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Windshear estimation along the trajectory of an aircraft

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WINDSHEAR ESTIMATION
ALONG THE TRAJECTORY OF AN AIRCRAFT

by

CHING-YAW TZENG

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IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF

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APPROVED, THESIS COMMITTEE

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Abstract

Windshear Estimation along the Trajectory of an Aircraft

by

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The application of the sequential gradient-restoration algorithm (SGRA) to the estimation of the windshear along the trajectory of an aircraft is studied. Based on the measured trajectory data obtained from the digital flight data recorder (DFDR) of Flight Delta 191 (August 2, 1985, Dallas-Fort Worth International Airport), a nonlinear least-square problem is formulated. The performance index being minimized measures the deviation of the experimental trajectory (the altitude, the relative velocity, and the pitch attitude angle) from the computed trajectory, obtained by integrating the equations of motion of an aircraft in a vertical plane. Since the thrust and the aerodynamic forces enter directly in this dynamic formulation, a clear picture of the forces acting on the aircraft can be seen. This leads to a good understanding of the behavior of the aircraft during the windshear encounter.

The angle of attack is treated as a control, and the power setting is regarded as a known input. By assuming that the manufacturer-supplied aerodynamic and thrust data are dependable, the dynamically estimated vertical wind shows reasonable agreement with that obtained with the kinematic approach. However, the results obtained for the horizontal wind are less satisfactory. Upon modifying the manufacturer-supplied thrust and aerodynamic data with unknown multiplicative factors, a better agreement between the measured and computed trajectory can be achieved. As a consequence, the estimated winds exhibit better accuracy. The inclusion of penalty terms in the performance index
being minimized forces the values of the unknown multiplicative factors to be close to unity. The estimation of these factors is important, because it might explain some unusual effects, such as the presence of rain.

Upon employing different combinations of the measured trajectory data, the relative importance of each data can be established. The horizontal distance data and the relative velocity data are found to have minor effect on the estimation results. The altitude data affect mostly the vertical wind, and the pitch attitude angle data are crucial to the estimation of both the horizontal and vertical winds.

**Key Words.** Windshear estimation, parameter estimation, flight mechanics, nonlinear least-square problems, dynamic approach, kinematic approach, optimal control theory, sequential gradient-restoration algorithm.
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1. Introduction

Low-altitude windshear is a threat to the safety of aircraft in take-off and landing (Ref. 1). Over the past 20 years, some 30 aircraft accidents have been attributed to windshear. The most notorious ones are the crash of PANAM Flight 759 of July 9, 1982 at New Orleans International Airport (Boeing B-727 in take-off, Ref. 2) and the crash of Delta Airlines Flight 191 of August 2, 1985 at Dallas-Fort Worth International Airport (Lockheed L-1011 in landing, Refs. 3-5). Aiming at overcoming this deadly threat, considerable research has been concerned with the effect of low-altitude windshear on flight safety (Refs. 6-14).

Windshear can be characterized as a wind with abrupt changes in magnitude and direction. Dangerous windshears are associated with downbursts beneath severe thunderstorms. A downburst whose size is less than 2.5 miles in diameter is called a microburst (Ref. 15). In turn, a microburst involves a descending column of air, which then spreads horizontally in the neighborhood of the ground (Fig. 1). This condition is hazardous, because an aircraft in take-off or landing might encounter a headwind coupled with a downdraft, followed by a tailwind coupled with a downdraft. The transition from headwind to tailwind engenders a transport acceleration, and hence a windshear inertia force (the product of the transport acceleration and the mass of the aircraft), which is equivalent to a drag increase or a thrust decrease. Hence, one can understand why an inadvertent encounter with a low-altitude windshear can be a serious problem for even a skilled pilot. Thus, to achieve a better understanding of windshear phenomena and flight strategies is of major importance.

To offset the windshear threat, there are two basic systems: windshear warning systems and windshear recovery systems (Ref. 16). A windshear warning system is designed to alert the pilot to the fact that a windshear encounter might take place; here,
the intent is the avoidance of a microburst. A windshear recovery system is designed to
guide the pilot in the course of a windshear encounter; here, the intent is to fly safely
across a microburst, if an inadvertent encounter takes place. Based on a windshear model
obtained from the combination of theory and experimental measurements (Refs. 17-18),
the Aero-Astronautics Group (AAG) of Rice University has conducted an extensive study
of optimal trajectories and guidance schemes for take-off, abort landing, and penetration
landing (Refs. 19-35).

The objective of this thesis is to reconstruct the windshear that caused the crash of
Delta 191 at Dallas-Fort Worth International Airport, while using only limited trajectory
data. It is known that, based on the reconstructed winds, safer flying procedures can be
established through optimal trajectory studies. As a result, this work is closely related to
windshear recovery systems. Previous research on windshear estimation can be found in
Refs. 36-47.

According to the cockpit voice recorder (CVR), the intention of the pilot of Delta 191
was to land at Dallas-Fort Worth International Airport, until a decision was taken to
initiate the go-around at about 7 seconds before impact. As the air traffic control (ATC)
radar data show nearly constant values in the y-coordinate (lateral direction), it is
reasonable to assume that flight took place in a vertical plane during the final approach.
Hence, use of the two-dimensional equations of motion is sufficient for this study. Also, for
trajectory studies, the description of the motion of the center of gravity of the aircraft is
adequate; thus, the aircraft is treated as a particle of constant mass. Here, we restrict our
study to the last 55 seconds before the impact occurred.

Estimation theory is often used in the reconstruction of the missing state variables
and/or parameters of a system from measured data (Refs. 48-49). The resulting problems
can be divided into two categories: stochastic and deterministic. The Kalman filtering
technique and the maximum likelihood method are stochastic in nature. These methods are complicated in implementation. However, if the noise distribution is available, better estimation results can be expected. The least-square method is deterministic in nature. This method is easier to implement and is often used in the case when noise information on the measurements and the system is not available.

In this thesis, the deterministic approach is adopted. By minimizing a performance index measuring the deviation of the experimentally determined flight trajectory from the computed flight trajectory, subject to the equations of motion as constraints, a nonlinear least-square problem is formulated. This is a Bolza problem of optimal control (Ref. 50), which can be solved via the sequential gradient-restoration algorithm (SGRA) for optimal control problems (Refs. 51-53).

Both the kinematic and dynamic equations of motion are introduced as differential constraints in the formulation. Based on the manufacturer-supplied aerodynamic data and the engine pressure ratio (EPR) data from the digital flight data recorder (DFDR), the lift, drag, and thrust are expressed as functions of the relative velocity \( \tilde{V} \), the angle of attack \( \alpha \), and the power setting \( \beta \). This method is called dynamic approach and differs from the kinematic approach used in Refs. 44-47. In the kinematic approach, information on the position, velocity, and acceleration of the aircraft is needed. By integrating the accelerometer data, the inertial velocity is obtained. The wind components are then computed as the difference between the inertial velocity and the relative velocity. The kinematic approach is straightforward, if all the data required are available. However, the accelerometer data might not be available on some airplanes.

In the dynamic approach, the data needed consist of the measured altitude \( \tilde{h} \), relative velocity \( \tilde{V} \), and pitch attitude angle \( \tilde{\theta} \), which are easier to obtain than the accelerometer data used in the kinematic approach. The angle of attack \( \alpha \) is treated as a control, while
the power setting $\beta$ is regarded as a known input. Hence, direct employment of EPR data is required. From the calculated thrust, drag, and lift forces, a better understanding of the dynamic effects during a windshear encounter can be obtained.

The digital flight data recorder (DFDR) of Flight Delta 191 is capable of sequentially recording 64 different quantities in one second. However, only one specific quantity can be recorded at one time instant. Thus, the different quantities are recorded at different time instants. An interpolation process is employed timewise on each quantity (Ref. 54). Then, the values obtained at integer times are taken as the basic data to be used in the computation.

Four problems are formulated in this thesis.

**Problem (P1).** Three physical quantities are employed, namely, the altitude $\bar{h}$, the relative velocity $\bar{V}$, and the pitch attitude angle $\bar{\theta}$, which are obtained from the DFDR. The angle of attack $\alpha$ is treated as a control variable, and the power setting $\beta$ is treated as a known input. Hence, EPR measurements are also needed. The manufacturer-supplied aerodynamic and thrust data are considered to be dependable and are treated as known inputs.

**Problem (P2).** This is the same as Problem (P1), but the inclusion of multiplicative factors for the thrust, drag, and lift is considered. Namely, the thrust, drag, and lift are written as $T=\pi_1 T_*$, $D=\pi_2 D_*$, $L=\pi_3 L_*$. Starred values correspond to manufacturer-supplied data. If one sets $\pi_1=\pi_2=\pi_3=1$, one obtains Problem (P1). By allowing the values of $\pi_1$, $\pi_2$, $\pi_3$ to vary, we are able to approximate the measured data with better precision. The estimated values of $\pi_1$, $\pi_2$, $\pi_3$, if different from unity, might be used in explaining some unusual effects, such as the presence of rain. In order to ensure a realistic assessment of the estimated parameters, penalty terms are introduced in the performance
index being minimized. This forces the estimated aerodynamic and thrust data to be close to the manufacturer-supplied data.

**Problem (P3).** This is the same as Problem (P2); however, in addition to the measurements $\bar{K}, \bar{V}, \bar{\theta}$, the horizontal distance $\bar{x}$ recovered from the kinematic approach is included (Ref. 44). In general, the more the measurements, the better the estimation results.

**Problem (P4).** This is the same as Problem (P2) except that either one or two of the measurements $\bar{K}, \bar{V}, \bar{\theta}$ are discarded. In general, the less the measurements, the worse the estimation results.
2. Notations

$C_D =$ drag coefficient;

$C_L =$ lift coefficient;

$D =$ drag force, lb;

$D_* =$ standard drag force, lb;

$g =$ acceleration of gravity, ft sec$^{-2}$

$h =$ altitude above the ground, ft;

$h =$ measured altitude above the ground, ft;

$L =$ lift force, lb;

$L_* =$ standard lift force, lb;

$m =$ mass, lb ft$^{-1}$sec$^2$;

$S =$ reference surface (basic wing area), ft$^2$;

$T =$ thrust force, lb;

$T_* =$ standard thrust force, lb;

$T_R =$ reference thrust force (EPR= 1.49, sea level, 100 deg F), lb;

$U_h =$ time derivative of vertical wind, ft sec$^{-2}$;

$U_x =$ time derivative of horizontal wind, ft sec$^{-2}$;

$V =$ relative velocity, ft sec$^{-1}$;

$\tilde{V} =$ measured relative velocity, ft sec$^{-1}$;

$V_e =$ absolute velocity, ft sec$^{-1}$;

$W =$ mg = weight, lb;

$W_h =$ vertical wind, ft sec$^{-1}$;

$W_x =$ horizontal wind, ft sec$^{-1}$;

$x =$ horizontal distance, ft;

$\tilde{x} =$ measured horizontal distance, ft;

$y =$ lateral distance, ft.
Greek Symbols

\( \alpha \) = relative angle of attack (wing), rad;
\( \alpha_e \) = absolute angle of attack (wing), rad;
\( \alpha^* \) = stick-shaker angle of attack, rad;
\( \alpha^{**} \) = angle of attack limiting the linear range for the lift coefficient, rad;
\( \beta \) = engine power setting;
\( \gamma \) = relative path inclination, rad;
\( \gamma_e \) = absolute path inclination, rad;
\( \delta \) = thrust inclination, rad;
\( \delta_F \) = flap setting, rad;
\( \Delta W_x \) = horizontal wind velocity difference, ft sec\(^{-1}\);
\( \Delta W_h \) = vertical wind velocity difference, ft sec\(^{-1}\);
\( \theta \) = pitch attitude angle (wing), rad;
\( \tilde{\theta} \) = measured pitch attitude angle (wing), rad;
\( \rho \) = air density, lb ft\(^{-4}\) sec\(^{2}\);
\( \tau \) = final time, sec;
\( \pi_1 \) = thrust multiplicative factor;
\( \pi_2 \) = drag multiplicative factor;
\( \pi_3 \) = lift multiplicative factor.

Abbreviations

AAG = Aero-Astronautics Group (Rice University);
ATC = air traffic control;
CVR = cockpit voice recorder;
DFDR = digital flight data recorder;
DFW = Dallas-Fort Worth;
EPR = engine pressure ratio;
IAS = indicated air speed;
JAWS = joint airport weather study;
LTP-BVP = linear two-point boundary-value problem;
MFR = metal foil recorder;
NASA = National Aeronautics and Space Administration;
NTSB = National Transportation Safety Board;
SAT = static air temperature;
SGRA = sequential gradient-restoration algorithm;
TAS = true air speed.
3. System Description

According to the ATC radar data, the final approach of Delta 191 took place nearly in a vertical plane. Thus, the two-dimensional equations of motion are adopted. Furthermore, the particle mass approach is considered adequate in describing the motion of the aircraft.

3.1. Equations of Motion. We make use of the relative wind-axes system in connection with the following assumptions (see Fig. 2): (a) the aircraft is a particle of constant mass; (b) flight takes place in a vertical plane; and (c) Newton's law is valid in an Earth-fixed system (Refs. 19, 55-56). With the above premises, the equations of motion include the kinematical equations

\[ \dot{x} = V \cos \gamma + W_x, \]  \hspace{1cm} (1a)

\[ \dot{h} = V \sin \gamma + W_h, \]  \hspace{1cm} (1b)

the dynamical equations

\[ \dot{V} = \frac{T}{m} \cos (\alpha + \delta) - \frac{D}{m} \sin \gamma - g \sin \gamma (U_x \cos \gamma + U_h \sin \gamma), \]  \hspace{1cm} (2a)

\[ \dot{\gamma} = \frac{T}{m V} \sin (\alpha + \delta) + L \sin V - \frac{g}{V} \cos \gamma + \frac{1}{V} (U_x \sin \gamma U_h \cos \gamma), \]  \hspace{1cm} (2b)

and the definition equations

\[ \dot{W}_x = U_x, \]  \hspace{1cm} (3a)

\[ \dot{W}_h = U_h. \]  \hspace{1cm} (3b)

These equations must be supplemented by the functional relations

\[ T = T(h, V, \beta), \]  \hspace{1cm} (4a)

\[ D = D(h, V, \alpha), \]  \hspace{1cm} \[ L = L(h, V, \alpha), \]  \hspace{1cm} (4b)
and by the analytical relations

\[ V_{ex} = V \cos \gamma + W_x, \]
\[ V_{eh} = V \sin \gamma + W_h, \]  \hspace{1cm} (5a)

\[ V_e = \sqrt{V_{ex}^2 + V_{eh}^2}, \]
\[ \gamma_e = \tan^{-1}(V_{eh}/V_{ex}), \]  \hspace{1cm} (5b)

\[ \theta = a + \gamma, \]
\[ \alpha_e = a + \gamma - \gamma_e. \]  \hspace{1cm} (5c)

For a given value of the thrust inclination \( \delta \), the differential system (1)-(4) involves six state variables [the horizontal distance \( x(t) \), the altitude \( h(t) \), the relative velocity \( V(t) \), the relative path inclination \( \gamma(t) \), the horizontal wind \( W_x(t) \), and the vertical wind \( W_h(t) \)] and four control variables [the angle of attack \( a(t) \), the power setting \( \beta(t) \), the auxiliary variable \( U_x(t) \), and the auxiliary variable \( U_h(t) \)]. However, the number of control variables reduces to three \( (a, U_x, U_h) \), if the power setting \( \beta \) is specified in advance. The quantities defined by the analytical relations (5) can be computed a posteriori, once the values of the state variables and the control variables are known.

