A STUDY OF URANIAN MAGNETOSPHERIC CONVECTION

by

GANG YE

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE
DOCTOR OF PHILOSOPHY

APPROVED, THESIS COMMITTEE:

Thomas W. Hill
Senior Research Scientist
Center for Space Physics
Chairman

Richard A. Wolf
Professor of Space Physics and Astronomy

Arthur A. Few
Professor of Space Physics and Astronomy

Ian M. Duck
Professor of Physics

Houston, Texas
November, 1989
ABSTRACT

A STUDY OF URANIAN MAGNETOSPHERIC CONVECTION

BY

GANG YE

In order to understand and explain the low-energy plasma structures observed by the PLS experiment on Voyager 2 in the Uranian inner magnetosphere, an analytic and self-consistent model of a time-dependent solar-wind driven convection system at Uranus has been developed in the corotating coordinate system. Many important results of this model agree with the observations very well.

Because of the unusual orientation of the planetary rotation and magnetic dipole axes, magnetic merging on the dayside magnetopause varies as a function of planetary spin, in response to the changing orientation of the planetary magnetic field relative to the upstream interplanetary magnetic field, which is assumed to have a fixed direction for many planetary rotations. Therefore the magnitude of the solar-wind driven convection electric field varies sinusoidally in time with the 17.2 hr planetary spin period, even though the field direction is fixed in the corotating frame in a direction analogous to the dawn-to-dusk direction in the Earth's magnetosphere.

By assuming conservation of the first adiabatic invariant we find that the "hot" (few keV) protons observed by the PLS experiment in the inner magnetosphere may be convected Sunward from a pick-up source provided by electron impact ionization of the
neutral torus of the outermost satellite Oberon. Under the time-dependent convection field this hot plasma forms a ring-current shielding layer in the region $L = 5 \sim 7$, similar to an Alfven layer because the hot plasma convection timescale (~20 days) is much larger than the 17.2 hr period of variation of the convection field. Inside of the shielding layer the time-averaged electric field is much smaller than the time average of the imposed field. The sinusoidal oscillation of the imposed electric field, however, is not significantly shielded by the shielding layer because the shielding timescale (~ 30 hr) is longer than the 17.2 hr oscillation period. A fraction of the hot plasma is therefore able to penetrate the shielding layer to form a trapped ring-current population. This trapped ring-current population is sufficiently long-lived to undergo charge-exchange and inelastic collisions with the widely distributed neutral hydrogen corona, resulting in the energy degradation of the "hot" component and the simultaneous appearance of the "intermediate" (few 100 eV) and "warm" (few 10 eV) components evident in the PLS results in the region between $L = 5$ and $L = 7$.

The region 2 Birkeland current system, in our model, is concentrated near the region of the ring-current shielding layer. By analogy with the Earth's magnetosphere, the lower boundary of the Uranian aurora is predicted by mapping the location of the shielding layer in the magnetic equatorial plane along the magnetospheric magnetic field lines onto the Uranian ionosphere.
ACKNOWLEDGEMENTS

I would like to thank my thesis director, Dr. T. W. Hill, for his constructive criticisms, suggestions and patient reading of my drafts which were essential to the completion of this dissertation. I also thank the other members of my committee, Drs. R. A. Wolf, A. A. Few, and I. M. Duck for their kind service and valuable review and assistance. Special thanks are due to Drs. G.-H. Voigt and R. A. Wolf, who have been exceedingly helpful and assisted me throughout all my graduate program at Rice. In spite of all this talented assistance none of them, of course, bears any responsibility for the remaining errors or omissions.

I am highly indebted to my wife, whose unending love, support and encouragement kept me sane and relatively productive. I also acknowledge the patience of my son, who received less attention than this dissertation on too many ways.

This work was supported by the Upper Atmosphere Research Section of the National Science Foundation under Grands ATM-8606843 and ATM-8911031.
TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 1</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>Chapter 2</td>
<td>Basic Theories of the Solar-Wind Driven Convection</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2.1 Boundary Conditions in Convection Models</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>2.2 Self-Consistent Steady-State Magnetospheric Convection</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>2.3 Time Dependent Convection</td>
<td>13</td>
</tr>
<tr>
<td>Chapter 3</td>
<td>Uranian Magnetosphere</td>
<td>17</td>
</tr>
<tr>
<td></td>
<td>3.1 Uranus — One of the Voyager's Targets</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>3.2 Pre-Encounter Expectations</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>3.3 In Situ Observations of the Voyager 2 Encounter</td>
<td>24</td>
</tr>
<tr>
<td></td>
<td>3.4 Plasma Convection in the Uranian Magnetosphere</td>
<td>32</td>
</tr>
<tr>
<td>Chapter 4</td>
<td>Quasi-Sinusoidal Convection Model for the Uranian Magnetosphere</td>
<td>35</td>
</tr>
<tr>
<td></td>
<td>4.1 Magnetospheric Magnetic Field</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>4.2 Domain and Reference Frame of the Model</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>4.3 Quasi-Sinusoidal Driving Field</td>
<td>39</td>
</tr>
<tr>
<td></td>
<td>4.4 Major Assumptions for the Convection Model</td>
<td>42</td>
</tr>
<tr>
<td></td>
<td>4.5 The Convection Model</td>
<td>44</td>
</tr>
<tr>
<td>Chapter 5</td>
<td>Distribution and Shielding of the Magnetospheric Convection Field</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>5.1 Steady State Field</td>
<td>51</td>
</tr>
<tr>
<td></td>
<td>5.2 Sinusoidally Time-Varying Field</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>5.3 Shielding Effect of the Steady Field</td>
<td>57</td>
</tr>
<tr>
<td></td>
<td>5.4 Shielding Effect of the Sinusoidally Time-Varying Field</td>
<td>61</td>
</tr>
<tr>
<td></td>
<td>5.5 Shielding Effect of the Total Field</td>
<td>65</td>
</tr>
<tr>
<td>Chapter 6</td>
<td>Plasma Convection Pattern and Plasma Shielding Boundary</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>6.1 Plasma Convection Time Scale</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>6.2 The Hot Plasma Convection Pattern</td>
<td>79</td>
</tr>
<tr>
<td></td>
<td>6.3 The Hot Plasma Inner Boundary (Shielding Layer)</td>
<td>94</td>
</tr>
<tr>
<td>Chapter 7</td>
<td>Plasma Sources and the Auroral Pattern</td>
<td>106</td>
</tr>
<tr>
<td></td>
<td>7.1 Plasma Loss Time Scale</td>
<td>106</td>
</tr>
<tr>
<td></td>
<td>7.2 Sources of Intermediate and Warm Ions</td>
<td>110</td>
</tr>
<tr>
<td></td>
<td>7.3 Sources of the Hot Plasma</td>
<td>112</td>
</tr>
<tr>
<td></td>
<td>7.4 Low-Latitude Boundary of the Uranian Aurora</td>
<td>119</td>
</tr>
<tr>
<td>Chapter 8</td>
<td>Discussion and Conclusions</td>
<td>123</td>
</tr>
<tr>
<td>Appendix A</td>
<td>Appendix A</td>
<td>126</td>
</tr>
<tr>
<td>Appendix B</td>
<td>Appendix B</td>
<td>128</td>
</tr>
<tr>
<td>Bibliography</td>
<td>Bibliography</td>
<td>135</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

Before the successful close encounter of the Voyager 2 spacecraft with the Uranus system on 24 January 1986, very little was known about the planet, even though it had been discovered for more than two hundred years. Similar to other planets in the solar system, such as Earth, Jupiter and Saturn, it was expected that Uranus should have its own intrinsic magnetic field and magnetosphere [Blackett, 1947; Hill et al., 1983; Desch and Kaiser, 1984; Hill and Dessler, 1985]. During the last twenty years, many predicted models of the Uranian magnetosphere, particularly a "pole-on" magnetosphere model, have been developed [Siscoe, 1971; Voigt et al., 1983; Isbell et al., 1984; Hill, 1984; Ip and Voigt, 1985]. The orbiting International Ultraviolet Explorer (IUE) spacecraft detected bright (and time variable) hydrogen Ly-α emissions from Uranus in 1981 and thereafter, providing indirect evidence for the existence of a magnetic field of Uranus, because the charged particle excitation of H and H₂ would explain the high H Ly-α albedo which is often associated with auroral precipitation and emissions in a planetary magnetosphere [e.g. Darius and Fricke, 1981; Fricke and Darius, 1982; Clarke, 1982; Durrance and Moos, 1982; Caldwell et al., 1983; Durrance and Clarke, 1984; Clarke et al., 1986].

With the successful accomplishment of its encounter with Uranus, Voyager 2 revealed a strong intrinsic planetary magnetic field and associated magnetosphere of Uranus [Ness et al., 1986; Bridge et al., 1986]. The most unexpected aspect of the
magnetic field is that the angle between the planetary offset magnetic dipole moment and the angular momentum has a large value of 60 degrees. The PLS experiment on Voyager 2 observed a very interesting structure of low-energy plasma (including hot, intermediate and warm components) in the inner magnetosphere. The hot (~ few keV) plasma is apparently convected Sunward from the (near) tail region and has a sharp inner boundary at \( L = 5 \), whereas both intermediate and warm components appear inside \( L = 7 \) [Bridge et al., 1986; Selesnick and McNutt, 1987]. (\( L \) is the equatorial crossing distance of a dipole field line in planetary radii.)

Because of the unusual orientation of the planetary rotation and magnetic dipole axes of Uranus at this epoch, it is believed that the solar-wind driven magnetospheric convection can penetrate deep into the inner magnetosphere unimpeded by corotation, and that a plasmasphere analogous to that in the Earth's magnetosphere does not exist [Hill, 1986; Selesnick and McNutt, 1986; Vasyliunas, 1986; Selesnick and Richardson, 1986]. Some steady state convection models for the Uranian magnetosphere have been developed [e.g. Vasyliunas, 1986; Selesnick and McNutt, 1987; Sittler et al., 1987]. As Hill [1986] pointed out, the solar-wind driven convection field and associated convection pattern in the Uranian magnetosphere would vary sinusoidally in time with a period of 17.2 hr, the planetary spin period, because the relative orientations of the planetary magnetic field and the upstream interplanetary magnetic field (IMF) changes periodically as the planet rotates, if the direction of the IMF remains fixed for many planetary rotation periods. Therefore it is interesting and necessary to develop a self-consistent and time-dependent solar-wind driven convection model for the Uranian magnetosphere. The major purpose of this study is to develop such a convection model to understand and explain the low-energy plasma structures observed by the PLS experiment on Voyager 2.

Large scale plasma convection in a dipole magnetic field has been studied since the 1930's [Alfvén, 1939, 1940, 1955, 1958], and the concepts of plasma convection in
planetary magnetospheres was first developed by Gold [1958]. Systematic and self-consistent convection models for the terrestrial magnetosphere have been developed since the beginning of the 1970's [e.g. Vasyliunas, 1970, 1972; Wolf, 1970, 1974, 1975, 1983; Chen and Wolf, 1972; Jaggi and Wolf, 1973; Southwood, 1977; Southwood and Wolf, 1978; Harel et al., 1980a, b; Spiro et al., 1981; Wolf et al., 1982; Chen et al., 1982; Senior and Blanc, 1984]. In Chapter 2, we review some important existing theories of solar-wind driven convection for Earth's magnetosphere. The logic chain of a self-consistent convection model, the determination of the boundary conditions for the model, and particularly the effects of periodically time-varying convection fields, are reviewed briefly.

In Chapter 3 we briefly describe the observations of Voyager 2 that are relevant to this study, and the knowledge of the planet before and after the Voyager 2 encounter with Uranus.

In order to obtain analytic solutions of the convection model, we use certain assumptions, which are described in Chapter 4, based on the observations of Voyager 2 and existing theories of planetary magnetospheres. Many of these assumptions may be replaced by more realistic ones if we seek numerical rather than analytical solutions. Using magnetosphere-ionosphere coupling theory and solar-wind-magnetosphere coupling theory, we set up equations to determine self-consistently the magnetospheric convection field distributions in Chapter 4.

The distribution and shielding effects of the time-dependent convection field in the Uranian magnetosphere are solved and discussed in Chapter 5. It is found that the time-averaged convection electric field is strongly shielded inside the ring-current shielding layer and the direction of the field inside the layer rotates nearly $\pi/2$ radians relative to the field outside the layer, similarly to the steady case of the terrestrial magnetosphere, whereas the sinusoidal oscillation of the imposed electric field is not
significantly shielded by the shielding layer and the field direction is not obviously changed because the shielding timescale is longer than the 17.2 hr oscillation period. Apparently, the distribution and the shielding effect of the total convection field depends on the ratio of the time-averaged component to the sinusoidally oscillating component. If the ratio is very large, then the convection field can be treated as a steady one to a very good approximation. If the ratio is close to unity, however, we have to deal with a time-dependent system. Convection field distributions for different ratios are given in Chapter 5.

Once the magnetospheric convection field is solved, the associated plasma convection pattern can be obtained easily. Since the hot plasma convection timescale is found to be much longer than the oscillation period of the convection field, the plasma convection orbits are given approximately by the Alfven layer theory [e.g., Alfven and Falthammar, 1963]. In chapter 6, we present the hot plasma convection orbits for different plasma energies and different convection fields; for comparison, the corresponding Alfven layers are also given. The most important characteristic of the plasma convection orbits in the time-dependent convection field, in comparison with the corresponding Alfven layers, is that some Sunward convected plasma may form a trapped ring-current population because of the sinusoidally oscillating field. Such a trapped ring-current population forming from the convected plasma is called a "modified" Alfven layer; whereas in the original Alfven layer theory the plasma either convects around the egg-shaped "forbidden region", or is permanently trapped inside this region. In this Chapter, we find that the predicted location of the hot plasma inner boundary agrees with the observations remarkably well.

It is widely considered that the "hot" component of the plasma has a source in the tail because of its convection characteristics, and that both "intermediate" and "warm" components of the plasma have local sources because they appear suddenly inside $L = 7$
[e.g. Bridge et al., 1986; McNutt et al., 1987; Selesnick and McNutt, 1987; Sittler et al., 1987]. Applying the concept of "modified" Alfven layers, we propose in Chapter 7 that both the intermediate and warm plasma are produced by charge exchange and inelastic collisions between the trapped hot plasma and the localized neutral hydrogen corona. At the same time, the energy decrease of the hot component plasma can be explained by the energy degradation resulting from the collision reactions. Assuming conservation of the first adiabatic invariant, we conclude that the hot component observed by the PLS experiment comes from a source at the neutral torus of the outmost satellite Oberon, where the protons are produced by electron impact and photo-ionization and are picked up with the local-corotation energy. In addition, from the location of the region 2 Birkeland currents on the magnetic equatorial plane, the lower boundary of the Uranian aurora is predicted by mapping along the magnetic field lines.

In Chapter 8, we present major conclusions of this study. Some possible applications and further extensions of this study are also discussed.
Chapter 2

Basic Theories of Solar-Wind Driven Convection

The idea of plasma convection in planetary magnetospheres, first developed by Gold [1958], is one of the major aspects of magnetospheric dynamics, and it is a very complex physical process because it involves the solar wind, the magnetosphere, the ionosphere, and even the atmosphere.

Generally, a magnetosphere can be considered as a vast region of space surrounded by an outer boundary -- the magnetopause -- and an inner boundary -- the ionosphere. In principle, if certain boundary conditions, such as the electric field distributions, the electrical conductivities, and the plasma sources, can be specified (by ground-based and/or spacecraft observations and relevant theories), then the magnetospheric plasma convection and the associated electric fields can be calculated [e.g. Vasyliunas, 1970, 1972; Wolf, 1970, 1974, 1975, 1983]. Basically, there are two different causes of magnetospheric plasma convection; one is fast planetary rotation, as in the magnetospheres of Jupiter and Saturn, and the other is the solar-wind-induced electric field (or current) as in the terrestrial and Uranian magnetospheres. In this chapter, we will briefly introduce the basic theory of solar-wind-driven magnetospheric plasma convection developed for Earth's magnetosphere.

2.1 Boundary Conditions in Convection Models

It is generally accepted that the magnetic field lines of the polar cap extend out
through the magnetopause to connect to the IMF, and that the electric field distribution on the magnetopause is mapped along equipotential field lines to the polar cap. For the Earth's magnetosphere, we know that the large scale electric potential drop over the polar cap results mostly from the interaction of the solar wind with the dayside magnetopause. Various mechanisms have been proposed for this interaction.

One major theory is the viscous or quasi-viscous model (known as the "closed model") proposed by Axford and Hines [1961]. The most prominent mechanism in this closed model for the case of Earth is the Kelvin-Helmholtz instability, which can plausibly provide the 10 ~ 30 kV potential drop across the polar cap observed during the most quite times [e.g., Pu and Kivelson, 1983; Miura, 1984]. In the case of the Kelvin-Helmholtz instability, although the coupling between the solar wind and the magnetosphere depends on the relative orientations of the interior and exterior fields, the results are symmetric with regard to the fields being parallel or antiparallel.

From correlative observations [e.g. Heppner, 1972; Fairfield, 1977; Reiff, 1984], we know that the polar cap and magnetospheric electric potential distributions are controlled to a large extent by the direction of the IMF. The more southward the IMF, the larger the potential drop. The closed model does not explain these observations.

The more efficient process is likely to be the magnetic merging (reconnection) mechanism, which was first proposed by Dungey [1961]. In his open model, the polar cap maps along open field lines into interplanetary space. In this case, the magnetic field lines within the polar cap are connected to the IMF, and are dragged along by the solar wind, resulting in the convection of plasma in the ionosphere and magnetosphere. The magnitude of the polar cap potential drop depends on the angle between the direction of the IMF and the planetary magnetic field in the reconnection model, unlike in the closed model. Much theoretical and numerical work has shown that reconnection occurs preferentially when the IMF is southward, and easily produces a 100 kV electric potential
across the polar cap. Though neither model has been completely developed, it is widely believed that the electric field in the magnetosphere is predominantly produced by magnetic merging.

From the viewpoint of the reconnection model, if the direction of the IMF remains unchanged, and if variations in the convection pattern are slow enough so that the induction field can be neglected, then the magnitude of the convection potential drop should be approximately time independent, because the Earth's rotation axis and magnetic dipole moment are nearly aligned, and both are nearly perpendicular to the solar wind flow.

In fact, the real magnetospheric convection potential inside the polar cap is quite uneven and variable. However, it is still often approximated by a constant field in order to simplify theoretical treatment of magnetospheric convection. The most widely used potential distribution is $\Phi = \Phi_0 \, e^{i \phi}$, where $\phi$ is the longitude and $\Phi_0$ is an averaged value of the observed potential drop.

The another important boundary condition is the ionospheric conductivity. The presence of electrons and ions in the ionosphere makes it electrically conducting. The concentrations of the charged particles and of the neutral particles govern the electrical conductivities, because collisions of charged particles restrict their movement under the action of any impressed electric field. The presence of a magnetic field greatly complicates the problem, as it restricts the motion of the charged particles across the magnetic field and therefore makes the conductivity anisotropic.

Theoretically, we can derive several different electric conductivities in different physical situations. First of all, the specific electric conductivity $\sigma_0$ is that which exists parallel to a magnetic field or in the absence of a magnetic field, so that it is often referred to as the zero-field conductivity. The Pedersen conductivity $\sigma_P$ is that which applies perpendicular to the direction of the magnetic field when one is present. It is sometimes
referred to as the reduced conductivity because the presence of the magnetic field lowers the specific electric conductivity for problems involving the component of an electric field which is perpendicular to the magnetic field. When an electric field $\mathbf{E}$ is applied in the presence of a magnetic field $\mathbf{B}$, the $\mathbf{E} \times \mathbf{B}$ drift and the collisions of the charged particles result in the Hall current which is perpendicular to both electric and magnetic fields. The conductivity used in computing this current is known as the Hall conductivity $\sigma_H$. Finally, there is the Cowling conductivity $\sigma_C$, a combination of the Pedersen and Hall conductivities, which is used in calculating energy dissipation associated with currents in plasma. Since $\mathbf{J} \cdot \mathbf{E} = J^2 / \sigma_C$, it is easy to get $\sigma_C = \sigma_P + \sigma_H^2 / \sigma_P$.

Ground-based and spacecraft observations indicate that, even though the conductivity parallel to the magnetic field ($\sigma_0$) is normally much larger than the perpendicular ones ($\sigma_P$ and $\sigma_H$), all these conductivities in the ionosphere vary with both time and location. Obviously, it is impossible to set up a model of the ionospheric conductivity to cover all situations. Fortunately, when we deal with magnetospheric convection, we are interested only in the large scale pattern, involving distances much greater than the thickness of the ionosphere. Under these circumstances, and based upon the idea that the magnetic field lines are approximately perfect conductors, we may treat the ionosphere as a thin layer, and effectively use the concept of the height-integrated conductivities. Even with this approximation, the latitude, longitude and local-time dependence of the height-integrated conductivities are, of course, highly uncertain.

Considering the known features of the ionospheric conductivities, such as the day-night effect, magnetic dip angle effect, solar zenith angle effect, and auroral precipitation enhancement effect, some "best-guess" models of the ionospheric conductivities have been developed for convection research [e.g., see Wolf, 1970; Spiro et al., 1982]. If only the dip-angle effect is considered, the time-independent and height-integrated conductivity tensor can be simply expressed as [see Wolf, 1983]
\[ \Sigma = \begin{bmatrix} \int \sigma_p \, dh / \sin^2 I & - \int \sigma_H \, dh / \sin I \\ \int \sigma_H \, dh / \sin I & \int \sigma_p \, dh \end{bmatrix} \]

where I is the magnetic dip angle, and the integration is along the magnetic field lines throughout the ionosphere.

