet must also be energetic enough to cause excess methane excitation in addition to and comparable to the excitation caused by Joule heating in the south. Yet the excess methane excitation must be negligible compared to the Joule heating in the north in order that the NPHS be fixed in System III. These requirements constitute the basis of the quantitative analysis in Chapter 4 where we will derive the group velocity of an atmospheric gravity wave and evaluate its heat loss. The analysis will be restricted within the observed SPHS longitude (i.e., 300°–90°) in which the wave packet is required to move mainly north-south, owing to the special geographic locations of the southern last-closed-field line boundary and the auroral disc.
Fig. 8. Illustrative sketch of the motion of the intersection point between two blades of a pair of scissors. The angle between the wave front (blade #1) and the auroral disc (blade #2) is designated by $\delta$. The line $a(0)b(0)$ represents the position of the wave front at $t = 0$. The arrow shows the direction that the wave front goes. At time $t$, the line moves to a new position denoted by $a(t)b(t)$ which is $Vt$ apart from $a(0)b(0)$ (shown as $OA$), $V$ being the velocity of the wave front. The distance that the intersection point has moved (shown as $OB$) during the time period $t$ can be derived, from the simple geometric view, as $Vt/\sin\delta$. The speed of the intersection point along the horizontal line is thus $V/\sin\delta$. 
Fig. 9. L=constant drift shell intersections with the planetary ionosphere in the southern hemisphere of Jupiter [from Roederer et al., 1977; Connerney et al., 1981].
3. SIMPLE MODEL OF LONGITUDINAL CONTROL OF THE NPHS

In this chapter, we use a perturbation method to derive the Joule heating power as a function of System III longitude. The zero-order situation will be adopted basically from Isbell et al. [1984] and from Section 2.2. The magnetic moment $M$ is assumed to be oriented parallel to the planetary spin vector $\Omega$. The annular ionosphere is simplified as a thin disc and is assumed to be magnetically mapped to the magnetotail boundary layer such that the Pedersen current in the disc-like ionosphere is closed by Birkeland currents flowing into and out of the disc (the Birkeland currents eventually connect with the current that sustains the helicity of the tail magnetic field lines). The inner and outer edges of the disc are represented by the colatitudes $\theta_1$ and $\theta_2$, respectively, mapping to the two tail cylinders illustrated in Figure 2 of Isbell et al. [1984] and in Figure 3 of this work. The zero-order height-integrated Pedersen conductivity of the disc is taken to be 5 mho, the value used by Isbell et al. [1984]. The disc geometry and the currents flowing into and out of the disc are shown in Figure 4. We start by deriving the zero-order electric potential utilizing the current continuity equation:

$$\nabla \cdot \vec{J} = 0$$  \hspace{1cm} (3.1)

which, when using cylindrical coordinates, can be written as:

$$\nabla_h \vec{J}_\perp + j_z = 0$$  \hspace{1cm} (3.2)

where $\nabla_h$ designates horizontal differentiation, $J_\perp$ is height-integrated current density perpendicular to the magnetic field (which is in the z-direction) and $j_z$ is the field-aligned Birkeland current density. Because:
\[ \vec{J}_x = \Sigma_p \vec{E} - \Sigma_H \vec{E} \times \frac{\vec{B}}{B} \] (3.3)

and

\[ \vec{E} = - \nabla \phi \] (3.4)

with \( \phi \) being the electric potential, \( \Sigma_p \) and \( \Sigma_H \) being height-integrated Pedersen conductivity and Hall conductivity respectively (for the sake of simplicity, we denote \( \vec{E}' \) as \( \vec{E} \)), we can write (3.2) as:

\[ \Sigma_p \frac{\partial \phi}{\partial \rho} + \Sigma_p \rho \frac{\partial^2 \phi}{\partial \rho^2} + \Sigma_p \rho \frac{1}{\rho} \frac{\partial^2 \phi}{\partial \phi^2} + \Sigma_p \frac{\partial \Sigma_p}{\partial \rho} \left( \frac{\partial \phi}{\partial \rho} + \frac{1}{\rho} \frac{\partial \phi}{\partial \phi} \right) - \rho j_z = 0 \] (3.5)

where we have used the condition \( \Sigma_p = \Sigma_H \) for the disc (because \( v_i = \omega_i \)) and assumed that they are both independent of the polar distance \( \rho \) (which is equivalent to assuming that they are independent of latitude because \( \rho = R J \theta \), with \( R_J \) being the planetary radius and \( \theta \) being the colatitude). In the paper of Isbell et al. [1984], these quantities are assumed to be also independent of System III longitude (or equivalently the azimuthal angle \( \phi \)). This situation is taken here to be our zero-order approximation, in which case the electric potential is also independent of \( \phi \), so that (3.5) becomes:

\[ \Sigma_o \frac{\partial \phi^{(o)}}{\partial \rho} + \Sigma_o \rho \frac{\partial^2 \phi^{(o)}}{\partial \rho^2} - \rho j_z^{(o)} = 0 \] (3.6)

where the superscript "o" represents zero-order quantities and \( \Sigma_o \) is the uniform Pedersen conductivity of the disc adopted by Isbell et al. [1984]. \( j_z^{(o)} \) is the zero-order Birkeland current injecting into the disc and is derived in Appendix A using the tail magnetic field configuration discussed by Zhan [1989] and is given in (A9). The latitudinal variation of the slippage rate is ignored in the following analysis and we simply take \( \Omega' = \Omega \). The
Fig. 10. Mapping between the auroral disc and the magnetopause boundary layer. Points A, B, C and D in the disc are connected to points A', B', C' and D' in the boundary layer by four different field lines. The total current and magnetic flux through the shaded area ABCD in the disc are equal to those through A'B'C'D' in the boundary layer. The arrows from A, B, C and D can be thought of as field lines with the arrows indicating the direction of the local field. The arrows into A', B', C' and D' can also be viewed as field lines in the boundary layer, although they are actually curved because of the helicity of the magnetic field in the boundary layer.
simple mapping relation between the auroral disc and the magnetotail is shown in Figure 10. Equation (3.6) is an inhomogeneous Euler equation and a solution is:

$$\phi^{(o)} = \frac{\rho^2}{4 \Sigma_o} J_z^{(o)}$$

(3.7)

This solution satisfies the boundary conditions at two edges of the disc:

$$J_\rho^{(o)}(\rho_1) = -\pi^{(o)}(\rho_1)$$

(3.8)

and

$$J_\rho^{(o)}(\rho_2) = \pi^{(o)}(\rho_2)$$

(3.9)

The two zero-order sheet currents $\pi^{(o)}(\rho_1)$ and $\pi^{(o)}(\rho_2)$ flowing into and out of the disc at the two edges, respectively, are derived in Appendix A. The corresponding zero-order electric field is:

$$E_\theta^{(o)}(\theta) = \frac{\Omega B}{\sigma_o} \rho$$

(3.10)

which is essentially the same as (2.2.1) with the simplification mentioned above. The zero-order electric field gives rise to a constant potential drop across the two edges of the disc and constant Joule heating rate in the disc. The first-order part of (3.5) will re-distribute the Joule heating power in the disc and cause a preferred longitude at which the power is larger than at other longitudes. Expanding (3.5) gives rise to the following first-order equation:

$$\Sigma_o \frac{\partial \phi^{(1)}}{\partial \rho} + \Sigma_1 \frac{\partial \phi^{(o)}}{\partial \rho} + \Sigma_o \rho \frac{\partial^2 \phi^{(1)}}{\partial \rho^2} + \Sigma_1 \rho \frac{\partial^2 \phi^{(o)}}{\partial \rho^2} + \Sigma_o \frac{\partial^2 \phi^{(1)}}{\partial \phi^2} + \Sigma_1 \frac{\partial^2 \phi^{(o)}}{\partial \phi^2} + \frac{\partial \Sigma_1}{\partial \phi} \frac{\partial \phi^{(o)}}{\partial \rho} = \rho \tilde{f}_z^{(1)}$$

(3.11)
where $\Sigma = \Sigma_p - \Sigma_o$. The first-order Birkeland current on the right hand side of (3.11) can be written in terms of (A9):

$$j_z^{(1)} = -\Sigma_o \frac{2\Omega}{\sigma_o} B^{(1)}$$

(3.12)

where $B^{(1)} = B_o \beta \sin \phi$. As pointed out earlier, this current is written under the assumptions of uniform and figure-8 type magnetotail, and is totally independent of the ionospheric conductivity. In a real situation, the feed-back effect of the conductivity would, from terrestrial experience, probably readjust the distribution of the Birkeland current and presumably increase the strength of the current at/near the longitude of large conductivity [Wolf, private communication, 1990]. Thus, (3.12) is written without considering the modulation of the conductivity. However, up to this point, we have no direct theoretical basis to write out an explicit dependence of $j_z$ on $B$ and/or $\Sigma_p$. We will, in the following analysis, derive such a dependence using an iterative method. With this current, we will be able to obtain the first-order conductivity-modulated electric field utilizing the current continuity equation (3.2) and Ohm's law (3.3). The longitude-dependent Joule heating power can then be calculated from the electric field and the conductivity.

