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Development of a large space robot: A multi-segment approach

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Rice University, 1991
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DEVELOPMENT OF A LARGE SPACE ROBOT: 
A MULTI-SEGMENT APPROACH

by

Reginald B. Berka

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ABSTRACT

A multi-segment robot is developed as a concept for use in space based construction operations. The multi-segment robot is envisioned as a member of a class of large space robots, or space cranes, used in the assembly of advanced spacecraft. The problems that arise when the requisite robot size becomes large are explored. The unique capabilities of the multi-segment robot are discussed.

The multi-segment robot involves a collection of common bodies, or segments, that are pinned together to form a snake-like, or train, configuration. A degree of freedom representing rotation is retained at each pinned connection. Reaction flywheels are suspended within each segment and provide the control necessary to position each body segment.

Algorithms are developed to position this serpentine robot to a prescribed location and orientation. The first algorithm is used to compute a general shape, based on a constrained polynomial function, that locates the robot tip at the proper position. Next, an algorithm is developed that is used to position the discrete bodies along the shape function and determines their relative positions. This
information is used as the target values in a control system that uses the reaction flywheels to position each body into the desired relative position.

An n-body simulation program is developed based on Newton-Euler equations of motion for the robot. The simulation is used to develop the robot control strategy, to verify its performance, and to size prototype hardware. Two cases are analyzed to investigate the dexterity of the proposed configuration.

Robot design issues are explored as they relate to the multi-segment robot. A prototype system is designed, fabricated, and tested. Motion tests are included that compare experimental results with pertinent analytical predictions.

Collectively, the present study demonstrates the viability of the proposed concept for addressing the unique problems associated with large robotic operations in space.
ACKNOWLEDGEMENTS

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# Table of Contents

1 INTRODUCTION ........................................ 1

2 PROPOSED CONFIGURATION .............................. 10

3 MATHEMATICAL MODELING .............................. 13
   3.1 Preliminary Remarks .............................. 13
   3.2 Kinematics of a Large, Multi-Segment Robot .... 15
   3.3 Kinetics of a Large, Multi-Segment Robot ...... 23

4 ALGORITHMS ............................................. 33
   4.1 Preliminary Remarks .............................. 33
   4.2 Kinematic Tip Positioning Algorithm .............. 33
      4.2.1 Shape Function .............................. 34
      4.2.2 Body Positioning ............................ 38
   4.3 Body Segment Positioning Control System ....... 45

5 NUMERICAL SIMULATION ............................... 48
   5.1 Preliminary Remarks .............................. 48
   5.2 Case 1 ............................................ 52
   5.3 Case 2 ............................................ 80

6 DEMONSTRATION ....................................... 100
   6.1 Preliminary Remarks .............................. 100
   6.2 Body Housing ..................................... 104
   6.3 Electric Motor .................................... 105
   6.4 Flywheel ......................................... 108
   6.5 Rate Gyro ........................................ 110
   6.6 Brakes - Electromagnetic Clutch .................. 111
   6.7 Control Computer .................................. 113
   6.8 Air Bearing Support System ....................... 120
   6.9 Testing ........................................... 121
   6.10 Correlation ..................................... 159

7 SUMMARY ............................................... 168

8 EPILOGUE .............................................. 175

9 REFERENCES ............................................ 177
Table of Figures

<table>
<thead>
<tr>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Anthropomorphich Robot</td>
<td>8</td>
</tr>
<tr>
<td>Geodesic Truss Beam Robot</td>
<td>8</td>
</tr>
<tr>
<td>Multi-segment Articulated System</td>
<td>9</td>
</tr>
<tr>
<td>Multi-segment Robot Coordinate System</td>
<td>14</td>
</tr>
<tr>
<td>Body Coordinate System and Vector Convention</td>
<td>17</td>
</tr>
<tr>
<td>Segment Free Body Diagram</td>
<td>26</td>
</tr>
<tr>
<td>First Body Segment Positioning</td>
<td>41</td>
</tr>
<tr>
<td>Second Body Segment Positioning</td>
<td>44</td>
</tr>
<tr>
<td>Control System Block Diagram</td>
<td>46</td>
</tr>
<tr>
<td>Coordinate System and Initial Robot Position</td>
<td>51</td>
</tr>
<tr>
<td>Shape Function, Case 1</td>
<td>54</td>
</tr>
<tr>
<td>Angular Position - Body 1, Case 1, Simulation</td>
<td>63</td>
</tr>
<tr>
<td>Angular Position - Body 2, Case 1, Simulation</td>
<td>63</td>
</tr>
<tr>
<td>Angular Position - Body 3, Case 1, Simulation</td>
<td>64</td>
</tr>
<tr>
<td>Angular Position - Body 4, Case 1, Simulation</td>
<td>64</td>
</tr>
<tr>
<td>Angular Position - Body 5, Case 1, Simulation</td>
<td>65</td>
</tr>
<tr>
<td>Angular Position - Body 6, Case 1, Simulation</td>
<td>65</td>
</tr>
<tr>
<td>Angular Rate - Body 1, Case 1, Simulation</td>
<td>67</td>
</tr>
<tr>
<td>Angular Rate - Body 2, Case 1, Simulation</td>
<td>67</td>
</tr>
<tr>
<td>Angular Rate - Body 3, Case 1, Simulation</td>
<td>68</td>
</tr>
<tr>
<td>Angular Rate - Body 4, Case 1, Simulation</td>
<td>68</td>
</tr>
<tr>
<td>Angular Rate - Body 5, Case 1, Simulation</td>
<td>69</td>
</tr>
<tr>
<td>Angular Rate - Body 6, Case 1, Simulation</td>
<td>69</td>
</tr>
<tr>
<td>Torque - Body 1, Case 1, Simulation</td>
<td>74</td>
</tr>
<tr>
<td>Torque - Body 2, Case 1, Simulation</td>
<td>74</td>
</tr>
<tr>
<td>Torque - Body 3, Case 1, Simulation</td>
<td>75</td>
</tr>
<tr>
<td>Torque - Body 4, Case 1, Simulation</td>
<td>75</td>
</tr>
<tr>
<td>Torque - Body 5, Case 1, Simulation</td>
<td>76</td>
</tr>
<tr>
<td>Torque - Body 6, Case 1, Simulation</td>
<td>76</td>
</tr>
<tr>
<td>Initial Position, Case 1, Simulation</td>
<td>78</td>
</tr>
<tr>
<td>Robot Position at t = 2.0 sec., Case 1, Simulation</td>
<td>78</td>
</tr>
<tr>
<td>Robot Position at t = 4.0 sec., Case 1, Simulation</td>
<td>79</td>
</tr>
<tr>
<td>Final Position, Case 1, Simulation</td>
<td>79</td>
</tr>
<tr>
<td>Shape Function, Case 2</td>
<td>81</td>
</tr>
<tr>
<td>Angular Position - Body 1, Case 2, Simulation</td>
<td>85</td>
</tr>
<tr>
<td>Angular Position - Body 2, Case 2, Simulation</td>
<td>85</td>
</tr>
<tr>
<td>Angular Position - Body 3, Case 2, Simulation</td>
<td>86</td>
</tr>
<tr>
<td>Angular Position - Body 4, Case 2, Simulation</td>
<td>86</td>
</tr>
<tr>
<td>Angular Position - Body 5, Case 2, Simulation</td>
<td>87</td>
</tr>
<tr>
<td>Angular Position - Body 6, Case 2, Simulation</td>
<td>87</td>
</tr>
<tr>
<td>Angular Rate - Body 1, Case 2, Simulation</td>
<td>89</td>
</tr>
<tr>
<td>Angular Rate - Body 2, Case 2, Simulation</td>
<td>89</td>
</tr>
<tr>
<td>Angular Rate - Body 3, Case 2, Simulation</td>
<td>90</td>
</tr>
<tr>
<td>Angular Rate - Body 4, Case 2, Simulation</td>
<td>90</td>
</tr>
<tr>
<td>Angular Rate - Body 5, Case 2, Simulation</td>
<td>91</td>
</tr>
<tr>
<td>Angular Rate - Body 6, Case 2, Simulation</td>
<td>91</td>
</tr>
<tr>
<td>Torque - Body 1, Case 2, Simulation</td>
<td>94</td>
</tr>
<tr>
<td>Torque - Body 2, Case 2, Simulation</td>
<td>94</td>
</tr>
<tr>
<td>Torque - Body 3, Case 2, Simulation</td>
<td>95</td>
</tr>
<tr>
<td>Topic</td>
<td>Page</td>
</tr>
<tr>
<td>--------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>Torque - Body 4, Case 2, Simulation</td>
<td>95</td>
</tr>
<tr>
<td>Torque - Body 5, Case 2, Simulation</td>
<td>96</td>
</tr>
<tr>
<td>Torque - Body 6, Case 2, Simulation</td>
<td>96</td>
</tr>
<tr>
<td>Initial Position, Case 2, Simulation</td>
<td>98</td>
</tr>
<tr>
<td>Robot Position at t = 2.0 sec., Case 2, Simulation</td>
<td>98</td>
</tr>
<tr>
<td>Robot Position at t = 5.0 sec., Case 2, Simulation</td>
<td>99</td>
</tr>
<tr>
<td>Final Position, Case 2, Simulation</td>
<td>99</td>
</tr>
<tr>
<td>Segment Assembly Cross-section</td>
<td>101</td>
</tr>
<tr>
<td>Segment Assembly Parts List</td>
<td>102</td>
</tr>
<tr>
<td>Complete Six Body Robot Assembly</td>
<td>103</td>
</tr>
<tr>
<td>Test Facility Schematic</td>
<td>123</td>
</tr>
<tr>
<td>Computer/Amplifier/Motor Schematic</td>
<td>127</td>
</tr>
<tr>
<td>Angular Position - Body 1, Test</td>
<td>143</td>
</tr>
<tr>
<td>Angular Position - Body 2, Test</td>
<td>143</td>
</tr>
<tr>
<td>Angular Position - Body 3, Test</td>
<td>144</td>
</tr>
<tr>
<td>Angular Position - Body 4, Test</td>
<td>144</td>
</tr>
<tr>
<td>Angular Position - Body 5, Test</td>
<td>145</td>
</tr>
<tr>
<td>Angular Position - Body 6, Test</td>
<td>145</td>
</tr>
<tr>
<td>Angular Rate - Body 1, Test</td>
<td>149</td>
</tr>
<tr>
<td>Angular Rate - Body 2, Test</td>
<td>149</td>
</tr>
<tr>
<td>Angular Rate - Body 3, Test</td>
<td>150</td>
</tr>
<tr>
<td>Angular Rate - Body 4, Test</td>
<td>150</td>
</tr>
<tr>
<td>Angular Rate - Body 5, Test</td>
<td>151</td>
</tr>
<tr>
<td>Angular Rate - Body 6, Test</td>
<td>151</td>
</tr>
<tr>
<td>Control Torque - Body 1, Test</td>
<td>153</td>
</tr>
<tr>
<td>Control Torque - Body 2, Test</td>
<td>153</td>
</tr>
<tr>
<td>Control Torque - Body 3, Test</td>
<td>154</td>
</tr>
<tr>
<td>Control Torque - Body 4, Test</td>
<td>154</td>
</tr>
<tr>
<td>Control Torque - Body 5, Test</td>
<td>155</td>
</tr>
<tr>
<td>Control Torque - Body 6, Test</td>
<td>155</td>
</tr>
<tr>
<td>Initial Position, Test</td>
<td>157</td>
</tr>
<tr>
<td>Robot Intermediate Position 1, Test</td>
<td>157</td>
</tr>
<tr>
<td>Robot Intermediate Position 2, Test</td>
<td>158</td>
</tr>
<tr>
<td>Final Position, Test</td>
<td>158</td>
</tr>
<tr>
<td>Position Test/Simulation Correlation - Body 1</td>
<td>165</td>
</tr>
<tr>
<td>Position Test/Simulation Correlation - Body 2</td>
<td>165</td>
</tr>
<tr>
<td>Position Test/Simulation Correlation - Body 3</td>
<td>166</td>
</tr>
<tr>
<td>Position Test/Simulation Correlation - Body 4</td>
<td>166</td>
</tr>
<tr>
<td>Position Test/Simulation Correlation - Body 5</td>
<td>167</td>
</tr>
<tr>
<td>Position Test/Simulation Correlation - Body 6</td>
<td>167</td>
</tr>
</tbody>
</table>
Table of Tables

<table>
<thead>
<tr>
<th>Table Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape Function Parameters, Case 1</td>
<td>53</td>
</tr>
<tr>
<td>Target Inertial Angles, Case 1</td>
<td>56</td>
</tr>
<tr>
<td>Target Relative Angles, Case 1</td>
<td>57</td>
</tr>
<tr>
<td>Control System Gains, Simulation</td>
<td>59</td>
</tr>
<tr>
<td>Shape Function Parameters, Case 2</td>
<td>80</td>
</tr>
<tr>
<td>Target Inertial Angles, Case 2</td>
<td>82</td>
</tr>
<tr>
<td>Target Relative Angles, Case 2</td>
<td>83</td>
</tr>
<tr>
<td>Control System Gains, Test</td>
<td>136</td>
</tr>
</tbody>
</table>
1 INTRODUCTION

Recent space exploration initiatives include a return to the Moon, and a manned mission to Mars [7]. These plans have identified space vehicle requirements that support these exploration missions. In addition, ambitious near-Earth missions have also been identified in the areas of remote sensing and communications. All of these missions are typified by systems and vehicles of unprecedented size. The large size of these systems dictates on-orbit assembly. Launching these systems directly from the Earth’s surface is neither possible nor practical.

The assembly of large space systems will require the design, fabrication, and placement of new assembly equipment in low Earth orbit (LEO). Special assembly facilities will be used to accomplish the final assembly of these vehicles. Current plans [39] for these facilities include the use of large robotic devices that position the subassemblies for mating to produce the final assembled vehicle. The positioning of subassemblies will require strict control of the manipulator’s end effector position and orientation.

As the size of robotic systems increase, technical problems that could be overlooked or ignored at conventional
sizes can become driving design considerations. New fundamental problems begin to surface when robotic manipulators grow in size.

Conventional industrial robots have reach envelopes of about five to ten feet. The Remote Manipulator System (RMS) on the Space Shuttle Orbiter has a maximum reach distance of close to fifty feet. To accomplish assembly of large space systems, robotic equipment will require reach capabilities of over two hundred feet. A design constraint for all space based equipment requires launch to low Earth orbit (LEO) by the Space Shuttle. This requires single launch system weights of less than 40,000 pounds. In addition, the single launch volume must fit into the 60 feet long by 15 feet diameter of the Space Shuttle’s payload bay [38].

To accomplish the required assembly of space vehicles or systems these large robotic devices must be able to position large subassemblies precisely for mating during final assembly. This precise positioning must be achieved through proper compensation for both kinetic and kinematic effects.

The large size of robotic arms used in space assembly leads to a new set of engineering challenges. The long arm lengths amplify the respective joint errors leading to tip static position errors. Typical industrial robots can locate the tip to within a quarter of an inch of a specified
three space coordinate [48]. The accuracy of this positioning is a function of the joint position encoding capability of the device. The Orbiter RMS has arm lengths an order of magnitude greater than the typical industrial robot, but this increase in size is accompanied by a similar increase in static tip positioning error [38]. The inability of the RMS to position the tip accurately has reduced its use and effectiveness in an automated operating mode. For assembly of advanced space systems, the large scale robotic arms must meet stringent positioning requirements. Before large space robots can be used, an effective means for negating tip position errors must be determined. If this problem is ignored, tip errors could be of the order of five to ten feet. Positioning errors of this magnitude are too large to support subassembly mating.