From the model of Hanson [1965], the height-integrated Pedersen and Hall conductivities are approximately equal for averaged conditions. In analytic models of magnetospheric convection, all the height-integrated conductivities are usually assumed approximately to be constants, which are determined by averaging the observational data.

The other important boundary condition, given by the plasma source at the outer boundary, has not been completely studied yet; however, there are sufficient observations to allow empirical specification of that boundary condition. Recently, Spiro et al. [1988] have done some systematic study on this problem.

### 2.2 Self-Consistent Steady-State Magnetospheric Convection

Once the solar-wind imposed convection potential and the conductivities in the ionosphere are specified, the magnetospheric convection of plasma and the associated electric field can be determined theoretically in a given magnetic field. The systematic approach to the calculation of self-consistent and steady-state magnetospheric convection was first employed by Vasyliunas [1970] and Wolf [1970] (with more realistic magnetic field and ionospheric conductivity) independently.

Fig. 1 shows the basic logic of the self-consistent calculation of magnetospheric convection first outlined by Vasyliunas [1970]. In general, by using the Boltzmann equation, the transport equation and the plasma density at the boundaries, one can determine the plasma pressure gradient in the magnetosphere if the electric and magnetic
field distributions are given. If the plasma convection is adiabatic, the plasma pressure can be obtained simply by calculating single particle trajectories. In steady-state, the plasma pressure gradient and the magnetic field determine the currents perpendicular to the magnetic field by the requirement of momentum conservation. This is true even if not in steady state, just requires that inertial effects are minor. These perpendicular currents are closed by Birkeland (field-aligned) currents, which are in turn closed by the diversion of the horizontal currents in the ionosphere, which are determined by the convection field and the electric conductivity there. The electric field in the magnetosphere can be obtained from that in the ionosphere by the generalized Ohm's law. Usually, with the perfect
MHD approximation, the magnetic field lines in the magnetosphere can be considered as perfect conductors; therefore, the electric field in the ionosphere can be mapped into the magnetosphere along the magnetic field lines. The fundamental equations of this self-consistent calculation for steady-state and adiabatic magnetospheric convection were derived systematically by Wolf [1983] both from MHD and particle-drift theories.

Many self-consistent theoretical models of steady-state magnetospheric convection imply that the magnetospheric charged particles should greatly shield the inner magnetosphere and low-latitude ionosphere from most of the time-average convection electric field. The basic principle of the shielding is that, under the effect of the solar-wind induced dawn-to-dusk convection electric field, the inner edge of the ring current is pushed toward the Sun, such that the azimuthal magnetospheric currents are interrupted at this boundary. The continuity of current requires a Birkeland (field-aligned) current away from the ionosphere on the dawn side and into the ionosphere on the dusk side. In other words, the inner edge of the ring current polarizes and creates a secondary potential drop which tends to cancel out the primary convection electric field.

The efficiency of the shielding can be expressed in terms of the shielding factor $\zeta$ and the shielding time scale $\tau_s$. The shielding factor indicates how strongly the convection electric field is shielded inside the inner edge of the ring current (or plasma sheet). Many theoretical studies have shown that the shielding factor is proportional to the ratio of the height integrated Pedersen conductivity $\Sigma_p$ in the ionosphere where it connects to the inner edge of the plasma sheet by magnetic field lines, and the effective Hall conductivity $\Sigma^*$ at the inner edge of the plasma sheet, i.e.

$$\zeta \propto \frac{\Sigma_p}{\Sigma^*} = \frac{\Sigma_p}{q \int \frac{n}{B} ds} = \frac{\Sigma_p}{q \int n / B ds}$$

(1)

where $q$ is the magnitude of the electron charge, $n$ is the plasma density, $B$ is the
magnetic field strength, and the integration is taken along magnetic field lines. The shielding time scale indicates how quickly the shielding layer forms once the convection field is applied, and it is proportional to the plasma magnetic curvature and gradient drift time scale and to the shielding factor. In the Earth's magnetosphere, the shielding factor is usually ~0.01 and the shielding time scale varies from minutes to hours.

Under the effect of the convection electric field, the magnetospheric plasma drifts Sunward from the tail. The minimum penetration distance of the plasma sheet in stationary state situations has been estimated by several authors from different viewpoints [Jaggi and Wolf, 1973; Southwood, 1977; Siscoe, 1982]. Generally the minimum geocentric distance is given by

\[ L_{\text{min}} = c_1 \left[ c_2 \frac{q \Phi_0}{K_0} \frac{\Sigma_p}{\Sigma^*} + 1 \right]^{c_3} \]  \hspace{1cm} (2)

where \( \Phi_0 \) and \( K_0 \) indicate the magnetospheric convection electric field potential and plasma kinetic energy respectively, and \( c_1, c_2, \) and \( c_3 \) are constants. Although these constants are not exactly the same in the different approaches, they are very close to 8, 1, and \(-1/3\) respectively.

### 2.3 Time Dependent Convection

We know that magnetospheric convection is determined not only by the driving fields or currents, but also by ionospheric conductivities, plasma pressure, magnetic field structure, and neutral atmospheric winds. Usually, the magnetospheric magnetic field is simply treated as approximately steady in convection problems. However, all other quantities are highly variable in both time and location. For instance, the ionospheric
conductivities vary with the latitude and local time because of the effects of solar illumination and auroral electron precipitation, and the driving convection field is time variable owing to the time variation of solar wind conditions.

Based upon the understanding of steady-state convection, several workers have studied magnetospheric convection for time dependent cases. With the help of rapid developments in computer science, more and more time-variable aspects can be taken into account in numerical models of magnetospheric convection. The most successful model to study time dependent convection in Earth's magnetosphere is the computer model developed at Rice University by Wolf and colleagues [Jaggi and Wolf, 1973; Harel et al., 1981a, b; Spiro et al., 1981; Wolf et al., 1982; Chen et al., 1982]. However, in analytical or semi-analytical models, it is difficult if not impossible to deal with all time variations of the system, and some approximations have to be made. For example, some authors have assumed that all quantities are time independent except the convection field [see Southwood, 1977], while others have treated the convection system as in a steady state except for ionospheric conductivity variations [e.g. Senior and Blanc, 1984].

It is widely realized that the superposition of the convection and corotation electric fields leads to the formation of the plasmasphere in the Earth's magnetosphere. Outside the plasmasphere the plasma convects Sunward from the tail under the influence of the convection field, while inside the plasmasphere the plasma corotates with Earth under the effect of the corotation field. Obviously, the stronger the convection field, the smaller the size of the plasmasphere. Chen and Wolf [1972] investigated theoretically the time evolution of the Earth's plasmasphere under a spatially uniform but time varying convection field. In their model, the magnetic field is assumed to be dipolar and time independent, and the convection field is allowed to vary with time in the form of periodic gusts (see Fig. 2). They found that if the period of the gusts is not close to the earth's rotation period (type 1 in Fig. 2), the gusty convection field only causes the plasmasphere
to rock back and forth, and distorts its shape slightly, in comparison to the plasmasphere under the effect of a constant convection field with a value equal to the time averaged value of the gusty field (see Fig. 3).

Fig. 2. Time varying convection electric fields. In a Type 1 convection field, all the short-duration enhancements (gusts) are identical; In a Type 2 convection field, every third gust has a larger peak convection field, while the other gusts have smaller peak convection fields. $E_{av}$ means the averaged value of the time varying convection fields. The period $p$ is set to be 8 hr, therefore the larger peaks in Type 2 have a period of 24 hr, which is the rotation period of the Earth. (From Chen and Wolf, 1972)

However, if there is a Fourier component of the time varying convection field with a period near the rotation period of the Earth (for example, type 2 in Fig. 2), then resonance effects result in a substantial loss of plasma from the plasmasphere, and consequently, the size of the plasmasphere is reduced remarkably (see Fig. 3). It is not difficult to understand the resonance effect. If the convection field varies with a 24-hr period, some of the plasma inside the plasmasphere moves radially either systematically outward, until escape to the magnetopause, or systematically inward, until balanced by the plasma pressure; both mechanisms tend to reduce the size of the plasmasphere. Thus, the closer the period of the convection field is to 24 hr, the greater is the loss from the
plasmasphere, or equivalently, the deeper the convection field penetrates into the inner magnetosphere (see Fig. 4).

Fig. 3. Illustration of the relative effects of Type 1 and Type 2 fields on the size of the plasmasphere. Contour (- - - - - ) corresponds to the plasmapause for a constant convection field with value of $E_{av}$, contour (- - - - - ) to the plasmapause for a Type 1 field, and contour (- - - - - ) to the plasmapause for a Type 2 field. (From Chen and Wolf, 1972)

Fig. 4. Illustration of the effect of the period of a Type 2 convection field on the shape of the plasmapause. Contour (- - - - - ) corresponds to the size of the plasmapause with the period $p$ of 6 hr. Contour (- - - - - ) to the size with the period of 8 hr, and contour (- - - - - ) to the size with the period of 10 hr. (From Chen and Wolf, 1972)
Chapter 3

Uranian Magnetosphere

Uranus, the seventh planet of the solar system, was discovered in 1781 by William Herschel during a telescopic survey of the sky. Even though it has been identified for more than two hundred years, the knowledge of this outer planet accumulated very slowly. In fact, very little was known about the planet prior to the successful Voyager 2 spacecraft close encounter with the Uranus system on 24 January 1986.

With a mass almost 15 times that of the earth \( M_{\text{Earth}} = 5.98 \times 10^{24} \text{ kg} \), Uranus is intermediate in size between the terrestrial planets and the large gas giants. Since Uranus is so far away, the planet's 51,200 km diameter subtends less than 4 arcsec, therefore it is very difficult to study its physical nature from the Earth. Uranus can be seen with the naked eye but is easily mistaken for a faint star. In a telescope it appears a faint bluish-green as a result of absorption due to methane above a reflecting cloud deck. Uranus orbits the sun with a period of 84 years at a mean orbital radius of 19.2 AU (1 AU = 1.496 \times 10^8 \text{ km}, the mean distance between the Sun and Earth), and it is unique in the solar system in that its rotation axis lies nearly in its orbital plane, which is inclined by less than 1 degree to the ecliptic plane. At present, its rotation axis is nearly aligned with the sun-planet vector. Unfortunately, its rotation rate was poorly known until recently.

In this chapter, we will briefly describe how the Voyager 2 spacecraft successfully encountered and observed Uranus, what was the human understanding of the Uranian magnetosphere before the Voyager encounter with Uranus, and what are the primary
results of the Voyager's observations relative to Uranian magnetospheric convection. Finally, we will overview some important studies of Uranian magnetospheric convection based upon the Voyager 2 observations.

3.1 Uranus — One of The Voyagers' Targets

Planetary exploration fills a very basic human need — the need to expand our horizons. But how? A flight to Jupiter would take 2 years, an acceptable journey for a robot explorer, but the time of a direct flight to six-times-more-distant Neptune would be 40 years, well beyond the lifetime of even the luckiest normal spacecraft, and probably even too long for human beings. We have to find some way to send an explorer to the outer planets in a reasonable time period. It is well known that when a comet passes over Jupiter, its speed and direction are changed by Jupiter's gravity field. Obviously, Jupiter may also be used to speed a spacecraft to the outer planets. Moreover, if the outer planets are in alignment, which rarely occurs only every 176 years, then a spacecraft can remarkably shorten its voyage to the outer planets by using Jupiter's gravity assist to reach Saturn, Saturn's gravity assist to reach Uranus, and Uranus' gravity assist to reach Neptune. Based upon this idea, the Grand Tour — a path to the outer planets discovered by Gary Flandro in 1966 [Kane and Kohlhase, 1983] — makes it possible that a spacecraft can fly to Neptune in only about 12 years, instead of 40 years.

The Voyager 2 spacecraft, carrying an array of scientific equipment (see Fig. 5a), left Earth to visit the outer planets in August 1977, nearly following the Grand Tour. Flybys of Jupiter on 9 July 1979, Saturn on 25 August 1981, and Uranus on 24 January 1986 have provided a great deal of information about each of these planetary systems as well as the whole solar system. Now Voyager 2 is flying toward Neptune for a flyby in August 1989. It will eventually pass through the orbit of Pluto (but not close to the planet) and then will continue out beyond the solar system.
The locations of the instruments on the Voyager 2 are shown in Fig. 5a. The trajectory of the Voyager 2 spacecraft is shown in Figure 5b, and the trajectory at Uranus is shown in Figure 5c.

Fig. 5a. A drawing of the Voyager spacecraft indicating the location of the science instruments and several spacecraft subsystems. (After E. C. Stone, J. Geophys. Res., 92, p.14873, 1987).
Fig. 5b. Voyager 2 trajectory projected onto the ecliptic plane (after Stone, [1987]).

Fig. 5c. A view normal to the trajectory plane of the Voyager 2 path through the Uranus system (after Stone [1987]).
3.2 Pre-Encounter Expectations

The studies of planetary magnetospheres — the regions in which planetary intrinsic magnetic fields govern local physical processes — are key objectives of planetary exploration. The first paper on Uranus' magnetosphere, as far as I know, by Siscoe [1971] proposed a "pole-on" magnetosphere, in which the planetary magnetic dipole moment, if any, is parallel to the rotation axis as is approximately the case for Mercury, Earth, Jupiter and Saturn, and parallel to the solar wind (at some certain time, e.g. 1984 and every 42 years thereafter). Such a "pole-on" model was popular before the Voyager 2 encounter and many authors extended it (e.g. Siscoe, 1975; Voigt et al., 1983; Hill et al., 1983; Isbell et al., 1984; and Hill, 1984). Ip and Voigt [1985] investigated magnetic field configurations of a plasma-loaded "pole-on" magnetosphere based on the assumption that Uranus' moons provide sources of plasma. Further, Ye and Voigt [1989] developed a stationary model of a rotating, axially-symmetric "pole-on" magnetosphere in MHD force balance.

In 1981, Darius and Fricke [1981] first pointed out significant although indirect evidence for the existence of a magnetic field of Uranus. By considering the bright (and perhaps time variable) hydrogen Ly-\(\alpha\) emissions from Uranus observed by the orbiting International Ultraviolet Explorer (IUE) spacecraft, they proposed that charged particle excitation of H and H\(_2\) could be responsible for the high H Ly-\(\alpha\) albedo which is often associated with auroral precipitation and emissions in a planetary magnetosphere, although such H Ly-\(\alpha\) emissions from the outer planets might be excited by other mechanisms, such as Rayleigh and resonant scattering of incident solar and interplanetary medium H Ly-\(\alpha\) (e.g. Fricke and Darius, 1982). Continued observations and analysis of such H Ly-\(\alpha\) emissions from Uranus [Clarke, 1982; Durrance and Moos, 1982; Caldwell et al., 1983; Durrance and Clarke, 1984; and Clarke et al., 1986] up until the time of the Voyager 2 encounter with Uranus led to the widely-held belief that Uranus
possesses an intrinsic magnetic field and an active magnetosphere.

On the assumption that Uranus possesses an active "pole-on" magnetosphere, many
people believed that most radio emissions would emanate from the dayside regions in
which the auroral power should be much larger than that on the nightside [e.g. Hill et al.,
contrary, radio-quiet-dayside model. He suggested that in a "pole-on" magnetosphere of
Uranus, nonthermal radio emissions would come predominantly from the nightside. In
comparison with the case of Earth, in which the generation of both extraordinary and
ordinary modes of auroral kilometric radiation (AKR) requires regions with field-aligned
fluxes of keV electrons in which the ratio of the electron plasma frequency to electron
gyrofrequency ($\omega_{pe}/\omega_{ce}$) is less than unity, in the Uranus dayside polar region, the
plasma density would be very high because of more direct access of magnetosheath
plasma to the ionosphere via the magnetospheric cusp and very strong photoionization.
The dayside plasma density may be high enough to let the ratio ($\omega_{pe}/\omega_{ce}$) be well larger
than unity, since the higher plasma density implies higher electron plasma frequency.
Therefore, the dayside nonthermal radiation may be greatly suppressed. In contrast, in
the nightside regions, the plasma density would be much smaller than that in the dayside
regions due to the lack of photoionization and direct interaction between solar wind
plasma and the ionosphere. Thus Curtis predicted that the dayside would be radio-quiet
and the nightside radio-active.

Hill et al. [1983] proposed a Faraday disk dynamo mechanism based on the
hypothetical "pole-on" magnetosphere to explain how the power of the observed Ly-\(\alpha\)
emissions from Uranus could be extracted from the kinetic energy of rotation of the
planet. From the model, they calculated that the power extracted from planetary rotation
by the mechanism is proportional to the 4/3 power of the planetary magnetic moment, and
inferred from the observed auroral power that Uranus' intrinsic magnetic moment is 4
Gauss-$R_u^3$ (1 $R_u = 25,600$ km, mean radius of Uranus), somewhat larger than the value of 1.8 Gauss-$R_u^3$ given by the empirical "magnetic Bode's law" based upon angular momentum considerations [Blackett, 1947]. Hill and Dessler [1985] predicted, based on the IUE results, that Uranus may have the largest or second-largest planetary magnetic field in the solar system—a minimum surface magnetic field of at least 0.6 Gauss, with the most probable value being about 4 Gauss according to their Faraday disc-dynamo model and about 13 Gauss according to an Earth-like model. Desch and Kaiser [1984] pointed out that all of the known magnetic planets are radio sources with the exception of Mercury, and the radio power is a function of the planetary magnetic field strength and the solar wind power input. From that they formulated a "Radiometric Bode's law" and expected that Uranus should have a magnetic moment of 0.14 Gauss-$R_u^3$ or larger, and they predicted that the measurement of the Uranus radio flux by the Voyager 2 spacecraft during its approach to the planet would be made in early to middle 1984.

Nevertheless, UV emissions from Uranus detected by IUE are still a disputable indicator of the existence of a magnetosphere. Shemansky and Smith [1986], for instance, adopted a new term "electroglow" to describe the spatially extended diffuse emissions, which are common to the outer planets. They pointed out that the ultraviolet emissions from Uranus observed by IUE can be explained by the direct excitation of atomic hydrogen by energetic electrons, even though the energy source of these electrons is unknown, without the need for a planetary magnetospheric interpretation. They argued that if the ingredient for production of an active magnetosphere is present, the nondetection of radio emission by the Voyager planetary radio astronomy experiment at a range of less than 0.7 AU from Uranus (Kaiser and Desch, 1985) then suggests that Uranus has a very weak or nonexistent magnetosphere. They also predicted that Uranus would be surrounded by a neutral hydrogen corona similar to that in the Saturn magnetosphere.
It was pointed out [Cheng and Lanzerotti, 1978; Cheng, 1984] that darkening and polymerization of methane and other organic ice surfaces under particle irradiation may explain the low albedo of the Uranus rings and the dark matter on the Uranian moons. Cheng [1984] and Eviatar and Richardson [1986] predicted that the icy satellites of Uranus would be a major contributor to the magnetospheric plasma population, and there is a heavy ion plasma torus from ionization and dissociation of $\text{H}_2\text{O}$ and other molecules sputtered from Uranus' icy moons. Cheng concluded that this mechanism would predict aurorae around both magnetic poles of Uranus.

### 3.3 In Situ Observations of the Voyager 2 Encounter

The first intensive observation of the Uranus system was made by the Voyager 2 spacecraft close encounter on 24 January 1986. The eleven groups of scientific equipment carried on Voyager 2 — Imaging system (ISS), Photopolarimetry (PPS), Infrared interferometer spectroscopy (IRIS), Ultraviolet spectroscopy (UVS), Radio science (RSS), magnetometer (MAG), plasma (PLS), Low-energy charged particle (LECP), Cosmic ray (CRS), Planetary radio astronomy (PRA), and Plasma wave (PWS) — were described by Stone and Miner [1986]. The initial results of the encounter are published in Science, Vol. 233, #4759, 4 July, 1986, and more detailed results are reported in J. Geophys. Res., Vol. 92, #A13, 30 December, 1987. Here, some important results of the Voyager 2 encounter with Uranus involved with the research of this thesis are briefly presented.

**Intrinsic Magnetic Field.**

As seen in section 2.2, the existence of an intrinsic planetary magnetic field was uncertain prior to the Voyager 2 encounter. Only five days before the close encounter, Voyager 2 detected the radio emission from Uranus at distance of $275\, \text{R}_\text{U}$ from the planet,
which indicates the probable presence of a planetary magnetic field and magnetosphere [Stone et al., 1986]. Subsequently, a well defined inbound bow shock with extremely high Mach number and a fully developed magnetosphere crossing occurred at 23.5 $R_u$ and 18 $R_u$ respectively [Bridge et al., 1986]. Figure 6 shows the planet, the Voyager 2 trajectory, and the location of the various inbound and outbound bow shock and magnetopause crossings projected onto the orbital plane of Uranus. From the observed solar wind ram pressure ($1.8 \times 10^{-10}$ dyne cm$^{-2}$) just before the inbound shock, the standoff distance of 18.4 $R_u$, and from the pressure balance between the solar wind ram and planetary magnetic field pressures, Bridge et al. [1986] inferred a planetary magnetic dipole moment of 0.21 Gauss-$R_u^3$.