The first-order electric potential under the influence of the longitude-variant Birkeland current (3.12) can be obtained from equation (3.11):

$$\Sigma_o \frac{\partial \varphi_1}{\partial \rho} + \Sigma_o \rho \frac{\partial^2 \varphi_1}{\partial \rho^2} + \Sigma_o \frac{\partial^2 \varphi_1}{\partial \phi^2} + \rho \Sigma_o \frac{2\Omega B_o}{\sigma_o} \beta \sin \phi = 0$$

(3.13)

where we have used (3.7) and let $\Sigma_1 = 0$. The general solution to (3.13) is derived in Appendix C and is given below:

$$\varphi_1 = \sum_{m=0}^{\infty} (A_m \cos m \phi + B_m \sin m \phi) \cdot (C_m \rho^m + D_m \rho^{-m}) - \frac{2\Omega B_o}{\sigma_o \beta \rho^2} \sin \phi$$

(3.14)
and the constants A, B, C and D are to be determined from the boundary conditions. The latitudinal current yielded from (3.7) and (3.14) is:

\[
J_\rho = \frac{\Omega B_o}{\sigma_o} \rho \\
- \Sigma_o \sum_{m=0} m[A_m(C_m \rho^{m-1} + D_m \rho^{-m-1}) + B_m(C_m \rho^{m-1} - D_m \rho^{-m-1})] \sin(m\phi) \\
+ \Sigma_o \sum_{m=0} m[B_m(C_m \rho^{m-1} + D_m \rho^{-m-1}) - A_m(C_m \rho^{m-1} - D_m \rho^{-m-1})] \cos(m\phi) \\
+ \frac{2 \Omega B_o}{3 \sigma_o} \beta \rho (2 \sin \phi - \cos \phi) \tag{3.15}
\]

To settle the unknown constants A, B, C and D, we need to match (3.15) with the sheet currents \(\pi(\rho_1)\) and \(\pi(\rho_2)\) at the inner and outer edges of the disc, as shown in (3.8) and (3.9). The two sheet currents are given in (A10-11) where B is given in (2.4.1). This procedure yields the following relations:

\[
\kappa_1 \equiv A_1 D_1 = -\frac{\rho_1^2 \rho_2^2}{2(\rho_1 + \rho_2) \sigma_o} \Omega B_o \beta \tag{3.16}
\]

\[
\kappa_2 \equiv B_1 C_1 = \frac{\rho_1^2}{2(\rho_1 + \rho_2) \sigma_o} \Omega B_o \beta \tag{3.17}
\]

\[
\kappa_3 \equiv B_1 D_1 = -\frac{\rho_1^2 \rho_2^2}{6(\rho_1 + \rho_2) \sigma_o} \Omega B_o \beta \tag{3.18}
\]

\[
\kappa_4 \equiv A_1 C_1 = -\frac{\rho_1^2 + \rho_2^2}{6(\rho_1 + \rho_2) \sigma_o} \Omega B_o \beta \tag{3.19}
\]

and \(A_m = B_m = C_m = D_m = 0\), for \(m > 1\). Thus, the electric field potential including the first-order perturbation due to (2.4.1) is:

\[
\varphi = -\frac{\Omega B_o}{6 \sigma_o} \rho^2 (3 + 4 \beta \sin \phi) + \frac{1}{\rho} (\kappa_1 \cos \phi + \kappa_3 \sin \phi) + \rho (\kappa_2 \sin \phi + \kappa_4 \cos \phi) \tag{3.20}
\]
As a reminder, we have ignored the dependence of the conductivity on longitude in the above calculation. Substituting (2.4.3) and (3.20) into (3.3) and (3.4), we get the first-order perpendicular current due to the nonuniform conductivity and to the simple-minded (unmodulated) Birkeland current (3.12). We can then combine such a perpendicular current with (3.2) to write the new Birkeland current which is considered to have been modulated by the nonuniform conductivity:

\[ j_z = - \sum_o \frac{\Omega B o}{\sigma_o} [2 + 2 \alpha \sin(\phi+\phi_o) + \alpha \cos(\phi+\phi_o) - \beta \cos\phi] \quad (3.21) \]

Using the modified Birkeland current, we can improve the incomplete potential (3.20) by substituting (3.21) into (3.11) and using (2.4.3), we can get the new \( \varphi \) using the same procedure from (3.13)-(3.20):

\[ \varphi = - \frac{\Omega B o}{6\sigma_o} \rho^2 (3 + 4 \beta \sin\phi) \]
\[ + \left[ \frac{1}{\rho} \left[ (\kappa_1 + \kappa_1') \cos\phi + (\kappa_2 + \kappa_2') \sin\phi \right] + \rho \left[ (\kappa_3 + \kappa_3') \sin\phi + (\kappa_4 + \kappa_4') \cos\phi \right] \right] \quad (3.22) \]

where

\[ \kappa_1' = - \frac{\rho_1^2 \rho_2^2}{4(\rho_1+\rho_2)\sigma_o} \Omega B_o(\alpha \sin\phi_o + \alpha \cos\phi_o - \beta) \quad (3.23) \]
\[ \kappa_2' = \frac{\rho_1^2 + \rho_2^2 + \rho_1 \rho_2}{4(\rho_1+\rho_2)\sigma_o} \Omega B_o(\alpha \sin\phi_o + \alpha \cos\phi_o - \beta) \quad (3.24) \]
\[ \kappa_3' = - \frac{\rho_1^2 \rho_2^2}{4(\rho_1+\rho_2)\sigma_o} \Omega B_o(\alpha \sin\phi_o - \alpha \cos\phi_o + \beta) \quad (3.25) \]
\[ \kappa_4' = \frac{\rho_1^2 + \rho_2^2 + \rho_1 \rho_2}{4(\rho_1+\rho_2)\sigma_o} \Omega B_o(\alpha \sin\phi_o - \alpha \cos\phi_o + \beta) \quad (3.26) \]

The corresponding field \( \mathbf{E} \) and current \( \mathbf{J}_\perp \) can be easily obtained from (3.4) and (3.3). We are now ready to calculate the height-integrated power \( P = \mathbf{J}_\perp \cdot \mathbf{E} \) as function of \( \theta \) and \( \phi \).
The parameters in the model are taken as follows:

\( V_0 \sim 400 \text{ km/s} \) so that \( \mu_0 V_0 \sim 0.5 \text{ Ohm} \). \( \Omega = 1.76 \times 10^{-4} / \text{sec} \). The parameter \( \Sigma_o \) has a large uncertainty: according to Strobel and Atreya [1983], \( \Sigma \sim 0.3 \text{ mho} \) in the absence of auroral precipitation, and it may reach as large as 10 mho in the auroral region. Huang and Hill [1989] derived \( \Sigma = 0.32 \text{ mho} \) and they stated that this is approximately the same as they obtained from the multilayered ionosphere structure deduced from the Pioneer 10 and 11 occultation measurements. Isbell et al. [1984] used a uniform \( \Sigma \) of about 5 mhos. In our simple model we adopt \( \Sigma_o \sim 5 \text{ mho} \) so that \( \sigma_o \sim 2.5 \) (different values of \( \sigma_o \) give different magnitudes for the total power but do not modify the power distribution). The outer edge colatitude (\( \theta_2 \)) of the auroral disc is chosen as 15° (i.e., at the latitude of 75°), which is about 15° poleward of the NPHS peak (~60° latitude). The reason that we choose this colatitude is the following: In our entire analysis, we have ignored certain asymmetric effects such as the offset-tilted-dipole (OTD), e.g., we have assumed that the magnetic dipole is aligned with the spin axis, which oversimplifies the problem. The auroral oval (see the area enclosed by the last-closed-field-line boundary in Figure 2) features an elliptical shape with a major axis of about 18° and minor axis of 13° [Livengood and Moos, 1990], and the ellipse is centered neither on the spin axis nor on the magnetic dipole axis. At 180° System III longitude, the edge of the oval is located at ~60° latitude where the NPHS is situated. To represent such an auroral oval in our simple model, we approximate the oval as a circle centered at the spin axis. The area of the circle is assumed to be the same as that of the ellipse so that the radius of the circle becomes \((18^\circ \times 13^\circ)^{1/2}\), which is about 15°. The width of the auroral disc is determined in such a way that it is equal to the observed width of the NPHS (about 9100 km). This corresponds to \( \theta_1 \sim 8^\circ \) for the inner edge colatitude of the annulus (i.e., at the latitude of 82°). Thus, \( \rho_1 \) and \( \rho_2 \) are about \( 1.83 \times 10^4 \text{ km} \) and \( 9.77 \times 10^3 \text{ km} \), respectively. The parameter \( \alpha \) is assumed to lie between 0 and 1. In terms of these parameters, we can calculate the height-integrated power as a
function of longitude and latitude. However, the latitude dependence of the power should not be taken too seriously. Since that we are interested only in the longitudinal control of the power, we have ignored latitudinal variations of any input physical quantity; thus, the latitudinal dependence of the power comes only from the geometry.

