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Field star interactions with globular clusters

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FIELD STAR INTERACTIONS WITH GLOBULAR CLUSTERS

by

WEI PENG

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ABSTRACT

Field Star Interactions with Globular Clusters

by

Wei Peng

We investigate a new interaction of globular clusters with galactic field stars. By dynamical friction, high-velocity field stars passing through individual globular clusters are decelerated. This frictional interaction contributes to cluster heating, and, in conjunction with disk shocking and other mechanisms, it helps regulate the evolution of globular clusters. Moreover, penetrating field stars with low relative velocities can even be captured by globular clusters. Our calculated rate of captures suggest that there is a substantial population of stars having an origin external to the globulars in which they now reside. Intriguing candidates for this "immigrant" population include some blue straggler stars and short-period pulsars.
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dedicated to my parents:

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Preface

This dissertation is divided into four parts. Chapter 1 introduces a number of basic observational facts about globular clusters, which are important to the understanding of this research. Chapter 2 summarizes theoretical concepts that are relevant to later chapters. The main parts of this thesis are Chapters 3 and 4, which present a detailed description of my research. Although Chapters 1 and 2 serve as two integral parts of the thesis, readers who have been exposed to the material therein may choose to begin at Chapter 3.
Chapter 1

Globular Clusters: Observational Facts

Globular clusters have been under study through much of this century. Early systematic studies of the subject trace back to Shapley (1930)'s book called "Star Clusters," and Sawyer's (1947) summary of cluster research on individual objects. Now, decades later, with the introduction of new observational tools as well as powerful theoretical methods, globular cluster research is very active and has become one of the main areas of modern astrophysics. As objects easily seen in galactic space, globular clusters are important clues to understanding how galaxies formed and evolved (Eggen, Lynden-Bell, and Sandage 1962; Searle and Zinn, 1978; Sandage, 1986). As ensembles of stars with no discernible gas, they have long been recognized as excellent laboratories for studying of the behavior of self-gravitating, N-body systems (Spitzer, 1987; Chernoff and Weinberg, 1990; Oh et al., 1992; Oh and Lin, 1992). Figure 1-1 is the picture of a well-known globular cluster 47 Tucanae in the Milky Way, and Figure 1-2 shows how an extragalactic globular cluster system looks. The tiny star-like images in Figure 1-2 are actually globulars in the giant elliptical galaxy M87.

Over the years, much knowledge has been accrued through the efforts of many astronomers studying individual globulars. Relevant aspects of that knowledge are summarized in this opening chapter.

Globular clusters are found throughout the Milky Way, from near the galactic center to remote regions in the halo. They have high stellar densities at their centers, and
Figure 1-1. A picture of the galactic globular cluster 47 Tucanae (NGC 104).
Figure 1-2. A picture of the globular cluster system in a giant elliptical galaxy (M87). The tiny, star-like images in this galaxy's halo are actually globular clusters.
have near spherical symmetry, with axial ratios in the range 0.9 to 1.0. Their sizes range from several tens of parsecs to a few hundred parsecs, but the typical size is around $10^2$ pc. Additionally, clusters are very luminous stellar systems. Their absolute visual magnitude $M_V$ is usually between -5 and -10. The luminosity distribution of globular clusters can be well fitted by a Gaussian function, with a mean near -8.5, and a dispersion near 1.2. The narrow width of the peak suggests that the globular clusters in the Galaxy are fairly uniform in luminosity (Figure 1-3). Moreover, it has been suggested that globular clusters in other galaxies have similar properties, regardless of the different environments in which they dwell. Therefore globulars clusters have been taken as standard candles in other galaxies in order to measure their distances.

Just as the Hertzsprung-Russell (HR) diagram gives clues to the evolution of individual stars, color-magnitude (CM) diagrams reveal much information about the evolutionary status of globular clusters. Extensive spectroscopic studies have shown that the intrinsic B-V colors for the Galactic globulars are typically between 0.4 and 0.8, with a peak near 0.57. Compared to the solar value, 0.62, these colors indicate that the observed light from globular clusters is dominated by stars that are somewhat cooler than the Sun. In fact, the average mass of stars in individual clusters is about $0.6 \sim 0.8 \, M_\odot$. The mass of a typical cluster is on the order of $10^5 \, M_\odot$, and therefore it has about $10^5$ stars.

Moreover, the distributions of cluster stars on CM diagrams for different globular clusters are quite similar (Figure 1-4), in the sense that they all have a main sequence, a well-defined turn-off point, a subgiant branch and a giant branch. The main sequence consists of stars forming a strip on a CM diagram that runs from the brighter, hotter end to the dimmer, cooler end. Turn-off points for most clusters are clear, while giant branches consist more or less of scattered stars. These trends are well explained by stellar evolution theory. According to theory, high mass stars exhaust their fuel on a timescale
Figure 1-3. The luminosity distribution of globular clusters in different galaxies. The abscissa here is visual absolute magnitude $M_V$. 
Figure 1-4. The Color-Magnitude (CM) diagram of the galactic globular cluster 47 Tuc. Plotted here is visual magnitude V versus B-V color.
much shorter than that of low mass stars, and then evolve much faster after turning off the main sequence. If one assumes that stars in globular clusters were born in the system approximately at same time, then the age of stars at the turn-off point should be representative of the age of a cluster. Theoretical CM curves at different ages, as predicted by stellar evolution theory, are called isochrones, and are widely used in estimating the age of globular clusters (Figure 1-5). For our Galaxy, it is estimated that the globular clusters were mostly born about $10^{10}$ years ago, when it was quite young.

It should be noted that theoretical isochrones are affected by many factors that are not known accurately. Metallicity, for example, plays a role in determining the location of the main sequence of a cluster and thus in the position of the turn-off point. Although most clusters individually show homogeneous metal abundances, the metallicities of different globular clusters cover a range of [Fe/H]-values from -2.2 to 0.0, that is, from extremely metal-poor to essentially solar abundances (Figure 1-6). There are, however, a few globular clusters which show some chemical inhomogeneities, e.g., ω Centauri and 47 Tucanae. The cause of such inhomogeneity is not yet clear, but is generally believed to have to do with chemical evolution of globular clusters.

Although isochrone curves closely match the trend in CM diagrams of globular clusters, there are some serious challenges to the standard stellar evolution picture. For example, in some globular clusters, there are stars bluer than the turn-off point and lying on an extension of the cluster main sequence line. These stars, which can be two or more times as massive as the turn-off mass and are called "blue stragglers," should already have evolved off the main sequence if they were original members of the cluster system. Plotted in figure 1-7 is the CM diagram of the globular cluster NGC 5053. Though the scatter in the main sequence is moderate, the straggler stars that lie on the extension are clearly seen. Blue stragglers were first noticed by Sandage (1953) in the globular M3 (NGC 5272). Since then, a number of blue straggler stars (BSS) have been discovered in
Figure 1-5. Isochrones for a family of stars at different ages. The metallicity of these stars is taken to be about 0.5 percent solar ([Fe/H]=−2.27).
Figure 1-6. The relation between the metal abundances \([\text{Fe}/\text{H}]\) of globular clusters and their galactocentric distances \(R\). See text for discussions.
Figure 1-7. Blue Straggler Stars. On the extension from the main sequence in the CM diagram of a globular cluster (here, NGC 5053), stars brighter and bluer than the turn-off point are found.
many other globular clusters, as well as in open clusters and even dwarf galaxies. The origin of blue straggler stars is not clear and how they fit into the history of these stellar systems is still under investigation. Several theories have been proposed to explain these peculiar objects, including single star scenarios that suggest unusual factors to prolong their lifetime, and binary star scenarios that require coalescence or mass transfer. Unfortunately none of the explanations is wholly satisfactory. (We will discuss them in detail in Chapter 4 and therein propose yet another scenario.)

In addition to blue stragglers, short period pulsars also are found in globular clusters. The first pulsar found in such an old stellar environment was reported by Lyne et al. (1987) for NGC 6626. Since then there have been dozens of pulsars discovered in globular clusters. A recent discovery by the Hubble Space Telescope reveals that there are more than 20 pulsars in 47 Tucanae alone (Paresce et al., 1991). The conventional theory of pulsars asserts that a short-period pulsar (SPP) is an early evolutionary stage of the neutron star with a large magnetic field, and that ordinary pulsars fade from view on timescales of order $10^8$ years after the supernova event, as their rotation rates and magnetic fields both decrease with time. But this theory is confronted with the challenge: how to explain a "young" pulsar in an old stellar environment. Several mechanisms have been proposed to accommodate these facts, including an accretion-induced collapse (AIC) model and a spin-up model. We shall also come back to this subject in Chapter 4, where we propose yet another way to explain at least some of these pulsars.

The total number of globular clusters in our Galaxy is estimated to be about 200. Since the Milky Way has a dimension on the order of 50 kpc, their distribution in the galaxy is quite sparse. The number density of clusters is matched by a power-law function of galactocentric distance, with an index of about -3. Zinn (1985, 1991) found that there is a correlation between the metallicity of a cluster and its location in the Milky Way. Generally, globulars that are far in the halo are metal-poorer than those near the
disk. But, because of errors in metallicity and distance measurement, this trend cannot be confirmed and remains controversial; Zinn (1985) in fact suggested that there exist two kinds of globular clusters in our Galaxy and that this may contribute to the metallicity variation. Halo globulars are generally metal-deficient and reside in halo most of their lifetime. Disk globulars have solar or near-solar metal abundance and reside near or in the galactic disk (Figure 1-8). Their kinematical properties are quite different, too. Halo globulars mainly move in circular orbits around the Galaxy's center. The orbital orientations are random and isotropic. A typical halo cluster velocity in the solar neighborhood is about 150 km/s. On the other hand, disk cluster movements also are about the Galaxy's center but are restricted near the plane. Velocities of disk globulars in the solar neighborhood are about the same as those of halo clusters. The velocity space distribution functions for both disk and halo globular clusters can be represented by Gaussian functions; the former has a small dispersion, but the latter has a large one.

Most observations have focused on star clusters in our own Galaxy. There are, however, strong interests in extragalactic star clusters. Identification of extragalactic star clusters used to be extremely difficult because their tiny images make it almost impossible to resolve them into stars. It is only within the past two decades that technology became sophisticated enough to make such observations feasible.

M31, the Andromeda galaxy, remains the one of the most observed galaxies. The first attempt to identify globular clusters in this galaxy was made by Hubble, who suggested that many of the small objects seen on the disk of M31 were its globular clusters. It is now estimated that M31 has about 400 globular clusters (Sargent et al., 1977; Harris and Racine, 1979), roughly twice the number in the Milky Way. Van den Bergh's (1969) results indicated that the integrated colors of the M31 globular clusters are redder, presumably due to a metal-richer environment. His conclusion was additionally supported by Searle (1978) who determined that the mean abundance for the outer M31
Figure 1-8. Two categories of globulars in the Milky Way: disk and halo clusters. Metal-richer clusters tend to reside near the disk, while metal-poorer ones reside in the halo.
sample is near [Fe/H] = -1.1 as opposed to -1.5 for the outer Galaxy halo. The kinematics of M31 globular clusters is similar to that of the Galactic cluster system (Hartwick and Sargent, 1974; Huchra et al. 1991). Construction of the luminosity function for the M31 clusters is impeded by incompleteness and by measurement errors in observation, as well as by somewhat uncertain reddening corrections. The result of an analysis of 114 clusters indicates that there is a peak at $M_V \sim -7.5$. The kinematics of M31's globular cluster system is believed to be similar to that of our Galaxy; its globular clusters have significant rotation and a large dispersion. Freeman (1983) divided these M31 clusters into disk and halo populations. Like their Galactic counterparts, as he suggested, the metal-rich disk clusters have a large rotational velocity ($\sim 200$ km/s) and a narrow dispersion of 90 km/s. However, the M31 halo clusters are quite different; they have a similar dispersion but show no evidence for systematic rotation. These results are supported by other analyses (e.g., Elson and Walterbos, 1988). In addition to old globular clusters, M31 has an identifiable population of intermediate-aged clusters (Hodge, 1988). These younger stellar systems are structurally closer to open clusters and associations in the Milky Way, and their kinematical properties have not been measured.

The globular cluster system of M33 has been observed in detail only recently, even though attempts date back three decades (Hiltner, 1960; Kron and Mayall, 1960). Unlike the Galactic cluster system, the M33 system consists of globular clusters with large age differences (Christian and Schommer, 1983; Cohen et al., 1984). A recent study (Christian and Schommer, 1988) indicates that there are massive clusters with ages younger than $10^8$ years. Moreover, globular clusters of different ages show different kinematical properties. According to Schommer et al (1991), the old globulars in M31 belong to a hot halo population with a thermal velocity about 70 km/s, but exhibit very little (if any) rotation. The young clusters, however, seem to rotate about the galactic center and have disk kinematics (i.e., large rotation and small dispersion). Their velocities
Figure 1-9. A picture of the Large Magellanic Cloud. Red dots represent old and intermediate-age globular clusters, and blue dots denote young globular clusters.
agree with the gas velocities at their position. The intermediate-age clusters show a slightly larger dispersion than the young ones, and may lag the young objects. The composite rotation curve for both young and intermediate-aged clusters is in harmony with that from gas velocity (21 cm) measurements.

Other frequently observed members of our Local Group include the Magellanic Clouds (Figure 1-9). Their globular clusters differ from those of our Galaxy in many respects. They are, generally, less massive but larger in size. Additionally, many of them are highly flattened, while Galactic clusters are mostly spherical. Age-wise, there are many young globular clusters in both Large Magellanic Cloud (LMC) and Small Magellanic Cloud (SMC). But, old clusters in the Magellanic Clouds are found to be more widely distributed than are the younger ones. (Interestingly, there are at least seven old globular clusters surrounding the LMC, and one surrounding the SMC.) These old globular clusters on average have the same luminosity ($M_V = -7.1$) as that of globulars in the Galaxy and M31. As Van den Bergh concluded (1991), the old globulars in the Magellanic Clouds are objects of the same type as those in giant spirals. However, since many Magellanic Cloud clusters are younger than those in our Galaxy, their luminosity function is expected to be quite different. As estimated by Elson and Fall (1985), the luminosity function of LMC clusters is similar to that for open clusters in the Galaxy. The kinematics of LMC clusters seems dependent on their age, as found for M31 clusters. Young clusters (age < $10^9$ years) have motions similar to that of the gas in their vicinity, with a line-of-sight velocity dispersion of ~ 15 km/s superposed on a rotation velocity about 37 km/s (Freeman, Illingworth and Oemler, 1983). Clusters a few billion years old belong to a flattened disk-like system with a narrow intrinsic line-of-sight dispersion (~17 km/s).

Aside from the spiral galaxies and the Magellanic Clouds, globular clusters also are found in other members of the Local Group, including the dwarf ellipticals NGC 147,
185, 205, and Fornax (Harris and Racine, 1979). Information about the clusters in these dwarf ellipticals is quite meager. Beyond the Local Group, globular clusters are seen in giant ellipticals like M87 in Virgo (Figure 1-2), NGC 3115, 3377, 3379, and 4278 (Sandage, 1961; de Vaucouleurs and de Vaucouleurs, 1965). Current knowledge about the globular clusters in extragalactic systems is detailed in the recent review by Harris (1991).

It was found more than a decade ago that there is a strong correlation between the luminosity of a galaxy and \( N_t \), its total number of globular clusters (Hanes, 1977; Harris and Racine, 1979); namely, brighter galaxies have more globular clusters. For many galaxies, including irregular, spiral and giant elliptical galaxies, the relation between \( N_t \) and the absolute magnitude of a galaxy can be approximated by a straight line in the log \( N_t - M_V \) plane (see Figure 1-10). Although it might be suggested by this trend that the total number of globular clusters in a galaxy is determined by its total mass (or luminosity), the data are not accurate enough to support this view. In fact, it is found that spirals and irregulars are somewhat too bright for their estimated \( N_t \). To account for this deviation, Harris and Van den Bergh (1981) defined a specific frequency \( S_N \) for the cluster population of a galaxy, normalized to the absolute magnitude \( M_V = -15 \):

\[
S_N = N_t \cdot 10^{-0.4(M_V + 15)} \tag{1 - 1}
\]

\( S_N \)-values differ for different galaxy types. In Table 1-I are listed mean \( S_N \)-values for five types of galaxies (Harris, 1991):
Figure 1-10. The total number $N_t$ of globular clusters of a galaxy correlates with its absolute magnitude. A straight line in log $N_t$-$M_V$ plane fits the data for elliptical, spiral and irregular galaxies.
### Table 1-I  Mean Specific Frequencies

<table>
<thead>
<tr>
<th>Galaxy Type</th>
<th>$&lt; S_N &gt;$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sc / Irr</td>
<td>0.5 ± 0.2</td>
</tr>
<tr>
<td>Sa / Sb</td>
<td>1.2 ± 0.2</td>
</tr>
<tr>
<td>E / S0</td>
<td>2.6 ± -0.5</td>
</tr>
<tr>
<td>E / S0 (Virgo, Fornax)</td>
<td>5.4 ± 0.6</td>
</tr>
<tr>
<td>dE</td>
<td>4.8 ± 1.0</td>
</tr>
</tbody>
</table>

$S_N$ systematically increases as one goes from late to early type galaxies. While cluster miscounting may be responsible for low $S_N$-values of some galaxies, a full understanding requires much better knowledge of galaxy formation and evolution.

The number of globulars per unit mass interval is given by their mass function. In practice, mass functions are obtained from luminosity functions by assuming a constant mass to luminosity ($\mathcal{M}/L$) ratio for globulars. Although it is still unclear whether globulars show a systematic variation in this ratio from galaxy to galaxy, $\mathcal{M}/L =$ constant is a good approximation considering that globular cluster data have large errors. The luminosity function $\Phi$ is usually represented in terms of absolute magnitude $M$, and the available data suggest that $\Phi(M)$ can be well described by a Gaussian function,

$$\phi (M) = \frac{N_t}{\sqrt{2\pi} \sigma} \exp \left[ - \frac{(M-M_0)^2}{2 \sigma^2} \right]$$

(1-2)
where $M_0$ is the peak magnitude and $\sigma$ is the dispersion of the distribution. Presumably, $M_0$ and $\sigma$ could be different for each galaxy, even though the Gaussian form fits all available observational results. However, up to the present, the data do not show much variation among different, observed galaxies; $M_0$ is usually about -7.1 and $\sigma$ about 1.5 (Harris, 1991), regardless of galaxy type. (We note that it is difficult to do faint-end observations for distant galaxies, and also for dwarf ellipticals, because the sample size is small. Therefore a Malmquist-type bias on bright globulars is possible.)

As we look at globular clusters in other distant galaxies, we also look back in time. Images of globular clusters at various cosmic epochs would yield a complete picture of cluster formation and evolution. However, observation of early-epoch (high redshift) globular clusters is impossible at our present level of technology, and so direct knowledge of how young clusters evolved is unavailable. On the other hand, high redshift quasistellar objects (QSOs) can be observed. In the spectra of QSO's, a forest of absorption lines is found to the blue of the principal Lyman series emission line (1216 Å) at the QSO's redshift (cf. Figure 1-11). Lynds (1971) suggested that most of these absorption lines are Lyman $\alpha$ lines at smaller redshifts, and called them the Lyman $\alpha$ forest lines. Sargent et al. (1980) later showed that Lyman $\alpha$ forest lines arise from clouds in intergalactic space; these clouds are generally metal-deficient. In addition to the Lyman $\alpha$ forest lines, some metal absorption features also are found in QSO spectra, some even from highly ionized species like CIV. To explain certain key features of the metal-lines, Peng and Weisheit (1991a, b) modeled the ionization structure of clouds and pointed out that clouds in high-redshift galaxies are a likely origin of the CIV lines. They also suggested that these clouds are the progenitor of globular clusters. If true, QSO absorption line spectroscopy is perhaps the only way to probe the early evolution of globular clusters.
Figure 1-11. A forest of Lyman $\alpha$ absorption lines are found in the spectra of most QSO's, to the blue side of their hydrogen Lyman $\alpha$ (1216Å) emission line. Plotted here is the spectrum of QSO 1151 + 068. See text for the relevance of QSO spectroscopy to globular clusters.
Finally, it must be recognized that globular clusters are influenced by their environments. For our Galaxy, a number of environment-related mechanisms are believed to have influenced the evolution of its globular system. These mechanisms, which can severely affect the survival of globular clusters (Fall, 1981), are discussed in Chapters 2 and 3 of this thesis. Also in Chapter 3, we identify and investigate in detail yet another important mechanism for cluster evolution, namely a diffusive process in which globular clusters are heated through dynamic friction involving high-velocity, galactic field stars. In Chapter 4, we show that globular clusters may be able to catch some (low-velocity) field stars. The significance of the capture process for blue stragglers, short-period pulsars, and other cluster "oddities" will also be discussed there.
Chapter 2

Globular Clusters: Theoretical Concepts

At the same time that observations on globular clusters have increased in both quantity and quality, there also has been considerable progress in the theoretical understanding of globular clusters. In this chapter, we present some of the major concepts and theories needed for our subsequent analytical and computational work. We first begin by discussing the structure of globular clusters. Then, we look at the interaction between pairs of stars before summarizing key results involving the dynamical evolution of globular clusters. We also discuss the structure of the Galaxy and how it is related to the demise of its globular clusters.