Fig. 6. Voyager 2 trajectory projected onto Uranus' orbit plane. Bow shock and magnetopause boundaries are indicated with the solid bars indicating those times magnetosheath plasma was observed. Thick marks are separated 6 hr apart (after Bridge et al. [1986]).
The Magnetic Field Experiment (MAG) detected an internal field directly when Voyager 2 entered the magnetopause. The investigators of the MAG group initially modelled the magnetic field of Uranus as an off-set tilted dipole (OTD), which provides a useful and easily visualized representation of the complex magnetic field [Ness et al., 1986]. The simple OTD model is relatively accurate up to 25 Ru from the surface of Uranus to deal with the plasma motions within the magnetosphere even though the magnitude of the quadrupole (and even higher order) contribution is substantial in comparison to the dipole contribution for the Uranian surface magnetic field. Such an OTD model is used as an input magnetic field in this research.

The dipole moment of the OTD, rather smaller than that expected, is 0.23 Gauss - Ru³. The most unexpected aspect is that the angle between Uranus' angular momentum and offset magnetic dipole moment has the large value of 60 degree. In the corotating planet-centered coordinates (in which the z axis is aligned with the rotation axis, the x and y axes lie on the geoequatorial plane, located at 0° W and 270° W longitude respectively), the OTD is located at x = - 0.02 Ru, y = + 0.02 Ru, and z = - 0.31 Ru, and the positive pole is tilted 60° from the rotation axis toward 48° W longitude, so that the positive pole intersects the Uranian surface at + 15.2°, 47.7° W and the negative pole at - 44.2° and 222.7° W.

Figure 7a shows the OTD field lines in the meridian plane containing the OTD axis and the rotation axis. In magnetic dipole coordinates, a meridional view of the Voyager 2 trajectory is shown in Fig. 7b. Fig. 7c shows the trajectory of Voyager 2 projected along the OTD magnetic field lines into the magnetic equatorial plane.
Fig. 7a. Diagram of the OTD field lines in the meridian plane containing the OTD axis and rotation axis, illustrating the effects of the large angular (and spatial) offset from the rotation axis (and the center) of Uranus (from Ness et al. [1986]).

Fig. 7b. Meridional view of Voyager 2 trajectory in magnetic dipole coordinates. The satellite minimum L shells for Miranda, Ariel and Umbriel are shown. Ring plane crossing, solar occultation ingress and egress are also shown. (from Ness et al., Science, 233, 1986)
Fig. 7c. The trajectory of Voyager 2 projected along dipole magnetic field lines into the magnetic equatorial plane. The magnetic field points into the plane of the figure. The negative x axis is the projection of the planetary spin axis into the magnetic equator. The y axis is in the direction analogous to the dawn to dusk direction in the Earth's magnetosphere which is the direction of the convection electric field. (from Selesnick and McNutt, JGR, 92, 1987)

**Plasma Environment**

The Voyager investigation shows that the Uranian magnetosphere is populated dominantly by protons with a minor fraction ($10^{-4}$) of heavy ions [Bridge et al., 1986; Krimigis et al., 1986], and the energy density of the magnetospheric plasma is much less than the magnetic field energy density ($\beta \leq 0.1$). In the inner magnetosphere, the Voyager plasma science (PLS) experiment discovered that the Uranus magnetosphere has three plasma components: a warm component ($\sim 10$ eV), an intermediate component (from about 30 to 200 eV) and a hot component ($\sim$ keV), which can be nearly described by Maxwellian cores [Bridge et al., 1986; Selesnick and McNutt, 1987]. From Selesnick
and McNutt [1987], the observed density and temperature of Uranian magnetospheric plasma versus dipole L are shown here in Figure 8. The observations of the PLS experiment indicate that there is a sharp boundary for hot and intermediate plasma (approximately 1 keV and 0.3 keV respectively) at L = 5.3 inbound and L = 4.8 outbound while the warm and intermediate components appear suddenly inside L ~ 7 inbound and L ~ 5.5 outbound. All the ions with energy larger than about 10 eV disappear within L ~ 4.5 both inbound and outbound. In Uranus' inner magnetosphere, there is a plasma trapping region analogous to the Earth's plasmasphere; however, the former is produced by gradient/curvature drift and the latter by the corotational electric field [see, e.g., Nishida, 1966; Wolf, 1983; Lyons and Williams, 1984; McNutt et al., 1987; Selesnick and McNutt, 1987; Sittler et al., 1987].

Near and within the Uranian magnetosphere, the low-energy charged-particle (LECP) instrument measured higher energy protons (4 MeV ≥ E ≥ 28 KeV) and electrons (1.2 MeV ≥ E ≥ 22 KeV) [Krimigis et al., 1986], and the Cosmic ray system (CRC) observed stably trapped and strongly pitch-angle dependent proton and electron spectra at energies above 2 ~ 3 MeV [Stone et al., 1986]. The electron fluxes observed by LECP have apparent satellite signatures - near the minimum satellite L-shell crossings, significant absorption of the electron spectrum appears. Figure 9 shows how the electron flux (≥ 1.1 MeV) decreases near the minimum L-shells of the satellites Miranda, Ariel, Umbriel and perhaps Titania. Unfortunately the outermost satellite Oberon is outside the region where significant fluxes of electrons above 1.1 MeV were observed. If the absorption does occur near the satellite Oberon, then, according to our study, Oberon might be the source of the plasma observed by PLS. In this thesis, the plasma distribution observed by PLS will be explained by our time-dependent magnetospheric convection model developed below.
Fig. 8. Density and temperature versus dipole $L$ for three (hot, intermediate, and warm) Maxwellian components of the magnetospheric convecting plasma used in modeling the data observed by the PLS experiment (from Selesnick and McNutt [1987]).
Fig. 9 Voyager 2 electron counter rates. From top to bottom, the three curves display rates for energies greater than 1.1, 3.1 and 7.7 MeV, respectively. The vertical ticks labeled T, U, A, and M mark the minimum L values of the satellites Titania, Umbriel, Ariel, and Miranda, respectively. The L coordinates on the upper scale were calculated from the OTD model (after Stone [1986]).

The Aurora

The UVS investigation identified a localized aurora in the Uranian magnetosphere. Although there is no apparent aurora detected by the UVS on Uranus' sunlit hemisphere because of the bright electroglow, an aurora was clearly observed in the darkside hemisphere. The initial analysis indicates that the nightside aurora is centered on the mid-latitude (nightside) OTD magnetic pole, and the polar cap is an approximate ellipse within a region 15° by 25° in diameter [Broadfoot et al., 1986]. The electroglow spectra at Uranus suggest atmospheric excitation by low energy electrons (~ eV), while the aurora require excitation by primary electrons with energies of approximately ~ keV. The
total power emitted from the nightside aurora is estimated to be $10^9 \sim 10^{10}$ W, and an input power of $10^{10} \sim 10^{11}$ W is inferred. Until now, the detailed location and shape of the auroral zone has not been given based on the Voyager 2 observations.

3.4 Plasma Convection in the Uranian Magnetosphere

The Voyager 2 encounter with Uranus revealed an intrinsic planetary magnetic field and associated magnetosphere of Uranus. The angle between Uranus' offset magnetic dipole moment and the angular momentum is very large, surprisingly. Because of the unusual orientation of the rotation and dipole axes, the magnetospheric convection pattern at Uranus would be quite different from that at Earth and Jupiter. Although the corotation speeds in the magnetosphere of Uranus may be larger than most estimated flow speeds associated with magnetospheric convection, the unique geometric configuration of a rotation axis in near alignment with the solar wind and a highly tilted magnetic axis at this epoch allows the convection to penetrate deep into the inner magnetosphere unimpeded by corotation. Unlike the magnetospheres of other outer planets, e.g. Jupiter and Saturn, solar wind driven magnetospheric convection is expected to be the dominant transport process in the Uranus magnetosphere, which is similar to that in the terrestrial magnetosphere. However, there would be no plasmasphere in the Uranus magnetosphere because the corotation and convection fields in the Uranus magnetosphere do not couple effectively [see e.g. Hill, 1986; Selesnick and Richardson, 1986; Vasyliunas, 1986].

Vasyliunas [1986] investigated a general convection model for a planetary magnetosphere with arbitrary $\theta_v$ (the angle between a planet's rotation axis and the direction of the solar wind flow) and $\theta_B$ (the angle between the planet's rotation and magnetic dipole axes), and showed that when $\theta_v$ is small, the maximum magnetic latitude of the plasmasphere varies approximately as $\sin \theta_v$; in the special case of the rotation axis perfectly aligned with the solar wind flow, the plasmasphere would
disappear and the corotation-dominated flow region would shrink to negligible size. Moreover, he pointed out that at present at Uranus, the spatial pattern of magnetospheric convection is steady in the corotating rather than in an inertial frame of reference; convection would be the dominant plasma transport mechanism and the plasma transport time scale would be comparable to 30 - 40 hours.

Selesnick and Richardson [1986] investigated the formation of a plasmasphere with the plasma $E \times B$ drift trajectories in planetary magnetospheres with arbitrary $\theta_v$ and $\theta_B$, where the electric field is the sum of the solar wind driven convection field and the corotational electric field [Nishida, 1966]. In the corotating frame with $\theta_B = 60^\circ$ and $\theta_v = 10^\circ$, the current values for Uranus, they found that the plasma convects sunward with small perturbations from straight lines due to the time dependence of the electric field. In other words, at the current epoch Uranus should not have a plasmasphere, because the near alignment of the planetary rotation axis and the solar wind flow results in effective decoupling of the corotation and convection electric fields, so that the plasma can move in helices as it travels through the magnetosphere from the tail to the day side when viewed in an inertial reference frame. Because the corotation electric field does not have to be superimposed on the convection electric field in the corotating reference frame, the Sunward convection of plasma from the tail will penetrate deep into the inner magnetosphere unimpeded by the corotation field, which is contrary to the case in the Earth's magnetosphere. People believe that such convection is limited only by a combination of gradient/curvature drift and partial cancellation of the steady convection electric field due to charge separation, or simply by the pressure gradient effects [e.g. McNutt et al., 1987; Selesnick and McNutt, 1987; Sittler et al., 1987]. Selesnick [1988] calculated the steady magnetospheric convection electric field and the shape of the polar caps with an asymmetric magnetic field model including the contributions of quadrupole and octupole moments. He found that the quadrupole moment strongly influences
magnetospheric convection and the octupole moment further alters the flow pattern, but the basic flow remains sunward in the inner magnetosphere. His time-dependent calculations of the convection field indicate that ring-current shielding of the electric field can be important and may have formed some of the dominant features in the PLS observations.

Hill [1986] has pointed out that since the magnetospheric magnetic field rotates with the 17.2-hr planetary rotation period with respect to the interplanetary magnetic field, which tends to maintain a given direction for many rotation periods, the convection electric field and associated convection pattern can not be quasi-steady, instead they should be quasi-sinusoidal with a 17.2-hr period owing to the rotation of Uranus. The major aim in this study is to develop a periodically time-varying convection model to understand and explain the low-energy plasma structures observed by the PLS experiment on Voyager 2 in the Uranian inner magnetosphere.
Chapter 4

Quasi-Sinusoidal Convection Model
for the Uranian Magnetosphere

Based on the knowledge of magnetospheric convection theory developed for the terrestrial magnetosphere [see, e.g., Vasyliunas, 1970; 1972; Southwood, 1977; Southwood and Wolf, 1978; Wolf, 1975, 1983], and the argument of Hill [1986] that in the Uranian magnetosphere, the convection field and associated plasma convection pattern should be quasi-sinusoidal with a 17.2 hr period owing to the rotation of Uranus and the special orientation of the planetary rotation and dipole axes, we develop an analytic, time-dependent, adiabatic convection model for present Uranus in this chapter. The basic outline of this model is to calculate self-consistently the magnetospheric convection field distribution inside the outer boundary, i.e. the boundary of the polar cap, where a quasi-sinusoidal solar wind driven convection field is assumed. Once the convection field distribution is determined, the associated plasma convection pattern can be obtained easily by plasma drift orbits since the convection is assumed to be adiabatic.

In this chapter, we first describe briefly the modeling region and reference system, the magnetic field model to start with, simplified boundary conditions and some assumptions in order to simplify the analytic model. It is noted that all these relatively simplified, probably oversimplified, conditions can be freed or replaced by more realistic ones if this analytic model is developed further toward a numerical one. Finally, according to the magnetosphere-ionosphere coupling relations, equations to determine
the magnetospheric convection field distribution are set up.

4.1 Magnetospheric Magnetic Field

After a preliminary analysis of the Voyager 2 observations, Ness et al. [1986] introduced a simple and intuitive offset tilted dipole (OTD) model of Uranus' magnetic field, in which the dipole of moment 0.23 G-R$_u^3$ is displaced antisunward along the rotation axis by 0.3 R$_u$ from the center of the planet and tilted 60° from the rotation axis (for details see section 3 in chapter 2). Connerney et al. [1987] developed a more accurate and, of course, more complicated model of Uranus' magnetic field -- the Goddard Space Flight Center "Q$_3$" model, by a spherical harmonic analysis of the magnetic field observations at Uranus. The "Q$_3$" model is characterized by a large dipole tilt (58.6°) relative to the rotation axis, a dipole moment of 0.228 G-R$_u^3$ and an unusually large quadrupole moment.

Even though the OTD model departs significantly from the observed magnetic field near Uranus' surface and even at radial distances up to 4 R$_u$ (initial analyses indicated that there are substantial quadrupole and octupole components of the magnetic field comparable to the dipole component near the planet's surface, see Ness et al. [1986] and Connerney et al., [1987]), the simple OTD model provides a convenient approximation to Uranus' magnetic field, sufficiently accurate for one to deal with the convection problem in Uranus' magnetosphere. In fact the observations on Voyager 2 have indicated that such an offset dipole magnetic field model is a very good approximation up to L ~ 25 R$_u$ [Ness et al., 1986; Behannon et al., 1987]. Thus in our convection model, the magnetospheric magnetic field $B$ is taken, for simplicity and convenience, as a time-independent offset and tilted dipole magnetic field.

4.2 Domain and Reference Frame of the Model
For the Earth, it is traditionally assumed that the magnetic dipole axis is aligned with the rotation axis, and the rotation axis is perpendicular to the solar wind flow. Then the magnetic field, the solar wind flow, the convection electric field and the spatial pattern of magnetospheric convection can be approximately treated as time-independent only in an inertial reference frame, and the electric equipotential contours are identified with the projection of the flowing plasma paths. In Uranus, however, there is no frame of reference, in general, in which the magnetic field, solar wind flow, electric field and the spatial pattern of magnetospheric convection are all static or quasi-static because of the unusual orientation of the dipole and rotation axes. At Uranus, the rotation axis is nearly in the ecliptic plane, therefore the angle between the rotation axis and the solar wind flow will vary through $2\pi$ radians during a Uranus year. Fortunately, at the present epoch, Uranus' rotation axis is almost aligned with the solar wind flow (only 8° between them), and this allows us to greatly simplify the complicated convection problem. Obviously, if it is assumed that Uranus' rotation axis is strictly aligned with the solar wind flow, then only in a reference frame which is corotating with the planet are both the solar wind flow and the magnetic field time-independent. Even though the antisunward plasma flow over the polar cap is time-independent in both the corotating and inertial reference frames, only in the corotating frame is the location of the polar cap time-independent.

Based upon the arguments above, in the corotating reference system we can investigate the Uranus magnetospheric convection problem with the aid of the convection theory developed for Earth with only one difference: that the solar wind driven convection field and associated convection pattern are quasi-sinusoidal with a 17.2-hr period resulting from the changing angle between the planet's magnetic dipole axis and the direction of the constant interplanetary magnetic field, if we adopt the reconnection model of solar wind - magnetosphere coupling and assume that the direction of the IMF is fixed for a long time relative to Uranus' rotation period. Since, in this particular case,
the convection and corotational flows are independent, we can obtain the convection pattern in an inertial reference frame by simple superposition of the corotation flow and the convection flow obtained in the corotating reference frame.

Fig. 10. Illustration of the modeling region for plasma convection on the magnetic equatorial plane. The origin of the two dimensional polar coordinates is located at the dipole instead of at the planetary center. Longitudes $\phi = 0^\circ$ and $180^\circ$ indicate the dawn and dusk sides respectively. The magnetic field is directed out of the plane, and the electric field at large distance is directed from dawn to dusk. The hot plasma convects from the outer boundary into the inner boundary.

Figure 10 shows the modeling region, boundaries and reference frame used in this thesis, viewed from above the south magnetic pole. The convection model is treated on the OTD magnetic equatorial plane in terms of simple polar coordinates $(r, \phi)$ corotating with the planet and the origin is at the OTD dipole. The magnetic field is directed out of the plane. The hot plasma ($\sim$ keV) is assumed to convect from the outer boundary at $r_u =$
25 $R_u$, approximately corresponding to the boundary of the polar caps in Uranus' ionosphere, inward to the impenetrable inner boundary for such hot plasma around $r_0 = 5 R_u$ based upon the observations of the PLS experiment. In fact, the outer boundary is adjustable according to the location of hot plasma sources and/or boundary of the polar cap.

4.3 Quasi-Sinusoidal Driving Field

If the solar wind driven convection electric potential over Uranus' polar caps is given, this distribution of electric potential can be mapped into the corresponding outer boundary ($r = r_u$) on the magnetic equatorial plane, by tracing along magnetic field lines. Because of the large magnetic dipole tilt to the rotation axis and the near alignment of the planetary spin axis and the solar wind flow in the Uranian magnetosphere, there will be enhanced merging on the dayside magnetopause for a portion of each planetary rotation period because the magnetic merging rate depends critically upon the angle between the direction of the magnetic field inside (i.e. planetary magnetic field) and outside (i.e. IMF) the magnetopause. Suppose the time scale of the direction change of the IMF is much larger than the planetary rotation period $\tau_r$ ($\tau_r = 17.2$ hr); then the solar-wind-driven convection potential $\Phi$ should be sinusoidally time variable. On the other hand, the direction of the convection field on the polar cap (or in the magnetosphere) should be unchangeable from dawn to dusk. For a simple time dependent convection model, the driving potential at the outer boundary ($r = r_u$) on the magnetic equatorial plane, as an input boundary condition, can be generally assumed as

$$
\Phi (r_u, \phi, t) = \Phi_s (r_u, \phi) + \Phi_t (r_u, \phi, t)
$$

$$
= \xi \Phi_0 e^{i k \phi} (e^s + e^t e^{i \omega t}) \quad (3)
$$
where \( k \) is a positive integer, usually (as in this study) taken to be 1. In this case, the spatial distribution of the convection potential (real part of (3)) leads to a uniform electric field pointing from dawn to dusk. \( \Phi_0 \) is the maximum available convective emf, and may be roughly estimated by the ideal MHD condition, that is \( 2 \Phi_0 = 2 R_m V_{sw} B_{sw} \), in which \( R_m \) is the effective radius of Uranus' magnetosphere, \( V_{sw} \) the solar wind flow speed, and \( B_{sw} \) the magnetic field magnitude in the solar wind at Uranus. The value of \( V_{sw} \) can be taken as a constant equal to the typical solar wind flow speed of \( \sim 400 \) km/s [e.g. Barnes and Gazis, 1984]. It is estimated that the magnitude of the IMF is about 0.1 nT at Uranus' magnetotail [McNutt et al., 1987] and 0.3 nT just upstream of the bow shock [Ness et al., 1986]; \( B_{sw} \) is taken to have average value of \( \sim 0.2 \) nT. It is observed that the tail radius of the magnetosphere is \( \sim 40 R_u \) [Behannon et al., 1987], so \( \Phi_0 \) is roughly 82 kV. \( \xi \) is the coupling coefficient; even though the total power dissipated in Uranus' magnetosphere, including such forms as aurora, joule heating, ionizing and energizing plasma, and so on, can not be well observed and estimated right now, we believe on the basis of terrestrial studies that the coupling coefficient \( \xi \) might be as large as 10%.

Equation (3) indicates that the total time dependent convection potential \( \Phi \) at the outer boundary \( r_u \) can be considered as a superposition of two parts: one is a steady potential \( \Phi_s(r_u, \phi) = \xi \Phi_0 e^{ik\phi} \epsilon_s \), the other is a sinusoidally time variable potential \( \Phi_t(r_u, \phi, t) = \xi \Phi_0 e^{ik\phi} \epsilon_t e^{i\omega t} \). In other words, the steady component is the time averaged value of the total time varying convection field. In equation (3), \( \omega = 2\pi / \tau \) is the frequency of the time dependent convection field; \( \epsilon_s \) and \( \epsilon_t \) are weight factors of the steady convection potential \( \Phi_s(r_u, \phi) \) and the pure sinusoidally time dependent convection potential \( \Phi_t(r_u, \phi, t) \) respectively. Obviously one can see that the larger/smaller is the ratio \( \epsilon_s/\epsilon_t \), the more steady/time-variable is the convection system. In this assumed convection field, there is a requirement that \( \epsilon_s + \epsilon_t = 1 \) since \( \xi \Phi_0 \) is the maximum possible convection potential
drop, and \( \varepsilon_s \geq \varepsilon_t \) in order to keep the direction of the solar wind driven convection field unchanged, so that we have \( 0 \leq \varepsilon_t / \varepsilon_s \leq 1 \).

