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Experimental investigation for constitutive modeling of fine sand under cyclic loading using hollow cylinder specimens

Sun, Yuanhui, Ph.D.

Rice University, 1991

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RICE UNIVERSITY

EXPERIMENTAL INVESTIGATION
FOR CONSTITUTIVE MODELING OF FINE SAND
UNDER CYCLIC LOADING
USING HOLLOW CYLINDER SPECIMENS

by

YUANHUI SUN

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

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March, 1991
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1991
ABSTRACT

Experimental Investigation for Constitutive Modeling of Fine Sand under Cyclic Loading using Hollow Cylinder Specimens

by

Yuanhui Sun

A comprehensive experimental program for the understanding of the behavior of fine cohesionless soil under monotonic and cyclic loading and for the development and refinement of constitutive models has been undertaken in this study. To this end, an entirely new state-of-the-art triaxial test system for both solid and hollow cylinder testing of soil, and specimen preparation devices have been designed and set-up during this study. The computer-automated experimental system for solid and hollow cylinder testing soil is capable of both stress and strain controlled tests, and simulating most stress paths encountered in the field. A number of computer programs for parameter study of constitutive modeling has also been developed. The specimen preparation procedures using Ladd's moist tamping undercompaction technique for both solid and hollow cylinder saturated sand samples have been developed and successfully used in this study.

A comprehensive experimental program for both loose and dense saturated fine Ottawa Silica sands under monotonic loading using both solid and hollow cylinder specimens has been conducted. The failure surfaces for both loose and dense sands have been established from twenty drained monotonic load tests. Results from the monotonic test program of loose and dense sands under drained conditions were found in good agreement with the failure surfaces incorporated in Lade's constitutive model.

The soil behavior and deformation characteristics under undrained conditions have been investigated under a cyclic experimental program for loose sand. Results from eight cyclic load tests indicated that the circular rotation of principal stress axes with a constant amplitude deviator stress, as well as the stress direction reversals have significant effects on the rate of pore water pressure buildup, the triggering of a liquefaction flow failure in contractive sand and the rate of accumulation of deformation.
The study indicated that the rate of excess pore water pressure buildup is faster during a cyclic test with circular rotation of principal stress axes than during a cyclic triaxial shear test or a cyclic torsional shear test having same amplitude of shear stress. The rate of excess pore water pressure buildup is faster during a cyclic triaxial extension test than in a cyclic triaxial compression test or a cyclic torsional simple shear test with the same amplitude of shear stress. Moreover, after the first cycle and before the specimen reaches failure, the unloading and reloading stress-strain curves result in more or less similar shapes of hysteresis loops but with gradually decreasing slopes at a rather slow rate. In the unloading and reloading process, irrecoverable strains are gradually induced. However, when failure states are approached, the hysteresis loops become wider, large plastic strains are produced and the slope of the hysteresis loops decreases remarkably. The deformation and significant pore water pressures are developed during the first cycle and more remarkably during the last cycle. The pore water pressures (and mean effective stress reduction) are developed fast when the stress path reaches the failure surface which was established in monotonic load tests.

The pore water pressure and deformation increase fast in the case of shear stress reversal. The pore water pressure buildup and mean effective stress reduction are more pronounced during extension loading than in compression loading. Moreover, the amplitude of shear stress has significant effects for the pore water pressure development and deformation. Significant pore water pressure and deformation can occur, during the rotation of the principal stress axes, even when the deviator stress is maintained unchanged.

The study presented here is the first step towards the experimental investigation of soil behavior and constitutive modeling. Recommendations for the continuation of this work are extended in this thesis.
ACKNOWLEDGEMENTS

The author wishes to express his sincere appreciation to Dr. Panos C. Dakoulas for his patient guidance and assistance throughout this work. His dedication is gratefully acknowledged.

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Specially, the author wishes to thank his wife, Xun, his parents and parents-in-law for their constant moral support and encouragement through his study at Rice University.

The author dedicates this dissertation to his motherland.
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<tr>
<td>A</td>
<td>cross section of specimen or function of hardening parameter</td>
</tr>
<tr>
<td>D_{10}</td>
<td>diameter at which 10% of the soil is finer</td>
</tr>
<tr>
<td>D_{50}</td>
<td>diameter at which 50% of the soil is finer</td>
</tr>
<tr>
<td>D_r</td>
<td>relative density</td>
</tr>
<tr>
<td>C_c</td>
<td>coefficient of uniformity</td>
</tr>
<tr>
<td>C_u</td>
<td>coefficient of concavity</td>
</tr>
<tr>
<td>C_{ijkl}</td>
<td>material compliance tensor</td>
</tr>
<tr>
<td>E</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>e</td>
<td>void ratio</td>
</tr>
<tr>
<td>e_{max}</td>
<td>void ratio of soil in loosest condition</td>
</tr>
<tr>
<td>e_{min}</td>
<td>void ratio of soil in densest condition</td>
</tr>
<tr>
<td>f</td>
<td>yield function</td>
</tr>
<tr>
<td>G</td>
<td>elastic shear modulus</td>
</tr>
<tr>
<td>g, g_p</td>
<td>plastic potential function</td>
</tr>
<tr>
<td>H</td>
<td>height of specimen or hardening parameter</td>
</tr>
<tr>
<td>I_1</td>
<td>the first invariant of stress tensor</td>
</tr>
<tr>
<td>I_2</td>
<td>second invariant of stress tensor</td>
</tr>
<tr>
<td>I_3</td>
<td>third invariant of stress tensor</td>
</tr>
<tr>
<td>J_1</td>
<td>first invariant of the deviator stress tensor</td>
</tr>
<tr>
<td>J_2</td>
<td>second invariant of the deviator stress tensor</td>
</tr>
<tr>
<td>J_3</td>
<td>third invariant of the deviator stress tensor</td>
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<tr>
<td>K</td>
<td>elastic bulk modulus</td>
</tr>
<tr>
<td>m</td>
<td>failure parameter</td>
</tr>
<tr>
<td>m, n</td>
<td>unit vectors</td>
</tr>
<tr>
<td>p'</td>
<td>mean effective stress</td>
</tr>
<tr>
<td>p_a</td>
<td>atmospheric pressure</td>
</tr>
<tr>
<td>q</td>
<td>deviator stress</td>
</tr>
<tr>
<td>q_f</td>
<td>deviator stress at failure state</td>
</tr>
<tr>
<td>r</td>
<td>radius</td>
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</table>
\( R \)  \hspace{1cm} \text{deviator length defined by Fig.2.1-4}

\( V \)  \hspace{1cm} \text{volume of specimen}

\( W_p \)  \hspace{1cm} \text{plastic work}

\( \beta \)  \hspace{1cm} \text{angle between the major principal stress direction and the vertical direction}

\( \tau_{cy} \)  \hspace{1cm} \text{cyclic shear stress}

\( \tau_{z\theta} \)  \hspace{1cm} \text{shear stress}

\( \tau_{o\!c\!t} \)  \hspace{1cm} \text{octahedral normal effective shear stress}

\( \sigma \)  \hspace{1cm} \text{stress}

\( \sigma_1, \sigma_2, \sigma_3 \)  \hspace{1cm} \text{principal stresses}

\( \sigma_z, \sigma_r, \sigma_\theta \)  \hspace{1cm} \text{normal stresses in polar coordinate system}

\( \sigma_{mc} \)  \hspace{1cm} \text{effective mean principal stress at consolidation}

\( \sigma_{o\!c\!t} \)  \hspace{1cm} \text{octahedral normal effective stress}

\( \varepsilon_1, \varepsilon_2, \varepsilon_3 \)  \hspace{1cm} \text{principal strains}

\( \varepsilon_{o\!c\!t} \)  \hspace{1cm} \text{octahedral normal effective strain}

\( \varepsilon_z, \varepsilon_r, \varepsilon_\theta \)  \hspace{1cm} \text{normal strains in polar coordinate system}

\( \varepsilon_v \)  \hspace{1cm} \text{volumetric strain}

\( \gamma_{d\!m\!a\!x} \)  \hspace{1cm} \text{dry unit weight of soil in densest condition}

\( \gamma_{d\!m\!i\!n} \)  \hspace{1cm} \text{dry unit weight of soil in loosest condition}

\( \gamma_{o\!c\!t} \)  \hspace{1cm} \text{octahedral normal effective shear strain}

\( \delta_{ij} \)  \hspace{1cm} \text{Kronecker delta}
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CHAPTER I
INTRODUCTION

1.1 OVERVIEW

The development of constitutive laws is one of the most essential and most difficult problems in geotechnical engineering. During the last two decades, considerable progress has been made in the area of constitutive modeling of soil behavior under monotonic and cyclic loading. A large number of sophisticated and powerful mathematical models has been developed to represent the complex soil behavior under general loading conditions. Unfortunately, although many different models have been proposed, there is not yet firm agreement among research results. Moreover, there is a significant gap between the theoretical modeling and the experimental investigation of soil behavior. A common shortcoming of many constitutive modes for soil is that they are applicable only to loading conditions of a rather specific nature. This is due, not only to the extreme complexity of soil behavior, but also to the substantial investment of capital and time usually required for refined experimental investigation.

More recently, the deformation characteristics of soil under cyclic loading conditions have been the subject of several investigations. Of particular theoretical and practical interest is the study of the effects of cyclic stresses and rotation of principal stresses on the pore water pressure buildup and the deformation characteristics. Such studies suggest that the deformation characteristics of sands under monotonic and cyclic loading are significantly affected by rotation of principal stresses.\(^{[6,7,40,41,79,88]}\) Although there is a generally agreement that the effects of principal stress rotation are significant and should be taken into account in modeling of soil behavior, the experimental procedures for such
testing are by no means conventional. Instead, they are complicated and costly. Thus, usually there are only limited sets of data for the development of constitutive models capable of predicting such unconventional stress paths.

The need to clarify our understanding of the phenomena during principal stress rotation was emphasized at the workshop on "Generalized Stress-Strain and Plasticity Theories for Soils" held in the Montreal, Canada, in 1980.\textsuperscript{[61]} In this workshop, the participants were provided with experimental data from conventional tests and were asked to make "class A" predictions of new data, including circular rotation of principal stresses. After evaluating a number of widely known plasticity models, Pooroshasb and Selig\textsuperscript{[61]} concluded that "a comparison of the predictions to test results showed good agreement for simple, monotonic loading stress paths, but poor agreement for the severe test of the circular stress path". Some progress has been achieved in understanding these phenomena through experimental investigations since then, but the level of understanding is still far less than desirable. It is widely recognized today that there is great need for good quality experimental data to verify the various assumptions employed in the mathematical modeling of soil.\textsuperscript{[61,78,70,71,40]}

Constitutive models for soil behavior are valuable tools in developing a better understanding of the response of complex systems with nonlinear inelastic behavior subjected to general loading conditions. The dynamic soil-structure interaction problem is a typical example in which such an analysis would allow a more realistic evaluation of the important parameters controlling the system response. Another example in which significant economy may be achieved by improving the design of the analysis with a constitutive soil model is the case of pile foundations for off shore structures and bridges. The success of a constitutive model depends significantly on the validity of its basic assumptions including the hardening (softening) rule and the flow rule. It is common
practice to use only a limited number of simple tests to derive the required input data and to rely for the more complex stress paths on the original assumptions of the model. Therefore, there is an imperative need to verify these assumptions with detailed measurements of the exact soil behavior under more general loading conditions. By developing a better understanding of soil behavior, sound experimental and numerical cost-effective procedures, it is expected that the use of constitutive soil models will soon become more attractive to the practicing engineer who will realize its significant benefit. Finally, the researcher could benefit from using such a model to study stress distributions within soil specimens and to improve the design of testing equipment, or to interpret results from indirect field tests in terms of more basic parameters.

1.2 OBJECTIVE

The main objective of this study is to assist in developing a unifying framework of understanding for the behavior of cohesionless soil under any loading condition with emphasis on cyclic loading. The study examines the failure and deformation characteristics of the cohesionless soil under both monotonic and cyclic loading conditions, and the effects of principal stress rotation and stress reversal during cyclic loads. The purpose of such a framework is to provide both insight and the specific information needed to develop a reliable constitutive model.

In addition to the theoretical interest for understanding the failure characteristics and for developing constitutive models, there are also some direct applications of the continuous rotation of principal stress directions at a constant deviator stress such as the cyclic stresses induced by the waves within the sea-bed soil, as well as the cyclic stresses induced by railway or motor vehicles within the underlying soil, etc.
In order to fulfil the above objective, a comprehensive experimental program has been performed. This test program includes monotonic load and cyclic load tests both for loose sand (with relative density, $D_r = 30\%$) and dense sand ($D_r = 75\%$) using both solid and hollow cylinder specimens. The main objective of monotonic load testing program is to establish the yield and failure surfaces for monotonic loading. The soil behavior under monotonic loading both for loose and dense sand is investigated in detail. A comparison of the failure surfaces, which are established in this study, with an existing constitutive model for monotonic load tests is presented. The main objectives of the cyclic testing program are (a) to establish the stress-strain and pore water pressure characteristics during cyclic loading; (b) to investigate the effects of principal stress axis rotation and stress reversal; (c) to modify the yield and failure surfaces as the results of repeated loading and reloading.

The results from the comprehensive experimental program will provide a deeper insight into the behavior of the chosen sand. The more general loading conditions or special stress paths chosen in the test program will help in establishing a solid set principles and common basis for developing sophisticated or simplified constitutive models for the sake of analysis and design. Thus, in general, the results of the proposed experimental program will be utilized in three different ways: (a) to expand our current knowledge of soil behavior for more general loading conditions and develop the general framework for understanding of soil behavior; (b) to develop and refine the constitutive model to be used for analysis and parametric studies and (c) to make the experimental results available to other investigators who would be interested in calibrating their models. Study of the experimental results in relation to the soil deformation characteristics will lead to the formulation of a set of general consistent postulates as well as quantitative expressions among the various variables, from which some of the characteristics of the constitutive model will be obtained. It should be noted that such a framework should be
able to describe qualitatively the behavior not only of the tested soil but also of a cohesionless soil in general.

It is also the objective of this study to set up the geotechnical research laboratory at Rice university and to make the first step in a defined research direction in the experimental investigation of soil behavior and constitutive modeling.

The specimens chosen for use in this study are both solid and hollow cylinder samples. The popularity attached to the hollow cylinder triaxial testing which have been used by numerous investigators to study the behavior of soil\[36,37,38,39,40,41,48,49,69,70,90\] stems from the fact that it allows control of the magnitude and direction of the principal axes in a continuous manner and almost at will. Independent variation of the intermediate principal stress is also possible by using different internal and external confining pressures (resulting however in undesirable stress and strain non-uniformities across the wall of the cylindrical specimens).

1.3 OUTLINE

Chapter II presents some materials which are necessary background for the study of soil behavior and constitutive modeling. In this chapter, a review of plasticity theory is provided in section 2.1. Two elastic-plastic constitutive models for soil are briefly reviewed in section 2.2. A overview of experimental techniques for constitutive modeling of soil is presented in section 2.3.

In Chapter III, experimental devices, test specimens and computer control programs for this study are described. Section 3.1 provides the experimental devices both for solid and hollow cylinder specimens. The set-up of test specimen both for solid and hollow
cylinder specimens are presented in section 3.2. Section 3.3 discusses briefly the computer control programs which are used in this study.

In Chapter IV, the experimental program and control procedure for this study are presented in detail. Section 4.1 provides the test program and control procedure for monotonic load tests. The experimental program and test control procedure for cyclic load tests are given in section 4.2.

Experimental results and analysis of monotonic load tests are presented in Chapter V. Section 5.1 provides the experimental results and discussion for monotonic load test. The failure surfaces which were established in this study and a comparison of the test results with Lade's constitutive model are presented in section 5.2.

Chapter VI presents the experimental results and analysis for the cyclic load tests.

Finally in Chapter VII, this study is summarized and further researches are recommended.
CHAPTER II

BACKGROUND AND LITERATURE SURVEY

The material presented in this chapter serves as background and reference material for later chapters. The fundamental concepts of elasto-plasticity theory are reviewed briefly in section 2.1. Section 2.2 provides the application of the elasto-plasticity theory to the constitutive modeling of soils. In section 2.3, a brief review of the experimental techniques often used in constitutive modeling of soil, with emphasis on the hollow cylinder apparatus and the stress and strain states in a hollow cylinder specimen, is presented.

2.1 Brief Review of Plasticity Theory

The history of plasticity theory as a science began in 1864. At that time, Tresca published his results dealing with the flow of metals under high pressure and formulated his famous yield criterion. A few years later, Saint-Venant and M. Levy established some of the foundations of the modern theory of plasticity using Tresca's results. Subsequently, the development of the theory of plasticity proceeded slowly, although important contributions were made by Von Mises, Hencky, Prandtl, and others. A unified theory began to form only since approximately 1945. Since that time, concentrated efforts by many researchers (e.g. Hill, Nadai, Prager and Drucker) have produced a voluminous literature which is growing at a rapid rate.\[19,21,11]\]

The development of soil plasticity, as the new field was called, was strongly influenced by the well-established theory of metal plasticity. The theory of metal plasticity began to be applied soil around 1952.\[19]\] Its fundamental concepts were formed based on physical observations from the simple uniaxial test on soil. With the help of geometrical
considerations, all these observations obtained from the simple unidimensional states, have been generalized to the combined state of stress in a tensorial form.\textsuperscript{[21]}

In this section a general formulation of elasto-plasticity for soils will be summarized. Plasticity theory requires consideration of three basic ingredients:\textsuperscript{[19,64,8]} (a) the yield criterion, (b) the flow rule and (c) the hardening rule. A brief description of these three basic concepts is given as follows.

2.1-1 Fundamental Concepts of Plasticity

A typical stress-strain relation with loading, unloading and reloading components obtained from a uniaxial test of most materials such as metal and soil, is shown in Fig.2.1-1. The axial components of stress (\(\sigma\)) and strain (\(\varepsilon\)) measured during such a test are both assumed to be uniformly distributed within the specimen. It is essential that the most significant characteristics of the complex nonlinear and inelastic stress-strain should be expressed in an analytical relation for constitutive modeling. On the other hand, less important features of the stress-strain behavior may at first be ignored in such analytical formulations for the sake of simplicity and effectiveness. A schematic illustration of the stress-strain behavior is shown in Fig.2.1-2 and is described by Bardet\textsuperscript{[11]} and Dafalias\textsuperscript{[21,22,23]} with the following remarks:

1. The response is reversible around the origin; it may be represented by a linear elastic, isotropic model as a first approximation.
2. A threshold for reversibility is reached when the stress exceeds a yield stress or an elastic limit. The yield stress is defined as the highest stress that can be reached before any permanent or unrecoverable deformation remains after removing the stress (Fig.2.1-2).
3. Following yielding, more apparent nonlinear effects manifest themselves, as the stress-strain curve bends towards the strain axis.
4. During unloading and reloading, the response is parallel to the initial response about the origin (if the hysteretic phenomenon is ignored).
5. The total strain $\varepsilon$ is the sum of an unrecoverable plastic strain $\varepsilon^p$ and a recoverable elastic strain $\varepsilon^e$, determined by generalized Hooke's law.

$$\varepsilon = \varepsilon^e + \varepsilon^p \quad (2.1-1)$$

6. The stress-strain relationship during a reloading phase is reversible until the current stress exceeds the previous yielding stress determined by the highest value taken by the stress state during the loading history. When this occurs, the material can sustain more load, or in other words, it "hardens". This phenomenon is known as "strain or work hardening". This remark also applies for successive unloading-reloading processes.

7. The material exhibits some memory of its previous loading history. When the material is reloaded to stresses greater than the past maximum stress, the stress-strain curve is essentially the same as if there never had been any loading.

8. The material fails when the stress state reaches a final failure stress. The failure stress is the highest possible stress that material can carry. It differs from the yield stress, which is the threshold for irreversible strain.

If one considers an infinitesimal increment of stress, $d\sigma$ (single component), as shown in Fig.2.1-2, the above remarks are also valid. For a stress $\sigma$, the total strain increment, $d\varepsilon$, corresponding to a stress increment, $d\sigma$, can be expressed as:

$$d\varepsilon = \begin{cases} 
\begin{align*}
\varepsilon^e \quad &\text{if} \quad \sigma < \sigma_y \\
\varepsilon^e + \varepsilon^p \quad &\text{if} \quad \sigma = \sigma_y \text{ or } \sigma = \sigma_y \text{ and } d\sigma < 0
\end{align*}
\end{cases}$$

$$d\varepsilon = \begin{cases} 
\begin{align*}
\varepsilon^e \quad &\text{if} \quad \sigma < \sigma_y \\
\varepsilon^e + \varepsilon^p \quad &\text{if} \quad \sigma = \sigma_y \text{ or } d\sigma \geq 0
\end{align*}
\end{cases} \quad (2.1-2)$$

where, $\sigma_y$ represents the yield stress, $d\varepsilon^p$ is the plastic strain increment and $d\varepsilon^e$ is the elastic strain increment.

