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Cartesian analysis in natural science as exposed through an investigation of his method in mathematics

Capistran, Michael D., Ph.D.
Rice University, 1992

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CARTESIAN ANALYSIS IN NATURAL SCIENCE AS EXPOSED THROUGH AN INVESTIGATION OF HIS METHOD IN MATHEMATICS

by

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A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE DOCTOR OF PHILOSOPHY

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ABSTRACT

CARTESIAN ANALYSIS IN NATURAL SCIENCE AS EXPOSED THROUGH AN INVESTIGATION OF HIS METHOD IN MATHEMATICS

by

Michael D. Capistran

Aspects of Descartes’ notion of analysis in his mathematics are looked at in detail with a view to relating this notion to Descartes’ work in natural science. Descartes’ method in his mathematics and science are subsumed beneath a more general method of analysis termed by Descartes ‘indirect problem investigation’. The distinction between Cartesian analysis and synthesis is discussed. In terms of natural science a specific interaction of (1) metaphysically deduced results with (2) empirical data, involving an attendant conversion of secondary qualities to primary qualities for the solution of such problems, is advanced using the magnet as an illustration. A possible relationship with Descartes’ methodology in his metaphysics is suggested.
For

My Parents

And For

Mark Alan Kulstad

For Their Patience

Although the conception of Descartes as the founder of modern philosophy is a true truism, it has done a lot of harm, particularly at the hands of those who are so fascinated by the hyperbolic scepticism of the first Meditation that they never see beyond it. They never ask why Descartes wanted to lead the mind away from the senses, and to what, but move off to their present-day concerns with their version of scepticism or their worries about mind and brain, or what you will. The chief change in emphasis in Cartesian scholarship that I want to call your attention to here consists in the stress on Descartes, not in patches and as our contemporary, but in his whole oeuvre and as a thinker of his time.

-- Marjorie Grene

*Descartes Among the Scholastics*
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Chapter 1: Analysis.

Close to the worst way of doing history of thought is by trying to substantiate one's own preconceived notions concerning a topic. The worst way, however, is the attempt to substantiate other peoples' entrenched beliefs. In this respect it is wise to follow the words of Galileo when he instructs us to not go along arbitrarily shaping our premises to fit the conclusions we mean to prove.¹

No field of discipline is more guilty of this sin than Cartesian scholarship. Whereas traditionally Descartes' method in both Science and Metaphysics has been viewed as entirely a priori in character, the relatively recent realization that Descartes' work involves considerable reference to experimentation has brought forward the question as to the role of such experimentation in his overall method.² Some have even proposed there is no important or great difference between, say, Descartes


I am indebted to Dr. Gregory Brown of the University of Houston for having introduced me to many of these issues.
as a rationalist and Locke as an empiricist. Often there are places in which Descartes refers to his method as a form of analysis, and it is my wish to examine this notion in the expectation that an understanding of what he means by this term, analysis, will lead then to an understanding of what he takes to be the relationship between reason and experimentation, what we moderns normally term *a priori* and *a posteriori*, in his method.

My thesis will be advanced in five Chapters.

In Chapter One I shall make some opening comments concerning the notion of analysis and Descartes’ understanding of this term; I shall also turn to matters having to do with Cartesian mathematics and suggest an interpretation of the term ‘analysis’ as it applies to this field.

In Chapter Two I shall discuss the relationship between Descartes’ early work the *Regulae* and his mature work *La Géométrie*. I wish primarily to do this because Descartes speaks not only of mathematics in the *Regulae*, but also of his method in natural science. A comparison of the methodology espoused by the two works will therefore, in my estimation, afford a bridge in the Cartesian handling of these two disciplines, mathematics and natural science. Also in Chapter Two I shall concentrate on the relationship between what Descartes has called in the *Regulae* ‘indirect problem solution’ and what I shall have termed ‘Cartesian mathematical analysis’ in Chapter One. My conclusion will be that indirect problem solution is a form of analysis.

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In the first part of Chapter Three I introduce a distinction developed by a mathematician named Polya, the distinction being between ‘problems to prove’ and ‘problems to find’; and I suggest this distinction fits what we know concerning what was supposed to be the differentiation in mathematical subjects between the second and third sections of the *Regulae*. Later in this Chapter I suggest an additional differentiation between the proposed subject material of the second and third sections of the *Regulae*, this one, however, in terms of natural science rather than mathematics. The differentiation I propose is one of the method of the solution of problems posed in terms of primary qualities in the case of the second section, and one of the solution of problems posed not in terms of primary qualities, that is, posed in terms of secondary qualities, in the third section.

In Chapter Four I introduce two additional topics not directly related to Cartesian scholarship, but topics helpful in understanding what I believe Descartes had in mind in his notion of analysis both in mathematics and natural science. The first topic is again an idea developed by Polya, that of a reasonably general method of discovery termed the ‘pattern of two loci’, and I try to show how this method may be construed to be similar to Descartes’ method in the solution of problems in mathematics. Next I introduce an example from modern day natural science. The example is that of Watson and Crick’s discovery of the structure of the DNA molecule. I then attempt to show how this example may be viewed as an example in natural science of Polya’s ‘pattern of two loci’, and I also point out, on the basis of this example, the importance of over-arching principles in the solution of some problems in natural science. In the last section of
this Chapter, I develop my general view of Cartesian analysis by first reducing Cartesian mathematical analysis to a broader form utilized by the ancient geometers; in passing, I also suggest a fashion in which this broader view of analysis may be considered to subsume certain arguments utilized by Descartes in his metaphysics. I then argue that Descartes' remarks concerning indirect problem solution may be viewed as depicting, from a Cartesian point of view, an even more general form both of this broader form of analysis already mentioned and of Polya's 'pattern of two loci' as applied to problems in natural science. I then relate the distinction of Cartesian indirect and direct problem solution to the distinction of Cartesian analysis and synthesis, respectively, and I try to give a general account of what this distinction means in terms of Cartesian natural science.

In Chapter Five I consider the specific example of the solution of the problem of the magnet both as an illustration and as an amplification of my general account of Cartesian science given at the end of Chapter Four.

To return now to my intentions in Chapter One, and speaking now more specifically, I intend to do the following four things: I shall first argue that Descartes conceived of his method as a universal one, one applicable not just to mathematics but to all fields of thought. Second I shall then review the statements with which I am familiar made by Descartes concerning the term analysis in an effort to motivate the topic in question. Third, to help understand what Descartes meant by this term analysis I shall give some background material concerning the term as the ancient Greeks understood it and as it might also be considered in
mathematics and geometry in general. And fourth, I shall examine an example taken from Descartes’ *Géométrie* to indicate the initial manner in which I believe Descartes’ notion of analysis should be understood in his mathematics.

Before plunging into this work, however, allow me to comment on my methodology. I believe there is much here the reader will find novel and even perhaps difficult at first to accept. The Descartes I am depicting is in many ways a new Descartes. Though the various opinions expressed here should not be considered frivolous, and I feel I can commit myself to them by and large, my project is as yet not altogether finished and again I wish to invoke Galileo concerning one of his reasonable opinions which turned out ultimately to be in error:

> That the comet was a mere image and appearance was never positively affirmed by us; it was merely raised as a question and offered for the consideration of philosophers, along with various arguments and conjectures that appeared suitable to show them this possibility.\(^4\)

I hope this work will also be accepted in such light -- offered for the consideration of the community together with arguments and conjectures put forward to assist one in drawing one’s own conclusions.

In this respect, the type of historiography utilized here may be considered broadly similar methodologically to science. After having read the selections and arrived gradually at my conclusions, I propose a number

\(^4\) Drake, p. 254.
of hypothetical assertions together with some textual evidence supporting these assertions. Or, again, my position may be viewed as similar to that of a defense lawyer, not however as one committed to an unrelenting defense, but rather as one with an additional interest in discerning the ultimate truth concerning his client. The test, or verdict, will be how well my hypotheses eventually fare with additional thought and additional textual references; and I feel confident the reader will not be idle in the provision of these.

Part I: Descartes' Conception of a Universal Method

In Descartes' *Regulae*, an early work published only posthumously, he clearly conceives his method as a universal one; that is, one useful not just for mathematics, but for other disciplines as well

[I]t is far better never to contemplate investigating the truth about any matter than to do so without a method. ...By 'a method' I mean reliable rules which are easy to apply, and such that if one follows them exactly, one will never take what is false to be true or fruitlessly expend one's mental efforts, but will gradually and constantly increase one's knowledge till one arrives at a true understanding of everything within one's capacity.\(^5\)

But if one attends closely to my meaning, one will readily see that ordinary mathematics is far from my mind here, that it is quite another discipline I am expounding, and that these illustrations are more its outer

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garments than its inner parts. This discipline should contain the primary rudiments of human reason and extend to the discovery of truths in any field whatever. Frankly speaking, I am convinced that it is a more powerful instrument of knowledge than any other with which human beings are endowed, as it is the source of all the rest.\(^6\)

When I considered the matter more closely, I came to see that the exclusive concern of mathematics is with questions of order or measure and that it is irrelevant whether the measure in question involves numbers, shapes, stars, sounds, or any other object whatever. This made me realize that there must be a general science which explains all the points that can be raised concerning order and measure irrespective of the subject-matter, and that this science should be termed *mathesis universalis* [i.e. 'universal mathematics'.] -- a venerable term with a well-established meaning -- for it covers everything that entitles these other sciences to be called branches of mathematics.\(^7\)

Although, as I mentioned, these passages are taken from the early work the *Regulae*, and some would argue that Descartes changed his position in his later works, some evidence would indicate otherwise.

Let us remember that the complete title of the *Discours* as it appears on the original, anonymously published, title page:

*Discours / de la Methode / Pour bien conduite sa raison, & chercher / la vérité dans les sciences. / Plus La Dioptrique. / Les Météores. / et / La Géométrie. / Qui sont des essais de cette Methode.*\(^8\)

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\(^6\) CSM 1, p. 17; AT X, 374.

\(^7\) CSM 1, p. 19; AT X, 377-8.

\(^8\) The title as it appears in AT is truncated; translated, it appears as: Discourse on the Method of rightly conducting one’s reason and seeking the truth in the sciences. Adam, Charles, and Paul Tannery, *Oeuvres de Descartes*, vol. VI, Cerf, Paris, 1897-1913 (Reprinted: Vrin 1957-8), p. 1.
This may be translated as:

*Discourse on the Method of rightly conducting one’s reason and seeking the truth in the sciences, and in addition the Optics, the Meteorology and the Geometry, which are essays in this Method*  

As pointed out in Cottingham, Stoothoff, & Murdoch, this title is a condensed version of the more extensive title which Descartes proposed to Mersenne in March 1636, where he speaks of ‘four treatises, all in French’, with the general title:

*The Plan of a universal Science which is capable of raising our nature to its highest degree of perfection. In addition, the Optics, the Meteorology and the Geometry, in which the Author, in order to give proof of his universal Science, explains the most abstruse Topics he could choose, and does so in such a way that even persons who have never studied can understand them.*

When Mersenne queried Descartes concerning the title of the published work, Descartes responded in a letter of February 1637:

I have not put *Treatise on the Method* but *Discourse on the Method*, which amounts to the same as Preface or Note concerning the Method, in order to show that I do not intend to teach the method but only to speak about it. For, as can be seen from what I say, it consists much more in practice than in theory. I call the treatises following it *Essays in*
this Method because I claim that what they contain could not have been discovered without it, and they enable us to recognize its value. And I have included a certain amount of metaphysics, physics and medicine in the introductory Discourse in order to show that the method extends to every kind of subject-matter.\footnote{CSM 1, p. 109.}

In fact he says in La Géométrie, one of the essays published together with the Discours, that the cultivation of the mind through the training of problem solution, rather than the specific solution of any given problem, is the greatest advantage to be gained from his mathematics:

...I shall not pause here to explain this [the reduction of unknown quantities to a single quantity] in greater detail, because I should be depriving you of the pleasure of learning it for yourself, as well as the advantage of cultivating your mind by training yourself in it, which is, in my opinion, the principal advantage we can derive from this science.\footnote{Oscamp, Paul J., tr & intro, Discourse on Method, Optics, Geometry, and Meteorology, Bobbs-Merrill, Indianapolis, 1965, p. 180. Cf. Polya, George, How to Solve it: A New Aspect of Mathematical Method, Princeton University Press, Princeton, New Jersey, 1948, pp. 3-4, 15-16, & 21.}

Descartes also goes on to say in a letter to a friend of Mersenne in 1637:

I want to tell you that the whole plan of what I propose to publish on this occasion is to prepare the road [for his treatise on physics] and see how the land lies. I am putting forward for this purpose a general method, which I do not in fact give instruction in, but which I attempt to prove by the three following treatises, attached to the above-mentioned
Discours. I discuss, in the first, a subject in which Philosophy and Mathematics are mixed; in the second, a pure problem of Philosophy, and in the third, a pure problem of Mathematics. In these treatises, I can state that I have avoided speaking of anything, except those things which can be known by the power of reasoning, because, with this exception, I believed that I had no knowledge; with the result that it seems to me that people can judge my use of a method by which I could just as well explain a quite different kind of problem, granted circumstances under which I might make the necessary experiments and the time to consider them. Beyond all this, in order to show that my method extends to all problems, I have inserted some metaphysical, physical, and medicinal problems into the opening Discours.\textsuperscript{13}

In his dedicatory letter for the Meditationes, Descartes elaborates a bit concerning a method attributed to him by others:

And finally, I was strongly pressed to undertake this task [of writing the Meditationes] by several people who knew that I had developed a method for resolving certain difficulties in the sciences -- not a new method (for nothing is older than the truth), but one which they had seen me use with some success in other areas; and I therefore thought it my duty to make some attempt to apply it to the matter in hand.\textsuperscript{14}

Descartes says here not just that some people think he has developed a method, but that these people know he has developed a method, and that they have seen him use this method with some success. The method is good for ‘resolving certain difficulties in the sciences’, ‘other areas’, and ‘the matter in hand’ or metaphysics. It is therefore safe to say that, at least

\textsuperscript{14} CSM 2, p. 4; AT VII, 3.
through the time of the Meditationes, Descartes believed he was using a
method of problem solution broadly general, if not universal, in character.

Part II: Introductory Material on Cartesian Analysis

Let us, in this section, review what Descartes has to say concerning
this term 'analysis'. There are four primary places in which Descartes
mentions it specifically: 1) there are at least two explicit references by
Descartes to analysis in his Regulae; 2) there is at least one mention in his
Discours; 3) there are two worth looking at in his responses to the
objections to his Meditationes \(^{15}\); and there is at least one additional
reference in his personal correspondence.

The two quotations from the Regulae are as follows:

[W]e are well aware that the geometers of antiquity employed a sort of
analysis which they went on to apply to the solution of every problem,
though they begrudged revealing it to posterity. At the present time a
sort of arithmetic called 'algebra' is flourishing, and this is achieving for
numbers what the ancients did for figures. These two disciplines are
simply the spontaneous fruits which have sprung from the innate
principles of this method. I am not surprised that, where the simplest
objects of these disciplines are concerned, there has been a richer
harvest of such fruits than in other disciplines in which greater obstacles

\(^{15}\) Descartes also makes reference to 'my method(s) of analysis' in two additional places in the
objections: CSM 2, p. 286 & p. 299; AT VII, 424 & 444, but the use in these instances is only the echoing or
mimicking of these words as already used by the objector, and its usage here is therefore highly inconclusive.
tend to stifle progress. But no doubt these too could achieve a perfect maturity if only they were cultivated with extreme care.²⁶

Slightly later Descartes says:

...I observed that in order to know [mathematical] proportions I would need sometimes to consider them separately, and sometimes merely to keep them in mind or understand many together. And I thought that in order the better to consider them separately I should suppose them to hold between lines, because I did not find anything simpler, nor anything that I could represent more distinctly to my imagination and senses. But in order to keep them in mind or understand several together, I thought it necessary to designate them by the briefest possible symbols. In this way I would take over all that is best in geometrical analysis and in algebra, using the one to correct all the defects of the other.

In fact, I venture to say that by strictly observing the few rules I had chosen, I became very adept at unravelling all the questions which fall under these two sciences.²⁷

From the first passage we find that ancient geometers employed a form of analysis. Algebra does for numbers what the ancient geometers did for figures. These two disciplines, algebra and analysis, are fruits of the same method. From the second passage we learn that by designating

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²⁶ CSM 1, 17; AT X, 373.
²⁷ CSM 1, p. 121; AT VI, 20. A bit later Descartes continues:
‘In short, the method which instructs us to follow the correct order, and to enumerate exactly all the relevant factors, contains everything that gives certainty to the rules of arithmetic.
‘But what pleased me most about this method was that by following it I was sure in every case to use my reason, if not perfectly, at least as well as was in my power. Moreover, as I practised the method I felt my mind gradually become accustomed to conceiving its objects more clearly and distinctly; and since I did not restrict the method to any particular subject-matter, I hoped to apply it as usefully to the problems of the other sciences as I had to those of algebra.’

This passage, I think, may be used as further evidence that Descartes believed his method to be universal.
lines by the briefest possible symbols to find mathematical proportions, Descartes believes he is taking over all that is best in both fields, analysis and algebra, and that by so doing he is able ‘using the one to correct all the defects of the other’.

Let us turn next, then, to the statement in Descartes’ Discours:

When I was younger, my philosophical studies had included some logic, and my mathematical studies some geometrical analysis and algebra. These three arts or sciences, it seemed, ought to contribute something to my plan. But on further examination I observed with regard to logic that syllogisms and most of its other techniques are of less use for learning things than for explaining to others the things one already knows.... As to the analysis of the ancients and the algebra of the moderns, they cover only highly abstract matters, which seem to have no use. Moreover the former [analysis of the ancients] is so closely tied to the examination of figures that it cannot exercise the intellect without greatly tiring the imagination; and the latter [algebra] is so confined to certain rules and symbols that the end result is a confused and obscure art which encumbers the mind, rather than a science which cultivates it.  

When Descartes is criticizing ‘algebra’ here he is criticizing the cossist algebra of his time, primarily that of of Vieté. I am not going to go into the details of the cossist art here; suffice it to say that algebra was a very rustic tool before the advent of Descartes and Fermat.  

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18 CSM 1, pp.119-20; AT VI, 17-8.
From the above quotation, however, we discover that analysis, at least the analysis of the ancient geometers, deals only with very abstract matters which seem to have no use and that it greatly tires the imagination because it is ‘so closely tied to the examination of figures’; we learn also that it is as useless as both logic and algebra are. Here the analysis of the ancient geometers is treated in a disparaging fashion, but then Descartes goes on to say:

For this reason I thought I had to seek some other method comprising the advantages of these three subjects [logic, the analysis of the ancients, and algebra] but free from their defects.\textsuperscript{20}

Immediately after this Descartes begins the discussion of his own method; so, clearly Descartes wishes to develop his own method embracing the best of the three disciplines and jettisoning the worst. This sits well with what he has already said earlier in the \textit{Regulae}.

Now let us turn also to Descartes’ response to the second set of objections to his \textit{Meditationes}. The first passage itself is quite lengthy and speaks concerning a differentiation Descartes draws between ‘analysis’ and ‘synthesis’. This is an important passage but a difficult one. The entire selection I have included as an Appendix, and I have here excerpted what I take to be the more salient aspects concerning our topic from it:

Analysis shows the true way by means of which the thing in question was discovered methodically and as it were \textit{a priori}, so that if the reader

\textsuperscript{20} CSM 1, p. 120; AT VI, 18.
is willing to follow it and give sufficient attention to all points, he will make the thing his own and understand it just as perfectly as if he had discovered it for himself. But this method contains nothing to compel belief in an argumentative or inattentive reader; for if he fails to attend even to the smallest point, he will not see the necessity of the conclusion. Moreover there are many truths which -- although it is vital to be aware of them -- this method often scarcely mentions, since they are transparently clear to anyone who gives them his attention.\textsuperscript{21}

Now it is analysis which is the best and truest method of instruction, and it was this method alone which I employed in my \textit{Meditations}.\textsuperscript{22}

Descartes here considers his work, the \textit{Meditations}, as a form of analysis. But here we are also informed that analysis is a pedagogical method which shows the manner something is discovered, and which allows us to understand a discovery as if we had made it ourselves -- as long as we are neither inattentive nor hostile. Descartes also speaks here of discovery and derivation, `as it were \textit{a priori}', `\textit{tanquam a priori}'; what must surely prove to be an important topic.

Let us also at this point look briefly at what Descartes has to say concerning `synthesis' for comparison:

Synthesis, by contrast, employs a directly opposite method [from analysis] where the search is, as it were, \textit{a posteriori} (though the proof itself is often more \textit{a priori} than it is in the analytic method). It demonstrates the conclusion clearly and employs a long series of definitions, postulates, axioms, theorems and problems, so that if

\textsuperscript{21} CSM 2, p. 110; AT VII, 155-6.
\textsuperscript{22} CSM 2, p. 111; AT VII, 156.
anyone denies one of the conclusions it can be shown at once that it is
contained in what has gone before, and hence the reader, however
argumentative or stubborn he may be, is compelled to give his assent.
However, this method is not as satisfying as the method of analysis, nor
does it engage the minds of those who are eager to learn, since it does
not show how the thing in question was discovered.

It was synthesis alone that the ancient geometers usually employed
in their writings. But in my view this was not because they were utterly
ignorant of analysis, but because they had such a high regard for it that
they kept it to themselves like a sacred mystery.

Now it is analysis which is the best and truest method of instruction,
and it was this method alone which I employed in my Meditations. As
for synthesis, which is undoubtedly what you are asking me to use
here, it is a method which it may be very suitable to deploy in geometry
as a follow-up to analysis, but it cannot so conveniently be applied to
these metaphysical subjects.23

Here we find that synthesis (1) employs definitions, postulates,
axioms, theorems and problems; that is, it may, I take it, correctly be
termed axiomatic in character; (2) that it has little pedagogical value; (3)
that it cannot be so conveniently applied to metaphysical matters as
analysis; and (4) that it is a search ‘tanquam a posteriori’ -- but -- just to
clarify matters perfectly, no doubt -- Descartes also adds here that ‘the
proof itself is often more a priori than it is in the analytic method’.

So much for the famous and extensive passage in the second set of
replies.

In the fourth set of replies there is a more cryptic and oblique
reference, not exactly to ‘analysis’ but to the ‘analytic style of writing’:

23 CSM 2, pp. 110-1; AT VII, 156.
The analytic style of writing that I adopted there [in the Meditationes] allows us from time to time to make certain assumptions that have not yet been thoroughly examined; and this comes out in the First Meditation where I made many assumptions which I proceeded to refute in the subsequent Meditations.\(^{24}\)

And the last reference to ‘analysis’ by Descartes of which I am aware, in a letter to De Beaune, 1639, is as follows:

\[\text{In the case of the tangents, I have only given a simple example of analysis, taken indeed from a rather difficult aspect and I have left out many of the things which could have been added so as to make the practice of the analysis more easy.}^{25}\]

This is an interesting quotation, and I shall be following up on it momentarily.

But the question at this point becomes: what is the nature of Cartesian analysis? For most commentators, an exposition of this topic usually also involves some notion of the concept of synthesis as well. Joachim, for example, states:

\[\text{The process of analysis is continuous and proceeds to the absolutely simple, while the synthesis is a progressive reconstruction beginning from the simple and going through the increasing degrees of}\]

\(^{24}\) CSM 2, p. 173; AT VII, 249.

\(^{25}\) From: Beck, Method, p. 179; letter to De Beaune, 1639, AT II, 511.
complexity until it ends in the original complex from which the analysis started. 26

This seems the general view. I have no conflict with analysis proceeding to the absolutely simple, as Joachim suggests -- though I am familiar with no statement on the part of Descartes expressing this -- but what is meant by 'simple'? Such an opinion as that given by Joachim here has not uncharacteristically produced views such as the following:

The significance of the doctrine of simple natures for Descartes's rationalist approach is this: by analyzing out a set of primitive or ultimate concepts and observing various necessary relations between them, he thought he could justify his claim to be able, in principle, to generate the whole body of knowledge on the basis of intuitive conviction alone. 27

The difficulty I have with the position expressed by this quotation is that it does not seem to allow for experimentation in Cartesian science, a known difficulty with the traditional view. 28

To recapitulate, then, in the Regulae and in the Discours analysis is the analysis of the ancient geometers, a method Descartes feels he has appropriated but in some fashion improved. In the responses to the Meditationes it is a pedagogical method proceeding 'tanquam a priori', and


28 Vide footnote 2 above.
it is a style of writing which allows us to make assumptions not yet thoroughly examined, assumptions which might ultimately be refuted. And in the letter to De Beaune, Descartes’ handling of the tangent to a curve is given as an example of it.

At this point, my hope, and expectation, is that certain difficulties concerning Cartesian method can be resolved by a look at Cartesian analysis in Descartes’ mathematics -- which may then shed light on his method in natural science (specifically his allowance for and utilization of experimentation) and perhaps also later shed light on his method in metaphysics -- though I am interested primarily in the former in this work. Thus I believe such an investigation will help show what Descartes took the relationship between reasoning and experimentation -- what we now call the relationship between the *a priori* and *a posteriori* -- to be. In the event, I believe Descartes’ method might very well be shown to be not an unreasonable, albeit not an entirely correct, first and rudimentary picture of the proper method of procedure in science.

Since Descartes makes reference to the analysis of the ancient geometers, let us first turn in our next section to what these people have had to say on the subject. We shall then afterwards follow up in the last section of this chapter with an investigation of Descartes’ mathematical method for finding the normal to a curve -- or, what is the same, his method for finding tangents.
Part III: The Analysis of the Ancient Geometers

In Euclid, propositions are presented and then given demonstration. This method was recognized as pedagogically deficient at about the time of the late Renaissance. At that time it was recognized that for the student to be simply handed the fully completed proofs of a geometer such as Euclid was, as a method of instruction, inferior; and that for educational purposes it was more important to challenge the student by posing a problem -- a theorem or a construction -- and by then having the student try to solve it. The student learned better by being challenged with a problem to be solved rather than by memorizing the completed solution by rote. This intuition has been carried over into our own modern curriculum in mathematics. Sometimes the problem posed to the student is sufficiently simple to where the student can simply follow out a line of reasoning similar to the Euclidean exposition and set forth the demonstration immediately. Such demonstrations in Euclidean form are today commonly considered 'synthetic' in nature. Often, however, the problem is too difficult to be solved in this fashion, and the student is then required to utilize a different method; a method, as opposed to the Euclidean exposition, commonly referred to now as 'analytic'. There is a sense in which this 'analytic' treatment might be characterized as 'reasoning backwards'.

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30 An example of reasoning backwards for the ancient Greeks was the reduction by Hippocrates of the problem of doubling the cube to that of finding two mean proportionals. Vide: Scott, J.F., The Scientific Work of René Descartes, Taylor & Francis, Ltd., London, 1952, p. 94; Scott in turn refers to Plato, Meno: 86 E - 87 A.
method familiar to all of us; it is the method which we all utilized to learn plane geometry in high school. Let me give the following illustrative example.

![Figure 1](image)

Given that ABCD is a rhombus and BD is a diagonal. Prove that \( m = n \).

Solution. First assume that \( m = n \). Now AB \parallel DC, since the figure is a rhombus, and consequently, \( m = p \). Therefore, \( p = n \) since we assumed that \( m = n \); but if \( p = n \), then DC must equal BC. We have now arrived at a known or a given, for we know that DC = BC since the figure is a rhombus. The backward movement is now ended and we are ready to give the final proof by reversing our steps as follows:

DC = BC since the figure is a rhombus; therefore \( \Delta DCB \) is isosceles and \( p = n \); but \( p = m \) since AB \parallel DC; therefore \( m = n \).³²

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This example is taken from a student introductory text to problems in plane geometry, and I think it worthwhile to quote verbatim the explanation of the example offered there to the student.

[W]e assume the conclusion and then draw deductions from the conclusion until we arrive at something known or something which can be easily proved. In other words this method is the opposite of the forward movement in which we begin with the given and proceed forward to the conclusion; in the backward movement we begin with the conclusion and move backward till we arrive at the given or the known. After we arrive at the given or the known, we then reverse the movement and proceed forward to the conclusion.

This method is sometimes called the analytic method. It is very important and deserves the student's closest study.33

That is to say, the problem is first set for us. After perhaps a cursory look at the given data and an attempt to solve the problem utilizing the most common theorems which seem applicable -- in the synthetic mode -- we then might attempt the analytic approach. First we assume what we are to prove is actually correct. Then we try to follow out any consequences from that ultimate conclusion, hoping, step by step, to lead ourselves back again to the data given. Once such a path has been secured, we then reverse the steps we have laid out for ourselves and write these down as the required proof.

33 Horblit, p. 6 & p. 8.
As already mentioned by Descartes, ancient geometers also adhered to a distinction they took to exist between the two methods of analysis and synthesis for the solution to a problem. Euclid was the post-Aristotelian geometer who, in his work *The Elements*, collated the different fundamental aspects of geometry into the form with which we are familiar today. Concerning analysis and synthesis he has the following to say:

Analysis is an assumption of that which is sought as if it were admitted and the passage through its consequences to something admitted to be true.

Synthesis is an assumption of that which is admitted and the passage through its consequences to the finishing or attainment of what is sought.\(^{34}\)

Although these statements sit well with our example of the rhombus earlier, they are not in themselves very revealing. Pappus, one of the last great Greek geometers before its subsequent demise, has something of a fuller account, however. Concerning analysis specifically he has some additional comments, some of which are germane to our discussion:

In *theoretical* analysis we assume what is sought as if it were existent and true, after which we pass through its successive consequences, as if they too were true and established by virtue of our hypothesis, to something admitted: then (a), if that something admitted is true, that which is sought will also be true and the proof will

---

correspond in the reverse order to the analysis, but (b), if we come upon something admittedly false, that which is sought will also be false.\textsuperscript{35}

This second type of theoretical analysis is called today \textit{reductio ad absurdum} in which a correct inference from a hypothetical proposition leads to a conclusion either (1) admittedly false or (2) contradictory to the hypothesis itself or to some one of its consequences. In this case we can conclude without further inquiry that the presumed hypothesis is false.

We have seen that Descartes speaks of his method as being a form of analysis. We have also seen that Euclid and Pappus speak of analysis in terms of assuming what is to be proved as if it were known. Concerning exactly this topic, Hintikka suggests Descartes’ method:

\ldots can be profitably considered as a variant of the method of analysis which was used in Greek mathematics and whose discovery was ascribed by some sources to Plato.\textsuperscript{36}

Hintikka believes the reason any recognition of Descartes’ method as analytic has had such little effect upon Cartesian scholarship is because of the various modern senses of the word ‘analytic’. My own belief, however, is that the conception of Descartes’ method as a kind of analysis has had so little effect because no sustained or systematic study has shown in what sense the various seemingly conflicting ways Descartes uses the term ‘analysis’ may be reconciled, firstly, and secondly no illustration of how

\textsuperscript{35} Euclid 1, p. 138.
such method can be exemplified in Descartes' metaphysics and natural science has been put forward. Hintikka sites Gerd Buchdahl as having ascertained three separate types of analysis in Descartes:

(i) Analysis as a technique of operating algebraically with unknowns, in the hope of finding equations that contain them, and then solving these equations for the unknowns.
(ii) Analysis as a literal or metaphoric "taking apart" of an actual physical or geometrical complex of phenomena.
(iii) The Pappian hypothetico-deductive procedure of "assuming what is to be proved as though it were known."

Type (i) is highly suggestive in itself, I think, but no more has been said concerning it than what has been stated here; and it is exactly this suggestion I wish to focus on and follow up in detail in the next section of this chapter. Hintikka, however, goes on to state that (i) may be thought of as a further technical development of the Pappian idea (iii). Unfortunately Hintikka does not go on to elaborate how or in what sense this is to be done.37

If we recall that, as mentioned above, Descartes considered the three essays -- *Dioptrique*, *Géométrie*, and *Météores* -- published along with his *Discours* as examples of his method, in consideration of the fact that Descartes considered his *Géométrie* an example of his method, and with the Hintikka-Buchdahl suggestion (i) in mind, let us now look at the

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37 Item (ii) seems misplaced. Though Descartes speaks of proceeding from or to simples, and this is an important topic, he does not to my knowledge speak of such directly in conjunction with the notion of analysis. Concerning the issue of reduction of (i) from (iii), vide Part III of Chapter Four below.
\textit{Géométrie} with an eye to whether it will shed some light on the Cartesian notion of analysis.

Part IV: Descartes’ Method of Finding the Normal to a Curve as an Example of his Analytic Method

Recall above the statement Descartes has made concerning the relationship between analysis and his example of finding tangents:

[I]n the case of the tangents, I have only given a simple example of analysis, taken indeed from a rather difficult aspect and I have left out many of the things which could have been added so as to make the practice of the analysis more easy.\textsuperscript{38}

In Book II of \textit{La Géométrie} Descartes claims the ability, utilizing his method, of discovering various properties of a curve: diameters, axes, foci and so forth. As an example to illustrate his claim, Descartes offers his method for finding normals to a curve.\textsuperscript{39} Since the normal to a curve at a given point is perpendicular to its tangent at that point, the solution of one provides the solution of the other. In Descartes’ own words:

\textsuperscript{38} Quoted above, footnote 25; From: Beck, \textit{Method}, p. 179; letter to De Beaune, 1639, AT II, 511, 18-20.

\textsuperscript{39} “Normal to Curve: The line at right angles to a tangent to a curve at the point of contact of the tangent.” From: Millington, T. Alaric & William Millington, \textit{Dictionary of Mathematics}, Barnes & Noble, N.Y., 1966, p. 156.
This is why I believe I shall have given here everything that is required for the elements of curved lines, when I have given a general method of drawing straight lines that fall at right angles on a curve at whichever points on it we wish to choose. And I daresay this problem in geometry is not only the most useful and the most general that I know, but even that I have ever desired to know.\footnote{Olscamp, p. 207.}

Indeed, this method is especially exemplary of Descartes’ mathematical method as a whole, I believe, and I would like to go into it in some detail.\footnote{I am indebted to Pashwar Sen for having helped me with an early version of this section.}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2}
\caption{figure 2.}
\end{figure}

Let CE be the given curve, he says, and let it be required to draw through point C a straight line which makes a right angle with CE, i.e. the normal (vide figure 2).
I assume the problem already solved,\(^{42}\)

Descartes states. The line we are seeking is CP, extended to point P where it meets straight line GA, which is the line he wishes to relate all the points of the curve CE. He does not expressly so mention, but the lines CM and BA are to be perpendicular to GA and thus parallel to themselves. He then lets MA = y, and CM = x. Thus the curve CE will be expressed in terms of what we would now call a dependent and an independent variable related to the lines GA, which we may, anachronistically, conceive in our own modern terms as the y axis, and BA, which may similarly be conceived as the x axes.\(^{43}\) Then he designates other lines; he lets PC = s and PA = v, thus PM = v - y. It is well to remember at this point and keep in mind that what we are solving for here is PC, the normal to the curve CE at point C, and that the length of PC has been designated as s.

Since our task will be to find s, or v which will determine s, Descartes notes certain relational properties for s. Because \(\triangle PMC\) is a right triangle, \(s^2\), which is the square of the hypotenuse, is equal to \(x^2\), the square of one side, plus \((v - y)^2\), the square of the other side. This relates distance s to distances x and y. Thus we have:

\[
s^2 = x^2 + (v - y)^2, \text{ or}
\]

\(^{42}\) Olscamp, p. 207.

