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The economics of undocumented immigration: Mexican participation in the U.S. labor market

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THE ECONOMICS OF UNDOCUMENTED IMMIGRATION:
MEXICAN PARTICIPATION IN THE U.S. LABOR MARKET

by

Héctor Alonso Olea

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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ABSTRACT

This study addresses the impact of Mexican illegal immigration on the U.S. labor market. It constitutes a first step towards developing rigorous structural econometric models that empirically analyze undocumented labor force dynamics. Structural estimation of the labor supply and the participation decision of illegal Mexican immigration requires the solution of intricate theoretical problems that have not been addressed in previous literature. The analysis developed here identifies those problems and proposes innovative solutions. In particular, undocumented participation in the U.S. labor market is studied in the context of life cycle theory and stochastic behavior. The empirical part of the analysis reviews the problems of sample selection and missing observations that characterize the available data on Mexican migration. The proposed empirical specification is evaluated employing limited dependent variables procedures, where a Tobit simultaneous equation model is solved using maximum likelihood methods.

According to the empirical results, Mexican undocumented immigration may be viewed as a transitory phenomenon. Individuals switch back and forth between Mexico and the U.S. reacting not only to income differentials, but also to social, family and economic attachments in their home-communities. Mexican workers seem to have little incentives to invest in human capital specific to the U.S., such as the ability to speak English. This behavior may be result of the partial transferability of Mexican skills, i.e. formal education, to the secondary market in the United States. Finally, contrary to conventional wisdom, the empirical evidence suggests that exogenous increases in U.S. wages, i.e. a non-expected hike in the legal minimum wage, may actually discourage Mexican undocumented participation in the U.S. labor market.

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I dedicate this dissertation to my Padre and my Madre, with all my love.
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CHAPTER 1

INTRODUCTION

The 1,952 mile Mexican-U.S. border is unique. No other border in the world separates two countries that differ so sharply in social, cultural and particularly economic circumstances. Yet these nations are linked by a symbiotic labor relationship. Mexico provides a source of mobile labor force, and the U.S. benefits by balancing excess labor demand disequilibriums, notably in its secondary market.

Since the beginning of modern Mexican labor participation in the U.S., when a network of railways linked both countries in the 1880s, Mexico-U.S. immigration relationships have not been easy. Characterized by periods of tolerance and marginal cooperation, labor dependency between these countries has persisted for more than a century. However, recent unilateral legislation by the U.S. and the lack of coordination between Mexican-U.S. authorities may threaten the mutual benefits derived from undocumented immigration.

Mexican migration to the U.S. is essentially an economic phenomenon. Nonetheless, current immigration policy, or the lack thereof, fails to address Mexican illegal participation in the context of rational economic behavior. Unfortunately, political interests rather than efficiency issues and special groups concerns rather than distributional gains have had substantial influence on recent legislation of international immigration.

Furthermore, Mexican migration is also a regional phenomenon observed solely in certain specific areas of the U.S. such as California, Texas and Illinois. This precludes nationwide involvement in economic and strategic consequences of this participation in the
low-skill labor market. Lastly, the lack of coordination and the apparent indifference of both the Mexican and the U.S. governments have inhibited negotiations that certainly could benefit both economies.

An extensive number of studies on demographic characteristics, regional distribution and social attachments of Mexican undocumented workers are generally available. Demographers, anthropologists and sociologists have accumulated a prolific literature on this topic. However, little is known about the economics of their migration decision and the labor markets in which they operate.

In this context, the foremost objective of this study is to assess the economic impact of Mexican immigration in the U.S. labor market. Consequently, the author expects, that by doing so a more accurate understanding of the mechanisms driving both the participation and the labor supply decisions will be attained. This will help policy makers to avoid irreversible mistakes in the design and instrumentation of further immigration policy.

The contribution of this research is twofold. First, it establishes the theoretical framework in which immigration decisions take place. It proposes a consistent dynamic model based on control theory and stochastic behavior, where international migration is addressed in a context of life cycle considerations. Here, the behavior of Mexican undocumented workers switching back and forth across countries is consistent with optimal lifetime behavior. This approach explores, in a rational economic framework, the elements affecting the participation decision beyond the conventional argument of net wage differentials between countries. Second, this study constitutes the first attempt in the literature to provide an efficient empirical methodology for the estimation of the labor supply and the participation rule of undocumented workers. In particular, the participation decision and the labor supply of Mexican undocumented workers are empirically estimated in the context of limited dependent variables methods. The econometric framework provides asymptotically unbiased estimates not only for earnings elasticities but also for other variables such as financial wealth, socio-demographic characteristics and human-capital investment.

The purpose of this introduction is to relate this study to the existing literature and the available information on undocumented immigration so that the reader is better able to understand the motivation for the following chapters, the problems, and the departure from conventional methods of analysis. The first section describes the background of Mexican migration to the United States. Here, particular attention is dedicated to recent legislation on
this subject. Section 2 describes the profile of undocumented workers. Here, some important questions are discussed: How large is the Mexican undocumented population?, In which sorts of markets do they operate?, Where do they come from? and Where do they locate in the U.S.? In order to introduce the working hypothesis of this research, Section 3 outlines some of the major controversies which highlight the undocumented immigration debate. Finally, section 4 presents a summary of the remaining chapters.

1.1 Background

Undocumented settlers are, by definition, immigrants with the desire to work in a foreign nation without the explicit consent of its authorities. In particular, undocumented migration to the U.S. commenced in the late 19th century with the introduction of a number of immigration laws that established quantitative and qualitative restrictions. From 1875 to 1929 Congress passed legislation containing exclusionist principles that would restrict Chinese immigrants, the first undocumented workers.

Ironically, none of these restrictions were aimed specifically at trans-border immigration. In the first decade of this century, immigration authorities in the U.S. manifested no serious concern about contracting unskilled labor from Mexico. Although the Mexican Revolution in 1910 drove a large number of refugees over the border, there was little evidence of modification in the tolerance of U.S. immigration authorities. On the contrary, in 1912 a “border asylum policy” was implemented under the impression that refugee migration was temporary [Corwin, 1978]. Indeed, the end of the Mexican conflict along with the Great Depression stopped and sometimes reversed international migration to the United States.

The contract-labor era of the Mexican Labor Program in 1942-64, commonly known as the bracero program, characterized a rare period of coordination between the Mexican and the U.S. governments. This agreement defined a number of safeguards, such as food, transportation and housing, for Mexican nationals who were employed as temporary agricultural workers in the United States.1 World War II and the Korean War provided the appropriate economic incentives that lead to the creation of the bracero program. In this

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1 The agreement expired in 1947, but continued on an informal basis until 1951, when it was reinstituted. [Greenwood and McDowell, 1986]
period, the U.S. government viewed the Mexican labor force as a welcome wartime emergency help.

However, incredible administrative exceptions, red-tape and bureaucratic controls led to major inefficiencies within the program. Employers tried to avoid government regulation by contracting free-lance Mexican workers [Corwin, 1978]. Furthermore, *braceros* were subject to backward conditions in their jobs. The rather novel contract-labor program soon became notorious for mismanagement and exploitation of Mexican workers. In 1964 the *bracero* program was unilaterally terminated by the U.S. government.

After this experience the U.S. authorities followed a policy of toleration of undocumented immigration. Labor requirements of agro-industries were met without disrupting institutional constraints. However, under the political proclamation “we need to restore control over our borders”, in 1986 U.S. Congress passed the Immigration Reform and Control Act, also known as the “Simpson-Rodino” bill. Provisions of the new immigration law grant amnesty to undocumented immigrants that could prove permanent uninterrupted residence in the U.S. since January 1982. Under a separate program (Seasonal Agricultural Worker), international immigrants could also qualify for amnesty if they worked in the agriculture sector for at least 90 days ending in 1986. In addition, a guest-worker program (Replacement Agricultural Workers) may be implemented in 1990 under which undocumented workers with three years in the fields are eligible for permanent-resident status. This will allow international contract-labor if there is a shortage of workers in perishable crop agriculture.

The Simpson-Rodino law includes a series of enforcement provisions that seem unlikely to reduce undocumented immigration. Employer sanctions, the heart and soul of the new immigration law, establishes that employers are liable for “knowingly” hiring workers without legal rights to work in the United States. They are intended to reduce the demand for undocumented workers by raising hiring costs.

According to Chiswick [1988], however, “employment sanctions are not likely to have a major impact.” [p.113] This perception is also shared by long time observers of international migration in the United States. Bustamante asserts that “[t]here is no sign the legislation has had any impact on the [immigration] flows.”2 Likewise, Cornelius admits

---

that "[t]hose who delayed migration to the U.S. during 1987 are now coming, having observed that work is still available even for new arrivals lacking papers."\textsuperscript{3} Two sorts of elements appear to justify these arguments: The lack of enforcement resources and the economic incentives that encourage permanent, rather than transitory, settlements.

First, scarce enforcement resources imply infrequent inspections. Chiswick [1988] recognized that

\begin{quote}
"[a]s result of minimal change in effective enforcement resources, the probability that employers will be detected in violation is very low... except perhaps for certain target or showcase establishments." [p. 113]
\end{quote}

In addition, Hill and Pearce [1987] evaluated the impact of employer sanction on the U.S. economy. They found that important re-allocation effects may be observed because enforcement is unlikely to eliminate undocumented workers from the U.S. labor force. A more likely outcome is that immigration authorities will concentrate their enforcement efforts in traditional industries and regions where international migration is common, e.g. landscaping in Texas. Consequently, industries and regions likely to face weak enforcement will absorb displaced immigrants, e.g. construction in North Carolina. "The incidence of employer sanctions will be uneven across factor groups as well as industries." [Ibid, p. 25]

Second, employer sanctions as well as tighter border controls significantly increase the costs of frequent border crosses. In order to obtain positive returns from migration activities, undocumented workers may lengthen the amount of work time in the U.S. per trip. Indeed, in a 1976 study conducted by North and Houstoun, respondents have made an average of 4.5 trips to the U.S. in a period of five years. In a more recent surveys Huddle \textit{et. al.} [1985] found that the typical illegal worker made 1.1 trips in the same amount of time. Furthermore, rising immigration costs may promote family immigration to the United States.\textsuperscript{4} In fact, the Simpson-Rodino law is designed to reverse transitory immigration patterns traditionally observed in Mexican undocumented workers to a permanent immigration.

\textsuperscript{3} Ibid.

\textsuperscript{4} "As a result [of the new immigration law], men who in the past came up north alone in the spring and returned to México for Christmas and New Year holidays now appear to be sending for their wives and children" [The New York Times, June 24, 1988: p. 1]
Surprisingly, the new immigration law was passed without addressing any economic issues, in particular those of efficiency and equity in the distribution of national income.\textsuperscript{5} Political rather than economic reasons motivated this legislation. Mexican undocumented migration is stimulated by economic incentives, where barriers between Mexican and U.S. markets introduce serious efficiency and distribution distortions in both economies.

"While the United States clearly needs to have voice in who enters to the country, this should not preclude an orderly and mutually beneficial approach to the particular complementarities that exist between the United States and Mexico..." [Reynolds and McCleery, p. 128]

Strong labor market interdependence links the economic policy of these countries. Future coordination between Mexican-U.S. authorities will be required to access potential benefits of immigration flows in a true partnership relation.

1.2 Profile of Undocumented Workers

Contrary to conventional wisdom, international immigration to the U.S. is not an homogeneous phenomenon. In particular, undocumented immigration from Mexico possesses distinctive characteristics not found in other sorts of international settlements in the United States. During the last twenty years illegal immigration to this country has been characterized by three distinctive groups: Political refugees, high-level manpower (brain drain) and Mexican low-skill workers. Each of these groups affects the U.S. economy in entirely different ways.

Immigration for political reasons and by high level manpower tend to be permanent. On one hand, political refugees are likely to break any possible family and economic attachments with their home-country. Family migration and the forfeiture of non-transferable capital characterizes political immigration. On the other hand, brain drain immigration is observed among high-skill individuals coming from low-income communities. Here, international migration has had the adverse effect of widening the inequalities between receiving

\textsuperscript{5} In contrast, legislation aimed at deregulation of the banking system and the airline industry were subject to intense economic review.
and home countries. Similar to political refugees, this group will tend to settle in the U.S. along with their immediate relatives.

In contrast, Mexican undocumented low-skill workers are deeply attached to their language and culture, and strongly rooted in their home-communities. Typically, they are individuals with few skills and little education, switching back and forth between Mexico and the United States. Transitory immigration is thus characterized by family, social and economic attachments with Mexico. “Their migration is not a sign of special attraction to the United States, but paradoxically of a commitment to their home community.” [Piore, 1986: p. 27]

Given the distinctive characteristics of international immigration flows, this study focuses solely on the impact of low-skill undocumented Mexican workers. However, a major problem confronting analyses of this sort is the reliability of available empirical data. No single source of information is likely to provide an unbiased assessment of the characteristics of Mexican immigrants. Traditionally, the Immigration and Naturalization Service (INS) has provided a basic set of statistics on deported individuals. Nonetheless, the serious methodological problems of this data is well documented, since a single immigrant may be apprehended several times in a short period of time.\(^6\)

Another source of information is found in independent surveys that attempt to characterize undocumented workers and their labor behavior. North and Houstoun [1976] made the first important effort in this field. More recently, Díez-Canedo [1980] used Mexican personal checks remittances to track undocumented workers sending money to their home-communities. De Maria [1983] analyzed the impact of Mexican workers in the Houston construction industry by direct interviews; likewise, Huddle conducted field surveys on non-union highway construction workers in 1985, and on immigrants applying for amnesty under provisions of the new immigration law in 1987 and 1988.

Lastly, recent studies have explored the potential of using data from the 1980 Census to acquire a fuller understanding of Mexican migration. Pearce and Gunther [1985] used 1980 Census data to describe the effects of undocumented workers in the Texas economy. Passei and Woodrow [1985] employed this statistical source to report the geo-

\(^6\) Chiswick [1988] calls this distortion the “revolving door” effect.
The number of undocumented Mexicans living in the U.S. have been traditionally overestimated. Johnsen [1980] suggested that immigrants without permission are probably in the 4 to 7 million range, with 2 to 5 million participating in the labor force. Similarly, Huddle et.al. [1985] established that the size of the undocumented population was close to 9 million in 1982, of which 5.5 million were working.7 However, studies based on more reliable methodologies seem to agree on a lesser number. Warren and Passel [1984] estimated that about 2 million undocumented aliens were counted in the 1980 census and that about 55 percent of this figure were Mexican settlers. In addition, Díez-Canedo [1980] found that around 0.8 million Mexican undocumented aliens were employed in the United States.

Demographic statistics suggest that Mexican illegal workers tend to be relatively young with a substantial concentration in the ages of 20 to 40, and a high proportion of immigrants are likely to be male and single. In addition, Mexican migrants to the U.S. do not come from the poorest regions of the country. According to Díez-Canedo [1980] and De Maria [1983], traditional sending states are located in the central part of the country, mainly Guanajuato, Zacatecas, Mexico City and Jalisco. Undocumented workers, in fact,

7 At the beginning of the Amnesty program on May 1987, the INS estimated that 4 million undocumented immigrants will come forward to apply for permanent-resident status. However, after the deadline, one year later, the INS received only 1.7 million applications for legal residence and 0.7 million through the Seasonal Agricultural Worker (SAW) program.
are not likely to migrate from the relatively poor southeast region, e.g. Oaxaca and Chiapas. Richer agro-industrial northern states like Sonora, Sinaloa and Chihuahua are also not an important source of undocumented workers.

Although traditionally, illegal immigration originated in rural areas, recently an increasing number of undocumented workers are coming from urban centers. The level of education is likely to be affected by this distinction. Urban immigrants are more likely to have longer periods of education than rural immigrants (Figure 1.1). Independent of their background, however, formal education is substantially low, ranging from five to ten years of school.\(^8\)

Finally, Mexican participation in the U.S. is a regional phenomenon. According to Census information processed by the author, only three states account for almost 90 percent of the total population of undocumented settlements from Mexico: California (57 percent), Texas (23 percent) and Illinois (8 percent). Therefore, the direct impact of Mexican nationals on the domestic labor market is highly concentrated in a few distinct areas.

1.3 Controversies on Mexican Participation in the U.S.

This section presents some of the major controversies that highlight the undocumented immigration debate. Several hypotheses discussed and tested in further chapters are introduced here. This section does not constitute, however, a comprehensive analysis of international immigration but rather focuses on particular issues relevant to the overall objective of this study.\(^9\)

1.3.1 The Assimilation Hypothesis

A dominant theme in studies on international immigration to the U.S. is the social adaptation and the economic integration of a foreign born population. Traditionally, theorists assume, either explicitly or implicitly, that immigrants confront similar socio-economic structures as domestic residents. The foreign born would eventually attain parity with their

\(^8\) Note that this argument does not imply that better educated residents have higher likelihood to become illegal workers.

\(^9\) See Greenwood and McDowell (1986) for a more comprehensive review.
native born counterparts [Tienda and Neidert, 1980]. Accordingly, much of the research on this subject has concentrated on documenting the adjustment experience or assimilation process of international flows in the United States.

Advocates of the *assimilation* perspective, e.g. Chiswick [1977 and 1978], and Hirschman [1978], sustained that because knowledge and skills are not perfectly mobile across countries, immigrants initially would have earnings significantly lower than domestic residents (see Figure 1.2). However, as time passes foreigners gain expertise on the characteristics of the economy. This is achieved through investment in U.S.-specific human capital.

![Figure 1.2 - Earnings in the Assimilationist Hypothesis](image)

The assimilation process and thus the foreign-native earnings gap are limited by the degree to which home-country skills may be transferred to the U.S. labor market. Adjustment time would be shorter for immigrants from Canada than immigrants from Mexico. Moreover, it is suggested that given long periods of immigration, earnings of the foreign born would approach, and even equal, those of local residents. In sum, the direct consequence of the *assimilation* perspective is that eventually international migration will displace domestic workers from the labor market.

A number of criticisms may be levelled upon this representation. Perhaps the most important one is the failure to address the assimilation argument in an explicit framework of economic theory. This leads to *ad-hoc* empirical hypotheses, lacking consistency with respect to theoretical rationale. This deficiency is even more evident when trying to address
the dynamics of the assimilation process in the absence of control theory and stochastic behavior.

In addition, this view implicitly assumes that international migration is a permanent phenomenon. In the context of legal settlements or political refugees, this representation may be accurate. However, Mexican undocumented immigration is more likely to have transitory characteristics. The empirical applicability of the assimilation hypothesis in the context of Mexican illegal workers is less obvious than it is for other sorts of immigration movements. The transitory elements of Mexican migration is essential to assess the impact of undocumented workers in the U.S. labor market.

Indeed, the composition of U.S.-specific human capital investment and thus the integration process is affected by the time horizon of illegal settlement. Transitory undocumented workers may have little incentives to invest in these activities. Three elements are likely to measure human capital formation of undocumented workers: Formal education, ability to speak English and length of time since immigration (on-the-job training). Notice that the latter two represent investments in U.S.-specific abilities, while the former embodies human capital acquired in Mexico. Contrary to the perception of assimilationists, Mexican-specific skills may be transferred to the secondary U.S. labor market. Cornelius [1978] indicated that Mexican immigrants "...require little or no technical skills, and only a rudimentary command of English, if any at all." [p. 5] The degree in which formal education acquired in Mexico impacts earnings of undocumented workers in the U.S. provide evidence in favor of the transferability of skills across regimes.

![Figure 1.3—Occupational Distribution in Mexico and in the United States](image-url)
Moreover, undocumented workers are likely to seek employment where further U.S.-specific human capital investment is not required. In this context, the occupational distribution of illegal immigrants is oriented to low-skill activities in both countries. De Maria [1983] suggested that over 60 percent of her sample included respondents working in agriculture related jobs in their home-communities. In contrast, according to Cer's information, the distribution of undocumented workers in U.S. industries tends to be broader, but is still somewhat concentrated in low-skill occupations: Manufacturing (automobile mechanics, brickmasons, carpenters, painters, etc.), agriculture (landscaping, farm and forestry workers, etc) and services (maids, waiters/waitresses, janitors and others). [See Figure 1.3]

The present study proposes the hypothesis that Mexican illegal workers are likely to be characterized by individuals with transitory migration patterns. Accordingly, they have little economic incentives to invest in human capital specific to the U.S., such as the ability to speak English and on-the-job training. This behavior is the result of partial transferability of home skills (e.g. formal education) to the secondary labor market of the United States.

1.3.2 Market Segmentation: An Alternative Hypothesis

An alternative proposition to the assimilationist view suggests that Mexican undocumented workers do not compete with native born workers. The domestic labor market, especially the low-skilled, is sufficiently segmented that native workers are insulated from the direct employment effects of immigrants. Segmented market analysts argue that the economy is characterized by major “market segments” which associate different reward systems and paths for mobility [Portes and Bach, 1978]. In the field of undocumented immigration, the segmentation hypothesis has been addressed in Cornelius [1978], Piore [1979 and 1986] and Tienda and Neidert [1980], among others.

The existing demand of a secondary market and the failure of the native labor force to supply its labor services contributes to the development of an undocumented labor market. According to Piore [1986], the immigration process tends to be governed by jobs which are relatively low paying, insecure and lacking of any career advancement opportunities, such jobs are not attractive to domestic residents. Undocumented workers undertake these jobs because they view it as a temporary situation. “They do not think of themselves
as staying long enough to take advantage of career opportunities and they obtain their self-definition from the work they perform at home.” [Ibid, p. 28]

Figure 1.4—Job Mobility Among Undocumented Workers

Figure 1.4 appears to support this hypothesis. It is shown that foreign workers may have a lesser degree of job mobility if they pursue permanent residence, e.g. respondents applying for amnesty under provisions of the new immigration law. Indeed, transitory Mexican workers are likely to be more mobile than permanent migrants; the former may work in a particular job an average of one to two years, while the latter may last from three to almost five years in an specific job.

In addition, the temporary characteristics of Mexican undocumented workers may be evident in the amount of money sent back home. Gamio asserts that “permanent residents who have their families and interests with them rarely remit money” [in Díez-Canedo, 1980: p. 33] According to De Maria [1983], undocumented workers in the construction industry send an average of 195 dollars/month. Similarly, North and Houstoun [1976] and Díez-Canedo [1980] reported average monthly payments of 189 and 129 dollars for Mexican illegal workers, respectively. In contrast, non-Mexican undocumented groups may have lower remittance levels: Western hemisphere (excluding Mexico) and Eastern hemisphere settlements send 76 and 37 dollars/month to their home countries, respectively [Díez-Canedo, 1980: p. 49].

1.3.3 Job Displacement
The current debate on Mexican illegal immigration bears direct relation to the issue of job competition between domestic residence and foreign workers. Do undocumented workers displace domestic workers from jobs? Notwithstanding international migration results in efficiency gains for the national economy, the displacement controversy has absorbed most of the research efforts in the literature. Conventional arguments assert that undocumented workers are close substitutes for native workers. This diminishes the employment opportunities for domestic residents either directly, by reducing employment prospects, or indirectly, by reducing wage levels. Bernard [1953], however, contested the displacement misrepresentation of undocumented workers:

“One of the most persistent and recurrent fallacies in popular thought is the notion that [undocumented] immigrants take away the jobs of native Americans. This rests on the misconception that only a fixed number of jobs exist in any economy and that any newcomer threatens the job of any old resident.” [p.57]

An implicit assumption in the displacement argument is the homogeneity of the labor force, in particular the substitutability between undocumented and native workers. Proponents of the assimilation hypothesis claim that foreign born workers acquire sufficient skills specific to the U.S. labor market so that any distinction with the native born will eventually vanish. Thus, complete displacement and depression of wage rates may be expected. In contrast, defenders of market segmentation argue that because undocumented and local workers operate in disparate labor markets, they do not compete for employment opportunities and therefore displacement is rejected. Whether these propositions provide an accurate representation of job competition in the low-skill labor market is an empirical question. Yet, this has not been systematically addressed in previous literature.

Under what set of conditions will the influx of Mexican undocumented workers leave domestic employment and wages unaffected? The number of native labor force displaced by immigrant workers depends on the conditions of the labor market, specifically upon their demand and supply functions. In particular, it is evident that an accurate mea-

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10 See Grossman [1984] and Hill [1985].
11 According to Greenwood and McDowell [1986], domestic employment and wage levels are unaffected by immigrants if their market demand is perfectly elastic. Nonetheless, Hamermesh [1976] estimated that the labor demand of the U.S. economy is rather inelastic (in the short run -0.32). Furthermore, Johnson [1980] used a demand elasticity of -1.5 for the low-skill market in a study of displacement effects. Since the scope
sure of the labor supply elasticity of undocumented workers is essential in accessing their impact on the U.S. labor market.

![Graph showing relative labor supply elasticities](image)

Figure 1.5—Relative Labor Supply Elasticities

In fact, the relative sensitivity of labor-leisure decisions among domestic and undocumented workers determinate the level of marginal displacement in the secondary market. Figure 1.5 describes this argument. If the labor supply elasticity of U.S. residents ($\varepsilon_{us}$) is equal to the supply elasticity of illegal Mexican workers ($\varepsilon_{mx}$), the rate of marginal job displacement is one-to-one. Thus, marginal displacement rates are low as long as $\varepsilon_{mx}$ is significantly low relative to $\varepsilon_{us}$. Unfortunately, no serious empirical analysis has addressed the magnitude of these parameters.

1.3.4 Taxes and Welfare Transfers

Another central issue commonly raised about Mexican immigration focuses on the question of whether they are a social welfare burden for the national economy. To what extent do they use public services? Do their tax contributions cover the cost of providing these services? The hard evidence that exists suggests that illegal workers contribute more in taxes than they receive in social benefits.

Some analysts argue that undocumented flows do not pay taxes. Huddle *et al.* [1985] asserted that "...about half of employers, contractors and subcontractors who em-
ploy illegals in the State of Texas do not deduct taxes from the worker’s gross pay.” [p. 16] However, North and Houstoun [1976] found that 73 percent of illegal workers had federal taxes withheld and 77 percent paid social security taxes but did not collect them. In addition, De Maria [1983] admitted that 81 percent of respondents in her sample paid taxes.

Furthermore, immigrants with permanent status show a similar behavior. Indeed, 75 percent of respondents applying for amnesty under the Simpson-Rodino law in 1987 have paid withholding taxes and 80 percent made social security contributions. However, only three percent of this sample received any sort of social benefits from the government (e.g. food stamps, medicare, housing, unemployment insurance). Ironically, almost 90 percent of respondents that filled tax returns to the IRS in the previous year (nearly half of the sample) had positive balances. Accordingly, Blau [1986] admitted that “…the evidence on transfer payments suggests that immigrants do not appear to overburden the transfer system. There is not evidence that they have done so in the past and no indication that there is any reason to be concerned about the future.” [p.107]

In sum, survey data consistently indicates a high incidence of tax withholding by employers of undocumented workers. It is evident that illegal immigrants either transitory or permanent do not represent a burden for society in the United States. Moreover, it may be shown that income flows to the U.S. fiscal system outstrip possible welfare transfers. Accordingly, undocumented workers are net welfare contributors. Following Hill [1985], there is little economic difference between admitting fewer low-skill immigrants and taxing imported goods that require in their production large amounts of low-skill labor. Then, immigration restrictions are similar to trade restrictions.

1.4 An Overview

The core of this study is distributed in three chapters. The first two chapters layout the theoretical elements of this research, while the last chapter reports the estimation analysis and the empirical results. The major conclusions and some policy recommendations are discussed in the last chapter. A summary of this study is presented below.

Chapter 2 proposes a theoretical model of life cycle information in an environment of perfect certainty and continuous time. The objective of this analysis is to describe the underlying elements determining the immigration process in a context of consumer maxi-
mization behavior. Principles of control theory and dynamic optimization are employed to achieve this goal.

This analysis compares favorably with traditional static models which rely exclusively on wage differentials between countries in order to explain international migration. Under these models, transitory participation in the U.S. labor market is inconsistent with large wage differentials (net of switching costs) that continuously favor the host country. In such conditions it is difficult to explain why the entire Mexican population does not immigrate to the United States or why undocumented workers return to their home-communities. The analysis presented here makes clear that theoretical limitations of conventional static models shapes the policy implications of many previous studies on Mexican undocumented workers.