The time varying convection field at the outer boundary is shown in Fig. 11. Obviously if the solar wind conditions are determined, then the maximum possible convection potential \( \xi \Phi_0 \) is fixed. Therefore, the ratio \( \varepsilon_t / \varepsilon_s \) characterizes the convection field and the convection pattern. If the ratio is close to zero, then the convection field and the pattern are quasi-static, and the time averaged convection field is nearly as strong as possible. If the ratio is close to 1, the convection is most time variable, and the convection field at the outer boundary will be close to zero once every planetary rotation period, and the time averaged field is only half of that in the former case.

\[
\phi(r_u, 0, t) = \varepsilon_t \xi \Phi_0 \quad \text{and} \quad \varepsilon_s \xi \Phi_0 = \Phi_s \text{ \text{max}}
\]

\[
\varepsilon_t \xi \Phi_0 = \Phi_t \text{ \text{max}}
\]

\[
\varepsilon_s \xi \Phi_0 = \Phi_s \text{ \text{max}}
\]

Fig. 11. Time variation of the convection potential at the outer boundary at the dusk side. \( t_{\text{max}}, t_{\text{min}}, \) and \( t_{\text{av}} \) represent the times when the convection field has maximum, minimum, and averaged values respectively. \( \tau_r \) is the period of the convection field as well as the planetary rotation period.
4.4 Major Assumptions for the Convection Model

Before setting up the convection model, it is necessary to overview major assumptions briefly in order to simplify the model and to allow analytic solutions.

(1) We assume that the electric potential is a constant along the magnetic field lines (\(E \cdot B = 0\)), so that the convection electric field in the ionosphere can be mapped into the magnetosphere along the magnetic field lines and visa versa. In fact, in this model, we deal with all variables in the magnetic equatorial plane, and once the convection pattern in this plane is determined, the pattern in the whole magnetosphere can be easily obtained with the frozen-in flux theory by tracing along the magnetic field lines.

(2) The magnetic field structure of the magnetosphere is time-independent and has a dipole form. This assumption greatly simplifies the geometry of the convection problem so that analytic solutions can be obtained. The Voyager 2 observations have shown that throughout the inner Uranian magnetosphere, the plasma energy density sampled by the PLS instrument is negligible compared to the energy density of the magnetic field (\(\beta < 0.01\)) [Bridge et al., 1986]; hence the magnetic field in the modeling region can safely be taken as a dipole field without considering the distortion due to plasma stresses.

(3) Voyager 2 observations by PLS have shown that inside the Uranian magnetosphere, protons are the dominant species of ions and the convecting plasma density varies as \(L^{-4}\) [see Selesnick and McNutt, 1987, or Fig. 8]. Normally, the ions are more energetic than the electrons in the plasma sheet of a planetary magnetosphere. In Uranus' magnetosphere, the observations of Voyager 2 (see Selesnick and McNutt [1987]) have shown that in the inner magnetosphere the number density of electrons \(n_e\) is nearly equal to the ion's density \(n_i\), but the electron's energy is much less, at least ten times, than the ion's energy. Because the Birkeland (field aligned) currents and the convection field shielding, if they exist, will be generated only in regions where there are large plasma pressure gradients, and the electrons' contribution to the total plasma
pressure is significantly smaller than the ions' contribution, the electrons here may be safely neglected in dealing with the convection mechanism. Moreover, Southwood and Wolf [1978] have shown that in the Earth's magnetosphere the electrons are precipitated into the ionosphere before the convection shielding occurs, which seems probably also true in Uranus' magnetosphere. Therefore in this model it is assumed that plasma-sheet plasma consists of only one ion specie (proton), and they are kept roughly isotropic by strong pitch angle scattering while adiabatically convecting Sunward with $\gamma = 5 \div 3$, so that the plasma number density $n$ and its single-particle kinetic energy $E_{\text{kin}}$ scale as $L^{-4}$ and $L^{-8/3}$ respectively throughout the modeling region of the magnetosphere, which is consistent with the observations.

(4) It is noted that for Earth, the spatial pattern and magnitude of the solar wind driven convection field on both northern and southern polar caps seem to be the same and the field is directed from dawn to dusk, and the southern and northern polar caps would have the same size, because of near alignment of the rotation and the magnetic dipole axes. However, because of the large offset between the magnetic dipole axis and the center of Uranus, and the large tilt angle between the rotation and magnetic dipole axes of Uranus, the magnitude of the solar-wind-driven convection electric field on the dayside and nightside polar caps should be different. In this study, we won't give a detailed discussion of those differences; rather, we assume an average effective polar cap convection field and the edge of the polar caps corresponds to $r = r_u$ on the equatorial plane.

(5). It is assumed that the height-integrated ionospheric Pedersen and Hall conductivities ($\Sigma_p$ and $\Sigma_H$) are uniform and equal in northern and southern ionospheres, respectively. Here we neglect the effects of electron precipitation enhancement and the magnetic field dip-angle; instead we assume that the magnetic field dip-angle in the ionosphere is $\pi / 2$ everywhere. In this case, the Birkeland current in the ionosphere is
only related to the Pedersen conductivity.

4.5 The Convection Model

Based on the above assumptions, we derive in this section the formula that determines the convection field distribution on the magnetic equatorial plane.

Since the magnetic field is independent of time \( t \) and longitude \( \phi \), the flux-tube volume \( \int ds / B \) is a function of \( r \) only; however, the flux tube content \( \eta = \int n / B \, ds \), i.e. the number of charged particles per unit magnetic flux, may be a function of \( r \) and \( \phi \), as well as time \( t \). In the above formula, \( B \) is the magnetic field magnitude, \( ds \) is a segment along the field line, and the integration path is along the magnetic field line. As Selesnick and McNutt [1987] pointed out, if there are sources or sinks for the convecting plasma, there is a mass continuity equation:

\[
\frac{\partial \eta}{\partial t} + V_d \cdot \nabla \eta = \eta_s'
\]  

(4)

where \( V_d \) is the bounce-averaged drift velocity, which is assumed to be the sum of the \( E \times B \) drift velocity \( V_{EB} \) and the magnetic gradient and curvature drift velocity \( V_{gc} \) in our model; \( \eta_s' \) denotes the flux tube sources and sinks, i.e. \( \eta_s' = \int n_s' / B \, ds \), where \( n_s' \) is the net creation of charged particle density. Obviously, if there are no plasma sources or sinks during plasma convection, as we assume, then \( \eta \) is conserved along the drift paths. Moreover, if we write \( E_{kin} = \lambda \cdot U \), where \( U^{-3/2} = [ \int ds / B ] \) is the flux tube volume per unit magnetic flux, it can be shown [e.g. Wolf, 1983] that the coefficient \( \lambda \) is an energy invariant for isotropic-pitch-angle convecting plasma, and the ion gradient/curvature drift velocity can be written as:
where q is the proton charge, the minus sign comes from the fact that $dU/dr$ is negative and the ion gradient and curvature drift is in the positive $e\phi$ direction.

From the discussion above we can see that the plasma can be represented by the energy invariant $\lambda$ and the flux tube content invariant $\eta$. According to the observations, it is assumed, in this model, that hot plasma adiabatically convects inward from the outer boundary $r = r_u = 25R_u$, roughly corresponding to the edge of the polar caps, to the inner sharp boundary around $r \sim r_0 = 5R_u$ with constant $\lambda$ and $\eta$, and the hot plasma can not penetrate the inner boundary. Therefore, the magnetospheric electric current per unit magnetic flux perpendicular to the magnetic field due to the gradient/curvature drift motion is

$$J_{g/c} = -\eta \frac{\lambda}{qB(r)} \frac{dU}{dr} e_\phi$$

The Birkeland (field aligned) current density flowing into the ionosphere can then be obtained from the negative divergence of the magnetospheric current. Based on the assumption (1), the induced current resulting from the time varying convection field is negligible, so that the magnetospheric current is approximately $J_{g/c}$.

It is obvious that if the sharp inner plasma-sheet boundary lines up exactly with contours of constant flux tube volume, i.e. with perfect circles in our model because of the dipole magnetic field, no Birkeland current can be generated, and of course no shielding effect can take place. Thus we assume that the plasma sheet inner boundary generally is at $r = r(\phi,t)$. In order to obtain analytic solutions, and consistent with the PLS observations, we assume the real inner boundary is located around the circle $r = r_0 =$
5 R\(u\) with a small perturbation, i.e. \(r = r_0 + \delta r(\phi, t)\), and require \(|\delta r(\phi, t)| \ll r_0\). In this case the net gradient and curvature drift current into the inner boundary, i.e. the Birkeland current, per unit length along the boundary on the magnetic equatorial plane is

\[
J_{B,e} = - \eta \frac{dU}{dr} \frac{1}{r_0} \frac{\partial r(\phi, t)}{\partial \phi} |_{r_0} b
\]

where \(b\) indicates the unit vector of the magnetic field at the magnetic equatorial plane, and \(b = -e_r \times e_\phi\).

Fig. 12 shows the Birkeland current flow direction on the magnetic equatorial plane. It is apparent that the Birkeland current will flow into the ionosphere at the boundary where \(\partial \delta r/\partial \phi > 0\), and from the ionosphere where \(\partial \delta r/\partial \phi < 0\).

Fig. 12. Diagram of the inner plasma boundary and associated Birkeland currents on the magnetic equatorial plane viewed from above the magnetic south pole. The concentric circles indicate the contours of the magnetic gradient and curvature drift currents.

From the ionosphere-magnetosphere coupling theory [e.g. Vasyliunas, 1972; Wolf,
or directly from the continuity of current, the Birkeland current on the magnetic equatorial plane should be continued by the Birkeland current at the ionosphere.

If we map the Birkeland current \( J_{B,e} \), which is evaluated at the magnetic equatorial plane, onto the ionosphere along the dipole magnetic field lines, and note that the total inner boundary length at the equatorial plane is \( (2 \pi r_0) \) and the associated boundary length in the ionosphere is \( (2 \pi R_u \cos \lambda_0) \), then the Birkeland current per unit length in the ionosphere \( J_{B,i} \) corresponding to the current (7) is

\[
J_{B,i} = \frac{1}{2} \frac{r_0}{R_u \cos \lambda_0} J_{B,e} = \frac{1}{2 \cos^3 \lambda_0} J_{B,e}
\]

where \( \lambda_0 \) is the latitude of the dipole field lines at the ionosphere \( (r = R_u) \) whose end points are at the inner boundary \( (r = r_0) \) on the equatorial plane, i.e. \( \lambda_0 = \cos^{-1} \sqrt{(R_u/r_0)} \), and the factor \( (1/2) \) comes from the assumption that the total Birkeland current in the magnetic equatorial plane is divided in an even fashion between the northern and southern ionospheres.

On the other hand, the Birkeland current in the ionosphere can be expressed in terms of ionospheric conductivities and the electric field distribution. On the basis of assumption (5), we simply have

\[
J_{B,i} = \Sigma_p \left[ ( - \frac{1}{R_u} \frac{\partial \Phi}{\partial \lambda} ) |_{\lambda_0}^+ - ( - \frac{1}{R_u} \frac{\partial \Phi}{\partial \lambda} ) |_{\lambda_0}^- \right] b
\]

where \( \Sigma_p \) is the height integrated effective Pedersen conductivity in the ionosphere; \( \Phi(r, \phi, t) \) is the convection electric potential in the ionosphere, and the gradient of \( \Phi \) is taken in the ionosphere.

From assumptions (1) and (2), the convection electric potential \( \Phi \) is a constant along
the magnetic field lines, therefore, by mapping along the dipole field lines, the electric potential gradient in the ionosphere can be transformed to that in the magnetic equatorial plane

\[
\frac{1}{R_u} \frac{\partial \Phi}{\partial \lambda} \mid_{\lambda_0} = \frac{1}{R_u} \frac{\partial r}{\partial \lambda} \frac{\partial \Phi}{\partial r} \mid_{r_0} = \frac{1}{r_0 \cos^2 \lambda_0} \frac{2 r_0 \sin \lambda_0}{\cos \lambda_0} \frac{\partial \Phi}{\partial r} \mid_{r_0} = \frac{2 \sin \lambda_0}{\cos^3 \lambda_0} \frac{\partial \Phi}{\partial r} \mid_{r_0} \]  

(10)

Equating the right sides of (8) and (9) with the aid of (7) and (10), we obtain

\[
- \eta \lambda \frac{dU}{dr} \mid_{r_0} \frac{1}{r_0} \frac{\partial \Phi}{\partial \phi} \mid_{r_0} = c_0 \Sigma_p \left[ ( - \frac{\partial \Phi}{\partial r} ) \mid_{r_0} - ( - \frac{\partial \Phi}{\partial r} ) \mid_{r_0} \right] \]  

(11)

where \( c_0 \) is a constant related to the location of the inner boundary on the magnetic equatorial plane

\[
c_0 = 4 \sin \lambda_0 = 4 \sqrt{r_0/R_u} \]  

(11a)

Equation (11) gives the convection potential jump condition at the plasma inner boundary which is located around \( L \sim 5 \), and completes the self-consistent loop shown in Fig. 1.

We know that in the ionosphere, \( \nabla \cdot J = 0 \). Based on the assumptions in our model, the magnetospheric convection potential \( \Phi \) in the ionosphere satisfies the Laplace...
equation. Moreover, the Laplace equation in the ionosphere can be transformed into the equatorial plane through the dipole field line equations (see Appendix A). The resulting potential equation in the equatorial plane is

\[ 4r \frac{\partial}{\partial r} (r \frac{\partial \Phi}{\partial r}) + \frac{\partial^2 \Phi}{\partial \phi^2} = 0 \]  

(12)

Combining equation (12), the jump condition (11), and the boundary condition (3), we can theoretically obtain the distribution of the magnetospheric convection electric field and the associated plasma convection pattern.
Chapter 5

Distribution and Shielding
of the Magnetospheric Convection Field

From equation (3) it is clear that the convection field at the outer boundary is a superposition of two parts: a steady potential $\Phi_s (r_u, \phi)$, and a sinusoidally time varying potential $\Phi_t (r_u, \phi, t)$. Since the magnetospheric convection field in our model is driven by the convection field at the outer boundary $r_u$, the convection field inside the outer boundary can also be regarded as a combination of two parts driven by the corresponding potentials on the outer boundary, i.e. $\Phi (r, \phi, t) = \Phi_s (r, \phi) + \Phi_t (r, \phi, t)$. Thus the problem of calculating the distribution of the magnetospheric convection field (eqs. (3), (11) and (12)) can be divided into two parts: one is the steady component,

\[
\begin{align*}
4 r \frac{\partial}{\partial r} (r \frac{\partial \Phi_s (r,\phi)}{\partial r}) + \frac{\partial^2 \Phi_s (r,\phi)}{\partial \phi^2} &= 0 \\
- \eta \lambda \left[ \frac{dU}{dr} \right]_{r_0}^{r^+} \left[ \frac{1}{r} \frac{\partial r(\phi)}{\partial \phi} \right]_{r_0}^{r^+} &= c_0 \Sigma_p \left[ - \frac{\partial \Phi_s (r,\phi)}{\partial r} \right]_{r_0}^{r^+} \\
\Phi_s (r_u, \phi) &= e_s \xi \Phi_0 e^{i k \phi}
\end{align*}
\]  

(13)

and the other is the sinusoidally time varying component
In this chapter, we find analytic solutions of these two boundary problems, and then discuss the shielding effects of the convection field.

5.1 Steady State Field

In equation (13), the driving field is time independent, so the system is in a steady state. In this case, the inner plasma boundary around $r_0$ is the ion drift path there. Obviously, if the $r$ component of the ion drift velocity is zero, then the ion drift paths are concentric circles, which means $\delta r = 0$. Using the assumption of small perturbations, i.e. $\delta r \ll r_0$, indicating that around the plasma inner boundary, the $r$ component of the ion drift velocity is much less than the $\phi$ component, we have

$$
\Phi_i(r, \phi, t) = \epsilon_i \xi \Phi_0 e^{ik\phi + i\omega t}
$$

where $V_{d,r}$ and $V_{d,\phi}$ indicate the $r$ and $\phi$ components of the bounce-averaged ion drift velocity, respectively. Obviously on the magnetic equatorial plane only the $E \times B$ drift has a radial component, so $V_{d,r} = -E_\phi / B$, where $E_\phi = ( - \partial \Phi / \partial \phi ) / r$. Since $\delta r \ll r_0$, which indicates that the magnetic gradient and curvature drift velocity (with only a $\phi$ component) is much larger than the $E \times B$ drift velocity (including both $r$ and $\phi$...
components), the \( \phi \) component of \( \mathbf{V}_{\text{ExB}} \) can be safely neglected compared with that of \( \mathbf{V}_{g/c} \), so that \( V_{d,\phi} \approx V_{g/c} = - (\lambda \frac{dU}{dr}) / q \mathbf{B} \). From the discussion above and equation (5), the jump condition in (13) can be rewritten as

\[
-q \frac{1}{r_0} \left( \frac{\partial \Phi_s(r,\phi)}{\partial \phi} \right) \bigg|_{r_0} = c_0 \sum_p \left[ \left( \frac{\partial \Phi_s(r,\phi)}{\partial r} \right) \bigg|_{r_0} - \left( \frac{\partial \Phi_s(r,\phi)}{\partial r} \right) \bigg|_{r_0} \right] \tag{16}
\]

In general, we can assume the steady state convection electric potential \( \Phi_s(r, \phi) \) has the following form which satisfies the first equation in (13)

\[
\Phi_s(r, \phi) = \begin{cases} 
\sum_{j=1}^{\infty} A_j \left( \frac{r}{r_0} \right)^{j/2} e^{ij\phi} & r \leq r_0 \\
\sum_{j=1}^{\infty} \left[ B_j \left( \frac{r_0}{r} \right)^{j/2} + C_j \left( \frac{r}{r_u} \right)^{j/2} \right] e^{ij\phi} & r_0 \leq r \leq r_u 
\end{cases} \tag{17}
\]

where \( A_j, B_j, \) and \( C_j \) are constant coefficients determined by the boundary conditions. Substituting equation (17) into (16), we have

\[
\sum_{j=1}^{\infty} \left\{ i \left( \frac{2c_0 \Sigma_p}{q \eta} \right) \frac{1}{4} \left[ B_j - \left( \frac{r_0}{r_u} \right)^{j/2} C_j + A_j \right] + A_j \right\} \frac{1}{r_0} e^{ij\phi} = 0 \tag{18}
\]

The continuity of \( \Phi_s \) at outer and inner boundaries gives another two equations

\[
A_j = B_j + \left( \frac{r_0}{r_u} \right)^{j/2} C_j \tag{19}
\]

\[
\left( \frac{r_0}{r_u} \right)^{j/2} B_j + C_j = \varepsilon_s \xi_0 \Phi_0 \delta_{jk} \tag{20}
\]
where

$$
\delta_{j,k} = \frac{1}{2\pi} \int_{0}^{2\pi} e^{i(k-j)\phi} d\phi
$$

Now for each \( j \), the three unknowns \( A_j, B_j \) and \( C_j \) can be easily solved from the corresponding three independent equations (18), (19), and (20). Substituting these coefficients back into equation (17) we obtain the steady state convection electric potential \( \Phi_s(r, \phi) \) on the magnetic equatorial plane:

$$
\Phi_s(r, \phi) = \begin{cases} 
\frac{\frac{i \zeta_s/2}{r_0^k}}{1 - \left(\frac{r_0}{ru}\right)^k + i \zeta_s/2} & r < r_0 \\
\frac{\frac{\varepsilon_s \Omega \Phi_0}{r_0^k} e^{i k \phi}}{1 - \left(\frac{r_0}{ru}\right)^k + i \zeta_s/2} \left[\left(1 + i \zeta_s/2\right)\left(\frac{r_0}{ru}\right)^{k/2} - \left(\frac{r_0}{ru}\right)\left(\frac{ru}{r}\right)^{k/2}\right] & r_0 \leq r \leq ru 
\end{cases}
$$

where

$$
\zeta_s = \frac{2c_0 \Sigma_p}{q \eta}
$$

is the shielding factor for a steady state convection field.