\(^{43}\) Descartes does not, strictly speaking, use what is now called the 'Cartesian coordinate system' of axes extending an infinite length from a point of origin. Descartes' relation to this system is somewhat complex and it is not best to enter into it here. Suffice it to say that he relates the curve in question to two finite lines which, insofar as possible, he tries to make perpendicular to one another.
\[ s^2 = x^2 + v^2 - 2v \cdot y + y^2. \]

Transposing and solving for \( x \) in terms of \( y \) we have,

\[ x^2 = s^2 - v^2 + 2v \cdot y - y^2, \text{ or} \]

\[ x = \sqrt{s^2 - v^2 + 2vy - y^2}. \]

or, solving for \( y \) in terms of \( x \) we have,

\[ y = v + \sqrt{s^2 - x^2}. \]

\[44\] Descartes seems to have used here a general solution formula for quadratic equations such as:

\[ y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \]

where the quadratic equation is expressed in the form, \( ay^2 + by + c = 0 \). This equation, in addition to the method of solution through completing the square, was familiar even to the Greeks. From our original equation, we have:

\[ s^2 = x^2 + v^2 - 2v \cdot y + y^2; \]

from which we derive:

\[ y^2 + (-2v)y + (x^2 + v^2 - s^2) = 0. \]

Then by the general solution formula given above we have:

\[ y = \frac{(-2v) \pm \sqrt{4v^2 - 4(x^2 + v^2 - s^2)}}{2}, \text{ or} \]

\[ y = v \pm \sqrt{s^2 - x^2}. \]

Descartes accepts the positive value, or, as he calls it elsewhere, the 'true' root.
In Descartes' words,

[B]y means of this equation, I eliminate one of the two indeterminate quantities \( x \) or \( y \) from the other equation, which explains to me the relation that all the points of the curved line CE have to those of the straight line GA. This is easy to do by substituting \( \sqrt{x^2 - v^2 + 2vy - y^2} \) throughout for \( x \), and the square of this sum for \( x^2 \), and its cube for \( x^3 \), and so on with the others, if it is \( x \) that I wish to eliminate; or else if it is \( y \), [I can do this] by substituting \( x + \sqrt{x^2 - x^2} \) for \( y \), and the square, the cube, etc., of this sum for \( y^2 \), \( y^3 \), etc. So that there always remains after this an equation in which there is only one unknown quantity: either \( x \) or \( y \).\(^{45}\)

That is, in the equation of the curve CE, once CE is known, we may substitute either the value derived here in terms of \( y \) for \( x \) or the value derived in terms of \( x \) for \( y \) to eliminate one variable. Descartes' discussion to this point has been quite general, applicable to any curve CE no matter what its equation.\(^{46}\) Then to illustrate the generality of his method Descartes gives three examples of different curves. In each of these three cases the indeterminate equation is derived immediately from the given data and one term, either \( x \) or \( y \), is substituted for the other.\(^{47}\) The result, then, is that the equation will be in terms no longer of \( x \) and \( y \), but rather of \( s, v \), and either \( x \) or \( y \). Notes Descartes,

\(^{45}\) Oiscamp, p. 208.
\(^{46}\) Descartes does not state, however, the requirement that curve CE must not loop back upon itself at exactly point C.
\(^{47}\) In one example he also introduces a third variable, \( z \).
Now after we have found such an equation, instead of using it to determine the quantities $x$ [or] $y$, which are already given because point C is given, we must use it to find $v$ or $s$, which determine the required point P. And to this end it is necessary to consider that if point P is such as we wish it to be [i.e. is in fact the normal], it will be the center of a circle which will pass through point C, where it will touch, but not cut, the curved line CE [vide figure 3]: but if this point P be ever so little nearer to, or farther from, point A than it should be, this circle will cut the curve not only at point C, but also necessarily at some other point.\footnote{Oiscamp, p. 210.}

Recall that point C and curve CE are given. Descartes imagines that as point P varies on line GA the distance CP will also vary; and if CP is considered the radius of a circle, such circle will also vary in circumference correspondingly. If the circle is sufficiently large to cut the
curve CE, it will either cut it at two places or touch it at one; and if it touches it at one, it will be equivalent to the tangent of the curve at that point. Thus for all values of P except that of the normal to the curve at C, there will be either two distinct roots for the equation, or there will be none. Only at just that one position where P lies upon the normal will the two roots coincide:

[T]he closer the two points C and E are to one another, the less difference there is between the two roots; and finally if the two points coincide, the roots are exactly equal, that is, the circle that passes through C will touch the curve CE there without cutting it.49

The point to be made is as follows. Take the equation \( x^2 - 7x + 12 = 0 \) for example. This is a quadratic equation and will have two roots; that is, there are two solutions which will satisfy the equation. In this case it happens that the roots are +4 and +3. Thus we are saying \( x = 4 \) and \( x = 3 \), or that \( x - 4 = 0 \) and \( x - 3 = 0 \). We are saying also, then, that the left side of the equation when that equation is set equal to zero (in this instance the left hand member, \( x^2 - 7x + 12 \)) is divisible by both \( x - 4 \) and by \( x - 3 \) (taking for granted of course that the divisor does not go to zero). Take, however, the equation \( x^2 - 8x + 16 = 0 \). In this case there are again two roots: \( x = 4 \), and also \( x = 4 \) once again; or \( x - 4 = 0 \) and, again, \( x - 4 = 0 \). Thus the equation is divisible by \( x - 4 \) twice. Leaving out some details having to do with the signs of the roots, and to make the point more general, Descartes notes that if in fact, as has been stated, the roots do coincide, the equation of the

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49 Osclamp, p. 211.
curve in question must be of such a nature that it is divisible by, to use a
rustic modern notation not utilized by Descartes, \( (x \ - \ \text{root}_i) \) once, and then
by the identical \( (x \ - \ \text{root}_i) \) once again; thus it is divisible by \( (x \ - \ \text{root}_i)^2 \).
In other words, if as in our example the equation of the given curve really
has two identical roots, then it must be of such a form as that \( (x \ - \ \text{root}_i)^2 \)
may be factored evenly from it. This will then leave after the factorization
a right-hand term of a polynomial of degree two less than that of the
polynomial in the original equation. As Descartes says:

\[
\text{[\text{If this last equation is not of as high a degree as the preceding one, we multiply it by another equation having as many dimensions as the second equation lacks... \text{\textsuperscript{50}}}]}\]

That is, that which we must multiply the square of the duplicate
factors by is just that polynomial which would be the remainder in form
after such factorization from the original equation; or, to utilize again a
rustic modern form of symbolization:

\[
(x \ - \ \text{root}_i)^2 \text{ times } ??? = \text{ form of original equation.}
\]

In contemporary notation, and if the original equation for curve CE
is of order \( n \), this would leave an equation of the form:

---
\textsuperscript{50} Oscamp, pp. 211-2.
\[(x - \text{root}_i)^2 (a_1 x^{n-2} + a_2 x^{n-3} + \ldots + a_{n-1}) = 0\]

But Descartes does not use this term 'root', instead he simply designates the term as \(\alpha\); in which case we have:

\[(\alpha - e)^2 (a_1 x^{n-2} + a_2 x^{n-3} + \ldots + a_{n-1}) = 0\]

Now we have two equations. One is for the curve CE. The other is the form any equation must be in if it satisfies the criteria for being the normal to the curve CE at point C. And as Descartes further notes:

[Each of the terms of the one equation will [now] correspond to each of the terms of the other.\textsuperscript{51}]

With this statement Descartes introduces for the first time a method which has come to be called the method (or principle) of indeterminate coefficients.\textsuperscript{52} For purposes of illustration, let us take the second, and most complicated, of the three examples given by Descartes mentioned above. From the data, the equation of the curve has been ascertained to be:

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\textsuperscript{51} Olsamp, p. 212.
\textsuperscript{52} 'Principle of Undetermined Coefficients: If two polynomial expressions are identically equal, the coefficient of a term of any power in one may be equated to the coefficient of the term of the same power in the other.' From: Millington, T. Alaric & William Millington, Dictionary of Mathematics, Barnes & Noble, N.Y., 1966, p. 186. (By 'identically equal' they seem to mean that one equation divided by the other will yield unity.)
According to Scott: 'The method [of indeterminate coefficients] was afterwards used with great effect by Newton, and Descartes himself was well aware of its power.' Scott, p. 117.
\[ y^6 - 2by^5 + (b^2 - 2cd + d^2)y^4 + (4bc - 2d^2)v^2 + (c^2d - 2b^2cd + d^2v^2 - dv^2s^2) y^2 - 2bc - 2d^2 y + bc = 2d^2. \]

This equation is of degree six. If we factor out a quantity of form \((y - e)^2\), or \((y^2 - 2ey + e^2)\), we shall have a polynomial in \(y\) of degree two less than degree six, that is of degree four. Which is to say the polynomial which will be the right-hand term on the left side of the equation when the equation is set equal to zero will be of the form:

\[(y^4 + fy^3 + gy^2 + hy + k^4),\]

where \(f\), \(g\), \(h\), and \(k\) are unknown constants. Thus the equation must have the same form as the expression obtained by multiplying:

\[(y^2 - 2ey + e^2) \quad \text{by} \quad (y^4 + fy^3 + gy^2 + hy + k^4),\]

or, the polynomial when expanded will have the form:

\[ y^6 + (f - 2e)y^5 + (g - 2ef + e^2)y^4 + (h^2 - 2eg + 2e^2f)y^3 + (k^4 - 2eh^3 + e^2g^2)y^2 + (e^2h^3 - 2ek^4)y + e^2k^4. \]

Again, we therefore have two equations. One is the given equation; the other is a general equation whose form satisfies certain characteristics ascertained specific to the normal to the curve; viz. that there should be just
two roots and that these roots should be equal. To repeat, Descartes has just said:

[Each of the terms of the one equation will [now] correspond to each of the terms of the other.\textsuperscript{53}

To recapitulate, the two equations in question are:

\begin{align*}
a) & \quad y^6 - 2by^5 + (b^2 - 2c d + d^2)y^4 + (4b c d - 2d^2) y^3 \\
& \quad + (c^2 d^2 - 2b^2 c d + d^2) y^2 + 2b c d^2 y + b^2 c^2 d^2,

b) & \quad y^6 + (f - 2e) y^5 + (g^2 - 2ef + e^2)y^4 + (h^3 - 2eg^2 + e^2 f) y^3 \\
& \quad + (k^4 - 2eh^3 + e^2 g^2) y^2 + (e^2 h^3 - 2ek^4) y + e^2 k^4.
\end{align*}

As Descartes says,

[From these two equations I can derive six others, which may be used to find the six quantities \(f, g, h, k, v,\) and \(s\).\textsuperscript{54}

In short, what we do is we set the coefficients of the variables to the same degree in the polynomials equal to one another. That is, the polynomials (\(a\)) and (\(b\)) are equal for all values of \(y\). Thus, the coefficients of \(y^5\) must be equal:

\textsuperscript{53} Olscamp, p. 212.
\textsuperscript{54} Olscamp, p. 213. Olscamp lists the unknown variables as \(f, g, h, k, o,\) and \(s\). The inclusion of \(o\) here instead of \(v\) would seem to be a misprint.
(I) \(-2b = (f - 2e)\);

and the coefficients of \(y^4\) must be equal:

(II) \((b^2 - 2c d + d^2) = (g^2 - 2ef + e^2)\);

and so on for the other coefficients:

(III) \((4bc d - 2d^2v) = (h^3 - 2eg^2 + e^2f)\);

(IV) \((c^2d^2 - 2b^2c d + d^2v^2 - d^2s^2) = (k^4 - 2eh^3 + e^2g^2)\);

(V) \(-2b c^2d^2 = (e^2h^3 - 2ek^4)\); and

(VI) \(b^2c^2d^2 = e^2k^4\).

And also, as Descartes indicates:

From this it is easy to understand that to whatever class the proposed curve belongs, by this procedure there will always be as many equations as we are obliged to assume there are unknown quantities. But in order to solve these equations in order, and finally to find the quantity \(v\) -- which is the only one we need, and which is the occasion of our looking for the others -- we must first look for the value of \(f\), the first of the unknown quantities of the last expression, through the second term….\(^{55}\)

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\(^{55}\) Olscamp, 213.
Briefly, we may determine, or solve for, $f$ from equation (I), $g$ from equation (II), $v$ from equation (III), $s^2$ from equation (IV); $h$ from equation (V); and $k$ from equation (VI). Since we seek $v$, we look to equation (III); but there we see we must first solve for $h^3$, $g^2$, and $f$. These only will be the quantities we must first solve for.\footnote{That is, we needn’t solve for $h$ and $g$ but only for $h^3$ and $g^2$.} So, Descartes proceeds -- in order:

\[
f = 2e - 2b \ ,
\]

\[
g^2 = 3e^2 - 4b - 2c \cdot d + b^2 + d^2 \ , \text{ and}
\]

\[
h^3 = \frac{2b^2c^2d^2}{e^3} - \frac{2b^2c^2d^2}{e^2} .
\]

From here by substitution we may now solve for $v$:

\[
4bcd - 2d^2 v = \frac{2b^2c^2d^2}{e^3} - \frac{2b^2c^2d^2}{e^2} - 2e (3e^2 - 4be - 2cd + b^2 + d^2 )
\]

\[
+ e^2 (2e - 2b ) , \quad \text{or}
\]
\[ v = \frac{2e^3}{d^2} - \frac{3be^2}{d^2} + \frac{b^2e}{d^2} - \frac{2ce}{d} + e + \frac{2bc}{d} + \frac{bc^2}{e^2} - \frac{b^2c^2}{e^3} . \]

**figure 2.**

Since \( e \) is the solution of the quadratic equation \((y - e)^2 = 0\) as noted earlier, \( e \) and \( y \) may be freely interchanged. Thus we may set \( y = e \), and then we have:

\[ v = \frac{2y^3}{d^2} - \frac{3by^2}{d^2} + \frac{b^2y}{d^2} - \frac{2cy}{d} + y + \frac{2bc}{d} + \frac{bc^2}{y^2} - \frac{b^2c^2}{y^3} ; \]

which gives us the length of AP which we seek, and the length of AP determines \( s \) in turn.

Be Descartes' discussion as it may, however, the importance of this example is to see that in order to determine the unknown quantities which
we seek, in this case $s$ or $v$, we designate them as if they were known; then treating both known and unknown quantities without discrimination we solve for the unknown. In Descartes' introductory remarks in his explanation of his method in Book I of the *Géométrie* he states:

Thus, if we wish to solve some problem, we should first of all consider it solved, and give names to all the lines -- the unknown ones as well as the others -- which seem necessary in order to construct it. Then, without considering any difference between the known and the unknown lines, we should go through the problem in the order which most naturally shows the mutual dependency between these lines, until we have found a means of expressing a single quantity in two ways. This will be called an equation, for the terms of one of the two ways [of expressing the quantity] are equal to those of the other.\(^{57}\)

'I assume the problem already solved,' states Descartes, from which point he proceeds to set up the proper equations and solve for the unknown. The equation or equations from which the solution emanates needs, in fact, to be in terms of the unknown for the method to work at all.

In any case, the primary point I am trying to make is that, for Descartes, mathematical analysis is in the setting up of an equation in terms of one or multiple unknowns with the other data of the given condition expressed in relation to those unknowns. The allocation of the unknowns assumes the problem as already solved. And this is the fundamental notion of analysis, at least in its mathematico-geometric sense, of which Descartes

\(^{57}\) Oslamp, p. 179.
speaks.\textsuperscript{58} It is unclear at this point, however, what part, this mathematico-
geometric sense of analysis is to play in Descartes’ overall conception of his 
‘universal Science’.

\textsuperscript{58} This point is, essentially, made by Mahoney, when he states: ‘Reflection on the matter makes it clear, on the one hand, why men such as Ramus, Viète, and Descartes considered algebra analytic in nature and, on the other, why in taking the further step of equating algebra and analysis they necessarily did violence to the classical doctrine of analysis. Algebra links one or more unknown quantities with known quantities in an equation and operates on that equation as if all the quantities were equally known. It aims to express the unknown quantities in terms of combinatorial products of the known and thereby to make the unknown quantities known.’ \textit{Fermat}, p. 33.

Mahoney also says: ‘A closer look … at the identification of algebra and analysis reveals an underlying lack of comprehension of the nature of Greek geometry. The analytic geometries of Fermat and Descartes alone suffice to show by contrast that Greek geometry was not algebraic in any essential way. Nor does Greek geometrical analysis rest on algebraic foundations. At heart, algebra and classical Greek geometry represent two substantially different approaches to mathematics and reflect different demands on mathematical knowledge.’ \textit{Fermat}, p. 33.

As insightful and as pleasurable to read as Mahoney is, I shall, in effect, argue against the consideration of algebra as non-analytic in Section Three of Chapter Four.
Chapter 2: Indirect and Reverse Problem Solution in the \textit{Regulae}.

Outside of Descartes’ claim, made above, that his \textit{Dioptrique}, \textit{Météores}, and \textit{Géométrie} are examples of his method, what evidence do we have of any methodological connection between Descartes’ mathematics and his approach to natural science? Something important which Descartes’ \textit{Regulae Ad Directionem Ingenii} or \textit{Rules for the Direction of the Mind} shows, I believe, is how closely Descartes considered his methods in these disciplines. In this chapter I intend to focus on some of the major concepts in the two works, \textit{La Géométrie} and the \textit{Regulae}, in some detail so as to draw out some strains from Descartes’ thought I shall develop later.

Specifically, my intent is the following. First I shall make a direct comparison of some of the material in the two works with the purpose in mind of showing an intrinsic similarity in approach and, importantly, method between the two works. I shall also argue that on the basis of this similarity, an affinity may be discerned in Descartes’ approach to the solution of problems in the two fields of mathematics and natural science. Because some statements concerning method in natural science occur in the \textit{Regulae}, I shall later, in Chapter 5, utilize the parallel methodology suggested in this Chapter between these two works as a bridge to the understanding of what I take Descartes to be expressing in his statements concerning both his general methodology overall and his approach
specifically to problem solution in these two disciplines. Second, in this chapter I shall inspect some comments in the Regulae regarding the difference between what Descartes terms ‘direct’ and ‘indirect’ problem solution. In this section I shall bring forth evidence that, from a Cartesian point of view, what Descartes calls ‘indirect’ problem solution may be considered a form of analysis. In the third part of this Chapter I relate Descartes’ claims concerning direct and indirect problem solution with some of his statements concerning arithmetical operations, and then make reference to a mechanical device mentioned by Descartes in La Géométrie with the purpose of indicating a manner in which the function of this device may be conceived as an example of the distinction between direct and indirect problem solution. These topics in turn will give greater depth to my claims both concerning the similarity between the Regulae and La Géométrie, and concerning the analytic nature of indirect problem solution. Finally, at the end of this Chapter, I shall have some summary remarks and conclusions.


In this section my intention is to point out a structural similarity between the Géométrie and the Regulae. If I can establish such a structural affinity between the two works, it is my expectation that Descartes’ statements concerning his method in natural science in the
Regulae can then be related to his method of mathematics in the Géométrie, hopefully to shed light on a method shared by the two works. This I expect will be worth our while as it will afford a bridge between Descartes’ mathematics and his natural science.

As I mentioned before, Descartes’ Géométrie was published along with two of his other works, the Dioptrique and the Météores, and was to have been, along with these others, an example of Descartes’ new method. The Géométrie is partitioned into three books. The first book is to be a general description of Descartes’ geometric method of algebraic analysis and is entitled Of Problems That Can Be Constructed Using Only Circles and Straight Lines. The second book is entitled, Of the Nature of Curved Lines, and the third book is entitled Of the Construction of Solid and Supersolid Problems. My claim, actually one put forward by Beck, will be that there is a structural relationship between the the first book of the Géométrie and the second section of the Regulae.

Let us start with the most striking and obvious similarity between the two works. Compare the following two statements, the first from the Géométrie, the second from the Regulae:

Thus, if we wish to solve some problem, we should first of all consider it solved, and give names to all the lines -- the unknown ones as well as the others -- which seem necessary in order to construct it. Then, without considering any difference between the known and the unknown lines, we should go through the problem in the order which
most naturally shows the mutual dependency between these lines, until
we have found a means of expressing a single quantity in two ways.¹

We should make a direct survey of the problem to be solved,
disregarding the fact that some of its terms are known and others
unknown, and intuiting, through a train of sound reasoning, the
dependence of one term on another.²

I have argued earlier that the former statement is central to the
method of the Géométrie. The similarity does not stop there, however.
There exist additional striking similarities between the two works. Beck
notes the following:

Rule 16 states that ‘when we come across matters which do not
require our attention at the moment, although they are necessary for our
conclusion, it is better to represent them by the briefest possible
symbols’. On the third page of La Géométrie is a small section devoted
to the use of symbols, and in particular the use of algebraic signs in
geometry.

Rule 17 advises us ‘that when a problem is put forward for
discussion, we should run over the problem in a direct course neglecting
the fact that some of its terms are known, others unknown’. On the
fourth page of La Géométrie we find the recommendation that when
faced with a problem, we should consider it as done ‘and give names to
all the lines, known or unknown, which are necessary for its
construction’.

Rule 18 lays down that to achieve the purpose of the previous rule,
‘only four operations are required, addition, subtraction, multiplication,

¹ Olscamp, p. 179.
² CSM1, p. 70; AT X, 459.
and division'. The second paragraph of the Géométrie contains an identical enumeration.

Rule 19 states that 'employing this method of reasoning we must find out as many magnitudes as we have unknowns terms, these latter treated as though they were known... for this will give us as many equations as there are unknowns'. The Géométrie contains a similar injunction: 'one must find as many equations as there are lines which have been assumed as unknowns'.

Rule 20 states that 'when we have obtained our equations, we must carry out such operations as we have omitted, taking care never to multiply when we can divide'. The Géométrie advises that in treating these equations, 'we should use division whenever this is possible and thus we will obtain infallibly the simplest terms to which the problem can be reduced'.

Finally, Rule 21 states that 'in the case of several equations of this kind, we should reduce them all to a single equation, namely, one whose terms do not occupy so many places in the series of magnitudes which are in continued proportion'. The Géométrie, on the fifth page, says that the equations should be reduced to one, or an equation of equations.³

And as Beck goes on to comment,

The identity of doctrine, and even of verbal expression, is striking....⁴

We are aware that the Regulae was to have been written in three sections of twelve rules each. As these similarities have been noted in Rules Sixteen through Twenty-one, we can conclude, I think, from even

³ Beck, Method, pp. 177-8.
⁴ Beck, Method, p. 178.
this brief review, that, at least portions of what was intended by Descartes to be the central, the second, section of the *Regulae* are related structurally to the opening section of *La Géométrie*. Clearly Descartes sees the method in the two works to be the same in some sense. But let us remember here the passages given above which imply not that the methods for solving these types of problems are identical, but that the method in mathematics is derivative from and in some sense dependent upon a higher and more comprehensive method: a ‘universal Science’. The similar statements in the two works might be attributed to the idea that Descartes is speaking of his method of mathematics in both works. And there is some truth to this notion. I shall point out at this time, however, that though Descartes’ method in the *Regulae* sounds as if he is speaking of his mathematical method in *La Géométrie*, the actual examples he uses are not taken from mathematics but are rather taken from natural philosophy, that is from science. These include examples in acoustics, cardiovascular function, and magnetism. Nor do the discussions of the mathematical and scientific content appear in different sections of the work; characteristically Descartes gives what appears to be an explication of his method in mathematics, but follows it up immediately with examples, not from mathematics, but from natural science.\(^5\) Since the proposed method of the *Regulae* is so similar to that of the *Géométrie*, and since the examples given in the *Regulae* are taken from natural science, my conclusion is that Descartes saw an affinity in problem solution in the two fields. What similarities Descartes feels he saw remains as yet an open question, though

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\(^5\) Vide Chapter Five, Section I, for a substantiation of this claim.
I shall have more to say on this later. But the point ultimately to be made here, then, is that the similarity in the two works indicates an affinity of method between the two derivative branches of mathematics and science. To some extent, what may be said of one may be said of the other.

In addition, an historical point which can also be made is that regardless whether we consider the *Regulae* an early work and the *Discours* a middle or mature work, a strong structural continuity holds between portions of the *Regulae* and *La Géométrie*, the latter a work published together with the *Discours*. This is circumstantial evidence for continuity in at least this aspect of Descartes' thought -- which is the aspect which most holds our interest at the moment: Cartesian analysis.

Part II: Direct vs. Indirect Solution of Problems.

We have seen that there exists a certain structural affinity between Descartes' *La Géométrie* and parts of his *Regulae*. Since in the *Regulae* Descartes makes a differentiation between what he calls the 'direct' and the 'indirect' solution of problems, I would like to look at this distinction for a moment with the expectation it may be related to his notion of analysis.

Before we begin, however, I would like to make a note on terminology. It will be remembered that in Euclidean mathematics, if a ratio is set such that a pair of terms are said to be proportional or similar to another pair, (e.g. \(a:b::c:d\)), then the two internal terms (in this case
terms b and c) are called the ‘means’ or ‘mean terms’, and the two external terms (in this case terms a and d) are called the ‘extremes’ or ‘extreme terms’. When the mean terms are identical, as in the ratio: a:b :: b:c, the ratio is called a continued proportion. A proportion or a continued proportion may be extended indefinitely; thus a continued proportion might also be of the form: a:b :: b:c :: c:d :: d:e, and so forth. An example of a continued proportion, and one Descartes uses, would be: 3, 6, 12, 24, 48; that is to say, the continued proportion itself would be: 3:6 :: 6:12 :: 12:24 :: 24:48. Thus, in the continued proportion: 3, 6, 12, 24, 48; the extreme terms would be: 3 and 48; and the mean terms would be: 6, 12, and 24.6

This having been said, to elucidate this distinction between ‘direct’ and ‘indirect’ solutions to problems, Descartes in Rule 6 introduces the series of numbers mentioned above: 3, 6, 12, 24, 48. As I have mentioned, he will speak of the first and the last terms in the series as the ‘extremes’ and the ones occurring in the middle as the ‘means’. He then maintains that if we are given two or more terms in a continued proportion and asked for the next term in the proportion, this is a different sort of problem than if we are given two extreme terms in a continued proportion and are asked to find the mean term?.

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6 According to Heath: ‘What we call a geometrical progression is with Euclid a series of terms “in continued proportion”’. Euclid 2, p. 346.

7 This whole discussion begins with the following passage: ‘[W]e should not begin our studies by investigating difficult matters. Before tackling any specific problems we ought first to make a random selection of truths which happen to be at hand, and ought then to see whether we can deduce some other truths from them step by step, and from these still others, and so on in logical sequence. This done, we should reflect attentively on the truths we have discovered and carefully consider why it was we were able to discover some of these truths sooner and more easily than others, and what these truths are. This will enable us to judge, when tackling a
[G]iven the magnitudes 3 and 6, I easily found a third magnitude which is in continued proportion, *viz.* 12, yet, when the extreme terms 3 and 12 were given, I could not find just as easily the mean proportional, 6. If we look into the reason for this, it is obvious that we have here a quite different type of problem from the preceding one. For, if we are to find the mean proportional, we must attend at the same time to the two extreme terms and the ratio between them, in order to obtain a new ratio by dividing this one. [trans. note: The problem: to find an \( x \) such that \( 3/x = x/12 \).] This is a very different task from that of finding a third magnitude, given two magnitudes in continued proportion. [trans. note: The problem: to find an \( x \) such that \( 3/6 = 6/x \).] 8

Altering the notation given in the Cottingham, Stoothoff, & Murdoch translation, the first problem given by Descartes here may be characterized:

\[
3:6 :: 6:x,
\]

whereas the second problem may be characterized:

\[
3:x :: x:12,
\]

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specific problem, what points we may usefully concentrate on discovering first. For example, say the thought occurs to me that the number 6 is twice 3: I may then ask what twice 6 is, *viz.* 12; I may, if I like, go on to ask what twice 12 is, *viz.* 24, and what twice 24 is, *viz.* 48, etc. It would then be easy for me to deduce that there is the same ratio between 3 and 6 as between 6 and 12, and again the same ratio between 12 and 24, etc., and hence that the numbers 3, 6, 12, 24, 48, etc. are continued proportionals." CSM 1, p. 23; AT X, 384.

Descartes was against the use of 'middle terms' in scholastic logic. Originally, however, the two -- middle and mean terms -- seem to have been related in the minds of the ancients.

8 CSM 1, pp. 23-4; AT X, 385.
with the difficulty being in each case to find the value of $x$. In the first example we have the extreme term 3 and the single mean term 6, but are missing the final extreme term -- which happens to be 12; whereas in the second example we have the extreme terms 3 and 12, but are missing the single, repeated mean term -- which happens to be 6. Descartes continues, stating that there is additionally a third type of problem: that of finding two proportional mean terms between a given pair of extreme terms (e.g. to find $x$ and $y$ in the continuing ratio $3:x :: x:y :: y:24$), and that this problem of finding two mean proportionals is 'even more complicated' than that of finding just one.\footnote{I can go even further and ask whether, given the numbers 3 and 24, it would be just as easy to find one of the two mean proportionals, viz. 6 and 12. Here we have another sort of problem again, an even more complicated one than either of the preceding ones. We have to attend not just to one thing or to two but to three different things at the same time, if we are to find a fourth. [trans. note: The problem: to find an $x$ and $y$ such that $3/x = x/y = y/24$.] We can go even further and see whether, given just 3 and 48, it would be still more difficult to find one of the three mean proportionals, viz. 6, 12 and 24. At first sight it does indeed seem to be more difficult. But then the thought immediately strikes us that this problem can be split up and made easier: first we look for the single mean proportional between 3 and 48, viz. 12; then we look for a further mean proportional between 3 and 12, viz. 6; then another between 12 and 48, viz. 24. In that way we reduce the problem to one of the second kind described above'. CSM 1, p. 24; AT X, 385-6. I suspect that this distinction between three types of problems is somehow structurally important to a full understanding of both the three sections of the *Géomdrie* and the three sections of the *Regulae*; but what exactly that structural relationship is, I have not yet discerned.} It may at first seem more difficult to find 3 mean proportionals, but Descartes states that this is no more difficult than finding one because the former problem may be reduced to the latter.\footnote{That is, the problem $3:x :: x:y :: y:24$ can be reduced first to the problem of finding $y$ in the continued proportion $a:y :: y:b$ and from there then finding $x$ and $z$.}

Using these examples, Descartes then goes on to draw his distinction between what he calls direct and indirect solution of problems:

Moreover, from these examples I realize how in our pursuit of knowledge of a given thing we can follow different paths, one of which
is much more difficult and obscure than the other. If, for example, we are asked to find the four proportionals, 3, 6, 12, 24, given any two consecutive members of the series, such as 3 and 6, or 6 and 12, or 12 and 24, it will be a very easy task to find the others. In this case we shall say that the proposition we are seeking is investigated in a direct way. But if two alternate numbers are given, such as 3 and 12, or 6 and 24, and we are to work out the others from these, in that case we shall say that the problem is investigated indirectly by the first method. Likewise, if we are to find the intermediate numbers, 6 and 12, given the two extremes, 3 and 24, then the problem will be investigated indirectly by the second method. I could thus go on even further and draw many other conclusions from this one example. But these points will suffice to enable the reader to see what I mean when I say that some proposition is deduced ‘directly’ or ‘indirectly’, and will suffice to make him bear in mind that on the basis of our knowledge of the most simple and primary things we can make many discoveries, even in other disciplines, through careful reflection and discriminating inquiry.\footnote{CSM 1, p. 24; AT X, 386-7. Italics added for emphasis.}

I would like to add at this point, based on the last sentence in this passage, the following similarity between indirect problem solution and analysis from Descartes’ point of view: both are methods for the making of discoveries, even in disciplines other than mathematics.

But in this passage Descartes is defining here by means of example; unfortunately the examples, though provocative and even simple in character, are not altogether perspicuous. We may note initially a kind of rough similarity between the second, or ‘indirect’ type of problem solution and Descartes’ injunction to ‘assume the unknown as if it were known precisely to solve for that unknown’ given in his \textit{La Géométrie} and echoed
in his *Regulae*. Descartes even makes reference to this edict when he talks about the conversion of an indirect method of inquiry into a direct one:

Now if, in order to deduce the way in which the last [in a sequence of interlinked propositions] depends on the first, we intuit the interdependence between the individual propositions without ever interrupting the order, we are going through the problem in a direct way. If on the other hand we know the first and last propositions to be interconnected in a definite way, and we wish to deduce from this the intermediate ones connecting them, the order we follow will be completely indirect and the reverse of the previous one. Now we are concerned here only with complicated questions where the problem is to discern, albeit in a complicated order, certain intermediate propositions, on the basis of our knowledge of the first and last propositions in the series. *So the trick here is to treat the unknown [propositions] as if they were known.* This may enable us to adopt the easy and direct method of inquiry even in the most complicated of problems. There is no reason why we should not always do this, since from the outset of this part of the treatise [trans. note: i.e. from Rule Thirteen] our assumption has been that we know that the unknown terms in the problem are so dependent on the known ones that they are wholly determined by them. Accordingly, we shall be carrying out everything this Rule prescribes if, recognizing that the unknown is determined by the known, we reflect on the terms which occur to us first and count the unknown ones among the known, so that by reasoning soundly step by step we may deduce from these all the rest, even the known terms as if they are unknown.\(^1\)

Notice there seems to be more of an air of generality about this statement than the earlier ones concerning continued proportions. For one

\(^1\) CSM 1, pp. 70-1; AT X, 460-1. Italics added for emphasis.
thing Descartes speaks here of 'propositions' whereas elsewhere he speaks primarily of 'magnitudes' or 'members'—though clearly the topic of the distinction between direct and indirect ways of solving problems is the same. But to recapitulate what Descartes has just said, Descartes believes here that in cases where we may assume the unknown terms to be wholly determined by the known terms in a series of interlinked propositions (that is, I take it, that the solution set of the unknown terms is determinate), we ought exactly to assume such unknown terms as if they were known so that we may reflect upon them and then deduce these unknown terms from the known. Also, Descartes says, this process enables us to adopt the easy and direct method of inquiry; and it even allows us to deduce ultimately the known terms as if they were unknown.

To provide only a bit of interpretation at this point, in the ratio: a:x::x:b, as a or b vary, so will x; this is what makes the unknown term x determinate. In the words of Descartes:

[T]he question becomes simply one of discovering certain magnitudes on the basis of the fact that they bear such and such a relation to certain given magnitudes.

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13 We may note here that a proportion may be thought of as a proposition or series of propositions insofar as it may be thought of as an equation or a series of equations. In the words of Descartes: 'We should note also that those proportions which form a continuing sequence are to be understood in terms of a number of relations...'. CSM 1, p. 68; AT X, 456. I shall refer back to this point in Chapter 4.

14 'Determine, vb. to be sufficient for the unique specification of (some entity). For example, any 2 points determine a straight line; a definite integral is determined to a constant.' Borowski, E.J. and J.M. Borwein, Dictionary of Mathematics, Collins, London, 1989.

15 CSM 1, p. 70; AT X, 459.
As I have alluded to above, this process also seems applicable to more than just ‘certain magnitudes’; ‘propositions’ have also been mentioned.

In any case, let us keep the passage above in mind as it relates direct and indirect problem solution with the utilization of known and unknown terms. This may also be taken as evidence, strong evidence, that from a Cartesian point of view the method of indirect problem solution may be considered a form of analysis.

Let us note in addition, however, that Descartes also speaks of the indirect method in the context of a ‘reverse movement of the imagination’:

We shall explain in due course how to find any number of mean proportionals between [the unit and the last magnitude to be divided]. For the moment we must be content to point out that we are assuming that we have not yet quite done with these operations [the extraction of roots], since in order to be performed they require indirect and reverse movements of the imagination, and at present we are dealing only with problems which are to be treated in the direct manner.16

Whatever Descartes has in mind, the analysis of the ancients was, of course, such a ‘reverse movement’. We have yet to examine the relation between the analysis of the ancients and Cartesian mathematical analysis, but let us here remember at this point Descartes’ cryptic comment concerning ‘the analytic style of writing’ which allows him to ‘make certain assumptions that have not yet been thoroughly examined’, many of which

16 CSM 1, p. 75; AT X, 467.
he proceeds to refute. The postulation of such assumptions and their subsequent acceptance or refutation may be considered such a process of ‘reverse movement of the imagination’.

An additional point can be made here. Descartes also says the reason indirect problem solution is more difficult is because it requires more than one act of conceiving:

But if only the first and the third [terms] are given, it will not be so easy for me to discern the intermediate magnitude, for this can be done only by means of an act of conceiving which simultaneously involves two of the [particular and distinct acts of conceiving] just mentioned. If only the first and the fourth magnitudes are given, it is even more difficult to intuit the two intermediate ones, for in this case three acts of conceiving are simultaneously involved. 17

Let us review the major points I have just made in this section; we now have three reasons for believing that by ‘indirect problem solution’

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17 This statement is taken from the following passage:

‘[B]y reflecting on the mutual dependence of simple propositions we acquire the habit of distinguishing at a glance what is more, and what is less, relative, and by what steps the relative may be reduced to the absolute. For example, if I run through a number of magnitudes which are continued proportionals, I shall be struck by the following points. It is just as easy for me to recognize the relation between the first and the second magnitude, as between the second and the third, the third and fourth, etc., and the act of conceiving is exactly similar in each case. But it is more difficult for me to form a simultaneous conception of the relation of the second magnitude to the first and the third; and it is much more difficult still to conceive the way in which it depends on the first and fourth magnitudes, etc. These considerations enable me to understand why it is that, given only the first and second magnitudes, I can easily find the third and fourth, etc.: the reason is that the discovery is made by means of particular and distinct acts of conceiving. But if only the first and the third are given, it will not be so easy for me to discern the intermediate magnitude, for this can be done only by means of an act of conceiving which simultaneously involves two of the acts just mentioned. If only the first and the fourth magnitudes are given, it is even more difficult to intuit the two intermediate ones, for in this case three acts of conceiving are simultaneously involved. So, as a logical consequence, it might seem even more difficult to find the three intermediate magnitudes given the first and fifth. Yet this is not the case, owing to a further reason, which is that, although four acts of conceiving are joined together in the present example, they can nevertheless be separated, since four is divisible by another number. So I can obtain the third magnitude alone on the basis of the first and the fifth, the the second on the basis of the first and the third, etc. If one is used to reflecting on these and similar matters, whenever one investigates a new problem one will immediately recognize the source of the difficulty and the simplest method for dealing with it. And that is the greatest aid to knowledge of the truth.’ CSM 1, pp. 38-9; AT X, 409-10.
Descartes means ‘analysis’. Firstly, from the passage given a bit earlier, Descartes refers to indirect problem solution as a method for making discoveries, even outside the field of mathematics. He has said this same thing concerning the method of analysis earlier. Secondly, he has said of indirect problem solution that the ‘trick’ is to treat of the unknown as if it were known. This I have regarded earlier as his central method of mathematical analysis. Thirdly, Descartes has spoken of indirect problem solution as a ‘reverse movement of the imagination’. Outside of this remark sitting well with the analysis of the ancients, more importantly it sits well with Descartes’ cryptic remark concerning the ‘analytic style of writing’.

The topic is not, however, closed, and I’m afraid I have more to say on it in the next section.

Part III: The Cartesian Compass.

In this section I am going to try to further draw out the relationship between Descartes’ notion of indirect problem solution and solving for unknowns as if they were known. There are some additional strands I wish to follow up on, however, having to do with Descartes’ statements in the Regulae concerning this distinction between direct and indirect problem solution in reference to proportions, particularly what were called by Descartes and by the ancients continued proportions.
To start out with, we have next an intriguing statement relating the operations of arithmetic with direct and indirect problem solution; this passage occurs later in the *Regulae*, in Rule Eighteen:

When we come to know one magnitude on the basis of our prior knowledge of the parts which make it up, the process is one of addition. When we discover a part on the basis of our prior knowledge of the whole and the extent to which the whole exceeds the part, the process is one of subtraction: there is no other possible way of deriving one magnitude from other magnitudes, taken in an absolute sense, which somehow contain it. But if we are to derive some magnitude from others which are quite different from it and which in no way contain it, it is necessary to find some way of relating it to them. If this relation or connection is to be made in a *direct* way, then we must use multiplication; if in an *indirect* way, then division.

In order to give a clear account of the latter two operations, we must be apprised of the fact that the unit, which we have spoken about earlier, [CSM 1, pp. 57f., 63, & 68] is here the basis and foundation of all the relations, and occupies the first place in a series of magnitudes which are in continued proportion. The given magnitudes occupy the second place in the series, while those to be discovered occupy the third, fourth, and the remaining places, if the problem in question is a *direct* one. If, however, the problem is an *indirect* one, the given magnitude comes last, and the magnitude sought comes in the second place or in other intermediate places.\(^{18}\)

Descartes speaks here about the 'unit' and I shall have something to say concerning this momentarily.

\(^{18}\) CSM 1, pp. 71-2; AT X, 462. Italics added for emphasis.
But in this passage the 'direct way' has been related in some fashion to multiplication, and the 'indirect way' has been related to division. Let us remember that the distinction between the arithmetical operations was important enough to warrant special mention in two of the Rules in the Regulae, Rules Eighteen and Twenty:

Rule Eighteen: For this purpose only four operations are required: addition, subtraction, multiplication, and division. The latter two operations should seldom be employed here, for they may lead to needless complication, and they can be carried out more easily later.\(^\text{19}\)

Rule Twenty: Once we have found the equations, we must carry out the operations which we have left aside, never using multiplication when division is in order.\(^\text{20}\)

Let us also recall once again both that Descartes has made general claims of utility for his method, and that the Regulae includes examples from natural science. As I have argued in Chapter One, Descartes has already stated he has something broader in mind than just mathematics or arithmetical operations when he speaks of these things.