In contrast with the strain or work hardening behavior of most materials (illustrated in Fig.2.1-1), soils, under some circumstances, may respond with decrease of its ability to sustain load. The phenomenon is known as "strain-softening" and shown in Fig.2.1-3. Note that, in this case, when the stress exceeds a certain value, the behavior is different from the response in the unloading shown in Fig.2.1-2. One may note that both strain and
stress increments are negative during the unloading, while during strain-softening these increments have opposite signs. If one uses $S$ to represent the slope of the stress-strain curve at stress $\sigma$, the sign of $\frac{1}{S} d\sigma$ can be used to distinguish the two different processes. During loading without or with strain-softening, $\frac{1}{S} d\sigma$ is positive, but for unloading $\frac{1}{S} d\sigma$ is negative.$^{[11,12,21]}$

In order to define the general stress and strain states, all the fundamental concepts which have been developed from the simple one dimensional state of stress and strain, must be generalized for all possible states of stress and strain, using either a strain-space or stress-space formulation. Usually, a stress space formulation is widely used to define the stress and strain states. With assistance of Euclidean geometrical considerations, all of the generalizations to the stress space can be conducted in Cartesian tensorial form.$^{[11,19,21,64]}

The stress state is given by the Cauchy stress tensor $\tilde{\sigma}$. The stress tensor $\tilde{\sigma}$ has six independent Cartesian components, $\sigma_{ij}$ $(i, j = 1, 2, 3)$. The strains, corresponding to $\varepsilon$, $\varepsilon^e$ and $\varepsilon^p$ in the unidimensional state, are represented by the total strain state $\tilde{\varepsilon}$ with components $\varepsilon_{ij}$ $(i, j = 1, 2, 3)$, the elastic strain $\tilde{\varepsilon}^e$ with components $\varepsilon^e_{ij}$ $(i, j = 1, 2, 3)$ and the plastic strain state $\tilde{\varepsilon}^p$ with components $\varepsilon^p_{ij}$ $(i, j = 1, 2, 3)$. In addition, the total strains are decomposed into elastic and plastic components by simple addition as in the single component case;

$$\tilde{\varepsilon} = \varepsilon^e + \varepsilon^p$$ (2.1-3a)

or in component form
\[ \varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p \quad (i, j = 1, 2, 3) \]  

(2.1-3b)

Similarly the increment of total strain is given by

\[ \ddot{\varepsilon} = \ddot{\varepsilon}^e + \ddot{\varepsilon}^p \]  

(2.1-4a)

or in component form

\[ \ddot{\varepsilon}_{ij} = \ddot{\varepsilon}_{ij}^e + \ddot{\varepsilon}_{ij}^p \quad (i, j = 1, 2, 3) \]  

(2.1-4b)

The increment of total strain, \( \ddot{\varepsilon} \), can be fully defined by its Euclidean norm and direction in stress space. The norm, \( \| \ddot{\varepsilon} \| \), can be defined as follows:\[11,19,21,64\]

\[ \| \ddot{\varepsilon} \| = \left[ \ddot{\varepsilon}_{11}^2 + \ddot{\varepsilon}_{22}^2 + \ddot{\varepsilon}_{33}^2 + 2\ddot{\varepsilon}_{12}^2 + 2\ddot{\varepsilon}_{13}^2 + 2\ddot{\varepsilon}_{23}^2 \right]^{1/2} \]  

(2.1-5a)

One can also define the norm, \( \| \ddot{\varepsilon} \| \), in Einstein's implied sum notation, such that

\[ \| \ddot{\varepsilon} \| = \left( \ddot{\varepsilon}_{ij} \ddot{\varepsilon}_{ij} \right)^{1/2} \quad \text{(sum on i, j = 1, 2, 3)} \]  

(2.1-5b)

The direction of total strain increment, \( \ddot{\varepsilon} \), is defined by the unit vector \( \mathbf{m} \) with the same direction as in \( \ddot{\varepsilon} \) and the following components:\[11,21,64\]

\[ m_{ij} = \frac{\ddot{\varepsilon}_{ij}}{\| \ddot{\varepsilon} \|} \quad (i, j = 1, 2, 3) \]  

(2.1-6)

The definitions of Eqs.(2.1-5) and (2.1-6) can be applied to any increments, such as, plastic and elastic strain increments \( \ddot{\varepsilon}^p \) and \( \ddot{\varepsilon}^e \).
The components $\tilde{e}_e$ and $\tilde{e}_p$ need to be determined in order to define the total strain increment, $\tilde{e}$, resulting from an increment of stress $\tilde{\sigma}$ at a stress state $\tilde{\sigma}$. The increment of elastic strain, $\tilde{e}_e$, is related to the increments in the stress components by using an elastic model, such as Hooke’s law for linear case. However, to fully define the plastic strain increment, $\tilde{e}_p$, three basic aspects are to be examined: a) the yield criterion, b) the flow rule and c) the hardening rules.[19,64,11,21,43] Accordingly, a brief description of these three points is presented as follows.

2.1-2 Yield Criterion

There must exist a yield surface in stress space such that, if the material is subjected to change in stress represented by points inside that surface, the material will deform elastically, whereas if the changes stress state lies on the yield surface, the material will yield plastically. The yield surface expands as the material is loaded to successively higher stress levels, and, at failure, coincides with the failure surface. In general, the yield condition of a material can be derived by loading specimens with different combinations of stress states.[19,21,64]

In the most general case, a yield criterion may be expressed mathematically by the following yield function $f$:

$$f = f(\sigma_{ij}, k)$$ (2.1-7)

where $k$ is a function of the stress-strain histories. Once the yield surface has been defined, the presence or absence of unrecoverable strain at a certain stress state, can be characterized by the “yield criterion”.[11,19,21]
If one introduces the loading path of the material by

\[ \dot{f} = \frac{\partial f}{\partial \sigma_{ij}} \sigma_{ij} \]  

(2.1-8)

then a precise definition of loading and unloading entering the yield criterion may be given by

\[ \varepsilon_{ij}^p = \begin{cases} 
\lambda \frac{\partial g}{\partial \sigma_{ij}} & \text{if } f = 0 \text{ and } \dot{f} > 0 \text{ (loading)} \\
0 & \text{if } f \leq 0 \text{ and } \dot{f} < 0 \text{ (unloading)} 
\end{cases} \]  

(2.1-9)

Plastic strain will occur only when \( \dot{f} \) is positive and \( f = 0 \). During unloading, as well as \( \dot{f} < 0 \), the material will behave elastically. In Eq.(2.1-9), \( \varepsilon_{ij}^p \) is the plastic strain rate and \( g \) is the plastic potential function.

Another general mathematical description of the yield surface for isotropic material\(^{19,21,90}\) can also be given by,

\[ f(I_1, J_2, J_3, H) = 0 \]  

(2.1-10)

or

\[ f(\sigma_{ij}, H) = 0 \]  

(2.1-11)

where \( I_1 = \sigma_{kk} \) is the first invariant of the stress tensor, \( J_2 = \frac{1}{2} S_{ij} S_{ij} \) is the second invariant of the deviator stress tensor and \( J_3 = \frac{1}{3} S_{ij} S_{jk} S_{ki} \) is the third invariant of the deviator stress tensor. The deviator stress tensor, \( S_{ij} \), is defined by
\[ S_{ij} = \sigma_{ij} - \frac{1}{3} I_1 \delta_{ij} \]  

(2.1-12)

where \( \delta_{ij} \) is the Kronecker delta defined as

\[ \delta_{ij} = \begin{cases} 
0 & \text{if } i \neq j \\
1 & \text{if } i = j 
\end{cases} \]  

(2.1-13)

Finally, \( H \) represents the hardening parameter which can be taken as the plastic work, i.e.,

\[ H = W_p = \int \sigma_{ij} \, d\varepsilon_{ij}^p \]  

(2.1-14)

It is convenient and customary to describe the yield criterion in the three principal stress space\(^{[64,19,8]} \). As shown in Fig.2.1-4a, in the principal stress space, there is a line, ON, that forms equal angles with all three coordinate axes. The line, ON is called as the hydrostatic axis, because any point on this line corresponds to a hydrostatic or isotropic stress state for which \( \sigma_1 = \sigma_2 = \sigma_3 \). The unit vector \( \mathbf{n} \) along the hydrostatic axis can be defined as

\[ n = \frac{1}{\sqrt{3}} \{ 1, 1, 1 \} \]  

(2.1-15)

Furthermore, any plane perpendicular to the hydrostatic axis is called a deviator plane (see Fig.2.1-5a). One of the features of the deviator plane is that the first stress invariant, \( I_1 \), or mean stress, \( p \), is kept constant, if the stress path moves in a given deviator plane. The deviator plane which passes through the origin \( (\sigma_1 + \sigma_2 + \sigma_3 = 0) \) is called the \( \pi \)-plane.

Consider an arbitrary state of stress at a given point with stress components \( \sigma_1, \sigma_2, \) and \( \sigma_3 \). This state of stress is represented by point \( P \) in the principal stress space
in Fig. 2.1-4a, corresponding to a stress vector \( \mathbf{OP} = \{ \sigma_1, \sigma_2, \sigma_3 \} \) The stress vector can be decomposed into two components,

\[
\mathbf{OP} = \mathbf{ON} + \mathbf{NP}
\]  \( (2.1-16) \)

where the vector \( \mathbf{NP} \) lies in the deviator plane passing through \( \mathbf{N} \) and \( \mathbf{P} \), and the vector \( \mathbf{ON} \) is in the direction of the unit vector \( \mathbf{n} \). Thus, the norm of \( \mathbf{ON} \) can be defined as

\[
|\mathbf{ON}| = \mathbf{OP} \cdot \mathbf{n} = \frac{1}{\sqrt{3}} (\sigma_1 + \sigma_2 + \sigma_3) = \sqrt{3} \ p
\]  \( (2.1-17) \)

in which

\[
p = \frac{1}{3} I_1 = \frac{1}{3} (\sigma_1 + \sigma_2 + \sigma_3) = \text{mean stress}
\]  \( (2.1-18) \)

Notice that the vector \( \mathbf{ON} \) relates to the isotropic consolidated stress state and has components \( \mathbf{ON} = \{ p, p, p \} \) along the three reference axes. The components of \( \mathbf{NP} \) represent the deviator stress (Eq.2.1-12) of the state of stress represented by a point \( \mathbf{P} \) in Fig.2.1-4. The vector \( \mathbf{NP} \) is given by:

\[
\mathbf{NP} = \mathbf{OP} - \mathbf{ON} = \{ s_1, s_2, s_3 \}
\]  \( (2.1-19) \)

in which \( s_1 = \sigma_1 - p = \text{principal values of the deviator stress tensor related to} \ \tilde{\sigma} \). The length of the vector \( \mathbf{NP} \) can be obtained by:

\[
R = |\mathbf{NP}| = \left( s_1^2 + s_2^2 + s_3^2 \right)^{\frac{1}{2}} = \sqrt{2J_2}
\]  \( (2.1-20) \)

where \( J_2 \) = the second invariant of stress tensor
As shown in Fig.2.1-4b, the coordinate axes $\sigma_1$, $\sigma_2$, and $\sigma_3$ are projected onto the deviator plane through $N$, so that the principal stress axes appear at angles of $2\pi/3$ to each other. The projections of $P^{[64,19]}$ onto the axes are obtained by

$$PH_1 = \sqrt{\frac{3}{2}} s_1$$  \hspace{1cm} (2.1-21)

As shown in Fig.2.1-4b, in order to define the stress state in the deviator plane, a parameter, $\theta$ needs to be introduced. The polar angle, $\theta$ is measured from the pure shear axis corresponding to $\sigma_2 = \left( \sigma_1 + \sigma_3 \right)/2$. From Eq.2.1-20 and Eq.2.1-21, the following expression can be derived$^{[64,19]}$

$$\sin\theta = \sqrt{\frac{3}{2}} \frac{s_2}{\sqrt{J_2}} \quad \frac{-\pi}{6} \leq \theta \leq \frac{\pi}{6}$$  \hspace{1cm} (2.1-22)

in which, the angle $\theta$ lies in the range of $\frac{-\pi}{6} \leq \theta \leq \frac{\pi}{6}$. Also from trigonometric identity:

$$\sin 3\theta = 3\sin\theta - 4\sin^3\theta$$

the following expression can be obtained

$$\sin 3\theta = \frac{3\sqrt{3}}{2} \frac{J_3}{J_2^{3/2}}$$  \hspace{1cm} (2.1-23)

Finally, the three principal stresses can be derived as$^{[64,19]}$

$$\sigma_1 = p + \frac{2}{\sqrt{3}} \sqrt{J_2} \sin(\theta + \frac{2\pi}{3})$$

$$\sigma_2 = p + \frac{2}{\sqrt{3}} \sqrt{J_2} \sin\theta$$  \hspace{1cm} (2.1-24)

$$\sigma_3 = p + \frac{2}{\sqrt{3}} \sqrt{J_2} \sin(\theta - \frac{2\pi}{3})$$
in which

\[ \sigma_1 \geq \sigma_2 \geq \sigma_3 \]

As shown in Eq.2.1-24, any stress state can be fully defined by \( p \), which relates to the isotropic consolidated state, \( J_2 \) (or \( R \)) and \( \theta \) which are used to define the position of the current stress state in the deviator plane corresponding to the isotropic consolidated state.

It is useful to introduce the concept of stress path to describe the successive states of stressing a material specimen. Stress paths can also be presented in various stress spaces.\(^{[64,19,8,48]}\) Several representations of stress paths in different stress spaces are shown in Fig. 2.1-5, including the principal stress space (Fig.2.1-5a), the triaxial stress space with \( \sigma_2 = \sigma_3 \) (Fig.2.1-5b), \( I_1 - J_2 \) stress space (where \( J_2 \) is the second deviator stress invariant and \( I_1 \) is the first stress invariant) and the deviator plane, \( \sigma_1 + \sigma_2 + \sigma_3 = \text{constant} \). The name of the stress paths and their abbreviations were used in this study are listed in Table 2.1-1.

2.1-3 Flow Rule

In the theory of plasticity, the flow rule is used to relate the direction of plastic strain increment to the current stress state \( (\sigma_1, \sigma_2, \sigma_3) \). During plastic flow, the total incremental strains, \( de_{ij} \), are composed of an elastic component, \( de_{ij}^e \) and a plastic, \( de_{ij}^p \) component,\(^{[19,8,90]}\) i.e.:

\[ de_{ij} = de_{ij}^e + de_{ij}^p \quad (i, j = 1, 2, 3) \]

(2.1-4b)
Although any other isotropic or anisotropic nonlinear elastic model may be used as well, the elastic strain increment is related to the stress component by the generalized Hooke's law. Thus, one may write the elastic strain increment, $\text{d}e_{ij}^e$, in terms of the stress increment, $\text{d}\sigma_{kl}$, as:

$$\text{d}e_{ij}^e = C_{ijkl} \text{d}\sigma_{kl}$$  \hspace{1cm} (2.1-25)

where $C_{ijkl} = \text{the compliance tensor, which is a function of the stress tensor} \ \sigma_{ij}$. For isotropic elastic materials, the strain incremental tensor has the form

$$\text{d}e_{ij}^e = \frac{1}{9K} dI_1 \delta_{ij} + \frac{1}{2G} dS_{ij}$$  \hspace{1cm} (2.1-26)

where

$G = \text{the elastic shear modulus}$

$K = \text{the elastic bulk modulus}$

$\delta_{ij} = \text{the Kronecker delta}$

and $I_1 = \sigma_{kk}$ is the first invariant of the stress tensor and $S_{ij}$ is the deviator stress given by

$$S_{ij} = \sigma_{ij} - \frac{1}{3} I_1 \delta_{ij}$$  \hspace{1cm} (2.1-27)

As described in Section 2.1-2, now that the loading criterion $f(\sigma_{ij}, k) = 0$ has been introduced and the elastic-strain increment tensor, $\text{d}e_{ij}^e$, has been specified, the plastic-strain increment tensor, $\text{d}e_{ij}^p$, needs to be determined.

One may note that there is not a obvious and necessary connection between yield function, $f$, and the plastic-strain increment vector, $\text{d}e_{ij}^p$. In general, one may introduce the
plastic-potential function, \( g(\sigma_{ij}) \), which enables to write the equations of the plastic flow in the form

\[
de_{ij}^p = d\lambda \frac{\partial g}{\partial \sigma_{ij}}
\]  

(2.1-28)

in which, \( d\lambda \) is a positive scalar factor of proportionality, which is dependent on the particular form of the loading function and is nonzero only when plastic deformation occurs. The equation, \( g(\sigma_{ij}) = \text{constant} \), defines the surface (hypersurface) of the plastic potential in the stress space. As shown in Fig.2.1-6, the incremental plastic strain, \( \tilde{\epsilon}^p \) is collinear to \( m \) which is the unit vector on plastic potential surfaces. It can be expressed as follows\(^{[11,19,21]} \):

\[
de_{ij}^p = \left\| \tilde{\epsilon}^p \right\| m_{ij}
\]  

(2.1-29a)

or

\[
de_{\epsilon}^p = \left\| \tilde{\epsilon}^p \right\| m
\]  

(2.1-29b)

in which, \( \left\| \tilde{\epsilon}^p \right\| \), amplitude of \( \tilde{\epsilon}^p \), is a positive scalar. Equation (2.1-28) implies, therefore, that the plastic flow vector \( \tilde{\epsilon}^p \) is directed along the normal to the surface of plastic potential.

Of great importance is the simplest case at which the yield function and plastic potential function coincide, \( f = g \). Thus,

\[
de_{ij}^p = d\lambda \frac{\partial f}{\partial \sigma_{ij}}
\]  

(2.1-30)
i.e., the plastic increment is along the normal to the yield surface $\frac{\partial f}{\partial \sigma_{ij}}$ (Fig. 2.1-7). Since the plastic potential function is connected (or associated) with the yield criterion, the relation expressed in Eq. (2.1-30) is called the associated flow rule. By considering yield surfaces of more complex form, the associated flow rule makes possible various generalizations of the plasticity equations.$^{[11,19,21,64,44]}

If one considers flow in the space of principal stresses and principal strains, in place of (Eq. 2.1-28), the following relation can be used

$$\text{d}e_{ii}^p = \lambda \frac{\partial g}{\partial \sigma_{ii}}$$

(2.1-31)

The relation in Eq. (2.1-28) with $f \ast g$ is called the non-associated flow rule.$^{[19]}

2.1-4 Hardening Rule

Finally, a strain or work-hardening rule is required to define the evolution of the yield surfaces. Naturally, depending on the assumption regarding the shape of the yield surfaces, the plastic flow rule and hardening rule, the above formulation can lead to a variety of different models.$^{[19,44,64,11]}$ In general, some of the work-hardening rules employed for soil constitutive modeling are the following:$^{[64,19,44]}

a. Isotropic Hardening Rule: It assumes that, during plastic flow, the initial yield surface expands or contracts uniformly without changing in shape as shown in Fig. 2.1-8(a).

b. Kinematic Hardening Rule: It assumes that, during plastic flow, the yield surface merely translates in the stress space without changing its size or shape
(Fig.2.1-8(b)). This type of hardening rule is particularly useful for modeling cyclic loading.

c. Combined Isotropic and Kinematic Rule: It assumes that the yield surface expands (or contracts) and translates (Fig.2.1-8(c)).

d. Combined Anisotropic and Kinematic Rule: It is the most general of all hardening rules, assuming that the yield surface translates, rotates, and changes in size and shape (Fig.2.1-8(d)) and therefore provides for more flexibility in describing the hardening behavior of the material.

As presented in the above, the plastic behavior of an ideal elastic-plastic material is specified by a yield surface, a flow rule and a hardening rule. A yield surface separates states of stress, which cause only elastic strains from states of stress which cause both plastic and elastic strains. A flow rule relates the direction of the vector of plastic strain increment to the yield surface. A hardening rule relates the magnitude of a plastic strain to the magnitude of an increment of stress.\textsuperscript{[19,11,21,44,64,90]}

In the following, a brief review of elastic-plastic constitutive modeling of soils and experimental technique for constitutive modeling is presented.

2.2 Elastic-Plastic Constitutive Modeling of Soils

An impressive number of sophisticated and promising models for the constitutive modeling of soil has been developed recently. Although verification of such models can be achieved only in the laboratory, the generality, the versatility and the potential for further development in some of them is very encouraging. These models generally fall into one of the following basic categories:\textsuperscript{[44,19,64]}

Deformation Theory of Plasticity
Incremental Theory with Work-Hardening Plasticity

Perfect and Incremental Plasticity

Endochronic Theory

Among the great variety of models, those that are qualified to represent soil behavior under cyclic loading fall in the broad categories of the incremental plasticity and the endochronic theory.\cite{44} The incremental plasticity is widely used by many researchers.\cite{44,64,48,43,11,23,24,25}

It is obviously impossible to review all the models that have been proposed to represent the constitutive behavior of soils. A brief review of two very important models for soils developed in the framework of incremental plasticity is presented in the following.

2.2.1 Single-Hardening Stress-Strain Model (Lade and Kim):

An elasto-plastic constitutive model for frictional materials under monotonic loading was developed by Lade and Duncan.\cite{46} This model has been frequently used to predict the behavior of frictional materials, such as sand, clay, concrete and rock. There is a general agreement that this model performs well, whenever applied within the range for which it was developed.\cite{19,11,64,44}

The model has proceeded through several stages of development. The most recent version presented by Lade and Kim includes a single yield surface and uses a non-associated flow rule. The model is briefly reviewed by Lade and Kim as follows.\cite{43,48,51,52,44}
**Failure Criterion:**

As shown in Fig. 2.2-1, the failure criterion is defined in terms of the first and third stress invariant of the stress tensor, $I_1$ and $I_3$:

$$
\left( \frac{I_1^3}{I_3} - 27 \right) \left( \frac{I_1}{p_a} \right)^m = \eta_1
$$

(2.2-1)

where $I_1$ and $I_3$ are given by

$$
I_1 = \sigma_{\text{s}1} = \sigma_x + \sigma_y + \sigma_z
$$

(2.2-2)

$$
I_3 = \sigma_{\text{s}ij} = \sigma_x \sigma_y \sigma_z + \tau_{xy} \tau_{yz} \tau_{zx} + \tau_{yx} \tau_{zy} \tau_{xz} - \left( \sigma_x \tau_{yz} \tau_{zy} + \sigma_y \tau_{zx} \tau_{xz} + \sigma_z \tau_{xy} \tau_{yx} \right)
$$

and

$p_a = \text{the atmospheric pressure}$

$\eta_1$ and $m$ are the failure parameters and represent the material properties.

The values of $\eta_1$ and $m$ in Eq. (2.2-1) can be determined from triaxial compression tests.

The failure surface is shown in Fig. 2.2-1. Like an asymmetric bullet in triaxial and deviator planes, it points at the origin of the stress axes. The shape of these cross sections is controlled by the two parameters of $m$ and $\eta_1$, and does not change with the value of $I_1$ when $m = 0$. The cross-sectional shape of failure surface changes from triangular to become more circular with increasing value of $I_1$ for $m > 0$. Notice that the failure surface is always concave towards the hydrostatic axis, and its curvature increases with the value $m$. These give Lade’s model in a fashion that confirms experimental evidence.

**Flow Rule:**

A non-associated flow rule, i.e., the plastic potential function is different from the yield function, is used in this model.$^{[48,51,52,43]}$ The plastic strain increments are defined by the flow rule:
\[ \text{de}^p_{ij} = d\lambda_p \frac{\partial g_p}{\partial \sigma_{ij}} \]  

(2.2-3)

where

\( d\lambda_p \) = a scalar factor of proportionality

\( g_p \) = a plastic potential function

A new plastic potential function, \( g_p \), is developed independently of the yield function in terms of the three invariant of the stress tensor. The plastic potential function is formulated through the observation of the directions of the plastic strain increments;\(^{[43,48]}\)

\[ g_p = \left[ \frac{I_1^3 I_3^3}{I_2^2} + \frac{I_1}{P_0} \right] \]  

(2.2-4)

where

\( I_2 \) = the second stress invariant.

\( \Psi_2 \) and \( \mu \) are dimensionless constants that may be determined from triaxial compression tests. The parameter \( \Psi_2 \) controls the intersection with the hydrostatic axis, and \( \mu \) determines the curvature of meridians. The parameter \( \Psi_1 \) is related to the curvature parameter \( m \) of the failure criterion as follows:

\[ \Psi_1 = 0.00155 \, m^{1.27} \]  

(2.2-5)

The parameter \( \Psi_1 \) acts as a weighting factor between the triangular shape (from the \( I_3 \) term) and the circular shape (from \( I_2 \) term).