### 5.2 Sinusoidally Time-Varying Field

Now let us find the time dependent part \( \Phi_t(r, \phi, t) \) to be determined by equation (14). In the time dependent problem, both the potential \( \Phi_t \) and the perturbation of the
inner boundary \( \delta r \) in equation (14) are functions of location, as well as time. In this case, the simple relation (15) is no longer valid, and we need to find another expression relating \( \delta r(\phi,t) \) to the convection potential and/or known quantities. Since

\[
\frac{D r(\phi,t)}{Dt} \bigg|_{r_0} = \left[ \frac{\partial}{\partial t} + \frac{\partial \phi}{\partial t} \right] r(\phi,t) \bigg|_{r_0}
\]

\[
= \frac{D \delta r(\phi,t)}{Dt} \bigg|_{r_0} = V_d \cdot r \bigg|_{r_0} = -\frac{E_\phi}{B} r \bigg|_{r_0}
\]

(23)

we first differentiate the jump condition in (14) with \( D/Dt \) and rewrite it as

\[
\frac{c_0 \Sigma_p r_0 B(r_0)}{\eta \lambda \frac{dU}{dr}} \left( \frac{\partial}{\partial t} + \frac{\partial \phi}{\partial t} \right) \left[ \frac{\partial q(t)}{\partial r} \right]_{r_0} = \frac{\partial}{\partial \phi} \left( \frac{1}{r_0} \frac{\partial q(t)}{\partial \phi} \right) \bigg|_{r_0}
\]

(24)

Because the driving field at the outer boundary \( r_u \) is a periodic one with the form \( e^{i \omega t} \), and because the frequency of a forced oscillation is determined by that of the driving force, it is natural to assume the convection potential \( \Phi_t \) has the same form as \( \Phi_s \) in (17), except for a time factor \( e^{i \omega t} \),

\[
\Phi_t(r,\phi,t) = \begin{cases} 
\sum_{j=1}^{\infty} A_j \left( \frac{r}{r_0} \right)^{j/2} e^{i j \phi + i \omega t} & r \leq r_0 \\
\sum_{j=1}^{\infty} B_j \left( \frac{r_0}{r} \right)^{j/2} + C_j \left( \frac{r}{r_u} \right)^{j/2} & \frac{r_0}{r} \leq r \leq r_u 
\end{cases}
\]

(25)

which apparently is a solution of the first equation in (14).
Obviously, we have

\[ \frac{\partial}{\partial t} = i \omega \quad \frac{\partial}{\partial \phi} = i j \]

and

\[ \frac{\partial \phi}{\partial t} \bigg|_{r_0} = \frac{1}{r_0} (r \frac{\partial \phi}{\partial t}) \bigg|_{r_0} = \frac{1}{r_0} V \phi \bigg|_{r_0} \equiv - \frac{\lambda}{r q B(r)} \bigg|_{r_0} \]

Using all these formulas we can rewrite (24) as

\[ \sum_{j=1}^{\infty} \left[ \frac{i (\omega \tau_j + \zeta_s)}{4} \left[ B_j - \left( \frac{r_0}{r_u} \right)^{j/2} C_j + A_j \right] \right] \frac{j}{r_0} e^{i(j \phi + \omega t)} = 0 \tag{26} \]

where $\zeta_s$ is the shielding factor for steady state convection (see equation 22) and $\tau_j$ is defined as

\[ \tau_j = \frac{2 c_0 \sum_p r_0 B(r_0)}{j \eta \lambda \frac{dU}{dr} |_{r_0}} \tag{27} \]

Of course the continuity of $\Phi_t$ at the outer and inner boundaries gives another two equations to be used together with (26) to determine the coefficients $A_j$, $B_j$, and $C_j$:

\[ A_j = B_j + \left( \frac{r_0}{r_u} \right)^{j/2} C_j \tag{28} \]

\[ \left( \frac{r_0}{r_u} \right)^{j/2} B_j + C_j = \epsilon \xi \Phi_0 \delta_{jk} \tag{29} \]
As in the solution of the steady potential \( \Phi_s \), we can find the three series coefficients \( A_j, B_j \) and \( C_j \), and substitute these coefficients back into the assumed form of \( \Phi_t \) to obtain

\[
\Phi_t(r, \phi, t) = \begin{cases} 
\frac{i \xi_t /2}{1 - \left( \frac{r_0}{r_u} \right)^k + i \xi_t /2} \varepsilon_t \xi \Phi_0 \left( \frac{r}{r_u} \right)^{k/2} e^{i k \phi + i \omega t} & r \leq r_0 \\
\frac{\varepsilon_t \xi \Phi_0 e^{i k \phi + i \omega t}}{1 - \left( \frac{r_0}{r_u} \right)^k + i \xi_t /2} \left[ (1 + i \xi_t /2) \left( \frac{r}{r_u} \right)^{k/2} - \left( \frac{r_0}{r_u} \right)^k \left( \frac{r}{r_u} \right)^{k/2} \right] & r_0 \leq r \leq r_u 
\end{cases}
\] (30)

where

\[
\zeta_t = \omega \tau_k + \zeta_s
\] (31)

is the shielding factor for a periodic time varying convection field with frequency \( \omega \). It is interesting to note that \( \Phi_t \) has the same form as \( \Phi_s \), except for different shielding factors, weight factors and the time variation factor \( e^{i \omega t} \). This implies that for a time dependent convection potential proportional to \( e^{i \omega t} \), we can use the results of the corresponding steady state convection potential, only replacing the steady shielding factor \( \zeta_s \) by the time dependent shielding factor \( \zeta_t \) and multiplying by the time variation factor \( e^{i \omega t} \). It is important to point out that the shielding factor for a steady convection field, \( \zeta_s \), is independent of the energy of the convecting ions; however, the shielding factor for a time dependent convection field, \( \zeta_t \), does depend upon the energy of the convecting ions since \( \tau_k \) is inversely proportional to \( E_{\text{kin}} \).
5.3 Shielding Effect of the Steady Field

Solution (21) is a typical result for a steady state convection electric potential distribution if there is a sharp inner plasma sheet boundary at \( r_0 \). Inside the inner plasma boundary, the convection electric potential \( \Phi_s \) is shielded by a shielding factor \( \zeta_s \). When the shielding factor \( \zeta_s \) is much less than 1, in other words, when the effective Hall conductivity \( q_\eta \) in the magnetosphere [Vasyliunas, 1972] is much greater than the height integrated Pedersen conductivity \( \Sigma_p \) in the ionosphere, the convection electric field inside the inner boundary \( (r \sim r_0) \), or equivalently inside the Birkeland current, is greatly reduced (roughly equal to the shielding factor times the field without Birkeland currents) and the direction of the shielded field will rotate nearly 90° relative to the unshielded field. However, if the shielding factor is much greater than 1, there is almost no shielding effect and no rotation of the convection field. The shielding factor is inversely proportional to the difference of plasma density between inside and outside the plasma inner boundary (one should note that in our model, we have assumed that the inner boundary is impenetrable, i.e. there is no hot plasma inside the boundary, then \( |\Delta \eta| = \eta \)). Obviously if other conditions are kept unchanged and the boundary is impenetrable, more convecting plasma will lead to larger Birkeland currents, and consequently a greater shielding of the convection field inside the Birkeland current. From the standpoint of current continuity, if the effective Hall conductivity \( q_\eta \) in the magnetosphere is larger than the Pedersen conductivity \( \Sigma_p \) in the ionosphere, the convection electric field near the Birkeland current (and inside this region because the field is driven from the outer region) has to be reduced in order to continue the Birkeland current in the ionosphere, and the reduction must be proportional to the ratio of the two conductivities, i.e. \( q_\eta / \Sigma_p \).

In 1984, Atreya proposed a model of Uranus' ionosphere from which he estimated \( \Sigma_p = 5 \sim 10 \) mho in auroral regions and \( \Sigma_p \sim 0.6 \) mho near the terminator [Atreya, 1984]. Based upon the quasi-static convection model and a strong shielding assumption, McNutt
et al. [1987] estimated $\Sigma_p = 0.4$ mho from initial analysis of the Voyager 2 observation that the hot proton density is 0.1 cm$^{-3}$ at the inner edge near $L = 6$. On the basis of the same model, Selesnick and McNutt [1987] re-estimated $\Sigma_p = 1.4 \sim 14$ mho from the total ion density of hot (~ keV) and intermediate (~ 100 eV) protons at $L = 5$. As we will see, the quasi-static convection model is not applicable to the Uranus magnetosphere, and $\Sigma_p$ cannot be estimated as McNutt et al. and Selesnick and McNutt did. Moreover, we have pointed out that the shielding factor for a sinusoidally-varying convection pattern depends not only on the ion density, but also on the ion energy at the sharp inner plasma boundary; thus, the value of $\Sigma_p$ estimated by Selesnick and McNutt [1987] might be less accurate than the earlier estimate of McNutt et al.. For lack of observations and reliable theoretical models of the conductivity of the Uranian ionosphere, we here choose $\Sigma_p$ to be 0.6 mho which probably represents a lower limit.

If the plasma density $n$ is assumed to be a constant along magnetic flux tubes (as implied by our assumptions of isotropy and $\mathbf{E} \cdot \mathbf{B} = 0$), then the flux tube content $\eta$ is

$$\eta = n \text{ (cm}^{-3}\text{)} \times 10^{18} \times L^4 \text{ (T}^{-1}\text{-m}^{-2}\text{)}$$

$$= 6.36 \times 10^{20} n_{r_0} \text{ (cm}^{-3}\text{)} \text{ (T}^{-1}\text{-m}^{-2}\text{)}$$

(32)

where $L = r/R_u$, $n$ is the plasma number density at $L$, and $n_{r_0}$ is the number density at $r_0$. Here we have used the assumptions that the convection is adiabatic ($\gamma = 5/3$) and that the plasma density is proportional to $L^{-4}$. According to the observations [Selesnick and McNutt, 1987], the hot plus intermediate plasma density just outside the inner boundary ($L = 5$) is 0.6 cm$^{-3}$, so that the constant $\eta$ is $3.81 \times 10^{20}$ (T$^{-1}$-m$^{-2}$). Substituting (32) into (22) we have
Using $\Sigma_p = 0.6$ mho, $n_{r0} = 0.6$ cm$^{-3}$, and $L = 5$, we obtain the shielding factor for steady convection $\zeta_s \approx 0.07$.

Formula (33) tells us that if the plasma convects Sunward adiabatically with $n_{r0} = 0.6$ cm$^{-3}$, and $\Sigma_p = 0.6$ mho, then the closer the plasma boundary forms to the planet, the stronger the shielding the convection field experiences.

Fig. 13 shows how the shielding factor for a steady convection field $\zeta_s$ varies with the location of the inner plasma boundary.

\[
\zeta_s = 0.08 \sqrt{1 - \frac{1}{L}} \frac{\Sigma_p \text{ (mho)}}{n_{r0} \text{ (cm}^{-3})}
\]
Figure 14 shows the shielding effect of the steady component of the convection electric field $\Phi_s$ with $\zeta_s = 0.07$. Similar results for Uranus' magnetospheric convection have been obtained by Selesnick and McNutt [1987]. Far from the inner boundary, the convection electric field approaches a uniform field with direction from dawn to dusk, which causes the plasma to drift Sunward from the tail. Inside the inner boundary, the convection field is strongly shielded and its direction is rotated nearly $\pi/2$ radians, directed nearly tailward. Obviously in Fig. 14, the $\phi$ component of the convection field is positive on the dawn side and negative on the dusk side; consequently, the $r$ component of the plasma drift velocity is negative on the dawn side and positive on the dusk side.

Fig.14. Distribution of the steady convection field potential (with $\zeta_s = 0.07$) on the magnetic equatorial plane, viewed from above the magnetic south pole. The plasma inner boundary is at $r_0 = 5R_u$. The sun is to the left.
Since the $\phi$ component of the plasma drift velocity is positive everywhere, then from equation (15), $\partial \delta r / \partial \phi$ is negative on the dawn side and positive on the dusk side as shown in Fig. 12. From equation (7), the Birkeland current at the inner boundary, in this case, flows from the ionosphere into the magnetosphere on the dawn side and toward the ionosphere from the magnetosphere on the dusk side. Corresponding to this Birkeland current pattern, a space charge separation forms around the inner boundary and tends to cancel the primary convection field.

5.4 Shielding Effect of the Sinusoidally Varying Field

In discussing shielding effects, particular in time dependent convection field shielding, an important parameter is the shielding time scale $\tau_{sh}$, which indicates how quickly the shielding forms once the convection field is applied. The shielding time scale $\tau_{sh}$ does not influence the strength of shielding for a steady convection field, though it does, as we discuss later, determine how strong the shielding is for a time dependent convection field, and also gives a criterion as to whether a given convection process should be dealt with as a quasi-static or a time-dependent one.

It is already known [e.g. Southwood, 1977; Southwood and Wolf, 1978; Siscoe, 1982] that the shielding time scale is proportional to the ion magnetic gradient and curvature drift time scale $\tau_{g,c}$ at the shielding layer, and the shielding factor $\zeta$. Using the definition of $\zeta_s$ and $V_{g/c}$ (equations 22 and 5), we can easily show that $\tau_k$ in equation (27) equals $\zeta_s \tau_{g,c} / 2 \pi k$, in which $\tau_{g,c}$ is the magnetic gradient and curvature drift time scale $2\pi r / V_{g/c}$. For the case $k = 1$, as discussed in 4.3, there is only one pair of Birkeland current sheets, one with flow into and the other with flow from the ionosphere. Generally, if the convection field is proportional to $e^{ik\phi}$, there are $k$ pairs of Birkeland current sheets, whose strengths are not related to the parameter $k$. Apparently, for the same strength of Birkeland current, the more pairs of current sheets, the faster the
shielding is completed. This is why the shielding time scale is inversely proportional to the parameter k. For simplicity, if the convection potential is chosen to vary cosinusoidally around the circular equatorial boundary at \( r = 25 \text{ R}_u \), following Vasyliunas [1970], in a sense which provides an anti-sunward plasma flow within the polar caps, then the parameter k in our model is 1, and the convection electric field is spatially uniform but time variable. In this case the shielding time scale in our model is \( \tau_1 \).

In our simple model, the magnetic gradient and curvature drift time scale for the ions is given by

\[
\tau_{g,c} = \frac{2\pi r}{V_{g,c}} = 1.06 \times 10^4 \frac{1}{L K(\text{keV})} \text{(hr)}
\]

\[
= 145.5 \frac{L^{5/3}}{K_{r_0}(\text{keV})} \text{(hr)} \tag{34}
\]

where \( K \) is the ion energy at radial distance \( L \) in units of keV, and \( K_{r_0} \) is the energy at \( r_0 \); here we have again used the assumption that the convection is adiabatic (\( \gamma = 5/3 \)) and the energy of convecting ions is proportional to \( L^{8/3} \). For instance, when \( L = 5 \), the observed energy of the hot plasma is about 1 keV, therefore, the magnetic gradient and curvature drift time scale at \( L = 5 \) is about 2128 hr.

Using the results of (33), (34) and (27), the shielding time scale in our model is given by

\[
\tau_{sh} = 1.85 \sqrt{1 - \frac{1}{L} L^{5/3} \frac{\Sigma_p(\text{mho})}{n_{r_0} (\text{cm}^{-3}) K_{r_0}(\text{keV})}} \text{(hr)} \tag{35}
\]

According to the observations of the PLS experiment [Bridge et al., 1986; McNutt et
al., 1987; Selesnick and McNutt, 1987], the hot and intermediate plasmas have the same density (0.3 cm⁻³) but different energies (1 keV and 0.3 keV respectively) at the inner boundary (r = r₀ = 5 R_u). If we take Σ_p = 0.6 mho, n_r₀ = 0.6 cm⁻³, K_r₀ = 0.65 keV as an average value for both hot and intermediate plasmas, the shielding time scale τ_sh is about 37.2 hours. Using the expressions (33) and (35), we obtain the shielding factor for a sinusoidally varying convection field

\[ \zeta_t = [0.68 \frac{L^{5/3}}{K_r^0(\text{keV})} + 0.08] \sqrt{1 - \frac{1}{L} \frac{\Sigma_p(\text{mho})}{n_{r_0}(\text{cm}^{-3})}} \]  

(36)

With the same parameters Σ_p, n_r₀, and K_r₀ used above, ζ_t = 13.8 at L = 5. Fig. 15 gives a curve of ζ_t vs. L, similar to Fig. 13. Unlike ζ_s, which indicates that the steady convection field in our model is highly shielded no matter where the plasma boundary is located, the large value of ζ_t indicates that the sinusoidal part of the convection field in our model is not significantly shielded at the inner plasma boundary.

Obviously from (30) and (31), if the shielding factor ζ_t is (i.e. both ζ_s and ωτ_j are) much smaller than 1, the time dependent convection electric field within the inner hot plasma boundary will experience strong shielding and rotate by nearly 90°, similar to the steady case. If ωτ_sh is less than or comparable to ζ_s, the time dependent shielding factor ζ_t can be approximately determined by the steady state shielding factor ζ_s. In other words, if the shielding time scale τ_sh is much less than the period of the convection field, such that ζ_s is larger than or even comparable to ωτ_sh, the shielding process for the time dependent convection field can be regarded as a steady state process to a good approximation. If ωτ_sh is much larger than the steady shielding factor ζ_s, then the value of ωτ_sh will determine how strongly the time dependent convection field will be reduced inside the inner plasma boundary, and the quasi-static shielding model does not work at
Fig. 15. The curve of the shielding factor for a sinusoidally varying convection field $\zeta_t$ varying with the location of the plasma inner boundary with $\Sigma_p = 0.6$ mho, $n_{ro} = 0.6$ cm$^{-3}$ and $K_{r0} = 0.65$ keV.

In general, if the shielding time scale $\tau_{sh}$ is much smaller than the period of the solar-wind-driven convection electric field, then the convection field can be approximately regarded as time independent, because the shielding layer can be maintained continually as the convection field changes. If the shielding time scale is comparable to, or even larger than the period of the convection field, then we have to deal with a time dependent convection process, and the shielding effect is relatively weak. In this case, before the shielding layer can be established, the convection field has changed both in direction and magnitude. This can be seen clearly from Fig. 12 and equation (11). We have shown that in Uranus' magnetosphere, the shielding time scale is larger than the
period of the driving convection electric field, so that the convection should be treated as time dependent.

Figure 16 shows how the sinusoidal part of the convection field $\Phi_t$ (with $k = 1$) penetrates through the inner boundary at $L \sim 5$ nearly without shielding, with the above parameters.

5.5 Shielding Effect of the Total Field

In our model, the total convection electric potential $\Phi(r, \phi, t)$ in Uranus' magnetosphere consists of a steady part $\Phi_s(r, \phi)$ and a (sinusoidally) time varying part $\Phi_t(r, \phi, t)$. These two parts have quite different behavior near the inner plasma sheet.
boundary; in particular, the steady convection potential $\Phi_s$ experiences strong shielding while the time dependent one, $\Phi_t$, experiences almost no shielding inside the inner boundary. Therefore the strength of shielding of the total convection field $\Phi$ depends on the ratio of the magnitudes of $\Phi_s$ and $\Phi_t$, i.e. the ratio $\varepsilon_s/\varepsilon_t$. According to our model, with $\zeta_s = 0.07$, $\zeta_t = 13.8$, Figures 17, 18, 19 and 20 show total convection potential distributions with ratios $\varepsilon_s/\varepsilon_t = 1, 2, 5, \text{and} 10$ respectively.

From Fig. 11 we can see that, at the outer boundary $r_u$ the total field, for a given ratio $\varepsilon_s/\varepsilon_t$, is $|\Phi_s| + |\Phi_t| = \Phi_{\text{max}}$ at time $t_{\text{max}}$, $|\Phi_s|$ at $t_{\text{av}}$, and $|\Phi_s| - |\Phi_t|$ at $t_{\text{min}}$ respectively. Obviously, at $t_{\text{av}}$, the shielding effect of the total field with any ratio $\varepsilon_s/\varepsilon_t$ is the same as that of the steady field shown in Fig. 14 because at this time, $\Phi_t = 0$. If the shielding time delay is considered, Fig. 14 can also be used as the total field distribution at time $t_{\text{av}} + \tau_{\text{sh}}$. Therefore, in Figures 17 to 20, we only give the convection field distribution at times $t_{\text{max}} + \tau_{\text{sh}}$ in panels (a) and $t_{\text{min}} + \tau_{\text{sh}}$ in panels (b).

In Fig. 17, $\varepsilon_s/\varepsilon_t = 1$. Panel (a) shows that after the shielding the convection field inside the plasma inner boundary is approximately half of the field outside $\Phi_{\text{max}}$ (and the field points from top to bottom), because of the strong shielding of the steady field. Panel (b) give a surprise result. At $t_{\text{min}}$, the total convection field $\Phi$ at the outer boundary is zero; therefore, it may seem that there would exist no convection field inside it at all. Actually, at the outer boundary, the total field at $t_{\text{min}}$ is a superposition of equal but opposite fields $\Phi_s$ and $\Phi_t$, and outside the shielding layer at $r_0$, the convection field is correspondingly small. The inner boundary, however, shields $\Phi_s$ but not $\Phi_t$, so that the field inside the inner boundary is no longer canceled, but is nearly $\Phi_t$, and the field direction is reversed (from bottom to top).

In Fig. 18, $\varepsilon_s/\varepsilon_t = 2$, which means that the magnitude of $\Phi_t$ is a third of $\Phi_{\text{max}}$. Hence in panel (a), the convection field has maximum value of $\Phi_{\text{max}}$ outside the inner plasma boundary. The shielded convection field inside the boundary has nearly the same
Fig. 17. Distribution of the total field $\Phi$ with $\varepsilon_y/\varepsilon_t = 1$, $\zeta_s = 0.07$ and $\zeta_t = 13.8$. (cf Fig. 14)
Fig. 18. Distribution of the total field $\Phi$ with $\varepsilon_s/\varepsilon_l = 2$, $\zeta_s = 0.07$ and $\zeta_l = 13.8$. (cf Fig. 14)
Fig. 19. Distribution of the total field $\Phi$ with $\varepsilon_p/\varepsilon_1 = 5$, $\zeta_s = 0.07$ and $\zeta_t = 13.8$. (cf Fig. 14)
Fig. 20. Distribution of the total field $\Phi$ with $\varepsilon_2/\varepsilon_1 = 10$, $\zeta_s = 0.07$ and $\zeta_l = 13.8$. (cf Fig. 14)
direction as the convection field outside, but with just over one third of the magnitude outside the boundary because of the effective shielding of $\Phi_s$. In panel (b), on the other hand, the convection field outside the boundary has minimum value $\Phi_s - \Phi_t = \Phi_{\text{max}}/3$; however, inside the boundary the convection field has the same magnitude but opposite direction. Similar results can be seen in Figures 19 and 20. It is noted that when the ratio $\varepsilon_s/\varepsilon_t$ becomes larger and larger, the direction of the shielded convection field inside the inner boundary rotates more and more relative to the unshielded convection field, because the shielded steady field inside the inner boundary becomes relatively larger and it is directed from left to right.