The long reach capability of large robots results in low frequency dynamic behavior. Lengthy arms exhibit low frequency beam dynamics when excited by joint torques or applied loads. Further, the large masses associated with vehicle subassemblies located at the arm tip contribute to this low frequency dynamic behavior. The tip control algorithm must incorporate some strategy to reduce the dilatory effects of dynamic tip motion.
Long arm lengths induce a reaction moment problem for the base of the robot. In space, robots are not attached to rigid bases with excessive strength capacity. Due to launch considerations, the structure supporting space robots reflects a minimum weight design. Forces applied to the robot tip are amplified by the long arm lengths to yield high base reaction moments. The base structure will not tolerate the excessive moments caused by these forces. Therefore, as robots increase in length, consideration must be given to the corresponding increase in base reaction moments.

As the distance from the base to the tip increases, the rotational inertia increases with the square of this distance. This rapid increase in rotational inertia requires special technical attention to ensure that sufficient base torque is provided to move the tip mass.

Identifying the important problems early in the design process allows high level configuration decisions that can minimize these detrimental effects. These difficult problems need to be solved irrespective of the robotic system utilized. Long reach arms with limited cross-section will always result in low frequency dynamics and kinematic positioning problems. However, configuration trade studies must be evaluated in the context of helping to control the
difficulty of these inherent problems. As with all robotic problems, the integration of system kinetics, computer programming, and hardware implementation is necessary to produce a workable design.

Several kinds of robotic configurations are possible to meet the sizing and performance requirements outlined earlier. There are three basic systems that will be evaluated for their suitability for large robotic applications. Each system will be considered for the inherent abilities of minimizing the kinematic and kinetic problems associated with large robotic systems.

Standard industrial robotic arms are modeled after their human equivalents [2, 11, 43]. Shoulder, elbow, and wrist joints are designed with sufficient degrees of freedom to enable the arm to position the tip at the proper location and orientation. This type of robot has been utilized extensively, almost exclusively, in industrial applications. Anthropomorphic robots have reached a level of maturity not achieved in other robotic configurations. Analytical methods for this robotic configuration are well documented in the literature [11, 12, 27, 43]. Mathematical modeling of these systems has been reduced to the manual entry of system configuration parameters into existing software procedures. Figure 1-1 provides an example of this kind of robot.
Truss members can be formed in such a way as to produce a geodesic beam [49]. By varying the length of the struts in this beam, the tip position can be changed. One advantage of this kind of system is that the robotic arm, which in this case is the tetrahedral beam, can be formed into curved shapes. This capability can be helpful under certain collision avoidance conditions. The strut extension device is common throughout the system and is a relatively simple control effector. Another advantage of this type of system is the redundancy that multiple, common actuators can provide. If an actuator should fail, other strut motions can compensate for the loss of motion in one cell. The tip can still be positioned properly with the remaining strut actuators. Figure 1-2 provides an example of this type of robot.

A segmented arm that is comprised of common body modules is another candidate for large robotic arms. In this configuration, each body module, or segment, is equipped with a rotational degree(s) of freedom at each end. Each section can be characterized as a rigid link connected to the next body section. The revolute degree of freedom between body sections controls the shape of the arm and ultimately the tip position. The body structural stiffness and relatively light weight result in high flex frequencies for the body
segment allowing it to be modeled as a rigid element. Figure 1-3 provides an example of this kind of robot. Some related work done in this area can be found in [9, 24, 29].
Figure 1-1  Standard Anthropomorphic Robot
(reprinted from [48])

Figure 1-2  Geodesic Truss Beam Robot
(reprinted from [49])
Figure 1-3 Multi-segment Articulated System
2 PROPOSED CONFIGURATION

For a large, space-based construction robot, the configuration proposed is the multi-segment robot. As mentioned earlier, this configuration includes a stiff body structure with a rotational degree of freedom at each end. Multi-segment robots have been proposed in several configurations [9, 24, 29]. For the large space robot, a new configuration is proposed using the multi-segment approach. This new kind of robot includes a bearing and brake assembly at each end of a diamond shaped rigid body segment. An inertial measuring system is integrated into the common body segments. These sensors are required to supply inertial position and rate information to the control computer. In addition, each segment has its own body mounted angular momentum variation device for exerting torques on the body segment. These momentum devices can be comprised of any circulating mass such as reaction wheels or fluid loops. In this case, a flywheel, driven by an electric motor, constitutes the angular momentum device. Changing the angular momentum of this device will produce a torque to the body segment that is proportional to the rate of this change. This acts as an externally applied torque to the robot body segment. If the base joint brake is released while every other joint brake is locked, the torques exerted on the individual bodies will
add up to provide increased torque about the base joint. Further, any joint can be released for rotation while keeping other joints locked and a similar increase in torque about the released joint will result. This yields a system of common body segments that provide a torque gradient that is largest at the base and smallest at the tip. This is the desired gradient to handle the larger rotational inertia at the base and the least rotational inertia at the body segment near the tip.

Further, this configuration has the unique capacity for solving the kinematic, kinetic, and base moment problems mentioned earlier. The ability to position outboard body segments while fixing the inboard body segments allows the robot to compensate for accumulated joint errors. Hence, there is an inherent ability to provide fine positioning control of the tip by adjusting the outboard body segments.

The multi-segment robot, designed with independent state vector sensors and torquers, provides an ideal collocation of sensors and effectors for damping low frequency vibration. Each body segment can sense angular rate changes and command the segment torquers to provide inputs out of phase with this sensed angular rate. With the joints locked, this provides active damping to the entire robotic system.
In space the robot base is attached to structure with limited strength due to weight considerations. For large space robots tip loads applied during assembly operations are multiplied by the long moment arm resulting in high base reaction moments. With the joint brakes locked, the multi-segment configuration can use its torquing devices to react these moments. This minimizes the structural requirements placed on the base support structure.

The capacity of the multi-segment robot configuration to solve the problems inherent in large space robots is the basis for its selection. This study includes the development of a tip positioning system, a dynamic computer simulation, a real time control system, and a scale model implementation of the proposed concept.
3 MATHEMATICAL MODELING

3.1 Preliminary Remarks

For this study the robot will be confined to operate in a two dimensional plane. This plane is parallel to the tangent of the Earth's surface. Restricting motion to this plane is necessary for eliminating static torques at the robot base due to gravity acting on the robot center of mass. Any static torque, when integrated over time, would quickly saturate the momentum torque effectors. Since a scale model implementation is planned, the computer simulation will be confined to a similar plane. The coordinate frame used in the analysis involves an inertial frame of orthogonal X and Y vectors originating at the base of the robot. At each body segment, a local x-y frame is in the same plane as the inertial frame. However, the local x-y frame is rotated with respect to the inertial X-Y frame as a function of the segment joint angles. Refer to Figure 3-1 for an illustration of the coordinate system.
Figure 3-1  Multi-segment Robot Coordinate System
3.2 Kinematics of a Large, Multi-Segment Robot

A general treatment of the kinematic and kinetic equations for multiple arm robots can be found in [35,51]. In many respects, the ensuing development of these equations follows this previous work.

Let \( \mathbf{r}_i \) represent the position vector from the inertial reference coordinate frame to the near end of the \( i \)th body segment (\( i \)th joint position). Let \( \mathbf{s}_i \) represent the position vector from the \( \mathbf{r}_i \) end of the body segment to the \( \mathbf{r}_{i+1} \) end of the segment. Then,

\[
\mathbf{r}_i = \mathbf{r}_{i-1} + \mathbf{s}_{i-1}
\]

The body segment is capable of rotating about a local z axis that is perpendicular to the local x-y frame. The angular velocity vector, along the local z-axis, is parallel to the inertial z-axis. Also, due to the planar motion constraints, changes in the angular velocity vector at each body involve only magnitude and not vector direction. In addition, all degrees of freedom are revolute (rotational). Therefore, local body position vectors, from one end of the body to the other, are constant within the body. This fact simplifies the equations of
motion considerably since derivatives involve only the changes due to rotational motion. The robot coordinate system and vector conventions are shown in Figure 3-2.
Figure 3-2  Body Coordinate System and Vector Convention
Taking the first derivative of the position vector equation leads to

\[ \mathbf{v}_i = \mathbf{v}_{i-1} + \mathbf{w}_{i-1} \times \mathbf{s}_{i-1}, \]  

(2)

where \( \mathbf{w}_{i-1} \) represents the angular velocity vector of the \( i-1 \) body. The second derivative leads to the recurrence relation for the acceleration of the joint positions. That is,

\[ \mathbf{a}_i = \mathbf{a}_{i-1} + \mathbf{w}_{i-1} \times \mathbf{s}_{i-1} + \mathbf{w}_{i-1} \times (\mathbf{w}_{i-1} \times \mathbf{s}_{i-1}) \]  

(3)

Similar recurrence relations can be found for the center of mass for each segment. Specifically,

\[ \mathbf{r}_{ci} = \mathbf{r}_i + \mathbf{s}_{ci} \]  

(4)

\[ \mathbf{v}_{ci} = \mathbf{v}_i + \mathbf{w}_i \times \mathbf{s}_{ci} \]  

(5)

\[ \mathbf{a}_{ci} = \mathbf{a}_i + \mathbf{w}_i \times \mathbf{s}_{ci} + \mathbf{w}_i \times (\mathbf{w}_i \times \mathbf{s}_{ci}) \]  

(6)

In the above relations the symbol \( \mathbf{w}_i \) denotes the total angular velocity of the body with respect to inertial space. Therefore, the body angular velocity is the vectorial sum of the angular velocity of the previous bodies in the segmented chain plus the angular velocity of the \( i \)th body relative to the previous body. Thus,
\[ \dot{\omega}_i = \dot{\omega}_{i-1} + \omega_{(i/i-1)} \]  

(7)

where \( \omega_{(i/i-1)} \) represents the angular velocity of the \( i \)th body relative to the \( i-1 \) body. The angular acceleration can be found by differentiating the above expression to yield

\[ \ddot{\omega}_i = \ddot{\omega}_{i-1} + \dot{\omega}_{(i/i-1)} + \omega_{(i-1)} \times \omega_{(i/i-1)} \]  

(8)

However, since all of the angular velocity vectors are parallel, the last term in the above equation vanishes. Thus, one obtains

\[ \ddot{\omega}_i = \ddot{\omega}_{i-1} + \dot{\omega}_{(i/i-1)} \]  

(9)

The above equations describe the position, velocity, and acceleration of the joints and center of mass for the \( i \)th body segment of the large space robot. These recursive relations can be represented in a matrix form. The matrix \( U \) is introduced as a matrix of unit vectors that describe the directions associated with the angular velocity of each body segment. With this matrix, the angular velocity vector for the complete robot can be represented as the product of the \( U \) matrix and a vector of scalars. The scalar vector represents the angular velocity at each joint in the robot.
This relation can be expressed by the equation

\[
\begin{pmatrix}
\omega_1 \\
\omega_2 \\
\omega_3 \\
\vdots \\
\omega_n
\end{pmatrix} =
\begin{bmatrix}
\mathbf{u}_1 & 0 & \cdots & 0 \\
\mathbf{u}_1 & \mathbf{u}_2 & 0 & \cdots & 0 \\
\mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\
\mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \cdots & \cdots & \cdots & \mathbf{u}_n
\end{bmatrix}
\begin{pmatrix}
\dot{\phi}_1 \\
\dot{\phi}_2 \\
\dot{\phi}_3 \\
\vdots \\
\dot{\phi}_n
\end{pmatrix}
\]

(10)

The above expression is used to find a similar equation for the angular acceleration vector. The angular velocity vectors are all parallel causing the cross-product terms to vanish. Therefore, the derivative of the above expression can be written as

\[
\begin{pmatrix}
\ddot{\omega}_1 \\
\ddot{\omega}_2 \\
\ddot{\omega}_3 \\
\ddots \\
\ddot{\omega}_n
\end{pmatrix} =
\begin{bmatrix}
\mathbf{u}_1 & 0 & \cdots & 0 \\
\mathbf{u}_1 & \mathbf{u}_2 & 0 & \cdots & 0 \\
\mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \ddots & \ddots \\
\mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 & \cdots & \cdots & \cdots & \mathbf{u}_n
\end{bmatrix}
\begin{pmatrix}
\ddot{\phi}_1 \\
\ddot{\phi}_2 \\
\ddot{\phi}_3 \\
\ddots \\
\ddot{\phi}_n
\end{pmatrix}
\]

(11)

To aid in representing the recursive kinematic relations the matrices \(\mathbf{S}, \mathbf{\Omega},\) and \(\mathbf{U}\) are introduced. They are defined by the equations
\[
\begin{bmatrix}
S_{ct} & 0 & \cdots & 0 \\
S_{1} & S_{ct2} & 0 & \cdots & 0 \\
S_{1} & S_{2} & S_{ct3} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
S_{1} & S_{2} & S_{3} & \cdots & S_{cn} & 0 \\
\end{bmatrix}
\] (12)

\[
\begin{bmatrix}
\omega_{1} & 0 & \cdots & 0 \\
0 & \omega_{2} & 0 & \cdots & 0 \\
0 & 0 & \omega_{3} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & \cdots & 0 & \cdots & 0 & \omega_{n} \\
\end{bmatrix}
\] (13)

and

\[
\begin{bmatrix}
u_{1} & 0 & \cdots & 0 \\
u_{1} & u_{2} & 0 & \cdots & 0 \\
u_{1} & u_{2} & u_{3} & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
u_{1} & \cdots & \cdots & \cdots & \cdots & u_{n} \\
\end{bmatrix}
\] (14)

The expressions for the angular velocity and acceleration can be substituted into the equations for the linear velocity and acceleration of the center of mass. Utilizing the definitions for \( S_c \) and \( U \) leads to the following matrix relation for the body linear velocity.
\[
\begin{pmatrix}
\dot{\nu}_{c1} \\
\dot{\nu}_{c2} \\
\vdots \\
\dot{\nu}_{cn}
\end{pmatrix} = -[S_c] \times [U] 
\begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\vdots \\
\dot{\theta}_n
\end{pmatrix} 
\]  
(15)

In a similar fashion, the equation for the vector of center of mass linear accelerations is given by the equation

\[
\begin{pmatrix}
\ddot{\nu}_{c1} \\
\ddot{\nu}_{c2} \\
\vdots \\
\ddot{\nu}_{cn}
\end{pmatrix} = -[S_c] \times [U] 
\begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\vdots \\
\dot{\theta}_n
\end{pmatrix} + ([S_c] \times [\Omega]) \times [U] 
\begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\vdots \\
\dot{\theta}_n
\end{pmatrix} 
\]  
(16)

The equations for linear and angular acceleration can be combined into a matrix representation in the following form

\[
\begin{pmatrix}
\dot{\nu}_c \\
\ddot{\nu}_c
\end{pmatrix} = 
\begin{bmatrix}
-([S_c] \times [U]) & ([S_c] \times [\Omega]) \times [U]
\end{bmatrix} 
\begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\vdots \\
\dot{\theta}_n
\end{pmatrix} 
\]  
(17)
Let

\[ [B] = \begin{bmatrix} -[S_c] \times [U] \\ [U] \end{bmatrix} \]  \hspace{1cm} (18)

and

\[ [C] = \begin{bmatrix} ([S_c] \times [\Omega]) \times [U] \\ 0 \end{bmatrix} \]  \hspace{1cm} (19)

Then, the equation for the linear and angular acceleration becomes

\[ \begin{bmatrix} \dot{V}_c \\ \dot{\omega}_c \end{bmatrix} = [B] \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \vdots \\ \dot{\theta}_n \end{bmatrix} + [C] \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \vdots \\ \ddot{\theta}_n \end{bmatrix} \]  \hspace{1cm} (20)

The above equations represent the kinematics of the center of mass for the robot body segments.