The plastic potential surfaces are shown in Fig.2.2-2. In principal stress space, the plastic potential surfaces look like asymmetric cigars with smoothly rounded triangular cross sections. One may note that the cross sections in plastic potential surfaces are similar but not identical to those for the failure surfaces shown in Fig.2.2-3.
In Eq.(2.2-3), the derivatives of $g_p$ with regard to the stresses are obtained as follows.

$$\frac{\partial g_p}{\partial \sigma_{ij}} = \left( \frac{I_1}{p_a} \right) \mu \left\{ \begin{array}{l}
G - (\sigma_y + \sigma_z)(I_1/I_2) - \Psi_1(\sigma_y\sigma_z - \tau_{yz})(I_1/I_3) \\
G - (\sigma_z + \sigma_x)(I_1/I_2) - \Psi_1(\sigma_x\sigma_z - \tau_{zx})(I_1/I_3) \\
G - (\sigma_x + \sigma_y)(I_1/I_2) - \Psi_1(\sigma_x\sigma_y - \tau_{xy})(I_1/I_3) \\
2(I_1/I_2)^2\tau_{yz} - 2\Psi_1(\tau_{xy}\tau_{zx} - \sigma_x\tau_{yz})(I_1/I_3) \\
2(I_1/I_2)^2\tau_{zx} - 2\Psi_1(\tau_{xy}\tau_{yz} - \sigma_y\tau_{zx})(I_1/I_3) \\
2(I_1/I_2)^2\tau_{yz} - 2\Psi_1(\tau_{xy}\tau_{zx} - \sigma_z\tau_{xy})(I_1/I_3) \end{array} \right\}$$

(2.2-6)

in which

$$G = \Psi_1 (\mu + 3) \frac{I_1}{I_3} - (\mu + 2) \frac{I_1}{I_2} + \frac{\mu}{I_1} \Psi_2$$

(2.2-7)

These derivatives can be used to obtain the plastic strain increment from Eq.(2.2-3).

**Yield Criterion and Hardening Rule:**

In the model, a single yield surface is used to define the boundary between the stress states where both elastic and plastic deformations take place and those where only elastic deformations occur\(^{[43,51,52]}\). The total strain increments, $\varepsilon_{ij}$, are divided into an elastic component, $\varepsilon_{ij}^e$, and a plastic component, $\varepsilon_{ij}^p$, such that

$$\varepsilon_{ij} = \varepsilon_{ij}^e + \varepsilon_{ij}^p \quad (i, j = 1, 2, 3)$$

(2.2-8)

As discussed earlier, these strain components are calculated separately. More specifically, the elastic strain increments, recoverable upon unloading, are determined
by Hooke's Law, using the concept of the nonlinear variation of Young's modulus with stress state. A stress-strain theory which involves a yield surface shaped as an asymmetric tear-drop with the pointed apex at the origin of the principal stress space (Fig. 2.2-3) is introduced to derive the plastic strain components.

In the model, the value of Poisson's ratio, $\nu$, is assumed to be constant and limited between zero and 0.5 for most materials. The Young's modulus is expressed in terms of a power law involving nondimensional material constants and functions as follows:\cite{48, 52}

$$E = M p_a \left[ \frac{I_1}{p_a} + R \frac{J_2'}{p_a} \right]^\lambda$$

(2.2-9)

in which $R$ is defined by

$$R = 6 \frac{1 + \nu}{1 - 2\nu}$$

(2.2-10)

where

$M$ = the modulus number

$\lambda$ = the constant dimensionless number

$p_a$ = the atmospheric pressure

and

$J_2'$ = the second invariant of the deviator stress tensor

In this model, a work-hardening/softening rule is proposed to describe the evolution of the yield surfaces.\cite{48, 43, 51, 52} The yield surfaces are intimately associated with and derived from surfaces of constant plastic work. The isotropic yield function is described as follows:

$$f_p = f_p^*(\sigma) - f_p^*(W_p) = 0$$

(2.2-11)

in which
\[ f'_p = \left[ \psi_1 \left( \frac{I_1^3}{I_3} - \frac{I_2}{I_3} \right) \right] \left[ \frac{\dot{I}_1}{p_a} \right]^h e^q \]  

(2.2-12)

and for hardening:

\[ f_p^* = \left[ \frac{1}{D} \right]^{1/p} \left[ \frac{W_p}{p_a} \right]^{1/p} \]  

(2.2-13)

in which

\[ I_1, I_2 \text{ and } I_3 = \text{the first, second and third stress invariant} \]

For a given material, the value of \( D \) and \( \rho \) are constant in Eq.(2.2-13). Thus, \( f_p^* \) changes with the plastic work only. For a given material, the parameter \( h \) is also constant and determined on the basis that the plastic work is constant along a yield surface. As in the expression for the plastic potential (Eq.2.2-4), the parameter \( \Psi_1 \) acts as a weighting factor between the circular shape (from the \( I_2 \) term) and the triangular shape (from the \( I_3 \) term). The parameter \( q \) varies with stress level \( S \) from zero at the hydrostatic axis to unity at the failure surface according to the following expression:

\[ q = \frac{\alpha S}{1 - (1 - \alpha) S} \]  

(2.2-14)

where \( \alpha \) is constant. In Eqs.(2.2-13) and (2.2-14), \( D \) is a function of \( C, \Psi_1 \) and \( \rho \), and the value of \( \alpha \) depends on \( p \) and \( h \).

The yield surfaces are shown in Fig.2.2-3. They are shaped as asymmetric teardrops with smoothly rounded triangular cross-sections. As the plastic work increases, the isotropic yield surface inflates until the current stress point reaches the failure surface. The relation between \( f_p \) and \( W_p \) is described by a monotonically increasing function whose slope decreases with increasing plastic work, as shown in Fig.2.2-4.

For softening, in order to describe isotropic deflation of yield surfaces, a exponential decay function is introduced such as:
\[ f_p^* = A e^{-B \left( \frac{W_p}{p_a} \right)} \] (2.2-15)

As shown in Fig.2.2-4, in terms of the slope of the hardening curve at the point of peak failure, the two positive constants A and B can be determined.

Based on the expression for the plastic potential in Eq.(2.2-4), the relation between the proportionality constant \( d\lambda_p \) and plastic work increment in Eq.(2.2-3) may be described as:

\[ d\lambda_p = \frac{dW_p}{\mu g_p} \] (2.2-16)

in which, by differentiation of the hardening and softening equations, the increment of plastic work can be obtained.

For hardening, Eq.(2.2-13) produces:

\[ dW_p = D p_{a,p} f_p^{p-1} df_p \] (2.2-17)

and for softening Eq.(2.2-13) yields:

\[ dW_p = -\left[ \frac{1}{B} \right] p_a f_p^{p-1} df_p \] (2.2-18)

where \( df_p \) is negative during softening.

Finally, the expression for the increment plastic strain increments in Eq.(2.2-6) can be obtained by combining Eqs. (2.2-17) and (2.2-18) with Eq.(2.2-16) and substituting this and Eqs.(2.2-9) into Eq.(2.2-6).

A comparison of Lade's failure criterion with the classic failure criterion is shown in Fig.2.2-5. One may note that Lade's model contains the essential features from the classical Mohr-Coulomb strength criterion, which distinguishes this scheme from many other elasto-plastic work-hardening models for soils.\(^{[44]}\)
The elastic-plastic model with single-hardening yield surface reflects quite well many important aspects of soil behavior encountered in laboratory investigations, such as stress path dependency, shear dilatancy, influence of the intermediate principal stress, and influence of mean effective stress levels. However, the model may not provide adequate predictions for situations where stress reversals or large changes in the stress path under loading or unloading take place. Inherent or induced anisotropy may not be handled by this model.\(^{19,44}\) To obtain the total strain increments, a total of eleven soil parameters are required (Table 2.2-1). As one may also note that this model is relatively complicated.

### 2.2.2 Bounding Surface Model (Dafalias and Popov, and Bardet):

The concept of bounding surface was originally introduced by Dafalias and Popov\(^{25}\) and Dafalias\(^{21}\) for constitutive modeling of metals. Later the model was adopted for modeling of cyclic behavior of soils by Mroz et al., Dafalias et al.,\(^{22,23,24}\) Anandarajah et al.,\(^{4}\) Hashiguchi and Ueno,\(^{34}\) Aboim and Roth,\(^{11}\) and Bardet.\(^{11,12}\)

The name and the concept of bounding surface were motivated by the observation that the stress-strain curves converge with specific “bounds” at a rate which depends on the distance of the stress point from the bounds.\(^{21,19,11,23,12}\) The bounding surface is conceived as a bounding envelop of the enclosed yield surface. In this model, a combined isotropic and kinematic rule, is proposed, in which the yield surface is allowed to translate, expand or contract within the bounding surface, while the latter can expand or contract isotropically.

The new concept introduced by Dafalias and Popov brings new features to conventional formulation. To some extent, this last adaptation can be perceived as a generalization of conventional plasticity material behavior. The new concepts are first
defined from the uniaxial test, and then generalized to the six-dimensional stress state space. The concepts of boundary plasticity are reviewed by Bardet,$^{[11,12]}$ Dafalias,$^{[21]}$ and Chen$^{[19]}$ as follows:

Fig.2.2-6 plots a typical stress-strain curve (such as metal, soil, or other materials) including a loading-unloading cycle in a uniaxial test. If one removes the elastic part of strain by transforming the $\sigma$-$\varepsilon$ space into the corresponding $\sigma$-$\varepsilon^p$ space (Fig. 2.2-7), it brings a new and different point of view for the stress-strain relation.

The response is linearly elastic, if the uniaxial applied stresses less than $\sigma_y$. If the material is loaded beyond a stress $\sigma_y$, the stress-strain curve approaches asymptotically and merges with the bound represented by the straight line $XX'$. As shown in Fig.2.2-7, one can define a function of $\delta$ which is the distance between the stress state and bound $XX'$ to describe the slope of the response curve at any point. Notice that the function decreases continuously and monotonically along the $\sigma$-$\varepsilon^p$ curve, from the infinite value it assumes at the initiation of yielding to absolutely smaller values as the curve approaches the bound. The transition between the elastic and elastic-plastic range becomes continuous. As shown in Fig.2.2-7, a point $A'$ on the bounding line $XX'$ is called the "image point". It moves to a new position, $B'$ during the plastic loading from $A$ to $B$. Notice that the following expression can be produced:

$$\frac{1}{S_A}d\sigma = \frac{1}{S_{A'}}d\sigma'$$  \hspace{1cm} (2.2-19)

in which

$S_A =$ the slope of stress-strain curve at $A$

$S_{A'} =$ the slope of bounding line $XX'$ at $A'$

and

$d\sigma$ and $d\sigma'$ = the infinitesimal changes of stress state at current and image points, respectively.
The image point may also be selected on the other bounding line YY’ shown in Fig. 2.2-7 during unloading. One may extend all the ideas defined during the previous loading to describe the unloading response.

Based on the bounding surface ideas from uniaxial test, a straightforward generalization of the bounding surface to six-dimensional stress space have been given by Dafalias and Popov.\textsuperscript{[11,12]} As shown in Fig. 2.2-7, based on the same geometrical considerations as in conventional plasticity, the yield stress $\sigma_y$ becomes a yield surface; similarly, the bounds XX’ and YY’ transform into a hypersurface called the “bounding surface”. The yield surface will translate towards the bounding surface along $\overrightarrow{PR}$ in Fig. 2.2-8. The two surfaces may come in contact with each other, but they do not intersect; this corresponds to the merging of stress-strain curve with the bounds in the uniaxial case. The state of the material is defined in terms of the stress $\sigma_{ij}$ and plastic internal variables $q_n$ accounting for the past loading history.

In Fig. 2.2-8, the bounding surface is described by

$$F(\overline{\sigma}_{ij}, q_n) = 0$$ \hspace{1cm} (2.2-20)

in which a bar over stress quantities indicates points on bounding surface, $\overrightarrow{F}=0$. The $q_n$ are usually scalar or second-order tensor quantities such as the plastic work, the plastic strains, etc.

In this model, the actual stress point $\sigma_{ij}$ lies always within or on the bounding surface. For any given $\sigma_{ij}$, a unique “image” point $\overline{\sigma}_{ij}$ on $\overrightarrow{F}=0$ is defined according to a specific rule which is part of the constitutive relations. In order to specify the plastic modulus at the actual stress state in terms of a bounding plastic modulus at the image stress states, the measure of the distance, $\delta$, between the actual and image stress points is used. The distance $\delta$ is provided by

$$\delta = [\overline{\sigma}_{ij} - \sigma_{ij})(\overline{\sigma}_{ij} - \sigma_{ij})]^{1/2}$$ \hspace{1cm} (2.2-21)
As in conventional plasticity, once these generalizations have been performed, the plastic strain increment can be defined by specifying successively its existence, direction and amplitude.

It is important to note that, with the bounding surface plasticity, two new fundamental ideas have been brought into the conventional plasticity theory such as: a) the response depend on the distance between the current stress state and these bounds. Recall that the material behavior is only described by quantities related to the stress state and its past loading history (such as the yield surface) in conventional plasticity. b) the response of the material also depends on some exterior bounds corresponding to the maximum admissible stress states.\[11,19,21,22,23]\n
As stated earlier, the verification of the constitutive models can be achieved only in the laboratory. Thus, the development of experimental techniques for constitutive modeling becomes more and more important in nowadays.

In the following, the experimental techniques for constitutive modeling are briefly reviewed.

2.3 Experimental Techniques for Constitutive Modeling

Saada and Townsend\[71\] in their state-of-the-art report, present a comprehensive review and evaluation of the advantages and limitations of traditional and new equipment for laboratory testing of soil strength. The equipment examined includes simple shear, triaxial, multiaxial, plane strain, hollow cylinder triaxial and directional shear devices. The reviewers recognized the value of the simple shear test for comparative studies, but concluded that this test has serious shortcomings due to stress non-uniformities and, therefore, should not be used to derive soil parameters for constitutive modeling. Regarding the popular standard triaxial test, they pointed out its versatility and simplicity,
provided that lubricated platens are used to avoid end effects and that membrane penetration during undrained testing of granular soil accounted for.

The multiaxial (or true triaxial) test allows the study of more general loading conditions on cubical soil elements. Sture and Desai\cite{80} summarized the advantages and disadvantages of the various multiaxial devices designed with rigid, flexible or mixed boundaries. For the device with rigid boundaries, strains are uniform and can be measured accurately, while control in complicated strain paths is performed easily. However, it is difficult to verify stress uniformity, and general test operation is usually complicated. Flexible boundaries assure uniform normal stress distributions and allow complicated stress paths to be achieved in stress control mode. Problems may occur, however, from the boundary interface and strain non-uniformities. Also, pore pressure measurements and test operations are usually complicated. The versatility allowed by the multiaxial cubical devices and by the conceptually similar plane strain and directional shear devices, makes them quite attractive for research in constitutive modeling of soil behavior.

In the following, a more detailed description of the triaxial testing on hollow cylinder specimens is given and its advantages and limitations for soil testing are evaluated.

2.3-1 Hollow Cylinder Testing

The application of hollow cylinder testing of soil started about fifty years ago and since then, numerous investigators have used it to study the behavior of soil, including Saada et al,\cite{69,70,71} Lade,\cite{47,48} Ishibashi et al,\cite{37,38} Ishihara et al,\cite{39,40} Hight, Gens and Symes\cite{36} and Symes, Gens and Hight.\cite{79}

The popularity attached to hollow cylinder triaxial testing stems from the fact that it allows control of the magnitude and direction of the principal stress axes in a continuous
manner and almost at will. Independent variation of the intermediate principal stress is also possible by using different internal and external confining pressures (resulting however to undesirable stress and strain non-uniformities across the wall of the cylindrical specimen). The most recent version of the hollow cylinder device developed by Hight, Gens and Symes\textsuperscript{[36]} at the Imperial College of London is a highly refined tool designed to ensure quality data. Fig.2.3-1 illustrates the stress state applied on the hollow cylinder and the definitions of average stresses and strains. Apparently, the measured average values of stresses and strains are meaningful only if the actual distributions are approximately uniform. In the following, some important aspects of nonuniformity are briefly addressed.

In hollow cylinder torsional testing, the need to apply torsion on the specimen requires either a rough platen surface or a smooth surface with fins. Apparently, both methods have serious shortcomings. Restriction of the radial displacements at the two ends of the specimen results in undesirable shear stresses $\tau_{zr}$ (and $\tau_{rz}$) and in circumferential stresses, $\sigma_\theta$, which decay with distance from the ends. In addition, undesirable bending moments are induced affecting the distributions of vertical stresses, $\sigma_z$. Also, rotations of the principal stresses occur out of the plane of cylinder wall.\textsuperscript{[36]} Saada and Townsend,\textsuperscript{[70,71]} in an effort to minimize the end effect by appropriate selection of the hollow cylinder geometry, used an elastic solution for a thin hollow cylinder to compute approximately the length over which end effects are important. They suggested that the cylinder should have a central zone free from end effects with a length at least equal to the length of the zone influenced by the platen. Accordingly, they proposed that the total length, $H$, of the hollow cylinder should be approximately

$$H \geq 5.44 \sqrt{r_0^2 - r_1^2}$$

(2.3-1)
in which \( r_0 \) and \( r_1 \) are the external and internal diameters respectively. Also, to reduce non-uniformity across the thickness of the wall, they suggested that \( \frac{r_1}{r_0} \geq 0.65 \).

The most recent comprehensive study of the stress non-uniformities within a hollow cylinder was conducted by Hight, Gens and Symes\(^{36}\) and Symes, Gens and Hight\(^{78}\). Linear elastic and non-linear elasto-plastic finite element analyses were carried out to investigate the influence of specimen geometry and material properties on the stress uniformity. All analyses demonstrated that the end effect is only local for the distribution of the radial stress, \( \sigma_r \). In contrast, specimen height, load and pressure combinations, Poisson's ratio and shear strength proved to have an important influence on the distribution and nonuniformity of the vertical and circumferential normal stresses, \( \sigma_z \) and \( \sigma_\theta \), respectively.

In addition to the non-uniformity caused by the boundary restraints, more non-uniformities occur due to curvature of the cylinder. Indeed, the shear stresses \( \tau_{\theta z} \) caused by the applied torque vary linearly across the thickness of the cylinder wall and may cause non-uniform distribution of \( \sigma_z \), \( \sigma_r \) and \( \sigma_\theta \) across the wall. To minimize such non-uniformities it is desirable to reduce the thickness of the wall and increase the mean radius \( r_m = \frac{(r_1 + r_0)}{2} \). Linear elastic and nonlinear finite element analysis, in which the material behavior was described by the Cam-clay model and strain hardening plasticity, showed that for radii ratio \( \frac{r_1}{r_0} \) varying from 0.7 to 1, the variations of \( \sigma_z \), \( \sigma_r \) and \( \sigma_\theta \) are small.\(^{36,70,71}\)

The above findings are used in this work to select the optimum specimen geometry by considering as criteria of the stress uniformity, easy specimen preparation, and minimum possible size.
2.3.2 Stress and Strain States in Hollow Cylinder Specimens

The average stresses and strains applied on a soil element on the wall of a hollow cylinder specimen is presented in Fig. 2.3-1. If stresses are assumed to be uniformly distributed in the radial direction across the wall of the sample, it can be proved that the stress in the circumferential direction is equal to the cell pressure when equal pressures are applied both in the inner and outer cells of the hollow cylinder sample. Therefore, although the horizontal stress, $\sigma_\theta$, can not be measured directly, its value may be taken to be equal to the applied cell pressure.\(^{[70,71,48,38,40]}\) If the shear stress, $\tau_{Z\theta}$, is applied to the sample along with other components of stress, the principal stresses in the plane of the cylindrical wall are given by

\[
\sigma_1 = \frac{\sigma_z + \sigma_\theta}{2} + \sqrt{\left[\frac{\sigma_z - \sigma_\theta}{2}\right]^2 + \tau_{Z\theta}^2}
\]

\[
\sigma_3 = \frac{\sigma_z + \sigma_\theta}{2} - \sqrt{\left[\frac{\sigma_z - \sigma_\theta}{2}\right]^2 + \tau_{Z\theta}^2}
\]

(2.3-2)

where $\sigma_1$ and $\sigma_3$ are major and minor principal stresses. The angle between the major principal stress direction and the vertical, $\beta$, is given by

\[
\tan 2\beta = \frac{2\tau_{Z\theta}}{\sigma_z - \sigma_\theta}
\]

(2.3-3)

Therefore, the difference of the major and minor principal stresses is given by

\[
\frac{\sigma_1 - \sigma_3}{2} = \sqrt{\left[\frac{\sigma_z - \sigma_\theta}{2}\right]^2 + \tau_{Z\theta}^2}
\]

(2.3-4)
Similarly, it is possible to define the principal strain components as follows:

\[
\varepsilon_1 = \frac{\varepsilon_Z + \varepsilon_\theta}{2} + \frac{1}{2} \sqrt{\left[\varepsilon_Z - \varepsilon_\theta\right]^2 + \gamma_{Z\theta}^2}
\]

\[
\varepsilon_3 = \frac{\varepsilon_Z + \varepsilon_\theta}{2} - \frac{1}{2} \sqrt{\left[\varepsilon_Z - \varepsilon_\theta\right]^2 + \gamma_{Z\theta}^2}
\]

where \( \varepsilon_Z \) and \( \varepsilon_\theta \) denote vertical and horizontal strains, respectively and \( \gamma_{Z\theta} \) is the shear strain in the torsional mode of deformation. The strains \( \varepsilon_1 \) and \( \varepsilon_3 \) are, respectively, the major and minor principal strains.