**Remark.** For the Lockheed L-1011 aircraft, the thrust inclination is \( \delta = 2.0 \) deg for the wing engines and \( \delta = 4.5 \) deg for the tail engine. In this thesis, a uniform thrust inclination \( \delta = 2.0 \) deg is assumed for all the engines. This causes a change in the total thrust of the order of 1/1000. This assumption is adopted, since it reduces the number of control variables of the problem.

**3.2. Approximation for the Forces.** Here, we discuss the approximations employed in the description of the forces acting on the aircraft, namely, the thrust, the drag, the lift, and the weight.

**Thrust.** The thrust is written in the form

\[ T = \beta T_R \exp[-(h+C)/k], \]  \hspace{1cm} (6a)
\[ T_R = A_0 + A_1 V + A_2 V^2, \] (6b)

where \( \beta \) is the engine power setting, \( h + C \) is the aircraft altitude above sea level, \( h \) is the aircraft altitude above ground, \( C = 495 \text{ ft} \) is the ground altitude above sea level, and \( k = 23,000 \text{ ft} \) is a constant associated with the pressure versus altitude profile.

The reference thrust \( T_R \) is the total thrust of the three engines of the aircraft at sea level and \( \beta = 1 \) (equivalently, EPR = 1.49). For a given relative velocity, \( T_R \) can be obtained by interpolating the manufacturer-supplied thrust map. The coefficients \( A_0, A_1, A_2 \) depend on the ambient temperature and are determined with a least-square fit of manufacturer-supplied data over a given interval of velocities. An ambient temperature of 100 deg F is assumed.

Drag. The drag \( D \) is written in the form

\[ D = (1/2) C_D \rho S V^2, \] (7a)

\[ C_D = B_0 + B_1 a + B_2 a^2 + B_3 a^3 + B_4 a^4, \quad a \leq a_*, \] (7b)

where \( \rho \) is the air density (assumed constant), \( S \) is a reference surface, \( V \) is the relative velocity, \( C_D \) is the drag coefficient, \( a \) is the angle of attack, and \( a_* \) is the stick-shaker angle of attack. The coefficients \( B_0, B_1, B_2, B_3, B_4 \) depend on the flap setting and the undercarriage position (gear up or gear down); they can be determined with a least-square fit of manufacturer-supplied data over the interval \( 0 \leq a \leq a_* \). Note that the flap setting of Flight Delta 191 was \( \delta_F = 33 \text{ deg} \) and that the gear was down.

Lift. The lift \( L \) is written in the form

\[ L = (1/2) C_L \rho S V^2, \] (8a)

\[ C_L = C_0 + C_1 a, \quad a \leq a_**, \] (8b)
\[ C_L = C_0 + C_1 a + C_2 (a - a_{**})^2, \quad a_{**} \leq a_*, \tag{8c} \]

where \( \rho \) is the air density (assumed constant), \( S \) is a reference surface, \( V \) is the relative velocity, and \( C_L \) is the lift coefficient. The coefficients \( C_0, C_1, C_2 \) depend on the flap setting and the undercarriage position (gear up or gear down); they can be determined with a least-square fit of manufacturer-supplied data over the intervals \( 0 \leq a \leq a_{**} \) and \( a_{**} \leq a_* \). Again, note that the flap setting was \( \delta_F = 33 \) deg and that the gear was down.

**Weight.** The mass \( m \) is regarded to be constant; namely, the fuel consumption during the final 55 seconds is neglected. Hence, the weight \( W = mg \) is regarded to be constant.

**Remark.** The coefficients \( A_i, B_i, C_i \) appearing in Eqs. (6)-(8) can be determined with a least-square fit of the manufacturer-supplied data (Table 1). Numerical experiments show that the resulting precision in the thrust function \( T_R(V) \), drag coefficient function \( C_D(a) \), and lift coefficient function \( C_L(a) \) is of order 1% or better in the range of velocities and angles of attack having interest in this study. These functions are plotted in Figs. 3A-3C.

### 3.3. Aircraft Data

The computations of the subsequent sections refer to a Lockheed L-1011 aircraft, which is powered by three Rolls Royce RB-211-22B engines (Fig. 3D). The ambient temperature is 100 deg F, the gear is down, and the flap setting is \( \delta_F = 33 \) deg.

Other data of interest are listed below:

\[ \begin{align*}
\rho & \quad \text{air density} = 0.002203 \text{ lb ft}^{-2} \text{ sec}^2, \tag{9a} \\
W & \quad \text{landing weight} = 324,802 \text{ lb}, \tag{9b} \\
S & \quad \text{basic wing area} = 3456 \text{ ft}^2, \tag{9c} \\
a_* & \quad \text{stick-shaker angle of attack} = 18.6 \text{ deg}, \tag{9d} \\
a_{**} & \quad \text{angle of attack limiting the linear range for the lift coefficient} = 6.0 \text{ deg}, \tag{9e} \\
\delta & \quad \text{thrust inclination} = 2.0 \text{ deg}. \tag{9f} 
\end{align*} \]

**Initial State.** The time instant corresponding to 55 seconds before impact is taken as
the initial point of the integration interval. The projection of the initial point on the ground is taken as the origin of the Earth-fixed coordinate system; hence, \( x(0) = 0 \). The initial values of the altitude and the relative velocity are obtained from the DFDR readout. Since the wind components are not recorded, their initial values are free. The relative path inclination is obtained from the relation \( \gamma = \theta - a \) [Eq. (5c)]. Since \( a \) is a control, the initial value of \( \gamma \) is free. As a result, we have the following initial conditions:

\[
\begin{align*}
    x(0) & = 0.00 \text{ ft}, \\
    h(0) & = 1060.16 \text{ ft}, \\
    V(0) & = 277.19 \text{ ft/sec}, \\
    \gamma(0) & = \text{free}, \\
    W_x(0) & = \text{free}, \\
    W_h(0) & = \text{free}.
\end{align*}
\]

**Final Time.** The final time is set at the value

\[
\tau = 55.00 \text{ sec},
\]

**Final State.** The data corresponding to impact are taken as final conditions. The final value of the relative velocity is obtained from the DFDR readout. At the final point, the horizontal distance can be measured directly and the altitude is the ground altitude, namely, \( h(\tau) = 0 \). Since the wind components are not recorded, they are free at the final point. The relative path inclination is obtained from the relation \( \gamma = \theta - a \) [Eq. (5c)]. Since \( a \) is treated as a control variable, the final value of \( \gamma \) is free. As a result, we have the following final conditions:

\[
\begin{align*}
    x(\tau) & = 14335.90 \text{ ft}, \\
    h(\tau) & = 0.00 \text{ ft}, \\
    V(\tau) & = 286.01 \text{ ft/sec}, \\
    \gamma(\tau) & = \text{free}.
\end{align*}
\]
\[ W_x(\tau) = \text{free}, \quad (12e) \]
\[ W_h(\tau) = \text{free}. \quad (12f) \]

3.4. DFDR and ATC Data. Most modern wide-body aircraft are equipped with a digital flight data recorder (DFDR), which is capable of recording sufficient trajectory data needed in windshear estimation. On the other hand, the ground-based air traffic control (ATC) radar records the aircraft position data, which can be used in verifying the correctness of the trajectory recorded by the DFDR.

**Digital Flight Data Recorder.** The measured trajectory data of Delta 191 are based on the DFDR readout released by the National Transportation Safety Board (NTSB). The recorder is capable of recording 64 different quantities in one second. However, only one specific quantity is recorded at each time instant. Thus, different quantities are recorded at different time instants. A timewise interpolation process is employed for each quantity (Ref. 54); then, the values obtained at integer times are used as the basic trajectory data. Most data of interest are recorded every second; however, the static air temperature (SAT) is recorded every two seconds and the engine pressure ratio (EPR) is recorded every four seconds.

In the dynamic approach, three measured physical quantities are employed, namely, the altitude \( \tilde{h} \), the relative velocity \( \tilde{V} \), and the pitch attitude angle \( \tilde{\theta} \). The DFDR relative velocity is the indicated air speed (IAS), which is related to the true air speed (TAS) through the following formula:

\[ \tilde{V} = \text{TAS} = \frac{\text{IAS}}{\sqrt{\rho/\rho_{\text{SL}}}}, \quad (13) \]

where the subscript \( \text{SL} \) denotes sea level. Note that the DFDR does not record the horizontal distance \( \tilde{x} \) and the lateral distance \( \tilde{y} \).
Other measured physical quantities include the EPR and the angle of attack $a$. Associated with each of the engines on board, there is a specific sequence of EPR data. Based on the EPR data, the power setting can be obtained via Eqs. (6). Four vanes placed on the aircraft wings are used in measuring the so-called local angle of attack $a_L$. The following correction formula, suggested by NTSB, is employed in converting the local angle of attack into the actual angle of attack:

$$a = \frac{a_L + c_1}{c_2},$$  \hspace{1cm} (14a)  

$$c_1 = 6.95 \text{ deg}, \quad c_2 = 1.87.$$  \hspace{1cm} (14b)  

Since the angle of attack is treated as a control, the recorded values of $a$ are not considered as known inputs, but only as nominal data. On the other hand, the EPR data, and hence the power setting $\beta$, are treated as known inputs.

**Air Traffic Control Radar.** The ATC radar is able to record three different physical quantities (the horizontal distance $\bar{x}$, the lateral distance $\bar{y}$, and the altitude $\bar{h}$) every 10 seconds. For the final period of 55 seconds, there are only 6 ATC measurements, which are much less in number than the 55 DFDR measurements. As a result, except for the initial value $\bar{x}(0)$, the ATC radar data are not employed in this thesis.
4. Problem Formulation

Given the measured trajectory data from the DFDR, it is desired to construct the wind distribution along the trajectory, while approximating the measured trajectory as closely as possible. This is a nonlinear least-square problem, in which one minimizes a performance index measuring the deviation of the measured trajectory from the computed trajectory, subject to the equations of motion as differential constraints. This is a Bolza problem of optimal control, which can be solved via the sequential gradient-restoration algorithm (SGRA) described in next section.

4.1. Performance Functional. Upon employing different measured physical quantities, four basic problems can be formulated. In each problem, a certain performance index is minimized, subject to differential constraints [Eqs. (1)-(3)], initial conditions [Eqs. (10)], and final conditions [Eqs. (11)-(12)].

Problem (P1). Standard Data Approach without Parameter Estimation. In this problem, the performance index to be minimized is given by

\[ I = \int_0^\tau \left( C_2 (h - \bar{h})^2 + C_3 (V - \bar{V})^2 + C_4 (\theta - \bar{\theta})^2 \right) dt. \]  \hspace{1cm} (15)

Here, \( C_2, C_3, C_4 \) are weighting coefficients, chosen in such a way that the residual terms are approximately of the same order; \( \tau \) is the final time of integration; \( \bar{h}, \bar{V}, \bar{\theta} \) are the measurements obtained from the DFDR; and \( h, V, \theta \) are the state variables generated by integration of the equations of motion. The manufacturer-supplied thrust and aerodynamic data are considered to be dependable and are treated as known data.

Concerning the horizontal distance \( \bar{x} \), as explained in Section 3, the initial value \( \bar{x}(0) \) is obtained from ATC data; the final value \( \bar{x}(\tau) \) is known from the measured impact point. The power setting \( \beta \) is treated as a known input; hence, direct employment of EPR data is required. Note that \( \bar{h}, \bar{V}, \bar{\theta} \) are recorded not only in wide-body airplanes, but also in
narrow-body airplanes; hence, the terminology "standard data" used in this thesis.

Other quantities used in defining Problem (P1) are listed below:

\[ \begin{align*}
\tau & = \text{final time} = 55 \text{ sec}, \\
n & = \text{number of state variables} = 6 \ (x, h, V, \gamma, W_x, W_h), \\
m & = \text{number of control variables} = 3 \ (a, U_x, U_h), \\
a & = \text{number of initial conditions} = 3 \ [x(0), h(0), V(0)], \\
b & = \text{number of final conditions} = 3 \ [h(\tau), V(\tau), x(\tau)], \\
p & = \text{number of parameters} = 0.
\end{align*} \] (16a)-(16f)

**Problem (P2).** Standard Data Approach with Parameter Estimation. In this problem, the inclusion of multiplicative factors for the thrust, the drag, and the lift is considered. Namely, one sets \( T = \pi_1 T^*, \ D = \pi_2 D^*, \ L = \pi_3 L^* \), where the starred values correspond to manufacturer-supplied data. If one sets \( \pi_1 = \pi_2 = \pi_3 = 1 \), one obtains the thrust, drag and lift of Problem (P1). By allowing the parameters \( \pi_1, \pi_2, \pi_3 \) to vary, the measured data can be approximated with better precision. The estimated values of the parameters \( \pi_1, \pi_2, \pi_3 \), if different from unity, might be used in explaining some unusual effects, such as the presence of rain.

The thrust, the drag, and the lift can be written explicitly as follows:

\[ \begin{align*}
T & = \pi_1 (\beta (A_0 + A_1 V + A_2 V^2) \exp[-(h + C)/k]), \\
D & = \pi_2 [(1/2) C_D \rho S V^2], \\
L & = \pi_3 [(1/2) C_L \rho S V^2].
\end{align*} \] (17a)-(17c)

Two particular cases of Problem (P2) are considered below.

**Problem (P2A).** In this problem, the performance index to be minimized is given by
\[ I = \int_0^\tau \left[ C_2 (h - \bar{h})^2 + C_3 (V - \bar{V})^2 + C_4 (\theta - \bar{\theta})^2 + C_5 ((\pi_2 - 1)^2 + (\pi_3 - 1)^2) \right] \, dt. \]  

Equation (18) differs from Eq. (15) because of the inclusion of two penalty terms, designed to prevent the drag and lift parameters \( \pi_2, \pi_3 \) from becoming physically unreasonable. The thrust parameter is set at the level \( \pi_1 = 1 \).

Other quantities used in defining Problem (P2A) are listed below:

\[ \tau = \text{final time} = 55 \text{ sec}, \]  
\[ n = \text{number of state variables} = 6 \, (x, h, V, \gamma, W_x, W_h), \]  
\[ m = \text{number of control variables} = 3 \, (a, U_x, U_h), \]  
\[ a = \text{number of initial conditions} = 3 \, [x(0), h(0), V(0)], \]  
\[ b = \text{number of final conditions} = 3 \, [h(\tau), V(\tau), x(\tau)], \]  
\[ p = \text{number of parameters} = 2 \, (\pi_2, \pi_3). \]

**Problem (P2B).** In this problem, the performance index to be minimized is given by

\[ I = \int_0^\tau \left[ C_2 (h - \bar{h})^2 + C_3 (V - \bar{V})^2 + C_4 (\theta - \bar{\theta})^2 + C_5 ((\pi_1 - 1)^2 + (\pi_2 - 1)^2 + (\pi_3 - 1)^2) \right] \, dt. \]

Equation (20) differs from Eq. (15) because of the inclusion of three penalty terms, designed to prevent the thrust, drag, and lift parameters \( \pi_1, \pi_2, \pi_3 \) from becoming physically unreasonable.