3.3 Kinetics of a Large, Multi-Segment Robot

There are many different methods for deriving the equations of motion of a robot. Since the vectorial nature of this multi-segment robot will lead to simplified recursive equations, the Newton-Euler method has
been selected for the present analysis. The mathematical
description of the robot starts with the vector descrip-
tion of the robot in a Newton-Euler form. Consistent
with the planar motion constraint (motion plane is
parallel to the Earth’s surface tangent), gravity forces
will be neglected. The robot is envisioned as an
orbital, space-based device. Therefore, the removal of
gravity from the equations is consistent with the problem
model. If $\mathbf{f}_i$ is used to describe the interactive shear
forces that exist between the bodies at the connection
joints, and $m_i$ represents the body mass, the $i$th segment
variation in body linear momentum is given by the equa-
tion

$$\mathbf{f}_i - \mathbf{f}_{i+1} = m_i \dot{\mathbf{v}}_i \quad (21)$$

The equation for body changes in angular momentum can
be written as

$$\mathbf{T}_i - s_i \times \mathbf{f}_i - (s_i - s_{i+1}) \times \mathbf{f}_{i+1} = I_i \ddot{\mathbf{\omega}}_i \quad (22)$$

In the above equation, $\mathbf{T}_i$ represents the control
torque provided by the momentum torque effector. The
$s_i \times \mathbf{f}$ form results from the position vector to the body
center of mass crossed into the interactive forces
between successive bodies. Since the motion of the body is restricted to a single plane, the inertia tensor becomes a scalar value depicted as $I$, in the equation. A free body diagram illustrating the vectorial description of these equations of motion is shown in Figure 3-3.
Figure 3-3  Segment Free Body Diagram
The equations for each body segment can be condensed into a single matrix equation. Representation in a matrix form enables the simultaneous solution of the equations of motion. Also, by examining the form, the matrix representation displays the structure of the coupled equations.

For the force equation, the matrix representation is given by the equation

\[
\begin{pmatrix}
\begin{bmatrix}
\dot{f}_1 - \dot{f}_2 \\
\vdots \\
\dot{f}_i - \dot{f}_{i-1} \\
\dot{f}_n
\end{bmatrix}
\end{pmatrix} =
\begin{bmatrix}
m_1 & 0 & \cdots & 0 \\
0 & \ddots & \ddots & \vdots \\
\vdots & \ddots & m_i & \ddots \\
0 & \cdots & 0 & m_n
\end{bmatrix}
\begin{pmatrix}
\dot{v}_{c1} \\
\vdots \\
\dot{v}_{ci} \\
\dot{v}_{cn}
\end{pmatrix}
\] (23)

For the moment equation, the matrix representation is given by the equation

\[
\begin{pmatrix}
\begin{bmatrix}
I_1 \\
\vdots \\
I_i \\
\vdots \\
I_n
\end{bmatrix} + \begin{bmatrix}
-\frac{\partial s_{cl} \times f_i}{\partial x} - (s_{cl} - s_{cl}) \times \frac{\partial f_i}{\partial x} \\
\vdots \\
-\frac{\partial s_{cl} \times f_i}{\partial x} - (s_{cl} - s_{cl}) \times \frac{\partial f_i}{\partial x}
\end{bmatrix}
\end{pmatrix} =
\begin{bmatrix}
l_1 & 0 & \cdots & 0 \\
0 & \ddots & \ddots & \vdots \\
\vdots & \ddots & l_i & \ddots \\
0 & \cdots & 0 & l_n
\end{bmatrix}
\begin{pmatrix}
\dot{\omega}_{c1} \\
\vdots \\
\dot{\omega}_{ci} \\
\dot{\omega}_{cn}
\end{pmatrix}
\] (24)
The force and moment equations can be combined into a single set of equations to obtain

\[
\begin{pmatrix}
0 \\
\vdots \\
0 \\
T_1 \\
\vdots \\
T_n
\end{pmatrix} + \begin{pmatrix}
\dot{f}_1 - \dot{f}_2 \\
\vdots \\
\dot{f}_i - \dot{f}_{i+1} \\
\dot{f}_n \\
-\mathbf{s}_{cl} \times \mathbf{f}_1 - (\mathbf{s}_1 - \mathbf{s}_{cl}) \times \mathbf{f}_2 \\
\vdots \\
-\mathbf{s}_{cl} \times \mathbf{f}_i - (\mathbf{s}_i - \mathbf{s}_{cl}) \times \mathbf{f}_{i+1} \\
-\mathbf{s}_{cn} \times \mathbf{f}_n
\end{pmatrix} = \begin{pmatrix}
\dot{\mathbf{v}}_1 \\
\vdots \\
\dot{\mathbf{v}}_i \\
\dot{\mathbf{v}}_n \\
\dot{\mathbf{\omega}}_1 \\
\vdots \\
\dot{\mathbf{\omega}}_i \\
0 \\
0 \\
0
\end{pmatrix}
\]

(25)

In the above equation, \(m_i\) is the mass of the body from the force equation and \(I_i\) is the rotational inertia of the body with respect to the body center of mass. The torque input, \(T_i\), from the angular momentum
devices, represents the only external force input in
the equations of motion. The last term represents the
contribution of the constraint forces on the dynamics
of the body segments.

Let

\[
[M_{eq}] = \begin{bmatrix}
m_1 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & m_i & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & m_n & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & l_i & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & l_i \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0
\end{bmatrix}
\]

(26)

\[
\begin{bmatrix}
\frac{f_1 - f_2}{f_i - f_{i+1}} \\
\frac{f_2 - f_3}{f_{i+1} - f_{i+2}} \\
\vdots \\
\frac{f_n}{-s_{el} \times f_1} - (s_i - s_{el}) \times f_2 \\
(-s_{el} \times f_i) - (s_i - s_{el}) \times f_{i+1} \\
\vdots \\
-s_{el} \times f_n
\end{bmatrix}
\]

(27)
\[
\{0\}
= \begin{pmatrix}
0 \\
\vdots \\
0 \\
T_1 \\
\vdots \\
T_l \\
\vdots \\
T_n 
\end{pmatrix}
\tag{28}
\]

and

\[
\begin{pmatrix}
\dot{\mathbf{v}}_1 \\
\vdots \\
\dot{\mathbf{v}}_l \\
\dot{\mathbf{v}}_n \\
\dot{\mathbf{\omega}}_1 \\
\vdots \\
\dot{\mathbf{\omega}}_l \\
\dot{\mathbf{\omega}}_n 
\end{pmatrix}
= \begin{pmatrix}
\dot{\mathbf{v}}_c \\
\dot{\mathbf{v}}_c \\
\vdots \\
\dot{\mathbf{v}}_c \\
\dot{\mathbf{\omega}}_c \\
\dot{\mathbf{\omega}}_c \\
\vdots \\
\dot{\mathbf{\omega}}_c 
\end{pmatrix}
\tag{29}
\]

Then, the equations of motion can be rewritten as

\[
\begin{pmatrix}
0 \\
\mathbf{T} 
\end{pmatrix} + \begin{pmatrix}
\frac{\mathbf{I}_c}{\mathbf{M}_c} 
\end{pmatrix} = [\mathbf{M}_{eq}] \begin{pmatrix}
\dot{\mathbf{v}}_c \\
\dot{\mathbf{\omega}}_c 
\end{pmatrix}
\tag{30}
\]
Substituting the kinematic expressions for linear and angular acceleration into the Newton-Euler equations of motion yields

\[
\begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\vdots \\
\dot{\theta}_n
\end{pmatrix}
+ \begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\vdots \\
\dot{\theta}_n
\end{pmatrix}
= \begin{pmatrix} 0 \\ T \end{pmatrix} + \begin{pmatrix} f_c \\ M_c \end{pmatrix}
\]  

(31)

The Newton-Euler equations can be reduced by eliminating the constraint forces term in the above equation. It is shown in [51] that the constraint force/torque vector vanishes if premultiplied by the \( B^T \) matrix. In other words

\[
[B]^T \begin{pmatrix} f_c \\ M_c \end{pmatrix} = 0
\]  

(32)

Using this relation, the matrix equation of motion is premultiplied by the \( B^T \) and, thereby, the constraint term is eliminated. This procedure leads to

\[
[B]^T [M_{eq}] [B] \begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\vdots \\
\dot{\theta}_n
\end{pmatrix}
+ [B]^T [M_{eq}] [C] \begin{pmatrix}
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\vdots \\
\dot{\theta}_n
\end{pmatrix}
= [B]^T \begin{pmatrix} 0 \\ T \end{pmatrix}
\]  

(33)
Further, rearranging terms one obtains

\[
[B]^T[M_{eq}][B]\begin{bmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2 \\
\vdots \\
\ddot{\theta}_n
\end{bmatrix} = [B]^T\begin{bmatrix}0 \\ T \end{bmatrix} - [B]^T[M_{eq}][C] \begin{bmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_2 \\
\vdots \\
\ddot{\theta}_n
\end{bmatrix}
\]

(34)

The above equation represents a simultaneous set of linear equations in $A\mathbf{x} = \mathbf{b}$ format. The equations can be solved at each time step in the simulation for the joint angular accelerations. These accelerations are integrated over the time step to obtain the angular velocity contributions in the right hand side of the equation. For each time step, the control torque $T$ is computed, the right side of the equation is calculated, and the system of equations is solved.
4 ALGORITHMS

4.1 Preliminary Remarks

Positioning the robot arm involves the conversion of a desired robot tip position and orientation into the required joint angles between body segments. Once these angles are determined, a control system is required that will properly position each body segment into the prescribed angular position. For this analysis the tip trajectory is not controlled. Instead, the robot is driven from an initial state to the desired state by eliminating the angular position errors in each of the body segments. Once each body segment has reached its proper spatial orientation, the desired tip position and orientation is guaranteed.

4.2 Kinematic Tip Positioning Algorithm

As with all robots, the forward and inverse mappings between joint space and physical space are the pivotal point for kinematic analysis. It is necessary to determine the tip position and orientation from a list of joint angles within the robot. This problem, referred to as the forward kinematic solution, is a simple bookkeeping of coordinate transformations and position vectors. More difficult, however, is the inverse kinematic problem
involving determination of the joint angles from the tip position and orientation. The inverse problem is more difficult because the physical space is of less order than the joint space leading to an infinite solution space. Of the many joint angle solutions available to produce the desired tip position, the one that is "best" for some prescribed set of criteria must be found.

For the multi-segment robot described in this analysis, the joint space order exceeds, by a significant margin, the order of the physical space. Therefore, the traditional approach of order reduction or least squares mapping is not as attractive. Instead, a different method of solving the inverse kinematic solution must be found.

4.2.1 Shape Function

To position accurately the tip of a multi-segment robot, the entire system must assume the proper overall shape. This notion leads to the concept of a function that describes the shape of the robot. This function, called the "shape function," is constrained by the end conditions for the base and tip. In addition, the shape function is constrained to have an arc length equal to the physical length of the robot.
The basic problem can be thought of as a string that is anchored at one end. To position the other end of the string at a certain point the string must be placed in a curvilinear form. In this form, one end of the string is anchored at the base and the other end is at the desired tip position and orientation. The curvilinear shape of the string that results in the proper end conditions is called the shape function.

Many different forms of a shape function are possible. However, due to limitations in the hardware, a shape function that is as smooth as possible is desired. With this in mind, a shape function of the following form is proposed

$$y = f(x) = A + Bx + Cx^2 + Dx^3$$  \hspace{1cm} (35)

It is subject to the following constraints. First, the base of the robot is at the coordinate origin. That is,

$$y(0) = 0$$  \hspace{1cm} (36)

Next, the robot tip position at $x=l$ is $y=h$. That is,

$$y(l) = h$$  \hspace{1cm} (37)

Finally, the tip slope at $x=l$ is given by
\[
\frac{dy(l)}{dx} = g
\]  \hspace{1cm} (38)

Since the base of the robot is at the origin of the workspace coordinate system, the constant \( A \) term vanishes, leading to

\[
y = f(x) = Bx + Cx^2 + Dx^3
\]  \hspace{1cm} (39)

This polynomial is forced to satisfy the tip endpoint position and orientation. The planar constraint requires \((x,y)\) and \(\frac{dy}{dx}\) to be given at the tip. The fourth constraint involves the arc length of the polynomial from the base to the prescribed tip position. This constraint ensures that the tip is reachable by the segmented robot arm. Also, the polynomial provides a smooth shape that uses the entire arm length.

The arc length constraint is given by the equation

\[
s = \int_0^l \sqrt{(1 + (dy/dx)^2)} \, dx
\]  \hspace{1cm} (40)

The solution algorithm uses an iterative method to find the proper values for \( B, C, \) and \( D \). The starting point for this iteration scheme is found by approximating the expression for the arc length. The integrand is expanded to first order in a binomial series given by the equation
\[ s = \int_0^l \left[ 1 + \frac{1}{2} (dy/dx)^2 \right] dx \quad (41) \]

The closed form determination of this integral provides the necessary third equation to solve for the three polynomial coefficients, \( B, C, \) and \( D. \) The three constraint equations with the three unknown coefficients are

\[ y(l) = h = Bl + Cl^2 + Dl^3 \quad (42) \]

for the tip position,

\[ \frac{dy(l)}{dx} = g = B + 2Cl + 3Dl^2 \quad (43) \]

for the tip slope, and

\[ s = l + B^2 l + 2BCl^2 + 2BDl^3 + \frac{4}{3} C^2 l^3 + 3CDl^4 + \frac{9}{5} D^2 l^5 \quad (44) \]

for the approximate arc length. These equations can be solved for the polynomial constants \( B, C, \) and \( D. \) Specifically, equations (42) and (43) can be used to express \( C \) and \( D \) in terms of \( B \) in the form

\[ C = \frac{3h}{l^2} - g \frac{2B}{l} \quad (45) \]

\[ D = -\frac{2h}{l^3} + g \frac{B}{l^2} \quad (46) \]
Then, these expressions for $C$ and $D$ can be substituted into the approximated arc length expression, equation (44). This equation is then solved for $B$ in terms of the arc length and tip constraints. The resulting equation is quadratic in $B$ and can easily be solved using the quadratic formula to obtain

$$B_1 = -\frac{\sqrt{15}\sqrt{16ls-(g^2+16)l^2+2ghl-9h^2-gl-3h}}{4l}$$  \hspace{1cm} (47)$$

$$B_2 = \frac{\sqrt{15}\sqrt{16ls-(g^2+16)l^2+2ghl-9h^2+gl+3h}}{4l}$$  \hspace{1cm} (48)$$

Notice that the value of $B$ is the slope of the curve at the coordinate origin. The two solutions for $B$ represent the two different slopes the curve can have at the origin and still meet the constraints. Selecting the slope at the root can be used to orient the arm away from obstacles in the workspace. The output of this algorithm is the polynomial description of the desired shape function that meets the boundary and length constraints.

4.2.2 Body Positioning

The second phase of the tip positioning solution involves the determination of the locations of the body segments along the shape function.
The goal of the body positioning algorithm is to find the best match of the segmented robot to the shape function. In other words, the final position of the body segments will result in the robot tip being at the endpoint defined by the shape function. The tip orientation will be approximated, but allowed to relax to achieve the proper tip position. This is a valid approach since for large space robots, a dexterous manipulator will be located at the tip for fine payload positioning. Therefore, strict adherence to location is provided, but tip orientation is approximated to achieve the proper end position. This approach results in a smooth overall shape to the segmented robot.