Also, the astute reader will notice Descartes has included something additional in this later account of continued proportionals he had not spoken of before. Here he makes reference to something he calls the 'unit', and concerning this 'unit' he has the following to say:

\(^{19}\) CSM 1, p. 71; AT X, 461.
\(^{20}\) CSM 1, p. 76; AT X, 469.
We should note also that those proportions which form a continuing sequence are to be understood in terms of a number of relations; others endeavor to express these proportions in algebraic terms by means of many dimensions and figures. The first of these they call 'the root', the second 'the square', the third the 'cube', the fourth 'the square of the square'. I confess that I have for a long time been misled by these expressions. For, after the line and the square, nothing, it seemed, could be represented more clearly in my imagination than the cube and other figures modelled on these. Admittedly, I was able to solve many a problem with the help of these. But through long experience I came to realize that by conceiving things in this way I had never discovered anything which I could not have found much more easily and distinctly without it. I realized that such terminology was a source of conceptual confusion and ought to be abandoned completely. For a given magnitude, even though it is called a cube or the square of the square, should never be represented in the imagination otherwise than as a line or a surface, in accordance with the preceding Rule. So we must note above all that the root, the square, the cube, etc. are nothing but magnitudes in continued proportion which, it is always supposed, are preceded by the arbitrary unit mentioned above. The first proportional is related to this unit immediately and by a single relation; the second proportional is related to it by way of the first proportional, and hence by way of two relations; the third proportional by way of the first and the second, and so by way of three relations, etc. From now on, then, the magnitude referred to in algebra as 'the root' we shall term 'the first proportional'; the magnitude referred to as 'the square' we shall call 'the second proportional', and the same goes for the other cases.\footnote{CSM 1, p. 68; AT X, 456-7. Elsewhere Descartes also says: 'Unity is the common nature which, we said above [CSM 1, pp. 57 f.], all the things which we are comparing must participate in equally. If no determinate unit is specified in the problem, we may adopt as unit either one of the magnitudes already given or any other magnitude, and this will be the common measure of all the others. We shall regard it as having as many dimensions as the extreme terms which are to be compared. We shall conceive of it either simply as something extended, abstracting it from everything else -- in which case it will be the same as a geometrical point (the movement of which makes up a line, according to the geometers), or as some sort of line, or as a square.' CSM 1, pp. 63-4; AT X, 450.}
What Descartes is alluding to here is: provided that in a continued proportion if the first term is the unit, or the number '1', then the second term will be the root of the progression. Thus, in the continued proportion: $1:x :: x:16$, the second, or as we have been calling it the mean, term $x$ is the square root of the final term 16; in the proportion $1:x :: x:y :: y:27$, the second term $x$ is the cube root of 27; in the proportion $1:x :: x:y :: y:z :: z:625$, the second term $x$ is the quadratic root of 625 -- and so on.

There is a statement at the beginning of *La Géométrie* which is similar in character:

[J]ust as all of arithmetic is composed of but four or five operations -- namely, addition, subtraction, multiplication, division, and the extraction of roots, which may be considered a species of division -- so in geometry, in order to find the lines for which we are looking, we need only add to them, or subtract from them, other lines; or else, by taking one line which I shall call unity, in order to relate it as closely as possible to numbers, and which usually can be chosen arbitrarily, and then by taking two others, [we may] find a fourth line which is to one of these two lines as the other is to the unity -- which is the same as multiplication; or else [we may] find a fourth line which is to one of the two as the unity is to the other -- which is the same as division; or finally, [we may] find one, or two, or several mean proportionals between the unity and some other line -- which is the same as extracting the square root, or cube root, etc. And I shall not hesitate to introduce these arithmetical terms into geometry, in order to make myself more intelligible.22

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22 Olscamp p. 177. To return to the *Regulae* again, Descartes also has the following to say: 'As for those divisions in which the divisor is not given but only indicated by some relation, as when we are required to extract the square root or the cube root etc. in these cases we must note that the term to be divided, and all the
What Descartes means by the last sentence will become apparent presently. But to gain some insight into what Descartes may be talking about here, let us look at Descartes’ definition of arithmetical operations in the *Géométrie* which immediately follow this passage. Addition and subtraction are simple enough and have already been handled in what Descartes has had to say here; Descartes follows with the operations of multiplication, division, and the finding of various roots. He defines multiplication as follows:

For example, let $AB$ be unity, and let it be necessary to multiply $BD$ by $BC$; I have only to join points $A$ and $C$, then to draw $DE$ parallel to $CA$, and $BE$ is the product of this multiplication.\(^{23}\)

![Figure 4](image)

Division he defines in similar fashion:

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\(^{23}\) OlsCamp, pp. 177-8.
Or else, if it is necessary to divide $BE$ by $BD$, having joined the points $E$ and $D$, I draw $AC$ parallel to $DE$, and $BC$ is the product of this division.\textsuperscript{24}

And Descartes defines the extraction of Roots as follows:

![Figure 5](image)

figure 5.

Or, if it is necessary to extract the square root of $GH$, I add to it, along the same straight line, $FG$, which is the unity; and, dividing $FH$ into two equal parts at point $K$, from the center $K$ I draw the circle $FHH$; then, if I construct from point $G$ to point $I$ a straight line at right angles to $FH$, $GI$ is the required root.\textsuperscript{25}

These definitions follow from Euclid’s theory of the comparison of ratios located primarily in Book VI. Specifically Proposition 13 of Book VI may be brought to mention. This reads:

Proposition 13: To two given straight lines to find a mean proportional.\textsuperscript{26}

\textsuperscript{24} Olscamp, p. 178.
\textsuperscript{25} Olscamp p. 178.
\textsuperscript{26} Euclid 2, p. 216. Proposition 14 of Book II may also receive mention here.
In this figure, $AB$ and $BC$ are the two given straight lines; $DB$, it turns out, is the mean proportional. As pointed out by Heath, ‘This proposition ... is equivalent to the extraction of the square root.’

Descartes’ assessment would seem more correct, however, when he says, in effect, that provided one or the other of the two segments is the unity, this is equivalent to the finding of the square root of the other segment.

As noted in the last sentence in the passage from page 177 of *La Géométrie* given above, for his warrant Descartes simply states:

I shall not hesitate to introduce these arithmetical terms into geometry, in order to make myself more intelligible.

That is, Descartes throws these definitions in at the beginning of his work without apology, explanation, or reference to either their source or justification..

It is easy to see at this point, however, that Descartes defines the extraction of square roots in terms of mean proportions, and thus in terms of continued proportions. A bit more complicated is the situation having to do with his definitions of multiplication and division. These by no means

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27 Euclid 2, p.216.
28 Oiscamp, p. 177.
always involve continued proportions; but they may be said to involve mean and extreme terms. Descartes is very explicit about this latter point in the *Regulae*, where he states:

Thus if we are told that as the unit stands to a given magnitude \(a\) (5, say), so \(b\) (7, say) stands to the number we are seeking, which is \(ab\) (i.e. 35), then \(a\) and \(b\) occupy the second place, and their product, \(ab\), comes in the third place. ...For whether a magnitude is multiplied by itself or by a quite different magnitude, the process of multiplication is the same.\textsuperscript{29}

Or, \(1:5 :: 7:x\); in this case \(x = 35\). This is not a continued proportion, it is a simple, or as I shall call it a ‘general’, proportion, where the magnitude (5) is not multiplied by itself but by a different number (7). In the case of Descartes’ definition of multiplication given above, we would have, where \(AB\) is the unit: \(AB\) (the number 1):\(BD :: BC:BE\). These same remarks apply *mutatis mutandis* to division as well. Now let us consider the following passage which I have already given once above:

When we come to know one magnitude on the basis of our prior knowledge of the parts which make it up, the process is one of addition. When we discover a part on the basis of our prior knowledge of the whole and the extent to which the whole exceeds the part, the process is one of subtraction: there is no other possible way of deriving one magnitude from other magnitudes, taken in an absolute sense, which somehow contain it. But if we are to derive some magnitude from others which are quite different from it and which in no way contain it,

\textsuperscript{29} CSM 1, p. 72; AT X, 463
it is necessary to find some way of relating it to them. If this relation or connection is to be made in a direct way, then we must use multiplication; if in an indirect way, then division.\(^{30}\)

The question here is whether multiplication and division are to be considered themselves forms of direct and indirect problem solution respectively. I confess from this passage I cannot tell -- though I suspect that they are.\(^{31}\) But I don't believe this to be of major concern for the topic presently under discussion. Suffice it to say that if they are, it would therefore appear that 'direct' relations of magnitudes, and thus 'direct' solution of problems, would have to do with the unknown magnitude appearing as an extreme term in a general proportion, whereas 'indirect' relations of magnitudes, and thus 'indirect' solution of problems, would have to do with the unknown magnitude appearing as a mean term in a general proportion. But this is what I would assume it to be even if Descartes did not consider the general cases of multiplication and division examples of direct and indirect problem solution anyway; the question then being the inconsequential one of whether the proportion is 'general' or 'continued'. As Descartes has said:

\(^{30}\) CSM 1, pp. 71-2; AT X, 462. Cited earlier in fn. 18.

\(^{31}\) I take it when Descartes speaks of 'the latter two operations' here he is speaking of the operations of multiplication and division as juxtaposed against the earlier operations of addition and subtraction. Indeed, the unit is the foundation of these operations, as is shown in their definitions in La Géométrie which are given above. If my estimation is correct, it sounds as if Descartes is attempting to give here 'a clear account' of these general operations or general proportions rather than a specific account of continued proportions. As I have mentioned, the difference between a general proportion and a continued proportion, I take it, is that in a continued proportion the mean terms will be identical, whereas in a general proportion they needn't be. The rest of the material in this passage provides no evidence against this more general view.
The given magnitudes occupy the second place in the series, while those to be discovered occupy the third, fourth, and the remaining places, if the problem in question is a direct one. If, however, the problem is an indirect one, the given magnitude comes last, and the magnitude sought comes in the second place or in other intermediate places.\textsuperscript{32}

So the point is moot.

Before his work in analytic geometry, Descartes had a strong interest in various mechanical compasses.\textsuperscript{33} One of these compasses he thinks important enough to include in his \textit{Géométrie} not just once, but twice. He seems to feel that this device shows particularly clearly his natural hierarchy in curves -- though I confess myself unable at this time to give a satisfactory account of what he may have in mind by such hierarchy.\textsuperscript{34} Let us first look at the operation of this device; we shall find that this compass will perform a number of functions.

\textsuperscript{32} CSM I, pp. 71-2; AT X, 462. This passage has already been quoted above, footnote. 18.

\textsuperscript{33} Galileo's invention of what he then called the 'geometric and military compass' or what we now term the 'sector' might be borne in mind here. Perhaps Descartes was hopeful of similar notoriety and commercial success through the invention of such a multi-purpose mechanical instrument. Vide: Drake, p. 16.

\textsuperscript{34} My suspicion at this time is that the 'higher order' curves of book three are used to construct three-dimensional objects.
Consider the lines $AB, AD, AF$, and so on, which I assume to have been described with the aid of an instrument, $YZ$, composed of several rulers. These are so joined that when the one marked $YZ$ is placed on the line $AN$, we can open and close the angle $XYZ$; and when it is completely closed, the points $B, C, D, E, F, H$ are all assembled at point $A$. But to the extent that we open it, the ruler $BC$, which is joined at right angles to $XY$ at point $B$, pushes the ruler $CD$ toward $Z$; $CD$ slides along $YZ$, always at right angles to it, and pushes $DE$, which slides all along $YZ$, remaining parallel to $BC$. Then $DE$ pushes $EF$, $EF$ pushes $FG$, $FG$ pushes $GH$, etc. And we can conceive of an infinity of others, which are pushed consecutively in the same way, half of which always maintain the same angles with $YX$ and the others with $YZ$. Now as we thus open the angle $XYZ$, the point $B$ describes the line $AB$, which is a circle, and the other points $D, F, H$, where the intersections of the other rulers occur, describe the other curved lines $AD, AF, AH$, of which the latter are successively more complex than the first, and thus more complex than the circle.\(^{35}\)

\(^{35}\) Olscamp, pp. 191-2.
First let's note some similarities here between this device and Descartes' definitions of arithmetical operations given above. Using the Cartesio-Euclidean principles given there, this device can multiply. We may see this by simply replacing the names of the points involved. In the diagram of the compass, replace $Y$ with $B$, $B$ with $C$, $C$ with $A$, $D$ with $E$, and $E$ with $D$, and we shall have the reverse of figure 4 above. For this same reason the device is capable of division as well.

But what the device is best at is analogically ascertaining the values of continued proportions, including especially the analogical values of roots of various sorts. In the words of Descartes:

[I] do not believe there is any easier or more clearly demonstrated method for finding however many mean proportionals you might wish, than to use the curved lines described by the instrument XYZ [Cartesian Compass], explained above. For if we wish to find two mean proportionals between $YA$ and $YE$, we have only to describe a circle whose diameter is $YE$; and because this circle cuts the curve $AD$ at point $D$, $YD$ is one of the required mean proportionals. The demonstration is obvious to the eye, merely by applying this instrument to the line $YD$: for, as $YA$, or $YB$ which is equal to it, is to $YC$, so $YC$ is to $YD$, and $YD$ to $YE$.

In the same way, in order to find four mean proportionals between $YA$ and $YG$, or to find six of them between $YA$ and $YN$, it is only necessary to trace the circle $YFG$, which, intersecting $AF$ at point $F$, determines the straight line $YF$, which is one of these four proportionals; or [else to trace] $YHN$, which, intersecting $AH$ at point $H$, determines $YH$, one of the six; and so with the others.36

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36 Olscamp, pp. 228-9.
Further, should it be required of us to find the square root of a number, we need only place the compass such that $BD$ is aligned upon the distance, and since $YB$ has been designated the unit, the distance $BC$, in accordance with Descartes' definition of the extraction of roots given above, will be the square root. A similar discussion applies to the quadratic root, and so forth.

The function of this instrument is, again, dependent on the Euclidean theory of ratios. The reason Descartes can say: $YB:YC :: YC:YD :: YD:YE$ is because $\Delta YBC$ is similar to $\Delta YCD$,\(^{37}\) which is in turn similar to $\Delta YDE$, and so on. Descartes can say this because of Euclid, Book VI, Proposition 4, the important part of which says:

> In equiangular triangles the sides about the equal angles are proportional....\(^{38}\)

Now to change the subject a bit and bring in some additional material, not only are the arithmetical foundations of Descartes' work in mathematics based on the theory of ratios derived from Euclid, but so are the specific solutions of individual problems.

Let us look at an example in which this is particularly apparent. In this example Descartes wishes to give an illustration of his classification system for curves. He is going to do so by giving an account of a particular curve. Unfortunately, and despite the example, his classification

\(^{37}\) 'Apparent to the eye' because both triangles are right and share one angle; thus the third angle will also be equal.

\(^{38}\) Euclid 2, p. 200.
system is again not altogether perspicuous. That's alright, however, because remember I am introducing this example to illustrate the characteristic dependence of Cartesian mathematics on certain well-known results of the Euclidean development of the theory of ratios. Sometimes, we shall see, the exact theorems in Euclid can be pinpointed explicitly -- though Descartes does not bother to do so.\textsuperscript{39}

The example is as follows.

We are to find the equation of a certain curve $ECG$ (vide figure 8). This curve is described by the intersection of a ruler $GL$ with 'the rectilinear plane figure' $CNKL$. $CNKL$ is apparently conceived as rigid. That is to say, $\angle NKL$ is given, and it is equal to $\angle CKL$. This rigid body $CNKL$ is to be conceived such that the side $KL$ slides along and always coincides with the line $BA$. Side $KN$ is produced indefinitely in the direction of $C$ and line $BA$ is produced indefinitely in both directions. Ruler $GL$ is hinged to figure $CNKL$ at $L$. The intersection of line $KN$ with line $GL$ at $C$ as figure $CNKL$ slides along $BA$ will produce curve $ECG$. This is the curve whose equation we are to find.

\footnote{In a letter to Princess Elisabeth (Nov. 1643) Descartes declares that \textit{La Géométrie} presupposes only two theorems demonstrated by Euclid. \textit{AT IV}, 38.}
Assume some arbitrary point $C$ on the curve. Through this hypothetical point is drawn line $CB$ parallel to $GA$. As Descartes says:

[B]ecause $CB$ and $BA$ are two indeterminate and unknown quantities, I name the one $y$ and the other $x$.\footnote{Olscamp, p. 194.}

These quantities are unknown as point $C$ was selected randomly; however, there are also known quantities which Descartes takes into consideration:

[S]uch as $GA$, which I call $a$; $KL$, which I call $b$; and $NL$ (parallel to $GA$), which I call $c$.\footnote{Olscamp, p. 194.}

Please attend, because next Descartes states:
Then I say that as $NL$ is to $LK$, or $c$ to $b$, so $CB$, or $y$, is to $BK$, which consequently is $\frac{b}{c} y - b$; and $BL$ is $\frac{b}{c} y - b$, and $AL$ is $x \frac{b}{c} y - b$.

The only reason Descartes can say this, obviously, is that because $CB$ has been drawn parallel to $NL$, $\triangle KNL$ and $\triangle KCB$ are similar triangles. And it is a theorem of Euclid, Book VI, Proposition 2, that:

If a straight line be drawn parallel to one of the sides of a triangle, it will cut the sides of the triangle proportionally; and, if the sides of the triangle be cut proportionally, the line joining the points of section will be parallel to the remaining side of the triangle.\footnote{Euclid 2, p. 194.}

This is what I mean when I say Descartes' solutions in his mathematics are characteristically dependent upon the Euclidean theory of ratios. Not only does Descartes take Euclid's theorem for granted, he assumes its familiarity on the part of the reader.

Next Descartes does essentially the same thing, but for different triangles:

Moreover, as $CB$ is to $LB$, or $y$ to $\frac{b}{c} y - b$, so $a$, or $GA$, is to $LA$ or $x + \frac{b}{c} y - b$.

Descartes is here using the same theorem, but in this case for similar triangles $\triangle CLB$ and $\triangle GLA$.\footnote{Euclid 2, p. 194.}
Now Descartes has just stated that \( y \) is to \( \frac{b}{c} \frac{y}{c} - b \) as \( a \) is to \( x + \frac{b}{c} \frac{y}{c} - b \) or, in other words,

\[
y : \frac{b}{c} \frac{y}{c} - b :: a : x + \frac{b}{c} \frac{y}{c} - b.
\]

Next he says:

Thus, multiplying the second by the third, we produce \( \frac{ab}{c} \frac{y}{c} - ab \), equal to \( xy + \frac{b}{c} y^2 - by \), which is produced by multiplying the first by the last; and so the equation which was found is:

\[
y^2 = cy + \frac{c}{b} x y + ay - ac.
\]

That is to say, he multiplies \( \frac{b}{c} \frac{y}{c} - b \) times \( a \) (obtaining \( \frac{ab}{c} \frac{y}{c} - ab \)) which he then equates without hesitation to the product of \( y \) times \( x + \frac{b}{c} \frac{y}{c} - b \) (or, \( xy + \frac{b}{c} y^2 - by \)). But the only reason he can do so is because the product of the means equals the product of the extremes; or, according to Euclid, Book VI, Proposition 16:

If four straight lines be proportional, the rectangle contained by the extremes is equal to the rectangle contained by the means; and, if the rectangle contained by the extremes be equal to the rectangle contained by the means, the four straight lines will be proportional.\(^{44}\)

\(^{43}\) Oliscamp, p. 194.
\(^{44}\) Euclid 2, p. 221.
All of these references to Euclid’s work, Descartes tacitly and gratuitously assumes. For a modern reader of La Géométrie not altogether familiar with Euclid, such tacit assumptions can at times prove quite puzzling.

We have seen a similar example in Descartes’ solution of finding the normal to a curve above in Chapter 1, where Descartes utilized the Pythagorean Theorem, also, of course, found in Euclid, Book I, Proposition 47:

In right-angled triangles the square on the side subtending the right angle is equal to the squares on the sides containing the right angle.\textsuperscript{45}

Also, in his solution of the elementary form of the Pappus four-line locus problem Descartes makes tacit reference to results also obtained from Euclid, in this case, Book I, Proposition 26:

If two triangles have the two angles equal to two angles respectively, and one side equal to one side, namely, either the side adjoining the equal angles, or that subtending one of the equal angles, they will also have the remaining sides equal to the remaining sides and the remaining angle to the remaining angle.\textsuperscript{46}

To show this at this point would be tedious; but examples may be multiplied, and I am familiar with no problem in Cartesian mathematics in which this is not the case. Suffice it to say that such tacit assumption on the

\textsuperscript{45} Euclid 1, p. 349.
\textsuperscript{46} Euclid 1, p. 301.
part of Descartes of the Euclidean theory of ratios is entirely characteristic of Cartesian mathematics.

Though never expressed explicitly, Euclid’s theory of ratios is always the invisible backbone of Descartes’ work in analytic geometry. Descartes sets up equations by first setting up ratios; and he does this by tacitly utilizing various theorems in Euclid. Since ratios are expressed in terms of proportions, and because Descartes characteristically sets up equations by means of conversion from ratios, solution of unknowns in equations may be expected to reduce to a solution of unknowns in proportions.

Generally speaking, Descartes tries to solve for an unknown by setting up a Euclidean ratio, something of the form: \( a:b :: b:c \), with the term \( b \) usually being the unknown. That is to say, for Descartes the assumption of unknowns is characteristically an example of indirect problem solution.

As for the mechanical Cartesian Compass itself, the reason this device can find roots is because it is also capable, in conformity with Euclidean principles, of finding missing terms in continued proportions;\(^{47}\) and this Descartes takes as its primary function. Thus, this device will perform both ‘direct and indirect problem solution’ of continued proportions. Earlier I have tried to argue an affinity between *La Géométrie* and the *Regulae*. With Descartes’ statements of continued proportions in the *Regulae* and with the intrusion of this compass capable of the solution of such problems in the Second and Third Books of *La

\(^{47}\) I.e. the ratio: \( a:x::x:b \) can be converted to the equation: \( x^2 = ab \).
*Géométrie*, we have yet another example of continuity between the two works.

All of this up to this point I feel pretty confident about; but now I shall wax a bit more speculative. I now wish to point out that all of Book VIII of Euclid and much of Book IX -- which are concerned primarily with propositions concerning continued proportions -- may be illustrated by this Cartesian compass visually and mechanically.\(^48\) Since Descartes considered the compass simple in nature, I believe he probably also considered it foundational in character: foundational in that since all of the later propositions having to do with the construction of the Platonic solids in Book XIII of Euclid are dependent upon the earlier theorems of continued proportions, such later theorems may be thought to be developed from the mechanical movements of Descartes’ compass in conjunction with the traditional apparatus of compass and ruler.\(^49\) In fact, Descartes may have had something more detailed in mind. The Third Book of *La Géométrie* is entitled ‘Of the Construction of Solid and Supersolid Problems*, and the Cartesian Compass is once again introduced immediately, for the second time, as an illustration of how to choose the simplest curve through which a problem may be solved. It is here that

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\(^{48}\) Examples from Euclid Book VIII are as follows:

Proposition 1: ‘If there be as many numbers as we please in continued proportion, and the extremes of them be prime to one another, the numbers are the least of those which have the same ratio with them.’ Euclid 2, p. 343.

Proposition 2: ‘To find numbers in continued proportion, as many as may be prescribed, and the least that are in a given ratio.’ Euclid 2, p. 346.

Proposition 3: ‘If as many numbers as we please in continued proportion be the least of those which have the same ratio with them, the extremes of them are prime to one another.’ Euclid 2, p. 348.

And so forth.

\(^{49}\) Descartes may be thought to imply this when he states his geometric compass should have been as acceptable to traditional geometry as were the ruler and the compass. Vide: Oiscamp, p. 190.
Descartes discussed the capacity of this instrument for finding mean proportionals.

Leibniz is known to have copied a manuscript of Descartes’ dealing with stereometric solids, including the Platonic ones. Certain discoveries were made in this work, some of which were original even at its date of eventual posthumous publication in 1860.50 Euler’s theorem, not published until 1750, is all but stated explicitly.51 Kepler’s concern with supposed relations between the orbits of the planets and certain properties of the Platonic Solids may be recalled here, and such statements had been neither proven nor disproven.52 In fact, it seems, the most recent work on 

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51 According to Polya: ‘These notes [the De Solidorum Elementis] treat of subjects closely related to Euler’s theorem: although the notes do not state the theorem explicitly, they contain results from which it immediately follows.’ Quoted in: Federico, p. 79.

52 Insofar as I am aware, Kepler’s conjecture has never been disproven; but the discovery of additional planets and the asteroid belt makes its veracity unlikely. Kepler himself always considered his ‘laws’ to be entirely secondary to what he took to be the importance of his attempts at discovering the dependence of the distance between the planets upon the properties of the perfect Platonic solids. Descartes doesn’t mention Kepler, but my opinion is that the *Principia* is written in such a fashion so as not to contradict any of Kepler’s seemingly outlandish claims -- just in case they might turn out to be true.

How much Descartes knew about Kepler is unclear. It will be remembered that the dogma of circular orbits of planets reigned supreme throughout the history of astronomy -- it was in fact never questioned -- until this period. Even Galileo continued to believe in circular orbits after reading Kepler’s arguments against them, and only after Newton deduced elliptical orbits from his laws did these enter the mainstream of astronomy. Yet in *Descartes’ Principia* we read arguments that the movements of the heavens are not perfectly circular (Part III, Proposition 34), that although the planes of the orbits of the planets deviate somewhat from the ecliptic, each intersects it in a line which passes through the center of the Sun (Proposition 35), and that the planets are not always equidistant from the Sun and do not always seem to move at the same speed in relation to it (Proposition 36) -- all of which were announced by Kepler. Although Descartes uses Kepler’s terms ‘aphelia’ and ‘perihelia’ to make this last point, the translators of the Principia into English make the following note concerning this: ‘This ... claim [that the planets do not always seem to move at the same speed in relation to the sun] is, of course true. In accordance with Kepler’s second law of planetary motion, planets move fastest when they are closest to the sun. The remainder of the text, however, seems to indicate that this is not what Descartes had in mind but that he was considering the longitude of the aphelia and perihelia (terms introduced by Kepler) of the planets. Further, there does not seem to be any evidence that Descartes knew Kepler’s laws. He certainly never attempts to deduce them from his assumptions (as Newion was able to do from his), and seems acquainted only with Kepler’s work in optics.’ *Principia*, p. 99, fn 31.

But how conversant Descartes was with the details of Kepler’s laws is a distinct question from whether Descartes read Kepler. In any case, Descartes would certainly have been familiar with Kepler’s *Rudolphine Tables*, published in 1627, which were vastly superior to Ptolemaic predictions of planetary positions and were used by virtually all astronomers of the period regardless of which physical system the astronomer might have believed in. In any case, regardless of Descartes’ familiarity with Kepler’s astronomical work, and though it makes little
polyhedra before Descartes' own was Kepler's work *Harmonice Mundi* (of 1619). It has been noted that in the manuscript as copied by Leibniz there are propositions interposed among others which do not seem to make sense. Prominent among these seemingly unrelated propositions are statements concerning the inscription of polyhedra (within spheres such that each angle of the figure touches the sphere). Much of Descartes' work throughout this manuscript is reason by analogy; that is, by analogy with the properties of plane figures Descartes derives the properties of polyhedra.

Let us recall the last Book of Euclid at this point, the Book in which the regular polyhedra receive their construction. The difficulty as posed by Euclid is not only to construct the 3-dimensional figure of each, but to construct it inscribed by a sphere so that the length of the edge of its figure may be known. The length of this side is very important, as we shall see in a moment. Thus, the penultimate propositions of Book XII in which the five perfect solids are finally constructed are as follows:

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53 Federico, pp. 51 and 60-1.

54 Federico, p. 59.

55 Mueller states: 'In propositions 13 to 17 of book XIII Euclid deals in succession with: each of the five so-called Platonic or regular solids: the pyramid, contained by four equilateral triangles; the octahedron, contained by eight; the cube, contained by six squares; the icosahedron, contained by twenty equilateral triangles; and the dodecahedron, contained by twelve regular pentagons. In each proposition Euclid does three things: (1) he constructs (*sunistasthai*) the solid; (2) he comprehends (*perilambanein*) it in a given sphere; and (3) he characterizes, sometimes quantitatively, sometimes qualitatively, what I shall call the edge value of the figure -- the mathematical relation between the length of the edge of the figure and that of the diameter of the comprehending sphere. In proposition 18, the final proposition of the *Elements*, Euclid "sets out the edges of the five figures and compares them," and concludes with an argument that no other regular solid can be constructed. The remainder of the book, propositions 1 to 12, consists -- with the exception of 2, the unused converse of 1 -- of lemmas for the principal propositions 13 to 17." Mueller, Ian, *Philosophy of Mathematics and Deductive Structure in Euclid's Elements*, MIT Press, Cambridge, Massachusetts, 1981., p. 251.
Proposition 13. To construct a pyramid, to comprehend it in a given sphere, and to prove that the square on the diameter of the sphere is one and a half times the square on the side of the pyramid.

Proposition 14. To construct an octahedron and comprehend it in a sphere, as in the preceding case; and to prove that the square on the diameter of the sphere is double of the square on the side of the octahedron.

Proposition 15. To construct a cube and comprehend it in a sphere, like the pyramid; and to prove that the square on the diameter of the sphere is triple of the square on the side of the cube.

Proposition 16. To construct on icosahedron and comprehend it in a sphere, like the aforesaid figures; and to prove that the side of the icosahedron is the irrational straight line called minor.

Proposition 17. To construct a dodecahedron and comprehend it in a sphere, like the aforesaid figures, and to prove that the side of the dodecahedron is the irrational straight line called apotome.\textsuperscript{56}

As Descartes points out in one of these seemingly unrelated short paragraphs, ‘To know if any solid body can be inscribed in a sphere, it is first necessary to know that all its faces necessarily can be inscribed in a circle’.\textsuperscript{57} In Euclid, such construction of 3-dimensional figures in spheres must be performed on the basis of the inscription of the 2-dimensional

\textsuperscript{56} Euclid 3, pp. 467-493.
\textsuperscript{57} Federico, p. 52.
figures in circles. Here, for example are Propositions 8, 9, and 10 leading up to Propositions 13 through 17:

Proposition 8: If in an equilateral and equiangular pentagon straight lines subtend two angles taken in order, they cut one another in extreme and mean ratio, and their greater segments are equal to the side of the pentagon.

Proposition 9: If the side of the hexagon and that of the decagon inscribed in the same circle be added together, the whole straight line has been cut in the extreme and mean ratio, and its greater segment is the side of the hexagon.

Proposition 10: If an equilateral pentagon be inscribed in a circle, the square on the side of the pentagon is equal to the squares on the side of the hexagon and on that of the decagon inscribed in the same circle.\textsuperscript{58}

As is apparent from these propositions, important in the construction of the perfect solids are theorems having to do with the cutting of a length in extreme and mean ratio. This is so not only in the case of the Propositions given immediately above, but also earlier in Book XIII:

Proposition 1: If a straight line be cut in extreme and mean ratio, the square on the greater segment added to the half of the whole is five times the square on the half.

Proposition 2: If the square on a straight line be five times the square on a segment of it, then, when the double of the said segment is

\textsuperscript{58} Euclid 3, pp. 453-7.
cut in extreme and mean ratio, the greater segment is the remaining part of the original straight line.

**Proposition 3:** If a straight line be cut in extreme and mean ratio, the square on the lesser segment added to the half of the greater segment is five times the square on the half of the greater segment.

**Proposition 4:** If a straight line be cut in extreme and mean ratio, the square on the whole and the square on the lesser segment together are triple of the square on the greater segment.

**Proposition 5:** If a straight line be cut in extreme and mean ratio, and there be added to it a straight line equal to the greater segment, the whole straight line has been cut in extreme and mean ratio, and the original straight line is the greater segment.

**Proposition 6:** If a rational straight line be cut in extreme and mean ratio, each of the segments is the irrational straight line called apotome.\(^{59}\)

It should be clear from the mention of the *apotome* in Proposition 6 and in Proposition 17 that a structural relationship exists between them -- and so with the other Propositions as well.\(^{60}\) The earlier propositions are needed to specifically designate the lengths of the sides of the perfect polyhedra. The material concerning extreme and mean ratio is included

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\(^{59}\) Euclid 3, pp. 440-9.

\(^{60}\) In dealing with irrational numbers, or incommensurables, the Greeks simply assigned names to certain ones which they found convenient to do so. For example, they had the 'Minor', the 'Major', the 'Lateral', the 'Medial' and the 'Bimedial', the 'Binomial', and so forth. The number we now call π is an additional example. Here is Euclid’s definition of the Apotome: 'If from a rational straight line there be subtracted a rational straight line commensurable with the whole in square only, the remainder is irrational; and let it be called an *apotome*.' Euclid 3, p. 158. It was possible also to find the first or second Apotome of a Medial, and so on.
because the construction of the regular pentagon is dependent on it;\textsuperscript{61} and the pentagon is necessary for the construction of dodecahedron. Again the reason why will be given presently. Euclid defines the term 'cut in extreme and mean ratio' as follows:

A straight line is said to have been \textit{cut in extreme and mean ratio} when, as the whole line is to the greater segment, so is the greater to the less.\textsuperscript{62}

As we have seen, to cut lengths in such ratios, and as a result the ability to find roots, is exactly what the Cartesian Compass is good at. And I shall include here Proposition 18, the last Proposition of the last Book of Euclid, and hope when looking at the accompanying figure the reader bears in mind certain other figures which have been shown up to now in conjunction with the Cartesian Compass:

\textsuperscript{61} According to Heath: '[T]he division of a straight line in extreme and mean ratio, on which the construction of the regular pentagon depends, comes in Euclid, Book II. (Prop. 11)....' Euclid 1, p. 414.

\textsuperscript{62} Euclid 2, p. 188.
Proposition 18: To set out the sides of the five figures and to compare them with one another.

![figure 9.]

But there's more; because what I have listed above is only the first half of the Proposition. Here is the second half, which is being led up to, and is the crown of the stereometric portion of Euclid's work on the Elements of Geometry.⁶³:

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⁶³ According to Heath: 'Proclus says... that [Euclid] was of the school of Plato and in close touch with that philosophy. But this was only an attempt of a New Platonist to connect Euclid with his philosophy, as is clear from the next words in the same sentence, "for which reason also he set before himself, as the end of the whole Elements, the construction of the so-called Platonic figures." It is evident that it was only an idea of Proclus' own to infer that Euclid was a Platonist because his Elements end with the investigation of the five regular solids, since a later passage shows him hard put to it to reconcile the view that the construction of the five regular solids was the end and aim of the Elements with the obvious fact that they were intended to supply a foundation for the study of geometry in general, "to make perfect the understanding of the learner in regard to the whole of geometry." To get out of the difficulty he says that, if one should ask him what was the aim... of the treatise, he would reply by making a distinction between Euclid's intentions (1) as regards the subjects with which his investigations are concerned, (2) as regards the learner, and would say as regards (1) that "the whole of the geometer's argument is concerned with the cosmic figures." This latter statement is obviously incorrect. It is true that Euclid's Elements end with the construction of the five regular solids; but the planimetric portion has no direct relation to them, and the arithmetical no relation at all; the propositions about them are merely the conclusion of the stereometrical division of the work." Euclid 1, p. 2.
I say next that no other figure, besides the said five figures, can be constructed which is contained by equilateral and equiangular figures equal to one another.\textsuperscript{64}

In short, these are the perfect solids, \textit{and there are no others}. This, which Euclid has done synthetically, is exactly what, according to Federico, Descartes proves analytically in the manuscript. According to Federico:

A geometric proof that there cannot be more than the five regular bodies is given by the last proposition on Book XIII of Euclid. Descartes' condition is essentially algebraic and may very well have been the first algebraic treatment.\textsuperscript{65}

Descartes is always trying to show how his work surpasses the work of others before his. The thrust of \textit{La Géométrie} itself is to show how his method is capable of solving a problem posed by Pappus supposedly unsolved by the ancients. As a research project, I suspect it would be worthwhile to investigate the relationship between the higher functions of Book III of \textit{La Géométrie} and the results of Descartes' work on Polyhedra. Perhaps if these are compared to Euclid Books VIII to XII,\textsuperscript{66} a further relationship might be discerned between Descartes' Compass and his statements on polyhedra via the method of 'indirect' problem solution of continued proportionals.

\textsuperscript{64} Euclid 3, p. 507.
\textsuperscript{65} Federico, p. 51.
\textsuperscript{66} Mueller states: 'There seems to be general agreement among scholars that the mathematical essentials of book XIII are due to Plato's younger contemporary Theatetus, who is also responsible for at least some part of book X. The close connection between these two books is apparent because book X is essential to the characterization of the edge value of the icosahedron and dodecahedron.' Mueller p. 251.
As I have already mentioned, and as the reader will have noticed, we have now reached the point of speculation, however. I am not sufficiently familiar with either Descartes’ work on polyhedra or his later work in geometry to relate the two or work out the details of a structural relationship between them. What would be helpful here would be an account of how the continued proportions of Book VIII in Euclid are instrumental, by means of Book X, to the construction of the regular polyhedra in Book XIII utilizing the cut in extreme and mean ratio (the ratio which has come to be called ‘the golden section’). This would then relate, theoretically, the Cartesian Compass to the composition of such figures and might give some insight into the Cartesian analytic handling of the Euclidean synthetic derivation.

I am not prepared to give such an account at this time, however.

Summary of Chapter Two.

We have seen a few things in this chapter.

Firstly we have seen a close association between the two works, the *Regulae* and the *Géométrie*, not only in terms of espoused method, but also in terms of basic definitions of arithmetical functions, and also in terms of remarks concerning continued proportions in the *Regulae* and the Cartesian Compass in *La Géométrie*. 
Secondly, we have seen three reasons for assuming that by ‘indirect solution of problems’ Descartes means ‘analysis’.

Thirdly, I have tried to show that by ‘direct’ problem solution Descartes means solution of a problem in which the unknown appears as an extreme term, and by ‘indirect’ problem solution Descartes means a problem in which the unknown appears as a mean term.