Section 2.1 Globular Cluster Structure

To simplify the problem, we assume that a globular cluster is an ideal stellar system, i.e., that the granularity of self-gravitating matter in a cluster can be ignored. As a result, the distribution of stars in a cluster can be described by a continuous function, and their collective gravitational potential $\phi$ smoothly depends on position. We further assume at this stage that, during a cluster's evolution, no mass loss occurs and that stellar evolution has only negligible effect on the dynamics of the cluster. (Indeed, stellar
evolution timescales tend to be much longer than the cluster dynamical timescales.) In this way, a cluster can be treated as an isolated N-body system.

Let \( f(\mathbf{r}, \mathbf{v}, t) \) describe the probability of finding a star at time \( t \) within the infinitesimal volume \( d\mathbf{r} \, d\mathbf{v} \) in phase space centered at \((\mathbf{r}, \mathbf{v})\). The dynamical evolution of an ideal cluster is then described by the collisionless Boltzmann equation

\[
\frac{Df}{Dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \mathbf{x}_i} \mathbf{v}_i + \frac{\partial f}{\partial \mathbf{v}_i} a_i = 0 \tag{2 - 1}
\]

where the \( x_i \) represent spatial coordinates, and the \( a_i \) the corresponding accelerations, defined by

\[
a_i = -\frac{\partial \phi}{\partial x_i} \tag{2 - 2}
\]

The density and mean square velocity of particles at a particular spatial point \( \mathbf{r} \) can be deduced from the probability function

\[
n(\mathbf{r}, t) = \int f(\mathbf{r}, \mathbf{v}, t) \, d\mathbf{v} , \tag{2 - 3}
\]

\[
\mathbf{v}_{\text{ms}}(\mathbf{r}, t) = \frac{1}{n(\mathbf{r}, t)} \int \mathbf{v} \cdot \mathbf{v} f(\mathbf{r}, \mathbf{v}, t) \, d\mathbf{v} . \tag{2 - 4}
\]

Moreover, the stellar number density, which is trivially related to the mass density \( \rho(\mathbf{r}, t) \), in turn determines the gravitational potential and, thus, cluster evolution, through Poisson's equation.
\[ \nabla^2 \phi (\vec{r}, t) = 4\pi G \rho . \] (2 - 5)

The set of the equations (2-1) through (2-5) completes the description of a cluster's structure at any stage in its evolution. Given appropriate boundary conditions, general solutions to the set are possible. However, only in a few extremely simple (idealized) cases have analytical solutions been found. The fact that actual globular clusters have as many as $10^5$ stars makes it tremendously difficult to work with the equations directly. To surmount this problem, we make further simplifications: (1) the evolution of a cluster is an extremely slow dynamical process, so the distribution function $f$ can be taken as independent of time; (2) all clusters have spherical symmetry, so $f$ has only a radial spatial dependence. With these simplifications, Poisson's equation becomes simpler

\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d\phi (r)}{dr} \right) = 4\pi G \rho . \] (2 - 6)

Based on the forms of $f$ various analytic models for clusters in equilibrium can be found. They are very useful both for theoretical investigations and for fitting to observed clusters. We now discuss some of these models.

*Planetary Model:* The simplest model is a sphere in which stars have no radial but only transverse velocities, $v_r = 0$ and $v_t = GM(r) / r$. Like a planetary system, stars in this class of clusters are rotating around the cluster center at a speed determined by the interior mass $M(r)$. Orbital planes may be randomly oriented.

*Polytropic Model:* Another class of spherical models is obtained by assuming an isotropic velocity distribution everywhere, of the form
\[ f = \text{const} \ (-E)^p, \quad (2 - 7) \]

where \( E=1/2 \ v^2 + \phi(r) < 0 \) is the total particle energy per unit mass. The mass density distribution has a similar form,

\[ \rho (r) = \text{const} \ (-E)^{p + 3/2}. \quad (2 - 8) \]

This is the basic equation for a polytropic sphere with index \( n=p+3/2 \). Analytical solutions for polytropic spheres are available only for systems with \( n=0, 1, \) or \( 5 \). Despite having infinite radii, the last class of models resembles actual clusters. These so-called Plummer models show a compact core and an extended outer envelope, as do real clusters:

\[ \rho(r) = \frac{3M}{4\pi R^3} \frac{1}{(1+r^2/R^2)^{5/2}}, \quad (2 - 9) \]

\[ M(r) = M \frac{r^3 / R^3}{(1+r^2/R^2)^{3/2}}, \quad (2 - 10) \]

\[ \phi(r) = -\frac{GM}{R} \frac{r^3 / R^3}{(1+r^2/R^2)^{3/2}}, \quad (2 - 11) \]

\[ \Sigma (r) = \frac{4\rho (0) R}{3} \frac{1}{(1+r^2 / R^2)^2}. \quad (2 - 12) \]

Here, \( M(r), \Sigma (r), \) and \( R \) respectively represent mass within \( r \), the surface density at \( r \), and a scaling factor. The mass within \( r=r_h =1.31 \ R \) is half the total mass \( M \).
Isothermal Model: This class of more realistic models assumes that cluster stars everywhere have same thermal velocity $v_m$, and that their distribution function $f$ is given by

$$f = \kappa_1 \cdot \exp(-BE), \quad B = 3\lambda_m^2,$$  \hspace{1cm} (2.13)

with $\kappa_1$ being a normalization constant. The density profile for isothermal models can be written as

$$\rho(r) = \kappa_1 \cdot m \left( \frac{2\pi}{B} \right)^{\frac{3}{2}} \exp(-B\phi(r)).$$  \hspace{1cm} (2.14)

Upon defining $\zeta = r/\kappa_1 = r \left[ 12\pi G \rho(0) / \lambda_m^2 \right]^{1/2}$, and solving Poisson equation numerically, we obtain

$$\frac{\rho}{\rho(0)} = 1 - \frac{\zeta^2}{6} + \frac{\zeta^4}{45} + ... \quad \zeta << 1,$$  \hspace{1cm} (2.15)

$$\frac{\kappa_1^2}{r^2} \quad \zeta >> 1,$$

and

$$M(r) = 8\pi \rho(0) \kappa_1^2 r.$$  \hspace{1cm} (2.16)

The infinite mass indicated by equation (2.16), as $r$ increases indefinitely, results from the tail of the Maxwellian velocity distribution for large $E$. Evidently, this is not the case in actual clusters, where particles with energy larger than certain threshold energy $E_e$ should be able to escape. Maintaining an infinite $E_e$, as implied by equation (2.13), naturally gives infinite radius and mass.
King Models: Another simple velocity distribution is the "lowered Maxwellian". It uses the Maxwellian distribution for most stars, but cuts off those stars whose kinetic energy exceeds the escape threshold $E_e$:

$$f = \kappa_2 \left( e^{-\frac{B}{E}} - e^{-\frac{B}{E_e}} \right), \quad E < E_e ;$$
$$= 0, \quad E > E_e.$$  \hspace{1cm} (2 - 17)

The cut-off is attributed to the fact that globular clusters dwell in the background of a galactic tidal force, which pulls some stars out if they are too distant from the cores of their host clusters. Quite naturally, the radius at which cut-off occurs is called the "tidal radius," and can be found from

$$r_t^3 = \frac{M_c}{2M_G} R^3,$$  \hspace{1cm} (2 - 18)

where $M_c$ is the mass of the cluster at galactocentric distance $R$, and $M_G$ is the galaxy mass within this radius.

Lowered Maxwellian models do resemble actual clusters. However, it was first found empirically by King (1962) that the surface density of a globular cluster can be described very accurately by a family of the curves, based on two length parameters, the tidal radius and the so-called core radius,

$$r_c = \left[\frac{3 v_m(0)^2}{4 \pi G \rho(0)}\right]^{1/2},$$  \hspace{1cm} (2 - 19)

which is defined as the radius where cluster surface density falls to half of its central value. King's formula is
\[ \Sigma(r) = \kappa_3 \left( \frac{1}{[1+r^2/r_c^2]^{1/2}} - \frac{1}{[1+r_0^2/r_c^2]^{1/2}} \right), \]  \hspace{1cm} (2.20)

where the constant \( \kappa_3 \) scales the central density. The spatial density can be then inferred as

\[ n(r) = \frac{\kappa_3}{\pi r_c \left[ 1+(r/r_c)^2 \right]^{3/2}} \frac{1}{z^2} \left[ \frac{1}{z \cos^{-1} z - (1-z^2)^{1/2}} \right]; \]  \hspace{1cm} (2.21)

here, \( \kappa_3 \) is the same constant as in equation (2-20), and \( z \) is defined by

\[ z = \frac{\left[ 1+(r/r_c)^2 \right]}{\left[ 1+(r_0/r_c)^2 \right]}. \]

The dimensionless quantity \( r_0/r_c \), which represents the size ratio of a cluster's halo and core, often shows up in structural studies. It quantifies the concentration of a globular cluster and therefore,

\[ c = \log \frac{r_0}{r_c} \]  \hspace{1cm} (2.22)

is called the concentration parameter. Any two of the three parameters, tidal radius, core radius and concentration parameter, give the complete description of a King model globular cluster. However, for the same globular cluster, the parameters from King's empirical model (Eqns. 2-20 to 2-22) may not be same as the ones from King dynamical model (Eqn. 2-17). Although the latter is more frequently used in literature, the former has analytical convenience and is used this dissertation.
It is instructive to contrast the King (empirical) model and the Plummer model, the two most frequently used models in globular cluster research. For various concentration parameters, Figure 2-1 plots the density profiles of a cluster of different concentrations, but with fixed \( r_t = 55 \text{pc} \), and \( M_c = 5.2 \times 10^5 \, M_\odot \) for both King and Plummer models. Evidently, the Plummer model does not have an abrupt cut-off at the tidal radius. Moreover, as a cluster becomes more concentrated, the difference in the inner portion is greater in that a King model gives a more distinct core-halo structure. This difference can be quantified. For clusters with \( c > 1.7 \), we find that

\[
\frac{\rho_o(\text{Plummer})}{\rho_o(\text{King})} = f(c) = 2a - \frac{1}{2} - a^2 \ln u - \frac{2a}{\cos u} + \frac{1}{\cos^2 u},
\]

(2.23)

where \( a = \sqrt{1 + 10^2c} \), \( \tan u = r_t/r_c = 10^c \) and \( c \), again, is the concentration parameter. The difference between the two models at high \( c \)'s is quite evident in this equation.

For a more thorough description of globular clusters models, readers are referred to monographs by Binney and Tremaine (1987) and by Spitzer (1987).

Section 2.2 Binary Encounters and Stellar Diffusion

A common form of interaction between stars in a cluster is binary collisions, which are regulated by the gravitational attraction between stars. In the following discussion, we assume that stars are mass points and do not evolve, and therefore that the interaction is entirely kinematical.

When two stars of masses \( m_f \) and \( m \) pass near each other, the change of velocity from the encounter can be quantified by the conventional Rutherford theory. Specifically, angular momentum conservation gives the orbit equation:
Figure 2-1. The density profile of globular clusters: Plummer model and King model. The solid lines represent King models for a cluster with $r_t=55\,\text{pc}, M_c = 5.2 \times 10^5 \, M_\odot$ but different concentrations, $c=1.3$, 1.7, and 2.0. The line plotted with the symbol 'o' is the Plummer model for the same cluster.
$$\frac{1}{r} = \frac{G (m_f + m)}{j^2} (1 + e \cos \theta) , \quad (2-24)$$

where $G$ is the gravitational constant, $J = r^2 \frac{d\theta}{dt}$ is the angular momentum per unit mass, and $\theta$ represents the angle measured from the direction of minimum separation $r$. When the orbits are hyperbolic, the eccentricity $e > 1$.

Energy conservation yields

$$\frac{d(r^2)}{dt} + \frac{j^2}{r^2} - \frac{2 G (m_f + m)}{r} = 2 E , \quad (2-25)$$

where here $E$ is the total energy divided by the reduced mass. The total deflection angle $\varphi$ is given by

$$\tan \varphi = \frac{b v^2}{G (m_f + m)} = \frac{b}{b_0} , \quad (2-26)$$

where $v$ is the initial relative velocity, $b$ is the impact parameter, and $b_0$ is the $b$-value for which the deflection angle is $90^0$. The transverse velocity change in one encounter is

$$\Delta v_t = \frac{2 m_f v \cos \varphi}{m_f + m} , \quad (2-27)$$

or

$$(\Delta v_t)^2 = \frac{4 m_f^2 v^2}{(m_f + m)^2} \frac{1}{1 + (b / b_0)^2} . \quad (2-28)$$

To take account of all particles that are influential, we multiply the above equation by the flux of stars, and then integrate over all impact parameters $b$. This yields
\[
\frac{d}{dt}(v_t^2) = \frac{4 \pi G^2 n_f m_f^2}{v} \ln \left[1 + \left(\frac{b_{\text{max}}}{b_o}\right)^2\right],
\]

where \(n_f\) is the number density of incident stars, and where, usually, we have the inequality \((b_{\text{max}}/b_o) >> 1\). Upon defining

\[
\Gamma \equiv 4 \pi G^2 \frac{m_f^2 \ln \left(\frac{b_{\text{max}}}{b_o}\right)}{m_f^2 \ln \Lambda},
\]

we obtain

\[
\frac{d}{dt}(v_t^2) = \frac{2 n_f \Gamma}{v}.
\]

This quantifies the velocity change in the direction perpendicular to the initial (relative) motion. We see that most deflection occurs when the velocity is small.

Section 2.2.2: Dynamical Friction

We now consider a test star with velocity \(\vec{v}_i\) shooting through the field star background. In order to simplify the problem, we first assume field stars are immobile and at fixed positions. This assumption is most plausible when the moving star has velocity much greater than the velocities of most background stars. Because of symmetry in the directions perpendicular to \(\vec{v}_i\), we can choose to denote the vector sum of \(\Delta v_y\) and \(\Delta v_z\) by \(\Delta v_{\perp}\) and denote \(\Delta v_X\) by \(\Delta v_{\parallel}\). Using equation (2-27) we obtain

\[
\Delta v_{\perp} = \frac{m_f}{m + m_f} v \sin \varphi = 2 \sqrt{\frac{m_f}{m + m_f} \frac{b/b_o}{1 + (b/b_o)^2}},
\]
\[ \Delta v_\parallel = - \frac{m_r}{m+m_r} v(1-\cos \varphi) = - 2 \frac{m_r}{m+m_r} \frac{1}{1+(b/b_0)^2}. \] (2-33)

Analogously, we first multiply these velocity changes by the "flux" of background field stars and then integrate the above two equations over \( b \) to get

\[ \frac{d\langle v_\parallel^2 \rangle}{dt} = - \frac{2 n_r \Gamma}{v}, \] (2-34)

\[ \frac{d\langle v_\parallel \rangle}{dt} = - \left[1 + \frac{m}{m_r}\right] \frac{n_r \Gamma}{v^2}, \] (2-35)

\[ \frac{d\langle v_\perp^2 \rangle}{dt} = \frac{n_r \Gamma}{v \ln \Lambda}. \] (2-36)

The set of equations (2-34) through (2-36) describes the kinematics of the test star moving in the background of field stars. As indicated by equation (2-35), this moving star will be slowed down, and the slowing down process is called "dynamical friction." It results from the test particle's gravitational interactions with other field particles. It can also be visualized by the following picture: field stars tend to congregate in the test star's wake, gravitationally attracting it and slowing it down. The diffusion coefficient \( d(\Delta v_\parallel)/dt \) is sometimes called the coefficient of dynamical friction. Equation (2-35) also implies that the test star will lose energy to (and thereby heat up) the cluster of field stars by passing through it.

However, this diffusion picture needs modification when field stars are significantly heated or have non-negligible thermal velocities to begin with. Although the above equations can be generalized through local coordinate transformations at each encounter, such rigorous derivations are tedious and beyond our scope. We list the
results for an isotropic velocity distribution. Interested readers are referred to Spitzer (1987) for a thorough discussion:

\[
\frac{d}{dt} \langle v \rangle = -2(1+ \frac{m}{m_f}) n_f \Gamma_j^2 G(x), \quad (2-37)
\]

\[
\frac{d}{dt} \langle v^2 \rangle = 2 n_f \Gamma_j \frac{G(x)}{x}, \quad (2-38)
\]

\[
\frac{d}{dt} \langle v^2 \rangle = 2 n_f \Gamma_j \frac{\Phi(x) - G(x)}{x}, \quad (2-39)
\]

\[
\frac{d}{dt} \langle E \rangle = \frac{n_f \Gamma_j}{x} \left\{ \frac{m}{m_f} \Phi(x) + (1+ \frac{m}{m_f}) x \Phi'(x) \right\}. \quad (2-40)
\]

In these equations, the quantity \( j = \sqrt{\frac{3}{2}} \frac{1}{v_m}, \) \( x = j v_m, \) and \( v_m \) is the thermal velocity of the background stars. \( \Phi(x) \) is the error function, defined as

\[
\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2} dy, \quad (2-41)
\]

and \( G(x) \) is defined by

\[
G(x) = \frac{\Phi(x) - x \Phi'(x)}{2x^2}; \quad (2-42)
\]

\( d\langle E \rangle/dt \) is the energy loss rate.

The functions \( \Phi(x) \) and \( G(x) \) do not have analytical forms. However, their numerical values are tabulated in many mathematical tables. Asymptotic expressions are available for limiting cases. When \( x \) is very large (\( x > 3 \)), \( \Phi'(x) \) falls below \( 10^{-4} \), while
\( \Phi(x) \) is almost unity and \( G(x) = \frac{1}{2}x^3 \); this is the special case we discussed above. When \( x \) approaches zero, series expansions give

\[
G(x) = \frac{2x}{3\sqrt{\pi}} \left( 1 - \frac{3}{5} x^2 \right), \tag{2-43}
\]

\[
\Phi(x) - G(x) = \frac{4x}{3\sqrt{\pi}} \left( 1 - \frac{1}{5} x^2 \right); \tag{2-44}
\]

for \( x < 0.5 \), these two expressions are highly accurate.

Section 2.3: Dynamical Evolution of Globular Clusters

The evolutionary course of globular clusters depends on many of their physical parameters. Before we briefly discuss the evolution of an isolated cluster in this section, we introduce several characteristic timescales:

**Relaxation Time:**

\[
t_r \equiv \frac{1}{3} \left\{ \frac{v_m^2}{\left( \frac{d/\langle v \rangle^2}{dt} \right)_{v=v_m}} \right\} = \frac{v_m^3}{1.22 n_f \Gamma}. \tag{2-45}
\]

This timescale measures the time required for deviations from a Maxwellian distribution to be significantly decreased.

**Crossing Time:**

\[
t_{cr} = \frac{R}{v}. \tag{2-46}
\]
This is approximately the time needed for a star with velocity $v$ to traverse the cluster. Again, since $v$ changes inside the cluster, it must be taken to be only a typical value.

**Collapse Time:**

$$t_{cl} \approx \frac{t_r}{P_{ev}} \frac{2}{7 - 3 \zeta} \leq \frac{t_r}{P_{ev}} .$$  \hspace{1cm} (2.47)

Here, $\zeta$ is the ratio of energy per unit mass carried away by evaporating stars to the mean energy per unit mass of all stars in the cluster, and $P_{ev}$ is the evaporation probability that will be discussed below.

**Mass Segregation Time:**

$$t_{eq} \approx \frac{m}{m^*} t_r .$$  \hspace{1cm} (2.48)

It represents the time that a minority of stars of mass $m^*$ in a multi-component globular cluster need to reach equipartition with the majority of cluster stars of mass $m$. It is also called the equipartition timescale. Here we see that heavy stars equilibrate and sink to the core in less than one relaxation time.

An isolated globular cluster evolves in a way similar to other gravitationally-bound systems. It loses mass and collapses. Mass loss occurs through the evaporation of lighter mass stars, which absorb some kinetic energy by encounters and then escape the cluster. If the cluster is in thermodynamic equilibrium, some stars at the high velocity end
of the Maxwellian distribution are bound to escape. The fraction of stars that escape per relaxation time (cf. Spitzer, 1987) is

\[ P_{ev} \geq 7.4 \times 10^{-3} \]  \hspace{1cm} (2.49)

It should be noted that equation (2.49) only gives a lower limit for real clusters. Actual values are higher because clusters in the Galaxy suffer a tidal force from the galactic potential, which makes it much easier for cluster stars to evaporate. From equation (2.47), we know that the faster evaporation is, the faster cluster collapse proceeds.

One interesting point relevant to the dynamical evolution of a cluster is its negative specific heat, which leads to the so-called gravothermal instability. This occurs in all self-gravitating systems. According to the Virial Theorem, in an isolated system, the kinetic energy \( T \) is half of the gravitational energy \( W \) with opposite sign, i.e., \( T = -W/2 \). Therefore the total energy of the system is \( E = T + W = W/2 < 0 \). Thus, if the system loses energy so that \( W \) becomes more negative, \( T \) increases. This means that stars in the system become "hotter" as it loses energy and "cooler" as it gains energy. With application to globular clusters, it implies a runaway core collapse; once a dense core forms inside the cluster, it will continue towards a catastrophic collapse.