Simply positioning the endpoints of the body segment along the defined polynomial will, in most cases, overshoot the endpoint. This occurs because the arc length of the polynomial will always be greater than, or equal to, the arc length of the line that connects the endpoints of the body segments. In other words, the body segment endpoints define a straight line, whereas the polynomial can have curvilinear shape between these points.

The individual body segments are aligned along the polynomial in such a way that the endpoint of the seg-
mented robot is coincident with the endpoint of the polynomial. In addition, the distance between the segment endpoints and the polynomial is minimized, achieving a close match to the target polynomial shape.

Starting at the base, where the shape function and the robot are coincident, the body positioning algorithm moves along the polynomial until the arc length is equal to twice the length of one body segment. This point, on the shape function, is used as an anchor point for a two segment length of the robot. Two isosceles triangles are created, each with the common base leg; a line that connects the base with the first anchor point. Of the two triangles, the one with the vertex between the two congruent sides that is closest to the shape function is selected. This triangle is used to specify the spatial orientation of the first body segment of the robot. This technique is shown in Figure 4-1.
Figure 4-1  First Body Segment Positioning
To determine the angular position of the second body segment, the polynomial is traversed outward from the anchor point for an arc length equal to one body segment. This point on the curve then becomes the new anchor point, discarding the last anchor point. A line is drawn from the endpoint of the first body segment to the new anchor point. The first body endpoint was the vertex of the isosceles triangle selected in the first iteration. The distance between these two points is equal to (if the polynomial is a straight line), or less than, the length of two body segments. This is because the distance between the first body endpoint and the previous anchor point is exactly one body segment length. This line segment was one of the legs of the isosceles triangle formed earlier. The curvilinear distance along the polynomial between the new anchor point and the previous anchor point is equal to one body length. Therefore, a straight line between the first body endpoint and the new anchor point will have a length less than, or equal to, the length of two body segments. This ensures that two isosceles triangles can be formed with the common base being the line that connects the first body endpoint and the new anchor point. Once again, of the two, the triangle that
results in the vertex between the congruent sides being closest to the polynomial is selected. The positioning of the second body segment is shown in Figure 4-2.

This process is repeated until the entire length of the polynomial has been traversed. This algorithm determines the spatial orientation of each body segment required to achieve the desired robot tip position.
Figure 4-2  Second Body Segment Positioning
4.3 Body Segment Positioning Control System

A single input-single output controller has been designed for each of the body segments. The controller provides the necessary torque to position the body segments into the commanded position. Each body is controlled independently into its final position, leading to the proper orientation of the arm.

A classical frequency domain controller was selected because of the emphasis on system stability. Also, the single input-single output classical design can be readily implemented into the real time control algorithm. The implementation of the control algorithm follows the block diagram shown in Figure 4-3.
Figure 4-3 Control System Block Diagram
The commanded reference signal for the control loop was based on relative angles between segments. This causes the control loop to act as an internal stiffness and damper to the overall system.

Gain values for rate and position feedback were set as constants. This approach, as opposed to adaptive methods that would calculate gains based on the reference inputs, was used to facilitate the analysis of the system.
5 NUMERICAL SIMULATION

5.1 Preliminary Remarks

Two cases were designed to test the positioning algorithms, the analytical simulation program, and the control system parameters. All simulations were started from an initial condition where the relative angles of each body were zero. This position corresponds to the robot lying on a straight line parallel to the inertial X coordinate axis.

The first test case involved moving the robot from the initial position to a point that positioned the tip at \( X = 114.00 \) and \( Y = 64.00 \). A tip slope of 0.57 was prescribed for this case. This case results in the robot moving to a slightly curved position about a line that is approximately 30 degrees from the X axis. This case was selected because it requires movement of the robot as a quasi-rigid body through a 30 degree angle from the starting orientation. The purpose of this test was to verify that the control system was capable of moving the robot as a quasi-rigid body without adverse control interactions.

A second test case was conceived to assess the dexterity of the robot. In this case, the robot positions itself in a more complicated shape than the nearly
straight line of the first case. This complex shape was derived by moving the desired tip position toward the coordinate origin. The second case locates the tip at the midpoint between the first case tip position and the origin. The resulting tip location was at $X = 57.0$ and $Y = 32.9$. The angular orientation of the tip was maintained along the 30 degree line resulting in a tip slope of 0.57.

The prototype model of the multi-segment robot was designed with a segment length of 22 inches. Six individual body segments were fabricated and linked together to form the prototype robot. Simulation parameters were set based on this design. One of these parameters, overall robot length, was set to 132 in. during the simulation to model the actual length of the prototype. This parameter is important because it sets the maximum envelope of the workspace.

A computer program was written to simulate the equations of motion that model the mechanics of the robot. The equations express the angular accelerations of the individual body segments that make up the robot. These accelerations are integrated using a trapezoidal integration scheme to produce the body angular rates and positions. The rate and position data is fed directly into
the control algorithm that computes the torque required to position the bodies properly. The simulations were conducted using a time step of 0.001 sec. The robot would position itself properly within 20 seconds. Position histories verified that the robot approached its desired position in the expected asymptotic fashion.

The torques from adjacent body controllers create a disturbance on the body and can be effectively cancelled by the body controller. Torque limits for the simulation were established by the specifications of the electric motor used to drive the flywheels in the body assembly. These simulations were important in sizing the prototype motor/flywheel assembly to guarantee sufficient torque and angular momentum capacity to perform the desired maneuvers.

A Cartesian coordinate system was established to support the mathematical and vectorial representation of the workspace. Initially, the robot was positioned along the X axis with zero position and velocity conditions. The initial robot position and coordinate system is shown in Figure 5-1.
Figure 5-1  Coordinate System and Initial Robot Position
5.2 Case 1

One of the issues raised during the development of this robotic concept was the interaction of the control system in each body with the controllers in adjacent bodies. As one body would attempt to move to its commanded orientation, the effect on the next body might cause it to move away from its commanded orientation. To study this problem, a case was designed to move the robot through a 30 degree arc. The resulting shape function required the body nearest the base to rotate 43 degrees from its initial position. The other bodies were determined to have diminishing inertial angles leading to a gentle arc shape about the 30 degree target line. Starting from a straight line orientation parallel to the X axis, the robot is commanded to move to the angular orientation prescribed for each body segment. In the starting position, all inertial angles are zero.

This first simulation case was designed to test the functionality of the complete algorithm set. Within the robot workspace, an endpoint and orientation was specified for the end effector. As mentioned earlier, the selected endpoint was at \( X = 114.0, \ Y = 64.0 \), with a slope at the tip of 0.57. A third order polynomial shape function was determined which meets the prescribed bound-
ary conditions. Next, the proper orientation of each body segment in the chain was determined which matches this shape function. These orientations are used as commanded inputs to the control system. The controller for each body calculates the error signal by differencing the present position with the desired position. The error signal was processed by the controller resulting in a commanded motor torque that reduces the positional error.

The third order polynomial describing the target shape of the robot was calculated. The shape function polynomial parameters are given in Table 5-1.

<table>
<thead>
<tr>
<th>SHAPE FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(x) = Ax + Bx + Cx^2 + Dx^3$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>A</th>
<th>0.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>1.092821</td>
</tr>
<tr>
<td>C</td>
<td>-0.009399</td>
</tr>
<tr>
<td>D</td>
<td>0.000042</td>
</tr>
</tbody>
</table>

Table 5-1 Shape Function Parameters, Case 1

The shape of this polynomial is shown in Figure 5-2
Figure 5-2  Shape Function, Case 1
The positive value for the root slope was taken for $B$ given by

$$B = \frac{15\sqrt{16l_5 - (g^2 + 16)l^2 + 2ghl - 9h^2 + gl + 3h}}{4l} \quad (49)$$

which in turn sets the values of $C$ and $D$, where

$$C = \frac{3h}{l^2} - \frac{g}{l} - \frac{2B}{l} \quad (50)$$

$$D = -\frac{2h}{l^3} + \frac{g}{l^2} + \frac{B}{l^2} \quad (51)$$

After the shape function had been determined, the next algorithm was executed that calculates the proper orientation of each body to approximate the shape function. The six angles, with respect to the inertial coordinate system, required to position the robot to meet the prescribed endpoint conditions are shown in Table 5-2.
<table>
<thead>
<tr>
<th>Joint #</th>
<th>Inertial Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43.61</td>
</tr>
<tr>
<td>2</td>
<td>36.47</td>
</tr>
<tr>
<td>3</td>
<td>25.18</td>
</tr>
<tr>
<td>4</td>
<td>24.96</td>
</tr>
<tr>
<td>5</td>
<td>21.46</td>
</tr>
<tr>
<td>6</td>
<td>24.31</td>
</tr>
</tbody>
</table>

Table 5-2 Target Inertial Angles, Case 1

Relative angles between the bodies are computed from the inertial angles. The relative joint angles are then submitted to the body control system. The controllers compare the present body position and angular rate to the commanded relative orientation and generate an error signal. The error signal is converted into a correcting torque to be applied by each motor/flywheel assembly. As the orientation errors for each body are reduced, the torque levels are likewise reduced. The relative angles used by the control system for each body are shown in Table 5-3.
<table>
<thead>
<tr>
<th>Joint #</th>
<th>Relative Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43.61</td>
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<tr>
<td>2</td>
<td>-7.14</td>
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<tr>
<td>3</td>
<td>-11.29</td>
</tr>
<tr>
<td>4</td>
<td>-0.22</td>
</tr>
<tr>
<td>5</td>
<td>-3.49</td>
</tr>
<tr>
<td>6</td>
<td>2.85</td>
</tr>
</tbody>
</table>

Table 5-3  Target Relative Angles, Case 1

Before beginning the simulation, control system gains must be selected. An analysis was conducted to understand the relationship between rate and position feedback gains and frequency response. Analytical experiments were conducted using various frequency separation methods. These methods were conceived to provide a variation in frequency response for each body and thereby reduce any adverse interaction. One of these methods used a root locus pole placement technique. Another investigated the use of relative versus inertial angles to set the control gains. An adaptive method was also investigated that computed the control gains as a direct function of the initial relative angle error. In gen-
eral, the system was found to be extremely stable and not sensitive to changes in control gains. However, if the gain values were increased beyond certain limits, non-linear behavior would result because of torque saturation in the simulated motor/flywheel system. Under these cases, the control torque would flip from maximum positive torque to maximum negative torque, producing erratic system response.

To achieve a better insight into the effects of gain parameters on the system response, a simple table of gains was entered into the computer. This allowed the analysis to proceed by simply entering new gain values and rerunning the simulation. The methods described earlier (pole placement, error based adaptive gains, etc.) were disregarded in favor of a more straightforward method for setting the control gains. The table implementation worked well throughout the analysis phase of the robot development. Table 5-4 shows the control gain values used in the analytical simulation.
<table>
<thead>
<tr>
<th>Body</th>
<th>Position Gain</th>
<th>Rate Gain</th>
<th>Control Gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60.0</td>
<td>100.0</td>
<td>20.0</td>
</tr>
<tr>
<td>2</td>
<td>60.0</td>
<td>100.0</td>
<td>20.0</td>
</tr>
<tr>
<td>3</td>
<td>60.0</td>
<td>100.0</td>
<td>20.0</td>
</tr>
<tr>
<td>4</td>
<td>60.0</td>
<td>100.0</td>
<td>20.0</td>
</tr>
<tr>
<td>5</td>
<td>60.0</td>
<td>100.0</td>
<td>20.0</td>
</tr>
<tr>
<td>6</td>
<td>60.0</td>
<td>100.0</td>
<td>20.0</td>
</tr>
</tbody>
</table>

Table 5-4  Control System Gains, Simulation

With the commanded angles determined and the control gains selected, the robot is set to begin its movement toward the desired endpoint. The simulation time length was varied between 20 and 40 seconds to view the entire damped behavior of the system. It was found that by using the proper control gains, the robot could move into the desired position within the 20 second period. Therefore, twenty seconds was selected as the time length for the simulation cases. Since the simulation program was executed on a microcomputer, minimizing simulation time was important in reducing computer turnaround time.
In Figures 5-3 thru 5-20 the angular position, angular rate, and control torque are shown for each body segment. The information in these plots was used to size the components of the large space robot.

The body one angular position, shown in Figure 5-3, shows a response with minimal overshoot of the target, highly damped, and convergent to the desired angular value. The character of this time response is desirable for a robot where large overshoot and lightly damped sinusoidal convergence is impractical. Sufficient damping must be maintained in the system at the expense of faster time response to avoid excessive overshooting and dynamics.

Slightly more overshoot is seen in the body two position history (Figure 5-4) compared to body one. This overshoot is minimal, however, and does not cause any appreciable positioning problems.

In Figure 5-5, body three initially moves in a direction opposite to the desired angular value. This is caused by the use of relative angles for generating the error function. The error signal causes the body to move to a negative relative angle, which initially is a negative inertial angle. However, as the inboard bodies move to assume the shape function, body three follows, while
holding the proper relative angle with respect to body two. During this time, the inertial angle increases. The overshoot of the target angle results from the dynamics of the robot as each body moves to the target angular position. Convergence to the proper position is achieved within the simulation period.

The interaction between the adjacent bodies is apparent from the body four position curve shown in Figure 5-6. The negative relative angle sent to the control system causes the body to move to a negative inertial orientation. As the inboard bodies move to their proper positions, the inertial angle approaches the target position. Due to the shape of the robot during the motion, the position curve shows an increased overshoot before achieving the desired position.

In Figure 5-7 the position history for body five is shown. This curve depicts a typical second order system response. The initial motion in the negative direction results from the control system driving the body to the proper relative orientation. As the inboard bodies achieve there proper position, this body moves toward its desired inertial orientation. Overshoot increases with segment number because the overall system shape is similar to a tip loaded cantilevered beam. The inertial
angle is highest at the tip and lowest at the root, or base. After this overshoot period, convergence to the desired position is achieved.

The body six position history is shown in Figure 5-8. This curve is typical of the other bodies in the robot segment chain. The initial motion of the body is in the negative direction. The body moves with the rest of the robot system to the proper inertial orientation. The overshoot sections of the curve have increased in comparison to the previous body segments. This is caused by the kinematic accumulation of relative angle motion throughout the body chain. In other words, the tip inertial angle orientation is dependent on the motion of the inboard bodies.
Figure 5-3 Angular Position - Body 1, Case 1, Simulation

Figure 5-4 Angular Position - Body 2, Case 1, Simulation
Figure 5-5 Angular Position - Body 3, Case 1, Simulation

Figure 5-6 Angular Position - Body 4, Case 1, Simulation
Figure 5-7 Angular Position - Body 5, Case 1, Simulation

Figure 5-8 Angular Position - Body 6, Case 1, Simulation
Another component that must be designed from the analytical results is the rate gyro. Figures 5-9 thru 5-14 show the inertial angular rates for each of the body segments in the robot chain. The maximum rate of each curve determines the range requirements on the rate sensors used by the prototype robot. The rate gyros selected for the prototype must be able to provide accurate output in this rate range.

