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A numerical study of vortex-shedding suppression in laminar flow about a cylinder near a plane boundary

Rothberg, Robert Hanks, Ph.D.

Rice University, 1989
RICE UNIVERSITY

A NUMERICAL STUDY OF VORTEX-SHEDDING SUPPRESSION IN LAMINAR FLOW ABOUT A CYLINDER NEAR A PLANE BOUNDARY

by

ROBERT H. ROTHBERG

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

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May, 1989
Abstract

A Numerical Study of Vortex-Shedding Suppression in Laminar Flow About a Cylinder Near a Plane Boundary

by

Robert H. Rothberg

The effect of a nearby plane boundary on vortex shedding from a circular cylinder is investigated for laminar flow. A two-dimensional finite difference numerical technique is used to solve the incompressible Navier-Stokes equations in primitive variable form on a computational mesh that is generated using a body-fitted coordinate transformation. Results are first presented for flows at Reynolds numbers of 80 and 100, based on the cylinder diameter, with the cylinder constrained against movement. Several cases of moving cylinder simulation are also presented for flow at a Reynolds number of 100.

Fixed cylinder cases were run initially at both Reynolds numbers for an unbounded cylinder to confirm agreement of the simulation results with experimental evidence. The influence of a nearby plane boundary was investigated through a traverse of gap ratios beginning with a value of 3.0 and concluding with 0.5. Additional attention was focused on the behavior of the flow at gap ratios in the vicinity of the vortex shedding suppression gap ratio. Nearing the plate, a maximum Strouhal period was observed for each flow as suppression was approached. Features of the more viscous flow at the lower Reynolds number occurred at larger gap ratios than for the higher Reynolds number, as expected.

The moving cylinder simulations were conducted primarily to demonstrate the capability of the simulator to accommodate the moving boundary of the cylinder. Initial gap ratios were chosen for examination based on the behavior of the fixed cylinder in the vicinity of the gap associated with vortex suppression.
Acknowledgements

This dissertation is dedicated to my wife, Jeanne, whose encouragement and support have enabled me to make the most of a mid-career opportunity to pursue advanced academic work. It is also dedicated to my parents, Irving and Vernita Rothberg, whose sacrifices made a better life for their children.

Special thanks are due to Dr. W. F. Walker and Dr. A. J. Chapman, who supported my admission and provided skillful guidance through each phase of the doctoral program. It is through their wisdom that my interests were focused on this research topic, and sustained through all of the inevitable hills and valleys encountered during a research effort. Deep gratitude is also due to Dr. R. D. Cohen and Dr. C. C. Wang for their participation as committee members and as friends; they always seemed to know when to lead and when to allow me the freedom to step forward on my own.

The historical role of numerical methods in hydrodynamics has been to develop a numerical technique and verify its results against experimental findings. The work in this area by Dr. D. W. Allen has enabled the numerical hydrodynamics research reported herein to advance from the verification role to that of a predictive tool.

Financial support during my graduate work was provided by a generous fellowship from the Amoco Foundation. I also wish to thank the Rice Graduate Programs Office for providing the $15,000 grant for computer funds used during the code development and numerical experimentation, and the staff of the Institute for Computer Services and Applications at Rice for their assistance as I followed the steep learning curve toward computer literacy.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section / Topic</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>ii</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>iii</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vi</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xv</td>
</tr>
<tr>
<td>Key to Symbols</td>
<td>xvi</td>
</tr>
<tr>
<td>1. INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>1.1 Overview of Flow-Induced Vibration</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Previous Investigations</td>
<td>6</td>
</tr>
<tr>
<td>1.3 Scope of The Current Investigation</td>
<td>14</td>
</tr>
<tr>
<td>1.4 Summary of Results</td>
<td>16</td>
</tr>
<tr>
<td>2. NUMERICAL SIMULATION TECHNIQUE</td>
<td></td>
</tr>
<tr>
<td>2.1 Overview</td>
<td>35</td>
</tr>
<tr>
<td>2.2 Generation of the Computational Mesh</td>
<td>38</td>
</tr>
<tr>
<td>2.3 Governing Equations and Boundary/Initial Conditions</td>
<td>49</td>
</tr>
<tr>
<td>2.4 Description of the Navier-Stokes Solver</td>
<td>56</td>
</tr>
<tr>
<td>2.5 Post-Processing Information</td>
<td>65</td>
</tr>
<tr>
<td>3. SEMI-BOUNDDED FLOW OVER A FIXED CYLINDER AT $\text{Re}_D = 80$</td>
<td></td>
</tr>
<tr>
<td>3.1 Reference Case of Unbounded Flow</td>
<td>74</td>
</tr>
<tr>
<td>3.2 Semi-bounded Cases at Gap Ratios $&gt; 1.7$</td>
<td>76</td>
</tr>
<tr>
<td>3.3 Semi-bounded Case at Gap Ratio $= 1.7$</td>
<td>80</td>
</tr>
<tr>
<td>3.4 Semi-bounded Cases at Gap Ratios $&lt; 1.7$</td>
<td>82</td>
</tr>
</tbody>
</table>
4. SEMI-BOUNDED FLOW OVER A FIXED CYLINDER AT $Re_D = 100$

4.1 Reference Case of Unbounded Flow 131

4.2 Semi-bounded Cases at Gap Ratios $> 1.5$ 133

4.3 Semi-bounded Case at Gap Ratio $= 1.5$ 137

4.4 Semi-bounded Cases at Gap Ratios $< 1.5$ 140

5. SEMI-BOUNDED FLOW OVER A MOVING CYLINDER AT $Re_D = 100$

5.1 Overview of Moving Cylinder Simulation 193

5.2 Summary of Moving Cylinder Investigations 199

Conclusions and Recommendations for Future Work 223

References 226
# List Of Figures

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-1</td>
<td>Reynolds Number Relationship to Vortex Shedding from a Stationary Circular Cylinder</td>
<td>22</td>
</tr>
<tr>
<td>1-2</td>
<td>General Configuration Examined</td>
<td>23</td>
</tr>
<tr>
<td>1-3</td>
<td>Starting Transient for h/D=2.0 at Re_D=100</td>
<td>24</td>
</tr>
<tr>
<td>1-4</td>
<td>Steady State Periodic Behavior for h/D=2.0 at Re_D=100</td>
<td>25</td>
</tr>
<tr>
<td>1-5</td>
<td>Vortex Shape at Re_D=100 (h/D=3.0 and 1.2)</td>
<td>26</td>
</tr>
<tr>
<td>1-6</td>
<td>Vortex Shape at Re_D=100 (h/D=0.7 and 0.5)</td>
<td>27</td>
</tr>
<tr>
<td>1-7</td>
<td>Velocity Profile Between Cylinder and Plate</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>(a) Gap Ratio 3.0, (b) Gap Ratio 2.0</td>
<td></td>
</tr>
<tr>
<td>1-8</td>
<td>Velocity Profile Between Cylinder and Plate</td>
<td>29</td>
</tr>
<tr>
<td></td>
<td>(a) Gap Ratio 1.0, (b) Gap Ratio 0.5</td>
<td></td>
</tr>
<tr>
<td>1-9</td>
<td>(a) Mean Drag Coefficient vs. Gap Ratio,</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>(b) Drag Coefficient Amplitude vs. Gap Ratio</td>
<td></td>
</tr>
<tr>
<td>1-10</td>
<td>(a) Mean Lift Coefficient vs. Gap Ratio,</td>
<td>31</td>
</tr>
<tr>
<td></td>
<td>(b) Lift Coefficient Amplitude vs. Gap Ratio</td>
<td></td>
</tr>
<tr>
<td>1-11</td>
<td>(a) Loading Period vs. Gap Ratio,</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>(b) Detail of Loading Period vs. Gap Ratio</td>
<td></td>
</tr>
<tr>
<td>1-12</td>
<td>(a) Mean Upper Separation Angle vs. Gap Ratio,</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>(b) Upper Separation Angle Amplitude vs. Gap Ratio</td>
<td></td>
</tr>
<tr>
<td>1-13</td>
<td>(a) Mean Lower Separation Angle vs. Gap Ratio,</td>
<td>34</td>
</tr>
<tr>
<td></td>
<td>(b) Lower Separation Angle Amplitude vs. Gap Ratio</td>
<td></td>
</tr>
<tr>
<td>2-1</td>
<td>Body-Fitted Coordinate Transformation</td>
<td>68</td>
</tr>
<tr>
<td>2-2</td>
<td>Discretized 40x40 Domain</td>
<td>69</td>
</tr>
</tbody>
</table>
2-3 Computational Mesh, Unbounded Cylinder
2-4 Computational Mesh, h/D=3.0
2-5 Computational Mesh, h/D=0.5
2-6 Extent of the Discretized Domain
3-1 Coefficient Time Histories for Unbounded Cylinder at $Re_D=80$,
     (a) Lift, (b) Drag
3-2 Separation Angle Time Histories for Unbounded Cylinder
     at $Re_D=80$
3-3 Streamlines for Unbounded Cylinder, $Re_D=80$
3-4 Streamlines for Unbounded Cylinder, $Re_D=80$
3-5 Pressure Coefficients for Unbounded Cylinder, $Re_D=80$
3-6 Coefficient Time Histories at $Re_D=80$, (a) Lift for h/D=3.0,
     (b) Lift for h/D=2.0
3-7 Coefficient Time Histories at $Re_D=80$, (a) Lift for h/D=1.8,
     (b) Drag for h/D=3.0
3-8 Coefficient Time Histories at $Re_D=80$, (a) Drag for h/D=2.0,
     (b) Drag for h/D=1.8
3-9 Separation Angle Time Histories at $Re_D=80$ and h/D=3.0
3-10 Separation Angle Time Histories at $Re_D=80$ and h/D=2.0
3-11 Separation Angle Time Histories at $Re_D=80$ and h/D=1.8
3-12 Streamlines for Semi-bounded Cylinder at $Re_D=80$, (a) h/D=3.0,
     Minimum Lift, (b) h/D=2.0, Minimum Lift
3-13 Streamlines for Semi-bounded Cylinder at $Re_D=80$ and h/D=1.8
3-14 Streamlines for Semi-bounded Cylinder at $Re_D=80$ and h/D=1.8
3-15 Pressure Coeff. at Cylinder and Plate for $Re_D=80$ and h/D=3.0
3-16 Pressure Coeff. at Cylinder and Plate for $Re_D=80$ and h/D=2.0
3-17 Pressure Coeff. at Cylinder and Plate for Re_D=80 and h/D=1.8 103
3-18 Pressure Contours for Semi-bounded Cylinder at Re_D=80 and h/D=1.8 104
3-19 Coefficient Time Histories at Re_D=80 and h/D=1.8,
(a) Lift, (b) Drag 105
3-20 Separation Angle Time Histories at Re_D=80 and h/D=1.7 106
3-21 Streamlines for Semi-bounded Cylinder at Re_D=80 and h/D=1.7 107
3-22 Pressure Coefficient at Cylinder and Plate for Re_D=80 and h/D=1.7 108
3-23 Pressure Contours for Semi-bounded Cylinder at Re_D=80 and h/D=1.7 109
3-24 Lift Coefficient Time Histories at Re_D=80, (a) h/D=1.5,
(b) h/D=1.2 110
3-25 Lift Coefficient Time Histories at Re_D=80, (a) h/D=1.0,
(b) h/D=0.7 111
3-26 Coefficient Time Histories at Re_D=80, (a) Lift at h/D=0.5,
(b) Drag at h/D=1.5 112
3-27 Drag Coefficient Time Histories at Re_D=80, (a) h/D=1.2,
(b) h/D=1.0 113
3-28 Drag Coefficient Time Histories at Re_D=80, (a) h/D=0.7,
(b) h/D=0.5 114
3-29 Separation Angle Time Histories at Re_D=80 and h/D=1.5 115
3-30 Separation Angle Time Histories at Re_D=80 and h/D=1.2 116
3-31 Separation Angle Time Histories at Re_D=80 and h/D=1.0 117

viii
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-32</td>
<td>Upper Separation Angle Time Histories at $\text{Re}_D=80$,</td>
</tr>
<tr>
<td></td>
<td>(a) $h/D=0.7$, (b) $h/D=0.5$</td>
</tr>
<tr>
<td>3-33</td>
<td>Streamlines for Semi-bounded Cylinder at $\text{Re}_D=80$,</td>
</tr>
<tr>
<td></td>
<td>(a) $h/D=1.5$, (b) $h/D=1.2$</td>
</tr>
<tr>
<td>3-34</td>
<td>Streamlines for Semi-bounded Cylinder at $\text{Re}_D=80$,</td>
</tr>
<tr>
<td></td>
<td>(a) $h/D=1.0$, (b) $h/D=0.7$</td>
</tr>
<tr>
<td>3-35</td>
<td>Streamlines for Semi-bounded Cylinder at $\text{Re}_D=80$ and</td>
</tr>
<tr>
<td></td>
<td>$h/D=0.5$</td>
</tr>
<tr>
<td>3-36</td>
<td>Pressure Coefficient at Cylinder and Plate for $\text{Re}_D=80$ and</td>
</tr>
<tr>
<td></td>
<td>$h/D=1.5$</td>
</tr>
<tr>
<td>3-37</td>
<td>Pressure Contours for Semi-bounded Cylinder at $\text{Re}_D=80$ and</td>
</tr>
<tr>
<td></td>
<td>$h/D=1.5$</td>
</tr>
<tr>
<td>3-38</td>
<td>Pressure Plots at Minimum Lift, $\text{Re}_D=80$ and $h/D=1.2$,</td>
</tr>
<tr>
<td></td>
<td>(a) Pressure Coefficients, (b) Pressure Contours</td>
</tr>
<tr>
<td>3-39</td>
<td>Pressure Coefficient at Cylinder and Plate for $\text{Re}_D=80$ and</td>
</tr>
<tr>
<td></td>
<td>$h/D=1.0$</td>
</tr>
<tr>
<td>3-40</td>
<td>Pressure Contours for Semi-bounded Cylinder at $\text{Re}_D=80$ and</td>
</tr>
<tr>
<td></td>
<td>$h/D=1.0$</td>
</tr>
<tr>
<td>3-41</td>
<td>Pressure Coefficient at Cylinder and Plate for $\text{Re}_D=80$ and</td>
</tr>
<tr>
<td></td>
<td>$h/D=0.7$</td>
</tr>
<tr>
<td>3-42</td>
<td>Pressure Contours for Semi-bounded Cylinder at $\text{Re}_D=80$ and</td>
</tr>
<tr>
<td></td>
<td>$h/D=0.7$</td>
</tr>
<tr>
<td>3-43</td>
<td>Pressure Coefficient at Cylinder and Plate for $\text{Re}_D=80$ and</td>
</tr>
<tr>
<td></td>
<td>$h/D=0.5$</td>
</tr>
<tr>
<td>3-44</td>
<td>Pressure Contours for Semi-bounded Cylinder at $\text{Re}_D=80$ and</td>
</tr>
<tr>
<td></td>
<td>$h/D=0.5$</td>
</tr>
</tbody>
</table>
4-1 Coefficient Time Histories for Unbounded Cylinder at Re_D=100
   (a) Lift, (b) Drag

4-2 Separation Angle Time Histories for Unbounded Cylinder at
   Re_D=100

4-3 Streamlines for Unbounded Cylinder at Re_D=100

4-4 Streamlines for Unbounded Cylinder at Re_D=100

4-5 Pressure Coefficient for Unbounded Cylinder at Re_D=100

4-6 Lift Coefficient Time Histories at Re_D=100
   (a) h/D=3.0, (b) h/D=2.0

4-7 Lift Coefficient Time Histories at Re_D=100
   (a) h/D=1.7, (b) h/D=1.6

4-8 Drag Coefficient Time Histories at Re_D=100
   (a) h/D=3.0, (b) h/D=2.0

4-9 Drag Coefficient Time Histories at Re_D=100
   (a) h/D=1.7, (b) h/D=1.6

4-10 Separation Angle Time Histories at Re_D=100 and h/D=3.0

4-11 Separation Angle Time Histories at Re_D=100 and h/D=2.0

4-12 Separation Angle Time Histories at Re_D=100 and h/D=1.7

4-13 Separation Angle Time Histories at Re_D=100 and h/D=1.6

4-14 Streamlines for Semi-bounded Cylinder at Re_D=100 and h/D=3.0

4-15 Streamlines for Semi-bounded Cylinder at Re_D=100 and h/D=2.0

4-16 Streamlines for Semi-bounded Cylinder at Re_D=100 and h/D=1.7

4-17 Streamlines for Semi-bounded Cylinder at Re_D=100 and h/D=1.6

4-18 Pressure Coefficient at Cylinder and Plate for Re_D=100 and
   h/D=3.0
4-19 Pressure Coefficient at Cylinder and Plate for Re_D=100 and h/D=2.0

4-20 Pressure Coefficient at Cylinder and Plate for Re_D=100 and h/D=1.7

4-21 Pressure Coefficient at Cylinder and Plate for Re_D=100 and h/D=1.6

4-22 Pressure Contour Plots for Semi-bounded Cylinder at Re_D=100 and h/D=1.6

4-23 Coefficient Time Histories at Re_D=100 and h/D=1.5

4-24 Separation Angle Time Histories at Re_D=100 and h/D=1.5

4-25 Streamlines for Semi-bounded Cylinder at Re_D=100 and h/D=1.5

4-26 Pressure Coefficient at Cylinder and Plate for Re_D=100 and h/D=1.5

4-27 Pressure Contour Plots for Semi-bounded Cylinder at Re_D=100 and h/D=1.5

4-28 Lift Coefficient Time Histories at Re_D=100
   (a) h/D=1.4, (b) h/D=1.2

4-29 Lift Coefficient Time Histories at Re_D=100
   (a) h/D=1.0, (b) h/D=0.7

4-30 Coefficient Time Histories at Re_D=100, (a) Lift at h/D=0.5,
   (b) Drag at h/D=1.4

4-31 Drag Coefficient Time Histories at Re_D=100
   (a) h/D=1.2, (b) h/D=1.0

4-32 Drag Coefficient Time Histories at Re_D=100
   (a) h/D=0.7, (b) h/D=0.5

4-33 Separation Angle Time Histories at Re_D=100 and h/D=1.4
Separation Angle Time Histories at $Re_D=100$ and $h/D=1.2$  
Separation Angle Time Histories at $Re_D=100$ and $h/D=1.0$  
Separation Angle Time Histories at $Re_D=100$ and $h/D=0.7$  
Upper Separation Angle Time History at $Re_D=100$ and $h/D=0.5$  
Streamlines for Semi-bounded Cylinder at $Re_D=100$ and $h/D=1.4$  
Streamlines for Semi-bounded Cylinder at $Re_D=100$  
(a) $h/D=1.2$, (b) $h/D=1.0$  
Streamlines for Semi-bounded Cylinder at $Re_D=100$  
(a) $h/D=0.7$, (b) $h/D=0.5$  
Pressure Coefficient at Cylinder and Plate for $Re_D=100$ and $h/D=1.4$  
Pressure Contour Plots for Semi-bounded Cylinder at $Re_D=100$ and $h/D=1.4$  
Pressure Coefficient at Cylinder and Plate for $Re_D=100$ and $h/D=1.2$  
Pressure Contour Plots for Semi-bounded Cylinder at $Re_D=100$ and $h/D=1.2$  
Pressure Coefficient at Cylinder and Plate for $Re_D=100$ and $h/D=1.0$  
Pressure Contour Plots for Semi-bounded Cylinder at $Re_D=100$ and $h/D=1.0$  
Pressure Coefficient at Cylinder and Plate for $Re_D=100$ and $h/D=0.7$  
Pressure Coefficient at Cylinder and Plate for $Re_D=100$ and $h/D=0.5$
Pressure Contour Plots for Semi-bounded Cylinder at $Re_D=100$ and $h/D=0.5$

5-1 Moving Cylinder Simulation Configuration

5-2 Cylinder Movement vs. Time, Initial Gap Ratio 2.0

5-3 Cylinder Movement vs. Time, Initial Gap Ratio 2.0
   (a) Position, (b) Lift Coefficient

5-4 Cylinder Movement vs. Time, Initial Gap Ratio 2.0,
   Impulsive Flow

5-5 Cylinder Movement vs. Time, Initial Gap Ratio 2.0,
   Impulsive Flow

5-6 Cylinder Movement vs. Time, Initial Gap Ratio 2.0,
   (a) Impulsive Start, (b) After Stabilization

5-7 Cylinder Movement vs. Time, Initial Gap Ratio 1.5

5-8 Cylinder Movement vs. Time, Initial Gap Ratio 2.0,
   (a) Cylinder Motion, (b) Lift Coefficient

5-9 Cylinder Movement vs. Time, Initial Gap Ratio 1.0

5-10 Cylinder Movement vs. Time, Initial Gap Ratio 1.0

5-11 Lift Coefficient vs. Time, Initial Gap Ratio 1.0

5-12 Lift Coefficient vs. Time, Initial Gap Ratio 1.0

5-13 Cylinder Movement vs. Time, Initial Gap Ratio 0.7

5-14 Cylinder Movement vs. Time, Initial Gap Ratio 0.7

5-15 Lift Coefficient vs. Time, Initial Gap Ratio 0.7

5-16 Lift Coefficient vs. Time, Initial Gap Ratio 1.0

5-17 Moving Cylinder Streamline Contour Plots, Initial Gap Ratio 2.0

5-18 Moving Cylinder Streamline Contour Plots, Initial Gap Ratio 2.0
5-19 Moving Cylinder Streamline Contour Plots,
(a) Initial Gap Ratio 1.5, (b) Initial Gap Ratio 1.0

5-20 Moving Cylinder Streamline Contour Plot, Initial Gap Ratio 0.7
## List of Tables

<table>
<thead>
<tr>
<th>Number</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2-1</td>
<td>Parameters Used in the Navier-Stokes Solver</td>
<td>64</td>
</tr>
</tbody>
</table>
Key to Symbols

\( a \) = acceleration
\( A_r \) = amplitude ratio
\( c \) = damping, structural damping
\( \text{cpu} \) = central processing unit
\( C_d \) = drag coefficient = drag force/\((0.5 \cdot \rho \cdot U_0^2 \cdot D)\)
\( C_l \) = lift coefficient = lift force/\((0.5 \cdot \rho \cdot U_0^2 \cdot D)\)
\( C_p \) = pressure coefficient = \((p - p_0)/(0.5 \cdot \rho \cdot U_0^2)\)
\( D \) = cylinder diameter
\( \text{Di} \) = dilatation
\( f \) = general function of \((x, y, t)\)
\( f_s \) = Strouhal frequency of vortex shedding
\( F_y \) = force in y direction (vertical)
\( h \) = gap between cylinder and plate
\( i \) = generic index
\( j \) = generic index
\( J, \sqrt{g} \) = Jacobian determinant
\( k \) = spring constant
\( m_c \) = mass of cylinder
\( n \) = index associated with the time step
\( p \) = pressure
\( p_0 \) = pressure at free stream boundary
\( P(x, y) \) = mesh generation control function
\( P \) = grid metric parameter
\( Q(x, y) \) = mesh generation control function
\( Q \) = grid metric parameter
\( r, r_o \) = radius from cylinder to outer boundary
\( \text{Re}_D \) = Reynolds number = \(U_0 D/\nu\) or equivalently \(\rho U_0 D/\mu\)
\( \text{Re}_{\text{mesh}} \) = mesh Reynolds number
\( \text{Re}_{\text{crit}} \) = critical mesh Reynolds number
\( St \) = Strouhal number
\( t \) = time
\( \text{TE} \) = truncation error
\( u \) = fluid velocity component in horizontal direction
\( U_0 \) = free stream inflow velocity
\( v \) = fluid velocity component in the vertical direction
\( x \) = horizontal coordinate direction in the physical plane
\( y \) = vertical coordinate direction in the physical plane
\( z \) = impulse loading
\( \alpha \) = grid metric parameter
\( \beta \) = grid metric parameter
\( \eta \) = "circumferential" coordinate direction in the computational plane
\( \gamma \) = grid metric parameter
\( \Gamma \) = flow boundary
\( \kappa \) = stretching parameter for sinh distribution function
\( \mu \) = dynamic viscosity
\( \nu \) = kinematic viscosity
\( \omega \) = frequency
\( \phi \) = phase angle
\( \rho \) = fluid density
\( \sigma \) = grid metric parameter
\( \tau \) = grid metric parameter
\( \xi \) = "radial" coordinate direction in the computational plane
\( \zeta \) = damping ratio
CHAPTER 1
Introduction

1.1 Overview of Flow-Induced Vibration

The research effort reported herein has been aimed toward the objective of improving the understanding of the fluid dynamics associated with flow-induced vibration. Specifically, the work is focused on the phenomenon of periodic vortex shedding in the wake of a circular cylinder, and the suppression of vortex shedding that has been observed to occur as a cylindrical body is brought into the proximity of a plane boundary. The periodic shedding of vortices contributes to alternating forces on the body, which can in turn lead to undesirable displacements and even failure of the member due to structural fatigue. Evidence of vortex shedding behavior is available in our everyday lives; most people have heard the strumming of overhead utility lines on a windy day, or felt the vibrations of a fishing line cast into the current of a stream or river.