A major contribution of the framework developed in this chapter is found in the identification of elements controlling the duration of Mexican participation in the U.S. labor force. Transitory migration is characterized by strong family, social and economic attachments to home-communities. This representation introduces the concept of household production as an important component of the immigration decision. First proposed by Mincer [1962] and then followed by Becker [1965], the "new" theory of allocation of time has become a major element of modern labor economics. Here undocumented workers participating in the U.S. are productive only in the labor market; however, in Mexico they are productive in both household or/and market activities. This dichotomy between household production across countries is an essential assumption reflecting the inability of the U.S. labor market to reproduce the social, cultural and economic environment of home-communities.

The theory underlying undocumented participation represents a natural extension of Friedman's [1957] permanent income theory and Heckman's [1976] life cycle model of female labor supply. At any point in time, consumers evaluate their potential benefits of being in the two possible states of the world. Mexican residents report pecuniary and non-pecuniary benefits as a result of their market and household activities. Illegal workers, on the contrary, earn higher relative wages than alternative activities in their home labor market (net of switching costs). Although market earnings are higher in the U.S., undocumented workers may have incentives to return to Mexico, even with large wages differentials. In this case, household productivity in Mexico becomes high enough to offset large net wage differentials across regimes. Household productivity induces the dynamic characteristics of
the model. Household marginal payouts decrease during residence in Mexico, but soars when immigration to the U.S. take place. This behavior leads to a cyclical pattern reflected along the entire lifetime of the individual.

Although the previous analysis represents a thorough theoretical examination, the lack of uncertainty and the inability to derive closed form solutions motivates the study of a stochastic discrete-time version of the original model. Indeed, Chapter 3 develops a life cycle analysis of undocumented participation and labor supply in which future events are uncertain. The objective of this analysis is to develop a tractable stochastic model that may be tested in a cross-section empirical estimation.

The study of a lifetime model under uncertainty is important because it identifies the way in which random components enter into the empirical specification. Stochastic behavior cannot be introduced simply by adding an error term, but rather it is the result of an optimal rational behavior. Here, individuals make efficient use of all available information at each period of time.

Although dynamic models are used extensively in recent economic literature, much of the empirical studies dealing with undocumented immigration does not recognize life cycle theory, and practically none of them formally introduce the possibility of uncertain behavior. Breaking with this tradition, the third chapter relies on important contributions in the fields of dynamic programming and stochastic behavior.

In addition, immigration costs constitute a major element in Mexican labor force participation. This chapter addresses the impact of fixed and job-searching costs on the participation decision. An important implication which emerged from this analysis is the existence of a link between domestic (U.S.) labor market disequilibrium conditions and searching costs. A very important consequence of this relationship is that exogenous increases in the domestic wage rate will actually discourage Mexican participation in the U.S. labor market. This hypothesis is tested empirically in the estimation chapter. Lastly, it is concluded that although the presence of stochastic behavior in the immigration process introduces new elements to the former life cycle model, this chapter shows that most of the results obtained in the deterministic model are carried over into the uncertainty framework.

Chapter 4 reports efficient estimates of the immigration decision and the labor supply functions of Mexican undocumented workers. Using the preceding economic theory as
a guide, this chapter addresses the specification and estimation of an empirical model in the context of limited dependent variables. The econometric model developed here provides a natural framework for interpreting life cycle estimates associated with cross-section analysis.

The objective of the fourth chapter is to test empirically the determinants of the Mexican immigration decision and its effects over the U.S. labor market. In particular, the empirical analysis focuses on the effects of U.S. earnings over the undocumented participation rule and their labor supply. Following the economic model outlined in former chapters, it is possible to decompose estimates of wage elasticities into their intertemporal and current multipliers. Lastly, the econometric framework provides estimates for analyzing the impact of other variables such as financial wealth and socio-demographic characteristics.

A major problem confronting the study of undocumented immigration is the reliability of empirical data. Two characteristics identify sample information in this field: Self-selection and missing information in the Mexican regime. First, the illegal population is not a random sample, since variables may only be observed in a limited range, i.e. when Mexican residents actually participate in the U.S. labor market. This yields a truncated sample, where standard estimation procedures are asymptotically biased. Second, observations of the Mexican regime are missing, because it is not feasible to identify potential undocumented workers, i.e. under certain conditions all of the Mexican population could become illegal immigrants. These characteristics provide an empirical estimation challenge not found in many other branches of econometrics.
This chapter presents a theoretical model of life cycle participation decisions for undocumented immigrants in an environment of perfect certainty and continuous time. Structural economic relationships between lifetime household production and market wages are examined. A cost-benefit analysis and switching regimes characteristics are integrated in a coherent continuous intertemporal framework. The identification and characteristics of undocumented immigration is considered in both a transitory and a permanent context. The objective of this model is to layout the underlying theory employed in following chapters.

The analysis focuses on the application of recent economic literature that allows for life cycle behavior in the individual's labor choice. The implementation of this approach to the field of undocumented immigration provides a richer analytical framework than traditional static models. The theory underlying undocumented participation represents a natural extension of Friedman's [1957] permanent income theory and Heckman's [1976] life cycle model of female labor supply. This theory is applied to a situation in which relative benefits of immigration varies over the individual’s lifetime. The intertemporal fluctuations in the participation rule leads to a model of switching regimes where consumers migrate from
Mexico to the U.S. depending on their relative net benefits.

The organization of Chapter 2 is as follows. The first section reviews the literature on the dynamics of the labor force participation. Section 2 develops a general life cycle model of undocumented participation. Here, transitory and permanent characteristics of the immigration decision are analyzed. Moreover, this section introduces the basic components of the Mexican and U.S. characteristics including their integration into a fully coherent model of switching regimes. The third section describes the elements of an immigration cycle and the decision rule; in addition, the theoretical procedure to obtain endogenous switching points is outlined here. Finally, some conclusions are discussed. There is a mathematical appendix supporting the results of section 3.

2.1 Previous Literature on the Dynamics of Labor Force Participation

The theoretical structure of labor-leisure choice models have been traditionally addressed in a static framework. Labor supply decisions are based upon the relation between the consumer’s reservation wage (marginal rate of substitution) and the market wage rate. In these models consumers consider the decisions on market participation and labor supply simultaneously. No further analytical burden is required since total available time may only be allocated to leisure and market activities.

In the last three decades, however, economists have made major contributions towards understanding labor supply decisions. A richer approach to labor economics has its origins in two significant articles: Mincer [1962] and Becker [1965].

Mincer argued that an individual’s labor force participation decisions are made in a family context. Labor choice is not only a function of market wages and fixed costs but also of variables that affect other household members. “An increase in one individual’s income may not result in a decrease in his hours of work but in those of other family members.” [1965, p. 66] In this context, a woman will work during periods when work is financially more profitable relative to other periods in her life cycle (e.g. spouse’s unemployment or fertility periods). Mincer introduces the concept of temporal distribution of

1 See Killingsworth [1983] for complete references on versions of these models.
work where “the timing of market activities during the working life may differ from one individual to another.” [Ibid, p. 68] Life cycle variations in family incomes and assets may affect the timing of labor force participation. According to Heckman [1977], Mincer’s approach implies a “state dependence” behavior in labor choice. In this sense, current work decisions are not independent of an agent’s life cycle.2

Moreover, Mincer stress the importance of household production. Labor supply functions cannot be derived in a residual fashion from hours of leisure, “...but also the demand of hours of work at home must be taken into account.” [Ibid, p. 65] Household work represents a productive activity independent of leisure time which in turn affects labor force participation.

“The total amount of work performed at home is, even more clearly, an outcome of family demand for home goods and leisure, given the production function at home....[T]he distribution of leisure, market work and home work for each family member as well as among family members is determined not only by tastes and biological and cultural specialization of functions, but by relative prices which are specific to individual members of the family.” [Ibid, p. 66]

In the “Theory of the Allocation of Time” Becker [1965] formalizes concepts stated by Mincer three years before. Following major contributions from Lancaster [1971] and Pollak and Wachter [1975] the “new” theory of household production become an important element of modern labor economics. However, it is not until Ghez and Becker [1975] and Heckman [1976] that Becker’s original one period model was extended to an intertemporal framework.

Most of the research featuring these elements can be found in the female labor supply literature, where women are more likely to participate in household production as well as market activities.3 Likewise, labor force participation of undocumented workers features similar characteristics to those of the female labor market. In this context, potential immigrants are productive in Mexico (either at the market or the household) and in the U.S. labor market. In terms of Mincer [1962], the immigration choice is not only determined by tastes and cultural specialization, but also by relative prices across home and host countries.

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2 This line of argumentation has its roots in the definitions of “permanent” and “transitory” components of income, stated by Friedman’s Consumption Theory. See Friedman [1957].

3 Female labor force participation is also affected by fertility, spouse unemployment, child care and so on.
To summarize, this chapter integrates recent developments in dynamic labor economics to the concept of undocumented labor force participation. Household production function, family ties and fixed costs are particular characteristics that govern illegal immigration. These elements may be addressed in a integral labor choice model where life cycle components are taken into account. The next section develops the basic elements of a dynamic model of this sort.

2.2 General Life Cycle Model of Undocumented Immigration

A major element driving immigration decisions in traditional static models rely solely in wage rates differentials. According to this approach, Mexican residents will participate in the U.S. labor force whenever earnings in the latter regime outstrip the former. Although positive wage differentials favoring the U.S. are continuously observed, transitory migration, i.e. switching back and forth across regimes, represents a typical behavior of undocumented workers. Consequently, continuous wage differentials and temporary immigration may not coexist in a traditional static framework.

Heckman and MacCurdy [1980] among others have emphasized that the relationship between current labor participation and current wage rates are difficult to interpret in a static model. In addition, these models cannot address issues that involve the response of market participation and labor supply to fluctuations over time in wage rates and property income. These issues involve intertemporal effects linking life cycle responses and current behavior. In order to address permanent as well as transitory undocumented migration, the participation decision may be modeled in a dynamic framework. A life cycle model of illegal labor force participation allows to explain the coexistence of permanent and transitory immigration flows in an coherent optimal control setting.

2.2.1 The Basic Framework

The representative individual faces two states of the world or regimes: Mexico (m) and the U.S. (u). Residents in the Mexican state may or may not choose to immigrate illegally to the receiving country, either temporarily or permanently. Mexican residents allocate their

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4 The word “regime” defines a feasible state of the world, and it is used in the same context as the world “country”.
time in leisure activities \([L_m(t)]\), household production \([I(t)]\) and market work \([H_m(t)]\). Following Mincer [1962] and Becker’s “new” theory of allocation of time, home residents are productive in their household as long as they invest some of their time to non-market activities \([L_m(t)+I(t) > 0]\). It is assumed that individuals receive their marginal productivity as payouts in each productive activity \([I(t)\) and \([H_m(t)]\). Therefore, residents get the home-market wage rate \((W_m)\) for working in the labor market and the marginal rate of household productivity \([S(t)]\) for time invested in non-market activities.

The latter concept, \(S(t)\), contains important dynamic information that makes immigrants switch back and forth across regimes. Household productivity is treated similarly to human capital accumulation in Heckman [1976], Heckman and Macurdy [1980] and others. In these articles, human capital introduces life-cycle characteristics to female labor participation. Likewise, in the case of undocumented migration household productivity features these dynamic elements.

The effects of the marginal rate of household productivity \([S(t)]\) on individuals is twofold: Throughout the utility function, by augmenting their leisure time, and throughout the budget set by affecting their household wealth. Firstly, similar to human capital accumulation, non-market productivity is embodied in the individual. Ben-Porath [1967] indicates that “human capital operates like Harrold neutral endogenous technical progress in augmenting time.” [Heckman, 1976: p. 512] In this context, consumer’s utility function is positively affected by augmenting home leisure time \((L_m)\). A simple description of household preferences which captures the notion that household production has non-market benefits is the instantaneous utility function at age \(t\) given by,

\[
U = U[C(t), S(t) L_m(t), L_u(t)] \quad \text{with} \quad U(0,0,0) = 0, \\
U_C, U_{L_m}, U_{L_u} > 0, \\
U_{CC}, U_{L_mL_m}, U_{L_uL_u} < 0, \\
U_{CL_i} = 0 \text{ for } i=m, u; \\
\text{and} \quad \text{if } L_m > (\leq) 0 \text{ then } L_u = (\leq) 0.
\]

\(C(t)\) is consumption of a Hick’s composite commodity in period \(t\), while \(L_m(t)\) and \(L_u(t)\) are instantaneous leisure time in the Mexican and the U.S. regimes, respectively. The consumer’s utility function is assumed to be strongly concave in its arguments with positive first partials derivatives. Note that if Mexican leisure is zero \([L_m(t)=0]\), household productivity plays no role in the utility function. The magnification effect that non-market produc-
tivity imposes over $L_m(t)$ is described by family and social attachments that characterize the domestic community. These cannot be transferred across regimes, making $L_m(t)$ and $L_u(t)$ far substitutes. The notion that tastes and cultural elements affect consumer preferences has been formerly addressed by Mincer [1962].

Secondly, household productivity also interacts with the consumer's budget set. Home residents are productive in household activities, thus investment in non-market time will generate income at each point in time. A perfect credit market is assumed with a riskless interest rate $r$ and a requirement that the terminal value of financial assets be non-negative. Commodity prices are normalized to unity, and the total financial wealth at period $t$ is $A(t)$. In the absence of taxes and revaluation of capital assets, total savings at age $t$ (i.e. total change in financial net worth) may be written as

\[ \dot{A} = rA(t) + [W_mH_m(t) + W_uH_u(t)] + S(t)I(t) - C(t) \]

where $A(0) = A_0$ and $A(T) \geq 0$,

and if $[H_m(t) + I(t)] > (=) 0$ then $H_u(t) = (> 0$.

The wage rate and the amount of time allocated to market activities in each regime are given by $W_i$ and $H_i$ (i.e. $i=m,u$) where in general, $W_u = W_m$. $I(t)$ is time invested in non-market activities that take place only in Mexico which is valued at price $S(t)$. The first three components of the RHS in (2.2) represents the individual’s sources of income: Revenues (or debt) due to interest payments [$rA(t)$], market earnings [$W_mH_m(t)$ or $W_uH_u(t)$] and non-market income [$S(t)I(t)$]. Similar to the utility function, if time invested in non-market activities is zero [$I(t)=0$], household productivity plays no role in the individual's budget constraint.

Another constraint affecting consumer behavior is the dynamic description of the marginal rate of household productivity, $S(t)$. Household activities in the Mexican regime are characterized by diminishing marginal returns in $S(t)$. Thus, increasing the time allocated to non-market activities, either home production [$I(t)$] or leisure time [$L_m(t)$], affects negatively consumers preferences and their net worth. In contrast, illegal workers in the U.S. observe increasing household productivity since home production is not feasible in this regime. A simple description consistent with this framework is the motion equation for

\[ 5 \text{ Capital letters with an overhead dot represents time derivatives, i.e. } \frac{\partial X}{\partial t} = \dot{X}. \]
the rate of growth of household productivity in the home-country, \( \dot{S} \):

\[
(2.3) \quad \dot{S} = \sigma S(t) - \delta [ I(t) + L_m(t) ] \quad \text{with} \quad S(0) = S_0 \quad \text{and} \quad S(T) \geq 0.
\]

Household productivity depreciates exponentially at rate \( \delta \) when individuals invest in non-market time \([I(t) + L_m(t) > 0]\). Accordingly, the magnification effects observed in the utility function and the budget constraint due to \( S(t) \), decrease over time, i.e. \( \dot{S} < 0 \). Moreover, notice that in the absence of household time \([I(t) + L_m(t) = 0]\), marginal productivity appreciates at rate \( \sigma \). This scenario is feasible in the case where individuals immigrate to United States. In such a case, the marginal productivity of non-market time rises monotonically over time.

Non-market productivity in the domestic community is the distinctive element that induces life cycle behavior to the participation model. Home consumers are productive at the market place and/or at the household. In contrast, undocumented participation in the U.S. does not allow for productive household activities since transitory immigrants are able to work only in the labor market. The dichotomy between household production across regimes is an essential assumption. This is reflected, on one hand, in the lack of family, social and cultural reproduction of the domestic environment and, on the other hand, in the economic attachments of the immigrant with the community of origin such as ownership and other non-tradable goods, e.g. real estate and cattle.

When consumers participate in the U.S. labor market as transitory workers, they leave behind family, social and economic attachments that anchor immigrants to their home-country. In this framework, transitory migration is distinguished by family, social and economic ties with Mexico, which is reflected through the consumer's household productivity. In contrast, permanent migration is characterized by mobility of family and economic environments, at least partially, to the host labor market. Under these circumstances, household production plays no role in the benefit function of illegal workers.

At any point in time consumers evaluate their potential benefits of being in the two possible states of the world, either permanently or temporarily. Mexican residences report pecuniary and non-pecuniary benefits as a result of their market and household activities. Illegal workers, on the contrary, earn higher relative wages than alternative activities in their home labor market (net of switching costs). Although market earnings are higher in
the U.S., undocumented workers may have incentives to return to Mexico, even in the presence of large wages differentials. This is the case where household productivity in Mexico becomes high enough to offset large wage differentials (net of fixed cost) across regimes. Such an argument has important implications since it denies the role of net wage differentials as the unique factor driving illegal immigration.

To illustrate the elements of this model, consider a representative peasant living in a rural community of Mexico. Given the socio-demographic characteristics of this environment, he shows strong attachments to his community. Furthermore, the peasant belongs to a cohesive family network who owns a small farm. At age $t$, he immigrates illegally to the U.S. so no further investment is made in household activities, i.e. $I(t) + L_m(t) = 0$. The migration decision causes a monotonic increase in the marginal productivity of non-market time in his home community at an exponential rate $\sigma$ so that $\dot{S} > 0$.

This behavior is observed because undocumented workers in the U.S. cannot reproduce the family and social environment of their rural communities. In terms of Sjaastad [1962], for every additional period in the host-country, psychic costs are weighed more heavily in the decision rule. Moreover, the family farm faces a decaying process since fewer family members are available to support it which in turn deteriorates the household wealth. At one point in time, the illegal worker confronts enough psychic and economic incentives in terms of household production that it is worth returning to Mexico. In this setting, immigrants will return to their home community when the marginal productivity of household production completely offsets wage differentials and switching costs.

When the peasant is back in Mexico, time is invested in household activities, i.e. $[I(t) + L_m(t)] > 0$. This depreciates the non-market productivity at an exponential rate $\delta(\dot{S} < 0)$. The relative importance of $S(t)$ inside the immigration decision deteriorates over time. Household productivity decreases monotonically, and other economic factors, such wage differentials, will weight heavier in the participation decision.

This cycling pattern of migration is described in figure 2.1. For a given level of wage rate differentials and switching costs, home residents observe deceeding household

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6 One important element of immigrants switching back to their home-communities is given by the magnitudes of $\sigma$ and $\delta$ which measure the degree of social and cultural attachment as well as the family distribution of household duties, e.g. parents will have higher values of $\sigma$ and $\delta$ than children.
productivity, i.e. portion $q_1$; at point $s$ non-market activities report low productivity so that individuals are better off switching to the U.S. labor market. The immigration decision induces the appreciation of household production due to family ties and economic attachments. $S(t)$ rises up to period $t_2$, where undocumented workers switch back to their home communities, initiating a new cycle. Notice that the amplitude of the each cycle depends of the rate of depreciation $\delta$ and the rate of appreciation $\sigma$.

It should be noted that figure 2.1 is related to an $(s, S)$ inventory policy. The $(s, S)$ policy is one of reordering at times when the inventory level falls below a certain critical level $s$, where $S$ is the maximum level of inventory. Intriligator and Sheshinski [1986] employ this inventory policy as an example of "event planning". Here the planning period may be influenced by the state of the system, where particular events trigger a revision plan.

![Figure 2.1 — Event Planning in Household Productivity](image)

Permanent residence in either country is a special case of equations (2.1)-(2.3). Individuals living in Mexico never immigrate if household productivity depreciates sufficiently slow (i.e. small values of $\delta$) and/or their stock of wealth $[A(0)]$ is sufficiently large, so as to offset positive wage differentials in the receiving country. In contrast, residents will immigrate permanently if they can break family and economic anchors to their home communities. Two consequences result from such a decision. Residents will migrate with the entire family, and they will sell all non-transferable property (e.g. farm, cattle, etc.). In terms of equations (2.1)-(2.3), permanent migration will make $L_m(t)$ and $L_u(t)$ closed sub-
stitutes at any point in time; moreover, non-market income will be integrated into the U.S. budget set.

Finally, an important feature of this framework represents the existence of state *dependency* across regimes. Former applications in the field of switching regimes do not recognize any feedback mechanisms between states. Here, the consumer’s decision to participate in one regime affects the opportunity costs of an alternative decision. This process is transmitted to the participation choice through the dynamic characteristics of non-market productivity in the home-country, $S(t)$. Residents in Mexico observe a progressive deterioration of their household wealth, since $S(t)$ depreciates at rate $\delta$ which constitutes an incentive for undocumented participation in the U.S. labor market. Conversely, household productivity increases monotonically at rate $\sigma$ as result of the immigration decision, which endorses switching back to the home-community. Therefore, permanence in one regime encourages mobility to the alternative state.

To summarize, the major contribution of this approach is that migration flows are not solely driven by wage rates and crossing fees as established in traditional static models, but also by non-market productivity. Life-cycle considerations introduce the possibility of transitory migration, a widely observed feature, and yet not modeled in undocumented immigration literature. Before analyzing the immigration behavior across regimes, the following two sections introduce an independent description of the dynamic elements of both the sending and receiving countries.

### 2.2.2 The Mexican Regime

Consider the life cycle profile of a typical permanent resident in Mexico. Individuals are assumed to operate in an environment of perfect certainty. Life is finite with horizon $T$. Utility at age $t$ is strongly concave and separable over time, where $C(t)$ and $L_{m}(t)$ are instantaneous consumption goods and leisure, respectively. Individuals face a wage market rate ($W_{m}$) assumed to be fixed over time and independent of their choices. The individual

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7 See Intriligator and Sheshinski [1987] for a particular example addressing the technological changes between oil-fueled and nuclear-fueled electricity generation plants.
8 Recall from conditions in (2.1) that consumption and leisure are gross substitutes, i.e. $U_{CL}=0$. This assumption is not required in the theoretical specification; however, it significantly simplifies the exposition.
9 This assumption introduces the intentions of this model to focus on the determinants of immigration factors rather than leisure-work decisions in each regime. Life cycle considerations of market wages are present in both countries, thus they cancel each other. In contrast, non-market productivity is observed only
starts life with an initial wealth \( A_0 \) and dies leaving no bequest \([A(T)=0]\). Credit markets are perfect at rate \( r \), and the rate of time preferences is given by \( \rho \). Permanent residents are assumed to maximize the time-preference-discounted stock of total utility over horizon \( T \). These elements are presented in the following optimal control problem,

\[
V_m = \max_{\{C,I,L_m\}} \int_0^T e^{-pt} U[C(t), S(t)L_m(t)] \, dt
\]

Subject to:

\[
\dot{A} = rA(t) + W_mH_m(t) + S(t)I(t) - C(t)
\]

with \( A(0)=A_0 \) and \( A(T)=0 \),

\[
\dot{S} = \sigma S(t) - \delta [L_m(t) + I(t)]
\]

with \( S(0)=S_0 \) and \( S(T)\geq 0 \),

\[
1 = H_m(t) + L_m(t) + I(t).
\]

The necessary conditions for an optimum are satisfied by\(^{10}\)

(i) State Equations: \((2.5)\)

\[
\dot{A} - rA = W_mH_m + SI - C
\]

\((2.6)\)

\[
\dot{S} - \sigma S = -(I + L_m);
\]

(ii) Costate Equations: \((2.7)\)

\[
\dot{\lambda} + r\lambda = 0
\]

\((2.8)\)

\[
\dot{\mu} + \sigma \mu = (\lambda I + U_2 L_m);
\]

(iii) Optimality Conditions:

\((2.9)\)

\[
U_1 = \lambda
\]

\((2.10)\)

\[
SU_2 = \gamma + \mu
\]

\((2.11)\)

\[
\lambda S = \gamma + \mu \quad \text{iff} \quad I > 0
\]

\[
H_m(\lambda W_m - \gamma) \geq 0
\]

\((2.12)\)

\[
\lambda W_m = \gamma \quad \text{iff} \quad H_m > 0.
\]

And the Hamiltonian function \( \mathcal{H}(t,C,L_m,I,A*,S*,\lambda,\gamma,\mu) \) is maximized by \( U=U^* \).\(^{11}\)

\(^{10}\) It is assumed \( p=0 \) and \( \delta=1 \). This simplifies the algebraic manipulation without altering any major conclusions.

\(^{11}\) Arrow and Kurz [1970] describe the conditions upon that necessary conditions \((2.5)-(2.12)\) also constitute sufficient conditions for an optimum (see Proposition 6, p.45).
From the Kuhn–Tucker Theorem, conditions (2.11) and (2.12) represent a full range of possible cases or corner solutions. Both expressions have interior solutions if consumers participate in labor market and household production, simultaneously; otherwise, individuals will specialize in one activity.

Without further assumptions, the dynamic characteristics of \( S(t) \) and its interaction with the market wage rate do not allow for a simultaneous coexistence between market time \([H_m(t)]\) and household production \([I(t)]\). Conditions (2.11) and (2.12) imply that consumers will specialize in one of the above activities. Suppose that an individual commences his productive life working at the household, i.e. \( H_m(t)=0 \) and \( I(t)>0 \). Successive investment in non-market time depreciates the consumer's productivity in this activity due to equation (2.3). Given compactness in the preference and the budget set, there exists a point where the consumer chooses to switch from household production to market activities, i.e. \( H_m(t) > 0 \) and \( I(t) = 0 \). Home productivity appreciates at rate \( \sigma \) up to the point where market activities are no longer profitable. This will make the consumer switch back to household production (see Figure 2.2). The process continues indefinitely since the agent will try to arbitrate any possible gains from switching activities.

![Diagram](image)

**Figure 2.2—Chattering Control in the Mexican Regime**

In the absence of fixed costs, specialization across productive activities (market and

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12 See Killingsworth [1983] for a complete description of leisure–work decision models. Recall that model (2.4) does not addresses directly this issue, but rather focuses on the immigration decision.
non-market time) results in infinitesimal zigzags along the t axis of the [t, Productive Time] plane or chattering control.\(^\text{13}\) Chattering is observed because it is optimal for the consumer to switch regimes for every infinitesimal difference between the market wage \((W_m)\) and the marginal household productivity \([S(t)]\). In the limit, an optimal path will imply an infinite number of switchings.

To avoid chattering control, economists introduce transaction costs or other type of fixed costs that restrict the number of switchings across regimes. These are feasible solutions when switching regimes take place between nations. However, in this section only the Mexican regime is analyzed, and fixed costs are not an important issue. Instead, let the home regime be characterized by two states: (i) A steady state equilibrium, where market and household activities coexist at the same time \([H_m(t), I(t) > 0]\), assuming a simultaneous interior solution for conditions (2.11) and (2.12); (ii) a non-stationary state where only non-market time is assigned by the consumer, letting condition (2.11) have an interior solution \([H_m(t)=0 \text{ and } I(t) > 0]\).

The previous approach guarantees that as long as the consumer remains in Mexico, household production is positive. Labor market time, in contrast, may or may not coexist with non-market activities which depending on the characteristics of the stationary state in each period. It is worth noting that the underlying force driving the simultaneous coexistence of market and household activities is found in the relationship between the wage rate \((W_m)\) and the marginal rate of non-market productivity \([S(t)]\). A detailed characterization of this relationship is reviewed next.