The maximum angular rate predicted by this analysis is nearly 10 degrees per second, taken from the body one data. Each successive body shows rate values less than the body one predictions. From these figures, a sensor capable of +/- 10 degrees per second is adequate for performing this motion case.
Figure 5-9 Angular Rate - Body 1, Case 1, Simulation

Figure 5-10 Angular Rate - Body 2, Case 1, Simulation
Figure 5-11  Angular Rate - Body 3, Case 1, Simulation

Figure 5-12  Angular Rate - Body 4, Case 1, Simulation
Figure 5-13 Angular Rate - Body 5, Case 1, Simulation

Figure 5-14 Angular Rate - Body 6, Case 1, Simulation
The torque history for body one is shown in Figure 5-15. Integrating this plot yields the maximum angular momentum required in the motor/flywheel assembly. A maximum angular momentum value of about 200 in-lb-sec is computed from this curve. Therefore, the design of the flywheel and the selection of a suitable motor will be driven by this angular momentum value.

Examining the remaining torque curves shows that the momentum requirements diminish as the body nears the robot tip. Hence, body one stores more angular momentum than body two, body two stores more momentum than body three, and so on. Body six has the least momentum storage requirements.

The torque history for body one (Figure 5-15) shows the maximum torque level is maintained for just under 4 seconds. The control system is requesting, based on the weighted errors and the controller gain, a torque that exceeds the maximum allowable torque. The requested value is reduced to the maximum torque level by the control program. Although this leads to a discontinuous curve, the control system is still capable of properly positioning the body. A reduction in the forward loop control gain would reduce or eliminate torque saturation. However, since the system performs properly, the gain
values were maintained at the mentioned levels.

Torque levels for the second body from the base is shown in Figure 5-16. This curve is very similar to the first body torque signal. However, due to the reduction in outboard inertia, the period of maximum torque is reduced. In this case, maximum torque is maintained for only two seconds. Again, torque saturation does not inhibit the body from moving to its proper angular position.

Torque levels for body three (Figure 5-17) achieve the maximum torque level, but only momentarily. The torque history is similar to the curves for bodies one and two.

Body four torque levels (Figure 5-18) are below saturation except for the initial negative torque pulse. This negative torque value results from the use of relative angles, as opposed to inertial angles, to drive the control system. The rapid change from negative to positive torque results from the dynamic interaction between bodies. This interaction is initiated by body one since it has the largest initial error. The remaining portions of the torque curve has a general character similar to the previous bodies in the chain.
The torque levels for body five (Figure 5-19) are less than the maximum torque throughout the simulation interval except for the initial torque pulse. The small variations in the curve at the 4.0 and 7.0 second mark are due to the adjacent bodies applying shear forces at the common structural interface. These shear forces create disturbance torques about the center of mass of the segment.

The torque curve for the most outboard body, body six, is shown in Figure 5-20. The torque values are smaller in comparison to the more inboard bodies. This occurs because body six is only connected to one other body, body five. Therefore, there is no inertial load, or interface force, occurring at the outboard tip to be overcome by the controller. The general shape of the curve is typical of the torque curves throughout the body segments.

The use of relative angles, instead of inertial angles, in the control loop was a choice made after comparison analysis. The relative angle method was adopted since it provided smaller errors in the control system. This would minimize the dilatory effects of one control system trying to overpower the adjacent bodies. Also, using relative angles provides the system with internal
stiffnesses that approximate cantilevered beam behavior. Simulation and tests using commanded inertial and relative angles did not establish a clear advantage in either of the two approaches. For the reasons cited above, the relative angle method was selected and successfully used.
Figure 5-15 Torque - Body 1, Case 1, Simulation

Figure 5-16 Torque - Body 2, Case 1, Simulation
Figure 5-17  Torque - Body 3, Case 1, Simulation

Figure 5-18  Torque - Body 4, Case 1, Simulation
Figure 5-19 Torque - Body 5, Case 1, Simulation

Figure 5-20 Torque - Body 6, Case 1, Simulation
The simulated motion of the robot can be seen in Figures 5-21 thru 5-24. These figures show the robot as it moves from its starting position to the final target position. Figure 5-21 shows the robot in the initial condition of zero angular position and rate. The robot is shown as a solid line with small circles at the robot joints. The target, or final position, of the robot is displayed with a dotted line. Actual values of the robot state are shown at the bottom of the display window. The target relative positions can be compared to the actual relative positions throughout the simulation. Current tip location and orientation is shown at the top of the display above the target tip specifications. Angular acceleration and rates are also shown in the bottom portion of these figures. Values for the control torques are shown to the right of the displayed robot.

The bending of the robot, similar to a low stiffness cantilevered beam, is apparent in these figures. This characteristic results from the beam stiffness being derived from relative deflections between the adjacent bodies. In Figure 5-24, at the end of the simulation, the robot has aligned itself with the target position.
Figure 5-21 Initial Position, Case 1, Simulation

Figure 5-22 Robot Position at t = 2.0 sec., Case 1, Simulation
Figure 5-23  Robot Position at t = 4.0 sec., Case 1, Simulation

Figure 5-24  Final Position, Case 1, Simulation
5.3 Case 2

A second analytical case was selected to test the dexterity of the robot and the associated positioning algorithms. This second case specifies an endpoint along the 30 degree line defined in the first case. The new target point is located at the midpoint between the first case endpoint and the origin. The second case endpoint is specified as \( X = 57.0, Y = 32.9 \), and a slope of 0.57 (same as the first case).

The endpoint specifications are entered into the shape function generation algorithm. The values for the shape function parameters that satisfy the boundary conditions is shown in Table 5-5.

<table>
<thead>
<tr>
<th>SHAPE FUNCTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y(x) = A + Bx + Cx^2 + Dx^3 )</td>
</tr>
<tr>
<td>\hline</td>
</tr>
<tr>
<td>A</td>
</tr>
<tr>
<td>B</td>
</tr>
<tr>
<td>C</td>
</tr>
<tr>
<td>D</td>
</tr>
</tbody>
</table>

Table 5-5 Shape Function Parameters, Case 2

The shape of this polynomial is shown in Figure 5-25
The shape function specification is entered into the segment positioning algorithm to determine the proper angular orientation for each body segment. The prescribed angles define the robot shape that approximates the shape function. In this case the shape function curvature near the tip is too great and does not allow the end body to position itself along the prescribed slope. The endpoint slope constraint is relaxed to allow the best approximation of the robot to the shape function. Relaxing this constraint does not cause any operational restriction since a real system of this type would
have an end effector at the tip for final angular positioning. The inertial angles for each of the bodies is shown in Table 5-6.

<table>
<thead>
<tr>
<th>Joint #</th>
<th>Inertial Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83.69</td>
</tr>
<tr>
<td>2</td>
<td>78.23</td>
</tr>
<tr>
<td>3</td>
<td>86.58</td>
</tr>
<tr>
<td>4</td>
<td>-2.26</td>
</tr>
<tr>
<td>5</td>
<td>-69.36</td>
</tr>
<tr>
<td>6</td>
<td>-30.03</td>
</tr>
</tbody>
</table>

Table 5-6  Target Inertial Angles, Case 2

As mentioned earlier, the body segment control system uses the relative angle between adjacent segments as the target angular position. The relative angles are computed directly from the inertial angles and are shown in Table 5-7.
<table>
<thead>
<tr>
<th>Joint #</th>
<th>Relative Angle (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>83.69</td>
</tr>
<tr>
<td>2</td>
<td>-5.46</td>
</tr>
<tr>
<td>3</td>
<td>8.35</td>
</tr>
<tr>
<td>4</td>
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<tr>
<td>5</td>
<td>-67.11</td>
</tr>
<tr>
<td>6</td>
<td>39.33</td>
</tr>
</tbody>
</table>

Table 5-7 Target Relative Angles, Case 2

The relative angles represent the reference target for the segment control system. The control system gains were maintained at the Case 1 values (Table 5-4). Within the 20 sec. simulation period, the segment controllers successfully position the bodies into the desired orientation. In Figures 5-26 thru 5-31 the body position histories are shown. The angular position plots shown are with respect to the inertial coordinate system and should be referenced against Table 5-6. Angular position responses similar to the first case are seen in these plots. Body one shows a highly damped history while moving to its target position. A highly damped characteristic is desirable at the base since overshoot of this body
will result in excessive tip overshoot. The body two
position history also exhibits a highly damped character-
istic as it achieves its desired inertial orientation.
The remaining bodies show angular position histories
typical of an underdamped cantilevered beam. This
results in a slight overshoot of the desired endpoint
before settling to the target position.
Figure 5-26  Angular Position - Body 1, Case 2, Simulation

Figure 5-27  Angular Position - Body 2, Case 2, Simulation
Figure 5-28 Angular Position - Body 3, Case 2, Simulation

Figure 5-29 Angular Position - Body 4, Case 2, Simulation
Figure 5-30 Angular Position - Body 5, Case 2, Simulation

Figure 5-31 Angular Position - Body 6, Case 2, Simulation
The angular rates for each body are shown in Figures 5-32 thru 5-37. The data displayed in these plots are used to size the rate sensor instruments of the robot. Each body must move to its desired relative angular position and maintain this position with a zero angular rate. Body interactions are also apparent in the rate data. The body one plot shows a typical rate response. The rate first builds as the body approaches the target position followed by a braking period as the target angle is achieved. The rate curves for each body reflect the interactions between adjacent segments. Each body overcomes these disturbances to achieve its final commanded position. The final state of the robot is a zero rate condition with the bodies arranged along the approximated shape function. This condition is shown by the zero rate levels in each of the bodies at the end of the simulation.

The inertial rates are predicted through this analysis to be as high as 70 degrees per second. However, most responses are contained within a +/- 30 degree per second range. This information is critical in specifying the range of the rate sensors to be implemented in the prototype design.
Figure 5-32 Angular Rate – Body 1, Case 2, Simulation

Figure 5-33 Angular Rate – Body 2, Case 2, Simulation
Figure 5-34  Angular Rate - Body 3, Case 2, Simulation

Figure 5-35  Angular Rate - Body 4, Case 2, Simulation
Figure 5-36  Angular Rate - Body 5, Case 2, Simulation

Figure 5-37  Angular Rate - Body 6, Case 2, Simulation
Figures 5-38 thru 5-43 show the control torque histories for each body throughout the simulation. The control torque plots reflect the required torque exerted by each body to position itself in the desired angular position. A peak torque value of 45 in-lb was set based on the specifications of the electric motors used in the prototype implementation. The body one torque plot shows a saturation at the peak torque value through the initial acceleration and braking phases of motion. Each of the torque plots reflect a different level of this saturation. However, this type of torque command does not result in adverse response of the bodies. Gain adjustments can be made to reduce or eliminate this type of saturation. Since no adverse impact to the system is attributable to this saturation, there was no need to make gain changes. The dynamic interactions between adjacent bodies is apparent in the torque plots. Shear forces acting at the connection points of the bodies produce disturbance torques about the body center of mass. Compensation for these disturbance torques must be made by the control torque. When the robot has achieved its desired final state, all torque levels should be approaching zero. The torque plots for all segments
reflect this zero torque condition at the end of the simulation. This indicates that all position errors and rates have been nullified.

A valuable design parameter, momentum storage requirements, can be determined from these plots. Integration of the torque curves measure the amount of momentum that must be stored by the motor/flywheel system in the prototype. In examining the body one torque curve it is seen that approximately 225 in-lb-sec of angular momentum capacity is required by the prototype system. This angular momentum value is similar to the values experienced in the first test case.
Figure 5-38 Torque - Body 1, Case 2, Simulation

Figure 5-39 Torque - Body 2, Case 2, Simulation
Figure 5-40 Torque - Body 3, Case 2, Simulation

Figure 5-41 Torque - Body 4, Case 2, Simulation
Figure 5-42 Torque - Body 5, Case 2, Simulation

Figure 5-43 Torque - Body 6, Case 2, Simulation
In this second case, a more complex shape must be achieved by the robot. Figures 5-44 thru 5-47 show the robot at different orientations as it moves from the starting position to the target shape. As the simulation proceeds, each body moves toward the commanded relative angle. This process can be seen by comparing the target and actual angular positions at the bottom of each figure. As the target position is achieved, the body angular rates and control torque approach zero (Figure 5-47).

Of special interest in this case is the final angular position of the tip. To approximate the shape function with the discrete line segments of the robot required a relaxation of the tip constraint. The shape function curvature near the tip is too great to be approximated by the last body in the robot chain. Therefore, this constraint is relaxed in favor of the proper spatial position. This does not constitute a significant problem since a dextrous end effector would be placed at the tip of a large space robot. The final robot configuration can be compared to the shape function in Figure 5-25 to see the effect of relaxing the end orientation constraint.
Figure 5-44 Initial Position, Case 2, Simulation

Figure 5-45 Robot Position at t = 2.0 sec., Case 2, Simulation
Figure 5-46 Robot Position at t = 5.0 sec., Case 2, Simulation

Figure 5-47 Final Position, Case 2, Simulation
6 DEMONSTRATION

6.1 Preliminary Remarks

Computer simulation is a valuable tool in assessing the benefits of a new concept. However, the development must ultimately lead to an actual hardware form. To achieve this next level of development, a prototype multi-segment robot was designed and fabricated.

The multi-segment robot utilizes changes in angular momentum to provide the torque necessary to position the individual bodies along the shape function. The segment design was not driven by strength considerations, therefore, detailed stress analysis was not performed. Instead, providing the necessary control authority to move the body segments in the uncertain environment of the test facility was the primary design issue. Addressing this problem, the dynamic simulation program was used extensively to size the electric motors and flywheels for the body segments. The design process for each of the major system components that comprise a body segment are detailed in the following sections. The segment cross-section is shown in Figure 6-1a. The corresponding parts list is given in Figure 6-1b. The complete six body assembly of the robot prototype is shown in Figure 6-2.
Figure 6-1a Segment Assembly Cross-section
<table>
<thead>
<tr>
<th>Part Number</th>
<th>Description</th>
<th>Quantity</th>
<th>Material</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>SLEEVE, 3/8&quot;-16UNC X 2.00 LE</td>
<td>30</td>
<td></td>
<td></td>
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| 2          | 5144-25                 | 4        | RETAINING RING | VELDES TWIN

Figure 6-1b Segment Assembly Parts List
Figure 6-2 Complete Six Body Robot Assembly
6.2 Body Housing

The most significant issue encountered in the design of the housing was whether to make a skin/frame design or an integral part machined from a single piece of aluminum. After a preliminary cost estimate, the integral machining method was selected. The results of this trade address a broader engineering issue that may be disguised by the subtlety of this decision. With the advent of computer based design and manufacturing systems, collectively referred to as CAD\CAM\CAE, the design through fabrication process has been streamlined. The housing was designed on a Computer Aided Design (CAD) terminal. This information was sent to a numerically controlled milling machine (CAM) where fabrication was completed. This process resulted in an exceptional machined part for less cost. The trade indicated that the man-hours spent providing assembly jigs and tools necessary to hold assembly tolerances, as well as the actual fabrication time, induced greater cost than the integral machined method.

The diamond shape of the housing resulted from two considerations. First, the housing must contain a circular flywheel with its center rotation axis coincident and parallel to the housing center line. Second, the angle
of the housing at the attachment point sets the maximum relative angle between two adjacent segments. The smaller the included angle, the greater the relative angular displacement. These considerations lead to the diamond shaped design that is 15.5 in. wide and 22.0 in. long between attach points. The interior integral ribs were designed for stiffness in longitudinal bending and control axis torsion. Provisions were made to support the electric motor, flywheel, and rate gyro on the housing. The body housing is the primary structural unit of the system. The body housing contributes 9.5 lbs. to the weight of each body segment.