The dynamical evolution is formally described by Boltzmann's equation:

\[ \frac{Df}{Dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x_i} v_i + \frac{\partial f}{\partial v_i} a_i = \frac{\partial f}{\partial t}_c , \]  \hspace{1cm} (2.50)

where now the collision term on the right hand side is non-zero. Interactions among stars contribute to the dynamical evolution. Once we know the exact form of the collision term, in principle it is possible to work out a precise solution. However, due to the fact
that globular clusters have a large number of stars, it is common practice to do a numerical analysis, e.g., by the Monte-Carlo method. Detailed discussions of numerical methods are beyond our scope here. We only summarize some results.

The evolutionary sequence for a cluster is mainly a process of core-halo structure formation, escape of stars, mass stratification and core collapse. Numerical calculations show that clusters, no matter what initial conditions they have, soon develop through relaxation two distinct regions: an inner core region and an outer halo region. The mass in the inner core is usually half of the total value and the velocity distribution is nearly isotropic. The average thermal velocity is a weakly decreasing function of \( r \) and the density profile can be approximately represented by an isothermal model. Within the sphere of radius \( r_c \), density is nearly constant. Out in the halo region, stars predominately move in radial orbits with the corresponding thermal velocity decreasing rapidly with \( r \).

Once the core-halo structure forms, the core collapse phase begins. The isothermal sphere experiences gravothermal instability as discussed above, and this run-away process happens on a core-collapse timescale. Evaporation of stars during collapse becomes faster, because stars are diffused to higher energy. Conservation of energy requires a simultaneous contraction of the core, and it thus helps accelerate the core collapse. Outer parts of the cluster start to expand at the same time. Figure 2-2 illustrates this dynamical process.

Numerical studies also reveal that clusters with several mass components evolve similarly; there is mass stratification in which the heaviest stars in particular tend to concentrate in the cluster core during a time comparable to the core relaxation timescale. This process occurs due to the stars' tendency toward energy equipartition, and is graphically shown in Figure 2-3. After the heavier stars become concentrated near the center, the contraction of the cluster proceeds in much the same way as in a single component cluster. The primary effect of this mechanism in multi-mass models is to
Figure 2-2. The structural evolution of globular clusters in time units $t_{rh}$, the half-mass relaxation time. The y-axis is the radius $r$ in units of half-mass radius, and the numbers to the right of the curves indicate the fraction of mass within $r$. 
Figure 2-3. The mass segregation process of globular clusters. Shown here is a Monte-Carlo result for a globular cluster with three mass components. The y-axis is the half mass radius of each component, and the x-axis is time. The numbers to the right of the dots are the mass of that component in units of the total mass of the globular cluster.
shorten the time needed from the birth of the cluster as a stellar system to the rapid
collapse of its central core. Consequently, the stratification leads to an earlier onset of
gravothermal instability.

Section 2.4: A Galaxy Model

Globular clusters dwell in galaxies, which are bound collections of stars with
masses $M_C \sim 10^6 M_\odot$. Although their internal evolution plays the single most important
role in their fate, globular clusters interact with various mass components in a galaxy as
they move within it. Such interactions, quite significant and sometimes even catastrophic
when integrated over time, depend on detailed kinematics of mass components in
galaxies. Although the earliest kinematical studies of galaxies trace back to the turn of the
century, knowledge of kinematics of even the Milky Way is still incomplete. Therefore,
many galaxy models have been proposed to explain the limited observations.

The spheroidal components of the Galaxy include the nucleus, the bulge, and the
halo. The space distribution of stars in these components is nearly spherical and is
strongly centrally concentrated, with number densities of all objects rising as $n \sim R^{-3}$ for
galactocentric distances $R$ in the range from 100 pc through 30 kpc. Metal-poor globular
clusters and RR Lyrae stars are the brighter constituents of the spheroidal components. In
the solar neighborhood, subdwarfs are the most common spheroidal-component stars. All
these objects are very old, with typical ages of $10^{10}$ years. Regardless of their subgroups,
these objects show similar properties kinematically: they all slowly rotate about the axis
perpendicular to the galactic plane at a typical velocity $v_{\text{rot}}$ of 60 km/s. The rms velocities
for them are roughly the same too, $\sigma \approx 130$ km/s, with a weak dependence on $R$. Their
distribution in velocity space can be best described by a Gaussian function:
\[ P(\vec{v}) = \frac{1}{(\sqrt{2\pi})^3 \sigma^3} \exp\left( -\frac{u^2}{2\sigma^2} - \frac{(v-v_{\text{rot}})^2}{2\sigma^2} - \frac{w^2}{2\sigma^2} \right), \quad (2.51) \]

where \((u,v,w)\) are the usual Cartesian velocity components of \(\vec{v}\) in the local rest frame. Apparently, the spheroidal component is quite "hot" because its rms velocity is large compared to its systematic rotational motion. This happens because both the constituent Population II stars and globular clusters largely move on highly eccentric orbits.

On the other hand, the disk component mostly consists of younger stars and clusters. These stars are generally metal-rich, as compared to their spheroidal counterparts. Perhaps this is only because disk stars formed from material pre-enriched by the ejecta of evolved spheroidal component stars. Although they are mostly younger than spheroidal stars, disk stars have a wide range of ages. Some disk stars, e.g., long-period variables, can be as old as \(10^{10}\) years. While intermediate-aged (\(\sim 5 \times 10^9\) years old) disk stars are stars like the Sun, most other G dwarfs and planetary nebulae, the young (\(\sim 10^9\) years old) disk population stars are A and F stars that escaped from loose associations and clusters. The youngest population (\(\sim 10^8\) years) consists of the stars that are found predominantly in the spiral arms. They were just recently formed from gas and dust-rich environments, and usually have above-solar metal abundances. All these stars of different ages are well mixed and spread out in the disk. Kinematically, these stars show a systematic rotation about the Galaxy center and an rms velocity about 30 km/s. This rms value for the disk component is smaller than that for spheroidal component, because interaction between stars is much stronger than in spheroidals. Figure 2-4 shows the rotation velocity curve as a function of galactocentric distance \(R\) in the disk. For stars very close to the core, the rotation velocity rises sharply with \(R\). For most other \(R\)-values (\(R > 1\) kpc) \(v_{\text{rot}}\) is roughly constant, but rises slightly with \(R\).
Figure 2-4. The measured rotation curve for the Milky Way: the rotation velocity of the disk as a function of galactocentric distance R.
This trend has been quite puzzling for years, because it indicates that the mass of the Galaxy within $R$ is actually an increasing function of $R$. Therefore, the total mass of the Galaxy would have no meaning if we do not truncate the rotation curve at a larger distance (say, $R > 30$ kpc). However, no truncation has been observed, partly because of a paucity of information beyond the solar position. On the other hand, the total mass inferred by counting stars only accounts for about 10% of what is implied by the rotation curve. The missing mass is now believed to come from dark matter, which is non-luminous, but has gravitational effects.

From the observed rotation curve, one can find the mass distribution $M_G(R)$ of the Galaxy. Two major theoretical models have been proposed to fit the rotation curve: the Bahcall, Soneira and Schmidt (BSS; 1983) model and the Ostriker and Caldwell (OC; 1983) model. The major disparity between these two models is that they have different disks. While both models are reasonable, the latter is used throughout this research for its convenience in theoretical investigations. Here we briefly present some of its results.

The OC galaxy model has three components. The disk is represented by the difference of two exponential disks, and has a density

$$\rho(R) = \rho_D \left[ \exp \left( -\frac{R}{R_D} \right) - \exp \left( -\frac{R}{R_G} \right) \right].$$

The mass density of the central bulge is

$$\rho = \rho_s \frac{3.75}{Z^2} \left[ \frac{3 - Z}{Z^{1/2}} \ln \frac{1 + Z^{1/2}}{(1 - Z)^{1/2}} - 3 \right], \quad R < R_s;$$

$$\rho = \rho_s \frac{3.75}{Z^2} \left[ \frac{3 + Z}{Z^{1/2}} \tan^{-1} \left( \frac{-1}{Z^{1/2}} \right) + \frac{\pi}{2} - 3 \right], \quad R > R_s;$$

(2.52)
where $Z = l (R/R_s)^2 - l$. The above formulae converge to $\rho_s$ when $R$ approaches $R_s$.

Asymptotic limits are

$$
\rho = \rho_s \cdot 3.75 \left[ 2 \ln \left( \frac{2R_s}{R} \right) - 3 \right], \text{ if } R \ll R_s \quad (2 - 54)
$$

$$
\rho = \rho_s \cdot 1.875 \pi \left( \frac{R_s}{R} \right)^3, \text{ if } R \gg R_s \quad (2 - 55)
$$

The third component, the halo, has a mass density of the form

$$
\rho = \frac{\rho_h}{1 + (R / R_h)^2} \quad (2 - 56)
$$

The parameters may vary for different versions of the OC model, but for this research they are the values tabulated in Table 2-I:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value 1</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_\odot$</td>
<td>8.151</td>
<td>kpc</td>
</tr>
<tr>
<td>$\rho_D$</td>
<td>$5.980 \times 10^4$</td>
<td>$M_\odot \text{ pc}^{-2}$</td>
</tr>
<tr>
<td>$R_D$</td>
<td>3.26033</td>
<td>kpc</td>
</tr>
<tr>
<td>$R_G$</td>
<td>3.25115</td>
<td>kpc</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>$1.018 \times 10^2$</td>
<td>$M_\odot \text{ pc}^{-3}$</td>
</tr>
<tr>
<td>$R_s$</td>
<td>$1.004 \times 10^{-1}$</td>
<td>kpc</td>
</tr>
<tr>
<td>$\rho_h$</td>
<td>$2.003 \times 10^{-1}$</td>
<td>$M_\odot \text{ pc}^{-3}$</td>
</tr>
<tr>
<td>$R_h$</td>
<td>2.332</td>
<td>kpc</td>
</tr>
</tbody>
</table>

To determine the Galaxy's rotation curve, one needs to compute the inclusive mass,
\[ M(\leq R) = \int_0^R 4\pi r^2 \rho(r) \, dr, \]  

(2.57)

where \( \rho = \rho(\text{disk}) + \rho(\text{bulge}) + \rho(\text{halo}) \). The rotation curve predicted by this model is shown in Figure 2-5. It adequately reproduces the observed curve. This model also includes the contribution of dark matter to both halo and disk densities, but the form of dark matter is unspecified. Many authors believe that dark matter is baryonic, i.e., matter that consists of non-luminous stars or planets (Adams and Walker, 1990). Others suggest that elementary (exotic) particles might be responsible for the missing mass. This is an active research area and interested readers are referred to the recent review by Trimble (1991).

Section 2.5: Environmental Effects on Globular Clusters

In the previous sections, we have discussed the dynamical evolution of an ideal globular cluster and a Galaxy model. However, actual globular clusters move in the background of galaxies whose gravitational potential much affects the cluster evolution. These environmental factors help accelerate the collapse of clusters, and in extreme cases destroy certain kinds of clusters. Such interactions, together with cluster's intrinsic evaporation, have been very effective over time at defining and modifying the system of globular clusters in a galaxy.

When perturbed by a tidal force from any massive object, whether a whole galaxy, a disk, a bulge, a massive black hole in the Galactic center, or the Magellanic Clouds, stars in a stellar system are accelerated by the perturbing potential. If the acceleration is impulsive, the average energy increment for the stars can be very significant. In this
Figure 2-5. A theoretical rotation curve for the Galaxy. Plotted here is the result for the Ostriker and Caldwell model parameterized in Table 2-I. The big dot denotes the solar position on the curve.
situation, the stellar system is said to have experienced a shock (which is similar only in its impulsiveness to the shocks in hydrodynamics). Some dwarf elliptical galaxies are found to have experienced tidal shocks when moving in orbits around a more massive galaxy (Keenan and Innanen, 1975). However, this shocking process is not limited to objects like dwarf galaxies: globular clusters orbiting in the Galaxy are subject to shocks from the central bulge and the disk. These two impulsive processes, called bulge shocking and disk shocking, respectively, have important consequences for the dynamical evolution of globular clusters.

_Bulge Shocking:_ Bulge shocking was first treated with an impulse approximation. In this approximation, stars are assumed immobile at the their positions in the cluster, which passes by the central bulge at a constant velocity. However, this simplification works well only when the cluster's internal motions are negligible in comparison with the cluster velocity. Spitzer (1958) introduced a first-order correction to the impulse approximation by assuming stars in a cluster are moving randomly in harmonic oscillator orbits. His result for a change in binding energy per unit mass produced by a single perigalactic passage is given by

\[
\Delta E = \frac{1}{2} \left( \frac{2GM_b}{V_c R_{peri}^2} \right)^2 \gamma^2(R_{peri}) \frac{1}{3} \langle r^2 \rangle \eta(\theta),
\]

where \( M_b \) is the mass of the perturbing object (here, the central bulge), \( V_c \) is the cluster's velocity at the perigalactic distance \( R_{peri} \), and \( \langle r^2 \rangle \) is the mean-square cluster radius. The numerical factor \( \gamma(R_{peri}) \) takes into account the fact that both the central bulge and the perturbed cluster are extended objects, not mass points. According to Aguilar and White (1985), this correction is...
\[
\gamma(R) = \int_1^R \frac{M_b^*(Rt)}{t^2(t^2-1)^{1/2}} \, dt,
\]

(2-59)

where \(M_b^*\) is the interior spheroid mass. When \(R\) tends to infinity, \(\gamma\) approaches unity, as suggested by Spitzer's (1958) point-mass model. Finally, in equation (2-58), \(\eta(\theta)\) represents the cluster's efficiency at absorbing energy from the tidal force; this depends on the ratio \(\theta\) of the encounter fly-by time to the internal crossing time in the cluster. Usually, \(\eta(\theta)\) is an exponential function of \((-\theta)\), and becomes very small when \(\theta > 2\). This is consistent with the fact that the member stars of a globular cluster are slow to respond to an external perturbation when it is changing fast. For stars moving in circular orbits in the cluster, it is easy to find that

\[
\theta = \frac{2R_{\text{peri}}}{V_c} \sqrt{\frac{G M_c}{r^3}},
\]

(2-60)

where \(M_c\) is the cluster's mass, and \(r\) is the radius of the stellar orbit in the cluster.

Since \(\theta\) depends on the orbital radius of stars, the absorption efficiency \(\eta(\theta)\) is clearly dependent on individual stars that move in random orbits in a cluster. This fact can be taken into account by defining a mean \(\eta(\theta)\) that is averaged over the cluster density profile

\[
\langle \eta \rangle = \frac{4\pi}{\langle r^{-2} \rangle M_c} \int_0^\infty \rho(r) \eta(\theta) r^4 \, dr;
\]

(2-61)

therefore equation (2-58) becomes
\[ \Delta E = \frac{1}{2} \left( \frac{2 \ GM_b}{V_c R_{\text{peri}}^2} \right)^2 \gamma^2 (R_{\text{peri}}) \frac{1}{3} \left( \frac{r^2}{\eta} \right) . \] 

In figure 2-6, we plot the dependence of \( \langle \eta \rangle \) on \( \theta_h (\theta \ at \ r_h) \) for a cluster of \( M_c=5 \times 10^5 \ M_\odot \), \( r_h=5 \text{pc} \), but various concentration parameters.

We note that the above equation corresponds to a single encounter with a perturber that moves along a straight line. However, globular clusters are mostly moving in bound elliptical orbits in the Galaxy. Knobloch (1976) found that Spitzer’s formula is correct within 15% only for small deflection angles, i.e., those not exceeding about one-fourth radian for hyperbolic encounters. To correct for this orbital effect on the straight-line approximation, Aguilar et al. (1988) suggested that equation (2-62) be multiplied by a dimensionless factor given by

\[ \lambda(e) \equiv \left[ \frac{M_b (R_{\text{apo}})}{M_b (R_{\text{peri}})} \left( \frac{1 - e}{1 + e} \right)^2 - 1 \right]^2 , \] 

where \( M_b (R_{\text{apo}}) \) and \( M_b (R_{\text{peri}}) \) represent the bulge mass interior to apogalacticon and perigalacticon respectively, and \( e \) again stands for the eccentricity. When \( e \sim 1 \), for very hyperbolic orbits, \( \lambda \sim 1 \), and the case is well described by Spitzer’s formula; when \( e=0 \), \( \lambda \) becomes zero, implying that there is no bulge shocking for globular clusters in circular orbits about the galaxy center.

The strength of this shocking process also can be quantified by defining a timescale \( t_{\text{bulge}} \) within which a cluster would be totally disrupted. If \( E_{\text{bind}} = 0.2GM_c/r_h \) is the mean binding energy per unit mass of the cluster, \( \Delta E \) is the change in binding energy when the cluster moves in an orbit with period \( T \), and \( \xi \) is a dimensionless factor,
Figure 2-6. The absorption efficiency $\langle \eta \rangle$ for bulge shocking as a function of $\theta_h$, a parameter dependent on cluster orbit (see text). Cluster concentration parameters are $c=1$, 1.25, 1.5, and 1.75 respectively.
which represents the ratio of relative binding energy change (induced by this gravitational shock) to corresponding cluster mass change, then

\[
\tau_{\text{bulge}} = \frac{\Delta E}{E_{\text{bind}}} = \frac{\xi}{\lambda(e)} \frac{3T}{20} \left[ \frac{R_{\text{peri}}}{M_b} \frac{V_c}{\gamma(R_{\text{peri}})} \right] \left[ \frac{M_c}{G m \langle \eta \rangle} \right], \quad (2-64)
\]

According to Spitzer and Chevalier (1972), \( \xi \) is about 1.5 to 2. It may be dependent on cluster parameters (Chernoff et al., 1990). Aguilar et al. (1988) calculated the bulge shocking timescale for several dozen globular clusters in the Galaxy and concluded that this disruption mechanism leads to the destruction of globular clusters on nearly radial orbits through the Galactic center. According to their results, the timescale for cluster disruption in the Milky Way by this mechanism is about \( 10^{12} \text{-} 14 \) years.

**Disk Shocking:** In a spiral galaxy like the Milky Way, when a globular cluster moves perpendicularly through the disk, its stars suffer a tidal acceleration resulting from the rapidly changing external force. This brief acceleration works in a way similar to bulge shocking: it increases the average stellar energy for the stars and thus provides yet another important way to heat up a cluster. Because of its impulsiveness, this mechanism is sometimes termed a compressive gravitational shock.

Similar to the case of bulge shocking, the energy increment (per unit mass) via disk shocking for stars in a cluster can be quantified through the impulse approximation, plus another correction factor

\[
\Delta E = 2 \left( \frac{m}{V_z} \right)^2 \langle z^2 \rangle \langle \eta \rangle; \quad (2-65)
\]
here, \( \eta_m \) is the maximum gravitational acceleration on the cluster, \( V_z \) is the cluster's velocity (perpendicular to the disk) and \( \langle z^2 \rangle \) is the mean square z-coordinate within the cluster, and is usually typical of cluster's dimension in z-direction. \( \eta_{z} \) is given by

\[
\langle \eta_z \rangle = \frac{4 \pi}{3 M_c \langle r^2 \rangle} \int_0^{r_h} \rho (r) \eta_z (\theta_z) r^4 \, dr ,
\]

(2-66)

where \( \theta_z \) has the same definition as that for bulge shocking and can be written as

\[
\theta_z = \sqrt{\frac{G M_c}{r^2}} \Delta t_z .
\]

(2-67)

where \( \Delta t_z \) is the disk crossing time for the cluster in consideration. Figure 2-7 plots the relations of efficiency \( \langle \eta_z \rangle \) with \( \theta \) at \( \eta_h \) for disk shocking, in comparison with those for bulge shocking.

Analogous to the treatment of bulge shocking, a destruction timescale for this dynamical process can be defined as (cf., Aguilar et al., 1988)

\[
t_{disk} \equiv \xi \frac{E_c}{\Delta E/\Delta t} = \left[ \frac{3 M_c \Delta T}{4 \pi^2 G r_i \langle r^2 \rangle} \right] \left[ \sum_{i=1}^4 \left( \frac{\sigma_d \eta_i^2}{V_z^2} \right) \right]^{-1} ,
\]

(2-68)

where \( \sigma_d \) is the surface density of the disk at the point where the cluster crosses, and to calculate this rate with a better accuracy, the summation is taken for 4 consecutive disk crossings, which in total take a timespan \( \Delta T \). The calculations of Aguilar et al. (1988) suggested the timescale \( t_{disk} \) is on the order of \( 10^{14} \) years. This mechanism has destroyed many clusters of small masses, and its major effect on individual globular clusters, according to Oh and Lin (1992), is to isotropize the orbits of their member stars.
Figure 2-7. The absorption efficiency $<n_z>$ for disk shocking as a function of $\theta_h$, a parameter dependent on cluster orbit (see text). Cluster concentration parameters are $c=1$, 1.25, 1.5 and 1.75 respectively. For comparison, the dotted curves for bulge shocking are also plotted here.
Dynamical Friction: This process has a substantial impact on the dynamics of the globular cluster system in the Galaxy. As derived by Chandrasekhar (1942), an object moving through a uniform medium suffers a momentum loss which corresponds to a frictional force

$$F_{DF} = -4\pi G^2 M_c^2 \rho_b \frac{F(V_c) \ln \Lambda}{V_c^3},$$

(2-69)

where $M_c$ is the object's (viz., the cluster's) mass, $\rho_b$ is the spatial density of the background particles (stars), and $F(V_c)$ is the probability of finding background stars moving slower than the cluster. The quantity $\Lambda$ has been introduced elsewhere; it represents the ratio of the limits in the integration over impact parameter.