Less evident, but potentially more serious, is the effect of vortex shedding in heat exchanger tube bundles and in the tubular elements of a nuclear reactor core. Flow-induced vibration is known to cause impact and wear of adjacent tubular components, and design procedures must include consideration of the motion or its suppression [Blevins (1977)]. Undersea transport pipelines and communication cables are subject to hydrodynamic loads that include the effects of vortex shedding. Irregular features of the seafloor lead to "spanning", or suspended regions of pipe or cable. Fatigue failure at the suspended section is possible unless the potential for flow-induced vibration is evaluated and addressed through installation of suppression devices or costly elimination of the span. Griffin (1984) notes that in air, the adjustment of structural parameters controlling the damping or mass per unit length of small diameter bodies such as cable is usually sufficient to control large amplitude
vibrations due to vortex-induced loads. Helical strakes are common vortex suppression devices used in air on larger diameter bodies such as industrial smokestacks.

Offshore platforms, with their closely grouped members, and other submerged structures must be designed with due consideration for the effects of vortex-induced loadings where the flow behavior is modified by the presence of adjacent members or nearby impermeable boundaries. Zdravkovich (1981) performed flow visualization studies that showed the effect of a nearby boundary on flow modification, and ultimately the suppression of vortex shedding as a cylinder was placed progressively closer to a plane boundary. Others such as Sarpkaya (1977), and Marumo, et. al. (1978) have shown experimentally that vortex shedding from a body is suppressed when the body is immersed in the boundary layer of an adjacent plate. In a review article, Grant (1986) states that the ocean bottom boundary layer can be as thick as 40 meters. The boundary layer velocity profile creates a range of flow conditions upstream of an immersed body dependent on the elevation of the body above the no-slip boundary. Grace and Zee (1981) report on a four year program undertaken to measure in situ loadings on a pipeline element located in a boundary layer region and elevated slightly above the ocean floor. Lambrakos (1985) describes an ocean boundary layer measurement program that led to the full scale, multi-million dollar, in situ pipeline force measurement program managed by this author. While these latter studies were not directly concerned with suppression of vortex shedding, the ocean boundary layer information obtained is of use in the application of the research reported herein.

The flow about a circular cylinder has been the subject of hundreds of investigations over the years, and each research effort appears to yield as many questions as are answered. Therefore, this bluff body configuration continues to offer fresh challenges for investigation. Absent the effects of any adjacent boundaries, the viscous flow around a stationary cylinder at low Reynolds numbers (based on cylinder diameter) results in a symmetric streamline pattern about the body with no flow separation. At a
Reynolds number of approximately 5 to 6, the viscous effects deplete enough momentum from the flow near the body to cause the onset of separation. Upon separation, two shear layers originating at the separation points are formed that feed vorticity into a pair of stable symmetric vortices in the cylinder wake. As the Reynolds number is increased to beyond approximately 40, instabilities in the flow develop and one shear layer is drawn across the wake as the opposite vortex grows in size. The presence of oppositely-signed circulation in the shear layer drawn across the wake limits the further supply of circulation to the vortex. The vortex is shed and the process is repeated on the opposite side of the wake in a periodic manner. Gerrard (1966) observes that when the shear layers are brought closer together, as would occur with motion of the separation points toward each other, the period of vortex shedding is reduced. Sarpkaya (1979) provides an essentially similar description of the shedding process and attributes the initiation of the process to Tollmien-Schlichting instabilities. Jackson (1987) has used a Galerkin finite element technique to successfully estimate the Reynolds number for onset of regular vortex shedding from a circular cylinder and other aspect ratio derivative shapes. Numerical simulations in a symmetric computational domain are able to maintain the stable wake region to much higher Reynolds number values [Fornberg (1985)] and must be destabilized externally to initiate the vortex shedding process.

The flow about the cylinder and throughout the wake remains laminar up to a Reynolds number of approximately 150, where a transition to turbulence in the wake is first apparent. As the Reynolds number increases, the transition to turbulence in the wake continues and moves further upstream toward the cylinder boundary layer. The flow regime ranging from the onset of wake turbulence to the transition to turbulence in the cylinder boundary layer is known as the subcritical range. Critical flow occurs at approximately $Re_D = 3 \times 10^5$ and is accompanied by a large reduction in the drag coefficient as the separation points move rearward on the cylinder. This movement is attributed to the presence of the turbulent kinetic energy in the boundary layer and its
effect in delaying flow separation [Savkar (1984)]. Flows between the critical value and a Reynolds number of approximately 3.5x10^6 show no organized pattern in the wake, and vortex shedding is not present. Depending on the reference, flow at Re_D greater than 3x10^5 is called either supercritical or transcritical. Re-establishment of a vortex shedding pattern in the wake is observed for Reynolds numbers greater than 3.5x10^6. **Figure 1-1** summarizes the Reynolds number relationship to vortex shedding from a circular cylinder.

The Strouhal number relates the vortex shedding frequency to the flow velocity and the diameter of the cylinder as in Equation (1.1-1):

\[ St = \frac{f_s D}{U_0} \]  

(1.1-1)

The Strouhal number is observed to be almost constant over the subcritical range, prompting many attempts to define a universal Strouhal number based on its Reynolds number functional dependence [Awbi (1981), Chen (1987)]. A cylindrical body that is not held stationary in the flow will experience displacements governed by the periodic loading of vortex shedding and by the structural parameters associated with mass, stiffness, and damping. When the frequency of vortex shedding excitation is much less than the natural frequency of the cylinder, small amplitude vibration of the cylinder occurs. As the vortex shedding frequency approaches to within ±25% of the natural frequency, a phenomenon known as lock-in, or flow synchronization results. At lock-in, the frequency of vortex shedding is controlled by the oscillation of the cylinder. In this mode, increases in lift, drag, and vibration amplitude are observed, along with an increase in the correlation length of vortex formation along the axis of the cylinder [Sarpkaya (1979)]. The correlation length is the distance along the axis of the cylinder over which the vortex maintains a uniform angular orientation. A large correlation length can produce higher loads than a more random vortex distribution. A
limiting amplitude occurs, beyond which further increase in the Reynolds number of the flow results in loss of synchronization and a return to low amplitude vibration.

The presence of a nearby body or boundary is known to modify the vortex shedding behavior and ultimately leads to the suppression of vortex shedding in the case of a stationary cylinder placed in close proximity to a plane boundary. The plane boundary provides a no-slip surface along which a boundary layer grows, and normal to which flow velocities are zero. In Section 1.2, several experimental investigations of this configuration are cited, each showing slight variations in behavior due to differences in Reynolds number and plate boundary layer thickness, among other considerations. The gap ratio h/D, is defined as the distance between the plate and cylinder divided by the cylinder diameter. As the gap ratio is reduced, the cylinder experiences the combined effects of downwash on the plate and the venturi effect of the flow between the plate and the cylinder. Moderating the magnitude of these effects in terms of effective upstream velocities is the velocity profile developed by the plate boundary layer. The downwash effect is best known to aircraft pilots as a "ground cushion" that provides increased lift and reduced drag during landings. This is due to the increase of pressure between the wing and ground resulting from flow washing down the front of the wing (cylinder) and stagnating on the boundary [Recant et. al. (1939)]. The venturi effect generates an attraction of the cylinder to the plate and is more pronounced in potential flow approximations such as those made by Yamamoto et. al. (1974) where viscous effects are not considered.

At very small gap ratios, a fully-viscous (Poiseuille) flow is developed in the gap and the pressure in this region increases substantially. The pressure increase is analogous to hydrodynamic lubrication in a bearing, and generates a large positive lift component. Thus lift behavior is governed by three distinct phenomena as a cylinder approaches a plane boundary. Initially, the downwash is predominant over the venturi effect. Then the venturi effect becomes dominant, until the gap narrows sufficiently for
establishment of Poiseuille flow.

As the cylinder approaches the plate, the period of vortex shedding rapidly progresses from that of a cylinder in a free stream to one that is essentially constant over the range of gap ratios. At a distinct gap ratio, apparently specific to each flow regime, vortex shedding is observed to cease. Further reduction of the gap ratio leads to fluctuations in lift and drag forces observed both experimentally [Roshko, et. al. (1975), Bearman and Zdravkovich (1978)] and in the current investigation. No evidence of vortex shedding is present at gap ratios smaller than that of initial suppression. Taneda (1965) performed experiments with a cylinder towed in the vicinity of a fixed plate, thereby eliminating the development of a plate boundary layer. He observed a vortex street at all gap ratios, however the street was composed of only vortices shed from the upper portion of the cylinder when the gap ratio was small. Thus it is evident that the presence of the plate boundary layer plays a significant role in the complete suppression of vortex shedding. Also of interest is the relatively sharp delineation of the gap ratio for which vortex shedding suppression is observed.

1.2 Previous Investigations

The current investigation of laminar flow about a circular cylinder near a plane boundary has foundations in numerous previous studies of unbounded and semi-bounded configurations. An unbounded configuration is one in which the body is placed into a uniform upstream flow and is free of the influence of other bodies or physical boundaries. A body whose flowfield is influenced by a nearby boundary or other body, but is not contained in an enclosure or subjected to free surface effects, will be classified as a semi-bounded configuration. Henceforth in this discussion, the phrase "semi-bounded cylinder" will refer to a cylinder placed near a plane boundary as shown in Figure 1-2. Previous investigations of unbounded and semi-bounded flow about a
circular cylinder fall into three major categories: experimental, analytical, and numerical/computational.

Experimental studies are typically performed in a wind tunnel or a liquid tow tank depending on the Reynolds number required and the degree of flow blockage that can be tolerated. Free stream turbulence introduced into the oncoming flow by the test apparatus is also an experimental consideration. Analytical investigations include wake oscillator models and potential flow models. Wake oscillator models do not attempt to characterize the flow field and are more closely related to a structural dynamics representation than a fluid dynamics investigation. The periodic wake motion is translated into a periodic loading on the cylinder, which in turn experiences motion governed by its structural properties. Potential flow models do not admit viscous effects such as boundary layer development or flow separation, and are therefore unable to account for boundary layer interaction, vortex formation, or dissipation. Potential flow models find use in prediction of order of magnitude behavior in the low velocity portion of accelerating flows, and have been used in the analysis of multiple body interactions as in flow over an array of tubular members. Numerical/computational investigations include discrete vortex methods and numerical approximations to the incompressible Navier-Stokes equations governing the fluid flow. Both techniques model the flow field and require a discretization of the physical continuum into a computational mesh. Bearman (1984) observes that the techniques capable of modeling the flowfield hold the greatest promise as successful predictive tools in the absence of experimental data. In the discrete vortex method, vortex sheets emanating from the separation points on the body are discretized as point vortices and subjected to mixing and dissipation. Discrete vortex methods are calibrated against experimental data to attain the necessary levels of vorticity distribution and dissipation. In the absence of experimental data for model calibration, however, it is difficult to view this method as a useful predictive tool for differing physical geometries such as for semi-bounded flow.
Numerical approximations to the incompressible Navier-Stokes equations are most commonly performed using finite difference, finite element, or finite volume methods. Computational effort increases greatly with the refinement of the discretized domain. Laminar flows can be directly simulated with a resolution of $10^3$ to $10^4$ mesh points. Direct simulation of turbulent flows requires a resolution fine enough to capture the length scale for dissipation of turbulent kinetic energy. Rogallo and Moin (1984) estimate that $10^{10}$ mesh points would be required to achieve this level of resolution. At present, turbulence modeling is required to circumvent this prohibitive level of computational effort. Numerical approximations are readily applied to most geometries; verification of the technique is readily made against experimental evidence, and experience with these methods suggest a high potential for use as predictive tools in the semi-bounded configuration investigated herein.

Experimental studies of vortex shedding in the unbounded cylinder configuration, both fixed and flexibly mounted, comprise a significant portion of previous investigations. Several comprehensive summaries of experimental works have been compiled, including those by King (1977), Sarpkaya (1979), Bearman (1984), and Griffin (1984). In the low subcritical range of Reynolds numbers, most works address the fixed cylinder configuration. Kovasznay (1949) examined the fully laminar flow range, followed by Tritton (1959), Tanida (1973), Gerrard (1978), and Friehe (1980). Bloor (1963) studied the transition of the fully laminar flow range to the onset of turbulence in the far wake. A cylinder subjected to a forced oscillation transverse to the flow at $Re_D$ from 120 to 350 was investigated by Griffin and Votaw (1972).

The balance of the subcritical range, where wake turbulence develops closer to the cylinder with increasing Reynolds number, has been widely studied beginning with the work by Bloor and Gerrard (1966) for a fixed cylinder in the Reynolds number
range of $10^3$ to $10^4$. Kiya, et. al. (1982) extended this work in the Reynolds number range of $3 \times 10^4$ to $10^5$ and the research by Cantwell and Coles (1983) at $Re_D=1.4 \times 10^5$ has yielded extensive information about turbulence in the wake region. The effect of free stream turbulence on reducing the separation angles at the cylinder has been the subject of studies by Symes and Fink (1977), Britter and Hunt (1979), Hanarp and Sunden (1982), and Mulcahy (1984). Roughness of the surface of the cylinder and its effect on the turbulent wake has been addressed by Achenbach and Heinecke (1981). Free oscillation of the cylinder was investigated by Bearman and Currie (1979) and by Howell and Novak (1979). Sarpkaya (1978) studied the forced oscillation of a cylinder in the Reynolds number range of $5 \times 10^3$ to $2.5 \times 10^4$, and later investigated a freely suspended cylinder in an oscillatory flow field [Sarpkaya et. al. (1981)]. More recently, Sin and So (1987) have examined the three dimensional spanwise correlation of the vortices forming at the cylinder for varying ratios of cylinder length to diameter.

The critical flow range, where a sharp reduction in the drag coefficient is observed as the separation points move rearward on the cylinder, has been bracketed by the fixed cylinder work of Bearman (1969), and Farrell and Blessman (1983). A configuration similar to a smokestack, where the cylinder has one end fixed on a plate and the other end in the free stream has been investigated by Ayoub and Karamcheti (1982) over the Reynolds number range of $8.5 \times 10^4$ to $7.7 \times 10^5$. Supercritical flow studies have been performed by Roshko (1961) for a fixed cylinder, by Szechenyi (1975) and James (1979) to study the effects of surface roughness, and by Adomaitis (1980) to observe the effects of free stream turbulence.

Analytical studies of the unbounded cylinder configuration include the elaborate order of magnitude analysis presented by Peregrine (1975) and the more concise order of magnitude analysis performed by Cohen and Walker (1989). Both utilize the
governing equations to develop a rationale for the physical phenomena observed. Ogawa and Nakagawa (1978) have modeled the inviscid flow about a fixed and a flexibly mounted cylinder in terms of a complex potential representation of the vortex street in the wake. The work differs from a discrete vortex method due to the absence of consideration for dissipation effects. Wehrle (1986) solved the laminar boundary layer equations for flow over a cylinder to attain a correlation between separation and the wall shear gradient in the flow direction, leading to prediction of drag behavior.

Numerical/computational investigations for the unbounded cylinder configuration are abundant in the literature, yet most of the effort has been concentrated in the last 10 years. Comprehensive summaries have been published by Rogallo and Moin (1984), Krause (1985), and Ferziger (1987). Sarpkaya and Schoaff (1979a,b) have reported discrete vortex method studies in the upper subcritical range for impulsively started flow and for improved simulation of the separation process in unsteady flow. Stansby (1981) used a discrete vortex method to investigate flow at $Re_D=3\times10^4$ and later reported on the simulation of secondary separation effects in the wake formation [Stansby and Dixon (1982)]. Early work in finite difference approximation was done by Ingham (1968) for starting flow at Reynolds numbers of 40 and 100. Hurlbut et. al. (1978) studied the synchronization of a flexibly mounted cylinder with vortex shedding at Reynolds numbers of 80 and 100 and later performed work with a flexibly mounted cylinder away from the region of flow synchronization [Hurlbut et. al. (1982)]. Swanson and Spaulding (1978) simulated three dimensional flow at $Re_D=100$ for a fixed cylinder in both steady and unsteady flow. A three dimensional analysis was also performed by Kwak et.al. (1986) investigating the onset of periodic vortex shedding. Fornberg (1980) used finite differences in the vorticity-stream function formulation to extend steady flow to Reynolds numbers of 150 and 300 [Fornberg (1985)]. The vorticity-stream function formulation was also used
by Loc (1980) along with a higher order differencing method to examine flow at a Reynolds number of 1000.

Finite difference simulations at increasing Reynolds numbers require a greater concentration of mesh points in the boundary layer and possibly higher order differencing schemes (as done by Loc) to couple more mesh points into the update of values at each individual mesh point. Lecointe and Piquet (1984) surveyed several differencing schemes for use in flows to Re_D=3000, covering impulsively started flow, fixed cylinder flow, and forced oscillation of a cylinder transverse to the flow direction. Kawamura and Kuwahara (1984) utilized a modified upwind differencing technique to achieve a simulation over the range of Re_D=10^3 to 10^5 without employing a turbulence model. They experienced difficulty at the upper end of this range but were able to accomplish a numerical investigation of the effects of varying surface roughness. Chilukuri (1987) utilized a concentrated mesh to investigate flow between Reynolds numbers of 60 and 1000 for both fixed and flexibly mounted cylinder configurations and this upper Reynolds number was also studied by Baba and Miyata (1987). The discontinuity in vortex shedding frequency at Re_D=100 reported in experimental studies was examined numerically by Allen and Walker (1989) to eliminate the effects of experimental errors. The numerical work showed a continuous function from above and below this Reynolds number. Jackson (1987) applied finite element analysis methods to the simulation of laminar flows about circular and other cylindrical bodies as verification of a tool for predicting onset of periodic vortex shedding. Karniadakis (1988) also reported successful results using finite element techniques for unbounded flows between Reynolds numbers of 50 and 200. Finite volume techniques have been used by Braza et. al. (1986) in the low subcritical range and by Majumdar and Rodi (1985) with a turbulence model to investigate Re_D=10^5 to 3.6x10^6.

Experimental studies of the semi-bounded configuration are prevalent in the
literature, although concentrated in the upper subcritical Reynolds number range. One laminar study was performed by Taneda (1965) using a cylinder towed near a plate boundary at gap ratios from 0.6 to 0.1. At the lower value, Taneda observed only a single-sided vortex street emanating from the top of the cylinder. No plate boundary layer was present. Clauser (1956) utilized a cylinder on a plate as a trip wire to induce turbulence in the plate boundary layer, and was followed by Okamoto (1983) in a similar investigation. Wilson and Caldwell (1973) performed a fixed cylinder study over the Reynolds number range of $10^3$ to $10^5$ and for gap ratios from 0.3 to 0.0. Roshko et. al. (1975) examined the gap ratio range from 2.0 to 0.0 and Göktun (1975) focused on the gap ratio range of 1.0 to 0.0. Bearman and Zdravkovich (1978) studied the gap ratio range from 3.5 to 0.0 and observed the suppression of vortex shedding at a gap ratio of 0.3 for this combination of Reynolds number and plate boundary layer development. Zdravkovich (1981) later repeated these results in a flow visualization study. Sarpkaya and Rajabi (1979) examined the effects of cylinder roughness at zero gap ratio and Knoll and Herbich (1980) followed with a zero gap ratio study at a single roughness value. The effect of varying levels of free stream turbulence was addressed by Marumo et. al. (1978) for gap ratios from 4.19 to 0.75. Angrilli et. al. (1982) traversed the gap ratio range of 6.0 to 0.5 and more recently, Figureido and Viegas (1988) examined the gap ratio range of 0.4 to 0.0 in conjunction with forced convection heat transfer research. Several investigations have also been conducted for oscillatory flow as would be experienced on the seafloor beneath a wave. Yamamoto et. al. (1974) examined oscillatory flow in the Reynolds number range of $2\times10^3$ to $3\times10^4$ over a gap ratio range of 0.82 to 0.10. Sarpkaya (1976) worked in a similar Reynolds number range and over gap ratios from 0.01 to 1.00, and was followed by Wright and Yamamoto (1979) working at $Re_D=1.8\times10^5$ over gap ratios from 2.0 to 0.0.

Interest in the critical flow range for the semi-bounded configuration began early as
a result of aircraft ground effect studies in a wind tunnel. Unable to eliminate the plate boundary layer, Pistolesi (1937) worked at Re_D = 3.7 \times 10^5 at gap ratios from essentially free stream to the computed thickness of the plate boundary layer. Buresti and Lanciotti (1979) observed suppression of vortex shedding during their study of varying cylinder roughness, covering a gap ratio range of 2.5 to 0.0. The same range was addressed by Bruschi et al. (1982) in another fixed cylinder investigation. Behavior of a flexibly mounted cylinder in the semi-bounded critical flow range was investigated over gap ratios from 50.0 to 1.0 by Tsahalis and Jones (1981). Recant et al. (1939) document another ground effect wind tunnel study, this time at the supercritical Reynolds number of 6 \times 10^5. More recently, Beattie et al. (1971) examined the effects of varied cylinder roughness in supercritical flow at zero gap ratio. Oscillating flow studies in the higher Reynolds number range have been conducted by Yamamoto and Nath (1976b) and by Sarpkaya (1977) for very small gaps where the effects of the plate boundary layer are present.

Evaluation of a ground cushion-supported high speed vehicle led Tuck (1971) to develop a complex potential analytical method for a semicircular cylinder moving near a plane boundary. Yamamoto and Nath (1976a,b) utilize potential flow analysis to model behavior of one or more cylinders arrayed adjacent to a plane boundary. Such analyses address inviscid flow and are necessarily limited to very high Reynolds number representations. At the opposite extreme, Davis and O'Neill (1977) examined low Reynolds number solutions to the problem of a cylinder approaching a plane boundary. Their work predicts alternate separation regions on the plate and cylinder that interweave to become the recirculating regions observed when the cylinder contacts the plate.

Careful examination of the literature for numerical/computational investigations of the semi-bounded configuration revealed this to be a novel area for research. Only three related studies have been performed, none of which directly address the semi-bounded configuration. Sesterhenn and Müller (1976) simulated free convection flow about
a cylinder at a fixed distance from a plane boundary using a finite difference representation in the vorticity-stream function formulation. Haussling and Coleman (1977) used finite difference techniques to model a low Reynolds number free surface flow induced by a submerged body, located near a plane boundary for some cases examined. A similar investigation of free surface flow induced by a submerged body was conducted by Thompson and Shanks (1977) using finite difference techniques and spanning Reynolds numbers from 20 to 100. Both free surface studies proved useful to the current investigation as references, providing support for the computational domain sizing rationale and guidance with regard to problems encountered by these investigators with the mesh transformations chosen for their work.

1.3 Scope of the Current Investigation

The review of previous investigations has shown that extensive experimental work has been done at the high subcritical Reynolds number range for both unbounded and semi-bounded configurations. Numerical work has been concentrated in the laminar unbounded flow configuration. Many practical engineering applications require consideration of the semi-bounded configuration, and the current research has been conducted as a step toward orderly development of a tool for numerical analysis of this configuration. The research described herein addresses the development of a finite difference simulation capability for the semi-bounded flow configuration that has been used to investigate the phenomenon of vortex shedding suppression. Two Reynolds numbers have been selected, 80 and 100, to establish the trends in lift, drag, and vortex shedding frequency with change in the viscous behavior of the flow. Primary emphasis has been placed on fixed cylinder simulations over a range of gap ratios bracketing the suppression of vortex shedding. However, several cases demonstrating the capability of the simulator to handle the dynamics of a flexibly mounted cylinder have also been
The two Reynolds numbers were selected to assure the validity of a two-dimensional flow assumption. Chen (1987) tabulates several investigations into spanwise correlation of vortices shedding from a cylinder and notes a correlation length of 15 to 20 diameters for unbounded flows in the fully laminar flow regime. Correlation length drops sharply to 2-3 diameters with the onset of turbulence in the far wake. The laminar Reynolds numbers permit the opportunity to capture the physics of the problem in a fundamental viscous flow. As noted by Cohen and Walker (1989), many investigations are undertaken before a firm footing in the physics has been established. At higher Reynolds numbers, the vortex suppression phenomenon has been observed at small gap ratios on the order of 0.3. In many experimental studies, the phenomenon has been overlooked due to emphasis on obtaining values for mean lift and drag coefficients. In others, gap ratios this small were not examined at all. For the laminar flows investigated herein, suppression was observed at gap ratios of between 1.5 and 1.7 due to the thicker boundary layer developed in a more viscous flow. Thus the selection of the laminar Reynolds numbers assured the ability to bracket the gap ratio range for vortex shedding suppression.

Because this research was planned as part of an orderly program to achieve a broad simulation capability, several features have been included to facilitate expansion of the capabilities. A fully-implicit Navier-Stokes solver developed by Allen (1987) has been improved to accept the semi-bounded flow geometry as well as to enhance the speed and accuracy of operation by several orders of magnitude. The solver is described in detail in Chapter 2, and has been extensively tested against experimental evidence in an unbounded configuration. The code has been written in primitive variable form (u, v, p are the dependent variables) to facilitate addition of the transport equations and mixing length model required for turbulent flow simulation. A body-fitted coordinate transformation has been used to permit consideration of alternate body and plate
geometries as well as admit the problem of multiple body interactions. For example, a hydrodynamicist may wish to model a wavy ocean floor and a non-circular pipeline shell in the presence of a parallel pipeline or cable. The body fitted coordinate transformation used herein requires only a description of the boundary points in Cartesian (x,y) space. Dynamic simulation capability has also been preserved in anticipation that future applications will require this capability.