6.3 Electric Motor

The electric motor is the primary control effector in the system. It must be sized with sufficient torque and speed to control the motion of the body segment. The dynamic simulation program was used to determine the torque and momentum requirements for the motor/flywheel system. Results from the simulation analysis indicated that a motor/flywheel system capable of 225 in-lb-sec of angular momentum would be sufficient to operate the robot. To meet these requirements, the Electro-Craft E-19 Series motor was selected. The motor is a DC-type servo motor that features accurate speed control in both
positive and negative directions. This motor has a peak torque capacity of 45 in-lb and a maximum operating speed of 3800 RPM. Excess motor capacity is important since test facility imperfections will lead to non-conservative forces that must be overcome by the motor.

Physically, the motor has a 3.25 in. diameter with an overall length of 8.42 in. The interface attachment plate is 3.50 in. square with four threaded holes equally spaced on a 4.03 diameter bolt circle. When attached, the motor contributes 10 lbs to the overall segment weight. The motor windings result in a $K_T$ of 15.5 V/KRPM. A tachometer is integrated into the motor housing allowing feedback of the motor speed.

A motor speed control linear amplifier was used to drive the speed of the motor. Although a torque control amplifier was preferred, availability considerations resulted in the use of the speed control amplifier. This meant that the real time control algorithm would have to convert desired torque commands into the proper motor speed commands to control the body segment. This amplifier received a +/- 10.0 volt DC signal as input from the control computer and output the proper voltage signal to the motor. The amplifier uses a closed loop control sys-
tem to maintain the commanded motor speed. Tachometer data from the motor is provided to the amplifier to furnish this feedback signal.

The amplifier was adjusted to provide a maximum motor speed of 3000 RPM. This maximum speed was selected because it was above the requirements determined from the dynamic simulation and below the maximum speed of the motor. With the maximum speed of the motor set, a requirement for the dynamic balance of the flywheel was established at 3600 RPM.

Final assembly of the robot hardware was completed after delivery of the individual components. This process was completed without an adverse incident. However, integration of the electronic components, particularly the electric motor system, proved challenging.

The DC motors are controlled by linear amplifiers that control the speed behavior of the motors. The amplifier gains were adjusted to limit the maximum motor speed to 3000 RPM when the maximum, +/- 10 VDC, output from the computer was received. This would minimize the chance of overspinning the motor and exceeding the 3600 RPM dynamic balance certification of the flywheel.
A final adjustment to the DC motor/amplifier was made to eliminate the zero offset in the motor controller. If not properly adjusted, a zero commanded speed to the motor resulted in small, non-zero motion of the motor. By making this adjustment, a zero commanded speed causes the motor to stop turning, which is the desired motor behavior.

6.4 Flywheel

The rate of change of the flywheel angular velocity provides the necessary control torque to position the robot body segments. The design objective for the flywheel is to maximize its rotational inertia within the housing envelope. This means concentrating the mass near the perimeter of the flywheel. The dynamic simulation program served as a valuable design tool in sizing the mass characteristics of the flywheel. Integrating the control torque curves from the simulations, a maximum required angular momentum value of 225 in-lb-sec was obtained. The flywheel was designed so that when spinning at a motor speed of 3000 RPM, a value in excess of this amount would be achieved.

A 12.0 in. diameter flywheel was selected after confirming that the design would lead to inertia characteristics in excess of the design requirement. A 1.0 in. x
1.375 in. cross-section is located at the perimeter of the flywheel to provide mass concentration at the largest radial distance from the center of mass. This thickened rim is supported in four places by 0.75 in. x 0.25 in. ribs. The central hub provides a fastening interface to the shaft extension. The shaft extension provides the axle functions of support in the housing and a means of transferring the motor torque into the flywheel. The shaft and flywheel, when mated, contribute 6 lbs. to the overall system weight.

The flywheel, designed to these specifications, results in a motor/flywheel system capable of storing approximately 500 in-lb-sec of angular momentum at a maximum motor speed of 3000 RPM. This is greater than the required 225 in-lb-sec determined in the analytical simulation.

The outer rim of the flywheel contains 24 equally spaced, threaded, holes. These holes were entered into the design to provide a means for adding weights to the perimeter if unmodeled, non-conservative forces necessitated additional control momentum.

To achieve the required momentum, the flywheel operates at a high angular velocity. Therefore, to minimize vibration, the flywheel was dynamically, or spin,
balanced. The balance requirement was established at 3600 RPM. This requirement was set above the operating limit established for the motor. The operating limit was set by adjusting the motor amplifier to yield a maximum speed of 3000 RPM.

6.5 Rate Gyro

The feedback loop in the robot control system requires rate and position information. Providing direct rate sensing and integrating to determine position satisfies these requirements. An aircraft quality rate gyro was selected to provide the rate information to the control computer. The Humphrey, Inc., Model No. RG51-0107-1, rate gyro was selected after considering products from several vendors. This instrument is capable of sensing angular rates in the range of +/- 30 degrees per second. The measurement requirement for angular rate was derived from analysis of computer simulation data. Simulated movements of the robot indicated that +/- 10 degrees per second (analytical case 1) was required to track the motion of the body segments. The +/- 30 degree per second capability of the selected gyro exceeds these requirements. Although higher rates may be experienced,
as shown in analytical case 2, cost considerations limited the selection of rate gyros to the +/- 30 degree capability.

The rate gyro is powered by a 24 VDC signal that is input through two pins of a nine pin connector located on the end cap of the gyro. Two other signal reference pins are held at +/- 10 VDC. This reference voltage sets the range of the output signal from the rate gyro. The +10 VDC reference limit corresponds to a +30 deg/sec rotation and the -10 VDC corresponds to a -30 deg/sec movement. The signal is sent to the control computer between these two limits for rate values between the +/-30 deg/sec capability of the gyro.

6.6 Brakes - Electromagnetic Clutch

Brakes are required at each joint to satisfy two requirements. First, after the robot has moved into the proper position, the brakes are activated to lock the joints and enable the robot to react tip loads. Second, any non-conservative forces encountered during robot motion will lead to residual momentum in each flywheel. Locking the joints after the proper position is achieved allows this residual momentum to be transferred from the
robot to the base support structure. Transferring this momentum requires the moment reaction capability in the joints that the brakes provide.

Serving the functions described above, the brakes can be of the on/off type and do not require any proportional braking. To meet these requirements a solenoid activated clutch was sought. The brakes are only activated when the proper position is achieved and body rates are small. Therefore, it is not required for the brakes to react the full torque capacity of the electric motors. When the brakes are applied, the torques in the system are approaching zero since the commanded robot state is achieved. With this in mind, a brake torque requirement of 15 in-lb was set. Several vendor products were reviewed that met the stated requirements. The Machine Components Corp, CEM series, electromagnetic clutch was selected. This clutch provides a maximum braking torque of 20 in-lb when energized. Its compact size makes it ideal for integrating into the joints of the robot body segments.
One disadvantage of the selected brake is the unidirectional braking force. The brake only holds for motion in a single rotational direction. Therefore, two brakes were installed at each joint, one to cover each direction of possible motion.

The brakes become energized when 24 VDC is applied across the input terminals. This signal is activated by a digital signal from the control computer that activates a digital relay. The relay closes the circuit that allows the proper voltage to be applied to the brakes.

The initial robot configuration does not allow the energizing/deenergizing of individual brakes. All joint brakes are turned on or off together. The ability to activate individual brakes are planned for future configurations of the robot.

6.7 Control Computer

A digital computer is used to analyze inputs, make appropriate calculations, and issue the proper outputs to the robot. A Northgate "Elegance" 386 microcomputer was selected to perform the required computational tasks. The computer was configured with 4 megabytes (Mb) of Random Access Memory (RAM), VGA Color monitor, 80 Mb Hard
Disk, a 3.5 in. 1.44 Mb disk drive, and a 5.25 in. 1.2 Mb floppy disk drive. The computer operates at a clock speed of 20 MHz.

The ability to read analog input signals from, and send output signals to, the robot electrical components resides in the data acquisition boards. These printed circuit boards are installed in the computer to provide the control function.

There are 12 analog inputs to the computer. Six are required to receive the rate gyro information and another six are used to feed back the electric motor speed to the torque controller. Six analog outputs are used to command motor speed which, in turn, provides the control torque to the body segments.

A single digital output channel is used to energize/deenergize the brake system. Signals from this channel open/close a relay that supplies the 24 VDC required to activate the brakes. The digital output channel is capable of switching up to 8 different devices. This is accomplished by the bit stream in the 8 bit word sent to the channel. For example, a "10000000" will activate the first device and "01000000" will activate the second, and so on. The initial test configuration of the robot ener-
izes the brakes when any of the bits are set high. A future configuration of the robot may use this bit stream to activate individual brakes selectively.

Meeting these signal input/output requirements necessitated the review of several different vendor products. The Scientific Solutions, Inc. Labmaster and DADIO boards were selected after assuring their ability to meet the system requirements. Each Labmaster board includes 16 analog input channels and two analog output channels. The DADIO boards have 4 analog outputs and 3 digital outputs. Each digital output is capable of controlling 8 separate devices. To meet the requirements, the test system was configured with two Labmaster boards and one DADIO board.

The data acquisition boards are driven by a special software package called "LABPAC" by Scientific Solutions, Inc. This software is a memory resident driver for executing the input/output and timing functions of the boards. As a memory resident program, the individual services must be accessed by writing data directly to the CPU registers and processing the proper interrupt. The LABPAC software reads the registers to determine which software services are being requested. Once determined, LABPAC drives the data acquisition boards to perform the
function and returns the result to the CPU registers. The control program can then read the data returned by LABPAC directly from the registers and interpret them according to the established control laws.

An interface software package was developed that enabled the control program to access the functions of the memory resident data acquisition software. This software provided the control program with a high level module for reading and writing data to the hardware boards. With this interface, the control program could request input/output services without concerning itself with items such as board and function addresses.

Moving the individual body segments into the proper orientation is the responsibility of the real time control system. The algorithms in this system are responsible for reading the proper system inputs and determining if corrective action is required. If so, the controller must issue the proper outputs to effect the desired result. A classical, single input, single output, controller was designed to perform these duties.

The commanded orientation is supplied to each body segment. An error signal is generated by differencing the current rate and position multiplied by their respective gains. The error signal is interpreted by the con-
controller and a torque signal is issued. As mentioned earlier, the linear amplifier used in the system was designed for speed control, not torque control. Therefore, the controller was required to convert the torque commands into speed commands. A relation between the time rate of change of the angular momentum and the externally applied torque is given by the equation

$$T = \dot{H},$$

(52)

where the symbol \( T \) represents the applied torque vector, and \( \dot{H} \) is the rate of change of the angular momentum vector for each body. Therefore, the control algorithm must read the current motor speed and, knowing the desired torque, calculate a new target motor speed. To calculate the target motor speed, the time step must be determined. The time step is determined by timing the control loop and calculating the difference between the present time and the previous time the torque command was issued. The rate of change of angular momentum is proportional to the change of angular velocity of the flywheel. The proportionality being derived from the mass and radius of the flywheel. By knowing the time interval and the required slope of the angular momentum the target motor speed is calculated by the equation
\[ \omega_2 = \omega_1 + \frac{T\Delta t}{r^2 m}, \]  

(53)

where \( \omega_1 \) represents the current motor speed read as an analog input from the motor and \( \omega_2 \) represents the target motor speed. The symbol \( \Delta t \) value represents the control loop cycle time, and \( m \) represents the mass in the rim of the flywheel. Also, the symbol \( r^2 \) represents the square of the radius of the flywheel. Together these quantities model the inertia properties of the flywheel.

The torque that results from the new motor speed acts on the body segment. The body parameters are sensed by the rate gyro and provides angular rate data to the control system. The rate is integrated to determine the current angular position of the body segment. The rate and position are multiplied by their respective gains and fed back for comparison with the commanded angular position. A new error signal is formed in the forward loop which, again, is multiplied by the control gain yielding a corrective torque. The control loop process repeats until a vector norm of the error is below a specified value and the body rates are acceptably low. At this time, the brakes are applied and the motors are commanded
to stop. Residual angular velocities in the segment flywheels are reacted as residual torques through the braking system.

In the final configuration, the control computer was set up at the test area. Each of the two Labmaster data acquisition boards were connected through ribbon cables to 40 pin connector plates. A special 20 pin connector plate was constructed for the DADIO board. From these connector plates, the DC motor outputs, the DC motor tachometer inputs, and the rate gyro inputs were attached. In addition to the data lines, power supplies for the rate gyros and electric motors were connected. Finally, the air bearing pads were connected to the pressurized air supply available in the test area.

Testing the electric motor connections required the development of a special purpose computer program. This program served two purposes: a means to drive the DC motors, and a verification of the software techniques that would be required in the real time control algorithm. Each analog output channel could be commanded separately allowing each DC motor/amplifier to be controlled and adjusted. Likewise, each analog input channel could be read to verify the tachometer input from the motors. The digital output channels were also tested
with this software for its suitability for running the brakes. Using this program proved to be a valuable integration tool since each component could be tested separately before being assembled into the final system.

6.8 **Air Bearing Support System**

This kind of robot depends on changes in angular momentum to provide torque to body segments. These momentum changes can occur within the performance limits of the electric motor/flywheel system. The maximum speed and torque of the motor are limiting factors on the control authority of the system. Non-conservative forces acting on the robot can cause saturation of the momentum system and the loss of control. Sources for non-conservative forces include friction at the interconnecting joints, friction at the ground contact, and forces exerted by the electrical cables and air supply hoses. These dissipative forces must be minimized.

An air bearing support system was designed for each body segment to minimize static and sliding friction. Precision air bearing pads were purchased from C & H Machine Co. to provide the air bearing support. Between the air bearing pad and the body segment is a height adjustment system. Each body segment has redundant support paths. The body can be supported by adjacent bodies
or vertically through the air bearing pad. The adjustment system enables the support system to be adjusted vertically until the air bearing pad is the primary body weight load path. This approach reduces joint friction between adjacent bodies. This vertical adjustment is provided by a threaded spindle that screws into the bottom of the segment housing. The amount the spindle is screwed into the housing changes the vertical height of the support system. On the bottom of the spindle is a spherical bearing that inserts into a mating hole on the air bearing pad interface plate. This feature allows the pad to float freely and transmit only vertical support forces. Once the proper height adjustment has been made, a lock nut is tightened against the housing to prevent the threaded spindle from additional motion.

6.9 Testing

Once the complete robotic system was assembled, tests to provide validation of the concept and comparison to simulation predictions were conducted. Prior to this testing, several startup problems were encountered. The air bearing system proved to be the most troublesome. The testing process is documented in this section.
Conducting a test of the motion of the robot required an area of 16 ft. x 20 ft. be cleared in the laboratory. In this area, an air bearing floor had to be constructed, an attachment system to hold the robot had to be installed, and all necessary power supplies had to be coordinated. The high bay structures laboratory at NASA/JSC was used for the test. A schematic of the test facility is provided in Figure 6-3.

The requirements on the facility to test the robot included 120 VAC and 208 VAC power to drive the motor and rate gyro power supplies. Also, compressed air at 120 psi was required, and available, at the test site.
The air bearing pads that are used to support the individual body segments are designed to operate on a precision air bearing floor. The nominal air gap for the type of air pads used for the test is 0.005 in. This small gap forced the air bearing floor to meet stringent flatness and smoothness requirements.

Initially, six 4 ft. x 8 ft. sheets of Plexiglass were laid to form a surface that was 12 ft. x 16 ft. The non-uniformity of thickness along the length of the Plexiglass led to inconsistent edge conditions between adjacent panels. To smooth these edge problems, a putty was smeared liberally along all mating edges of the Plexiglass. After the putty was dry, floor buffers with sanding pads were used to smooth the rough edge interfaces. This induced a minor improvement. However, the air bearing pads, when weighted with the equivalent mass of one body segment, would not slide properly over many of these areas. A survey of the floor using a 3 ft. straight edge showed problems with high spots and low depressions in the Plexiglass. The low areas were filled with additional putty and the high spots were sanded down. Again, after the putty had dried, these areas were sanded using the floor sander. The air bearing pads continued to hang up on certain areas of the floor. The
process of surveying, filling, and sanding was repeated four times. Although each application improved the floor, the problem persisted.