**2.2.2a Steady State Equilibrium**

Here optimality conditions (2.11) and (2.12) are simultaneously satisfied. Therefore, it must hold

\[
(2.13) \quad S(t) = W_m + \frac{\mu(t)}{\lambda(t)}.
\]

Differentiating (2.13) with respect to time and using costate equation (2.7) yields the dy-

---

\(^\text{13}\) See Young [1969] for a detailed description of chattering problems in an optimal control framework. The author gives a very intuitive explanation of "chattering" by comparing it with the problem faced by a sailing boat and the optimal number of tacks (pp. 155-57).
namic behavior of household productivity as a function of “shadow prices” $\lambda(t)$ and $\mu(t)$,

$$\dot{S} = \frac{\dot{\mu} + \mu(t)r}{\lambda(t)}.$$  

(2.14)

Some algebraic manipulation of the functions (2.8), (2.10) and (2.11) result in an expression for the motion equation of the costate variable for non-market productivity

$$\dot{\mu} + \sigma \mu(t) = -\lambda(t)[I(t) + L(t)];$$

according to state condition (2.6), the above equation may be written as

$$\dot{\mu} + \sigma \mu(t) = \lambda(t)[\dot{S} - \sigma S(t)].$$

In order to obtain an equilibrium condition for the costate variable associated with household productivity ($\mu^*$), the values of $S(t)$ and $\dot{S}$ in equations (2.13) and (2.14) may be plugged into the previous condition. Solving for $\mu$, the equilibrium marginal utility of non-market production is given by

$$\mu^* = -\frac{\sigma W_m \lambda(t)}{2 \sigma - r}.$$  

(2.15)

Substituting (2.15) in equation (2.13) and solving for $S(t)$ leads to the equilibrium relation between non-market productivity and wage rates given by

$$S(t)^* = \frac{W_m}{R}, \quad \text{where} \quad R = \frac{2 \sigma - r}{\sigma - r} > 1 \text{ for } \sigma > r.$$  

(2.16)

Inspection of the steady state equilibrium condition (2.16) reveals that the optimal rate of household productivity is a monotonic increasing function of the market wage rate. This condition must hold whenever market and non-market time are simultaneously positive at each point of time. Moreover, notice that $S(t)^*$ is time invariant; thus, no external forces within the Mexican regime may deviate consumers from a steady-state equilibrium.

Solving the first order homogeneous equation in (2.7) and assuming no bequest
function \( A(T)=0 \), then the costate variable associated with the budget constraint has an equilibrium solution,

\[
(2.17) \quad \lambda(t)^* = \lambda_0 e^{-rt} \quad \text{with } \lambda(0) = \lambda_0 .
\]

Under special circumstances, \( \lambda_0 \) may be interpreted as the marginal utility of wealth.\(^{14}\) In the absence of unforeseen circumstances, \( \lambda_0 \) stays fixed over the consumer's lifetime. Furthermore, notice that its value is a function of all the parameters of the model including the vector of lifetime wage rates and the household initial stock of wealth. In fact, \( \lambda_0 \) summarizes the individual's life cycle information.

Given the assumptions of strong concavity and twice differentiability of preferences, a straightforward application of the Implicit Function Theorem over conditions (2.9) and (2.10) yield the "\( \lambda_0 \)-demand" functions for consumption and leisure time that satisfy optimal paths in (2.16) and (2.17):\(^{15}\)

\[
(2.18) \quad C(t)^* = C[ \lambda_0 e^{-rt} ],
\]

\[
(2.19) \quad L_m(t)^* = L_m[ \lambda_0 e^{-rt} ] \frac{R}{W_m} .
\]

As a consequence of strict concavity in the utility function, these demand functions satisfy \( C' < 0 \) and \( L_m' < 0 \). The equilibrium household supply can be obtained by plugging the leisure function into the state condition (2.6),

\[
I(t)^* = \frac{\sigma W_m}{R} - L_m[ \lambda_0 e^{-rt} ] \frac{R}{W_m} .
\]

Note that \( \dot{S}^* = 0 \), since \( S(t)^* \) represents a steady state path. The equilibrium labor supply at time \( t \) is

\[
H_m(t)^* = 1 - \frac{\sigma W_m}{R} .
\]

\(^{14}\) See Heckman and MaCurdy [1980] for a discussion of these conditions (p. 50).

\(^{15}\) Heckman [1976] and MaCurdy [1981] refer to these equations as "Constant-\( \lambda_0 \)" Demand Functions. This terminology may be misleading, and it is avoided throughout the exposition.
Lastly, given that condition (2.16) holds for every \( t \), the consumer's total labor supply, market and non-market, is obtained by adding the previous two equations

\[
[\text{I}(t)^* + \text{H}_m(t)^*] = 1 - L_m[\lambda_0 e^{-\pi t}] \frac{R}{W_m}.
\]

The \( \lambda_0 \)-demand functions (2.19)–(2.21) decompose market and household decisions into two different elements: (i) An indirect life cycle component, \( \lambda_0 \), that contains all past and future information relevant to the consumer's current choices, and (ii) a direct element, \( W_m \), that represents the variables actually observed in the decision period.

The "\( \lambda_0 \)-demand" for leisure shows negative direct and indirect effects. Increases in the marginal utility of wealth or in the market wage rate generate reductions in current leisure time

\[
\frac{\partial L_m(t)^*}{\partial \lambda_0} < 0 \quad \text{and} \quad \frac{\partial L_m(t)^*}{\partial W_m} < 0.
\]

In contrast, the equilibrium supply for market and non-market time has positive direct and indirect effects. Larger marginal utilities of wealth, as well as increases in wage rates, expand the time dedicated to productive activities

\[
\frac{\partial [\text{I}(t)^* + \text{H}_m(t)^*]}{\partial \lambda_0} > 0 \quad \text{and} \quad \frac{\partial [\text{I}(t)^* + \text{H}_m(t)^*]}{\partial W_m} > 0.
\]

Following MaCurdy [1981], the "\( \lambda_0 \)-demand" functions represent an extension of the permanent income hypothesis in Friedman [1957] to a situation in which consumption and productive time varies over the individual's life cycle. According to these functions, current consumption and labor supply decisions depend on a permanent component and a current wage rate. The value \( \lambda_0 \) acts as permanent income in the theory of the consumption function. Equations (2.18) and (2.20) fully characterize the consumer's dynamic behavior in a world of perfect certainty.

The expected lifetime wage rate (\( W_m \)) and the initial household wealth (\( A_0 \)) are two factors underlying the life cycle component (\( \lambda_0 \)). In order to access the indirect life cycle considerations of \( W_m \) and \( A_0 \) on the "\( \lambda_0 \)-demand" functions, a comparative dynamic exercise is presented. This can be addressed by solving the state condition (2.5) associated with
the budget constraint. Given the equilibrium values in system (2.18)-(2.20) and under boundary conditions \(A(0)=A_0\) and \(A(T)=0\), the first order differential equation (2.5) has a solution

\[
A_0 = \frac{W_m}{r} \int \left[ 1 - \frac{\sigma^2 W_m}{R(2\sigma - r)} \right] e^{\gamma T} \left( e^{-\gamma r} - 1 \right) + \int_0^T e^{-\gamma t} \left[ L(\lambda_0 e^{-\theta}) + C(\lambda_0 e^{-\theta}) \right] dt .
\]

**Proposition 2.1:** A concave and twice differential utility function implies

\[
\frac{\partial \lambda_0}{\partial A_0} < 0 \quad \text{and} \quad \frac{\partial \lambda_0}{\partial W_m} < 0 .
\]

**Proof:**

(i) Differentiate equation (2.21) with respect to \(A_0\),

\[
1 = \frac{\partial \lambda_0}{\partial A_0} \int_0^T e^{-\gamma t} (L' + C') dt .
\]

Concavity of preferences implies \((L' + C') < 0\). Hence, \(\partial \lambda_0/\partial A_0\) must be negative in order to hold the equality.

(ii) Likewise, differentiation of (2.21) with respect to \(W_m\) yields

\[
0 = \frac{1}{r} \left[ 1 - \frac{2\sigma^2 W_m}{R(2\sigma - r)} \right] e^{\gamma T} \left( e^{-\gamma r} - 1 \right) + \frac{\partial^2 \lambda_0}{\partial W_m^2} \int_0^T e^{-\gamma t} (L' + C') dt .
\]

Given a concave utility function, necessary and sufficient conditions for \(\partial \lambda_0/\partial W_m < 0\) is that \(R(2\sigma - r) > 2\sigma^2 W_m\).

\[\text{[Q.E.D.]}\]

Under proposition 2.1, it is easy to verify that the "\(A_0\)-demand" for consumption is an increasing function of the household stock of initial wealth \((A_0)\) and the market wage rate \((W_m)\),

\[
\frac{\partial C(t)}{\partial A_0} = C e^{-\theta} \frac{\partial \lambda_0}{\partial A_0} > 0 \quad \text{and} \quad \frac{\partial C(t)}{\partial W_m} = C e^{-\theta} \frac{\partial \lambda_0}{\partial W_m} > 0 .
\]
Furthermore, the demand for leisure time is a positive function of the initial wealth. However, it declines monotonically with market wages, if the direct (substitution) effect is greater than the indirect (income) effect:

\[
\frac{\partial L_m(t)}{\partial A_0} = \frac{R}{W_m} L \frac{\partial \lambda_0}{\partial A_0} > 0 \quad \text{and} \quad \frac{\partial L_m(t)}{\partial W_m} = \frac{R}{W_m} L \frac{\partial \lambda_0}{\partial W_m} - \frac{R}{W_m^2} L_m(t) < 0.
\]

Inspection of the above expressions indicates that consumption and labor supply decisions in period \( t \) are related to the initial household wealth solely through changes in the marginal utility of wealth, \( \lambda_0 \). The market wage rate, in contrast, influences the leisure demand in two ways: A direct effect that accounts for current changes in the wage rate and an indirect effect which induces changes in the marginal utility of income. These two elements have opposite signs, and the net effect on the demand for leisure is ambiguous.

2.2.2b Non-Stationary State

The non-stationary equilibrium is described by the consumer's specialization in household production where only condition (2.11) yields an interior solution. Individuals allocate the entire available time to household activities, either leisure or productive work \([I(t) + L_m(t) = 1]\), while labor market time is assumed to be zero \([H_m(t) = 0]\).

Accordingly, under the initial condition \( S(0) = S_0 \), the expression associated with non-market productivity (2.6) has a straightforward solution:

\[
(2.22) \quad S(t)^* = S_0 e^{\sigma t} + \frac{1}{\sigma} (1 - e^{\sigma t}).
\]

Inspection of the equilibrium condition (2.22) reveals that non-market productivity is a decreasing function of time (\( \dot{S}^* < 0 \)), if \( \sigma S_0 < 1 \); this result contrasts with the stationary case where \( \dot{S}^* = 0 \). Furthermore, under the non-stationary state, household productivity and wage rate maintain no relation. The dynamics of this variable is described in figure 2.3. Notice that \( \dot{S}^* < 0 \) is required for stability purposes, since non-stationary processes converge to steady-state equilibriums.
Similar to the stationary case, concavity and twice differential preference imply that "$\lambda_0$-demand" functions can be obtained by means of the Implicit Function Theorem. Thus, optimality conditions (2.9) and (2.10) may be written as

\begin{align*}
(2.23) \quad C(t)^* &= C[ \lambda_0 e^{-rt} ] , \\
(2.24) \quad L_m(t)^* &= L_m[ \lambda_0 e^{-rt} ] \frac{1}{S(t)} ;
\end{align*}

where $S(t)^*$ is given by (2.22). Notice that $C' < 0$ and $L' < 0$ continues to apply in this case.

A comparative dynamics exercise can be obtained by substituting the equilibrium demand functions (2.23) and (2.24) into the state condition associated with the budget constraint (2.5),

$$
\dot{A} - \tau A(t) = S(t)^* - [L_m(t)^* + C(t)^*].
$$

Under boundary conditions $A(0) = A_0$ and $A(T) = 0$, this difference equation has the following solution:
\begin{equation}
A_0 = \frac{1}{\sigma r} \left[ e^{-\tau r} - 1 \right] - \frac{\sigma S_0 - 1}{\sigma (\sigma - r)} \left[ e^{(\sigma - r) T} - 1 \right] + \int_0^T \left[ C(\lambda_0 e^{-\tau t}) + L(\lambda_0 e^{-\tau t}) \right] e^{-\tau t} \, dt.
\end{equation}

It is easy to show from (2.25), that the marginal utility of wealth, $\lambda_0$, is a decreasing function of the initial household wealth ($A_0$) and the initial level of non-market productivity ($S_0$),

\[ \frac{\partial \lambda_0}{\partial A_0} < 0 \quad \text{and} \quad \frac{\partial \lambda_0}{\partial S_0} < 0. \]

A non-stationary state requires no further restrictions in order to obtain well behaved results. In this context, Proposition 1.2 also applies to this case. A formal proof follows the same procedure outlined in section 2.2.2a.

2.2.3 The U.S. Regime

The life cycle model that describes the host labor market is a special case of the framework presented in the previous section. Here, home production is not feasible [I(t)=0] and $L_m(t)$ is not substituted for leisure time in this regime, $L_u(t)$. Furthermore, the market wage rate is, in general, larger than the corresponding wage rate in the home labor market. Consumption, however, is homogeneous across regimes as well as financial assets and interest rates. It is assumed that the typical consumer maximizes the discounted value of his lifetime utility flows,\(^{16}\)

\begin{equation}
V_u = \max_{\{C, L\}} \int_0^T U[C(t), L_u(t)] e^{pt} \, dt.
\end{equation}

Subject to:

\[
\begin{align*}
\dot{A} &= \tau A(t) + Wu(t) - C(t); \text{ with } A(0)=A_0 \text{ and } A(T)=0, \\
\dot{S} &= sS(t); \text{ with } S(0)=S_0 \text{ and } S(T)\geq0, \\
1 &= Hu(t) + Lu(t).
\end{align*}
\]

\(^{16}\) For exposition purposes, it has been assumed that the immigration process takes place in period 0, and it will last until time T. The determination of the immigration limits is an important feature of the model, and it will be addressed at the end of this chapter.
Notice that household productivity, $S(t)$, is independent of the optimization problem faced by the undocumented resident. Here, home production payoffs appreciate monotonically at rate $\sigma$ over time.

Without loss of generality let $\rho=0$. The system (2.26) is maximized by the following conditions:

(i) State Equations: \begin{align*}
\dot{A} - rA(t) &= W_uH_u(t) - C(t) \\
\dot{S} - \sigma S(t) &= 0;
\end{align*}

(ii) Costate Equations: \begin{align*}
\dot{\lambda} + r\lambda &= 0 \\
\dot{\mu} + \sigma \mu &= 0;
\end{align*}

(iii) Optimality Conditions:

\begin{align*}
U_1 &= \lambda \\
U_2 &= \gamma \\
H_u(\lambda W_u - \gamma) &\geq 0 \\
\lambda W_u &= \gamma \quad \text{if} \quad H_u > 0;
\end{align*}

and the Hamiltonian function $\mathcal{H}(t,C,L_u,\lambda^*,S^*,\lambda,\gamma,\mu)$ is maximized by $U=U^*$.

Solving the costate condition using a similar procedure as in (2.17) and assuming an interior solution for (2.33), the optimality conditions (2.31) and (2.32) may be written as

\begin{align*}
U_1 &= \lambda_0 e^{-rt} \quad \text{and} \quad U_2 = W_u(t)\lambda_0 e^{-rt}.
\end{align*}

Under concave preferences, twice differential utility function and gross substitutes, the "$\lambda_0$-demand" equations can be obtained by inverting the previous functions,

\begin{align*}
C(t)^* &= C[\lambda_0 e^{-rt}] \quad \text{with} \ C < 0; \\
L_u(t)^* &= L_u[W_u\lambda_0 e^{-rt}] \quad \text{with} \ L' < 0.
\end{align*}

The comparative dynamics of this model can be evaluated by (i) substituting the previous equilibrium values into the budget constraint (2.27) and (ii) solving for the entire life cycle period. Consequently, under the boundary conditions $A(0)=A_0$ and $A(T)=0$ and equilibrium values (2.34) and (2.35), the first order differential equation (2.27) has a solu-
tion given by

\[ A_0 = \frac{W_u}{r}(e^{-\tau T} - 1) + \int_0^T [W_u L_u(t)^* + C(t)^*] e^{-\tau t} dt. \]

Assuming a well behaved utility function and by means of Proposition 2.1, it is easy to show that the life cycle component of the U.S., \( \lambda_0 \), preserves the same dynamic characteristics outlined in the previous section where \( \partial \lambda_0 / \partial A_0 < 0 \) and \( \partial \lambda_0 / \partial W_u < 0 \). It is hardly surprising that concavity implies a diminishing marginal utility of wealth. From the assumption that consumption goods and leisure time are normal goods, increments in the initial stock of household wealth increase the level of consumption expenditure as well as leisure at all ages:

\[ \frac{\partial C(t)^*}{\partial A_0} = e^{-\tau} \frac{\partial \lambda_0}{\partial A_0} > 0 \quad \text{and} \quad \frac{\partial L_u(t)^*}{\partial A_0} = e^{-\tau} W_u L_u \frac{\partial \lambda_0}{\partial A_0} > 0. \]

Note that initial wealth induces its current effects only through the marginal utility of income (\( \lambda_0 \)) for every period \( t \). The consumption and leisure decisions, in this case, are related to the immigrant's stock of wealth solely by lifetime considerations.

Similar to the Mexican case, the functional relation between \( W_u \) and the demand for leisure is ambiguous. The main element that characterizes this functional form is the relationship between lifetime considerations and current events,

\[ \frac{\partial C(t)^*}{\partial W_u} = e^{-\tau} C \frac{\partial \lambda_0}{\partial W_u} > 0 \quad \text{and} \quad \frac{\partial L_u(t)^*}{\partial W_u} = e^{-\tau} \left[ W_u L_u \frac{\partial \lambda_0}{\partial W_u} + L_2 \lambda_0 \right] > 0. \]

If the current (direct) effect of \( W_u \) dominates its life cycle (indirect) element, then leisure time observes a negative relationship with the wage rate at age \( t \). Recall that in traditional static models where only current events affect the demand for leisure, this relation is unambiguously negative. The introduction of life cycle considerations, therefore, does not only reduce the actual leisure demand as a result of expansions in the wage rate, but also may yield a positive function. Accordingly, the possibility of highly inelastic labor supplies or even negative functions are more likely to occur in a dynamic framework relative to conventional static models.
2.2.4 Integrated Model of Switching Regimes

In previous sections the analysis of the Mexican and the U.S. regimes has been addressed as separate states without any interaction. This section, however, integrates the results presented above to the case where individuals may choose alternative states of the world at any point in time. In this framework, there exists a choice of alternative migration status at each instant, but fixed costs are imposed in switching from one state to another. Immigrants have the option of (i) permanent residence in Mexico, (ii) temporary immigration switching back and forth across countries or (iii) permanent migration to the United States.

The model is an extension of a cost-benefit analysis. Specifically, it focus on monetary and psychic costs, as well as the respective benefits of one regime versus another, treating the possibility of switching processes at each instant. The framework allows for internal optimization along the path, where each regime is characterized by a different set of preferences and equations of motion.

Following the methodology presented in Intriligator and Sheshinski [1987], the immigrant's life cycle plan is stated in two different stages. In the first stage, the planner selects an optimal path over a given period of time for a set of control variables. Here, the period of a given immigration plan is simply defined as the time interval between successive migrations,

\[ P_\tau = t_{\tau+1} - t_\tau \quad \text{where } \tau = 0..T-1. \]

The second stage is characterized by the aggregation of different immigration periods into an overall life cycle plan. Under this methodology, the planner determines the optimal paths for control variables, i.e. consumption, leisure and productive time, over a given period of time. Then a sequence of decision times (switching points) are calculated within the maximization problem, i.e. \( t_0, t_1, \ldots, t_\tau, t_{\tau+1}, \ldots \). In terms of Intriligator and Sheshinski, this procedure treats the maximization problem through a "complexity-reduction" process, which breaks the overall problem into a sequence of subproblems. This approach arises as a result of the complexity of formulating an optimal trajectory over the entire lifetime interval \([t_0, T]\).

Let regime \( m \) (Mexico) occur when period \( P_\tau \) is even, i.e. \( t_0 \leq t_1, t_2 \leq t_3, \ldots, \)
while regime $u$ (U.S.) happens when $P_u$ is odd, i.e. $t_1 < t_2, t_3 < t_4, \ldots$. Accordingly, even numbered periods $P_0, P_2, P_4, \ldots$ are those which consumers have chosen the former regime, and odd numbered periods $P_1, P_3, P_5, \ldots$ are ones in which they prefer immigration to the U.S.

There are, however, implicit and explicit costs of switching regimes. The former are psychic costs that reflect the lack of substitution between Mexican and U.S. leisure time; while the latter reflects direct monetary costs, which are treated as fixed, and they are incurred at the beginning of the immigration ($F_m$) or emigration ($F_u$) period.\(^{17}\)

In the first stage, agents face the problem of maximizing the net discounted value of their utility flows in each feasible state:

(i) Home-country:

$$V_m = \max_{(C, L), P \text{ Even}} \int_{t}^{t_{c+1}} U[C(t), S(t)L_m(t)] e^{-\rho t} dt - F_m$$

Subject to:

$$\dot{A} = rA(t) + W_mH_m(t) + S(t)I(t) - C(t),$$
$$\dot{S} = \sigma S(t) - [L_m(t) + I(t)],$$
$$1 = H_m(t) + L_m(t) + I(t);$$

(ii) Host-country:

$$V_u = \max_{(C, L), P \text{ Odd}} \int_{t}^{t_{c+1}} U[C(t), L_u(t)] e^{-\rho t} dt - F_u$$

Subject to:

$$\dot{A} = rA(t) + W_uH_u(t) - C(t),$$
$$\dot{S} = \sigma S(t),$$
$$1 = H_u(t) + L_u(t);$$

where $A(t)$ and $S(t)$ are state variables, subject to the following boundary conditions,

$$A(t_c) = \begin{cases} 
A_0 & \text{if } t_c = t_0 \\
0 & \text{if } t_c = T - 1 \\
\text{Free} & \text{elsewhere}
\end{cases}$$

and

$$S(t_c) = \begin{cases} 
S_0 & \text{if } t_c = t_0 \\
S(T) \geq 0 & \text{if } t_c = T - 1 \\
\text{Free} & \text{elsewhere}.
\end{cases}$$

\(^{17}\) See the next chapter for more on immigration costs.
Note that residents never immigrate if \( t_\tau = t_0 \) and \( t_{\tau+1} = T \). With such conditions, the analysis replicates a standard life cycle model without switching regimes. In addition, the general specification also handles the case of permanent migration, where only two periods are required, i.e. \( P_0 \) and \( P_1 \), as well as \( F_0 = 0 \).

In contrast, elements of transitory illegal immigration are replicated by the individual if at least three different periods are observed: (i) Home residents described by a steady-state equilibrium, (ii) U.S. labor market participation and (iii) home residents characterized by a non-stationary equilibrium. These basic components constitute an immigration cycle which, in general, can be reproduced any number of times during the consumer's lifetime. Furthermore, an immigration cycle requires three independent maximization problems, given by systems (2.4) with its stationary and non-stationary versions and system (2.26) which are linked together by their transversality conditions.

The second stage of the optimization problem is characterized by the aggregation of different migration periods into an overall lifetime immigration plan, \( V_I \),

\[
V_I \equiv \max_{\{t_\tau\}} \left\{ \sum_{\tau=0}^{T-1} [V^*_m(t_\tau)] + \sum_{\tau=1}^{T-1} [V^*_u(t_\tau)] \right\};
\]

where \( \{t_\tau\} \) is the sequence of switching times and \( V^*_i \) is the optimal value-function in each regime with \( i = m, u \). For an optimal sequence of immigration periods, the solution involves a set of switching times which take into account differences in benefit functions and immigration costs. In the case of perfect certainty, differential equations of the Mexican system can be integrated forward from the initial time \( t_0 \), starting with the initial \( m \) process and continuing to the next switching time \( t_I \). Then, the U.S. regime is integrated using the \( u \) process, from the new initial time \( t_I \) to the upper limit given by \( t_2 \); this procedure continues up to time \( T \).

Finally, under a certainty model, the optimal path results in switching regimes at known dates. In terms of Intriligator and Sheshinski, this result is characterized by time planning where "[the] nature of the optimal path and, in particular, the switches between processes is predictable at the initial time \( t_0 \)." [p. 14] Furthermore, the authors argued that by integrating the systems forward and optimizing the correspondent process at each point
of time, the entire past and future immigration behavior can be determined at the beginning of the life cycle, including all switching points.\footnote{The assumption of perfect certainty is relaxed in the following chapter. With uncertainty the qualitative nature of the solution is completely different. In such a case, optimal switching decisions depend on the current state of the system for every period of time.}

### 2.3 Structural Model of the Participation Decision Rule

The general specification of the model presented in the previous section does not allow for a full description of the participation decision and its interaction with life cycle effects. This limitation is alleviated in this section by introducing further structure to the original model. Assuming a specific utility function that may be solved for a closed form solution, it is possible to obtain a tractable mathematical specification. In this context, assume that utility function (2.1) follows a log-linear monotonic transformation of the form,

\[
U[t, C, S, L_m, L_u] = \left\{ \begin{array}{ll}
a[\log C(t)] + [\log S(t)][\log L_m(t)] & \text{if Mexico} \\ 
a[\log C(t)] + b[\log L_u(t)] & \text{if U.S.} \\
\end{array} \right.
\]  

where coefficients \(a\) and \(b\) are positive productivity parameters which are set to one, without loss of generality.\footnote{Although contemporaneous strong separability is not required in the general framework, this assumption is maintained here because it will be used in the empirical specification of the next chapter.}

#### 2.3.1 The Immigration Cycle

Formally the consumer's problem is to choose the control variables so as to maximize the migration lifetime value function

\[
V_1 = V_0 + V_1 + V_2 - F.
\]  

\(V_0\) and \(V_2\) are the stationary and non-stationary benefit functions in the Mexican regime, respectively. \(V_1\) is the corresponding value function in the U.S., while \(F\) represents the aggregate fixed monetary costs derived from the immigration decision. Expression (2.37) defines an entire immigration cycle where consumers switch back and forth across regimes. This cycle can be reproduced any number of periods during the individual's life-
time. Assuming that agents engage in only one immigration cycle over their entire life, the maximization problem (2.37) can be rewritten as

\[
V_1 = \max_{\{C, I, L_m, L_u\}} \int_0^{t_1} \left[ \int_0^{t_2} \left( \log C(t) + \log S(t) L_m(t) \right) + \int_{t_1}^{t_2} \left( \log C(t) + \log L_u(t) \right) + \int_{t_2}^T \left( \log C(t) + \log S(t) L_m(t) \right) \right] dt - F.
\]

subject to:

\[
\begin{align*}
\dot{A} &= rA(t) + [W_m H_m(t) + S(t) I(t)] J + W_u H_u(t)(1 - J) - C(t), \\
\dot{S} &= \sigma S(t) - \delta[I(t) + L_m(t)] J, \\
1 &= [H_m(t) + L_m(t) + I(t)] J + [H_u(t) + L_u(t)] (1 - J).
\end{align*}
\]

Consumers immigrate to the U.S. at age \(t_1\) and return to their home-country at time \(t_2\). \(\lambda(t)\), \(\mu(t)\) and \(\gamma\) are costate variables associated with the budget, the household productivity and time constraints, respectively. The boundary conditions are given by

\[
A(t) = \begin{cases} 
A(0) = A_0 \\
A(t_1) \text{ Free} \\
A(t_2) \text{ Free} \\
A(T) = 0
\end{cases}, \quad S(t) = \begin{cases} 
S(0) = S_0 \\
S(t_1) \text{ Free} \\
S(t_2) \text{ Free} \\
S(T) \geq 0
\end{cases}
\]

and

\[
J = \begin{cases} 
1 & \text{if } V_0 \text{ or } V_2 \\
0 & \text{if } V_1.
\end{cases}
\]

Following the methodology proposed in Section 2.2.4, \(V_1\) is solved by integrating forward the optimization problems \(V_0\), \(V_1\) and \(V_2\). The detailed derivation of the equilibrium conditions for each of these regimes is developed in the Appendix A.