The fixed cylinder cases examined at each Reynolds number cover the gap ratio range from 3.0 to 0.5, with one unbounded flow case included at each Reynolds number to provide a tie point to experimental verification. Flexibly mounted cylinder cases were run at a Reynolds number of 100 and for single values of the structural parameters governing vibrational behavior. Gap ratios for the flexibly mounted cylinder cases were selected based on the lift characteristics at each gap ratio that were displayed by a fixed cylinder. The moving cylinder experiences a different net lift with each change in the gap ratio. Thus a cylinder placed in an impulsively started flow with a developing plate boundary layer can be expected to exhibit a different behavior than for one that is released at a particular gap ratio in an established flow field. A comprehensive study of the flexibly mounted cylinder would require an extensive case matrix at each gap ratio covering a range of Reynolds numbers and structural parameters as well as consideration of the effects of starting flow. Clearly, such work would have been a substantial digression from the objective of studying vortex shedding suppression in laminar flow for a semi-bounded configuration.

All computational work, including code development and modification was performed on the Rice University AS/9000 mainframe computer.

1.4 Summary of Results

This section provides a summary of the results obtained for the gap ratio traverses
conducted at the two Reynolds numbers for the fixed cylinder in the semi-bounded configuration. More detailed documentation, including results for the flexibly mounted cylinder investigations, is presented in Chapters 3 through 5. Qualitatively, the time history data plots for each gap ratio case begin with an impulsive start of the flowfield from initial values set by the simulator's velocity field generator. As an approximation, this initial estimate lacks the features of the wake, or of the boundary layers at the cylinder and plate. Until the simulation has run long enough to establish these features, the starting transient behavior is evident. Figure 1-3 shows a typical starting transient for lift and drag coefficient time histories over the first thousand simulation intervals. Each simulation timestep, or record, corresponded to a nondimensional time interval of 0.05 and each time history was run for 3000 records or until steady state periodic behavior was achieved. A rough estimate for the minimum number of records to pass the transient flow out of the computational domain is made by observing that an undisturbed particle traveling at $U_0$ would have required at least 800 records to pass through the domain employed. In all cases, more than 800 records were required due to the disturbances presented by the cylinder and the plate, wake features, and the diffusive effects of the flow in the upstream direction. Figure 1-4 illustrates steady state periodic lift and drag coefficient time histories taken from records near the end of a simulation. During a complete vortex shedding cycle, the alternate shedding of the upper and lower vortices creates two drag maxima while generating a lift maximum and minimum over the same cycle. This accounts for the different shape of the drag coefficient plot from the lift coefficient plot. As the gap ratio is reduced toward that of vortex shedding suppression, the drag coefficient time history plot becomes smoother as a result of the decrease in separation point motion.

Vortex patterns forming in the wake of the cylinder under steady state conditions differ markedly in their appearance as the gap ratio is reduced. In this study, vortex
shedding suppression was observed for gap ratios at or below 1.5 for flow at \( \text{Re}_D = 100 \) and at or below the gap ratio of 1.7 for flow at \( \text{Re}_D = 80 \). The vortices formed in an unbounded configuration are similar in shape whether originating from the upper or the lower shear layer. The semi-bounded cases showed a progressive elongation of the vortices as the gap ratio was reduced toward that of vortex shedding suppression (Figure 1-5). Upon cessation of vortex shedding, an elongated lower vortex remains attached to the cylinder. The pressure on the plate and in the wake region aft of the cylinder is higher than on the cylinder itself. This prevents the upper shear layer from being drawn across the wake, and in the absence of the oppositely-signed circulation provided by the upper shear layer, the lower vortex is not shed. As the gap ratio is further reduced, the cylinder is located deep in the plate boundary layer where upstream velocities are small and the lower vortex is greatly reduced in size (Figure 1-6).

The velocity profile in the gap between the plate and the cylinder yields information on the degree of interaction between the cylinder boundary layer and the plate boundary layer as the gap ratio is reduced. Figures 1-7 and 1-8 show the progression from two essentially distinct boundary layer profiles (cylinder boundary layer and plate boundary layer) to the formation of a parabolic Poiseuille profile as \( h/D \) is reduced. The cylinder boundary layer is much thinner initially because it has less distance over which to develop relative to the length over which the plate boundary layer has developed.

A quantitative summary of the results of this investigation is presented in Figures 1-9 through 1-13 where fixed cylinder data for both Reynolds numbers is plotted against the gap ratio. The mean drag coefficient and drag coefficient amplitude are shown in Figure 1-9. The amplitude represents the magnitude of the departure from the mean over each cycle. It is provided along with the mean to identify potentially large loadings that could average to a less significant value. The general shape of each mean drag coefficient curve appears to be governed by the progression of the cylinder into the region
of reduced upstream velocity as the gap ratio is reduced. This trend is consistent with experimental findings at higher Reynolds numbers for the semi-bounded studies of Bruschi et. al. (1982), Figureido and Viegas (1988), and Roshko et. al. (1975), among others. Examination of the shear and pressure components of the drag for each Reynolds number indicates that the offset of the two curves is primarily due to a larger shear component at the lower Reynolds number associated with the more viscous flow. Both curves appear to approach the experimentally-determined values for an unbounded configuration as the gap ratio is increased. The drag coefficient amplitude is linked to the relative movement of the separation points. The amplitude exhibits a small magnitude, slowly decreasing as the gap ratio is reduced toward the suppression of vortex shedding. Upon further decrease of the gap ratio, a significant increase in the magnitude is observed. This feature will be noted again for the lift coefficient amplitude as well as for the loading period and separation point behavior. Experimental studies by Bearman and Zdravkovich (1978) and Roshko et. al. (1975) indicate the onset of periodic disturbances at small gap ratios in the absence of vortex shedding. Briefly, the behavior described has been attributed by Bearman and Zdravkovich to the pressure disturbances associated with Tollmein-Schlichting waves preceding the transition to turbulence in the wake region, and more generally by Roshko et. al. to flow instabilities in the wake. Göktun (1975) cites a 1946 study by Spivak of flow between two closely spaced cylinders that also resulted in oscillatory forces when the gap was smaller than 0.5 diameters. Vortex shedding was not present at this small gap, but switching of the wake bias from the side of one cylinder to the other was observed.

The mean lift coefficient and the lift coefficient amplitude are shown in Figure 1-10. The mean lift coefficient for an unbounded configuration ($h/D=\infty$) is zero, hence the positive mean lift coefficient observed at the upper end of the gap ratio range examined must return to the zero value as the cylinder moves further from the plate. The experimental works for the semi-bounded configuration that have previously been cited
are concentrated in the high subcritical and critical Reynolds number flows. For these flows, the boundary layers tend to be thin and show significant interaction at much smaller gap ratios than determined herein for the low Reynolds number flows. Sarpkaya (1977) differentiates between "wall proximity effects" and "boundary layer effects" although the presence of a boundary layer will always contribute to the overall wall proximity. The wall proximity effects include the interference of the boundary on the flowfield about the cylinder. He observes that the venturi effect of attraction to the wall is always present and is fairly independent of the gap ratio. The lift away from the wall, caused by the presence of flow separation, is strongly influenced by the gap ratio in the range of boundary layer interactions. Experimental studies in the subcritical flow regime performed by Bruschi et. al. (1982), Figureido and Viegas (1988), and Roshko et. al. (1975), show a mean lift coefficient that has a large positive value for small gap ratios, decreasing rapidly to a zero value for an unbounded cylinder. Figure 1-10 shows that the current research in a more viscous flow captures the large mean lift coefficient at small gap ratios reported for the higher Reynolds number experimental flows, but also admits a range of gap ratios in which the attraction to the plate boundary exceeds the lift away from the plate. This latter behavior is reasonable in view of the relationship of the lift away from the plate to the amplitude (and phasing) of the separation angles on the cylinder that are at minimum values in this same range of gap ratios.

The period of loading on the cylinder is plotted in Figure 1-11. The period of oscillatory loading rapidly approaches a maximum value as the gap ratio is reduced toward the suppression of vortex shedding. Maximum periods of 12.25 seconds and 11.85 seconds were determined for flow at ReD of 100 and 80, respectively. This phenomenon was also observed during the semi-bounded flow investigations of Bearman and Zdravkovich (1978), Göktun (1975), and Buresti and Lanciotti (1979). In contrast to this maximum value, the vortex shedding period of an
unbounded cylinder is 6.3 seconds and 7.0 seconds at Re_D of 100 and 80, respectively [Tanida (1973)]. Further reduction of the gap ratio leads to the onset of long period loading on the cylinder that is attributed to changes in the pressure field of the wake.

Behavior of the mean upper and lower separation angles as the gap ratio is reduced (Figures 1-12 and 1-13) is guided by the pressure field in the wake of the cylinder. A smaller absolute value of the separation angle (note that angles are measured from the rear of the cylinder with counterclockwise as the positive direction) corresponds to rearward movement of the separation points on the cylinder. Such rearward movement is consistent with delayed separation due to the favorable pressure gradient provided by viscous flow in the gap. Zdravkovich (1981) observed from flow visualization studies that the lower separation point moved aft on the cylinder as the gap ratio was reduced. This is consistent with the data presented in Figure 1-13 for the current investigation. In the present study, the separation angle is determined by numerical computation of the velocity gradient in curvilinear coordinates at the surface of the cylinder, and a search around the cylinder boundary for the change in sign of the gradient indicative of flow separation. At small gap ratios, the routine was unable to determine a unique lower separation angle. One possible explanation is provided by Davis and O’Neill (1977) whose low Reynolds number analysis of the semi-bounded configuration predicted the formation of recirculation regions alternating on the plate and cylinder. Upon contact between the cylinder and plate, these regions coalesce into the recirculating flows observed in flow visualization studies. The amplitude of the upper and lower separation point motion is small in magnitude until the gap ratio is reduced to the point where long period loading begins. This latter effect on separation angle movement is attributed to the response of streamline curvature to the long period variation of pressure in the wake noted above.
Figure 1-1 Reynolds Number Relationship to Vortex Shedding from a Stationary Cylinder
Figure 1-3 Starting Transient for h/D=2.0 at ReD=100. (a) Lift Coefficient, (b) Drag Coefficient
Figure 1-4 Steady State Periodic Behavior for h/D=2.0 at ReD=100.
(a) Lift Coefficient, (b) Drag Coefficient
Figure 1-5 Vortex Shape at ReD=100. (a) h/D=3.0, (b) h/D=1.2
Figure 1-6  Vortex Shape at ReD=100.  (a) h/D=0.7,  
(b) h/D=0.5
Figure 1-7 Velocity Profile Between Cylinder and Plate
(a) h/D=3.0, (b) h/D=2.0. ReD=100.
Figure 1-8 Velocity Profile Between Cylinder and Plate
(a) h/D=1.0, (b) h/D=0.5. ReD=100.
Figure 1-9 (a) Mean Drag Coefficient, (b) Drag Coefficient Amplitude
Figure 1-10 (a) Mean Lift Coefficient, (b) Lift Coefficient Amplitude
Figure 1-11 (a) Loading Period, (b) Detail of Loading Period
Figure 1-12 (a) Mean Upper Separation Angle, (b) Upper Separation Angle Amplitude
Figure 1-13 (a) Mean Lower Separation Angle, (b) Lower Separation Angle Amplitude
CHAPTER 2
Numerical Simulation Technique

2.1 Overview

The most widely used numerical techniques for the analysis of fluid-structural interactions are finite difference methods and finite element methods. Finite element analysis techniques have been widely applied to structural problems and are just beginning to attain acceptance in fluid mechanics and heat transfer. Finite element methods hold the primary advantage of accepting arbitrary flowfield boundary geometries in a manner that is effectively transparent to the user. Thus considerations associated with the problem geometry comprise a much smaller portion of the overall computer code than is required for a finite difference code. The disadvantages of finite element methods reside in reduced historical experience with their use; in time, greater numbers of applications will foster interest in the quantification of error analysis and computational mesh refinement requirements that have long been established for finite difference methods. The latter enjoy a long history of application in structural and fluid mechanics problems.

Finite difference methods yield a more intuitive problem formulation that, for coarse mesh discretization, can be solved without the use of a computer. The development of the body-fitted coordinate transformation for use with finite difference methods makes possible the application of this technique to problems of arbitrary geometry, including those involving interaction of multiple bodies. However, the enhanced capability comes at a cost of substantially increased dependence of the computer code on the particular transformation of coordinates selected. This results in the need to create customized code for each specific application, however portions of the code related to the actual solution of the governing equations share many common features.
The body-fitted coordinate transformation represents a tradeoff between the use of simple forms of the governing equations with extremely complicated boundary conditions, and the use of extended forms of the governing equations with simplified boundary conditions. For a non-rectangular body geometry, the former requires special differencing techniques at the boundary due to the uneven spacing of grid points in this area. The body-fitted coordinate transformation maps the physical domain onto a uniformly-spaced rectangular computational domain for which conventional differencing techniques are appropriate. The current research has been accomplished through use of the body-fitted coordinate transformation with the finite difference method.

In the numerical simulation of a fluid flow problem, approximations to the physical continuum must be made. Finite difference techniques seek solutions to the governing equations at each discrete point of the computational mesh. The quality of the overall approximation is in part dependent upon the degree of mesh refinement and upon the relative distribution of the mesh points in physical space. Practical considerations of cost and available computational capability also limit the size of the physical space that can be considered. As an example, flow about an unbounded cylinder originates at an infinitely distant upstream location and moves toward an infinitely distant downstream location, carrying with it the slowly-decaying velocity defect caused by the presence of the cylinder. Above and below the cylinder, infinite distances also prevent any disturbance to the flow. In a numerical simulation, the infinity boundaries are replaced by computational domain boundaries placed far enough from the body to limit the effect on the flow, but close enough to present a reasonable region for discretization by a finite number of grid points.

Much thought must be given to the sizing of the computational domain, the number and distribution of the grid points, and the effect of inflow/outflow boundary conditions on the accuracy of the numerical simulation. These considerations, along with the presentation of the governing equations and simulator operation, are addressed in this
chapter. As an aid in the understanding of the material to follow, the operation of the simulator used in the current investigation is summarized below:

1. Enter the geometry and flow parameters to be used by the simulator, including structural parameters required for flexibly-mounted cylinder investigations.

2. Generate a computational mesh based on input geometry information that includes the initial location of the center of the cylinder and the size of the computational domain.

3. Generate an estimated velocity field based on input boundary values.

4. Generate an estimated pressure field based on input boundary values.

5. Solve the governing equations at each point in the computational mesh for the current timestep.

6. Output data for the current timestep and prepare for the next timestep.

7. (Moving cylinder only): Compute pressure and shear loads on the cylinder.

8. (Moving cylinder only): Update the cylinder position and velocity based on the loads computed in Step 7.

9. (Moving cylinder only): Generate a new computational mesh reflecting the updated position of the cylinder.

10. Return to the solution of the governing equations (Step 5) and repeat Steps 5 through 10 until the desired number of timesteps have been completed.
2.2 Generation of the Computational Mesh

A body-fitted coordinate transformation is used in the current research to map the curved geometry of the physical plane onto a uniformly-spaced computational plane. This transformation is generally attributed to the work of Thompson (1974) and results in a curvilinear coordinate system that has coordinate lines coincident with the boundaries in the physical plane. In doing so, the need for special differencing techniques at the irregular portions of the boundaries is eliminated. Figure 2-1 illustrates the transformation of the inner boundary \( \Gamma_2 \) comprising the cylinder in physical space to the curvilinear coordinate line corresponding to \( \eta_{\text{max}} \) in the computational plane. Similarly, the outer boundary \( \Gamma_1 \) comprising the free stream boundary and plate boundary in the physical plane is mapped onto the curvilinear coordinate line corresponding to \( \eta=1 \) in the computational plane. Because the physical boundaries are continuous, points a and d on the outer boundary must have the same value and points c and b on the inner boundary must have the same value. The lines shown as \( \Gamma_3 \) and \( \Gamma_4 \) in the physical plane define a branch cut along which this multi-valued relationship exists. In the computational plane, a correspondence exists between the values along \( \Gamma_3 \) (the re-entrant line) and \( \Gamma_4 \) (the end line) comprising the branch cut. This figure represents a type-O coordinate transformation that is particularly well suited for cylindrical bodies. Thompson (1982) and Thompson et. al. (1985) detail several alternative transformation configurations.

The specific mapping of interior mesh points in the physical plane onto the uniform rectangular mesh of the computational plane is, in general, neither known nor required. The elements of the tensor transformation between the two coordinate systems are computed numerically at each grid point using \((x,y)\) coordinate information from the
distribution of grid points in physical space. This distribution is guided by the desire to concentrate grid points in the boundary layer regions and in other areas of large gradients. Limitations on the distribution are also imposed by truncation error that can be introduced by skewness of the \((\xi,\eta)\) coordinate curves in physical space and by regions of large spacing between the grid points. The distribution of grid points in the physical plane can be achieved by use of several techniques, ranging from manual placement to elaborate adaptive grid methods.

The technique used in the current research is known as an algebraic method and begins (see Figure 2-2) with establishment of lines of constant \(\xi\) connecting the inner boundary (cylinder) to the outer boundary (free stream boundary and plate boundary). These lines begin at the branch cut and proceed counterclockwise in constant angular increments from the center of the cylinder. Lines of constant \(\eta\) run from the re-entrant line to the end line of the branch cut beginning with the outer boundary and proceeding inward to the inner boundary. The intersection of a \(\xi\) and an \(\eta\) line defines the location of a grid point. The algebraic method relies on an explicit functional realtionship to generate the \((x,y)\) coordinates for each intersection [Gordon and Hall (1973), Thompson, et. al. (1985)]. Figures 2-3 through 2-5 illustrate the algebraically-generated computational meshes used for simulation of an unbounded cylinder and of a semi-bounded cylinder at gap ratios of 3.0 and 0.5 respectively.

Another widely used technique requires solution of a system of elliptic partial differential equations subject to the boundary values established for grid points on the inner and outer boundaries. The relations in physical space take the form of Poisson equations

\[
\nabla^2 \xi = P(x,y) \quad (2.2.1a) \\
\nabla^2 \eta = Q(x,y) \quad (2.2.1b)
\]
where \( P(x,y) \) and \( Q(x,y) \) are control functions that are used to concentrate grid points in desired regions. Thompson cautions that not all forms of \( P(x,y) \) and \( Q(x,y) \) will yield meshes free of overlapping coordinate trajectories. Note that if the control functions are set to zero, Laplace's equation results for each coordinate and the placement of grid points tends toward an equidistribution in spacing. Adaptive grid techniques using elliptic generation methods will link the form of the control functions to a parameter in the solution such as a pressure or velocity gradient, such that spacing is reduced in regions of high gradient values. The primary disadvantage of the elliptic generation technique is the large consumption of computational time in the mesh generation. The numerical solution of an elliptic equation results in a large system of simultaneous equations that must be solved by iterative techniques. In a dynamic simulation, as in the case of a moving cylinder or in an adaptive grid investigation, the mesh must be regenerated many times. In addition, the numerical solution of the elliptic problem requires a "first guess" for the grid point distribution, usually provided by an algebraic generation method. Shanks and Thompson (1977) report use of as much as 6 cpu minutes for elliptic mesh generation at each timestep of their dynamic simulation, exclusive of the actual governing equation solution. The algebraic mesh generation used in the current research required less than 0.3 cpu seconds and the entire problem solution required less than 3.0 cpu seconds per timestep. Steger and Sorensen (1980) report on a fast hyperbolic mesh generation technique that permits an explicit (marching) numerical solution and has been applied in compressible flow simulation.

While Equations 2.2-1a,b are expressed in the physical plane, it is desired to conduct all numerical work in the uniform spacing of the computational plane. Therefore these equations, like the governing equations to follow, must be transformed to the computational plane. The basis for the transformation from physical to computational space is:
\[
\begin{align*}
\begin{bmatrix}
\xi = \xi(x, y) \\
\eta = \eta(x, y)
\end{bmatrix}
\end{align*}
\] (2.2.2)

and from computational space to physical space is

\[
\begin{align*}
\begin{bmatrix}
x = x(\xi, \eta) \\
y = y(\xi, \eta)
\end{bmatrix}
\end{align*}
\] (2.2.3)

The Jacobian of the transformation from physical to computational space is given by

\[
J_{p\rightarrow c} = \begin{bmatrix}
\xi_x & \xi_y \\
\eta_x & \eta_y
\end{bmatrix}
\] (2.2.4)

and the Jacobian of the transformation from computational to physical space is given by

\[
J_{c\rightarrow p} = \begin{bmatrix}
x_\xi & x_\eta \\
y_\xi & y_\eta
\end{bmatrix}
\] (2.2.5)

Because the transformation from physical to computational space is the inverse of the reverse transformation,

\[
J_{p\rightarrow c} = J_{c\rightarrow p}^{-1}
\] (2.2.6)

the following hold:

\[
\begin{align*}
\xi_x &= \frac{y_\eta}{J} & (2.2.7a) \\
\xi_y &= -\frac{x_\eta}{J} & (2.2.7b)
\end{align*}
\]
\[ \eta_x = -\frac{y_\xi}{J} \quad (2.2.7c) \]
\[ \eta_y = -\frac{x_\eta}{J} \quad (2.2.7d) \]

where
\[ J = x_\xi y_\eta - x_\eta y_\xi \quad (2.2.8) \]

The position of each grid point in physical space is known in terms of its \((x,y)\) coordinates resulting from the mesh generation. The corresponding \((\xi,\eta)\) coordinates are also known from the mesh generation. Thus it is possible to obtain the grid spacing parameters (left hand side of Equations 2.2-7a through d) in terms of readily available \((x,y)\) coordinate information. The following parameters will be required for the future transformation of physical space equations for use in the computational space [Thompson (1974)]:

\[ \alpha = x_\eta^2 + y_\eta^2 \quad (2.2.9a) \]
\[ \beta = x_\xi x_\eta + y_\xi y_\eta \quad (2.2.9b) \]
\[ \gamma = x_\xi^2 + y_\xi^2 \quad (2.2.9c) \]
\[ P = \alpha x_\xi - 2\beta x_\xi y_\eta + \gamma x_\eta y_\eta \quad (2.2.9d) \]
\[ Q = \alpha y_\xi - 2\beta y_\xi x_\eta + \gamma y_\eta x_\eta \quad (2.2.9e) \]
\[ \tau = (x_\eta Q - y_\eta P)/J \quad (2.2.9f) \]
\[ \sigma = (y_\xi P - y_\eta Q)/J \quad (2.2.9g) \]

Note that the expressions for \(P\) and \(Q\) are not related to the elliptic generation control functions previously described; this is an unfortunate coincidence of common terminology. An example of a first spatial derivative transformation of a function \(f(x,y,t)\) is provided through chain rule differentiation:
\[ f_x = f_{\xi} \xi_x + f_{\eta} \eta_x \quad (2.2-10) \]

but Equations 2.2-7 and 2.2-8 allow this to be expressed in terms of readily computed grid spacing parameters as

\[ f_x = (y_{\eta} f_{\xi} - y_{\xi} f_{\eta}) / J \quad (2.2-11) \]

A similar chain rule transformation would hold for the \( y \) derivative. The first temporal derivative requires consideration of the convective effects of a moving mesh in physical space and the mapping of that mesh onto a fixed domain in the computational space. This is the case in the moving cylinder simulations conducted as part of the current investigation. Additional terms due to convective effects are present as follows:

\[ f_t = [f_t]_{(x, y)} = [f_{\xi}]_{(\xi, \eta)} \frac{(y_{\eta} f_{\xi} - y_{\xi} f_{\eta})}{J} \left[ x_t \right]_{(\xi, \eta)} \]

\[ - \frac{(x_{\xi} f_{\eta} - x_{\eta} f_{\xi})}{J} \left[ y_t \right]_{(\xi, \eta)} \quad (2.2-12) \]

Note that \( x_t \) and \( y_t \) are zero in a fixed cylinder simulation where the body does not move relative to the mesh outer boundary.

In any finite difference approximation of the governing equations for fluid flow, truncation error related to the spacing between the mesh points will occur. In general, the accuracy of the approximation will increase as the number of mesh points employed is increased, provided that proper consideration is given to their relative distribution. The number of grid points chosen for this investigation was 1600, with grid points located at the intersection of the curvilinear coordinate trajectories. The 1600 points are the result of a domain described by 40 radial (\( \xi \)) lines and 40 circumferential (\( \eta \)) lines as shown in Figure 2-2 and hereafter described as a 40x40 mesh. The assessment of the adequacy of the number of grid points is typically determined by repeating a test case on grids of
successively increasing density until changes in the results are minimal [Ghia et. al. (1982), Ferziger (1987)]. The number of grid points is limited by the practical considerations of data storage and retrieval, as well as the increased run time expense to perform computations at a greater number of points. Investigators addressing the unbounded flow over a cylinder have found good correlation with experimental results by using meshes of 40x80 [Chilukuri (1987)], 56x43 [Swanson and Spaulding (1978)], and 54x30 [Shanks and Thompson (1977)]. A direct relationship between accuracy and the mesh sizes used in these references cannot be inferred due to the different mesh point distributions, boundary conditions, domain sizes, and finite difference approximations used by each investigator. The following text will address the effect of these considerations on the results of the current investigation.