This deceleration in our case comes from two sources: the disk and the spheroid. Because of the disk contribution, the deceleration generally will not be opposite to the cluster's motion. When a cluster moves very close to the disk (say, within a few scale heights), dynamical friction due to the disk is the dominant effect and works to circularize the cluster orbit, which gradually causes the cluster to spiral towards the Galactic center. However, if a cluster moves only in the spheroid, where disk's effect is negligible, dynamical friction tends to slow it down by dissipating its kinetic energy. The cluster then gradually spirals towards the center. On its way towards the Galactic center, the cluster will experience mass loss due to the increasing strength of the Galactic potential that strips cluster stars. Eventually the mass loss becomes catastrophic.

As identified by Aguilar et al. (1988), the mass-loss timescale for this mechanism is

$$t_{df} = \frac{M_c}{dM_c/dt}.$$

(2-70)
Further discussion of this process, which is sensitive to the cluster's concentration $c$, is provided by Chernoff et al. (1986).

All of these environmental destruction mechanisms have influenced the galactic globular cluster system. However, just how globular clusters reached their present states requires knowledge of their orbital histories, information that is not available. But, extrapolations based on the present parameters of globular clusters indicate that evaporation of light stars represents the dominant destruction mechanism in individual globular clusters, and its strength is nearly independent of cluster position (Aguilar et al., 1988). For the OC galaxy model, disk shocking is the next most important process; it wanes only gradually as clusters get more distant from the center, where the disk potential becomes less deleterious. Dynamical friction, which mainly drags massive globulars towards the galactic center, is the third most destructive process. Its variation with environment nearly mimics that of the spheroid density profile itself. Finally, bulge shocking becomes fatal only for globular clusters on essentially radial orbits that pass very close to the galactic nucleus. This shocking mechanism must have contributed very much to the present kinematics of globular cluster system, if the clusters were in randomly distributed orbits at birth.
Chapter 3

Diffusive Heating by Field Stars

As outlined in Chapter 2, several mechanisms have been identified over the years as being important to the dynamical evolution of globular clusters. Aguilar et al. (1988) estimated the destruction rates of all these processes, for about half of the globular clusters in the Milky Way, and demonstrated that evaporation is the dominant destruction mechanism for the surviving globulars, but that the galactic environment has influenced the evolution of the original cluster system.

Fall (1980) conveniently parameterized the timescales for evaporation, disk shocking and dynamical friction (both in solar neighborhood):

\[ t_{ev} = 2 \times 10^7 \left( \frac{M_c}{M_\odot} \right)^{1/2} \left( \frac{r_h}{pc} \right)^{3/2} \text{ yr} , \]  
\[ t_{ds} = 1 \times 10^8 \left( \frac{M_c}{M_\odot} \right) \left( \frac{r_h}{pc} \right)^{-3} \text{ yr} , \]  
\[ t_{df} = 1 \times 10^{18} \left( \frac{M_c}{M_\odot} \right)^{-1} \text{ yr} , \]

where \( M_c \) and \( r_h \) represent the total mass and the half-mass radius of a globular cluster. Only those clusters for which the various destruction mechanisms all have timescales greater than the Hubble time ( \( \sim 10^{10} \) years ) have survived. These dynamical timescales are consistent with the narrow ranges observed for the masses and dimensions of
surviving globular clusters, namely, $M_c = 10^5 \pm 1$ $M_\odot$ and $n_h = 10^1 \pm 0.5$ pc. (Figure 3-1). Clearly, all globular clusters will eventually be disrupted through a combination of these internally and externally driven processes.

In this chapter, we investigate yet another constraint on the clusters' survival. This new process involves the gravitational interaction of individual globular clusters with field stars, which represent a significant mass component of the Galaxy. Globular clusters orbit in this background of field stars and encounter many of them. Viewed in the rest-frame of a globular cluster, its stars collectively scatter and slow down (by dynamical friction) all field stars that diffuse through it. Field stars thereby lose some of their energy to the random motions of cluster stars and heat up the cluster. Over time, this diffusive heating disrupts clusters at a rate we now determine.

We first consider a simple situation. A field star moving through a background of identical, fixed cluster stars, whose number density is $n_c$ and total number is $N$, feels a drag force. As reviewed in Chapter 2, the field star's mean deceleration due to dynamical friction in the direction parallel to its motion is given by

$$\langle \Delta v \rangle = - \left(1 + \frac{m_f}{m_c} \right) \frac{n_c \Gamma}{v^2} \Delta t,$$

where $v$ is the velocity of the field star during the time interval $\Delta t$, and where $m_f$ and $m_c$ are the masses of the incident (field) star and each background (cluster) star, respectively. The quantity $\Gamma$ in this equation is $4\pi G^2 m_c^2 \ln \Lambda$, where $\ln \Lambda = \ln(0.4 \, N)$ is about 10 for most globular clusters. In the standard diffusion picture, dynamical friction also causes a mean energy loss,
Figure 3-1. The triangle formed by requiring globular clusters to survive evaporation, disk shocking and dynamical friction over a Hubble time. The circles represent certain galactic globular clusters.
\[ \langle \Delta E \rangle = -\frac{m_f^2}{m_c} \frac{n_c \Gamma}{v} \Delta t \]  

(3 - 5)

during this same time interval.

The total energy which is lost by a field star during its encounter with a globular cluster depends on its actual path inside the cluster. The globular's gravitational potential does not directly contribute to this net loss; but, it does influence the diffusing star's kinetic energy at each point inside the cluster, and it thereby indirectly affects the local energy loss rate. All globulars are sufficiently spherical that an incident star's trajectory can be characterized by just its impact parameter \( b \) and incident relative velocity \( v_0 \). Hence, the net loss of energy by one field star is

\[ \Delta E_f(b, v_0) = \int_l^{\infty} \frac{\langle \Delta E \rangle}{\Delta t} \, dt = \int_l^{\infty} \frac{\langle \Delta E \rangle}{\Delta t} \frac{ds}{v}, \]  

(3 - 6)

where \( ds \) is an infinitesimal displacement along the route \( L \) (not necessarily a straight line) and \( v \) is the instantaneous velocity. The effective heating by field stars of this relative velocity can be easily identified as an integral over all possible impact parameters,

\[ \Xi(v_0) = 2 \pi \int_0^{\infty} b \left| \Delta E_f(b, v_0) \right| \, db. \]  

(3 - 7)

Even though, theoretically, an isolated globular cluster can interact with field stars at any distance, globular clusters actually do not have much influence on stars outside their own
tidal radius. For this reason, the uppermost impact parameter will later be set equal to the cluster's tidal radius.

So-called gravitational focusing plays a role in determining the heating rate. Because of gravitational attraction, the path of a penetrating star inside the globular cluster is bent towards the cluster center. This effect, which strongly depends on the star's velocity, tends to drag it closer to the core where the background star density is higher. Thus the actual heating caused by field star penetrations will be higher than that computed with straight line trajectories. To estimate the importance of the focusing effect in our calculation, we first assume that an incident star moving in a cluster has an impact parameter $b$. If there is no focusing effect -- in other words, if the straight line approximation holds -- this impact parameter will equal to the distance $b'$ (with respect to the cluster center) of the tangent line at any point on its route. But when the focusing effect is taken into account, the straight line approximation fails, and the equality breaks as the star proceeds within the cluster (See figure 3-2). To simplify the problem and place an upper limit on the effect, we assume that all of the cluster's mass $M_c$ is at its center. As a result, the incident star's path is described by a Keplerian orbit equation [see Eqn.(2-24)]. At a distance $r$ from the center, the tangent line distance $b'$ is then

$$b' = \frac{b}{\sqrt{1 + \frac{2GM_c}{rv^2}}}.$$  \hspace{1cm} (3 - 8)

Notice that $b'$ would be the impact parameter if the incident star were on a straight line trajectory at that point. For actual events, we have

$$\frac{2GM_c}{rv^2} \approx \frac{2GM_c}{r_t v^2} = \left(\frac{v_{esc}}{v}\right)^2.$$  \hspace{1cm} (3 - 9)
Figure 3-2. The effect of gravitational focusing on the path of an incident star inside a globular cluster: the trajectory is now bent towards the cluster center.
Most globulars have surface escape velocities of at most only a few tens km/s, while the majority of field stars have velocities of order $10^2$ km/s. Evidently, for heating purposes, where field stars with significant kinetic energy are important, this factor is practically negligible. In the following work, we simply ignore the focusing effect.

To determine the rate of energy input to a cluster at a particular galactocentric position $\vec{R}$, equation (3-7) must be multiplied by the flux of impinging field stars. Frenk and White (1980) concluded that the Milky Way's globular cluster system has a Gaussian velocity distribution, with an isotropic dispersion $\sigma = 130$ km/s only weakly dependent on $R$, and a systematic rotation speed $v_{\text{rot}}$ near 60 km/s. According to Carney (1986), no significant difference has been found between the kinematical distributions of globular clusters and (Population II) field stars in the halo of our Galaxy. For these reasons, we adopted the product

$$f(\vec{R}, \vec{v}) = n_f(\vec{R}) P(\vec{v})$$

(3 - 10)

for the phase space distribution of field stars in the halo, where $P(\vec{v})$ is the same as the velocity distribution of the globular cluster system in the Galaxy [cf. Eqn (2-51)], and $n_f(\vec{R})$ is the number density of field stars at a given galactocentric position. Unfortunately, this last quantity is not well known. The mass density determined by counting stars in the solar neighborhood is quite discrepant with the total density inferred from the galactic rotation curve (Ostriker and Caldwell 1983; Bahcall et al. 1983),

$$\rho_{\text{tot}} = 0.5 \left( \frac{1 \text{ kpc}}{R} \right)^2 M_\odot \text{pc}^{-3}, \quad R \geq 1 \text{ kpc}.$$  

(3 - 11)
This total, however, includes the contribution of sub-stellar objects (e.g., brown dwarfs, Adams and Walker 1990) as well as stellar remnants (Ryu, Olive, and Silk 1990) and other possible forms of dark matter. Moreover, because $\Delta E_f$ depends on $m_f^2$, not $m_f$, it does not scale linearly with the local mass density of stars and sub-stellar objects in the field. We therefore computed the total rate of heating from field stars plus any sub-stellar component of the dark matter by making the following plausible assumptions (see, e.g., Binney and Tremaine 1987; Trimble 1987; Gilmore et al. 1989): (1) all condensed objects have the same, spherically symmetric density profile as $\rho_{\text{tot}}$, and all sub-stellar objects have the same velocity distribution as the field stars; (2) field stars themselves all have masses $m_f = 1/2 \ M_\odot$, and contribute 1/10 of the Galaxy's total mass density; (3) unseen sub-stellar objects have masses 1/10 $m_f = 1/20 \ M_\odot$ and contribute a fraction $\alpha_D$ of the total matter density.

It follows from these assumptions that the total diffusive heating rate $H_{\text{dh}}$ is simply $(1+\alpha_D)$ times the rate from known field stars, and that it varies with the cluster's position simply as $R^{-2}$. For a cluster moving with velocity $\vec{V}_c$ this rate is

$$H_{\text{dh}}(R, \vec{V}_c) = (1 + \alpha_D) n_f(R) \int_0^\infty d\vec{v} P(\vec{v}) |\vec{v} - \vec{V}_c| \Xi (|\vec{v} - \vec{V}_c|) . \quad (3 - 12)$$

We want to further simplify equation (3-12). For the simple case in which cluster stars have negligible thermal velocities, we use equation (3-5). Then it follows from equation (3-6) that

$$\Delta E_f(b, v_0) = \int L \frac{\langle \Delta E \rangle}{\Delta t} \frac{ds}{v} = \frac{m_f^2 \Gamma}{m_c} \int L \frac{n_c}{v^2} ds$$
\[ \frac{m_f^2 \Gamma}{m_c v_0^2} \int n_c \, ds = \frac{m_f^2 \Gamma}{m_c v_0^2} N_c(b) \quad . \]  

where the approximation \( v = \text{constant} \) has been made and \( N_c(b) \) is the projected density along the route. To estimate the error that might be caused by this approximation, we calculated the fractional velocity change \( \Delta v / v \) on a radial route for a range of incident velocities and plotted the relation in Figure 3-3. Velocity changes on other routes are smaller than the one through the diameter, since elsewhere incident stars encounter fewer cluster stars. Evidently, there is little change of velocity for fast incident stars; they just penetrate through without being slowed much by dynamical friction from cluster members. Now equation (3-7) becomes

\[ \Xi (v_0) = \frac{m_f^2 \Gamma}{m_c v_0^2} \int_0^{r_h} 2 \pi b N_c(b) \, db = \frac{m_f^2 \Gamma N}{m_c v_0^2} \quad , \]  

where \( N \) is the total number of stars of the cluster in consideration.

Next we use equation (3-12) in the rest frame of the cluster to remove \( V_c \). We have, after some simple algebra, that

\[ \Xi (v_0) v_0 = \frac{m_f^2 \Gamma N}{m_c v_0} = E_{\text{bind}} 20 \pi \left( \frac{m_f}{m_c} \right)^2 \ln \Lambda \left( \frac{v_h}{v_0} \right) v_h r_h^2 \quad , \]

where \( v_h \) is the escape velocity at half mass radius \( r_h \). Therefore, the heating rate per unit time is easily found to be
Figure 3-3. The maximum fractional velocity changes of an incident star passing radially through a cluster of \( r_t = 55 \text{pc} \), \( M_c = 5.2 \times 10^5 \text{ M}_\odot \), and \( c = 1.3 \). The top curve represents the contribution from gravitational acceleration, and the bottom curve, from dynamical friction.
\[ H_{dh}(R, \vec{V}_c) = E_{\text{bind}} (1 + \alpha_D) n_f(R) 20 \pi \left( \frac{m_r}{m_c} \right)^2 \frac{\ln \Delta}{N^2} \nu_h r_h^2 \int_0^\infty P_c(\vec{v}_0) \left( \frac{\nu_h}{\nu_0} \right) d\vec{v}_0 , \quad (3-15) \]

where \( P_c(\vec{v}) \) denotes the velocity distribution of field stars as seen by the cluster.

As seen by a cluster with some typical velocity, the field stars would have a velocity distribution modified from Gaussian function. For analytical purposes, we assume that \( P_c(\vec{v}) \) is a Gaussian with same parameters in equation (3-9). This should be a good approximation for halo clusters with velocities up to 130 km/s. For disk clusters, we make a similar assumption. But, since the velocity dispersion \( \sigma \) is smaller (i.e., a thinner peak), the approximation will be good for a smaller fraction of disk clusters.

We also have to modify the lower limit of the integration, because equation (3-5) cannot be used for incident stars moving too slowly relative to the cluster stars [which would otherwise cause divergence in equation (3-15)]. Intuitively, it makes sense to use the mean thermal velocity \( \nu_m \) \([<< \sigma(\text{disk}) \text{ or } \sigma(\text{halo})]\) of cluster stars as the actual lower limit; this is about half the mean star escape velocity of the cluster. Then, equation (3-15) becomes

\[ H_{dh}(R) = E_{\text{bind}} (1 + \alpha_D) 20 \sqrt{2\pi} \left[ n_f(R) \nu_h r_h^2 \right] \left( \frac{m_r}{m_c} \right)^2 \frac{\ln \Delta}{N^2} \left( \frac{\nu_h}{\nu_m} \right) \exp \left( -\frac{\nu_m^2}{2 \sigma^2} \right) . \quad (3-16) \]

Following Aguilar et al. (1988), we then use the equation

\[ H_{dh} = \nu_{dh} E_{\text{bind}} , \quad (3-17) \]

to convert our calculated diffusive heating rates at different galactocentric radii \( R \) to destruction rates (per unit time) \( \nu_{dh} \):
\[ v_{\text{dh}}(R) = (1 + \alpha_D)2\sqrt{2\pi} \left[ n_f(R) v_h r_h^2 \right] \left( \frac{m}{m_e} \right)^2 \left( \frac{\ln \Lambda}{N^2} \right) \left( \frac{v_h}{\sigma} \right) . \quad (3 \cdot 18) \]

The timescale for a halo cluster to be dissolved by this mechanism is
\[ t_{\text{dh}}(R) = \frac{1}{v_{\text{dh}}} = 3 \times 10^{12} \left( \frac{R}{1 \text{ kpc}} \right)^2 \left( \frac{M_c}{10^5 M_\odot} \right) \left( \frac{1 \text{ pc}}{r_h} \right) \text{ yr.} \quad (3 \cdot 19) \]

For a typical halo cluster in the Milky Way, at a galactocentric distance \( R = 10 \) kpc, this formula predicts a heating rate about \( 10^{-15} \text{ E}_{\text{bind}} \) per year. In other words, the average halo cluster would be disrupted via diffusive heating by incident stars on a timescale \( t_{\text{dh}} \) around \( 10^{15} \) years, which far exceeds the Hubble time.

However, it must be emphasized that the destruction timescale in equation (3-19) for individual clusters is a strong function of their orbits. If a cluster moves in the disk, for example, it would be disrupted on a much shorter timescale. It is also interesting to see that, according to the above equation, this effect is strongest near the galactic core where the number density of field stars is relatively high. In addition, it is more effective at disrupting less massive clusters since \( v_{\text{dh}} \) is proportional to \( M_c^{-1} \). Figure 3-4 shows the distribution of timescales for globular clusters listed by Webbink (1984). Of course, all the present globular clusters have so far survived the heating; those that did not survive are not seen.

We have made use of several approximations to achieve these simple formulae. Working analytically with equation (3-9) for a general situation is much more difficult. Hence we have written a computer code to numerically calculate this rate for globular clusters whose internal structure may be characterized by King models (King 1962). This code is described in some detail in Appendix A. Below, we describe results for our
"standard" cluster, whose parameters are average values for the uncollapsed clusters listed in Table 3-1 of Aguilar et al. (1988): tidal radius $r_t = 55$ pc (independent of galactocentric distance $R$), half-mass radius $r_h = 7.3$ pc, core radius $r_c = 1.9$ pc, concentration parameter $c = \log(r_t/r_c) = 1.45$, and total mass $M_c = 5.2 \times 10^5 M_\odot$. By fitting a King curve to these parameters we obtained a central density $\rho_c(0) = n_c(0)m_c = 3 \times 10^3 M_\odot/pc^3$ for this representative cluster.

The left panel of Figure 3-5 shows a comparison of $v_{dh}$ with rates computed by Aguilar et al. (1988) for other globular dissipation processes, in the limiting case that $\alpha_D = 0.9$. For these computations we assumed that our "standard" globular is in a circular orbit, with a velocity $V_c = 100$ km/s, also independent of $R$. All of the results in this panel correspond to the galactic $\rho(R)$ model of Ostriker and Caldwell (1983), but it should be noted that the model of Bahcall, Schmidt, and Soneira (1983) yields generally similar destruction rates. In both models, evaporation is the dominant destruction mechanism outside the nuclear region ($R \geq 1$ kpc), and this process depends only weakly on galactocentric radius. The other destruction processes all have a strong environmental dependence, and disk shocking in particular is sensitive to details of the Galaxy’s mass distribution. At all $R$-values shown here, $v_{dh}$ exceed $v_{bs}$, the rate of destruction via bulge shocking; near the galactic center, $v_{dh}$ even exceeds the rate of destruction via disk shocking. Note that the numerical results agree well with analytic predictions in order of magnitude.

The right panel of Figure 3-5 compares various destruction rates for globular clusters that happen to reside in the galactic disk. Recall that, according to Zinn (1985, 1991), such globulars form a distinct, chemically enriched, and highly flattened subgroup that appears to participate in the Galaxy’s rotation. Therefore, for disk globulars, there is no disk shocking (or bulge shocking, if the orbits are nearly circular). On the other hand, both dynamical friction involving background (disk) stars and diffusive heating are
Figure 3-4. The distribution of approximate timescales for disruption via diffusive heating, for all the Milky Way's globular clusters.
Figure 3-5. Dependence of globular cluster destruction rates on galactocentric distance $R$. Curves labeled by ev, df, dh, ds, and bs represent the processes of evaporation, dynamical friction, disk shocking, diffusive heating, and bulge shocking, respectively. The left panel applies to halo globulars and the right panel, to disk globulars in the Milky Way.
markedly enhanced by the higher ambient density and lower relative velocity of stars in the Galactic plane (Wielen and Fuchs 1983; Binney and Tremaine 1987): $n_f$ (disk) = 0.1 pc$^{-3}$, and $\sigma$ (disk) = 30 km/s.

Uncertainty in the make-up of the Galaxy's non-luminous halo carries over to all destruction processes that rely on dynamical friction. For the halo globulars, this uncertainty translates to a reduction in the values of $v_{dh}$ plotted in Figure 3-5 by a factor of two [see the discussion following equation (3-11)], but for heating by dynamical friction involving background (i.e., non-penetrating) field stars, the reduction could be as large as a factor of ten if none of the halo's dark matter is in condensed objects. Present uncertainties concerning the amount of dark matter near the Galactic plane are small (Trimble 1987; Gilmore et al, 1989) and so the possible reductions in these destruction rates for disk globulars also are small.