The next approach to solving this problem consisted of laying a Formica layer on top of the Plexiglass/putty surface. A glossy Formica was selected to help smooth the irregularities still existing in the Plexiglass floor. The Formica was laid with special attention to the mating edges of the individual sheets. The Formica was available in 5 ft. x 16 ft. sheets. This size reduced the number of seams that had to be guaranteed flush. Also, the 5 ft. width meant that the Formica seams would not line up with the Plexiglass seams. Pad testing after the Formica was laid indicated a significant improvement over the Plexiglass floor. However, some pad dragging problems were still encountered. The remaining problems were solved by increasing the volume of air supplied to the air bearing pads. This increased the air gap beneath the pad sufficiently to reduce the dragging problem to acceptable levels.

Once the problems with the air bearing support system were solved, testing the complete system could begin. During the initial startup phase, all wiring connections were tested and verified by using the test software pro-
gram mentioned earlier. With this software each input/output channel could be tested along with the component connected to that channel.

The amplifier circuitry is responsible for changing the motor speed based on a speed command input. A schematic of the control computer/amplifiers/motors assembly is shown in Figure 6-4. The electronics of this system results in a second order behavior as the motor changes speed. The motor tachometer is fed back to the amplifier as part of the speed control loop. To adjust the damping characteristics of this system, a step pulse generator was connected to the amplifier input channel. This input emulated a step increase, and decrease, in commanded motor speed. An oscilloscope was connected to the output of the amplifier to measure the time response of the fly-wheel/motor/amplifier system. The amplifier gains were adjusted until a near critically damped response was achieved. This adjustment minimized the speed overshoot and settling time. Therefore, it assured that a commanded speed would be achieved without unstable or lightly damped, sinusoidal motor response.
Figure 6-4  Computer/Amplifier/Motor Schematic
With the amplifier stability properly adjusted, the motor speed for each DC motor was tuned from a single reference motor. The reference motor was established through a special verification process. An electric motor, from body segment 5, was set up in a dynamometer to measure the motor's speed accurately. This motor was commanded through the test software to a speed of 1200 RPM. The motor command was adjusted until the dynamometer verified the speed at 1200 RPM. Body number six, completely assembled, was selected as the reference body. A command was issued to this motor that should result in 1200 RPM. Since this motor is under the load of the flywheel, the command initially caused the motor to run at a speed lower than 1200 RPM (approximately 1150 RPM). Using a strobe light to determine motor speed, the amplifier gains were adjusted until the commanded 1200 RPM resulted in 1200 RPM as the speed of the motor/flywheel assembly.

With body number six as the new reference, the motor in the dynamometer was returned to body number five. Each body was commanded to 1200 RPM. The strobe light was used to adjust each amplifier until the output for each body matched the speed of reference body number six. This speed matching was accomplished at 1200, 2400, and
3000 RPM. Speed matching was considered satisfactory at each test level. The final speed adjustment to the amplifiers was to nullify any zero offset conditions. With the motors commanded to zero, some motors still had small residual rotations. An adjustment to the amplifier eliminated this problem.

The rate gyro instruments were calibrated by the manufacturer before shipment. For this reason, no further calibration or testing was conducted on this system.

The real time control software proved to be the key element in the entire system. The control software drives the entire system. If it is not operating properly, the robot will not perform satisfactorily. The software could not be checked out or verified until the robot had been assembled and testing could begin. Therefore, the early testing focused on the software performance instead of the robot motion.

Control loop timing seemed to play a critical role in the behavior of the system. If the control loop was operating too fast, interactions with the second order dynamics in the amplifier electronics and the body segments caused unpredictable, sometimes unstable, behav-
iors. Program statements that purposefully delayed certain aspects of the control loop proved beneficial in obtaining the proper system performance.

The initial tests were conducted with a single body segment. This approach simplified the dynamics of the robot into a single input, single output system without interactions from adjacent body segments. With this configuration the real time control software was tested. The single body caused the control loop to be performed so rapidly that the system was only marginally stable. Placing a 50 millisecond (msec) delay in the control loop stabilized the system. Once stable behavior was achieved, the position and rate feedback gains were adjusted until a classical second order response was exhibited by the body segment.

The next step was to add a body segment and demonstrate control of the robot comprised of two body segments. Understanding the behaviors and interactions of the complete electro-mechanical system obtained from the experiments with a single body segment proved invaluable in this set of experiments. Again, the delay software statement was adjusted to ensure control loop stability. With the control loop acting predictably, the feedback loop gains were altered to achieve the desired system
response.

After achieving the desired response from the robot comprised of two body segments, a third segment was added. The process of adjusting control loop timing parameters and gains was repeated until satisfactory performance was achieved. The desired system behavior was quickly attained with three body segments attached. The speed of this solution led to confidence in the control loop and the complete system. Based on this confidence, a decision was made to connect the other bodies to form the complete six body chain.

With the six body chain in place, motion tests were conducted. These tests demonstrated the value in building the assembly one body segment at a time. This build-up method enabled early problems with the system to be identified and eliminated. The six body system moved in a predictable fashion to the prescribed shape function after a few minor gain adjustments. On balance, the robot moved in a more subtle, controlled fashion with six bodies than with three, or less. This is undoubtedly due to the effects of increased inertia in the entire system.

Once the complete system was operating properly, a series of initial tests were conducted. These tests were completely successful and quickly verified the design and
control algorithms. Experiments were conducted that examined the effect of control system gain changes on the system performance. These tests demonstrated the robot's ability to move to different tip locations and orientations. The experimental tests were run over a period of three days.

The initial tests were successful to the extent that it was decided to choose a nominal set of control gains, run the robot, and save the test data. The test data were to be used for follow up test/analysis correlation studies. After selecting the nominal gains, the robot motion was initiated. The nominal gains were values that, through the previous tests, resulted in the desired time response. In this test, for the first time, unexpected behavior was seen. Successive tests involved varying the control gains and other system parameters to see if the earlier success could be restored. After many tests without success, a troubleshooting process was begun. Each component of the system was electronically tested. The outcome of this search was the conclusion that the rate gyros on several body segments were not functioning properly. Out of the six rate gyros, two were completely dead, two showed marginal performance, and two were working properly. This created a twofold
problem. First, the control system was not supplied with the proper rate signal, corrupting this part of the feedback loop. Second, the rate signal is integrated by the control software to produce the position component of the feedback loop. With the rate data corrupted, the position information was also degenerate. These two effects, caused by failed rate information, forced a termination of the tests until repairs could be made.

The two failed gyros were returned to the manufacturer for repair. It was two and a half months before the gyros were returned to the lab for re-installation into the Large Space Robot. During this time, environmental conditions (Houston summer) caused the air bearing floor to warp. This reduced the area of the floor that could be used for testing. Moving the robot into areas degraded by the warping caused the air bearing pads to drag on the floor. In addition, in some areas of the floor the warping produced a sloping surface. This causes the robot, supported by the air bearing pads, to slide down the sloping surfaces. These effects saturate the motor/flywheel torque assemblies leading to degenerate results.

It became necessary to survey the floor to determine which areas were least effected by the floor warping.
Several large ball bearings were used to conduct this survey. The ball bearings were placed statically on the floor in different areas. If the ball remained motionless, a level surface was indicated. If the ball bearings began to roll, the motion indicated a sloping surface. Once the surface had been surveyed in this fashion, future tests were designed to keep the robot in the most level areas for the duration of the test. To maintain this restriction, only tip locations that kept the root body segment from moving more than 50 degrees were specified. Although a limiting factor, this restriction did not impair the value of the tests. All objectives for testing the design and analysis were achieved within the restrictions caused by the degraded air bearing surface.

To test the complete six body assembly, a case previously analyzed was selected. This test case (analytical case 1) requires the robot to move such that the final tip position is at $X = 114.00$, $Y = 64.00$, and at a slope of 0.57. This test is designed to validate all aspects of the robot positioning system. With this tip specification, the shape function that meets these boundary conditions are calculated. The shape function parameters used in the test are identical to those shown in
Table 5-1.

The shape function, with these parameters, meets the end conditions of the robot. The arc length of the shape function is 132.022 in., within 0.02% of the length of the robot.

Once the shape function parameters have been determined, the individual body segment angles are calculated. These angles are derived by approximating the shape function with the discrete body elements that make up the robot. The algorithm that determines the body angles along the shape function is discussed in section 4.2.2. The inertial angles resulting from this calculation are given in Table 5-2.

The case described was tested using a variety of control system gains to investigate the sensitivity of the system to these parameters. Analysis provided the starting point for the gain selection. However, test data was used to finalize the gain selection. Table 6-1 provides the rate, position, and control gains selected for the test.
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<td>80.0</td>
<td>80.0</td>
</tr>
<tr>
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<td>160.0</td>
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<tr>
<td>5</td>
<td>160.0</td>
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</tr>
<tr>
<td>6</td>
<td>160.0</td>
<td>80.0</td>
<td>80.0</td>
</tr>
</tbody>
</table>

Table 6-1 Control System Gains, Test

With these gains programmed into the real time control algorithm, the robot motion tests were conducted. Although many tests were conducted using a variety of control gains, runs made with these gains provided the test data included in this report.

To begin the test, the robot is extended along the x-axis and steadied to remove any angular rates from the system. This procedure provides the zero position and zero rate initial conditions expected by the control system.

It is important to consider when examining the time histories that the control loop derives its commanded position based on relative angles between adjacent
bodies, not inertial angles. This is important because the initial angular error determines which bodies will dominate the system dynamics through high initial control torques. For a better understanding of the control loop, Table 5-3 can be referenced for the commanded relative angles.

To position the tip at the prescribed location takes about 15 seconds. The angular position time response for each body shows the robot successfully achieving the desired position. In Figures 6-5 through 6-10, the angular position histories are shown. By the end of the test, each body has moved to its targeted angular position.

As the first body moves toward its target of 43.6 degrees, it receives a disturbance force from body two acting at the shared interface. This disturbance interrupts the motion and causes body one to remain virtually motionless for the period between 4 and 8 seconds. The position offset from the target angular position yields a torque that, in effect, is supplying a reaction force at the body one/two interface. This allows body two to move toward its desired position. As body two nears its target, the torque in the body is reduced allowing body one to continue toward its final position.
Since the control system is based on relative angles, body one exerts the largest initial torque. This is responsible for initially forcing body two in the negative inertial angular direction. However, body two is trying to move in this direction based on the prescribed negative relative angle. The interaction between adjacent bodies is shown in the body two position time history. For each body, the actual behavior caused by this interaction depends on the control gains and whether this effect produces disturbances that aid or hinder the body from achieving its control objective.

Bodies three, four, and five all exhibit a predictable second order behavior. They all exhibit initial negative inertial angles. This results from the accumulation of negative relative angles. This accumulation, plus the dynamic interaction, contributes to the negative inertial response seen in these bodies.

Initially, body six moves to a positive inertial angle because its controller is trying to achieve a positive relative angle with respect to body five. However, this small positive relative angle is overshadowed by the accumulation of negative relative angles in the inboard bodies. This explains the initial positive motion in the time history followed by the negative angular response.
As the inboard bodies achieve their control objective, the inertial angle in body six builds to the desired value.

The angular motion described in the position plots gives the overall system a response that appears as a low stiffness cantilevered beam that is being rotated. This type of behavior is expected where the beam, or robot arm, derives its internal stiffness from the control system. The damping capability of the control system provides a highly damped response, not possible in a completely passive structural system.

Some of the bodies show sinusoidal curves that are flattened in the peak amplitude region, particularly on the negative side. This is caused by stiction between the body and the air bearing floor. This drag effect was a significant non-conservative force that acted on different bodies at different times throughout the test. Another source of non-conservative forces is attributed to the air bearing pad supply hoses. These hoses are composed of a stiff plastic material. The stiffness of the hose is derived from its strength requirement to handle the 120 psi air supply. To a lesser degree, but still a significant contributor, is the actual power/data cable running the length of the robot arm. This cable
supplies the electric power to the motors, rate gyros, and solenoid brakes. In addition, this cable bundle provides all data communications between the control computer and the robot. Although efforts were taken during installation to reduce its stiffness, this cable bundle contributes to the non-conservative forces acting on the robot.

The resulting effect of these non-conservative forces is the residual angular momentum in the system when the desired tip position is achieved. If the system was conservative, the flywheels would come to a stop as the robot arm stopped in the desired position along the shape function. Instead, significant residual RPM is recorded in each motor. In some cases, this residual flywheel rate was as high as 1000 RPM. This residual momentum is equivalent to the time integral of the non-conservative forces acting on the robot.

At the end of each test, when the desired position had been achieved, the motor/flywheel speed continued to increase. Measurements had indicated that the plane of the air bearing floor was not perpendicular to the gravity vector. The resulting gravity force caused the robot to slide toward one end of the floor. The control system would attempt to counter this force by applying a static
torque. This torque would increase the speed of each body motor until saturation or termination of the test. This effect can be seen in the position time histories toward the end of the test. Near the end of the test data, each body gradually begins to slide down the air bearing floor. This effect is dependent on the body's position on the floor.

There are two types of dynamic interaction apparent from the test data. The first interaction occurs as a low frequency dynamic response in the 0.075 Hz range. This interaction is caused by the control gains and the inertial properties of the body segment. This type of interaction can be predicted and modified through control gain variation. This interaction between bodies was expected and did not lead to any dilatory behavior with respect to the system achieving its control target.

The second interaction occurs as a higher frequency, approximately 3 Hz, vibration. This interaction was not predicted by analysis prior to the test. Post-test analysis indicated that this vibration was rooted in the fly-wheel/motor control loop. This interaction is caused by noise in the control loop. Electromagnetic interference that intrudes into the control system through unshielded cabling is one source of this noise. Another source of
noise is generated internally by the method used to provide torque through motor speed control. These topics will be considered in more detail in later sections. These vibrations were small and did not grow in amplitude. Although this effect produced some jitter in the response, it did not keep the overall system from achieving the target point.
Figure 6-5  Angular Position - Body 1, Test

Figure 6-6  Angular Position - Body 2, Test
Figure 6-7 Angular Position - Body 3, Test

Figure 6-8 Angular Position - Body 4, Test
Figure 6-9  Angular Position - Body 5, Test

Figure 6-10  Angular Position - Body 6, Test
The angular rate data are shown in Figures 6-11 through 6-16. The rates show no unexpected trends since they all start at zero, build to a maximum value, and then diminish as the body approaches its target position. The item of interest in this data is the amount of noise being seen by the control computer in the rate signal. Frequency analysis of this signal indicated a prominent 3 Hz component. As mentioned earlier, this noise results from the speed control method for applying torque.

Due to funding limitations, the robot was constructed using speed control amplifiers to operate the motors. The control system computes the required torque. Therefore, the control software must convert the control torque into a commanded speed signal to the motor. To do this, the software keeps track of the previous time cycle in the control system. Since the computer was allowed to operate in a control loop without constraint, the cycle time would vary slightly. The variation results because, in addition to controlling the robot, the control computer is recording test data to the storage disk. The writing of data to the disk is a buffered process within the computer. Therefore, on some passes through the control cycle no data are written to the disk and on some passes data are written. This induces time variation
between control cycles. In general, the control cycle was about 30 milliseconds. Converting the torque into a commanded motor speed required, in effect, differentiation of the speed curve. This differentiation caused noise to be generated by the control computer and imposed on the commanded speed output. This introduced speed variations in the motor, which resulted in a high frequency torque variation. The torque variation, through its effect on body rotational acceleration, is sensed by the rate gyro. This signal, when fed back into the control loop, resulted in additional noise in the control commands further contributing to the noise in the overall system.