The validation of the 40x40 computational domain chosen for this study is derived from the unbounded cylinder work performed by the above investigators and by the prior study at Rice [Allen (1987), Allen and Walker (1989)]. A 20x20 mesh was tested and insufficient resolution of the cylinder boundary layer occurred. Cases using 40x40 and 60x60 mesh sizes provided results consistent with experimental evidence, however the three hour cpu requirement of the 40x40 mesh increased to the economically impractical requirement of 182 cpu hours for the 60x60 mesh. The key result of the mesh sizing investigation was the quantification of the grid point density required in the vicinity of the cylinder boundary layer. Through use of grid point distribution controls, the current investigation utilized a boundary layer grid point spacing comparable to the 60x60 case, while maintaining the computational cost practicality of the 40x40 mesh.

The truncation error of a finite difference technique also includes the effect of grid point distribution when a body fitted coordinate transformation is used [Thompson, et. al. (1985), Thompson and Mastin (1985)]. A complementary analysis of spacing effects is also provided by Braaten and Shyy (1986). In Thompson's analysis, mathematical tools are developed to examine the effect of changes in grid point spacing
and of skewness (non-orthogonality) in the curvilinear coordinate lines. Kalnay de Rivas (1972) asserts that any smooth function with continuous derivatives and zero slope at the boundaries may be used to distribute grid points in physical space. Thompson shows that the choice of distribution function has an impact on the maintenance of order of the differencing technique under the transformation to body-fitted coordinates. Specifically, Thompson shows the unsuitability of simple polynomial functions, logarithmic, and certain trigonometric functions. The hyperbolic sine function recommended by Thompson for problems requiring resolution of a boundary layer has been used in the current investigation to determine the spacing of the \( \eta \) coordinate lines along a line of constant \( \xi \). For a specific \( \eta \) line, associated with the index \( j \) (\( j=1 \) is the outer boundary, \( j=40 \) is the inner boundary), the hyperbolic sine distribution function takes the following form:

\[
\begin{align*}
    r &= r_0 \left( \frac{\sinh \left( \kappa \left( \frac{40 - j}{40 - 1} \right) \right)}{\sinh \kappa} \right) \\
    & \quad \text{ (2.2-13)}
\end{align*}
\]

where \( r_0 \) is the distance along a specific \( \xi \) line from the inner to outer boundary and \( \kappa \) is a stretching parameter that can be determined from a specified minimum spacing between points.

Skewness of the mesh is an obvious concern for the type-O mesh used in this study. The mesh becomes increasingly skew between the cylinder and the plate as the gap ratio \( h/D \) is decreased. Simultaneously the grid point spacing is substantially reduced, an effect associated with reduction in truncation error. Thompson et. al. (1985) offer the following expression for the leading term of truncation error of a first derivative of the general function \( f(x,y) \) in body-fitted coordinates:
\[ TE_x = \frac{1}{2fg} (y_\xi x_\eta x_\eta - x_\xi y_\eta y_\eta^2) f_{xx} + \frac{1}{2fg} (y_\xi y_\eta y_\eta y_\xi^2) f_{yy} \\
\quad + \frac{1}{2fg} [y_\xi y_\eta (x_\eta x_\eta - x_\xi^2) + x_\eta x_\xi y_\eta y_\eta - x_\xi y_\eta y_\eta] f_{yy} \quad (2.2-14) \]

where \( TE_x \) is the leading term of the truncation error, \( \sqrt{g} \) is the Jacobian determinant at the point in question, the first derivatives in \( \xi \) and \( \eta \) represent the grid point spacing, and the second derivatives in \( \xi \) and \( \eta \) represent change in spacing in the neighborhood of the grid point. The \( x \) and \( y \) derivatives apply to the function in physical space. Note that as the skewness increases, the curvilinear first derivatives (related to spacing) grow in magnitude and the truncation error can become large. However, each spacing is multiplied by a factor related to the change in spacing, which can be very small in a compacted region as found between the cylinder and the plate for the problem investigated. Maintenance of a small truncation error in a region of high skewness therefore requires a very small change in spacing between grid points to offset the skewness.

The most highly skewed case investigated herein utilized a gap ratio of 0.50, whereas the mesh generator was capable of a gap ratio as small as 0.10. Using second order central differences for the spacing derivatives, and mesh \((x,y)\) coordinates for the gap ratio of 0.10, the truncation error term of Equation 2.2-14 was calculated at the mesh location of maximum skewness for comparison to that associated with a uniform mesh. The truncation error term reduced to:

\[ TE_x = -0.0002 f_{xx} - 0.0001 f_{yy} - 0.000052 f_{xy} \quad (2.2-15) \]

Provided that the function gradients are not extreme (as in a shock wave), the evaluation indicates that the extremely small change of spacing in the compacted region beneath the cylinder can offset the effect of skewness. Markatos (1978) made use of this effect in his three-dimensional numerical analysis of ship hydrodynamics and Raitby (1976) addressed the effects of skewness through use of special upwind differencing techniques.
that change the form of the truncation error term itself.

Resolution of the boundary layer on both the cylinder and the plate must also be considered in the selection of a computational mesh. Kalnay de Rivas (1972) estimates that a minimum of three grid points need be immersed in the boundary layer to effectively couple the viscous effects to an otherwise inviscid flow. The spacing of the grid points near the cylinder is adjustable in the hyperbolic sine distribution function through use of the stretching parameter included in the function. Sensitivity studies performed in this investigation for the unbounded cylinder case showed the need for more than three points in the boundary layer of the cylinder and a stretching parameter of 5.4 was used to achieve the required concentration for the semi-bounded cases addressed. The maximum cell spacing at the cylinder was 0.025 diameters for all cases examined. Moving away from the cylinder, the hyperbolic sine function generates increased spacing and care must be taken that the gap ratio under examination provides sufficient resolution at the plate boundary. For both Reynolds numbers selected, the undisturbed laminar boundary layers of the cylinder and of the plate would intersect at gap ratios less than 3.0; this anticipated interaction was the basis for selecting the 3.0 gap ratio to begin the current investigation. At the gap ratios studied (ranging from 3.0 to 0.50), half of the computational mesh points were located in the region beneath the cylinder to resolve this boundary layer interaction. Grid point spacing sufficient to resolve the boundary layer effects on the upper portion of the cylinder inherently led to spacing on the underside of the cylinder capable of accurately resolving this region. For gap ratios significantly greater than 3.0, it would be necessary to consider a distribution function capable of providing grid point concentration at both boundaries.

The discretization of continuous space into a computational domain requires the determination of how far the limits of the domain should extend from the region of interest. The form of the governing equations solved in this simulation includes elliptic behavior in pressure. Therefore, any error in the boundary approximation for pressure
immediately affects the entire computational domain. Error in the boundary approximations for the velocity variables is subject to the effects of viscous diffusion from both physical and numerical sources, and is therefore dependent on the distance between the region of interest and the outer boundary approximation (Figure 2-6).

One effect, known as flow field constriction or lateral confinement error [Shaw (1971)], is dominated by the spacing between the body and the boundaries above and below. This effect is analogous to wind tunnel blockage [West and Apelt (1982)] and appears as an increased effective velocity. Separation points on a cylinder experiencing such effects are found to advance into the flow, increasing the pressure drag on the body [Richter and Naudascher (1976)]. The Strouhal number is observed to increase, thereby decreasing the period of oscillation. In the current investigation, sensitivity studies were performed for an unbounded cylinder configuration (due to availability of experimental data for comparison) at spacing above and below the cylinder of 10 and 20 diameters to the free stream boundary. Results at 10 diameters showed the anticipated increase in the mean drag coefficient and associated advance of the separation points into the flow. A test at 20 diameters yielded results well within the range of experimental data of Tritton (1959) for an unbounded cylinder at Re_D = 100. A value of 20 diameters above the cylinder was selected for all cases run in this investigation while the gap between the plate and the cylinder was varied according to the case under investigation. This is contrasted to the value of 50 diameters needed by Chilukuri (1987) for an unbounded cylinder where the differencing technique did not provide supplemental viscous effects in regions of large mesh spacing.

The distance to the outflow boundary required to minimize error is affected by the type of outflow boundary condition chosen. A number of approximations are commonly used, including zero velocity gradient conditions, impressed potential flow, and most commonly, the free stream conditions. The latter, combined with a boundary layer
approximation, was chosen for this investigation because the Navier-Stokes solver used provided substantial viscous damping effects in regions where the mesh spacing is large. Sensitivity studies were conducted at distances of 10, 20, and 40 diameters between the cylinder and the outflow boundary. Results at the 10 diameter spacing showed a tendency of the simulation to underestimate drag loads due to the forced recovery of the flow to free stream conditions imposed by the outflow boundary conditions. It is of interest to note that Shanks and Thompson (1977) used a value of 10 diameters in semi-bounded hydrofoil studies for the U. S. Navy. Results at both 20 and 40 diameter spacings were comparable to each other and in close agreement with the experimental data of Tritton (1959). The 20 diameter spacing to the outflow boundary was selected because it allowed improved resolution of the overall flow field for a given number of grid points. Fuchs (1984) observes that the error introduced by the outflow approximation is propagated upstream in proportion to the reciprocal of the Reynolds number. At a Reynolds number of 100, outflow boundary-induced error would be diffused upstream at only one percent of the rate at which flow features are convected downstream.

2.3 Governing Equations and Boundary/Initial Conditions

The investigation of laminar incompressible flow about a circular cylinder near a plane boundary was accomplished using the primitive variable (u,v,p) form of the Navier-Stokes equations and the incompressible form of the mass conservation (continuity) equation. The Navier-Stokes equations for two-dimensional incompressible flow of a Newtonian fluid are expressed in physical (x,y) space as follows:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \tag{2.3-1}
\]
\[ \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) \]  

(2.3-2)

and the incompressible form of the continuity equation is expressed as:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(2.3-3)

In the numerical approximation of the governing equations, the dilatation represented by the continuity equation is not necessarily zero. An objective is to minimize the magnitude of the dilatation as an approximation to a truly incompressible (zero dilatation) solution. The dilatation is defined as:

\[ D_l = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \]  

(2.3-4)

The Navier-Stokes and continuity equations are placed in nondimensionalized form by using the free stream velocity, the cylinder diameter, and the dynamic pressure head as scaling parameters. The following expressions represent the nondimensionalized variables that will appear in the governing equations:

\[ \tilde{u} = \frac{u}{U_0} \quad \tilde{v} = \frac{v}{U_0} \quad \tilde{x} = \frac{x}{D} \quad \tilde{y} = \frac{y}{D} \quad \tilde{p} = \frac{p}{\rho U_0^2} \quad \tilde{t} = \frac{t U_0}{D} \]  

(2.3-5a) (2.3-5b) (2.3-5c)

\[ \text{Re}_D = \frac{\rho U_0 D}{\mu} \quad \text{or equivalently,} \quad \frac{U_0 D}{v} \]  

(2.3-5d)

The tilde (\(\sim\)) is traditionally dropped in the notation once the nondimensionalization is defined. Reverting to subscripts to denote differentiation with respect to the subscripted variable, the nondimensionalized governing equations without the tilde symbols appear as:
\[ u_x + u_x + v_y = - p_x + \frac{1}{Re_D} (u_x + u_y) \]  
(2.3-6)

\[ v_x + u_x + v_y = - p_y + \frac{1}{Re_D} (v_x + v_y) \]  
(2.3-7)

\[ u_x + v_y = 0 \]  
(2.3-8)

The momentum equations are nonlinear coupled partial differential equations requiring a means of decoupling before a numerical solution can proceed. It is this decoupling that leads some researchers to use a vorticity-stream function formulation [Anderson et al. (1984), Abdallah (1987a, b)] and others to use an artificial compressibility approach [Chorin (1967)]. The procedure used in this investigation is known as "iterative lagging" for the values of \( u, v, \) and \( p \) at each timestep. This requires a means by which the pressure solution may be obtained from an interim velocity field. 

Equation 2.3-6 is differentiated with respect to \( x \), Equation 2.3-7 is differentiated with respect to \( y \), and the results are added. Application of Equation 2.3-8 to the sum results in a Poisson equation for pressure that can be used in place of the continuity equation in the lagging process:

\[ p_{xx} + p_{yy} = - \left( u_x^2 + 2u_y v_x - v_y^2 + D_i \right) \]  
(2.3-9)

Note that the time derivative of the dilatation has been retained in this expression; ideally, it would be zero if there were no numerical error. The work of Hirt and Harlow (1967) has shown that the convergence of the solution of this Poisson equation can be enhanced if the nonzero dilatation term (representing error in the incompressibility) is retained as a corrective measure.

The lagging process begins with an initial estimate of the velocity and pressure fields supplied by initial condition generators subject to the boundary conditions. Equations 2.3-6, 2.3-7, and 2.3-9 are solved for updated values at the current timestep. The dilatation (Equation 2.3-4) is computed and examined for approach to a zero value approximating incompressibility. During the course of this approach, the lagging procedure is repeated several times to improve the accuracy of the solution; in the
current study, the lagging procedure was repeated a minimum of 10 cycles for each nondimensionalized timestep of 0.05. Upon achieving 10 cycles, a convergence test was performed. Additional cycles, including adjustment of the pressure at the cylinder and plate boundaries, were required if the incompressibility tolerance criterion was not satisfied. In this convergence test the absolute value of the dilatation at each boundary point was summed and the cycling was ended when the sum no longer exceeded $10^{-5}$. In many of the simulations the procedure led to dilatation residuals at individual points of $10^{-8}$ or less. This would occur during intervals of flow between vortex shedding events. In contrast, timesteps early in the impulsive start of the flowfield often required 500 or more cycles to address the rapidly changing flowfield and resolve the correct boundary pressure distribution from the initial estimate.

Examination of the dilatation elsewhere in the computational domain reflected the accuracy of the solution at the cylinder boundary to within one order of magnitude, still a highly accurate level. Of particular interest for this investigation was the dilatation at the plate boundary as a measure of the accuracy of the pressure solution. The plate dilatation was initially reviewed in a similar manner to the cylinder dilatation as a criterion for cessation of the boundary pressure adjustment process. Examination of the plate dilatation showed that whenever the cylinder dilatation criterion was satisfied, the plate dilatation would be of the same order of magnitude. This was assured by the technique used to update boundary pressure values as explained in the next section.

The discussion of the body-fitted coordinate transformation in the previous section focused on the admission of simple boundary conditions while introducing complexity into the governing equations. Using the grid metric parameters given in Equations 2.2-9a through g, the nondimensionalized form of Equations 2.3-6, 2.3-7, and 2.3-9 transform as follows for use in the computational plane:
\[ u_t - \frac{X}{J} (y_\eta u_\xi - y_\xi u_\eta) - \frac{Y}{J} (x_\xi u_\eta - x_\eta u_\xi) \]
\[ + \frac{u}{J} (y_\eta u_\xi - y_\xi u_\eta) + \frac{v}{J} (x_\xi u_\eta - x_\eta u_\xi) \]
\[ = -\frac{1}{J} (y_\eta p_\xi - y_\xi p_\eta) + \frac{1}{\text{Re} J^2} (\alpha u_{\xi \xi} - 2 \beta u_{\eta \eta}) \]
\[ + \gamma u_{\eta \eta} + \sigma u_\eta + \tau u_\xi \] (2.3-10)

\[ v_t - \frac{X}{J} (y_\eta v_\xi - y_\xi v_\eta) - \frac{Y}{J} (x_\xi v_\eta - x_\eta v_\xi) \]
\[ - \frac{u}{J} (y_\eta v_\xi - y_\xi v_\eta) - \frac{v}{J} (x_\xi v_\eta - x_\eta v_\xi) \]
\[ = -\frac{1}{J} (x_\xi p_\eta - x_\eta p_\xi) + \frac{1}{\text{Re} J^2} (\alpha v_{\xi \xi} - 2 \beta v_{\eta \eta}) \]
\[ + \gamma v_{\eta \eta} + \sigma v_\eta + \tau v_\xi \] (2.3-11)

\[ \alpha p_{\xi \xi} - 2 \beta p_{\eta \eta} + \gamma p_{\eta \eta} + \sigma p_\eta + \tau p_\xi = \]
\[ -(y_\eta u_\xi - y_\xi u_\eta)^2 - 2 (x_\xi u_\eta - x_\eta u_\xi) (y_\eta v_\xi - y_\xi v_\eta) \]
\[ -(x_\xi v_\eta - x_\eta v_\xi)^2 - J^2 D_t \]
\[ + x_t J (y_\eta D_{\xi \xi} - y_\xi D_{\eta \eta}) + y_t J (x_\xi D_{\eta \eta} - x_\eta D_{\xi \xi}) \] (2.3-12)

During each simulation timestep the grid coordinates are held constant, permitting each of the grid metric parameters in the above equations to be computed only once per timestep.

Inner boundary conditions on velocity at the cylinder are straightforward no-slip conditions. Velocity boundary conditions are considerably more involved at the outer boundary because this region represents the plate, the free stream, and the inflow/outflow approximations to a plate boundary layer for the portions of the outer boundary that
would be immersed in the undisturbed plate boundary layer. The velocity boundary conditions used in this investigation are as follows:

**Inner Boundary (cylinder):**
- \( u = 0.0 \) no-slip condition
- \( v = 0.0 \) no-slip condition [fixed cyl]
  - \( [v = v(cyl) \text{ if moving cylinder}] \)

**Outer Boundary (infinity boundary portion):**
- \( u = 1.0 \) free stream condition
- \( v = 0.0 \) free stream condition

**Outer Boundary (plate boundary portion):**
- \( u = 0.0 \) no-slip condition
- \( v = 0.0 \) no-slip condition

**Outer Boundary (inflow/outflow boundary layer):**
- \( u = \text{function of } (y/6) \)
- \( v = \text{function of } (y/6) \)

Note that the outer boundary velocities were not functions of time in this investigation. The simulator is capable of addressing time-dependent boundary conditions as might be found in studies of wave-induced current [Phillips and Ackerberg (1973)] or in unidirectional transient flows.

A fully-implicit solver for the three governing equations was written by Allen (1987) and modified for this investigation. The fully-implicit solver requires not only the specification of the boundary values but also an initial estimation of the remaining field values for each dependent variable \( u, v, \) and \( p \). The estimated velocity field is generated by first computing values for \( u \) and \( v \) at each grid point as though it were immersed in an undisturbed plate boundary layer. A second order boundary layer
approximation [Shames (1962)] was used in combination with the continuity equation to estimate the field values for velocity at each grid point as a function of the grid point position in the field. All semi-bounded cases investigated herein utilized a plate beginning at coordinates (0,0), although provisions were made in the code to examine the effects of a thicker plate boundary layer resulting from placement of the cylinder further downstream. The portion of the outer boundary immersed in the plate boundary layer was maintained at an outflow condition computed from the second order boundary layer approximation. Error introduced by this outflow approximation was minimal due to the effects of fluid viscosity and numerical dissipation over the long distance to the outer boundary.

Following the plate boundary layer computation, a potential (inviscid) flow solution was calculated at each grid point on the line of constant \( \eta \) mid-way between the inner and outer boundaries. Here,

\[
\begin{align*}
  u &= - u_0 \left( \frac{\xi^2}{r^2} \right) \cos (2\theta - \pi) + u_0 \quad (2.3-13a) \\
  v &= - u_0 \left( \frac{\xi^2}{r^2} \right) \sin (2\theta - \pi) \quad (2.3-13b)
\end{align*}
\]

where \( U_0 \) is the free stream velocity, \( a \) is the cylinder radius, \( r \) is the distance from the cylinder center to the grid point, and \( \theta \) is the angle to the point measured from the \( \xi=0 \) coordinate line under the cylinder. This was linearly interpolated along each line of constant \( \xi \) radially inward to the no-slip conditions at the inner boundary (cylinder) based on the distance of the grid point in question from the inner boundary. Next, the potential flow solution was linearly interpolated outward along each line of constant \( \xi \) to the outer boundary where values were determined by whether the portion of the outer boundary represented the free stream (above the cylinder), the inflow or outflow approximation of the plate boundary layer (upstream and downstream of the cylinder), or the plate boundary itself (below the cylinder).
The pressure boundary conditions used in this study were:

Inner Boundary (cylinder): \[ P = P \text{ such that } D_i \text{ is zero on boundary} \]

Outer Boundary (infinity boundary): \[ P = 0.0 \]

Outer Boundary (plate portion): \[ P = P \text{ such that } D_i \text{ is zero on boundary} \]

Because the pressure distribution on the solid boundaries of the cylinder and the plate is a variable quantity in the flow solution, a method must be employed to find the unique combination of velocity field and boundary pressures that comprise an incompressible flow. An initial pressure distribution was provided by setting the pressure to 0.1 at the solid boundaries and linearly interpolating along the lines of constant \( \xi \) for the field point values. Three methods were investigated for use in the update of the pressure at the solid boundaries from this initial estimate. These, along with the operation of the Navier-Stokes solver, will be described in the following section.

2.4 Description of the Navier-Stokes Solver

Following generation of the mesh and establishment of the estimated pressure and velocity fields, incompressible flow simulations enter a process generally known as a Navier-Stokes solver. In the Navier-Stokes solver, the transformed governing equations 2.3-10 through 2.3-12 are differenced and solved at each grid point for updated values of \( u, v, \) and \( p \). Several forms of Navier-Stokes solvers have been developed, one of which is the fully-implicit Marker and Cell (MAC) solver written by Harlow and Welch (1965) that uncouples the governing equations by computing values of \( u, v, \) and \( p \) on a staggered mesh. An alternative form is provided by the mixed explicit/implicit "SIMPLE" solver written by Patankar and Spalding (1972) in which the
momentum equations are differenced in explicit form and the pressure Poisson equation is solved implicitly. Quianshun (1987) offers a heuristic stability analysis for this method that shows the run time advantages of the explicit portion easily offset by limitations on the timestep size in situations where a fine mesh is required. The solver used in the current research is derived from that written by Allen (1987) for the unbounded cylinder configuration, modified herein to accept multiple solid boundaries in the flowfield, improve accuracy, and reduce run time requirements.

The solver utilizes an iterative lagging process to decouple the governing equations, and a fully-implicit differencing scheme that is theoretically stable (in the von Neumann sense) for all timestep sizes [Steger (1978)]. The iterative lagging process becomes more accurate than a simple lagging process as the timestep size is increased. As an example, consider the x-direction momentum equation (Equation 2.3-6), where the desired quantity is an updated value of the u velocity component at an arbitrary grid point for the (n+1) timestep. In the simple lagging process, the values of v and p that appear in the x-direction momentum equation would be supplied by the prior timestep (n) values at this grid point. A similar procedure would be followed for the y-direction momentum equation (Equation 2.3-7) and for the pressure Poisson equation (Equation 2.3-9) until computations were completed for all grid points. The simulation would then proceed to the next timestep. In the iterative lagging process, the values for u, v, and p computed at the (n+1) level are substituted for the (n) level and a refined (n+1) level is computed; the iterative process continues at a single timestep for a fixed number of cycles or until a user-specified convergence criterion is satisfied.

A fully-implicit solver generates large systems of simultaneous equations that must be solved during each iteration for the desired values at the (n+1) level. For a mesh containing 1600 grid points, the three governing equations generate three sets of 1600 simultaneous equations to be solved, less the equations associated with the known boundary values. Early efforts such as by Ghia et. al. (1979) used the solution
techniques of successive overrelaxation (SOR) or alternating direction implicit (ADI). A number of fast iterative techniques are now available for solution of these systems. **Khosla and Rubin (1981)** reported success with a hybrid conjugate gradient/strongly implicit technique. **Jacobs (1983)** performed a comparison study and identified two schemes as most promising for this class of problems, the preconditioned conjugate gradient method (now available as part of the IMSL Scientific Subroutine Library) and the Modified Strongly-Implicit (MSIP) method developed by **Schneider and Zedan (1981)**. Allen chose to use the MSIP technique based on the results of Jacobs' studies and the ready availability of documentation. Due to the availability of such documentation [**Schneider and Zedan (1981)**, **Zedan and Schneider (1985)**, **Anderson et. al. (1984)**, and **Allen (1987)**], it will not be repeated in this text.