In summary, Figure 21 shows the directions and magnitudes of the shielded convection electric field inside the plasma sheet inner boundary ($r = r_0$) with different ratios ($\varepsilon_s/\varepsilon_t$). For a given ratio $\varepsilon_s/\varepsilon_t$, the height (the vectors at $t_{\text{av}} + \tau_{\text{sh}}$) of the shaded triangle indicates the shielded steady component ($\Phi_s$), and the shaded vectors indicate the shielded (sinusoidally) time dependent components ($\Phi_t$) at $t_{\text{max}} + \tau_{\text{sh}}$ and $t_{\text{min}} + \tau_{\text{sh}}$ respectively. The total shielded convection field vectors are restricted within the shaded triangle, and oscillate between the two legs of the isosceles triangles with period $\tau_r = 17.2$ hr. As a comparison, the vectors of the convection field at the outer boundary at time $t_{\text{av}}$, $t_{\text{max}}$ and $t_{\text{min}}$, which have magnitudes of $\varepsilon_s \Phi_0$, $(\varepsilon_s + \varepsilon_t) \Phi_0$ and $(\varepsilon_s - \varepsilon_t) \Phi_0$ respectively (see Fig. 11), are also given. Obviously, as the ratio $\varepsilon_s/\varepsilon_t$ decreases/increases, the magnitude of the shielded convection field inside the plasma inner boundary increases/decreases, and the variable range of the field direction expands/shrinks. However, the plasma inner boundary will shield at least half of the total convection field no matter what the ratio $\varepsilon_s/\varepsilon_t$ is. The most interesting result is that the solar wind driven convection field in the polar caps (high latitude region), whose magnitude may be time variable, has a constant direction from dawn to dusk, whereas, both the direction and magnitude of the convection field inside the plasma inner boundary
Fig. 21. Vectors of the shielded and unshielded convection field with $\zeta_s = 0.07$ and $\zeta_t = 13.8$. 
(low latitude region), which is driven by the high latitude convection field, may be time dependent.
In the last chapter, we obtained the self-consistent distribution of the convection field and discussed the shielding effects of the inner plasma boundary observed by the PLS experiment on Voyager 2. Once the convection field structure is known, the associated plasma convection pattern is easy to derive, particularly for the case of adiabatic convection. In this chapter, we evaluate the plasma convection time scale and compute the convection pattern by tracing ion drift orbits. We find that in the Uranian magnetosphere, the plasma convection time scale is much larger than the oscillation period of the convection field, so that the convection can be dealt with approximately by using a time-averaged convection field, while the shielding effect has to be treated as time dependent as discussed in the last chapter. Another problem addressed in this chapter is how the observed inner plasma boundary forms self-consistently with the convection field distribution.

6.1 Plasma Convection Time Scale

In dealing with a time dependent convection problem, it is important to determine some characteristic time scales that play a significant role in the problem. In the last chapter, we discussed the shielding time scale \( \tau_{sh} \) and the ion magnetic gradient and curvature drift time scale \( \tau_{g,c} \). Here we will discuss another important time scale: the \( \mathbf{E} \times \mathbf{B} \)
B convection time scale $\tau_{E \times B}$.

For a time dependent convection electric field, the $E \times B$ drift velocity is a function of both time and location. From solutions (21) and (30), we can find the electric field distribution from $E = -\nabla \Phi$. Fig. 22 shows, on the OTD magnetic equatorial plane, the maximum values (at $t = t_{\text{max}}$) of the $r$ component (solid lines) and the $\phi$ component (dotted lines) of the steady convection field vs. radial distance $L$ at given values of the azimuthal angle $\phi$. Fig. 23 shows the same thing for the time-varying part of the convection field. Fig. 24 shows the same thing for the total (steady plus time-varying) field components. Obviously both components of the time varying field penetrate almost smoothly into the shielding layer at $L = 5$, while those of the steady field decrease sharply inside the shielding layer and undergo a significant rotation. The convection field outside the shielding layer is nearly uniform and directed from dawn to dusk.

Fig. 22. The maximum values of the $r$ component (solid lines) and $\phi$ component (dotted lines) of the steady convection field vs. $L$ at given values of the azimuthal angle $\phi$. 
Fig. 23. The same thing as Fig. 22 but for time varying convection field.

Fig. 24. The same thing as Fig. 22 but for total (steady plus time varying) convection field.
Apparently the average electric field strength outside the shielding layer is approximately a constant, with $E_{\text{max}} = 1.25 \times 10^{-5}$ V/m for both components, and the electric field is directed from dawn to dusk (Figs. 14 to 16). This average field strength can also be obtained from the uniform-field assumption, which is approximately consistent with the solutions (21) and (30), i.e. $E_{\text{max}} = \xi \left( 2 \Phi_0 \right) / \left( 2 \times 25 R_{\text{u}} \right) = 1.25 \times 10^{-5}$ V/m.

Generally the space uniform convection field in our model can be written as

$$E = E_{\text{max}} \left[ \varepsilon_s + \varepsilon_t \cos(\omega t + \alpha) \right]$$

$$= E_{\text{max}} \frac{1}{1 + \frac{\varepsilon_s}{\varepsilon_t}} \left[ \varepsilon_s + \cos(\omega t + \alpha) \right] \quad (37)$$

where $\omega = 2\pi/\tau$, and $\alpha$ is an arbitrary initial phase. Hence the Sunward $E \times B$ drift velocity is

$$V_{E \times B} = \frac{E}{B} = 0.54 L^3 \frac{\varepsilon_s}{\varepsilon_t} + \cos(\omega t + \alpha) \quad (m/s) \quad (38)$$

If it is assumed that hot plasma drifts Sunward from $L = 25$, then the convection time scale $\tau_{\text{conv}}$, i.e. the time for the plasma drift from $L = 25$ to any $L$ smaller than 25, without considering the shielding effect, is determined by

$$\frac{\varepsilon_s}{\varepsilon_t} \tau_{E \times B} (hr) + \frac{1}{\omega} \sin(\omega \tau_{E \times B} + \alpha) = 6.54 \times 10^3 \left( 1 + \frac{\varepsilon_s}{\varepsilon_t} \right) \left( \frac{1}{L^2} - \frac{1}{625} \right) \quad (39)$$

Figure 25 shows the logarithm of the convection, magnetic gradient/curvature drift,
and shielding time scales (in unit of hr) as functions of \( L \) according to the expressions derived above, in comparison with the constant oscillation period of the convection electric field (\( \tau_r = 17.2 \) hr). Here the plasma energy near the inner plasma boundary (\( r = 5 \text{ R}_u \)) \( K_{r0} \) is taken to be 1 and 3 keV for the two \( \tau_{g,c} \) curves. Considering the fact that the plasma at the shielding layer consists of both hot (1 keV) and intermediate (0.3 keV) components, we use an averaged plasma energy \( K_{r0} \) of the two components (i.e. 0.65 keV) for the \( \tau_{sh} \) curve. For the \( \mathbf{E} \times \mathbf{B} \) convection time scale curves, we show two extreme limiting cases: \( \varepsilon_s/\varepsilon_t = 1 \) and \( \varepsilon_s/\varepsilon_t = \infty \).

![Diagram showing logarithm of time scales vs. radial distance L.](image)

Fig. 25. The logarithm of time scales vs. radial distance \( L \).

Obviously near the plasma inner boundary, we have \( \tau_{g,c} > \tau_{E \times B} >> \tau_{sh} \geq \tau_r \). As discussed in the last chapter, since the shielding time scale (~ 38 hr) is larger than the period of the convection field (~17.2 hr), the quasi-steady approximation is no longer valid, and we must use the time dependent model to deal with the shielding problem near
L = 5, unless the shielding layer is formed by plasma of higher energy (say, ~ 10 keV) and/or larger particle density (~ several protons/cm\(^3\)), in which case the shielding time scale (~ 2 hr) might be much less than the period of the convection field. Another important conclusion from Fig. 25 is that in the Uranian magnetosphere, although the convection field is treated as a time dependent one in dealing with the shielding process at the inner plasma boundary (L = 5), a time averaged convection electric field can be used approximately when we investigate the plasma convective motion from L = 25 to the inner plasma boundary, because \( \tau_{E \times B} \) (~ several hundred hours) is much longer than \( \tau_r \), so that during the plasma convection from L = 25 Sunward to L = 5, the convection field will oscillate many (at least 20) times.

From Figure 25 it is noted that at L = 5 the magnetic drift time scale is about 80 days for 1 keV plasma, and the \( E \times B \) convection time scale is about 21 days (if \( \varepsilon_s/\varepsilon_t = 1 \)), which is roughly consistent with the plasma transport rate \( \tau_D \sim 30 \) days estimated by Cheng [1987] and the plasma residence time \( \tau_D \leq 50 \) days suggested by McNutt et al. [1987]. This is considerably longer than the original estimate of the convection time (~ 40 hours) by Siscoe [1975; see also Bridge et al. 1986, and Vasyliunas 1986]. One difference is that we have adopted a coupling efficiency \( \xi = 0.1 \) whereas Siscoe assumed \( \xi = 0.25 \). A more important difference is that Siscoe's estimate applies to the convection time across the polar cap, whereas our estimate is for the longer trip from L = 25 to L = 5.

6.2 The Hot Plasma Convection Pattern

Nearly a half century ago, Alfven [1939, 1940, 1955, 1958] showed that, when a uniform, static electric field is imposed in the equatorial plane of a magnetic dipole field, trapped charged particles are forced either to drift to infinity or to be stably trapped inside an egg-shaped "forbidden region" (e.g. see Fig. 26a), the size of which is determined by
the value of the length unit $L_A$ which depends on the magnetic dipole moment, the strength of the electric field and the plasma energy. According to Alfven's calculation [e.g. 1963]

$$L_A = \left( \frac{\mu M_d}{qE} \right)^{1/4}$$  \hspace{1cm} (40)

where $\mu = K / B$ denotes the magnetic moment invariant, $M_d$ denotes the magnetic dipole moment, $E$ is the imposed electric field and $q$ is the ion charge. The farthest and nearest points of the "forbidden region" are located at

$$L_F = 1.32 L_A \quad L_N = 0.74 L_A$$  \hspace{1cm} (41)

It may seem inappropriate to directly apply the Alfven layer model here because the electric field in Uranus' magnetosphere is time dependent. However, for a large scale description of plasma convection in Uranus' magnetosphere, the Alfven layer model is a good approximation if we use the time-averaged electric field because the convection time scale $\tau_{E \times B}$ is much larger than the oscillation period $\tau_r (\sim 17.2$ hours) of the convection electric field, as shown in the last section. In our model, the time-averaged convection electric field is $\langle E \rangle_t = E_{\text{max}} \epsilon_s$, and $M_d = 0.23$ Gauss-R$_d^3$. Substituting this into (40) and (41), we obtain the dimensions of the Alfven layer for various situations indicated in Table 1. Note that an ion energy of 6.8 keV at $r_0$ is equivalent to an ion energy of 3 keV at $L = 6.8$ as observed by the PLS experiment.
If the time variation of the convection field is taken into account, the standard Alfven forbidden region will be disturbed somewhat. Under the effect of different convection fields, the convection orbits of hot ions (with energies of 1 keV and 3 keV at \( r_0 \), respectively) in the magnetic equatorial plane are shown in Figs. 26 - 31. The negative horizontal axis is Sunward and the vertical axis is dawnward. In these figures, panel (a) shows the standard Alfven layer computed from the first adiabatic invariant (magnetic moment) with a uniform, time independent convection field \( \langle E \rangle_t = E_{\text{max}} \epsilon_s \); the other panels show the drift orbits of the same ions under a time variable convection field \( E = \langle E \rangle_t \times [1 + (\epsilon_s/\epsilon_t) \cos (\omega t + \alpha)] \) starting at \( t = 0 \) from the same points as in panel (a); the initial phase parameter \( \alpha \) is taken to be 0, \( \pi/2 \), and \( \pi \) for panels (b), (c), and (d), respectively.

<table>
<thead>
<tr>
<th>( \epsilon_s/\epsilon_t )</th>
<th>( K_{r0} = 1 \text{ keV} )</th>
<th>( K_{r0} = 3 \text{ keV} )</th>
<th>( K_{r0} = 6.8 \text{ keV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>L_\text{N}</td>
<td>L_\text{A}</td>
<td>L_\text{F}</td>
</tr>
<tr>
<td>3.9</td>
<td>5.3</td>
<td>7.0</td>
<td>5.2</td>
</tr>
<tr>
<td>2</td>
<td>3.6</td>
<td>4.9</td>
<td>6.5</td>
</tr>
<tr>
<td>3</td>
<td>3.6</td>
<td>4.8</td>
<td>6.3</td>
</tr>
<tr>
<td>5</td>
<td>3.5</td>
<td>4.8</td>
<td>6.2</td>
</tr>
<tr>
<td>10</td>
<td>3.4</td>
<td>4.6</td>
<td>6.1</td>
</tr>
</tbody>
</table>

Table 1. The dimensions of the Alfven layer for different ion energies and different convection fields.
Fig. 26 (a). Alfvén layer with $e_y/e_t = 1, K = 1 \text{ keV}$.

Fig. 26 (b). Ion drift orbits with $e_y/e_t = 1, K = 1 \text{ keV}$, and $\alpha = 0$. 
Fig. 26 (c). Ion drift orbits with $\varepsilon_y/\varepsilon_z = 1$, $K = 1$ keV, and $\alpha = \pi/2$.

Fig. 26 (d). Ion drift orbits with $\varepsilon_y/\varepsilon_z = 1$, $K = 1$ keV, and $\alpha = \pi$. 
Fig. 27 (a). Alfven layer with $\varepsilon_y/\varepsilon_t = 1$, $K = 3$ keV.

Fig. 27 (b). Ion drift orbits with $\varepsilon_y/\varepsilon_t = 1$, $K = 3$ keV, and $\alpha = 0$. 
Fig. 27 (c). Ion drift orbits with $\varepsilon / \varepsilon_t = 1$, $K = 3$ keV, and $\alpha = \pi/2$.

Fig. 27 (d). Ion drift orbits with $\varepsilon / \varepsilon_t = 1$, $K = 3$ keV, and $\alpha = \pi$. 
Fig. 28 (a). Alfvén layer with $\epsilon_y/\epsilon_t = 2$, $K = 1$ keV.

Fig. 28 (b). Ion drift orbits with $\epsilon_y/\epsilon_t = 2$, $K = 1$ keV, and $\alpha = 0$. 
Fig. 28 (c). Ion drift orbits with $\varepsilon_0/\varepsilon_1 = 2, K = 1 \text{ keV},$ and $\alpha = \pi/2.$

Fig. 28 (d). Ion drift orbits with $\varepsilon_0/\varepsilon_1 = 2, K = 1 \text{ keV},$ and $\alpha = \pi.$
Fig. 29 (a). Alfvén layer with $\varepsilon_0/\varepsilon_t = 2$, $K = 3$ keV.

Fig. 29 (b). Ion drift orbits with $\varepsilon_0/\varepsilon_t = 2$, $K = 3$ keV, and $\alpha = 0$. 
Fig. 29 (c). Ion drift orbits with $\varepsilon_3/\varepsilon_1 = 2$, $K = 3$ keV, and $\alpha = \pi/2$.

Fig. 29 (d). Ion drift orbits with $\varepsilon_3/\varepsilon_1 = 2$, $K = 3$ keV, and $\alpha = \pi$. 
Fig. 30 (a). Alfen layer with $\epsilon_2/\epsilon_1 = 5$, $K = 1$ keV.

Fig. 30 (b). Ion drift orbits with $\epsilon_2/\epsilon_1 = 5$, $K = 1$ keV, and $\alpha = 0$. 
Fig. 30 (c). Ion drift orbits with $\varepsilon_2/\varepsilon_1 = 5$, $K = 1$ keV, and $\alpha = \pi/2$.

Fig. 30 (d). Ion drift orbits with $\varepsilon_2/\varepsilon_1 = 5$, $K = 1$ keV, and $\alpha = \pi$. 
Fig. 31 (a). Alfvén layer with $e_y/e_t = 5$, $K = 3$ keV.

Fig. 31 (b). Ion drift orbits with $e_y/e_t = 5$, $K = 3$ keV, and $\alpha = 0$. 
Fig. 31 (c). Ion drift orbits with $\varepsilon_p/\varepsilon_t = 5$, $K = 3$ keV, and $\alpha = \pi/2$.

Fig. 31 (d). Ion drift orbits with $\varepsilon_p/\varepsilon_t = 5$, $K = 3$ keV, and $\alpha = \pi$. 
Evidently the drift orbits for hot ions (≥ 1 keV) in a time dependent convection field are almost the same as those in a time-averaged convection field because their convection time scale is much larger than the oscillation period of the convection field. However, it is important to note that the time variable convection field allows some of the Sunward drifting hot ions to penetrate into the Alfven forbidden regions, and some of these stay inside and drift along nearly closed orbits for a long time, while others quickly escape from the forbidden regions. Nevertheless, even under the effect of a time dependent convection field, the convecting ions form a layer which is analogous to, but smaller than, the corresponding standard Alfven layer; we call this layer an "modified" Alfven layer. Another important difference between the standard and "modified" Alfven layers is that, for the standard layer, the convecting ions outside the layer and those with concentric orbits inside the layer can not exchange places; the "modified" layer, however, is penetrable because of the time dependent convection field.

6.3 The Hot Plasma Inner Boundary (Shielding Layer)

In this section let us discuss how the observed inner plasma boundary forms. The inbound PLS observations (see Fig. 8) show a complicated character of the plasma distribution. Evidently the hot ions adiabatically convect Sunward from the tail in to L = 6.8, where they have energy K ∼ 3 keV and density n = 0.1 cm⁻³. Between L = 6.8 and L = 5.3 (where the plasma shielding layer is located), the energy of the hot plasma drops from 3 keV to 1 keV while the density increases continuously up to 0.6 cm⁻³ in accordance with the adiabatic law; in addition, the intermediate (40 ∼ 300 eV) and warm (∼ 10 eV) ions appear suddenly inside L = 6.8 and their densities are roughly 0.4 cm⁻³. Inside L = 5.3, there is a sharp decrease in the density of both hot and intermediate ions, meanwhile, the warm plasma spreads without apparent change in to L = 4.6.

Figures 32 and 33 show the "modified" Alfven layers according to our convection
field model for 1 keV ions at $L = 5.3$ and for 3 keV ions at $L = 6.8$ with different convection field parameters $e_s$ and $e_t$ on the OTD magnetic equatorial plane. In these Figures, the negative x axis is in the direction of the projection of the planetary spin axis onto the magnetic equator, the negative y axis is in a direction analogous to the dawn-dusk direction in the Earth's magnetosphere, which is the direction of the convection field which has a shielding effect at the inner boundary ($L \sim 5$) as we described in chapter 5. The U-shaped solid line in each picture represents the trajectory

Fig. 32 (a). "Modified" Alfven layer for 3 keV ions (at $L = 6.8$) with the $e_s/e_t = 1$ convection field.
of Voyager 2 projected onto the magnetic equatorial plane along the OTD magnetic field lines (from Selesnick and McNutt, [1987]), and the shaded region indicates the inbound region where both hot and intermediate ions are observed. (The outbound region is neglected because of the Voyager 2 spacecraft charging event [see, e.g., Bridge et al. 1986; McNutt et al. 1987]).

It is clear that the region where both hot and intermediate ions were observed is, to a good approximation, located between the 3 keV (at $L = 6.8$) and 1 keV (at $L = 5.3$)

Fig. 32 (b). "Modified" Alfven layer for 3 keV ions (at $L = 6.8$) with the $\varepsilon_{z}/\varepsilon_{t} = 2$ convection field.
ions' "modified" Alfven layers in our model. As the ratio $\varepsilon_s/\varepsilon_t$ increases, the time averaged convection field increases, and the "modified" Alfven layers become smaller, consistent with the formulas for the standard Alfven layer (see equations 40 and 41). In addition, the smaller the ratio $\varepsilon_s/\varepsilon_t$ becomes, the better our model fits the observational data. We obtain the best fit for $\varepsilon_s/\varepsilon_t = 1$, consistent with the idea that dayside merging is the dominant source of the convection electric field.

The adiabatic compression of the hot ions can simply explain their temperature and

Fig. 32 (c). "Modified" Alfven layer for 3 keV ions (at $L = 6.8$) with the $\varepsilon_s/\varepsilon_t = 5$ convection field.
density increase as they convect in from the tail. The temperature decrease of the hot ions inside the 3 keV (at L = 6.8) ions' Alfvén layer can be generally explained by the existence of a sink of higher energy plasma (1 - 3 keV) and a source of relatively lower energy plasma. As we mentioned in the last section, under the influence of a time dependent convection field, some hot ions can penetrate into their own forbidden region and drift along nearly closed orbits for such a long time that they may be lost by resonant charge exchange reactions within the neutral hydrogen corona. Such a charge exchange

Fig. 33 (a). "Modified" Alfvén layer for 1 keV ions (at L = 5.3) with the $\varepsilon_g/\varepsilon_t = 1$ convection field.
reaction may produce protons and hydrogen atoms within a large energy range (≤ 3 keV).

The resulting higher energy hydrogen atoms (up to 3 keV) collide inelastically with the hydrogen corona to produce new protons with energies less than that of the colliding hydrogen atoms. In other words, these reactions (H+ - H and H - H) decrease the density of higher energy ions and increase the density of lower energy ions. In fact, as we will discuss in chapter 7, this is why, just inside the 3 keV ions' Alfvén layer, the

Fig. 33 (b). "Modified" Alfvén layer of 1 keV (at L = 5.3) with ε/ε_l = 2 convection field.
intermediate and warm components appear suddenly.