The fluctuations in the commanded speed also causes an additional source of noise in the rate signal. The changes in current supplied by the amplifiers to the motor to deliver the commanded speed also contributed to the rate signal noise. The motor cabling and the rate gyro cabling were all unshielded and bundled together. The variations in the current running in the motor cables induced noise into the nearby rate gyro signal cables. A simple test was conducted to verify this phenomenon. The rate gyro output was connected to an oscilloscope. With the motors powered off the signals were examined. No
significant noise in the signal was reported. The motors were powered on and the signal was reexamined. Significant noise was seen in the rate gyro output signal.

The effect of rate gyro noise on the control system was more apparent if the rate gain was increased. If the rate gain was increased, a more damped response resulted as expected. However, more jittering in the bodies was also experienced. To minimize this effect, the rate gains were kept low. This caused restricted control over the damped nature of the response. However, since this effect was stable, some jittering was allowed such that excessive overshoot was eliminated. Without sufficient rate gain, the system response becomes a lightly damped sinusoid, with significant position overshoot.
Figure 6-11 Angular Rate - Body 1, Test

Figure 6-12 Angular Rate - Body 2, Test
Figure 6-13 Angular Rate - Body 3, Test

Figure 6-14 Angular Rate - Body 4, Test
Figure 6-15 Angular Rate - Body 5, Test

Figure 6-16 Angular Rate - Body 6, Test
The control torque determined by the control computer was converted to a commanded speed to the motor. As stated earlier, this was necessary due to the limitation on available motor amplifiers. The torque command was calculated in units of in-lbs before being converted to a desired motor speed. The desired torque sets the slope required in the motor speed curve. In Figures 6-17 through 6-22, the control torque is shown. The values in these plots result from the multiplication of the error signal by the forward loop controller. The values shown are above the 45 in-lb peak torque available in the motors. This was allowed to occur because this results in a high slope speed curve over the control interval. If the resulting commanded speed is greater than the maximum motor speed of 3000 RPM, the control software will truncate this value to the maximum speed. This method guarantees that the maximum available torque will be requested from the motor amplifiers without overspinning the motors. Although this method worked well enough to complete the tests, it was not an ideal arrangement. The differentiation that occurs in the conversion from torque to motor speed was the primary source of noise in the control loop. This noise can be seen in the output from the control computer to the motor amplifiers.
Figure 6-17 Control Torque - Body 1, Test

Figure 6-18 Control Torque - Body 2, Test
Figure 6-19 Control Torque - Body 3, Test

Figure 6-20 Control Torque - Body 4, Test
Figure 6-21  Control Torque - Body 5, Test

Figure 6-22  Control Torque - Body 6, Test
Photographs of the test were made to compare the intermediate positions and shapes with the simulation. The desired tip position is depicted in the photographs as a circular shaded target. The target location is measured and placed on the air bearing floor prior to the test. Figure 6-23 shows the robot in the initial position. The test is started and the robot begins to move in the predicted fashion. Figure 6-24 shows the robot in the curved cantilevered shape predicted in the analysis and displayed in Figure 5-22. Similarly, the shape shown in Figure 6-25 resembles the simulation shape in Figure 5-23. As the test is completed, the robot has assumed the desired shape and placed the tip coincident with the target.
Figure 6-23 Initial Position, Test

Figure 6-24 Robot Intermediate Position 1, Test
Figure 6-25 Robot Intermediate Position 2, Test

Figure 6-26 Final Position, Test
6.10 Correlation

When the tests were completed, the parameters for the "as tested" configuration of the robot were entered into the simulation program. This analysis was conducted to investigate the correlation between the analytical predictions and the test results. Control system gains identical to the test case described earlier were entered into the simulation program (reference Table 6-1). Mass property data were entered based on the "as built" configuration. The mass of each body was derived from measurements of the weight. Rotational inertia, an important component, was calculated from the engineering drawings.

Although the mass properties parameters were entered to match the test conditions, other important considerations were not modeled. Namely, the stiction between the air bearing pads connected to the body segments and the air bearing floor. This phenomenon was apparent in nearly every test despite efforts to eliminate it. This problem resulted from the use of a nonprecision floor for the test. Another important factor not considered in the mathematical simulation model was the effect of electrical cables and the air bearing supply hoses. The simulation also assumed a flat surface whose normal vector
was parallel to the gravity vector. As mentioned before, the test area only approximated this condition. Each of these components exerted small, but noticeable, forces on the body segments during the test. The effects of each of these non-conservative sources was apparent by the high residual motor rates in the system when the arm had quit moving. Possibly the most significant difference between the test configuration and the analysis is the torque versus the motor speed control system. The simulation body controller exerts torque on the body directly. The prototype robot used forward differentiation of the motor speed to produce torque. The deficiencies and drawbacks of this type of system has been outlined previously.

In Figures 6-27 through 6-32, a comparison between the analytical predictions and the test is shown. The position time history for body one shows an initial trajectory very similar to the test data. The angular rate is nearly identical between the test and analysis. The position curve flattens in the test before achieving the target position. At this point the curves begin to differ. Both curves show signs of body two interacting with body one. However, both bodies achieve the desired target position by the end of the test/analysis.
In Figure 6-28, body two exhibits a generally correlated result except for the predicted overshoot in the 5 to 12 second time interval. Although difficult to verify, this deviation in the test data may have resulted from floor stiction. The test and analysis show a convergence to the target position at the end of the time history. The saw tooth nature of the test curve, especially at the end of the test, is the effect of the differentiation noise in the motor speed control loop. Frequency analysis of the test data showed this waveform to have a 3 to 4 Hz component. This correlated with the predicted frequency of the motor speed component of the prototype control system.

Simulation data of body three appears to have similar frequency content as the test data. This similarity is shown in Figure 6-29. The movement of the body in the opposite direction of the desired position is predicted by the analysis and experienced during the initial part of the test. The flattened character of the test curve in the 2 to 4 second interval is due to floor stiction. This was verified visually during the test when the body moved in the negative direction. After this stiction delay, the slopes of the recovery portions of the two curves are very similar. The time difference between the
two curves is due to the stiction that occurs early in the test data. The test curve does recover, as the analysis predicts, and achieves the desired position by the end of the test. The saw tooth signal at the end of the test data results, as mentioned for body two, from noise generated by the motor speed control loop.

Body four exhibits a significant overshoot character in the opposite direction of the target position (Figure 6-30). The general negative trend is predicted by the analysis, however, to a lesser extent. The excessive negative overshoot may be cause by gravity acting through a sloping floor or bending in the air bearing supply hoses. Again, however, the system does recover to its final target position. The recovery sections of each curve, where the position trend is toward the target, is of a similar slope. At the end of the test, the saw tooth waveform is again apparent. This occurs because the position component of the error signal is small when the target position is achieved. However, the noise in the rate signal is causing a contaminated error signal to be passed to the controller. This noise manifests itself in a noisy steady state response about the target position.
Correlation in bodies five and six (Figures 6-31 and 6-32) are similar to the plot comparisons of body four. Each shows the initial negative movement in both the analysis and test. This interval of the curve is followed by a recovery period that achieves the final target position. The outer bodies were moving on an area of the floor that exhibited pad stiction. This problem was verified visually during the test. Weights were placed along the edge of the air bearing floor in this area to reduce this problem.

On balance, the correlation was moderately successful. The non-linearities and non-conservatives forces inherent to the test configuration make better correlation unlikely. The motor speed control versus torque control introduces another variable in the correlation studies. Additional correlation analyses were conducted using different control gain values. The curves generated during these subsequent analyses showed better correlation in some areas and less in others. In particular, higher rate gains were used to model the effects of the non-conservative forces acting on the system. This caused some portions of the analysis to correlate better with the test data by reducing the overshoot, and other portions of the plot to worsen. The values within this
report show the results with a consistent set of parameters between the analytical simulation and the test data.
Figure 6-27  Position Test/Simulation Correlation, Body 1

Figure 6-28  Position Test/Simulation Correlation, Body 2
Figure 6-29  Position Test/Simulation Correlation, Body 3

Figure 6-30  Position Test/Simulation Correlation, Body 4
Figure 6-31  Position Test/Simulation Correlation, Body 5

Figure 6-32  Position Test/Simulation Correlation, Body 6
7 SUMMARY

This study has addressed the conceptual design of a multi-segment robot for use as an assembly aid in low Earth orbit. The tip positioning algorithm for this unique robotic configuration has been demonstrated. A prototype of the proposed configuration has been built and tested. The body of this dissertation has described the analysis, design, fabrication, and test of the multi-segment robot.

The successful completion of the proposed test and development phase has proven the viability of this concept. From these results several conclusions can be drawn. One of these focuses on the specific design and test of the multi-segment robotic configuration. Lessons can be learned from the specific design and the methods used to carry the concept from the CAD computer screen to the actual implementation. Perhaps the most important summary of this study deals with the broader issues of the concept. Primarily, how this kind of system could be used in space and what benefits and advantages it would provide over existing configurations. The essential virtue of conclusions is that they serve as a point of departure for future work. Additional development is required to take full advantage of the benefits of this configuration and move it to the next level of flight readiness, a flight experiment.
The design developed and tested in this study demonstrated several important features. First, an unconventional robot could be developed based on shape functions and discrete bodies. Also, momentum torquers could be used to control the position of the discrete bodies. The successful completion of the positioning tests marks a milestone in the development of this concept. The ability to position the robot tip to a prescribed location is fundamental to the development of any new robotic concept.

The next phase of development will capitalize on the benefits of the configuration. As robots grow in size, new problems emerge. Namely, the kinematic positioning errors increase, structural dynamic frequencies are lowered, and tip loads result in high base moments. This configuration has the unique capability to solve each of these problems.

Once the tip positioning system has properly located the end effector/payload, the joint brakes are applied. At this point, the robot is an oddly shaped, cantilevered beam. The high tip mass and low cantilevered stiffness will produce low frequency dynamic behavior. The structural dynamics can be excited by stopping the motion of the tip mass or by any tip load. With the brakes applied, the rate gyros, previously used by the position control system, are now used to sense bending rates in the robot. The torquers can be used
to provide torque out of phase with the angular rate and
damp the vibrational motion. The collocated sensor/effector
pairs inherent to this design enables the easy implementa-
tion of this feature.

After the vibratory motion has been adequately damped,
kinematic position errors may result in tip placement
errors. To negate these errors, the joint brakes are
released for the outboard three joints. Only the last three
segments are repositioned to nullify the initial tip place-
ment errors. Two solution approaches can be used. The
first simply uses the same shape function method used to
generate the shape of the complete robot. In this case, the
problem is solved with the base located at the \( n-3 \) joint
and solved for the last three body segments. A second
approach would use the conventional robotic methods involv-
ing a map between the physical space and the joint space.
In this solution a square coefficient matrix comprised of
trigonometric terms is used to define the relationship
between the physical and joint spaces. The correcting joint
positions that nullify the position errors are computed
directly from this map. The joints are repositioned based
on this computation resulting in the proper position of the
robot tip.
Unlike robots attached to rigid bases, large robots in space will be attached to minimum weight, load limited structures. Consideration must be given to the potentially large moment reaction that occurs when tip shear loads are applied. For large robots, this tip shear, multiplied by the long moment arms, can result in base moments sufficient to cause failure of the base structure. With the brakes locked and the large robot properly positioned, the multi-segment robot is capable of reducing the moment seen by the base structure. If tip shears are applied, a sensor at the base will detect the increase in moment. This information is used to drive the torquers in the direction of the base moment reaction. This, in effect, allows the robot to react the base moment with the torquers, minimizing the moment reaction at the base.

When a robot of this type is attached to a space station, or any orbiting construction facility, the momentum torquers can augment the station control system. On the station, movement of mass with velocity components orthogonal to the axis of rotation results in changes in station angular velocity due to conservation of angular momentum. To maintain the proper station angular velocity, the station control system must compensate for these changes in angular momentum. Therefore, the station control system must be
designed for this capacity. For the kind of robots described in this report, the torquers themselves can be used to store changes in angular momentum. This capability of the robot minimizes the impact of large mass motions on station systems.

Another virtue of this design is its ability to perform as a backup for the control moment gyros of a space station. Control moment gyros, or CMG's, are used to control the attitude of the space station. These devices utilize spinning gyroscopes to produce torques that control the orientation of the station. They are rated, and limited, by the total angular momentum stored by the device. The robot torquers can be commanded to provide control torque to the station if necessary due to malfunction or repair of the CMG system.

Space based equipment must remain safe, if not functional, in the event of system failures. The multi-segment robot has many levels of redundancy as a product of its many body segments. If one of the body segments becomes inoperable, a single joint brake that attaches the body to an adjacent body is locked. The algorithms are adjusted to operate with \( n-1 \) body segments. One of the bodies, the one adjacent to the failed body, is double in length. In spite of the failure, the shape function method remains the same.
The algorithm that determines the placement of the bodies on the shape function is adjusted to account for one of the segments being twice as long. This modification poses no problem for the geometric nature of this algorithm.

Consideration of a large space robot must include attention to the practical aspects of on-orbit assembly. In this sense, the assembly of the robot itself must be considered. The limited capacity of extra-vehicular (EVA) crew to perform complex assembly must be accounted for in any legitimate design. In this robot, several design features are suited to expedite on-orbit assembly. First, the common nature of all of the body segments simplifies the integration into launch vehicles. Once the body segments are in orbit the common body segments can be joined using a single tooling jig or assembly platform. Perhaps most important is that body segments can be joined by simply inserting pins between body segments. This connection joins the bodies and allows for the angular degree of freedom between adjacent bodies. The simplicity of this connection allows the design to be readily assembled by an EVA crewman.

The ability of this kind of robot to solve the problems associated with large space robotics is unique. It is easily launched and assembled on-orbit and provides the levels of redundancy important to space based systems. In
operation, this large robot is capable of manipulating large vehicle components to support the construction of advanced spacecraft. In addition, the ability of this robotic system to solve the dynamic vibration problem has been outlined. Further, two approaches were proposed for the negation of residual tip positioning errors. A control method for minimizing the moment reacted at the base has also been discussed. In each case, the unique characteristics of this kind of robot were exploited to achieve the solutions.
8 EPILOGUE

Since this report was written, improvements have been made to the prototype multi-segment robot. The initial tests demonstrated that the method for supplying the control torque, differentiation of the speed curve, caused noise generation in the control loop. The dilatory effects of this approach made improvements in this system a priority.

Additional funds were sought, and found, to procure new torque amplifiers for the motors. This eliminated the need to convert torque commands into speed commands within the control loop. The use of the torque amplifiers eliminated a major contributor to rate signal noise. With these new torque amplifiers in the system, the motion of the robot was much smoother and could tolerate higher damping levels in the control loop.

This system implementation matched the system modeled in the analysis. Correlation between test and analysis was significantly improved by the addition of the torque amplifiers.

Additional work is needed to filter, or reduce, the noise in the angular rate signal caused by the unshielded data lines being bundled with the motor power lines. A shielded data cable, separation of the data cables from the power bundles, or both may be employed as a way of reducing
this source of noise. If these methods prove to be impractical, a low-pass software filter can be implemented into the control software to screen out the unwanted noise.
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