Second order central finite difference approximations to the governing equations have been used except in temporal derivatives, where first order backward differencing was applied, and in those terms where central differencing would introduce "wiggles" to the solution. **Anderson et. al. (1984)** explain wiggles as insufficiently damped oscillations in the flow solution that can occur in regions of large mesh spacing even though von Neumann stability criteria are satisfied for a differencing scheme. Although some investigators have approached the problem through higher order differencing [**Hirsh (1975)**, **Kawamura et. al. (1984)**], prevention of these oscillations generally requires the addition of artificial viscosity (damping). This is most commonly achieved through use of upwind differencing techniques for the convective terms of the transformed governing equations. The hybrid differencing scheme described by Anderson et. al. was used in this investigation. In the hybrid differencing scheme, the mesh Reynolds number

\[
Re_{\text{mesh}} = \frac{(\text{Velocity}_{\text{grid point}})}{\text{Viscosity}_{\text{grid point}}} \frac{(\text{Mesh Spacing}_{\text{grid point}})}{\text{ }}
\]

(2.4-1)
serves as a criterion for interpolating between use of the more accurate central differencing in regions where mesh spacing is small and a third order upwind differencing formulation (attributed to Leonard (1979)) used to prevent wiggles in regions where mesh spacing is large. Let an interpolation function \( \phi \) be defined as

\[
\phi = \frac{Re_{\text{crit}}}{Re_{\text{mesh}}} \tag{2.4-2}
\]

where \( Re_{\text{crit}} \) is a critical mesh Reynolds number that can be set from a value of 0.0 to 2.0 in order to prevent wiggles. A value of 1.0 was selected for this simulation, and \( \phi \) was constrained not to exceed 1.0 as the mesh spacing became small. A sample transformed convective term is interpolated as follows:

\[
u \frac{\partial v}{\partial \xi} = \nu \left[ \phi \left( \frac{\partial v}{\partial \xi} \right)_{\text{central}} + (1-\phi) \left( \frac{\partial v}{\partial \xi} \right)_{\text{upwind}} \right] \tag{2.4-3}
\]

where the central difference takes the form of

\[
u \frac{\partial v}{\partial \xi} = \nu \left[ \frac{v_{i+1} - v_i}{2 \Delta \xi} \right] \tag{2.4-4}
\]

and the third order upwind difference by Leonard takes the form of

\[
u \frac{\partial v}{\partial \xi} = \frac{1}{12 \Delta \xi} \left[ u \left( -v_{i+2} + 8 v_{i+1} - 8 v_i + v_{i-2} \right) \right.
\]

\[
+ \left| u \right| \left( v_{i+2} - 4 v_{i+1} + 6 v_i - 4 v_{i-1} + v_{i-2} \right) \right] \tag{2.4-5}
\]

Anderson notes that the streamwise pressure gradient \( \partial p/\partial \eta \) represents an additional means by which instability may be propagated upstream to the solution region of interest. A first order backward difference is preferred to a second order difference for this quantity to prevent instability in the streamwise direction.
The progression of the simulation through the iterative lagging procedure yields an accurate solution for a problem having pressure boundary values corresponding to those of the initial estimate at the solid bodies of the cylinder and the plate. However, the pressure boundary values at the cylinder and plate are unknowns in the problem and do not remain constant in an unsteady flow. Therefore a means must be employed to achieve an update of the solid boundary pressure values consistent with changes in the velocity field and the objective of maintaining a divergence-free (incompressible) flow. In the current research, three methods were investigated for use in updating the pressure boundary conditions between iteration cycles. These were the method of Viecelli (1969), a modified Viecelli approach developed by the author, and the PUMPIN method of Raithby and Schneider (1979). Ultimately, the modified Viecelli approach was employed and found to yield significant advantages in improved accuracy and computational cost reduction. Because this process is repeated at each timestep, it can be the most costly phase of the entire simulation. Viecelli reasoned that the update of pressure at a boundary point on the grid should respond to the dilatation at the point. A flow having a positive dilatation (expanding) at the boundary point exhibits the effect of excessive pressure at the point and requires a reduction in pressure. Conversely, flow exhibiting a negative dilatation shows the effect of underpressure and requires an increase in pressure. A zero dilatation flow at a grid point requires no correction. Viecelli developed the following relationship, transformed to curvilinear coordinates by Hodge, as reported in Shanks and Thompson (1977):

\[
p^{k+1} = p^k - \frac{[2\varepsilon I^2 Di]}{[(\alpha + \gamma)\Delta t]} \quad (2.46)
\]

The underrelaxation coefficient \(\varepsilon\) is included to prevent the pressure update process from destabilizing due to overcorrection. A value of 0.95 was used throughout this investigation. The grid metric quantities that appear along with the timestep essentially
represent the discretized portion of the mesh affected by the dilatation at the boundary grid point. Hence, on a finer mesh the pressure correction would proceed in smaller steps, effectively proportional to the square of the spacing. Herein lay the disadvantage of the Viecelli method; the wide variation of mesh spacing around the cylinder (dictated by boundary layer resolution between the cylinder and the plate, and economy of grid point deployment above the cylinder) led to rapid achievement of incompressible flow above the cylinder, while below the cylinder the extremely small spacing led to even smaller values of $J^2$ and a very slow approach toward incompressible flow in this region. Also, the presence of two boundaries (cylinder and plate) at which this pressure correction process was occurring posed an additional opportunity for destabilization of the simulation. Envision a boundary pressure change at each point on the plate that in turn modifies the flow field, creating a new dilatation at each boundary point of the cylinder. The cylinder boundary pressure distribution responds to the new dilatation, creating a new flow field to which the plate pressure distribution must respond. This form of instability was observed upon attempts to increase the parameter $\varepsilon$ as a means of increasing the rate of convergence.

The author's modification to the Viecelli method was obtained by observing the approach toward zero dilatation of each boundary point during a simulation timestep and noting the relative values of $J^2$ at the locations where the approach was most rapid. The strong dependence of the correction on the $J^2$ term caused the boundary points adjoining the coarsest mesh to progress most rapidly toward zero dilatation. By using the values of $J$, $\alpha$, and $\gamma$ of the coarsest mesh near the cylinder for all points at the cylinder, the convergence process was substantially accelerated. Stability of the process was maintained in accordance with Viecelli's analysis because the mesh at the plate was more widely-spaced than at any location on the cylinder. Thus the plate proceeded rapidly to an incompressible flow pressure distribution and the cylinder experienced a minimal
influence from pressure changes at the plate during the update process.

A flow was considered as converged to incompressibility when the sum of cylinder dilatation values did not exceed $10^{-5}$, a value readily achieved and consistent with the following order of magnitude analysis (note that the nondimensionalizations of Equations 2.3-4, and 2.3-5a and b revert to the tilde notation):

$$\tilde{D} = \frac{\tilde{u}}{\partial x} + \frac{\tilde{v}}{\partial y} \ll 1 \text{ for incompressibility} \quad (2.4-7a)$$

Then,

$$\frac{D}{U_0} \left( \frac{\tilde{u}}{\partial x} + \frac{\tilde{v}}{\partial y} \right) = \tilde{D} \ll 1 \quad (2.4-7b)$$

or,

$$\frac{D}{U_0} (\tilde{D}) \ll 1 \quad (2.4-7c)$$

which yields upon rearrangement:

$$\tilde{D} \ll \frac{U_0}{D} \quad (2.4-7d)$$

In this simulation, the quantities $U_0$ and $D$ were set to 1.0 and 1.0 respectively, with Reynolds number variations accomplished through change of the viscosity. Thus the nondimensionalization would permit any dilatation much smaller than the order of 1.0 to be representative of incompressible flow.

While investigating the boundary pressure update alternatives, the PUMPIN method (Pressure Update through Multiple Path INtegration) offered by Raithby and Schneider (1979) was adapted to the curvilinear coordinate transformation. Briefly, the PUMPIN method is based on the fact that the Navier-Stokes equations hold at all locations in the flow field. The pressure distribution along a boundary is obtained by
solution of a momentum equation for the pressure gradient, using the current velocity field. The gradient is numerically integrated starting with a known boundary value such as that for the free stream portion of the outer boundary. Because the current velocity field is not necessarily incompressible, the pressure at any point achieved by integration along one path will not necessarily equal the pressure at that point obtained by integration along an alternative path. Raithby and Schneider assert that by doing this over multiple paths intersecting each respective boundary point, differences in the pressure field can be averaged out, permitting a new velocity field to be computed. After several iterations, the procedure converges to an incompressible flow. In applications on rectangular meshes, they found that as few as two paths would suffice to attain convergence. The PUMPIN method did not converge as rapidly as the modified Viecelli approach when applied to the semi-bounded cylinder configuration, but was faster than the original Viecelli method. Use of the modified Viecelli method relative to the original Viecelli method resulted in a run time improvement from an average of 90 seconds per timestep to 2.9 seconds per timestep. Concurrently, the cylinder dilatation residual was reduced from $10^{-2}$ to $10^{-5}$.

In a fixed cylinder simulation, the Navier-Stokes solver proceeds to the next timestep after satisfying the governing equations at each grid point for the current timestep. Each timestep in a 40x40 mesh yields a computational record consisting of five vectors, each having 1600 elements. The five vectors contain the x-coordinates, y-coordinates, u-velocity components, v-velocity components, and pressure data for each point in the mesh. The x and y coordinates for the grid points do not change in time for a fixed cylinder simulation. In the moving cylinder simulations, only y-direction motion was admitted for consistency with most experimental setups. The forces acting on the cylinder will tend to displace the center of the cylinder relative to the plate and the outer boundary of the computational domain. Accordingly, the amount of the cylinder displacement occurring over the past timestep must be calculated, and a new set of mesh coordinates must be prepared before the start of the next timestep. Also, the boundary
values of velocity at the surface of the cylinder must be revised to reflect the velocity of the body. The force on the cylinder is obtained from numerical integration of the shear and pressure loads over the surface of the cylinder. Once this force is available, the following algorithm is used to obtain the acceleration, velocity, and displacement of the cylinder over a timestep.

\begin{align*}
\text{Acceleration:} & \quad \mathbf{a}^n = \left[ F_y^n - k (y^{n+1} - y^n) - c v^n \right] / m_c \\
\text{Impulse:} & \quad z^n = \frac{(a^n - a^{n-1})}{\Delta t} \\
\text{Velocity:} & \quad v^{n+1} = v^n + a^n (\Delta t) + \frac{1}{2} z^n (\Delta t)^2 \\
\text{Displacement:} & \quad y^{n+1} = y^n + v^n (\Delta t) + \frac{1}{2} a^n (\Delta t)^2 + \frac{1}{6} z^n (\Delta t)^3
\end{align*}

As a closure to this section, Table 2-1 summarizes the parameters used in the simulation routines.

Table 2-1

<table>
<thead>
<tr>
<th>Parameter and Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 0.9$</td>
<td>acceleration parameter for MSIP iterative solver</td>
</tr>
<tr>
<td>$\varepsilon = 0.95$</td>
<td>underrelaxation parameter for Viecelli pressure update</td>
</tr>
<tr>
<td>$\kappa = 5.40$</td>
<td>stretching parameter for grid point distribution function</td>
</tr>
<tr>
<td>$\rho = 1.0$</td>
<td>fluid density</td>
</tr>
<tr>
<td>$\nu = 0.01$</td>
<td>kinematic viscosity used in $Re_D=100$ cases</td>
</tr>
</tbody>
</table>
\( \nu = 0.0125 \) \hspace{1cm} \text{kinematic viscosity used in Re}_D=80 \text{ cases} \\
\Delta t = 0.05 \hspace{1cm} \text{timestep} \\
D = 1.0 \hspace{1cm} \text{cylinder diameter} \\
U_0 = 1.0 \hspace{1cm} \text{free stream velocity} \\
\text{TOL} = 10^{-5} \hspace{1cm} \text{dilatation residual tolerance for convergence} \\

2.5 Post-processing

The vectors of data generated at each timestep were post-processed for time histories of the lift and drag coefficients, upper and lower separation angle movement, pressure coefficient distributions at the solid boundaries of the cylinder and plate, and relative streamline contours. Output from the individual post-processing routines was converted to graphic format by the mainframe computer and preserved on diskette through the graphics terminal emulation capabilities of the Macintosh computer. The mainframe graphics support has been focused on simple x-y plotting. Accordingly, the reader will note a lack of smoothness in contour plots for streamlines; this is due to the simple linear search method employed by the software.

Computation of the lift and drag coefficients requires calculation of the pressure and shear forces on the cylinder from the pressure and velocity data. Pressure forces are obtained from integration of boundary grid point pressures around the curve of constant \( \eta \) (equal to 40 on a 40x40 mesh) that comprises the cylinder, assuming a unit depth. Differential areas are computed in transformed coordinates by noting that:

\[
\begin{align*}
\mathrm{d}x &= \frac{\partial x}{\partial \xi} \delta \xi + \frac{\partial x}{\partial \eta} \delta \eta \\
\mathrm{d}y &= \frac{\partial y}{\partial \xi} \delta \xi + \frac{\partial y}{\partial \eta} \delta \eta
\end{align*}
\]  

(2.5-1a)  

(2.5-1b)
where, along a curve of constant $\eta$, $\delta \eta = 0$. In a uniformly spaced computational plane, $\delta \xi = 1$. As an example, the y-direction pressure component of force is numerically integrated as a summation over the cylinder surface:

$$F_{y\text{\,press}} = \sum_{\xi = 1}^{\xi = 0} p(\xi) \frac{\partial x}{\partial \xi} (1) \text{ (unit depth)} \quad (2.5.2)$$

The shear component of force is an integration of shear stress evaluated at the surface of the cylinder over the associated differential area obtained from transformed coordinates. The shear stress is calculated from the computed velocity distribution in the vicinity of the cylinder. As an example, the y-direction shear component of force is given by

$$F_{y\text{\,shea}} = \sum_{\xi = 1}^{\xi = 0} \mu \frac{\partial y}{\partial \xi} \left[ \frac{\partial u}{\partial \eta} \frac{\partial x}{\partial \xi} + \frac{\partial v}{\partial \eta} \frac{\partial y}{\partial \xi} \right] (1) \text{ (unit depth)} \quad (2.5.3)$$

The lift and drag coefficients are based on the free stream velocity as follows:

$$C_l = \frac{2 (F_{y\text{\,press}} + F_{y\text{\,shea}})}{\rho U_0^2 A} \quad (2.5.4)$$

$$C_d = \frac{2 (F_{x\text{\,press}} + F_{x\text{\,shea}})}{\rho U_0^2 A} \quad (2.5.5)$$

where the area, $A$, is unity. During the computation of the shear stress, the velocity gradient at the surface of the cylinder is surveyed for a change in sign indicative of flow separation. The gradient is evaluated at the surface and at one coordinate curve immediately above the surface to enable interpolation for the upper and lower separation angles.

The pressure coefficient plots for the cylinder and the plate were taken at periods of locally extreme lift force in association with the streamline contours plotted at the same timestep. The pressure coefficient is also based on the free stream velocity:
\[ C_p = -\frac{2p}{\rho U_0^2} \quad (2.5-6) \]

The streamline contours are calculated relative to the zero stream function value occurring at the plate and the cylinder, \( \psi_0 \). Between any two points A and B in the flow,

\[ \psi - \psi_0 = \int_A^B (u \, dy - v \, dx) \quad (2.5-7a) \]

and upon transformation the integral is approximated by:

\[ \psi - \psi_0 = \frac{1}{2} \left[ \left( \frac{\partial v}{\partial \eta} \right)_B - \left( \frac{\partial v}{\partial \eta} \right)_A \right] + \left( \frac{\partial u}{\partial \eta} \right)_A - \left( \frac{\partial u}{\partial \eta} \right)_B \left( \delta \eta \right) \quad (2.5-7b) \]

where \( \delta \eta = 1 \) in the uniformly-spaced computational plane.
Figure 2.1 Body-Fitted Coordinate Transformation
Figure 2-2: Discretized 40x40 Domain
Ordinate and Abscissa Scaled to Cylinder Diameter Units

Figure 2-3 Computational Mesh, Unbounded Cylinder, (a) Full Field, (b) Detail
Figure 2-4 Computational Mesh, h/D=3.0, (a) Full Field, (b) Detail
Ordinate and Abscissa Scaled to Cylinder Diameter Units

Figure 2-5 Computational Mesh, h/D=0.5, (a) Full Field, (b) Detail
Figure 2-6 Extent of the Discretized Domain
3.1 Reference Case of Unbounded Flow

A reference case was examined at Re_D=80 for an unbounded flow configuration to verify the overall operation of the simulator and to assure the adequacy of boundary layer resolution in the vicinity of the cylinder. The computational domain was set at 20 diameters from the cylinder upstream and downstream, as well as above and below. This relatively large spacing beneath the cylinder (compared to gaps of 3.0 to 0.5 diameters in the semi-bounded cases) led to the largest spacing of grid points near the cylinder of all the cases studied. The close agreement of the results of the simulation with experimental evidence gave indication of the suitability of the grid point distribution in the boundary layer of the cylinder. The unbounded configuration was achieved using the same mesh generator as for the semi-bounded configuration. The boundary pressure iteration at the plate was replaced with the constant free stream value of p=0.0 and all outer boundary velocities, including those on the plate, were set to the free stream conditions of u=1.0, v=0.0. Thus the plate behaved like a free stream boundary. Initial pressure and velocity fields were generated based on the modified boundary conditions for the unbounded configuration.

The mean drag coefficient developed in the steady state simulation was 1.328 with a range about the mean of ± 0.006. Tritton (1959) reported a mean drag coefficient of 1.294 at this Reynolds number, and noted that the varying component was too small to be significant. Tanida et. al. (1973) show a range of 1.28 to 1.31 for the mean drag coefficient and cite a value of 1.4 as reported by Relf and Simmons. A mean value of zero was obtained herein for the lift coefficient, consistent with the unbounded
configuration. A value of ±0.275 for the varying component of lift was determined and is much larger than the estimate of ±0.06 offered by Tanida et. al. However, Tanida et. al. note the difficulty in obtaining consistent values for the varying component of lift and acknowledge a wide range of possible results. The Strouhal frequency obtained for this investigation was 0.143 which falls into the range between 0.12 provided by Tanida et. al. and the value of 0.15 provided by Tritton. Tritton also cites an early effort by Roshko that provides a value of 0.156.

The mean separation angle location is symmetric on the upper and lower portion of the cylinder, computed as ±58.1 degrees from the rearmost point on the cylinder. The varying component ranges over ±2.1 degrees. The extrema in upper and lower separation angle locations were found to occur in phase, with the widest net separation in direct correlation with the maximum drag and the smallest net separation in direct correlation with the minimum drag.

Figure 3-1 shows the time histories of lift and drag for the unbounded cylinder at ReD=80 as steady state conditions are approached. The lift trace proceeded rapidly into the characteristic pattern of vortex shedding behavior. The drag trace is at a much finer scale and shows a longer approach to steady state behavior, approximated by the last two cycles. Further resolution was not deemed necessary and the simulation was stopped to avoid waste. Figure 3-2 shows the time histories of the upper separation angle and the lower separation angle. The stream function contour plots for four segments of a vortex shedding cycle are presented in Figures 3-3 and 3-4. The pressure coefficient around the surface of the cylinder is plotted in Figure 3-5 for the maximum and minimum lift portions of the shedding cycle. The shift in pressure distribution from the lower aft portion of the cylinder (0° to -90°) to the upper aft portion (0° to +90°) over a cycle is evident, and in agreement with the overall lift behavior. The highest pressure is found at the front of the cylinder in the flow stagnation region.
3.2 Semi-bounded Cases at Gap Ratios > 1.7

The semi-bounded case studies were begun at a gap ratio of 3.0. This gap ratio approximates the intersection of the undisturbed boundary layer thicknesses for the plate and the cylinder. Also, it was desired to keep the starting transient interval to less than 3000 records (150 nondimensional time units) in order to avoid waste of computer funds. The transient interval increases with the gap ratio. Examination of the starting transient behavior showed the applicability of a penetration depth analysis [Batchelor (1974)], with the starting transient interval obtained by solving for $t$ in the following relationship:

$$h = \sqrt{4vt}$$ (3.2-1)

In the case of a gap ratio such as 3.0 where the cylinder is not immersed in the plate boundary layer, a good estimate of the starting transient duration was obtained by setting the gap $h$ to the undisturbed plate boundary layer thickness. In the case of a small gap ratio such as 0.5 where the cylinder is fully immersed in the plate boundary layer, transient behavior was observed to cease when the cylinder boundary layer grew across the gap $h$. From the gap ratio of 3.0, successive cases were investigated at smaller gap ratios down to a value of 0.5.

Suppression of vortex shedding was observed at a gap ratio of 1.7 for the Reynolds number of 80. Continued reduction of the gap ratio past 1.7 led to the onset of fluctuating loading on the cylinder as observed experimentally by Roshko et. al. (1975) and Bearman and Zdravkovich (1978). Göktun (1975) also cites work by Spivak in 1946 where this loading was observed in the absence of vortex shedding. The behavior of cases at gap ratios greater than 1.7 were similar to each other and are presented in this section. Gap ratios of 3.0, 2.0, and 1.8 were examined in the regular vortex shedding regime.
The lift coefficient time histories for these gap ratios are presented in Figures 3-6 and 3-7a. Three traces appear on each plot, the pressure component of lift, the shear component of lift, and the lift coefficient trace itself. The pressure component accounts for approximately 75% of the lift loading and the shear component accounts for approximately 25%, consistent with the numerical simulation results of Allen (1987) who applied this Navier-Stokes solver to an unbounded cylinder configuration. The mean lift for the cylinder was positive at each of these gap ratios but decreased as the plate was approached. Values of mean lift obtained were 0.040, 0.015, and 0.010 respectively. The downwash of flow ahead of the cylinder aids in the development of a positive pressure component for each case. The range of the lift coefficient also decreased as the plate was approached; this effect on the range and on the mean lift coefficient is attributed to the progression of the cylinder deeper into the plate boundary layer where the effective upstream velocity encountered by the cylinder is lower. This is consistent with the findings of Bruschi et. al. (1982), Figureido and Viegas (1988), and Roshko et. al. (1975). Values of the lift coefficient range were ±0.059, ±0.017, and ±0.009 for gap ratios of 3.0, 2.0, and 1.8 respectively. Maximum lift occurs just prior to the shedding of the upper vortex and minimum lift occurs just prior to the shedding of the lower vortex.

The steady state drag coefficient for these gap ratios is presented in Figures 3-7b and 3-8. The trace of the drag coefficient shows the effect of two vortex shedding events per cycle, with the large peak accompanying shedding of the upper vortex and the smaller peak accompanying the shedding of the lower vortex. The distinction between the two events was diminished as the cylinder approached the plate. As with the lift, both the mean drag coefficient and the range of the drag coefficient decreased with reduction of the gap ratio. Values of 1.180, 1.07, and 1.039 were obtained for the mean drag coefficient at gap ratios of 3.0, 2.0 and 1.8, respectively. The range of the drag coefficient yielded decreasing values of ±0.004, ±0.002, and ±0.001 for the same gap
ratios.

The loss of symmetry in the mean values of the upper and lower separation angles, relative to an unbounded cylinder configuration, becomes evident in the time histories of Figures 3-9, 3-10, and 3-11. At gap ratios greater than that for vortex shedding suppression, Bearman and Zdravkovich (1978) observed earlier separation on the lower side of the cylinder than on the upper side. They attribute this to the influence of the plate boundary on the pressure field in the wake of the cylinder. Such behavior was also observed in the current investigation. The magnitude of the mean values, as well as the range about each mean, decreases as the cylinder approaches the plate. At a gap ratio of 3.0 the mean upper and lower separation angles were 61.407 and -62.964 degrees, measured from zero located at the rearmost point on the cylinder. The ranges about the mean values were ±0.610 and ±0.560 degrees respectively. At a gap ratio of 2.0 the values of the mean upper and lower separation angles were 60.337 and -61.517 degrees. The ranges about the mean values were ±0.173 and ±0.174 degrees respectively. At a gap ratio of 1.8 the mean upper and lower separation angles were 60.008 and -60.453 degrees. The ranges about the mean values were ±0.088 and ±0.089 degrees respectively.

For each of these gap ratios, the maximum lift was found to lead the maximum drag by approximately one third of a cycle due to the asymmetry of the separation points on the cylinder. The maximum value of the upper separation angle was observed to lead the minimum (least negative) value of the lower separation angle by approximately five percent of a cycle, almost in phase.

The period of vortex shedding was found to increase toward a limiting value in a manner consistent with the experimental findings of Bearman and Zdravkovich (1978), Göktun (1975), and Buresti and Lanciotti (1979). Periods of 10.05, 11.80, and 11.90 seconds were computed for the gap ratios of 3.0, 2.0, and 1.8, respectively. The value of 11.9 seconds proved to be the limiting (maximum) period also
observed with vortex shedding suppression at h/D=1.7 for this Reynolds number.

Stream function contour plots for the three gap ratios are presented in Figures 3-12, 3-13, and 3-14. In Figure 3-12 the streamline contours at the time of minimum lift over a cycle are shown for h/D=3.0 and h/D=2.0. A time series of vortex shedding is shown for the gap ratio of 1.8 in Figures 3-13 and 3-14, with the minimum lift configuration present in Figure 3-14b.

Pressure coefficient plots for the cylinder and the plate at gap ratios of 3.0, 2.0, and 1.8 are shown in Figures 3-15, 3-16, and 3-17. Each pressure coefficient plot is referenced against the angle in degrees over the surface of the cylinder, beginning with the value of zero at the rear of the cylinder and measured positive in the counter-clockwise direction. On the outer boundary/plate (dashed line) the free stream outer boundary condition of p=0.0 is found from 0° to 180°, running from the aft of the cylinder to the front of the cylinder. The portion of the plate from the leading edge to directly underneath the cylinder corresponds to angles from 180° to 270° (note: 270° also equal to -90°). The plate portion beneath the cylinder, running to the trailing edge corresponds to angles from -90° back to 0°. The high pressure at the front portion of the plate (180° to 270°) is due to the stagnation of downwash from the cylinder causing a high pressure between the front of the cylinder and the plate. From -90° to 0° the pressure drops as the velocity increases through the venturi formed by the plate and cylinder, recovering as the velocity decreases aft of the cylinder, and then dropping to the free stream boundary pressure (0° to 180°). The venturi effect is most noticeable in the -90° to 0° region as the cylinder is placed closer to the plate.