In summary, the equilibrium values for the maximization problems in (2.37) are given by

\[
\begin{align*}
V_2^* &= (T - t_2)[r(T - t_2) - 2\log \lambda_m(t_2)], \\
V_1^* &= -(t_2 - t_1)[r(t_2 - t_1) + \log W_u + 2\log \lambda_u(t_2)], \\
V_0^* &= -t_1[r t_1 + 2\log \lambda_m(t_1)].
\end{align*}
\]

---

\(^{20}\) Recall that it has been assumed that the rate of time preference \(\rho\) is zero.

\(^{21}\) Here the switching points are assumed to be known. This assumption is relaxed in the following section.
The vector of marginal utility of wealth is obtained by equations:

(A.14) \[ A(t_2) = \frac{1}{r \sigma} \left[ e^{-r(T-t_2)} - 1 \right] - \left[ \frac{\sigma S(t_2) - 1}{\sigma (\sigma - r)} \right] \left[ e^{(\sigma - r)(T-t_2)} - 1 \right] + \frac{2}{\lambda_m(t_2)} (T-t_2), \]

(A.27) \[ A(t_1) = A(t_2) e^{-r(t_2-t_1)} + \frac{W_m}{r} \left[ e^{-r(t_2-t_1)} - 1 \right] + \frac{2(t_2-t_1)}{\lambda_u(t_2)} e^{-r(t_2-t_1)}, \]

(A.42) \[ A(0) = A(t_1) e^{-r t_1} + \frac{W_m}{r} \left[ 1 - \frac{\sigma^2 W_m}{R(2 \sigma - r)} \right] [e^{-r t_1} - 1] + \frac{2 t_1}{\lambda_m(t_1)} e^{-r t_1}. \]

It has been shown that components of the immigration cycle may be expressed in terms of their boundary points. Under an optimal control framework, current time \( t \) has been integrated in the boundary conditions. Equilibrium equations are expressed as a function of end-point conditions. These conditions are given in terms of known parameters in different periods: 0, \( t_1 \), \( t_2 \) and \( T \).\(^{22}\) Time 0 and time \( T \) represent the outset and end of the immigration cycle, respectively, while \( t_1 \) and \( t_2 \) are switching points within regimes.

2.3.1a Transversality Conditions

Although the system of equilibrium equations are expressed in terms of boundary conditions, only those in periods 0 and \( T \) are assumed to be known by consumers at the outset. Boundary conditions in periods \( t_1 \) and \( t_2 \) have to be determined inside the optimization model. Accordingly, two transversality conditions may be incorporated into the framework. In fact, transversality is required in order to link the three independent problems developed in the last section.

Under the special case of known switching points, \( t_1 \) and \( t_2 \), the transversality conditions are given by

\[ (2.38) \frac{\partial V^*_1}{\partial A(t_1)} = \lambda_m(t_1), \]

\(^{22}\) This set of conditions is given by the following vectors: Endowment \( A = \{ A_0, A(t_1), A(t_2), A(T) \} \), household productivity \( S = \{ S(0), S(t_1), S(t_2), S(T) \} \) and marginal utility of wealth \( \lambda = \{ \lambda_m(t_1), \lambda_u(t_2), \lambda_m(t_2) \} \).
\[
\frac{\partial V^*_2}{\partial A(t_2)} = \lambda^*_u(t_2) .
\]

The partial differentials required by the (2.38) and (2.39) are obtained from equations (A.15) and (A.28). For instance, the partial derivative of (A.15) with respect to the stock of wealth in \(t_2\) is

\[
\frac{\partial V^*_2}{\partial A(t_2)} = -\frac{2(T-t_2)}{\lambda^*_u(t_2)} \frac{\partial \lambda^*_u(t_2)}{\partial A(t_2)}
\]

where, \(\frac{\partial \lambda^*_u(t_2)}{\partial A(t_2)} = -\frac{\lambda^*_u(t_2)^2}{2(T-t_2)}\) from (A.14).

Likewise, equation (A.28) follows a similar procedure using (A.27). Some algebraic manipulation on both conditions leads to

\[
\frac{\partial V^*_2}{\partial A(t_2)} = \lambda^*_m(t_2) \quad \text{and} \quad \frac{\partial V^*_1}{\partial A(t_1)} = \lambda^*_u(t_2) e^{-\tau(t_2-t_1)} .
\]

The relation within the vector of marginal utilities of income across regimes (\(\lambda^*\)'s) is found by the substitution of these expressions into the transversality conditions (2.38) and (2.39),

\[
\lambda^*_m(t_2) = \lambda^*_u(t_2) ,
\]

\[
\lambda^*_u(t_2) = \lambda^*_m(t_1) e^{-\tau(t_2-t_1)} .
\]

Not surprisingly condition (2.40) states that undocumented workers will switch back to their home-country, if "shadow prices" of monetary assets in both regimes are equal at any point of time.\(^{23}\) Furthermore, under condition (2.41) the marginal utility of income at age \(t_2\) is the discounted value of the marginal utility of wealth in the beginning of the immigration period \(t_1\). It is clear that \(\lambda^*_u(t_2)\) observes a depreciation process during the time of migration, \(t_2-t_1\). This result and Proposition A.1 imply that undocumented workers

\(^{23}\) Net of immigration costs.
are net savers during their residence in the receiving country. Under their undocumented status, immigrants will accumulate assets to the point where \( \lambda_u(t_2) \) depreciates enough to offset monetary gains in the U.S. labor market.

Non-market productivity, on the contrary, does not require the imposition of further restrictions since equations (A.24) and (A.38) fully characterize vector \( S \). Direct substitution of these conditions leads to

\[
S(t_2) = \frac{W_m}{R} e^{-\sigma(t_2-t_1)}; \quad \text{with } S(0) = S_0 = S(t_1)
\]

Introducing the transversality conditions (2.40) to (2.42) into the equilibrium equations in section 2.2 permits to summarize the major results obtained by the model:

(i) The equilibrium value functions:

\[
(A.15') \quad V_2^* = (T - t_2)[r(T - t_2) + 2r(t_2-t_1) \cdot 2\log \lambda_m(t_1)],
\]

\[
(A.28') \quad V_1^* = (t_2 - t_1)[r(t_2-t_1) - \log W_u - 2\log \lambda_m(t_1)],
\]

\[
(A.43) \quad V_0^* = t_1 [-r t_1 - 2\log \lambda_m(t_1)];
\]

(ii) End point conditions of the net worth vector \( A \):

\[
(A.14') \quad A(t_2) = \frac{1}{\tau \sigma} \left[ e^{-\pi(t_2-t_1)} - 1 \right] - \frac{[\sigma(W_m/R)e^{-\sigma(t_2-t_1)} - 1]}{\sigma(\sigma-\tau)}[e^{(\sigma-\tau)(T-t_2)} - 1] + \frac{2(T-t_2)}{\lambda_m(t_1)} e^{-\tau(t_2-t_1)},
\]

\[
(A.27') \quad A(t_1) = A(t_2) e^{-\pi(t_2-t_1)} + \frac{W_u}{\tau} \left[ e^{-\pi(t_2-t_1)} - 1 \right] + \frac{2(t_2-t_1)}{\lambda_m(t_1)},
\]

\[
(A.42) \quad A(0) = A(t_1) e^{-r t_1} + \frac{W_m}{\tau} \left[ 1 - \frac{\sigma^2 W_m}{R(2\sigma-\tau)} \right] \left[ e^{-\pi t_1} - 1 \right] + \frac{2t_1}{\lambda_m(t_1)} e^{-r t_1}.
\]

\[24\] Recall from Proposition A.1 that the marginal utility of income and the stock of wealth in the receiving country are negatively correlated.
Finally, notice that equations (A.14'), (A.27') and (A.42) constitute a recursive fully identifiable simultaneous system with three endogenous variables, i.e. A(t), A(t2) and \( \lambda_m(t1) \).

### 2.3.1b Comparative Dynamics

The immigration model presented in equation (3.28) has an optimal solution given by the aggregation of equilibrium value functions in (A.15'), (A.28') and (A.43). This aggregation yields

\[
(2.38') \quad V^*_1 = rT(T - 2t1) - (t2 - t1)\log W_u - 2T[\log \lambda_m(t1)] - F;
\]

where \( \lambda_m(t1) \) is obtained by the simultaneous system of end-point conditions, i.e. equations (A.14'), (A.27') and (A.42). Inspection of (2.38') reveals that only the wage rate in the receiving labor market \( W_u \) directly affects the equilibrium benefit function of the immigration process, \( V^*_1 \). The other two exogenous values, \( A_0 \) and \( W_m \), are related to the migrant's value function solely through \( \lambda_m(t1) \).

Accordingly, there are two sorts of elements affecting benefits obtained from undocumented participation: (i) Current events, such as the wage rate in the U.S., the immigration period \( t2 - t1 \) and the interest rate, and (ii) lifetime effects. The initial endowment of consumers \( [A(0)] \) and the Mexican wage \( W_m \) influence the benefit function through the marginal utility of income at age \( t_f \), which in turn relates life cycle considerations to migration decisions.

In order to address a comparative dynamics exercise, one has to characterize the behavior of the life cycle component, \( \lambda_m(t1) \), with respect to changes in the vector of independent variables \( X \). A generalization of the results described in Appendix A leads to:

**Remark 2.1:** Under certain regularity conditions, the lifetime component of the immigration cycle is a decreasing function of the initial wealth and the market wage rates in each country.

\[\text{\textsuperscript{25}} \text{ Where vector } X=[A_0, W_m, W_u].\]
\[ \frac{\partial \lambda_m(t_1)}{\partial X} < 0. \]

**Proof:**

(i) Initial endowment \((A_0)\):

Recursive differentiation of the end point conditions yields

\[ \frac{\partial A(t_2)}{\partial A_0} = -\frac{2(T-t_2)}{\lambda_m(t_1)^2} e^{-r(t_2-t_1)} \frac{\partial \lambda_m(t_1)}{\partial A_0}, \]

\[ \frac{\partial A(t_1)}{\partial A_0} = \frac{\partial A(t_2)}{\partial A_0} e^{-r(t_2-t_1)} - \frac{2(t_2-t_1)}{\lambda_m(t_1)^2} \frac{\partial \lambda_m(t_1)}{\partial A_0}, \]

\[ 1 = \frac{\partial A(t_1)}{\partial A_0} e^{-r_1} - \frac{2t_1}{\lambda_m(t_1)^2} \frac{\partial \lambda_m(t_1)}{\partial A_0}. \]

Then

\[ 1 = -K \frac{2}{\lambda_m(t_1)^2} \frac{\partial \lambda_m(t_1)}{\partial A_0}, \]  

with \(K \equiv [(T-t_2)e^{-2r(t_2-t_1)} + (t_2-t_1)e^{-r_1+t_1}] > 0.\)

Hence, \(\partial \lambda_m(t_1)/\partial A_0\) must be negative in order to hold the equality.

(ii) Domestic wage rate \((W_m)\):

\[ 0 = \frac{1}{t} \left[ 1 - \frac{2\sigma^2 W_m}{R(2\sigma-r)} \right] (e^{-r_1} - 1) - \frac{2K}{\lambda_m(t_1)^2} \frac{\partial \lambda_m(t_1)}{\partial W_u}; \]

under the regularity condition \(R(2\sigma-r) > 2\sigma^2 W_m\), the first term is negative. Therefore, \(\partial \lambda_m(t_1)/\partial W_u\) must be negative.

(iii) U.S. wage rate \((W_u)\):

\[ 0 = \frac{1}{t} \left[ e^{-r(t_2-t_1)} - 1 \right] - \frac{2K}{\lambda_m(t_1)^2} \frac{\partial \lambda_m(t_1)}{\partial W_u}. \]
where the term in brackets is negative, so $\partial \lambda_m(t_1)/\partial W_u < 0$.

[Q.E.D.]

Under Remark 2.1, it is easy to verify that the present value of the life cycle utility streams in expression (2.38), $V_1^*$, is a positive function of the initial endowment ($A_0$) and the domestic wage rate ($W_m$):

$$\frac{\partial V_1^*}{\partial A_0} > 0 \quad \text{and} \quad \frac{\partial V_1^*}{\partial W_m} > 0.$$  

However, the relation of (2.38's) with respect to the host wage rate ($W_u$) is ambiguous. In particular,

$$\frac{\partial V_1^*}{\partial W_u} = -\frac{T \lambda_m(t_1)}{Kr} \left[ e^{-r(t_2-t_1)} - 1 \right] - \frac{(t_2-t_1)}{W_u} < 0.$$  

Current and life cycle effects offset each other. In this expression current events show a negative relationship with respect to the net benefit function, but such a relation is positive when lifetime components are considered. Consequently, an increase in $W_u$ will lead to a temporary contraction in migrants' benefits since household activities (leisure and non-market production) are valued by consumers. Over the long run, the immigration process matures and accumulation of wealth increases their net benefits. Lastly, notice that distinctions of transitory and permanent effects introduce the possibility of immigrants becoming worse off due to U.S. labor participation, even in the presence of large wage differentials, $W_u W_m$.

2.3.2 The Non-Immigration Model

Economic behavior of non-immigrant residents is a particular case of the dynamic framework described in former sections given that $t_1=t_2=T$. Analysis of the non-immigrant case is important because consumers will compare their net discounted benefits of migration,

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26 Family ties and economic attachments in the home-community are important elements considered in this argument.
$V^*_1$, against the alternative gains of the non-immigration process, $V^*_{NI}$.

Permanent residents in the home-country are assumed to maximize their time-preference-discounted stock of total utility over horizon $T$:

\[
V_{NI} = \max_{\{c, i, L_m\}} \int_0^T [(\log C(t) + \log S(t)L_m(t))e^{-\rho t}] \, dt
\]

Subject to:

\[
\dot{A} = rA(t) + W_m H_m(t) + S(t)I(t) - C(t)
\]

\[
A(0)=A_0 \text{ and } A(T)=0
\]

\[
\dot{S} = \sigma S(t) - \delta[I(t)-L_m(t)]
\]

\[
S(0)=S_0 \text{ and } S(T)\geq 0
\]

\[
1 = H(t) + L_m(t) + I(t)
\]

Under the simplifying assumptions $\rho=0$ and $\delta=1$, the Hamiltonian function resulting from system (2.43) is

\[
\mathcal{H}(c, i, L_m, A, S, \lambda_{ni}, \mu_{ni}, \gamma) = \log C + \log S L_m + \lambda_{ni}(rA + W_m H + S I - C) + \\
\mu_{ni}(\sigma S - I - L_m) + \gamma(1-H-I-L).
\]

Sufficient and necessary conditions are given by (A.29) to (A.36); notice that these conditions characterize a stationary process.

Although model (2.43) has a similar structure as the one described in the steady-state equilibrium (Appendix A.3), the solution differs in terms of boundary conditions in each case. The non-immigration problem is integrated under the terminal condition $A(T)=0$, which implies that the marginal utility of income at age $T$ is positive $[\lambda_{ni}(T)>0]$. In contrast, the stationary process, $V_0$, is identified by a terminal condition where the stock of wealth is positive $[A(t_f)>0]$ and the "shadow price" of income equals zero at age $t_f$ $[\lambda_{ni}(t_f)=0]$.

The solution of the first order homogeneous difference equation (A.30) over the end points $0, T$ yields

\[
\lambda^*_{ni}(t) = \lambda_{ni}(0)e^{-\rho t}.
\]
\( \lambda_{ni}(0) \) represents the marginal utility of income at the outset for a non-immigrant consumer; in contrast, \( \lambda_{m}(t) \) in (A.37) is the marginal utility of income at the beginning of the immigration period. It is evident that these values bear no direct relation.

Assuming an interior solution, the steady-state equilibrium implies \( S^*(t) = W_m/R \) and \( \dot{S} = 0 \). Accordingly, the "\( \lambda_{ni}(0) \)-demand" functions are given by

\[
(2.45) \quad C^*(t) = \frac{1}{\lambda_{ni}(0)} e^{\tau t},
\]

\[
(2.46) \quad L^*(t) = \frac{R}{W_m \lambda_{ni}(0)} e^{\tau t};
\]

using equation (2.46) and condition \( \dot{S} = 0 \), the supply for productive non-market time is

\[
(2.47) \quad I^*(t) = \frac{\sigma W_{in}}{R} - \frac{R}{W_m \lambda_{ni}(0)} e^{\tau t}.
\]

In order to access the lifetime effect of \( \lambda_{ni}(0) \), equilibrium conditions (2.45) to (2.47) may be substituted into the state condition associated with the budget constraint,

\[\dot{A} - rA(t) = W_m \left[ 1 - \frac{\sigma^2 W_m}{R(2\sigma - r)} \right] e^{\tau t} - \frac{2}{\lambda_{ni}(0)} e^{\tau t}.\]

Under the boundary conditions \( A(0) = A_0 \) and \( A(T) = 0 \), the equation above has a solution given by

\[
(2.48) \quad A(0) = \frac{W_m}{r} \left[ 1 - \frac{\sigma^2 W_m}{R(2\sigma - r)} \right] e^{\tau T} + \frac{2T}{\lambda_{ni}(0)}.\]

It is easy to show under Proposition A.1 that \( \partial \lambda_{ni}(0)/\partial A_0 < 0 \); in addition, \( \partial \lambda_{ni}(0)/\partial W_m < 0 \) holds if \( R(2\sigma - r) > \sigma^2 W_m \).

The equilibrium benefit function for the non-immigrant case, \( V_{NI}^* \), may be derived by substituting "\( \lambda_{ni}(0) \)-demand" functions (2.45) and (2.46) into (2.43), and integrating
over the lifetime interval \([0, T]\)

\[
V_{N1}^* = T[rT - 2\log\lambda_{ni}(0)];
\]

with \(\lambda_{ni}(0)\) given by expression (2.48). Inspection of (2.43') reveals that \(\partial V_1^*/\partial A_0\) is un-
ambiguously negative; in contrast, \(\partial V_1^*/\partial W_m\) is negative only if the lifetime effects domi-
nate current events.

2.3.3 The Decision Rule

Undocumented participation in the U.S. labor market is an economic decision. A cost-ben-
efit analysis of immigration addresses the elements driving the participation decision. Con-
sumers evaluate benefits and migration costs against permanent residence in their home-
community. In this context, undocumented participation is chosen upon the realization of
the lifetime trajectory that reports higher net utility.

Transitory immigration represents a straightforward application of model (2.38),
where undocumented workers switch back and forth across regimes. Permanent residents
in Mexico, in contrast, is addressed in system (2.43). The immigration decision is driven
by the comparison of these two processes. The difference between the equilibrium values
of lifetime utilities in each state of the world, GAP, is defined as

\[
\text{GAP} = V_1^* - V_{N1}^*;
\]

plugging the equilibrium values derived in expressions (2.28') and (2.43'), the GAP equa-
tion may be written as

\[
(2.49) \quad \text{GAP} = -2rt_1 - (t_2-t_1)\log W_u + 2T[\log\lambda_{ni}(0) - \log\lambda_m(t_1)] - F.
\]

\(\lambda_{ni}(0)\) is given by expression (2.48), and \(\lambda_m(t_1)\) is obtained by the simultaneous equations
(A.14'), (A.27') and (A.42). Note that the term in brackets represents the life-cycle ele-
ments of the immigration decision. Furthermore, lifetime components are induced by dif-
fierentials in the marginal utility of wealth in each process. If the value of \(\text{GAP}\) is positive,
then home residents will participate in the U.S. labor market as transitory workers, while a
negative value of GAP implies no Mexican undocumented participation.

Permanent immigration to the U.S. constitutes a special case of the model (2.38) where \( t_2 = T \). In this case, the GAP equation may be written as

\[
(2.49') \quad \text{GAP} = -2rTt_1 - (T - t_1) \log W_u + 2T[\log \lambda_{ni}(0) - \log \lambda_m(t_1)] - F,
\]

and \( \lambda_m(t_1) \) given only by expressions \((A.27')\) and \((A.42)\); \( \lambda_{ni}(0) \) is still obtained by \((2.48)\).

### 2.3.4 Endogenous Switching Points

It has been assumed in the previous analysis that the migration periods in which undocumented workers switch regimes are known at the outset. This assumption constitutes a complexity-reduction procedure. However, switching points may be determined within the optimization model. This section presents an outline of the methodology required in order to obtain endogenous switching points.

Recall that the simultaneous equations \((A.14')\), \((A.27')\) and \((A.42)\) are a recursive system with three endogenous variables, i.e. \( A(t_2), A(t_1) \) and \( \lambda_m(t_1) \). If the assumption of predetermined switching points is relaxed \((t_1 \text{ and } t_2 \text{ are set free})\), two new equations are required in order to obtain an identifiable system. This new set of equations is provided by the transversality conditions of the free points. Following Kamien and Schwartz [1983], if \( t_1 \text{ and } t_2 \) are free, then it must hold that

\[
(2.50) \quad -\frac{\partial V_2^e}{\partial t_2} = \mathcal{H}_u(t_2, C, L_u, A, S, \lambda_u, \mu_u) =
\]

\[
\log C + \log L_u + \lambda_u[rA + W_u(1 - L_u) - C] + \sigma \mu_u S,
\]

\[
(2.51) \quad -\frac{\partial V_2^*}{\partial t_2} = \mathcal{H}_m(t_1, C, I, L_m, A, S, \lambda_m, \mu_m) =
\]

\[
\log C + \log (S\lambda_m) + \lambda_m[rA + W_m H_m(1 - L_m - I) + S\lambda - C] + \mu_m(\sigma S - I - L_m).
\]

Notice that the Hamiltonian functions, \( \mathcal{H}_u \) and \( \mathcal{H}_m \), are evaluated at the free points \( t_2 \) and \( t_1 \), respectively. In this context, the entire set of variables in conditions \((2.50)\) and \((2.51)\)
can be determined by the previous results presented throughout the model. Proper substitution of the variables required for such conditions yields two transcendental equations that may be solved in terms of the free points. The resultant expressions, in terms of $t_2$ and $t_f$, are incorporated into the original set of end point conditions, i.e. $A(t_2), A(t_1)$ and $\lambda_m(t_1)$. The simultaneous interaction of these five conditions will provide the life cycle information required by the GAP equations in (2.49) and (2.49').

**Concluding Remarks**

This chapter develops a life cycle model of Mexican undocumented labor force participation. Unlike traditional static models, illegal immigration is analyzed in a dynamic framework where transitory migration is an optimal solution. It is shown that household productivity in the Mexican regime, but not in the U.S. labor market, constitutes a major consideration to explain the coexistence of temporary migration and large positive wage differentials across countries.

The objective of this analysis is to obtain a decision rule that describes undocumented participation, either transitory or permanent, in an environment of perfect certainty. Accordingly, life cycle components of the migration process are integrated into a maximization process and solved for their boundary conditions. Assuming a specific set of consumer preferences, close forms solutions are obtained in terms of values known at the outset. In this framework, a cost-benefit analysis is developed in the context of switching regimes. Domestic residents will participate in the U.S. labor market if the discount value of their net lifetime benefits is greater than the discount value of their Mexican opportunities.

A major contribution of this model is found in the identification of the elements controlling the duration of Mexican undocumented migration. Transitory immigration is characterized by strong family, social and economic attachments with the home-community. In this context, home residents are productive in household activities and in the labor market, while undocumented workers in the U.S. are productive only in the labor force. Moreover, leisure in Mexico is far substitute of leisure in the United States. Permanent migration, in contrast, is characterized by the imperfect transferability of household environment. Permanent residence in the U.S. required breaking family, social and economic anchors with the home-regime which in turn results in family rather than individual immigra-
The analysis of undocumented participation is addressed in an optimal control framework, where residents switch back and forth across regimes. The consumer's benefit function, as well as the dichotomy between home-production possibilities in each country, are the factors underlying the behavior of switching regimes. Furthermore, household production introduces the dynamic characteristics of the model. The marginal productivity of household activities decreases during residence in Mexico, but soars when immigration to the U.S. takes place. This behavior leads to a cyclical pattern over the individual's lifetime.

Not surprisingly, the life cycle specification resulting from this chapter predicts that consumers are willing to migrate to the U.S. because of a deterioration in their stock of wealth. A standard result found in traditional models somewhere else. However, the theoretical contribution of the present analysis is that depreciation of wealth is originated by diminishing marginal returns of household production, rather than a direct comparison of market wage rates across regimes. Participation in the U.S. labor market induces a monotonic increase in the marginal rate of household production, since no time is allocated to non-market activities in Mexico. Eventually, consumers switch back to their home-communities because productivity have grown enough to offset net monetary gains of their undocumented status. This initiates a new immigration cycle which reproduces the lifetime behavior of undocumented immigrants.
CHAPTER 3

STOCHASTIC MODEL OF MEXICAN PARTICIPATION AND UNDOCUMENTED LABOR SUPPLY

"Life can only be understood going backwards, but it must be lived going forwards."

Kierkegaard

This chapter presents a discrete life cycle model of undocumented participation and labor supply in which future events are uncertain. The objective of this analysis is to develop a tractable stochastic model that can be tested in cross-section empirical estimation. This model relies on the basic set of results discussed in the deterministic framework presented in the previous chapter.

A major modification in this chapter is the introduction of stochastic behavior within undocumented migration. Here, uncertainty about the future and the discrepancy between anticipated future values of random variables and their realization are important elements that determine Mexican undocumented participation. An important consequence of admitting uncertainty into the analysis concerns the way in which randomness enters the empirical mode. Three sources of uncertainty are introduced: Incomplete information, household productivity and wage rates in both countries.

Although dynamic models are used extensively in recent economic literature, much of the empirical studies dealing with undocumented immigration does not recognize life cycle theory, and practically none of them formally introduce the possibility of uncertain behavior. Breaking with this tradition, the present chapter relies on important contributions in the fields of dynamic programming and stochastic behavior. Here, the interaction between
lifetime effects and state variables is studied in a context of rational expectation models.

Contrary to traditional static models, Mexican intertemporal immigration is the result of two independent factors: The participation decision and the actual allocation of market time. Labor supply choices in the U.S. are observed only if undocumented participation takes place at any point in time. In this context, immigration costs and the process of forming future expectations are two major elements in determining these decisions.

The chapter proceeds as follows. Section 1 provides a brief review of the economic literature on life cycle models under uncertainty and immigration costs. Sections 2 and 3 develop the core elements of the participation rule. First, a general stochastic model is derived under dynamic programming techniques, and second a search model is introduced in the context of variable immigration costs. Section 4 proposes an empirical specification for Mexican undocumented participation; while section 5 discusses the labor supply and the determination of wage rates in an integrated simultaneous equation model. The chapter concludes discussing some final remarks.

3.1 Literature Review

The study of undocumented population flows has long been of interest to social scientists. Demographers, sociologists and anthropologists focus on illegal migration as the dominant factor in shifts in the composition of domestic population and the effects of those changes in the social environment. In contrast, the economics of illegal immigration, is viewed by some authors as an efficient reallocation of resources. Thus, the existence of Mexican participation in the U.S. is conceived as an indication of the proper functioning of market structure within the economy.

The treatment of undocumented population flows as a consequence of market efficiency is not new in the literature. In a seminal paper Sjaastad [1962] introduced a model of spatial separation markets and lifetime approach to regional migration. Sjaastad argued, in a dynamic framework, that individuals choose locations at each point of their life cycle so as to maximize the present value of their lifetime utility. The treatment of "...migration as an investment increasing the productivity of human resources" is viewed as an optimal resource allocation [Ibid, p. 82]. An efficient allocation is obtained since individuals will not migrate until their productivity elsewhere is sufficiently high to compensate for rent differ-
MaCurdy [1985] provides a study that formulates a tractable empirical model of labor supply that addresses decision making in a multiperiod setting where the future is uncertain. The author not only develops the underlying theory required for the specification of a stochastic model but also introduces conditions upon which life cycle components may be interpreted in an empirical cross-section model.

In addition, the process of forming future expectations is discussed in Hall [1978] and Altonji [1986]. Although Hall’s paper addresses the life cycle permanent hypothesis in a consumption model, the implication for undocumented immigration are rather relevant to the understanding of rational expectation behavior. The author argues that if deviations of consumption from its trend are unexpected and permanent, then the best forecast of future consumption is the current level adjusted for its trend. “Forecasts of future changes in income are irrelevant, since the information used in preparing them is already incorporated in today’s consumption.” [Ibid, p. 973] Furthermore, Altonji explores in a life cycle model with uncertainty the sensitivity of the labor supply function to intertemporal variations in the wage rate. Following MaCurdy, the author introduces a model of rational expectation in order to predict lifetime behavior of consumers. He concludes that temporary changes have little effect over the lifetime component of labor supply.