The distinction between halo and disk globulars, at least insofar as environmental influences are concerned, raises a related issue, namely, the fact that systems of globular clusters in elliptical and in spiral galaxies evolved in response to very different galactic perturbations, and therefore are likely to have different mass (and luminosity) functions. For instance, there is no disk shocking in elliptical galaxies; hence, as can be seen by reference to Figure 3-6, diffusive heating should play a dominant role in the evolution of globular cluster systems in these galaxies. At present we are investigating the extent to which this complication affects the use of globular cluster luminosity distributions to make extragalactic distance estimates.
Figure 3-6. The triangle formed by requiring globular clusters to survive evaporation, diffusive heating, and dynamical friction over a Hubble time. The dashed line represents the condition from disk shocking. The circles denote certain galactic globular clusters.
Chapter 4

Capture of Field Stars

The integrated velocity change of a field star during its passage through a cluster usually is quite small, at most a few km/s [see Eqn. (3-4) and Fig. 3-3]. However, some penetrating stars will be moving slowly enough that their initial energy of relative motion is completely dissipated by dynamical friction. These stars cannot escape and so, in addition to heating the globular cluster, they become new members of it. This phenomenon is quite distinct from and, as we now show, it can be far more effective than the scenario proposed by Leonard (1985), who suggested that clusters could capture field stars by means of exchange collisions involving binary systems in a cluster's core. Our mechanism also differs from the process by which gas clouds capture field stars (Whitman et al. 1991).

Let $\vec{v}_{\text{crit}}$ be the maximum initial relative velocity of incident field stars that results in capture. Clearly, this velocity depends on the density of globular cluster stars encountered on route, which in turn is a strong function of impact parameter. By definition, field stars outside a cluster's tidal radius respond mainly to the Galaxy's gravitational field. Therefore we assumed that any incident field star with impact parameter $b>r_t$ is not gravitationally focused by the cluster and simply misses it. Thus, $b$ effectively represents the impact parameter of a field star at the cluster's tidal radius (instead of at infinity), and the number of stars captured by a cluster per unit time depends on the cluster's size and its velocity through the equation
\[
\frac{dN^*}{dt} = 2 \pi n_f (R) \int_0^R db \cdot b \int_0^{v_{\text{max}}(b)} d\vec{v} \cdot P(\vec{v}) |\vec{v} - \vec{v}_c| \\
(4 - 1)
\]

where \( \vec{v}_{\text{max}}(b) = \vec{v}_{\text{crit}}(b) - \vec{v}_c \).

All field stars likely to be captured must be moving slowly relative to the cluster. Hence, the critical velocity for capture is essentially equal to the velocity loss of such a star during its transit of the cluster,

\[
\Delta v (b) = \left(1 + \frac{m_f}{m_c}\right) \int_L \frac{n_c \Gamma}{v^3} \mu ds \\
(4 - 2)
\]

where \( m_f \) and \( m_c \) are the masses of field and cluster stars, \( v \) is the instantaneous velocity on the route \( L \), and, again, \( n_c \) is the local number density of cluster stars. The factor

\[
\mu = \frac{3v^2}{v_m^2} G\left(\sqrt{\frac{3}{2}} \frac{v}{v_m}\right),
\]

where \( G(x) \) is a function defined in terms of error function \( \Phi(x) \) (see Chapter 2), represents the correction factor needed when cluster stars have a thermal velocity \( v_m > 0 \): We find that for \( v > v_{\text{esc}} \) (which is the case in this capture process), \( \mu \) is essentially unity.

It is difficult to perform analytical calculations with equation (4-2). All stars that move into a cluster experience some gravitational focusing, and their orbits may not be easy to describe. The focusing effect for high velocity stars is negligible but, realistically, these stars are never captured because their energy loss during passage through the cluster is much too small. On the other hand, the motion through the cluster of stars with small incident velocity is nearly a free-fall through the cluster center. Thus, impact parameters
for very low velocity stars are essentially zero and the straight-line approximation for their paths again holds. (In other words, regardless of its impact parameter at \( r_t \), an incident star of low relative velocity tends to pass radially through the cluster.)

For this reason, we find it useful to introduce the dimensionless quantity

\[
\beta = \frac{v_{esc}^3}{n_c(0) r_t} \int_0^r \frac{n_c}{v^3} \, ds. \tag{4-3}
\]

Then, we can write the critical velocity as

\[
v_{crit}(b) = \Delta v(b) = (1 + \frac{m_c}{m_r}) \frac{\beta \Gamma n_c(0) r_t}{v_{esc}^3}. \tag{4-4}
\]

The value of \( \beta \) varies from cluster to cluster, but, since the central density, total mass and tidal radius have all been scaled out, we expect \( b \) to be principally a function of concentration parameter \( c=\log(r_t/r_c) \) and/or \( r_c \). To determine this functional dependence, we again used a King model for cluster structure and numerically integrated the quantity \( n/v^3 \) along the (radial) route, under the assumption that all slow stars \( v<v_{esc} \) have acquired an additional velocity \( v_{esc} \) by the time they enter the cluster. As illustrated in figure 4-1, for a range of concentration parameters \( 0.5 \leq c \leq 2.6 \), we found that \( \beta \) is independent of core radius and given accurately by the expression

\[
\log \beta = -0.68 - 0.091 c - 0.077 c^2. \tag{4-5}
\]

Because \( \Delta v(b) \) is much smaller than the velocity dispersion \( \sigma \) of either halo or disk field stars and in fact does not depend on \( b \), the actual capture rate, equation (4-1), can now written as
Figure 4-1. Parameter $\beta$ as a function of concentration $c$, independent of other structural parameters.
\[
\frac{dN_*}{dt} = \left[ n_r (\text{pc}^{-3}) \sigma (\text{km/s}) A_{\text{capt}} \text{ (pc)}^2 \right] \text{ captures /10^6 yr },
\]
(4 - 6)

where
\[
A_{\text{capt}} = 2.0 \beta^2 \left( 1 + \frac{m_c}{m_r} \right)^2 \left( \frac{\sigma}{30 \text{ km/s}} \right)^2 \left( \frac{M}{5.2 \times 10^5 M_\odot} \right)^3 \times \left( \frac{m_c}{0.5 M_\odot} \right)^2 \left( \frac{\rho_c(0)}{3 \times 10^3 M_\odot/\text{pc}^3} \right)^2 \left( \frac{r_t}{55 \text{ pc}} \right)^7 \left( \frac{\ln A}{13} \right)^2 \text{ pc}^2
\]
(4 - 7)

is an effective cross section for capture events. The ratio \(A_{\text{capt}}/\pi r_t^2\) is roughly the fraction of field stars encountered by a globular that are captured, and quantities in equation (4-7) have been scaled to the values for our "standard" cluster of Chapter 3.

Figure 4-2 illustrates this effective cross sections versus central concentration for the Galactic globular clusters listed in Webbink (1984)'s compilation. Note that there is significant scatter in the figure. Equation (4-7), apparently, favors globulars with large core densities, but the \(\beta\)-value is a strong, decreasing function of central concentration. Figure 4-3, which plots \(c\) versus \(\rho(0)\) for all the globulars in the Milky Way, suggests that there is a correlation between these two cluster parameters. It is also interesting to see in Figure 4-4 that more concentrated clusters tend to be located near the disk, where the field star density is higher. Therefore, we expect that these clusters have a higher probability to capture field stars.

Over the age of the Galaxy (here, taken to be \(10^{10}\) years) its globular clusters evidently accrue a population of stars that we shall call "immigrants." However, to calculate the total number of immigrant stars we should take the evolution of a cluster into consideration. As explained in Chapter 2, because of the gravothermal instability, a cluster's inner part gradually contracts while its outer part expands. Consequently, the
Figure 4-2. The empirical relation between the capture cross section $A_{\text{capt}}$ and the concentration parameter $c$. 
Figure 4-3. A plot of globular cluster central density $\rho(0)$ as a function of $c$. Clusters of higher $\rho(0)$ tend to be more centrally concentrated.
Figure 4-4. The distribution of globular clusters with different concentrations $c$ with respect to distance $z$ from the disk. Top and bottom graphs are on two different scales of $z$. High-$c$ clusters tend to reside near the disk.
central density, core radius and tidal radius are all functions of time. In addition, the evolution of a cluster is strongly influenced by internal and external disruption mechanisms (see Chapter 2). The complete picture of how a cluster evolves is simply not known. Fortunately, the overall evolution of a cluster is usually a very slow process, generally operating on the Hubble timescale. According to Spitzer (1987), who studied dynamical evolution using Monte-Carlo methods, the central density \( \rho(0) \) of a globular cluster can be approximated as a function of time by the following expression:

\[
\left( \frac{\rho_c(t)}{\rho_c(t=0)} \right)^{0.86} = (1 - \frac{t}{t_{\text{coll}}})
\]

where \( \rho_c(t) \) represents the cluster central density at time \( t \), and the cluster collapse time \( t_{\text{coll}} \), approximately 15 times the half-mass relaxation time, is generally about \( 10^{10} \) years. His result agrees with a systematic study of Duncan and Shapiro (1982). Some globular clusters observed today probably have calculated collapse times shorter than \( 10^{10} \) years, but binary formation in their cores might have prevented them from collapsing into black holes. Figure 4-5 indicates that the core density may oscillate with time and exhibits brief spikes, but overall the evolution is still slow. Because of this, the actual numbers

\[
N_{\text{im}} = \int_{0}^{10^0} \frac{dN_*}{dt} \, dt
\]

of captured stars should not differ much from the numbers estimated by using a constant capture rate.

Table 4-I lists the number of immigrants for all the globular clusters in Webbink's (1984) compilation having calculated \( N_{\text{im}} \)-values larger than one; all cluster parameters
Figure 4-5. The core density evolution of a globular cluster. Subsequent evolution after first spike depends on assumed central energy sources to avoid catastrophic collapse. This additional energy is proportional to $Sp^3/v_m$, where $\rho$ is the core density, $v_m$ is the thermal velocity of stars in the core region, and $S$ is an adjustable constant.
### Table 4-I  Globular Cluster Immigrants

<table>
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<tr>
<th>Cluster</th>
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<th>β</th>
<th>Q</th>
<th>N_{im}</th>
<th>$\theta_{\text{max}}$ (deg)</th>
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Table 4-I  Globular Cluster Immigrants
(continued)

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* The number of halo immigrants is proportional to fraction of the halo's mass in field stars, which here is assumed to be $\zeta_f = 1/10$. 
needed to compute these results were taken from his comprehensive data set, which we reproduce in Appendix B. The effect of core evolution is measured by the factor

\[
Q \equiv N_{\text{im}} \int \left[ \left( \frac{\text{d}N_{\ast}}{\text{d}t} \right)_{\text{now}} \cdot 10^{10} \text{ yrs} \right] ,
\]

which is also listed in Table 4-I. For those globular clusters whose collapse timescale is shorter than $10^{10}$ years (indicated by $Q=1$), we just list the $N_{\text{im}}$-values obtained under the constant capture rate assumption. The positions of the globulars in the Galaxy with at least one captured star are plotted in Figure 4-6. Again, we see that the clusters with the most captures tend to reside in the disk.

Most known field stars in the halo are similar to the stars in halo globular clusters (e.g., Population II), and this fact has been used to argue that the Galaxy's halo is, at least in part, debris from disrupted clusters (Fall and Rees 1977, 1985; Sandage 1986). Of course, Population II immigrants in a globular would be indistinguishable from the cluster's original stars. On the other hand, some modest, but unknown number of immigrants (the actual number depends on unknown details of a cluster's orbit) could be Population I objects acquired during disk passages. In this regard, Table 1 also lists $\theta_{\text{max}}$, an estimate of the maximum angle of inclination of a halo cluster's orbit relative to the galactic plane such that at least one Pop I immigrant also is captured in $10^{10}$ years. For this estimate, circular orbits and a disk thickness of 500 pc were assumed. Moreover, disk stars can be launched into the halo through the disruption of binaries in supernova events [neutron stars are obvious candidates (see Lyne and Graham-Smith 1990 and references cited therein)]. Both of these situations give halo globulars some chance to capture young, chemically enriched objects. These stars would be spectroscopically distinct from the halo cluster's original, lower-metallicity stars. And, if the standard
Figure 4-6. The positions of globular clusters predicted to have $N_{im} \geq 1$ immigrant in the galactic coordinates. Clusters near the disk tend to capture more field stars.
Salpeter mass function is assumed, about one-third of all captures should be stars more massive than the Sun. In contrast, most immigrant stars in disk globulars should be younger than their host cluster, but composition differences may not be very large (Zinn 1985).

What evidence, then, do we have that an immigrant class really exists? First of all, there are a few true rarities -- a planetary nebula in M15 (NGC 7078, O'Dell et al., 1964; Adams et al., 1984) and one in M22 (Cohen and Gillett, 1989; a nova in M14 (NGC 6402, Shara et al., 1986); and an anomalous Cepheid in NGC 5466 (Zinn and Dahn 1976). All of these can understood as particular stages of captured field stars. With regard to the anomalous Cepheid, for example, we note that NGC 5466 has, within the lifetime of that variable star (as estimated from its mass, cf. Cox and Proffitt, 1988), undergone ~10 disk passages where the field star density is much higher. The number of immigrants accrued from disk passages strongly depends on the details of the cluster orbit. Any captured Pop I star with $m_f \sim 1.5 M_\odot$ could have been this Cepheid's progenitor. Similar arguments also apply to the planetary nebulae in M15 and M22, and to the nova in M14; however, simple arguments involving evolution timescales indicate that these could also be evolved, original cluster members. In addition to these exceptional cases, there are two categories of unusual cluster stars that present major difficulties for current theories of stellar and cluster evolution. These are the blue stragglers and the short-period pulsars, introduced in Chapter 1, which we now consider in some detail.

**Blue Stragglers:** Sandage (1953) first noted that a few stars in the globular M3 (NGC 5272) were bluer than the stars at the main sequence turn-off point of that cluster's C-M diagram. In the decades since then, these so-called blue straggler stars (BSS) have been discovered in several other globulars, as well as in several open clusters and even some dwarf galaxies (e.g., Abt 1985; Nemec and Cohen 1989; Stetson et al. 1985; Carney and Seitzer 1986). Since globular clusters, in particular, are usually old stellar
environments, the existence of blue and presumably young stars presents a serious challenge to the standard stellar evolution picture. Several theories have been advanced to explain the presence of the BSS, whose (apparent) masses sometimes exceed twice the turn-off mass (see Leonard 1989, for a recent summary); these include mixing, binary, and merger scenarios.

Among all the scenarios, binary mass transfer perhaps has been the most favored recently. Several authors (Hoyle 1964; McCrea 1964; Collier and Jenkins 1984) suggested that blue stragglers might be close binary systems in which mass transfer has occurred. The original secondary star, according to the theory, has received mass from the original primary star that now is in an advanced evolutionary phase, and therefore re-appears on the main sequence (Pritchett and Glaspey 1991; King 1991). Model calculations predict that blue stragglers ought to be found only up to 2.5 magnitude above the main-sequence turn-off point in a cluster's CM diagram, and have total mass no larger than twice the mass of the turn-off stars of the same age. Observations of several globular BSS's well agree with these predictions. In particular, findings of eclipsing binary stars in globular clusters seem to indirectly support this view. However, there are serious challenges to this hypothesis, too. For instance, blue stragglers are found to have a steep mass function (Wheeler, 1979), which is difficult to produce in binary systems. Moreover, the fact that blue stragglers also are found in high density environments (cf. Paresce et al., 1991), where binary systems are prone to disruption, poses further difficulties for this hypothesis.

Another mechanism involving binary stars to produce blue stragglers is stellar coalescence (Zinn and Searle, 1976). This concept has been attractive for years, even though one cannot point to a single proved coalesced binary system. Like the mass transfer hypothesis, this idea does explain the mass and the brightness of blue stragglers and is consistent with the fact that blue stragglers do not show radial velocity variations.
However, because of angular momentum conservation, blue stragglers born this way must be rapid rotators, whereas majority of observed blue stragglers are found not to be so. A few BSS even demonstrate masses more than twice the turn-off mass, which threatens both binary-related scenarios.

Yet another hypothesis that involves multiple stars (not necessarily binaries) is the so-called stellar merger theory, which proposes that BBS are results of stars that have merged into each other due to physical stellar collisions in star clusters (Hills and Day 1976). This can happen in high density environments (Benz and Hills 1987; Leonard 1989) especially in the core of a globular cluster, for example. However, it apparently fails to explain the large number of BSS observed in low central density clusters like NGC 5053 (Nemec and Cohen, 1989) and NGC 5466 (Nemec and Harris, 1987).

Contrary to suggestions that BSS are binary stars, single star scenarios also are possible answers to the formation of globular cluster blue stragglers. Wheeler (1979) proposed that BSS are isolated stars with extended lifetimes. Due to internal mixing, these stars that should have evolved off the main sequence get refueled by mixing hydrogen into their burning cores and thus look young. The mixing might be the result of a rapid rotation or of a strong magnetic field (Finzi and Wolf, 1968; Schild and Berthet 1986; Mader 1987; Deliyannis et al. 1989). Bizarre as they seem, these ways to produce blue stragglers are actually consistent with many observations. That BSS have low surface lithium abundance, for example, may be merely the consequence of mixing. Also, they do not rule out the possibility that BBS can have mass more than twice the turn-off mass, nor do they reject existence of yellow stragglers (Hesser et al., 1984).

There are other theories, too. Roberts (1960) suggested that BBS are simply young stars, born from the intracluster gas or interstellar medium. Since globular clusters do not show much gas content, it would be unlikely that BBS observed in globulars are actually born this way. It also has even been suggested that early-type field stars directly
captured (via exchange) into binary systems in a cluster might be the stragglers although the probability of such captures is much too small (Leonard, 1985).

To summarize, Figure 4-7 lists some of the important facts about blue stragglers, and compares them against what would be predicted from the theories of binary mass transfer, stellar coalescence, mixing, and our "immigrant" scenario. It seems to us that more than one mechanism probably will be required to explain all the stars that have been put into the blue straggler category. In particular, captures of field stars into globular clusters by dynamical friction may explain some blue stragglers in high density environments. We believe that systematic observations of globulars with a range of core densities can help establish the relative importance of the capture process proposed here. In this regard, we emphasize that the information gathered to date may be misleading, because searches have focused on less dense clusters (or the outer portions of dense ones). For example, a thorough, ground-based CCD study of 47 Tuc (NGC 104) by Hesser et al. (1987) found no BSS in fields of view that avoided the cluster core, whereas new observations of the core, obtained with the Hubble Space Telescope (Paresce et al. 1991), revealed more than 20 such objects.

**Short-period Pulsars:** Conventional pulsar theory asserts that a short-period pulsar (SPP) with a large magnetic field is an early evolutionary stage of the neutron star, and that ordinary pulsars fade from view on timescales \( \sim 10^8 \) years after the supernova event, as their rotation rates and magnetic fields both decrease with time. Accordingly, when Lyne et al. (1987) first reported a rapidly rotating pulsar (PSR 1821-24) in the globular cluster NGC 6626 (M28), the theory was confronted with a serious challenge: how to explain a *young* pulsar in an *old* stellar environment. Moreover, this pulsar's position on the period derivative -- period diagram did not fall in the expected area, a fact that has been interpreted as being due to the neutron star having a fairly weak \(< 10^9 \) G)
<table>
<thead>
<tr>
<th>Observations / Facts</th>
<th>Binary</th>
<th>Coalesce</th>
<th>Mixing</th>
<th>Capt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Most BS have M/Mt ≤ 2 and are -2.5 brighter than turn-over point.</td>
<td></td>
<td></td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>Yellow stragglers found</td>
<td>●</td>
<td>●</td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>Some have M/Mt &gt; 2</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>BS have steep MF</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>BS have low surface Li</td>
<td></td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Most have no radial velocity variations</td>
<td>●</td>
<td></td>
<td></td>
<td>●</td>
</tr>
<tr>
<td>BS are not rapid rotators</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>BS found in dwarfs</td>
<td></td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Many BS found in low C, low density environments</td>
<td></td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
<tr>
<td>Some found in high density environments</td>
<td>●</td>
<td>●</td>
<td>●</td>
<td>●</td>
</tr>
</tbody>
</table>

Figure 4-7. Major scenarios for blue stragglers compared with observational facts. In this "Consumer's Guide," filled circles mean "inconsistent", shaded circles mean "inconclusive", and open circles mean "consistent."
magnetic field. Since the original discovery, pulsars have been found in several other globular clusters, and the majority of them have periods less than 50 msec (Manchester et al. 1991, and references therein). To explain these new facts, two quite different scenarios have been proposed. In the so-called accretion-induced collapse model, an old white dwarf in a globular cluster collapses to a neutron star after accreting mass from a binary companion (Michel, 1987; Bailyn and Grindlay 1990; Ruderman 1991). In the spin-up model, an old neutron star captures a white dwarf to form a binary, and is eventually rejuvenated as a pulsar by mass accretion from the new companion (Kulkarni et al. 1990). These latter authors argued that we are seeing only "the tip of the iceberg," and that individual clusters could contain literally thousands of neutron stars. Michel's (1992) model calculations for magnetic field decay of globular cluster pulsars also support the contention that there are many unseen neutron stars in globular clusters. Principal concerns with the spin-up scenario are: (1) whether a globular could have so many neutron stars (Chernoff and Weinberg, 1990), or could retain them, if (as in the disk) they are born with high spatial velocities; indeed (2) whether a young cluster could have survived so many disruptive supernova events; and (3) how to avoid large-scale chemical enrichment in globular clusters (Smith and McClure 1987; Peng and Weisheit 1991). To explain the fact that several globular cluster pulsars have no companions, both scenarios require strong pulsar winds to completely evaporate the low-mass object.