Fourthly, since Cartesian equations are set up by means of reference to the Euclidean theory of ratios and proportions, and characteristically involve the unknown or unknowns set as mean terms in proportions, any such problem in Cartesian mathematics will, again characteristically, be a problem in ‘indirect’ problem solution.

On the basis of these points, we may say then, I think, that insofar as Cartesian mathematical analysis may be considered a form of analysis, so can indirect problem solution also be considered a form of analysis.
Chapter 3: Determinate Problem Solution in the *Regulae*.

Descartes' early work, the *Regulae* was never finished. It was to have been a work expressing a new method. Of a proposed thirty-six rules, only twenty-one are known to have been written. Descartes makes it clear that the work was to have consisted of three sections of twelve rules each. But in the secondary literature and commentaries upon the *Regulae* what each of the three sections was to deal with is not altogether clear.\(^1\) Since the *Regulae* is an explicit attempt on the part of Descartes of the working out of his method, it is my intention in this Chapter to look at certain aspects of his espoused methodology in this work.\(^2\) Let us begin with an assessment of what Descartes actually says the structure of the *Regulae* was to be. We shall deal with the primary sections in which Descartes discusses, within the text of the *Regulae* itself, the tripartite structure of that work. In the course of our discussion I shall make two distinct suggestions which shall derive from these passages.

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\(^1\) For example, the following statement is from the introductory material from Haldane and Ross: 'The work was to have been complete in thirty-six rules falling into three parts containing twelve rules each. The first part gives the general nature of Descartes' new Method; while in the second a transition is made to its application in the field of Mathematics. Unfortunately the treatise, which was never completed, breaks off after Rule XXI, and indeed the explanation of the last three rules is also omitted. The third part was to have shown the application of the Method to the general problems of Philosophy.' Haldane, Elizabeth S. and G.R.T.Ross, *The Philosophical Works of Descartes*, 2 vols., Cambridge University Press, London, 1972 (1911, 2nd ed 1931), p. x.

\(^2\) I believe the following discussion will show the latter part of this statement to be incorrect.

I should alert the reader that there is some question how relevant this early work is to Descartes' mature beliefs; my own opinion is that the *Regulae* is a mine of rich ore in Cartesian methodology -- unrefined to be sure. To disregard this important work on this topic would be to throw the baby out with the bathwater.
1. I suggest that in terms of mathematics the distinction between the three sections of the Regulae is, first, the proper comportment of the mind towards its objects of concern; second, the proper method of solution for 'problems to prove'; and third, the proper method of solution of 'problems to find'.

2. I suggest that in terms of natural science the distinction between the three sections of the Regulae is, first, as above, the proper comportment of the mind towards its objects; second, different from above, the proper method for solution of problems posed in terms of primary qualities; and third, also different from above, the proper method for solution of problems not yet posed in terms of primary qualities -- that is, posed still in terms of secondary qualities.

Part I: Problems to Prove and Problems to Find.

Rule Twelve sits as a central part of the Regulae. It recapitulates what Descartes feels he has accomplished in the first section of that work and expounds upon what he expects to do in the next two sections. It is therefore worthwhile, I think, to review what it has to say.

When we look at Rule Twelve, it states:

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3 The distinction between 'problems to prove' and 'problems to find' will be delineated below.
Finally we must make use of all the aids which intellect, imagination, sense-perception, and memory afford in order, firstly, to intuit simple propositions distinctly; secondly, to combine correctly the matters under investigation with what we already know, so that they too may be known; and thirdly, to find out what things should be compared with each other so that we make the most thorough use of all our human powers.\(^4\)

The first twelve rules (i.e. the first section of the *Regulae*) have to do with the comportment of the human faculty of comprehension; primary is the mental faculty of comprehension, the intellect.\(^5\) Juxtaposed against this mental faculty of the intellect are the functions of the body having to do with human comprehension: sense, imagination, and memory. Thus, the faculties of intellect, imagination, sense and memory read like a personal or private liturgy in the *Regulae*; they represent, from Descartes' point of view, a complete enumeration of our cognitive faculties -- a type of magical formula for Descartes.\(^6\)

In reviewing Rule Twelve, let us note that there are three parts to it. We ought to employ our faculties, firstly, to intuit simple propositions distinctly; secondly, to combine correctly the matters under investigation with matters already known; and thirdly, to find out what things should be compared with each other.

I am going to suggest an interpretation of this rule; an interpretation which I believe will go far in explaining the second and third sections of

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\(^4\) CSM 1, p. 39; AT X, p. 410.
\(^5\) Termed the 'understanding' in HR.
\(^6\) Descartes feels the derivation of these faculties to be an example of his method of 'enumeration'. CSM 1, pp. 29-30; AT X, 395-6.
the *Regulae* and give us insight into what Descartes had in mind for his overall method. Let us first review a problem in geometry which we shall solve by recourse to the utilization of algebra. Here is the problem.

If the right triangle $XYZ$ with sides of lengths $x$ and $y$, and hypotenuse of length $z$, has an area of $z^2/4$, then the triangle $XYZ$ is isosceles.

![Figure 10](image)

The data, or what we already know, is as follows:

The right triangle $XYZ$ with sides of lengths $x$ and $y$, and hypotenuse of length $z$, has an area of $z^2/4$.

And this is what we are to prove:

The triangle $XYZ$ is isosceles.
We have seen in Chapter 1 that we can start with that which is to be proved and reason backwards. If we are to convert our conclusion in geometry to one with an algebraic representation, we may assert,

\[ B1: x = y. \]

That is to say, if we can prove that \( x = y \), then we have proven that triangle \( XYZ \) is isosceles. Next we ask ourselves how we can show that the two lengths, \( x \) and \( y \) are equal. We might do this by proving that the difference between the two numbers is zero; that is,

\[ B2: (x - y) = 0. \]

So this is what we are now looking to prove.

Next we can also reason forwards by beginning with the data and reasoning to this new conclusion. How can we show that \((x - y) = 0\) ?

We already know from the data that the area of the triangle is to be \( z^2/4 \). Because the area of a right triangle (in this case a right triangle with a hypotenuse \( z \) opposite angle \( Z \)) is one-half the base times the height, we note that \( xy/2 = z^2/4 \), . Thus,

\[ F1: xy/2 = z^2/4. \]

We may also note that, by the Pythagorean theorem, we have,
F2: \((x^2 + y^2) = z^2\).

We may now combine F1 with F2 by replacing \(z^2\) in F1 by \((x^2 + y^2)\) from A2, thereby obtaining the statement,

F3: \(xy/2 = (x^2 + y^2)/4\).

Continuing now, we attempt to write F3 so as to look more like B2. We multiply both sides of F3 by 4 and subtract \(2xy\) from both sides, to obtain,

F4: \((x^2 - 2xy + y^2) = 0\).

By factoring, we obtain,

F5: \((x - y)^2 = 0\).

Our final step, and the step by which we bridge the forward and backward movements, is to take the square root of both sides of the equality in F5, thus obtaining the statement in B2: \(x - y = 0\). The demonstration is now complete, and we can formalize the proof as follows:
<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of XYZ is ( z^2/4 )</td>
<td>Given</td>
</tr>
<tr>
<td>F1: ( xy/2 = z^2/4 )</td>
<td>Area = (base)(height)/2</td>
</tr>
<tr>
<td>F2: ( (x^2 + y^2) = z^2 )</td>
<td>Pythagorean theorem</td>
</tr>
<tr>
<td>F3: ( xy/2 = (x^2 + y^2)/4 )</td>
<td>Substitute F2 into F1</td>
</tr>
<tr>
<td>F4: ( (x^2 - 2xy + y^2) = 0 )</td>
<td>Algebra</td>
</tr>
<tr>
<td>F5: ( (x - y)^2 = 0 )</td>
<td>Factoring F1</td>
</tr>
<tr>
<td>B2: ( (x - y) = 0 )</td>
<td>Take square root in F5</td>
</tr>
<tr>
<td>B1: ( x = y )</td>
<td>Add ( y ) to both sides of B2</td>
</tr>
<tr>
<td>XYZ is isosceles</td>
<td>Since B1 is true</td>
</tr>
</tbody>
</table>

This example is interesting because it is simultaneously an example of geometry done in the traditional style of synthetic or deductive proof, and also one in which algebra is utilized to solve for simultaneous equations. It is not, strictly speaking, traditional geometry, nor is it strictly speaking analytic geometry.

Let us note in passing here a certain rough similarity between the solution to this problem and the solution to a problem in continued
proportions by the 'indirect' method proposed by Descartes in Chapter 2 above. That is to say, in this problem, we have proceeded forward from the given and backward from what is to be proven to achieve an intermediate connecting bridge between the two. In Descartes’ conception of an 'indirect' solution of a continued proportion, we also have, as he has stated, two movements of mind. Again, this is only a rough analogy at this point, and I shall have a bit more to say concerning it in Chapters Four and Five.

But to return once again to the proof at hand, notice in this proof that we are already familiar with the proposition we are trying to prove: that XYZ is isosceles. However, on the basis of the data given, it is possible to prove a number of different conclusions, not just this one; for example, it is possible to prove that, 'angle X is less than 90 degrees'. New theorems are discovered by mathematicians all the time; some are 'interesting' and some are not. Also, some proposed propositions, such as Goldbach's and Fermat's conjectures for example, have never been proven or disproven. In 1750 Euler proposed his theorem and stated publicly he was incapable as yet of providing a proof; the proof was forthcoming in the following year.\textsuperscript{7} There is a difference between a discovery of a

\textsuperscript{7} According to Federico, 'Euler's main theorem is stated as follows:

1. In every solid body bounded by plane faces the sum of the number of solid angles and the number of faces exceeds the number of edges by two.

2. Then, taking the number of solid angles as \( S \), the number of faces as \( H \), and the number of edges as \( A \), the theorem is expressed by the formula:

\[ S + H = A + 2. \]

Euler then states that he must confess that he was not yet able to give a definite proof, but that its truth will be recognized for all types of solids for which it is considered. The theorem is then demonstrated as true for:

(1) all pyramids; (2) wedge-shaped bodies, that is pyramid type bodies having a line instead of a point for apex; (3) all prisms; (4) prism type bodies with the two bases having different numbers of sides (prismoids); (5), (6) and (7) combinations of two of type (4) joined base to base; and (8), the five regular solids treated individually. The truth of the general proposition is left to be inferred from the individual cases.
rigorous demonstration of a proposition already selected, as we have in the case above, and the discovery of a new proposition previously unknown. It is possible to try to construct a proof without yet knowing what the final conclusion will inevitably be; construction problems are sometimes like this. In these cases, it may even be a major feat to demonstrate that the construction is possible at all; or, alternatively, that the construction is in fact impossible; or, that the construction is possible only under certain conditions. In ancient geometry such questions as how to double a given cube, or how to trisect an angle -- both by using no more than ruler and compass -- were ongoing research questions never resolved (until the modern era, in which their solutions were proven to be impossible given the constraints). The discovery of the following theorem on the part of Archimedes, for example, is considered by historians astounding:

The area of any segment of a section of a right-angled cone (i.e. a parabola) is four-thirds of that of the triangle which has the same base and height.

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"Since analogy had proved fruitless, it is very probable that Euler discovered his theorem by induction. Pólya repeatedly uses this as a classic example of induction in mathematical discovery." Federico, p. 67.

The proof for the theorem was delivered in Euler's second paper in 1751.

8 In Euclidean Geometry there are two basic types of problems. One is the proof of a theorem; the other is the construction of a figure. These latter are called construction problems. For example, Book I, Proposition 1 of Euclid is a construction problem which states: 'On a given finite straight line to construct an equilateral triangle'. As we have seen, Book XIII, Proposition 17 is, in part, another construction problem which states: 'To construct a dodecahedron and comprehend it in a sphere...'. An example of a theorem would be Book I, Proposition 6: 'If in a triangle two angles be equal to one another, the sides which subtend the equal angles will also be equal to one another'. For examples of construction problems whose conclusion is as yet unknown, vide the reference to some traditional problems in ancient geometry below.
Consider the following words on the part of Heath concerning Archimedes' achievements:

There is... a certain mystery veiling the way in which [Archimedes] arrived at his results. For it is clear that they were not discovered by the steps which lead up to them in the finished treatises. If the geometrical treatises stood alone, Archimedes might seem, as Wallis said, 'as it were of set purpose to have covered up the traces of his investigation, as if he had grudged posterity the secret of his method of inquiry, while he wished to extort from them assent to his results'. And indeed (again in the words of Wallis) 'not only Archimedes but nearly all the ancients so hid from posterity their method of Analysis (though it is clear that they had one) that more modern mathematicians found it easier to invent a new Analysis than to seek out the old'. A partial exception is now furnished by The Method of Archimedes, so happily discovered by Heiberg. In this book Archimedes tells us how he discovered certain theorems in quadrature and cubature, namely by the use of mechanics, weighing elements of a figure against elements of another simpler figure the mensuration of which was already known.

At the same time he is careful to insist on the difference between (1) the means which may be sufficient to suggest the truth of theorems, although not furnishing scientific proofs of them, and (2) the rigorous demonstrations of them by orthodox geometrical methods which must follow before they can be finally accepted as established.\(^9\)

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The reader will notice a certain similarity between the words of Wallis and what Descartes has said here. Wallis wrote *Arithmetica Infinitorum* which appeared in 1656, some twenty years after the publication of *La Géométrie*, in which the Cartesian notation of writing exponents with Hindu-Arabic numerals is utilized, but -- according to Scott -- with Wallis we have the first clear use of indices other than positive whole numbers. Vide Scott, p. 93. The relationship between Wallis and Descartes has not, to my knowledge, been clarified.
Bearing in mind the distinction just made on behalf of Archimedes by Heath, however, let us return to the statement made by Pappus, given in Chapter 1, concerning the nature of analysis. I am not going to use the Heath translation utilized earlier; rather I am going to use the more suggestive and colloquial paraphrase offered by Polya\textsuperscript{10}:

The so-called Heuristic is, to put it shortly, a special body of doctrine for the use of those who, after having studied the ordinary Elements, are desirous of acquiring the ability to solve mathematical problems, and it is useful for this alone. It is the work of three men, Euclid, the author of the Elements, Apollonius of Perga, and Aristaeus the elder. It teaches the procedures of analysis and synthesis.

In analysis, we start from what is required, we take it for granted, and we draw consequences from it, and consequences from the consequences, till we reach a point that we can use as starting point in synthesis. For in analysis we assume what is required to be done as already done (what is sought as already found, what we have to prove as true). We inquire from what antecedent the desired result could be derived; then we inquire again what could be the antecedent of that antecedent, and so on, until passing from antecedent to antecedent, we come eventually upon something already known or admittedly true. This procedure we call analysis, or solution backwards, or regressive reasoning.

But in synthesis, reversing the process, we start from the point which we reached last of all in the analysis, from the thing already known or admittedly true. We derive from it what preceded it in the analysis, and go on making derivations until, retracing our steps, we

finally succeed in arriving at what is required. This procedure we call
synthesis, or constructive solution, or progressive reasoning.

Now analysis is of two kinds; the one is the analysis of the
‘problems to prove’ and aims at establishing true theorems; the other is
the analysis of the ‘problems to find’ and aims at finding the unknown.

If we have a ‘problem to prove’ we are required to prove or
disprove a clearly stated theorem $A$. We do not know yet whether $A$ is
true or false; but we derive from $A$ another theorem $B$, from $B$ another
$C$, and so on, until we come upon a last theorem $L$ about which we have
definite knowledge. If $L$ is true, $A$ will be also true, provided that all
our derivations are convertible. From $L$ we prove the theorem $K$ which
proceeded $L$ in the analysis and, proceeding in the same way, we retrace
our steps; from $C$ we prove $B$, from $B$ we prove $A$, and so we attain our
aim. If, however, $L$ is false, we have proved $A$ false.

If we have a ‘problem to find’ we are required to find a certain
unknown $x$ satisfying a clearly stated condition. We do not know yet
whether a thing satisfying such a condition is possible or not; but
assuming that there is an $x$ satisfying the condition imposed we derive
from it another unknown $y$ which has to satisfy a related condition; then
we link $y$ to still another unknown, and so on, until we come upon a last
unknown $z$ which we can find by some known method. If there is
actually a $z$ satisfying the condition imposed upon it, there will be also
an $x$ satisfying the original condition, provided that all our derivations
are convertible. We first find $z$; then, knowing $z$, we find the unknown
that preceded $z$ in the analysis; proceeding in the same way, we retrace
our steps, and finally, knowing $y$, we obtain $x$, and so we attain our
aim. If, however, there is nothing that would satisfy the condition
imposed upon $z$, the problem concerning $x$ has no solution.\footnote{Polya, HSI, pp. 141-3. Poly has more to say concerning this distinction between problems ‘to prove’ and problems ‘to find’, though I don’t believe what he says there will add greatly to our discussion here. Vide: HSI, pp. 33 & 154 ff.}
I am about to suggest that this distinction between 'problem to prove' and 'problem to find' seems to be an aspect of the distinction Descartes is trying to get at in Rule Twelve. First, consider figure 10 in the example above and the two distinct problems given below in reference to it; firstly the same problem as previously stated:

If the right triangle XYZ with sides of lengths \(x\) and \(y\), and hypotenuse of length \(z\), has an area of \(z^{2/4}\), then the triangle XYZ is isosceles.

Next, a slightly different problem:

If the right triangle XYZ with sides of lengths \(x\) and \(y\), and hypotenuse of length \(z\), is isosceles, then what is its area?

I think clearly the latter proposition may be considered a 'problem to find' whereas the former proposition may be considered a 'problem to prove'. That is to say, the conclusion we are seeking to prove is determined and apparent in the case of proposition A, but not of the case of proposition B. Now, my suggestion is that there is a certain similarity between this quotation of Pappus offered by Polya and the expression of Rule Twelve. Recall that Rule Twelve was stated in three parts. The distinction between 'problems to prove' and 'problems to find' seems, if I am correct, to fit the latter two sections of this Rule:

Finally we must make use of all the aids which intellect, imagination, sense-perception, and memory afford in order, firstly, to intuit simple propositions distinctly; secondly, to combine correctly the matters under
investigation with what we already know, so that they too may be known; and thirdly, to find out what things should be compared with each other so that we make the most thorough use of all our human powers.¹²

So I am suggesting that when Descartes says ‘to combine correctly the matters under investigation with what we already know, so that they too may be known’, in mathematical terms he has in mind ‘problems to prove’; and that when he says ‘to find out what things should be compared with each other so that we make the most thorough use of all our human powers’, he means, again in mathematical terms, problems to find. I am also suggesting here that, mathematically speaking, such ‘problems to prove’ are the subject topic of the second section of the Regulae, and that ‘problems to find’ were, by implication, to have been the subject matter of the third section which was apparently never finished.

Furthermore, this distinction also seems to correspond to what is stated by Descartes to be the subject material for sections two and three of the Regulae if we consider the following quote which we have already seen in Chapter One of this work

For the rest, in case anyone should fail to see the interconnection between our Rules, we divide everything that can be known into simple propositions and problems. As for simple propositions, the only rules we provide are those which prepare our cognitive powers for a more distinct intuition of any given object and for a more discerning examination of it. For these simple propositions must occur to us

spontaneously; they cannot be sought out. We have covered simple
propositions in the preceding twelve Rules, and everything that might in
any way facilitate the exercise of reason has, we think, been presented
in them. As for problems, however, some can be understood perfectly,
even though we do not know the solutions to them, while others are not
perfectly understood. We shall deal solely with the former sort of
problems in the following twelve Rules, and shall postpone discussion
of the latter until the final set of twelve Rules.\footnote{CSM 1, pp. 50-1; AT X, 428-9.}

Again, this is given in three parts: (1) precepts for correctly
comporting our cognitive faculties for the solution of problems (i.e. the
first section of the Regulae -- in this case to 'prepare our cognitive powers
for a more distinct intuition of any given object and for a more discerning
examination of it'); (2) precepts for the correct solution of problems some
of which 'can be understood perfectly, even though we do not know the
solutions to them' (i.e. the second section of the Regulae); and (3)
precepts for the correct solution of problems which 'are not perfectly
understood' (i.e. the third section of the Regulae).

Of course, what Descartes means by problems 'understood perfectly'
and those 'not perfectly understood' is open to interpretation. Still, the
interpretation I am proposing here seems to fit in with this quotation as
well as the other given above. Again, to clarify my suggestion --
presumably through the correct usage of memory, sense and imagination --
the types of problems addressed by the second section of the Regulae (i.e.
problems which 'can be understood perfectly') are what Polya's Pappus
calls 'problems to prove', and that the third unwritten section of the
Regulae (i.e. problems which ‘are not perfectly understood’) was to have dealt with ‘problems to find’. Again, I stress this suggestion applies in my view to mathematical types of problems only at this point.¹⁴

La Géométrie is designed primarily as a work to illustrate the solution of a single problem: what is called the ‘four-line locus problem’ in Pappus. There are many digressions in the work, but concerning this one central problem, Descartes says:

Then, since there are always an infinite number of points which can fulfill the requirement here, it is always necessary to know and to trace the line in which they are to be found; and Pappus says that, when there are only three or four given straight lines, this line is one of the three conic sections; but he does not undertake to determine it, nor to describe it, nor to do any more than explain those where all these points must be found, when the problem involves a greater number of lines. He only adds that the ancients had recognized one of them which they had shown to be useful, but which seemed the most obvious, and yet was not the most important. This gave me occasion to try and discover whether, through my method, I could go as far as they had gone.¹⁵

Again, the salient portion of this passage is:

...Pappus says that, when there are only three or four given straight lines, this line is one of the three conic sections; but he does not undertake to determine it, nor to describe it....¹⁶
This seems rather disingenuous, for, as Descartes admits for his own solution:

[W]hen there are only three or four given lines, the required points are to be found, not only on one of the three conic sections, but sometimes also on the circumference of a circle or a straight line.\(^{17}\)

Descartes is anxious to show that his method can solve a problem which the ancients apparently did or could not. Since Pappus stated that the solution is a conic section, Descartes may have originally utilized this knowledge in the solution of the problem -- though he needn't have -- and did not use it in his actual proof at the end of Book 1; he even seems strangely intent on denying that the solution was even available. In any case, the more difficult solution of the problem in the case of more than four lines (or four lines which are parallel) would certainly be an example of the type of problem in which the solution is not yet known -- the type of problem which, if I am right, it seems the third section of the *Regulae* was to have addressed. This is true of the solution in both cases. It is also true that in both cases the proof is given in the new Cartesian form of mathematical analysis. The more traditional Euclidean synthetic form of solution was not given before Newton’s *Principia Mathematica*, in which it appears as Lemma XIX in Section V of Book 1.

\(^{17}\) Olscamp, p. 185.
Though he does not use the same terminology, Descartes may be construed to be discussing this distinction between problems to prove and problems to find in the following passage:

[W]e should realize that, on the basis of their method, dialecticians are unable to formulate a syllogism with a true conclusion unless they are already in possession of the substance of the conclusion, i.e. unless they have previous knowledge of the very truth deduced in the syllogism. It is obvious therefore that they themselves can learn nothing new from such forms of reasoning, and hence that ordinary dialectic is of no use whatever to those who wish to investigate the truth of things. Its sole advantage is that it sometimes enables us to explain to others arguments which are already known. It should therefore be transferred from philosophy to rhetoric.\textsuperscript{18}

Under this interpretation, one of the deficiencies of the traditional syllogistic is its inability to handle any problem except those in which the conclusion is already known; that is, it is unable to handle problems to find. Notice also the similarity here between this type of thought and what Descartes has called synthesis before now -- both are good for explaining things to others, but not for discovering things themselves.

There are, however, a couple of extensive quotations which are seemingly difficult to reconcile with this interpretation. These quotations are important because they are exactly the ones, in addition to the one given above, which are the primary discussions in the \textit{Regulae} for what

\textsuperscript{18} CSM 1, pp. 36-7; AT X, 406.
subjects the three different sections of the *Regulae* are to deal with. Let us handle these here one at a time.

Part II: Primary and Secondary Qualities in Natural Philosophy.

Here is the first of these passages; for ease of reference I shall refer to it as Quotation (A):

(A) [I]t will be very useful if we transfer what we understand to hold for magnitudes in general to that species of magnitude which is most readily and distinctly depicted in our imagination. But it follows from what we said in Rule Twelve that this species is the real extension of a body considered in abstraction from everything else about it save its having a shape. In that Rule we conceived of the imagination, along with the ideas existing in it, as being nothing but a real body with a real extension and shape. That indeed is self-evident, since no other subject displays more distinctly all the various differences in proportions. One thing can of course be said to be more or less white than another, one sound more or less sharp than another, and so on; but we cannot determine exactly whether the greater exceeds the lesser by a ratio of 2 to 1 or 3 to 1 unless we have recourse to a certain analogy with the extension of a body that has shape. Let us then take it as firmly settled that perfectly determinate problems present hardly any difficulty at all, save that of expressing proportions in the form of equalities, and also that everything in which we encounter just this difficulty can easily be, and ought to be, separated from every other subject and then expressed in terms of extension and figures. Accordingly, we shall dismiss
everything else from our thoughts and deal exclusively with these until we reach Rule Twenty-five.\(^{19}\)

To start with, this quotation lends a bit of corroborating evidence to my interpretation insofar as it speaks of 'perfectly determinate problems'. This may be construed as 'problems to prove'. But on the other hand, it is unclear how reference to figure, extension, and magnitude and the reduction to equations is to fit into this interpretation. Recall his statement above:

Let us then take it as firmly settled that perfectly determinate problems present hardly any difficulty at all, save that of expressing proportions in the form of equalities, and also that everything in which we encounter just this difficulty can easily be, and ought to be, separated from every other subject and then expressed in terms of extension and figures.\(^{20}\)

Let us look a bit deeper into it. Descartes here speaks of things being more or less one color than another and one sound being sharper or flatter than another, but 'we cannot determine exactly whether the greater exceeds the lesser by a ratio of 2 to 1 or 3 to 1 unless we have recourse to a certain analogy with the extension of a body that has shape'. Descartes goes on to add that it is concerning these types of problems that the work up to Rule Twenty-five is to be about (I take it Descartes is referring to the second section of the Regulae here). Descartes speaks of sounds in one other place in the Regulae. There he says the following:

\[^{19}\text{CSM 1, p. 58; AT X, 441.}\]
\[^{20}\text{CSM 1, p. 58; AT X, 441.}\]
Or again I may be asked to determine what the nature of sound is, solely and precisely from the following data: three strings, A, B, and C emit the same sound; B is twice as thick as A, but no longer, and is tensioned by a weight which is twice as heavy: C is twice as long as A, though not so thick, and is tensioned by a weight four times as heavy. It is easy to see from such examples how imperfect problems can all be reduced to perfect ones -- as I shall explain at greater length in the appropriate place.\textsuperscript{21}

Descartes does not in fact address such conversion again; I assume it was to have been addressed in the unwritten third section. But we notice that what Descartes is doing here is converting observations which are not expressed in quantitative terms (observation in terms of affective qualities, in this case sounds) to those which are so expressed. He is converting sounds to measurements of sounds and expressing such sounds, somewhat vaguely, in terms of ratios or equations. We may, I think, rewrite the above passage as follows:

\begin{align*}
\text{thickness } B &= 2 \times \text{thickness } A \\
2 \times \text{weight } B &= \text{weight } A \\
\text{length } C &= 2 \times \text{length } A \\
\text{thickness } C &< \text{thickness } A \\
4 \times \text{weight } C &= \text{weight } A.
\end{align*}

My suggestion here is that, in terms of natural science, Descartes wishes to convert affective qualities to qualities expressed in terms of

\textsuperscript{21} CSM 1, p. 52; AT X, 431.
extension and figure (optimally expressed as equations) in order to solve problems having to do with such qualities. There is a distinction drawn by philosophers called the distinction between primary and secondary qualities. What exactly this distinction is to be taken to mean is not altogether agreed upon. Most philosophers become acquainted with the distinction through Locke. There we learn primary qualities are ‘solidity, extension, figure, motion or rest, and number’; secondary qualities are ‘colours, sounds, tastes &c.’. Whereas primary qualities we perceive exactly resemble the corresponding attributes in their representative objects; secondary qualities such as color, taste, smell, etc., though caused by, do not resemble such attributes. Also, though it is clear that secondary qualities are not true qualities of matter but are merely powers in the objects to produce sensory effects in us by means of the attributes of its minute parts, Locke is equivocal concerning the nature of primary qualities. Sometimes he speaks as if they reside exclusively in the object; sometimes he speaks as if they may also reside in the mind. Some such distinction between primary and secondary qualities has long been recognized to be a part of Descartes’ thought, though he does not use these terms himself. I am going to suggest here that Descartes’ proposal is for secondary qualities to be reduced to primary qualities in order for problems dealing with them to achieve proper solution.22 Hence, in addition to my suggestion above that the difference between section two and section three of the Regulae is that between problems to prove and

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22 Cottingham hints at this when he says: ‘One feature of the mathematical approach which clearly appealed to Descartes was its rejection of loose qualitative notions in favour of precisely measurable quantities.’ Cottingham, John, Descartes, Basil Blackwell, New York, 1986, p. 88.
problems to find, I am also here proposing a second suggestion. My second suggestion is that, now in terms of natural science, the second section of the *Regulae* is to deal with problems in which the phenomenological data is properly expressed in the determinate form of primary qualities, and the third section of the *Regulae* is to treat of problems in which such reduction has not yet occurred, the data being expressed in terms of secondary qualities. The *Regulae* discusses the solution of problems both in the fields of mathematics and natural science. In general, at this point, it seems the distinction between problems to find/problems to prove has to do with mathematics, whereas the distinction between problems expressed in terms of secondary as opposed to primary qualities has to do with problems in natural science. What they seemingly have in common is that the third section of the *Regulae* was to have dealt with problems ‘not yet determined’ or ‘imperfect’. Recall what Descartes has said above:

Let us then take it as firmly settled that perfectly determinate problems present hardly any difficulty at all, save that of expressing proportions in the form of equalities, and also that everything in which we encounter just this difficulty can easily be, and ought to be, separated from every other subject and then expressed in terms of extension and figures. Accordingly, we shall dismiss everything else from our thoughts and deal exclusively with these until we reach Rule Twenty-five.\(^{23}\)

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\(^{23}\) CSM 1, p. 58; AT X, 441. Cited in fn 19 above.
Recall also what Descartes has said concerning perfect and imperfect problems in Part I of this Chapter above:

As for problems, however, some can be understood perfectly, even though we do not know the solutions to them, while others are not perfectly understood. We shall deal solely with the former sort of problems in the following twelve Rules, and shall postpone discussion of the latter until the final set of twelve Rules.

How all this is to fit together will become more apparent, I trust, in Chapter Five of the present work, where I shall argue that this conversion from affective qualities to qualities expressed in terms of figure fits in with my overall interpretation of Descartes’ method.

Up to this point in Part II of this Chapter we have considered a particular, lengthy passage from the Regulae, a passage which has prompted my suggestion of the reduction of secondary to primary qualities as a subject matter of the natural science aspect of the third section of the Regulae. Now let us consider another lengthy and problematic quotation from the Regulae. I shall refer to it as Quotation (B). It is taken from Rule Eight, and is as follows:

(B)  [W]e shall first divide into two parts what ever is relevant to the question; for the question ought to relate either to us, who have the capacity for knowledge, or to the actual things it is possible to know. We shall discuss these two parts separately.
   Within ourselves we are aware that, while it is the intellect alone that is capable of knowledge, it can be helped or hindered by three other
faculties, viz. imagination, sense-perception, and memory. We must therefore look at these faculties in turn, to see in what respect each of them could be a hindrance, so that we may be on our guard, and in what respect an asset, so that we may make full use of their resources. We shall discuss this part of the question by way of a sufficient enumeration, as the following Rule [Rule Nine] will make clear.

We should then turn to the things themselves; and we should deal with these only in so far as they are within the reach of the intellect. In that respect we divide them into absolutely simple natures and complex or composite natures. Simple natures must all be either spiritual or corporeal, or belong to each of these categories. As for composite natures, there are some which the intellect experiences as composite before it decides to determine anything about them: but there are others which are put together by the intellect itself. All these points will be explained at greater length in Rule Twelve, where it will be demonstrated that there can be no falsity save in composite natures which are put together by the intellect. In view of this, we divide natures of the latter sort into two further classes, viz. those that are deduced from natures which are the most simple and self-evident (which we shall deal with throughout the next book), and those that presuppose others which experience shows us to be composite in reality. We shall reserve the whole of the third book for an account of the latter.24

In the opening portion of this passage Descartes speaks of a distinction between questions which relate to us as knowers or to actual things it is possible to know. Questions of the latter type are then later in the passage broken down into two different kinds, with Descartes saying he will deal with one type in the second section of the Regulae, but with the other type in the third section. We may note here again a pattern similar to

24 CSM 1, pp. 31-2; AT X, 395-6.
one we have seen before: there are three sections to the *Regulae*. The first twelve Rules have to do with the proper comportment of the mind towards its objects of investigation. And there is also here some additional talk about the content of the second and third sections of the work.

Let us look a bit closer at this later distinction; it is quite complex. This later class is further divided into ‘absolutely simple natures and complex or composite natures’. Then the later class, the complex or composite natures, is further divided into ‘some which the intellect experiences as composite before it decides to determine anything about them’, and ‘others which are put together by the intellect itself’. Now then, after all these distinctions, it is this, the final one, which should hold our interest. Of complex or composite natures put together by the intellect itself, there are two kinds: (1) ‘those [natures] that are deduced from natures which are the most simple and self-evident’, and (2) ‘those [natures] that presuppose others which experience shows us to be composite in reality’.

Let me first point out that in the case of (1) an important catchphrase has been lost in the Cottingham, Stoothoff, and Murdoch translation. In Latin the section of the sentence in question reads as follows: ‘*quae ex simplicissimis naturis & per se cognitis deducuntur*’.25 I shall translate this literally as: ‘which from the most simplest of natures & known *per se* deducible’. Let us refer to the Haldane & Ross translation for a moment. There we read: ‘which are deducible from natures which are of the

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25 AT X, 399.
maximum simplicity and are known per se'. In this instance the Haldane and Ross translation seems closer to the original.

'Simple natures known per se' seem, in this context, to be those expressed in terms of 'figure, extension, motion, etc.'; 'all others we conceive to be in some way compounded out of these'.

In short, from the above lengthy passage we may say the Regulae is to be divided into three sections: 1. An assessment of the proper use of our faculty of understanding itself, 2. an assessment of how it is that we should go about solving questions having to do with complex or composite natures put together by the intellect 'which are deducible from natures which are of the maximum simplicity and are known per se' (i.e. things expressed in terms of 'figure, extension, motion, etc.'), and 3. an assessment of how it is that we should go about solving questions having to do with complex or composite natures which 'presuppose others which experience shows us to be composite in reality'.

With these preliminary comments, allow me to introduce some additional material I believe germane to the proper interpretation of this passage. 'All these points will be explained at greater length in Rule Twelve,' says Descartes. Remember, we are trying to figure out the meaning of this quotation (what I have called Quotation (B)); in order to do so, I suggest we turn now to the three different sections of Rule Twelve itself.

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26 HR 1, p. 27; AT X, 399.
THREE SECTIONS OF RULE TWELVE

Before we start on our little excursion which is to help us in our interpretation of the above passage, notice that in the problematic quotation under consideration here there is a reference to Rule Twelve. I take this as corroborating evidence that Rule Twelve has to do with the tripartite partitioning of the Regulae -- something which is not explicit in the Rule itself. Not surprisingly, the content of the appended explanation or scholium for Rule Twelve is given in three parts as well. I believe we shall see that these three parts are to conform to the proposed sections of the Regulae as Descartes envisioned them. Recall that Rule Twelve read as follows:

Finally we must make use of all the aids which intellect, imagination, sense-perception, and memory afford in order, firstly, to intuit simple propositions distinctly; secondly, to combine correctly the matters under investigation with what we already know, so that they too may be known; and thirdly, to find out what things should be compared with each other so that we make the most thorough use of all our human powers.27

Immediately after the statement of Rule Twelve itself, Descartes says:

27 CSM 1, p. 39; AT X, 410.
This Rule sums up everything that has been said above, and sets out a general lesson the details of which remain to be explained as follows.28

That is to say, Rule Twelve is his summation of the first eleven rules, or the first section of the Regulae. Again, Rule Twelve is itself given in three parts. The first twelve rules are to ‘...explain as briefly as possible what, for my purposes, is the most useful way of conceiving everything within us which contributes to our knowledge of things’.29

We shall deal with each of the sections of this Rule in the following three sections of this work.

First Section: firstly, to intuit simple propositions distinctly.

The first part of the content of the appended explanation for Rule Twelve revolves around the suggestion or hypothesis that: 'sense-perception occurs in the same way in which wax takes on an impression from a seal'.30 Descartes’ statement continues:

This [the sameness of the seal and wax] is the case, we must admit, not only when we feel some body as having a shape, as being hard or rough

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28 CSM 1, p. 39; AT X, 411.
29 CSM 1, p. 40; AT X, 412.
30 CSM 1, p. 40; AT X, 412. Descartes’ introductory remark is: ‘[I]n so far as our external senses are all parts of the body, sense-perception occurs in the same way in which wax takes on an impression from a seal. It should not be thought that I have a mere analogy in mind here: we must think of the external shape of the sentient body as being really changed by the object in exactly the same way as the shape of the surface of the wax is altered by the seal.’
to the touch etc., but also when we have a tactile perception of heat or cold and the like. The same is true of the other senses: thus, in the eye, the first opaque membrane receives the shape impressed upon it by multi-coloured light; and in the ears, the nose and the tongue, the first membrane which is impervious to the passage of the object thus takes on a new shape from the sound, the smell and flavour respectively.\textsuperscript{31}

Notice he is speaking here about ‘heat, cold, and the like qualities’, and also of ‘the shape impressed upon’ the sensory membrane. Descartes continues:

This is a most helpful way of conceiving these matters, since nothing is more readily perceivable by the senses than shape, for it can be touched as well as seen. Moreover, the consequences of this supposition are no more false than those of any other. This is demonstrated by the fact that the concept of shape is so simple and common that it is involved in everything perceivable by the senses. Take colour, for example: whatever you may suppose colour to be, you will not deny that it is extended and consequently has shape. So what troublesome consequences could there be if -- while avoiding the useless assumption and pointless invention of some new entity, and without denying what others have preferred to think on the subject -- we simply make an abstraction, setting aside every feature of colour apart from its possessing the character of shape, and conceive of the difference between white, blue, red, etc. as being like the difference between the following figures or similar ones?