The cylinder pressure traces show a peak at 180° corresponding to the forward stagnation point. At these gap ratios, the movement of the forward stagnation point toward the plate is not yet evident. This movement was more apparent at smaller gap ratios and was observed by both Bearman and Zdravkovich and by Göktun in their respective experimental studies. The largest component of lift and drag is due to
pressure, and Figure 3-15 provides a clear view of the shift in the cylinder pressure over a lift cycle. At maximum lift, higher pressure on the cylinder is present from -90° to 0° (lower aft) and at minimum lift the higher pressure is present from 0° to 90° on the upper aft portion of the cylinder. Decreasing the gap ratio results in a general reduction of the forward stagnation pressure due to a lower effective upstream velocity.

Pressure contour plots are shown in Figure 3-18 for the gap ratio of 1.8 at maximum and minimum lift events. A pressure contour plot assists in visualization of the pressure field about the cylinder. Comparison of the two contour plots is done by focusing on one or more specific iso-pressure lines near the cylinder wake. In the maximum lift configuration, the iso-pressure lines are closer to the lower aft portion of the cylinder and the upper iso-pressure lines are further from the cylinder, providing net lift. In the minimum lift configuration, the reverse situation is evident. The pressure contours in this figure above and upstream of the cylinder are almost indicative of potential flow and are consistent with the smooth streamline contours found in these regions in Figures 3-13 and 3-14.

3.3 Semi-bounded Case at Gap Ratio = 1.7

The periodic shedding of vortices was not observed at gap ratios of 1.7 or smaller for Re_D=80. By comparison the gap ratio for suppression of vortex shedding in the vicinity of Re_D=40,000 was found to be approximately 0.3 by Bearman and Zdravkovich as well as Buresti and Lanciotti. At the higher Reynolds number, the boundary layers on the plate and cylinder are considerably thinner, summing to approximately 0.13 diameters compared to 2.95 diameters at the lower Reynolds number (Blasius boundary layer). Based on the thickness of the undisturbed boundary layers, it is reasonable to expect that the gap ratio for vortex shedding suppression would increase as the Reynolds number
decreases. Intuitively, one would expect a constant lift and drag behavior in the absence of vortex shedding. However, the previously-cited works by Bearman and Zdravkovich, Roshko et. al., and Göktun/Spivak indicate that a periodic loading should be expected due to wake instability. This was observed in the current research for all gap ratios less than or equal to the gap ratio for vortex shedding suppression.

**Figure 3-19a** shows the lift coefficient time history at the critical gap ratio of 1.7. A positive mean lift coefficient of 0.009 with a range of ±0.004 was computed at the limiting period of 11.9 seconds. The drag coefficient time history is shown in **Figure 3-19b** and had a mean value of 1.023 with a range of ±0.0004. The upper and lower separation angle time histories are presented in **Figure 3-20**. The mean upper separation angle was 59.836 degrees with a range of ±0.041 degrees, while the mean lower separation angle was -59.747 degrees with a range of ±0.034 degrees. The lower separation point has now moved further aft on the cylinder than the upper separation point. Both separation points can be expected to move rearward due to the favorable pressure gradient in the wake region. The minimum lift led the maximum drag by approximately one quarter cycle while the maximum upper separation angle led the minimum lower separation angle by less than one tenth of a cycle.

Although vortex shedding was no longer present, changes in the pressure field downstream of the cylinder were reflected in changing streamline contours. **Figure 3-21** shows the difference in lower vortex shape during maximum and minimum lift events over a cycle. At maximum lift the lower vortex is larger, permitting less of the upper rear surface of the cylinder to experience the reattaching streamline pressure. At minimum lift, the lower vortex is smaller, exposing more of the cylinder to downward pressure. Examination of the corresponding pressure coefficient plots in **Figure 3-22** shows a slight difference in the 0° to +90° region (upper aft portion of the cylinder) with pressure higher in the case of minimum lift. The upstream portion of the plate (180° to 270°) shows the effect of stagnating downwash continuing to occur at the lower front
portion of the cylinder. The downstream portion of the plate (-90° to 0°) shows the venturi pattern observed in previous cases. The small kink in the curve from -10° to 0° is representative of the far field influence of the outflow boundary; occurring at almost 40 diameters, it is not viewed as having significance.

Pressure contours for the maximum lift and minimum lift events over a cycle are presented in Figure 3-23. The upper aft portion of the cylinder can be seen experiencing lower pressure at maximum lift and the reverse is true at minimum lift.

3.4 Semi-bounded Cases at Gap Ratios < 1.7

Cases having gap ratios smaller than 1.7 were examined at h/D=1.5, 1.2, 1.0, 0.7, and 0.5. At these smaller gap ratios the fluctuating loading in the absence of vortex shedding progressed beyond the maximum period for vortex shedding to a long period behavior. The results remained stable over the period of as many as 4000 records. The pattern was repeated for the higher Reynolds number of 100, but at gap ratios closer to the plate. The magnitude of the range in lift and drag coefficients about their mean value at each gap ratio increased markedly as the gap ratio was decreased.

Figures 3-24, 3-25, and 3-26a show the lift coefficient time histories over this range of gap ratios. At h/D=1.5 the mean lift coefficient in the last cycle was 0.005; the mean was still changing slightly at this time but the change is magnified on the expanded plot scale. The range of the lift coefficient is ±0.0003 over this cycle. It appears that the shorter limiting period of 11.90 seconds may be superimposed on a longer period feature. At h/D=1.2 the lift coefficient trace became negative, showing a negative pressure component for the first time. The period of the loading was over 200 seconds and a full cycle was not obtained within 4000 records. An estimate of the mean lift coefficient is -0.027 with a range of ±0.002, based on a partial cycle. The negative mean lift appeared again at the gap ratio of 1.0 with a value of -0.033 and a range of
±0.014. The period of loading on the cylinder was 89.65 seconds. In these two cases, the downwash is not capable of overcoming the venturi effect causing attraction to the wall. At the next two smaller gap ratios, establishment of Poiseuille flow in the gap reduced the velocity beneath the cylinder and restored a positive mean lift. At h/D=0.7, the mean lift coefficient was 0.029 with a range of ±0.021. The gap ratio of 0.5 led to an even higher mean lift coefficient of 0.129 with a range of ±0.058. A period of 88.35 seconds was calculated for the loading at h/D=0.5. The rapid increase in the mean lift coefficient as the cylinder approaches the plate is consistent with the experimental results of Wilson and Caldwell (1971), Roshko et. al. (1975), Göktun (1975), and Bruschi et.al. (1982) at low subcritical Reynolds numbers. The increase in the range is an effect of the variations occurring in the pressure field in the wake and the increase in its magnitude relative to the lift coefficient could be a significant design consideration.

The drag coefficient time histories are presented in Figures 3-26b, 3-27, and 3-28 for the range of gap ratios between 1.5 and 0.5. The drag coefficient trace at h/D=1.5 gives evidence of long period behavior. It has a mean value of 0.988 and a range of ±0.0001. The mean drag coefficient decreases with the reduction in gap ratio that brings the cylinder into a region of progressively lower effective upstream velocity. Based on a partial cycle at h/D=1.2, the estimated mean drag coefficient dropped to 0.865 but the range increased to ±0.003. The trend of reduction in mean value and increase in range with decreasing gap ratio continued at h/D=1.0 with a mean drag coefficient of 0.716 and a range of ±0.047. For h/D=0.7 this became 0.574 and ±0.121 respectively, and at the final gap ratio of 0.5, the mean drag coefficient decreased to 0.489 while the range increased to ±0.126. At the gap ratio of 1.5 the maximum lift led the maximum drag by two-thirds of a cycle. However, at gap ratios of 1.0 and smaller the lead decreased to approximately one-fifth of a cycle.

The behavior of the upper and lower separation angles as the gap ratio is reduced
follows the same pattern as found for the drag coefficient. The magnitude of the mean value for each separation angle decreased as the cylinder approached the plate, while the magnitude of the range about each mean increased. The mean lower separation angle moved further aft on the cylinder than the upper separation point, consistent with the rise in the mean lift as the cylinder approached the plate. This was noted by Zdravkovich (1981) in his flow visualization studies. Figure 3-29 shows on an expanded scale the upper separation angle time history for h/D=1.5. The mean upper separation angle, taken from the last cycle, was 59.413 degrees with a range of ±0.003 degrees. The mean lower separation angle was -57.963 degrees with a range of ±0.004 degrees. Separation angle motion associated with the long period behavior reported above is evident in Figure 3-30 showing the results of records 3000 through 4000 for the gap ratio of 1.2. The estimated mean upper separation angle was 57.432 degrees with a range of ±0.066 degrees. The estimated mean lower separation angle was -53.395 degrees with a range of ±0.008 degrees. As the gap ratio decreased to 1.0, the magnitude of the range in separation point motion increased substantially (Figure 3-31). The mean upper separation angle was 55.275 degrees with a range of ±1.033 degrees. The lower separation angle had a mean of -45.316 degrees with a range of ±1.727 degrees. The onset of Poiseuille flow in the gap is associated with the large reduction in the magnitude of the mean lower separation angle; the favorable pressure gradient through the gap leads to longer travel of each fluid particle around the cylinder before encountering the adverse pressure gradient in the wake of the cylinder.

Figure 3-32 shows only the time histories of the upper separation points for gap ratios of 0.7 and 0.5. The subroutine used to compute the velocity gradients and search for the separation points was unable to resolve a unique lower separation point for these cases. This is consistent with the presence of a large region of separated flow very close to the surface of the cylinder as shown in Figures 3-34b and 3-35. The mean upper separation angle at h/D=0.7 was 52.923 degrees with a range of ±3.135 degrees. At
h/D=0.5 this had changed to 52.113 degrees with a range of ±3.480 degrees.

The difference in appearance of streamline contour plots taken at maximum lift and minimum lift for cases at gap ratios less than 1.7 proved to be small in the absence of vortex shedding activity. Figure 3-33 shows the streamline pattern for the gap ratios of 1.5 and 1.2. The lower vortex is reduced in size as the cylinder approaches the plate and encounters a lower effective upstream velocity profile. The same trend is observed in Figure 3-34 for gap ratios of 1.0 and 0.7. Figure 3-34b shows more of the upstream flow pattern to illustrate the lowering of the forward stagnation point toward the plate as observed by Bearman and Zdravkovich and by Göktun in their experimental works. Figure 3-35 shows both a closeup and an extended view of the flow field about the cylinder at h/D=0.5. At the smaller gap ratios of 0.7 and 0.5 the range of stream function values plotted did not include sufficient resolution to show a closed vortex in the separated region aft of the cylinder. Streamlines passing between the cylinder and the plate tend to curve upward around the rear of the cylinder in response to the pressure gradient in this region. Here the cylinder and plate both carry negative pressure values but the aft plate pressure is always higher than on the rear of the cylinder. Of particular interest in Figure 3-35b is the streamline closest to the plate and its behavior downstream. In this region the stream function plotting routine began to encounter multiple locations of stream function values, potentially indicating the onset of turbulence in the wake. In an unbounded configuration, the onset of turbulence in the wake can occur at a diameter Reynolds number of as small as 150. While this observation is speculative, the effect is not without precedent. A wire (cylinder) placed on a plate is frequently used to trip the onset of turbulence [Clauser (1956)] as a means of thickening the boundary layer in experimental studies.

Figures 3-36 and 3-37 provide the pressure coefficient and pressure contour information for the case at h/D=1.5. Both figures are taken from the last full cycle of the simulation at the local maximum and local minimum lift events. The plots at maximum
and minimum lift are virtually indistinguishable, consistent with the small range of lift amplitude observed at this gap ratio. Upon reduction of the gap ratio to 1.2 the short period lift features decayed, leaving a long period change in the loading on the cylinder. Due to the length of the period, only the minimum lift value was available after 4000 records. The peak pressure shown in Figure 3-38 has dropped relative to the case at h/D=1.5, and the plate pressure coefficients have become more negative in the -90° to 0° region aft of the cylinder. The trend toward lower stagnation pressure at the front of the cylinder as the body is moved closer to the plate continues at h/D=1.0. Pressure coefficient plots at this gap ratio are presented in Figure 3-39 for the maximum lift and minimum lift. The difference in lift pressure contours is more clearly evident in the 0° to +90° portion of the cylinder shown in Figure 3-40.

As the gap ratio is further reduced, the pressure coefficient plots for the plate and cylinder become increasingly similar in shape and magnitude, however the pressure on the plate remains higher than on the cylinder (except at the far field locations on the plate). At h/D=0.7 (Figure 3-41) the pressure over the 60° to 110° portion of the cylinder appears to provide the most significant contribution to the difference in lift over a long period cycle. In Figure 3-42 more of the upstream pressure contour information is shown than at larger gap ratios to show the changes in the pressure field ahead of the cylinder that are more evident than at the larger gap ratios. The magnitude of the range of lift about its mean was greatest at the smallest gap ratio examined, 0.5. The close proximity of the cylinder to the plate, as well as the Poiseuille flow occurring in between, is largely responsible for the close approach of plate and cylinder pressures in the -90° to 0° and 220° to 270° regions of Figures 3-43 and 3-44. With the cylinder deeply immersed in the plate boundary layer, the front stagnation pressure on the cylinder is small. The plots of pressure contours at the maximum and minimum lift events clearly show the shift in lines of constant pressure consistent with the large magnitude of the lift range about its mean value for this gap ratio.
Figure 3-1 Coefficient Time Histories for Unbounded Cylinder at ReD=80, (a) Lift, (b) Drag
Figure 3-2 Separation Angle Time Histories for Unbounded Cylinder at ReD=80, (a) Upper, (b) Lower
Figure 3-3 Streamlines for Unbounded Cylinder, ReD=80
(a) t=86.00, (b) t=87.15
Figure 3-4 Streamlines for Unbounded Cylinder, ReD=80
(a) t=88.30, (b) t=89.50
Figure 3-5 Pressure Coefficient for Unbounded Cylinder, ReD=80
(a) Maximum Lift, \( t=89.50 \), (b) Minimum Lift, \( t=86.00 \)

Note: Angle measured from zero at rear of cylinder
Figure 3-6 Coefficient Time Histories at ReD=80, (a) Lift for h/D=3.0, (b) Lift for h/D=2.0
Figure 3-7 Coefficient Time Histories at ReD=80, (a) Lift for h/D=1.8, (b) Drag for h/D=3.0
Figure 3-8 Coefficient Time Histories at ReD=80, (a) Drag for h/D=2.0, (b) Drag for h/D=1.8
Figure 3-9 Separation Angle Time Histories at ReD=80 and h/D=3.0
(a) Upper, (b) Lower
Figure 3-10 Separation Angle Time Histories at ReD=80 and h/D=2.0
(a) Upper, (b) Lower
Figure 3-11 Separation Angle Time Histories at ReD=80 and h/D=1.8
(a) Upper, (b) Lower
Figure 3.13 Streamlines for Semi-bounded Cylinder at ReD=80 and h/D=1.8
(a) I=141.5, (b) I=144.10
Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 3-14  Streamlines for Semi-bounded Cylinder at ReD=80 and h/D=1.8
(a) $t=145.70$, (b) $t=147.75$, minimum lift

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 3-15 Pressure Coefficient at Cylinder and Plate for ReD=80 and h/D=3.0
(a) maximum lift, (b) minimum lift
Figure 3-16 Pressure Coefficient at Cylinder and Plate for ReD=80 and h/D=2.0
(a) maximum lift, (b) minimum lift
Figure 3-17 Pressure Coefficient at Cylinder and Plate for ReD=80 and h/D=1.8
(a) maximum lift, (b) minimum lift
Figure 3.18 Pressure Contours for Semi-Bounded Cylinder at ReD=80 and h/D=1.8
(a) maximum lift, (b) minimum lift
Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 3-19 Coefficient Time Histories at ReD=80 and h/D=1.7
(a) Lift, (b) Drag
Figure 3-20 Separation Angle Time Histories at ReD=80 and h/D=1.7
(a) Upper, (b) Lower
Figure 3.2-1 Streamlines for Semi-bounded Cylinder at ReD=80 and h/D=1.7

(a) maximum lift, (b) minimum lift
Figure 3-22 Pressure Coefficient at Cylinder and Plate for ReD=80 and h/D=1.7
(a) maximum lift, (b) minimum lift
Figure 3-23 Pressure Contours for Semi-bounded Cylinder at ReD = 80 and h/D = 1.7
(a) maximum lift, (b) minimum lift
Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 3-24 Lift Coefficient Time Histories at ReD=80, (a) h/D=1.5, (b) h/D=1.2
Figure 3-25 Lift Coefficient Time Histories at ReD=80, (a) h/D=1.0, (b) h/D=0.7
Figure 3-26 Coefficient Time Histories at ReD=80, (a) Lift at h/D=0.5, (b) Drag at h/D=1.5
Figure 3-27 Drag Coefficient Time Histories at ReD=80, (a) h/D=1.2, (b) h/D=1.0
Figure 3-28 Drag Coefficient Time Histories at ReD=80, (a) h/D=0.7, (b) h/D=0.5
Figure 3-29 Separation Angle Time Histories at ReD=80 and h/D=1.5
(a) Upper, (b) Lower
Figure 3-30 Separation Angle Time Histories at ReD=80 and h/D=1.2,
(a) Upper, (b) Lower
Figure 3-31 Separation Angle Time Histories at ReD=80 and h/D=1.0,
(a) Upper, (b) Lower
Figure 3-32 Upper Separation Angle Time Histories at ReD=80, (a) h/D=0.7, (b) h/D=0.5
Figure 3.33: Streamlines for Semi-bounded Cylinder at ReD=80
(a) h/D=1.5, (b) h/D=1.2
Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 3-35 Streamlines for Semi-bounded Cylinder at ReD=80 and h/D=0.5
(a) Closeup, (b) Extended View

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 3-36 Pressure Coefficient at Cylinder and Plate for ReD=80 and h/D=1.5
(a) local maximum lift, (b) local minimum lift
Figure 3-37 Pressure Contours for Semi-bounded Cylinder at ReD=80 and h/D=1.5
(a) local maximum lift, (b) local minimum lift
Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 3-38 Pressure Plots at Minimum Lift, ReD=80 and h/D=1.2
(a) Pressure Coefficients, (b) Pressure Contours
Figure 3-39 Pressure Coefficient at Cylinder and Plate for ReD=80 and h/D=1.0
(a) maximum lift, (b) minimum lift
Figure 3.40 Pressure Contours for Semi-bounded Cylinder at ReD=80 and h/D=1.0
(a) maximum lift, (b) minimum lift
Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 3-41 Pressure Coefficient at Cylinder and Plate for ReD=80 and h/D=0.7
(a) maximum lift, (b) minimum lift
Figure 3.42: Pressure Contours for Semi-bounded Cylinder at ReD=80 and h/D=0.7
(a) maximum lift, (b) minimum lift
Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 3-43 Pressure Coefficient at Cylinder and Plate for ReD=80 and h/D=0.5
  (a) maximum lift, (b) minimum lift
Figure 3.44: Pressure Contours for Semi-bounded Cylinder at ReD=80 and h/D=0.5
(a) maximum lift, (b) minimum lift
Ordinate and Abscissa Scaled to Cylinder Diameter Units
CHAPTER 4

Semi-bounded Flow Over a Fixed Cylinder
at $Re_D = 100$

4.1 Reference Case of Unbounded Flow

As in the fixed cylinder case at $Re_D=80$, a reference case was examined at $Re_D=100$ to verify the overall operation of the simulator and to assure the adequacy of the boundary layer resolution in the vicinity of the cylinder. In order to simulate the unbounded configuration, a computational domain with a gap ratio of 20.0 was used. The boundary pressure iteration at the plate was replaced with the constant free stream value of $p=0.0$ and all outer boundary velocities, including those on the plate, were set to the free stream conditions of $u=1.0$, $v=0.0$. Initial pressure and velocity fields were then generated based on the modified boundary conditions. This geometry gave the maximum grid point spacing in the vicinity of the cylinder of all the cases studied. The close agreement of the results of the simulation with experimental data is indicative of a suitable level of boundary layer resolution by the mesh generator.

The mean drag coefficient developed in the steady state portion of the simulation was 1.236 with a maximum varying component of ±0.005. This compares favorably with a value of 1.24 obtained experimentally by Tritton (1959) and a value of 1.19-1.20 obtained experimentally by Tanida et. al. (1973). Tanida et. al. also cite a higher value of 1.30 from work by Relf and Simmons, later observed in a finite difference numerical simulation by Chilukuri (1987). Allen (1987) obtained a value of approximately 1.13 from his numerical simulation. Allen's result is somewhat low, but consistent with the smaller value of mean separation angles obtained in his study. A mean value of the lift coefficient of zero was obtained herein, consistent with the symmetry of the unbounded flow geometry. A value of ±0.212 was obtained for the
varying component of the lift. This is higher than the value of ±0.08 obtained by Tanida et al., but in closer agreement with the value of 0.23 obtained by Allen. Tanida et al. note the difficulty in obtaining consistent experimental values for the varying component of lift and acknowledge a wide range of possible results.

The Strouhal frequency obtained in this investigation was 0.159, which falls well into the range of values produced by experimental studies. Tanida et al. (1973) report a value of 0.14, which corresponds to the lower end of the range 0.145-0.167 reported by Friese (1980). Tritton (1959) obtained a value of 0.164 and Roshko (1954) reported a value of 0.167. In other numerical simulations, Allen (1987) obtained a value of 0.154, Chilukuri (1987) obtained a value of 0.155, and Karniadakis (1988) obtained a relatively high value of 0.18 on a coarser finite element mesh. The mean separation angle location, measured from zero at the aft of the cylinder, is symmetric on the upper and lower portion of the cylinder. A mean separation angle of ±63.02° was obtained in the current research, with a varying component of ±1.99°. As was found in the previous study at ReD=80, the extrema in upper and lower separation angle locations were observed to occur in phase. The widest net separation was in direct correlation with the maximum drag and the smallest net separation was in direct correlation with the minimum drag.

Figure 4-1 shows the steady state lift and drag coefficients for the unbounded cylinder configuration at ReD=100. The lift trace shows a single event per vortex shedding cycle, while the drag trace shows two events per vortex shedding cycle. The expanded scale of the drag trace shows the effects of the interaction between the upper and lower vortices over a shedding cycle. Figure 4-2 illustrates the time histories of the upper and lower separation angles in unbounded flow. Figures 4-3 and 4-4 show the streamline contour plots over four segments of a vortex shedding cycle.

The pressure coefficient around the surface of the cylinder is plotted in Figure 4-5
for the maximum and minimum lift portions of the vortex shedding cycle. The effects are most notable over the aft half of the cylinder, from angular coordinates of -90° to +90°. At maximum lift, pressures are higher over the -90° to 0° (lower aft) region of the cylinder, while the pressure is higher over the 0° to +90° (upper aft) region at minimum lift. The front of the cylinder (180°) experiences the highest pressure due to flow stagnation.

4.2 Semi-bounded Cases at Gap Ratios >1.5

The investigation of the semi-bounded configuration at $Re_D=100$ was begun at a gap ratio of 3.0. This value is slightly greater than the sum of the undisturbed boundary layer thicknesses for the plate and the cylinder. Regular vortex shedding was present at this gap ratio, as well as for cases run at gap ratios of 2.0, 1.7, and 1.6. Regular vortex shedding was not observed at gap ratios of 1.5 or less for this Reynolds number. Each case was initiated as an impulsive start of the flow field, and therefore passed through a transient period en route to steady state behavior. The duration of the transient period was related to the Reynolds number (effect of viscosity) and the gap ratio through a boundary layer penetration depth analysis presented in Chapter 3. Upon reduction of the gap ratio from the value of 1.5, a periodic loading was evident although vortex shedding from the cylinder had been suppressed. This phenomenon was observed in the experimental studies of Roshko et. al. (1975), Bearman and Zdravkovich (1978), and Göktun (1975) and attributed to instabilities in the wake region. The following text describes the results of the current investigation for gap ratios greater than 1.5, at which regular vortex shedding was clearly evident.

Recalling the mean lift coefficient of an unbounded cylinder to be zero, the mean lift coefficient at each of the gap ratios of 3.0, 2.0, 1.7, and 1.6 were found to be positive
and to decrease as the cylinder approached the plate. The positive value of the lift is attributed to the downwash flow ahead of the cylinder stagnating on the plate and creating an increased pressure in this region. The upward lift due to downwash pressure is offset in part by the venturi effect of higher velocity flow between the cylinder and the plate. At the gap ratio of 3.0, the mean lift coefficient was 0.063 with a range about the mean of ±0.0787. The net lift is a combination of pressure and shear forces integrated over the perimeter of the cylinder. The pressure component of lift is the dominant force, becoming more significant as the Reynolds number is increased. The mean lift coefficient at the gap ratio of 2.0 was 0.023 with a range of ±0.030 and for the gap ratio of 1.7, a mean lift coefficient of 0.018 with a range of ±0.017 was observed. At the gap ratio of 1.6, a mean lift coefficient of 0.017 with a range of ±0.013 resulted from the simulation. Figures 4-6 and 4-7 show the time histories of the lift coefficient during steady state vortex shedding behavior. The maximum lift occurs just prior to the shedding of the upper vortex and the minimum lift occurs just prior to the shedding of the lower vortex.