One can also use the octahedral normal effective stress, \( \sigma_{\text{oct}} \) and the octahedral shear stress, \( \tau_{\text{oct}} \) to define the stress states.\(^8\) Note that one can assume \( \tau_\theta (= \tau_{r\theta}) \) and \( \tau_z (= \tau_{rz}) \) are zero in the soil element. Therefore, the octahedral normal effective stress, \( \sigma_{\text{oct}} \), and the octahedral shear stress, \( \tau_{\text{oct}} \), are defined by

\[
\sigma_{\text{oct}} = \frac{1}{3} \left( \sigma_Z + \sigma_r + \sigma_\theta \right)
\]

\[
\tau_{\text{oct}} = \frac{1}{3} \sqrt{\left[\sigma_Z - \sigma_r\right]^2 + \left[\sigma_r - \sigma_\theta\right]^2 + \left[\sigma_\theta - \sigma_Z\right]^2 + 6\tau_{Z\theta}^2}
\]

or, in terms of principal stress

\[
\sigma_{\text{oct}} = \frac{1}{3} \left( \sigma_1 + \sigma_2 + \sigma_3 \right)
\]

\[
\tau_{\text{oct}} = \frac{1}{3} \sqrt{\left[\sigma_1 - \sigma_2\right]^2 + \left[\sigma_2 - \sigma_3\right]^2 + \left[\sigma_3 - \sigma_1\right]^2}
\]
Accordingly, it is also possible to define the octahedral normal strain, $\varepsilon_{oct}$, and the octahedral shear strain, $\gamma_{oct}$, where

$$
\varepsilon_{oct} = \frac{1}{3} (\varepsilon_z + \varepsilon_r + \varepsilon_\theta)
$$

(2.3-8)

$$
\gamma_{oct} = \frac{2}{3} \sqrt{\left[\varepsilon_z - \varepsilon_r\right]^2 + \left[\varepsilon_r - \varepsilon_\theta\right]^2 + \left[\varepsilon_\theta - \varepsilon_z\right]^2 + \frac{2}{3} \gamma_{z\theta}^2}
$$

If the axes are rotated so that the faces of the element are principal planes, Eq.(2.3-8) becomes

$$
\varepsilon_{oct} = \frac{1}{3} (\varepsilon_1 + \varepsilon_2 + \varepsilon_3)
$$

(2.3-9)

$$
\gamma_{oct} = \frac{2}{3} \sqrt{\left[\varepsilon_1 - \varepsilon_2\right]^2 + \left[\varepsilon_2 - \varepsilon_3\right]^2 + \left[\varepsilon_3 - \varepsilon_1\right]^2}
$$

The volumetric strain, $\varepsilon_V$ is defined as

$$
\varepsilon_V = -\frac{\Delta V}{V} = \varepsilon_z + \varepsilon_r + \varepsilon_\theta
$$

(2.3-10)

where $\Delta V$ is a small increment in a volume $V$. In the theory of elasticity, it can be shown that $\varepsilon_r = \varepsilon_\theta$ in the soil element of hollow cylinder sample.\textsuperscript{70,71} In the plasticity, the relationship of $\varepsilon_r = \varepsilon_\theta$ is approximately correct. In the case of an undrained test, all three components $\varepsilon_Z$, $\varepsilon_r$, $\varepsilon_\theta$ are known because $\varepsilon_z + \varepsilon_r + \varepsilon_\theta = 0$. In the case of a drained test, the volumetric strain can be measured using the volume change measurement device. If one assumes that the sample preserves the shape of a hollow cylinder and, if the initial across-section area of the unstrained sample is $A_0$, its initial volume $V_0$ and its height $H_0$, the current area $A$ of the specimen is given by\textsuperscript{8,53}
where $\Delta V$ and $\Delta H$ are the current changes of the volume and height of the sample, $\varepsilon_V$ and $\varepsilon_Z$ are the current volumetric and axial strains.
Fig. 2.1-1  Typical Stress-Strain Curve in Uniaxial Test

Fig. 2.1-2  Idealized Stress-Strain Curve in Uniaxial Test for Development of Plasticity Theory (after Bardet, 1983)
Fig. 2.1-3 Typical Stress-Strain Curve in Uniaxial Test on a Strain-Softening Soil
Fig. 2.1-4  Principal Stress Space and Deviatoric Stress Plane
(after Prevost, 1987)
\[ \text{ON} = \frac{I_1}{\sqrt{3}} = \sqrt{\frac{3}{3}} \sigma_{oct} \]
\[ \text{NP} = \sqrt{2J_2} = \sqrt{\frac{3}{3}} \tau_{oct} \]

Current State \((\sigma_1, \sigma_2, \sigma_3)\)

Hydrostatic Stress Axis \((\sigma_1 = \sigma_2 = \sigma_3)\)

Initial State \((\sigma_c \sigma_c \sigma_c)\)

Deviatoric Plane \(I_1 = \sigma_1 + \sigma_2 + \sigma_3 = \text{constant}\)

Triaxial Plane \(\sigma_2 = \sigma_3\)

a. Principal Stress Space

\[ \sqrt{2\sigma_2} = \sqrt{2}\sigma_3 \]

b. Triaxial Stress Plane

c. \(I_1 - \sqrt{J_2}\) Plane

d. Deviator Plane

Fig. 2.1-5 Stress Paths in Different Stress Spaces
Fig. 2.1-6 Yield Surface and Plastic Potential Surface in Stress Space
Fig. 2.1-7 Pictorial Representation of Yield and Potential Surfaces with Associated Flow Rule
Fig. 2.1-8 Comparison of Yield Surface Motion According to Different Hardening Rules
Fig. 2.2-1 Characteristics of Lade's Model Failure Surfaces Shown in Principal Stress Space: Traces of Failure Surfaces Shown in (a) Triaxial Plane; and (b) Deviator Plane (Lade, 1977)
Fig. 2.2-2 (a) Schematic Illustration of Plastic Potential Surface in Principal Stress Space and (b) Contours in Triaxial Plane (Lade and Kim, 1988)
Fig. 2.2-3  (a) Schematic Illustration of Yield Surface in Principal Stress Space and (b) Contours in Triaxial Plane (Lade and Kim, 1988)
Fig. 2.2-4 Modeling of Work Hardening and Softening in Lade's Model
(Lade and Kim, 1988)
Fig. 2.2-5 Projection of Lade's Failure Criterion in the Deviator Plane
Fig. 2.2-6  A Typical Material Response in $\sigma$ - $\varepsilon$ Space
(Bardet, 1983)

Fig. 2.2-7  Idealized Material Response (in $\sigma$ - $\varepsilon^p$ space)
for Developing Bounding Surface Plasticity
(Bardet, 1983)
Fig. 2.2-8  Yield and Bounding Surfaces in Stress Space
(Chen, 1985)
\[
\begin{align*}
\sigma_z &= \frac{F}{\pi (r_o^2 - r_1^2)} + p \\
\sigma_\theta &= p \\
\sigma_r &= p \\
\tau_{\theta z} &= \frac{3M}{2\pi (r_o^3 - r_1^3)} \\
\varepsilon_z &= \frac{u_z}{H} \\
\varepsilon_\theta &= -\frac{u_{\text{ro}} - u_{\text{r1}}}{r_o + r_1} \\
\varepsilon_r &= -\frac{u_{\text{ro}} + u_{\text{r1}}}{r_o + r_1} \\
\end{align*}
\]
where \(u_z, u_r, u_\theta\) are displacements in the \(z, r\) and \(\theta\) directions.

Fig. 2.3-1 Average Stresses and Strains Applied on a Soil Element on the Wall of a Hollow Cylinder Specimen.
<table>
<thead>
<tr>
<th>Stress Path</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isotropic Compression</td>
<td>IC</td>
</tr>
<tr>
<td>Triaxial Compression</td>
<td>TC</td>
</tr>
<tr>
<td>Triaxial Extension</td>
<td>TE</td>
</tr>
<tr>
<td>Torsional Simple Shear</td>
<td>SS</td>
</tr>
<tr>
<td>Compression and Torsion</td>
<td>CT</td>
</tr>
<tr>
<td>Combination of TC and SS</td>
<td>TC, SS</td>
</tr>
<tr>
<td>Model Component</td>
<td>Parameter</td>
</tr>
<tr>
<td>--------------------------</td>
<td>--------------------------</td>
</tr>
<tr>
<td>Elastic Behavior</td>
<td>Modulus Number, $M$</td>
</tr>
<tr>
<td></td>
<td>Exponent, $\lambda$</td>
</tr>
<tr>
<td></td>
<td>Poisson's Ratio, $\nu$</td>
</tr>
<tr>
<td>Failure Criterion</td>
<td>Intercept, $\eta_1$</td>
</tr>
<tr>
<td></td>
<td>Exponent, $m$</td>
</tr>
<tr>
<td>Plastic Potential</td>
<td>Intercept, $\Phi_1$</td>
</tr>
<tr>
<td></td>
<td>Exponent, $\mu$</td>
</tr>
<tr>
<td>Yield Criterion</td>
<td>Exponent, $h$</td>
</tr>
<tr>
<td></td>
<td>Constant, $\alpha$</td>
</tr>
<tr>
<td>Hardening Function</td>
<td>Intercept, $C$</td>
</tr>
<tr>
<td></td>
<td>Exponent, $p$</td>
</tr>
</tbody>
</table>
CHAPTER III

EXPERIMENTAL DEVICES, TEST SPECIMENS AND COMPUTER CONTROL PROGRAMS

This chapter deals with the details of the experimental devices and test specimens. Section 3.1 describes the experimental devices used in this study. Section 3.2 discusses the set-up of test specimens. It includes the selection of sand, specimen geometry and specimen preparation. The computer control programs developed during this study are briefly described in Section 3.3.

3.1 Experimental Devices

An entirely new state-of-the-art soil triaxial test system for both solid and hollow cylinder tests has been designed and set-up during this research. The computer-automated experimental system for both solid and hollow cylinder testing soils is capable of both stress and strain controlled tests. It consists of an MTS axial/torsional servo-hydraulic frame, two MTS electronic control units, a Hewlett-Packard 9000/360 workstation, a HP-3852A data acquisition and control unit, a soil triaxial testing cell with two pressure transducers, a pressure control panel and a volume change measurement device, etc. The details of the system are provided by a block diagram and a photo in Figs.3.1-1 & 3.1-2. The key features of the test system and experimental devices are summarized as the follows.

MTS Frame and Control Units

The axial/torsional servo-hydraulic MTS frame has the capacity of 5.5 kips in axial load and 2500 in-lbs in torque. The frame is connected with the hydraulic pressure pump.
Four transducer conditioners were used as a part of a closed loop to control independently both axial and torsional actuators, using either load or displacement control. Two feedback selection modules were provided to make the possibility of changing from load to displacement control mode (or vise-versa) during a test either in axial or torsional directions.

**Hewlett-Packard 9000/360 Workstation**

The Hewlett-Packard 9000/360 workstation can operate in UNIX system. It has 4 megabytes random access memory, 152 megabytes mass storage and speed about 4-5 million instructions per second. The computer monitor is an ultra-high resolution 16 in color monitor for real time plotting of several graphics on the same screen. The UNIX operating system can also offer useful windows and communication capabilities.

**HP-3852A Data Acquisition and Control Unit**

The HP-3852 data acquisition and control unit consists of a HP-44702A high speed voltmeter (maximum speed is 100,000 measurements per second), a HP-44711A 24-channel fet multiplexer for data acquisition and 2-channel voltage output for control of the servovalves of the axial and torsional actuators at any desired loading history used in this study. To perform cyclic axial/torsional triaxial tests, the HP-44726A 2-channel arbitrary waveform DAC accessory supplies the sine waveforms which with defined amplitudes, frequencies, phases and offsets to the axial/torsional actuators.

**Soil Triaxial Cell**

A soil triaxial testing loading cell (Fig.3.1-3 and Fig.3.1-4) both for solid and for hollow cylinder specimens has been successfully designed and set-up during this study. The cell is made of clear cast acrylic tubing. It is 19 inches (48.2 cm) high, 13 inches
(33.0 cm) in inner diameter and with wall thickness in 1.2 (3 cm) inches. The exteriors of top plate and base plate annulus are grooved for O-rings and provide a seal for the pressure chamber. The cell is equipped with appropriate instrumentation. It allows easier installation and specimen preparation compared to other designs found in the literatures. [35,36,70,71]

**Specimen Pedestal and Cap**

In order to ensure the torque transmission for hollow cylinder specimen, the porous stones are used both for the specimen pedestal and cap (Figs. 3.1-3 and 3.1-4). The lower section of the pedestal and upper section of the cap are used to seal the inner membrane with O-rings so that no special inner specimen clamps are necessary. [35,36,71]

**Loading Piston and Bearing House**

A Teflon coated stainless steel rod with a 1.5 inches (3.8 cm) diameter is used as a vertical and torque loading piston. The interior of the spacer in the bearing house is grooved for a rubber Teflon O-ring. The function of the rubber Teflon O-ring is to seal for the pressure chamber and reduce the friction between the piston and the air bushing. The calibration tests shown that the friction is less than one per cent at any stress level measurement both for the axial and torsional directions. The test results are corrected for any piston friction effects.

**Piston Locker**

A piston locker is necessary to avoid any disturbance of the specimen when lowering the piston rod, tightening the connecting rod and connecting the torque and axial loading system to the piston. During the test, the piston locker is completely loosened from its air
bushing and tightened on the piston rod in order to eliminate the friction between the piston locker and piston rod. \cite{35,70,71}

**Elimination of Electrical Noise**

In order to eliminate the electrical noise, which is induced from the MTS control system and from radio waves, a set of custom-made electrical analog filters has been designed and made. The selection of analog filters, as opposed to digital filters, was made by considering the fact that the later requires about 10 times more data, and consequently if used, they would slow down the speed of the system during dynamic load testing. As shown in Fig.3.1-5, the results of the noise elimination turn out quite satisfactory.

**Pressure Control Panel and Volume Change Measurement**

A control panel for applying both cell and back pressures built with precision instrumentation and for operating in a range of 0-400 psi has been designed and set-up. The control panel can be used to apply both cell and back pressure simultaneously and keep the difference between cell and back pressure as a constant. The volume change measurement device is also used in drained test to measure the volumetric change of the soil sample (Fig. 3.1-6).

**Inner and Outer Specimen Molds**

The inner split specimen mold consists of four sections (Fig.3.1-8). A steel cylinder with its outer diameter same as the inner diameter of the split mold is used to keep the position of the inner mold. The bottom sections of the mold are engaged in the pedestal. The outer split specimen mold consists similarly of four sections. The mold cap and the pedestal of the specimen are used to keep the position of the outer mold. After the
specimen is formed and the vacuum is applied to the specimen, the inner mold is removed from the beneath of the base plate of the soil triaxial cell (Fig. 3.1-4).[35,36,70,71]

**Taping Cylinders for Solid and Hollow Cylinder Specimens**

In order to obtain a uniform specimen, both the solid and hollow cylinder specimens were prepared using fifteen layers of equal mass of sand. To make each sublayer, a known amount of moist sand is poured between the outer and inner specimen molds from a controlled height for hollow cylinder specimen (or is pored into the specimen mold for the solid specimen). A tamping cylinder (Fig.3.1-7 and Fig.3.1-8) is used to obtain the desired height. The position of the collar can be adjusted to create the desired depth between the shoulder of the outer specimen mold of the hollow cylinder specimen (or the shoulder of the specimen mold of solid cylinder specimen) and the specimen surface.[35,36,70,71]

**3.2 Set-up of Test Specimens**

This section describes the soil sample, specimen geometry and the specimen preparation.

**3.2-1 Selection of the Sands**

The sand used in this study is a very fine and commercially available sand presently sold by the U.S. Silica Co. and mined from Ottawa, Illinois under the trade name “F-125”. The manufacture product data sheet and the grain size distribution of the sand are presented in Fig.3.2-1 and Fig. 3.2-2.

This type of sand is a rather typical and has been widely used in previous studies of sand behavior.[8,52,53] This may provide some convenience for the comparison of the experimental results. The grain size of the sand is fine enough to eliminate or reduce to
practically negligible error levels caused by membrane penetration. Membrane penetration\textsuperscript{[33,35]} is the deformation of the membrane surrounding the soil specimen, caused by the high confining pressure and led to incorrect volumetric strain measurements. Research results here shown that the membrane penetration effect can be neglected for the average particle size of the soil is less than 0.1 mm.\textsuperscript{[33,35]} The particle shape was also another factor considered in the selection of the sand. The roundness of the chosen sand will let the soil particles break harder than the sharper surface shape of soil particles.\textsuperscript{[53]} The sands have been well re-sieved and remixed based on the mean values of the manufacture data sheet.

As shown in Table 3.2-1, the chosen sand has a specific gravity of 2.65. It consists of soil particles in size range of 0.053 to 0.30 mm. The mean diameter, $D_{50}$ and the uniformity coefficient, $C_u$, were 0.11 mm and 1.16 mm respectively. For such a fine sand the effects of membrane penetration are very small and can be neglected.\textsuperscript{[33,35]} The maximum void ratio, $e_{\text{max}}$, is 0.95 and the minimum void ratio, $e_{\text{min}}$, is 0.57. The maximum dry unit weight $\gamma_{d_{\text{max}}}$, and minimum dry unit weight $\gamma_{d_{\text{min}}}$ determined by ASTM D 4253 and 4254,\textsuperscript{[2,3]} were equal to 105 pcf and 85 pcf respectively.

3.2.2 Specimen Geometry

The solid cylinder specimens used in this study have a nominal diameter of 4 inches (10.16 cm) and height of 8 inches (20.32 cm). The hollow cylinder specimens have the nominal dimensions of an outer diameter of 5.5 inches (13.97 cm), an inner diameter of 4 inches (10.16 cm) and height of 8 inches (20.32 cm). The primary consideration of the specimen geometry is to minimize the effects of end restraint.\textsuperscript{[36,70]}
3.2-3 Specimen Preparation

The details of preparing the triaxial sand specimen are provided in Appendix A. In general, all specimens tested in this investigation were solid and hollow cylinder reconstituted sand specimens. Each specimen is made by using a split mold with the "moist tamping undercompaction" technique developed by Ladd,[45] and by performing the compaction in fifteen layers. This technique is chosen to ensure a high degree of uniformity both within each specimen and among all specimens. In this investigation, two different relative densities ($D_r$) of sand were used to build the specimens: loose sand with $D_r=30\%$ and dense sand with $D_r=75\%$. All specimens were made using 6% water content needed to facilitate the compaction process.[45,28,29,91] Each layer was compacted to the required density results in further compaction of the lower layers. However, in order to achieve uniform density throughout the specimen, only the top layer is compacted to the desired dry density while all other layers are compacted to some lower value. The degree of undercompaction is taken to vary linearly from the bottom layer to the top layer, being a maximum at the bottom layer and zero at the top layer.[45,28,29,91]

In order to assure a good contact between the specimen and the top porous stone, the top layer was only partially compacted. After scarifying the top surface to increase the surface roughness, the specimen cap with the porous stone was positioned. The final density (height) of specimens was achieved by hitting lightly the top of the loading piston. To avoid any disturbance of the specimens, the loading piston was locked into place after the proper specimen height was achieved. A vacuum of 3 psi was applied to keep the shape of the specimen, and then the molds (both inner and outer molds for hollow cylinder specimens or outer mold for solid cylinder specimens) were carefully removed. The
diameter of the specimens was measured at the bottom, middle and top with a "\(\pi\)" tape.\(^{[28,29,45,91]}\)

To prevent from the migration of air through the specimen membrane, the cell was filled with desired water. Then, a confining pressure, \(\sigma'_{30}\), of 3 psi, all around the specimen, was applied, while the applied vacuum is gradually removed. The specimen was then percolated upwards with \(\text{CO}_2\) for a period at least one hour. This was done to ensure that any air in the voids of the specimen was replaced by \(\text{CO}_2\), which dissolved much easier in the pore water added during saturation. As a result, 100% saturation can easily be achieved with the use of \(\text{CO}_2\).\(^{[28,29,91]}\)

Then desired water was flushed through the specimen. The system was back pressured overnight at an effective confining pressure of 3 psi and a total confining pressure of 33 psi. The specimen is assumed to be fully saturated and ready for testing if the pore pressure response (B parameter) is at least 0.95.\(^{[28,29,70,71]}\) The B parameter\(^{[8,53]}\) is defined as the ratio of the pore water pressure increment, \(\Delta u\), caused by an undrained isotropic total stress increment, \(\Delta \sigma\). In all performed tests the value of B was larger than 0.98.

The specimen was consolidated to the next day. All changes in volume and height were noted during the consolidation process. After the consolidation, the triaxial cell was transferred to the MTS testing frame.

### 3.3 Computer Control Programs

During this study, a set of efficient custom-made computer program for the control of the entire experimental system has been developed.\(^{[92]}\) The programs were written by HP technical BASIC. In general, the main program allows the generation of any arbitrary
loading functions applied independently in the axial and torsional frame actuator. For a given stress path, the testing can be performed by controlling the load or the displacement of each actuator, while changing from one mode to the other during the test is also possible. The test monitoring graphics both for monotonic and cyclic load tests are provided in Fig.3.3-1 and Fig.3.3-2 respectively. Some of the computer program for parameter study of constitutive modeling of soil has also been developed in this study.
Fig. 3.1.1 Overview of the Soil Triaxial Test System
Fig. 3.1-2  Block Diagram of the Soil Triaxial Test System
Fig. 3.1-3  Overview of the Triaxial Test Loading Cell of Solid Specimen
Fig. 3.1-4 Overview of the Triaxial Test Loading Cell of Hollow Cylinder Specimen
a. before use the Electronic Filters

b. after use the Electronic Filters

Fig. 3.1-5 Elimination Results of Electrical Noise
Fig. 3.1-7 Schematic Diagram of Taming Device for Solid Specimen
Fig. 3.1-8 Schematic Diagram of Tamping Device for Hollow Cylinder Specimen
Fig. 3.2-1 Grain Size Distribution of Ottawa Illinois Sand (F-125)
### Typical Size Analysis:

<table>
<thead>
<tr>
<th>U.S.A. Std. Sieve</th>
<th>Millimeter Designation</th>
<th>% Retained On Sieve</th>
<th>% Retained Cumulative</th>
<th>% Passing</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>.300</td>
<td>0.2</td>
<td>0-0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>70</td>
<td>.212</td>
<td>1.6</td>
<td>0.2-10</td>
<td>1.8</td>
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<td>100</td>
<td>.150</td>
<td>12.2</td>
<td>0.2-20</td>
<td>14.0</td>
</tr>
<tr>
<td>140</td>
<td>.106</td>
<td>38.2</td>
<td>0.2-49</td>
<td>52.2</td>
</tr>
<tr>
<td>200</td>
<td>.075</td>
<td>34.6</td>
<td>0.2-40</td>
<td>86.8</td>
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<tr>
<td>270</td>
<td>.053</td>
<td>10.0</td>
<td>0.2-16</td>
<td>96.8</td>
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<tr>
<td>Pan</td>
<td></td>
<td>3.2</td>
<td>0.3-7</td>
<td>100.0</td>
</tr>
</tbody>
</table>

### Typical Physical Properties:

- **Mineral**: Quartz  
- **Color**: White  
- **Grain Shape**: Rounded  
- **Sphericity (Krumbein)**: .6/.7  
- **Roundness (Krumbein)**: .6/.7  
- **Hardness (Moh)**: 7.0  
- **Specific Gravity**: 2.65  
- **Bulk Density — Compacted (lbs/ft³)**: 104  
- **Uncompacted (lbs/ft³)**: 87

- **Melting Point**: 3100°F  
- **Theoretical Surface Area (cm²/gm)**: 227  
- **Actual Surface Area (cm²/gm)**: 266  
- **Coefficient of Area**: 1.17  
- **Base Permeability**: .26  
- **Moisture Content Dry (Max.)**: <1  
- **Acid Demand (pH-7)**: <1  
- **AFS Grain Fineness (Range)**: 115-130  
- **pH**: Neutral

### Typical Chemical Analysis (Percent Reported as Oxide):

- **SiO₂ (Silicon Dioxide)**: 99.280  
- **TiO₂ (Titanium Dioxide)**: .145  
- **Fe₂O₃ (Iron Oxide)**: .095  
- **CaO (Calcium Oxide)**: <.01  
- **Al₂O₃ (Aluminum Oxide)**: .360  
- **MgO (Magnesium Oxide)**: <.01

LOI (Loss-on-Ignition): .15

---

**Fig. 3.2-2** Product Data Sheet of Ottawa Illinois Sand (F-125)
Fig. 3.3-1 A Typical Monitoring Graphics for Monotonic Load Test
Fig. 3.3-2 A Typic Monitoring Graphics for Cyclic Load Test
### Table 3.2-1

**Selected Properties of Ottawa, Illinois Sand**

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Specific Gravity, $G_s$</td>
<td>2.65</td>
</tr>
<tr>
<td>$D_{50}$(mm)</td>
<td>0.11</td>
</tr>
<tr>
<td>$C_c = \frac{D_{30}^2}{(D_{60}+D_{10})}$</td>
<td>1.11</td>
</tr>
<tr>
<td>$C_u = \frac{D_{60}}{D_{10}}$</td>
<td>1.66</td>
</tr>
<tr>
<td>$e_{max}$</td>
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</tr>
<tr>
<td>$e_{min}$</td>
<td>0.57</td>
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<tr>
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<td>105 pcf</td>
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<tr>
<td>$\gamma_{dmin}$</td>
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CHAPTER IV

EXPERIMENTAL PROGRAM AND CONTROL PROCEDURE

This chapter provides the details of the experimental program and control procedure. Section 4.1 describes the experimental program and control procedure for monotonic load tests. In section 4.2, the experimental program and control procedure for cyclic load tests will be discussed.