Other quantities used in defining Problem (P2B) are listed below:

\[ \tau = \text{final time} = 55 \text{ sec}, \]  
\[ n = \text{number of state variables} = 6 \, (x, h, V, \gamma, W_x, W_h), \]  
\[ m = \text{number of control variables} = 3 \, (a, U_x, U_h), \]  
\[ a = \text{number of initial conditions} = 3 \, [x(0), h(0), V(0)], \]  
\[ b = \text{number of final conditions} = 3 \, [h(\tau), V(\tau), x(\tau)], \]  
\[ p = \text{number of parameters} = 3 \, (\pi_1, \pi_2, \pi_3). \]

**Problem (P3).** Full Data Approach with Parameter Estimation. This problem is
obtained from Problem (P2A) by adding a term measuring the deviation of the measured horizontal distance \( \tilde{x} \) from the computed horizontal distance \( x \). Hence, the performance index to be minimized is given by

\[
I = \int_0^\tau \left( C_1(x-\tilde{x})^2 + C_2(h-\tilde{h})^2 + C_3(V-\tilde{V})^2 + C_4(\theta-\tilde{\theta})^2 \\
+ C_5[(\pi_1-1)^2 + (\pi_2-1)^2 + (\pi_3-1)^2]\right) dt.
\]

(22)

Note that \( \tilde{x} \) is not recorded by the DFDR; instead, it is obtained from the kinematic approach (Ref. 44). Hence, the terminology "full data" used in this thesis.

Other quantities used in defining Problem (P3) are listed below:

\[
\begin{align*}
\tau &= \text{final time} = 55 \text{ sec}, \quad \tag{23a} \\
n &= \text{number of state variables} = 6 \ (x, h, V, \gamma, W_x, W_h), \quad \tag{23b} \\
m &= \text{number of control variables} = 3 \ (a, U_x, U_h), \quad \tag{23c} \\
a &= \text{number of initial conditions} = 3 \ [x(0), h(0), V(0)], \quad \tag{23d} \\
b &= \text{number of final conditions} = 3 \ [h(\tau), V(\tau), x(\tau)], \quad \tag{23e} \\
p &= \text{number of parameters} = 2 \ (\pi_2, \pi_3). \quad \tag{23f}
\end{align*}
\]

**Problem (P4).** Reduced Data Approach with Parameter Estimation. Problem (P4) is obtained from Problem (P2A) by removing either one or two of the measurements \( \tilde{h}, \tilde{V}, \tilde{\theta} \). Hence, the terminology "reduced data" used in this thesis. Several particular cases are presented below.

**Problem (P4A).** This problem is obtained from Problem (P2A) by removing the altitude measurements. Hence, the performance index is given by

\[
I = \int_0^\tau \left( C_3(V-\tilde{V})^2 + C_4(\theta-\tilde{\theta})^2 + C_5[(\pi_2-1)^2 + (\pi_3-1)^2]\right) dt.
\]

(24)

In this problem, Eqs. (19) still hold.

**Problem (P4B).** This problem is obtained from Problem (P2A) by removing the relative velocity measurements. Hence, the performance index is given by
\[ I = \int_0^\tau \left[ C_2 (h - \bar{h})^2 + C_4 (\theta - \bar{\theta})^2 + C_5 [ (\pi_2 - 1)^2 + (\pi_3 - 1)^2 ] \right] \, dt. \] (25)

In this problem, Eqs. (19) still hold.

**Problem (P4C).** This problem is obtained from Problem (P2A) by removing the pitch attitude angle measurements. Hence, the performance index is given by

\[ I = \int_0^\tau \left[ C_2 (h - \bar{h})^2 + C_3 (\bar{V} - \bar{V})^2 + C_5 [ (\pi_2 - 1)^2 + (\pi_3 - 1)^2 ] \right] \, dt. \] (26)

In this problem, Eqs. (19) still hold.

**Problem (P4D).** This problem is obtained from Problem (P2A) by removing both the relative velocity and the pitch attitude angle measurements. Hence, the performance index is given by

\[ I = \int_0^\tau \left[ C_2 (h - \bar{h})^2 + C_5 [ (\pi_2 - 1)^2 + (\pi_3 - 1)^2 ] \right] \, dt. \] (27)

In this problem, Eqs. (19) still hold.

**Problem (P4E).** This problem is obtained from Problem (P2A) by removing both the altitude and the pitch attitude angle measurements. Hence, the performance index is given by

\[ I = \int_0^\tau \left[ C_3 (\bar{V} - \bar{V})^2 + C_5 [ (\pi_2 - 1)^2 + (\pi_3 - 1)^2 ] \right] \, dt. \] (28)

In this problem, Eqs. (19) still hold.

**Problem (P4F).** This problem is obtained from Problem (P2A) by removing both the altitude and the relative velocity measurements. Hence, the performance index is given by

\[ I = \int_0^\tau \left[ C_4 (\theta - \bar{\theta})^2 + C_5 [ (\pi_2 - 1)^2 + (\pi_3 - 1)^2 ] \right] \, dt. \] (29)

In this problem, Eqs. (19) still hold.

### 4.2. Nominal Conditions

Before the iterative procedure of the sequential gradient-restoration algorithm (SGRA) can be initiated, certain nominal conditions need to be supplied. This refers to the state variables \([x(t), h(t), V(t), \gamma(t), W_x(t), W_h(t)]\), the control variables \([u(t), U_x(t), U_h(t)]\), and the parameters \((\pi_1, \pi_2, \pi_3)\).
For the angle of attack $\alpha$, the DFDR measurements are used as nominal data. Direct employment of the EPR data is required, since the power setting $\beta$ is treated as a known input. For the auxiliary variables $U_x$ and $U_h$ (the time derivative of the wind components $W_x$ and $W_h$), the nominal functions $U_x(t)=0$ and $U_h(t)=0$ are assumed. During the iteration procedure, the control variables [$\alpha(t)$, $U_x(t)$, $U_h(t)$] are allowed to vary so as to lead toward the minimal value of the performance index being considered.

The nominal values of the state variables ($x$, $h$, $V$, $\gamma$, $W_x$, $W_h$) are obtained by integrating the equations of motion [Eqs. (1)-(3)] with the nominal controls given above and the initial conditions given by Eqs. (10). The initial value $\gamma(0)$ is obtained by subtracting the measured $\alpha(0)$ from the measured $\theta(0)$. The initial values of the wind components are assumed to be $W_x(0)=0$ and $W_h(0)=0$, since no a priori information is available.
5. Sequential Gradient-Restoration Algorithm

Problems (P1)-(P4) of Section 4 are Bolza problems of optimal control (Ref. 50). They can be solved with the sequential gradient-restoration algorithm (SGRA) in either the primal formulation (Ref. 50) or the dual formulation (Refs. 52-53).

Regardless of whether the primal formulation is used or the dual formulation is used, SGRA involves a sequence of two-phase cycles, each cycle including a gradient phase and a restoration phase. In the gradient phase, the value of the augmented functional is decreased, while avoiding excessive constraint violation. In the restoration phase, the value of the constraint error is decreased, while avoiding excessive change in the value of the functional. In a complete gradient-restoration cycle, the value of the functional is decreased, while the constraints are satisfied to a preselected degree of accuracy. Thus, a succession of suboptimal solutions is generated, each new solution being an improvement over the previous one from the point of view of the value of the functional being minimized.

In this thesis, the primal formulation is used. The method of particular solutions is employed in solving the linear two-point boundary-value problems (LTP-BVP) arising in the gradient phase and the restoration phase of SGRA.

5.1. Optimization Problem. The optimization problem can be formulated as follows.

Minimize the functional

\[ I = \int_0^1 f(x,u,\pi,t)dt + [h(x,\pi)]_0 + [g(x,\pi)]_1, \]  

(30)

with respect to the n-vector state \( x(t) \), the m-vector control \( u(t) \), and the p-vector parameter \( \pi \), which satisfy the constraints

\[ \dot{x} + \phi(x,u,\pi,t) = 0, \quad 0 \leq t \leq 1, \]  

(31a)

\[ \omega(x,\pi) = 0, \]  

(31b)

\[ \psi(x,\pi) = 0. \]  

(31c)
In the above equations, \( f \) is a scalar; \( h \) is a scalar; \( g \) is a scalar; \( \phi \) is an \( n \)-vector; \( \omega \) is an \( a \)-vector, \( a \leq n \); and \( \psi \) is a \( b \)-vector, \( b \leq n \). We assume that the \( n \times a \) matrix \( \omega_\chi \) has rank \( a \) at the initial point, that the \( n \times b \) matrix \( \psi_\chi \) has rank \( b \) at the final point, and that the constrained minimum exists.

### 5.2. First-Order Optimality Conditions

From calculus of variations (Ref. 50), it is known that the above problem is of the Bolza type. It can be recast as that of minimizing the augmented functional

\[
J = I + L,
\]

subject to (31), where \( L \) denotes the Lagrangian functional

\[
L = \int_0^1 \lambda^T (x + \phi) dt + (\sigma^T \omega)_0 + (\mu^T \psi)_1.
\]

In Eq. (33), \( \lambda(t) \) denotes an \( n \)-vector Lagrange multiplier, \( \sigma(t) \) denotes an \( a \)-vector Lagrange multiplier, and \( \mu \) denotes a \( b \)-vector Lagrange multiplier.

The first-order optimality conditions for the above problem take the form

\[
\dot{\lambda} - f_x - \phi_x \lambda = 0, \quad 0 \leq t \leq 1, \quad (34a)
\]

\[
f_u + \phi_u \lambda = 0, \quad 0 \leq t \leq 1, \quad (34b)
\]

\[
\int_0^1 (f_\pi + \phi_\pi \lambda) dt + (h_\pi + \omega_\pi \sigma)_0 + (g_\pi + \psi_\pi \mu)_1 = 0, \quad (34c)
\]

\[
(\lambda + h_x + \omega_\chi \sigma)_0 = 0, \quad (34d)
\]

\[
(\lambda + g_x + \psi_\chi \mu)_1 = 0. \quad (34e)
\]

Summarizing, we seek the functions \( x(t) \), \( u(t) \), \( \pi \) and the multipliers \( \lambda(t) \), \( \sigma \), \( \mu \) such that the feasibility conditions (31) and the optimality conditions (34) are satisfied.

### 5.3. Performance Indexes

Since the differential system (31)-(34) is generally nonlinear, approximate methods are employed to find a solution iteratively. In this connection, let the norm squared of a vector \( y \) be defined as \( N(y) = y^T y \). Then, the following performance indexes are useful in computational work:

\[
P = \int_0^1 N(x + \phi) dt + N(\omega)_0 + N(\psi)_1, \quad (35a)
\]
\[
Q = \int_0^1 N(\frac{\dot{\lambda}}{\lambda} - f_x - \phi_x \lambda) dt + \int_0^1 N(f_u + \phi_u) dt + N[I(\frac{1}{2} f_{\pi} + \phi_{\pi}) dt + (\pi + \omega_{\pi} \sigma)] + (g_{\pi} + \psi_{\pi} u) \mu_1 + N(- \lambda + h_x + \omega_x \sigma_0 + N(\lambda + g_x + \psi_x u) \mu_1. \tag{35b}
\]

Here, \( P \) denotes the error in the constraints and \( Q \) the error in the optimality conditions. For the exact optimal solution, one must have \( P = 0 \) and \( Q = 0 \). For a numerical approximation to the optimal solution, convergence can be characterized by the relations

\[ P \leq \epsilon_1, \tag{36a} \]
\[ Q \leq \epsilon_2. \tag{36b} \]

where \( \epsilon_1 \) and \( \epsilon_2 \) are preselected, small, positive numbers (for instance, \( \epsilon_1 = 10^{-10} \) and \( \epsilon_2 = 10^{-8} \)).

\textbf{5.4. Algorithm Description.} The sequential gradient-restoration algorithm (SGRA) is an iterative procedure, which includes a sequence of two-phase cycles, each cycle including a gradient phase and a restoration phase. This technique is designed to achieve the decrease in the functional \( I \) between the endpoints of each cycle, while the constraints are satisfied to a predetermined accuracy. The two phases of a cycle are called the gradient phase and the restoration phase.

The gradient phase is started only when Ineq. (36a) is satisfied. It involves one iteration and is designed to decrease the value of the augmented functional \( J \), while satisfying the linearized constraints.

The restoration phase is started only when Ineq. (36a) is violated. It involves one or more iterations, each designed to decrease the constraint error \( P \), while the constraints are satisfied to the first order and the norm squared of the variations of the control vector \( u(t) \), the parameter vector \( \pi \), and the initial state vector \( x(0) \) is minimized. The restoration phase is terminated whenever Ineq. (36a) is satisfied.

The algorithm as a whole is terminated whenever Ineqs. (36) are both satisfied.
Let \( x(t) \), \( u(t) \), \( \pi \) denote the nominal functions; let \( \tilde{x}(t) \), \( \tilde{u}(t) \) \( \tilde{\pi} \) denote the varied functions; and let \( \Delta x(t) \), \( \Delta u(t) \), \( \Delta \pi \) denote the perturbations of \( x(t) \), \( u(t) \), \( \pi \) about the nominal values. Assume that the perturbations \( \Delta x(t) \), \( \Delta u(t) \), \( \Delta \pi \) are linear in the stepsize \( a \), where \( a > 0 \); and let \( A(t) \), \( B(t) \), \( C \) denote the perturbations per unit stepsize. Then, the following relations hold:

\[
\begin{align*}
\tilde{x}(t) &= x(t) + \Delta x(t) = x(t) + aA(t), \\
\tilde{u}(t) &= u(t) + \Delta u(t) = u(t) + aB(t), \\
\tilde{\pi} &= \pi + \Delta \pi = \pi + aC.
\end{align*}
\] (37a) (37b) (37c)

Therefore, each iteration of the gradient phase and the restoration phase includes two distinct operations: (i) the determination of the basic functions \( A(t) \), \( B(t) \), \( C \); and (ii) the determination of the stepsize \( a \).