\textsuperscript{31} CSM 1, p. 40; AT X, 412-3.
The same can be said about everything perceivable by the senses, since it is certain that the infinite multiplicity of figures is sufficient for the expression of all the differences in perceptible things.\textsuperscript{32}

Descartes is here arguing, obliquely, that differentiation in color (and, by extension, sounds and savors) are attributable to differentiation in figure. Again, using philosophers' talk, he is saying secondary qualities are caused by (and by implication, may be reduced to) primary ones. We have seen this is what he has done in the case of harmonics above. Affective sounds are reduced, in principle, to quantifiable, measurable terms -- though we may not be directly familiar with what exactly these primary qualities are.

Interestingly, in Rule Twelve, Descartes goes on to say:

We can see how, by following this Rule, we can abstract a problem which is well understood from every irrelevant conception and reduce it to such a form that we are not longer aware of dealing with this or that subject-matter but only with certain magnitudes in general and the comparison between them.\textsuperscript{33}

\textsuperscript{32} CSM 1, pp. 40-1; AT X, 413.
\textsuperscript{33} CSM 1, p. 52; AT X, 431.
I take it what Descartes is saying here is that, in terms of natural science, if a problem is well-understood, the irrelevant conceptions of the affective qualities have been reduced to the more manageable form of magnitudes and the comparison between magnitudes; or, that qualitative expressions are to be reduced to quantative ones. The rest of the first part of Rule Twelve is a description of how the intellect is informed by the intrusion upon the sensory apparatus of the body by the bodily figures in question, and how the sensation is conveyed through the nerves to the brain, and so forth, with reference to how the faculties of sense, memory, imagination, and intellect are, optimally, to function.

Second Section: *secondly, to combine correctly the matters under investigation with what we already know, so that they too may be known.*

Descartes opens this section:

Let us now take up the second factor. Our aim here is to distinguish carefully the notions of simple things from those which are composed of them...34

However, let us bear in mind he goes on to say:

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34 CSM 1, p. 43; AT X, 417.
[A]nd in both cases to try to see where falsity can come in, so that we may guard against it, and to see what can be known with certainty, so that we may concern ourselves exclusively with that.  

[W]hen we consider things in the order that corresponds to our knowledge of them, our view of them must be different from what it would be if we were speaking of them in accordance with how they exist in reality. If, for example, we consider some body which has extension and shape, we shall indeed admit that, with respect to the thing itself, it is one single and simple entity. For, viewed in that way, it cannot be said to be a composite made up of corporeal nature, extension and shape, since these constituents have never existed in isolation from each other. Yet with respect to our intellect we call it a composite made up of these three natures, because we understood each of them separately before we were in a position to judge that the three of them are encountered at the same time in one and the same subject. That is why, since we are concerned here with things only in so far as they are perceived by the intellect, we term ‘simple’ only those things which we know so clearly and distinctly that they cannot be divided by the mind into others which are more distinctly known. Shape, extension and motion, etc. are of this sort; all the rest we conceive to be in a sense composed out of these.

One important thing to notice here, I think, is that there are two kinds of ‘simple’ natures or things: those which are real and those which are relative to the mind — or, things we consider in the order that correspond to our knowledge of them and things we consider in accordance with how they exist in reality. With some interpretation this clears up to

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35 CSM 1, p. 43; AT X, 417.
36 CSM 1, P. 44; AT X, 418.
an extent a problem I’m sure the reader may have already noticed. Are the simple natures to which affective qualities are to be reduced (1) presumed ‘metaphysical’ primary qualities in the objects which account for the secondary qualities we perceive; or (2) perceived ‘epistemic’ primary qualities constituting empirical phenomena susceptible to being handled in terms of quantitative science? That is, are primary qualities (1) in the world, or (2) in the mind? The former kinds of primary qualities may be considered ‘real’ in the sense of being actually existent in the physical object itself, whereas the latter might be considered ‘relative’ in the sense of being existent only as mental or phenomenal representations of perceived aspects of the object. The example taken from harmonics above argues for the latter. But let us reconsider the following:

> [W]hen we consider things in the order that corresponds to our knowledge of them, our view of them must be different from what it would be if we were speaking of them in accordance with how they exist in reality.\(^{37}\)

Later he speaks of things ‘simple with respect to our intellect’.\(^{38}\) It is worth indicating here it has been pointed out that Descartes uses the term ‘simple natures’ equivocally:

That some of Descartes’ simple natures have a sort of ontological independence seems to me incontrovertible; it is equally obvious also

\(^{37}\) CSM 1, p.44; AT X, 418.

\(^{38}\) CSM 1, p. 44; AT X, 419.
that many of them do not. It is natural to object that this makes of simple natures something rather odd: a strange and worthless class of entities some of which are 'in the world,' and others of which are just 'ideas.' I am afraid this is the truth of the matter....39

Thus for Descartes things may be simple in accordance with how they exist in reality or simple with respect to our intellect. From these considerations I draw the conclusion that, in terms of natural science, for Descartes there are two types of 'simple natures': (1) the physical attributes of the metaphysical building blocks of the universe, the 'minute corpuscles', and (2) the basic and direct phenomenal data by means of which we know the world.40 Why or how these both might retain the same or similar ontological status I shall take up in Chapter Five; suffice it to say here they may both be considered primary qualities.

So the answer to the question posed above, are the secondary qualities we perceive to be reduced to the primary qualities of real objects which produce them, or are they to be converted to quantative data for problem solution, is that Descartes wants both. As I read this passage, in real terms secondary qualities are to be reduced to the primary qualities (microscopic and/or mesoscopic) of objects which produce them; but in relative terms secondary qualities are to be converted to phenomenal primary qualities at the mid-range level so as to be susceptible to solution

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39 O'Neil, Epistemological Direct Realism in Descartes' Philosophy, University of New Mexico Press, 1974, p. 20.
40 I suspect this distinction corresponds, generally, to the Cartesian distinction between 'absolute' and 'relative' terms, though this is a highly complex issue and I have not worked it out in detail.
in terms of science. As Descartes points out in the passage above, however, the two should be considered in some sense a different order, and we are primarily concerned for the moment with the latter -- precisely because we are concerned in the *Regulae* with the nature of problem solution.

Also, he is saying here that corporeal nature, extension, figure (and motion) are not to be considered complex or composite natures. They are only compound relative to our understanding. Also, relatively speaking, they are the ultimate simples, 'which we know so clearly and distinctly that they cannot be divided by the mind into others which are more distinctly known'.

Descartes later goes on to add, '[T]hese simple natures are all self-evident and never contain any falsity'. Again I would like to have recourse to the Latin here, which states: 'Dicimus tertió, naturas illas simplices esse omnes *per se* notas, & nunquam ullam falsitatem continere.' The Haldane and Ross translation states at this point, 'we assert that all these simple natures are known *per se* and are wholly free from falsity.' The reference to 'per se' here seems important.

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41 That Locke was familiar with, and even inspired by, Descartes is documented. In his introductory material to Locke's *Essay*, Fraser states: '[Locke] was strongly attracted to Descartes. 'The first books, as Mr. Locke has told me,' Lady Masham writes, 'which gave him a relish of philosophical things were those of Descartes. He was rejoiced in reading these, because, though he very often differed in opinion from this writer, whence he was encouraged to think that his not having understood others had possibly not proceeded from a defect in his understanding.' Descartes, often named in Locke's letters to Stillingfleet, probably influenced him more than any metaphysical philosopher, not only by his analytic intrepidity, but by his introspective method.' Locke, John, *An Essay Concerning Human Understanding*, Dover, New York, 1959.

Insofar as Locke was familiar with Descartes, Locke's equivocation on this topic may ultimately derive from a misunderstanding of Descartes. In any case, if I am correct in my assessment here, it would seem Descartes was much more clear in this distinction than was Locke.

42 CSM 1, p. 45; AT X, 420.
43 HR 1, p. 42; AT X, 420.
Schouls, in his book *The Imposition of Method*, speaks of what he calls a doctrine of 'knowledge per se'.\(^{44}\) He seems of the opinion that certain 'simple' concepts like 'unity' and 'equality' or 'thought' and 'existence' in addition to certain fundamental principles which serve to link these concepts are 'known per se'. As Schouls says:

In [Descartes'] metaphysics, it is not just 'simple' concepts like 'unity' and 'equality' (or more to the point, concepts like 'thought' and 'existence') which play a fundamental role. There are also certain fundamental principles which serve to link these concepts. Such principles are, according to Descartes, known *per se*, and also in this doctrine of 'knowledge per se' a limitation of the process of reduction is implied.\(^{45}\)

I suggest this catch-phrase 'known per se' corresponds to knowledge of the relative sort, which I am reading as knowledge expressed in terms of observed or phenomenal primary qualities as opposed on the one hand to secondary qualities and on the other hand to the actual or real primary qualities which are the presumed cause of secondary qualities. I have already indicated that if we substitute the Haldane and Ross translation for that of Cottingham, Stoothoff, and Murdoch, at the end of quotation (B), we obtain the following:


This is why we further subdivide these into the class of those which are (1) deducible from natures which are of the maximum simplicity and are known per se, of which we shall treat in the whole of the succeeding book; and into (2) those which presuppose the existence of others which the facts themselves show us to be composite. To the exposition of these we destine the whole of the third book.46

Descartes has also told us in our problematic Quotation (B) from Rule Eight that the answer is to be found in Rule Twelve, that the answer has something to do with human error, and that it also has something to do with how the intellect makes its own compounds. I am going to give the passage from Rule Twelve to which he is referring, but let me first give what I take to be its opening line:

[T]hose natures which we call ‘composite’ are known by us either because we learn from experience what sort they are, or because we ourselves put them together.47

Recall that above in Quotation (B) this is exactly the distinction that Descartes has made. Sections Two and Three of the Regulae were both to be about ‘composites’ put together by the intellect. Section Two was to be about composite natures constructed by ourselves from simple natures; Section Three was to be about composite natures which presuppose others which are themselves composite. Initially it appears we are on the right track. Now for the rest of the passage:

46 HR 1, p.27; AT X, 399. Numbers in parentheses added for emphasis.
47 CSM 1, p. 46; AT X, 422.
Our experience consists of whatever we perceive by means of the senses, whatever we learn from others, and in general whatever reaches our intellect either from external sources or from its own reflexive self-contemplation. We should note here that the intellect can never be deceived by any experience, provided that when the object is presented to it, it intuits it in a fashion exactly corresponding to the way in which it possesses the object, either within itself or in the imagination. Furthermore, it must not judge that the imagination faithfully represents the objects of the senses, or that the senses take on the true shapes of things, or in short that external things always are just as they appear to be. In all such cases we are liable to go wrong, as we do for example when we take as gospel truth a story which someone has told us; or as someone who has jaundice does when, owing to the yellow tinge of his eyes, he thinks everything is coloured yellow; or again, as we do when our imagination is impaired (as it is in depression) and we think that its disordered images represent real things. But the understanding of the wise man will not be deceived in such cases: while he will judge that whatever comes to him from his imagination really is depicted in it, he will never assert that it passes, complete and unaltered, from the external world to his senses, and from his senses to the corporeal imagination, unless he already has some other grounds for claiming to know this. But whenever we believe that an object of our understanding contains something of which the mind has no immediate perceptual experience, then it is we ourselves who are responsible for its composition. In the same way, when someone who has jaundice is convinced that the things he sees are yellow, this thought of his will be composite, consisting partly of what his corporeal imagination represents to him and partly of the assumption he is making on his own account, viz. that the colour looks yellow not owing to any defect of vision but because the things he
sees really are yellow. It follows from this that we can go wrong only when we ourselves compose in some way the objects of our belief.\textsuperscript{48}

Error is due to us, not to God; this we know from \textit{Meditationes} Chapter IV. There Descartes informs us that error occurs in those cases when the infinite will surpasses the finite judgment. I shall say more on that in a moment. Here, however, I believe a similar argument is being given. The argument, like that of the \textit{Cogito}, is from Augustine.\textsuperscript{49}

Couched in terms of the person with jaundice -- and assuming, obviously, that the objects being called yellow are in fact not yellow -- the mistake being made does not occur when the person says the objects around him appear yellow: they certainly do appear yellow. Rather, the mistake occurs when the person says such objects \textit{are} yellow. That is, the error occurs when the person says, 'that the colour looks yellow not owing to any defect of vision but because the things he sees really are yellow'. As Descartes indicates here, the reason the thought is considered composite by him is because it consists: (1) 'partly of what [the person's] corporeal imagination represents to him', and (2) 'partly of the assumption he is making on his own account,' that the object actually is as his corporeal

\begin{footnotesize}
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\item[48] CSM 1, pp. 46-7; AT X, 422-3.
\item[49] In the words of Augustine: 'It now remains for us to inquire whether the senses report the truth when they give information. Suppose that some Epicurean should say: 'I have no complaint to make in regard to the senses; for it is unjust to demand more of them than they can give; moreover whatever the eyes can see they see in a reliable manner.' Then is what they see in regard to an oar in the water true? It certainly is true. For when the reason is added for its appearing thus, if the oar dipped in the water seemed straight, I should rather blame my eyes for the false report. For they did not see what should have been seen when such causes arose. What need is there of many illustrations? This can also be said of the movement of towers, of the feathers of birds, of innumerable other things. 'And yet I am deceived if I give my assent,' someone says. Do not give assent any further than to the extent that you can persuade yourself that it appears true to you, and there is no deception.' St. Augustine, \textit{Against the Academicians}, III, tr. Sister Mary Patricia Garvey, Milwaukee, Wisconsin, Marquette University Press, 1957, pp. 23-6.
\end{itemize}
\end{footnotesize}
imagination represents to him. This is what he means when he says, ‘It follows from this that we can go wrong only when we ourselves compose in some way the objects of our belief,’ the faulty composition being the conjunction of the idea together with the assumption the idea correctly resembles the corporeal object. This is also what he means when he says, ‘[The intellect] must not judge that the imagination faithfully represents the objects of the senses, or that the senses take on the true shapes of things, or in short that external things always are just as they appear to be’. And, though this is a difficult passage, when he says, ‘the intellect can never be deceived by any experience, provided that when the object [I take it here Descartes means the mental object] is presented to it, it intuits it in a fashion exactly corresponding to the way in which it possesses the object, either within itself or in the imagination,’ he means, I believe, that as long as we do not attribute an experience in the mind as actually existing external to itself in physical reality, we cannot go wrong or be deceived. It is important to point out at this point, however, that such attribution does not automatically lead to error -- the object might be correctly referred to as yellow -- it leads only to the possibility of error. In any case, this is why, in this and in similar passages, Descartes believes we cannot be in error if we turn away from the senses to look for the truth.

Similarly, in the example where we take as gospel truth what someone has told us: we are perfectly free to say someone has told us this story, the possibility of error arises when we pronounce the story true.
Let me follow up on this point. It is well known that Descartes demands we depart from the senses in the opening pages of the *Meditationes*, and turn our attention inwards:

I will suppose then, that everything I see is spurious. I will believe that my memory tells me lies, and that none of the things that it reports ever happened. I have no senses. Body, shape, extension, movement and place are chimeras.\(^{50}\)

But Descartes also doubts, provisionally, the existence of God at the beginning of the *Meditations*, and it would be as erroneous to say that Descartes distrusts all senses as it would be to say that he disbelieves in God. It is not always remembered that by the end of the *Meditations* we get our senses back, and external objects are said, on the basis of Descartes’ argumentation, to exist. But, for Descartes, which senses in the end are to be trusted? Well, those which are clear and distinct, we are informed:

If, however, I simply refrain from making a judgement in cases where I do not perceive the truth with sufficient clarity and distinctness, then it is clear that I am behaving correctly and avoiding error. But if in such cases I either affirm or deny, then I am not using my free will correctly. If I go for the alternative which is false, then obviously I shall be in error; if I take the other side, then it is by pure chance that I arrive at the truth, and I shall still be at fault since it is clear by the natural light that the perception of the intellect should always precede the determination of the will. In this incorrect use of free will may be found the privation which constitutes the essence of error. The privation, I

\(^{50}\) CSM 1, p. 16; AT X, 24.
say, lies in the operation of the will in so far as it proceeds from me, but
not in the faculty of will which I received from God, nor even in its
operation, in so far as it depends on him.\footnote{CSM 2, p. 41; AT VII, 59-60. Italics added for emphasis.}

As long as we limit our active will to the affirmation of just and only
those ideas which are found to be 'clear and distinct' in our passive
understanding, we shall avoid error. Good. But which ideas are clear and
distinct?\footnote{In reference to Newton and Descartes, J.E. McGuire states: 'Both thinkers draw a distinction between
the "faculties" of imagination and understanding, together with the objects appropriate to these ways of
conceiving. Though hedged with many qualifications by different thinkers, the distinction is essentially this:
the imagination represents the sensuously based features of a given object, but the understanding is capable of
producing a non-sensuous representation of an object' [sic] essential nature. On this conception, the mind has the
capacity of knowing one and the same object in different ways, and, correspondingly, there are levels of knowing
appropriate to each way of conceiving. Moreover, we can be said to have confused as well as distinct
representations of things.' McGuire, J.E., 'Space, Geometrical Objects and Infinity: Newton and Descartes on

Apparently McGuire is suggesting here, though he does not expressly state it, that the distinction
between clear vs. confused ideas applies properly to the contents of the faculty of the understanding, whereas the
distinction between distinct and obscure ideas applies to the contents of the faculty of the imagination -- or vice
versa. McGuire is of the opinion that for Descartes in the Meditations, the understanding can apprehend figures
such as the chiliagon and the essential nature of a piece of wax whereas the imagination cannot. Later in the same
article, however, McGuire says: '...the essential dispositional nature of the wax is conceived by the power of the
understanding; for it grasps distinctly and clearly that the wax is an extended, flexible and changeable nature; in
contrast, that fact is conceived confusedly in terms of the sensuously-based and occurrent representations,
the contents of which are present in the imagination.' McGuire, p. 79. This seems to contradict the distinction
implicit in his earlier statement.

The suggestion, however, that clear ideas have to do with the understanding whereas distinct ideas have
to do with the imagination is borne out to some extent by the following passage from the Principia: 'Yet who
ever doubted that bodies are moved, and are moved variously according to their various sizes and figures; or that as
a result of the collision of these bodies, the larger ones are divided into many smaller ones, and change their
figures? We do not observe this through only one sense, but through several: through sight, touch, and hearing
[sic ]; and we also (very) distinctly imagine and (clearly) understand this. This cannot be said of the remaining
qualities (perceived by our senses), like colors, sounds, and the rest, which are perceived not by means of several
senses, but only by means of individual ones: for their images in our minds are always confused, and we do not
know what they may be.' Principia, p. 283.
which, viewed in general terms, are comprised within the subject-matter of pure mathematics.\(^53\)

From this passage I read that, for Descartes, quantative qualities ('properties comprised within the subject-matter of pure mathematics') are clearly and distinctly understood -- i.e. are veridical and to be trusted. Presumably these are the qualities of figure and extension.

[T]hese sensations of hunger, thirst, pain and so on are nothing but confused modes of thinking which arise from the union and, as it were, intermingling of the mind with the body.\(^54\)

The problem here, I take it, is that the sensations in question -- 'confused modes of thinking' because they are secondary qualities to begin with -- are attributed to the body; rather for Descartes they should be considered sensations in the mind.

And from the fact that I perceive by my senses a great variety of colours, sounds, smells and tastes, as well as differences in heat, hardness and the like, I am correct in inferring that the bodies which are the source of these various sensory perceptions possess differences corresponding to them, though perhaps not resembling them.\(^55\)

I can attribute the causes of my sensations to certain objects; I err when I attribute the sensations themselves to the objects.

\(^{53}\) CSM 2, p. 55; AT VII, 80.
\(^{54}\) CSM 2, p. 56; AT VII, 81.
\(^{55}\) CSM 2, p. 56; AT VII, 81.
Heat is mentioned here. Lists of secondary qualities of the period are pretty standard, if not altogether enlightening. They include commonly the qualities of colors, sounds, smells, and tastes, with the list usually ending by saying 'etc.' or 'and so forth'. One quality usually considered paradigmatically secondary by these people -- but, it seems, normally forgotten by us moderns -- is that of heat. For certain Medieval scholastics heat was considered a property of the physical object. The new view in the Seventeenth Century -- that proposed by Boyle, Locke, Galileo, Malebranche, Descartes and others\textsuperscript{56} -- was that heat was now considered a mental affect produced by the quick motion of tiny bodies -- corpuscles or atoms depending on the thinker. It was now considered a phenomenal object only -- like the rainbow.

\textit{[A]}lthough I feel heat when I go near a fire and feel pain when I go too near, there is no convincing argument for supposing that there is something in the fire which resembles the heat, any more than for supposing that there is something which resembles the pain. There is simply reason to suppose that there is something in the fire, whatever it may eventually turn out to be, which produces in us the feelings of heat or pain.\textsuperscript{57}

The problem here, the error, again is that the secondary quality heat is said to exist in the object, not in the mind. Sensory perceptions can be useful, however:

\textsuperscript{56} My suspicion is that most probably Descartes’ own distinction in this respect derives from Galileo.  
\textsuperscript{57} CSM 1, p. 57; AT X, 83.
For the proper purpose of the sensory perceptions given me by nature is simply to inform the mind of what is beneficial or harmful for the composite of which the mind is a part; and to this extent they are sufficiently clear and distinct. But I misuse them by treating them as reliable touchstones for immediate judgments about the essential nature of the bodies located outside us; yet this is an area where they provide only very obscure information.\(^{58}\)

The perceptions of our senses do not teach what really exists in things, but only what can harm or benefit that union.\(^{59}\)

The *Principia* follows a near identical argumentation as that of the *Meditationes*; and the same is true also for the appropriate sections of the *Principia*, where -- after the arguments for the Cogito and for the existence of God -- we are told we can know with certainty that material objects exist.\(^{60}\) What Descartes goes on to say about error in the *Principia* is much more detailed, though not perhaps quite so general, as what we find in the *Meditations*. Before speaking there about truth and error, he distinguishes between ideas which are clear and ideas which are distinct. Distinct ideas, according to Descartes, are always clear, whereas clear ideas needn't always be distinct. The example he uses is that of pain:

\(^{58}\) CSM 1, p. 58; AT X, 84.

\(^{59}\) *Principia*, p. 40.

\(^{60}\) *Principia*, p. 39. As Descartes says: "[I]f God were Himself directly presenting the idea of this extended matter to our mind, or even merely causing it to be presented by something which lacked extension, shape, and movement; it would be impossible to devise any reason for not thinking Him a deceiver. ...Moreover, we seem to see clearly that the idea of it comes from external things, which it perfectly represents; and, of course, as has already been noticed, it is completely contrary to God's nature to be a deceiver." But the manner in which our idea of an external thing 'perfectly represents' that thing will require a bit more discussion below.
[B]y the example of pain, it is shown that a perception can be clear even though it is not distinct; but that it cannot be distinct unless it is clear. 61

Thus, when someone feels some great pain, the perception of pain is indeed very clear in him, but is not always distinct; for commonly men confuse that perception with their uncertain (false) judgment about its nature; because they believe something resembling the feeling of pain to be in the painful part. And thus a perception which is not distinct can be clear; but no perception can be distinct unless it is clear. 62

The problem with pain for Descartes is that it exists properly speaking only in the mind, not in the part of the body in distress. Any other secondary quality may be substituted into this passage and retain as much sense as the original. Let us substitute color, for example:

Thus, when someone sees some color, the perception of the color is indeed very clear in him, but is not always distinct; for commonly men confuse that perception with their uncertain (false) judgment about its nature; because they believe something resembling the color to be in the corporeal object perceived. And thus a perception which is not distinct can be clear; but no perception can be distinct unless it is clear. 63

61 Principia, p. 20.
62 Principia, pp. 21-1.
63 Descartes also has the following to say: ‘And exactly the same is true of all the other sensations which are felt, even pleasure and pain. For although these are not thought to be outside us; they are however not usually regarded as being solely in our mind or perception, but as being in our hand, or our foot, or some other part of our body. And it is definitely as uncertain that a pain which we feel as if in the foot, say, is something existing outside our mind, in the foot; as it is uncertain that the light which we see as if in the Sun exists outside us, in the Sun (in the way it is in us): but both these prejudices belong to our childhood....’ Principia, p. 30.

There is a problem with this interpretation, however. Cf.: ‘[W]hen we observe some body, although we are as certain that it exists insofar as it appears to have color as we are insofar as it appears to have figure; yet we know much more clearly what it is for that body to have figure than what it is for it to have color.’ This quotation is given in context below. I suspect Descartes was not altogether consistent in this distinction between clear and distinct ideas. Perhaps it occurred to him as an afterthought. I do not see that this particular distinction occurs in any of the early or middle works. Leibniz was, however, to adhere forcefully to a distinction between clear and distinct ideas.
Descartes goes on, however, to say:

[T]he nature of body does not consist in weight, hardness, color, or other similar properties; but in extension alone. [I]t can be shown that weight, color, and all the other properties of this kind which are experienced in material substance, can be taken away; leaving that substance intact. From this it follows that the nature of matter does not depend on any such properties, [but consists solely in the fact that it is a substance which has extension].

There is, however, an additional section on error in the *Principia* worth looking at:

[Part I Article 68:] How, in these matters, that which we clearly know must be distinguished from that in which we can be deceived.

[Pl]ain, and color, and the remaining things of this kind, are clearly and distinctly perceived when regarded as only sensations of thoughts. …[W]hen someone says that he sees color in some body, or feels pain in some limb, it is exactly as if he were to say that he sees or feels there something of whose nature he is completely ignorant, that is, that he does not know what he is seeing or feeling. …[I]f however he examines what it is that this sensation of color or pain (considered as if existing in the colored body, or in the painful part) represents [to him], he will certainly notice that he is entirely ignorant of it.

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64 *Principia*, p. 41. There is a footnote in the *Principia* by the translators on this page which shows they appreciate to some extent the point I am trying to make in this section: ‘An important consideration, which is not made explicit here [in Part II, Article 4], is that only extension, figure, and motion are capable of being clearly and distinctly perceived by the understanding. They are also the only properties which can be directly represented geometrically; see Part I, Article 69. Consequently, only those properties are capable of generating necessary truths about bodies.’ Part I, Article 69, is given here below.

65 *Principia*, p. 31.
Again:

[W]hen we say that we perceive colors in objects, this is in fact the same as if we were to say that we perceive something in objects of whose nature we are ignorant, but by means of which a certain very manifest and evident sensation is created in us, which is called the sensation of colors. However, there is a very great difference in the ways of judging [associated with these two remarks]: for as long as we merely judge that there is something in objects (that is, in the things from which a sensation comes to us, of whatever exact kind those things may be) the nature of which we do not know; we will be so far from being deceived that we will instead avoid error; because when we notice that we are ignorant of something, we are less inclined to judge rashly of it.⁶⁶

Of primary qualities, Descartes has a different evaluation, however:

[Part I Article 69:] That size, figure, etc., are known in a very different manner from colors, pains, etc.

[I]f he considers that size, in a body which has been observed, or figure, or motion..., or situation, or duration, or number, and other similar things which we have already stated are perceived clearly in bodies; are known by him in a manner very unlike that in which he knows, in the same body, what color is, or pain, or odor, or flavor, or any of the other things which I have said must be referred to the senses. For when we observe some body, although we are as certain that it exists insofar as it appears to have color as we are insofar as it appears to have figure; yet we know much more clearly what it is for that body to have figure than what it is for it to have color.⁶⁷

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⁶⁶ Principia, pp. 31-2.
⁶⁷ Principia, p. 31.
Sensation for Descartes -- which I believe is to be distinguished from the faculty of the mind he terms the Senses -- consists only in the effects on the mind of movements transmitted by means of the neurological system. These are internal and external in nature; the internal senses are for Descartes the passions with the external senses being the ones we normally refer to as 'the five senses': touch, taste, smell, hearing, and sight. In any case, the problem is that our thoughts do not (always) resemble their objects of reference.

What Descartes has said here in the *Principia* seems to fit in well with what he has said in the *Regulae* concerning jaundice. Descartes' analogy with the basket of apples is to be taken seriously. In the beginning of the *Meditationes*, and the *Principia*, Descartes metaphorically dumps all the bad apples of every type of perception out of the basket. Later the good apples, i.e. the Primary Qualities and those Secondary Qualities not assumed to be representative, are allowed back in.

The reader will at this point recall that this entire discussion on primary and secondary qualities was prompted by an examination of the second section of Rule Twelve of the *Regulae*, and that we have already considered the first section, but have yet to consider the third. We shall now do so.

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Third Section: *thirdly, to find out what things should be compared with each other so that we make the most thorough use of all our human powers.*

To recapitulate, in the first section of the *Regulae*, the faculty of intuition is to inform us how to properly comport the mind towards its objects of investigation 'to intuit simple propositions distinctly'. The second section was to inform us how, 'to combine correctly the matters under investigation with what we already know, so that they too may be known'. The third part of the appended explanation, however, does not seem to have to do with the third part of the Rule itself. Rather it is a list of conclusions drawn from the second part of the appended explanation. In addition, it uses the example of the magnet:

[If the question concerns the nature of the magnet, foreseeing that the topic will prove inaccessible and difficult, [almost everyone] turns his mind away from everything that is evident, and immediately directs it at all the most difficult points, in the vague expectation that by rambling through the barren field of manifold causes he will hit upon something new. But take someone who thinks that nothing in the magnet can be known which does not consist of certain self-evident, simple natures: he is in no doubt about how he should proceed. First he carefully gathers together all the available observations concerning the stone in question; then he tries to deduce from this what sort of mixture of simple natures is necessary for producing all the effects which the magnet is found to have. Once he has discovered this mixture, he is in a position to make the bold claim that he has grasped the true nature of the]
magnet, so far as it is humanly possible to discover it on the basis of given observations.\textsuperscript{69}

This advice is remarkably similar to that given in Rule XIII, that is to say the second -- not third -- section of the \textit{Regulae}:

But if the problem is to be perfect, we want it to be determinate in every respect, so that we are not looking for anything beyond what can be deduced from the data. For example, someone may ask me what conclusions are to be drawn about the nature of the magnet simply from the experiments which Gilbert claims to have performed, be they true or false.\textsuperscript{70}

We can also see how, by following this Rule, we can abstract a problem which is well understood from every irrelevant conception and reduce it to such a form that we are no longer aware of dealing with this or that subject-matter but only with certain magnitudes in general and the comparison between them. For example, once we have decided to investigate specific observations relating solely to the magnet, we no longer have any difficulty in dismissing all other observations from our mind.\textsuperscript{71}

In addition, after the eighth point made in the second section, as I read this scholium, there is an obvious and universally recognized lacuna -- just at the place we would expect the third section of the explanation.

It seems probable, therefore, that the third section of Rule Twelve, which was to conform to the third section of the \textit{Regulae}, was either never written or, at least, not included in the work we have today.

\textsuperscript{69} CSM 1, pp. 49-50; AT X, 427.
\textsuperscript{70} CSM 1, p. 52; AT X, 431.
\textsuperscript{71} CSM 1, p. 52; AT X, 431.
Probably the reader has gotten the point by now. I attribute some sort of distinction between primary and secondary qualities to Descartes. I suggest that Descartes wishes us to reduce secondary qualities to primary qualities in order to utilize them in problems in the field of natural science. In terms of natural science, the second section of the *Regulae* was to educate us in how to solve problems already expressed in terms of primary qualities, and the third section was to show how to solve the conversion of secondary qualities to primary qualities and thereby solve them presumably in conformity with the second section. The importance of this will become clear later when we discuss Descartes’ procedure in natural science.

Summary.

In this Chapter I have argued that by ‘perfect’ problems in mathematics Descartes means problems to prove, and by ‘imperfect’ problems in mathematics Descartes means problems to find. I have also argued that by ‘perfect’ problems in natural science Descartes means problems expressed in terms of primary qualities, and by ‘imperfect’ problems in natural science Descartes means problems not yet expressed in terms of primary qualities. I have also argued that ‘perfect’ problems are dealt with in the second section of the *Regulae*, Rules Thirteen through Twenty-Four, and that ‘imperfect’ problems were to have been dealt with by Descartes in the
third section of the *Regulae*, the unwritten Rules Twenty-Five through Thirty-Six.
I have used the term 'Cartesian Analysis' in the heading of this chapter but only the Chapter's third and last section will actually address this topic. The first two sections will pursue very little historical investigation concerning Descartes. Instead, in these two sections I review some material I have found helpful in understanding what I take to be Cartesian analysis, specifically Cartesian analysis in natural science. Firstly, I am going to set out some opinions concerning the nature of discovery by a mathematician by the name of Polya. Secondly, I am going to take an example of discovery from modern science -- the discovery of the physical structure of the DNA molecule by Watson and Crick -- and discuss some of what I take to be its more salient aspects, aspects I believe illucidatory, broadly speaking, of Cartesian science. Thirdly, I shall discuss a proposed relationship between what I have been calling Cartesian mathematics and the analysis of the ancients. The results of this discussion will give a clarification, in my opinion, of what we may take Descartes to mean by the term 'analysis' in his scientific and metaphysical pursuits.

Allow me to insert a cautionary note at this juncture, however. I have stated that the example of Watson and Crick's discovery may be utilized as an illustrative example to help in the understanding of what Descartes was trying to discuss in his expressed views concerning analysis, discovery, and natural philosophy; but I wish to stress also that the fit
between Watson and Crick and Descartes on this topic will not be perfect. This will become more apparent, however, in the next Chapter.\textsuperscript{1} Additionally I should stress that Descartes is not Polya and Polya is not Descartes. I have introduced these two discussions concerning Watson and Crick and Polya because I have found them helpful in understanding Descartes, and I am hopeful the reader will find them helpful as well. It will be remembered that an analogy is only good for as far as it extends, and that no analogy is perfect lest it cease to be an analogy and become an identity.


George Polya is a mathematician who has addressed himself to the question of how it is that mathematicians make discoveries and solve problems. We have met him before; he is the one who translated Pappus so as to draw the distinction between ‘problems to find’ and ‘problems to prove’. His purpose is to understand the activity of problem solution, to propose means to teach problem solution, and to improve the problem-solving ability of persons interested in that topic. He does this generally by presenting, as it were, what may be called case histories of solutions. He does not believe there is a single all-encompassing or general method; he

\textsuperscript{1} Simplistically put, the difference I shall delineate there is that whereas Watson and Crick's principles are empirically derived, Descartes’ are metaphysically deduced.
seems rather of the opinion that there are various types of methods which might be thought of as one single method only in a figurative sense.² (One is reminded of a Wittgensteinean ‘family resemblance’ here.) Still, he feels that it is very productive to think about the general character of problem-solving in mathematics and other fields, because the development of such general forms of understanding can be very helpful on the more specific level in the approach to solving particular problems.

Polya believes that problem solution is a practical art like swimming, or skiing, or playing the piano: the best way to learn it is by imitation and practice.³ According to Polya it is beneficial to the learning of the solution of various sorts of problems to go back and review problems once they are solved. As he says:

The best time to think about methods may be when the reader has finished solving a problem, or reading its solution, or reading a case history. With his task accomplished and his experience still fresh in mind, the reader, in looking back at his effort, can profitably explore the nature of the difficulty he has just overcome. He may ask himself many useful questions: “What was the decisive point? What was the main difficulty? What could I have done better? I failed to see this point: which item of knowledge, which attitude of mind should I have had to see it? Is there some trick worth learning, one that I could use the next time in a similar situation?” All these questions are good, and there are

² In the words of Polya: ‘Descartes meditated upon a universal method for solving all problems, and Leibnitz [sic] very clearly formulated the idea of a perfect method. Yet the quest for a universal perfect method has no more succeeded than did the quest for the philosopher’s stone which was supposed to change base metals into gold; there are great dreams that must remain dreams. Nevertheless, such unattainable ideals may influence people: nobody has attained the North Star, but many have found the right way by looking at it. This book cannot offer you (and no book will ever be able to offer you) a universal perfect method for solving problems, but even a few small steps toward that unattainable ideal may clarify your mind and improve your problem-solving ability.’ MD 1, p. vi.

³ MD 1, p. v. Descartes seems to be of a similar opinion; vide Rule Ten of the Regulae.
many others -- but the best question is the one that comes spontaneously to mind.  

On this topic, Polya approvingly quotes Descartes:

Each problem that I solved became a rule which served afterwards to solve other problems.

If I found any new truths in the sciences, I can say that they all follow from, or depend on, five or six principal problems which I succeeded in solving and which I regard as so many battles where the fortune of war was on my side.

Before, however, we discuss Polya’s approach to discovery in mathematics it would be convenient to have some grasp of what mathematicians have meant by the term ‘locus’. Consider the following statement from a textbook in analytic geometry written in 1924:

[T]here is a correspondence between points of the plane and pairs of real numbers. A correspondence may be established between certain geometrical figures and certain equations in $x$ and $y$. An equation is not satisfied by arbitrary values of $x$ and $y$, that is, by the coordinates of arbitrary points. The points whose coordinates do satisfy the equation form a geometrical figure which is called the \textit{locus of the equation}, and the equation is said to be the \textit{equation of the figure}. \textit{Definition: The locus of an equation is the totality of points whose coordinates satisfy the equation.} The coordinates of any point on the locus satisfy the

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4 MD 1, p. xii.
5 MD 1, p. 1; the quotations themselves are taken from the \textit{Discours}, AT VI, 20-1 and 67.
equation, and any point whose coordinates satisfy the equation lies on
the locus.⁶

Example. The circle whose center is (2, -3) and whose radius is 3 is
the locus of the equation...

\[(x - 2)^2 + (y + 3)^2 = 9\]

That is to say, in this example, if we were to map the solution set of
this equation, the result would be a circle whose center is the point (2, -3)
and whose radius is 3. This solution set is termed the locus of the equation.

Now consider, suggests Polya, the solution to a simple problem in
constructive geometry. It is a variation of the first problem of Book I of
Euclid’s Elements:

Describe (or construct) a triangle being given its three sides.⁷

The solution is simple but instructive, and is given by Polya as
follows

The solution is to set down one segment length as a base, then rotate
another segment length from the end point of the first segment -- this
gives a locus of possible solution points for the point which constitutes
the third apex of the triangle -- the third point must be on this circle if the
point exists at all. Then rotate the third segment length from the
opposite end point of the first segment -- this also gives a different locus
of possible solution points for the point which constitutes the third apex
of the triangle -- the third point must be on this circle if the point exists

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given is also from Brink, p. 15.

⁷ MD 1, p. 3.
at all. Where the two loci intersect designates the point of the third angle of the triangle.