Figure 11 shows a series of power distributions with respect to System III longitude for several different $\alpha$ at $15^\circ$ colatitude. We see that in a relatively large range of $\alpha$ (from 0.2 to 0.8), the Joule heating peaks at $175^\circ$–$180^\circ$, which is in fairly good agreement with the observed hot-spot longitude of $180^\circ$. The peak moves towards smaller longitudes, by about $10^\circ$, at smaller colatitude, for any $\alpha$. Therefore, the Joule heating peak position is not very sensitive to the choice of $\alpha$ and we can not determine, from the maximum power longitude, the value of $\alpha$ that best represents the real situation. To determine the value of $\alpha$, one would probably need to combine the magnetic-anomaly model [Dessler and Hill, 1979] with an aeronomy calculation to derive the electron and ion concentration distribution in System III longitude as a function altitude. Because these calculations involve uncertainties related to the observations such as the energy spectra of ions and electrons before they penetrate into the atmosphere, the resultant value of $\alpha$ may not be better than the one given here.

The height-integrated power shown in Figure 11 is equivalent to the energy flux deposited over the entire ionospheric altitude range. Because of the altitude-dependent feature of the conductivity shown in Figure 6, most of the power is dissipated at the level where the conductivity is a maximum (this argument is valid if and only if the parallel conductivity is sufficiently large). We have already shown in Section 2.3 that the maximum conductivity altitude is close to the upper end of the hot-spot altitude range, and thus we see in Figure 6 that nearly half of the total height-integrated power is dissipated within the hot-spot altitude range ($1\mu b$-$1mb$ pressure level). In other words, as much as $4.0 \times 10^3$ ergs/cm$^2$-sec energy flux is put into the hot-spot range at the peak longitude, as can be
Fig. 11. Illustration of the height-integrated Joule heating power distribution in System III longitude for $\alpha = 0.2, 0.4, ..., 0.8$ (at $\theta = \theta_2$). All the curves have maximum near $180^\circ$, in accordance with the observed NPHS longitude.
calculated from Figure 11. This would coincidentally assign 10.4 ergs/cm²-s to methane, which would agree with the observed radiation power of 9 ergs/cm²-s, if we use methane's mixing ratio of $2 \times 10^{-3}$ [Kim et al., 1985] and the ratio of the numbers of degrees-of-freedom (of methane to that of molecular hydrogen at 200°K) of 1.3. We attempt no further investigation of the detailed energy allocation because we do not have enough data on the radiation power in other wavelengths. The total energy supplied here by Joule heating is, in any event, larger than the direct particle precipitation contribution in the same altitude range by a factor of $\sim 4 \times 10^4$. One should keep in mind that this surprisingly large ratio does not represent the overall input-power ratio (which is of the order of 10–100, compared to 6 for Earth [Hultqvist, 1989]) because the particle precipitation power is restricted mainly to higher altitudes than the Joule heating power.

Figure 12 shows the modulated Birkeland current represented by (3.21) subject to the modulation of Pedersen conductivity, for several different values of $\alpha$. Although the detailed features of the modified current for small $\alpha$ differ slightly from those for greater $\alpha$, the general form of the current is quite insensitive to the value of $\alpha$: each current is distributed in longitude as a sinusoidal function, and has a phase shift of about 90° towards larger longitude relative to the original Birkeland current. The maximum magnitude of the modified Birkeland current occurs at about 240°, consistent with our previous qualitative analysis.
Fig. 12. Longitude-dependence of the modified Birkeland current (amps/cm$^{-2}$) under the modulation of the Pedersen conductivity [Equation (3.21)]. The strength of the current with System III longitude increases as $\alpha$ increases from 0.2 to 0.8. The modulation causes the maximum current to move $\sim 90^\circ$ towards larger System III longitude.
4. SOUTHERN POLAR HOT SPOT (SPHS)

In order to investigate the possibility that atmospheric gravity waves cause the motion of the SPHS, we first study the dispersion relation of such waves in the Jovian atmosphere. The dispersion relation will tell us whether or not the wave speed is large enough to be responsible for the motion of the SPHS. We then evaluate the heat production due to dissipation of the wave through processes such as diffusion.

4.1 Dispersion Relation of the Gravity Wave Under the $\beta$-Plane Approximation

We begin by considering small-amplitude motion in an isothermal atmosphere and adopting the $\beta$-plane approximation [Lindzen, 1967], which accounts for the effect of Coriolis force. To do so, we establish a Cartesian coordinate system on the planetary surface with $x$, $y$, and $z$ being westward (longitudinal), southward (latitudinal) and upward (radial) directions. Other symbols are defined as follows:

- $p, \rho, T$ pressure, density, and temperature
- $t$ time
- $u, v, w$ westward, southward, and upward velocities
- $f$ Coriolis frequency
- $\beta$ \([=-df/dy]\)
- $R$ gas constant for molecular hydrogen
- $c_p$ heat capacity at constant pressure
- $\gamma$ ratio of specific heats
- $g$ acceleration of gravity
- $\nu$ diffusivity

All dependent fields are written in the form of a basic state plus a perturbation; for
example, \( p = p_o + p_1 \). The basic state is motionless and isothermal \((T_o \sim 200^\circ K)\). \( p_o \) and \( p_o \) decay exponentially with height with the atmospheric scale height \( H=RT_o/g \). For the Jovian atmosphere, \( R = 4.2 \times 10^7 \) ergs gm\(^{-1}\) (°K\(^{-1}\)), \( g = 2600 \) cm sec\(^{-2}\), the scale height \( H \sim 32 \) km, \( \beta = -3.26 \times 10^{-4} \) sec\(^{-1}\). \( \beta = -1.71 \times 10^{-14} \) sec\(^{-1}\) cm\(^{-1}\) at the southern latitude of about \( 70^\circ \).

The linearized equations for hydrostatic perturbations on a polar beta plane read (following French and Giersch [1974])

\[
\rho_o \frac{\partial u_1}{\partial t} - \rho_o \nu v_1 + \rho_o R J \beta w_1 + \frac{\partial p_1}{\partial x} = \rho_o \nu \nabla^2 u_1 \tag{4.1.1}
\]

\[
\rho_o \frac{\partial v_1}{\partial t} + \rho_o u_1 + \frac{\partial p_1}{\partial y} = \rho_o \nu \nabla^2 v_1 \tag{4.1.2}
\]

\[
\rho_o \frac{\partial w_1}{\partial t} - \rho_o R J \beta u_1 - \frac{\partial p_1}{\partial z} = \rho_o \nu \nabla^2 w_1 - g \rho \tag{4.1.3}
\]

\[
\frac{\partial p_1}{\partial t} + \rho_o \frac{\partial u_1}{\partial x} + \rho_o \frac{\partial v_1}{\partial y} + \rho_o \frac{\partial w_1}{\partial z} + w_1 \frac{\partial \rho_o}{\partial z} = 0 \tag{4.1.4}
\]

\[
\rho_o c_p \frac{\partial T_1}{\partial t} - \frac{\partial p_1}{\partial z} - w_1 \frac{\partial \rho_o}{\partial z} - \nu \nabla^2 p_1 = c_p \rho_o \nu \nabla^2 T_1 - \nu \nabla^2 p_1 \tag{4.1.5}
\]

\[
\frac{p_1}{p_o} = \frac{\rho_1}{\rho_o} + \frac{T_1}{T_o} \tag{4.1.6}
\]