While the lift coefficient time histories show one event per vortex shedding cycle, the drag coefficient time histories plotted in Figures 4-8 and 4-9 show two events per vortex shedding cycle, along with evidence of the interaction of the upper and lower vortices. The larger drag coefficient peak is associated with the presence of the upper vortex and the smaller peak is associated with the presence of the lower vortex. As the gap ratio is reduced, the effect of the lower vortex is less evident. Also, the magnitude and range of the drag coefficient decreases as the cylinder approaches the plate. Bruschi et. al. (1982), Figureido and Viegas (1988), and Roshko et. al. (1975) each observed this experimentally, confirming the decrease in the pressure component of drag as the cylinder moves toward the cylinder. As the cylinder progresses deeper into the plate boundary layer, the effective upstream velocity is lower, decreasing the associated pressure effects. The mean drag coefficient was 1.185, 1.011, 0.971, and
0.957 for gap ratios of 3.0, 2.0, 1.7, and 1.6 respectively. Associated ranges of the drag coefficient about the mean values were \( \pm 0.003, \pm 0.003, \pm 0.002, \) and \( \pm 0.002. \)

Figures 4-10 through 4-13 provide the time histories of the upper and lower separation angles over the range of gap ratios where regular vortex shedding was observed. Zdravkovich (1981) performed flow visualization studies on a semi-bounded cylinder that showed forward movement of the mean upper separation angle (zero degrees measured at the aft of the cylinder) relative to the mean position for an unbounded cylinder. Further, his work showed two effects for the mean lower separation angle; an initial forward movement, followed by a rearward movement at smaller gap ratios where viscous flow in the gap creates a favorable pressure gradient that delays the effects of flow separation on the cylinder. The mean upper separation angle at a gap ratio of 3.0 was 65.11° with a range of \( \pm 0.635°. \) The mean lower separation angle was -64.50° with a range of \( \pm 0.515°. \) These values are contrasted to the symmetric positions on an unbounded cylinder of \( \pm 63.02° \) and are directionally in agreement with the findings of Zdravkovich.

The mean upper separation angle at a gap ratio of 2.0 was 63.924° with a range of \( \pm 0.396°. \) The mean lower separation angle was -64.674° with a range of \( \pm 0.239°. \) At a gap ratio of 1.7 the mean upper separation angle was 63.583° with a range of \( \pm 0.249°. \) The mean lower separation angle was -63.595° with a range of \( \pm 0.142°. \) Similarly, at h/D=1.6, the mean upper separation angle was 63.418° with a range of \( \pm 0.193° \) and the mean lower separation angle was -63.062° with a range of \( \pm 0.172°. \)

Experimental studies by Göktun (1975), Bearman and Zdravkovich (1978), and Buresti and Lanciotti (1979) showed that the period of regular vortex shedding from a semi-bounded cylinder would differ from that of an unbounded configuration and tend toward a maximum value as the gap ratio for vortex shedding suppression was approached. This behavior was also observed in the current investigation. The vortex shedding periods for gap ratio cases of 3.0, 2.0, 1.7, and 1.6
were 8.25 sec, 11.95 sec, 12.25 sec, and 12.25 sec respectively. The period of 12.25 seconds is the maximum period for this Reynolds number and is contrasted to the value of 11.9 seconds determined for $Re_D=80$. Gerrard (1966) explained this in simple terms related to the overall width of the wake region; a wider wake requires a longer period of time for the crossing of the respective shear layers necessary for vortex shedding to occur. In each of the gap ratio cases greater than 1.5, the maximum drag preceded the maximum lift by an amount decreasing from a third of a cycle at $h/D=3.0$ to less than a tenth of a cycle at $h/D=1.6$. In each case, the minimum value of the upper separation angle occurred at the same time as the most negative value of the lower separation angle.

Plots of the stream function contours at maximum and minimum lift events for each gap ratio greater than 1.5 are provided in Figures 4-14 through 4-17. It is of interest to note the contrast in the size and shape of the lower vortex at the minimum lift condition between gap ratios of 2.0 and 1.6. The lower vortex is considerably elongated as the plate is approached. The overall size will begin to decrease as the gap ratio is further reduced and the cylinder progresses into the lower effective upstream velocities near the plate.

Corresponding to the maximum and minimum lift events shown in the streamline contour plots are the pressure coefficient plots illustrated in Figures 4-18 through 4-21. Each pressure coefficient plot contains pressure information for both the cylinder surface and the plate boundary. The information is presented by location referenced from the zero degree location at the aft of the cylinder. Thus the portion of the plate from the leading edge to the point directly underneath the cylinder is associated with the 180° to 270° (also equal to -90°) segment of the cylinder. Similarly, the portion of the plate running from the point directly underneath the cylinder to the trailing edge of the plate is associated with the angles of -90° to 0°. In Figure 4-18 the effect of the change in
pressure distribution on lift is most clearly seen in the relative shift of the cylinder pressure distribution from -90° to 0° and from 0° to +90°. The pressure in the region ahead of the cylinder is large and positive, reflecting the effect of the stagnating downwash flow from the front of the cylinder. The plate pressure aft of the cylinder shows some wake effects but does not show the venturi effect as clearly as is present in the figures for smaller gap ratios such as 2.0, 1.7, and 1.6. By following through each of these pressure coefficient plots, an overall decrease in pressure magnitudes is observed. This is due to the progression of the cylinder into the lower effective upstream velocities near the plate and can be most readily seen by comparing the cylinder forward stagnation pressure (near 180°) at each gap ratio.

**Figure 4-22** shows the contours of constant pressure at the maximum and minimum lift events for h/D=1.6. The relative proximity of a curve of constant pressure can be related to the pressure felt at the surface of the cylinder. In **Figure 4-22a** the iso-pressure curves over the 0 to +90° portion of the cylinder are more distant than in **Figure 4-22b**, indicative of lower pressure over this region and representative of higher lift. Recalling that maximum drag was almost in phase with maximum lift at this gap ratio, note also the greater spacing of the contour at the 0° location in (a) than in (b), indicative of greater drag.

4.3 Semi-bounded Case at Gap Ratio = 1.5

Examination of the streamline contour plots in steady state behavior showed no vortex shedding at or below the gap ratio of 1.5 for Re_D=100. This gap ratio is analogous to the value of 1.7 observed for Re_D=80 and consistent with the thinner boundary layers developed on the cylinder and plate with an increase in the Reynolds number. The periodic loading on the cylinder that persisted in the absence of regular
vortex shedding at $Re_D=80$ was also observed to occur at the maximum period for this Reynolds number. The existence of this phenomenon is attributed to instability in the wake flow in the experimental work of Roshko et. al. (1975), and in Göktun (1975) who cites the 1946 research by Spivak providing these results. Bearman and Zdravkovich (1978) observed periodic fluctuations in their experiments after suppression of regular vortex shedding and interpreted these as either indicative of the transition to turbulence in the wake or of conditions surrounding the "awakening" of the vortex shedding process. Bloor (1963) showed the transition to turbulence in the wake of a cylinder to be accompanied by fluctuations in the pressure field of the wake. Further, the transition to turbulence in the wake of an unbounded cylinder can occur at Reynolds numbers as low as 150. The transition to turbulence in the wake of a semi-bounded cylinder remains open to future investigation. Variation in the pressure field as reported by Bloor is evident in the results of the numerical investigation conducted herein.

A limited analogy between the semi-bounded flow over a cylinder and flow over two cylinders placed side-by-side appears frequently in the literature. Zdravkovich (1977) prepared a review of experimental studies addressing the latter configuration. It is of interest to note that a region of bistable wake flow is observed for small gaps between the cylinders. Zdravkovich cites a 1972 publication by Ishigari, et. al. that attributed the fluctuating forces to alternating degrees of boundary layer attachment (Coanda effect). This was disproven in the experimental work of Andreopoulos (1972) cited in Bearman and Wadcock (1973). Andreopoulos examined the wake generated by two flat plates set normal to the flow, with varying gaps in between them. The edge definition of the plates fixed the separation points, yet the wake instability was observed over a greater range of gaps than for a pair of cylinders.

The mean lift coefficient at $h/D=1.5$ was 0.015 with a range of ±0.008. The shear
component of lift at this gap ratio (Figure 4-23a) is negative and is dominated by the variation in the pressure component of lift. Figure 4-23b shows the time history plot of the drag coefficient. The mean drag coefficient is 0.943 with a range of ±0.001. Examination of the trace at this scale shows no secondary features such as were associated with upper and lower vortex interaction at larger gap ratios.

The upper and lower separation time histories for this gap ratio are presented in Figure 4-24. The mean upper separation angle was 63.207° with a range of ±0.122°. This location is still advanced farther into the flow than that for an unbounded cylinder configuration (63.02°). The mean lower separation angle was -62.141° with a range of ±0.107°. The location of the mean lower separation point has moved aft of the mean location for an unbounded cylinder at this gap ratio, indicative of sufficient viscous effects present in the gap to create a favorable pressure gradient and delay separation. This behavior is consistent with the experimental findings of Zdravkovich (1981) at the gap ratio of vortex shedding suppression.

The period of the pressure field loading on the cylinder was determined to be 12.25 seconds, which proved to be the maximum period for vortex shedding as the cylinder approached the plate. Such a maximum period phenomenon was reported in the works of Buresti and Lanciotti (1979), Göktun (1975), and Bearman and Zdravkovich (1978). At this period, the maximum drag leads the maximum lift by a third of a cycle. The maximum forward motion of the upper separation point leads the maximum forward motion of the lower separation point by a tenth of a cycle.

Figure 4-25 shows the change in streamline shape between the maximum and minimum lift portions of a loading cycle. Because the curvature of the streamlines is determined by the pressure gradient, the difference in curvature of the lower vortex at the lift extremes can be related to the pressure fluctuations occurring in the wake region. The pressure coefficient plots for the cylinder and plate are presented in Figure 4-26. At both maximum and minimum lift conditions, the plots show pronounced effects of
downwash from the cylinder (180 to 250 degrees) and the venturi effect in the gap (-90 to 0 degrees). Careful examination of the plate pressure near the lower vortex (-90 to -20 degrees) in both plots shows the difference in pressure responsible for the change in the shape of the streamlines between the lift extrema.

The pressure contour plots for these extrema are provided in Figure 4-26. The pressure contours near the lower aft portion of the cylinder are closer at maximum lift than at minimum lift conditions. Also of interest is the contour touching the cylinder at the aft location. Maximum drag occurred one third of a cycle ahead of maximum lift, so the effect of pressure contour location on drag should also be evident in these two plots. The higher drag on the cylinder near maximum lift is indeed represented by a greater distance of the aft pressure contour from the cylinder than at the minimum lift condition. Lower pressure in the aft region of the cylinder, with all other regions approximately equal, can be interpreted as a higher drag.

4.4 Semi-bounded Cases at Gap Ratios < 1.5

The behavior of the flow at gap ratios smaller than necessary for suppression of regular vortex shedding was examined at h/D values of 1.4, 1.2, 1.0, 0.7, and 0.5. As previously discussed, the periodic loading remaining in the absence of regular vortex shedding at h/D=1.5 was also found at each of these smaller gap ratios. This effect was accompanied by a progressive increase in the period of the loading from the limiting period for vortex shedding.

Over this series of gap ratios, the mean lift coefficient passed from a positive value (lift away from the plate) to a negative value, and back to a positive value as the gap ratio was reduced (Figures 4-28 through 4-30a). This marks the progression from the effect of downwash pressure at the lower front of the cylinder, to the venturi effect of higher velocity flow in the gap, and then to establishment of fully viscous (Poiseuille)
flow at the smaller gap ratios. This progression was encountered previously in the study at $Re_D=80$. The mean lift coefficients were 0.018, 0.015, -0.043, 0.001, and 0.08 for the gap ratios of 1.4, 1.2, 1.0, 0.7, and 0.5, respectively. Experimental studies such as Göktun (1975) show a characteristic rapid increase in the mean lift coefficient as the cylinder approaches the plate at small gap ratios. Jones (1970) notes the inability to maintain turbulent flow in the gap region due to establishment of Poiseuille flow in the gap, leading to reduced flow velocity in the gap relative to the flow velocity over the top of the cylinder. The rapid increase in the mean lift coefficient at the small gap ratios is attributed to the increased pressure in the gap. The range about the mean lift coefficients was ±0.004, ±0.000, ±0.004, ±0.002, and ±0.040 for gap ratios of 1.4, 1.2, 1.0, 0.7, and 0.5, respectively. With the onset of fully viscous flow in the gap, the magnitude of the range approaches that of the mean.

The progression of the cylinder into the lower effective upstream velocity of the plate boundary layer is accompanied by a uniform decrease in the mean drag coefficient. Figures 4-30b through 4-32 show the time histories of the drag coefficient taken from the last records established for each case. The mean drag coefficients were 0.929, 0.898, 0.685, 0.557, and 0.455 for the gap ratios of 1.4, 1.2, 1.0, 0.7, and 0.5. The magnitude of the range of the drag coefficient about the mean also increased at the smaller gap ratios. The drag coefficient range values were ±0.001, ±0.000, ±0.017, ±0.011, and ±0.199 for this set of gap ratios.

Figures 4-33 through 4-37 provide the time histories of the upper and lower separation angles for each of these gap ratios. The mean upper separation angle for these gap ratios is smaller (farther aft on the cylinder) than for an unbounded cylinder configuration. The delay in separation is due to the reduced strength of the wake pressure field deep in the plate boundary layer. Mean upper separation angles were 62.968°, 62.476°, 57.415°, 56.089°, and 53.423° for the gap ratios of 1.4, 1.2, 1.0, 0.7,
and 0.5, respectively. The ranges about the mean upper separation angles were ±0.060°, ±0.004°, ±0.184°, ±0.179°, and ±4.606°. The relatively large range of upper separation point motion for the 0.5 gap ratio is consistent with the increased range in lift and drag coefficient values at this gap ratio. The mean value of the lower separation angle is also found farther aft on the cylinder than in the case of an unbounded cylinder, consistent with the flow visualization studies of Zdravkovich (1981). The mean lower separation angle experiences a significant reduction in magnitude upon establishment of fully viscous flow in the gap region. Mean lower separation angles were -61.089°, -58.485°, -52.330°, and -38.314° for gap ratios of 1.4, 1.2, 1.0, and 0.7, respectively. A unique lower separation angle could not be resolved for the gap ratio of 0.5, where problems were encountered in locating a single point of zero velocity gradient at the lower aft surface of the cylinder. This is similar to the difficulty experienced at $Re_{D}=80$ for both the 0.5 and 0.7 gap ratios; such correspondence would be expected due to the thicker boundary layers on the plate and cylinder at the lower Reynolds number.

The period of the loading on the cylinder in the absence of regular vortex shedding began at a value of 12.25 sec for $h/D=1.4$ and was not discernable for the gap ratio of 1.2 where the range of lift and drag about the mean values was essentially zero. The periods for smaller gap ratios were 66.70 sec at $h/D=1.0$, 73.70 sec at $h/D=0.7$ and 87.20 sec at $h/D=0.5$. At the gap ratio of 1.4, the maximum drag leads the maximum lift by approximately one quarter of a cycle. The maximum forward motion of the upper separation angle leads the most rearward movement of the lower separation angle by less than a tenth of a cycle. Lift and drag variation, along with separation point motion, were essentially zero for the gap ratio of 1.2. Further reduction of the gap ratio restored the phase difference between maximum drag and maximum lift, with maximum drag leading maximum lift by one quarter cycle at $h/D=1.0$, then trailing by one quarter of a cycle and one third of a cycle at $h/D=0.7$ and 0.5, respectively. The change to a trailing mode
corresponds to changes in lift and separation point movement associated with establishment of fully viscous flow in the gap region.

Streamline contour plots for these gap ratios are presented in Figures 4-38 through 4-40. The elongated lower vortex found at gap ratios of 1.4 and 1.2, where downwash dominates the lift, was shortened considerably at the gap ratio of 1.0. The lower vortex continues to shrink in size as the gap ratio is reduced through values of 0.7 and 0.5 in response to the low velocity flow between the cylinder and the plate at these gaps. Pressure at the plate in the region aft of the cylinder is negative at these small gap ratios but is greater than the pressure on the aft of the cylinder at all times. The upward curvature of the streamlines in the region aft of the cylinder is in response to this pressure gradient.

Figure 4-41 shows the pressure coefficient plots for the cylinder and the plate at maximum and minimum lift events for h/D=1.4. The difference in pressure distribution between these two events is small in these plots but they serve to illustrate the downwash pressure on the plate near the lower front of the cylinder (180° to 260°). The pressure drop in the gap due to venturi effects is evident at the plate over the angles of -90° to -20°. Figure 4-42 shows a contour plot of the pressure field around the cylinder at this gap ratio for the same events. Figures 4-43 and 4-44 illustrate the pressure coefficient and pressure contour plots for maximum and minimum lift events at h/D=1.2. Recalling that the lift and drag behavior was essentially constant at this gap ratio, the plots for both events are virtually identical and consistent with the coefficient data. Figures 4-45 and 4-46 provide the same information for the gap ratio of 1.0. This gap ratio experienced a negative mean lift attributed to the dominance of the venturi effect in the gap. This is evident in the plate pressure coefficient plots in terms of the low positive downwash pressure (180° to 260°) and the strong negative pressure located throughout the gap (-90° to 0°). The approach of the plate pressure to the cylinder pressure at small
gap ratios was noted in the work at $Re_D=80$. Similar behavior at $Re_D=100$ is observed in the pressure coefficient plots for $h/D=0.7$ and $0.5$ shown in Figures 4-47 and 4-48. Note also the overall reduction in the cylinder front stagnation point pressure that occurred as the cylinder progressed deeper into the plate boundary layer, encountering a reduced effective upstream velocity. At the smallest gap ratio, $0.5$, Figures 4-48 and 4-49 illustrate significant changes in the pressure field about the cylinder between the maximum and minimum lift events. Most interesting is the change in the cylinder pressure coefficient distribution over the $90^\circ$ to $270^\circ$ range, comprising the front half of the cylinder. This change in pressure distribution is consistent with the increase in lift and drag ranges about the mean values that were observed at these smaller gap ratios.
Figure 4-1 Coefficient Time Histories for Unbounded Cylinder at ReD=100
(a) Lift, (b) Drag
Figure 4-2 Separation Angle Time Histories for Unbounded Cylinder at ReD=100, (a) Upper, (b) Lower
Figure 4.4 Streamlines for Unbounded Cylinder at ReD=100
(a) t=98.60, t=99.90

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-5 Pressure Coefficient for Unbounded Cylinder at ReD=100
(a) Maximum Lift, (b) Minimum Lift
Figure 4-6 Lift Coefficient Time Histories at ReD=100, (a) h/D=3.0, (b) h/D=2.0
Figure 4-7 Lift Coefficient Time Histories at ReD=100
(a) h/D=1.7, (b) h/D=1.6
Figure 4-8 Drag Coefficient Time Histories at ReD=100
(a) h/D=3.0, (b) h/D=2.0
Figure 4-9 Drag Coefficient Time Histories at ReD=100
(a) h/D=1.7, (b) h/D=1.6
Figure 4-10 Separation Angle Time Histories at ReD=100 and h/D=3.0
(a) Upper, (b) Lower
Figure 4-11 Separation Angle Time Histories at ReD=100 and h/D=2.0
(a) Upper, (b) Lower
Figure 4-12 Separation Angle Time Histories at $ReD=100$ and $h/D=1.7$
(a) Upper, (b) Lower
Figure 4-13  Separation Angle Time Histories at ReD=100 and h/D=1.6
(a) Upper, (b) Lower
Figure 4.14 Streamlines for Semi-bounded Cylinder at ReD=100 and h/D=3.0

(a) Maximum Lift, (b) Minimum Lift

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-15 Streamlines for Semi-bounded Cylinder at ReD=100 and h/D=2.0
(a) Maximum Lift, (b) Minimum Lift

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-16 Streamlines for Semi-bounded Cylinder at ReD=100 and h/D=1.7
(a) Maximum Lift, (b) Minimum Lift

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-18 Pressure Coefficient at Cylinder and Plate for ReD=100 and h/D=3.0, (a) Maximum Lift, (b) Minimum Lift
Figure 4-19 Pressure Coefficient at Cylinder and Plate for ReD=100 and h/D=2.0, (a) Maximum Lift, (b) Minimum Lift
Figure 4-20 Pressure Coefficient at Cylinder and Plate for ReD=100 and h/D=1.7, (a) Maximum Lift, (b) Minimum Lift
Figure 4-21 Pressure Coefficient at Cylinder and Plate for ReD=100 and h/D=1.6, (a) Maximum Lift, (b) Minimum Lift
Figure 4.22 Pressure Contour Plots for Semi-Bounded Cylinder at ReD=100 and h/D=1.6, (a) Maximum Lift, (b) Minimum Lift

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-23 Coefficient Time Histories at ReD=100 and h/D=1.5
(a) Lift, (b) Drag
Figure 4-24 Separation Angle Time Histories at ReD=100 and h/D=1.5, (a) Upper, (b) Lower
Figure 4-25 Streamlines for Semi-bounded Cylinder at ReD=100 and h/D=1.5
(a) Maximum Lift, (b) Minimum Lift

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-26 Pressure Coefficient at Cylinder and Plate for ReD=100 and h/D=1.5, (a) Maximum Lift, (b) Minimum Lift
Figure 4-27: Pressure Contour Plots for Semi-bounded Cylinder at ReD=100 and h/D=1.5; (a) Maximum Lift, (b) Minimum Lift

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-28 Lift Coefficient Time Histories at ReD=100, (a) h/D=1.4, (b) h/D=1.2
Figure 4-29 Lift Coefficient Time Histories at ReD=100, (a) h/D=1.0, h/D=0.7
Figure 4-30 Coefficient Time Histories at ReD=100, (a) Lift, h/D=0.5, (b) Drag, h/D=1.4
Figure 4-31 Drag Coefficient Time Histories at ReD=100, (a) h/D=1.2, (b) h/D=1.0
Figure 4-32 Drag Coefficient Time Histories at ReD=100, (a) h/D=0.7, (b) h/D=0.5
Figure 4-33 Separation Angle Time Histories at ReD=100 and h/D=1.4
(a) Upper, (b) Lower
Figure 4-34 Separation Angle Time Histories at ReD=100 and h/D=1.2
(a) Upper, (b) Lower
Figure 4-35 Separation Angle Time Histories at ReD=100 and h/D=1.0
(a) Upper, (b) Lower
Figure 4-36 Separation Angle Time Histories at ReD=100 and h/D=0.7
(a) Upper, (b) Lower
Figure 4-37 Upper Separation Angle Time History at ReD=100 and h/D=0.5

Figure 4-38 Streamlines for Semi-bounded Cylinder at ReD=100 and h/D=1.4
Figure 4.39 Streamlines for Semi-bounded Cylinder at ReD=100
(a) h/D=1.2, (b) h/D=1.0
Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-40 Streamlines for Semi-bounded Cylinder at ReD=100
(a) h/D=0.7, (b) h/D=0.5

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-41 Pressure Coefficients at Cylinder and Plate for ReD=100 and h/D=1.4, (a) Maximum Lift, (b) Minimum Lift
Figure 4-42 Pressure Contour Plots for Semi-bounded Cylinder at ReD=100 and h/D=1.4, (a) Maximum Lift, (b) Minimum Lift

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-43 Pressure Coefficients at Cylinder and Plate for ReD=100 and h/D=1.2, (a) Maximum Lift, (b) Minimum Lift
Figure 4.44 Pressure Contour Plots for Semi-bounded Cylinder at ReD=100 and h/D=1.2, (a) Maximum Lift, (b) Minimum Lift. Ordinate and Abscissa Scaled to Cylinder Diameter Units.
Figure 4-45 Pressure Coefficients at Cylinder and Plate for ReD=100 and h/D=1.0, (a) Maximum Lift, (b) Minimum Lift
Figure 4.46 Pressure Contours for Semi-bounded Cylinder at ReD=100 and h/D=1.0

(a) Maximum Drag  (b) Minimum Drag

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 4-47 Pressure Coefficients at Cylinder and Plate for ReD=100 and h/D=0.7, (a) Maximum Lift, (b) Minimum Lift
Figure 4-48  Pressure Coefficients at Cylinder and Plate for ReD=100 and h/D=0.5, (a) Maximum Lift, (b) Minimum Lift
Figure 4.49 Pressure Contours for Semi-bounded Cylinder at ReD=100 and h/D=0.5
(a) Maximum Lift, (b) Minimum Lift
Ordinate and Abscissa Scaled to Cylinder Diameter Units
CHAPTER 5

Semi-bounded Flow Over a Moving Cylinder
at \( \text{Re}_D = 100 \)

5.1 Overview of Moving Cylinder Simulation

The simulator developed for the current research includes the capability to model the flow over a moving cylinder under the influence of a nearby boundary. Movement was permitted only in the \( y \)-direction to focus attention on the effect of lift. It was anticipated that the movement of a cylinder in the semi-bounded configuration would reveal an influence of both the initial gap ratio (lift is a function of \( y \)-distance from the plate) and of the lift conditions at the start of the simulation. The gradual startup of many industrial processes and the periodic flow reversal of wave-induced currents provide conditions similar to the former case for heat exchanger or structural tubular design. The latter case is representative of a body entering a fluid with a "slam" loading.