A major element in Mexican labor force participation constitutes immigration costs. Fixed costs in the context of leisure-labor decision models have been widely studied in recent literature.¹ In particular, Cogan [1980] introduces fixed costs as externalities in a Neoclassical sense. These distortions impose non-convexities in the budget and time constraints, which result in discontinuous labor supply functions. The contribution of Cogan, however, is found in the distinction between time and monetary costs, e.g. migration experience and crossing fees, respectively. These concepts are inversely related, since immigrants will trade-off time and monetary resources in order to minimize their total investment.²

Hill [1985] proposed a model on undocumented migration where the sole distortion

² For instance, an experimented illegal worker will eventually offset crossing fees, relative to recent immigrants. The “known-how” may be acquired by self-experimentation or by allocating monetary resources.
in the system is due to border enforcement and employer interdiction. The author argued that restrictions on immigration are similar to restriction on trade.\footnote{Net of welfare losses since domestic low-skill workers may be displaced. Nonetheless, efficiency gains obtained as result of undocumented migration are more likely to offset negative redistribution effects.} There is little of economic differences between admitting fewer low-skill immigrants and taxing imported goods that required large amounts of low-skill labor. Each policy tend to reduce the domestic national product. Hill concludes that a policy of more vigorous border enforcement will trade-off economic efficiency for an income redistribution to a relatively small sector, i.e. from high-skilled workers and farmland, to the low-skilled sector.

Finally, Ethier [1986] and a follow up paper by Bond and Chen [1987], present a stochastic model of undocumented participation where border enforcement and employer sanctions are feasible. Although the model is restricted to the analysis of labor demand, the interaction between immigration costs and employment decisions is relevant for the present analysis. The authors concluded that individuals who bear the burden of immigration controls (e.g. tax payers) are not, in general, the ones who collect the benefits of the enforcement policy (e.g. low-skilled workers).

### 3.2 Intertemporal Preferences

#### Under Uncertainty

This section presents a modified version of the theoretical framework developed in the previous chapter. Two new elements are introduced in the present discussion. First, this study considers time $t$ as the individual’s age. This variable is required to be an integer so that the theoretical model reflects the discrete characterization of time. Second, stochastic behavior introduces a framework that underlies the empirical specification of the immigrants' decision rule and their labor supply.

This section introduces the basic ideas and methods of dynamic programming. The analysis displays the objective function and restrictions in a life cycle model with uncertainty which is applicable to empirical estimation. Here, undocumented Mexican workers confront future labor participation decisions which are a realization of stochastic processes partially controlled by their own strategies. According to Burdett and Mortensen [1978] the appropriate technique for deriving and characterizing optimal strategies under uncertainty is dynamic programming. In general, this approach permits breaking a single large dimen-
sional problem into a collection of smaller optimization problems that can be solved sequentially. A formal derivation of the stochastic maximization model is presented below.

3.2.1 A Dynamic Programming Model

The study of the undocumented labor force participation requires the specification of consumer preferences and constraints governing asset accumulation and household production. Similar to the continuous time model, the consumer's lifetime utility function is assumed to be strongly separable over time and within its arguments. Formally, at each age \( t \) the consumer's problem is to choose policies \( C(\tau), L_m(\tau), L_u(\tau) \) and \( I(\tau) \) for \( \tau \geq t \), to maximize the expected utility of the time-preference-discounted sum of lifetime utility

\[
V(t) = \max_{\{C,L,I\}} \quad E_t \left\{ \sum_{\tau=t}^{T} \frac{1}{(1+\rho)^{T-\tau}} U[C(\tau), S(\tau) L_m(\tau), L_u(\tau)] \right\}
\]

where \( E_t \) is the mathematical expectation operator that indicates the information set available at time \( t \). Recall that according to the utility specification in equation (2.1), \( L_m(t) \) and \( L_u(t) \) constitute exclusive elements in the utility function, i.e. \( L_m(\tau)=0 \) if \( L_u(\tau)=0 \) for every \( \tau \). By virtue of Bellman's principle of dynamic optimality [Bellman, 1957] the value function \( V(t) \) may be rewritten as a function of recursive substitutions for the closed interval \([t,T]\)

\[
V(t) = \max_{\{C,L,I\}} \left\{ U[C(t), S(t) L_m(t), L_u(t)] + E_t \sum_{\tau=t+1}^{T} \frac{1}{(1+\rho)^{T-\tau}} U[C(\tau), S(\tau) L_m(\tau), L_u(\tau)] \right\}
\]

The first term of the R.H.S. is the present value of utility flows realized in period \([t, t+\Delta]\). The second term is the expected value of the maximum end point utility \((t+1,T]\) at date \( t \) given that an optimal strategy is pursued subsequent to \( t+\Delta \). The previous two expressions may be combined in a simpler representation of the Bellman functional form,

\[
(3.1) \quad V(t) = \max_{\{C,L,I\}} U[C(t), S(t) L_m(t), L_u(t)] + V(t+1).
\]

Expression (3.1) indicates that the maximization problem in period \([t, t+\Delta]\)

\[4\] See Altonji [1986] for a discussion on these assumptions and the implication on labor supply estimation.
summarizes all past and present information available at age \( t \). This implies that optimal behavior in each period guarantees an optimal path during the consumer’s life cycle. Dynamic programming decomposes the overall optimal problem in sequences of independent maximization problems that are carried over the set of control variables. Each one of these problems is far more simple than the original. In terms of Bertsekas [1987], “dynamic programming is the only general approach for sequential optimization under uncertainty.” (p.15)

The first element of the Bellman equation in (3.1) is maximized subject to three constraints: Asset accumulation, household production and allocation of time.\(^5\) The discrete time versions of these conditions are the following:

Asset accumulation,
\[
A(t+1) = (1+r)A(t) + [W_m H_m(t) + S(t)I(t)]J + W_u H_u(t)[1 - J] - C(t),
\]
with \( A(t)=A_0 \) and \( A(T)=0; \ 0 < r < 1 \).

Household production,
\[
S(t+1) = \sigma S(t) - \delta I(t+1) + L_m(t+1)J + \varepsilon(t+1),
\]
with \( S(0)=S_0 \) and \( S(T)\geq 0; \ 0 < \sigma < 1, \varepsilon(t+1) \sim (0, \sigma^2_\varepsilon) \).

Time allocation,
\[
1 = [H_m(\tau) + L_m(\tau) + I(\tau)]J + [H_u(\tau) + L_u(\tau)](1-J),
\]
for all \( \tau = 0...T \).

The dichotomy \((0, 1)\) variable \( J \) indicates two mutually exclusive regimes. The Mexican regime \((m)\) is characterized by \( J=1 \), while undocumented participation in the U.S. labor market \((u)\) is represented by \( J=0 \).

So far, uncertainty is introduced to the model throughout two distinctive mechanisms: Preferences and home productivity.\(^6\) First, concavity in the utility function is equivalent to risk aversion. If the utility function is strictly concave in income and leisure in both regimes, the optimal choice of control variables \([C(t), L_m(t), L_u(t), I(t)]\) is unique and con-

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\(^5\) The rationalization of these set of constraints and the way in which affect the immigration process is discussed in detail in section 2.1.

\(^6\) Wage rates in both countries constitute a third source of stochastic behavior which is developed in the next section.
tinuous with respect to the vector of state variables, i.e. wage rates \(W_m(t)\) and \(W_U(t)\), financial wealth \(A(t)\) and marginal productivity of household work \(S(t)\). Following Burdett and Mortensen [1978], “concavity of the utility function [and thus of the value function \(V(t)\)] is a cardinal property of preferences and as such has no meaning in the standard theory of household demand under conditions of certainty.” (p.127) However, under uncertainty, this property may be interpreted in the sense of Arrow-Pratt's risk aversion.7

Second, the marginal productivity of household activities \(S(t)\) follows a stochastic process described by condition (3.3). A basic hypothesis is that the serially uncorrelated random term \(\varepsilon(t)\) satisfies

\[
E[\varepsilon(t)]=0, E[\varepsilon_t^2]=\sigma_\varepsilon^2 \text{ and } E[\varepsilon_t\varepsilon_{t-s}]=0, \quad \text{for all } t \text{ and } s \neq 0.
\]

The assumption of \(\varepsilon(t)\) following a white-noise process implies that only non-expected change will affect home productivity payouts in the future. However, alternative specification may describe more accurate undocumented immigration. For instance, farmer workers may follow a Markov process of the type \(\varepsilon(t)=b\varepsilon(t-1)+\eta(t)\), where \(|b|<1\) and \(\eta(t)\) is white noise. In this case, consumers are highly productive in their home communities during particular seasons [e.g. harvesting], while during periods of low Mexican productivity they will engage in temporary migration to the U.S.

3.2.2 The Optimal \(\lambda\)-Demand Functions

Given that consumers participate in the Mexican labor force, i.e. \(J=1\), optimization of the benefit function (3.1) subject to motion equations (3.2)-(3.4) implies the following first-order conditions:

(i) For \(\tau=t\),

\[
U_C(C(t), S(t)L_m(t)) = \lambda_m(t), \tag{3.5}
\]

\[
S(t)U_L[C(t), S(t)L_m(t)] \geq \gamma_m(t) - \delta \mu_m(t), \tag{3.6}
\]

\[
\text{equal if } L_m(t)>0,
\]

\[
\lambda_m(t)S(t) \geq \gamma_m(t) - \delta \mu_m(t), \tag{3.7}
\]

\[
\text{equal if } I(t)>0,
\]

---

7 See Varian, [1984] for a formal treatment for the Arrow-Pratt measure of risk aversion.
(3.8) \[ \lambda_m(t) W_m(t) \geq \gamma_m(t) \]
equal if \( H_m(t) > 0 \).

(ii) For \( t = t+1 \),

(3.5') \( E_t \frac{1}{1+r} U_C[C(t+1), S(t+1) L_m(t+1)] = \frac{1}{1+r} \lambda_m(t) \).

(3.6') \( E_t \frac{S(t+1)}{1+r} U_L[C(t+1), S(t+1) L_m(t+1)] \geq \gamma_m(t) - \delta \sigma_m(t) \)
equal if \( L_m(t+1) > 0 \).

(3.7') \( \lambda_m(i) S(t+1) \geq [\gamma_m(t) - \delta \sigma_m(t)](1+r) \)
equal if \( I(t+1) > 0 \).

(3.8') \( \lambda_m(i) W_m(t+1) \geq \gamma_m(t)[1+r] \)
equal if \( H_m(t+1) > 0 \).

Where, in particular, \( \lambda_m(t) \) represents the Lagrange multiplier associated with the budget constraint or the marginal utility of income at age \( t \). Moreover, note that the previous set of Kuhn-Tucker conditions may or may not hold with an equality; this introduces the possibility of corner solutions within the Mexican regime. Accordingly, condition (3.6) and (3.6') determines the choice of leisure across time. Consumers choose not to work when this relationship becomes a strict inequality. However, it is assumed that they always allocate a positive amount of productive time either in the market or/and the household, i.e. \( H_m(t) + I(t) > 0 \). Consequently, the lack of time resources allocated to productive activities in the Mexican regime will imply undocumented participation in the U.S. labor market.

Conversely, given that Mexican undocumented workers immigrate to the U.S., i.e. \( J = 0 \), the first-order conditions of the maximization problem in (3.1) may be expressed as

(i) For \( t = t \),

(3.9) \( U_C[C(t), L_u(t)] = \lambda_u(t) \).

(3.10) \( U_L[C(t), L_u(t)] \geq \gamma_u(t) \)
equal if \( L_u(t) > 0 \).

(3.11) \( \lambda_u(i) W_u(t) \geq \gamma_u(t) \)
equal if \( H_u(t) > 0 \).

(ii) For \( t = t+1 \),
\[(3.9')\] 
\[E_t \frac{1}{1+\rho} U_C[C(t+1),L_u(t+1)] = \frac{1}{1+r} \lambda_u(t),\]

\[(3.10')\] 
\[E_t \frac{1}{1+\rho} U_L[C(t+1), L_u(t+1)] \geq \gamma_u(t) \quad \text{equal if } L_u(t+1)>0,\]

\[(3.11')\] 
\[\lambda_u(t)W_u(t+1) \geq \gamma_u(t)[1+r] \quad \text{equal if } H_u(t+1)>0.\]

Notice that in this case, household production becomes a non-binding constraint with \(S(t+1)\) following a first order Markov process.

The equilibrium demand functions for both regimes may be obtained given a straightforward application of the implicit-function theorem over conditions (3.5)-(3.6) and (3.9')-(3.10'). Consequently, consumption and leisure demands in each regime are function of their marginal utility of income \([\lambda_{m,u}(t)]\), the payoff rate for household production \([S(t)]\) and the market wage rate \([W_u(t)]\)

\[
\begin{align*}
\text{Regime:} & \quad \text{Mexico} & \quad \text{U.S.} \\
(3.12) & \quad C^*(t) = C[\lambda_m^*(t)] & \quad C^*(t) = C[\lambda_u^*(t)] \quad \text{with } C'<0; \\
(3.13) & \quad L_m^* = \frac{L[\lambda_m^*(t)]}{S^*(t)} & \quad L_u^* = \frac{L[\lambda_u^*(t)]}{W_u(t)} \quad \text{with } L'<0.
\end{align*}
\]

According to expressions in (3.13), wage rates in each country affect the demand for leisure in different ways. Market wages in the U.S. regime appear as an explicit argument in the functional form, while in the Mexican case the wage rate and the marginal productivity of home production interact in the determination of its leisure demand. This argument becomes more evident in the derivation of \(S^*(t)\) that follows.

The optimal payoff value of household activities in the Mexican regime \([S^*(t)]\) may be obtained from conditions (3.7)-(3.8) and their intertemporal equivalence (3.7')-(3.8') where\(^8\)

\(^8\) This argument assumes the existence of a stationary equilibrium in the sense defined in section 2.2. In such a case, consumers allocate time to both productive activities: household and market activities. The
\[ \lambda_m(t)[W_m(t) - S(t)] = \delta \mu_m(t), \]
\[ \lambda_m(t)[W_m(t+1) - S(t+1)] = (1+r)\delta \mu_m(t). \]

Some algebraic manipulation of both conditions yields a first order difference equation of the form

\[ S(t+1) = (1+r)\sigma S(t) + W_m(t+1) - (1+r)\sigma W_m(t). \]

Notice that solution of this expression is stationary if its characteristic roots fall outside of the unitary circle of the complex plane. This condition is met when \( p = (1+r)\sigma \) is less than one. Let \( \Delta W_m(t) = W_m(t) - pW_m(t-1) \) and given the boundary condition \( S(0) = S_0 \), then \( S^*(t) \) may be solved backwards for the time horizon \( \tau = 0 \ldots t \),

\[ S^*(t) = p^tS_0 + \sum_{\tau=1}^{t} p^{t-\tau} \Delta W_m(\tau). \]

The optimal condition (3.14) relates the current value of household production with the evolution of the market wage in the Mexican economy. Indeed, any change in current levels of wage rate in the domestic market (e.g. an economic recession) will affect the \textit{a-priori} composition between market and household activities. Furthermore, expression (3.14) shows that temporary changes in the Mexican wage rate has little effect over the labor-leisure decision obtained in (3.13). Conversely, permanent effects in the market wage will not only influence the leisure demand through changes in \( S^*(t) \), but also will affect the marginal utility of income \( \lambda_m \).

Lastly, the basic analysis is completed with the specification of the optimal amount of time allocated to home production \( [I^*(t)] \). From constraint (3.3), \( I(t) \) may be expressed as a function of current and past home-productivity payoffs, current leisure demand and a stochastic term that characterizes non-expected family and economic events

\[ I(t) = -\frac{1}{\delta} [S(t) - \sigma S(t-1) - \varepsilon(t)] - L_m(t). \]

generalization for a non-stationary case with only household production follows straightforward.
Using the optimal values in equations (3.13) and (3.14), the previous expression may be written as

\[
I^*(t) = \frac{\delta(t)}{\delta} - \frac{1}{\delta} \left[ p^{t-1} S_0(\rho-\sigma) + \Delta W_m(t) + (1-\sigma) \sum_{\tau=1}^{l-1} p^{t-1-\tau} \Delta W_m(\tau) \right] - L^*_m(t).
\]

Finally, recall from the discussion in Chapter 2 that the vector of control variables \([C, L_m, L_u, I]\) is formed by two major components: Current direct effects and life cycle information. "λ-Demand" functions (3.12), (3.13) and (3.15) decompose decisions of consumption and allocation of time into variables actually observed in period \(t\), e.g. \(W_m(t)\) and \(W_u(t)\), and a lifetime element \(\lambda(t)\) that summarizes all past and future information relevant to consumers. The marginal utility of income changes only when new information becomes available, therefore anticipated shifts in state variables like wage rates and asset accumulation held \(\lambda(t)\) constant. In fact, temporary changes in the state of the world has little effect on this variable.

To summarize, this section presents a general discrete model where future events are uncertain. Using dynamic programming techniques, the maximization problem is solved for the control variables, where current and lifetime elements interact in the determination of optimal consumption and labor supply functions. The basic consumption-leisure choice model constitutes one element of the immigration decision, which characterizes the individual's benefit function in each country. However, switching regimes is costly and immigration costs constitute a major element in the Mexican participation decision. This issue is reviewed in the following section.

### 3.3 Immigration Costs

In previous theoretical and empirical work, immigration costs have been treated in much the same way as a fixed license fee in traditional consumer theory [Chiswick, 1977; Reimers, 1982; Borjas, 1987]. Undocumented workers are assumed to incur fixed time and/or money costs upon entry in the U.S. labor force. These components of the total cost are assumed to be invariant to changes in the economic environment. This section, in contrast, studies two sources of immigration costs, firstly in the context of time and monetary fixed
costs, and secondly in relation to variable or search costs. An important implication that emerges from this analysis is the existence of a link between domestic (U.S.) labor market disequilibrium conditions and search costs. A direct consequence of this relationship is that exogenous increases in the domestic wage rate will actually discourage Mexican participation in the U.S. labor market.

3.3.1 Fixed Costs

In order to participate in the U.S. labor force undocumented workers engage in a series of private costs of migration. They are, in particular, monetary or fixed-entry costs and non-monetary costs. The former include out-of-pocket expenses of switching regimes, while the latter include forgone earnings and "psychic" costs of changing the individual's environment. These concepts are reviewed below.

3.3.1a Monetary Costs

This category includes expenditures for transportation, food, lodging and crossing fees. The magnitude of those costs are by no means insignificant considering that, typically, undocumented workers come from rural or blue-collar urban regions. Browning and Rodriguez [1985] pointed out that "even the young unattached man who hitchhikes rides to the border, swims across the river on his own, and then walks several hundred miles to his destination, needs a stake" (p. 287). However, the do-it-yourself approach has become rare in recent times, even among the young men. Now, an efficient crossing technology has developing a profitable industry. Virtually everyone use the services of a coyote (those who guide illegal aliens across the border). This "agent" provides a full range of services, from motorist delivery to the destination to finding housing and even an initial job.

In 1981 the rate for an adult to cross the border from Mexico and to be delivered in a Texas destination (San Antonio or Houston) was about 300 dollars [Browning and Rodriguez,1985], while in 1985 the same service ranged from 500 dollars to 1000 dollars [Huddle,1985]. For a family of four the tour would require more than 2500 dollars. Quite sizable amounts considering that a Mexican urban blue-collar worker earns about three dollars per day.

Aside from the fixed costs involved in reaching the labor market location, undocumented immigrants need some financial resources to defray costs while they settle and find
jobs. Most of the time this is not a major consideration, since about everyone has some relative or friend already working in United States. In addition, a social network (charity associations, chapels and so on) will provide food and shelter to the newly arrived. They will be critical contacts in finding the first job.

3.3.1b Non-Monetary Fixed Costs

The consideration of this sort of costs are, in terms of Sjaastad [1962], probability far more significant than fixed-entry costs. The opportunity costs related with earnings forgone while traveling and learning a new job are concepts included in this category. However, Browning and Rodriguez [1985] pointed out that most jobs available to undocumented immigrants rarely demand skills not acquired already on the job in their source country. Hence, on-the-job training and learning costs are not a major obstacle.

Nonetheless, the most important learning process faced by the illegal workers is their inability to speak English. Most Mexican immigrants do not learn the language in any systematic fashion. They acquire, instead, a minimal basic vocabulary of words and phrases enabling them to perform adequately on the job and routine shopping situations. This provides the rudimentary communication skills needed to move around the U.S. labor market.

A second form of non-monetary costs considered are the "psychic" costs reflected in terms of the reluctance to leave familiar surroundings, family, friends and so on. These sort of costs have special consideration in social environments that involve strong family ties. Sjaastad recognized that "psychic" costs do not affect resource allocation, since they do not involve real resources for the economy and should not be included as part of the immigration process.

In order to analyze correctly undocumented mobility, two different categories need to be distinguished: Individual and family immigration. Browning and Rodriguez [1985] stress this point by arguing that individuals working in the U.S. for only a short time can engage in all kinds of "unnatural" behavior (e.g. working 70 hours per week, sharing a room with two or more friends, saving and sending back home half or more of their income), because their time horizon imply temporary permanence. Family migration, however, requires a settlement process with different strategy. In the latter case, a time horizon set the basis for a permanent residence.
3.3.2 Search Costs

Labor force participation literature has long recognized the importance of measures of labor market conditions in its efforts to explain participation rates. In particular, undocumented labor force participation is affected by the so-called "discouraged worker" hypothesis. The argument stresses that market participation is less likely to occur when jobs in the U.S. are more difficult to find. Although this is a notable idea in the context of undocumented immigration, it has not been formally documented in the economic literature. A simple model of linear "search technology" is developed below.

The basic proposition suggests that search costs are negatively correlated with the domestic excess-demand in the U.S. labor market. Potential undocumented workers perceive a-priori probabilities of finding jobs in the domestic low-skill market. The likelihood of a successful immigration by Mexican workers, ceteris paribus, is mapped in a measure of time and resources required in search for jobs. When the labor market shows large net domestic demand, jobs are easy to locate since employment competition is relatively low among participants. Conversely, requirements of undocumented labor force are reduced when local residents are able to clear the domestic labor market. In this context, illegal immigration acts as a "reservation army" that responds to contractions and expansions of domestic labor supply and demand. Accordingly, search costs \( F_s \) will decrease as result of a higher excess-demand in the domestic labor market

\[ F_s = f(L^D - L^S_d), \quad \text{with} \quad f^{*}\leq 0. \]

A simple description of this argument is presented in figure 3.1. Total market supply \( L^S \) is given by the horizontal sum of labor supplies in both sectors: Domestic residents \( L^S_d \) and immigrant workers \( L^S_u \). The joint equilibrium in the low-skill labor mar-

---

9 This subsection was stimulated in part by Angel O'Dogherty. Of course, he is not responsible for any errors in the argument.
10 See Burdett and Mortensen [1978] for an example of this literature.
11 A little insight is introduced to the general analysis by using some notion of "diminishing returns" to searching costs.
12 Some readers will be tempted to characterize this argument as "pull" factors of undocumented immigration. Chiswick [1977 and 1978], among others, developed a series of hypotheses related to "push-pull" elements of migration. However, the search model presented here does not depart from such an approach, which is known by its lack of economic theory underlying its basic conclusions.
ket is achieved at the wage rate $W_e$, with total employment $H^D$ and a excess-demand given by the distance $H^D H_d^S$. Notice that, the amount of labor supplied by domestic workers is $H_d^S$, while Mexican workers provide $H^D - H_d^S$.

Figure 3.1—Excess-Demand in the Domestic Low-Skill Labor Market

Consider a non-expected contraction in the domestic excess-demand caused by an exogenous event. Here, the new market disequilibrium shows a contraction in the domestic excess-demand, which in turn increases the equilibrium wage rate and reduces the total labor force. Consequently, search costs soar as results of higher competition in the low-skill labor market.

Notice that the probability of undocumented labor force participation in the U.S. is significantly small when the domestic labor market is in equilibrium at wage $W_d$, i.e. excess-demand is zero. In such a case, search costs become prohibitive for Mexican immigrants, and most of the entire labor demand will be supplied by resident workers.\textsuperscript{13} Accordingly, disequilibrium conditions in the domestic labor market is the compelling force driving sectorial composition of the low-skill labor force: Illegal immigration is encourage by large domestic excess-demand, while equilibrium conditions promote domestic partici-

\textsuperscript{13} Hill [1985] studied the welfare implications of such a policy. The author concludes that efficiency gains and tax revenues overshadow the possible welfare loss due to domestic job displacement. This conclusion is supported, in a partial equilibrium context, by the identification of “Harberger” triangles of consumer surplus in figure 3.1.
It is easy to show in a demand-supply framework the relation between excess-demand and market wage rates. The effective wage rate is depressed with disequilibrium conditions in the domestic low-skill market which are a result of external shocks, e.g. minimum wage legislation, enforcement of employer sanctions, etc.,

\[ W_u = g(L^D - L^S_d), \quad \text{with} \quad g' < 0; \]

assuming that \( g(\cdot) \) is concave and twice differentiable, this condition may be inverted as

\[ L^D - L^S_d = g^{-1}(W_u). \]

(3.16) \[ F_s = f[g^{-1}(W_u)] = F(W_u) \quad \text{with} \quad F' > 0. \]

Although conventional wisdom has suggested that undocumented immigration is motivated by large domestic wages, expression (3.16) indicates that Mexican participation is actually dissuaded as a result of higher wage rates in the United States. Figure 3.2
presents an integrated analysis that incorporates this result. Labor supply decisions (figure A) are independent of undocumented participation decisions (figure B). The north-west quadrant on both figures represents equation (3.16), where domestic wage rates ($W_d$) and search costs ($F_s$) observe a positive relation; in addition, a linear accounting function is described in the south-west quadrant, where the probability of Mexican participation (GAP) decreases with higher costs. Finally, the resulting function is presented in the south-east quadrant, where the participation decision is positively correlated with market wages.

To summarize, it is shown that exogenous increases in the U.S. market wage, given that leisure is a normal good, will raise the number of hours supplied by undocumented workers already participating in the labor market (figure A). However, because the domestic excess-demand falls as a result of a wage hike, ceteris-paribus, the likelihood of further Mexican participation also falls (figure B). In this case, available jobs become scarce and competition drives up search costs.

### 3.4 An Empirical Specification

The previous two sections have independently addressed two different components of the Mexican participation rule: Benefit functions and immigration costs. This section formulates an integrated empirical model of undocumented participation and labor supply based on the concepts described above. In particular, the analysis focuses in the effects of wage rates on immigration flows to the U.S. and the decision process in which Mexican participation take place. Furthermore, this section imposes a stronger structure than the initial model in an attempt to derive testable implications.

Undocumented workers switch across regimes at each point in their life cycle so as to maximize the present value their net lifetime utility. This concept summarizes the economic content of the theory of migration proposed by Sjaastad [1962]. In this context, “Migration is viewed as a rational investment process carried out at each stage of the life cycle.” [Polachek and Horvath, 1977 p.103] Illegal immigration is addressed as one of resource allocation. An efficient allocation is achieved since undocumented workers will not migrate until their productivity in the U.S. is sufficiently high to compensate for net benefit differentials with respect to Mexico.

Consider the optimal value function [$V^*(t)$] in which switching regimes is a feasible
alternative. Then, the initial Bellman equation in (3.1) may be modified as

\[(3.17) \quad V^*(t) = \text{MAX} \{ U[C^*_u(t), L^*_u(t)], U[C^*_m(t), S^*(t) L^*_m(t)] - F(t) \} + V^*(t+1). \]

Where \(U(\cdot)\) constitutes the indirect utility function of the U.S. and the Mexican regimes, respectively; they are optimal solutions of the maximization problems in (3.1). \(F(t)\) represents costs of migration that may be written as a convex combination of fixed (monetary and non-monetary) costs \((F_x)\) and search costs \((F_s)\). Recall that, by virtue of expression (3.16), the latter concept is an increasing function of the wage rate in the U.S.:

\[
\log F(t) = \log F_x + \log F_s = \log F_x + f \log W_u(t), \quad \text{with} \quad 0 < f \leq 1.
\]

The optimal benefit function in (3.17) reveals that Mexican workers will choose the regime that provides higher degree of net utility in each period of their life cycle. By doing this, consumers will optimize their behavior with respect to all available information at each \(t\). The implications of this rational expectation behavior is that participation decisions may be revised as a result of new information in the future.\(^{14}\) The dynamic characteristics of the model provides the ability to incorporate unexpected changes in the economic environment at any point in time.