4.1 Monotonic Load Tests

A comprehensive experimental program under the category of monotonic load was conducted on the selected soil both for loose sand (relative density, $D_r = 30\%$) and dense sand ($D_r = 75\%$) in order to investigate the mechanical behavior under more general loading conditions. The stress paths of the experimental program both for loose and dense sand, and under monotonic loading are shown in Fig.4.1-1. The objective of the monotonic load tests is to establish the yield and failure surfaces and to understand the stress-strain characteristics for monotonic loading. Each test started at a given isotropically consolidated state (at which the stress path moves along with the hydrostatic axis); then the stress path moved within a chosen deviator plane (i.e., $I_1$ was kept as a constant) along a predetermined stress path as shown in Fig.4.1-1. Totally, 22 tests under monotonic loadings were performed. The experimental program under monotonic loading consists of the following types of tests.

a. Isotropic Compression (IC) Tests

Two isotropic compression tests on drained solid cylinder specimen prepared at two different relative densities ($D_r=30\%$ and $D_r=75\%$) were conducted. In order to simulate the isotropic compression condition, the tests were performed using a triaxial cell with a
light-weighted plastic specimen cap as shown in Fig. 4.1-2. Each test included some of the unloading and reloading in order to obtain the unloading Young's modulus. The stress path corresponding to the IC tests remains along the hydrostatic axis ($\sigma_1 = \sigma_2 = \sigma_3$), as shown in Fig. 2.1-5. In this path, the change in the stress invariant $I_1$ and $J_2$ is given by

$$\Delta I_1 = 3\Delta \sigma_c, \quad \Delta J_2 = 0$$

(4.1-1)

where $\Delta \sigma_c$ is the increment of hydrostatic (or mean) stress ($\Delta \sigma_1 = \Delta \sigma_2 = \Delta \sigma_3 = \Delta \sigma_c$).

The isotropic compression tests can provide the information on the volumetric behavior of soil. For example, the relationship between the specific volume and effective mean stress and the expressions for the bulk modulus $K$ can be determined.

b. Triaxial Compression (TC) / Extension (TE) Tests

Nine triaxial compression/extension tests on drained solid specimens (Fig. 4.1-3) prepared at two different relative densities and consolidated isotropically at various stress levels were performed. The tests included a unloading and a reloading. A triaxial compression tests on drained hollow cylinder specimens with relative density, $D_t=30\%$, was conducted also to evaluate the consistency between the results from solid and hollow cylinder specimens. In order to achieve good measurement accuracy and to obtain the information of the strain softening characteristics, each of this kind of test started by controlling the displacements in the torsional direction and the load control in the axial direction, but before the softening, continued by controlling the displacements in both actuators until the end of test. In each of these tests, the stress increments $\Delta \sigma_1, \Delta \sigma_2, \text{ and } \Delta \sigma_3$ are applied such that $I_1$ remains always constant (equal to the initial value $3\sigma_c$). In other words, the condition $\Delta \sigma_1 + \Delta \sigma_2 + \Delta \sigma_3 = 0$ is always satisfied. For the TC tests, this condition is satisfied as follows: $\sigma_1$ is increased by $\Delta \sigma_1$, whereas both
\( \sigma_2 \) and \( \sigma_3 \) are decreased such \( \Delta \sigma_2 = \Delta \sigma_3 = \frac{1}{2} \Delta \sigma_1 \). In the TE tests, \( \sigma_1 \) is decreased by \( \Delta \sigma_1 \), whereas both \( \sigma_2 \) and \( \sigma_3 \) are equally increased such \( \Delta \sigma_2 = \Delta \sigma_3 = \frac{1}{2} \Delta \sigma_1 \) (\( \Delta \sigma_1 \) is negative in this case). Notice also that, in TC tests, \( \sigma_1 \) is the major principal stress, and \( \sigma_2 \) and \( \sigma_3 \) are the intermediate and minor principal stresses (\( \sigma_2 = \sigma_3 \)). As has been demonstrated in Section 2.1-2, the parameter \( \theta \), (Eqs.2.1-22 and 2.1-23) can be used to determine the direction of the stress path in the deviator plane (Figs.2.1-4 and 4.1-1). Thus, corresponding to the TC stress path (\( \sigma_1 > \sigma_2 = \sigma_3 \)), Eq.2.1-22 gives \( \theta = -30^\circ \). For the TE stress path, in which \( \sigma_1 \) is the minor principal stress (\( \sigma_2 = \sigma_3 > \sigma_1 \)), Eq.2.1-22 gives \( \theta = 30^\circ \).

In this type of tests, the failure and deformation characteristics, and the Young's modulus at the used stress range can be evaluated.

c. Torsional Simple Shear (SS) Tests

Two torsional simple shear tests on drained hollow cylinder specimens (Fig.4.1-4) prepared at two different relative densities and consolidated isotropically at B stress level were performed (see test SS in Fig.4.1-1). Each of this type of test was conducted by controlling the load on the axial actuator and the displacement on the torsional actuator. As in the case of TC and TE tests, during the SS tests, the stress invariant \( I_1 \) is kept constant. In other words, the stress path for SS tests lies always in the chosen deviator plane, which corresponds to the initial state of isotropic consolidation. In the SS tests, one of the principal stresses, for example, \( \sigma_2 \), remains constant (\( \Delta \sigma_2 = 0 \)), while \( \sigma_1 \) and \( \sigma_3 \) are increased and decreased, respectively, by the same amount; that is, \( \Delta \sigma_3 = - \Delta \sigma_1 \), in
which, $\Delta \sigma_1 > 0$. As shown in Fig.4.1-1, for any stress state along the SS stress path, Eq.2.1-22 gives $\theta = 0^\circ$. The failure and deformation characteristics can be evaluated from this type of test.

d. Compression and Torsion (CT) Test

A test under the combination of both compressional and torsional load on drained hollow cylinder specimen (Fig.4.1-4) prepared at a relative density $D_r=30\%$ and isotropically consolidated to stress state B, was conducted. In the CT test, the stress increments $\Delta \sigma_1$, $\Delta \sigma_2$, and $\Delta \sigma_3$ are applied such that $I_1$ remains always constant (equal to the initial value $3\sigma_c$). To achieve this, the cell pressure and the vertical and torsional loads were adjusted simultaneously in a relation satisfying Eq.2.1-24. Thus, the stress path in the deviator plane moved to the failure surface along a direction following an angle ($\theta = -15^\circ$). Notice that, in the CT test, $\sigma_1$, $\sigma_2$ and $\sigma_3$ were all changed according to Eq.2.1-24. By considering the difficulty and the accuracy of the test control, both the axial and torsional actuators were under load control. Since the failure of the specimen happened very fast, the stress-strain softening behavior could not be completely obtained, even though the maximum feedback collection speed was used. However, the failure stress state could be obtained.

e. Combination of Triaxial Compression and Simple Shear (TC,SS) Tests

Four tests consisting of a sequence of triaxial compression and simple shear tests on hollow cylinder specimens prepared at two different relative densities and isotropically consolidated at B stress levels were performed under drained conditions (see tests TC,SS in Fig.4.1-1b). In each of this type of test, the stress path in the B deviator plane moved first along the stress path of TC to a predetermined stress state, and then parallel to the direction of the SS test until the failure surface reached. During this test, the axial actuator
was under load control and the torsional actuator was under the displacement control. In fact, the stress path of this type of tests is similar with the SS test which started at anisotropic consolidation state. This type of test was easier to control than the CT test described above. The failure and deformation characteristics under the loading conditions can be evaluated.

4.2 Cyclic Load Tests

A comprehensive experimental program under the category of cyclic load was conducted. The cyclic test program was restricted on the behavior of loose sand by using isotropically consolidated specimens at the stress state B.

In contrast with the monotonic program, the cyclic program was performed entirely under undrained conditions, during which no volumetric strains are allowed. During undrained testing, instead of measuring manually the volumetric strains at certain time intervals, the pore water pressures developed during loading are measured continuously with electronic instrumentation. This is necessary because it is impossible to have manual measurements during cyclic loading. It should be noted that the failure surface of a soil are the same regardless of the type of test (drained or undrained) but, of course, the corresponding effective stress paths will be quite different in drained and undrained conditions.[8,53]

All cyclic load tests were run at a frequency of 0.1 HZ. This low frequency was selected so that there was complete equalization of pore water pressures within the specimen. All specimens were initially isotropically consolidated to a confining stress ($\sigma^c$) is 300 KPa, where they had the same void ratio, e, equal to 0.819.
Eight successful cyclic load tests were conducted corresponding to six distinct types of loading. The tests were performed by changing the two components of shear stress, $\tau_{Z0}$ and the stress difference $(\sigma_Z - \sigma_\theta)/2$ either singly or in combination\cite{40,41}. The loading schemes of these tests are illustrated in the stress space shown in Fig. 4.2-1, in which the shear stress and the stress difference are represented in a rectangular coordinate system. Table 4.2-1 provides the abbreviations and notations for the cyclic load tests.

a. Cyclic Triaxial Shear (CTrS) Tests

The cyclic triaxial shear tests are performed on three solid specimens. In this type of test, only the stress difference $(\sigma_Z - \sigma_\theta)/2$, is cyclically applied to the specimen, while the other component of the shear stress, $\tau_{Z0}$, is kept equal to zero throughout the cyclic phase of loading. A sinusoidal axial stress of constant amplitude is superimposed on the initial stress state of a solid cylinder specimen along the direction of the major or minor principal stresses (Fig. 4.2-1a). Three types of cyclic triaxial shear tests were performed. In the first test, the stress difference, $(\sigma_Z - \sigma_\theta)/2$ was applied only in one direction from zero to a negative constant value, $\tau_{cy}$ (one-way cyclic triaxial extension shear test, Fig. 4.2-1a-1). In the second test, the stress difference, $(\sigma_Z - \sigma_\theta)/2$ was applied only in one direction from zero to a positive constant value, $\tau_{cy}$ (one-way cyclic triaxial compression shear test, Fig. 4.2-1a-2), and in the third test the stress difference was applied in both directions to the same constant value of $\tau_{cy}$ (two-way cyclic triaxial shear test, Fig. 4.2-1a-3).
The stress ratio of \( \frac{\tau_{cy}}{\sigma'_{mc}} \) chosen in this type of test equals to 0.15, in which

\[
\frac{\tau_{cy}}{\sigma'_{mc}}
\]

is the cyclic shear stress amplitude normalized by the effective mean principal stress at the consolidation state (Fig.4.2-2). For \( \sigma'_{mc} = 300 \text{ KPa} \), the cyclic shear stress amplitude, \( \tau_{cy} \), is 45 KPa.

During the test, the soil specimen was first consolidated isotropically to the stress state B and then subjected to cyclic changes in the vertical stress, \( \sigma_z \), under undrained conditions. During the cyclic loading, the axial actuator was under load control and the torsional actuator was under displacement control. In the one-way triaxial extension test (Fig.4.2-1a-1), the axis of the major principal stress is directed horizontally, while in the one-way triaxial compression test (Fig.4.2-1a-2), the axis of the major principal stress is directed vertically. However, in the two-way test (Fig.4.2-1a-3), the axis of the major principal stress in directed vertically in the course of increasing the vertical stress \( (\sigma_z > \sigma_\theta) \) and horizontally for decreasing the vertical stress \( (\sigma_z < \sigma_\theta) \). Therefore, the axis of the major principal stress is suddenly rotated by 90 degrees at the instance of stress difference reversal. The effects of a sudden principal stress rotation by 90 degrees should be evaluated by comparison of the results from the two types of tests.

b. Cyclic Torsion Shear (CToS) Tests

In this type of test, only the shear stress, \( \tau_{z\theta} \), is applied cyclically to a hollow cylindrical specimen. First, the test specimen was isotropically consolidated to B consolidated state and then subjected to the cyclic torsional shear stress under undrained conditions, while holding the vertical stress, \( \sigma_z \), unchanged. During the procedure of the
test set-up and the specimen consolidation, the axial actuator was under the load control and the torsional actuator was under the displacement control. Before applying the cyclic load to the test specimen, the torsional actuator was changed from the displacement control to the load control.

The same stress ratio of \( \frac{\tau_{cy}}{\sigma'_{mc}} \) was chosen in this type of test and equals to 0.15, corresponding to \( \tau_{cy} = 45 \) KPa.

As shown in Fig.4.2-1b, two types of the cyclic load test were performed: in one test, the torsional shear stress, \( \tau_{Z \theta} \), was applied only in one direction, from zero to a constant value, \( \tau_{cy} \) (one-way test, Fig.4.2-1b-1), and in the other test the shear stress was applied in both directions to the same constant value of \( \tau_{cy} \) (two-way test, Fig.4.2-1b-2). In the one-way test, the axis of the major principal stress is rotated to a direction of 45 degrees from the vertical towards the left in the course of increasing the shear stress, \( \tau_{Z \theta} \), towards the left. In the two-way test (Fig.4.2-1b-2), the axis of the major principal stress is rotated to a direction of 45 degrees from the vertical towards the left in the course of increasing the shear stress, \( \tau_{Z \theta} \), towards the left. However, when the shear stress, \( \tau_{Z \theta} \), is applied to the right direction, the axis of the major principal stress is oriented 45 degrees from the vertical towards the right. Hence, a sudden 90 degrees rotation in the axis of the major principal stress occurs each time the direction of shear stress is reversed during the cyclic loading. The difference of soil behavior caused by the sudden changes in the direction of major principal stress should be evaluated by the comparison of the two test results.
c. Cyclic Tests with Circular Rotation of the Principal Stress Axes (CRPS)

In this type of test, both the shear stress, $\tau_{26}$, and the stress difference, $\left(\sigma_z - \sigma_\theta\right)/2$, were changed so that the half deviator stress, $\left(\sigma_1 - \sigma_3\right)/2$, defined by Eq.(2.3-4) is held constant throughout the cyclic load test. In the test, a stress difference, $\left(\sigma_z - \sigma_\theta\right)/2$, was first applied from zero to a constant value, $\tau_{cy}$, to anisotropically consolidated test specimen under drained conditions (i.e., keeping $I_1$ constant), and then two predetermined sine waves (i.e., the cyclic vertical shear stress, $\left(\sigma_z - \sigma_\theta\right)/2 \cos(2\pi ft) = \tau_{cy}\cos(2\pi ft)$ and the cyclic torsional shear stress, $\tau_{26}\sin(2\pi ft) = \tau_{cy}\sin(2\pi ft)$) with same amplitude of $\tau_{cy}$, same frequency, $f$ and 90 degree difference in phase were independently applied to the specimen through both axial and torsional actuators under undrained conditions.

Three cyclic load tests of this type were conducted under the stress ratio $\frac{\tau_{cy}}{\sigma_{mc}}$, equal to 0.15, 0.10 and 0.06, in which $\frac{\tau_{cy}}{\sigma_{mc}}$ is the cyclic shear stress amplitude normalized by the effective mean principal stress at consolidation state. This results to cyclic shear stress amplitudes, $\tau_{cy}$, are 45 KPa, 30 KPa and 18 KPa, respectively.

As shown in Fig.4.2-1c, the shear stress, $\tau_{26}$, is plotted versus the stress difference, $\left(\sigma_z - \sigma_\theta\right)/2$, in a rectangular coordinate system. In the CRPS test, the stress path first moves from zero to point A during the initial application of the stress difference (under drained conditions). Then, the stress path turns counterclockwise to point B where the
torsional stress was increased to a maximum value, $\tau_{cy}$, with the stress difference, $(\sigma_Z - \sigma_0)/2$, vanishing. Then, the vertical stress, $\sigma_Z$, was reduced below the existing horizontal stress, $\sigma_0$, while decreasing the torsional shear stress. During this phase of loading, the stress point turns along the circle (shown in Fig.4.2-1c) and eventually reaches point C. As shown in the figure, loading from point C to point D involves an increase in the torsional stress, $\tau_{z0}$, in the opposite direction and decrease in the absolute value of the stress difference. A reduction in the torsional shear stress with a simultaneous increase in the stress difference constitutes the stress path from point D back to point A. One may note that in the course of a round excursion of the stress change as above along a circular stress path, the direction of the principal stress rotates continuously through an angle of 180 degrees according to Eq.(2.3-3), while the half deviator stress as defined by Eq.(2.3-4) is kept constant throughout the loading process.

In order to identify the effect of the intermediate principal stress (e.g., $\sigma_2$) on the behavior characteristics of soils, let's introduce a parameter $b$ to define the relative position of $\sigma_2$ with respect to $\sigma_1$ and $\sigma_3$ by the expression:[40,41]

$$b = \frac{\sigma_2 - \sigma_3}{\sigma_1 - \sigma_3} = \frac{1}{2}\left(1 - \frac{\sigma_Z - \sigma_0}{\sigma_1 - \sigma_3}\right)$$

(4.2-1)

In the CRPS test, because the intermediate stress, $\sigma_2 = \sigma_0$, remains unchanged, the magnitude of the major and minor principal stress changes relative to the intermediate principal stress during the cyclic loading. In other words, the $b$-value as defined by Eq.4.2-1 changes between 0 and 1.0 in the loading process of CRPS test. Notice also that since the change of the b-value is under the condition of $\sigma_1 - \sigma_3 = \text{constant}$ as shown in Fig.4.2-3, the stress path in the deviator plane consists of a back and forth movement.
between point A and B. However, in the loading process of CRPS tests used in the study, the deviator stress components, $\sigma_1 - \sigma_2$, and $\sigma_2 - \sigma_3$ are cyclically changing in the sample.\cite{40,41}
Fig. 4.1-1 Test Program of Monotonic Loading in Different Stress Space
Fig. 4.1-2  Overview of the Isotropic Compression Test Specimen
Fig. 4.1-3 Overview of the Solid Specimen for TC and TE Tests
Fig. 4.1-4  Overview of the Hollow Cylinder Specimen for SS, CT and CT.SS Tests
a. Cyclic Triaxial Shear (CTrS) Test

b. Cyclic Torsion Shear (CToS) Test

c. Cyclic Test with Circular Rotation of Principal Stress (CRPS)

Fig. 4.2-1 Lading Schemes Adopted in the Cyclic Load Tests
Fig. 4.2-2  Schematic Effective Stress Path for the Specimens Subjected to Cyclic Shear Stresses
Fig. 4.2-3  Stress Path Showing the Change in b-value During the Cyclic Loading

with \( \sigma_1 - \sigma_3 = \text{constant} \)

( Ishihara and Towhata, 1983 )
### Table 4.2-1

**Abbreviations and Notations of Cyclic Load Tests**

<table>
<thead>
<tr>
<th>Type of Test</th>
<th>Abbreviation</th>
</tr>
</thead>
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<tr>
<td>Cyclic Triaxial Shear</td>
<td>CTrS</td>
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<tr>
<td>Cyclic Triaxial Compression Shear</td>
<td>CTCS</td>
</tr>
<tr>
<td>Cyclic Triaxial Extension Shear</td>
<td>CTES</td>
</tr>
<tr>
<td>Cyclic Torsional Shear</td>
<td>CToS</td>
</tr>
<tr>
<td>Circular Rotation of Principal Stress</td>
<td>CRPS</td>
</tr>
</tbody>
</table>

**Notations in Cyclic Load Tests:**

- **Stress Ratio or Type of Sine-Wave**
- **Consolidated State**
- **Type of Specimen**
- **Type of Test**
CHAPTER V

EXPERIMENTAL RESULTS AND ANALYSIS
OF MONOTONIC LOAD TESTS

This chapter provides the details of the experimental results and analysis under the
category of monotonic load both for the loose sand (relative density, $D_r = 30\%$) and dense
sand ($D_r = 75\%$). The experimental results and discussion of monotonic load test are
presented in section 5.1. Section 5.2 provides the analysis of the experimental results for
the monotonic load tests.

5.1 Experimental Results and Discussion of Monotonic Load Tests

5.1-1 Isotropic Compression (IC) Tests

In this type of the test, the stress state moved along the hydrostatic axis, where
$\sigma_1 = \sigma_2 = \sigma_3$ (Fig.4.1-1). Two isotropic compression test results on drained solid cylinder
specimen prepared at two different relative densities ($D_r=30\%$ and $D_r=75\%$) are presented
in Fig.5.1-1. The figure shows the isotropic consolidation pressure, $\sigma_c / p_a$, normalized with
respect to the atmospheric pressure, versus the volumetric strain, $\varepsilon_v$. Each test includes
five unloading and reloading branches. It is apparent from these figures that, the loose and
dense sand exhibit a nonlinear “locking” behavior without any yielding under hydrostatic
loading and reloading. Upon unloading to the initial stress state at small stress levels, a
significant part of the volumetric strains were recovered; whereas, at high stress levels,
more permanent strain was resulted for each loading-relading cycle. The comparison of
Fig.5.1-1a with Fig.5.1-1b gives that for a given stress ratio, the volumetric strain in dense
sand ($D_r=75\%$) is approximately three times bigger than the volumetric strain in the loose
sand ($D_r=30\%$). Fig.5.1 also provides the necessary information for the later parameter study of constitutive model of soil.

5.1.2 Triaxial Compression (TC) Tests

The stress-strain curves of eight triaxial compression test results prepared at two relative densities and consolidated at four stress states (B, C, D and E) are provided in Fig.5.1-2 and Fig.5.1-3. Fig.5.1-4 plots the normalized stress difference, $\frac{\sigma_1 - \sigma_3}{\sigma_c}$, versus the major principal strains both for loose and dense specimens. A typical comparison of triaxial compression test for loose and dense specimens consolidated at the stress state B is shown in Fig.5.1-5. It shows the effects of the initial void ratio (or relative density) on the stress-strain curves. As expected, for dense sand, the deviator stress shows a clear peak and a gradual decreases following the peak. On the other hand, for loose sand, the deviator stress does not develop a peak, but remains essentially constant with further straining once the compressive strength is reached. For the dense sand, the peak is reached at axial strain, $\varepsilon_z \approx 3-4\%$. Notice also in Fig.5.1-5b, that the dense specimen first contracts a little in volume, and then expands until finally approaches asymptotically a certain value of $\varepsilon_z$ at large strains. On the other hand, the loose specimen contracts in volume as the specimen is strained, and until finally at large strains its density approaches a constant value. Typical shapes of failed specimens both for loose and dense sands under triaxial compression loading condition are shown in Fig.5.1-6. The reason for this change is the fact that “specimens fail under shear stress” is obvious in soil mechanics. Note that the shear plane could be seen clearly in the failing specimens during the tests. Fig 5.1-6a shows a typical failed loose specimen at axial strain, $\varepsilon_z$, equal to about 20%. Fig 5.1-6b plots a typical failed dense specimen at axial strain, $\varepsilon_z$, equal to about 15%. 
Comparison of Results from Hollow and Solid Cylinder Specimens:

Fig.5.1-7 gives a comparison of results of otherwise identical TC tests on a hollow cylinder specimen and a solid cylinder specimen of loose sand consolidated at the stress state B. The figure shows that there is a good agreement in the stress-strain relationship between the results from solid and hollow cylinder specimens which were built under the same specimen preparation technique. This may suggest that the moist tamping undercompaction technique (Ladd, 1978) which was used in this study can ensure a high degree of uniformity between the solid and hollow cylinder specimens.