We follow the work of French and Giersch [1974] and neglect the terms involving diffusivity momentarily in order to solve the dispersion relation. Assuming time and spatial dependencies of the form \( \exp[i(kx+\lambda y+\mu z-\omega t)] \) together with the following transformation:

\[
(u',v',w',T') = \rho_o^{1/2}(u_1,v_1,w_1,T_1)
\]

\[
(\rho',p') = \rho_o^{-1/2}(\rho_1, p_1) \tag{4.1.7}
\]
gives rise to six simultaneous, homogenous algebraic equations:

\[-i\omega u' - fv' + R_\beta w' + ikp' = 0 \] (4.1.1')

\[-i\omega \nu' + fu' + i\lambda p' = 0 \] (4.1.2')

\[-i\omega w' - R_\beta u' - (i\mu - \frac{1}{2H}) p' + g\rho' = 0 \] (4.1.3')

\[-i\omega p' + iku' + i\lambda \nu' - (i\mu - \frac{1}{2H}) w' = 0 \] (4.1.4')

\[-i\omega c_p T' + i\omega p' + gw' = 0 \] (4.1.5')

\[\frac{Hg}{T_o} T' - p' + Hg\rho' = 0 \] (4.1.6')

Setting the determinant of the coefficient matrix equal to zero yields the following dispersion relation:

\[\gamma HgR_\beta^2 + (\gamma - 1)g^2 - \gamma Hg \omega^2] \lambda^2 + [(\gamma - 1)g^2 - \gamma Hg \omega^2] k^2 + 2R_\beta \gamma gH \mu H \lambda - (\gamma - 2)R_\beta g \omega k + \omega^4 - [(\gamma + R_\beta)^2 + \gamma Hg \mu^2 + \frac{2\beta}{4H}] \omega^2 + \gamma Hg \mu^2 + \frac{1}{4H^2} \gamma^2 = 0 \] (4.1.8)

For given \(\omega\) and \(\mu\), (4.1.8) is a quadratic equation for both \(k\) and \(\lambda\). We provisionally let \(\lambda\) be real, because we are interested in waves that propagate mainly in the north-south direction, i.e., waves that have large \(\lambda\). The periodicity condition for the \(x\)-wavenumber is:

\[k = \frac{2\Omega}{|\beta R_\beta^2|}, \quad s = \pm 1, \pm 2, \ldots, \] (4.1.9)

which requires that \(k\) be real, too. (No such periodicity condition is imposed on the \(y\)-wavenumber, in fact, the \(\beta\)-plane approximation we adopted here works only in and near
the mid-latitude region). We are interested in solutions to (4.1.8) for parameters and their upper and lower limits chosen in the following way: the vertical wavenumber \(\mu\) is determined from the pronounced oscillations in the temperature profile derived from the occultation of \(\beta\)-Scorpii by Jupiter [Veverka et al., 1974] which showed a large amplitude (~5K) component with wavelength about 13 km. Thus, \(\mu = 2\pi/(13\text{km})\), or \(4.8 \times 10^{-6} \text{ cm}^{-1}\) [French and Gierasch, 1974]. We set the upper and lower limits for \(k\) and \(\lambda\) as \(1 \times 10^{-4} \text{ cm}^{-1}\) and \(1 \times 10^{-9} \text{ cm}^{-1}\). The choice for the upper limits is based on the fact that the atmospheric waves of interest should have wavelength at least of the order of 1 km. The choice for the lower limits is made in order to comply with the \(\beta\)-plane approximation which does not allow the wave to have wavelength longer than the planetary radius. We choose wave frequencies larger than \(1 \times 10^{-1} \text{ sec}^{-1}\), because a wave with frequency less than \(10^{-1} \text{ sec}^{-1}\) can be shown to have a group velocity that is too small to be related to the SPHS and/or has imaginary \(k\) or \(\lambda\).

Each pair of \((\lambda, \omega)\) yields two different \(k\)'s, two different \(x\)-components of the group velocity \((d\omega/dk_1\) and \(d\omega/dk_2\)) and two corresponding \(y\)-components of the group velocity \((d\omega/d\lambda_1\) and \(d\omega/d\lambda_2\)), as can be seen from the quadratic equation (4.1.8). For those \(\lambda\) that are less than \(1.0 \times 10^{-7} \text{ cm}^{-1}\), we find that \(|k_1| > \lambda, |k_2| > \lambda\) and that \(d\omega/dk_1 > d\omega/d\lambda_1\) and \(d\omega/dk_2 > d\omega/d\lambda_2\) (Figure 13). These inequalities prohibit the wave from propagating primarily in the north-south direction. Therefore waves within this part of the phase space are not considered in this work. Typical values of some important quantities are listed in Table 2 for waves that have larger northward propagation components than E-W components \((\lambda > 0, \lambda >> k)\), and in Table 3 for waves that have larger southward propagation components than E-W components \((\lambda < 0, |\lambda| >> k)\). Because of the double-valued property, we conclude that for each pair of \((\lambda, \omega)\) in the table, two wave modes and wave packets can be expected. The two conjugate wave packets have nearly the same velocity component in the \(y\)-direction, but opposite components in the \(x\)-direction:
Fig. 13. Plot showing the phase space in which the wave can propagate with real east-west component of the wave vector (k) (as shown by the dotted areas). The thick curves at the boundaries of the areas indicate that the wave vector has a larger north-south component than the east-west one ($|\lambda| > |k|$).
TABLE 2. Typical values of the important quantities described in the text with positive north-south wavenumbers. The speed of the SPHS is calculated assuming the angle between the auroral disc and the constant-latitude circle (70°S) is ~8.5°.

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TABLE 3. Typical values of the important quantities described in the text with negative north-south wavenumbers. The speed of the SPHS is calculated assuming the angle between the auroral disc and the constant-latitude circle (70°S) is ~8.5°.

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one towards the east and the other towards the west. The difference in the x-component of
the group velocities between the two modes gives rise to two different drift speeds of the
SPHS. The following section shows a schematic derivation for the drift speeds, using the
concept of the "scissors" hypothesis and in terms of the variables discussed here.

4.2 The Speed of the SPHS

Because our discussion has been restricted to waves with wavelengths no longer
than the Jovian radius, we can approximately take the line of constant latitude of 70°S as a
straight line along the x-direction, and take the line of constant longitude as a straight line
along the y-direction on the β-plane. As mentioned earlier, the equatorial edge of the
auroral disc is almost parallel to the constant latitude line of (70°S) between 300°–90°
longitude, with a small angle between them designated as α₀ which is presumably less than
10°. The simplified geometry is shown in Figures 8 and 14 where the x-axis represents the
constant-latitude line (70°S) and the y-axis represents any constant-longitude line. Let
α^±(1,2) denote the angle between the auroral disc and the x-axis, where the superscript +(−)
indicates that the wave has a large southward (northward) component of wave vector, and
the subscript 1 (2) stands for the 1st (2nd) wave mode. For example:

\[ \tan \alpha^*_1 = \frac{\left| \frac{d\omega}{dk_1} \right|}{\left( \frac{d\omega}{d\lambda} \right)_{k_1}} \]  \hspace{1cm} (4.2.1)

The speed of the intersection point between the wave front and the auroral disc is
\( V_{\text{group}}/\sin \delta \) (refer to Figure 8). From Figure 14a, we find \( \delta=\alpha_1+\alpha_0 \). Thus, the speed of
the SPHS (the intersection point) due to the wave mode #1 whose y-wavenumber is
Fig. 14. Sketches illustrating variations in the speed and direction of the SPHS produced in the following situations:

(a) $\lambda > 0, k_1 < 0$, $V_{SPHS}^{(1)} = - |V_{group}^{(1)}| \sin^{-1}(\alpha_1 + \alpha_o)$;
(b) $\lambda > 0, k_2 > 0$, $V_{SPHS}^{(2)} = |V_{group}^{(2)}| \sin^{-1}(\alpha_2 - \alpha_o)$;
(c) $\lambda < 0, k_1 < 0$, $V_{SPHS}^{(1)} = - |V_{group}^{(1)}| \sin^{-1}(\alpha_1 - \alpha_o)$;
(d) $\lambda < 0, k_2 > 0$, $V_{SPHS}^{(2)} = |V_{group}^{(2)}| \sin^{-1}(\alpha_2 + \alpha_o)$.