Integration of the pressure and shear loadings over the surface of the cylinder yield a net lift force that can be used in a force balance on the cylinder to predict the acceleration at a particular timestep. Recall the expression given as Equation 2.4-8 in Chapter 2 that solved for the acceleration in terms of the forces acting on the cylinder and the mass of the cylinder itself:

\[
a^n = F_y^n - k (y^{n+1} - y^n) - c v^n / m_c
\]

(2.4-8)

The acceleration can be integrated numerically to obtain the cylinder displacement over a particular timestep:

Impulse:

\[
z^n = \frac{(a^n - a^{n-1})}{\Delta t}
\]

(2.4-9)
Velocity:
\[ v^{n+1} = v^n + a^n (\Delta t) + \frac{1}{2} z^n (\Delta t)^2 \]  \hspace{1cm} (2.4-10)

Displacement:
\[ y^{n+1} = y^n + v^n (\Delta t) + \frac{1}{2} a^n (\Delta t)^2 + \frac{1}{6} z^n (\Delta t)^3 \]  \hspace{1cm} (2.4-11)

The displacement becomes a change in the gap ratio, which is then passed to the grid generator for generation of a new computational mesh at the next timestep. Figure 5-1 illustrates the dynamic spring-mass-damper system modeled.

In Chapter 4 the behavior of the lift coefficient at \( Re_D = 100 \) was described as being positive at gap ratios down to approximately 1.0. At \( h/D = 1.0 \) a negative mean lift was determined, followed by a return to a positive value by \( h/D = 0.7 \). In a semi-bounded configuration, movement of the cylinder in the y-direction places the cylinder in a region of differing mean lift properties than at the previous location. The amount of movement is related to the net lift as well as the spring, structural damping, and inertia forces acting on the cylinder. A large matrix of cases for study can be formed from combinations of these parameters, repeated over a traverse of gap ratios. Completion of such a case matrix was not the intent of the current investigation, so parameters were fixed at zero structural damping, a mass ratio of 5.0 for the cylinder, and a spring stiffness sufficient to generate an undamped natural period equal to that of the 6.289 second vortex shedding period of an unbounded cylinder. This period was selected because it was far enough removed from the 12.25 second maximum vortex shedding period of a semi-bounded cylinder to prevent lock-in from occurring. The lock-in phenomenon is described later in this section. The mass ratio is defined as:

\[ \text{Mass Ratio} = \frac{m_c}{\rho D^2} \]  \hspace{1cm} (5.1-1)

and ranges from 1-5 for a metallic cylinder in water to 15-500 for a cable in air [Griffin (1984)]. The mass ratio is a convenient form of nondimensionalization and has great
importance to lift and drag force analyses that attempt to relate these loads to a uniform flow condition. It was of lesser importance to this study because the computed pressure and velocity fields lead directly to computation of forces on the cylinder.

Several initial gap ratios were considered for simulation with a dynamic system having the above properties. An initial gap ratio of 2.0 offered regular vortex shedding in a region of positive mean lift. Two simulations were run at this initial gap ratio, one with the cylinder released from fixity in an established flow and one with the cylinder subjected to the transient of an impulsively-started flow. Reduction of the initial gap ratio to 1.5 placed the cylinder into a region of suppressed vortex shedding, but with a net lift away from the plate. It was of interest to determine whether the cylinder displacement due to the lift force would increase sufficiently to initiate vortex shedding. Further reduction of the initial gap ratio to 1.0 placed the cylinder at the location where a net lift toward the plate was observed for the fixed cylinder. Of interest was whether motion toward the plate would be limited by the passage of the cylinder back into the region of lift away from the plate found at small gap ratios. A final case was run at an initial gap ratio of 0.7 to determine the effect of a cylinder initially subjected to lift away from the plate, potentially entering the region of negative lift near the gap ratio of 1.0.

Experimental and numerical studies of flow about an unbounded cylinder show that as the vortex shedding frequency approaches the natural frequency of cylinder vibration, a phenomenon called lock-in occurs. Under lock-in conditions, the frequency of vortex shedding is controlled by the frequency of the oscillating cylinder. This behavior spans a "capture range" of ±25 to 30 percent of the cylinder's natural frequency [Sarpkaya (1979)]. While under the influence of lock-in, the correlation length of vortex shedding along a cylinder has been observed to increase [King (1977)] and lift coefficient behavior is altered in accordance with the amplitude of cylinder motion. Low amplitude motion results in an increased lift coefficient, while large amplitude motion results in a decreased lift coefficient. A self-limiting effect prevents motion greater than 1 to 1.5
diameters. As flow velocity is increased from an existing lock-in condition, the cylinder continues to oscillate at its natural frequency while the vortex shedding frequency jumps to the expected Strouhal frequency for this velocity. The effect of a semi-bounded configuration on the lock-in phenomenon has not been addressed quantitatively in experimental studies, but Sarpkaya has qualitatively observed that the approach toward a plate boundary reduces the width of the capture range.

Achievement of lock-in during the current investigation was not attempted due to the wide difference between the limiting period of vortex shedding at $Re_D=100$ (12.25 seconds) and the natural period of a cylinder having a higher spring constant (6.29 seconds for the spring constant and mass ratio noted above) consistent with the vortex shedding frequency of an unbounded cylinder. An investigation of the lock-in behavior of a cylinder approaching a plate boundary would require a lower spring constant for a given mass ratio than would be predicted by setting the natural frequency of the cylinder to the unbounded cylinder vortex shedding frequency. Selection of the stiffer spring was made in an effort to limit displacement of the cylinder to a region near a particular initial gap ratio. A softer spring mounting could permit movement of the cylinder from the predominant lift direction of one gap ratio region to the predominant lift direction of an adjacent region; while this development would be of interest, it was not viewed as a priority item in the allocation of available computing funds.

The moving cylinder simulations conducted in the current research were performed with primary emphasis on observing the displacement of the cylinder relative to the plate boundary, and correlating the displacement to the time history of the lift coefficient. Further, the studies permit comparison of the periodic loading frequency of a moving cylinder to that of a fixed cylinder at a comparable gap ratio. The amplitude of cylinder motion under the influence of a nearby boundary can also be compared to that of an unbounded moving cylinder having the same structural parameters. Some insight can be
gained into the dynamic behavior exhibited by the cases examined herein through a review of the forced oscillation of a simple spring-mass-damper system. Two immediate differences between the forced oscillation model and the system being simulated must be highlighted. First, the forced oscillation model assumes sinusoidal motion. The traces of the lift coefficient time histories presented in the previous chapters show that the lift loading is periodic, but only approximately sinusoidal in nature. Second, the amplitude of the forcing function is assumed constant in the forced oscillation model, whereas the amplitude in a numerical simulation is itself a function of both time and the y-direction spatial coordinate. This is because all hydrodynamic loadings, including viscous damping, are integrated into the lift coefficient and hence into the amplitude of the forcing function.

Consider the equation of motion under a sinusoidal forcing function [Meirovitch (1967)]:

\[ m_c \ddot{y} + c \dot{y} + k y = F_y \sin \omega t \]  
\[ (5.1-2) \]

and assume a solution of the form

\[ y = Y \sin (\omega t - \phi) \]  
\[ (5.1-3) \]

which yields after substitution:

\[ Y = \frac{F_y}{\sqrt{(k - m_c \omega^2)^2 + (c \omega)^2}} \]  
\[ (5.1-4) \]

\[ \phi = \tan^{-1} \left( \frac{-c \omega}{k - m_c \omega^2} \right) \]  
\[ (5.1-5) \]

After the following definitions,
Natural Frequency \( \omega_n = \sqrt{\frac{k}{m_c}} \) (5.1-6a)

Critical Damping \( c_c = 2 m_c \omega_n \) (5.1-6b)

Damping Ratio \( \zeta = \frac{c}{c_c} \) (5.1-6c)

the familiar forms of the nondimensional amplitude response and the phase angle are obtained:

\[
\frac{Y_k}{F_y} = \frac{1}{\sqrt{1 - \left(\frac{\omega}{\omega_n}\right)^2 + \left[2 \zeta \left(\frac{\omega}{\omega_n}\right)\right]^2}}
\]

(5.1-7)

\[
\phi = \tan^{-1} \left[ \frac{2 \zeta \left(\frac{\omega}{\omega_n}\right)}{1 - \left(\frac{\omega}{\omega_n}\right)^2} \right]
\]

(5.1-8)

Thus for a simple spring-mass-damper system subjected to a sinusoidal excitation, the amplitude response depends on the damping ratio and the ratio of the excitation frequency to the natural frequency of the spring-mass system. The presence of damping prevents unlimited amplitude motion when the excitation frequency equals the natural frequency of the system. Damping also influences the phase angle between the excitation and the response; in the absence of damping the motion of the mass is in phase for excitation frequencies below resonance, 90° out of phase at resonance, and 180° out of phase for excitation frequencies above resonance. Damping tends to increase the phase difference for excitation frequencies below resonance and to reduce the phase difference for excitation frequencies above resonance.

As noted above, all of the moving cylinder simulations were conducted at excitation frequencies lower than the natural frequency of the cylinder spring-mass-damper system.
The presence of viscous damping indicates that the results of the simulation should yield out-of-phase behavior, and this was in fact observed at the gap ratio where regular vortex shedding was present. As the period of excitation increased with reduction in the gap ratio, the excitation frequency became small and displacements were almost in phase with the loading. The following section summarizes the results of the moving cylinder cases investigated in the current research.

5.2 Summary of Moving Cylinder Investigations

Two significant results were obtained from the cases investigated at initial gap ratios of 2.0, 1.5, 1.0, and 0.7. The first concerns the influence of the flow conditions at the start of the simulation. At an initial gap ratio of 2.0, two cases were studied, one in which the cylinder was released from fixity in a regular vortex shedding pattern and one in which the cylinder was subjected to the transient of an impulsively started flow. In the former case, the cylinder moved rapidly to a new equilibrium position and continued regular vortex shedding. In the latter case, the cylinder tracked the large lift transient and then proceeded toward the same equilibrium position as in the former case. The second and perhaps most interesting result is the apparent reduction in the magnitude of wake-induced periodic loading at gap ratios smaller than that of vortex shedding suppression for a fixed cylinder.

Recall that for a fixed cylinder at Re₅=100, vortex shedding was not observed at gap ratios of 1.5 or less. However, fluctuations in the lift and drag loading on the cylinder continued with increasing periods and with the range about each mean loading value approaching the magnitude of the mean loading as the gap ratio was further reduced. References citing experimental evidence for such loading in the absence of periodic vortex shedding have been presented, with wake instability at small gaps offered
as the explanation. The cases investigated herein show that the ability of the cylinder to move even slightly is sufficient to minimize or prevent the feedback of wake instability to the cylinder. Lift and drag loading on the flexibly-mounted cylinder is essentially constant in the absence of regular vortex shedding, with the magnitude of wake instability loadings at least a full order of magnitude smaller than experienced for the fixed cylinder cases at these gap ratios. These and other findings will be expanded upon in the following case summaries.

Figures 5-2 and 5-3a show the movement of a cylinder released from an initial gap ratio of 2.0 into an initially-stabilized flow. At a gap ratio of 2.0, the mean lift is positive, and regular vortex shedding is present. The cylinder seeks an equilibrium position balancing the mean lift against the spring tension of the "mounting" and continues regular oscillation at the new location. The maximum displacement was only 0.093 diameters from the initial position and the peak-to-peak amplitude was 0.01 diameters. Figure 5-3b shows the trace of the lift coefficient over the final time period simulated, and comparison with the displacement over the same interval indicates the lift force to be approximately 180° out of phase with the displacement. The period of oscillation increased to 12.2 seconds, compared to a period of 11.95 seconds for a fixed cylinder at h/D=2.0. The mean lift coefficient was 0.024 with a range about the mean of ±0.011, representing a slight increase over the fixed cylinder mean of 0.023 and a slight decrease in the range from ±0.030. The mean drag coefficient was 1.016 with a range of ±0.003, comparable to fixed cylinder values of 1.011 and ±0.003 respectively.

Figures 5-4 and 5-5 illustrate the movement of a cylinder placed initially at a gap ratio of 2.0, but subjected to an impulsive start of the flow field. Note that the initial movement of the cylinder is toward the plate, under the influence of the large lift coefficients found during an impulsive start. The corresponding lift coefficient trace is provided in Figure 5-6a. The combination of lift force and spring stiffness experienced by the cylinder was sufficient to retain the cylinder in a region of net positive lift and of
regular vortex shedding. Accordingly, the cylinder moved toward an equilibrium position that was the essentially the same after 4000 records as for the cylinder released into a stabilized flow. The trace of the lift coefficient for this equilibrium behavior is shown in Figure 5-6b. At the equilibrium location, the lift and drag coefficient values were found to be equal to those for the cylinder released into the stabilized flow.

The movement of a cylinder released from fixity into a stabilized flow at an initial gap ratio of 1.5 is presented in Figures 5-7 and 5-8a. This gap ratio represented a region of positive mean lift in the fixed cylinder simulation, and this positive lift resulted in a maximum displacement of 0.059 diameters from the initial position for the moving cylinder simulation. At the large magnification of each position trace, a periodic loading effect is apparent even though the streamline contour plots for this case show no vortex shedding activity. The period for these small fluctuations was 12.75 seconds, compared to 12.25 seconds found in the fixed cylinder simulation. Minimum lift led maximum displacement for a slight reduction in the 180° phase lag observed in the previous case. In perspective, however, the peak-to-peak amplitude of motion after 3000 records was less than a thousandth of a diameter. The mean lift coefficient was 0.017 with a range of ±0.0007, compared to the values of 0.015 and ±0.008 for the fixed cylinder at this initial gap ratio. Figure 5-8b illustrates the time history of the lift coefficient during the last 1000 records of simulation. The mean drag coefficient was 0.950 with a range of ±0.000, for comparison to the fixed cylinder values of 0.943 and ±0.001 respectively. For all practical purposes, the cylinder is stationary and experiencing negligible fluctuation in the lift and drag forces at the conclusion of the simulation run. This reduction by over an order of magnitude in the fluctuating component of the lift and drag loads indicates that the flexible mounting of the cylinder either reduces the feedback of wake-generated loading to the cylinder, or aids in the suppression of wake instability at small gap ratios.

Figures 5-9 and 5-10 illustrate the displacement time history of a cylinder
released from fixity into a stabilized flow at an initial gap ratio of 1.0. Recall that the region surrounding this gap ratio yielded a negative mean lift (attraction toward the plate) in the fixed cylinder simulations. The moving cylinder simulation led to establishment of an equilibrium position closer to the plate, however, examination of the lift coefficient traces in Figures 5-11 and 5-12 show that the movement of the cylinder was not sufficient to enter the region of net positive lift found closer to the gap ratio of 0.7 in the fixed cylinder simulations. Thus equilibrium was established between the attraction to the plate and the restoring force of the spring mounting without substantial influence from the variation of mean lift with gap ratio. The magnification of the plotting scale used in the displacement and lift coefficient curves shows a long period (approximately 70 seconds) variation in the displacement, essentially in phase with the variation in the lift coefficient. The amplitude of the cylinder motion appeared to be decreasing as the simulation progressed through 3000 records, however, the simulation was stopped at this point due to negligible changes in the lift and drag coefficients. The maximum displacement from the starting position was 0.1162 diameters, but the peak-to-peak amplitude was only 0.006 diameters. The mean lift coefficient at the equilibrium gap ratio was -0.031 with a range of ±0.0006 for comparison to the values of -0.043 and ±0.004 obtained for a fixed cylinder held at h/D=1.0. The mean drag coefficient was 0.639 with a range of ±0.008 in the moving cylinder simulation, while respective values obtained at h/D=1.0 for the fixed cylinder were 0.685 and ±0.017. Both values for the mean coefficients in the moving cylinder case are consistent with the fixed cylinder values that would be expected at the nominal equilibrium gap ratio of 0.89, i.e. the lift is more positive and the drag is reduced as the cylinder nears the plate. Again, the cylinder is virtually motionless, and the ranges for the lift and drag loading fluctuations are an order of magnitude smaller than for the fixed cylinder case.

Figures 5-13 and 5-14 present the displacement time history of a cylinder released from fixity into a stabilized flow from an initial position at h/D=0.7. The fixed
cylinder simulation found the region surrounding this gap ratio to yield a positive mean lift coefficient that was small in magnitude. Accordingly, the moving cylinder underwent a very small displacement of 0.003 diameters from the starting position and experienced loading fluctuations leading to a peak-to-peak amplitude of 0.0006 diameters. These values were decreasing in magnitude when the simulation was stopped after 3000 records. Absent the large magnification of the displacement plots, the cylinder was effectively motionless at this time. The small fluctuations in loading led the displacement only slightly, leaving the cylinder response essentially in phase with the loading. The mean lift coefficient was 0.001 with a range of ±0.0001 for comparison with fixed cylinder values of 0.001 and ±0.002 respectively [Figures 5-15 and 5-16]. The mean drag was 0.556 with a range of ±0.002 which can be compared to the fixed cylinder values of 0.557 and ±0.011 respectively. The ranges about the mean for both the lift and the drag coefficients are again found to be an order of magnitude smaller for the moving cylinder simulation than for the fixed cylinder case.

Figures 5-17 and 5-18 present four sequential plots of the streamline contours that illustrate the regular vortex shedding for the moving cylinder case at h/D=2.0. The 0.09 diameter motion of the semi-bounded cylinder at this gap ratio is smaller than the 0.162 diameter lock-in motion for an unbounded cylinder as reported by Allen (1987). Figures 5-19 and 5-20 show the streamline contours for the flexibly-mounted cylinders at gap ratios of 1.5, 1.0, and 0.7. The appearance is unchanged from the fixed cylinder streamline plots, consistent with the negligible fluctuation in loading reported above for these gap ratios.

Summarizing the moving cylinder case results, the current research yielded lift and drag coefficients that were slightly higher than their fixed cylinder counterparts, consistent with the experimental observations of Sarpkaya (1979). The range of fluctuating loading on the cylinder, attributed to wake instability at small gap ratios, was at least an order of magnitude smaller than the corresponding fixed cylinder results. In
the absence of regular vortex shedding, these fluctuating loads were evident only under extreme magnification of scale. The flexibility of the cylinder mounting appears to either reduce the feedback of wake-generated loading to the cylinder, or aid in the suppression of wake instability at small gap ratios.
Figure 5-2 Cylinder Movement vs. Time, Initial Gap Ratio 2.0, Release Into Stabilized Flow
Figure 5-3 Cylinder Movement vs. Time, Initial Gap Ratio 2.0, Release Into Stabilized Flow, (a) Position, (b) Lift Coefficient
Figure 5-4  Cylinder Movement vs. Time, Initial Gap Ratio 2.0, Impulsive Flow
Figure 5-5 Cylinder Movement vs. Time, Initial Gap Ratio 2.0, Impulsive Flow
Figure 5-6 Lift Coefficient vs. Time for Cylinder at Initial Gap Ratio 2.0
(a) Impulsive Start, (b) After Stabilization
Figure 5-7 Cylinder Movement vs. Time, Initial Gap Ratio 1.5, Release Into Stabilized Flow
Figure 5-8 Cylinder Movement vs. Time, Initial Gap Ratio 1.5
(a) Cylinder Motion, (b) Lift Coefficient
Figure 5-9 Cylinder Movement vs. Time, Initial Gap Ratio 1.0
Release Into Stabilized Flow
Figure 5-10 Cylinder Movement vs. Time, Initial Gap Ratio 1.0
Release Into Stabilized Flow

Figure 5-11 Lift Coefficient vs. Time, Initial Gap Ratio 1.0,
Release Into Stabilized Flow
Figure 5-12 Lift Coefficient vs. Time, Initial Gap Ratio 1.0
Release Into Stabilized Flow
Figure 5-13 Cylinder Movement vs. Time, Initial Gap Ratio 0.7 Release Into Stabilized Flow
Figure 5-14 Cylinder Movement vs. Time, Initial Gap Ratio 0.7
Release into Stabilized Flow

Figure 5-15 Lift Coefficient vs. Time, Initial Gap Ratio 0.7
Release into Stabilized Flow
Figure 5-16 Lift Coefficient vs. Time, Initial Gap Ratio 0.7
Release Into Stabilized Flow
Figure 5-17  Moving Cylinder Streamline Contour Plots, Initial Gap Ratio 2.0
Release Into Stabilized Flow

Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 5.19 Moving Cylinder Streamline Contour Plots. (a) Initial Gap Ratio 1.5, (b) Initial Gap Ratio 1.0
Ordinate and Abscissa Scaled to Cylinder Diameter Units
Figure 5-20  Moving Cylinder Streamline Contour Plot, Initial Gap Ratio 0.7
Release Into Stabilized Flow

Ordinate and Abcissa Scaled to Cylinder Diameter Units
Conclusions and Recommendations for Future Work

The flow about a circular cylinder near a plane boundary has been simulated numerically for two Reynolds numbers, 80 and 100, over a range of gap ratios. The experimentally-observed suppression of vortex shedding at small gap ratios, reported in the literature, was also observed during the numerical simulation herein. Experimental investigations of this flow geometry have been performed at Reynolds numbers of approximately $10^4$ (the low subcritical range). The results of the current investigation in the fully laminar range were consistent with the experimental investigations, with features occurring at larger gap ratios due to the greater thickness of the plate and cylinder boundary layers at the lower Reynolds numbers. The current research determined the gap ratio for vortex shedding suppression to be approximately 1.7 at $Re_D=80$ and 1.5 at $Re_D=100$.

The reduction of the gap ratio at each Reynolds number was accompanied by a reduction in the mean drag on the cylinder. This was due to the progression of the cylinder into the region of lower effective upstream velocity in the plate boundary layer. The mean lift behavior resulted from the combination of lift away from the plate due to downwash from the front of the cylinder and lift toward the plate due to the venturi effect. At large gap ratios, the mean lift on a cylinder is approximately zero. As the gap ratio was decreased in the simulation, the mean lift became positive under the influence of the downwash pressure field. Further reduction of the gap led to higher velocities between the cylinder and the plate, enhancing the venturi effect and ultimately creating a lift toward the plate. At very small gap ratios, establishment of Poiseuille flow in the gap was accompanied by increased pressure in the gap that generated a strong component of lift away from the plate.

The period of loading on the cylinder due to regular vortex shedding tended toward
a maximum value as the gap ratio was reduced. Upon suppression of vortex shedding, loading on the cylinder did not become constant, but remained at the period of the maximum value. As the gap ratio was further reduced, the period increased substantially. The increase in period was accompanied by an increase in the range of the lift and drag about the mean value. The presence of a fluctuating loading in the absence of regular vortex shedding has been observed in experimental studies and attributed to instabilities in the far wake.

The analysis of vortex shedding from a cylinder has stimulated research for decades; its endurance as a subject for continued study is an indication of the complexity of the flow posed by this simple configuration. The research reported herein has provided additional insight into the suppression of vortex shedding from a cylinder as it approaches a nearby boundary. This is a configuration that occurs frequently in engineering design. The technique can also be extended to geometries other than the plate/cylinder and can admit multiple bodies.

The research has also opened several areas for future investigation. An extension of the numerical technique to include turbulence modeling would allow numerical investigation to extend into higher Reynolds number flows that are more consistent with engineering applications. The choice of an appropriate turbulence model will prove to be a challenge, as the problem offers a diverse range of flow regimes in a compact region. The so-called "two-equation" turbulence models currently in wide use are known to lack accuracy in regions of separating flow, yet it is the flow separation that is responsible for most of the behavior of interest in this geometry.

The fluctuating loading behavior observed in the absence of regular vortex shedding activity is also a subject for future investigation. First observed in experimental studies, and again in the current research, this phenomenon could potentially be investigated by a substantial increase in grid refinement in the far wake region. Similar behavior has been observed in experimental studies of two closely-spaced cylinders in uniform flow. The
multiple body problem could also be investigated on a refined grid to determine similarities in the far wake flow. Such work would likely require code conversion for use on supercomputer resources due to the increased execution time that would accompany an increase of mesh density in the far wake.

The ability of the numerical simulation to address a cylinder permitted to translate in the y-direction near a plane boundary has been demonstrated for a single set of mass ratio, structural damping, and spring stiffness parameters. The initial conditions for the flow simulation and the initial gap ratio were identified as additional relevant parameters for moving cylinder simulation. Also, the ability of the cylinder to move even slightly appeared to reduce or eliminate the fluctuating loading on the cylinder at small gap ratios. Whether this is due to the attenuation or the prevention of wake instabilities is a subject for future investigation on a high density mesh. The need exists to develop an extensive matrix of simulation parameters and perform these simulations if the dynamic behavior is to be fully understood. Modifications to the code to permit movement in the x-direction as well as the y-direction would allow a more realistic description of "galloping" phenomena observed in the motions of heat exchanger and reactor tubulars.

A body-fitted coordinate transformation was used to enable the future investigation of alternate geometries for the cylinder and the plate. For example, a rippled lower boundary or an elliptical cylinder might be more relevant to a particular investigation. Inflow and outflow boundary conditions need not be held constant over a simulation, admitting the possibility of investigating the returning wake effects in oscillatory (wave) flow. Combined with a valid turbulence model, the simulator used in this investigation could provide a reliable, cost-effective means for performing hydrodynamic research.
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