5.1-3 Triaxial Extension (TE) Tests

The stress-strain curves of four triaxial extension tests both for loose and dense specimens and consolidated at two stress states (B and D) are shown in Fig.5.1-8 and Fig.5.1-9. Fig.5.1-10 provides the stress paths of the triaxial extension tests in the two deviator planes. The comparison of triaxial extension tests for loose and dense specimen consolidated at the stress state B is shown in Fig.5.1-11. The curves of deviator stress versus axial strain show a pronounced peak both for the dense specimen, and a much slight peak for the loose specimen. Notice that in these figures, the axial strains are negative because the specimens were subjected to extension in the axial direction. The peak strength is observed at about 5% axial strain for dense specimen and at about 10% axial strain for loose specimen. During the test, the shear plane could be observed when the peak was reached. Furthermore, the dense specimen first contracts a little, then gradually expands until the volume reduces a certain value at large strains. At this stage, the deviator stress remains essentially constant. On the other hand, the loose specimen contracts until its volume reaches a certain value.
A typical failed specimen in the triaxial extension test is shown in Fig. 5.1-12. It is clear that the specimen is failed under shear stress.

5.1-4 Torsional Simple Shear (SS) Tests

The stress-strain curves (shear stress, $q$, versus octahedral shear strain, $\gamma_{oct}$) of two torsional simple shear tests both for loose and dense specimens and consolidated at the stress state B are shown in Fig. 5.1-13. Fig. 5.1-14 shows the stress paths of the two torsional simple shear tests. Notice that the stress-strain curves do not show any peaks for either the loose or the dense specimens. As expected, the shear stress, $q$, is developed faster in dense specimen than the shear stress in the loose specimen. In dense specimen, the shear stress, $q$, remains essentially constant after the octahedral shear strain, $\gamma_{oct}$, reaches 10% approximately. A typical failed specimen under torsional simple shear loading condition is shown in Fig. 5.1-15.

5.1-5 Compression and Torsion (CT) Test

In the compression and torsion (CT) test, because the failure of the specimen happened very fast, the stress-strain softening behavior could not be completely obtained even though the maximum speed of data acquisition was used. The peak point, however, corresponding to failure, was obtained and the failure stress state under compression and torsion load condition is shown in Table 5.2-1. Two more tests under same loading conditions were performed to confirm this failure stress state and showed good agreement.

5.1-6 Combination of Triaxial Compression and Simple Shear (TC.SS) Test

The stress-strain curves (shear stress, $q$, versus octahedral shear stain, $\gamma_{oct}$) and the volumetric strain-octahedral shear strain curves of four tests under the combination of triaxial compression with simple shear loading condition and consolidated at the stress state
B are shown in Fig.5.1-16, 5.1-17, 5.1-19 and 5.1-20. Notice that the stress-strain curves of the loose specimens as shown in Fig.5.1-16 and Fig.5.1-17 do not show peaks. In the dense specimens, there is no peak in the TC.SS-1 test, but there is a peak in the TC.SS-2 test as shown in Fig.5.1-19 and Fig. 5.1-20. Of course, the shear stress, \( q \), is developed faster in dense specimen than that in the loose specimen. In dense specimens, \( q \) remains essentially constant after the octahedral shear strain, \( \gamma_{ocel} \), reaches about 15% in TC.SS-1 test and about 5% in TC.SS-2 test. Notice also that again the loose specimens contract to a large extent, and finally their volume remains essentially constant as specimens deform at large strains (Fig.5.1-16 and Fig.5.1-17). On the other hand, the dense specimens first contract a little in volume, and then gradually expand at large strains (Fig.5.1-19 and Fig.5.1-20). The stress paths of the four tests under the combination of triaxial compression and simple shear loading conditions are shown in Fig.5.1-18 and Fig.5.1-21 respectively.

A typical failed specimen in the combination of triaxial compression and simple shear tests is shown in Fig.5.1-22. Notice that the specimen was failed under shear stress.

5.2 Analysis of Monotonic Load Test Result

5.2-1 Failure Criteria and Failure Surfaces

As presented in section 5.1, a series of monotonic load tests was performed at various stress levels using both solid and hollow cylinder specimens and under different loading conditions. In order to establish the failure surfaces for both loose and dense sands, it is necessary to investigate the failure characteristics and set the criteria of specimen failure.

In general, the failure criterion for each sample should be evaluated on the basis of soil stress-strain behavior and the characteristics of the deformed specimen. The failure criteria which were applied in this study are as follows:[53,35,8]
1. Reaching a peak value of the deviator stress (for dense sand) or a constant value of the deviator stress at large strains (for loose sand).

2. Visible failure, e.g. the development of a slip surface in the sample (used if criterion 1 is not conclusive).

3. Reaching a predetermined strain, 20\% (used if criterion 1 is not conclusive).

4. The volume change at large deformation remains relatively constant (used only as additional evidence together with one of the above criterion).

Based on the above failure criteria, the stress states at failure were determined for all of the monotonic tests and summarized as in Table 5.2-1. The friction angle, $\phi$, corresponding to the given failure state is also shown in Table 5.2-1. In Table 5.2-1, $\sigma'_{1f}$, $\sigma'_{2f}$ and $\sigma'_{3f}$ represent the principal stresses at the failure state, $\varepsilon'_{1f}$, $\varepsilon'_{2f}$ $\varepsilon'_{3f}$ and $\varepsilon'_{vf}$ represent the principal strains and volumetric strain at the failure state, respectively. Finally, $R$ is the length of the vector $NP$ as shown in Fig.2.1-5.

The failure envelops from triaxial compression (TC) and triaxial extension (TE) tests of loose and dense specimens in $q-p$ space are shown in Fig.5.2-1 and Fig.5.2-2 respectively. Fig.5.2-3 plots the failure surfaces for both loose and dense sand in triaxial stress plane. All of monotonic tests results provide the whole picture of failure surfaces both for loose and dense sands. It is clear that the failure surfaces of the sand sample is shaped like an asymmetric bullet with the principal apex at the origin of the stress axes. The failure surface of dense sand ($D_f=75\%$) has bigger apex angle than the loose sand ($D_f=30\%$). For a sand sample with a given relative density, one may interpolate the failure surface for the sample using the failure surfaces which were established from the monotonic load tests.
5.2-2 Comparison of Experimental Failure Data with Lade's Model Failure Surfaces

In this study, the Lade's model failure parameters were investigated based on the monotonic load test results both for loose and dense sands. As mentioned in section 2.2-1, the failure surface in Lade's model is shaped as an asymmetric bullet with a pointed apex at the origin of the stress axes and is expressed by

$$\left( \frac{I_1^3}{I_3 - 27} \right) \frac{I_1}{\rho_a}^m = \eta_1$$  \hspace{1cm} (2.2-4)

where

$$I_1 = \sigma_1' + \sigma_2' + \sigma_3'$$

$$I_3 = \sigma_1' \sigma_2' \sigma_3'$$

$$\rho_a = \text{the atmospheric pressure}$$

and

$$m \text{ and } \eta_1 \text{ are the failure parameters representing material properties}$$

Fig.5.2-4 provide the Lade's model failure parameters for loose and dense sands. Note that the parameters were derived form all of the monotonic load tests. A regression analysis of the data in Fig.5.2-4 gives $m = 0.12$ and $\eta_1 = 18.03$ for loose sand and $m = 0.24$ and $\eta_1 = 37.17$ for dense sand.

The comparison of experimental data at failure with Lade's model failure surface in B, C, D and E deviator plane for loose sand is shown in Fig.5.2-5 and Fig. 5.2-6. Fig.5.2-7 and Fig.5.2-8 provide the comparison of experimental failure data with Lade's model failure surface in B, C, D, and E deviator plane for dense sand.
In those figures, the open circles correspond to the test results, whereas, the solid curve represents the failure surface according to Lade's model for frictional materials. Notice that Lade's model is in good agreement with the experimental data both for loose and dense sands.

In Lade's model, the failure parameters, \( m \) and \( \eta_1 \), can be derived from triaxial compression tests only instead of using all of the test data in failure stress states. In order to see the difference between these two ways, a parallel study was made to obtain the failure parameters by using triaxial compression (TC) test data only. Fig.5.2-9 gives the Lade's model failure parameters both for loose and dense sands. Note that the parameters were derived from eight TC tests only. For the data in Fig.5.2-9, it gives \( m = 0 \) and \( \eta_1 = 11.90 \) for loose sand and \( m = 0.21 \) and \( \eta_1 = 35.0 \) for dense sand.

The comparison of experimental data at failure with Lade's model failure surfaces for loose and dense sands in B deviator plane with parameters derived from TC tests only is shown in Fig.5.2-10. In Fig.5.2-10, the solid circles correspond to test results, while the solid curves represents the Lade's model failure surfaces. Notice that Lade's model is in good agreement with the experimental data both for loose and dense sands. The comparison of Fig.5.2-10 with Fig.5.2-5 and Fig.5.2-7 indicates that there is no significant difference in the results, regardless of whether Lade's model failure parameters derived from TC tests only or from all of the monotonic tests. It is clear that Lade's model can provide good prediction of the failure surface for the sand sample used in this study subjected to monotonic loading conditions.
Fig. 5.1-1 Isotropic Compression of Ottawa, Illinois Sand with Primary Loading, Unloading and Reloading Branches
Fig. 5.1-2 Experimental Results of Triaxial Compression (TC) Tests in Different Deviator Plane
(Loose Sand, Dr = 30 %)
Fig. 5.1-3 Results of Triaxial Compression (TC) Tests in Different Deviator Plane (Dense Sand, Dr = 75%)

Major Principal Strain, $\varepsilon_1$ (%)
Fig. 5.1-4  Normalized Triaxial Compression (TC) Test Results in Different Deviator Plane.
Fig. 5.1-5 Comparison of Triaxial Compression (TC) Test for Loose and Dense Specimens in B Deviator Plane.
a. Loose Sand

b. Dense Sand

Fig. 5.1-6 Typical Failed Specimens in Triaxial Compression Tests
Fig. 5.1-7  Comparison of Hollow Cylinder Specimen with Solid Cylinder Specimen in B Deviator Plane (Loose Sand, Dr = 30 %)
Fig. 5.1-8 Triaxial Extension (TE) Tests in Different Deviator Plane
(Loose Sand, $D_r = 30\%$)
Fig. 5.1-9 Triaxial Extension (TE) Tests in Different Deviator Plane
(Dense Sand, Dr = 75%)
Fig. 5.1-10  Stress Paths of Triaxial Extension (TE) Test in Different Deviator Plane
Fig. 5.1-11 Comparison of TE Tests for Loose and Dense Specimens in B Deviator Plane
Fig. 5.1-12  Typical Failure Specimen in Triaxial Extension Tests
Fig. 5.1-13  Torsional Simple Shear (SS) Test in B Deviator Plane
Fig. 5.1-14 Stress Paths of Simple Shear (SS) Test in B Deviator Plane
Fig. 5.1-15  Typical Failure Specimen in Torsional Simple Shear Tests
Fig. 5.1-16  TC. SS-1 Test in B Deviator Plane
(Loose Sand, Dr = 30%)
Fig. 5.1-17  TC . SS -2 Test in B Deviator Plane  
( Loose Sand, Dr = 30% )
Fig. 5.1-18 Stress Paths of TC-SS Test in B Deviator Plane (Loose Sand, Dr = 30%)
Fig. 5.1-19 TC SS-1 Test in B Deviator Plane
    (Dense Sand, Dr = 75%)
Fig. 5.1-20  TC SS-2 Test in B Deviator Plane  
(Dense Sand, Dr = 75%)
Fig. 5.1-21 Stress Paths of TC.SS Test in B Deviator Plane
(Dense Sand, Dr = 75 %)
Fig. 5.1-22 Typical Failure Specimen in TC.SS Tests
Fig. 5.2-1 Stress Paths of Triaxial Compression (TC) and Triaxial Extension (TE) Tests
(Loose Sand, Dr = 30%)
Fig. 5.2-2 Stress Paths of Triaxial Compression (TC) and Triaxial Extension (TE) Tests
(Dense Sand, Dr = 75%)
Fig. 5.2-3 Failure Surfaces in Triaxial Stress Plane
Fig. 5.2-4 Failure Parameters of Lade's Model
(derived from all of the monotonic tests)
Fig. 5.2-5 Comparison of Experimental Failure Data with Lade's Failure Surface in B and D Deviator Planes (Loose Sand, Dr = 30%)
Fig. 5.2-6  Comparison of Experimental Failure Data with Lade's Failure Surface in C and E Deviator Planes (Loose Sand, Dr = 30 %)
Fig. 5.2-7 Comparison of Experimental Failure Data with Lade's Failure Surface in B and D Deviator Planes (Dense Sand, Dr = 75%)
Fig. 5.2-8 Comparison of Experimental Failure Data with Lade's Failure Surface in C and E Deviator Planes
(Dense Sand, Dr = 75 %)
Fig. 5.2-9 Failure Parameters of Lade's Model
(derived from TC tests only)
Fig. 5.2-10 Comparison of Experimental Failure Data with Lade's Failure Surface in B Deviator Plane  
(Parameters Derived from TC Tests only)
<table>
<thead>
<tr>
<th>Test</th>
<th>Stress States at Failure (KPa)</th>
<th>Strain States at Failure (%)</th>
<th>Friction Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{1f}'$</td>
<td>$\sigma_{2f}'$</td>
<td>$\sigma_{3f}'$</td>
</tr>
<tr>
<td>TC-S-B-L</td>
<td>518.0</td>
<td>187.2</td>
<td>187.2</td>
</tr>
<tr>
<td>TC-H-B-L</td>
<td>523.7</td>
<td>186.8</td>
<td>186.8</td>
</tr>
<tr>
<td>TC-S-C-L</td>
<td>1154.2</td>
<td>387.0</td>
<td>387.0</td>
</tr>
<tr>
<td>TC-S-D-L</td>
<td>1767.0</td>
<td>614.8</td>
<td>614.8</td>
</tr>
<tr>
<td>TC-S-E-L</td>
<td>2264.5</td>
<td>811.2</td>
<td>811.2</td>
</tr>
<tr>
<td>TE-S-B-L</td>
<td>390.4</td>
<td>390.4</td>
<td>116.8</td>
</tr>
<tr>
<td>TE-S-D-L</td>
<td>1269.1</td>
<td>1269.1</td>
<td>474.6</td>
</tr>
<tr>
<td>SS-H-B-L</td>
<td>464.5</td>
<td>299.3</td>
<td>134.3</td>
</tr>
<tr>
<td>TC.SS-H-B-L-1</td>
<td>524.3</td>
<td>243.3</td>
<td>133.1</td>
</tr>
<tr>
<td>TC.SS-H-B-L-2</td>
<td>532.4</td>
<td>218.4</td>
<td>138.5</td>
</tr>
<tr>
<td>CT-H-B-L</td>
<td>497.0</td>
<td>242.0</td>
<td>156.0</td>
</tr>
</tbody>
</table>

TC-S-B-D  | 601.4            | 164.5            | 164.5          | 356.7 | 2.32           | -1.55           | -1.55          | -0.78        | 34.8   |
| TC-S-C-D  | 1222.2           | 361.4            | 361.4          | 702.9 | 3.02           | -1.82           | -1.82          | -0.63        | 32.9   |
| TC-S-D-D  | 1738.9           | 568.5            | 568.5          | 955.7 | 5.01           | -2.97           | -2.97          | -0.93        | 30.5   |
| TC-S-E-D  | 2417.7           | 736.0            | 736.0          | 1373.1| 3.77           | -2.26           | -2.26          | -0.74        | 32.2   |
| TE-S-B-D  | 397.0            | 397.0            | 105.3          | 238.2 | 1.93           | 1.93            | 4.44           | -0.57        | 35.5   |
| TE-S-D-D  | 1309.8           | 1309.8           | 376.5          | 762.1 | 1.62           | 1.62            | -3.31          | *           | 33.6   |
| SS-H-B-D  | 495.0            | 299.9            | 104.7          | 276.0 | 9.74           | -1.12           | -9.18          | -0.56        | 40.6   |
| TC.SS-H-B-D-1 | 548.8     | 230.7            | 97.4           | 328.0 | 13.86          | -5.08           | -10.15         | -1.38        | 44.3   |
| TC.SS-H-B-D-2 | 569.5     | 200.9            | 116.7          | 340.5 | 6.46           | -3.02           | -4.79          | -1.36        | 41.3   |

* -- Lost Data due to Improper Test Control

Tabel 5.2-1 Failure States in Monotonic Load Tests
CHAPTER VI

EXPERIMENTAL RESULTS AND ANALYSIS

OF CYCLIC LOAD TESTS

This chapter provides the details of the experimental results and analysis under the category of cyclic loadings. As discussed in section 4.2, in order to focus better on the investigation of soil behavior under the cyclic load, all of the cyclic tests were performed on loose sand specimens. Eight successful cyclic load tests, corresponding to six distinct types of loading, were conducted (Fig.4.2-1). The abbreviations and notations of the cyclic load tests are given in Table 4.2-1. The eight performed cyclic tests are: (a) three cyclic triaxial shear (CTrS) tests performed on the solid specimens; (b) two cyclic torsional simple shear (CToS) tests performed on the hollow cylinder specimens and (c) three cyclic tests with a circular rotation of principal stress axes and at a constant deviator stress, \( \sigma_1 - \sigma_3 \), performed on hollow cylinder specimens at three different shear stress ratios.

The specimen preparation techniques, the initial density and the stress state were identical in all eight tests. Recall that although CTrS tests were performed for convenience on solid cylinder specimens, while CToS and CRPS tests on hollow cylinder specimens, comparison of monotonic test results from solid and hollow cylinder specimens showed good agreement (Section 5.1-2). All of the eight specimens were isotropically consolidated to a confining stress as 300 KPa (B consolidated state), where they had the same void ratio, \( e \), equal to 0.819.

For the sake of convenience of discussion, the experimental results and discussion of cyclic load tests are presented in the following order. The experimental results and discussion of one-way cyclic triaxial shear tests are presented in section 6.1. Section 6.2
provides experimental results and analysis for the two two-way cyclic triaxial shear tests and a circular rotation of principal stress axes test with a constant deviator stress at a stress ratio as 0.15. Results and discussion of the three cyclic tests with circular rotation of the principal stress axes (CRPS) and stress ratios of 0.06, 0.10 and 0.15, are presented in section 6.3. A summary of the eight cyclic load test results is presented in Tables 6.1, 6.2, 6.3 and 6.4.

In all of the figures which are presented in this chapter, the solid lines with open circles or solid squares indicate the experimental results. The one-way and two-way cyclic loading histories, and the terminologies which were used in the cyclic tests and the discussion are shown in Fig.6.1. Note that the two types of sine-wave load have the same shear stress ratio and frequency, but they have two differences: (a) the one-way test experiences no rotation of the principal stress directions, as opposed to a 90° rotation occurring at the moment of stress reversal at the two-way test; and (b) the mean value of the cyclic shear is \( \tau_{cy}/2 \) for the one-way test and zero for the two-way test.

6.1 One-way Cyclic Triaxial Shear Tests

Results from three different types of one-way cyclic tests are presented and discussed in this section. The three performed one-way cyclic tests are: (a) a cyclic triaxial compression shear test performed on a solid specimen (CTCS-S-B-1) (b) a cyclic triaxial extension shear test performed on a solid specimen (CTES-S-B-1) and (c) a cyclic torsional simple shear test performed on a hollow cylinder (CToS-H-B-1) The stress ratio for all three cyclic tests was 0.15 which corresponds to a cyclic shear stress, \( \tau_{cy} \), from 0 to 45 KPa.

The orientation and amplitude of the shear stress for the three cyclic tests are shown in Fig.6.1-1. Fig.6.1-2 plots the cyclic shear stress versus the effective mean stress,
\( p' = p - u \), during the three cyclic tests, where \( p \) is the total mean stress and \( u \) is the excess pore water pressure. Fig.6.1-3 plots the pore water pressure buildup, \( u \), during the three one-way cyclic tests.

As shown in Fig.6.1-2, in the one-way cyclic tests, the amplitudes of shear stress, \( \tau_{z6} \) (for the CToS-H-B-1 test) and of the shear stress, \( (\sigma_z - \sigma_0)/2 \) (for CTCS-S-B-1 and CTES-S-B-1 tests) remained constant, while the pore pressures gradually developed with the number of cycles.

One of the most important conclusions found from the three one-way cyclic tests as shown in Figs.6.1-2, 6.1-3 and Table 6.1, is that significant pore water pressures, \( u \), are generated during the first half cycle and more remarkably during the last half cycle. The pore water pressure buildup, \( \Delta u \), is 42.61 KPa during the first half cycle (path ABC) for the CTCS-S-B-1 test, i.e. 3.76 time larger than the pore pressure developed during the second half cycles. The pore water pressure buildup is 46.40 KPa during the first half cycle for the CTES-S-B-1 test, i.e. 2.80 times larger than the pore pressure developed during the second half cycles. Similarly, the pore water pressure buildup is 48.53 KPa during the first half cycle for the CToS-H-B-1 test, i.e. 3.26 times larger than the pore pressure developed during the second half cycles. In the first half cycle, the test specimen is in an initial state before which it was subjected only to volumetric strains, caused by isotropic loading. The subsequent application of shear strains, is resulting to some significant initial rearrangements of the soil skeleton which appears to have a more stable structure after the first half cycle of load, in which this significant development of plastic yielding is observed. For this reason, the generation of the pore water pressure is greater in the first cycle as compared to the subsequent cycles. In the last half cycle, the ratio of the stress difference or the shear stress to the existing mean effective stress exceeds a
threshold value which is corresponds to the "angle of phase transformation" at which a flow failure is triggered. Similar findings have been also reported by Ishihara and Towhata\textsuperscript{[40]}, Yamada and Ishihara\textsuperscript{[89]}, Mohamad and Dobry\textsuperscript{[56]}, etc.

As shown in Table 6.1, the absolute value of the mean effective stress reduction, $\Delta p'$, at the end of any full half cycle should be equal to the pore water pressure buildup, $\Delta u$, at the end of the same half cycle (in the one-way tests). Results show that significant mean effective stress reductions, $\Delta p'$, occur during the first half cycle and more remarkably during the last half cycle.