### 5.4.1. Gradient Phase (GP)

Minimize the quadratic functional

\[
I_{GP} = \int_0^1 (f_x^T A + f_u^T B + f_{\pi}^T C) dt + \left( h_x^T A + h_{\pi}^T C \right)_0 + \left( g_x^T A + g_{\pi}^T C \right)_1 \\
+ \frac{1}{2} [ B^T B dt + C^T C + (A^T A)_0 ]
\] (38)

with respect to the vectors \( A(t) \), \( B(t) \), \( C \) which satisfy the linearized constraints

\[
\begin{align*}
\dot{A} + \phi_x^T A + \phi_u^T B + \phi_{\pi}^T C &= 0, & 0 \leq t \leq 1, \\
(\omega_x^T A + \omega_{\pi}^T C)_0 &= 0, \\
(\psi_x^T A + \psi_{\pi}^T C)_1 &= 0.
\end{align*}
\] (39a) (39b) (39c)

From calculus of variations, it is known that this is an optimal control problem of the Bolza type. It can be recast as that of minimizing the augmented functional

\[
J_{GP} = I_{GP} + L_{GP}
\] (40)

subject to (39), where \( L_{GP} \) denotes the Lagrangian functional

\[
L_{GP} = \int_0^1 \lambda^T (A + \phi_x^T A + \phi_u^T B + \phi_{\pi}^T C) dt + \sigma^T (\omega_x^T A + \omega_{\pi}^T C)_0 \\
+ \mu^T (\psi_x^T A + \psi_{\pi}^T C)_1.
\] (41)

In Eq. (41), \( \lambda(t) \) denotes an \( n \)-vector Lagrange multiplier, \( \sigma \) an \( a \)-vector Lagrange multiplier, and \( \mu \) a \( b \)-vector Lagrange multiplier.
The first-order optimality conditions for the above problem take the form

\[
\dot{\lambda} - f_x - \phi_x \lambda = 0, \quad 0 \leq t \leq 1, \tag{42a}
\]

\[
f_u + \phi_u \lambda + B = 0, \quad 0 \leq t \leq 1, \tag{42b}
\]

\[
\int_0^1 (f_\pi + \phi_\pi \lambda) dt + (h_\pi + \omega_\pi \sigma) 0 + (\varepsilon_\pi + \psi_\pi \mu)_1 + C = 0, \tag{42c}
\]

\[
(- \lambda + h_x + \omega_x \sigma + A)_0 = 0, \tag{42d}
\]

\[
(\lambda + \varepsilon_x + \psi_x \mu)_1 = 0. \tag{42e}
\]

Summarizing, we seek the functions A(t), B(t), C and the multipliers λ(t), σ, μ such that the feasibility equations (39) and the optimality conditions (42) are satisfied. Equations (39) and (42) constitute a linear two-point boundary-value problem (LTP-BVP), which can be solved with the method of particular solutions.

### 5.4.2. Restoration Phase (RP).

Minimize the quadratic functional

\[
I_{RP} = (1/2) \int_0^1 \dot{B}^T B dt + C^T C + (A^T A)_0, \tag{43}
\]

with respect to the vectors A(t), B(t), C which satisfy the linearized constraints

\[
\dot{A} + \phi_x A + \phi_u B + \phi_\pi C + (\dot{x} + \phi) = 0, \quad 0 \leq t \leq 1, \tag{44a}
\]

\[
(\omega_x A + \omega_\pi C + \omega)_0 = 0, \tag{44b}
\]

\[
(\psi_x A + \psi_\pi C + \psi)_1 = 0. \tag{44c}
\]

From calculus of variations, it is known that this is an optimal control problem of the Bolza type. It can be recast as that of minimizing the augmented functional

\[
J_{RP} = I_{RP} + L_{RP}, \tag{45}
\]

subject to (44), where \( L_{RP} \) denotes the Lagrangian functional

\[
L_{RP} = \int_0^1 \lambda^T (\dot{A} + \phi_x A + \phi_u B + \phi_\pi C + \dot{x} + \phi) dt + \sigma^T (\omega_x A + \omega_\pi C + \omega)_0 + \mu^T (\psi_x A + \psi_\pi C + \psi)_1. \tag{46}
\]

In Eq. (46), \( \lambda(t) \) denotes an n-vector Lagrange multiplier, \( \sigma \) an a-vector Lagrange multiplier, and \( \mu \) a b-vector Lagrange multiplier.

The first-order optimality conditions for the above problem take the form
\[
\dot{\lambda} - \phi_x \lambda = 0, \quad 0 \leq t \leq 1, \quad (47a)
\]
\[
\phi_u \lambda + B = 0, \quad 0 \leq t \leq 1, \quad (47b)
\]
\[
\int_0^1 (\phi_x \lambda) dt + (\omega_x \sigma)_0 + (\psi_x \mu)_1 + C = 0, \quad (47c)
\]
\[
(-\lambda + \omega_x \sigma + A)_0 = 0, \quad (47d)
\]
\[
(\lambda + \psi_x \mu)_1 = 0. \quad (47e)
\]

Summarizing, we seek the functions \(A(t), B(t), C\) and the multipliers \(\lambda(t), \sigma, \mu\) such that the feasibility equations (44) and the optimality conditions (47) are satisfied. Equations (44) and (47) constitute a linear two-point boundary-value problem (LTP-BVP), which can be solved with the method of particular solutions.

5.4.3. First-Variation Properties. The basic functions \(A(t), B(t), C\) solving Problems (GP) and (RP) are endowed with some first-variation properties, which are explained below.

For the gradient phase, it can be shown that
\[
\delta I = \delta J = aQ, \quad (48a)
\]
\[
\delta P = 0, \quad (48b)
\]
where \(I\) is the original functional, \(J\) is the augmented functional, \(P\) is the constraint error, and \(Q\) is the error in the optimality conditions. Clearly, the decrease of the functionals \(I\) and \(J\) is guaranteed for a gradient stepsize \(a\) sufficiently small.

For the restoration phase, it can be shown that
\[
\delta P = -2aP, \quad (49)
\]
where \(P\) is the constraint error. Clearly, the decrease of the constraint error is guaranteed for a restoration stepsize \(a\) sufficiently small.

5.4.4. Linear Two-Point Boundary-Value Problem. Both the gradient phase and the restoration phase of SGRA involve solving a linear two-point boundary-value problem.
(LTP-BVP), which can be recast in a very general form. The generalized LTP-BVP includes the feasibility equations

\[
\begin{align*}
\dot{\lambda} + \phi_x^T A + \phi_u^T B + \phi_{\pi}^T C + K_1(x + \phi) &= 0, & 0 \leq t \leq 1, \\
(\omega_x^T A + \omega_{\pi}^T C + K_1\omega) &= 0, \\
(\psi_x^T A + \psi_{\pi}^T C + K_1\psi) &= 0.
\end{align*}
\]

(50a) \hspace{1cm} (50b) \hspace{1cm} (50c)

and the optimality conditions

\[
\begin{align*}
\dot{\lambda} - K_2f_{x} - \phi_x\lambda &= 0, & 0 \leq t \leq 1, \\
K_2 f_u + \phi_u\lambda + B &= 0, & 0 \leq t \leq 1, \\
\int_0^1 (K_2f_{\pi} + \phi_{\pi}\lambda)dt + (K_2h_{\pi} + \omega_{\pi}\sigma)_0 + (K_2g_{\pi} + \psi_{\pi}\mu)_1 + C &= 0, \\
(-\lambda + K_2h_x + \omega_x\sigma + A)_0 &= 0, \\
(\lambda + K_2g_x + \psi_x\mu)_1 &= 0.
\end{align*}
\]

(51a) \hspace{1cm} (51b) \hspace{1cm} (51c) \hspace{1cm} (51d) \hspace{1cm} (51e)

with \( K_1 = 0, K_2 = 1 \) for the gradient phase and \( K_1 = 1, K_2 = 0 \) for the restoration phase.

\section*{5.4.5. Method of Particular Solutions.}
The technique used in solving the LTP-BVP (50)-(51) requires the execution of \( b+1 \) independent backward sweeps of the differential system, each characterized by a different value of the multiplier \( \mu \). We recall that \( b \) is the number of final conditions.

First, we consider Eqs. (51a), (51c), (51e). We observe that, if \( \mu \) is assigned, \( \lambda(1) \) can be computed with Eq. (51e), \( \lambda(t) \) can be computed by backward integration of (51a), and \( B(t) \) can be computed with Eq. (51b). With \( \lambda(t) \) given, \( \lambda(0) \) is known. Therefore, Eqs. (50b), (51c), (51d) constitute a system of \( a+n+p \) linear equations in which the unknowns are the \( a+n+p \) components of the vectors \( \sigma, A(0), C \). For this system to have a unique solution, the following condition must hold:

\[
\text{det}[\omega_x^T \omega_x + \omega_{\pi}^T \omega_{\pi}] = 0.
\]

(52)

With \( A(0) \) known, \( A(t) \) is obtained by forward integration of (50a). The sweep is now completed and leads to satisfaction of the system (50)-(51), except (50c).
In order to satisfy Eq. (50c) and because the system (50)-(51) is nonhomogenous, b + 1 independent sweeps must be executed employing b + 1 different multiplier vectors \( \mu_i \), \( i = 1, \ldots, b + 1 \). The first b sweeps are performed by choosing the vectors \( \mu_1, \ldots, \mu_b \) to be the columns of the identity matrix of order b. The last sweep is executed by choosing \( \mu_{b+1} \) to be the null vector. As a result, one generates the functions and multipliers

\[
A_i(t), B_i(t), C_i, \lambda_i(t), \sigma_i, \mu_i, \quad i = 1, \ldots, b + 1.
\]

(53)

Now, we introduce the b + 1 undetermined, scalar constants \( k_i \) and form the linear combinations

\[
A(t) = \sum k_i A_i(t),
\]

(54a)

\[
B(t) = \sum k_i B_i(t),
\]

(54b)

\[
C = \sum k_i C_i,
\]

(54c)

and

\[
\lambda(t) = \sum k_i \lambda_i(t),
\]

(55a)

\[
\sigma = \sum k_i \sigma_i,
\]

(55b)

\[
\mu = \sum k_i \mu_i,
\]

(55c)

where the summations are taken over the index \( i \). The \( b + 1 \) coefficients \( k_i \) are obtained by forcing the linear combinations (54) to satisfy Eq. (50c), together with the normalization condition

\[
\sum k_i = 1.
\]

(56)

Once the constants \( k_i \) are known, the solution of the LTP-BVP is given by Eqs. (54)-(55).

5.4.6. Gradient Stepsize. With the functions \( A(t), B(t), C \) known, the one-parameter family of varied functions (37) can be formed. For this family, the functionals I, J, P take the following form:

\[
\tilde{I} = \tilde{I}(a),
\]

(57a)

\[
\tilde{J} = \tilde{J}(a),
\]

(57b)
\[ \tilde{P} = \tilde{P}(a). \] 

Then, the gradient stepsize \( a \) is computed by a one-dimensional search on the augmented functional \( \tilde{J}(a) \) until the following relations are satisfied:

\[ \tilde{J}(a) < \tilde{J}(0), \] 
\[ \tilde{P}(a) \leq P_* . \]  

Inequality (58a) ensures that the augmented functional decreases at each iteration. Inequality (58b) avoids excessive violation of the constraints. Note that \( P_* \) is a preselected number, not necessarily small (for instance, \( P_* = 10 \)).

The simplest way of ensuring satisfaction of Ineqs. (58) is to employ a bisection process, starting from the reference stepsize \( a = a_0 \). In turn, the reference stepsize \( a_0 \) can be obtained by the combination of a scanning process and a cubic interpolation process. With the scanning process, one brackets the minimum point of the function \( \tilde{J}(a) \). This operation is then followed by a cubic interpolation process, which is stopped when theoretically

\[ |\tilde{J}_{a}(a)| = 0, \]  

and when practically

\[ |\tilde{J}_{a}(a)/\tilde{J}_{a}(0)| \leq \epsilon_3, \]  

subject to an upper limit for the number of search steps \( N_s \). In this thesis, \( N_s = 5 \) and \( \epsilon_3 = 10^{-3} \). Once the stepsize \( a_0 \) has been selected, Ineqs. (58) must be checked. If satisfaction occurs, then the stepsize \( a_0 \) is accepted. If any violation occurs, then the stepsize \( a_0 \) must be bisected as many times as needed until satisfaction of Ineqs. (58) is finally achieved.

\textbf{5.4.7. Restoration Stepsize.} With the functions \( A(t) \), \( B(t) \), \( C \) known, the one-parameter family of varied functions (37) can be formed. For this one-parameter family, the constraint error (35a) becomes a function of the form
\[ \tilde{\gamma} = \tilde{\gamma}(\alpha). \]  

Then, the restoration stepsize is computed by a one-dimensional search on the constraint error \( \tilde{\gamma}(\alpha) \) until the following relation is satisfied:

\[ \tilde{\gamma}(\alpha) < \gamma(0). \]  

The simplest way of ensuring satisfaction of Ineqs. (60) is to employ a bisection process, starting from the reference stepsize \( a_0 = 1 \). This reference stepsize has the property of yielding one-step restoration for the case where the constraints are linear.

5.4.8. Descent Property of a Cycle. A descent property exists for a complete gradient-restoration cycle under the assumption of small stepsizes. Let \( a_g \) denote the gradient stepsize; and let \( a_r \) denote the restoration stepsize. It can shown that the gradient corrections are of \( O(a_g) \), while the restoration corrections are of \( O(a_r a_g^2) \). Hence, for \( a_g \) sufficiently small, the restoration corrections are negligible with respect to the gradient corrections. Therefore, the restoration phase preserves the descent property of the gradient phase.

More specifically, let \( I, \tilde{I}, I^* \) denote the values of the functional (30) at the beginning of the gradient phase, at the end of the gradient phase, and at the end of the subsequent restoration phase. Note that \( I \) and \( \tilde{I} \) are not comparable, since the constraints are not satisfied to the same accuracy. On the other hand, \( I \) and \( I^* \) are comparable, and the gradient stepsize \( a_g \) must be selected so that

\[ I^* \leq I. \]  

This inequality constitutes the descent property of a complete gradient-restoration cycle. In order to enforce it, one proceeds as follows. At the end of the restoration phase, one must verify Ineq. (61). If it is satisfied, the next gradient phase is started; otherwise, one returns to the previous gradient phase and bisect the gradient stepsize as many times as needed until, after restoration, Ineq. (61) is satisfied.
6. Numerical Results on Windshear Estimation, Standard Data Approach

For the numerical results in Sections 6-8, the primal formulation of SGRA was used. The method of particular solutions was employed in solving the linear two-point boundary-value problems (LTP-BVP) associated with the gradient phase and the restoration phase. The differential systems were integrated using Hamming's predictor-corrector method with a Runge-Kutta starting procedure (Ref. 57). Definite integrals were computed using a modified Simpson's rule. A standard Gaussian elimination routine was used in solving the linear algebraic systems.

In this section, numerical results on Problems (P1) and (P2) of Section 4 are given. The following quantities are described: the wind components \( W_x, W_h \); the state variables \( x, h, V, \gamma \) and the pitch attitude angle \( \tilde{\theta} \); the angle of attack \( \alpha \) and the power setting \( \beta \); and the multiplicative factors \( \pi_1, \pi_2, \pi_3 \) for thrust, drag, and lift.

In the standard data approach, the measurements of the altitude \( \bar{h} \), the relative velocity \( \tilde{V} \), and the pitch attitude angle \( \tilde{\theta} \) are employed. Since \( \bar{h}, \tilde{V}, \tilde{\theta} \) are data obtainable from both the DFDR (digital flight data recorder) of wide-body airplanes and the MFR (metal foil recorder) of narrow-body airplanes, the terminology "standard data approach" is adopted in this thesis. Use of EPR data is necessary, since the power setting \( \beta \) is treated as a known input.