The wave front moves poleward in (a) and (b), equatorward in (c) and (d). The wave front and the auroral disc are approximated as straight lines. The intersection point of the auroral disc and the wave front always moves eastward in (a) and westward in (d); it moves westward in (b) if $\alpha_2 > \alpha_o$ and eastward if $\alpha_2 < \alpha_o$; it moves westward in (c) if $\alpha_1 < \alpha_o$ and eastward otherwise.
positive whereas the x-wavenumber is negative is given by the following equation:

\[ V_{SPHS}^{(1)} = -\frac{|V_{group}^{(1)}|}{\sin(\alpha_1 + \alpha_2)} \]  \hspace{1cm} (4.2.2)

The negative sign here comes from the simple geometric analysis of Figure 14a and it indicates that the SPHS moves eastward (in the direction of decreasing System III longitude). For mode #2 with positive x and y wavenumbers, the speed of the SPHS (note that \( \delta = \alpha_2 - \alpha_0 \), as can be seen from Figure 14b) is given by

\[ V_{SPHS}^{(2)} = \frac{|V_{group}^{(2)}|}{\sin(\alpha_2 - \alpha_0)} \]  \hspace{1cm} (4.2.3)

It can be seen that the speed of the SPHS due to mode #2 can either be positive (westward) if \( \alpha_2 > \alpha_0 \) or be negative (eastward) if \( \alpha_2 < \alpha_0 \). Similar conclusions can be drawn for waves with equatorward wave vectors, see Figures 14c and 14d. Some numerical results for the SPHS speed are listed in Tables 2 and 3, assuming \( \alpha_0 = 8.5^\circ \). A positive \( V_{SPHS} \) represents a subcorotating SPHS which drifts in the direction of increasing System III longitude, while a negative \( V_{SPHS} \) represents a supercorotating SPHS which drifts in the direction of decreasing System III longitude. The observed drift speeds of the SPHS (see Figure 7) are all positive [Caldwell et al., 1988, Clarke et al., 1989]. This may be explained by some additional unknown mechanism that suppresses wave modes that would otherwise cause the SPHS to move towards lower longitude (supercoretationally). However, it is too early now to exclude the existence of a supercorotating SPHS. There are only three sets of consecutive observations that favor subcorotational SPHS motion [Caldwell et al., 1988]. It is still possible that a supercorotational SPHS exists but has not been reported.

The x-wavenumber \( (k) \) and any relevant quantities (such as the group velocity) are
in fact discrete, owing to the periodicity restriction (Equation 4.1.9). However, this
discrete feature has negligible effect because of the large values of the integer s. Sub-
stituting the minimum k in Tables 2 and 3, we get the minimum |s| which is about 2×10^3.
The variation of s, which is of the order of unity, is small compared to s itself. Therefore
we could view k and any other related quantities as continuous, as we did in this work.

4.3 Heat Dissipation from the Waves

The dissipation of gravity waves in Jovian atmosphere may be an important source
of local energy [Chamberlain and Hunten, 1987]. The observations and theory regarding
with the proposal that there is anomalous heating of the Jovian atmosphere due to the
dissipation of the waves are discussed by Veverka et al. [1974] and French and Giersch
[1974]. Were it not for dissipative forces, the wave amplitude would grow with altitude as
the inverse square of density, as can be seen from Equation (4.1.7). This can be
understood by considering that, for example, the kinetic energy density \(1/2\rho |v_1|^2\), must
remain approximately constant in the absence of dissipation. It follows that \(|v_1(z)| \sim \rho^{-1/2}\).
Since \(\rho\) varies as \(\exp(-z/H)\), then \(|V(z)| \sim \exp(z/H)\) [Brasseur and Solomon, 1984].
However, molecular viscosity and turbulence dissipate wave energy and tend to destroy the
vertical wave structure [Rees, 1989]; the organized dynamic energy of the wave system as
a whole gradually dissipates into disorganized thermal energy and radiation [Hines, 1977].
In this section, we investigate the rate at which the wave packet dissipates heat, hoping that
the dissipation is comparable to what Joule heating can supply. The reason for this concern
is simple: if the wave does not provide enough power, we would not expect to see a
moving hot spot even though the intersection point has a large enough drift speed, because
the maximum emission power would be controlled by the more localized Joule heating
rather than the wave. On the other hand, if the wave dissipates too much power so that the Joule heating becomes negligible, we would expect to see a similar drift motion of the NPWS. If either of the two situations happens, it is not adequate to relate the observed drift of the SPWS to gravity waves, despite of the high drift speed of the SPWS that might be explained in terms of the waves. It is the main task of this section to locate the portion of the phase space of the wave within which the wave packet can propagate at a speed of the order of km/s and dissipate heat at a rate of the order of several $10^3$ ergs/cm$^2$-s in the southern polar region but less in the northern polar region.

In the following, we derive the dissipation heat by first, following the work of French and Gierasch [1974], asking for the value of the diffusivity $v$ just sufficient to suppress the amplitude growth. This critical value may be an indication of the wave-generated eddy diffusivity if the wave amplitude is constrained by internal instabilities. This value also gives an indication of the molecular diffusivity which would begin to affect the wave importantly [French and Gierasch, 1974]. The critical value is obtained by making the following transformation:

\[ \mu = \mu_0 (1+\delta), \quad \hat{\omega} = \omega (1+\epsilon) \quad (4.3.1) \]

The growth is suppressed if we require $\delta = i/(2H\mu_0)$. It follows [French and Gierasch, 1974] that $\epsilon = -i v (k^2+\lambda^2+\mu^2)/\omega$, or if we plug (4.3.1) into (4.1.8) and solve for $\epsilon$ to first order in $\delta$, it becomes

\[ \epsilon = \frac{2\gamma H g \beta [\mu_0 \lambda + (f^2-\omega^2_0)\mu_0^2] \delta}{\omega_0 [2\gamma H g \omega_0 (k^2+\lambda^2+\mu^2) + \frac{1}{4H^2} + R_0 \beta g (\gamma-2)k + \omega_0 (\frac{R}{H} + 2f^2 - 4\omega_0^2 + 2R^2 \beta^2) }] \quad (4.3.2) \]

It can be shown that $\epsilon$ is small compared to unity a posteriori. The heat production through the wave dissipation can be obtained from the first law of thermodynamics:
\[
\frac{D(u/p)}{Dt} + p \frac{D(1/p)}{Dt} = \frac{Q}{\rho} \tag{4.3.3}
\]

where \(D/Dt = \partial/\partial t + v \cdot \nabla\). We can rewrite (4.3.3) by noting that the density of internal energy \(u = (3+f_0)p/2\) \((f_0\) being the number of the non-translational degrees of freedom per molecule) [see for example, Wolf, 1979]:

\[
Q = \frac{1}{2} (3+f_0) \left( \frac{\partial p}{\partial t} + \vec{v} \cdot \nabla p \right) - \frac{1}{2} (5+f_0) \frac{p}{\rho} \left( \frac{\partial \rho}{\partial t} + \vec{v} \cdot \nabla \rho \right) \tag{4.3.4}
\]

In order to compare \(Q\) with the height-integrated Joule heating derived in Chapter 3, we need to integrate (4.3.4) with respect to altitude. The time average of the height-integrated heat is given by:

\[
\langle Q \rangle_z = \frac{3+4f_0}{4} H \rho_0(0) \Re \left[ -i(ku' + \lambda v' + \mu w')p' \right] \\
+ \frac{5+4f_0}{4} H^2 g \rho_0(0) \Re \left[ (iku' + i\lambda v' + \frac{1}{H} w')p'' \right] \\
- \frac{5+4f_0}{4} H \rho_0(0) \Re \left[ i\hat{\omega} p' \rho'' - \frac{P'}{H} w'' \right] \\
+ \frac{5+4f_0}{4} H^2 g \rho_0(0) \Re \left[ i\hat{\omega} p'' \rho' - \frac{P'}{H} w' \right] \tag{4.3.5}
\]

where the asterisk (*) designates complex conjugate and "Re" designates real part.

Numerical values of \(\langle Q \rangle_z\) for each pair of \((\omega, \lambda)\) can be obtained utilizing Equations (4.1.1–4.1.6) and the dispersion relation (4.1.8). In Figure 15, we illustrate the heat dissipation from the wave as functions of \(\lambda\) and \(\omega\), respectively, by requiring the heating rate in the southern auroral disc (upper curve) to be no less than \(2.5 \times 10^3\) ergs/cm²-s, and the heating rate in the peak longitude of the northern auroral disc (lower curve) to be no greater than \(2.5 \times 10^3\) ergs/cm²-s. The lower limit for the south is set in this way for the unique purpose of letting the total power output (corresponding to the sum of the Joule heating and the heat from the wave) be comparable to that observed in the north (the SPHS is not found to be significantly weaker than the NPHS). The upper limit for the north, on the other hand, is
Fig. 15. Sketches showing the heat dissipation of the wave as functions of the north-south component of the wave vector (a) and the wave frequency (b). The two upper curves represent the heat deposited in the southern auroral disc by a wave capable of causing the SPHS to drift at the observed speed; whereas the two lower ones represent the heat deposited in the northern one. Note that the curves do not include those waves that dissipate heat less (more) than $2.5 \times 10^3$ ergs/cm$^2$-s in the south (north). In evaluating the heat depositions, we have assumed constant temperatures of $210^\circ$K in the southern auroral disc and $230^\circ$K in the northern one.
set for the following two reasons: (1) avoiding the total output in the north being so large that it may completely overshadow the output in the south; (2) avoiding the Joule heating power at the northern peak being equal to or even lower than the total output at other longitudes. The upper and lower limits (of $2.5 \times 10^3$ ergs/cm$^2$-s) are compared with the average Joule heating rate within the hot-spot altitude range.