Perhaps surprisingly, the pore pressure buildup, $\Delta u$, during the first half cycle (path ABC in Figs.6.1-2a, 6.1-2b and 6.1-3c) did not show a significant difference among the CTCS-S-B-1, CTES-S-B-1 and CToS-H-B-1 tests. However, starting at the second half cycle, the pore water pressure buildup, $\Delta u$, in the CTES-S-B-1 test is more pronounced than it is in the CToS-H-B-1 and CTCS-S-B-1 tests. In the last half cycles for the cyclic torsional shear (CTES-S-B-1) test, the pore water pressure buildup, $\Delta u$, is approximately 120 KPa which is 33.3% larger than the pore pressure buildup in CToS-H-B-1 test and even much more than $\Delta u$ in the CTCS-S-B-1 test. In all of the three tests, the development of excess pore pressure led to a liquefaction flow failure (Dobry, R. et al, 1982), which occurred after 51.5 cycles for the cyclic triaxial compression shear (CTCS-S-B-1) test, 10.5 cycles for the cyclic triaxial extension shear (CTES-S-B-1) test, and 15.5 cycles for the cyclic torsional shear (CToS-H-B-1) test. The life time of CTES-S-B-1 test specimen is 32% shorter than the life time of CToS-H-B-1 test specimen, and 80% shorter than the life time of CTCS-S-B-1 test specimen.

The stress-strain relationships for the three one-way cyclic tests are shown in Fig.6.1-4 through Fig.6.1-6. Fig.6.1-4 plots the stress difference, $\sigma_2 - \sigma_0$, or shear stress,
\( \tau_{20} \) versus octahedral strain, \( \gamma_{\text{oct}} \), for the one-way cyclic tests. The stress difference or shear stress versus vertical strain, \( \varepsilon_z \), of the one-way cyclic tests are shown in Fig.6.1-5. Fig.6.1-6 plots the shear stress, \( \tau_{20} \), versus shear strain, \( \gamma_{20} \), for CToS-H-B-1 test.

An important conclusion derived from Figs.6.1-4, 6.1-5 and 6.1-6 is that after the first half cycle and before the specimen reaches failure, the unloading and reloading curves result to more or less similar shape of hysteresis loops but with gradually decreasing slopes at a rather slow rate. During the unloading and reloading process, irrecoverable (or plastic) strains are gradually induced. However, when failure states are approached, the hysteresis loops become wider, large plastic strains are produced and the slope of the hysteresis loops decreases remarkably. The plastic strains are caused by the deformations resulting mainly from sliding, rearrangement and perhaps some crushing of soil particles. These plastic deformations cause a change in the internal structure of the soil element.

In general, as discussed in the pore water pressure buildup and reduction of mean effective stress, relatively large strains are induced during the first half cycle and even more remarkably during the last half cycles. With respect to the strain increments developed in each of the three tests during the first two half cycles of loading, during the first half cycle, test CTCS-S-B-1 induced the increment of octahedral strain, \( \Delta \gamma_{\text{oct}} = 0.056\% \), i.e. 4.60 times larger than the increment of octahedral strain induced during the second half cycle. The increment of octahedral strain, \( \Delta \gamma_{\text{oct}} \) induced during the first half cycle of CTES-S-B-1 test is 0.048\%, i.e. 3.36 times larger than the increment of octahedral strain induced during the second half cycle. Similarly, the increment of octahedral strain, \( \Delta \gamma_{\text{oct}} \) induced during the first half cycle of CToS-H-B-1 test is 0.067\%, i.e. 4.58 times larger than the increment of octahedral strain induced during the second half cycle (Fig.6.2-4 and Table 6.3).
As shown in Fig.6.1-5 and Table 6.2 for the CTCS-S-B-1 test (Fig.6.1-5a), the vertical strain increment, $\Delta \varepsilon_2$, developed during the loading of the first half cycle is 0.039% which is 2.9 times larger than the vertical strain increment, $\Delta \varepsilon_2$, developed in the second half cycle. The vertical strain increment, $\Delta \varepsilon_2$, induced during the loading of the first half cycle in the test CTES-S-B-1 is 0.034%, i.e. 3.25 times larger than the vertical strain increment, $\Delta \varepsilon_2$, developed in the second half cycle. The vertical strain increment, $\Delta \varepsilon_2$, developed in the loading of the first half cycle of CToS-S-B-1 test is 0.011%, i.e. 1.75 times larger than the vertical strain increment, $\Delta \varepsilon_2$, developed during the second half cycle. Similarly, as shown in Fig.6.1-6 and Table 6.2, the shear strain increment, $\Delta \tau_{ze}$, developed during the simple shear loading of the first half cycle of CToS-H-B-1 test is 0.080%, i.e. 4.0 times larger than the vertical strain increment, $\Delta \tau_{ze}$, induced in the second half cycle.

As shown in Fig.6.1-2, the pore water pressure is developed fast when the stress path reaches the failure surface which was established in the monotonic test program. Following this, the specimen failed immediately (although the application of the cyclic stress continued after failure for few moments, before stopping the MTS testing device) Notice also that the octahedral strain, $\gamma_{oct}$, increases during the loading phase, but decreases in the unloading phase. As the pore water pressure was gradually developed, the specimen failed when $\gamma_{oct}$ was approximately 5.0% for the CTCS-S-B-1 test, 0.7% for the CToS-H-B-1 test and 0.5% for the CTES-S-B-1 test. In the one-way cyclic tests (Fig.6.1-5), the vertical strain, $\varepsilon_2$, also increases during the loading phase, but decreases in the unloading phase in the CTCS-S-B-1 and CTES-S-B-1 tests. However, $\varepsilon_2$, increases continuously both in the loading and unloading phases of CToS-H-B-1 test. The specimen failed when $\varepsilon_2$ was approximately 3.5% for the CTCS-S-B-1 test, 0.07% for the
CToS-H-B-1 test and -0.03% for the CTES-S-B-1 test. Notice that the negative sign means the specimen was stretched in vertical direction. As shown in Fig.6.1-6, in the one-way CToS-H-B-1 test, the shear strain, $\gamma_{x0}$, also increases during the loading phase, but decreases in the unloading phase. Finally, the specimen failed when $\gamma_{x0}$ reached about 0.9%.

6.2 Two-way Cyclic Triaxial Shear and CRPS Tests

Results from two different types of two-way cyclic tests and a cyclic test with a circular rotation of principal stress axes and a constant deviator stress $(\sigma_1 - \sigma_3)/2 = 45$ KPa, are presented and discussed in this section. The three performed cyclic tests are: (a) a cyclic triaxial shear test performed on a solid specimen (CTrS-S-B-2) (b) a cyclic torsional simple shear test performed on a hollow cylinder specimen (CToS-H-B-2) and (c) a cyclic test with a circular rotation of principal stress axes at a constant deviator stress, $\sigma_1 - \sigma_3$, performed on a hollow cylinder specimen (CRPS-H-B-0.15). The stress ratio for all of the three cyclic tests was also 0.15 yielding a the shear stress, $\tau_{cy}$, equal to 45 KPa.

The orientation and amplitude of the shear stress for the three cyclic tests are shown in Fig.6.2-1. Fig.6.3-2 plots the cyclic shear stress $\tau_{x0}$ or $(\sigma_2 - \sigma_3)/2$ versus the mean effective stress, $p'$ during the three tests. Finally, Fig.6.2-3 plots the pore water pressure buildup, $u$, in the three cyclic tests.

The most important conclusion derived by comparison of the results in Figs.6.2-3a, 6.2-3b and 6.2-3c and Table 6.1, is that circular rotation of principal stress axes with a constant deviator stress, $\sigma_1 - \sigma_3$, may induce excess water pressures, which appear to be more significant than those developed in the cyclic triaxial shear and torsional simple shear
tests. For the presented test results, as shown in Fig.6.2-3 and Table 6.1, the amount of pore water pressure buildup, \(\Delta u\), during the first three quarters of the first cycle (path ABCD in Figs.6.2-2a, 6.2-b and 6.2-2c) is 153.20 kPa for the CRPS-H-B-0.15 test, i.e. 1.49 times larger than it was developed in the first three quarters of the first cycle of CTrS-S-B-2 test and 0.94 times larger than the pore pressure, \(u\), induced during the first three quarters of the first cycle of CToS-H-B-2 test. It shows that the pore pressure buildup in the CRPS-H-B-0.15 test occurs at a faster rate than the other two tests. In all of these tests the development of excess pore pressure led to a liquefaction flow failure, which occurred after 1.75 cycles for the cyclic triaxial shear test, 2.25 cycles for the cyclic torsional simple shear test, and 0.75 cycles for the CRPS-H-B-0.15 test. (Note that, although the application of cyclic load continued after the occurrence of the liquefaction flow failure, for clarity, the presented results end at the steady state of deformation, which can be easily defined from the significant deformation occurred at constant stresses during the sudden liquefaction flow failure.) The above conclusion is in complete agreement with results from a similar study by Ishihara and Towhata (1983). In their study, Ishihara and Towhata performed the same three types of tests on Toyoura sand with \(D_{50} = 0.17\) mm, a coefficient of uniformity \(C_u = 2\) consolidated at 294 kPa and at a void ratio \(e = 0.800\) for the cyclic triaxial shear test, \(e = 0.784\) for the torsional simple shear test, and \(e = 0.811\) for the test with the circular rotation of the principal stress axes. For an amplitude of cyclic shear stress \(\tau_{cy} = (\sigma_1 - \sigma_3)/2 = 55.5\) kPa, the corresponding numbers of cycles sufficient to cause liquefaction were 6 cycles for CTrS test, 38 cycles for CToS test and 2 cycles for CRPS test. Although these numbers of cycles or shear stress amplitude are higher than those in the present study, the trends are consistent for the two sands. (The significantly higher number of cycles for the case of the torsional simple shear test was attributed by Ishihara and Towhata partly to the smaller value of void ratio of this specimen.) Comparison of results from this study and other sources demonstrates that the circular
rotation of principal stress axes results in a more effective reorganization of the soil structure which leads to a more rapid development of excess pore pressure, compared to those in the cyclic triaxial shear test (CTrS) and cyclic torsional simple (CToS) test.

As observed in the case of the one-way cyclic tests, significant pore water pressures are generated during the first cycle. As shown in Figs.6.2-2, 6.2-3 and Table 6.1, the pore pressure buildup is 90.45 KPa during the first cycle (path ABCDE) for the CToS-H-B-2 test, which is about equal to the pore pressure developed in the second cycles. Because in this test failure occurred at 2.25 cycles, the second cycle is almost the last one and, therefore, in both cycles there is significant pore water pressure buildup.

Two important factors controlling the rate of pore pressure development are the magnitude of the cyclic stress ratio and the rotation of stress directions. Comparison of results in one-way and two-way cyclic tests shows the different effects of cyclic loading with and without shear stress reversal. It is clear that the pore water pressures increase faster in the case of shear stress reversal than the pore water pressures develop without shear stress reversal. In the one-way CTCS-S-B-1 test, the specimen failure happened after 51.5 cycles, as opposed to 1.75 cycles in the two-way CTrS-S-B-2 test (see Figs.6.1-2 and 6.2-2). Similarly, in the one-way CToS-H-B-1 test the specimen failure happened after 15.5 cycles, as opposed to only 2.25 cycles for the CToS-H-B-2 test. Moreover, the pore water pressure buildup in the CRPS-H-B-0.15 test (with the same shear stress ratio equal to 0.15) was developed even faster than the two other tests. The specimen failure happened after 0.75 of a cycle only. Similar findings, regarding the difference with or without shear stress reversal, have been reported not only for sands but also for clays, e.g., by Hicher and Lade (1987) for the investigation of rotation of principal directions in Kc-consolidated clay. In their study, a specimen failed after 10 cycles in the two-way cyclic test, while in a similar one-way test the pore pressure development during the last
cycles (the 5th cycle) was very slow, showing even no tendency to reach failure within a number of cycles comparable to that in two-way test. The comparison clearly indicates the important role of stress reversal and stress direction. It shows that pore water pressures increase faster when shear stresses are reversed (involving a sudden rotation of principal stresses) as well as when principal stress axes are rotated continuously.

Similar results were found as in the case of one-way cyclic tests, the pore water pressure buildup, Δu, in the CTrS-S-B-2 test is more pronounced during extension loading/unloading (path CDE in Fig.6.2-2a) than during compression loading/unloading (path ABC in Fig.6.2-2a). Indeed, during the second half cycle of the first cycle, the extension loading/unloading induced a pore pressure Δu = 57.95 KPa, i.e. 35.0% larger than the pore water pressure buildup, Δu, in the compression loading/unloading (path ABC in Fig.6.2-2a and Table 6.1). This is also observed in the case of the CRPS-H-B-0.15 in Fig.6.2-2c and Table 6.1, where the part of the shear stress, σ₂ - σ₀, (path CD) involving extension unloading produces Δu = 114.5 KPa which is much larger than the Δu corresponding to the unloading compression. Similar findings, regarding to the difference between compression and extension loading, have been reported by Ishihara and Towhata,40 Yamada and Ishihara,189 Mohamad and Dobry,56 etc. This difference could be the result of three factors: (a) the greater proximity towards the failure envelope in the extension loading, compared to compression loading; (b) the accumulation of excess pore water pressure results in smaller effective stresses and, therefore, a more softening behavior of the sand during path CD than during path AB. For the torsional simple shear test (Fig.6.2-2b), in which there is no effects from the difference shown in the triaxial test between compression and extension loading, the effective stress paths AB and CD produce Δu = 28 KPa and 41 KPa, respectively (i.e. Δu is 46% higher in CD than it is in AB). This difference may partly reflect the softening effects of the accumulated pore pressure;
however, the smaller pore water pressures during EF compared to that CD in Fig.6.2-2b suggests the third factor: (c) the reorganization of the soil structure occurring more effectively during the first cycle of the loading. Experimental results from this study and other sources, corresponding to more cycles of loading triggering flow failure, show that generally more water pressure develops during the first cycle of loading than in subsequent cycles, except the last one [Yamada and Ishihara,[40] Mohamad and Dobry,[56] etc.]. This may be attributed to a rearrangement of the mineral skeleton during the first cycle of loading, in which deformation or partial collapse of local loose structure of soil occurs, leading to a more stable structure with less tendency for contraction, during the subsequent cycles of loading. Note, however, that, this rearrangement of soil structure, may sometimes affect more than one loading cycles, depending on the specimen contractiveness, non-homogeneity and the level of cyclic shear stress.

If one considers the circular rotation of principal stress axes with a constant shear stress $\tau_{cy} = (\sigma_1 - \sigma_3)/2 = 45$ KPa as a superposition of a cyclic triaxial shear stress and a cyclic torsional shear stress, the pore pressure buildup must be the combined effect of a simultaneous application of the two cyclic waves. In practice, this is of little significance because the pore pressure buildup is not a linear phenomenon and therefore it is not possible to predict the results of (CRPS-H-B-0.15) test from the results of (CTrS-S-B-2) and (CToS-H-B-2) tests.

The stress-strain relationships for the three cyclic tests are shown in Fig.6.2-4 through Fig.6.2-6. Fig.6.2-4 shows the stress difference or shear stress versus octahedral strain, $\gamma_{oct}$, for the three cyclic tests. The stress difference or shear stress versus vertical strain, $\varepsilon_z$, of the three cyclic tests are shown in Fig.6.2-5. Fig.6.2-6 plots the shear stress, $\tau_z$, versus shear strain, $\gamma_z$, for the CToS-H-B-2 and CRPS-H-B-0.15 tests.
As discussed in Section 6.1, the strain components are induced remarkably during the last cycle (Fig.6.2-4). As shown in Fig.6.2-4 and Table 6.3, with respect to the strain increments developed in each of the three tests, it was found that during the first cycle, the CTrS-S-B-2 test induced an increment of octahedral strain $\Delta \gamma_{\text{oct}} = 0.063\%$. The specimen failed when the second cycle started when $\gamma_{\text{oct}}$ was approximately 0.2%. The increment of octahedral strain, $\Delta \gamma_{\text{oct}}$, reached 0.076% during the first cycle of CToS-H-B-2 test. The specimen failed after 2.25 cycles when $\gamma_{\text{oct}}$ reached about 0.4%. As one may note that the increment of octahedral strain, $\Delta \gamma_{\text{oct}}$, induced in the last cycle is much larger than it is in the last cycle but one.

As shown in Fig.6.2-5 and Table 6.2 for the CTrS-S-B-2 test (Fig.6.2-5a), the vertical strain increment, $\Delta \varepsilon_z$, developed during the third quarter of the first cycle (extension unloading) was 0.113% which is 66% larger in absolute value than the vertical strain increment, $\Delta \varepsilon_z$, developed in the first quarter of the first cycle (compression unloading). At the end of the first cycle, the vertical strain increment, $\Delta \varepsilon_z$, reached -0.045%. Again, the negative sign means the specimen is under extension deformation in vertical direction. The specimen failed after 1.75 cycles under extension loading. Notice also that, in the three cyclic tests (Fig.6.2-5), the vertical strain, $\varepsilon_z$, increases also during loading phase, but decreases in the unloading phase in the CTrS-S-B-2 and CRPS-H-B-0.15 tests. However, in the CToS-H-2 test, the vertical strain, $\varepsilon_z$, increases during loading phase, but essentially keeps constantly during the unloading phase. As shown in Fig.6.2-6, in the CToS-H-B-2 and CRPS-H-B-0.15 tests, the shear strain, $\gamma_{2\theta}$, increases also during the loading phase, but decreases during the unloading phase. Finally, the specimen failed when $\gamma_{2\theta}$ reached about 0.3% and -0.75%, respectively.
6.3 Cyclic Rotation of Principal Stress Axes (CRPS) Tests

Results from the three circular rotation of principal stress axes tests with a chosen constant deviator stress are presented and discussed in this section. The three performed cyclic tests are: (a) a CRPS test with a constant shear stress ratio, $\tau_{cy}/\sigma_{mc}'=0.15$ which yields an amplitude of the cyclic shear stress, $\tau_{cy}$, equal to 45 KPa (CRPS-H-B-0.15); (b) a CRPS test with a constant shear stress ratio, $\tau_{cy}/\sigma_{mc}'=0.10$ which corresponding to an amplitude of the cyclic shear stress, $\tau_{cy}$, equal to 30 KPa (CRPS-H-B-0.10) and (c) a CRPS test with a constant shear stress ratio, $\tau_{cy}/\sigma_{mc}'=0.06$ which yields an amplitude of the cyclic shear stress, $\tau_{cy}$, as 18 KPa (CRPS-H-B-0.06). All of the three CRPS tests were preformed on hollow cylinder specimens.

The orientation and amplitude of the shear stress for the three cyclic tests are shown in Fig.6.3-1. As one may note, the three CRPS tests were controlled as expected. The effective stress paths $q$ versus $p'$ during the three cyclic tests are shown in Figs.6.3-2 and 6.3-3. Fig.6.3-4 plots the pore water pressure buildup, $u$, during the three cyclic tests. As shown in Fig.6.3-1, the interchange of the torsional stresses, $\tau_{z\theta}$, and the stress differences, $(\sigma_z - \sigma_\theta)/2$, were continued cyclically, thereby producing the rotation of the principal stress axes along a circular path. All of the three specimens were failed at the phase where both the torsional stress, $\tau_{z\theta}$, and the stress difference, $(\sigma_z - \sigma_\theta)/2$, were applied. Notice that in the course of round excursion of the stress state as shown in Fig.6.3-1 along the circular stress path, the direction of the principal stress rotates continually through an angle of $180^\circ$, while the deviator stress, $\sigma_1 - \sigma_3$, is kept constantly throughout the loading precess (Fig.6.3-2).
As found in one-way and two-way cyclic tests, *significant pore water pressure, u*, are generated during the first cycle and more remarkably during the last cycle. As shown in Fig.6.3-3 and Table 6.1, the pore pressure buildup is 44.33 KPa during the first cycle of CRPS-H-B-0.10 test, i.e. 56% larger than the pore pressure developed in the second cycle. The pore pressure buildup is 8.00 KPa in the first cycle of CRPS-H-B-0.06 test, i.e. 14.9% larger than the pore pressure induced during the second cycles. Moreover, the pore pressure buildup is approximately 100 KPa during the last cycle of the CRPS-H-B-0.10 test and 35 KPa during the last cycle of CRPS-H-B-0.06 test. Again, the findings from CRPS tests confirm the conclusion which derived from both one-way and two-way test results.

Furthermore, notice that the amplitude of shear stress, $\tau_{cy}$ (or stress ratio, $\tau_{cy}/\sigma'_{mc}$) has also significant effects on the pore pressure development. The pore pressure buildup during the first cycle of CRPS-H-B-0.10 test is 44.33 KPa i.e. 4.54 times larger than the pore pressure developed during the first cycle of CRPS-H-B-0.06 test. Similarly, the pore pressure buildup during the second cycle is 28.34 KPa for CRPS-H-B-0.10 test, i.e. 3.07 times larger than the pore pressure developed in the second cycle of CRPS-H-B-0.06 test. The pore water pressure buildup in the CRPS-H-B-0.15 test is much faster than pore pressure buildup in the other two tests. In all three tests the development of excess pore pressure led to liquefaction flow failure, which occurred after about 0.75 cycles for the CRPS-H-B-0.15 test, 7 cycles for the CRPS-H-B-0.10 test and 76 cycles for the CRPS-H-B-0.06 test. The relationship of cycles to failure, N, with stress ratio, $\tau_{cy}/\sigma'_{mc}$, from the CRPS tests is provided in Fig.6.3-8.
The stress-strain relationships for the three cyclic tests are shown in Figs.6.3-5, 6.3-6 and 6.3-7. Fig.6.3-5 shows the shear stress, $\tau_{z\theta}$, versus octahedral strain, $\gamma_{oc}$, for the three cyclic tests. The stress difference, $(\sigma_z - \sigma_{\theta})/2$, versus vertical strain, $\varepsilon_z$, of the three cyclic tests are shown in Fig.6.3-6. Fig.6.3-7 plots the shear stress, $\tau_{z\theta}$ versus shear strain, $\gamma_{z\theta}$, for the three CRPS tests.

As found in the one-way cyclic tests shown in Figs.6.3-5, 6.3-6, 6.3-7, after the first cycle and before the specimen reaches failure states, the unloading and reloading curves of the two separate shear stress components result to more or less similar shapes of hysteresis loops but with gradually decreasing slopes at rate depending on the amplitude of the applied shear stress. During the unloading and reloading, the irrecoverable strains are gradually induced. As expected, when failure states are approached, the hysteresis loops become wider, large plastic strains are produced and the slope of the hysteresis loops decreases remarkably.