In this framework, the undocumented participation rule (GAP) follows from a straightforward application of equation (3.17)

\[(3.18) \quad \text{GAP}(t) = U[C^*_u(t), L^*_u(t)] - \{ U[C^*_m(t), S^*(t) L^*_m(t)] + \log F(t) \} \geq 0.\(^{15}\)
\]

Mexican immigration to the U.S. is determinate by the sign of condition (3.18). Illegal participation will occur only when GAP(t) is positive.

In order to explicitly access the net effects of the state variables \((W_u, W_m, S, \text{and } A)\) over the participation decision, additional structure may be imposed into condition (3.18). Consider a specific separable and additive utility function of the type

\(^{14}\) See Hall [1978], MaCurdy [1985] and Altonji [1986] for further discussions on rational expectation models and empirical analysis of labor supply behavior over the life cycle.

\(^{15}\) Note that condition (3.18) is invariant to monotonic transformations, i.e. GAP(t) = U_w/U_mF(t).
\[ U_m(t) = a \log C_m(t) + b \log [S(t) L_m(t)], \]

(3.19) \[ U_u(t) = a \log C_u(t) + b \log L_u(t), \quad \text{for } t = 0 \ldots T; \]

where \( a \) and \( b \) are time invariant productivity parameters that are homogeneous across regimes. Therefore, under the set of preferences (3.19), the GAP condition may be rewritten as

\[ \text{GAP}(t) = a[\log C_u''(t) - \log C_m'(t)] + b[\log L_u''(t) - \log S^*(t) L_m'(t)] - \log F(t). \]

According to the stochastic model of life cycle behavior outlined in Section 2, the optimal demand functions (3.12)-(3.13) are determined by lifetime indirect and current direct elements. Assuming internal solutions in the first-order conditions (3.5)-(3.11), the "\( \lambda \)-demand" functions under the preferences specification (3.19) may be expressed as

\[
\begin{align*}
\text{Mexico} & : & C_m^*(t) &= \frac{a}{\lambda_m^*(t)} \\
\text{U.S.} & : & C_u^*(t) &= \frac{a}{\lambda_u^*(t)} \\
\text{U.S.} & : & S^*(t) L_m^*(t) &= \frac{b}{\lambda_m^*(t)} \\
\text{U.S.} & : & L^*_u(t) &= \frac{b}{W_u(t) \lambda_u^*(t)}.
\end{align*}
\]

Taking logarithms, and using condition (3.18) and the definition of immigration costs, the Mexican participation rule may be obtained as a function of lifetime and current events,

(3.18') \[ \text{GAP}(t) = (a + b)[\log \lambda_m^*(t) - \log \lambda_u^*(t)] - b(1 + f) \log W_u(t) - \log F_X. \]

The permanent component of the GAP condition \( [\lambda(t)] \) is a function of every variable relevant to the immigration process over time. In particular, it depends on wage rates in each country, home productivity and asset accumulation. When undocumented workers acquire new information in the form of non-expected shocks, they review their present and future participation plans. The precise form of this revision process depends on the relation between the marginal utility of income and the state variables. A stochastic specification for
\( \lambda(t) \) follows.

### 3.4.1 Determination of the Marginal Utility of Income

\( \lambda(t) \) is the result of the optimization process which leads to an important behavioral interpretation regarding to lifetime information. Contrary to the analysis in the previous chapter, life cycle components of the immigration decision \( \lambda_i(t) \) [for \( i=m, u \)] are random variables that are determined using the information set available at age \( t \).

It is easy to show, using first-order conditions (3.5) and (3.5'), the intertemporal characteristics of the marginal utility of income in the Mexican regime

\[
E_t \{ \lambda_m(t+1) \} = R \lambda_m(t),
\]

where \( R = [(1+\rho)/(1+r)]<1 \) (i.e. \( r > \rho \)) is required for stationary purposes. According to MacCurdy [1985] condition (3.20) represents the consumer's decision rule in terms of time allocation of resources. Potential immigrants choose to allocate time to alternative activities so that the marginal utility of income at age \( t \) equals the discounted expected value of next period marginal utility of income. The author argues that the marginal utility of income follows a martingale process, that is \( E_t \{ \lambda_m(t+1) \} \) only depends on \( \lambda_m(t) \). "The consumer controls the time path of \( \lambda(t) \) through his accumulation of financial wealth." [Ibid, p. 117] Condition (3.20) dictates how immigrants allocate their financial and time resources to stochastic shocks.

By definition, the mathematical expectation operator may be expressed in terms of a stochastic term. In fact, let the left hand side of (3.20) obey the following transformation

\[
E_t \{ \lambda_m(t+1) \} = \lambda_m(t+1) \exp[\eta_m(t+1)], \quad \text{where} \quad \eta_m(t+1) \sim (0, \sigma^2_\eta).
\]

The disturbance \( \eta_m(t+1) \) summarizes the impact of all new unexpected information that becomes available in period \( t+1 \). Substituting the stochastic definition of \( E_t \{ \lambda_m(t+1) \} \) into (3.20) and taking logarithms, it can be shown that the marginal utility of income may be expressed as a first order Markov process of the form,\(^{16}\)

---

\(^{16}\) Formally a Markov process is sensitive to the specification of the stochastic term \( \eta(t+1) \) which is assumed to be an iid random term. Empirically, this may or may not hold, in particular homocedasticity.
\[(3.21) \quad \log \lambda_m(t+1) = \log R + \log \lambda_m(t) - \eta_m(t+1).\]

Equation (3.21) represents a simple difference equation with a forward-looking solution for the interval \([t, T]\). This leads to the optimality condition for the marginal utility of income in the Mexican regime:

\[(3.22) \quad \log \lambda_m^*(t) = (t-T)\log R + \log \lambda_m(T) + \eta_m(t), \quad \text{where} \quad \eta_m(t) = \sum_{\tau=t}^{T} \eta_m(\tau).\]

Notice that this result requires the boundary condition \(A(T) = 0\), given that \(\lambda_m(T)\) is greater than zero; recall that \(\partial V_m(t) / \partial A(t) = \lambda_m(t)\) represents the transversality condition for the state variable \(A(t)\).\(^{17}\) In addition, a similar procedure ensures that the derivation of the marginal utility of income that characterizes the U.S. regime,

\[(3.22') \quad \log \lambda_u^*(t) = (t-T)\log R + \log \lambda_u(T) + \eta_u(t), \quad \text{where} \quad \eta_u(t) = \sum_{\tau=t}^{T} \eta_u(\tau).\]

Expressions (3.22) and (3.22') describe the stochastic properties of the life cycle component regarding consumption and allocation of time decisions in each regime. These processes are consequences of rational economic behavior which requires that undocumented workers revise their participation decisions upon the realization of non-anticipated events. Moreover, notice that these conditions are expressed in terms of the marginal utility of income in the last period \(\lambda(T)\), which is unknown at period \(t\). A simple procedure to compute this value is presented below.

### 3.4.1a The Mexican Regime

To formulate an empirical model capable of predicting variations in undocumented labor force participation, the life cycle component must be derived as a function of past and current information. An explicit solution for expression (3.22), in the case of Mexico, requires that the budget constraint (3.2) takes the form of recursive substitutions upon the boundary conditions \(A(t)=A_t\) and \(A(T)=0\).

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problems may arise. See MaCurdy [1985] for further discussion.

\(^{17}\) In general, optimal control techniques suggest that state variables are determined from past periods, e.g. \(A_t\) and \(A_0\), while costate variables are solved forward in time, e.g. \(\lambda_m(t)\).
\[ A_t = \sum_{\tau=t}^{T-1} \frac{1}{(1+r)^{\tau-t}} \left[ C_m^*(\tau) - W_m(\tau)H_m^*(\tau) - S^*(\tau)I^*(\tau) \right]. \]

Using optimal values for consumption and household production functions given in (3.12') and (3.15), respectively, the asset accumulation constraint may be rewritten as

\[ A_t = \sum_{\tau=t}^{T-1} \frac{1}{(1+r)^{\tau-t}} \left[ \left( \frac{\left( a+bW_m(\tau) \right)R^{T-\tau}}{\lambda_m(T)S^*(\tau)} \right) - W_m(\tau) - \left[ S^*(\tau) - W_m(\tau) \right] \frac{\varepsilon(\tau-1) - \Delta S^*(\tau)}{\delta} - \frac{br^{T-\tau}}{\lambda_m(T)S^*(\tau)} \right]. \]

Recall that \( \Delta S^*(t) = S^*(t) - \sigma S^*(t-1) \), where the optimal marginal productivity of household activities \( S^*(t) \) is governed by equation (3.14). The objective of this derivation is to obtain a closed form solution of \( \lambda_m(T) \) in terms of the information set available to immigrants. Inspection of the previous expression, however, reveals that an explicit solution only may be obtained by numerical analysis techniques. Macurdy [1985] indicated that "only rarely is it possible to obtain an analytical solution for \( \lambda_m(t) \) in terms of these variables." (p. 121) Instead, the above equation may be expressed in reduced form,

\[ A_t = A_m \left[ \lambda_m(T), S_0', \sum_{\tau=t}^{T-1} W_m(\tau), v_m(t) \right]; \]

where \( v_m(t) \) is a vector of serially uncorrelated stochastic terms that incorporate prediction errors generated through the marginal utility of income \( [\lambda_m(t)] \) and non-expected fluctuations in home productivity \( [S(t)] \):

\[ v_m(t) = \left[ \sum_{\tau=t}^{T-1} \hat{r}_m(\tau) + \sum_{\tau=t}^{T-1} \varepsilon(\tau-1) \right]. \]

Assuming that the inverse function \( A_m^{-1} \) exists, a straightforward application of the implicit function theorem yields the reduced form for the end-point condition for \( \lambda_m \)

\[ \lambda_m(T) = \lambda_m \left[ A_t, S_0', \sum_{\tau=t}^{T-1} W_m(\tau), v_m(t) \right]; \]
from discussion in Chapter 2, it was clear that the partial derivatives of \( \lambda_m(T) \) with respect to \( A_t, S_0 \) and \( W_m \) are all negative. Accordingly, consider a log-linear approximation of function \( \lambda_m(T) \) given by

\[
\log \lambda_m(T) = l_0 + l_1 A_t + l_2 S_0 + l_3 \sum_{t=1}^{T-1} \log W_m(t) + \phi_m(t).
\]

(3.23)

The empirical use of equation (3.23) requires an assumption concerning the formation of expected wages in the future. In order to generate unbiased predictions of these expectations, two approaches are widely used in labor supply estimation. Firstly, Macurdy [1981 and 1986] employed quadratic approximations of time as instruments for expected wage rates

\[
\log W(t) = \pi_0 + \pi_1 t + \pi_2 t^2 + e(t), \quad \text{for } t=0 \ldots T.
\]

Secondly, Heckman and Macurdy [1980], Altonji [1986] and Borjas [1987] among others, generate wage predictions using time invariant socio-demographic characteristics of individuals, like education, marital status, sex and so on. This approach permits the use of elements such as work experience and human capital investment in a convenient specification.\(^{18}\)

A modified version of these approaches are adopted in this model. Expected wages in the Mexican regime are described with the following log-linear process

\[
\log W_m(t) = \bar{W}_m(t) + X\phi_0 + \epsilon_m(t);
\]

where \( \epsilon_m(t) \) is a disturbance term representing the contribution of unobserved variables that shows the usual stochastic properties. Intertemporal elements that characterized Macurdy's specification are described by the average productivity of market activities \( [\bar{W}_m(t)] \) in equation (3.24). This variable captures transitory changes in the wage rate through a first order Markov process of the form

\(^{18}\) See Heckman [1976] for a study that formally explores the empirical implications of human capital investment in a dynamic model of labor supply. The present study does not persuade this line of research.
\[ \bar{w}_m(t) = \alpha_0 + \alpha_1 \bar{w}_m(t-1) + \zeta(t), \quad \text{with} \quad \zeta(t) \sim (0, \sigma^2_\zeta). \]

Assuming an stationary process, i.e. \(|\alpha_1|<1\), the transitory component of the Mexican wage equation collapses to \( \bar{w}_m(t) = \alpha + \zeta(t) \), with \( \alpha = \alpha_0 / (1 - \alpha_1) \).

In addition, expression (3.24) preserves time-invariant, permanent characteristics that may describe job experience and human capital investment by means of a vector of socio-demographic variables given by \( X \phi_0 \). It is easy to show that the process generating wage expectations is governed by

\[ \sum_{t=1}^{T-1} \log W_m(t) = \alpha + X \phi_0 + \tilde{\epsilon}_m(t), \quad \text{with} \quad \tilde{\epsilon}_m(t) = \sum_{t=1}^{T-1} [e_m(t) + \zeta(t)]. \]

Accordingly, equation (3.23) and (3.24) describe the life cycle component of the participation decision \( \lambda_m(T) \) in terms of the information set available to immigrants at time \( t \),

\[ \log \lambda_m(T) = \lambda_0 + \lambda_1 A_1 + \lambda_2 S_0 + X \phi + \bar{v}_m(t); \]

note that the stochastic component of (3.23), i.e. \( \bar{v}_m(t) = \bar{v}_m(t) + \lambda_3 \tilde{\epsilon}_m(t) \), aggregates three different sources of prediction errors: Household productivity \([e(t)]\), market wage \([e(t)]\) and intertemporal preferences introduced by variations in the marginal utility of income \([\eta(t)]\).

Finally, according to the theoretical model developed here, parameters in (3.23') are expected to have the following signs: \( \lambda_0 = (\lambda_0 + \alpha)^2 > 0 \), \( \lambda_1 < 0 \), \( \lambda_2 < 0 \) and \( \phi = \lambda_3 \phi_0 > 0 \).

### 3.4.1b The U.S. Regime

The reduced form of the end-point condition \( \lambda(T) \) for the U.S. follows a similar procedure than the one outlined in the Mexican case. In this context, equation (3.23) may be rewritten in terms of the U.S. regime,

\[ \log \lambda_u(T) = \lambda_{u0} + \lambda_{u1} A_1 + \lambda_{u2} \sum_{t=1}^{T-1} \log W_u(t) + \bar{v}_u(t). \]
Notice, that household productivity is not included in (3.25), since $S(t)$ is exclusive of the Mexican regime. Furthermore, the disturbance term $\hat{\theta}_u(t)$ reflects only non-expected variations in the marginal utility of wealth $[\eta_u(t)]$. The stochastic characteristics in the U.S. are a result, mainly, of prediction errors in the market wage $[W_u(t)]$; while in the Mexican case, these errors may originate either from changes in wage rates $[W_m(t)]$ or home-productivity payouts $[S(t)]$.

The determination of the stochastic behavior that generate consistent U.S. wages predictions is described by the following auto-regressive process:

$$
\log W_u(t) = \bar{w}_u + \beta \log W_u(t-1) + e_u(t);
$$

where $|\beta|<1$ and $e_u(t)$ is assumed to be white noise. Here, the average productivity of labor market $[\bar{w}_u]$ is time-invariant and describes permanent characteristics attached to undocumented workers. Moreover, $\bar{w}_u$ is a function of a vector of socio-economic variables that reflect U.S. specific human capital investment and undocumented experience ($Z$), e.g. ability to speak English and time since immigration, respectively. The solution of the earlier first-order difference equation for the interval $[t, T-1]$ yields

$$
\log W_u(t) = \frac{Z\theta_0}{1-\beta} + \sum_{t=1}^{T-1} (1-\beta)^t e_u(t).
$$

(3.26)

In addition, note that wages in expression (3.25) may be decompose in current and future values,

$$
\sum_{\tau=t}^{T-1} \log W_u(\tau) = \log W_u(t) + \sum_{\tau=t+1}^{T-1} \log W_u(\tau);
$$

applying this definition in conjunction with equation (3.26) leads to the expression of expected wages as a function of current information

$$
\sum_{\tau=t}^{T-1} \log W_u(\tau) = \log W_u(t) + \frac{Z\theta_0}{1-\beta} + \hat{e}_u(t).
$$

This expression may be plugged in condition (3.25) in order to obtain a complete
specification of $\lambda_u(T)$ that can be use in empirical estimation procedures,

\[(3.25') \quad \log \hat{\lambda}_u(T) = l_u0 + l_u1A_t + l_u3\log W_u(t) + Z\theta_{\bar{w}} + \hat{v}_u(t); \]

where $\hat{v}_u(t)=l_u3\hat{\phi}_u(t)+\hat{\theta}_u(t)$ incorporates non-expected variation in the $\lambda_u(T)$. Lastly, similar to the Mexican case, the life cycle model presented in the former chapter guaranties the following parameter signs: $l_u0<0$, $l_u1<0$, $l_u3<0$ and $\frac{\theta_{\bar{w}}}{1-\beta} > 0$.

### 3.4.2 The Estimation Model

A complete empirical model of Mexican undocumented participation and labor supply is proposed in this subsection. The various modifications of the basic model presented in equations (3.13') and (3.18) may now be assembled into a system of simultaneous linear equations defined for a cross-section sample of $I$ individuals. Next, the major components of the empirical model are discussed.

#### 3.4.2a The Participation Rule (GAP)

The decision rule (GAP) establishes the circumstances in which undocumented workers are encouraged to migrate. Equation (3.18') summarizes the elements that determine the decision process

\[(3.18') \quad \text{GAP}(t) = (a+b)[\log \hat{\lambda}_m^*(t) - \log \hat{\lambda}_u^*(t)] - b(1+f)\log W_u(t) - \log F_x. \]

Notice that in contrast with traditional models of undocumented migration, wage differentials across regimes do not appear as an explicit argument in the participation rule. Furthermore, expression (3.18') decomposes the immigration decision in current and life cycle components. In order to access lifetime effects of undocumented participation in an usable empirical context, the marginal utility of income in each regime is derived in terms of past and current information available to individuals. The following four equations outline the life cycle elements of the participation function:

(i) Mexico

\[(3.22) \quad \log \hat{\lambda}_m^*(t) = (t-T)\log R + \log \hat{\lambda}_m(T) + \hat{\eta}_m(t), \quad \text{where} \quad \hat{\eta}_m(t) = \sum_{\tau=t}^{T} \eta_m(t); \]
\[ (3.23) \quad \log \lambda_m(T) = l_m + l_{m1} A_t + l_{m2} S_0 + X\phi + \tilde{v}_m(t). \]

(ii) The U.S.

\[ (3.22) \quad \log \lambda_u^*(t) = (t-T) \log R + \log \lambda_u(T) + \tilde{f}_u(t), \quad \text{where} \quad \tilde{f}_u(t) = \sum_{\tau=t}^{T} \eta_u(\tau); \]
\[ (3.25) \quad \log \lambda_u(T) = l_u + l_{u1} A_t + l_{u2} \log W_u(t) + Z\theta_w + \tilde{v}_u(t). \]

Some algebraic manipulation of these expressions in conjunction with condition \( (3.18') \), yields the variable GAP(t) as a function of current variables and stochastic terms,

\[ \text{GAP}(t) = (a+b)[(l_{m-l_u}) + (l_{m1-l_u1}) A_t + l_{m2} S_0 - l_{u3} \log W_u(t) + (X\phi - Z\theta_w) + [\tilde{v}_m(t) - \tilde{v}_u(t)] + [\tilde{f}_m(t) - \tilde{f}_u(t)]] - b(1+f) \log W_u(t) - \log F_x. \]

Grouping terms, it is easy to show that \( (3.18') \) leads to the following estimation function

\[ (3.27) \quad \text{GAP}(t) = \gamma_0 + \gamma_1 A_t + \gamma_2 \log W_u(t) + [X-Z] \gamma + \zeta(t); \]

\[ \text{where} \quad \gamma_0 = (a+b)[l_{m-l_u} + l_{m2} S_0] - \log F_x, \]
\[ \gamma_1 = (a+b)[l_{m1-l_u1}], \]
\[ \gamma_2 = -[(a+b)l_{u3} + b(1+f)], \]
\[ [X-Z] \gamma = (a+b)[X\phi - Z\theta_w], \]
\[ \zeta(t) = (a+b)[\tilde{v}_m(t) - \tilde{v}_u(t) + \tilde{f}_m(t) - \tilde{f}_u(t)]. \]

Without further restrictions, coefficients in \( (3.27) \) are ambiguously determined. Their magnitude and direction of change are empirical questions. Nonetheless, a theoretical interpretation base on the previous discussion follows below.

The constant term \( \gamma_0 \) is composed of lifetime parametric elements given by home production outlays, real interest and time-preference rates; in addition, the coefficient \( \gamma_0 \) includes the fixed component of migration costs. There is not a conventional reason to predict

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19 All the results presented in this section rest on the important assumption that immigrants confront known, constant time-preferences and real interest rates, i.e. \( R(t) = R(t+1) \). However, MacCurdy [1985] introduces a dynamic interest rate as an additional source of stochastic behavior. In the present framework, deterministic variation in the real interest rate may be introduced with minor amendments through changes in the marginal utility of income. Hall [1978] argues that \"the importance of known variations in interest rates depends on the elasticity of substitution between the present and future\" (p. 976). Given low elasticities the influences of stochastic interest rates are inconsequential.
the dimension nor the sign of this parameter.

The financial net-worth coefficient ($\gamma_1$) is defined in terms of the life cycle differential effects of the asset accumulation constraint. Note that $l_{m1}$ and $l_{u1}$ captures lifetime responses to changes in the property income in Mexico and in the U.S., respectively. Not surprisingly, a decline in Mexicans' stock of wealth will encourage undocumented participation to the U.S. only if $l_{m1}>l_{u1}$. This condition implies that residents are more sensitive to changes in property income than migrant workers. Such a result has important policy implications, since substantial reductions in the net-worth of illegal settlements due to strict enforcement policy may have small effect over the immigration decision. In contrast, improvements in Mexico's economy may tend to be more effective in retaining domestic residents from leaving their country.

Perhaps the more relevant coefficient in the participation rule (3.27) is the uncompensated wage elasticity $\gamma_2$. This coefficient is composed of three elements: Life cycle $[l_{u3}(a+b)]$, current [b] and search cost [bf] effects. Lifetime permanent characteristics are positive since $l_{u3}<0$ and in general, tend to dominate the negative contribution of current direct factors, i.e. $l_{u3}(a+b)>b$.

In the absence of search costs, $\gamma_2$ is more likely to be positive which implies that undocumented migration is motivated by expansions in the U.S. wage rate. This is the theoretical reason underlying the conventional wisdom in Mexican undocumented participation. As noted in section 3.2, however, search costs are a major factor in the determination of the decision rule. In a more realistic setting, wage hikes may, in fact, discourage undocumented workers from participating in the U.S. labor force. This scenario will apply to the case where search costs and current effects dominate life cycle factors $[b(1+f)>l_{u3}(a+b)]$.

The vector of socio-demographic characteristics represents a linear combination of time-invariant attributes intrinsic to immigrants regardless their location ($X$) and features that distinguish U.S. labor force participation ($Z$). In addition, the transformation vector $\gamma$ permits a direct comparison of these characteristics.

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20 Recall that both $l_{m1}$ and $l_{u1}$ are negative.
21 See the discussion at the end of section 2.3.1b.
22 See figure 3.2 (B) for an direct relation between wage rate in the U.S. and the probability of undocumented participation.
Finally, the unobserved random vector $\xi(t)$ denotes additive latent components of the following three sources of uncertainty: Wage rate in the U.S. and Mexico [$e_u(\tau)$ and $e_m(\tau)$], household productivity [$e(\tau)$] and life cycle behavior [$\eta_u(\tau)$ and $\eta_m(\tau)$] for $\tau=\ldots T$. It is assumed that $\xi$ has a nondegenerate distribution function where the two first moments exist, i.e. $E[\xi]=0$ and $E[(\xi^2)]=\Sigma_{\text{Gap}}$. Furthermore, $\xi$ is distributed independently of the endogenous variable $\text{GAP}(t)$ for every $t$ as well as across individuals. Note that, in general, $\Sigma_{\text{Gap}}$ is not likely to be homoskedastic across time nor is this assumption required for estimation purposes.

3.4.2b The U.S. Labor Supply

Most cross-section studies on labor supply in the literature ignore life cycle theory. Traditionally, current wages and property income appear as regressors for annual hours of work. However, the empirical assessment of these estimates may lead to erroneous conclusions if lifetime considerations are not taken into account. Following MaCurdy [1985] two questions may be addressed regarding labor supply estimation: (i) The behavioral interpretation of cross-section labor supply estimates and (ii) the possibility of modifying cross-section models so that life cycle factors are taken into account.

To answer the latter question, consider a preferences specification for the U.S. regime that obeys a monotonic transformation of the utility function (3.19)

$$U_u(t) = G[C(t)] + b\log[1-H_u(t)];$$

where $G(\cdot)$ is an increasing function of current consumption and $b$ is a positive productivity parameter that sets hours of work [$H_u(t)$] as a decreasing function of preferences. Given the set of constraints (3.2)-(3.4) applicable to the U.S., the first order condition (3.10) may be rewritten in terms of the latter expression,

$$-\frac{b}{1-H_u(t)} \geq -\lambda_u(t)W_u(t),$$

equal if $\text{GAP}(t)>0$ for all $t$.

Taking logarithms and solving for the amount of hours worked yields
\[ \log[1-H_u(t)] \geq \log b - \log \lambda_u(t) - W_u(t); \]

assume that individuals participate in the U.S. labor market (GAP$>0$). Using optimal conditions for $\lambda_u^*(t)$ in (3.22) and (2.25) the former expression may be written as

\[ \log \left[ 1 - H_u^*(t) \right] = \log b + (T_0)\log R - \left[ \lambda_0 + \lambda_1 + \lambda_2 \log W_u(t) + Z\theta_h + \beta_u(t) \right] - \log W_0(t) + \tilde{\Gamma}_{10}. \]

Collecting terms, the optimal U.S. labor supply obeys the following structural form,

\begin{equation}
\log \left[ 1 - H_u^*(t) \right] = h_0 + h_1 t + h_2 A + h_3 \log W_u(t) - Z\theta_h + \nu(t); \tag{3.28}
\end{equation}

where

\[ h_0 = \log b + T_0 \log R - l_{u0}, \]
\[ h_1 = -\log R, \]
\[ h_2 = -l_{u1}, \]
\[ h_3 = -(1 + l_{u3}), \]
\[ \nu(t) = \tilde{\nu}_u(t) + \tilde{\Gamma}_{10}(t). \]

The question concerning the behavioral interpretation of the labor supply estimates is addressed next. Assessment of coefficients in equation (3.28) suggests that the variable age $t$ enters to the labor supply function as a result of life cycle consideration. Age is linked in the labor supply only through the marginal utility of income $\lambda_u(t)$. Typically, however, cross-section specifications fail to include age as a regressor in the labor supply ($h_1=0$) which results in misspecification problems due to omitted variables. In the present model, the exclusion of age would imply that the time-preference and the real interest rate are equal ($p=r$). Nonetheless, $h_1$ is expected to be positive since, in general, $R$ is less than one ($\log R<0$). This theoretical result indicates that older undocumented workers are more likely to dedicate less hours to market activities than relatively younger immigrants.

Similar to the age regressor, property income $A$ is introduced to the labor supply function only by means of $\lambda_u(t)$. In general, $l_{u1}$ is expected to be negative, so that higher stock of wealth will tend to discourage hours of work, i.e. $h_2>0$.