The reason that each pair of $(\lambda, \omega)$ may correspond to two different heat deposition rate in the two polar regions can be attributed to the difference in surface magnetic fields. The average (zero-order) B-field in the northern auroral disc ($B_0$) is relatively larger than the southern one (10.5 Gauss compared to 9 Gauss, see the two curves in Figure 5). From Equation (3.10), we can see that the average Joule heating power is roughly proportional to $B_0^3$ since $\Sigma_0 \sim 1/B_0$, which means that the power is larger in the north than in the south. Also, the smooth features of the surface magnetic field in the southern polar region leads to weaker auroral activities in this region (as is true for UV emissions) compared to its northern counterpart. Correspondingly, the temperature in the south is lower than in the north, which is confirmed by Voyager 1 CH$_4$ brightness data [Kim et al., 1985]. Caldwell et al. [1979] also found that at higher latitudes the Jovian stratosphere is warmer in the northern hemisphere than in the southern hemisphere, a finding consistent with Voyager 1 observations. (The hemispheric asymmetry in stratospheric temperature was suggested by them to be due to seasonal effects caused by Jupiter's small but finite obliquity). It can be shown that lower base temperature corresponds to higher dissipation of heat from the wave if the first-order perturbation in temperature remains unchanged. This can be understood intuitively: a higher temperature relates to higher mobility of the atmospheric gases, and thus requires lower power supply from the source of the wave in order to excite a gravity wave in the atmosphere with a fixed perturbation temperature. In Figure 15, we also constrain the group velocity to be such that it causes the intersection point between the wave front and the auroral disc in the south to fall into the range [2.5 km/s, 6 km/s]. Thus, the curves
in the figure represent those waves that may be responsible for the drift motion of the SPHS by dissipating sufficient heat in the southern auroral disc while dissipating insufficient heat to overshadow the Joule heating in the north. Figure 16 shows the phase space of the wave with the overlapped region satisfying all the requirements stated above and thus only this part of the phase space may be viewed as being consistent with the drift motion of the SPHS and the simultaneous localization of the NPHS.
Fig. 16. The area covered by the circular points indicates the phase space in which any gravity wave can dissipate heat that is less than $2.5 \times 10^3$ ergs/cm$^2$-s in the northern auroral disc. The area covered by the squares indicates the phase space in which the wave can not only dissipate heat that is larger than $2.5 \times 10^3$ ergs/cm$^2$-s in the southern auroral disc but also propagate at a sufficient high speed to be consistent with the motion of the SPHS. Only those waves enclosed within the overlapped region of phase space are potentially able to account for the observed features (i.e., the localization of the NPHS and the drift of the SPHS).
5. CONCLUSION AND DISCUSSION

The primary objective of this study has been the search for reasonable mechanism(s) in interpretation of the interesting infrared-emission phenomena in Jovian polar regions, specifically, searching for quantitative theories to account for the localization of the NPHS and the longitudinal drift of the SPHS. First, we tested, by quantitative modeling, our hypothesis that Joule heating by auroral Pedersen currents (instead of particle precipitation) causes the IR hot-spot emissions. This includes (a) rejecting particle precipitation as a power source, because not enough energetic particles can penetrate down to the hot-spot altitude to produce the observed radiation intensity (the energy flux provided by precipitating particles accounts for only 1/80 of the radiated power); (b) qualitatively demonstrating the possibility that the longitude confinement of the NPHS is caused by the irregularity of the surface field in System III longitude; (c) quantitatively determining the altitude of maximum Joule heating power which is found only 10km above the estimated hot-spot altitude range. Thus, we conclude that Joule heating is not only capable of confining the NPHS to a certain longitude but also capable of providing a major part of its total power to feed the hot spot. Secondly, we introduced a mathematical model to give a quantitative explanation of the localization of the NPHS at 180° when the Joule heating hypothesis is applied to the highly asymmetric magnetic field in the northern hemisphere. We found that maximum Joule heating averaged over the entire perturbation range is about 175°–180° System III longitude, in reasonable agreement with the observed hot-spot longitude of 180°±10°. Furthermore, we calculated the energy flux at hot-spot longitude and found it to be larger than the radiated energy flux. Third, we tested our hypothesis that the drift motion of the SPHS may be due to a gravity-wave modulation that essentially decouples the SPHS from the weak confinement of the Joule heating. We found that within a particular portion of the wave's phase space, the group velocity of the wave can be
large enough to match the observed SPHS' drift speed of up to several km/s and generate heat comparable to Joule heating in the south while less than Joule heating in the north.

Although we can not rule out other mechanisms that may also be able to interpret the longitudinal confinement of the NPHS and/or the drift motion of the SPHS, the Joule heating hypothesis and the gravity-wave modulation do seem to be plausible and successful interpretations, consistent with all the observations.

In applying the Joule heating hypothesis to the hot spot, we have made the assumption that the electron/ion concentration has a smooth appearance, as described in (2.3.2). The resultant Pedersen conductivity is thus singly peaked at an altitude of ~400km above the ammonia cloud top. This idealized single-peak conductivity profile should be replaced by a multi-peaked one if the multilayer structure of the electron concentration is taken into account. Because the estimated hot-spot altitude range includes part of the multilayer structure, we speculate that the IR hot spot region may also have a similar layered structure although no such observation has been reported. If this is true, each layer acts like a small Faraday disc dynamo, and the connection between two adjacent layers is the lower electron/ion concentration region (or a concentration gap region). Because of the high parallel conductivity that allows the Birkeland current to flow almost freely even at the concentration gap region where the Pedersen conductivity and the corresponding Pedersen current are low, these Faraday disc dynamos can be considered to constitute an electric generator consisting of several smaller generators that are electrically connected in parallel.

The calculated Joule heating rate that is dissipated in the hot-spot range at the longitude of peak heating is about $4.0 \times 10^3$ ergs/cm$^2$-sec (see Chapter 3), which is substantially larger than the observed radiation power. It is possible that the extra energy goes to other wave channels that we know little about. In the actual circumstances, however, it is also possible that the Joule heating rate is less than the above value by a factor of 10 or more, mainly because of the following reasons: (1) The electric field in
Equation (3.10) is overestimated by first ignoring the latitudinal variation of the slippage rate and then simply taking $\Omega' = \Omega$. Comparing Equation (3.10) with Equations (2.2.1-2), we see that nearly 6 times as much power as in Equation (2.2.2) (which takes into account the latitudinal variation of the slippage) can be derived from Equation (3.10) (which is written under the simplification procedure). (2) The uncertainty in the Pedersen conductivity, especially its distribution in altitude, contributes an uncertainty in the Joule heating rate. For instance, if the Pedersen conductivity peak in the high latitudinal region is actually several 10 km higher than shown in Figure 6, the fraction of the total power that is deposited in the hot-spot altitude range can decrease significantly from 50% as used in this thesis to, say, 10%. The combined effect of (1) and (2) can therefore cause the actual Joule heating power available for the hot-spot altitude range to be less than $4.0 \times 10^9$ ergs/cm$^2$-sec. If this happens, the phase space in which gravity waves are consistent with the observed features of the SPHS and the NPHS must be relocated.

We have assumed that the hot spots are located at different latitudes from the UV aurora. Specifically, the UV aurora is in the closed field-line region mapping to the torus and/or larger L shells while the IR hot spots are poleward of the last-closed-field-line boundary. This assumption constitutes the base line of the thesis but, in any event, is not a requirement. It is known that the huge production rate of fresh ions ($\sim 10^3$ kg/sec) from Io will probably generate considerably large pick-up current and transport current in the plasma sheet [Dessler and Hill, private communication, 1991]. The injection of these currents in the ionosphere at Io foot prints and larger L shells through Birkeland currents may also be powerful enough to generate Joule heating that is adequate for the hot-spot emission. Therefore, the hot spots can be located in the same region as the UV aurora. The analysis of the total power supply and power distribution for this new Joule heating mechanism is quite different from the one given in this thesis: the hot-spot-nested ionosphere becomes a load while the Io torus and the middle magnetosphere are now the
sources. We defer detailed discussion to future work, but we would like to predict a possible feature of the hot spots: there may be some observations that show more than one hot spot in each hemisphere, corresponding to the polar hot spot (as discussed in this thesis), the pick-up current associated hot spot and the transport current associated hot spot.