Here, again, we believe immigrants may play a role. Old neutron stars (with relatively weak magnetic fields) could be captured from the disk or halo, then become part of a binary system -- via stellar exchange with an existing binary (Leonard 1985) or a three-body reaction -- and thereafter be spun up through accretion. This sequence of events mitigates problems associated with a cluster experiencing too many supernova events. In addition, the single pulsars with periods $\sim$0.1 sec that have been found in globulars (e.g., in NGC 7078; Anderson et al. 1990) could simply be (younger) immigrant
pulsars that have not become part of a binary system. Unfortunately, we expect that the importance of these various possibilities will be difficult to sort out, because our capture process and both the accretion-induced-collapse and the spin-up schemes all strongly favor clusters with high central densities. Perhaps the radial distribution of cluster pulsars will help clarify their origins: on the basis of mass segregation timescales (e.g., Chernoff and Weinberg 1990), neutron stars formed from old binary systems should be more centrally condensed than those formed via schemes involving immigrants.
Appendix A  About the Codes

The computer codes for diffusive heating by field stars are written in Fortran. Shown here is the flowchart of the codes, followed by a listing of all the routines including an input file and a corresponding output file. The results of other output channels opened at the beginning of the main program are not shown, since they are mostly for the purpose of data transfer and/or monitoring.
PROGRAM GLOBULAR

C

C

C

DIMENSION EE(299), R(100), RR(299), VV(299), SS(299), ALPHA(10),
1 M(10), M2(10), TT(299), VR(100), VT1(100), VT2(100), F1(100),
2 DENS(100), TH(100), D(83), E(83), BE(83), VR0(83), MV(83), C(83),
3 RO(83), R0(83), RH0(83), RTH(83), MC(83), ROC(83), NUB(83,2),
4 NUD(83,2), NUDF(83,2), NUE(83,2), ID(83), R(140), EM(140),
5 TOB(100), RMAS(41), DFLCT(101,21)

REAL J, INT, M, MT, N, M1, M2, MF, MC, BC, MV, NUB, NUD, NUDF, NUE, LOM
CHARACTER*7 CHAR

COMMON EPS, INUM
COMMON /AMASS/ALPHA, M1, M2, NIMF
COMMON /CONST/PI, G
COMMON /CONTRO/DS, BC, BCRIT, ID2D
COMMON /DISTR/RSIGMR, SIGMR, SIGMT1, SIGMT2, RROT
COMMON /GALX/RSM, VM, N, RT, RC, RH, MT, ROC, Eb, COR, RROT
COMMON /GRID/R1, L, EM
COMMON /MESCH/V(101), BGRID(21), E(101, 21), VCRIT(21)
COMMON /ORBIT/IOBT, DENS, VR, VT1, VT2, R, TH, FL, EL, DT, K, DVDT, TOBT
COMMON /OUTPUT/EE, VV, RR, SS, TT, KT, ICAP
COMMON /PARTK/V0, B, M
COMMON /REL/V, RVR, RV, VT1, VT2
COMMON /VBASE/VB, VT1B, VT2B

EXTERNAL GEE, GLOBRO, GLOBMAS, ROE, AMASS, EB, EV, F1, F2

OPEN (UNIT=55, FILE='TRANSFILE', STATUS='NEW')
OPEN (UNIT=66, FILE='DFLCT-TABLE', STATUS='NEW')
OPEN (UNIT=77, FILE='GLOBPUT', STATUS='OLD')
OPEN (UNIT=88, FILE='OUTPUT', STATUS='NEW')
OPEN (UNIT=99, FILE='PARAOUT', STATUS='NEW')

PIE=3.141592635
G=6.67E-8

READ(77,5) M, IMF, ALFA, HIM, LOM, IOBT, INUM, IREAD, IMESH, KAPT
WRITE(88,6) M, IMF, ALFA, HIM, LOM, IOBT, INUM, IREAD, IMESH, KAPT
1, 6X, 7X, 17, 5X, 18
6 FORMAT(1X, 'M=', F11.2, ', IMF=', I7, ', ALFA=', F7.2, ', HIM=', F8.2,
1 'LOM=', F8.2, ', IOBT=', I8, ', INUM=', I8, ', IREAD=', I7, ', IMESH=', I7,
2 'KAPT=', I8)
READ(77,10) EPS, AMV, NTO, NIMF, DS, BCR, ID2D, DT, SIGMR,
1 SIGMT1, SIGMT2, RROT
WRITE(88,20) EPS, AMV, NTO, NIMF, DS, BCR, ID2D, DT, SIGMR,
1 SIGMT1, SIGMT2, RROT
IF(NIMF.GT.10.0, NIMF.LE.0) STOP
READ(77,30) (ALPHA(I), M1(I), M2(I), I=1, NIMF)
WRITE(88,40) (ALPHA(I), M1(I), M2(I), I=1, NIMF)
30 FORMAT(1X,6X,F7.2,3X,E10.2,3X,E10.2)
40 FORMAT(1X,'ALPHA=',E10.2,'M1=',E10.2,'M2=',E10.2)
READ(77,50) RSUN,ROD,RD,RO,RS,RX,ROCEE,RCOR,RTOT
WRITE(88,60) RSUN,ROD,RO,RS,RX,ROCEE,RCOR,RTOT
1 3X,F10.3,6X,F7.3,5X,F8.3,5X,F8.3)
60 FORMAT(1X,'RSUN=',F8.2,'ROD=',F8.2,'RO=',F8.2,'RG=',F9.3,1
1 'ROSE=',F8.3,'RX=',F10.3,'RC=',F8.3,'R=',F8.3)
65 READ(77,70) IOWN,IK,CHAR
WRITE(88,80) IOWN,IK,CHAR
70 FORMAT(1X,5X,I8,3X,I8,2X,5X,7,1X)
80 FORMAT(1X,'YOUR OWN CHOICE?',I8,'IK=',I3,'NAME=',I7A)
IF(IOWN.LE.0) STOP
READ(77,90) VM,RT,RC,RH,MT,ROC
CALL GCMESS
TOTMAS=RMAS(41)
IF(ABS(TOTMAS-MT)/MT.GT.0.05) MT=TOTMAS
WRITE(88,100) VM,RT,RC,RH,MT,ROC
90 FORMAT(1X,3X,E10.2,3X,E10.2,3X,E10.2,3X,E10.2,4X,E9.2)
100 FORMAT(1X,'VM=',E10.2,'RT=',E10.2,'RC=',E10.2,1
1 'RH=',E10.2,'MT=',E10.2,'ROC=',E10.2)
READ(77,105) VVR,VV1,VV2,RR0,TH0,F10
WRITE(88,106) VVR,VV1,VV2,RR0,TH0,F10
106 FORMAT(1X,'TO START WITH THE CLUSTER OF/')
1 IX,'VR=',F10.2,'VP=',F10.2,3X,F10.2,'RG=',F10.2,'B=',F10.2,2
2 'EL=',F10.2)
110 CALL MASG
CALL MESH(IOWN,IREAD,DFLCT)
IF(12D,EQ.1) THEN
WRITE(66,1101) ((DFLCT(IJ),J=1,10),I=1,101)
WRITE(66,1101) ((DFLCT(IJ),J=11,21),I=1,101)
1101 FORMAT(1X,10(E8.2))
ELSE
ENDIF
IF(IMESH.EQ.1) STOP
CALL TRANSF(VVR,VV1,VV2,RR0,TH0,F10)
WRITE(88,111) RR0
111 FORMAT(1X,'THE GALACTOCENTRIC DISTANCE IS',1PE12.4)
CALL ORBIT(VVR,VV1,VV2,RR0,TH0,DT)
IC=1
112 UM=0.
JKM=K
A=AMV
IF(IMF.NE.0) THEN
PROC=(HIM**((-ALFA+2)-LOM**(-ALFA+2)))/(-ALFA+2.)
ELSE
PROC=1.
ALFA=1.
ENDIF
S=0.
DO 500 JK=1,JKM
CALL VINE(PR(JK),VT1(JK),VT2(JK),A,S)
C CONVERT DENSITY (MSUN/PC**3) TO ONE IN UNITS OF MSUN/CM**3. S IS IN CGS UNITS.
C NOTICE AMP IS IN MSUN.
CONST=DENS(IK)/PROC
FORM=S-3.*ALOG10(3.08E18)
DIF=1.
IF(JK.GT.1) DIF=TOBT(JK)-TOBT(JK-1)
SUM=SUM+DIF*CONST*M**(-ALFA)*10.**((FORM)
IF(JK.GT.1) SUM=SUM/TOBT(JK)
RATE=SUM*3.15*17
IF(KAPT.EQ.0) WRITE(9,600) JK, RATE,DENS(JK)
IF(KAPT.EQ.0) WRITE(88,600) JK, RATE,DENS(JK)
IF(KAPT.EQ.1) WRITE(9,601) JK, RATE,DENS(JK)
IF(KAPT.EQ.1) WRITE(88,601) JK, RATE,DENS(JK)
600 FORMAT(IX,' CONGRATULATION ! THE FINAL DESTRUCTION RATE
1 IN HUBBLE UNITS ON ,13,' TH STEP IS', E12.4//
2 1X,' BY THE WAY, THE DENSITY AT THIS POSTION IS',E12.4/)
601 FORMAT(IX,' CONGRATULATION ! THE FINAL CAPTURE RATE
1 IN HUBBLE UNITS ON ,13,' TH STEP IS', E12.4//
2 1X,' BY THE WAY, THE DENSITY AT THIS POSTION IS',E12.4/)
500 CONTINUE
IC=IC+1
IF (OWN.GE.NTOT) STOP
GO TO 65
END

SUBROUTINE INTE(X,Y,Z,A,RES)
DIMENSION ALPHA(10),M1(10),M2(10),RMASS(41)
REAL M,N,MT,M1,M2

COMMON EPS, INUM
COMMON /CONST/Pi,E,G
COMMON /CONTRO/DS,BCRT,12D
COMMON /AMASS/ALPHA,M1,M2,NIMF
COMMON /COMMAND/KAPT
COMMON /PARTK/V0,B,M
COMMON /GLOB/RMASS,VM,N,RT,RC,RH,MT,ROC,EBIND
COMMON /VBASE/VRB,VT1B,VT2B
COMMON /DISTR/RSIGMA,RSIGM,A,SIGMT1,SIGMT2,VROT
COMMON /INTLIM/AA,BB,CC

EXTERNAL FUNC,F1,F2,F3,F4,RATE
RSIGMA=R SIGMA
AA=A/SIGMA
SIGMT1=SIGMT1/SIGMA
SIGMT2=SIGMT2/SIGMA
VROT=VROT/SIGMA
VRB=X/SIGMA
VT1B=Y/SIGMA
VT2B=Z/SIGMA
SIGMA=1.
IF(KAPT.NE.0) THEN
CALL QTREP(RATE,0.01,0.99,S)
ELSE
S=F4(AA)
ENDIF

RES=(S*1.E42*SIGMA*1.E5)*RT**2*(3.08)**2 TIMES DENSITY WILL BE
C DESTRUCTION RATE.

IF(S.NE.0.) THEN
RES=ALOG10((S*RSIGMA)+41.42*ALOG10(RT*3.08)
ELSE
RES=0.
ENDIF
WRITE(9,10) RES
FORMAT(1X,'CONGRATULATION! LOG RES=',E12.4)
SIGMT1=SIGMT1*RSIGMR
SIGMT2=SIGMT2*RSIGMR
VROT =VROT *RSIGMR
VRB = X
VT1B = Y
VT2B = Z
SIGMR=RSIGMR
RETURN
END

SUBROUTINE CONVERT()

C THIS PART IS FOR CONVERSION OF ALL THE CONVENTIONAL UNITS FROM/TO
C THEIR RESPECTIVE CALCULATIONAL UNITS. ONLY B-PARAMETER IS ENTERED
C AS RELATIVE TO RT, FOR CONVENIENCE. IN ORDER TO AVOID NUMERIC
C OVERFLOW, MASSES ARE IN SOLAR UNITS.

DIMENSION EE(299),VV(299),RR(299),SS(299),TT(299),RMAS(41)
REAL M, N, MT

COMMON /OUTPUT/EE,VV,RR,SS,TT,KT,ICAP
COMMON /CONTROL/DS,BCRT,DT
COMMON /HALX/RSUN,ROD,RD,RG,ROS,RS,RX,ROCEE,RCOR,RTOT
COMMON /GLOB/RMAS,VM,N,RT,RC,RH,MT,ROC,EBIND
COMMON /PARTK/V0,B,M

IF(I.GE.1) GO TO 10
XV0=V0*1.1E5
M=M
MT=MT
XVM=VM*1.1E5
XRC=RC/RT
XRH=RH/RT
XRT=RT*3.08E18
C
XR0C=ROC*2.153/(3.08E18)**3
XR0C=ROC*6.84E-23
VM=XVM
RC=XRC
RH=XRH
RT=XRT
ROC=XR0C
V0=XV0

C DR IS ALREADY IN UNITS OF RT

RETURN

10 XV0=V0*1.1E-5
M=M
XVM=VM*1.1E-5
XRT=RT/3.08E18
XRC=RC*XRT
XRH=RH*XRT
MT=MT
XROC=ROC/6.84E-23
VM=XM
RC=XRC
RH=XRH
RT=XRT
ROC=XROC
V0=VX0
RETURN
END

FUNCTION RATE(B)
COMMON /CONST/PIE,G,
COMMON /DISTR/RSIGMR,SIGMR,SIGMT1,SIGMT2,VROT
EXTERNAL F4,VCAP

A=VCAP(B)/RSIGMR
FORM=F4(A)
RATE=FORM*2.*PIE*B
RETURN
END

FUNCTION F4(AA)
EXTERNAL F3

CALL QTRAP(F3,-AA,AA,S)
FORM=S
F4=S
RETURN
END

FUNCTION F3(Z)
COMMON /INTLIM/AA,BB,CC
COMMON /RELV/BLOC1,BLOC2,BLOC3
EXTERNAL F2

BB=SQRT(AA**2-Z**2)
BLOC3=Z
IF(BB.LT.1.E-25) THEN
S=0.
ELSE
IF(BB.LE.1.E-5) THEN
S=F2(0.)*2.*BB
ELSE


CALL QTRAP(F2,-BB,BB,S)
ENDIF
ENDIF
F3=S
RETURN
END

FUNCTION F2(Y)
COMMON /INTLIM/AA,BB,CC
COMMON /RELV/BLOC1,BLOC2,RVT2
EXTERNAL F1

BLOC2=Y
CC=SQRT(BB**2-Y**2)
IF(CC.LE.1.E-25) THEN
  S=0.
ELSE
  IF(CC.LE.1.E-5) THEN
    S=F1(0.)*2.*CC
  ELSE
    CALL QTRAP(F1,-CC,CC,S)
  ENDIF
ENDIF
ENDIF
F2=S
RETURN
END

FUNCTION F1(RVR)
COMMON /INTLIM/AA,BB,CC
COMMON /RELV/BLOC1,RVT1,RVT2
COMMON /COMMAND/KAPT
EXTERNAL EB,VISO

BLOC1=RVR
V=SQRT(RVR**2+RVT1**2+RVT2**2)
S=1.
IF(KAPT.LE.0) CALL QTRAP(EB,0.01,0.99,S)
FORM=VISO(BLOC1,RVT1,RVT2)
F1=S*FORM*V
RETURN
END

FUNCTION VISO(X,Y,Z)
COMMON /VBASE/VRB,VT1B,VT2B
COMMON /DISTR/RSIGMR,SIGMR,SIGMT1,SIGMT2,VROT

FORM=-(X+VRB)*SIGMR)**2-((Y+VT1B-VROT)/SIGMT1)**2
((Z+VT2B)/SIGMT2)**2
FORM=EXP(FORM/2.)/SIGMR/SIGMT1/SIGMT2
VISO=FORM/(2.*3.1415926535)**1.5
RETURN
END

FUNCTION VCAP(B)
COMMON /MESCH/V(101),BGRID(21),E(101,21),VCRIT(21)
IF(B.GT.BGRID(21)) STOP
KB=B/0.05+1
DERIV=(VCRIT(KB+1)-VCRIT(KB))/(BGRID(KB+1)-BGRID(KB))
FORM=DERIV*(B-BGRID(KB))+VCRIT(KB)
VCAP=FORM
RETURN
END

FUNCTION EB(BB)
DIMENSION EE(299),VV(299),RR(299),SS(299),TT(299)
REAL M
COMMON /DISTR/RSIGMR,SIGMR,SIGMT1,SIGMT2,VROT
COMMON /PARTK/V0,B,M
COMMON /RELV/RVR,RVT1,RVT2
COMMON /OUTPUT/EE,VV,RR,SS,TT,KT,Icap
EXTERNAL ELOSS
B=BB
V0=Sqrt(RVR**2+RVT1**2+RVT2**2)*RSIGMR
FORM=ELOSS(V0,B,M)
EB=2.*3.1415926535*B*FORM
RETURN
END

SUBROUTINE TRAPZD(FUNC,A,B,S,N,IT)
EXTERNAL FUNC
IF(N.EQ.1) THEN
S=0.5*(B-A)*(FUNC(A)+FUNC(B))
IT=1
ELSE
TNM=IT
DEL=(B-A)/TNM
X=0.5*DEL+A
SUM=0.
DO 11 J=1,IT
SUM=SUM+FUNC(X)
X=X+DEL
11 CONTINUE
S=0.5*(S+(B-A)*SUM/TNM)
IT=2*IT
10 FORMAT(1X,TT=',I8)
   IF(IT.GT.1024) WRITE(9,10) IT
   IF(IT.GT.1024) STOP
   ENDIF
   RETURN
   END

SUBROUTINE QTRAP(FUNC,A,B,S)
COMMON EPS, INUM
EXTERNAL FUNC
JMAX=20
OLDS=.1E30
DO 11 J=1,JMAX
   CALL TRAPZD(FUNC, A,B,S,J,IT)
   IF(ABS(S-OLDS),LE,EPS*ABS(OLDS)) RETURN
   OLDS=S
11 CONTINUE
   PAUSE ' TOO MANY INTEGRATION STEPS!'
   END

SUBROUTINE MESH(IOWN,IREAD,DFLCT)

DIMENSION RMAS(41), DFLCT(101,21)
REAL M1,M1,MT,N
COMMON /CONST/PIE,G
COMMON /COMMAND/KAPT
COMMON /MESCH/V(101),BGRID(21),E(101,21),VCRIT(21)
COMMON /PARTK/V0,B,M
COMMON /GLOB/RMAS,VM,N,RT,RC,RH,MT,ROC,EBIND
COMMON /OUTPUT/EE(299),VV(299),RR(299),SS(299),TT(299),

1 DO 1 I=2,101
   V(I)=FLOAT(I-1)*10.
   IF(I.LE.20) BGRID(I)=FLOAT(I-1)*0.05
1 CONTINUE
   BGRID(1)=0.01
   BGRID(21)=0.99
   V(1)=0.0001
   M1=M
   M=1.
   CALL CONVERT(0)
   VESC=SQRT(2.*G*MT/(RT/2.*E33))
   DO 5 I=1,5
      V0=0.1*I*VESC
   DO 3 J=1,21
      B=BGRID(J)
      CALL GLOB(ANGLE)
      E(I,J)=EE(KT)
      KSIGN=-1
      IF(ICAP.EQ.0) KSIGN=1
      DFLCT(I,J)=KSIGN*ANGLE
3 CONTINUE
   WRITE(9,119) V(I),I
119 FORMAT(1X,'ANOTHER 20 GLOBS PASSED';,V=,'E12.4,I6')
5 CONTINUE
130 IF(1.GT.-5) STOP
10 CALL CONVERT(1)
M=M1
130 VESC=SQR(1.2G*MT/(RT*3.0E18/2.E33))/1.E5
DO 300 I=1,12
10 KOUNT=0
10 B=BGRID(I)
10 V0=2.*VESC
10 VOL=0.
10 V0H=0.
10 EPS=1.E30
10 CALL CONVERT(0)
10 KOUNT=KOUNT+1
10 CALL GLOB(ANGLE)
11 WRITE(9,11) ICAP, V0, TT(KT)
11 FORMAT(1X,'ICAP V0 TT(KT),18.2X,1PE12.4,2X,1PE12.4)
11 IF(V0H.NE.0.) EPS=ABS(1.-VOL/V0H)
11 IF(ABS(VV(KT)-VESC)/VESC).LE.15.0E-2.AND.ICAP.NE.1) GO TO 200
11 IF(EPS.LE.0.02) GO TO 200
11 IF(ICAP.EQ.1) THEN
11 V0L=V0
11 IF(V0H.EQ.0.) V0=M0*2.
11 IF(V0H.NE.0.) V0=(V0+V0H)*0.5
11 ELSE
11 V0H=V0
11 V0=(V0+V0L)*0.5
11 ENDIF
11 IF(KOUNT.LE.100) GO TO 10
11 WRITE(9,150)
150 FORMAT(1X,'TOO MANY ITERATIONS FOR A SINGLE CRITIC V!')
STOP
200 VCRIT(I)=V0/1.E5
200 WRITE(9,249) I
249 ORMAT(1X,'ANOTHER CRITICAL VELOCITY FOUND I='I4)
249 CALL CONVERT(1)
300 CONTINUE
END