Therefore, even after the first Alfvén layer (corresponding to 3 keV ions) forms, the convection field is not strongly shielded because the flux tube content of hot ions does not change appreciably (see Plate A, Selesnick and McNutt [1987] and Fig. 8 above). However, where the 1 keV ions' Alfvén layer forms, very little hot plasma is observed within this layer because the penetrating hot ions (with energy of $\sim$ 1 keV) lose energy to become lower energy (< 1 keV) ions which no longer contribute to the hot plasma.

Fig. 33 (c). "Modified" Alfvén layer for 1 keV ions (at L = 5.3) with the $\epsilon_0/\epsilon_1 = 5$ convection field.
population; thus a sharp inner boundary forms for hot plasma. Consequently, the convection field experiences a considerable shielding as discussed previously. On the other hand, after the 3 keV ions' Alfven layer forms, the intermediate and warm components (including the low energy end of the hot component, ~ several 100 eV) are produced by the resonant charge-exchange and inelastic collision reactions. In Figure 8, we see that the density of the intermediate component is nearly linearly proportional to the density of the hot component outside the inner boundary of the hot ions, while the density of the warm component is nearly constant through the region in which they are observed. The reason is that the production of the intermediate and warm components depends upon the loss of the hot components; thus it is natural that the density of the intermediate component (with higher energy relative to the warm component) has a stronger relation to the hot plasma density than does the warm component.

The PLS experiment on Voyager 2 observed an inbound plasma boundary (at 1736 SCET, L = 5.3) at which the hot and intermediate proton densities decreased by an order of magnitude from an upstream value of 0.6 cm\(^{-3}\) in a time interval of less than the 48 second sampling period of the instrument, which implies that the thickness of the inbound plasma boundary \(\Delta L_{\text{in}}\) is less than 500 km. The corresponding outbound plasma boundary was less distinct in hot proton spectra than the inbound boundary (starting at about 1848 SCET, \(L \approx 4.6\), inside Miranda's minimum L shell, and ending at about 1890 SCET, \(L \approx 4.9\), i.e. \(\Delta L_{\text{out}} \leq 0.3 \text{ R}_\oplus\)); at this boundary the hot proton density increases to a peak value of 0.2 cm\(^{-3}\) (see Fig. 8 and Bridge et al. [1986]; McNutt et al. [1987]; Selesnick and McNutt [1987]; Sittler et al. [1987]).

Obviously, if \(\varepsilon_e >> \varepsilon_t\) or \(\varepsilon_t \equiv 0\), i.e., if the convection field is quasi-steady (as shown in Figure 14), as assumed by Selesnick and McNutt [1987], one can see that all particles within a large energy range drift inward by \(E \times B\) drift outside the boundary (following equipotentials), while inside the boundary the particle motions are dominated
by magnetic gradient/curvature drifts because of the strong convection field shielding. In this case the boundary layer would be very sharp. However, if \( \varepsilon_s = \varepsilon_t \), i.e., the convection field is quasi-sinusoidally time variable, as we have shown above, a considerable fraction of the convection field (\( \Phi_t \)) can penetrate the boundary that causes strong shielding of the quasi-static part (\( \Phi_s \)). Even in this case, the observed sharp boundary can still be explained because the convection electric field inside the boundary has almost a purely sinusoidal oscillation (see Figs. 17–21) and the oscillation period (\( \tau_r = 17.2 \text{ hr} \)) is much less than the convection and drift time scales. More importantly, the \( \mathbf{E} \times \mathbf{B} \) drift velocity is much less than the magnetic gradient/curvature drift velocity for \( \sim \) keV plasma near the inner boundary. It is easy to see that the thickness of the boundary layer depends on the magnitude of the quasi-sinusoidal convection field and the local magnetic field. Using the parameters quoted before in our model, we calculate that the maximum thickness of the boundary layer (consisting of only one content, say plasma of 1 keV) is of the order of \( \Delta L (L = 5) \sim v_{\mathbf{E} \times \mathbf{B}} \cdot \tau_r / 2 \sim 200 \text{ km} \).

Inside the shielding layer, because the steady component of the convection field is greatly reduced, the intermediate and warm ions have very little systematic convection like that of the hot ions outside. That is why the plasma data are not clearly distinguishable inside the layer (see Fig. 8). For the warm (10 eV) ions, the magnetic gradient/curvature drift velocity is much less than the \( \mathbf{E} \times \mathbf{B} \) drift velocity, so the orbits of those ions oscillate back and forth under the influence of the oscillating convection field. However, the orbits of intermediate (500 eV) ions are much more similar to those of the hot ions (see Figs. 34 and 35). In these figures, the orbits are cut off when the ions' residence time is much larger than the loss time scale for the resonant charge exchange reactions between the ions and the neutral hydrogen corona (see next chapter).
Fig. 34 (a). Warm (10eV) ion orbits with $e_0/e_t = 1$.

Fig. 34 (b). Warm (10eV) ion orbits with $e_0/e_t = 2$. 
Fig. 34 (c). Warm (10eV) ion orbits with \( \varepsilon_y / \varepsilon_t = 5 \).

Fig. 35 (a). Intermediate (500 eV) ion orbits with \( \varepsilon_y / \varepsilon_t = 1 \).
Fig. 35 (b). Intermediate (500 eV) ion orbits with $\epsilon_g/\epsilon_t = 2$.

Fig. 35 (c). Intermediate (500 eV) ion orbits with $\epsilon_g/\epsilon_t = 5$. 
Chapter 7

Plasma Sources and Auroral Pattern

On the basis of our analytic model of a time-dependent solar-wind-driven convection system at Uranus, we have offered an explanation, in chapter 6, of the low-energy plasma structures observed by Voyager 2 in the inner magnetosphere. A remaining question is where those ions, particularly the hot ions, come from. In this chapter, we first examine the local production of the intermediate and warm ion populations from the neutral hydrogen corona. Assuming conservation of the first adiabatic invariant (consistent with the PLS observations of the hot ions), we conclude that electron-impact-ionized protons from the neutral torus of the outermost satellite Oberon are the dominant source of the hot plasma. Finally, we offer a prediction of the pattern of the Uranian aurora associated with the shielding layer of our convection model.

7.1 Plasma Loss Time Scale

One of the basic assumptions of our convection model is that plasma losses are negligible during the convection, which requires that the plasma loss time scale \( \tau_{\text{loss}} \) is much larger than the time scale of the convection \( \tau_{\text{E\times B}} \) and of the magnetic gradient and curvature drift \( \tau_{\text{g,c}} \). However, as pointed out by Krimigis et al. [1986], there exists a potentially important loss mechanism for convecting protons through resonant charge-exchange interactions with a non-thermal corona of neutral atomic hydrogen, the existence of which was originally inferred by Shemansky and Smith [1986] from
International Ultraviolet Explorer (IUE) observations and confirmed by the observations of H Ly α in the Uranian system by the Voyager 2 Ultraviolet Spectrometer (UVS) [Broadfoot et al., 1986].

The loss time scale for convecting protons due to resonant charge exchange is

\[
\tau_{\text{loss}} = \left[ n_H \sigma_{H^+H^-} V_r \right]^{-1}
\]  \hspace{1cm} (42)

where \( V_r \) is the relative velocity between the collision partners (H\(^+\) and H), \( \sigma_{H^+H^-} \) denotes the charge exchange cross section for H\(^+\) - H interactions, and \( n_H \) denotes the number density of the neutral hydrogen corona.

From the H Lyman α intensities observed by Broadfoot et al. [1986] and the model of Shemansky and Smith [1986], the inferred particle density of the neutral hydrogen corona ranges from 10 \(-\) 1000 cm\(^{-3}\). However, as Herbert [1988] pointed out, the multiple scattering effect, and more importantly, solar H Lyman α backscattering from the ISM [Yelle and Sandel, 1986] followed by re-scattering off the corona into the UVS observations' line of sight, may result in the observed high intensities, and consequently the corona particle density \( n_H \) may have been estimated to be much higher than the actual values. Herbert [1988] redetermined an upper limit for \( n_H \) (5 cm\(^{-3}\) on the nightside and 50 cm\(^{-3}\) on the dayside) at \( L = 5 \) by using models of collisional reactions between the hot protons and the neutral atomic hydrogen corona to explain the observed hot plasma inner boundary. Here we use the nightside upper limit to calculate a minimum value for \( \tau_{\text{loss}} \). It is important to note that \( \tau_{\text{loss}} \) would be decreased by ten times if the dayside upper limit were used.

Two models have been proposed for the neutral hydrogen corona density profile: \( n_H \propto (L/L_H)^{-2} \) [Herbert, 1988] or \( n_H \propto \exp \left(-L/L_H\right) \) [Shemansky and Smith, 1986; Sittler et
Within the interesting ion energy range (10 eV ~ 3 keV), we have roughly (see, e.g., Hasted, 1964)

\[ \sigma_{H^+H} = 42.9 - 3.9 \ln(K) \times 10^{-16} \text{ cm}^2 \]  

(44)

where K denotes the plasma energy in unit of eV. Because the thermal energy of the protons is much larger than that of the neutral hydrogen atoms, \( V_r \) approximately equals the hot protons' thermal velocity

\[ V_r = V_{th} = 1.38 \sqrt{K(\text{eV})} \times 10^6 \text{ cm/s} \]  

(45)

Substituting (43), (44) and (45) into (42), we obtain the plasma loss timescale as a function of L and K. The ion loss timescale and magnetic gradient/curvature drift timescale are compared in Fig. 36. In this figure, solid curves denote the magnetic drift time scales, while dotted and dashed curves indicate the plasma loss timescales with Herbert's model and Shemansky and Smith's model of the neutral hydrogen corona, respectively. In the hot ion adiabatic convection region (L > 6.7), we use \( K = 3000 \ (6.8 \ / \ L)^{8/3} \) eV to calculate both time scales. In the region L < 6.8, since ion energy losses are no longer negligible, representative constant plasma energies are used to calculate the
Fig. 36a. The loss and magnetic drift time scales for adiabatically convecting hot ions.

Fig. 36b. The loss and magnetic drift time scales for hot ions (with constant energies 1 and 2 keV).
time scales. Clearly the ion loss timescale $\tau_{\text{loss}}$ is much larger than the ion magnetic drift time scale $\tau_{g,c}$ in the adiabatic convection region; however, $\tau_{\text{loss}}$ is comparable to, and even less than, $\tau_{g,c}$ in the region $L < 6.7$.

7.2 Sources of Intermediate and Warm Ions

The intermediate and warm ion components are observed only inside $L = 6.8$ (1650 SCET) on the inbound leg and inside $L = 5.7$ (1935 SCET) on the outbound leg, and disappear inside $L \sim 4.5$ both inbound and outbound (see Fig. 8). The energy of these two components ranges from about 10 eV to 500 eV and their number density ranges from about 0.02 to 0.5 cm$^{-3}$. The rather diffuse distributions in both energy and space indicate that these ions are not convected from an external source in the tail, but are probably produced within the region where they were observed. McNutt et al. [1987] and
Selesnick and McNutt [1987] discussed two possible sources for these ions: electron-impact-ionization of the neutral hydrogen corona, and ion outflow from the ionosphere. There is a third possible source for these ions: electron-impact-ionization of the neutral tori of the inner satellites. However none of these hypothetical sources can explain the following two observed phenomena: (1) both the intermediate and warm components appear suddenly at the point where the hot ion adiabatic convection stops, and (2) within the region where they exist, the ions' energy ranges from 10 eV to several 100 eV everywhere.

From Fig. 36, it is clear that the loss timescale $\tau_{\text{loss}}$ for those hot ions that adiabatically convect Sunward from the tail is much larger than their magnetic drift time scale $\tau_{\text{g,c}}$, whereas their $E \times B$ drift time scale $\tau_{\text{ExB}}$ is smaller than $\tau_{\text{g,c}}$ (see Fig. 25), so that losses are indeed negligible as we assumed in our convection model. However, for $L < 6.8$, the ion loss timescale $\tau_{\text{loss}}$ is comparable to and even smaller than the ion magnetic drift time scale $\tau_{\text{g,c}}$, so that charge-exchange reactions between the ions and the neutral hydrogen atoms should be significant in that region. Such reactions, together with inelastic collisions between the energetic hydrogen atoms resulting from the charge-exchange reactions, lead to the energy degradation of the hot ions and simultaneously the appearance of the intermediate and warm ions.

As we mentioned in the last chapter, when the "modified" Alfven layer of the highest energy ions (3 keV at $L = 6.8$) forms, not all of the 3 keV ions are excluded from the "forbidden" region, as they are in a steady convection field. Under the influence of a time-dependent convection field, some of the 3 keV ions will penetrate their own Alfven layer and drift along nearly closed orbits for a long time. From Fig. 36, we can see that the magnetic drift timescale for 3 keV ions at $L = 6.8$ is about 515 hr, while the loss timescale is 843 hr if we use Shemansky and Smith's model or 1070 hr if Herbert's model is used for the neutral hydrogen corona. Obviously, if the 3 keV ions drift around
these nearly closed orbits two or more times, they will be significantly depleted by resonant charge exchange reactions. From these elastic collisions, high-energy hydrogen atoms (≤ 3 keV) and low energy (∼ eV) protons are produced. It is clear that the time dependent convection field causes the hot plasma ions to form a trapped ring current population, and this ring current population is sufficiently long-lived to undergo charge exchange and inelastic collisions with the local distribution of the neutral hydrogen corona. We propose that such collisions result in the sudden appearance of the intermediate and warm ion components just inside the first trapped ring current loop (the 3 keV ions' "modified" Alfvén layer).

Inside the hot plasma boundary (corresponding to the "modified" Alfvén layer of 1 keV ions), newly produced protons cannot have energy above 1 keV. Moreover, the loss timescale τ_{\text{loss}} becomes smaller than the magnetic drift time scale τ_{g,c} when the ion energy decreases to 1 keV, and the difference increases with decreasing distance L (see Fig. 36); therefore, there is a sharp drop in the hot ion density and consequently, in the flux-tube content there.

Because both the intermediate and warm ion populations are produced at the expense of the hot ion energy, the density of the intermediate component depends more sensitively upon the density of the hot plasma; this explains why the intermediate component density is linearly proportional to the hot plasma density until the hot plasma has a sharp boundary at r = r_0. Because the quasi-static convection field is greatly shielded by the hot and intermediate plasma boundaries, convection in the region occupied by the intermediate and warm ions is weak and variable, and the densities and temperatures of these two components have no clear dependence on L.

7.3 Sources of the Hot Plasma

The observed density and temperature gradients of the hot protons strongly suggest
that they are convected inwards from the magnetotail. The existence of solar-wind driven magnetospheric convection implies that the solar wind is a potential source of magnetospheric plasma, although not for the ion populations (Fig. 8) considered here (solar-wind protons convected from the vicinity of the magnetopause or the magnetotail would be heated by adiabatic compression to energies in excess of 10 keV in this distance range, well above the energy range of the PLS instrument). At higher energies, however, the observations of Krimigis et al. [1986] seem to rule out a significant solar-wind source for magnetospheric plasma because of the absence of high-energy $\alpha$ particles. Another possible source of the hot plasma may be photo- or electric-impact-ionization of the neutral hydrogen corona in the magnetotail [see Bridge et al., 1986; McNutt et al., 1987; Selesnick and McNutt, 1987; Sittler et al., 1987; Herbert, 1988]. The most difficult point with this source is that such ionizations should take place everywhere, and should result in a diffuse hot ion energy distribution, which is contrary to the observations (see Fig. 8).

If the hot ions derive from a remote source by pick-up and subsequent adiabatic compression, the source must be located between 20 and 25 $R_u$ from Uranus, near the orbit of the outermost satellite Oberon. This indicates that the satellite Oberon may be the dominant source of the hot ion component if there exists a suitable ionization process and a source of hydrogen and hydrogen compounds.

Many observations and theoretical studies have shown that there is an abundance of water ice ($H_2O$) and methane ice ($CH_4$) on the surface of Oberon [see, e.g., Anderson et al., 1987; Johnson et al., 1987; Nelson et al., 1987; Thompson et al., 1987; Lanzerotti et al., 1987]. The high energy ions ($\geq 28$ keV) observed by the Low-Energy Charged Particle (LECP) instrument on the Voyager 2 spacecraft will impact these icy surfaces and produce considerable sputtering of water and methane ices and related molecules such as $OH$, $H$, $O$, and $CH$ to form a neutral torus around Oberon. The average production rate is
estimated to be about $10^5 \sim 10^6$ cm$^{-2}$ s$^{-1}$ [Harrison and Schoen, 1967; Eviatar and Richardson, 1986; Lanzerotti et al., 1987]. Photoionization and electron-impact ionization of the sputtered molecules is provided by solar UV photons and high energy electrons respectively. The ionization products, including H$^+$ and heavy ions such as OH$^+$, O$^+$, CH$_4^+$ and H$_2$O$^+$, then convect inwards from the source location (around the orbit of Oberon) with magnetic moments corresponding to the pick-up energy - the kinetic energy of corotation relative to the satellite orbital velocity.

From a steady-state model of the heavy neutral tori and the thermal heavy-ion and proton plasmas in the inner Uranian magnetosphere [Cheng, 1987], it is expected that the ratio of heavy ions to protons is about $10^{-4}$, which is in agreement with the measurements by the LECP [Krimigis et al., 1986]. In fact, there are many ionization and recombination processes for both protons and heavy ions in Uranian magnetosphere. Even though the total ionization rate of protons is nearly the same as that of heavy ions, the total recombination rate of protons is tremendously smaller than that of the heavy ions; for instance, the electron impact recombination rate is about $10^{-14}$ s$^{-1}$ for protons and $10^{-8} \sim 10^{-9}$ s$^{-1}$ for heavy ions (see Cheng [1987] and references therein). This is why both observations [Krimigis et al., 1986] and theoretical estimates [e.g., Cheng, 1987] give a heavy-ion to proton ratio of about $10^{-4}$. In this thesis we need not discuss the details of the sputtering, ionization and recombination processes. Instead, we are interested in showing, by assumption of conservation of the first adiabatic invariant, that the population of H$^+$ around Oberon, produced principally by electron-impact ionization, may be the major source of the hot ion component observed by the PLS experiment in the inner magnetosphere.

Because of the very small ratio of heavy ions to protons, we assume that all protons are electron-impact ionized directly from the neutral hydrogen co-orbiting with Oberon. Just after ionization, the pick-up energy of the protons is the same as the kinetic energy of
the neutral hydrogen atoms before ionization, relative to the (approximately corotating) plasma reference frame. The protons' magnetic moment after pick-up is then \( \mu_{p,u} = \frac{W_{p,u}}{B_{p,u}} = \frac{1}{2} m_p |\mathbf{v}_{p,u} \times \mathbf{B}_{p,u}|^2 / B_{p,u}^3 \), where \( m_p \) denotes the proton mass, \( \mathbf{v}_{p,u} \) is the velocity of protons when picked up, which is equivalent to the velocity of Oberon relative to the corotating magnetosphere, and \( \mathbf{B}_{p,u} \) indicates Uranus' magnetic field strength at the place of pick-up.

Oberon is in synchronous rotation with orbital period \( \tau_{\text{Oberon}} = 323.1 \) hr in Uranus equatorial plane, and the orbit is nearly circular with radius \( r_{\text{Oberon}} = 23 \) R\(_u\) [Stone and Miner, 1986]. Since Uranus' spin vector is parallel to Oberon's orbital rotation axis (see Fig. 1c), the speed of Oberon (and its neutral torus) relative to the plasma reference frame (assumed to be corotating with Uranus with \( \tau_r = 17.2 \) hr) is

\[
V_{p,u} = \frac{2\pi}{3600} \left[ \frac{1}{\tau_{\text{r}}} - \frac{1}{\tau_{\text{Oberon}}} \right] r_{\text{Oberon}} = 6 \times 10^4 \text{ m/s}
\]  

(46)

Because of the large tilt (60°) of the magnetic dipole axis relative to the rotation axis, all the satellites sweep across broad ranges of L values and magnetic latitude \( \lambda \) (between -60° and +60°) as Uranus rotates. Figure 37 presents curves of magnetic latitude (degree) versus dipole L for the major satellites of Uranus. Here the offset of the dipole from the center of the planet (0.3 R\(_u\)) is neglected. While the L value of Oberon varies through a remarkably large range (from about 23 to 90), the sine of the angle between Oberon's orbital velocity \( \mathbf{v}_{p,u} \) and the OTD field \( \mathbf{B}_{p,u} \) also varies between 0.5 and 1, corresponding to the location of Oberon at OTD magnetic latitude \( \lambda = 0° \) and 60°, respectively.
From the Appendix B we have

\[ V_{p,u\perp} = \left| \frac{V_{p,u} \times B_{p,u}}{B_{p,u}} \right| = V_{p,u} F(\lambda) \]  \hspace{1cm} (47)

where

\[ F(\lambda) = \frac{2}{\sqrt{1 + 3 \sin^2(\lambda)}} \left\{ \frac{\sin^2 \lambda}{3} + \frac{\cos^2 \lambda}{12} (3 - \tan^2 \lambda) \right\}^2 \]

\[ + \left[ \frac{2 \sin^3 \lambda}{3 \cos \lambda} + \frac{\cos \lambda \sin \lambda}{6} (3 - \tan^2 \lambda) \right]^2 + \frac{\sin^2 \lambda}{4} (3 - \tan^2 \lambda) \right\}^{1/2} \]  \hspace{1cm} (48)
Therefore the magnetic moment of the pick-up protons $\mu_{p,u}$ can be written as

$$
\mu_{p,u} = \frac{W_{p,u} \perp}{B_{p,u}} = \frac{W_{p,u}}{B(r = 23 \text{ Ru})} \frac{F^2(\lambda)}{\sqrt{1 + 3 \sin^2(\lambda)}}
$$

$$
= 17 \times 10^{-10} \frac{F^2(\lambda)}{\sqrt{1 + 3 \sin^2(\lambda)}} \text{ (J/T)} \quad (49)
$$

According to the observations of the PLS experiment (Figs. 8 and 7b), the hot ion energy is about 3 keV at $L = 6.8$ and $\lambda = 32^\circ$. Therefore the magnetic moment of the hot ions $\mu_{\text{hot}}$ is

$$
\mu_{\text{hot}} = \chi \frac{W(=3 \text{ keV})}{B(L=6.8, \lambda=32^\circ)} = \chi \cdot 16.3 \times 10^{-10} \text{ (J/T)} \quad (50)
$$

where $\chi$ depends on the ratio of the perpendicular energy to the total energy of the hot ions observed at $L = 6.8$.