Income and substitution effects are captured by coefficient $h_3$. Unlike $h_1$ and $h_2$, the

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23 Recall that $R=(1+p)/(1+r)$ and, in general, $r>p$ such that $R<1$.  

uncompensated wage elasticity (h₃) integrates present and intertemporal effects. The net impact of wage rates over the amount of hours worked will depend on the relative sensitivity of these two components. In the absence of life cycle information (l₃=0), transitory expansions of the domestic wage rate unambiguously increase the amount of hours supplied (h₃<0); temporary changes in wages have a full impact on Hₚ(t). However, when lifecycle components are considered, permanent responses in expected earnings reduce the initial wage elasticity (since, in general, l₃<0). Consequently, according to the labor supply specification in (3.28), small absolute values of h₃ will imply that lifetime elements play an important role in the determination of wage elasticities. In contrast, estimated values of |h₃|→1 will be evidence in favor to traditional static labor supply models where only current effects are taken into account.

Furthermore, wage elasticity h₃ responds to “anticipated” and “unexpected” changes. This distinction is important because the former constitutes movements along the labor supply function while the latter represents shifts of the labor supply profile. In this context, the h₃ estimator confuses between transitory and permanent effects. Temporal changes have little effect on the unanticipated component of h₃ (l₃) and they will tend to lead large absolute values of the wage elasticity. This result has important implications in terms of identifying permanent and transitory migration. Illegal workers switching back and forth across regimes may be described by relatively low (absolute) wage elasticities relative to settlers with permanent residence horizons.

The participation decision rule (GAP) and the U.S. labor supply functions presented in equations (3.27) and (3.28) assume estimation using simultaneous equation techniques. An obvious source of simultaneous bias is observed in the determination of the U.S. wage rate, which is likely to be correlated with the endogenous variables GAP and Hₚ.

Endogenous wages may be rationalized in terms of human capital investment. Undocumented labor force participation represents a rational investment decision that enhance market productivity and therefore wages. A simple specification of the U.S. wage rate has been described before in expression (3.26):

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24 Notice that substitution and intertemporal terms are not equivalents. The latter responds to permanent changes in the wage rate but not to transitory effects.

25 The magnitude of this adjustment will depend on the sensitivity of lifetime effects with respect to transitory changes in the economic environment.
(3.26) \[ \log W_u(t) = Z\theta_w + \hat{\theta}_u(t); \]

where \[ \theta_w = \frac{\theta_0}{1-\beta}, \]

\[ \hat{\theta}_u(t) = \sum_{\tau=t}^{T-1} (1-\beta)^{\tau}[e_u(\tau)]. \]

Recall that vector \( Z \) represents a set of socio-economic characteristics intrinsic, but not exclusive, to the U.S. labor market. For instance, the ability to speak English will solely enhance the market productivity of undocumented workers if they participate in the U.S. labor force; in contrast, such a investment is useless in the Mexican regime.

Finally, the system of simultaneous equations given by expressions (3.26)-(3.28) fully characterize the behavior of undocumented Mexican workers in the U.S. labor market. The empirical model developed here provides a natural framework for interpreting estimates in cross-section specification. Moreover, it indicates how a cross-section model may be modified so that empirical estimates are interpreted in a life cycle context. Although lifetime planning has been incorporated in the underlying optimization process, these equations are described in terms of solely contemporaneous variables. Similar to traditional rational expectation models, migrant workers do not only use the relevant optimization model, but also make efficient use of their incomplete information.

**Concluding Remarks**

The discussion in this chapter focuses on the theoretical interpretation of cross-section estimates derived from the optimization of a dynamic programming problem. In particular, it is shown that wage differentials across countries are, by far, a crude simplification of the underlying elements defining Mexican participation. Rather, according to the decision rule GAP, illegal workers are sensitive to differences in marginal utilities of income \( \Lambda(t) \) between regimes, wage rates in the U.S. and fixed and search costs of migration.

The developed lifetime model under uncertainty is important because it relates the way in which random components enter into the empirical specification of Mexican participation and undocumented labor supply in the United States. Although the presence of
stochastic behavior in the immigration process introduces new elements to the former life cycle model, this chapter shows that much of the results obtained in the deterministic model are carried over into the uncertainty framework.

Lastly, an important conclusion derive from the preceding analysis is that expansions in U.S. wages may, in fact, dissuade participation of further Mexican settlers. Contrary to conventional wisdom, the undocumented labor force may be reduced as result of expansions in local wages that contribute to close the domestic excess demand for low-skill labor. This result is a direct consequence of the methodological differentiation between participation and labor supply decisions. Domestic wage rate increases will effectively reduce the number of new Mexican immigrants, while undocumented workers already participating in the U.S. will boost their amount of hours allocated to market activities.
CHAPTER 4

ESTIMATION OF A
TOBIT SIMULTANEOUS EQUATION MODEL
WITH TRUNCATED DEPENDENT VARIABLES

"... It is not the figures themselves, it's what you do with them that matters."  
K.A.C. Manderville

This chapter reports efficient estimates of the immigration decision and the labor supply functions of Mexican undocumented workers. Using the preceding economic theory as a guide, this chapter addresses the specification and estimation of an empirical model in a context of limited dependent variables. The econometric model develops here provides a natural framework for interpreting life cycle estimates associated with cross-section analysis.

A major problem confronting the study of undocumented immigration is the availability of empirical data. Two characteristics identify sample information in this field: Self-selection and unobservable Mexican regime. First, undocumented population is not a random sample, since variables may only be observed in a limited range, i.e. when Mexican residents actually participate in the U.S. labor market. This yields a truncated sample, where standard estimation procedures are asymptotically biased. Second, observations of the Mexican regime are missing, because it is unfeasible to identify potential undocumented workers, i.e. under certain conditions all the population could become illegal immigrants. These characteristics provide an estimation challenge not found in many other branches of econometrics.

The objective of this chapter is to test empirically the determinants of the Mexican
immigration decision and its effects over the U.S. labor market. In particular, the empirical analysis focuses on the effects of U.S. wages over the undocumented participation rule and their labor supply. Following the economic model outlined in former chapters is possible to decompose estimates of wage elasticities into their intertemporal and current effects. Furthermore, the econometric framework provides estimates for analyzing the impact of other variables such as financial wealth and socio-demographic characteristics.

Section 1 outlines the basic economic relations of the simultaneous system and discusses the limited characteristics of its dependent variables. Section 2 examines the misspecification problems observed in standard procedures due to self-selection. Here, the motivation for the use of alternative estimation methods is addressed. The proposed econometric model is featured in Section 3, where the actual derivation of the maximum likelihood function and its asymptotic properties are presented. The sample data and the variables are described in section 4. Section 5 and 6 report the estimation technique and the economic interpretation of empirical estimates for the U.S. wage equation, the undocumented labor supply and the Mexican participation rule. Here, some immigration policies are reviewed in light of the empirical results.

4.1 Estimation Theory

This section introduce the theoretical estimation framework which provides the basis of the empirical specification developed in subsequent sections. In addition, it indicates the modifications required to the original model in order to adapt a cross-section analysis. The self-selected (nonrandom) sample and the exclusion of one regime transform a simple simultaneous equation model into a Tobit specification with truncated dependent variables.

4.1.1 The Basic Model

The economic specifications developed in the previous chapter are solutions to an intertemporal optimization problem. Life cycle considerations are integrated to the participation rule and the labor supply. Illegal workers are assumed to plan their lifetime behavior making optimal use of the available information at age $t$. In this context, Mexican residents choose an optimal pattern that may result in transitory immigration or permanent residence either in Mexico or in the United States.
An empirical challenge of this analysis is to infer the life cycle behavior of immigrants using a set of exogenous contemporaneous variables. Let us summarize the theoretical framework in the following system of equations:\(^1\)

U.S. wage equation:
(4.1a) \[ \log W_{ui} = Z\theta + e_{ui}, \quad \text{with } i = 1...N_u. \]

U.S. labor supply:
(4.1b) \[ \log \left[ 1 - H_{ui}^* \right] = h_0 + h_1 t_i + h_2 A_i + h_3 \log W_{ui} - Z\theta + \nu, \]
\[ \text{with } i = 1...N_u. \]

Participation rule:
(4.1c) \[ GAP_i = \gamma_0 + \gamma_1 A_i + \gamma_2 \log W_{ui} + [X-Z]_T + \xi = YT + \xi, \]
\[ \text{with } i = 1...N. \]

Where
\[
\begin{pmatrix}
e_{ui} \\
\nu \\
\xi
\end{pmatrix}
\sim \begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix},
\begin{pmatrix}
\sigma_e & \sigma_{ev} & \sigma_{e\xi} \\
\sigma_{ev} & \Sigma_v & \sigma_{v\xi} \\
\sigma_{e\xi} & \sigma_{v\xi} & \Sigma_{\xi}
\end{pmatrix}
\]

\(N\) is the total Mexican population, and \(N_u\) is the number of actual undocumented workers already participating in the U.S. labor market.\(^2\) Note that equations (4.1a) and (4.1b) are specified only for the latter sample, while the participation rule (4.1c) is defined for the entire population \(N\). This system of equations constitutes a partially recursive model since the hours worked (\(H_{ui}\)) and the participation variable (\(GAP_i\)) do not appear as explanatory variables in any other equation. However, system (4.1) maintains its simultaneous properties because the variance-covariance matrix may not be diagonal.

An alternative specification for system (4.1) involves the use of earnings (\(E\)) in-

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\(^1\) In contrast with the equivalent simultaneous system given in (3.26)-(3.28), where dependent variables were a function of time, here the assumption of cross-section estimation has been explicitly incorporated in these expressions. Recall that bold capital and non-capital letters represent matrix and vectors, respectively.

\(^2\) This definition assumes that anyone living in México may be a potential immigrant. Although for some Mexican residents the likelihood of working in the U.S. may be rather small, it does constitute a feasible option.
stead of wage rates. Such a model may provide a more accurate description of the economic behavior in the Mexican undocumented population. Consider the following empirical specification

U.S. earnings equation:
\[(4.1a')\quad \log E_{ui} = \log W_{ui} + \log H_{ui}^* = Z\theta_w + e_{ui}.\]

U.S. labor supply:
\[
\log H_{ui}^* = h_0 + h_{1i} + h_2A_i + h_3\log W_{ui} + (h_3\log H_{ui}^* - h_3\log H_{ui}^*) - Z\theta_B + \nu,
\]
\[
(1+h_3)\log H_{ui}^* = h_0 + h_{1i} + h_2A_i + h_3(\log W_{ui} + \log H_{ui}^*) - Z\theta_B + \nu,
\]
\[(4.1b')\quad \log H_{ui}^* = h_0' + h_1' + h_2'A_i + h_3'\log E_{ui} - Z\theta_B' + \nu'.\]

Participation rule:
\[(4.1c')\quad GAP_i = \gamma_0 + \gamma_1A_i + \gamma_2\log E_{ui} + [X-Z]\gamma + \xi = \gamma T + \xi.\]

In particular, notice that in this model the labor supply elasticity \((h_3)\) requires a simple transformation given by

\[
h_3' = \frac{h_3}{1 + h_3} \quad \text{or} \quad h_3 = \frac{h_3'}{1 - h_3'}.\]

Whether the original system of the equations \((4.1)\) or the alternative specification \((4.1')\) represent a better description of the immigration process is an empirical question. In the former model wage rates, a primitive variable, interact explicitly with labor and participation decisions. The latter specification, in contrast, combines not only the level of wages but also the availability of jobs in the U.S. for a given period of time.

4.1.2 Limited Dependent Variables

An obvious problem with the estimation of \((4.1)\) is that the participation variable GAP is not available. Moreover, the wage rate \([W_{ui}]\) and the number of hours worked \([H_{ui}]\) are observed only if Mexican residents actually participate in the U.S. \((\text{GAP}_i > 0)\). The class of
observed only if Mexican residents actually participate in the U.S. (GAP$_2$>$0$). The class of regression models where endogenous variables can only be observed in a limited range have been documented in the econometric literature in the context of limited dependent variables with truncated samples.

According to Judge, et. al. [1985] a truncated sample results when the knowledge of independent variables is available solely when the dependent variable is also observed. In contrast, censored samples apply to observations on the dependent variable corresponding to known sets of exogenous variables which are not observed. Following Heckman [1980], a truncated sample differs from a censored sample in that the probability of sample selection cannot be estimated from observed data. Kendall and Stuart [1973] argues that censoring is a property of the sample while truncation is a property of the distribution (p. 541). As expected, dealing with truncation problems is more difficult than censoring problems since less information is available.

The statistical theory of limited dependent variables has its foundations in the notion that discrete endogenous variables are generated by continuous latent variables crossing thresholds. In a seminal paper Tobin [1958] estimates the demand for consumer durables goods. The author proposed the analysis of limited dependent variables by stating:

“...In Economic surveys of household, many variables have the following characteristics: The variable has a lower, or upper, limit and takes on the limiting value of the substantial number of respondents. ...As specific example, many—indeed, most—households would report zero expenditures in automobiles or major household durable goods during any given year. Among those households who made such expenditure, there will be wide variability in amount.” [Ibid, p. 24]

In his model the intensity of demand for durable goods is limited. Although consumers’ demands are continuous, actual purchases of goods are discrete. Consequently, some consumers will show corner solutions at the time of the survey. Observations will be missing if purchases are not made, so that the sample is censored at zero. This will result in large variations among those households that make any expenditures, with many observations concentrated around zero.$^3$

$^3$ According to Maddala [1983], Tobin was the first to address this problem in a regression model. "Because he related his study with the literature on probit analysis, his model was nicknamed Tobit model (Tobin’s probit) by Goldberger (1964)." [Ibid, p. 151]
Censored and truncated regression models have also been developed in other scientific fields. Biometrics and engineering, for instance, have directed particular efforts to the study of survival analysis (survival time of a patient) and duration models (analysis of the time to machinery failure), respectively. According to Amemiya [1985], these examples belong to the same general class of Tobit models. The introduction of Tobit analysis into econometric applications is due to the publication of Tobin’s paper 30 years ago. Nonetheless, it is not until the last decade when numerous researches focused on the applicability of these models into a wide area of economics. This phenomenon is explained by the increased availability of longitudinal sample data and by recent advances in computing technology that have made estimation of Tobit models less expensive.

Single equation models with limited dependent variables have received considerable attention in the econometric literature. However, little is known about the specification and estimation of simultaneous equation models. Amemiya [1974] discusses a system of simultaneous equations in which all of the dependent variables are truncated at zero. Following this contribution, Sickles and Schmidt [1978] develop a similar procedure where some, but not all, of the dependent variables are truncated. In particular, they analyzed the estimation of a two-equation model with one truncated variable using maximum likelihood procedures. Likewise, Nelson and Olson [1978] proposes an alternative specification where some endogenous variables of the structural equation are assumed to depend not on the observed values of truncated variables, but on their unobserved latent indexes.

In a Tobit model dependent variables which are smaller than a certain threshold are not observed. The estimation model considered here, however, follows the characteristics of a simultaneous Tobit model with a truncated sample. The wage rate and the hours worked in the U.S. are observed only if GAP is positive. Furthermore, the decision rule (GAP), which aggregates the individual’s labor behavior in both countries, is totally unobserved. Accordingly, the system of simultaneous equations (4.1) may be defined in terms of the following conditions,

\[
\log W_i = \begin{cases} 
\log W_{ui} & \text{if } Y \Gamma > -\xi \\
\text{Unobservable} & \text{otherwise;}
\end{cases}
\]

4 See Maddala [1983] for an extensive list of articles.
5 Tobit models are censored by construction.
Notice that the participation rule, i.e. $Y_{1} \geq \xi_{i}$, is the condition driving the truncation characteristics of both dependent variables ($W_{j}$ and $H_{j}$). The wage equation and the labor supply are observed only if Mexican workers participate in the U.S. labor force. System (4.2) represents a convenient representation because it integrates the unobservable variable GAP into the equations (4.1a) and (4.1b).

Finally, the simultaneous characteristics of the model respond to two different components: (i) The recursive characteristics of wage rates in the labor supply and the participation rule, and (ii) the stochastic dependency across equations. Although the former source of autocorrelation is evident, the latter concept may require further explanation. Recall from section 3.4 that the latent (random) term in the participation rule $\xi_{i}$ aggregates three sources of stochastic behavior: Wage rates in each country, home production in Mexico and limited information about future events. In this context, $\xi_{i}$ may be approximated as a linear combination of the vector $[e_{u_{1}}, e_{m_{i}}, e_{i}, \eta_{u_{i}}, \eta_{m_{i}}]$. Thus, the participation rule (4.1c) is contemporaneously correlated with the wage equation (4.1a) not only as a RHS regressor, but also by means of its stochastic term, i.e. the variance-covariance matrix is not diagonal. The implication of this correlation in terms of misspecification of the regression model are studied in the next section.

4.2 Misspecification Bias in Self-Selection Models

The basic model (4.1) and its modification (4.2) resulting from insufficient information lead to important misspecification problems under standard estimation procedures. Direct application of least squares (LS) may generate asymptotically bias estimators. Figure 4.1 shows the relation of wage rate in the U.S. and hours worked in both countries. Each dot represents an observation of latent and actual undocumented workers. Indeed, an important characteristic of the sample is that potential immigrants (those in the lower quadrant) are

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6 Notice that similar specification applies for model (4.1')
not observed.\textsuperscript{7} This feature leads to a truncated sample, where the linearity assumption of least squares methods is clearly inappropriate.

![Figure 4.1—Misspecification Error in Truncated Models](image)

Self-selection is a problem related specifically to the issue of how observations on a given economic phenomenon are generated. In particular, how dependent variables may be observed or not depending on another set of variables or decision rules. Dhrymes [1986] suggests that observation of $W_1$ in (4.2) is a function of the stochastic process that generates $\text{GAP}_1$, as well as the stochastic structure that governs $W_{ui}$ in expression (4.1a). Recall that the participation rule ($\text{GAP}_1$) is assumed to be inherently unobservable, although the RHS variables are known, at least in part.

The system of equations (4.1) and (4.2) have been extensively reviewed in the literature, which takes two major approaches. On one hand, Heckman [1974], Hanoch [1980], Lee [1981] and Heckman and MaCurdy [1986], among others exploit two-stage estimation procedures. The first stage requires that the estimation of a Probit model be used in the least squares regression model of the second stage.\textsuperscript{8} On the other hand, Tobin [1958], Lee, Maddala and Trost [1980], Perloff and Sickles [1987] among others solved self-selection misspecification using maximum likelihood Tobit procedures. The main difference between both approaches is that the computational burden is reduced in the for-

\textsuperscript{7} The lack of data in this sector of the sample is due to the impossibility of defining the elements that characterize latent immigrants. Mexican residents will reveal their migration preferences only by actual participation in the host country.

\textsuperscript{8} This procedure is also known as Heckit models. See Killingsworth [1983] p. 156-57.
mer method, but it leads to a heteroskedastic covariance matrix. LS estimation is consistent but not efficient.

Overlooking the statistical differences of these procedures, the important feature of both methods is that the selection rule (GAP>0) generates a sample of observed data. Following Heckman [1980], this feature characterizes self-selection specifications, and it is not a representation of a prototype of limited dependent variables models, as most economists have interpreted it.

Yet another source of confusion in the literature is the difference between self-selection and sample-selection. In the previous section, it was shown that undocumented workers choose participate in the U.S. labor market depending on the state of the index function GAP. From an initial random sample, self-selection implies that the sample of actual undocumented workers is not random since it implies GAP>0. In contrast, a general concept is implied in sample-selection models, where self-selection is a particular case. Here, the researcher imposes a selection rule to a random sample that generates a particular sample employed in empirical analysis. This rule may or may not be the consequence of a choice made by each individual in the whole sample.

According to Heckman [1980], the underlying idea in selection problems is that nonrandom self-selection arises due to missing data. Similar to standard omitted variables, this is a problem of misspecification of the error term. "Thus, a sample selection bias, initially viewed as a missing dependent variable problem, may be reformulated as an ordinary omitted explanatory variable problem." [Ibid, p. 210]

It is easy to show the misspecification bias that arises from the use of least squares estimators in self-selection models. Consider the wage equation in (4.1) and (4.2).

\[
\log W_i = \begin{cases} 
\log W_{ui} = Z_{\theta} + e_{ui} & \text{if } Y \Gamma \geq -\xi \\
\text{Unobservable} & \text{otherwise};
\end{cases}
\]

where the error component for the wage equation \((e_{ui})\) is assumed to follow conventional stochastic properties, i.e. \(e_{ui} \sim (0, \sigma_u)\). Similarly, \(\xi\) is an iid random term with known distribution function and with its first two moments given by \(E[\xi]=0\) and \(E[\xi^2]=\Sigma\xi\). Recall, that \(\xi\) is a vector of latent variables that aggregates, among other elements, the
stochastic behavior of the wage rate, i.e. $E[e_{ui}|\xi]=\sigma_{e}\xi\neq 0$. Therefore, the conditional expectation of the wage equation is

$$E[\log W_{i}| Z, \ Y\bar{T}>\xi] = Z\theta_{w} + E[ e_{ui}| Z, \ Y\bar{T}>\xi].$$

Assuming orthogonal regressors, i.e. $E[Ze_{ui}]=0$, and using results well known in the literature [see Killingsworth, 1983 and Maddala, 1983], the conditional expectation of $e_{ui}$ may be written as

$$E[ e_{ui}| Z, \ Y\bar{T}>\xi] = -\frac{\sigma_{e}\xi}{\Sigma_{\xi}^{1/2}} \left[ \frac{\phi(\frac{Y\bar{T}/\Sigma_{\xi}^{1/2}}{\xi})}{\Phi(\frac{Y\bar{T}/\Sigma_{\xi}^{1/2}}{\xi})} \right];$$

$$= -\frac{\sigma_{e}\xi}{\Sigma_{\xi}^{1/2}} \left[ \frac{\phi(M)}{\Phi(M)} \right];$$

where $\phi$ and $\Phi$ are the density and the distribution functions of the standardized normal variable $M=Y\bar{T}/\Sigma_{\xi}^{1/2}$, respectively. The conditional expectation for the wage equation is

$$(4.5) \quad E[\log W_{i}| Z, \ Y\bar{T}>\xi] = Z\theta_{w} - \frac{\sigma_{e}\xi}{\Sigma_{\xi}^{1/2}} \left[ \frac{\phi(M)}{\Phi(M)} \right].$$

Direct application of LS implies that the second term in the previous expression is omitted from the RHS variables. Hence, least squares estimators are, in general, asymptotically biased.

However, note that LS estimation may be consistent if $\sigma_{e}\xi=0$. Here, the disturbance term of the U.S. wage rate is orthogonal to the self-selection rule. Consequently, the omitted terms banish from the estimation procedure of the wage equation, and its expected value is given by $Z\theta_{w}$.

---

9 Such a situation may be observed if wage rates across regimes are identical. Then, the economic essence of undocumented Mexican participation is neglected.
Furthermore, LS bias over equation (4.5) may be monotonically reduced if the size of the variable GAP increases. This argument is important since it identifies the conditions in which least squares may be employed as a reasonable estimation procedure, although not efficient. Following Maddala [1983], the expected value of \( W_{ui} \) in the sample may be systematically understated with respect to the whole population. To show this, denote \( B \) the biased term resulting from expression (4.5),

\[
B \equiv -\frac{\sigma_{\epsilon \xi}}{\Sigma_{\xi}^{1/2}} \left[ \frac{\phi(M)}{\Phi(M)} \right] = -\frac{\sigma_{\epsilon \xi}}{\Sigma_{\xi}^{1/2}} \left\{ \frac{\phi[(\text{GAP} - \xi)/\Sigma_{\xi}^{1/2}]}{\Phi[(\text{GAP} - \xi)/\Sigma_{\xi}^{1/2}]} \right\},
\]

since \( M \equiv \frac{YT}{\Sigma_{\xi}^{1/2}} = \frac{\text{GAP} - \xi}{\Sigma_{\xi}^{1/2}} \).

A comparative statics exercise shows that,

\[
\frac{\partial B}{\partial \text{GAP}} = -\frac{\sigma_{\epsilon \xi}}{\Sigma_{\xi}^{1/2}} \left[ \frac{\phi' + \phi^2}{\Phi} \right] < 0
\]

since \( \Phi' = -\phi \) and \( \sigma_{\epsilon \xi} > 0 \).

Hence, a monotonic increase in the participation decision variable (GAP) will tend to reduce the bias introduced by LS procedures. In the limit (GAP→∞), the self-selection bias approaches to zero, and LS estimators become consistent. This exercise shows that under special conditions, i.e. very high values of GAP and positive covariance \( \sigma_{\epsilon \xi} \), LS may be asymptotically unbiased even when \( \sigma_{\epsilon \xi} = 0 \). The case of undocumented immigration is particularly sensitive to this argument since large immigration costs and wage differentials may lead to high values of the decision index GAP. In theory, asymptotic unbiased LS estimators are more likely to be observed in the context of illegal migration than in a context of traditional leisure-work choice models, where fixed costs are not a major concern.

Another source of least squares inefficiency is found in the failure to identify the estimates in the participation rule, i.e. expression (4.1c). Because GAP is completely unobservable the vector of parameters \( \Gamma \) is not estimable under standard regression procedures. Accordingly, elements driving Mexican participation decisions are not empirically estimable.\(^{10}\) Failure to estimate such parameters will deny one of the most important objectives of this study.

\(^{10}\) In particular, the uncompensated wage elasticity of the GAP equation (\( \gamma_2 \)).
To summarize, misspecification problems arise as a result of omitted variables. Then, least squares procedures are not efficient and may yield asymptotic biased estimates. The magnitude of this bias is a function of the correlation between the decision rule and other limited dependent variables in the system. Moreover, traditional econometric methods do not allow for empirical identification of parameters in the decision rule. In the case of undocumented migration, these estimates provide fundamental information in the analysis of Mexican population flows. In order to achieve asymptotic consistency and to be able to identify the elements affecting the undocumented decision maximum likelihood methods may be used. Such procedures represent an appropriate alternative to estimate the specific characteristics addressed in Mexican undocumented immigration in United States.

4.3 Maximum Likelihood Estimation and Asymptotic Efficiency

In this section, a general estimation strategy for the regression model (4.1) and (4.2) is proposed based on full-information maximum likelihood methods. In the presence of limited endogenous variables and simultaneous regression models, maximum likelihood procedures are conceptually possible but complicated in practice. Furthermore, they are more expensive to compute than standard least squares procedures. However, potential sources of misspecification bias and efficiency losses may be evaded when using these techniques.

Estimation models similar to the one presented here are not new in econometric literature. Roberts, Maddala and Enholm [1978] develop a simultaneous equation Tobit model to explain how utility rates are determined. In addition, Hauserman and Wise [1979] use a version of these models to analyze the labor supply of participants in negative income tax (NIT) experiments. Their model is truncated because they use observations only for those actually participating in the experiment. According to Amemiya [1985] these models may be generalized under his Type 3 Tobit Model classification, where the likelihood function is described by $P(Y_1<0):P(Y_1,Y_2)$.

4.3.1 The Likelihood Function

Maximum likelihood (ML) estimation implies the maximization of the likelihood function
of the following linear model\textsuperscript{11}

\begin{equation}
Y \mathbb{B} + X \Gamma = U;
\end{equation}

where, in particular, \(Y\) is a \(1 \times 3\) vector of dependent variables and \(X\) is a \(1 \times K\) vector of independent variables; \(\mathbb{B}\) and \(\Gamma\) are conformable matrices of coefficients with dimension \(3 \times 3\) and \(K \times 3\), respectively. Moreover, it is required that \(\mathbb{B}\) may be nonsingular in order to obtain a unique solution of the dependent variables with respect to predetermined and random components. The stochastic term \(U\) is assumed to follow a normal distribution with mean zero and constant variance-covariance \(\Psi\).\textsuperscript{12} Under usual completeness assumptions, the reduced form of (4.3) is given by

\[ Y = X \Pi + V; \]

where \(\Pi = -\Gamma \mathbb{B}^{-1}\) and \(V = UB^{-1}\). In particular, the random term \(V\) is characterized by a trivariate normal distribution\textsuperscript{13}

\[
\begin{bmatrix}
    v_{11} \\
    v_{21} \\
    v_{31}
\end{bmatrix}
\sim N
\begin{bmatrix}
    0 \\
    0 \\
    0
\end{bmatrix}, \Omega;
\]

the variance-covariance matrix of the reduced-form residuals \(\Omega\) may be expressed with respect to the following partitions

\[
\Omega = \begin{bmatrix}
\omega_{11} & \omega_{12} & \omega_{13} \\
\omega_{12} & \omega_{22} & \omega_{23} \\
\omega_{13} & \omega_{23} & \omega_{33}
\end{bmatrix} = \begin{bmatrix}
\Omega_{11} & \Omega_{12} \\
\Omega_{12} & \Omega_{22}
\end{bmatrix}.
\]

Accordingly, the vector of dependent variables \(Y\) conforms a distribution function given by

\textsuperscript{11} Recall that ML procedures requires the following assumptions: (i) The complete specification of all equations is known, and (ii) the random variables of the structural equations are normally distributed.