The unknown is a point (the third vertex of the required triangle); the data are two points (B and C) and two lengths ($b$ and $c$); the condition requires that the desired point be at the distance $b$ from the given point C and at the distance $c$ from the given point B.

All that remains is to use a straight edge to draw the two lines from the two endpoints of the segment first laid out to the point of intersection of the two curves.\(^8\) In reference to such problems in general, Polya suggests as follows:

In any problem there must be an unknown -- if everything is known, there is nothing to seek, nothing to do. In our problem the unknown (the thing desired or required, the quesitum) is a geometric figure, a triangle.

Yet in any problem something must be known or given (we call the given things the data) -- if nothing is given, there is nothing by which we could recognize the required thing: we would not know it if we saw it. In our problem the data are three “finite straight lines” or line segments.

Finally, in any problem there must be a condition which specifies how the unknown is linked to the data. In our problem, the condition specifies that the three given segments must be the sides of the required triangle.\(^9\)

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\(^8\) There will, of course, be two such intersections, and thus two possible solutions to the problem.

\(^9\) MD 1, pp. 2-4.
Concerning what he has called here the condition Polya has the following to say:

The condition is an essential part of the problem. Compare our problem with the following: "Describe a triangle being given its three altitudes." In both problems the data are the same (three line segments) and the unknown is a geometric figure of the same kind (a triangle). Yet the connection between the unknown and the data is different, the condition is different, and the problems are very different indeed (our problem is easier).\(^{10}\)

His general observations here are somewhat similar to those of Descartes in the *Regulae*. Descartes states in Rule 13:

We view the whole matter in the following way. First, in every problem there must be something unknown; otherwise there would be no point in posing the problem. Secondly, this unknown something must be delineated in some way, otherwise there would be nothing to point us to one line of investigation as opposed to any other. Thirdly, the unknown something can be delineated only by way of something else which is already known.\(^{11}\)

\(^{10}\) MD 1, p. 4.
\(^{11}\) CSM 1, pp. 51-2; AT X, 430.

Later Descartes adds:

'Frequently people are in such a hurry in their investigation of problems that they set about solving them with their minds blank -- without first taking account of the criteria which will enable them to recognize distinctly the thing they are seeking, should they come across it. They are thus behaving like a foolish servant who, sent on some errand by his master, is so eager to obey that he dashes off without instructions and without knowing where he is to go.

'In every problem, of course, there has to be something unknown -- otherwise the inquiry would be pointless. Nevertheless this unknown something must be delineated by definite conditions, which point us decidedly in one direction of inquiry rather than another. These conditions should, in our view, be gone into right from the very outset. We shall do this if we concentrate our mind's eye on intuiting each individual condition distinctly, looking carefully to see to what extent each condition delimits the unknown object of our inquiry. For in this context the human mind is liable to go wrong in one or other of two ways: it may assume something beyond the data required to define the problem, or on the other hand it may leave something out.

'We must take care not to assume more than the data, and not to take the data in too narrow a sense.'
Polya is familiar with Descartes' mathematics and has read some Leibniz as well; and considers the *Regulae* an important forerunner to the task he has set himself. His approach is not that of a historian of ideas, however, but rather of someone interested in how it is that problems, especially mathematical problems, get solved.

In our example, as Polya indicates, we have reduced the problem of finding a triangle to that of finding a single point.

A point of the plane that has the given distance \( b \) from the given point \( C \) is neither completely determined nor completely free: it is restricted to a "locus"; it must belong to, but can move along, the periphery of the circle with center \( C \) and radius \( b \). The unknown point must belong to two such loci and is found as their intersection.

We perceive here a pattern (the "pattern of two loci") which we can imitate with some chance of success in solving problems of geometric construction:

*First, reduce the problem to the construction of ONE point.*

*Then, split the condition into TWO parts so that each part yields a locus for the unknown point; each locus must be either a straight line or a circle.*

\[12\]

Polya believes that the solution to our problem is a specific example of a general pattern. The general pattern may be extended to construction

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CSM 1, p. 54; AT X, 434-5.

\[12\] MD 1, pp. 4-5. Polya also adds: 'Almost all the constructions which traditionally belong to the high school curriculum are straightforward applications of the pattern of two loci.' MD 1, p. 5.

The restriction that the locus be either a straight line or a circle seems simply to be the traditional restriction that geometry be performed using no more than ruler and compass.
problems in three dimensions as well, or might involve more than just two loci.\textsuperscript{13} That is to say, the solution of problems through the use of multiple loci is conceivable.

In view of the earlier example of the construction of a simple triangle from its three sides it is perhaps easier now to see in what sense a locus is a curve which sweeps out a solution-set of points. Thus, if we are solving for simultaneous equations we are solving for the intersection of solution sets for such curves. One curve is a set of possibilities with another curve being another set of possibilities; and their intersection is a narrowing down of the total possible solution set.

In analytic geometry, the unknowns would be in terms of the unknown parameters (x & y; or x, y, & z, etc.), but the principle would be the same. According to Polya:

\textsuperscript{13} An example involving three loci is given by Polya as follows:

*The pattern of three loci. A concept of plane geometry may have various analogues in solid geometry. For instance, ... we could ... regard a tetrahedron as analogous to an ordinary triangle....

*Circumscribe a sphere about a given tetrahedron.

*Let us work out the analogy in some detail. We reduce the problem to obtaining the center of the required sphere. In the so reduced problem

the unknown is a point, say \(X\);

the data are four points (the vertices of the given tetrahedron), say \(A, B, C, \) and \(D\);

the condition consists in the equality of four distances

\[XA = XB = XC = XD\]

*We may split this condition into three parts:

First \(XA = XB\)

Second \(XA = XC\)

Third \(XA = XD\)

*To each part of the condition corresponds a locus. If the point \(X\) satisfies the first part of the condition, its locus is (it can vary on) a plane, the perpendicular bisector of the segment \(AB\); to each other part of the condition there corresponds an analogous plane. Finally, the desired center of the sphere is obtained as the intersection of three planes.

*Let us assume that we have instruments with which we can determine the points of intersection of three given surfaces when each of these surfaces is either a plane or a sphere. (In fact, we have made this assumption implicitly in the foregoing. By the way, ruler and compasses are such instruments -- we can determine with them those points of intersection if we know enough descriptive geometry.) Then we can propose and solve problems of geometric construction in space. The foregoing problem is an example and its solution sets an example: with the help of analogy, we can disentangle from it a pattern for solving problems of construction in space, the 

*pattern of three loci.* MD 1, p. 19.
In dealing with geometric constructions, we considered "loci." Such a locus is really just a set of points. In what follows we shall call a set a locus if it intervenes in the solution of a problem in a certain characteristic manner.... As the term "set", ... has already so many synonyms (class, aggregate, collection, category) it may seem wanton to add one more. Yet the term "locus" may remind us of our experience with certain elementary geometric problems and so it may suggest, by analogy, useful steps when we are dealing with other, perhaps more difficult, problems.\(^\text{14}\)

This suggests a vast generalization of the pattern of two loci, a scheme that could work in an inexhaustible variety of cases, could solve almost any problem: first, split the condition into appropriate clauses, then form the loci corresponding to the various clauses, finally find the solution by taking the intersection of those loci.\(^\text{15}\)

For convenience of reference, I shall refer to this, as does Polya, as the 'pattern of two loci' (as opposed, for example, to the four-line locus problem of Pappus mentioned earlier), though it should perhaps properly be referred to as the 'pattern of many loci' or the 'pattern of more than one locus'. However, though this appellative 'the pattern of two loci' may be a bit misleading because locus problems often utilize more than just two loci for determination of the solution set, in principle such loci may receive first an original pairing of two loci, followed by the pairing of the result with an additional locus, and so forth with each subsequent locus paired with the result in a recursive fashion. In fact sometimes this may be helpful for the solution of a problem. We have seen this in the example of

\(^\text{14}\) MD 1, p. 133.  
\(^\text{15}\) MD 1, p. 135.
the normal to a curve given in Chapter One above. Recall that the problem was ultimately reduced to the solution of the following system of equations:

(I) \[-2b = (f - 2e)\ ;\]

(II) \((b^2 - 2c + d + 2) = (g^2 - 2ef + e^2)\ ;\)

(III) \((4b + 2d - 2g^2) = (h^3 - 2eg + d^2 + e^2)\ ;\)

(IV) \((c^2d^2 - 2c + d + c + 2v - 2s + 2) = (k^4 - 2eh + 2e^2 - 2g^2)\ ;\)

(V) \(-2bc + 2d^2 = (e^2h^3 - 2ek^4)\ ;\) and

(VI) \(b^2c + 2d^2 = e^2k^4\ .\)

Each of these equations may be considered as defining a locus. Ultimately we wished to find \(v\), but in order to do so we had first to find \(f\), \(g^2\), and \(h^3\), in that order. This type of solution is what Polya means by recursion.\(^{16}\) In any case, it is in this sense, and to this extent, that Polya's pattern of two loci may be taken as equivalent to Cartesian method in analytic geometry.

Polya suggests that solution of problems by the utilization of the pattern of two loci is applicable to other types of problems outside of

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\(^{16}\) This is also, I believe, what Descartes means when he speaks of proceeding in proper order-- at least in terms of mathematics -- though I cannot show that here.

Also, Polya has more to say on the notion of recursion. Vide: MD 1, Chapt. 3.
mathematics as well.\textsuperscript{17} He suggests it to be applicable to the solution of crossword puzzles, for example. Consider the space for a letter which is the intersection of a series of horizontal and vertical spaces for two words. Consider also the clauses which are the clues for those words. Now consider the two different cases: one in which neither of the words has been discovered, the other in which one of the words has been discovered. Polya states:

The [first] clause selects from the vast range of all words a small set of words, one among which is the solution. The second clause does the same, but there is a difference: the selection is easier in one case than in the other, we can handle the first clause more efficiently than the second. We have used the more manageable clause for a first selection and the less manageable clause for a subsequent second selection. It is more necessary to be efficient in the first selection: we select elements the first time from the immense reservoir of all words, the second time from the very much more restricted first locus, obtained by the first selection.

The moral is simple: To each clause corresponds a locus. \textit{Begin with the clause, for which the locus can be more fully, or more efficiently, formed}. Doing so, you may avoid forming the loci corresponding to the other clauses: you use those other clauses in selecting elements from the first locus.\textsuperscript{18}

\footnote{Polya states: "[T]he practical importance of geometric constructions is negligible and their theoretical importance not too great. Still, the place of such constructions in the [high school] curriculum is well justified: they are most suitable for familiarizing the beginner with geometric figures, and they are eminently appropriate for acquainting him with the ideas of problem solving." MD 1, p. 3. He also states: "This book [\textit{Mathematical Discovery}] deals most of the time with mathematical problems. Nonmathematical problems are rarely mentioned, but they are always present in the background. In fact, I have carefully taken them into consideration and have tried to treat mathematical problems in a way that sheds light on the treatment of nonmathematical problems whenever possible." MD 1, p. vii.}

\footnote{MD 1, p. 139.
That, in brief, is what Polya has to say concerning the matter.

Part II: Watson & Crick’s Discovery of the Structure of the DNA Molecule as an Example of the Pattern of Two Loci Method of Discovery.

I am going, here, to introduce into our discussion an example of discovery taken from modern science. It is the example of Watson and Crick’s discovery of the structure of the DNA molecule. I am introducing this example for two reasons. Firstly, I believe it can be construed as an illucidatory example from science of Polya’s two-loci (or, in this case, multi-loci) solution process. Secondly, I also feel it illuminates the importance of over-arching principles in the making of some scientific discoveries. Both aspects I feel important to an understanding of Cartesian science -- though, and let me be clear about this, I do not believe this example to be identical in structure to Descartes’ own endeavors in this field.19

First let us appraise the following document:

Stimulated by the results presented by the workers at King’s College, London, at a colloquium given on 21st Nov. 1951, we have attempted to see if we can find any general principles on which the structure of D.N.A. might be based. We have tried, in this approach, to

incorporate the minimum number of experimental facts, although certain results have suggested ideas to us. Among these we may include the probable helical nature of the structure, the dimensions of the unit cell, the number of residues [nucleotides] per lattice point, and the water content. Having arrived at a tentative structure this way, we have generalized what we regard as the important features, and now present these as postulates. [Our attack is] to write down systematically all the possible topological combinations. This is done by writing down all the possible linkage schemes... for an infinite plane surface. A strip of this surface [is] then isolated and folded round into an infinite cylindrical sheet. The number of ways of doing this can then be explored systematically.

Having thus obtained all possible linkage schemes, the next step is to build models of them all.... The model is then subjected to a series of tests, in which the experimental data from simple structures is increasingly used. For example,... the number of residues per 27Å length of helix may be quite wrong, and so on. In doing this it is particularly important not to use as a basis of rejection rather vague criteria, such as not quite seeing how it fits in with possible preconceived ideas about the X-ray pattern, and not quite seeing how it would stretch. If great care is not taken the right model may be rejected.20

20 Eighth Day, pp. 125 and 127.

Crick's words are similar here to those of Garber in his 'Science and Certainty in Descartes'. There, Garber believes Descartes' approach, in theory but ultimately failing in practice, was to set out a 'complete enumeration' of all possible candidates for an explanation of a given set of data which are in conformity with Descartes' accepted principles. Provided Descartes was sure he had listed all possible candidates, de-selection of candidates on the basis of 'crucial experiments' would ensure the certainty of the final solution. That is to say, if I read Garber correctly, Descartes requires additional 'crucial experiments' beyond those with which he begins the de-selection of his candidates.

If Garber is correct, I am puzzled why Descartes would regard the problem of the magnet as 'perfect' if we are dealing with no more than just those observations provided by Gilbert. Vide Quotation (D) in Chapter Five below. In a moment, I shall associate Crick's words with Polya's account of the 'pattern of two looi'. Though I am more indebted to Garber than my footnotes might suggest, I believe Polya's approach is probably closer to the mark of what Descartes had in mind. Vide: Garber, Daniel, 'Science and Certainty in Descartes', Descartes: Critical and Interpretive Essays, Michael Hooker, ed, Johns Hopkins University Press, Baltimore, Maryland, 1978.
These are the words of Francis Crick and are taken from a lengthy memo penned by him some months before his and Watson's discovery of the correct structure of the DNA molecule. We see here, I think, an actual enactment in science of Polya's proposed method for the general solution of certain types of problems. The postulated structures proposed by Crick are structures in conformity with a certain set of minimum empirical experimental data available in the hopes of de-selecting all but the correct structure. Graphically, the situation may be expressed in conformity with the following suggestive figure:

![](image)

Datum 1 through Datum 4 suggest a set of possible stereometric structures in conformity with these data -- Theories x through z. Each datum suggests a locus or possible solution set; the solution to the problem is the intersection of multiple loci, in this case Theory y, with Theories x and z being deselected.
In the event, the specific structure proposed by Watson and Crick at the time on the basis of this memo was incorrect. It was incorrect because the amount of the water listed as a requirement in one of the original datum was wrong; this was quickly pointed out to Watson and Crick by their colleague Rosiland Franklin before their premature publication of the incorrect structure. Their proposed structure just couldn’t be right because it wasn’t in conformity with one of the essential data. As a result of this failure, the two were told by the head of their lab, Laurance Bragg, to forget their speculations on DNA and get back to work on the research they were supposed to be working on.

Implicit to Crick’s memo is also the requirement that the structures postulated must be in conformity with not only the empirical, observational data, but also the known principles of molecular biology as expressed by Linus Pauling in his book *The Nature of the Chemical Bond*. Thus we may depict the situation figuratively as follows:

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21 *Eighth Day*, pp. 71 and 86.
Sometime after Watson and Crick's first-proposed solution to the problem, Linus Pauling made a stab himself at the solution and published a proposed structure for the molecule. Watson and Crick read through the proposal and recognized immediately that it couldn't be right: Pauling had not correctly taken into account his own principles for molecular combination; that is, the proposed model was not in conformity with such principles. Pauling's proposed solution, however, galvanized Watson and Crick into making one more attempt to solve the structure themselves. This time, in conformity with the correct observational data and the correct principles, they postulated the correct structure. The data in
conformity with the principles left essentially only one simple structural possibility which satisfied them all. In a sense, each datum left open only certain structural possibilities (or loci, as we have been terming the general solution sets from which the final solution is to be selected) satisfying its conditions in conformity with the principles; the intersection of these loci turns out to have been the correct structure.

At the time, Watson and Crick’s proposed structure was, and still is, impressive. It sat well with the known empirical data and also with Pauling’s principles. In the words of Watson, ‘it was just too pretty to be wrong’. Pauling himself at the time stopped by Watson and Crick’s lab personally to inspect the model; he examined it for only fifteen minutes before pronouncing it to be correct.

Though Pauling accepted the model almost immediately, the general community was a bit more hesitant. From the point of view of the biologists, substantiation and acceptance of the theory was forthcoming from the following experiment:

The bacterium E. Coli was grown in a soup of nutrients containing only the (non-radioactive) isotope of heavy Nitrogen (N^{15}). After 14 generations, the soup of nutrients was abruptly changed to one containing only normal Nitrogen (N^{14}). The structure proposed by Watson and Crick predicts a particular outcome to such an experiment. Replication is expected through the splitting of the central base pairs and the reception on

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22 ‘Simple’ must be stressed here. There are, in principle, an infinite number of possible solutions to the structure; but the tacit assumption that the solution would be the simplest possible is in keeping with the normal assumption of scientists and seems to have led Watson and Crick, in the event, to the correct solution.

23 Described as ‘Classic’ by Watson.
the part of each strand of new complementary components to form two complete new strands from the single old one. Thus, the first generation will contain components themselves composed only of heavy Nitrogen. In the next generation, the original half-strands will be composed of heavy Nitrogen, but the new half-strands will be composed of normal Nitrogen. Thus, when the DNA is harvested, we shall expect it to be lighter than that composed exclusively of the older, heavier, strands. In the next generation, as the strands split and accept new material, the theory predicts that the old half-strands, still composed of heavy Nitrogen, will again accept components made up of normal Nitrogen, producing DNA of the lighter sort of the second generation. But the half-strands of the second generation containing only normal Nitrogen will also accept components only of normal Nitrogen as well, producing DNA of an additionally lighter sort. Thus, the theory predicts there will be two types of DNA in the third generation: the DNA of the second generation, lighter than that of the first, and an additional sort lighter than that in either the first or second generations.

‘Clean as a whistle!’ wrote Meselson to Watson about the results. The DNA was harvested in each instance, spun 44,770 times a minute for 20 hours in a solution of cesium chloride and the DNA separated out into three distinct bands: first generation lowest, second generation in the middle, and third generation both in the middle and highest.24

In view of this example we may now alter out suggestive illustration:

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24 *Eighth Day*, pp.190-2.
Upon the postulation of a theory, the theory must be shown to be in conformity with (1) the principles, (2) the known data, and (3) additional data predicted by the theory.

Part III: Cartesian Analysis in Natural Science.

Earlier in Chapter 1, we cited Buchdahl as having ascertained the following two types of analysis in Descartes. They were as follows:
Analysis as a technique of operating algebraically with unknowns, in the hope of finding equations that contain them, and then solving these equations for the unknowns.

The Pappian hypothetico-deductive procedure of "assuming what is to be proved as though it were known.

As mentioned by Buchdahl, the analysis of the ancients has about it a 'hypothetico-deductive' quality of postulating what is to be proven to see if it holds true. I have tried to show in Part III of Chapter One how, if we can trace a trail back by 'reasoning backwards' to previously proven theorems or assumed axioms -- and if the steps are convertible\textsuperscript{25} -- then we may state that the postulated proposition is also ascertained to be true -- or false if we reach a contradiction. Thus we may postulate what we like, and reason backwards until we find a proposition deemed to be either true or false within the system.

We moderns are familiar with the fact that the type of geometry in which we work is structured by the kinds of original assumptions from which we begin. Hintikka stated that the first of the above senses of analysis might be thought of as a further technical development of the Pappian idea expressed in the second sense of analysis expressed above -- though he did not elaborate how this might be done. Allow me to do my best to show this now.\textsuperscript{26}

\textsuperscript{25} Recall, however that the actual 'reasoning backwards' process is the analysis, and the conversion or 'reasoning forward' process is the synthesis.

\textsuperscript{26} I am greatly indebted to Dr. Frank Jones of the Mathematics Department of Rice University for having suggested this reduction to me. Errors in the expression of this reduction are, of course, attributable to myself. I am also indebted to Dr. Jones for examining a draft of the mathematical sections of Chapters One and Two of this work, and for indicating to me a mathematical faux pas in an early version of Chapter One.
Insofar as an equation may be considered as having the same status as a theorem, the postulation of an equation, involving unknowns, may be considered the same status as the postulation of a proposed theorem to be proven. But if this is true, in what sense are we ‘reasoning backwards’, and to what types of theorems and postulates?

Let us take an example. We are to find $x$ in the following equation: $x^2 + 2x - 4 = 0$. Now we do not know the solution to this problem at this time, and thus the status of this equation is that of a proposition whose veracity is unknown. The solution set to this equation may be determinate, it may be infinite, or it may be the null set. Let us now solve for the unknown:

\[
\begin{align*}
x^2 + 2x - 4 &= 0 \\
x^2 + 2x &= 4 \\
x^2 + 2x + 1 &= 5 \\
(x + 1)^2 &= 5 \\
x + 1 &= \pm\sqrt{5} \\
x &= -1 \pm\sqrt{5}
\end{align*}
\]

Modern mathematicians would have no difficulty with this answer. However, due to the irrational nature of one of the terms of the solution, for an ancient pythagorean the solution to this equation would be the null set -- as such a mathematician would not admit the existence of incommensurable numbers. Because for Descartes positive roots were ‘true’ and negative roots were ‘false’, presumably here we have an intermediate case and this equation would have only one ‘true’ solution. To those for whom complex numbers are problematic, similar remarks would
obtain if we were to change the equation from \( x^2 + 2x - 4 = 0 \) to \( x^2 + 2x + 4 = 0 \). Thus we may say that through our manipulations of the equation we are reasoning backwards to such theorems and postulates as are acceptable within the context of our system. In the case of a *reductio ad absurdum* we would be reasoning backwards to a contradiction; for our pythagorean friend the contradiction might be the unacceptable conclusion, 'there exists and does not exist the irrational number the square root of five'. The solution says the number in question does exist, the postulates of the system say it does not. Thus, not only is the solution to the equation the null set for this person, the proposition in question itself, the original equation, is a false proposition within the context of our friend's preferred system. For these reasons the process of following out the equation and reasoning to our conclusion we may consider a specific form of the more general method of the 'reasoning backwards' process of the ancients.\(^{27}\)

From the point of view of modern formal logic this should not be surprising as one of its projects has been to express equations as propositions utilizing formal notation. Such propositions might then be labeled true or false within the context of a given system.\(^{28}\)

Suppose now we are to solve simultaneous equations. Again whether a solution is possible will be dependent upon the system we prefer. For example, let us take the two following equations: a circle, \( x^2 + y^2 = 1 \) and a straight line, \( y = 2 \). Because we are told \( y = 2 \) in the second equation, we

\(^{27}\) Dr. Jones is of the opinion that this is in fact a type of reasoning utilized extensively by mathematicians today, and that my reference to 'the analysis of the ancients', though perhaps in keeping with Descartes' own usage, is misleading insofar as being too limited in scope.

\(^{28}\) I say they might be; Gödel's first proof informs us that within any formal system adequate for number theory, there exist propositions within that system which can be labeled neither true nor false.
may substitute 2 for y in the first equation, and we shall receive \( x^2 + 4 = 1 \),
or \( x^2 = -3 \), or \( x = \pm \sqrt[3]{3} \). In the traditional plane of real numbers this result is
unacceptable and the simultaneous solution of the equations will be the null
set -- that is, the circle does not intersect the line. But if our system is that
of complex numbers the solution set would have two members. In this
example our postulation is that the equations may receive a simultaneous
solution. In the latter case we reach no contradiction; in the former we
reach the contradiction, there exists and does not exist the number \( \sqrt[3]{3} \). Or,
the solution tells us the solution set is: \( \{ \sqrt[3]{i}, -\sqrt[3]{i} \} \), and the system dictates
whether such members are acceptable or not.

One might originally expect that because we are dealing with
equations, the process of ‘reasoning backwards’ is identical to that of
‘reasoning forwards’, but this is not quite the correct. In cases such as
those given above involving quadratics the solution set may include more
than one member, and we must take care that all solutions are included in
the ‘reasoning forward’ process, especially in cases involving auxiliary
variables or constructions.\(^{29}\)

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\(^{29}\) Polya gives us an example of such analysis and synthesis from an algebraic point of view, which I
include here in the following passage:

‘Find x satisfying the equation

\[ 8(4^x + 4^y) - 54(2^x + 2^y) + 101 = 0. \]

‘This is a “problem to find,” not too easy for a beginner. He has to be familiar with the idea of analysis;
not with the word “analysis” of course, but with the idea of attaining the aim by repeated reduction. Moreover, he
has to be familiar with the simplest sorts of equations. Even with some knowledge, it takes a good idea, a little
luck, a little invention to observe that, since \( 4^x = (2^x)^2 \) and \( 4^y = (2^y)^2 \), it may be advantageous to introduce

\[ y = 2^x. \]

Now, this substitution is really advantageous, the equation obtained for \( y \)

\[ 8(y^2 + \frac{1}{y}) - 54 (y + \frac{1}{y}) + 101 = 0 \]

appears simpler than the original equation; but our task is not yet finished. It needs another little invention,
another substitution

\[ z = y + \frac{1}{y} \]

which transforms the condition into
How much Descartes may have been aware of all this is open to question, of course. We have, obviously, no reason to believe Descartes entertained in any way the notion of geometries or numerical domains outside the traditional Euclidean system; but we do have two reasons for believing Descartes adhered to a view of analysis similar to the one expressed above. Firstly we have the statement by Descartes concerning 'the analytic style of writing', the one which reads:

The analytic style of writing that I adopted there [in the Meditationes] allows us from time to time to make certain assumptions that have not yet been thoroughly examined; and this comes out in the First Meditation where I made many assumptions which I proceeded to refute in the subsequent Meditations. 30

As for the second reason, Descartes has told us:

[I]t is analysis which is the best and truest method of instruction, and it was this method alone which I employed in my Meditations. 31

\[8x^2 - 54x + 85 = 0.\]

Here the analysis ends, provided that the problem-solver is acquainted with the solution of quadratic equations.

'What is synthesis? Carrying through, step by step, the calculations whose possibility was foreseen by the analysis. The problem-solver needs no new idea to finish his problem, only some patience and attention in calculating the various unknowns. The order of calculation is opposite to the order of invention; first \( z \) is found (\( z = 5/2, 17/4 \)), then \( y \) (\( y = 2, 1/2, 4, 1/4 \)), and finally the originally required \( x \) (\( x = 1, -1, 2, -2 \)). The synthesis retraces the steps of the analysis, and it is easy to see in the present case why it does so.' HSI, pp. 144-5.

A potential difficulty which surfaces here is that Polya has called this problem a 'problem to find' whereas my claim is that the equation is itself of the same status as a stated proposition or a 'problem to prove'. In my estimation, the problem is a problem to find because it involves finding auxiliary problems -- that \( y \) may be profitably substituted for \( 2^z \) and that \( z \) may be substituted for \( y + 1/y \). The equation itself may still be considered a problem to prove.

30 CSM 2, p. 173; AT VII 249; quoted above in Chapter One, fn. 24.
31 CSM 2, p. 111; AT VII, 156.
And, as has been pointed out by Curley, this is exactly the form of argumentation utilized by Descartes in his *Meditationes*. As Curley states:

What I should now stress is Descartes’ use of what I have come to call a dialectical method. By a dialectical method I mean the essentially Platonic procedure of beginning with a conjecture, considering what can be said against the conjecture, and then revising the conjecture in whatever ways the objections suggest. The initial conjecture may be (typically, will be) a false start, in the sense that it will ultimately be rejected in the form in which it is first proposed. But typically it will also be a proposition which recommends itself to common sense. The process of conjecture, refutation, and revision may be repeated indefinitely until the inquirer reaches a result to which he can find no further objection. If the initial conjecture was not a totally false start (and normally it will not have been), something of what it asserted will survive in the final result.32

At this point Curley quotes, for his original warrant, the passage on ‘analytic style of writing’ given above. He then proceeds to give some convincing examples of such argumentation on the part of Descartes in the


Garber makes a similar statement: "[A]n examination of the six Meditations shows one clear sense in which they can be read analytically, showing how one might actually come to discover for oneself the conclusions reached. Actual discovery involves false steps as well as true, bad arguments considered and rejected as well as good arguments that ultimately lead to enlightenment. This, I claim, is an important aspect of the expository strategy of the *Meditations*.' From: Garber, Daniel, 'Semil in vita: The Scientific Background', in *Essays on Descartes’ Meditations*, Amélie Oksenberg Rorty, ed., University of California Press, Berkeley, California, 1986, pp. 98-9.

We may compare this example with what Descartes has to say concerning cyphers:

'S[ay] we want to read something written in an unfamiliar cypher which lacks any apparent order: what we shall do is to invent an order, so as to test every conjecture we can make about individual letters, words, or sentences, and to arrange the characters in such a way that by an enumeration we may discover what can be deduced from them. Above all we must guard against wasting our time by making random and unmethodical guesses about similarities.' CSM 1, pp. 35-6; AT X, 404-5.

This remark is itself quite cryptic. But when Descartes speaks of ‘inventing an order’ he is suggesting, I believe, a postulation of a specific hypothetical system to be checked as a translation or decoding of the unfamiliar cypher.
**Meditationes.** Let us remember here that the *reductio ad absurdum* is a form of analysis. If we substitute the term ‘analysis’ for ‘dialectical method’ in the above passage, Curley’s work dovetails well with my own.\(^{33}\)

We should remember Descartes’ claims to a universal method; I believe it’s correct to say he had this sort of ‘analysis’ or ‘reasoning backwards’ in mind in both his mathematics and his metaphysics.\(^{34}\)

At this point I feel I have shown in what sense Cartesian mathematical equations, or a technique of operating algebraically with

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\(^{33}\) To give an illustration of Curley’s notion of dialectical method in Descartes, let us take the example of the soul. Curley states:

‘Descartes goes through [such a dialectical] procedure with his pre-philosophic concept of mind (*mens*) or soul (*anima*). Here the very choice of terminology is significant. Descartes uses both *mens* and *anima* to refer to the same thing, a thinking thing. But though both terms, in that sense, mean the same, Descartes generally prefers the term *mens* because he feels that the term *anima* has unfortunate connotations. It suggests something corporeal. [fn.: ‘In good Latin *anima* signifies air or breath, from which usage I believe it has been transferred to signify the mind [*mens*]; that is why I said it is often taken for something corporeal.’ Letter to Mersenne, 21 April 1641 (Alquié 2:327), cf. AT VII, 161.] So it is no accident that, in Descartes’ first self-conscious discussion of the nature of the mind in the *Meditations*, he should choose to designate it by the term *anima*. In this exceptional context, he wants to suggest those usually unfortunate connotations:

‘It occurred to me also that I am nourished, that I walk, sense, and think, all of which actions I referred to the soul (*anima*). But what this soul was, I either did not consider, or else (FV: if I did consider it) I imagined it to be a something-I-know-not-what, something very subtle, like wind, or fire, or air, which was infused throughout the grosser parts of me.’ (AT VII, 26)

‘One of the things Descartes must do, in order to clarify this prephilosophic concept, is to recognize that not all the activities he had previously ascribed to the soul are necessarily to be ascribed to it. Nutrition and motion clearly presuppose the existence of a body (AT VII, 27). But Descartes is proceeding, at this point, on the assumption that he has no body. So he cannot ascribe nutrition or motion to the soul. The case of sensation is more difficult. At first the meditator is inclined to say that sensation requires a body, and hence is not to be ascribed to the soul (AT VII, 27). Later he realizes that it is possible to conceive of sensation as not necessarily involving a body, but only a peculiar kind of thought, the kind of thought in which we perceive corporeal things as if through the senses (AT VII, 29). And conceived this way, sensation is properly attributed to the soul, as are all thought processes. Thinking is a property he cannot deny to himself, since any hypothesis he might entertain in an attempt to cast doubt on his thinking would imply that he thinks.’

To interpret Curley’s comments into my own framework, Descartes starts with a proposed problem to prove: that I am nourished, that I walk, sense, and think are all actions to be referred to the soul. One by one an analysis is performed on each of these attributes, and each supposed reference to the soul is shown to be false by means of a *reductio ad absurdum* except for the last one, thought. Let me continue with Curley’s remarks at this point:

‘So the only thing that is left of [Descartes’] prephilosophic conception of himself is that he is a soul conceived as a thing that thinks (AT VII, 27, line 13). And at this point he introduces the term *mens* as a synonym for “thinking thing.” Henceforth the term *mens* will displace the term *anima* as the preferred prephilosophic term for referring to the thinking things.’ Rorty, pp. 158-9.

Curley provides similar discussions for the the Cartesian handling of the concepts *Idea*, *substance*, and *God*. Again, the point to be made here is that due to the ‘reasoning backward’ process involved, each of these cases may be considered a form of argument by analysis.

\(^{34}\) As is evident from the above passage, similar remarks concerning Plato and analysis are in order, but to the best of my knowledge these have not as yet been adequately worked out.
unknowns, in the hope of finding equations that contain them, and then solving these equations for the unknowns, may be considered a specific case of the more general form of the Pappian hypothetico-deductive procedure of 'assuming what is to be proved as though it were known'. Both may be considered 'problems to prove' insofar as the statement or equation in question is proposed and we reason to accepted postulates in the system -- either (1) to perform a *reductio ad absurdum* with the result of rejecting the proposed proposition, or (2) to perform an analysis to accepted postulates with the result of reversing the process and performing a synthesis which shows the acceptance of the proposition within the system.

If I am right, there seem, then, to be two aspects to analysis considered from a Cartesian point of view. Firstly there is the need and difficulty of finding the original proposition to be postulated; secondly there is the need and difficulty of subsequently refuting or substantiating the postulated proposition. In short, we have both problems to find and problems to prove. Let us ask now, again from a Cartesian point of view, what the two have in common that might make Descartes consider both forms of analysis.

Descartes has made the distinction between direct and indirect solution of problems, and I have argued that, insofar as we mean by Cartesian analysis the assumption of the unknown in order precisely to solve for that unknown, 'analysis' may be considered a form of 'indirect problem solution' in Cartesian thought. If we consider exclusively the
example of continued proportions,\textsuperscript{35} we have two known extremes, and one unknown mean term (e.g. $A:x :: x:B$), and, according to Descartes, the solution is a 'reverse movement of the imagination' requiring 'two acts of conceiving'.\textsuperscript{36} We may depict the attempted solution of such problems in the following fashion:

$$A ----- > \ x \ < ------ B$$

Here the unknown term $x$ is caught in a certain tension between the known terms $A$ and $B$. First we have the difficulty of proposing a postulation for the unknown term within the context of this tension, a problem to find; and second we have the demonstration of the solution set for this term, also within the context of this tension, a problem to prove. Thus, firstly, we proceed easily from the Euclidean dictum that the product of the means equals the product of the extremes, we have our postulate: $x^2 = AB$. And, secondly, we resolve our postulation through appropriate manipulation or 'reasoning backwards'; thus:

\textsuperscript{35} Similar remarks apply in other cases, however. Thus, in the simple proportion: $A:x :: B:C$, the postulated equation would then be: $xB = AC$.

\textsuperscript{36} As Descartes has stated:

'We shall explain in due course how to find any number of mean proportionals between [the unit and the last magnitude to be divided]. For the moment we must be content to point out that we are assuming that we have not yet quite done with these operations [the extraction of roots], since in order to be performed they require indirect and reverse movements of the imagination, and at present we are dealing only with problems which are to be treated in the direct manner.' CSM 1, p. 75; AT X, 467.

Descartes also states: 'But if only the first and the third [terms] are given, it will not be so easy for me to discern the intermediate magnitude, for this can be done only by means of an act of conceiving which simultaneously involves two of the [particular and distinct acts of conceiving] just mentioned. If only the first and the fourth magnitudes are given, it is even more difficult to intuit the two intermediate ones, for in this case three acts of conceiving are simultaneously involved.' CSM 1, pp. 38-9; AT X, 409-10.
Turning now to natural science, in the case of the illustrative example from Watson and Crick given above, we may express the situation as follows:

Principles \rightarrow \text{unknown geometric structure} \leftarrow \text{Data}

The unknown geometric structure is held in a certain tension between the known data and the accepted principles. On the basis of this tension we postulate, firstly, a possible solution to the difficulty. Then we reason out, secondly, the acceptability of the proposition within the context of that tension. In the case of natural science, the reasoning is not reciprocal in character and we feel it is best for us to have recourse to additional data predicted by the proposed solution.\textsuperscript{37} Thus, the secondary process of 'reasoning backwards' will involve a reasoning in two directions -- that of the acceptance of the proposition in terms of the postulates or principles of the system on the one hand, and that of its acceptance in terms of the data involved on the other. Optimally the data will include additional tests postulated by the proposition. We may express the situation as follows:

Principles \leftarrow \text{Postulated Structure} \rightarrow \text{Data}

\textsuperscript{37} Descartes' opinion is a bit different, I believe. Though he does seem to allow for empirical testing of explanatory propositions in natural science, as I touch on at the end of Chapter Five, he does, I shall argue in Part IV of that Chapter, view his \textit{Principia} as a synthetic account.
Thus, I am suggesting the two forms of analysis, problems to prove and problems to find, may be viewed as having a similar structure within the context of Cartesian natural science. The similarity is that the unknown structure in the case of problems to find and the postulated structure in the case of problems to prove are both caught in a certain tension between the principles and the data of the problem in question. Should the reader feel dissatisfied with this talk of a supposed similarity between problems to prove and problems to find in Cartesian natural science, that this notion of being caught in a tension between principles and data is a bit too vague and general, at the very least we may say these two forms of analysis stand on a different side of the conceptual fence from any subsequent reverse movement of thought, synthesis -- and may be considered similar in at least that sense.

Let us at this point recall what Heath has had to say concerning Archimedes:

At the same time [Archimedes] is careful to insist on the difference between (1) the means which may be sufficient to suggest the truth of theorems, although not furnishing scientific proofs of them, and (2) the rigorous demonstrations of them by orthodox geometrical methods which must follow before they can be finally accepted as established.38

Analysis, in this context and, I take it, from a Cartesian point of view, is the method of discovery; synthesis is the method of proof.