The gravity-wave-related heat dissipation derived in Chapter 4 is actually confined near the excitation altitude because the number density of the atmospheric molecules decreases exponentially with altitude. This is consistent with the fact that no wave speed of up to several km/s has been reported at altitudes higher than the hydrocarbon layer. This is also consistent with Lindzen’s result [1967] showing that midlatitude disturbances of sufficiently high frequency and large spatial scale are vertically trapped. [The derivation made here is simpler than that of Lindzen who solved the differential equations similar to (4.1.1-4.1.6) with certain boundary conditions, using substantial mathematical manipulation]. The effect of the trapped wave-induced heat on Jovian aeronomy may be quite interesting: the heat becomes less and less important compared to Joule heating with increasing altitude, because the decrease of the number density of the atmospheric particles with altitude goes faster than the Pedersen conductivity within the 400–700 km altitude range, owing partially to the increase in the ion/electron number density [compare Equations (2.3.2) and (2.3.4)]. Consequently, anything that occurs at altitudes higher than the hydrocarbon layer should be controlled mainly by Joule heating and thus should have less tendency to drift, unless strong gravity waves can be excited at higher altitudes.

Another way that gravity waves may increase the local IR emission rate is by changing methane’s number density. This effect can only be second order because the time-average of the first-order effect is negligible. One can show that the change of methane’s density is not as important as the heat dissipation (4.3.5) because a typical value of $\rho_I/\rho_o$ is of the order of $10^{-3}$. 
In constructing the overlapped region in Figure 16, we have required that the wave should dissipate heat at a rate less than $2.5 \times 10^3$ ergs/cm$^2$-s in the north but greater than $2.5 \times 10^3$ ergs/cm$^2$-s in the south. We have also required that the wave should be able to propagate at a sufficient high speed to cause the drift of the SPHS. This is, however, overly restrictive and probably not necessary to require that the same wave should simultaneously apply to both hemispheres. The known asymmetry between the two hemispheres probably generates different gravity waves in the two hemispheres. It is possible that nature favors waves in the north that dissipate less heat than the Joule heating, while favoring waves in the south that dissipate heat comparable to or even larger than the Joule heating does. Therefore, other combinations of $\omega$ and $\lambda$ may be available to play the same roll as those in the overlapped region of Figure 16.

To calculate the height-integrated heat dissipation, we have integrated $Q$ in (4.3.4) over the entire ionospheric altitude. This can actually be replaced by an integral over several scale heights because of the exponentially decreasing density. The smaller scale height ($H$) in the southern polar region corresponding to the lower zero-order temperature ($T_0$) there allows most of the heat to deposit within the hydrocarbon layer (we have adopted, from Figure 1 of French and Gierasch [1974], the source altitude of $\sim 5 \times 10^{14}$ cm$^3$/cm which is also within the hydrocarbon layer). The larger scale height in the north may cause some of the heat deposition to occur above the major hydrocarbon layer, i.e., above the 1 mb pressure level or $1 \times 10^{14}$ cm$^3$/cm$^3$ density level. This heat will not directly contribute to the IR emission or the drift motion. Furthermore, the total wave-induced heat may be weaker in the north than in the south, as discussed in Section 4.3, which makes the heating rate per unit height smaller in the northern polar region and thus further reduces the heat in the northern hydrocarbon layer. Therefore, the asymmetry in temperature between the two polar regions allows more gravity waves (than described in the overlapped phase space in Figure 16) to contribute to the localization of the NPHS and the drift of the SPHS.
A wave packet, unlike a solitary wave, is not absolutely steady. It can be shown that a wave packet will diffuse (distort) and finally collapse with time if the second-order term is included in the expansion of $\omega$ with respect to $\lambda$, or vice versa, especially when the medium is highly dispersive and/or the packet is sharp and narrow (i.e., $\lambda_o \Delta x \leq 1$, $\lambda_o$ being the central wavenumber of the initial packet and $\Delta x$ being the length of the initial wave packet). The three sets of SPHS observations (Figure 7) were made at different times, and each SPHS might have been produced after the preceding one had collapsed. The atmospheric conditions change from time to time and so do the initial conditions with which each SPHS is produced. Therefore, it is reasonable to believe that each SPHS may be associated with a different wave length and/or different frequency, as long as it is within the overlapped part of the phase space in Figure 16.

Note that the gravity-wave model presented in Chapter 4 does not require the motion of the SPHS. It only shows how gravity waves might cause the SPHS to move, if the assumptions made here are acceptable and if the observational phenomena and the related physical quantities (including the drift of the SPHS itself) we adopted are real. For example, if the temperature oscillation profile derived from the occultation of $\beta$-Scorpii by Jupiter [Veverka et al., 1974; French and Gierasch,1974] should occur at different altitudes (below the hydrocarbon layer, for example) and/or the oscillation amplitude should be smaller, the waves' effect on the SPHS could be negligible compared to Joule heating, which causes the SPHS to be stationary. If so, some other mechanism would have to be invoked to explain the drift of the SPHS. Furthermore, an observer might see a fixed SPHS (either in System III longitude or with respect to the subsolar point), even if the observations from the occultation and the drift are real, if the observation is made at the time when one gravity wave has just been annihilated and a second wave has not yet been created. Conversely, if the oscillation temperature amplitude should occasionally be larger, the waves' effect could overwhelm the Joule heating even in the northern polar region,
which would lead to a drift of the NPHS. The extent of the accuracy of the magnetic field map derived from the O₄ model is another important matter that could change our quantitative and qualitative conclusions of the localization of the NPHS and the drift of the SPHS.
6. APPENDIX

(A). Ionospheric Birkeland Currents

The currents that flow in and out of the auroral disc at ionospheric altitude can be calculated using the current continuity condition: \( j/B = \text{constant} \), assuming that these currents are essentially field-aligned Birkeland currents and are magnetically mapped to Birkeland currents in the magnetotail. The magnetotail of Jupiter in the distant region features a spiral magnetic field configuration in each tail boundary layer due to the quasi-viscous interactions between the solar wind and planetary rotation. Under the assumption that each tail lobe is of circular cross-section which results in a figure-8 (instead of a more realistic theta-like) cross-section for the magnetospheric tail, the spiralling magnetic field in the magnetopause boundary layer can be described in terms of (for the northern hemisphere)

\[
\vec{B}^{(r)} = B_o^{(r)} [\cos \alpha(r) \, \vec{e}_z - \sin \alpha(r) \, \vec{e}_\phi ] \quad R_1 < r < R_2 \tag{A1}
\]

where \( B_o^{(r)} \) is a constant, \( \alpha = \tan^{-1}(q \tau) \) is the helical angle between the vector \( B^{(r)} \) and \( \vec{e}_x \), q is assumed to be small and is a constant only if the latitudinal variation of the slippage of the polar ionosphere is negligible, and \( R_1 \) and \( R_2 \), the radii of the coaxial inner and outer surfaces of the boundary layer, are assumed to map magnetically to \( \theta_1 \) and \( \theta_2 \) respectively in the auroral disc. A cylindrical coordinate system is chosen here, with \( z \) axis aligned with the tail and pointing in the antisolar direction. This field configuration was first chosen by Zhan [1989]. One thing is worth mentioning: the current in the boundary layer derived from (A1) \( j^{(r)} = \nabla \times B^{(r)}/\mu_0 \) is not strictly a Birkeland current, rather \( j^{(r)} \times B^{(r)} \sim q^2 \). However, for small pitch angle \( \alpha \) (or q), \( j^{(r)} \) can be approximately viewed as field-aligned.
because its nonparallel component $j_{\perp}^{(T)}$ (which is of the order of $q^2$) is small compared to its parallel component $j_{ll}^{(T)}$ (which is of the order of $q$).

The magnetic field inside the vacuum-like tail lobe is taken to be a constant and along the $z$ direction

$$\vec{B}_{lobe} = B' \hat{e}_z \quad r < R_1$$

where the value of $B'$ is determined by requiring that the total pressure across the boundary $R_1$ be balanced (see Zhan [1989] for details).