6.1. Problem (P1). Standard Data Approach without Parameter Estimation. With reference to Problem (P1) of Section 4, it is assumed that the manufacturer-supplied thrust, drag, and lift data are reliable. Hence, no parameter estimation is involved here. The measured trajectory data employed include the altitude \( \bar{h} \), the relative velocity \( \tilde{V} \), the pitch attitude angle \( \tilde{\theta} \), and the engine pressure ratio (EPR). In the computations, the angle of attack \( \alpha \) is treated as a control, while the power setting \( \beta \) is treated as a known input.
The estimated wind components are given in Figs. 4A-4B. The results obtained employing the dynamic approach of this work are labeled "AAG", denoting Aeronautics Group of Rice University; the results obtained by Bach and Wingrove (Ref. 44) employing the kinematic approach are labeled "NASA" and are included for comparison. Along the first half of the trajectory, the AAG estimated horizontal wind \( W_x \) lies above the NASA estimated wind by about 20-30 ft/sec. However, for the second half of the trajectory, a good agreement is reached. The difference between the present results and the NASA results might be due to the inaccuracy of the manufacturer-supplied engine data and aerodynamic data. The horizontal wind velocity difference obtained here is \( \Delta W_x = 95 \) ft/sec, which is less than the value \( \Delta W_x = 122 \) ft/sec obtained by NASA. The AAG estimated vertical wind \( W_h \) shows better agreement with the NASA estimated wind; in particular, the sequence of updrafts and downdrafts is preserved. The vertical wind velocity difference obtained here is \( \Delta W_h = 52 \) ft/sec, which is close to the value \( \Delta W_h = 62 \) ft/sec obtained by NASA.

Figures 4C-4F show the profiles of the state variables, including the horizontal distance \( x \) (Fig. 4C), the altitude \( h \) (Fig. 4D), the relative velocity \( V \) (Fig. 4E), and the relative path inclination \( \gamma \) (Fig. 4F). For the horizontal distance, even though only two measured data are employed [the values \( \tilde{x}(0) \) and \( \tilde{x}(\tau) \)], the estimated horizontal distance, obtained by integrating the equations of motion, shows good agreement with that of the kinematic approach. Both the altitude profile and the relative path inclination profile approximate the DFDR data nicely. Note that there is no direct measurement of the relative path inclination; instead, \( \gamma \) can be recovered with the relation \( \gamma = \theta - \alpha \). The estimated relative velocity profile is quite different from that supplied by the DFDR, and the difference is especially noticeable in the first half of the trajectory. From this, we surmise that there is some inaccuracy in the estimated horizontal wind \( W_x \) during that period.
Figure 4G shows the pitch attitude angle $\theta$. As can be seen, the AAG estimated pitch approximates the trend of the DFDR data with good precision.

Figure 4H shows the angle of attack $\alpha$. It is clear that the AAG estimated angle of attack is almost the same as that obtained from the DFDR. This shows that the measured data on the angle of attack are believable.

Figure 4I shows the power setting $\beta$, which is computed from the given EPR data. Since $\beta$ is treated as a known input, only one curve is shown.

Figures 4J, 4K, 4L respectively describe the thrust, drag, and lift forces acting on the aircraft along the trajectory. From these diagrams, a clear understanding of the dynamic behavior of the aircraft during the windshear encounter can be achieved.

In Problem (P1), no parameter estimation is included. This means that we assume that the manufacturer-supplied engine data and aerodynamic data are dependable. However, the estimated horizontal wind is not sufficiently precise, and this can be attributed to poor estimation of the relative velocity. Better results can be achieved by including multiplicative factors for the thrust, the drag, and the lift. This is described next.

6.2. Problem (P2). Standard Data Approach with Parameter Estimation. In this section, numerical results on Problem (P2) are discussed; namely, Problem (P1) is modified with the inclusion of multiplicative factors for the thrust, drag, and lift. As explained in Section 4, two particular cases are studied. First, the manufacturer-supplied thrust data are considered to be dependable; hence, only the drag parameter $\pi_2$ and the lift parameter $\pi_3$ appear in the Problem (P2A). Then, a more general problem is studied, in which the presence of three parameters is considered: the thrust parameter $\pi_1$, the drag parameter $\pi_2$, and the lift parameter $\pi_3$; the resulting problem is called Problem (P2B). Penalty terms are added to the performance index being minimized, in order to ensure that the
estimated parameters $\pi_1$, $\pi_2$, $\pi_3$ are close to unity.

6.2.1. **Problem (P2A).** In this case, the manufacturer-supplied thrust data are considered dependable. The measured trajectory data include the altitude $\tilde{h}$, the relative velocity $\tilde{V}$, and the pitch attitude angle $\tilde{\theta}$. The angle of attack $\alpha$ is treated as a control, while the power setting $\beta$ is regarded as a known input.

The estimated wind components are given in Figs. 5A-5B. For the horizontal wind $W_h$, the AAG estimated result approximates the NASA estimated result with good precision. A horizontal wind velocity difference $\Delta W_h = 109$ ft/sec is obtained here, which is close to the value $\Delta W_h = 122$ ft/sec obtained by NASA. The AAG estimated vertical wind $W_\theta$ shows reasonable agreement with the NASA estimated wind; in particular, the sequence of updrafts and downdrafts is preserved, albeit with smaller amplitude. Here, a vertical wind velocity difference $\Delta W_\theta = 46$ ft/sec is obtained, which is less than the value $\Delta W_\theta = 62$ ft/sec obtained by NASA.

Figures 5C-5F show the profiles of the state variables, including the horizontal distance $x$ (Fig. 5C), the altitude $h$ (Fig. 5D), the relative velocity $V$ (Fig. 5E), and the relative path inclination $\gamma$ (Fig. 5F). For the horizontal distance, even though only two measured data are employed [the values $\tilde{x}(0)$ and $\tilde{x}(\tau)$], the estimated horizontal distance, obtained by integrating the equations of motion, shows reasonable agreement with that of the kinematic approach. Both the altitude profile and the relative path inclination profile approximate the DFDR data nicely. Note that there is no direct measurement of the relative path inclination; instead, $\gamma$ can be obtained by using the relation $\gamma = \theta - \alpha$. The estimated relative velocity profile approximates the DFDR data in the least-square sense. This result is different from that of Problem (P1), where the estimation of the relative velocity is not satisfactory. The reason for the improvement is the inclusion of the multiplicative factors for drag and lift in this problem. Hence, a satisfactory simulated
trajectory is obtained.

Figure 5G shows the pitch attitude angle $\theta$. As can be seen, the AAG estimated pitch follows the trend of the DFDR data nicely until 20 sec before impact. The discrepancy apparent in the last 20 sec induces weaker updrafts and downdrafts in the AAG estimated vertical wind during that period.

Figure 5H shows the angle of attack $\alpha$. It is clear that the AAG estimated angle of attack is almost the same as that obtained from the DFDR. This shows that the measured data on the angle of attack are believable.

Figure 5I shows the power setting $\beta$, which is computed from the given EPR data. Since $\beta$ is treated as a known input, only one curve is shown.

Figures 5J, 5K, 5L respectively describe the thrust, drag, and lift forces acting on the aircraft along the trajectory. From these diagrams, a clear understanding of the dynamic behavior of the aircraft during the windshear encounter can be achieved.

In Problem (P2A), the multiplicative factors for drag and lift are treated as unknown parameters to be estimated. Namely, the drag and the lift are written as $D = \pi_2 D_\ast$, $L = \pi_3 L_\ast$. The numerical values of the parameters are found to be

$$\pi_2 = 0.854, \quad (62a)$$

$$\pi_3 = 0.803. \quad (62b)$$

This means that the drag is 85% of the value calculated from the manufacturer-supplied data, while the lift is 80%. Since the manufacturer-supplied thrust data are considered to be dependable, the thrust parameter is assumed to be $\pi_1 = 1$.

After including the multiplicative factors for drag and lift, the trajectory can be
approximated with better accuracy, and the AAG estimated wind components come closer to those of the kinematic approach. Hence, the numerical values (62) are considered reasonable.

As can be seen from Table 2, the estimated values of the parameters are nearly independent of the value of the weighting factor \( C_5 \) appearing in performance index (18). Indeed, the numerical results given in Fig. 5 correspond to the limiting case \( C_5 = 0 \).

6.2.2. Problem (P2B). In this case, the multiplicative factors \( \pi_1, \pi_2, \pi_3 \) for the thrust, drag, and lift are considered. The measured trajectory data include the altitude \( \tilde{h} \), the relative velocity \( \tilde{V} \), and the pitch attitude angle \( \tilde{\theta} \). The angle of attack \( \alpha \) is treated as a control, while the power setting \( \beta \) is regarded as a known input.

The estimated wind components are given in Figs. 6A-6B. For the horizontal wind \( W_x \), the AAG estimated result approximates the NASA estimated result with good precision. Here, we have a horizontal wind velocity difference \( \Delta W_x = 106 \text{ ft/sec} \), which is close to the value \( \Delta W_x = 122 \text{ ft/sec} \) obtained by NASA. The AAG estimated vertical wind \( W_h \) shows reasonable agreement with the NASA estimated wind; in particular, the sequence of updrafts and downdrafts is preserved, albeit with smaller amplitude. Here, we have a vertical wind velocity difference \( \Delta W_h = 46 \text{ ft/sec} \), which is less than the value \( \Delta W_h = 62 \text{ ft/sec} \) obtained by NASA.

Figures 6C-6F show the profiles of the state variables, including the horizontal distance \( x \) (Fig. 6C), the altitude \( h \) (Fig. 6D), the relative velocity \( V \) (Fig. 6E), and the relative path inclination \( \gamma \) (Fig. 6F). For the horizontal distance, even though only two measured data are employed [the values \( \tilde{x}(0) \) and \( \tilde{x}(\tau) \)], the estimated horizontal distance, obtained by integrating the equations of motion, shows reasonable agreement with that of the kinematic approach. Both the altitude profile and the relative path inclination profile
approximate the DFDR data nicely. Note that there is no direct measurement of the relative path inclination; instead, $\gamma$ can be obtained by using the relation $\gamma = \theta - a$. The estimated relative velocity profile approximates the DFDR data in the least-square sense. This result is different from that of Problem (P1), where the estimation of the relative velocity is not satisfactory. The reason for the improvement is the inclusion of the multiplicative factors for thrust, drag, and lift. Hence, a satisfactory simulated trajectory is obtained.

Figure 6G shows the pitch attitude angle $\theta$. As can be seen, the AAG estimated pitch follows the trend of the DFDR data nicely until 20 sec before impact. The discrepancy apparent in the last 20 sec induces weaker updrafts and downdrafts in the estimated vertical wind during that period.

Figure 6H shows the angle of attack $a$. It is clear that the AAG estimated angle of attack is almost the same as that obtained from the DFDR. This shows that the measured data on the angle of attack are believable.

Figure 6I shows the power setting $\beta$, which is computed from the given EPR data. Since $\beta$ is treated as a known input, only one curve is shown.

Figures 6J, 6K, 6L respectively describe the thrust, drag, and lift forces acting on the aircraft along the trajectory. From these diagrams, a clear understanding of the dynamic behavior of the aircraft during the windshear encounter can be achieved.

In Problem (P2B), the multiplicative factors for thrust, drag, and lift are treated as unknown parameters to be estimated. Namely, the thrust, the drag, and the lift are written as $T = \pi_1 T_\star$, $D = \pi_2 D_\star$, $L = \pi_3 L_\star$. The numerical values of these parameters are found to be
\[ \pi_1 = 1.019, \]  
\[ \pi_2 = 0.888, \]  
\[ \pi_3 = 0.822. \]  

This means that the drag is 89% of the value calculated from the manufacturer-supplied data, while the lift is 82% and the thrust is 102%. This justifies the assumption made in Problem (P2A), namely, trust the manufacturer-supplied thrust data are dependable.

The weighting factor \( C_5 \) [see Eq. (20)] has an important effect on the value of the estimated parameters. In Table 3, the estimated values of the parameters \( \pi_1, \pi_2, \pi_3 \) are given for different values of \( C_5 \). The value \( C_5 = 4.5 \times 10^5 \) yields a satisfactory trajectory approximation and wind estimation; indeed, this particular value yields the results (63) and is employed in Fig. 6.

6.3. Summary. The numerical values of the estimated multiplicative factors \( \pi_1, \pi_2, \pi_3 \) and the wind velocity differences \( \Delta W_x, \Delta W_y \), are summarized in Table 4. Based on the above numerical results, the following conclusions can be inferred.

(i) Without considering the multiplicative factors for thrust, drag, and lift [Problem (P1)], neither the relative velocity nor the wind components can be estimated satisfactorily.

(ii) With two parameters (namely, the multiplicative factors for drag and lift), the relative velocity can be estimated properly, the estimated results for the wind components are satisfactory, and the estimated results are independent of the values of the weighting coefficient \( C_5 \).

(iii) With three parameters (namely, the multiplicative factors for thrust, drag, and lift), the relative velocity and the wind components can be properly estimated if \( C_5 \) is in a suitable range; however, the estimated lift parameter \( \pi_3 \) is almost independent of the
value of the weighting factor $C_5$.

(iv) The numerical results for the two-parameter problem [Problem (P2A)] are close to those of the three-parameter problem [Problem (P2B)]. This justifies the assumption that the manufacturer-supplied thrust data are dependable. For the two-parameter problem, the results are almost independent of the value of the coefficient $C_5$ appearing in the performance index (18). Hence, the two-parameter problem, along with $C_5 = 0$, yields reasonably good estimated results, even though less computing effort is expended.
7. Numerical Results on Windshear Estimation, Full Data Approach

In this section, we give the numerical results for Problem (P3) of Section 4. In addition to the measured data $\bar{h}$, $\bar{V}$, $\bar{\theta}$, the inclusion of the horizontal distance $\bar{x}$ obtained from the kinematic approach is considered. Direct employment of EPR data is required, since the power setting $\beta$ is treated as a known input. The angle of attack $\alpha$ is treated as a control.

We recall one of the results of Section 6, dealing with the standard data approach: Problem (P2A), involving the multiplicative factors $\pi_2$, $\pi_3$ for drag and lift, yields almost the same wind estimation as Problem (P2B), involving the multiplicative factors $\pi_1$, $\pi_2$, $\pi_3$ for thrust, drag, and lift. Therefore, in Problem (P3) below, we set $\pi_1 = 1$ and we employ the same formulation as in Problem (P2A), except for fact that different collections of measured trajectory data are considered.

7.1. Problem (P3). Full Data Approach with Parameter Estimation. In this problem, the manufacturer-supplied thrust data are considered to be dependable. The measured trajectory data include the horizontal distance $\bar{x}$, the altitude $\bar{h}$, the relative velocity $\bar{V}$, and the pitch attitude angle $\bar{\theta}$. The angle of attack $\alpha$ is treated as a control, while the power setting $\beta$ is regarded as a known input. Compared with the standard data approach [Problem (P2A) of Section 6], the full data approach is characterized by the inclusion of the horizontal distance $\bar{x}$. Note that $\bar{x}$ is not available from the DFDR; instead, it is obtained from the kinematic approach.