38 Heath, 2, pp. 20-1.
Analysis is indirect, according to Descartes, in that it involves a 'reverse movement of the imagination' and 'two acts of conceiving'. Synthesis, in contrast, is a direct movement of the imagination involving a single act of conceiving. An example of direct problem solution involving continued proportions, it will be recalled, is $A:B :: B:x$. We may depict this graphically as follows:

$$A \longrightarrow B \longrightarrow x$$

In terms of natural science, the situation may be depicted analogously as follows:

Principles $\longrightarrow$ Postulated Structure $\longrightarrow$ Data

The proof flows in a single direction.

To flesh this out, let us turn to Descartes’ original statements concerning synthesis in the second set of objections noted in Chapter One.

(1) ‘Synthesis employs definitions, postulates, axioms, theorems and problems’; that is, it is axiomatic in character. This should be reasonably obvious from what has been said before now. Whereas in analysis the solution is proposed and we ‘reason backwards’ to the original axioms, in synthesis we start with the original axioms and ‘reason forwards’ to the proposed solution. The only difficulty here is that we in fact also utilize, ultimately, ‘definitions, postulates, axioms’ and so forth in analysis as well, though I feel that Descartes was speaking here relatively in the sense that such axioms are more prominently displayed in synthesis and are more
proximally distant in analysis. That is, in synthesis the emphasis is upon
the premises, whereas in analysis the emphasis is upon the conclusion.39

(2) 'Synthesis has little pedagogical value.' I have already indicated
what Descartes might have meant by this in Part III of Chapter One where
I noted that the method of synthesis was recognized in and before the time
of Descartes as pedagogically deficient and it was recognized that the
student learned better by being challenged with a problem to be solved
rather than by memorizing the completed solution by rote.

One of the things which makes Kepler difficult to read is that he
presents his discoveries autobiographically and, in the sense Curley
expressed above, dialectically. By that I mean Kepler sets out the progress
of his investigations chronologically as they occurred to him, with his
account even including all the dead ends he runs into. He explains, for
example, his futile attempts to make the orbit of Mars conform to a
circular pattern. In contrast, modern scientists generally present their
discoveries so as to optimally illustrate the positive aspects of their
proposed theory. That is, a modern scientist would simply state the theory
of elliptical orbits and give that scientist's reasons for believing in it. Such
exposition on the part of Kepler may be termed pedagogical in the sense of
introducing the reader to a deeper understanding of the difficulties
involved with the theory.

(3) 'Synthesis cannot be so conveniently applied to metaphysical
matters as analysis.' Here I think Descartes is simply expressing his

39 In terms of Euclidean Geometry, for synthesis the situation would be as follows:
Axioms and Definitions -------> Theorems and Constructions --------> Additional Theorems and Constructions.
opinion that in questions similar to those posed in the *Meditationes*, he believes results are more readily obtainable through the method of proposing possible solutions and reasoning concerning their acceptability in the fashion suggested by Curley.

(4) Synthesis is a search *tanguam a posteriori*. This is a tough nut to crack. Any association between synthesis and things *a posteriori* will seem unsatisfactory because such association appears so counter-intuitive. Still, we might say synthesis is epistemically posterior in the sense that the discovery process has already occurred. And if this is the case, analysis proceeds *tanguam a priori* because the discovery is, generally, prior epistemically to the proof. What Descartes meant when he added, cryptically, ‘though the proof itself is often more *a priori* than it is in the analytic method’ is probably more in conformity with our own modern usage of this term.

One additional point can be made here, however. If Descartes happened to consider a mathematical equation, including unknowns, the same or similar status as a theorem as I have suggested above, then in terms of discovery it could be classified, as I have indicated, along with other ‘problems to prove’. And if this is the case, the extant Rules Thirteen through Twenty-One of the *Regulae*, insofar as these are concerned with the setting up and solution of problems involving equations, may be considered injunctions on the setting up and solution of problems to prove. Also, taking our cue from Part II of this chapter above, the setting out of the data in a problem in natural science in quantitative terms, or the conversion of secondary qualities to primary qualities, may be considered a
preliminary procedure in a 'problem to find'. Thus we may speculate at this point that the third section of the \textit{Regulae}, insofar as it was, as I have argued, to have dealt with both mathematical problems to find and scientific conversion of secondary to primary qualities, may be regarded as a series of injunctions for the setting up and solution of problems to find. In other words, my speculation is that Rules Thirteen through Twenty-Four were to have dealt with the first sort of analysis proposed by Polya's Pappus, and Rules Twenty-Five through Thirty Six were to have dealt with the second sort. The myriad difficulties in the differentiation and explication of these methods probably explain why the work was never finished.\textsuperscript{40}

Summary.

I submit that there are two forms of analysis in Cartesian natural science. The first form of analysis attempts to find an explanatory proposition in keeping with the accepted principles and the known data. The second form of analysis seeks to show how the proposed proposition accounts for or explains the data in keeping with the principles. In the next Chapter I shall argue Descartes views synthesis in natural science as a proof which proceeds from principles, through the explanatory proposition, to the known data.

\textsuperscript{40} This opinion is not without its problems. Vide footnote 29 above where Polya refers to a problem involving an equation as a 'problem to find'. 
Chapter 5: The Magnet as an Example of Cartesian Analysis.

In Descartes' dedication of the *Principia* to Princess Elizabeth, he has the following to say:

> It would not be appropriate for me either to flatter, or to affirm anything which I had not sufficiently investigated; especially here, where I am about to attempt to lay the foundations of truth. And I know that the unaffected and simple judgment of a Philosopher will be more pleasing to your noble modesty than the more elaborate praises of more ingratiating men. Accordingly, I shall write only those things which I know to be true either from reason or experience.\(^1\)

In Descartes' conversations with Burman, when queried how it is that we arrive at Descartes' elementary particles as being just three kinds, Descartes responds:

> Through reasoning, and then through experience, which confirms the reasoning.\(^2\)

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It is time for me to try to show just how I believe Descartes actually used both thought and observation, reason and experience, in his natural science.

Recall that Descartes in his *Regulae* suggests we look at the ‘nature of the magnet’ if we wish to see his method. His actual explanation of the nature of the magnet does not appear in his early work, the *Regulae*, however; nor does it appear in the middle work which was supposed to have been expository of his method, the *Discours*. Only much later does it appear in his work the *Principia*. In this Chapter I am going to give my account of what I take to be Descartes’ method in science using the central example of the magnet. Again, I have chosen the magnet because of Descartes’ explicit references to it in the *Regulae* and because of its later appearance in the *Principia*. In Part I of this Chapter I shall deal with what Descartes has to say concerning the enumeration of the empirical data having to do with this subject. In Part II I shall review what Descartes has to say concerning the principles involved. In Part III I shall discuss Descartes’ explanation itself. And in Part IV I shall review a general account of what I take to be Descartes’ method in these matters.

Part I: Empirical Data in the *Regulae* and in the *Principia*. 
This part of this Chapter will consist essentially of three sections: The ‘nature of the magnet’ in the *Regulae*, the ‘nature of the magnet’ in the *Principia*, and some discussion of the relation between the two.

First, let us discuss Descartes’ empirical approach to the question of the magnet in the *Regulae*. Specifically, I am going to review some material concerning the manner in which Descartes believes empirical data to be important to any solution to this problem in that work. To do this, we shall consider in detail an extensive quotation from the appended explanation for Rule Thirteen of the *Regulae*. The reader will recall that Rule Thirteen was to be the first Rule of the second section of that work. I have broken the quotation down into three distinct sections which I have taken the liberty of labeling (C), (D), and (E) for convenience of reference. In each of these passages is the magnet mentioned; and though some of this material has been dealt with before -- specifically passage (D) in Chapter Three -- I think it is important to review the entire quotation as a whole at this juncture:

(C) We view the whole matter in the following way. First, in every problem there must be something unknown; otherwise there would be no point in posing the problem. Secondly, this unknown something must be delineated in some way, otherwise there would be nothing to point us to one line of investigation as opposed to any other. Thirdly, the unknown something can be delineated only by way of something else which is already known. These conditions hold also for imperfect problems. If, for example, the problem concerns the nature of the magnet, we already understand what is meant by the words ‘magnet’
and ‘nature’, and it is this knowledge which determines us to adopt one line of inquiry rather than another, etc.\textsuperscript{3}

(D) But if the problem is to be perfect, we want it to be determinate in every respect, so that we are not looking for anything beyond what can be deduced from the data. For example, someone may ask me what conclusions are to be drawn about the nature of the magnet simply from the experiments which Gilbert claims to have performed, be they true or false. Or again I may be asked to determine what the nature of sound is, solely and precisely from the following data: three strings, A, B and C emit the same sound; B is twice as thick as A, but no longer, and is tensioned by a weight which is twice as heavy: C is twice as long as A, though not so thick, and is tensioned by a weight four times as heavy. It is easy to see from such examples how imperfect problems can all be reduced to perfect ones -- as I shall explain at greater length in the appropriate place.\textsuperscript{4}

(E) We can also see how, by following this Rule, we can abstract a problem which is well understood from every irrelevant conception and reduce it to such a form that we are no longer aware of dealing with this or that subject-matter but only with certain magnitudes in general and the comparison between them. For example, once we have decided to investigate specific observations relating solely to the magnet, we no longer have any difficulty in dismissing all other observations from our mind.\textsuperscript{5}

In (C) here, we have reference to (1) the unknown, (2) the designation or delineation of the unknown, and (3) the delineation of the unknown in terms of something else already known. These are all in keeping with Descartes’ method in mathematics, and we have seen in Chapter Two an affinity between such statements in the \textit{Regulae} and \textit{La
Géométrie. But what is of interest here is not only the similarity to what seems to be a general account of Descartes' mathematical method; but also the surprising example given by Descartes, not from any of his numerous mathematical discoveries, but rather from natural science. Specifically the magnet is mentioned. The same may also be said of, and is I believe even a bit more clear in, passage (E) where Descartes states we can abstract the problem from every irrelevant conception and reduce it to a form in which we are dealing only with certain magnitudes in general and the comparison between them. In both passages, (C) and (E), Descartes clearly sees a strong affinity between his method in mathematics and his method in natural science.

I shall not again belabor the reader with my views that Descartes believed his mathematical method to be but an example of an overall general method of problem solution, and that such general method is to somehow ultimately include, or in some sense deal with, the data of experience and experimentation. But because the magnet specifically is here under discussion, I shall draw attention to the following quotation which appears in Rule Fourteen:

In the same way, if the magnet contains some kind of entity the like of which our intellect has never before perceived, it is pointless to hope that we shall ever get to know it simply by reasoning; in order to do that, we should need to be endowed with some new sense, or with a divine mind. But if we perceive very distinctly that combination of familiar entities or natures which produced the same effects which appear in the magnet, then we shall credit ourselves with having
achieved whatever it is possible for the human mind to attain in this matter.\textsuperscript{6}

In each of the above passages is the importance of empirical content apparent.\textsuperscript{7}

I wish to note that when ‘conclusions are to be drawn about the nature of the magnet simply from the experiments which Gilbert claims to have performed, be they true or false’, or when ‘we are not looking for anything beyond what can be deduced from the data’, the problem of the ‘nature of the magnet’ is considered ‘perfect’. Otherwise the problem is an imperfect one and one to be dealt with after Rule Twenty-Four -- that is to say, in the proposed third section of the \textit{Regulae}. Here is also found the example in the ‘nature of sound’ in which the sounds made by three strings are rendered comparable to one another by reducing this imperfect problem to a perfect one. We have seen above that I consider this an example of a reduction of secondary qualities (sound) to primary qualities (qualities quantifiable or expressed in terms of figure and extension). Again, the point seems to be here that by means of such reduction, ‘imperfect problems can all be reduced to perfect ones’. Apparently for Descartes, Gilbert’s enumeration of the properties of the magnet is sufficient -- somehow -- to render the imperfect problem of the ‘nature of the magnet’ to a perfect problem. In this case we are ‘drawing conclusions’ from ‘experiments’.

\textsuperscript{6} CSM 1, p. 57; AT X, 439.
\textsuperscript{7} We may also make note to the reference to ‘cause and effect’ in these passages.
However, the most important two points I wish to make concerning these passages are the following: (1) In passage (E), I wish to note Descartes' explicit designation of the magnet as an example of the abstraction of a problem from every irrelevant conception, and also as an example of a reduction to such a form as to be dealing with magnitudes and the comparison between them. And (2) I also wish to note that we may assume here, since passage (E) appears in the same quotation as that of passage (D), that the observations in question are, at least, those attributed by Descartes to Gilbert. These two points will prove important in the following material.

So far we have been dealing with the 'nature of the magnet' in the Regulae; let us now turn to the 'nature of the magnet' in the Principia.

I have suggested that for 'imperfect' problems to be rendered 'perfect' in natural science, secondary qualities are to be rendered in terms of primary or measurable qualities, qualities expressed in terms of figure and extension. To some extent this seems to be the case in the example of the magnet as depicted in the Principia. Consider the qualities listed by Descartes in that work which require explanation in order to state what Descartes calls the 'nature of the magnet': Article 145 of Book IV of the Principles is cited as: 'An enumeration of the properties of magnetic force'. I draw attention to the usage of the term 'enumeration' which is a term utilized extensively in the Regulae. This would show some continuity

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8 If we consider an 'enumeration' as a setting out of the data for review in the solution to a problem -- which is certainly one of its meanings for Descartes -- then the usage of this term is in general conformity with a new view of the meaning of that term on the part of Bacon. Vide: Urbach, Peter, Francis Bacon's Philosophy of Science: An Account and a Reappraisal, Open Court, La Salle, Illinois, 1987.
between the two works. Beneath this heading, Descartes lists the various known physical properties of magnets, which, not surprisingly, read as a compendium of the results obtained by Gilbert in his *De Magnete*:

(1) That there are, in a magnet, two poles, one of which everywhere turns toward the North pole of the Earth, while the other turns toward the South.
(2) That, according to the diverse places on the Earth which they occupy, these poles of the magnet incline diversely toward the Earth's center.
(3) That, if two magnets are spherical (and close), each turns toward the other in the same way as either does toward the Earth.
(4) That after they have thus turned, they approach each other.
(5) That if they are restrained in a contrary situation, they repel each other.

This is quite an 'enumeration', and it continues on in this vein up through number 34:

(34) That, finally, this force is also diminished by rust, humidity, and moisture, and removed by fire; but not by any other cause known to us.\(^9\)

I have already mentioned that these are the same qualities listed by Gilbert in his *de Magnete*. These did not change between the time of the *Regulae* and the *Principia*; that is, these must have been the same properties

\(^9\) *Principia*, p. 250. Cf also Descartes' listing of the principle phenomena of comets (*Principia*, p. 157) and his subsequent explanations of the reasons for these phenomena.
Descartes had in mind in his comments in the *Regulae* as espoused in Passage (D) above.

It may now be questioned, however, whether or in what sense these may be considered primary qualities. At this point I shall again refer the reader to Passage (E) where Descartes is explicit that the magnet is an example of a reduction to the comparison of magnitudes. We note that the apparent properties of the magnet listed here are not expressed in terms of equations like those in the case of sounds given above; let us return to Descartes' injunction in this matter:

One thing can of course be said to be more or less white than another, one sound more or less sharp than another, and so on; but we cannot determine exactly whether the greater exceeds the lesser by a ratio of 2 to 1 or 3 to 1 unless we have recourse to a certain analogy with the extension of a body that has shape. Let us then take it as firmly settled that perfectly determinate problems present hardly any difficulty at all, save that of expressing proportions in the form of equalities, and also that everything in which we encounter just this difficulty can easily be, and ought to be, separated from every other subject and then expressed in terms of extension and figures. Accordingly, we shall dismiss everything else from our thoughts and deal exclusively with these until we reach Rule Twenty-Five.\(^{10}\)

This is a difficult passage.\(^{11}\) The question here is how are Gilbert's phenomena, listed here in the *Principia*, to be made to fit in with these

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\(^{10}\) CSM 1, p. 58; AT X, 441. Quoted above, Chapter Three, fn 19.

\(^{11}\) The difficulty of interpretation is compounded by a difficulty in translation. Whereas AT reads here: 'in proportionibus in aequalitates evolvendis'; the Leibniz MS. reads: 'in aequalitatibus'; and the Amsterdam ed. reads: 'inaequalitatis'.
strictures? In what sense might we say that the phenomena listed in the
*Principia* is given in terms of extension and figure? We may note that
these phenomena are given in a form which might *in principle* be measured
or rendered in terms of equalities. Thus for phenomenon number (1), the
direction indicated by the pole of the magnet might be measured by
comparing it with, say, angle of divergence from the North Star. For
number (2) the specific angle of declination might be measured in an
appropriate fashion. And presumably the others might be similarly
measured. But these are not so measured or rendered in terms of equalities
here.

Concerning this issue, Cottingham has the following to say:

> This apparent discrepancy [that the frequency of precise
> mathematical analysis is markedly less than one might be led to expect,
> given the explicit connection which Descartes makes between the study
> of physics and the ‘subject matter of pure mathematics’[^12] is explained
> fairly simply. Despite his stress on the mathematical notion of *quantity*
> as the key to physics, Descartes never commits himself to the thesis
> that, in order to do good science, one must be able actually to supply
detailed formulae for calculation and measurement in respect of each
> phenomenon to be explained. All that he seems to require is that the
> properties involved be in principle *capable* of being quantified.[^13]

[^12]: Descartes for example makes such claims in the *Principia*:
> ‘64. That I do not accept or desire in Physics any other principles than in Geometry or abstract
> Mathematics; because all the phenomena of nature are explained thereby, and certain demonstrations concerning
> them can be given.’ *Principia*, p. 76.

[^13]: Cottingham, p. 89. According to Cottingham, for Descartes, ‘The tie-up between mathematics and
mechanics would thus seem to be this: Cartesian science deals only with the shape, size, and motion of particles;
since these are exactly quantifiable, the laws determining the interaction of the particles will be suitable for
description by means of precise mathematical formulae.’ *Burman*, p. 110.
In any case, Descartes says in passage (E) above that in the example of the Magnet we are dealing with ‘certain magnitudes in general and the comparison between them’. And clearly these phenomena have passed muster for Descartes in some sense. How this is may not be altogether perspicuous to us; nevertheless I suggest we accept Descartes’ words here at face value to see where they lead.\footnote{Descartes seems to recognize a distinction between the manner in which magnitudes in quantity are handled in geometry and physics when he says: ‘This [following certain Cartesian suppositions concerning what the human mind is, what the body is, and how it is informed by the mind] is just what you do in geometry when you make certain assumptions about quantity, which in no way weaken the force of the demonstrations, even though in physics you often take a different view of the nature of quantity.’ CSM 1, p. 40; AT X, 412.}

Part II: Principles.

I am going to argue in this section that Descartes’ principles are important in his solution of the ‘nature of the magnet’, and I am going to show what I take those principles to be. Specifically, I am interested in the derivation of these principles. In the first section, Section A., I discuss Descartes’ deduction of his natural laws; in the second section, Section B., I discuss his derivation of his elementary particles.

A. THE DEDUCTION OF THE LAWS OF NATURE.

Let us first consider the derivation of Descartes’ laws in the

\textit{Principia}. Descartes’ \textit{Principles} is his \textit{Magnum Opus}. It is written in
four parts. The first part begins with what is essentially the Cogito, proceeds to a proof for the existence of God, and from there advances to considerations of the physical world -- all more or less along the lines of the Meditationes -- but from there goes on to discuss the fabric of the cosmos and the material causes of natural occurrences.

In a letter to Mersenne, 28 January, 1641, Descartes states:

I may tell you, between ourselves, that these six Meditations contain the entire foundations for my physics. But it is not necessary to say so, if you please, since that might make it harder for those who favor Aristotle to approve them. I hope that those who read them will gradually accustom themselves to my principles and recognize the truth in them before they notice that they destroy those of Aristotle.\textsuperscript{15}

It is correct to say that Descartes actually feels he has deduced the laws of motion from his metaphysical foundations. Here's how he feels he does it. He first differentiates between the general cause and the particular cause of movement in the world,\textsuperscript{16} and then continues with a very important idea to his thesis:

[And God now maintains in the sum total of matter, by His normal participation, the same quantity of motion and rest as He placed in it at that time.\textsuperscript{17}]


\textsuperscript{16} In the words of Descartes: 'As far as the general (and first) cause is concerned, it seems obvious to me that this is none other than God Himself, who, [being all-powerful] in the beginning created matter with both movement and rest...' Principia, p. 58.

\textsuperscript{17} Principia, p. 58.
In other words, the quantity of motion in the universe is constant.
This is because, we are told:

[I]t is one of God's perfections to be not only immutable in His nature,
but also immutable and completely consistent in the way He acts.
...From this it follows that it is completely consistent with reason for us
to think that, solely because God moved the parts of matter in diverse
ways when He first created them, and still maintains all this matter
exactly as it was at its creation, and subject to the same law as at that
time; He also always maintains in it an equal quantity of motion.\textsuperscript{18}

For Descartes, two consequences follow from this principle of the
conservation of the quantity of motion in the universe, which I shall label
in the following quotation (1) and (2):

(1) when one part of matter moves twice as fast as another twice as
large, there is as much motion in the smaller as in the larger
(2) whenever the movement of one part decreases, that of another
increases exactly in proportion.\textsuperscript{19}

With the principle, the conservation of the quantity of motion in the
universe, together with these two derivative principles of local motion, in

\textsuperscript{18} Principia, p. 58.
\textsuperscript{19} The entire quotation reads: "For although motion is only a mode of the matter which is moved,
nevertheless there is a fixed and determined quantity of it; which, as we can easily understand, can be always the
same in the universe as a whole even though there may at times be more or less motion in certain of its individual
parts. That is why we must think that when one part of matter moves twice as fast as another twice as large, there
is as much motion in the smaller as in the larger; and that whenever the movement of one part decreases, that of
another increases exactly in proportion." Principia, p. 58.
hand, Descartes goes on immediately to deduce his three laws of physical motion. The first is as follows:

37. The first law of nature: that each thing, as far as is in its power, always remains in the same state; and that consequently, when it is once moved, it always continues to move.\textsuperscript{20}

This law, when combined with the second law given below, is the first expression of the law of inertia.\textsuperscript{21} From the first sentence of the explanation for the law, it is clear Descartes feels this law is to be deduced from the immutability of God.\textsuperscript{22} He goes on here in his appended scholium to give an explanation of what he means by this law and points out that it is borne out by experience; but offers no further proof that it is derived from the immutability of God than what he has already offhandedly expressed.

The next, the second, law is:

39. The second law of nature: that all movement is, of itself, along straight lines; and consequently, bodies which are moving in a

\textsuperscript{20} \textit{Principia}, p. 59. Compare this law with Newton's First Law: 'Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it'. Newton's came later. Newton's work was often a direct reaction to the work of Descartes, not the least example of which is the one before us here. Remember Newton's statement, 'If I have seen so far, it is because I have stood on the shoulders of giants'. For an assessment of Newton's great debt to Descartes, vide: Koyré, \textit{Newtonian Studies}, University of Chicago, Chicago, 1965, Chapter 3: 'Newton and Descartes', pp. 53-114. Vide also: McGuire, J.E., 'Space, Geometrical Objects and Infinity: Newton and Descartes on Extension', in \textit{Nature Mathematized}, William R. Shea (ed.), D. Reidel, Dordrecht, Holland, 1983, pp. 69-112; especially the section 'Imagining and Understanding', pp. 76-87.

\textsuperscript{21} According to Hanson, Descartes' first and second laws are both important for the expression of the law of inertia, and it was Descartes who first formulated this notion explicitly. Descartes' proteges Beeckman, Hobbes, and the younger Galileo were all of the opinion objects continued in motion unless impeded, but believed the motion was naturally circular in character. Vide: Hanson, N.R., \textit{Patterns of Discovery}, Cambridge University Press, 1972 (1958), fn 1, p. 187.

\textsuperscript{22} Descartes says: 'Furthermore, from this same immutability of God, we can obtain knowledge of the rules or laws of nature, which are the secondary and particular causes of the diverse movements which we notice in individual bodies.' \textit{Principia}, p. 59.
circle always tend to move away from the center of the circle which they are describing.\textsuperscript{23}

Descartes gives his reasoning for accepting this law as follows:

This rule, like the preceding one, results from the immutability and simplicity of the operation by which God maintains movement in matter; for He only maintains it precisely as it is at the very moment at which He is maintaining it, and not as it may perhaps have been at some earlier time.\textsuperscript{24}

Once again, Descartes has based one of his laws of nature on a metaphysical aspect of the Deity; this also holds for the third and final law. Here he states he believes his first law is derived from this source as well:

40. The third law: that a body, upon coming in contact with a stronger one, loses none of its motion; but that, upon coming in contact with a weaker one, it loses as much as it transfers to that weaker body.\textsuperscript{25}

This law is given in two parts: (1) a body, upon coming in contact with a stronger body, loses none of its motion, and (2) upon coming in contact with a weaker body, a body loses as much as it transfers to that weaker body; Descartes proves each part separately:

41. The proof of the first part of this law.

\textsuperscript{23} Principia, p. 60.
\textsuperscript{24} Principia, p. 60.
\textsuperscript{25} Principia, p. 61.
[A]s has been stated above, each thing which is not complex but simple, as motion is, always continues to exist as long as it is not destroyed by any external cause. And in an encounter with an unyielding body, there certainly appears a cause which prevents the movement of the body which strikes the other from maintaining its determination in the same direction. However, there is no cause which would remove or decrease the motion itself, (since none is taken from it by this body or any other cause and) since movement is not contrary to movement. From which it follows that its motion must not be diminished.\(^{26}\)

That is to say, because motion continues to exist as long as it is not destroyed by any external cause, as specified in the first law, motion must not be diminished. Thus, the first part of this, the third, law is derived from the first law, and therefore from God's immutability. Now for the proof of the second part of the third law:

42. The proof of the second part.

Similarly, the second part is proved by the immutability of God's manner of working in always uninterruptedly maintaining the world by the same action by which He created it.\(^{27}\)

Descartes goes on to argue in this section that, 'Thus, this continuous changing in created things is an argument for the immutability of God'. The English translators comment at this point, 'This is either a straightforward case of affirming the consequent or a vicious circle'. This

\(^{26}\) Principia, p. 62.
\(^{27}\) Principia, p. 62.
does not change, however, the fact that Descartes clearly feels this second half of his third law is derived from the immutability of God.

Though Descartes is often careful to point out these laws are borne out by experience, I think these may be taken at this point as no more than incidental illustrations, and I believe we ought to take Descartes at his word here: he really feels he has deduced the general laws of physical motion, with some sort of rigor acceptable to himself, from the nature of God. Later he will even say these laws should be considered correct even if not borne out by experience. Despite Descartes' verbiage, I take it his reasoning is as follows: Because of the immutable and completely constant nature of God, and because God created and constantly preserves the universe, the sum total of the quantity of motion throughout the universe is necessarily constant. And, because the quantity of motion in the universe remains constant, (1) everything moved continues to move unless impeded, (2) this movement has no reason to be other than along straight lines, and (3) bodies do not transfer motion to stronger bodies, but do transfer motion to weaker ones. From his principle of the conservation of the quantity of motion in the universe, Descartes feels, his three laws of motion follow. He then proceeds from these laws to deduce his specific laws of impact.28

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28 His laws of impact are as follows:

'First, if these two bodies, for example B and C, were completely equal in size and were moving at equal speeds, B from right to left, and C toward B in a straight line from left to right; when they collided, they would spring back and subsequently continue to move, B toward the right and C toward the left, without having lost any of their speed. ...

'Second, if B were slightly larger than C, and everything else were as previously described, then only C would spring back, and both would move toward the left at the same speed. ...

'Third, if the two bodies were equal in size, but if B were moving slightly more rapidly than C; after their collision not only would C alone spring back and both continue their movement toward the left, (that
B. THE DERIVATION OF THE ELEMENTARY PARTICLES.

In this section I am continuing to pursue Descartes’ derivation of his principles; in this case I am concerned with the derivation of his elementary particles.

Firstly, from the concept of the nature of matter, Descartes deduces that the nature of matter is extension:

4. That the nature of body does not consist in weight, hardness, color, or other similar properties; but in extension alone.

By so doing [i.e. laying aside those prejudices which arise solely from our senses, and using instead only our understanding], we shall perceive that the nature of matter, or of body considered in general, does not consist in the fact that it is hard, heavy, colored, or affects the senses in any other way; but only in the fact that it is a thing possessing extension in length, breadth, and depth. For as far as hardness is

is, in the direction from which C came), but also one half of B’s additional speed would be transferred from it to C, [since B could not move more rapidly than C which would be ahead of it]. ...

‘Fourth, if the body C were entirely at rest, [that is, if it not only had no apparent motion but also were not surrounded by air or any other fluid (which makes the hard bodies immersed in such a fluid very easily movable, as I shall show)], and if C were slightly larger than B; the latter could never {have the force to} move C. Rather, B would be driven back by C in the opposite direction. ...

‘Fifth, if the body C were at rest and {even very slightly} smaller than B; then, no matter how slowly B might advance toward C, it would move C with it by transferring to C as much of its motion as would permit the two to travel subsequently at the same speed. ...

‘Sixth, if the body C were at rest and exactly equal in size to body B, which was moving toward it; necessarily, C would be to some extent driven forward by B and would to some extent drive B back in the opposite direction. ...

‘Finally, if B and C were travelling in the same direction, C more slowly than B, so that B (which would be following C) would eventually strike it; and if C were larger than B but B’s speed exceeded C’s by a greater extent than C’s size exceeded B’s: then B would transfer to C as much of its speed as would be required to permit them both to travel subsequently at the same speed and in the same direction. However, if, on the contrary, B’s speed exceeded C’s by a smaller extent than C’s size exceeded B’s; B would be driven back in the opposite direction, and would retain all its movement.”

Principia, pp. 64-9.

These laws were objected to by Huygens and Leibniz, and, according to the translators of the Principia into English, the correct laws of elastic impact were submitted by Huygens to the Royal Society in 1669. Vide Principia, fn. 43, p. 64.
concerned, our senses tell us nothing about it except that the parts of hard bodies resist the movement of our hands when they encounter them. Besides, if whenever our hands moved in a certain direction, the bodies situated there were to move back at the speed at which our hands approach; we would certainly never feel any hardness. Yet it cannot in any way be understood that the bodies which would thus move back would thereby cease to have the nature of a body. Therefore, the nature of body does not consist in hardness. In the same way, it can be shown that weight, color, and all the other properties of this kind which are experienced in material substance, can be taken away; leaving that substance intact. From this it follows that the nature of matter does not depend on any such properties, but consists solely in the fact that it is a substance which has extension.\footnote{Principia, pp. 40-1.}

Other properties may be stripped from a substance, leaving that substance intact; extension alone cannot be so stripped.\footnote{The translators add here: 'An important consideration, which is not made explicit here, is that only extension, figure, and motion are capable of being clearly and distinctly perceived by the understanding. They are also the only properties which can be directly represented geometrically....' Principia, p. 41, fn 2.} Thus the nature of matter is that of a substance whose only necessary quality is extension. And from this notion of the nature of body as extension, Descartes deduces the uniformity of matter:

\[\text{All the matter in the whole universe is of one and the same kind; since all matter is identified [as such] solely by the fact that it is extended. Moreover, all the properties which we clearly perceive in it are reducible to the sole fact that it is divisible and its parts movable; and that it is therefore capable of all the dispositions which we perceive can result from the movement of its parts.}\footnote{Principia, p. 50.} \]
Descartes deduces the impossibility of the existence of a void:

[T]he extension of space, or of internal place, does not differ from the extension of body. From the sole fact that a body is extended in length, breadth, and depth; we rightly conclude that it is a substance: because it is entirely contradictory for that which is nothing to possess extension. And the same must also be concluded about space which is said to be empty: that, since it certainly has extension, there must necessarily also be substance to it.  

Then, because there exists no void, the particles of the universe must fill all its space, and are therefore infinitely divisible:

It must, however, be admitted that there is in this movement [of the displacement of particles by others] something which our mind cannot [fully] understand, even though we perceive it to be true: namely, a division of certain parts of matter to infinity, or an indefinite division into so many particles that we cannot conceive of any so small that we do not understand that it is in fact divided into others even smaller.

Thus, Descartes seems to feel he is able to deduce, on the basis of his metaphysics alone, that the world is entirely filled with minute particles, all composed of the same matter.

Now, however, when it comes time to state what kinds of particles there are, Descartes has the following to say:

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32 Principia, p. 47.
33 Principia, pp. 56-7. When Descartes says 'perceive' here, I take it he means perception on the part of our understanding, not sensory perception.
We noticed earlier that it is certain that all the bodies which compose the universe are formed of one [sort of] matter, which is divisible into all sorts of parts and already divided into many which are moved diversely and the motions of which are in some way circular, and that there is always an equal quantity of these motions in the universe: but we have not been able to determine in a similar way the size of the parts into which this matter is divided, nor at what speed they move, nor what circles they describe. For, seeing that these parts could have been regulated by God in an infinity of diverse ways; experience alone should teach us which of all these ways He chose. That is why we are now at liberty to assume anything we please, provided that everything we shall deduce from it is (entirely) in conformity with experience.\footnote{Principia, p. 106. Cf. also the following letter to Mersenne, 17 May 1638: 'You ask if I regard what I have written about refraction as a demonstration. I think it is, in so far as one can be given in this field without a previous demonstration of the principles of physics by metaphysics -- that is something I hope to do some day but it has not yet been done -- and so far as it is possible to demonstrate the solution to any problem of mechanics, or optics, or astronomy, or anything else which is not pure geometry or arithmetic. But to ask for geometrical demonstrations in a field within the range of physics is to ask the impossible. And if you will not call demonstrations anything except geometers' proofs, then you must say that Archimedes never demonstrated anything in mechanics, nor Vitellio in optics, nor Ptolemy in astronomy. But of course nobody says this. In such matters people are satisfied if the authors' hypotheses are not obviously contrary to experience and if their discussion is coherent and free from logical error, even though their hypotheses may not be strictly true. I could demonstrate, for instance, that even the definition of a centre of gravity given by Archimedes is false, and that there is no such centre; and the other hypotheses he frames elsewhere are not strictly true either. The hypotheses of Ptolemy and Vitellio are much less certain again; but that is not a sufficient reason for rejecting the demonstrations which they have based on them. ... I say that there are only two ways to refute what I have written. One is to prove by experience or reason that the hypotheses I have made are false; the other is to show that what I have deduced from them cannot be deduced from them. M. de Fermat understood this very well; for he tried to refute what I wrote about refraction by attempting to prove that it contained a logical error. But if people simply say that they do not believe what I have written, because I deduce it from certain hypotheses which I have not proved, then they do not know what they are asking or what they ought to ask.' K, p. 56; AT II, 134.}

Descartes deduces the existence of minute particles on the basis of metaphysical considerations, but when it comes to the descriptions of the specific shapes of the particles he has resort to experience -- not to perceive them sensually, but to derive them.

In a letter to Mersenne dated April 5, 1632, Descartes says:
[I]n the treatise which I now have in hand \textit{[Le Monde]}, after the general description of the stars, the heavens and the earth, I did not originally intend to give an account of particular bodies on the earth but only to treat of their various qualities. In fact, I am now discussing in addition some of their substantial forms, and trying to show the way to discover them all in time by a combination of experiment and reasoning. This is what has occupied me these last days; for I have been making various experiments to discover the essential differences between oils, ardent spirits, common and strong waters, salts, etc.\textsuperscript{35}

Also, though Descartes feels that his most general principles are deduced upon metaphysical considerations alone, just as Euclid’s postulates, definitions, and common notions all taken as principles are able to produce an infinite number of theorems and constructions, so Descartes sees his own principles as capable of producing an infinite myriad of effects:

The power of nature is so ample and so vast, and my principles so simple and so general, that I notice hardly any particular effect of which I do not know at once that it can be deduced from my principles in many different ways, and my greatest difficulty is usually to discover in which of these ways it depends on them. I know of no other way to discover this than by seeking further observations whose outcomes vary according to which of them provides the correct explanation.\textsuperscript{36}

\textsuperscript{35} K, p. 22; AT I, 242.
\textsuperscript{36} CSM 1, p. 144; AT VI, 64.
That is, it takes a consideration of both observations and principles to discover exactly how it is that the effects, the observations, are to be produced by the causes, the principles.

Again:

4. Of phenomena or experiments and of their use in philosophy.

However, the principles which we have already discovered are so vast and fertile that many more things follow from them than we see included in this visible universe, and even many more than we could mentally examine (in our entire lives).\(^{37}\)

Let us now see how Descartes goes about determining the natures of the minute particles which are to account for the observable phenomena.

After discussing the three candidates for celestial cosmologies in Part III of the *Principia*,\(^ {38}\) Descartes makes the surprising move of introducing his opinion as an ‘hypothesis’. His primary motivating factor for using this term, however, appears to be that he is expressing himself contrary to scripture. For things to have developed as Descartes expects (that is, for the particles of earth to have been broken down into particles of the second and first element) would have taken considerably longer to have occurred than the Bible seems to allow. He therefore states:

44. That I nevertheless wish those [causes] I am proposing here to be taken only as hypotheses.\(^ {39}\)
45. That I shall even assume here some which it is certain are false.\textsuperscript{40}

This is a very peculiar ploy. On the one hand, according to Descartes, we cannot understand the causes of all natural things except on the basis of the hypothesis he is presenting here alone -- but on the other hand, the hypothesis is without question false.