In Figure 38, the solid curve presents the magnetic moment (in units of $10^{-10}$ J/T) of protons ionized from the neutral torus of Oberon, versus the location (expressed by the OTD magnetic latitude $\lambda$) where the ionization takes place. The dotted lines denote the magnetic moment of the observed hot ions when the ratio $\chi$ is taken to be 0.5 and 1 respectively.

Remember that in our model, the plasma pressure is assumed to be isotropic as the result of pitch angle scattering, while the first two adiabatic invariants $\mu = W_{\perp}/B$, and $J = \int p_\parallel ds$ are not conserved, but rather the plasma energy invariant $\lambda$ and the flux-tube content $\eta$. However, Southwood and Kivelson [1975] proved that in a dipole magnetic field, the descriptions of hydromagnetic flow are similar for the two different approaches.
In fact, Selesnick and McNutt [1987] pointed out that the two approaches are indistinguishable in the data observed by Voyager 2 (Fig. 8). Moreover, if the parallel energy does not exceed the perpendicular energy (i.e., if $\chi > 0.5$), the kinetic energy depends on $L$ through an exponent between $8/3$ and $3$, the extremes corresponding to the isotropic case and the equatorially mirroring case respectively. From the discussion above and Fig. 38 one can conclude that the source of the hot ions observed by the PLS experiment is those protons that are ionized and picked up at high magnetic latitudes. However, this does not mean that only part of the Oberon neutral torus is a potential source of hot ions; every segment of the neutral torus has the same probability of being located at high magnetic latitude because of Uranus' spin. From the trajectory of the Voyager 2 spacecraft (see Fig. 7b and 7c) and the plasma convection orbits (Fig. 26 to 31)
as well as the figure in the Appendix.B, we can see that the observed hot ions in Fig 8 come from a source located within a cylinder centered at the dipole with a rough radius of 15 $R_u$, since only ions from that region can convect into the inner magnetotosphere up to $L = 6.8$, and the ions picked up from the neutral torus at high magnetic latitudes are located just at the right place. Therefore we believe that the neutral torus of the satellite Oberon is the major source of the hot ions observed in the inner magnetosphere.

7.4 Low-Latitude Boundary of the Uranian Aurora

During the Voyager 2 encounter with Uranus, the resolution of the UVS experiment was sufficient to identify localized aurora. Because of the bright dayside electroglow, the aurora on the darkside is much clearer than that on the sunlit side. The observations clearly show a nightside aurora within a region of 15° ~ 20° in diameter, apparently centered on the mid-latitude OTD magnetic pole. The possible existence of a dayside aurora is not excluded by the observations.

Both observational and theoretical studies of Earth's magnetosphere [e.g. Iijima and Potemra, 1976 a, b, 1978; Wolf et al., 1982] indicate that field-aligned currents are concentrated in two principal areas which encircle the geomagnetic pole. The so-called Region-1 Birkeland currents are located near the poleward boundary of the auroral oval, and the Region-2 Birkeland currents are located near the equatorward boundary of the auroral oval. Thus the location of the Region-2 Birkeland currents basically gives the lower latitude boundary of the auroral distribution.

Further study [e.g. Ashour-Abdalla and Thorne, 1978; Hill, 1982 and references therein] have shown that, as plasma is convected inward from the tail, its rate of transport diminishes, whereas the rate of removal is expected to increase. When the timescale for precipitation into the atmosphere becomes shorter than or comparable to that for convective transport, a sharp precipitation boundary is established. The anticipated
location of this precipitation boundary agrees well with observations of the inner edge of the plasma sheet. The precipitation loss of ring current particles owing to wave-particle interactions not only reduces the effectiveness of the shielding process around the inner edge of the plasma sheet, but also produces the diffuse aurora and the concomitant widespread heating of the thermosphere. Thus the location of the plasma sheet inner edge should map to the equatorial boundary of the diffuse electron aurora.

At present there is no comprehensive model of the Uranian aurora based on observations. We don't yet know what processes are involved in the Uranian aurora; for instance, are there diffuse aurora at Uranus, or discrete auroral arcs, or both? Also, we don't know the detailed spatial distribution of the Uranian aurora. Nevertheless, we may predict the equatorial boundary of the Uranian aurora by analogy with the Earth's magnetosphere.

In our simplified model, the Region-2 Birkeland currents are located at the inner edge of the plasma sheet on the magnetic equatorial plane. According to the above discussion for the Earth's magnetosphere, the low-latitude boundary of the Uranian aurora can be obtained in our model either from the field-aligned current location or from the plasma sheet inner edge location. Mapping this location (approximately a circle at \( L = 5 \)) along the dipole magnetic field lines indicates that the low-latitude auroral boundary is located at a constant OTD latitude \( \lambda = \cos^{-1} 0.2 = 63.4^\circ \). Because of the large angle between the OTD moment and the planetary spin axis, this boundary should be transformed into geographical latitude/longitude coordinates for comparison with observational data. This is shown in Figures 39a and 39b for the nightside and dayside auroral zones, respectively.
Fig. 39a. Lower boundary of nightside aurora.
Fig. 39b. Lower boundary of dayside aurora.
Chapter 8

Discussion and Conclusions

For most planets in the solar system the rotation axis is almost perpendicular to the ecliptic plane and nearly aligned with the magnetic dipole axis. However, for Uranus the rotation axis is nearly in the ecliptic plane, which means that for Uranus the angle between the rotation axis and the solar wind flow will change $2\pi$ during a Uranus year, while for all other planets the angle is limited to values near $\pi/2$. In addition the Uranus magnetic dipole axis and the spin axis are separated by a surprisingly large angle. These factors make the study of magnetospheric convection for Uranus more complicated than for other planets.

At Jupiter and Saturn, the influence of planetary rotation is very strong and dominates the plasma motion throughout most of the magnetosphere. In the Earth's magnetosphere, both the solar-wind driven convection field and the corotational electric field are of importance to the plasma motion; in the inner magnetosphere the superposition of these electric fields leads to the formation of the plasmasphere within which cold plasma is trapped by corotation and outside of which the plasma moves through the magnetosphere under the influence of the convection field. Because of the particular characteristics and unique orientation of the rotation and dipole axes at Uranus, plasma motion in Uranus' magnetosphere is more complicated and always time-dependent. At the current epoch, the rotation axis is nearly aligned with the solar wind flow, and the main influence of corotation is merely to twist the convection
trajectories throughout the magnetosphere, without forming a plasmasphere. A quarter Uranus year from now, the angle $\theta_s$ (between the solar wind flow and the planetary rotation axis) will increase from near zero to ninety degrees. The influence of rotation will become gradually more important, and the plasmasphere will become more and more evident. However, unlike in the Earth's plasmasphere, the resulting plasma orbits are not closed because of the 60° dipole tilt angle. Thus plasma motion in Uranus' magnetosphere is quite different from that at Jupiter and Saturn, and is instead analogous to that at Earth in some aspects at some times.

When Voyager 2 encountered the Uranian system, the planetary rotation axis was, fortunately, nearly aligned with the solar-wind flow, which greatly simplifies the Uranian magnetospheric convection problem at this epoch. In this case, we may choose a corotating coordinate system to deal with the convection problem, in which the magnetic field structure, the solar-wind flow, and the convection system boundary locations (polar cap and inner plasma boundary) are time-independent.

In order to explain the plasma data observed by the PLS experiment, we have developed a time dependent convection model for Uranus' magnetosphere. We assume that Uranus' magnetospheric convection is driven by the solar wind. As in the terrestrial magnetosphere, solar wind-magnetosphere coupling produces an electric potential drop over the polar caps. For ideal MHD, the electric potential distribution along the edge of the polar caps can be mapped into the magnetic equatorial plane by tracing along the magnetic field lines. In our model, we deal with plasma convection and associated shielding effects in the magnetic equatorial plane. Because of the unusual orientation of Uranus' rotation and magnetic dipole axes, the solar-wind-driven convection field oscillates in time with the planetary rotation period of 17.2 hr.

To simplify the convection model and obtain analytic solutions, some additional assumptions are necessary. First, the plasma pressure is assumed to be isotropic and the
convection adiabatic with $\gamma = 5/3$. With this assumption, we conserve flux tube content $\eta$ and ion energy invariant $\lambda$, instead of the first and second adiabatic invariants $\mu$ and $J$. The plasma density $n$ and energy $W$ are proportional to $L^{-4}$ and $L^{-8/3}$ respectively, which agrees with the observations very well. Secondly, we assume that the electric potential is constant along the magnetic field lines, so that we need only solve the convection electric field distribution and associated plasma convection pattern on the magnetic dipole equatorial plane, and by mapping along the magnetic field lines, the field distribution and the convection pattern in the whole magnetosphere are easily obtained. The third assumption is that the ionospheric conductivities are uniform, and the same in the northern and southern hemispheres. The fourth assumption is a small-perturbation linear approximation: the inner plasma boundary is located at $r = r_0 + \delta r$, where $r_0$ is specified at $L = 5$ according to the PLS observations and $\delta r << r_0$. This approximation indicates that the $E \times B$ drift speed is much smaller than the magnetic gradient/curvature drift speed for the hot ions near the inner boundary, which is true in the Uranian magnetosphere.

With these assumptions, the main conclusions are described as follows.

(1) Three timescales, the convection timescale $\tau_{E\times B}$, the shielding timescale $\tau_{sh}$, and the oscillation period of the convection field $\tau_r$, determine whether the magnetospheric convection can be approximately dealt with as steady or not. If $\tau_{sh} << \tau_r$ and $\tau_r << \tau_{E\times B}$, a steady-state convection model would be a very good approximation. At Uranus, the second condition is satisfied, so the plasma convection pattern can be approximately obtained by using a time-averaged convection field. However, $\tau_{sh}$ is larger than $\tau_r$ at Uranus, so the steady model for convection-field shielding fails.

(2) For a periodic oscillation of the convection field, the shielding effect, if there is a shielding layer, depends not only on the ratio of the ionospheric conductivity to the "effective" conductivity of the magnetosphere $\Sigma/q\eta$, as in the case of the steady model, but also on the ratio of the shielding timescale to the period of the oscillating field $\tau_{sh}/\tau_r$. At
Uranus, since $\tau_{sh}$ is longer than $\tau_r$, even though the ratio $\Sigma/q\eta$ is very small, the sinusoidal part of the convection field penetrates through the ring-current shielding layer without significant shielding.

(3). Because the time-varying part of the convection field is not significantly shielded at the shielding layer, a fraction of the convecting plasma is able to penetrate the shielding layer to form a trapped ring-current population. This is a very important feature of the time dependent convection model; in a steady model, plasma either convects outside the shielding layer, or is trapped inside the layer.

(4). With our time-dependent convection model, the low-energy plasma structures observed by the PLS experiment can be well explained. The hot ions are able to penetrate the shielding layer to form a trapped ring-current population that is sufficiently long-lived to undergo charge-exchange and inelastic collisions with the local neutral hydrogen corona, resulting in the energy degradation of the hot ions and the simultaneous appearance of the intermediate and warm ions evident in PLS results in the region between $L = 5$ and $L = 7$ (see Fig. 8).

(5). Based on the conservation of the first adiabatic invariant and the restriction that the ion perpendicular energy is larger than the parallel energy (with this restriction, the approach based on conservation of $\mu$ and $J$ and the approach based on conservation of $\eta$ and $\lambda$ are similar in theory and indistinguishable in the data observed by PLS [Southwood and Kivelson, 1975; Selesnick and McNutt, 1987]), our analysis indicates that the hot ions are convected Sunward from a source in the neutral torus of Oberon, where the protons are produced by electron-impact and photo-ionozation, and picked up with the local-corotation energy.

In our model, the imposed convection field at the outer boundary ($r = r_u$) is specified to have a special form (3). In fact, analytic solutions of our model can be obtained for any reasonable form of $\Phi(r_u, \phi, t)$ because any such function of $\phi$ and $t$ can generally be
expanded in terms proportional to $e^{ik\phi}$ and $e^{i\omega_j t}$, where $k$ and $j$ are integers and $\omega_j = j \omega_0$. For each $k$ and $j$, the solution has the form of (21) or (30), and the total solution is a summation over $k$ and $j$. Some of the assumptions and simplifications in this analytic model can be eliminated or replaced by more realistic physical conditions if one wishes to pursue this study by numerical methods. Of course, a more comprehensive understanding of the plasma convection system in the Uranian magnetosphere requires more study in both theory and observation. For example, the formation of the warm and intermediate ion populations by charge-exchange between the hot ions and the neutral hydrogen corona requires detailed modeling.
Appendix A

In the ionosphere, we can use spherical polar coordinates \((\theta, \varphi)\), in which the Laplace equation for the convection electric potential \(\Phi\) is

\[
\sin \theta \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial \Phi}{\partial \theta} \right) + \frac{\partial^2 \Phi}{\partial \varphi^2} = 0 \tag{A-1}
\]

On the other hand, we use polar coordinates \((r, \phi)\) in the equatorial plane.

Fig. A-1. The x-y plane is the equatorial plane, while the ionosphere is located on the surface of Uranus whose radius is normalized to 1. In the ionosphere a spherical coordinate system \((\theta, \varphi)\) is used, while a polar coordinate system \((r, \phi)\) is used in the equatorial plane. The dipole field lines connect the two coordinate systems.
These two coordinate systems are connected by dipole field lines

\[
\begin{align*}
1 &= r \sin^2 \theta \\
\varphi &= \phi 
\end{align*}
\] (A-2)

where the radial distance \( r \) is normalized in unit of Uranus' radius \( R_u \).

Thus

\[
\begin{align*}
\mathrm{d}r &= -2 r \sqrt{r-1} \, \mathrm{d}\theta \\
\mathrm{d}\phi &= \mathrm{d}\phi
\end{align*}
\] (A-3)

Since the convection field potential is constant along the magnetic dipole field lines, we can map the Laplace equation for the convection potential in the ionosphere (A-1) down to the equatorial plane along the dipole field lines. From equations (A-2) and (A-3), we have

\[
\begin{align*}
\sin \vartheta \frac{\partial}{\partial \theta} &= \sin \theta \frac{\partial}{\partial r} \cdot \frac{\mathrm{d}r}{\mathrm{d}\theta} = -2 \sqrt{r^2 - r} \frac{\partial}{\partial r} \\
\frac{\partial}{\partial \phi} &= \frac{\partial}{\partial \phi}
\end{align*}
\] (A-4)

Combining (A-1) and (A-4), we obtain the equation of the convection potential \( \Phi \) in the equatorial plane

\[
4 \sqrt{r^2 - r} \frac{\partial}{\partial r} \left( \sqrt{r^2 - r} \frac{\partial \Phi}{\partial r} \right) + \frac{\partial^2 \Phi}{\partial \phi^2} = 0
\] (A-5)
This equation can be analytically solved by separating variables and using the Sturm-Liouville theory. However, since the region in which we are interested in this study is \( r \geq 5 R_u \), we can make a reasonable approximation of equation (A-9) by setting \( (r^2 - r)^{1/2} \sim r \).

Finally we have

\[
4r \frac{\partial}{\partial r} (r \frac{\partial \Phi}{\partial r}) + \frac{\partial^2 \Phi}{\partial \phi^2} = 0
\]  

(A-6)

which is equation (12) in Chapter 4.
Appendix B

In Chapter 7, when discussing the hot plasma sources, it is important to determine the magnetic moment $\mu$ of the protons that are ionized and picked up at the orbit of the satellite Oberon. When they are ionized, the protons have the corotation energy $(1/2 m_p V^2_{p,u})$ relative to the Uranian magnetosphere, as calculated in Chapter 7. In this Appendix, the perpendicular component of the energy (i.e. $V_{p,u,\perp}$) is to be determined.

Fig. B-1. Configuration of the OTD equatorial plane ($\Pi$-plane) and the satellite Oberon's orbit plane ($\Sigma$-plane).
In Fig. B-1, the $\Pi$ plane represents the magnetic equatorial plane in the Uranian magnetosphere, the $\Sigma$ plane represents Oberon's orbit plane, and the two planes intersect at a 60° angle. For simplicity, we neglect the offset of the dipole here. On the $\Sigma$ plane, the location of Oberon is specified by the radial distance $r$ and the azimuthal angle $\phi$. In the corotating reference frame, we set up a cartesian coordinate system $x$-$y$-$z$, where the $x$-$y$ plane is the $\Pi$ plane and the $z$ axis is aligned with the magnetic dipole moment.

The three components of $\mathbf{V}_{p,u}$ in $x$-$y$-$z$ coordinates are

$$
\begin{align*}
V_{p,u,x} &= -V_{p,u} \sin \phi \\
V_{p,u,y} &= -V_{p,u} \cos \phi \cdot \cos 60^\circ \\
V_{p,u,z} &= -V_{p,u} \cos \phi \cdot \sin 60^\circ
\end{align*}
\quad \text{(B-1)}
$$

The velocity $\mathbf{V}_{p,u}$ can also be expressed in spherical coordinates ($r, \lambda, \psi$) by using the following coordinate transformation:

$$
\begin{bmatrix}
  \mathbf{r} \\
  -\lambda \\
  \psi
\end{bmatrix} =
\begin{bmatrix}
  \cos \lambda \cos \psi & \cos \lambda \sin \psi & -\sin \lambda \\
  -\sin \lambda \cos \psi & -\sin \lambda \sin \psi & -\cos \lambda \\
  -\sin \psi & \cos \psi & 0
\end{bmatrix}
\begin{bmatrix}
  \mathbf{x} \\
  \mathbf{y} \\
  \mathbf{z}
\end{bmatrix}
\quad \text{(B-2)}
$$

where $\lambda$ is the magnetic latitude, and $\psi$ is the azimuthal angle on the magnetic equatorial plane. Thus we have
\[
\begin{align*}
V_{p,u,r} &= V_{p,u}( -\sin\phi \cos\lambda \cos\psi + \cos 60^\circ \cos\phi \cos\lambda \sin\psi - \sin 60^\circ \cos\phi \sin\lambda ) \\
V_{p,u,\lambda} &= V_{p,u}( \sin\phi \sin\lambda \cos\psi - \cos 60^\circ \cos\phi \sin\lambda \sin\psi - \sin 60^\circ \cos\phi \cos\lambda ) \\
V_{p,u,\psi} &= V_{p,u}( \sin\phi \sin\psi + \cos 60^\circ \cos\phi \cos\psi ) 
\end{align*}
\]
(B-3)

Because the location of the satellite can be determined by magnetic latitude \(\lambda\) alone, the velocity \(V_{p,u}\) can be also determined by \(\lambda\) alone. In fact, from Fig.B-1, we have the following relations:

\[
\begin{align*}
\cos\phi &= \cos\lambda \cos\psi \\
\sin\lambda &= \sin\phi \sin 60^\circ
\end{align*}
\]
(B-4a)

and

\[
\begin{align*}
\cos\psi &= \frac{\sqrt{3}}{3} \sqrt{3 - \tan^2 \lambda} \\
\sin\psi &= \frac{\sqrt{3}}{3} \tan \lambda
\end{align*}
\]
(B-4b)

Using (B-4a,b) we can simplify (B-3) to

\[
\begin{align*}
V_{p,u,r} &= 0 \\
V_{p,u,\lambda} &= \frac{1}{2} \sqrt{3 - \tan^2 \lambda} \\
V_{p,u,\psi} &= \frac{2 \sin^2 \lambda}{3 \cos \lambda} + \frac{1}{6} \cos \lambda (3 - \tan^2 \lambda)
\end{align*}
\]
(B-5)
On the other hand, the dipole magnetic field in \((r, \lambda, \psi)\) coordinates is

\[
\begin{align*}
B_r &= B_p \sin \lambda \\
B_\lambda &= -\frac{1}{2} B_p \cos \lambda \\
B_\psi &= 0
\end{align*}
\]  

(B-6)

with \(B_p = 2 M_d / r^3\), and \(B = 0.5 B_p (1 + 3 \sin^2 \lambda)^{1/2}\).

Finally, the perpendicular component of \(V_{p,c}\) is easily obtained by

\[
V_{p,c, \perp} = \frac{|V_{p,c} \times B|}{B} = V_{p,c} F(\lambda)
\]  

(B-7)

where the function \(F(\lambda)\) is given by (48) in Chapter 7.
BIBLIOGRAPHY


Iijima, T., and T.A. Potemra, Field-aligned currents in the dayside cusp observed by


Nishida, A., Formation of plasmapause, or magnetospheric plasma knee, by the combined action of magnetospheric convection and plasma escape from the tail, *J.


Spiro, R. W., et al., Quantitative simulation of a magnetospheric substorm, 3.


Wolf, R. A., et al., Coupling between the solar wind and the earth's magnetosphere: Summary comments, in *The proceedings of the Chapman conference on*