The estimated wind components are given in Figs. 7A-7B. For the horizontal wind $W_x$, the AAG estimated result approximates the NASA estimated result with better precision than the standard data approach [Problem (P2A)]. Here, we have a horizontal wind velocity difference $\Delta W_x = 117 \text{ ft/sec}$, which is close to the value $\Delta W_x = 122 \text{ ft/sec}$ obtained by NASA. The AAG estimated vertical wind $W_h$ shows reasonable agreement
with the NASA estimated wind; in particular, the sequence of updrafts and downdrafts is preserved, albeit with smaller amplitude. Here, we have a vertical wind velocity difference $\Delta W_h = 44$ ft/sec, which is less than the value $\Delta W_h = 62$ ft/sec obtained by NASA.

Figures 7C-7F show the profiles of the state variables, including the horizontal distance $x$ (Fig. 7C), the altitude $h$ (Fig. 7D), the relative velocity $V$ (Fig. 7E), and the relative path inclination $\gamma$ (Fig. 7F). For the horizontal distance, a very good approximation is reached; this is due to the employment of $\bar{x}$ obtained from the kinematic approach. Both the altitude profile and the relative path inclination profile approximate the DFDR data nicely. Note that there is no direct measurement of the relative path inclination. Instead, $\gamma$ is obtained by using the relation $\gamma = \theta - \alpha$. The estimated relative velocity profile approximates the DFDR data in the least-square sense.

Figure 7G shows the pitch attitude angle $\theta$. As can be seen, the estimated pitch follows the trend of the DFDR data nicely until 20 sec before impact. For the last 20 sec, the apparent discrepancy induces weaker updrafts and downdrafts in the AAG estimated vertical wind $\bar{W}_h$ during that period.

Figure 7H shows the angle attack $\alpha$. It is clear that the estimated angle of attack is almost the same as that obtained from the DFDR. This shows that the measured data on the angle of attack are believable.

Figure 7I shows the power setting $\beta$, which is computed from the given EPR data. Since $\beta$ is treated as a known input, only one curve is shown.

Figures 7J, 7K, 7L respectively describe the thrust, drag, and lift forces acting on the aircraft along the trajectory. From these diagrams, a clear understanding of the dynamic behavior of the aircraft during the windshear encounter can be achieved.
In Problem (P3), the multiplicative factors for drag and lift are treated as unknown parameters to be estimated. Namely, the drag and the lift are written as $D = \pi_2 D_\ast$, $L = \pi_3 L_\ast$. The numerical values of the parameters are found to be

\begin{align*}
\pi_2 &= 0.797, \\
\pi_3 &= 0.809.
\end{align*}

This means that the drag is 80% of the value calculated from the manufacturer-supplied data, while the lift is 81%.

7.2. **Summary.** The numerical values of the estimated multiplicative factors $\pi_2$, $\pi_3$ and the wind velocity differences are summarized in Table 4. Based on the above analyses, it can be inferred that adding the measured horizontal distance $\tilde{x}$ improves the accuracy of the estimated horizontal wind $W_x$, albeit by only a limited amount. This can be seen by comparing the results of the full data approach [Problem (P3)] with those of the standard data approach [Problem (P2A)].
8. Numerical Results on Windshear Estimation, Reduced Data Approach

In this section, we discuss Problem (P4) of Section 4, which is obtained from Problem (P2A) by removing either one or two of the measurements $\bar{h}, \bar{V}, \bar{\theta}$. Direct employment of EPR data is required, since the power setting $\beta$ is treated as a known input. The angle of attack $\alpha$ is treated as a control.

Once more, we recall one of the results of Section 6, dealing with the standard data approach: Problem (P2A), involving the multiplicative factors $\pi_2, \pi_3$ for drag and lift, yields almost the same wind estimation as Problem (P2B), involving the multiplicative factors $\pi_1, \pi_2, \pi_3$ for thrust, drag, and lift. Therefore, in Problem (P4), we set $\pi_1 = 1$ and we employ the same formulation as in Problem (P2A), except for the fact that different collections of the measured trajectory data are considered.

8.1. Problem (P4). Reduced Data Approach with Parameter Estimation. In this problem, the manufacturer-supplied thrust data are considered to be dependable. The measured trajectory data include either one or two of the measurements $\bar{h}, \bar{V}, \bar{\theta}$. With this approach, the relative importance of each measured quantity on the identifiability of the wind components can be assessed.

8.1.1. Problem (P4A). In this case, the measured trajectory data include the relative velocity $\bar{V}$ and the pitch attitude angle $\bar{\theta}$; by comparison with the standard data approach (Problem (P2A)), the altitude $\bar{h}$ is discarded. The angle of attack $\alpha$ is treated as a control, while the power setting $\beta$ is regarded as a known input.

The estimated wind components are given in Figs. 8A-8B. For the horizontal wind $W_x$, the AAG estimated result approximates the NASA estimated result with good precision. A horizontal wind velocity difference $\Delta W_x = 116$ ft/sec is obtained here, which is close to the value $\Delta W_x = 122$ ft/sec obtained by NASA. The AAG estimated vertical
wind $W_h$ lies above the NASA estimated wind for the first half of the trajectory. For the second half, the sequence of updrafts and downdrafts is preserved, albeit with smaller amplitude, and the vertical wind profile shifts downward with respect to that of the standard data approach [Problem (P2A)]. Here, a vertical wind velocity difference $\Delta W_h = 49$ ft/sec is obtained, which is less than the value $\Delta W_h = 62$ ft/sec obtained by NASA.

Figures 8C-8F show the profiles of the state variables, including the horizontal distance $x$ (Fig. 8C), the altitude $h$ (Fig. 8D), the relative velocity $V$ (Fig. 8E), and the relative path inclination $\gamma$ (Fig. 8F). For the horizontal distance, even though only two measured data are employed [the values $\bar{x}(0)$ and $\bar{x}(\tau)$], the estimated horizontal distance, obtained by integrating the equations of motion, shows reasonable agreement with that of the kinematic approach. The estimated altitude profile is quite different from the DFDR data, except near the initial and final points. The explanation for this result is that the measured altitude $\bar{h}$ has been ignored everywhere, except at the initial and final points. The relative path inclination profile approximates the DFDR data reasonably. Note that there is no direct measurement of the relative path inclination angle; instead, $\gamma$ can be obtained using the relation $\gamma = \theta - a$. The estimated relative velocity profile approximates the DFDR data in the least-square sense.

Figure 8G shows the pitch attitude angle $\theta$. As can be seen, the estimated pitch follows the trend of the DFDR data with reasonable precision, except for some oscillations, which in turn induce weak updrafts and downdrafts in the AAG estimated vertical wind $W_h$.

Figure 8H shows the angle of attack $a$. It is clear that the estimated angle of attack is almost the same as that obtained from the DFDR. This shows that the measured data on the angle of attack are believable.
Figure 8I shows the power setting $\beta$, which is computed from the given EPR data. Since $\beta$ is treated as a known input, only one curve is shown.

Figures 8J, 8K, 8L respectively describe the thrust, drag, and lift forces acting on the aircraft along the trajectory. From these diagrams, a clear understanding of the dynamic behavior of the aircraft during the windshear encounter can be achieved.

In this formulation, the multiplicative factors for drag and lift are treated as unknown parameters to be estimated. Namely, the drag and the lift are written as $D = \pi_2 D^*$, $L = \pi_3 L^*$. The numerical values of these parameters are found to be

\[
\pi_2 = 0.848, \quad (65a)
\]

\[
\pi_3 = 0.797. \quad (65b)
\]

This means that the drag is 85% of the value calculated from the manufacturer-supplied data, while the lift is 80%.

To sum up, the effect of removing the measured altitude $\overline{H}$ is the following. The AAG estimated horizontal wind stays close to the NASA estimated wind; however, this is not the case for the AAG estimated vertical wind. Except for the poor approximation of the altitude profile, other estimated trajectory data are reasonably consistent with the measured ones.

8.1.2. Problem (P4B). In this case, the measured trajectory data include the altitude $\overline{H}$ and the pitch attitude angle $\overline{\theta}$; by comparison with the standard data approach [Problem (P2A)], the relative velocity $\overline{V}$ is discarded. The angle of attack $\alpha$ is treated as a control, while the power setting $\beta$ is regarded as a known input.

The estimated wind components are given in Figs. 9A-9B. For the horizontal wind $W_x$, the AAG estimated result approximates the NASA estimated result with good
precision, but there is some deterioration in the first half of the trajectory. A horizontal wind velocity difference $\Delta W_x = 103$ ft/sec is obtained here, which is slightly less than the value $\Delta W_x = 122$ ft/sec obtained by NASA. The AAG estimated vertical wind $W_h$ shows reasonable agreement with the NASA estimated wind; in particular, the sequence of updrafts and downdrafts is preserved, albeit with smaller amplitude. Here, a vertical wind velocity difference $\Delta W_h = 46$ ft/sec is obtained, which is less than the value $\Delta W_h = 62$ ft/sec obtained by NASA. Overall, the estimated results deteriorate slightly with respect to those of the standard data approach (Problem (P2A)).

Figures 9C-9F show the profiles of the state variables, including the horizontal distance $x$ (Fig. 9C), the altitude $h$ (Fig. 9D), the relative velocity $V$ (Fig. 9E), and the relative path inclination $\gamma$ (Fig. 9F). For the horizontal distance, even though only two measured data are employed [the values $\bar{x}(0)$ and $\bar{x}(\tau)$], the estimated horizontal distance, obtained by integrating the equations of motion, shows reasonable agreement with that of the kinematic approach. Both the altitude profile and the relative path inclination profile approximate the DFDR data nicely. Note that there is no direct measurement of the relative path inclination; instead, $\gamma$ can be obtained using the relation $\gamma = \theta - \alpha$. For the relative velocity profile, even though only the terminal values $\bar{V}(0)$ and $\bar{V}(\tau)$ are employed, a satisfactory approximation is reached. However, there is some deterioration of the estimated relative velocity for the first 15 sec; in turn, this causes a deterioration in the estimated horizontal wind during that period.

Figure 9G shows the pitch attitude angle $\theta$. As can be seen, the estimated pitch follows the trend of the DFDR data with reasonable precision, except for some oscillations, which in turn induce weak updrafts and downdrafts in the AAG estimated vertical wind $W_h$. 
Figure 9H shows the angle of attack $\alpha$. It is clear that the estimated angle of attack is almost the same as that obtained from the DFDR. This shows that the measured data on the angle of attack are believable.

Figure 9I shows the power setting $\beta$, which is computed from the given EPR data. Since $\beta$ is treated as a known input, only one curve is shown.

Figures 9J, 9K, 9L respectively describe the thrust, drag, and lift forces acting on the aircraft along the trajectory. From these diagrams, a clear understanding of the dynamic behavior of the aircraft during the windshear encounter can be achieved.

In this formulation, the multiplicative factors for drag and lift are treated as unknown parameters to be estimated. Namely, the drag and the lift are written as $D=\pi_2D^*$, $L=\pi_3L^*$. The numerical values of these parameters are found to be

$$\pi_2=0.909, \quad (66a)$$

$$\pi_3=0.864. \quad (66b)$$

This means that the drag is 91% of the value calculated from the manufacturer-supplied data, while the lift is 86%.

To sum up, the effect of removing the measured relative velocity $\tilde{V}$ is the following. The AAG estimated horizontal wind is satisfactory, except for some slight deterioration for the first 15 sec. The AAG estimated vertical wind follows closely the NASA estimated wind. Hence, we infer that, if the measured relative velocity is removed, reasonable estimation results can still be obtained.

8.1.3. Problem (P4C). In this case, the measured trajectory data include the altitude $\tilde{h}$ and the relative velocity $\tilde{V}$; by comparison with the standard data approach [(Problem (P2A))], the pitch attitude angle $\tilde{\theta}$ is discarded. The angle of attack $\alpha$ is treated as a
control, while the power setting $\beta$ is regarded as a known input.

The estimated wind components are given in Figs. 10A-10B. For the horizontal wind $W_x$, the AAG estimated result approximates the NASA estimated result with reasonable precision only for the first half of the trajectory; for the second half, the AAG estimated wind is totally different from the NASA estimated wind. A horizontal wind velocity difference $\Delta W_x = 43$ ft/sec is obtained here, which is much smaller than the value $\Delta W_x = 122$ ft/sec obtained by NASA. The AAG estimated vertical wind $W_h$ follows closely the NASA estimated wind in the first half along the trajectory; for the second half, the correct sequence of updrafts and downdrafts is totally lost; however, a vertical wind velocity difference $\Delta W_h = 54$ ft/sec is obtained here, which is close to the value $\Delta W_h = 62$ ft/sec obtained by NASA.

Figures 10C-10F show the profiles of the state variables, including the horizontal distance $x$ (Fig. 10C), the altitude $h$ (Fig. 10D), the relative velocity $V$ (Fig. 10E), and the relative path inclination $\gamma$ (Fig. 10F). For the horizontal distance, even though only two measured data are employed [the values $\tilde{x}(0)$ and $\tilde{x}(\tau)$], the estimated horizontal distance, obtained by integrating the equations of motion, shows reasonable agreement with that of the kinematic approach; however, there is some slight deterioration vis-a-vis the standard data approach [Problem (P2A)]. The altitude profile approximates the DFDR data nicely. The relative velocity profile approximates the measured data in the least-square sense. The relative path inclination profile is not acceptable; this is a consequence of having ignored the measured pitch attitude angle $\tilde{\vartheta}$. Note that there is no direct measurement of the relative path inclination; instead, $\gamma$ can be obtained using the relation $\gamma = \vartheta - \alpha$.

Figure 10G shows the pitch attitude angle $\vartheta$. Since the measured pitch is ignored, the estimated pitch is not satisfactory; in particular, this is true for the last 20 sec. The discrepancy induces the loss of the correct sequence of updrafts and downdrafts in the
estimated vertical wind as well as the erroneous result in the estimated horizontal wind during that period.

Figure 10H shows the angle of attack \( \alpha \). It is clear that the estimated angle of attack is almost the same as that obtained from the DFDR. This shows that the measured data on the angle of attack are believable.

Figure 10I shows the power setting \( \beta \), which is computed from the given EPR (engine pressure ratio). Since \( \beta \) is treated as a known input, only one curve is shown.

Figures 10J, 10K, 10L respectively describe the thrust, drag, and lift forces acting on the aircraft along the trajectory. From these diagrams, a clear understanding of the dynamic behavior of the aircraft during the windshear encounter can be achieved.

In this formulation, the multiplicative factors for drag and lift are treated as unknown parameters to be estimated. Namely, the drag and the lift are written as

\[
D = \pi_2 D_s, \\
L = \pi_3 L_s.
\]

The numerical values of these parameters are found to be

\[
\pi_2 = 0.808, \quad (67a) \\
\pi_3 = 0.793. \quad (67b)
\]

This means that the drag is 81% of the value calculated from the manufacturer-supplied data, while the lift is 79%.