FUNCTION ELOSS(VX,BX,MX)

DIMENSION RMAS(41)
REAL M,MT,N,MX
DIMENSION A(10), BA(10), BBA(10)
COMMON EPS, INUM
COMMON /CONST/PI,G
COMMON /MESCH/(V(101),BGRID(21),E(101,21),VCRIT(21)
COMMON /GLOB/RMAS,VM,N,RT,RC,RH,MT,ROC,EBIND
COMMON /PARTK/V0,B,M
COMMON /OUTPUT/EE(299), VV(299), RR(299), SS(299), TT(299)
1 KT,ICAP

VESC=SQR(2.G*MT/(RT*3.0E18/2.E33))
IF(VX.GT.V(101).OR.BX.GT.BGRID(21)) STOP
X=SQR(1.5)*(VX+VESC)/VM
IF(X.GT.3.) GO TO 100
V0=VX
B=BX
M=MX
CALL CONVERT(0)
CALL GLOB(ANGLE)
CALL CONVERT(1)
ELOSS=EE(KT)
RETURN

100 KV=VX/10.+1
    KB=BX/0.05+1
    IF(INUM.LE.1.OR.INUM.GT.10) INUM=2
    J1=KV
    J2=J1+INUM-1
    DO 120 I=1,110
       A(I)=0.
       BA(I)=0.
       BBA(I)=0.
    120 CONTINUE
    DO 150 I=J1,J2
       BA(I+1-J1)=E(I,KB)
       BBA(I+1-J1)=E(I,KB+1)
       A(I+1-J1)=V(I)
    150 CONTINUE
    KKI=0
    DO 200 I=1,1101
       IF(ABS(VX-V(I)),NE.0.) GO TO 200
       KKI=I
       GO TO 210
    200 CONTINUE
    210 DIF=BX-BGRID(KB)
       IF(KKI.EQ.0) THEN
          CALL POLINT(A,BA,INUM,VX,Y,DY)
          EB1=Y
          CALL POLINT(A,BBA,INUM,VX,Y,DY)
          EB2=Y
          ELSE
          EB1=E(KKI,KB)
          EB2=E(KKI,KB+1)
          ENDIF
       FB=(EB2-EB1)/(BGRID(KB+1)-BGRID(KB))*DIF+EB1
    1000 ELOSS=FB*MX**2
    RETURN
END

SUBROUTINE GCMESH

C THIS SUBROUTINE IS TO COMPUTE THE MASS WITHIN CERTAIN RADIUS IN A
C GLOBULAR CLUSTER

DIMENSION XR(41), RMAS(41)

COMMON /GLOB/RMAS, VM,N,RT,RC,RH,MT,ROC,EBIND
COMMON /PARTK/V0,B,M
EXTERNAL GLOBRO,GLOBMAS

V0=0.
XR(1)=0.01
DO 1 I=2,40
   XR(I)=(I-1)*0.025
1 CONTINUE
XR(41)=0.999
CALL CONVERT(0)
DO 2 I=1,41
IF(I.LT.1) THEN
CALL QTREP(GLOBMAS,0.001,XR(I),S)
ELSE
CALL QTREP(GLOBMAS,XR(I-1),XR(I),S)
ENDIF
RMAS(I)=S
IF(I.GE.2) RMAS(I)=RMAS(I-1)+RMAS(I)
2 CONTINUE
CALL CONVERT(1)
DO 3 I=1,41
FORM=ALOG10(RMAS(I))+3.*ALOG10(RT*3.08E18)-33.301
RMAS(I)=10.**(FORM)
3 CONTINUE
RETURN
END

SUBROUTINE IF(DVDT,EDDT,ESIGN,V,VM,ROR,MT,MF)

REAL  J, M, MT, MF
COMMON /CONST/PIE,G
COMMON /PARTK/V0,B,M
EXTERNAL GEE

C THE OUTPUT VALUE OF EDDT IS IN LOGARITHM UNLESS IT IS ZERO.
C THE SIGN OF IT IS ESIGN.

SUBS=0.
J=SQR(T(1.5)/VM
FACT=ALOG10(12.56)+2.0*ALOG10(G)
GAMMA=FACT+2.0*ALOG10(MF)+66.602+ALOG10(ALOG0.4*MT/MF)
X=J*V
RAT=M/MF
IF(ROR.GT.0.0) SUBS=GAMMA+ALOG10(ROR)-ALOG10(MF)-33.301
DVDT=-2*(1.0+RAT)*GEE(X,0.,0.)*J**2*10.**SUBS
IF(SUBS.EQ.0.) DVDT=0
DEDDT=J*(-RAT*GEE(0.,X,0.)+(1.+RAT)*X*GEE(0.,0.,X))
IF(ABS(DEDDT).GT.1.E-35) THEN
ESIGN=DEDDT/ABS(DEDDT)
ELSE
ESIGN=0.
DEDDT=0.
RETURN
ENDIF
IF(ROR.GT.0.0) SUBS=GAMMA+ALOG10(ROR)+ALOG10(RAT/X)

C NOTICE HERE SUBS WOULD HAVE BEEN SUBS=GAMMA+ALOG10(ROR/MF/2.E33).
C HOWEVER, SINCE THE ENERGY EDDT WE NEED IS ONE PER STAR (NOT AS DEFINED
C PER UNIT MASS) MULTIPLYING M MAKES SUBS=GAMMA+ALOG10(ROR)+ALOG(RAT).

DEDDT=ALOG10(ABS(DEDDT))+SUBS
IF(SUBS.EQ.0.) DEDDT=0.
RETURN
END
SUBROUTINE GLOB(ANGLE)

C THIS ROUTINE CALCULATES THE PHYSICAL QUANTITIES OF THE INPUT STAR ALONG
C ITS TRAJECTORY. THE PARALLEL AND VERTICAL DISPLACEMENT IS DS, AND DP.

DIMENSION EE(299), RR(299), VV(299), SS(299),
ALPHA(10), M1(10), M2(10), TT(299), RMAS(41)
REAL J, INT, M, MT, N, M1, M2, MF, BCRIT

COMMON /AMASS/ALPHA,M1,M2,NIMF
COMMON /CONST/PI,G
COMMON /CONTRO/DS,BCRIT,12D
COMMON /GLOB/RMAS,VM,N,RT,RC,RH,MT,ROC,EBIND
COMMON /OUTPUT/EE,VV,RR,SS,TT,KT,ICAP
COMMON /PARTK/V0,B,M

EXTERNAL GLOBRO, AMASS, GCMASS

C NOTICE ALL THE PHYSICAL QUANTITIES ARE IN THEIR CONVENTIONAL UNITS.
C DIMENSIONS ARE IN UNITS OF TIDAL RADIUS. VESC IS THE ESCAPE VELOCITY.
C NOTICE HERE THE DENSITY IS IN GRAM/CM**3 AND MF IS IN GRAM. EBIND IS
C IN ITS LOG UNITS (BASE 10).

VESCO=SQR(2.*G*MT/(RT**2,E33))
IF(B.LE.BCRIT) B=BCRIT
BIM=B
COSIN0=SQR(1-B**2)
COSIN=COSIN0
EE(1)=0.
RR(1)=1.
VV(1)=V0+VESCO
TT(1)=0.
SS(1)=0.
KT=1
EBIND=ALOG10(0.2*G*MT**2/RH/RT)+66.602
XDS=DS
V=V0+VESCO
VP=0.
MF=AMASS(ALPHA,M1,M2,NIMF)
R=1.
S=0.
P=0.
ISIGN=1
I=0
ICAP=0
E=0.
ESIGNP=0.
ESIGN=0.
DEDTP=0.
DEDT=0.
ANGLE=0.
I=I+1
ROR=GLOBRO(R)
GMASS=GCMASS(R)
DFIDR=0.
IF(GMASS.GT.0.) THEN
DFIDR=10.**2*(ALOG10(G)+ALOG10(GMASS)+33.301-2.*ALOG10(RT))
DFIDR=DFIDR/R**2
ELSE
ENDIF
IF(V*VP.LT.0.) GO TO 3
CALL DF(DVDT,DEDT,ESIGN,V,VM,ROR,MT,MF)
DVD=DVD+DFIDR*COSIN

IF(12D.GT.0) THEN
CALL DF(DVDT,DEDTP,ESIGNP,VP,VM,ROR,MT,MF)
DVDTP=DVDTP+DFIDR*SQR1(-COSN*COSIN)
ELSE
ENDIF

C CHOOSE THE TIME STEP SUCH THAT INCREMENT IN V IS NO LARGER THAN 10% OF V.

IF(DVDT.NE.0.) XDS=0.1*V*V/ABS(DVDT)
IF(DS.LT.XDS) XDS=DS
IF(I.EQ.1.AND.ESIGN.GT.0.) GO TO 3
IF(ESIGN.GT.0.) GO TO 3

15 DT=(XDS*RT)/ABS(V)
V=V+0.25*DVD*DT
VP=0.25*DVDT*DT*12D
IF(DEDT.NE.0.) E=E+ESIGN**.10.*(DEDT+ALOG10(DT)-40.)
IF(DEDTP.NE.0.AND.ESIGN.NE.0.)
E=E+ESIGN**.10.*(DEDTP+ALOG10(DT)-40.)
VESC=SQR1((2.*G*GMMASS/(R*RT/2.E33))
IF(SQR1(V*V+VP*VP).LE.0.99*VESC.AND.R.LE.0.99) THEN
ICAP=1
I=I-1
GO TO 3
ELSE
ENDIF
S=S+0.25*V*DT
P=(P+0.25*VP*DT)*12D

2 ROLD=R
FAC=ACOS(COSIN)
IF(ABS(1.-COSN).LE.1.E-5) FAC=SQR1(2.-2.*COSN)
FACT=FAC-ATAN(VP/V)
DFLCT=SQR1(V*V+VP*VP)*DT*0.25/RT
R=SQR1(R*R-2.*COS(FACT)*DFLCT*R+DFLCT*DFLCT)

C BIM=ROLD*SQR1(1-COS(FACT)**2). HOWEVER, SINCE FACT IS VERY SMALL,
C WE USE THE FOLLOWING EXPANSION.

XBIM=ROLD*ABS(FACT)
IF(ABS(FACT).GT.1.E-2) XBIM=ROLD*SQR1(1-COS(FACT)**2)
IF(FACT.LE.0.) THEN
WRITE(*,25) I

25 FORMAT(1X,'WE ARE PAUSING AT I=',I)
PAUSE
ELSE
ENDIF

BIM=XBIM
IF(ABS(BIM-R).LE.0.05.AND.ISIGN.EQ.1) ISIGN=-1
COSIN=ISIGN*SQR1(R*R-BIM*BIM)/R
IF(COSN.LT.-1.) COSIN=-1.
ANGLE=ANGLE-ATAN(VP/V)
V=SQR1(V*V+VP*VP)*12D
T=T+0.25*DT
EE(I)=ESIGN*E**10.*(-EBIND)
VV(I)=V/I.5
RR(I)=R
SS(I)=S/RT
TT(I)=T/3.15E7
IF(BIM.LE.0.5*BCRIT) ICAP=1
IF(BIM.LE.0.5*BCRIT) GO TO 3
IF(R.GT.0.99.AND.DVDT.LT.0.) GO TO 3
IF(LGE.299) GO TO 3
GO TO 1
3
KT=I
IF(ESIGN.GT.0.OR.V*VP.LE.0.) ICAP=1
IF(ESIGN.GT.0.OR.V*VP.LE.0.) KT=I-I
C VV(KT)=VV(KT)-VEJC/I.5
C IF WE CARE ABOUT THE OUTGOING VELOCITY OF THE STAR, WE HAVE TO INVOKE
C THE LINE.
RETURN
END

FUNCTION GEE(X,Y,Z)
DIMENSION XX(17), YY(17), ZZ(17)
DATA YY(I),I=1,17)/0.183,0.198,0.208,0.213,0.214,0.211,0.205,
0.196,0.186,0.175,0.163,0.152,0.140,0.129,0.119,0.080,0.056/
DATA ZZ(I),I=1,17)/0.421,0.480,0.534,0.584,0.629,0.669,0.706,
0.738,0.766,0.791,0.813,0.832,0.849,0.863,0.876,0.920,0.944/
C TWO OF THE THREE VARIABLES HAVE TO BE ZERO AT A TIME.
X1=X
X=X+Y+Z
DO 1 I=1,15
XX(I)=0.5+FLOT(I)*0.1
1 CONTINUE
XX(16)=2.5
XX(17)=3.0
IF(X-0.6.LE.0.) GO TO 10
IF(X-3.0.LE.0.) GO TO 15
5 G=1/(-_X**X)
PHI=1.
PHIP=0.
GO TO 20
10 G=2.*X*(1.-_X**X/5.)/5.31
PHI=G+.0.*X*(1.-_X**2/5.)/5.31
PHIP=2./5.31*(3.-_X**2)
GO TO 20
15 I=INT(10.*X)-5
IF(X.GT.2.) I=INT(2.*(X-2.))+15
CONST1=(YY(I+1)-YY(I))/(XX(I+1)-XX(I))
CONST2=(ZZ(I+1)-ZZ(I))/(XX(I+1)-XX(I))
G=CONST1*(X-XX(I))+YY(I)
PHI=G+CONST2*(X-XX(I))+ZZ(I)
PHIP=CONST1+CONST2
IF(X.EQ.3.) PHP=-(2.*X**2-G-PHI)/X
20     GEE=G
       IF(X1.EQ.0.0.AND.Z.EQ.0.0.AND.Y.NE.0.0) GEE=PHI
       IF(X1.EQ.0.0.AND.Y.EQ.0.0.AND.Z.NE.0.0) GEE=PHI
       RETURN
       END

FUNCTION GLOBMAS(X)

EXTERNAL GLOBRO
GLOBMAS=4.*3.1415926535*GLOBRO(X)**2
RETURN
END

FUNCTION GCMASS(AR)

DIMENSION RMAS(41)
COMMON /GLOB/RMAS,VM,N,RT,RC,RT,MT,ROC,EBIND

XR=AR
IF(XR.GE.0.01) GO TO 1
IF(XR.LT.0.05) THEN
   GCMASS=0.
   RETURN
ELSE
   XR=0.01
ENDIF
1     IF(XR.GE.0.025) GO TO 2
     FORM=(RMAS(2)-RMAS(1))/1.4625E-5
     GCMASS=FORM+(XR*X*R*X-1.E-6)+RMAS(1)
     RETURN
2     IF(XR.GT.0.999) XR=0.999
     I=INT(XR/0.025)
     J=I+1
     FORM=(RMAS(J)-RMAS(I))/(J*I-1.I)*1.5625E-5
     GCMASS=FORM+(XR*X*R*X-1.I)**1.5625E-5+RMAS(I)
     RETURN
END

FUNCTION GLOBRO(X)

DIMENSION RMAS(41)
REAL K, N, MT
COMMON /GLOB/RMAS,VM,N,RT,RC,RT,MT,ROC,EBIND

C     ROC'S UNITS MUST BE CONSISTENT WITH THOSE OF RT AND RC

C=1./RC
Z1=1./SQRT(1.+C*C)
Z=Z1
FORM=(ACOS(Z)/Z-SQRT(1.-Z**2))*Z
K=ROC**3.14*RC/FORM
Z=SQRT(1.+(X/RC)**2)*Z1
FORM=(ACOS(Z)/Z-SQRT(1-Z*Z))/Z**2
FORM=FORM*Z**3
GLOBRO=K*FORM/3.14/RC
RETURN
END

FUNCTION AMASS(ALPHA,M1,M2,ICON)

REAL M1, M2, K, L
DIMENSION ALPHA(10), M1(10), M2(10), BETA(10), K(10), L(10),
1       WEIT(10), RATIO(10)
C GLOBRO HERE IS THE MASS DENSITY THE IMF'S HAVE TO BE NORMALIZED TO

DO 10 I=1,ICON
  BETA(I)=ALPHA(I)-2.
  K(I)=M1(I)**(-BETA(I))-M2(I)**(-BETA(I))
  IF(BETA(I).EQ.0.) GO TO 5
  K(I)=K(I)/(-BETA(I))
  GO TO 10
5  K(I)=ALOG(M2(I)/M1(I))
10 CONTINUE
DO 15 I=1,ICON
  BETA(I)=ALPHA(I)-1.
  L(I)=M1(I)**(-BETA(I))-M2(I)**(-BETA(I))
  IF(BETA(I).EQ.0.) GO TO 14
  L(I)=L(I)/(-BETA(I))
  GO TO 15
14  L(I)=ALOG(M2(I)/M1(I))
15 CONTINUE
  WEIT(ICON)=1.
  COEF=1.
  DO 20 I=1,ICON-1
    J=ICON-I
    RATIO(I)=M1(J+1)**(-ALPHA(J+1))/M2(J)**(-ALPHA(J))
    COEF=COEF*RATIO(I)
    WEIT(ICON-I)=COEF
20 CONTINUE
  SUMA=0.
  SUMB=0.
  DO 30 I=1,ICON
    SUMA=SUMA+K(I)*WEIT(I)
    SUMB=SUMB+L(I)*WEIT(I)
30 CONTINUE
  AMASS=SUMA/SUMB
RETURN
END

SUBROUTINE QROMB(FUNC,A,B,SS)
C RETURNS AS S THE INTEGRAL OF THE FUNCTION FUNC FROM A TO B. INTEGRATION IS
C PERFORMED BY ROMBERG'S METHOD OF ORDER 2K, WHERE, E.G., K=2 IS SIMPSON'S
C RULE. HERE EPS IS THE FRACTIONAL ACCURACY DESIRED, AS DETERMINED BY THE
C EXTRAPOLATION ERROR ESTIMATED; JMAX LIMITS THE TOTAL NUMBER OF STEPS;
C K IS THE NUMBER OF POINTS USDEJD IN THE EXTRAPOLATION.
PARAMETER (JMAX=20,JMAXP=JMAX+1,K=5,KM=K-1)
DIMENSION S(JMAXP),H(JMAXP)
COMMON EPS, INUM
EXTERNAL FUNC

H(1)=1,
DO 11 J=1,JMAX
CALL TRAPZD(FUNC, A,B,S(J),J,IT)
IF(J.GE.K) THEN
CALL POLINT(H(J-KM),S(J-KM),K,0.,SS,DSS)
IF(ABS(DSS).LE.EPS*ABS(SS)) RETURN
ENDIF
S(J+1)=S(J)
H(J+1)=0.25*H(J)
11 CONTINUE
PAUSE 'TOO MANY STEPS'
END

SUBROUTINE POLINT(XA,YA,N,X,Y,DY)
C GIVEN ARRAYS XA AND YA,EACH OF LENGTH N, AND GIVEN A VALUE X,
C THIS ROUTINE RETURNS A VALUE Y, AND AN ERROR ESTIMATE DY. IF P(X) IS THE
C POLYNOMIAL OF DEGREE N-1 SUCH THAT P(XAI)=YA(I), I=1,...,N, THEN
C THE RETURN VALUE Y=P(X).
PARAMETER (NMAX=10)
DIMENSION XA(10),YA(10),C(10), D(10)

NS=1
DIF=ABS(X-XA(1))
DO 11 I=1,N
DIFT=ABS(X-XA(I))
IF(DIFT.LT.DIF) THEN
NS=1
DIF=DIFT
ENDIF
C(I)=YA(I)
D(I)=YA(I)
11 CONTINUE

Y=YA(NS)
NS=NS-1
DO 13 M=1,N-1
DO 12 I=1,N-M
HO=XA(I)-X
HP=XA(I+M)-X
W=C(I+1)-D(I)
DEN=HO*HP
IF(DEN.EQ.0.) PAUSE
DEN=W/DEN
D(I)=HP*DEN
C(I)=HO*DEN
12 CONTINUE
IF(2*NS.LT.N-M) THEN
DY=C(NS+I)
ELSE
DY=D(NS)
NS=NS-1
ENDIF
Y = Y + DY
CONTINUE
RETURN
END

SUBROUTINE TRANSF(VR, VT1, VT2, R, TH, FI)

C THIS ROUTINE IS DESIGNED TO TRANSFER THE HELIOCENTRIC COORDINATES TO
C THE GALACTOCENTRIC COORDINATES, WHICH WE USE THROUGHOUT THE
C CALCULATIONS. NOTICE IN THE LATTER COORDINATES THE DIFFERENCE BETWEEN
C THETA AND BEE. THE INPUTS ARE RADIAL VELOCITY AND PROPER MOTION
C VELOCITIES, HELIOCENTRIC DISTANCE, LATTITUDE B AND LONGITUDE EL. HOWEVER,
C THE OUTPUTS USING THE SAME CHANNEL AND SAME CHARACTERS REPRESENT
C EVERY RESPECTIVE THING FROM THE GALAXY CENTER. TH COUNTS FROM
C THE Z-AXIS.