As discussed in the results of one-way and two-way cyclic tests, the plastic strains which are induced during the first cycle, appear to be larger than these developed in the second cycle. Of course, significant strains are developed during failure. As shown in Figs.6.3-5 and Table 6.3, with respect to the strain increments developed in each of the three tests, the increment of octahedral strain, $\Delta\gamma_{oc}$ induced during the first cycle of CPRS-H-B-0.10 test is $\Delta\gamma_{oc}=0.029\%$ which is 1.23 times larger than the increment of octahedral strain, $\Delta\gamma_{oc}$ in the second cycle. Again, the $\Delta\gamma_{oc}$ created in the last cycle is much larger than those in the rest of cycles. The specimen failed at $\gamma_{oc}$ was approximately $0.5\%$. The same conclusion was also derived for strain components of vertical strain, $\varepsilon_z$, and shear strain, $\gamma_{z\theta}$, and shown in Figs.6.3-6, 6.3-7 and Table 6.2.
The results of CRPS tests also indicated that significant deformation and pore water pressure can occur, during the rotation of principal stress axes, even when the deviator stress, $\sigma_1 - \sigma_3$, is maintained unchanged. [40,41]
Fig. 6.1  Schematic Effective Stress Paths in the (a) One-way Cyclic Tests and (b) Two-way Cyclic Tests
Fig. 6.1-1 Orientation and Amplitude of the Shear Stress for the One-way Cyclic Tests \( (\tau_{cy} = 0.15\sigma'_{mc} = 45 \text{ KPa}) \)
Fig. 6.1-2 Effective Stress Paths During the One-way Cyclic Tests
\( \tau_{cy} = 0.15 \sigma_{mc} = 45 \text{ KPa} \)
Fig. 6.1-3  Pore Water Pressure Buildup during the One-way Cyclic Tests

\( \tau_{cy} = 0.15\sigma'_m = 45 \text{ kPa} \)
Fig. 6.1-4 Stress-strain Curves (Stress: $\gamma_{\text{oct}}$) for the One-way Cyclic Tests

($\tau_{cy} = 0.15\sigma_{mc}^* = 45\text{ KPa}$)
Fig. 6.1-5 Stress-Strain Curves (stress: $\varepsilon_z$) for the One-way Cyclic Tests

($\tau_{\text{ref}} = 0.15\sigma_{\text{mc}} = 45$ KPa)
Fig. 6.1-6 Stress - Strain Curves (τ_{2θ} vs γ_{2θ}) of One-way and Two-way CToS Tests

(τ_{cy} = 0.15σ'_{me} = 45 KPa)
Fig. 6.2-1 Orientation and Amplitude of the Shear Stress for the Two-way Cyclic Tests ($\tau_{cy} = 0.15\sigma_{mc} = 45$ KPa)
Fig. 6.2-2 Effective Stress Paths During the Two-way Cyclic Tests

\( \tau_{cy} = 0.15\sigma'_{mc} = 45 \text{ KPa} \)
Fig. 6.2-3  Pore Water Pressure Buildup during the Two-way Cyclic Tests
\[ \tau_{cy} = 0.15\sigma_{me} = 45 \text{ KPa} \]
Fig. 6.2-4 Stress-strain Curves (Stress: $\gamma_{oct}$) for the Two-way Cyclic Tests

($\tau_{cy} = 0.15\sigma_{mc}' = 45$ KPa)
Fig. 6.2-5 Stress-Strain Curves ($\sigma_z - \sigma_0$) for the Two-way Cyclic Tests
($\tau_{cy} = 0.15\sigma'_{mc} = 45$ KPa)
Fig. 6.2-6 Stress-strain Curves ($\tau_{2\theta} : \gamma_{2\theta}$) of Two-way CToS and CRPS Tests
($\tau_{cy} = 0.15 \sigma_{mc}' = 45$ KPa)
Fig. 6.3-1 Orientation and Amplitude of the Shear Stress for the Cyclic Rotational Principal Stress Tests
Fig. 6.3-2  Effective Stress Paths ($q$: $p$) During the CRPS Tests
Fig. 6.3-3 Effective Stress Paths (stress difference : p) During the CRPS Tests
Fig. 6.3-4 Pore Water Pressure Buildup during the CRPS Tests
Fig. 6.3-5 Stress-Stain Curves (τ_{ζ0}, γ_{oct}) of the CRPS Tests
Fig. 6.3-6 Stress-Strain Curves (stress difference: $\varepsilon_z$) for the CRPS Tests
Fig. 6.3-7 Stress - Strain Curves (τθθ, γθθ) of CRPS Tests
Fig. 6.3-8 Relation of Cycles to Failure with Stress Ratio in CRPS Tests
<table>
<thead>
<tr>
<th>Test</th>
<th>Pore Pressure Buildup, $\Delta u$ (KPa)</th>
<th>Effective Strass Reduction, $\Delta p'$ (KPa)</th>
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<tr>
<td></td>
<td>AB</td>
<td>BC</td>
</tr>
<tr>
<td></td>
<td>1st half cycle</td>
<td>2nd half cycle</td>
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<tr>
<td>CTrS-S-B-2</td>
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Tabel 6.1 Summary of Cyclic Load Test Results (1)
### STRAIN INCREMENT COMPONENTS

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<th>Test</th>
<th>( \Delta \varepsilon_z ) (%)</th>
<th>( \Delta \gamma_{2g} ) (%)</th>
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<td></td>
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<td>BC</td>
</tr>
<tr>
<td></td>
<td>1st half cycle</td>
<td>2nd half cycle</td>
</tr>
<tr>
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<tr>
<td>CTES-S-B-1</td>
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<td>-0.031</td>
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<tr>
<td>CToS-H-B-1</td>
<td>0.008</td>
<td>0.003</td>
</tr>
<tr>
<td>CTrS-S-B-2</td>
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<td>-0.028</td>
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<tr>
<td>CToS-H-B-2</td>
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<td>0.000</td>
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Tabel 6.2 Summary of Cyclic Load Test Results (2)
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<th>COMPONENTS</th>
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<td>CD</td>
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Tabel 6.3 Summary of Cyclic Load Test Results (3)
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<td>290</td>
<td>16</td>
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Tabel 6.4 Summary of Cyclic Load Test Results (4)
CHAPTER VII

SUMMARY AND CONCLUSIONS

The main objective of this study is to assist in developing a unifying framework of understanding of the behavior of cohesionless soil under any loading condition with emphasis on cyclic loadings. The purpose of such a framework is to provide both insight and specific information needed to develop a reliable constitutive model.

To this end, an entirely new state-of-the-art triaxial test system for both solid and hollow cylinder testing of soil, and specimen preparation devices have been designed and set-up during this study. The computer-automated experimental system for solid and hollow cylinder testing soil is capable of both stress and strain controlled tests, simulating most of stress paths encountered in the fields. A number of computer programs for parameter study of constitutive modeling has also been developed during this study. The specimen preparation procedures using Ladd's\textsuperscript{45} moist tamping undercompaction technique both for solid and hollow cylinder saturated sand samples have been successfully used in this study.

A comprehensive experimental program for both loose and dense saturated fine Ottawa Silica sands using solid and hollow cylinder specimens under monotonic loading has been conducted. The failure surfaces for both loose and dense sands have been established from twenty drained monotonic load tests. Also, the soil behavior and deformation characteristics have been investigated. A comparison of experimental failure data with Lade’s model failure surfaces has also been made.

The soil behavior and deformation characteristics have been investigated under a cyclic experimental program for loose sand under undrained conditions. During this program, the
effects of circular rotation of principal stress axes and the stress reversal have been investigated

From this study, the following conclusions may be drawn:

1. The failure criterion provided by Lade's constitutive model for frictional materials is in good agreement with the experimental failure data obtained for the loose and dense fine Ottawa Silica Sand under undrained conditions.

2. The experimental results presented in this study show that the circular rotation of principal stress axes with a constant amplitude deviator stress has a significant effects on the development of excess pore water pressure and the deformation characteristics of saturated sands. This is in agreement with previous finding from the literatures.\[40,42\]

3. The results indicated that the rate of excess pore water pressure buildup is faster during a cyclic test with circular rotation of principal stress axes than during a cyclic triaxial shear test or a cyclic torsional shear test having the same amplitude of cyclic shear stress.

4. The rate of excess pore water pressure is faster during a cyclic triaxial extension test than in a cyclic triaxial compression test or a cyclic torsional simple shear test with same amplitude of cyclic shear stress.

5. Significant pore water pressures and deformations are developed mostly during the first and the last cycles.

6. The pore water pressure buildup and the mean effective stress reduction are developed very quickly when the stress path approaches the failure surface which was established in monotonic load tests.

7. The pore water pressure and deformation increase fast in the case of shear stress reversal, during which a sudden \(90^\circ\) rotation of the major and minor principal stresses takes place.
8. After the first cycle and before the specimen reaches failure, the unloading and reloading stress-strain curves result to more or less similar shapes of hysteresis loops but with gradually decreasing slopes at a rather slow rate. In the unloading and reloading process, irrecoverable strains are gradually induced. However, when failure states are approached, the hysteresis loops become wider, large plastic strains are produced and the slope of the hysteresis loops decreases remarkably.

9. The amplitude of shear stress has significant effects on the development of pore water pressure and deformation.

10. Also, in the two-way tests, the number of cycles to trigger a liquefaction flow failure in contractive sand is smaller during a circular rotation of principal stress axes test than during a triaxial shear or triaxial torsional shear test. In the one-way tests, the number of cycles to trigger failure in contractive sand is smaller during a triaxial extension test than during a triaxial compression or triaxial torsional shear test.

11. The findings regarding the effects of stress reversal, the principal stress axis rotation and shear stress amplitude on the pore water pressure and deformation characteristics should be taken into the consideration in the development of constitutive modeling of soils.

12. Finally, the experimental results indicate that the soil triaxial test system, the specimen preparation procedure and the specimen preparation devices developed during this study are quite satisfactory. The system is capable of simulating most of stress paths encountered in the field.

The study presented in here is the first important step in towards the experimental investigation of soil behavior and constitutive modeling. Clearly, further research needs to be done to extend this work as follows:

1. A parallel study on the dense sand under a similar cyclic experimental program is recommended to compare the soil behavior and deformation characteristics of the loose and the dense sands.
2. A simulation and prediction of static and cyclic behavior from one or two existing general constitutive models need to be conducted. The predicted soil behavior needs to be compared based on results from independent monotonic and cyclic tests. The accuracy of the predictions and the overall efficiency of the analysis should be used as the criteria to decide possible modification of the model.

3. An analytic study is recommended to develop the constitutive model in the form of a computer program subroutine for use in finite element analysis.
REFERENCES


[26] Dakoulas, P. and Sun, Y. (1991), "Behavior of Fine Sand under Cyclic Rotation of Principal Stress Using the Hollow Cylinder Apparatus". The Second International Conference on Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics. St. Louis, Missouri, USA.


[92] These are unpublished custom-made computer programs for the control of the entire soil triaxial testing system. The programs were developed during this study with efforts from Shinyuan Yu and Panos C. Dakoulas. Details of the programs could be obtained upon request to PCD.
APPENDIX A

A PROCEDURE FOR THE SET-UP OF TRIAXIAL SAND SPECIMEN

In this appendix, details of preparing the triaxial sand specimen\textsuperscript{[29,45,35,70,81,90,91]} which were used in the study are given. The initial preparation of specimen set-up is presented in section A.1. Section A.2 discusses the set-up of the specimen both for solid and hollow cylinder specimens. Section A.3 provides the procedures of specimen saturation.

A.1 Initial Preparation of Specimen Set-up

A.1-1 Fill a water container with water to be used to fill the triaxial cell. Close the proper valves and apply a vacuum of approximately 10 psi to the water container for at least 30 minutes of cavitation, and then lock in the vacuum until use.

A.1-2 Boil additional water using a pressure canner. In order to get a good B parameter measurement, it is necessary to boil the water at least 1 hour and let the air-bubbles come out completely. Place the cover and the cap to the release valve on the pressure canner and let the water cool, then siphon the water from the bottom of the pressure canner to the water reservoir on the control panel. A vacuum of approximately 10 psi could also be used in this procedure. This water will be used to saturate the specimen and apply the pore pressure.

A.1-3 According to the prescribed testing program, decide upon:

--- dry unit weight of specimen ($\gamma_d$) for a given relatively density ($D_r$).

--- the volume of the specimen ($V_s$) and the weight of the dry sand ($W_s$) for a given specimen.

--- average water content of sand prior to building-in (usually about 6%).

--- number of compaction layers, etc.
A.1-4 Thoroughly mix the dry sand with the water. Next, weigh the sand for each layer with the weight scalar, then carefully put the weighed sand for each layer into a plastic bag and seal it. Once all layer's sands are weighed, the sand will be ready for compaction.

A.2 Specimen Preparation

A.2-1 Preparation of Solid Specimen

A.2-1-1 Clean the base of the cell and fasten the pedestal to the base plate of the cell. Take a rubber membrane with the proper diameter, and examine it too see if it is in good condition. Next, measure the length of the membrane to about 12 inches. Install the rubber membrane over to the upper portion of the pedestal, and then use rubber strips (O-rings) to hold the membrane securely in place. To provide a more impervious joint, the side of pedestal may be lightly coated with silicone grease prior to attaching the membrane; this will increase the seal between the membrane and the pedestal.

A.2-1-2 In this step, you will place a specimen mold around the rubber membrane, and, then, using a vacuum, seal the two together. First, however, coat the inner portion of the mold with powder, to assure easy removal once the specimen has been completed. Next, place the mold around the rubber membrane, then fold the top portion of the membrane over the top of the mold. Hook up the specimen mold to the vacuum pump, and make sure the vacuum seals the membrane to the specimen mold properly. Minimum vacuum should be used (approximately 10 psi). It is necessary to apply silicone grease along the splits where the two halves of the mold connect to ensure a vacuum seal.

A.2-1-3 Carefully place sand into the membrane and try to spread the sand evenly in order to get a fairly uniform composition. A tamper is used to maintain the sample shape and proper density for the specimen. One should use care to avoid puncturing the rubber membrane during compaction. In this study, fifteen layers are divided for each specimen when the specimen is built. A rotational bearing which is put at the bottom of the base plate is used. During compaction, one can rotate the whole base plate of the cell. Try to obtain a layer as uniform as possible
by very gently tamping all around the whole layer and working your way uniformly up to the final height of the specimen.

A.2-1-4 Carefully install step by step the tie rods, the specimen cap, the loading piston, the head plate of the cell, the bearing house and the position locker of the loading piston.

A.2-1-5 Loosen the loading piston locker and lower down the specimen cap. It is important to make sure the specimen cap and the specimen attach securely. Attach the upper part of the rubber membrane over the specimen cap using rubber strips (O-rings). Again, to provide a more impervious joint, the side of the specimen cap needs to be lightly coated with silicone grease prior to attaching the membrane; this will increase the seal between the membrane and the side of specimen cap.

A.2-1-6 Install the slide cylinder. It is necessary to coat the upper portion of the slide cylinder with lubricating grease to assure easy installation of the pressure chamber.

A.2-1-7 Disconnect the vacuum of seal. Apply the vacuum to the specimen through the pore pressure outlets of the base plate so that the shape of the specimen is maintained.

A.2-1-8 Wait about 15 minutes for a complete application of the vacuum. Now remove the specimen mold and observe the membrane for holes and obvious leaks. If any is found, the sample must be rebuilt using a new membrane.

A.2-1-9 Obtain four height measurements approximately 90° apart and use the average value for the initial specimen height. Take three diameter readings at the top, the midheight, and the base by using a "π" tape. Take these measurements to the nearest 0.1 cm or 0.01 in. Compute final average specimen diameter as, 

$$d_{av} = \frac{d_t + d_m + d_b}{3},$$

where \(d_t\) is the average diameter based on top measurements, etc.
A.2-1-10 Install the seal O-rings to the head plate and base plate. In order to provide impervious joints, it is necessary to coat the grooves that are used to hold the O-rings in place with silicone grease.

A.2-1-11 Carefully install the pressure chamber. After this procedure, take off the slide cylinder.

A.2-1-12 Fill the pressure chamber with the water obtained in Section A.1-1 of this appendix. Fill approximately \( \frac{14}{15} \) of the chamber's total volume with this water. In order to avoid air-bubbles, it is necessary to fill the cell through the outlet in the base plate.

A.2-1-13 Apply a predetermined lateral pressure \( \sigma_3 \) to the cell using compressed air and simultaneously reduce the vacuum on the interior of the sample to zero. The purpose of this procedure is to keep the shape of the specimen.

A.2-1-14 Now the specimen is ready to be saturated.

A.2-2 Preparation of Hollow Cylinder Specimen

A.2-2-1 Clean the base of the cell and the positional cylinder for the inner specimen mold. Take an inner membrane and examine it to make sure it is a good one and to see if the length is 15 inches. Install the inner membrane over the inner mold. It is necessary to use powder on the outside surface of the inner mold and the inside surface of the inner membrane to ensure easy installation and removal of the inner mold. Apply silicone grease to the splits where the sections of the inner mold connect to effect a vacuum seal.

A.2-2-2 Install the inner mold to the pedestal and fold the bottom portion of the membrane over the inner mold and the pedestal using rubber strips (O-rings). Fasten the pedestal to the base plate of the cell (Fig. A.1). To provide a more impervious joint, the inside and outside surfaces of the pedestal may be lightly coated with silicone grease prior to attaching the membrane; this will improve the seal between the membrane and the pedestal.
A.2-2-3 Take an outer membrane, examine it to make sure it is a good one, and measure the length to 13 inches. Attach the outer membrane to the pedestal using rubber strips (O-rings). Install the outer mold to the proper position and fold the top portion of the membrane down over the mold (Fig. A.2). To assure easy removal of the mold, it is necessary to use powder on the inside surface of outer mold. In order for the vacuum seal to be effective, it is also necessary to apply the silicon grease to the splits in the sections and the bottom of the outer mold.

A.2-2-4 Hook up the inner mold and outer mold to the vacuum and make sure the inner and outer membranes seal securely to the surfaces of the inside and outside molds. The minimum vacuum which can properly seal the membrane to the specimen mold should be (approximately) 10 psi.

A.2-2-5 Carefully place a small amount of sand from the bag containing the bottom layer material into the mold, and fill in the porous stone by tamping and spreading the sand with a metal tamper. After compaction, scratch the sand surface to ensure better contact with the following layer.

A.2-2-6 Put the rest of the sand from the bag into the mold, spread it evenly in order to get a fairly uniform composition, and then tamp it with the tamper. In this study, a rotational bearing, which is put at the bottom of the base plate is used. During the compaction, one can rotate the whole base plate. Try to obtain a layer as uniform as possible by very gently tamping all around the whole layer and working your way uniformly down to the final height. Tamping gently is necessary to assure the membrane will not be broken (Fig. A.3).

A.2-2-7 Adjust the tamper reading, i.e., the tamper rod height to the height of the second layer (Fig. A.4).

A.2-2-8 Scratch the sand surface to ensure better contact with the following layer. Carefully dump the sand from the second layer bag into the mold, spread it evenly to get a fairly uniform composition and then tamp it with the tamper. Try to obtain a layer as uniform as possible.

A.2-2-9 Scratch the sand surface to ensure better contact with the following layer. Use the same procedure while compacting the following layers. In this research,
fifteen layers are divided for each specimen in order to get a uniform specimen. The top of the last layer must be scratched before it is attached with the specimen cap.

A.2-2-10 Carefully install the tie rods, the specimen cap, loading piston, head plate of the cell, the bearing house and the position locker of the loading piston, etc. in proper position by steps.

A.2-2-11 Loosen the loading piston locker and lower down the specimen cap. It is necessary to make sure the porous stone and the top of the specimen attach securely.

A.2-2-12 Fold the top portion of the inner membrane down over the inside of the specimen cap and fasten it using rubber strips (O-rings). Attach the upper part of the outer membrane to the specimen cap using rubber strips (O-rings). Again, to provide a more impervious joint, the inside and the outside surfaces of the specimen cap need to be lightly coated with silicone grease prior to attaching the inner and outer membranes (Fig.A.5).

A.2-2-13 Install the slide cylinder. It is necessary to coat the upper portion of the slide cylinder with lubricating grease in order to ensure easy installation of the pressure chamber (Fig.A.5).

A.2-2-14 Disconnect the vacuum seal. Apply the vacuum to the specimen through the pore pressure outlets in order to maintain the shape of the specimen.

A.2-2-15 Wait about 15 minutes for a complete application of the vacuum; then, carefully remove the inner mold from the bottom of the base plate (Fig.A.6). Observe the inner membrane for holes and obvious leaks. If any is found, the specimen must be remade using a new membrane.

A.2-2-16 Carefully remove the outer mold of the specimen. Again, observe the membrane for holes and obvious leaks. If any is found, the sample must be remade using a new membrane (Fig.A.7).
A.2-2-17 Install the seal O-rings to the head plate and base plate. In order to provide impervious joints, it is necessary to coat the grooves that hold the O-rings in place with silicone grease (Fig.A.7).

A.2-2-18 Carefully install the pressure chamber, then remove the slide cylinder (Fig.A.8).

A.2-2-19 Fill the pressure chamber with the de-aired water obtained in Section A.1-1 of this appendix. Fill approximately $\frac{14}{15}$ of the chamber's total volume with this water. In order to avoid air-bubbles, it is necessary to fill the cell through the outlet in the base plate.

A.2-1-20 Apply a predetermined lateral pressure $\sigma_3$ to the cell using compressed air and simultaneously reduce the vacuum on the interior of the sample to zero. This procedure is executed to keep the shape of the specimen.

A.2-1-21 Now the specimen is ready to be saturated (Fig.A.9).

A.3 Specimen Saturation

A.3-1 Make sure that all the valves of the CO$_2$ system are closed. Connect the CO$_2$ tube to the bottom pore pressure outlet. Once this valve is opened, CO$_2$ will percolate upwards through the specimen.

A.3-2 Connect a tube to the top pore pressure outlet and put the other end of it into a jar full of water.

A.3-3 Open the CO$_2$ valve connected to the specimen pedestal just slightly, then let the CO$_2$ percolate through the specimen slowly so that only about 1 to 2 bubbles per second are coming into the jar from the top of the specimen.

A.3-4 Percolate the CO$_2$ for at least 1 hour.

A.3-5 Close the CO$_2$ output from the tank and then close the CO$_2$ input valve to the specimen. Afterwards, close the CO$_2$ output from the specimen, then, disconnect the CO$_2$ tubes from the cell.
A.3-6 Connect the water tube from the water reservoir on the control panel to the bottom pore pressure outlet. Open the top cap valve and then open the valve to the specimen pedestal. Water from this water reservoir will now slowly percolate upwards through the specimen and replace the CO₂ in the specimen.

A.3-7 Approximately one third of the water from the water reservoir has to percolate for a full saturation. In order to obtain a fully saturated specimen in a reasonable amount of time, one can use a slight vacuum (less than 5 psi) on the sample to speed up the saturation process.

A.3-8 Increase simultaneously the cell pressure and the back pressure while maintaining a stress difference of 3 psi. Increase the pore pressure and cell pressure to the desired values (usually a pore pressure of 30 psi and a cell pressure of 33 psi).

A.3-9 Leave the cell overnight to ensure full saturation. Trapped air under a pore pressure of 30 psi should be dissolved completely during this period of time.

A.3-10 After sitting overnight, the sample should be fully saturated and is ready to be tested.
Fig. A.1  Specimen Preparation (1)
Fig. A.2  Specimen Preparation (2)
Fig. A.3  Specimen Preparation (3)
Fig. A.4 Specimen Preparation (4)
Fig. A.5  Specimen Preparation (5)
Fig. A.6  Specimen Preparation (6)
Fig. A.7  Specimen Preparation (7)
Fig. A.8 Specimen Preparation (8)
Fig. A.9 Specimen Preparation (9)