To sum up, the effect of neglecting the measured pitch attitude angle \( \tilde{\theta} \) is the following. The AAG estimated horizontal wind is characterized by less than half the wind velocity difference obtained by NASA. The AAG estimated vertical wind loses the correct sequence of updrafts and downdrafts. Hence, we conclude that the availability of the measured pitch attitude angle is crucial to the correctness of the windshear estimation results.
8.1.4. Problem (P4D). In this case, the measured trajectory data employed include only the altitude $\tilde{h}$; by comparison with the standard data approach [Problem (P2A)], the relative velocity $\tilde{V}$ and the pitch attitude angle $\tilde{\theta}$ are discarded. The angle of attack $\alpha$ is treated as a control, while the power setting $\beta$ is regarded as a known input.

The estimated wind components are given in Figs. 11A-11B. For the horizontal wind $W_x$, the AAG estimated result approximates the NASA estimated result with reasonable precision for only the first 15 sec; elsewhere, the AAG estimated wind is totally different from the NASA estimated wind. A horizontal wind velocity difference $\Delta W_x = 41$ ft/sec is obtained here, which is much smaller than the value $\Delta W_x = 122$ ft/sec obtained by NASA. The AAG estimated vertical wind $W_h$ is quite different from the NASA estimated wind. The sequence of updrafts and downdrafts is totally lost; however, a vertical wind velocity difference $\Delta W_h = 50$ ft/sec is obtained here, which is close to the value $\Delta W_h = 62$ ft/sec obtained by NASA.

Figures 11C-11F show the profiles of the state variables, including the horizontal distance $x$ (Fig. 11C), the altitude $h$ (Fig. 11D), the relative velocity $V$ (Fig. 11E), and the relative path inclination $\gamma$ (Fig. 11F). For the horizontal distance, even though only two measured data are employed [the values $\hat{x}(0)$ and $\hat{x}(\tau)$], the estimated horizontal distance, obtained by integrating the equations of motion, shows reasonable agreement with that of the kinematic approach; however, there is some slight deterioration vis-a-vis the standard data approach [Problem (P2A)]. The altitude profile approximates the DFDR data nicely; this is no surprise, since the measured altitude is employed. The relative velocity profile approximates the measured data in a reasonable way, even though only two measured data are employed [the values $\hat{V}(0)$ and $\hat{V}(\tau)$]. However, the velocity profile shows deterioration when compared with that of the standard data approach [Problem (P2A)]. The relative path inclination profile is not acceptable; this is a consequence of having
ignored the measured pitch. Note that there is no direct measurement of the relative path inclination; instead, \( \gamma \) can be obtained using the relation \( \gamma = \theta - a \).

Figure 11G shows the pitch attitude angle \( \theta \). Since the measured pitch is ignored, the estimated pitch is not satisfactory.

Figure 11H shows the angle of attack \( a \). It is clear that the estimated angle of attack is almost the same as that obtained from the DFDR. This shows that the measured data on the angle of attack are believable.

Figure 11I shows the power setting \( \beta \), which is computed from the given EPR data. Since \( \beta \) is treated as a known input, only one curve is shown.

Figures 11J, 11K, 11L respectively describe the thrust, drag, and lift forces acting on the aircraft along the trajectory. From these diagrams, a clear understanding of the dynamic behavior of the aircraft during the windshear encounter can be achieved.

In this formulation, the multiplicative factors for drag and lift are treated as unknown parameters to be estimated. Namely, the drag and the lift are written as \( D = \pi_2 D_* \), \( L = \pi_3 L_* \). The numerical values of these parameters are found to be

\[
\pi_2 = 1.167, \quad (68a) \\
\pi_3 = 0.848. \quad (68b)
\]

This means that the drag is 117% of the value calculated from the manufacturer-supplied data, while the lift is 85%.

To sum up, the effect of removing the measured relative velocity \( \tilde{V} \) and the measured pitch attitude angle \( \tilde{\theta} \) is the following. The AAG estimated horizontal wind is characterized by one-third of the wind velocity difference obtained by NASA. The AAG estimated
vertical wind loses the correct sequence of updrafts and downdrafts. The drag multiplicative factor $\pi_2 = 1.167$ obtained here is too high when compared with the value $\pi_2 = 0.854$ obtained with the standard data approach [Problem (P2A)]. Hence, we conclude that using the measured altitude does not yield good estimation results.

8.1.5. Problem (P4E). In this case, the measured trajectory data include only the relative velocity $\tilde{V}$; by comparison with the standard data approach [Problem (P2A)], the altitude $h$ and the pitch attitude angle $\theta$ are discarded. The angle of attack $a$ is treated as a control, while the power setting $\beta$ is regarded as a known input.

The estimated wind components are given in Figs. 12A-12B. For the horizontal wind $W_x$, the AAG estimated result approximates the NASA estimated result with reasonable precision for only the first 40 sec; for the remaining 15 sec, the AAG estimated horizontal wind is totally different from the NASA estimated wind. A horizontal wind velocity difference $\Delta W_x = 46$ ft/sec is obtained here, which is much less than the value $\Delta W_x = 122$ ft/sec obtained by NASA. The AAG estimated vertical wind $W_h$ lies below the NASA estimated wind, and the sequence of updrafts and downdrafts is missing. As a result, the AAG estimated vertical wind is not satisfactory; however, a vertical wind velocity difference $\Delta W_h = 60$ ft/sec is obtained here, which is quite close to the value $\Delta W_h = 62$ ft/sec obtained by NASA.

Figures 12C-12F show the profiles of the state variables, including the horizontal distance $x$ (Fig. 12C), the altitude $h$ (Fig. 12D), the relative velocity $V$ (Fig. 12E), and the relative path inclination $\gamma$ (Fig. 12F). For the horizontal distance, even though only two measured data are employed [the values $\tilde{x}(0)$ and $\tilde{x}(\tau)$], the estimated horizontal distance, obtained by integrating the equations of motion, shows reasonable agreement with that of the kinematic approach; however, there is a deterioration vis-a-vis the standard data approach [Problem (P2A)]. Poor approximation of the altitude profile occurs near the
middle part of the trajectory. The estimated relative velocity profile approximates the measured data nicely; this is no surprise, since the measured relative velocity is employed. The approximation for the relative path inclination is not good; this is a consequence of having ignored the measured pitch. Note that there is no direct measurement of the relative path inclination; instead, $\gamma$ can be obtained using the relation $\gamma = \theta - \alpha$.

Figure 12G shows the pitch attitude angle $\theta$. Since the measured pitch is ignored, the estimated pitch is not satisfactory.

Figure 12H shows the angle of attack $\alpha$. It is clear that the estimated angle of attack is almost the same as that obtained from the DFDR. This shows that the measured data on the angle of attack are believable.

Figure 12I shows the power setting $\beta$, which is computed from the given EPR data. Since $\beta$ is treated as a known input, only one curve is shown.

Figures 12J, 12K, 12L respectively describe the thrust, drag, and lift forces acting on the aircraft along the trajectory. From these diagrams, a clear understanding of the dynamic behavior of the aircraft during the windshear encounter can be achieved.

In this formulation, the multiplicative factors for drag and lift are treated as unknown parameters to be estimated. Namely, the drag and the lift are written as $D = \pi_2 D_*$, $L = \pi_3 L_*$. The numerical values of these parameters are found to be

$$\pi_2 = 0.480,$$  
(69a)

$$\pi_3 = 0.796.$$  
(69b)

This means that the drag is 48% of the value calculated from the manufacturer-supplied data, while the lift is 80%.
To sum up, the effect of removing the measured relative velocity $\tilde{V}$ and the pitch attitude angle $\tilde{\theta}$ is the following. The AAG estimated horizontal wind is characterized by less than half the wind velocity difference obtained by NASA. The AAG estimated vertical wind loses the correct sequence of updrafts and downdrafts. The drag multiplicative factor obtained here, $\pi_2 = 0.480$, is too low when compared with the value $\pi_2 = 0.854$ of the standard data approach [Problem (P2A)]. Hence, we conclude using only the relative velocity does not yield good estimation results.

8.1.6. Problem (P4F). In this case, the measured trajectory data include only the pitch attitude angle $\tilde{\theta}$; by comparison with the standard data approach [Problem (P2A)], the altitude $\tilde{h}$ and the relative velocity $\tilde{V}$ are discarded. The angle of attack $\alpha$ is treated as a control, while the power setting $\beta$ is regarded as a known input.

The estimated wind components are given in Figs. 13A-13B. For the horizontal wind $W_x$, the AAG estimated result approximates the NASA estimated result with reasonable precision. A horizontal wind velocity difference $\Delta W_x = 108$ ft/sec is obtained here, which is close to the value $\Delta W_x = 122$ ft/sec obtained by NASA. The AAG estimated vertical wind $W_h$ lies above the NASA estimated vertical wind for the first 35 sec; for the remaining 20 sec, even though the sequence of updrafts and downdrafts is preserved, the AAG estimated vertical wind profile lies below that of NASA. A vertical wind velocity difference $\Delta W_h = 59$ ft/sec is obtained here, which is almost the same as the value $\Delta W_h = 62$ ft/sec obtained by NASA. However, the overall behavior of the AAG estimated vertical wind is not satisfactory.

Figures 13C-13F show the profiles of the state variables, including the horizontal distance $x$ (Fig. 13C), the altitude $h$ (Fig. 13D), the relative velocity $V$ (Fig. 13E), and the relative path inclination $\gamma$ (Fig. 13F). For the horizontal distance, even though only two measured data are employed [the values $\tilde{x}(0)$ and $\tilde{x}(\tau)$], the estimated horizontal distance,
obtained by integrating the equations of motion, shows reasonable agreement with that of
the kinematic approach. The estimated altitude profile is quite different from the DFDR
data, except near the initial and final points; the explanation for this results is that we
have ignored the measured altitude, except for two measured data [the values \( \tilde{h}(0) \) and
\( \tilde{h}(\tau) \)]. The relative path inclination profile approximates the DFDR data nicely. Note that
there is no direct measurement of the relative path inclination; instead, \( \gamma \) can be obtained
using the relation \( \gamma = \theta - a \). Concerning the relative velocity, even though only the
terminal values \( \tilde{V}(0) \) and \( \tilde{V}(\tau) \) are used, the estimated relative velocity profile
approximates the DFDR data reasonably.

Figure 13G shows the pitch attitude angle \( \theta \). As can be seen, the estimated pitch
follows the trend of the DFDR data nicely; this is no surprise, since the measured pitch is
employed.

Figure 13H shows the angle of attack \( a \). It is clear that the estimated angle of attack
is almost the same as that obtained from the DFDR. This shows that the measured data
on the angle of attack are believable.

Figure 13I shows the power setting \( \beta \), which is computed from the given EPR data.
Since \( \beta \) is treated as a known input, only one curve is shown.

Figures 13J, 13K, 13L respectively describe the thrust, drag, and lift acting on the
aircraft along the trajectory. From these diagrams, a clear understanding of the dynamic
behavior of the aircraft during the windshear encounter can be achieved.

In this formulation, the multiplicative factors for drag and lift are treated as unknown
parameters to be estimated. Namely, the drag and the lift are written as \( D = \pi_2 D^* \),
\( L = \pi_3 L^* \). The numerical values of these parameters are found to be
\( \pi_2 = 0.912, \)  

\( \pi_3 = 0.848. \)  

This means that the drag is 91\% of the value calculated from the manufacturer-supplied data, while the lift is 85\%.

To sum up, the effect of removing the measured altitude \( \bar{h} \) and relative velocity \( \bar{V} \) is the following. The AAG estimated horizontal wind is reasonably precise. However, the AAG estimated vertical wind has limited accuracy; this is a direct consequence of poor altitude approximation.

8.2. Summary. The numerical values of the estimated multiplicative factors \( \pi_2, \pi_3 \) and the wind velocity differences are summarized in Table 4. Based on the above analyses, the following conclusions can be inferred.

(i) Removing the measured altitude \( \bar{h} \) has no effect on the estimated horizontal wind. However, the estimated vertical wind deteriorates, which is a consequence of poor altitude approximation.

(ii) Removing the measured relative velocity \( \bar{V} \) has almost no effect on the estimated wind components. Namely, by employing the measured altitude \( \bar{h} \) and pitch attitude angle \( \bar{\theta} \), reasonable estimation results are obtained.

(iii) Removing the measured pitch attitude angle \( \bar{\theta} \) results in poor estimates of the wind components. Hence, the availability of the measured pitch attitude angle \( \bar{\theta} \) is crucial to good estimation results.

(iv) Employing only the measured altitude \( \bar{h} \) results in poor estimation results of the wind components. This is due to poor approximation of the pitch attitude angle.
(v) Employing only the measured relative velocity \( \tilde{V} \), causes poor estimation of both the altitude \( h \) and the pitch attitude angle \( \theta \). Consequently, the estimated wind components are not satisfactory.

(vii) Employing only the measured pitch attitude angle \( \tilde{\theta} \), a good estimate of the horizontal wind is obtained. However, the estimated vertical wind is less satisfactory. This is due to poor altitude approximation.

Finally, the relative importance of each physical quantity on the identifiability of the wind components can be established as follows.

(a) The availability of the measured relative velocity \( \tilde{V} \) has only minor effect on the estimation results.

(b) The availability of the measured altitude \( \tilde{h} \) is essential to good estimation of the vertical wind.

(c) The availability of the measured pitch attitude angle \( \tilde{\theta} \) is essential to good estimation of the wind components.
9. Concluding Remarks

Hazardous low-altitude windshears have been blamed for many aircraft accidents. Hence, it is important to determine the windshears that caused these accidents and establish safe recovery procedures during a windshear encounter.

In this thesis, limited trajectory data are employed in determining the windshear along the aircraft trajectory. By minimizing a functional measuring the deviation of the measured trajectory from the computed trajectory, a nonlinear least-square problem is formulated. This is a Bolza problem of optimal control, and the primal formulation of the sequential gradient-restoration algorithm (SGRA) is used to solve this problem.

We assume that flight takes place in a vertical plane and that the aircraft is a particle of constant mass. Both the kinematic equations and the dynamic equations are included as differential constraints. In the dynamic approach of this thesis, the thrust and the aerodynamic forces enter directly in the formulation. The measured trajectory data include the altitude \( \tilde{h} \), the relative velocity \( \tilde{V} \), and the pitch attitude angle \( \tilde{\theta} \), which are easier to obtain than the accelerometer data needed in the kinematic approach. In the computations, the angle of attack \( \alpha \) is treated as a control, while the power setting \( \beta \) is regarded as a known input. As a result, direct employment of the EPR data is required.

If one assumes that the manufacturer-supplied thrust data and aerodynamic data are dependable, the dynamically estimated vertical wind is close to that obtained with the kinematic approach. However, the estimated horizontal wind is less satisfactory.

The inclusion of multiplicative factors for the thrust and the aerodynamic forces results in both better trajectory approximation and more accurate winds. Unitary values of the multiplicative factors correspond to the case where the manufacturer-supplied thrust data and aerodynamic data are considered dependable. As a result, the estimated values of
the multiplicative factors, if different from unity, might be used in explaining some unusual effects, such as the presence of rain.

Different collections of measured trajectory data are analyzed to assess the relative importance of each measured physical quantity on the identifiability of the wind components. The measured horizontal distance $\tilde{x}$ and the relative velocity $\tilde{V}$ are found to have only minor effects on the estimated wind components. The measured altitude $\tilde{h}$ appears to have important effects on the identifiability of the vertical wind; and the measured pitch attitude angle $\tilde{\theta}$ is crucial to the identifiability of both the horizontal wind and the vertical wind.