COMMON /CONST/PI, G
COMMON /GALX/R5UN, ROD, RD, RG, ROS, RS, RX, ROCEE, RCR, RTOT

RATIO = 180.0 / PI
EL = FI / RATIO
BEE = TH / RATIO
FORM = 1.0 + R5UN * R5UN / (2.0 - 0.5 * R5UN / COS(BEE) * COS(EL)
FORM = SQRT(FORM)
TANBP = TAN(BEE) / FORM
BP = ATAN(TANBP)
FORM = FORM * R * COS(BEE)
R = R * SIN(BEE) / SIN(BP)
SINA = R5UN * SIN(EL) / FORM
AL = ASIN(SINA)
ELP = EL + AL
FORM1 = 0.
FORM2 = 0.
FORM3 = 0.
IF(VR .EQ. 0.) GO TO 2
FORM1 = FORM1 + SIN(BEE) * SIN(BP) + COS(BEE) * COS(BP) * COS(AL) * VR
FORM2 = FORM2 + SIN(BEE-BP) * VR
FORM3 = FORM3 - COS(BEE) * SIN(AL) * VR
GO TO 3
2
IF(VT1 .EQ. 0.) GO TO 3
FORM1 = FORM1 + VT1 * SIN(BP) * COS(BEE) - COS(BEE) * COS(BP) * COS(AL)
FORM2 = FORM2 + VT1 * COS(BEE) * COS(BP) + SIN(BEE) * SIN(BP) * COS(AL)
FORM3 = FORM3 + VT1 * SIN(BEE) * SIN(AL)
GO TO 4
3
IF(VT2 .EQ. 0.) GO TO 4
FORM1 = FORM1 + VT2 * COS(BP) * SIN(AL)
FORM2 = FORM2 + VT2 * SIN(BP) * SIN(AL)
FORM3 = FORM3 + VT2 * COS(AL)
4
VR = FORM1
VT1 = FORM2
VT2 = FORM3
TH = (0.5 * PI - BP) * RATIO
FI = ELP * RATIO
RETURN
END

SUBROUTINE ORBIT(XML, XV1, XV2, XR, XTH, XFI, XDT)
DIMENSION VR(100), VT1(100), VT2(100), FI(100), R(100)
DIMENSION TH(100),TOBT(100),DENS(100)
REAL M, L

COMMON /CONST/PIE,G
COMMON /ORBIT/TOBT,DENS,VR,VT1,VT2,R,TH,FI,EL,DT,K,DVDT,TOBT

EXTERNAL PHI,ROE

CONST=PIE/180.
VR(1)=XVR
VT1(1)=XVI
VT2(1)=XV2
R(1)=XR
TH(1)=XTH
FI(1)=XFI
DT=XDIT
I=1
RDT=DT
FII=FI(1)*CONST
THH=TH(1)*CONST
RR=R(1)
DENS(1)=ROE(RR,THH,FI)
TOBT(1)=0.
IF(IOBT.EQ.0) GO TO 50
COSTH=VR/V
V2=VR(1)**2+VT1(1)**2+VT2(1)**2
V=SQR(V2)
COSTH2=VR(1)**2/V2
SINTH=SQR1(1.-COSTH2)
THT=ASIN(SINHT)
EL=R(1)*V*SINHT
E=0.5*V2*1.E10+PHI(R(1),THH,FI,0)
CALL ZERO(A1,A2,B1,B2,B3,FI,THH,RR)
ELY=A1*VT1(1)+B1*VT2(1)
ELY=A2*VT1(1)+B2*VT2(1)
ELY=0. B3*VT2(1)
THET=THH
FIE=FI
ICON=1
20 CALL AUTOT(RDT,1)
DFI=VT2(2)*DT/R(1)/SIN(TTHH)*3.15E-9/3.08
FIE=FIE+DFI
DTHET=VT1(1)*DT/R(1)*3.15E-9/3.08
C IF(CON.EQ.1.AND.,THH-DTHET,LT.0.) ICON=1
THET=THET+FLOAT(YICON)*DTHET
CALL ADJUST(FII,THH,FIE,THET,ELY,ELY,ELZ,ICON)
RR=RR+VR(1)*DT*3.15E-9/3.08+0.5*DVT*DT*3.15E7)**2
1 /3.08E21
FORM=PHI(RR,THH,FI,0)
VNEW=SQR(E-FORM)*1.414/1.5E
VSN=EL/RR
I=1+1
TOBT(I)=TOBT(I-1)+DT
VR(I)=VR(I-1)+DVDT*DT*3.15E2
CALL ZERO(A1,A2,B1,B2,B3,FI,THH,RR)
IF(ABS(B3),NE.0.0) GO TO 30
D=A1*B2-A2*B1
DVT1=ELY*B2-ELY*B1
DVT2=A1*ELY-A2*ELY
VT1(I)=DVT1/D
VT2(I)=DVT2/D
GO TO 40
30  D=B3*A2
     DVT1=ELY*B3-ELZ*B2
     DVT2=ELZ*A2
     IF(A2,NE.0.0) GO TO 35
     VT2(0)=ELZ/B3
     VT1(0)=(ELX-B1*VT2(0))/A1
     GO TO 40
35  VT2(0)=DVT2/D
     VT1(0)=DVT1/D
40  R(I)=RR
     DENS(I)=ROE(RR,THH,FII)
     TH(0)=THH/CONST
     FI(0)=FII/CONST
     CRIT1=ABS(FI(I)-FI(1))
     CRIT2=ABS(TH(I)-TH(1))
     IF(TH,EQ.100.) GO TO 50
     IF(ABS(CRIT1-360.),LE.5.) GO TO 50
     IF(ABS(FI(I)-100.,AND.,1.,GE.10.,AND.
1  ABS(DTH,EQ.0.0) GO TO 50
     IF(CRIT2.IE.10.0,AND.),GE.10.,AND.
1  ABS(DTH,EQ.0.0) GO TO 50
     GO TO 20
50  K=I
     WRITE(99,60) K,TOBT(K),R(K),TH(K),FI(K)
     WRITE(99,70) VR(K),VT1(K),VT2(K)
60  FORMAT(1X,I=13,'T=',E12.4,' R,TH,THET,PHI= ',E12.6,2X,
1  E12.6,2X,E12.6)
70  FORMAT(5X,' VR, VTHETA, VPHI= ',2X,E12.6,2X,E12.6,2X,E12.6)
RETURN
END

SUBROUTINE ADJUST(FII,THH,FIE,THET,ELX,ELY,ELZ,ICON)

C  THIS SUBROUTINE IS DESIGNED TO ADJUST ANGLES FOR SOME VERY SPECIAL ORBITS,
C  E.G. THE ORBITS WITH ELX=ELZ=0, TO THE EXTENT THAT 0 < THETA < + PIE, AND
C  0 < FII < 2.*PIE.

THH=THET
     FII=FIE
     IF(THET.LE.0.0) THH=-THET
     IF(THH,GT.3.1415926535) ICON=-1
     IF(ICON,EQ.-1) THH=2*3.1415926535-THH
     IF(ELZ,EQ.0.0,AND.,ICON,EQ.-1) FII=FIE+3.1415926535
     IF(ABS(FII),GT.2.*3.1415926535
1  FII=FII-2.*3.1415926535*FII/ABS(FII)
RETURN
END

SUBROUTINE AUTOT(RDT,I)

C  THIS SUBROUTINE IS DESIGNED TO AUTOMATICALLY CHOOSE TIME-STEP.
DIMENSION VR(100), VT1(100), VT2(100), FI(100), R(100)
DIMENSION TH(100), DENS(100), TOB(100)

COMMON /ORBIT/IOBT, DENS, VR, VT1, VT2, R, TH, FI, EL, DT, K, DVDT, TOBT

EXTERNAL PHI

RR=R(I)
X=TH(I)
Y=FI(I)
DVDT=-PHI(RR, X, Y, 1)+(EL*1.E5/RR)**2/(RR*3.08E21)
IF(ABS(DVDT), LT, 1.E-25, OR, VR(I), EQ, 0.0) GO TO 10
DT=ABS(VR(I))/ABS(DVDT)/3.15E7
DT=DT/100.
IF(DT, LT, 1.E2, OR, DT, GT, 1.E10) DT=RD
GO TO 50
10 DT=RT*ABS(SIN(TH(I)))*2./180.*3.14/VT1(I)*3.08E16/3.15E7
DT=ABS(DT)
IF(DT, LT, 1.E2, OR, DT, GT, 1.E10) DT=RD
GO TO 50
20 IF(ABS(VT1(I)), LT, 1.E-25) GO TO 30
DT=RT*2./180.*3.14/VT1(I)*3.08E16/3.15E7
DT=ABS(DT)
IF(DT, LT, 1.E2, OR, DT, GT, 1.E10) DT=RD
GO TO 50
30 DT=RD
GO TO 50
RETURN
END

SUBROUTINE ZERO(A1, A2, B1, B2, B3, FI, TH, RR)

C THIS PROGRAM IS DESIGNED TO ADJUST THE ZERO POINTS IN SINE-COSINE
C FUNCTIONS. IT MAKES A BIG DIFFERENCE IF SIN(PIE)=COS(0.5*PIE)≠0.0

DATA PIE/3.1415926535/

SINF=ABS(FI)
COSF=ABS(TH)
SINT=ABS(FI/TH)
COST=ABS(TH/FI)
CRITF=ABS(FI/PIE)
CRITT=ABS(TH/PIE)
INTF=INT(CRITF)
INTT=INT(CRITT)
CRITF=CRTF-INTF
CRITT=CRITT-INTT
IF(ABS(CRITF), GT, 1.E-5) GO TO 1
SINF=0.
COSF=(-1)**INTF
1 IF(ABS(CRITF-0.5), GT, 1.E-5) GO TO 2
SINF=(-1)**INTF
COSF=0.
2 IF(ABS(CRITT), GT, 1.E-5) GO TO 3
SINT=0.
COST=(-1)**INTT
3 IF(ABS(CRITT-0.5), GT, 1.E-5) GO TO 4
SINT=(-1)**INTT
COST=0.
4 A1=SINF*RR
A2= COSF*RR
B1= COST*COSF*RR
B2= COST*SINF*RR
B3= SINT*RR
RETURN
END

FUNCTION PHI(RR,TH,EL,K)

COMMON /CONST/PIE,G
COMMON /GALX/R/SUN,ROD,RD,RG,ROS,RS,RX,ROCEE,RCOR,RTOT

EXTERNAL DENS,R2
EXTERNAL FORCE,BS10,BS11,BSK0,BSK1,SPHMAS

C RATIO=2.E33/3.08E21=0.67E12
C RATIO=0.67E12
Z=ABS(RR*COS(TH))
ZR=ABS(RR*SIN(TH))
XD=ZR/RD
XG=ZR/RG
IF(K,N.E.0) GO TO 10
FORM=4.*PIE*G*ROSS*1.9*RS**3*15.*PIE/8.
IF(RR.LT.RX) THEN
CALL QTRAP(FORCE,0.01*RS,RX,S)
CORE=S
PHISPH=CORE+G*SPHMAS(RX)/RX+FORM*(RTOT-RX)/RTOT
ELSE
PHISPH=G*SPHMAS(RR)/RR +FORM*(((ALOG(RR/RS)+1)/RR)-1./RTOT)
ENDIF

1 PHID=G*PIE*ROD*1.65*ZR*(BS10(XD)*BSK1(XD)-
  BS11(XD)*BSK1(XD)-BS10(XG)*BSK1(XG)+BS11(XG)*BSK0(XG))
C THIS IS TO THICKEN THE DISK, ORIGINALLY DESIGNED TO BE 2-D MODEL.
FORM=4.*PIE*G*ROD*1.65*0.2*1.-(5.*Z+1)*EXP(-5.*Z)*
1 (EXP(-XD)-EXP(-XG))
PHID=PHID-FORM
FORM=SQRT(1.+(RTOT/RCOR)**2)/SQRT(1.+(RR/RCOR)**2)
PHICOR=4.*PIE*G*ROCEE*1.9*RCOR**2*(1.+ALOG(FORM)-
1 ATAN(RR/RCOR)**RCOR/RR)
PHI=PHID+PHISPH+PHICOR
PHI=PHI*RATIO
RETURN
10 PHID=PIE*G*ROD*1.65*(XD*(BS10(XD)*BSK0(XD)-BS11(XD)*BSK1(XD))
  XG*(BS10(XG)*BSK0(XG)-BS11(XG)*BSK1(XG)))
PHISPH=SPHMAS(RR)*G/RR**2
X=RR/RCOR
PHICOR=4.*PIE*G*ROCEE*1.9*RCOR*(X-ATAN(X))/X**2
PHI=(PHID+PHISPH+PHICOR)*RATIO/3.08E21
RETURN
END

FUNCTION ROE(RR,TH,EL)

C THIS IS THE GALAXY DENSITY AT THE INPUT POSITION. UNITS=MSUN/PC**3
C THE THICKENING OF THE DISK IS TO ADD A PDF EXP(-Z/2)**2, ZD=0.2KPC

COMMON /GALX/R/SUN,ROD,RD,RG,ROS,RS,RX,ROCEE,RCOR,RTOT
EXTERNAL ROSPH

Z = ABS(RR*COS(TH))
ZR = ABS(RR*SIN(TH))
ROSF = ROSPH(RR)
RODISK = ROD*EXP(-ZR/RD)-EXP(-ZR/RS)
RO = ROSF+RODISK
RETURN
END

FUNCTION FORCE(RR)
COMMON /CONST/PIE,G
EXTERNAL SPHMAS

FORCE = G*SPHMAS(RR)/RR**2
RETURN
END

FUNCTION SPHMAS(RR)

C UNITS=MSUN
REAL M
COMMON /GRID/R1(140),EM(140)

K=1
DO 10 I=1,140
IF(R1(I),GT,RR) GO TO 20
10 CONTINUE

20 K=I-1
M = (EM(K+1)-EM(K))/(R1(K+1)-R1(K))
M = M*(RR-R1(K))+EM(K)
SPHMAS = M
RETURN
END

SUBROUTINE MASG

COMMON /CONST/PIE,G
COMMON /GAL,X/RSUN,ROD,RD,RS,RX,ROCE,ROC,RG,RTOT
COMMON /GRID/R1(140),EM(140)

EXTERNAL DENSR2

SUM = 0.
S = 0.
DO 10 I=1,140
IF(LGT.100) GO TO 1
R1(I) = 0.001*I
10 CONTINUE
GO TO 5
IF(L.GT.120) GO TO 2
R1(I)=(I-100)*0.5
GO TO 5
2 IF(L.GT.128) GO TO 3
R1(I)=(I-120)*5.+10.
GO TO 5
3 IF(L.GT.133) GO TO 4
R1(I)=(I-128)*10.+50.
GO TO 5
4 R1(I)=(I-133)*25.+100.
5 IF(L.GT.1) CALL QTRAP(DENSR2,R1(I-1),R1(I),S)
SUM=SUM+S
EM(I)=SUM*4.*PIE*1.E9
10 CONTINUE
RETURN
END

FUNCTION DENSR2(R)
EXTERNAL ROSPH

DENSR2=ROSPH(R)*R**2
C NOTICE HERE ROSPH IS IN MSUN/PC**3 AND R IS IN KPC. CONVERTING UNITS IS DONE
C IN MASS ROUTINE.
RETURN
END

FUNCTION ROSPH(R)
C UNITS=MSUN/PC**3

COMMON /GALX/RSUN,ROD,RD,RG,ROS,RS,RX,ROCEE,RCOR,RTOT
COMMON /CONST/PIE,G

IF(R.NE.RS) GO TO 1
RO=ROS
GO TO 3
1 Z=ABS((R/RS)**2-1.)
IF(R.GT.RS) GO TO 2
RO=(3.-Z)/SQRT(Z)*ALOG((1.+SQRT(Z))/SQRT(1.-Z))-3.
RO=RO*ROS*3.75/Z**2
GO TO 3
2 RO=(3.+Z)/SQRT(Z)*(ATAN(-1./SQRT(Z))+0.5*PIE)-3.
RO=RO*ROS*3.75/Z/Z
3 ROSPH=RO
RETURN
END

FUNCTION BSI0(X)
C BSI0,BSK0,BSI1,BSK1 ARE MODIFIED BESSEL FUNCTIONS. UNITS=DIMENSIONLESS

DATA A1,A2,A3,A4,A5,A6/3.5156229,3.0899424,1.2067492,0.2659732,
1 0.0360768,0.0045813/
DATA B1,B2,B3,B4,B5,B6,B7,B8,B9/0.39894228,0.01328592,
1 0.00225319,-0.00157565,0.00916281,-0.02057706,0.02635537,
2 0.01647633,0.00392377/

T=X/3.75
IF(X.LE.3.75) THEN
ELSE
1 +B9/T**8
BSIO=BSIO*EXP(X)/SQRT(X)
ENDIF
RETURN
END

FUNCTION BSII(X)
C UNITS=DIEMSIONLESS

DATA A1,A2,A3,A4,A5,A6,A7/0.5,0.87890594,0.51498869,0.15084934,
1 0.02658733,0.00301532,0.00032411/

DATA B1,B2,B3,B4,B5,B6,B7,B8,B9/0.39894228,-0.03988024,
1 0.00362018,0.00163801,-0.01031555,0.02282967,-0.02895312,
2 0.01787654,-0.00420059/

T=X/3.75
IF(X.LE.3.75) THEN
BSII=A1+A2*T**2+A3*T**4+A4*T**6+A5*T**8+A6*T**10+A7*T**12
ELSE
BSII=BSII*X
BSII=B1+B2/T+B3/T**2+B4/T**3+B5/T**4+B6/T**5+B7/T**6
1 +B8/T**7+B9/T**8
BSII=BSII*EXP(X)/SQRT(X)
ENDIF
RETURN
END

FUNCTION BSKO(X)
C UNITS=DIEMSIONLESS

EXTERNAL BSIO

DATA A1,A2,A3,A4,A5,A6,A7/-0.57721566,0.42278420,0.23069756,
1 0.03488590,0.00262698,0.00010750,0.00000740/

DATA B1,B2,B3,B4,B5,B6,B7/1.25331414,-0.07832358,0.02189568,
1 0.01062446,0.00587872,-0.00251540,0.00053208/

T=X/2.
IF(X.LE.2.) THEN
BSKO=A1+EXP(T)*BSIO(X)+A1+A2*T**2+A3*T**4+A4*T**6+A5*T**8
1 +A6*T**10+A7*T**12
ELSE
BSKO=B1+B2/T+B3/T**2+B4/T**3+B5/T**4+B6/T**5+B7/T**6

BSK0=BSK0*EXP(-X)/SQRT(X)
ENDIF
RETURN
END

FUNCTION BSK1(X)
  C
  UNITS=DIMENSIONLESS
  EXTERNAL BS11
  DATA A1,A2,A3,A4,A5,A6/0.15443144,-0.67278579,-0.18156897,
  0.01919402,-0.00110404,-0.00004686/

  DATA B1,B2,B3,B4,B5,B6,B7/1.25331414,0.23498619,0.03655620,
  0.01504268,-0.00780353,0.00325614,-0.00068245/
  T=X/2.
  IF(X LE 2.) THEN
    BSK1=X*ALOG(T)*BS11(X)+1.+A1*T**2+A2*T**4+A3*T**6+A4*T**8+
    A5*T**10+A6*T**12
  ELSE
    BSK1=BSK1/X
  ENDIF
ENDIF
RETURN
END
Appendix B Globular Cluster Parameters

Listed in this appendix are the globular cluster parameters used in this research. They are adapted from the Webbink's (1984) compilation of globular clusters. Interested readers are referred to this paper for more details.

Table 6-I Globular Cluster Parameters

<table>
<thead>
<tr>
<th>Cluster</th>
<th>R(kpc)</th>
<th>r_e (pc)</th>
<th>r_l (pc)</th>
<th>logp_0(M_☉/pc^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 104</td>
<td>8.10E+00</td>
<td>5.20E-01</td>
<td>6.03E+01</td>
<td>5.02E+00</td>
</tr>
<tr>
<td>NGC 288</td>
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</tr>
<tr>
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</tr>
<tr>
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<tr>
<td>Lil 1</td>
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<td>6.30E-01</td>
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<tr>
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<td>7.39E+01</td>
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<tr>
<td>AM 1</td>
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<td>8.12E+01</td>
<td>-1.00E-03</td>
</tr>
<tr>
<td>Eri 2</td>
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<td>8.55E+01</td>
<td>5.84E-01</td>
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<td>E 3</td>
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<td>6.36E+01</td>
<td>5.77E-01</td>
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</table>
Table 6-I  Globular Cluster Parameters

(continued)

<table>
<thead>
<tr>
<th>Cluster</th>
<th>R(kpc)</th>
<th>$r_c$(pc)</th>
<th>$r_1$(pc)</th>
<th>$\log \rho_0$(M$_\odot$/pc$^3$)</th>
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<td>6.57E+01</td>
<td>3.35E+00</td>
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<tr>
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Table 6-I Globular Cluster Parameters

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Bibliography


Paresce, F, Shara, M., Meylan, G, Macchetto, Baxter, D., Blades, J. C., Greenfield, P.