The solar wind outside the boundary layer ($r > R_2$) is assumed unmagnetized so that $B_{\text{solar wind}} = 0$. In terms of these magnetic fields, we can easily derive the sheet currents flowing on the two boundary surfaces and the volume current in the boundary layer:

$$\pi^{(T)}(R_1) = \frac{1}{\mu_o} \left( B' - B_o^{(T)} \right) \hat{e}_\phi - \frac{B_o^{(T)} \Omega'(R_1) R_1}{\mu_o V_o} \hat{e}_z$$

$$\pi^{(T)}(R_2) = \frac{B_o^{(T)}}{\mu_o} \hat{e}_\phi + \frac{B_o^{(T)} \Omega'(R_2) R_2}{\mu_o V_o} \hat{e}_z$$

$$j_{l}^{(T)}(r) = -\frac{2 \Omega' B^{(T)}(r)}{\mu_o V_o}$$

where we have used

$$\tan \alpha(r) = qr = \frac{r \Omega'}{V_o}$$

$V_o$ being the solar wind velocity and $\Omega'$ being the angular frequency of the ionospheric plasma. $j_z^{(T)}(r)$, $\pi_x^{(T)}(R_1)$ and $\pi_x^{(T)}(R_2)$ are current components in the boundary layer that eventually flow into and out of the auroral disc, in other words, they are magnetically connected with the Birkeland currents $j_\phi(\rho)$, $\pi(\rho_1)$ and $\pi(\rho_2)$ respectively above the disc.
The geometric relationship between these currents is shown in Figure 10. The shaded area ABCD in the disc is mapped to the shaded area A'B'C'D' in the boundary layer so that \( j_z(\rho) \) within ABCD comes exclusively from \( j_z^{(\tau)}(r) \) within A'B'C'D'. Therefore (note the difference in the definitions of coordinate \( z \) at the two places)

\[
 j_z(R_2^2 \theta d\theta d\phi) = j_z^{(\tau)}(r \ dr \ d\phi^{(\tau)}) \quad \theta_1 < \theta < \theta_2, \quad R_1 < r < R_2 \quad (A7)
\]

On the other hand, the magnetic flux through ABCD is equal to that through A'B'C'D':

\[
 B_z(R_2^2 \theta d\theta d\phi) = B_z^{(\tau)}(r \ dr \ d\phi^{(\tau)}) \quad \theta_1 < \theta < \theta_2, \quad R_1 < r < R_2 \quad (A8)
\]

(A7) and (A8) imply

\[
 \frac{j_z}{B_z} = \frac{j_z^{(\tau)}}{B_z^{(\tau)}} \quad (A9)
\]

Substituting the already known expression for \( j_z^{(\tau)} \) yields

\[
 j_z(\rho) = -\frac{2\Omega B}{\mu_0 V_o} \quad (A10)
\]

Likewise, the two sheet currents \( \pi(\rho_1) \) and \( \pi(\rho_2) \) can be derived in a similar way to read

\[
 \pi(\rho_1) = -\frac{\Omega B}{\mu_0 V_o} \rho_1 \quad (A11)
\]

\[
 \pi(\rho_2) = \frac{\Omega B}{\mu_0 V_o} \rho_2 \quad (A12)
\]
(B). Slippage of the Polar Cap

The production and maintenance of the helical field lines in the magnetopause boundary layer requires a torque which balances the solar-wind's tendency to straighten the field lines. Suppose that at time $t = 0$ the planetary spin angular velocity is zero. The field lines in the magnetotail are therefore approximately straight and stretched out to great distances, owing to the solar wind flow. If, at $t > 0$, the planet starts spinning, so do the feet of the field lines in the ionosphere. (It is not strictly proper to refer to the magnetic field lines as corotating for an axially symmetric field; for the purpose of demonstration, however, it may be helpful to think of the field lines as being frozen in the ionosphere owing to the high ionospheric conductivity.) The field lines in the tail now behave like twisted rubber bands which, on one hand, are trying to corotate with respect to the tail axis in correspondence to their feet in the ionosphere and, on the other hand, are being straightened by the solar wind flow, which is allowed only to slide along the tail field line by MHD. (The inertial moment of the solar wind is assumed to be so large that any azimuthal motion of solar wind plasma in response to the corotation of the field lines is negligible.) To sustain the helicity of the field lines, field-aligned Birkeland currents in the magnetopause boundary are needed and they are closed by Pedersen currents in the ionosphere [Hill et al., 1983b; Isbell et al., 1984]. The field-aligned current system closes in the distant tail and/or solar wind where the associated $\mathbf{J}^{(T)} \times \mathbf{B}^{(T)}$ torque tends to spin up the tail and/or solar wind. The torque is transmitted by Birkeland currents from the ionosphere and, in a steady state, is balanced by the viscous torque exerted by ion-neutral collisions in the atmosphere/ionosphere. The viscous torque in the atmosphere/ionosphere requires some departure from rigid corotation, i.e., some difference between the average rotation velocities of the ionospheric plasma and of the un-ionized atmosphere. This difference can be derived from the following equation given by Isbell et al. [1984]:
\[
\frac{d}{d\theta} [(\Omega - \Delta \Omega) \theta^2] = \sigma_o \frac{d}{d\theta} [\Delta \Omega \theta^2] \tag{B1}
\]

where \(\sigma_o = \mu_o V_o \Sigma\), \(\Sigma\) being the height-integrated Pedersen conductivity. In writing (B1), the difference in the field strengths between the tail lobe \((r < R_1)\) and the boundary layer \((R_1 < r < R_2)\) has been ignored. The magnetic field \(B\) in the polar cap is also assumed to be constant provided that the size of the polar cap is small compared to the planetary radius \(R_p\).

The differential equation can be solved using the boundary condition:

\[
\Delta \Omega(\theta \leq \theta_1) = 0 \tag{B2}
\]

which can be understood intuitively: the torque required to spin up the magnetic field lines in the vacuum-like tail lobe is essentially zero and thus the plasma in the polar cap within \(\theta_1\) can corotate as fast as the planet does (i.e., \(\Omega(\theta \leq \theta_1) = \Omega\)). The so-obtained solution to (B1) is

\[
\Delta \Omega(\theta) = \frac{1}{1 + \sigma_o} \left(1 - \frac{\theta_1^2}{\theta^2}\right) \tag{B3}
\]

which, when combined with Equation (9) of Isbell et al. [1984], yields the electric field in the frame corotating with the planet:

\[
E_\theta'(\theta) = \frac{BR_1}{\sigma_o + 1} \theta \left(1 - \frac{\theta_1^2}{\theta^2}\right) \tag{B4}
\]
(C). Derivation of the First-Order Electric Potential in Chapter 3

We define a new variable

\[ \zeta_1 = \varphi_1 + \frac{2\Omega B_o}{3\sigma_o} \beta \rho^2 \sin \phi \]  

(C1)

so that (3.13) becomes

\[ \frac{\partial \zeta_1}{\partial \rho} + \rho \frac{\partial^2 \zeta_1}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial^2 \zeta_1}{\partial \phi^2} = 0 \]  

(C2)

This equation can be solved by variable separation:

\[ \zeta_1(\rho, \phi) = \Phi(\phi) \chi(\rho) \]  

(C3)

which leads to the following two independent equations

\[ \dot{\Phi} + m^2 \Phi = 0, \quad m = 0, \pm 1, \pm 2, \ldots \]  

(C4)

and

\[ \rho^2 \ddot{\chi} + \rho \dot{\chi} - m^2 \chi = 0, \quad m = 0, \pm 1, \pm 2, \ldots \]  

(C5)

(C4) gives the solutions

\[ \Phi = \{ \cos m\phi, \sin m\phi \}, \quad m = 0, 1, 2, \ldots \]  

(C6)

(C5) is the Euler equation and its solutions are

\[ \chi = \{ \rho^m, \rho^{-m} \}, \quad m = 0, 1, 2, \ldots \]  

(C7)

Therefore, the general solution to (C3) is

\[ \zeta_1 = \sum_{m=0}^{\infty} \left( A_m \cos m\phi + B_m \sin m\phi \right) \left( C_m \rho^m + D_m \rho^{-m} \right) \]  

(C8)
and the first-order electric potential in the disc is given by

\[ \varphi_i = \sum_{m=0} \left( A_m \cos m\phi + B_m \sin m\phi \right) \left( C_m \rho^n + D_m \rho^{-n} \right) - \frac{2 \Omega B_0}{3 \sigma_0} \beta \rho^2 \sin \phi \]  

(C9)
7. REFERENCES


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