Better estimation result can be obtained when the measured data on the angle of attack $\alpha$, the power setting $\beta$, the relative velocity $\tilde{V}$, and the pitch attitude angle $\tilde{\theta}$ are treated as fixed quantities; namely, these quantities are no longer control variables or state variables. Hence, a simpler least-square problem can be formulated. Details can be found in Ref. 58.

Extensions of this research can be made via the inclusion of the rotational motion of the aircraft. In this formulation, the elevator angle $\delta_e$ replaces the angle of attack $\alpha$ as a control. Extensions to three-dimensional flight can also be made, if flight does not take place in a vertical plane. In that case, one must estimate three wind components, includes the horizontal wind, the lateral wind, and the vertical wind.
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<td>Vertical wind $W_h$, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 6C.</td>
<td>Horizontal distance $x$, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 6D.</td>
<td>Altitude $h$, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 6E.</td>
<td>Relative velocity $V$, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 6F.</td>
<td>Relative path inclination angle $\gamma$, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 6G.</td>
<td>Pitch attitude angle $\theta$, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 6H.</td>
<td>Angle of attack $\alpha$, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 6I.</td>
<td>Power setting $\beta$, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 6J.</td>
<td>Thrust force, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 6K.</td>
<td>Drag force, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 6L.</td>
<td>Lift force, Problem (P2B).</td>
</tr>
<tr>
<td>Fig. 7A.</td>
<td>Horizontal wind $W_x$, Problem (P3).</td>
</tr>
<tr>
<td>Fig. 7B.</td>
<td>Vertical wind $W_h$, Problem (P3).</td>
</tr>
<tr>
<td>Fig. 7C.</td>
<td>Horizontal distance $x$, Problem (P3).</td>
</tr>
<tr>
<td>Fig. 7D.</td>
<td>Altitude $h$, Problem (P3).</td>
</tr>
<tr>
<td>Fig. 7E.</td>
<td>Relative velocity $V$, Problem (P3).</td>
</tr>
<tr>
<td>Fig.</td>
<td>Description</td>
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<tr>
<td>------</td>
<td>--------------------------------------------------</td>
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<tr>
<td>7F</td>
<td>Relative path inclination angle $\gamma$, Problem (P3)</td>
</tr>
<tr>
<td>7G</td>
<td>Pitch attitude angle $\theta$, Problem (P3)</td>
</tr>
<tr>
<td>7H</td>
<td>Angle of attack $\alpha$, Problem (P3)</td>
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<tr>
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<td>Thrust force, Problem (P3)</td>
</tr>
<tr>
<td>7K</td>
<td>Drag force, Problem (P3)</td>
</tr>
<tr>
<td>7L</td>
<td>Lift force, Problem (P3)</td>
</tr>
<tr>
<td>8A</td>
<td>Horizontal wind $W_x$, Problem (P4A)</td>
</tr>
<tr>
<td>8B</td>
<td>Vertical wind $W_h$, Problem (P4A)</td>
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<td>8C</td>
<td>Horizontal distance $x$, Problem (P4A)</td>
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<td>8F</td>
<td>Relative path inclination angle $\gamma$, Problem (P4A)</td>
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<tr>
<td>8G</td>
<td>Pitch attitude angle $\theta$, Problem (P4A)</td>
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<td>8H</td>
<td>Angle of attack $\alpha$, Problem (P4A)</td>
</tr>
<tr>
<td>8I</td>
<td>Power setting $\beta$, Problem (P4A)</td>
</tr>
<tr>
<td>8J</td>
<td>Thrust force, Problem (P4A)</td>
</tr>
<tr>
<td>8K</td>
<td>Drag force, Problem (P4A)</td>
</tr>
<tr>
<td>8L</td>
<td>Lift force, Problem (P4A)</td>
</tr>
<tr>
<td>9A</td>
<td>Horizontal wind $W_x$, Problem (P4B)</td>
</tr>
<tr>
<td>9B</td>
<td>Vertical wind $W_h$, Problem (P4B)</td>
</tr>
<tr>
<td>9C</td>
<td>Horizontal distance $x$, Problem (P4B)</td>
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<tr>
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<td>Altitude $h$, Problem (P4B)</td>
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<tr>
<td>Fig.</td>
<td>Description</td>
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<td>------</td>
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<tr>
<td>9E</td>
<td>Relative velocity $V$, Problem (P4B).</td>
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<tr>
<td>9F</td>
<td>Relative path inclination angle $\gamma$, Problem (P4B).</td>
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<tr>
<td>9G</td>
<td>Pitch attitude angle $\theta$, Problem (P4B).</td>
</tr>
<tr>
<td>9H</td>
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</tr>
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<td>Power setting $\beta$, Problem (P4B).</td>
</tr>
<tr>
<td>9J</td>
<td>Thrust force, Problem (P4B).</td>
</tr>
<tr>
<td>9K</td>
<td>Drag force, Problem (P4B).</td>
</tr>
<tr>
<td>9L</td>
<td>Lift force, Problem (P4B).</td>
</tr>
<tr>
<td>10A</td>
<td>Horizontal wind $W_x$, Problem (P4C).</td>
</tr>
<tr>
<td>10B</td>
<td>Vertical wind $W_h$, Problem (P4C).</td>
</tr>
<tr>
<td>10C</td>
<td>Horizontal distance $x$, Problem (P4C).</td>
</tr>
<tr>
<td>10D</td>
<td>Altitude $h$, Problem (P4C).</td>
</tr>
<tr>
<td>10E</td>
<td>Relative velocity $V$, Problem (P4C).</td>
</tr>
<tr>
<td>10F</td>
<td>Relative path inclination angle $\gamma$, Problem (P4C).</td>
</tr>
<tr>
<td>10G</td>
<td>Pitch attitude angle $\theta$, Problem (P4C).</td>
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<td>10H</td>
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</tr>
<tr>
<td>10I</td>
<td>Power setting $\beta$, Problem (P4C).</td>
</tr>
<tr>
<td>10J</td>
<td>Thrust force, Problem (P4C).</td>
</tr>
<tr>
<td>10K</td>
<td>Drag force, Problem (P4C).</td>
</tr>
<tr>
<td>10L</td>
<td>Lift force, Problem (P4C).</td>
</tr>
<tr>
<td>11A</td>
<td>Horizontal wind $W_x$, Problem (P4D).</td>
</tr>
<tr>
<td>11B</td>
<td>Vertical wind $W_h$, Problem (P4D).</td>
</tr>
<tr>
<td>11C</td>
<td>Horizontal distance $x$, Problem (P4D).</td>
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</table>
Fig. 11D.  Altitude $h$, Problem (P4D).
Fig. 11E.  Relative velocity $V$, Problem (P4D).
Fig. 11F.  Relative path inclination angle $\gamma$, Problem (P4D).
Fig. 11G.  Pitch attitude angle $\theta$, Problem (P4D).
Fig. 11H.  Angle of attack $\alpha$, Problem (P4D).
Fig. 11I.  Power setting $\beta$, Problem (P4D).
Fig. 11J.  Thrust force, Problem (P4D).
Fig. 11K.  Drag force, Problem (P4D).
Fig. 11L.  Lift force, Problem (P4D).
Fig. 12A.  Horizontal wind $W_x$, Problem (P4E).
Fig. 12B.  Vertical wind $W_h$, Problem (P4E).
Fig. 12C.  Horizontal distance $x$, Problem (P4E).
Fig. 12D.  Altitude $h$, Problem (P4E).
Fig. 12E.  Relative velocity $V$, Problem (P4E).
Fig. 12F.  Relative path inclination angle $\gamma$, Problem (P4E).
Fig. 12G.  Pitch attitude angle $\theta$, Problem (P4E).
Fig. 12H.  Angle of attack $\alpha$, Problem (P4E).
Fig. 12I.  Power setting $\beta$, Problem (P4E).
Fig. 12J.  Thrust force, Problem (P4E).
Fig. 12K.  Drag force, Problem (P4E).
Fig. 12L.  Lift force, Problem (P4E).
Fig. 13A.  Horizontal wind $W_x$, Problem (P4F).
Fig. 13B.  Vertical wind $W_h$, Problem (P4F).
Fig.13C.  Horizontal distance $x$, Problem (P4F).

Fig.13D.  Altitude $h$, Problem (P4F).

Fig.13E.  Relative velocity $V$, Problem (P4F).

Fig.13F.  Relative path inclination angle $\gamma$, Problem (P4F).

Fig.13G.  Pitch attitude angle $\theta$, Problem (P4F).

Fig.13H.  Angle of attack $\alpha$, Problem (P4F).

Fig.13I.  Power setting $\beta$, Problem (P4F).

Fig.13J.  Thrust force, Problem (P4F).

Fig.13K.  Drag force, Problem (P4F).

Fig.13L.  Lift force, Problem (P4F).
<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>Thrust $A_0$</td>
<td>$0.112272 \times 10^6$ lb</td>
</tr>
<tr>
<td></td>
<td>$A_1 = -0.110173 \times 10^3$ lb ft$^{-1}$sec</td>
</tr>
<tr>
<td></td>
<td>$A_2 = 0.108268 \times 10^0$ lb ft$^{-2}$sec$^2$</td>
</tr>
<tr>
<td>Drag $B_0$</td>
<td>$0.113300 \times 10^0$</td>
</tr>
<tr>
<td></td>
<td>$B_1 = -0.608200 \times 10^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$B_2 = 0.511360 \times 10^1$</td>
</tr>
<tr>
<td></td>
<td>$B_3 = -0.142185 \times 10^2$</td>
</tr>
<tr>
<td></td>
<td>$B_4 = 0.265618 \times 10^2$</td>
</tr>
<tr>
<td>Lift $C_0$</td>
<td>$0.486960 \times 10^0$</td>
</tr>
<tr>
<td></td>
<td>$C_1 = 0.630049 \times 10^1$</td>
</tr>
<tr>
<td></td>
<td>$C_2 = -0.423739 \times 10^1$</td>
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Table 2. Effect of the weighting factor on the estimated parameters, Problem (P2A).

<table>
<thead>
<tr>
<th>$C_5$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>$\pi_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.0 \times 10^0$</td>
<td>(1.0)</td>
<td>0.854</td>
<td>0.803</td>
</tr>
<tr>
<td>$1.0 \times 10^1$</td>
<td>(1.0)</td>
<td>0.854</td>
<td>0.803</td>
</tr>
<tr>
<td>$1.0 \times 10^2$</td>
<td>(1.0)</td>
<td>0.854</td>
<td>0.802</td>
</tr>
<tr>
<td>$1.0 \times 10^3$</td>
<td>(1.0)</td>
<td>0.854</td>
<td>0.802</td>
</tr>
<tr>
<td>$1.0 \times 10^4$</td>
<td>(1.0)</td>
<td>0.846</td>
<td>0.806</td>
</tr>
<tr>
<td>$1.0 \times 10^5$</td>
<td>(1.0)</td>
<td>0.861</td>
<td>0.798</td>
</tr>
<tr>
<td>$5.0 \times 10^5$</td>
<td>(1.0)</td>
<td>0.870</td>
<td>0.818</td>
</tr>
<tr>
<td>$1.0 \times 10^6$</td>
<td>(1.0)</td>
<td>0.883</td>
<td>0.826</td>
</tr>
</tbody>
</table>

$T = \pi_1 T_\pi$, $D = \pi_2 D_\pi$, $L = \pi_3 L_\pi$; thrust parameter $\pi_1 = 1$ is assumed.
Table 3. Effect of the weighting factor on the estimated parameters, Problem (P2B).

<table>
<thead>
<tr>
<th>$C_5$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>$\pi_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0.0 \times 10^0$</td>
<td>0.166</td>
<td>0.441</td>
<td>0.814</td>
</tr>
<tr>
<td>$1.0 \times 10^5$</td>
<td>0.485</td>
<td>0.607</td>
<td>0.818</td>
</tr>
<tr>
<td>$2.0 \times 10^5$</td>
<td>0.788</td>
<td>0.769</td>
<td>0.818</td>
</tr>
<tr>
<td>$3.0 \times 10^5$</td>
<td>0.678</td>
<td>0.716</td>
<td>0.826</td>
</tr>
<tr>
<td>$4.0 \times 10^5$</td>
<td>0.990</td>
<td>0.861</td>
<td>0.813</td>
</tr>
<tr>
<td>$4.5 \times 10^5$</td>
<td>1.019</td>
<td>0.888</td>
<td>0.822</td>
</tr>
<tr>
<td>$4.6 \times 10^5$</td>
<td>0.922</td>
<td>0.843</td>
<td>0.824</td>
</tr>
<tr>
<td>$4.7 \times 10^5$</td>
<td>0.904</td>
<td>0.835</td>
<td>0.825</td>
</tr>
<tr>
<td>$9.0 \times 10^5$</td>
<td>1.025</td>
<td>0.899</td>
<td>0.827</td>
</tr>
<tr>
<td>$1.0 \times 10^6$</td>
<td>1.002</td>
<td>0.897</td>
<td>0.831</td>
</tr>
<tr>
<td>$5.0 \times 10^6$</td>
<td>1.035</td>
<td>0.953</td>
<td>0.867</td>
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</tbody>
</table>

$T = \pi_1 T_*$, $D = \pi_2 D_*$, $L = \pi_3 L_*$. 
Table 4. Summary of the estimated parameters and the wind velocity differences.

<table>
<thead>
<tr>
<th>Problem</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>$\pi_3$</th>
<th>$\Delta W_x$</th>
<th>$\Delta W_h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
<td>.</td>
</tr>
<tr>
<td>NASA</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>122</td>
<td>62</td>
</tr>
<tr>
<td>(P1)</td>
<td>[1.0]</td>
<td>[1.0]</td>
<td>[1.0]</td>
<td>95</td>
<td>52</td>
</tr>
<tr>
<td>(P2A)</td>
<td>[1.0]</td>
<td>0.854</td>
<td>0.803</td>
<td>109</td>
<td>46</td>
</tr>
<tr>
<td>(P2B)</td>
<td>1.019</td>
<td>0.888</td>
<td>0.822</td>
<td>106</td>
<td>46</td>
</tr>
<tr>
<td>(P3)</td>
<td>[1.0]</td>
<td>0.797</td>
<td>0.809</td>
<td>117</td>
<td>44</td>
</tr>
<tr>
<td>(P4A)</td>
<td>[1.0]</td>
<td>0.848</td>
<td>0.797</td>
<td>116</td>
<td>49</td>
</tr>
<tr>
<td>(P4B)</td>
<td>[1.0]</td>
<td>0.909</td>
<td>0.864</td>
<td>103</td>
<td>43</td>
</tr>
<tr>
<td>(P4C)</td>
<td>[1.0]</td>
<td>0.808</td>
<td>0.793</td>
<td>43</td>
<td>54</td>
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<tr>
<td>(P4D)</td>
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<td>50</td>
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<td>(P4E)</td>
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<td>0.796</td>
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<tr>
<td>(P4F)</td>
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<td>0.912</td>
<td>0.848</td>
<td>108</td>
<td>60</td>
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</table>

Brackets denote assigned values; asterisk indicates values not available.