\textsuperscript{41}If, finally, as I hope to do, I clearly show that the causes of all natural things can be understood by means of that hypothesis, though by no other; it will thence be justly concluded that their nature is the same as if they had indeed been formed in such a way, \{although the world was not formed in that way in the beginning, but was created directly by God\}.\textsuperscript{41}"

Be this theological consideration as it may, however, the importance of the intrusion of this hypothesis from the point of view of Descartes’ method of science at this juncture is that he feels the hypothesis needs to be in conformity with -- and explain -- the empirical phenomena of the world:

\textsuperscript{42}If we can devise some principles which are very simple and easy to know and by which we can demonstrate that the stars and the Earth, and indeed everything which we perceive in this visible world, could have sprung forth as if from certain seeds (even though we know that things did not happen that way); we shall in that way explain their nature much better than if we were merely to describe them as they are now, \{or as we believe them to have been created\}.\textsuperscript{42}

\textsuperscript{40}Principia, p. 105.
\textsuperscript{41}Principia, p. 181.
\textsuperscript{42}Descartes states: ‘[I]n order to better explain natural things, I may even retrace their causes here to a stage earlier than any I think they ever passed through. \{For example\}, I do not doubt that the world was created in
Now let us look at the specific assumptions Descartes puts forward on the basis of this peculiar ploy:

Let us therefore suppose, if you please, that God, in the beginning, divided all the matter of which He formed the visible world into parts as equal as possible and of medium size, that is to say that their size was the average of all the various sizes of the parts which now compose the heavens and the stars. And let us suppose that He endowed them collectively with exactly that amount of motion which is still in the world at present. And, finally, that He caused them all to begin to move with equal force (in two different ways, that is), each one separately around its own center, by which means they formed a fluid body, such as I judge the heaven to be; and also several together around certain other centers equidistant from each other, arranged in the universe as we see the centers of the fixed Stars to be now; and also around other somewhat more numerous points, equal in number to the Planets [and the Comets].

the beginning with all the perfection which it now possesses; so that the Sun, the Earth, the Moon, and the Stars existed in it, and so that the Earth did not only contain the seeds of plants but was covered by actual plants; and that Adam and Eve were not born as children but created as adults. The Christian faith teaches us this, and natural reason convinces us that this is true; because, taking into account the omnipotence of God, we must believe that everything He created was perfect in every way. But, nevertheless, just as for an understanding of the nature of plants or men it is better by far to consider how they can gradually grow from seeds than how they were created [entire] by God in the very beginning of the world; so, if we can devise some principles which are very simple and easy to know and by which we can demonstrate that the stars and the Earth, and indeed everything which we perceive in this visible world, could have sprung forth as if from certain seeds (even though we know that things did not happen that way); we shall in that way explain their nature much better than if we were merely to describe them as they are now, [or as we believe them to have been created] Principia, pp. 105-6.

Cf also the following: "76. That divine authority is to be preferred to our perception: but that, apart from divine authority, it does not become a philosopher to assent to things other than those which have been perceived." Principia, p. 35. Again I take it Descartes means 'perceived by the intellect'.

43 Principia, pp. 106-7.
The primary assumption here, as I read this passage, is that God set a world of mid-sized globules of matter into motion. Once Descartes has this in hand, he is then able to deduce, to his satisfaction and in conformity with his laws of motion, the natures of the three basic types of elements in the world. That is, he doesn't assume the shapes of the particles outright as an hypothesis themselves; rather he feels he deduces them from his principles in conjunction with this hypothesis concerning God's movement of the universe.

I shall now attempt to show this.

Descartes' third element is of those particles of gross matter which are still of the original mid-sized range and which have not as yet been broken down to more minute particles. His second element is of a nature which is minute and spherical. Such particles must exist, according to Descartes, because of -- as I shall term it -- a tumbling effect upon the third element which wears it down into spheres:

[I]n order that we may begin to show the efficacy of the laws of nature in the proposed hypothesis, let us consider that, since all the matter of which the world is composed was in the beginning divided into many equal parts, these could not at first have been spherical; for several spheres joined together do not (form a completely solid and continuous body, like this universe, in which, as I demonstrated earlier, there can be no void). However, no matter what shape these parts may have had at that time, it was impossible for them not to become spherical with the passing of time because of their various circular motions. And because the force by which they were moved in the beginning was sufficient to separate them from one another; that same force, enduring (in them subsequently), was also undoubtedly great enough to break off all their
angles as they came in contact with one another, for this effect required less force than the previous one had. And solely from the fact that all the angles of a body are thus worn down, we easily understand that each at length became spherical: because every part of that body which protrudes beyond the spherical figure is here referred to as an angle.44

Again, Descartes imagines the world as a conglomerate of lumps which God sets into motion. Due to the natural motion of the lumps, it was 'impossible', in his view, for these not to be rendered spherical -- with additional smaller bits and pieces being formed from the 'angles' being broken off the larger lumps of gross matter.

However, inasmuch as there cannot be any empty space anywhere (in the universe), and because the parts of matter, being spherical, cannot unite closely enough to avoid leaving certain little intervals (or spaces) around themselves: these spaces must be filled by certain other scrapings of matter which must be extremely tiny and able to change their shapes at any moment in order to conform to those of the places they enter. For in fact, that which is detached from the angles of those particles of matter which are becoming spherical, by being gradually worn down, is so tiny, and acquires such great speed, that the sole force of its motion can divide it into innumerable scrapings which, (being of no determined size or shape, easily) fill all the angles (or spaces) into which the other parts of matter cannot enter.45

Things must be this way, says Descartes, and according to Descartes, it is impossible for things to be otherwise -- granting the original

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44 Principia, p. 108.
45 Principia, p. 109.
hypothesis of the movement of the universe. I wish to point out, however, the necessity involved here is of a different sort than that of Descartes' earlier metaphysical considerations. To adopt a distinction from Leibniz, the breaking down of the third element into the spherical bits of the second element and concomitant pieces or angular bits of the first element,\(^\text{46}\) is necessarily hypothetically, not absolutely.\(^\text{47}\) That is to say, if the presumed hypothesis of God's setting into motion of gross particles of matter obtains, then the necessity of the tripartite character of the particles follows. Thus I believe Descartes' deduction of the types of corpuscles existent in the world is of a different deductive status from his earlier metaphysical considerations -- but still retains a deductive status in Descartes' eyes. The earlier considerations are necessary under any circumstances; the nature of the three types of elements are only necessary assuming Descartes' hypothesis. To illustrate the difference, as I indicated before, whereas Descartes believes his laws may be contrary to experience and still be maintained as true,\(^\text{48}\) the nature of the minute particles, on the other hand,

\(^{46}\) For further discussion of the nature of these particles, vide: *Principia*, p. 110, Article 52.

\(^{47}\) I am not suggesting Leibniz' distinction is a particularly perspicuous one as expressed by him, nor do I wish here to enter into the difficulties of what he meant by these terms. I wish only to indicate what I take to be a difference Descartes might have adopted in deductive structure between his derivation of his most general principles and his derivation of the natures of his three types of basic elementary particles.

\(^{48}\) After the expression of his laws, Descartes has the following to say:

'[Indeed, experience often seems to contradict the rules I have just explained). However, because there cannot be any bodies in the world which are thus separated from all others, and because we seldom encounter bodies which are perfectly solid; it is very difficult to perform the calculation to determine to what extent the movement of each body may be changed by collision with others. Since, (before we can judge whether these rules are observed here or not), we must simultaneously calculate the effects of all those bodies which surround the bodies in question and which affect their motion.' *Principia*, p. 69.

Immediately subsequent to this, however, Descartes begins to make serious reference to experience as warrant when he speaks of the difference between solid and fluid:

'[F]rom the testimony of our senses, (for these properties [of solid and fluid] are in their domain), we recognize this difference to consist simply in the fact that the parts of fluid bodies easily move out of their places, and consequently do not resist the movement of our hands into those places; while, on the contrary, the parts of solid bodies adhere to one another in such a way that, without sufficient [external] force to overcome their cohesion, they cannot be separated.' *Principia*, p. 70.
must be in conformity with experience. I believe that for Descartes, however, the existence and nature of these particles are, nevertheless still, principles -- but principles of a hypothetical character. Thus I believe Descartes still feels he is entitled at this point in speaking as if his results are still being deduced metaphysically -- although one of the metaphysical principles is the hypothetical assumption of the setting into motion of the universe on the part of God.

Part III. The Explanation of the Magnet

Requisite to Descartes' explanation of the magnet are certain particles of the first, angular, nature with a screw-like configuration. The deduction of these particles is both complex and flawed;\textsuperscript{49} it is also, to my way of thinking, highly contrived.

We may say that due to the deflection of particles in motion, though these strive to move linearly, such particles are restrained into patterns of a curved or more circular character, and thus vortices. And because, according to Descartes: 'It is a law of nature that all bodies which are moved circularly attempt to recede from the centers around which they revolve,'\textsuperscript{50} the spherical particles of the second element recede from the

\textsuperscript{49} Vide translators' notes in Part III: footnotes. 55, 57, 60, and 62.

\textsuperscript{50} Principia, p. 111.
center of the vortex leaving only the smaller, angular particles of the first element -- as the sun.

What I have just said about the stone rotating in a sling around the center E [that a stone which is in a sling makes the rope more taut as the speed at which it is rotated increases, and that what makes the rope taut is nothing other than the force by which the stone strives to recede from the center of its movement], ... can easily be similarly understood of all the globules of the second element: i.e., each of these strives to recede from the center of the vortex in which it rotates; for it is restrained by the other globules beyond it in the same way as the stone is {restrained} by the sling.51

The circular movements of these particles will produce poles in the spherical configuration of the vortices.

From these things, it is clearly perceived how that action which I take to be light is emitted equally in all directions from the body of the Sun or of any fixed Star, and how it is transmitted in the shortest space of time to the greatest distance: and how this [occurs] along straight lines, drawn not only from the center of the luminous body, but also from any other point on its surface. From this, all the other properties of light can be deduced. And, though this may perhaps seem a paradox to many people, all these things would exist in the heavenly matter, even if there were absolutely no force in the Sun or other star around which it was rotating. So that, if the body of the Sun were nothing other than an empty space, nevertheless, its light (which would admittedly not be as strong, but which would otherwise not differ from what it now is) would still be perceived by us. At least, {this would be true} in the

51 Principia, p. 115.
circle along which the matter of the heaven is rotated; for we are not yet considering here the other dimensions of the sphere (which extend toward the poles). 52

Whereas particles of the first element in the vicinity of the system’s equator continue out and into the next star system, those entering from other star systems at our pole continue toward our sun and are there recycled to proceed outward again.

90. What the shape of these particles, which from now on we shall call grooved, is.

Of course, they must be triangular in cross-section, because they frequently pass through those narrow triangular spaces which are created when three globules of the second element touch. As for their length, it is not easy to determine, since it seems to depend solely on the quantity of matter of which these small bodies consist; but it suffices that we conceive of them as small (fluted) cylinders with three grooves (or channels) which are twisted like the shell of a snail. This enables them to pass in a twisting motion through the little spaces which have the form of [a] curvilinear triangle… and which are always found between three contiguous globes of the second element. For, since these [grooved particles] are oblong and pass very rapidly between the particles of the second element (while these are themselves being rotated around the axis of the heaven by another movement), we can easily understand that the grooves of each one must be twisted like the grooves in a snail’s shell; and that these [three grooves] are more or less twisted according to the distance which separates the spaces through which they are passing from that axis; because the parts of the second element

52 Principia, pp. 117-8.
revolve more rapidly when further from the axis than when closer to it....

Moreover, because they approach the center of the heaven from opposite directions, that is, some from the South (pole) and some from the North, while the whole vortex rotates in one direction on its axis, it is obvious that those coming from the South pole must be twisted in exactly the opposite direction from those coming from the North. And this fact seems to me very noteworthy, because the force of the magnet, which is to be explained later, mainly depends upon it.

Furthermore, while this matter of the first element is being carried along in a straight line by its own motion from A and B toward d and f, it is also being carried along circularly by the movement of the entire vortex around the axis AB; so that these individual scrapings describe lines which are spiral, or twisted like a cochlea. And after these spirals have reached d and f, they are turned back from there, on both sides, toward the equator eg.

53 Principia, pp. 133-4.
54 Principia, p. 134.
55 Principia, pp. 122-3.
It is not necessary to go into great detail concerning Descartes' actual explanation of the magnet. Suffice it to say that it is the existence and motion of the screw particles 'deduced' in Descartes' account of the motions of the particles of the universe throughout the various solar
systems which account for all of the entries on Descartes’ list of phenomena having to do with the magnet.

The general explanation for all these phenomena is very clever and very quaint. It must be said, however, that Descartes does account for how both the magnet and the screw-particles develop from other minute corpuscles; and for how the screw-particles combine to form magnets:

139. What the nature of the magnet is.
And those scrapings [of the particles of earth] which have been frequently turned, (sometimes in one direction, sometimes in another) while thus ascending through the veins of the exterior earth, whether they have accumulated alone or have been driven into the pores of other bodies, form a lump of iron. On the other hand, those which have either always retained the same position, or else, if sometimes forced to change it in order to reach the mines, have at least subsequently remained immobile there for many years after having been firmly driven into the pores of a rock or other body, form a magnet. And thus there is scarcely any lump of iron [ore] which does not in some way approach the nature of a magnet, and there is absolutely no magnet in which there is not some iron contained. Though perhaps this iron may sometimes adhere so closely to some other bodies that it can more easily be damaged than detached from these bodies by fire. (And often the stone in which these magnetic fragments are trapped is very hard, so it is sometimes almost impossible to melt magnets to make iron.)

And I shall point out here that Descartes refers to this explanation of the formation of the magnet as the ‘nature of the magnet’. Again we see

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56 Principia, p. 246.
the same terminology in both the *Principia* and the *Regulae*; and thus a
certain continuity between the two works.

Also, from this passage we may ascertain, I believe, that the ‘nature
of the magnet’ is an intermediate explanation between the principles of the
three basic sorts of elementary particles on the one hand -- principles, as I
am terming them-- and the empirical phenomena on the other -- the data:

All of these things follow from the principles of Nature expounded
above, in such a way that, even if I were not considering those magnetic
properties which I have undertaken to explain here, I nonetheless would
not judge these things to be otherwise. However, we shall see,
successively, that with the help of these principles, a reason for all those
properties (which the most careful experiments... have been able to
discover up to this time) is furnished so clearly and perfectly that this
fact also would seem sufficient to convince us of the truth of these
things, even if we did not know that they followed from the principles
of Nature.\textsuperscript{57}

Again, according to Descartes, the conical screw particles which are
to pass through the magnet are to account for the disparate phenomena
recorded on Descartes’ list.

Meticulously Descartes explains each one of the thirty-four
phenomena in terms of his principles.

For example, he explains that poles of magnets repel because the
supposed screw particles emerging from one magnet, O (see figures 15 and

\textsuperscript{57} *Principia*, p. 250. This is the introductory paragraph before Descartes introduces his list of thirty-
four phenomena of the magnet.
16), are unable to enter the pole of the other magnet, P, as their grooves twist in the improper direction.

![Figure 15](image)

And as the grooved particles require space to move to the side of the magnet and from thence to its opposite end, the agitated screw particles force the the other magnet away. In addition, this activity is duplicated on the part of the screw particles emerging from magnet P.\(^{58}\) In such fashion

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\(^{58}\) Proposition 154 of Part IV of the *Principia* reads as follows:

'154 Why [magnets] sometimes recede from one another.'
as this Descartes demonstrates how his supposed screw particles account for each of the phenomena on a case-by-case basis.


Descartes claims a distinction between the method of discovery and the method of exposition. Referring to himself in the third person, and speaking of the differences between the Principia and the Meditationes, he states he (Descartes):

is speaking of the sort of argument that can take some effect of God as a premiss from which the existence of a supreme cause, namely God, can subsequently be inferred. In fact, however, he discovered no such effect: after a most careful survey of all the effects, he found none which would serve to prove God's existence except for the idea of God. By contrast, the other argument in the Fifth Meditation proceeds a priori and does not start from some effect. In the Meditationes that argument comes later than the one here; the fact that it comes later, while the proof in this Meditation comes first, is the result of the order in which the author

*However, the like-named poles of two magnets do not thus approach each other, but on the contrary, if they are brought too close, recede instead. For the grooved particles emerging from that pole of one magnet which is facing [the like pole of] the other magnet are unable to enter that other magnet, and require some space between these two magnets through which they may pass in order that they may return to the other pole of the magnet from which they emerged. Specifically, since those coming out of O (in figures 15 and 16) through pole A cannot enter P through its pole a, they require some space between A and a through which they may pass toward V and B; and with the force which they have acquired by being moved from B to A, they drive magnet P. And similarly, those coming out of P drive magnet O; at least when their axes BA and ab are on the same straight line. However, when their axes are even very slightly inclined at an angle to one another, these magnets turn in the way explained a little earlier; or if their turning is impeded (but not their movement in a straight line), they once again recede from one another along a straight line.* Principia, p. 257.
discovered the two proofs. In the *Principles*, however, he reverses the order; for the method and order of discovery is one thing, and that of exposition another. In the *Principles* his purpose is exposition, and his procedure is synthetic.\(^{59}\)

We may here recall Descartes' response to the Second Objections to the *Meditationes* where he speaks of the distinction between 'analysis' and 'synthesis'.

Analysis shows the true way by means of which the thing in question was discovered methodically, ... so that if the reader is willing to follow it and give sufficient attention to all points, he will make the thing his own and understand it just as perfectly as if he had discovered it for himself. But this method contains nothing to compel belief in an argumentative or inattentive reader...\(^{60}\)

Synthesis, by contrast, ... demonstrates the conclusion clearly and employs a long series of definitions, postulates, axioms, theorems and problems, so that if anyone denies one of the conclusions it can be shown at once that it is contained in what has gone before, and hence the reader, however argumentative or stubborn he may be, is compelled to give his assent.

Comparing these two passages, and in keeping with all we have considered before, it would not be unreasonable to suppose that by analysis Descartes means his method of discovery, and by synthesis he means his method of exposition.

\(^{59}\) *Burman*, p. 12.
\(^{60}\) CSM 2, p. 110; AT VII, 155-6.
In view of this supposition, let me draw up now my estimation of the situation as regards the discovery and expositions of such problems as that of the magnet in Cartesian science. I shall address first the notion of discovery and second the notion of exposition.

First, in terms of discovery, I see the situation as follows. Descartes actually believes he has deduced his laws of motion on the basis of metaphysical considerations alone. His three types of basic element, however, are derived on the basis of a confluence of his metaphysical principles on the one hand, and the empirical data on the other. The problem at this point becomes a problem to find: what kinds of natures are simultaneously in conformity with Descartes’ accepted principles on the one hand, and the perceived data on the other. Once Descartes has supplied his preferred answer -- the three types of elementary particles -- the question then becomes a problem to prove. The postulation -- theorem -- of the existence and nature of these types of presumed entities needs to be linked up with the principles and the data. At that point it is incumbent upon the Cartesian scientist to consider each of the different various problems in natural philosophy requiring explanation: the rainbow, the movement of comets, the flow of blood through the heart, the nature of the magnet, and so on. So the problem now becomes a number of problems to find. In each case, the difficulty is that of reconciling the new, enhanced set of principles -- including the natures of the elementary particles -- with

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61 That Descartes derives some of his principles with reference to empirical data before or simultaneous with considering specific explanations of the data should not appear horrific to a modern reader; Pauling derived his principles primarily on the basis of empirical considerations as well, and this was a major step in the geometric structural analysis of biochemical substances.
a ‘complete’ enumeration of the data for each problem. In the words of Descartes:

This is not a matter of drawing a single deduction from a single, simple fact, for, as we have already pointed out, that can be done without the aid of rules; it is, rather, a matter of deriving a single fact which depends on many interconnected facts, and of doing this in such a methodical way that no greater intellectual capacity is required than is needed for the simplest inference.

In the specific case of the magnet, the postulation of the screw-like nature of the particles of the first element is proposed as the solution of this problem to find. Now the problem becomes a problem to prove, the task being to show how these postulated screw particles link up with the principles on the one hand and the accepted data on the other. This process is continued until each one of the various problems is solved. The working out of all this may be called the analysis.

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62 When Descartes speaks about ‘complete enumeration’ I am reminded of a sign outside a restaurant stating ‘All You Can Eat -- $2.95’. But after the first helping when you ask the waiter for more, he says, ‘No’. ‘Why?’ you ask. ‘Because that’s all you can eat,’ he says indicating your plate, meaning ‘that’s all we’ll allow you to eat’. A ‘complete enumeration’ is only as complete as it is humanly possible to compile. Remember when Descartes says in Quotation (D): ‘But if the problem is to be perfect, we want it to be determinate in every respect, so that we are not looking for anything beyond what can be deduced from the data. For example, someone may ask me what conclusions are to be drawn about the nature of the magnet simply from the experiments which Gilbert claims to have performed, be they true or false.’ Assuming ‘perfect problems’ to achieve a ‘complete enumeration’, the enumeration provided by Gilbert is ‘complete’ -- as complete as we can get it under the circumstances.

63 CSM 1, p. 51; AT X, 429.

64 There is a potential problem with this account, however. Whereas I have intimated earlier that the second section of the Regulae seems to have been concerning problems to prove and the third section seems supposed to have been concerning problems to find, in the case as I have depicted it here, the problems to find already utilize a listing or ‘enumeration’ of the data and thus would, presumably, be considered by Descartes perfect.

I suspect the solution to this difficulty is that an imperfect problem is one in which: (1) the solution is unspecified (as in a problem to find), or (2) the known (either data or principles) are unspecified, or (3) the conditions are unspecified. Thus, a problem to find would have been but one of the types of problems discussed in the third section.
I have just discussed the process of discovery; now let me discuss the process of exposition.

Once all of the data have been linked with the principles by means of such auxiliary hypotheses as that of the postulated existence and nature of the screw particles, the process is reversed and the data, or 'effects', are derived from the principles, or 'causes'. Whereas in Euclid the deduction of the postulated theorems and constructions, and the auxiliary theorems and constructions, from the accepted principles proves those propositions, Descartes seems to feel that the subsequent direct deduction from the principles of the data by means of the auxiliary material provided by him proves the correctness of his intermediate propositions and of the entire system.\footnote{The situation as I have depicted it here is a reconstruction, of course. If I am right, we do not know the actual steps of the discovery process, or the analysis as I am terming it, except insofar as such steps have been expressed in the general stipulations of the \textit{Regulae} and the \textit{Discours}, and, plausibly, the specific examples in the body of the three essays accompanying the \textit{Discours}. Outside of these, and the occasional oblique reference to specific examples in the \textit{Regulae}, we have only the steps of the \textit{Principia} to follow; that is, we have only the expository or, as I am terming it here, the synthetic presentation of the material.}

\textit{Cf.} 'We must note that a problem is to be counted as perfectly understood only if we have a distinct perception of the following three points: first, what the criteria are which enable us to recognize what we are looking for when we come upon it; second, what exactly is the basis from which we ought to deduce it; third, how it is to be proved that the two are so mutually dependent that the one cannot alter in any respect without there being a corresponding alteration in the other.'\footnote{Although each of these points taken by itself gives only probability to the conclusion, taken together they amount to a proof of it.} CSM I, p. 51; AT X, 429.

\textit{Cf.} also the following from a letter from Descartes to Marin dated July 13, 1638:

'Finally, you say that there is nothing so easy as adapting some cause to an effect. But although there certainly are several effects to which it is easy to adapt diverse causes, one [cause] to each [effect], yet it is not so easy to adapt one identical cause to several different effects, unless it is the true cause from which they originate. There are even often some [effects] which are such that to give a cause from which they can be clearly deduced is sufficient proof of their true cause.'\footnote{AT II, 197-99; taken from \textit{Principia}, p. 104, fn 42.}
[L]et us now set forth a brief description of the principal natural phenomena whose causes are to be investigated here, though not in order that we may use them to prove anything. For we wish to deduce the effects from their causes rather than the causes from their effects. Rather, [we do this] only so that we can consider some, rather than others, of the innumerable effects which we judge can be produced by those causes.66

Compare the above with the following two quotations, each from the Principia:

And certainly, if anyone will consider how marvelous are the properties of the magnet and of fire, and how different they are from those which we commonly observe in other bodies; how huge and forceful a fire can be ignited from the tiniest spark in an instant; to what an immense distance the fixed stars send their light on every side; and the remaining things whose causes (sufficiently obvious, in my judgement) I have deduced in this piece of writing from principles known and accepted by all (namely, from the figure, magnitude, situation, and motion of particles of matter): he will be easily persuaded that there are, in rocks or plants, no forces so secret, no marvels of sympathy or antipathy so astounding, and finally, no effects in all of nature which are properly attributed to purely physical causes or causes lacking in mind and thought; the reasons for which cannot be deduced from these same principles. Consequently, it is unnecessary to add anything else to them.67

66 Principia, p. 85.
67 Principia, p. 275. Garber and Cohen believe there is little evidence outside Descartes' statement to Burman for believing the Principia to be synthetic in character. They state: '[T]he direct evidence that Descartes wrote the metaphysical part of the Principles synthetically is very weak. The only textual evidence for this claim comes from the Conversations with Burman. But, it must be remembered, these words are not from Descartes's own hand. They are filtered through Burman and almost certainly through Clauberg, and clearly contain a number of mistakes. Thus it is difficult to be sure that the particular wording of any given passage represents Descartes's intentions, particularly when the remarks relate to such an obscure point as the distinction between analysis and synthesis. It is defensible to use that document to
And certainly, if the principles which I use are very obvious, if I
deduce nothing from them except by means of a Mathematical sequence,
and if what I thus deduce is in exact agreement with all natural
phenomena; it seems (to me) that it would be an injustice to God to
believe that the causes of the effects which are in nature and which we
have thus discovered are false. For we would then be accusing Him of
having made us so imperfect as to be liable to make mistakes, even
when correctly using our reason (which He has given us).\textsuperscript{68}

The primary similarity between these three passages is that Descartes
feels he is deducing the phenomena (or effects) from the principles (or
causes). It seems correct to say that as the \textit{Principia} was to have been his
final word in these matters, the \textit{Principia} was expressed in the expository,
or synthetic, or deductive mode. Although the \textit{Discours}' material was to
have illustrated Descartes' method, in his \textit{magnum opus}, the \textit{Principia},
Descartes did not take a chance with the method which, from his point of
view, might allow for disbelievers (i.e. the method of analysis); rather he
chose to use the method which, again from his point of view, would not
broach of dissension (i.e. the method of synthesis). Consider what
Cottingham has to say on the subject:

\textsuperscript{68} \textit{Principia}, pp. 104-5.
At a level of high generality, then, we can deduce certain abstract structural principles *a priori*; these function, as it were, as the scaffolding of Cartesian physics. But as soon as we wish to construct the detailed explanations of specific phenomena, Descartes moves away from apriorism and towards something which is much closer to the modern idea of 'hypothetico-deductive' approach. A hypothesis is advanced, and is then tested by seeing whether its observable consequences square with experience.\(^{69}\)

In view, however, of the difference between a Euclidean deduction and a Cartesian one -- that in Euclid the theorems are proven whereas in Descartes the auxiliary hypotheses are 'proven' -- why did Descartes feel his direct deduction proved his results? We hear, here, I think, the distant echo of Watson's 'it was just too pretty to be wrong' (i.e. that the explanation accounts for all the available data to such a high degree of accuracy that it 'simply couldn't be wrong'). From Descartes' point of view, his explanation of the magnet, and his other explanations, were too finely reasoned, and accounted for the phenomena much too well to be incorrect. Unfortunately for Descartes, however, these explanations were generally wrong -- like any scientific theory could possibly be. In

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\(^{69}\) Cottingham, pp. 91-2.

The question arises -- at the end of the *Principia* after he has offered all his explanations -- how Descartes can know of the existence of such minute and unobserved entities. His answer, which I take to be inconclusive, is as follows:

'203. How we know the figures and movements of imperceptible particles.

But I attribute determined figures, and sizes, and movements to the imperceptible particles of bodies, as if I had seen them; and yet I acknowledge that they are imperceptible. And on that account, some readers may perhaps ask how I therefore know what they are like.' *Principia*, p. 285.

We're all ears:

'To which I reply: that I first generally considered, from the simplest and best known principles (the knowledge of which is imparted to our minds by nature), what the principal differences in the sizes, figures, and situations of bodies which are imperceptible solely on account of their smallness could be, and what perceptible effects would follow from their various encounters. And next, when I noticed some similar effects in perceptible things, I judged that these things had been created by similar encounters of such imperceptible bodies; especially when it seemed that no other way of explaining these things could be devised.' *Principia*, p. 285.
addition, Descartes' principles turn out to be wrong as well -- as Huyghens and Leibniz were subsequently to point out.

I have indicated previously that what I have been calling here 'analysis', has a certain affinity with what Descartes has called the indirect method of problem solution, insofar as by that we mean an unknown mean term of a proportion in conjunction with a 'reverse movement of the imagination' together with 'two acts of conceiving' involving two known mean terms. The latter process, however, what I have called the 'synthesis', has a certain affinity with the direct method of problem solution. We can express this in the following manner:

(A) Direct Method:

[Elementary] Particles ----> Nature of Magnet ----> Gilbert's Phenomena
Principles ----> Explanation ----> Data

(B) Indirect Method:

[Elementary] Particles ----> Nature of Magnet <---- Gilbert's Phenomena
Principles ----> Explanation <---- Data

I have said that the methods of Descartes and of Watson and Crick are not identical. The important difference I see between the two is that for Descartes the principles are derived, as I have previously indicated, primarily upon the basis of metaphysical considerations, whereas Pauling's principles are derived on the basis of experimental evidence.
In the example of Watson and Crick's discovery of the structure of the DNA molecule in Chapter Four, I have argued that the puzzle which they solved was that of coming up with a structure which would at one time satisfy all the known phenomena and accepted principles. I have here argued that Descartes' approach to discovery was essentially similar insofar as requiring accepted principles, but different insofar as the genesis of the principles of the systems are different.

Recall our conclusions concerning 'simple natures' at the end of Chapter Three: that, in terms of natural science, for Descartes there are two types of 'simple natures': (1) the physical attributes of the metaphysical building blocks of the universe, the 'minute corpuscles', and (2) the basic and direct phenomenal data by means of which we know the world. I said at that time that both might be considered primary qualities.

One important thing to notice here, I think, is that there are two kinds of 'simple' natures or things: those which are real and those which are relative to the mind -- or, things we consider in the order that correspond to our knowledge of them and things we consider in accordance with how they exist in reality.

[W]hen we consider things in the order that corresponds to our knowledge of them, our view of them must be different from what it would be if we were speaking of them in accordance with how they exist in reality.\(^{70}\)

\(^{70}\) CSM 1, p. 44; AT X, 418.
In the previous chapter I suggested that the situation surrounding Watson and Crick's discovery might be expressed as follows:

Principles -------> unknown geometric structure ------ Data

Though Descartes speaks of the differentiation of the order that corresponds to our knowledge of things versus how the things exist in reality, considering the remarks made at the end of Chapter Three concerning two types of simple natures, I shall assume some literary license here and speak of the difference between the 'order of things' and the 'order of knowledge'; and I shall suggest that the situation surrounding Descartes' exposition of the magnet might be expressed as follows:

Order of Things -------> Explanation of Magnet ------ Order of Knowledge

In the *Meditationes* Descartes differentiates between the sun as perceived and sun as reasoned by astronomers. He is clearly of the opinion that the 'reasoned' sun of the astronomers is a more correct characterization than that of the perceived sun. So it is with what I am calling the order of things vs. the order of knowledge. I believe it is Descartes' opinion -- it is not my own opinion, it is probably not yours, but I believe it is Descartes' opinion -- that we can be more certain concerning the nature of the elementary particles which make up the universe than we can of other natures and of, at least many of, our observations -- though both the natures of the elementary particles and observations expressed in terms of primary qualities are to be considered 'simple natures'. 
Concerning the problem of certainty, I have not worked out the
details of Descartes’ handling of the rainbow as yet, but a couple of
comments would be in order here. Firstly, the conversion of colors to
(measurable) angles may be construed as the conversion of secondary to
primary qualities. Secondly, Descartes’ \textit{calculation} of the paths of rays of
light through the rain droplets \textit{predicts} the existence of Alexander’s Band;
and this prediction may be viewed as a test for the theory.\footnote{On this latter point, vide: Dales, Richard C., \textit{The Scientific Achievement of the Middle Ages}, University of Pennsylvania Press, Philadelphia, PA, 1973, pp. 98-101.}

Consider what Descartes has had to say concerning such situations:

\begin{quote}
[W]e shall know that we have correctly determined these causes [of
more general things] when we observe that we can explain, by their
means, not only those phenomena which we have considered up to
now, but also everything else about which we have not previously
thought.\footnote{\textit{Principia}, p. 104.}
\end{quote}

In the words of Cottingham:

So in order to make the transition from general principles to the
explanation of particular events, we need, according to Descartes, to
rely on ‘observations’ or, in the original French, \textit{expériences}. This term
is frequently used by Descartes to refer to observations made by the
scientist. In the Latin texts the corresponding term is \textit{experimenta}; and
in his use of both the Latin and the French terms Descartes sometimes
comes close to the notion of an ‘experiment’ in the modern sense. It is
worth noticing that the idea of putting something to the \textit{test} is contained
in the meaning of the original Latin verb *experior* from which the Latin
*experimentum* and the French *expérience* are both derived.73

**SUMMARY**

I recall to the reader that my original purpose was to examine the
relationship between reason and experimentation, between *a priori* and *a
posteriori*, in Cartesian method. I believe that question is posed in
improper terms as for Descartes there seems to be, as there was for
Aristotle and for certain of the Medievals, two kinds of *a priori*. For
Descartes there are things which are prior in reality -- metaphysically
prior -- and there are also things which are prior in knowledge --
epistemically prior. In the corporeal realm the former are ‘simple natures’
which are the corpuscular components of the universe; the latter are
‘simple natures’ of phenomena known in terms of primary qualities. If I
am right, the task in natural science seems to be to link up the metaphysical
‘simple natures’ satisfactorily with the epistemological ‘simple natures’ in a
fashion along the lines I have indicated. In this limited sense, then, it
would seem traditional scholarship is correct in its claim that Cartesian

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73 Cottingham, p. 91.
Cottingham himself might disagree, however, that Descartes’ explanations were to be substantiated by
additional tests. He says:
‘When Descartes says that his explanations are ‘confirmed by reliable everyday experience’ *(Principles
IV 200)*, this is certainly not a matter of his being able to specify experiments designed to test whether the
predicated results of a given hypothesis actually obtain.’ Cottingham, p. 94.
science is completely *a priori* in nature; but the nature of the claim has been entirely misunderstood.

Descartes says his results are *as certain as* those in Geometry; that he achieves mathematical certainty. Perhaps this is because he is using, just as some of modern science seems to, what can be considered a method of analysis in problem solving. It happens that Descartes’ principles are wrong, and methodologically improperly derived, and that thus his results are vitiated. Nevertheless, he believes (incorrectly) he has achieved mathematical certainty. But I believe his claim is not a deceptive one. Much more importantly, his *method*, insofar as attempting to link up phenomenal data with accepted principles, is, in more ways I think than have been traditionally appreciated, the correct *method* regardless of his results -- and it is this primarily for which Descartes is, in my opinion, justly to be venerated.
Appendix: Analysis and Synthesis from the Second Set of Replies.

I now turn to your proposal that I should set out my arguments in geometrical fashion to enable the reader to perceive them ‘as it were at a single glance’. It is worth explaining here how far I have already followed this method, and how far I think it should be followed in future. I make a distinction between two things which are involved in the geometrical manner of writing, namely, the order, and the method of demonstration.

The order consists simply in this. The items which are put forward first must be known entirely without the aid of what comes later; and the remaining items must be arranged in such a way that their demonstration depends solely on what has gone before. I did try to follow this order very carefully in my Meditations, and my adherence to it was the reason for my dealing with the distinction between the mind and the body only at the end, in the Sixth Meditation, rather than in the Second. It also explains why I deliberately and knowingly omitted many matters which would have required an explanation of an even larger number of things.

As for the method of demonstration, this divides into two varieties: the first proceeds by analysis and the second by synthesis.
Analysis shows the true way by means of which the thing in question was discovered methodically and as it were \textit{a priori},\textsuperscript{1} so that if the reader is willing to follow it and give sufficient attention to all points, he will make the thing his own and understand it just as perfectly as if he had discovered it for himself. But this method contains nothing to compel belief in an argumentative or inattentive reader; for if he fails to attend even to the smallest point, he will not see the necessity of the conclusion. Moreover there are many truths which -- although it is vital to be aware of them -- this method often scarcely mentions, since they are transparently clear to anyone who gives them his attention.

Synthesis, by contrast, employs a directly opposite method where the search is, as it were, \textit{a posteriori} (though the proof itself is often more \textit{a priori} than it is in the analytic method). It demonstrates the conclusion clearly and employs a long series of definitions, postulates, axioms, theorems and problems, so that if anyone denies one of the conclusions it can be shown at once that it is contained in what has gone before, and hence the reader, however argumentative or stubborn he may be, is compelled to give his assent. However, this method is not as satisfying as the method of analysis, nor does it engage the minds of those who are eager to learn, since it does not show how the thing in question was discovered.

\textsuperscript{1} Translators' note: 'Descartes' use of the term \textit{a priori} here seems to correspond neither with the modern, post-Leibnizian sense (where \textit{a priori} truths are those which are known independently of experience), nor with the medieval, Thomist sense (where \textit{a priori} reasoning is that which proceeds from cause to effect). What Descartes may mean when he says that analysis proceeds 'as it were \textit{a priori} (\textit{tanquam a priori}) is that it starts from what is epistemically prior, i.e. from what is prior in the 'order of discovery' followed by the meditator.' CSM 2, p. 110, fn 2.
It was synthesis alone that the ancient geometers usually employed in their writings. But in my view this was not because they were utterly ignorant of analysis, but because they had such a high regard for it that they kept it to themselves like a sacred mystery.

Now it is analysis which is the best and truest method of instruction, and it was this method alone which I employed in my *Meditations*. As for synthesis, which is undoubtedly what you are asking me to use here, it is a method which it may be very suitable to deploy in geometry as a follow-up to analysis, but it cannot so conveniently be applied to these metaphysical subjects.

The difference is that the primary notions which are presupposed for the demonstration of geometrical truths are readily accepted by anyone, since they accord with the use of our senses. Hence there is no difficulty there, except in the proper deduction of the consequences, which can be done even by the less attentive, provided they remember what has gone before. Moreover, the breaking down of propositions to their smallest elements is specifically designed to enable them to be recited with ease so that the student recalls them whether he wants to or not.

In metaphysics by contrast there is nothing which causes so much effort as making our perception of the primary notions clear and distinct. Admittedly, they are by their nature as evident as, or even more evident than, the primary notions which the geometers study; but they conflict with many preconceived opinions derived from the senses which we have got into the habit of holding from our earliest years, and so only those who really concentrate and meditate and withdraw their minds from corporeal
things, so far as is possible, will achieve perfect knowledge of them. Indeed, if they were put forward in isolation, they could easily be denied by those who like to contradict just for the sake of it.

This is why I have wrote ‘Meditations’ rather than ‘Disputations’, as the philosophers have done, or ‘Theorems and Problems’, as the geometers would have done. In so doing I wanted to make it clear that I would have nothing to do with anyone who was not willing to join me in meditating and giving the subject attentive consideration. For the very fact that someone braces himself to attack the truth makes him less suited to perceive it, since he will be withdrawing his consideration from the convincing arguments which support the truth in order to find counter-arguments against it.²

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