\textsuperscript{12} Note that in contrast with the model in section 4.1, expression (4.3) requires the assumption of normality.

\textsuperscript{13} For further references on trivariate normal distributions see Johnson and Kotz (1972).
\[
\begin{bmatrix}
Y_{1i} \\
Y_{2i} \\
Y_{3i}
\end{bmatrix} \sim \mathcal{N}
\begin{bmatrix}
\Pi_1 \\
\Pi_2 \\
\Pi_3
\end{bmatrix}, \Omega.
\]

Now, suppose that the sample is drawn from distributions of $Y_{1i}$ and $Y_{2i}$ truncated at the point $Y_{3i}=0$, e.g. $\text{GAP}_i=0$, so that no observations are recorded for $Y_{3i}<0$. In fact, all observations come from the shaded area in Figure 4.2. It is recognized that a continuous density function may not be employed to explain the conditional distribution of $Y_{1i}$ and $Y_{2i}$.

![Figure 4.2—Truncated Normal Distribution](image)

In this context, let $Y_{3i}$ be an unobservable index function upon which $Y_{1i}$ and $Y_{2i}$ are limited (truncated) in their range. The latter variables are observed only if $Y_{3i}>0$. In particular, notice that the marginal distribution of $Y_{3i}$ is given by $\mathcal{N}(X\Pi_3, \Omega_{22})$. Therefore, the conditional probability distribution function (pdf) for system (4.3) may be expressed as

\[
pdf_i[Y_{1i}, Y_{2i} | Y_{3i}>0] = \frac{\int_{0}^{\infty} \pdf(Y_{1i}, Y_{2i}, Y_{3i}) \, dY_{3i}}{1 - \text{CDF}_i(Y_3=0)};
\]

where $\text{CDF}_i(Y_3=0)$ is the marginal cumulative distribution function of the decision rule evaluated at zero. This represents the probability that the decision rule is greater than zero. In addition, notice that the trivariate $pdf_i$ given by the numerator of (4.4) may be expressed in terms of the following probability and cumulative distribution functions.
pdf(Y_1, Y_2, Y_3) = h_1(Y_1, Y_2) \cdot h_2(Y_3 | Y_1, Y_2) \, dY_3,

where \( h_1 \) is a bivariate probability function and \( h_2 \) is the conditional cumulative distribution function of the decision rule given that \( Y_1 \) and \( Y_2 \) are observed. Expression (4.4) may be combined with the former equation to yield the conditional pdf of the truncated system in (4.3),

\[
\text{pdf}[Y_1, Y_2 | Y_3 > 0] = \frac{\int_0^\infty h_1(Y_1, Y_2) \cdot h_2(Y_3 | Y_1, Y_2) \, dY_3}{1 - CDF(Y_3 = 0)};
\]

note that \( h_1 \) as well as CDF are independent of the participation rule \( Y_3 \). Then some algebraic manipulation leads to

\[
\text{pdf}[Y_1, Y_2 | Y_3 > 0] = \frac{h_1(Y_1, Y_2)}{1 - CDF(Y_3 = 0)} \int_0^\infty h_2(Y_3 | Y_1, Y_2) \, dY_3.
\]

Inspection of (4.4) reveals that three elements characterize the behavior of the conditional pdf for every observation. Each of these components are reviewed next.

1. \( h_1 \) follows a bivariate normal distribution of the type

\[
h_1(Y_1, Y_2) \sim N \left[ \begin{pmatrix} \Pi_1 \\ \Pi_2 \end{pmatrix}, \Omega_{11} \right];
\]

given that \( v_s = Y_s - \Xi \Pi_s \) for \( s = 1, 2 \) the density function of \( h_1 \) may be expressed as

\[
h_1(Y_1, Y_2) = \frac{1}{(2\pi)^{2/2} |\Omega_{11}|^{1/2}} \exp \left[ -\frac{1}{2} \begin{pmatrix} Y_1 - \Xi \Pi_1 \\ Y_2 - \Xi \Pi_2 \end{pmatrix} \Omega_{11}^{-1} \begin{pmatrix} Y_1 - \Xi \Pi_1 \\ Y_2 - \Xi \Pi_2 \end{pmatrix} \right].
\]

A version of the normal density function (4.5) may yield the likelihood function of model (4.3) if \( Y_{1i} \) and \( Y_{2i} \) are treated as standard (non-truncated) dependent variables, and if \( Y_{3i} \)
is observable. Since this is not the case, the characterization of the appropriate likelihood function requires a broader description.

2. The conditional density function \( h_2 \) describes the statistic behavior of the index function \( Y_{3i} \) since this is completely missing. While actual data for \( Y_{3i} \) is not available, its distribution function may be evaluated indirectly imposing the theoretical structure of the model.\(^{14}\) The following definition and lemma are required to describe the conditional density \( h_2 \),

**Definition 4.1:** Let \( X \sim N(\mu, \sigma^2) \)

\[
\int_{-\infty}^{a} f(x)dx = CDF(a) = \Phi\left[ \frac{a - \mu}{\sigma} \right];
\]

then,

\[
\int_{b}^{\infty} f(x)dx = 1 - CDF(b) = 1 - \Phi\left[ \frac{b - \mu}{\sigma} \right].
\]

**Lemma:** Let

\[
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} \sim N\left( \begin{bmatrix}
    \mu_1 \\
    \mu_2
\end{bmatrix}, \begin{bmatrix}
    \Sigma_{11} & \Sigma_{12} \\
    \Sigma_{21} & \Sigma_{22}
\end{bmatrix} \right);
\]

then, the conditional distribution for \( x_1 \) given \( x_2 \) is

\[
[x_1|x_2] \sim N\left( \mu_1 + \Sigma_{12} \Sigma_{22}^{-1}(x_2 - \mu_2), \Sigma_{11} - \Sigma_{12} \Sigma_{22}^{-1} \Sigma_{12} \right).
\]

**Proof:** [See Dhrymes, 1970 p. 16].

Consequently, \( h_2 \) may be expressed as

\(^{14}\) \( Y_{3i} \) may not be observable, but its empirical effects over the estimation model are.
\[
\int_0^\infty h_2(Y_3 \mid Y_1, Y_2) \, dY_{3i} = 1 - \Phi \left[ \frac{0 - X\Pi_3 - \Omega_{12}^{-1} \Omega_{11}^{-1} (Y_1 - X\Pi_2)}{\sqrt{\Omega_{22}^{-1} \Omega_{12}^{-1} \Omega_{11}^{-1} \Omega_{12}^{-1}}} \right].
\]

Notice that the conditional density function of the decision rule is a function of the all available information in the system. According to expression (4.6) only observable variables (at least partially due to truncation) may appear explicitly in the estimation model.

3. Lastly, the denominator of (4.4') is the result of the truncated characteristics of dependent variables \(Y_{1i}\) and \(Y_{2i}\). Here, a direct application of definition 4.1 leads to the function

\[
CDF_i(Y_3=0) = \Phi \left[ \frac{0 - X\Pi_3}{\sqrt{\Omega_{22}^{-1}}} \right].
\]

The cumulative standard normal distribution function \(\Phi\) in (4.7) denotes the limited range of the observable dependent variables. Note that in contrast to censored models, this function depends on elements within the decision rule.

It is easy to verify that the appropriate log likelihood function resulting from (4.4) is the summation of conditional probability functions over the entire observable sample \(i=1\ldots N\),

\[
\log L = \sum_{i=1}^N \log pdf_i[Y_1, Y_2 \mid Y_3 > 0].
\]

Note that the likelihood function is a mixture of two normal CDF's (discrete probabilities) and a density function. Combining the components of the conditional pdf given by expressions (4.5)-(4.7) and taking logarithms, the log likelihood function may be rewritten as

\[\text{footnote text: The assumption of an iid sample is required in this proposition.}\]
\[
\log L = -N \log(2\pi) - \frac{N}{2} \log |\Omega_{11}| - \sum_{i=1}^{N} \frac{1}{2} \left( \frac{\Omega_{11}^{-1} \left( Y_{ii} - X_i \Pi_1 \right) \Omega_{11}^{-1} \left( Y_{2i} - X_i \Pi_2 \right)}{\sqrt{\Omega_{22} - \Omega_{12} \Omega_{11}^{-1} \Omega_{12}}} \right) \]

Not surprisingly, a feature of the log likelihood function in (4.8) is that assuming a diagonal covariance matrix, i.e. $\Omega_{12}=0$, transforms a Tobit truncated model into a standard full information maximum likelihood (FIML) estimation model. Under such a condition, the last two rows of expression (4.8) vanished. Then, the likelihood function is solely given by a bivariate normal density function of the type given in (4.5).

Expression (4.8) shows that the log likelihood function is highly non-linear, and thus a root must be obtained numerically. Therefore, consistent estimators may be obtained using iterative search processes. Olsen [1978] shows that if standard iterative methods are employed, e.g. Newton-Ramson or Scoring, global concavity of $\log L$ in terms of the parameters $\Pi$ and $\Omega$ always converge to a global maximum under a Tobit ML procedure.

### 4.3.2 Properties of Tobit ML Estimators

Under fairly general assumptions, the method of maximum likelihood yields estimators which are consistent, asymptotically normal and/or asymptotically efficient in the sense that their limiting distribution has the smaller variance-covariance matrix possible. This is the case where the covariance of ML estimates is equal to the Cramer-Rao lower bound.\(^\text{17}\)

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\(^\text{16}\) A diagonal variance-covariance matrix implies that the decision rule bears no relation to the limited dependent variables within the system. In this case, the link between the unobservable index function and the partially observable variables is broken. Consequently, $Y_{3i}$ is not estimable and the rest of the system is unaffected by the index function.

\(^\text{17}\) For a complete specification of the Cramer-Rao inequality see Dhrymes [1970] and Chow [1983].
However, ML procedures are particularly sensitive to various types of nonstandard assumptions that may lead to important misspecification errors. This result contrasts with the classical regression model where least square estimators are generally asymptotically consistent under heteroscedasticity, serial correlation and nonnormality. Tobit ML estimators, on the contrary, remain consistent under serial correlation, but not under heteroscedasticity and nonnormality.

According to Amemiya [1985], there are two cases where heteroscedasticity is present in simple truncated Tobit models: (i) The case of a regressor consisting only of a constant term, and (ii) the case of a constant term plus one independent variable. Extremely large asymptotic bias may be expected in these cases. In addition, heteroscedasticity problems may be less likely to appear in cross-section analysis than times-series or panel data. Lastly, under the present circumstances, i.e. cross-section estimation, serial correlation does not constitute an important source of misspecification errors. The ML estimates obtained from the maximization of the likelihood function in (4.8) are expected to be asymptotically efficient.

4.4 The Data Set

Elaborate theoretical analysis on Mexican migration has its major limitation in the availability of empirical data. Sample information complicates the estimation procedure because (i) the illegal status of undocumented workers and (ii) the impossibility to identify potential immigrants in the Mexican regime. Hence, data requirements transform an essential switching regression model into a self-selection specification where one regime is completely unobservable.

4.4.1 Sample Selection

The sample data of this study is based on the 1980 U.S. Census of Population. The Census Bureau provides public-use microdata samples on magnetic tapes that contain information on the characteristics of each survey person. In particular, this study uses the one percent microdata sample (B sample) that comprises 2,172,293 personal records in all of the 50 states and the District of Columbia. Mexican undocumented workers are identified from

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18 The estimation model presented in (4.3) does not resemble such models.
this survey by imposing a series of constraints that are more likely to describe them.

A similar methodology has been proposed by Passel and Woodrow [1984] and Pearce and Gunther [1985]. These articles explore the potential of using data from the 1980 Census to acquire a better understanding of how immigration from Mexico affects the U.S. economy. Their methodology suggests that the ability to speak English provides an important element in distinguishing undocumented settlers. Although information on place of birth and citizenship has traditionally been collected by the Census Bureau in previous years, the 1980 Census introduces an additional dimension by including language elements.\(^{19}\)

In particular, Pearce and Gunther characterize Mexican immigrants as respondents born in Mexico who speak no English. The present study, in contrast, introduces further constraints that provide a more restricted sample than previous research and which may be used in actual estimation analysis. Table 4.1 summarizes the effects of different restrictions over the complete Census sample. Respondents born in Mexico constitute 1.02 percent of the original sample; given an overall U.S. population of approximately 223 million in 1980, this percentage implies that 2.275 million Mexicans were living legally or illegally in this country.

<table>
<thead>
<tr>
<th>Constraint</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1% MicroData Sample B Mexican Born</td>
<td>2,172,293</td>
</tr>
<tr>
<td>Spanish Spoken at Home</td>
<td>22,219</td>
</tr>
<tr>
<td>Mexican Origin</td>
<td>21,218</td>
</tr>
<tr>
<td>16-70 Age Range</td>
<td>21,034</td>
</tr>
<tr>
<td>Not a U.S. Citizen</td>
<td>17,690</td>
</tr>
<tr>
<td>Ability to Speak English: Low or Null</td>
<td>15,468</td>
</tr>
<tr>
<td>Positive Hours Worked in 1979</td>
<td>11,282</td>
</tr>
<tr>
<td>Positive Wage Earnings in 1979</td>
<td>11,811</td>
</tr>
<tr>
<td>All Combined</td>
<td>11,395</td>
</tr>
<tr>
<td></td>
<td>4,662</td>
</tr>
</tbody>
</table>

* Includes data from 20 states of the Union. This accounts for 96 percent of the original sample.  

\(^{19}\) Notice that undocumented settlers have incentives to lie about their citizenship status, since it bears direct relation with their illegal condition. In contrast, the ability to speak English is an objective representation that may not be distorted by respondents.
Additional constraints, however, are required to characterize Mexican undocumented respondents. According to Table 4.1 the restrictions that more likely to identify the Mexican undocumented settlers are age range, U.S. citizenship and, in particular, the ability to speak English. Furthermore, in order to access the labor force behavior of this group, unemployed respondents during 1979 were deleted. This factor turns out to be an important binding constraint. A joint imposition of all eight restrictions in Table 4.1 yields the actual sample employed in the estimation model.

This sample represents 0.22 percent of the original Census data. Which implies that apparently less than a half million undocumented Mexican workers were employed in the U.S. labor market during 1979. In order to compare the accuracy of this estimation consider the study of Warren and Passel (1987). Using a less restricted methodology, they estimate that in 1980 the total number of Mexican-born undocumented aliens was 1.13 million [Ibid, p. 382]. This estimate, however, does include working and non-working immigrants, and it does not restrict the age range. Consequently, it may establish that almost half of the Mexican undocumented population is under-age or/and does not participate in the labor market.

![States with Highest Mexican Population](image)

*Rest indicates a percentage of the 20 states sampled.*
*Primary Source: Public-Use Sample B, 1980 U.S. Census of Population.*

*Figure 4.3—Regional Concentration of Mexican Settlements*

20 The actual amount is 498,516. Four percent more than the number calculated in Table 4.1, which conveys information on only 20 states (96 percent of the total sample).
21 Where only English ability was considered.
Mexican undocumented immigration is a regional phenomenon where highly concentrated settlements are observed in a few parts of the United States. According to Figure 4.3, only three states capture up to 88 percent of Mexican born immigrants, i.e. California (57 percent), Texas (23 percent) and Illinois (8 percent). However, in order to obtain a nationwide representative sample, the 20 states with the largest Mexican population were included in the data base, i.e. the shaded regions in Figure 4.4. This aggregation corresponds to 96 percent of the overall national sample.

![Figure 4.4—Geographic Distribution of the Sample](image)

Lastly, even if many members of the proxy group identified in Table 4.1 are not actual undocumented Mexican workers, the use of Census data can provide the information required for the empirical analysis. According to Pearce and Gunther, Census respondents have comparable labor skills and socio-demographic characteristics to typical illegal workers, and they are likely to reproduce their economic behavior. Moreover, Census information is probably a more nearly random and methodologically accurate than direct regional surveys of Mexican immigrants.

4.4.2 Variables
In addition to the ability to identify Mexican undocumented settlers, the Census data provides further advantages. It furnishes detailed descriptions of economic, social and demographic characteristics of immigrants. Using the coded information provided in the original files, the following variables are defined and computed for each observation as follows:

(i) Economic Variables:

EARNING = Earnings, salary or self-employment income in 1979 (dollars);
NDATA(I, 22).²²
HOURS = Hours worked in 1979, which is the product of usual hours worked per week and total number of weeks worked in 1979 (hours);
[8769–NDATA(I, 23)].
ASSETS = Income from all sources but WAGE in 1979 (dollars); NDATA(I, 24).

(ii) Personal Variables:²⁴

AGE = (years); NDATA(I, 3).
SEX = 0 if male, 1 female; NDATA(I, 2).
MARITAL = 0 if married, 1 otherwise; NDATA(I, 4).
SCHOOL = Formal education (years); NDATA(I, 6).
CHILDREN = 0 if NA or male, 1 if none, 2 if one children, ..., 13 if twelve or more; NDATA(I, 8).
ENGLISH = Ability to speak English: 0 if not well, 1 if not at all; NDATA(I, 7).
IMMIGR = Years of immigration: 1 if 1975-80, 2 if 1970-74, 3 if 1965-69,
4 if 1960-64, 5 if 1950-59 and 6 if before 1950; NDATA(I, 5).
POVERTY = Poverty status in 1979 (ratio of family income to poverty cutoff):
0 if NA, below poverty level if 1 and 2, above poverty level otherwise;
NDATA(I, 9).

(iii) Demographic Variables:

HOUSELD = 1 if household supporter, 0 otherwise; NDATA(I, 10).
FAMILY = 1 if family member within the household, 0 otherwise; NDATA(I, 11).
URBAN = 1 if SMSA residence, 0 otherwise; NDATA(I, 12).
INDAGR = 1 if agriculture industry, 0 otherwise; NDATA(I, 13).
INDCONST = 1 if construction industry, 0 otherwise; NDATA(I, 14).
INDMANUF = 1 if manufacture industry, 0 otherwise; NDATA(I, 15).
INDSERV = 1 if service industry, 0 otherwise; NDATA(I, 16).
OCCUTECH = 1 if technical, sales and administrative occupation, 0 otherwise;
NDATA(I, 17).
OCCUSERV = 1 if service occupation, 0 otherwise; NDATA(I, 18).
OCCUFARM = 1 if farming, forestry and fishing occupations, 0 otherwise;
NDATA(I, 19).
OCCUPROD = 1 if production, craft and repair occupations, 0 otherwise;
NDATA(I, 20).
PRIVATE = 1 if employee of private company, 0 otherwise. NDATA(I, 21).

²² Variable units are in parentheses.
²³ Represents the name of the variable used in the Fortran subroutine, see Appendix B.
²⁴ This set of characteristics is intrinsic to the individual, and they are not exclusive to the U.S. environment.
Summary statistics of these variables are presented in Table 4.2. According to such information Mexican undocumented workers in the sample typically tend to be male, married and in their early thirties. The age structure of the sample suggests a relatively young working population characterized by recently arrived immigrants, typically with no more than ten years of immigration. Their formal education is almost eight years, and 42 percent of the sample does not speak English at all.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Deviation</th>
<th>Min. Value</th>
<th>Max. Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>EARNING</td>
<td>7,148.9</td>
<td>5,174.4</td>
<td>15.0</td>
<td>75,000.0</td>
</tr>
<tr>
<td>HOURS</td>
<td>1,642.7</td>
<td>722.3</td>
<td>1.0</td>
<td>5,148.0</td>
</tr>
<tr>
<td>ASSETS</td>
<td>261.8</td>
<td>1,628.8</td>
<td>-3,265.0</td>
<td>67,005.0</td>
</tr>
<tr>
<td>AGE</td>
<td>33.00</td>
<td>11.60</td>
<td>16.0</td>
<td>70.0</td>
</tr>
<tr>
<td>SEX</td>
<td>0.33</td>
<td>0.47</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>MARITAL</td>
<td>0.22</td>
<td>0.46</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>SCHOOL</td>
<td>7.90</td>
<td>4.00</td>
<td>0.0</td>
<td>22.0</td>
</tr>
<tr>
<td>CHILDREN</td>
<td>1.20</td>
<td>2.30</td>
<td>0.0</td>
<td>13.0</td>
</tr>
<tr>
<td>ENGLISH</td>
<td>0.58</td>
<td>0.49</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>IMMIGR</td>
<td>2.10</td>
<td>1.30</td>
<td>1.0</td>
<td>6.0</td>
</tr>
<tr>
<td>POVERTY</td>
<td>4.90</td>
<td>2.20</td>
<td>0.0</td>
<td>7.0</td>
</tr>
<tr>
<td>HOUSEHLD</td>
<td>0.49</td>
<td>0.50</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>FAMILY</td>
<td>0.40</td>
<td>0.49</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>URBAN</td>
<td>0.30</td>
<td>0.46</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>INDAGR</td>
<td>0.19</td>
<td>0.39</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>INDCONST</td>
<td>0.08</td>
<td>0.27</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>INDMANUF</td>
<td>0.41</td>
<td>0.49</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>INDSERV</td>
<td>0.31</td>
<td>0.46</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>OCCUTECH</td>
<td>0.04</td>
<td>0.21</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>OCCUSERV</td>
<td>0.15</td>
<td>0.36</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>OCCUFARM</td>
<td>0.18</td>
<td>0.38</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>OCCUPROD</td>
<td>0.61</td>
<td>0.49</td>
<td>0.0</td>
<td>1.0</td>
</tr>
<tr>
<td>PRIVATE</td>
<td>0.96</td>
<td>0.20</td>
<td>0.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Primary Source: Public Use Sample B, 1980 U.S. Census of Population.

The typical Mexican illegal worker earns more than seven thousand dollars annually. These earnings account for an average wage rate of 4.35 dollars/hour, which represents an amount 45 percent higher than the minimum wage in 1980, i.e. 3.10 dollars/hour, but it is 34 percent smaller than the average hourly earnings of legal workers, i.e. 6.55 dollars/hour.\textsuperscript{25} The property income reported (ASSETS) provides little information on the

\textsuperscript{25} Legal minimum wage and average wage rate of all nonsupervisory workers in the private sector. Sources: IMF International Financial Statistics and A.F.L.-C.I.O, respectively.
net worth of Mexican workers. This is expected since respondents are asked to report financial, social security and public assistance earnings. None of these items are likely to characterize the actual wealth, if any, of a typical undocumented immigrant who tends to keep assets in their home-communities, e.g. real estate and cattle.

The basic social unit among Mexican workers sampled is the family. Almost 90 percent of the household is composed of direct family members, i.e. parents, children and siblings. The other 10 percent corresponds to persons not related to the head of the household, e.g. roommates, friends, paid employees and so on. Although only 30 percent of the sample may be described as urban residents, a small number of respondents are employed in agriculture related jobs (18 percent). In contrast, 72 percent of respondents worked in manufacturing and services industries. This may suggest that illegal workers tended to locate outside the SMSA commuting to the urban centers for their jobs. Construction workers in turn are not represented in this sample since only 8 percent of respondents worked in this industry. Lastly, 96 percent of the sample were employed by private corporations.

4.5 The Econometric Model

This section outlines the identification conditions and the estimation procedure of the proposed Tobit ML model. First, the identification conditions are imposed on the empirical model. Here two versions of the original model are identified allowing the derivation of robust inference analysis. These are distinguished by the number of exclusion restrictions inflicted on the original framework. Second, a detailed estimation procedure is presented based upon the general econometric model described in section 4.3.

Consider a more convenient specification of the simultaneous system (4.1), where the truncated dependent variables are observed for the sample size $N_u$, i.e. $W_i = W_{ui}$ and $H_i = H_{ui}^*$ [see conditions in (4.2)]:

---

26 Given the large standard deviation observed in Table 4.2, this variable does not provide a reliable description of its first moment.
27 Such an argument assumes that manufacturing and service jobs are more likely to be found in urban areas.
Observed wage equation:
\[(4.1a) \quad \log WAGE_i = d_0 + Zd + u_{1i}, \quad \text{with } i = 1 \ldots N_u;\]

observed labor supply:
\[(4.1b) \quad \log [1 - HOURS_i] = h_0 + h_1 \text{AGE}_i + h_2 \text{ASSETS}_i + h_3 \log WAGE_i - Zh + u_{2i}, \quad \text{with } i = 1 \ldots N_u;\]

unobserved participation rule:
\[(4.1c) \quad \text{GAP}_i \equiv \gamma_0 + \gamma_1 \text{ASSETS}_i + \gamma_2 \log WAGE_i + Z\gamma + u_{3i}, \quad \text{with } i = 1 \ldots N \text{ and } \gamma = g - d;\]

where \(d\), \(h\) and \(\gamma\) are conformable vectors of socio-demographic characteristics. Moreover, notice that \(\gamma\) represents a linear combination of parameters affecting the wage equation in both countries the U.S. (d) and Mexico (g). In particular, consider the following vectors:

\[
Z = \begin{bmatrix}
\text{SEX} \\
\text{MARITAL} \\
\text{SCHOOL} \\
\text{CHILDREN} \\
\text{ENGLISH} \\
\text{IMMIGR} \\
\text{POVERTY} \\
\text{HOUSEHLD} \\
\text{FAMILY} \\
\text{URBAN} \\
\text{INDAGR} \\
\text{INDCONST} \\
\text{INDMANUF} \\
\text{INDSERV} \\
\text{OCCUTECH} \\
\text{OCCUSERV} \\
\text{OCCUAGR} \\
\text{OCCUPROD} \\
\text{PRIVATE}
\end{bmatrix}, \quad d = \begin{bmatrix} d_1 \\
d_2 \\
d_3 \\
d_4 \\
d_5 \\
d_6 \\
d_7 \\
d_8 \\
d_9 \\
d_{10} \\
d_{11} \\
d_{12} \\
d_{13} \\
d_{14} \\
d_{15} \\
d_{16} \\
d_{17} \\
d_{18} \\
d_{19} \end{bmatrix}, \quad h = \begin{bmatrix} h_1 \\
h_2 \\
h_3 \\
h_4 \\
h_5 \\
h_6 \\
h_7 \\
h_8 \\
h_9 \\
h_{10} \\
h_{11} \\
h_{12} \\
h_{13} \\
h_{14} \\
h_{15} \\
h_{16} \\
h_{17} \\
h_{18} \\
h_{19} \end{bmatrix}, \quad \gamma = \begin{bmatrix} \gamma_1 - d_1 \\
\gamma_2 - d_2 \\
\gamma_3 - d_3 \\
\gamma_4 - d_4 \\
-d_5 \\
-d_6 \\
-d_7 \\
-d_8 \\
-d_9 \\
\gamma_10 \\
-d_{11} \\
-d_{12} \\
-d_{13} \\
-d_{14} \\
-d_{15} \\
-d_{16} \\
-d_{17} \\
-d_{18} \\
-d_{19} \end{bmatrix};
\]

where dotted subscripts represent coefficients associated with socio-demographic characteristics \(Z\). Moreover, recall that according to model (4.1) the labor and immigration decision may be better described by using an earnings function rather than the explicit average hourly wage rate as suggested in the previous specification:
Observed wage equation:
\[(4.1a') \quad \log EARNING_i = d_0 + zd + u_{1i}, \quad \text{with} \ i = 1...N_u;\]

observed labor supply:
\[(4.1b') \quad \log[1-HOURS_i] = h'_0 + h'_1 \text{AGE}_i + h'_2 \text{ASSETS}_i + h'_3 \log EARNING_i - Zh' + u_{2i},\]

with \(i = 1...N_u;\) \[\quad \text{where } h'_2 = \frac{h_3}{1+h_3}.\]

unobserved participation rule:
\[(4.1c') \quad \text{GAP}_i = \gamma_0 + \gamma_1 \text{ASSETS}_i + \gamma_2 \log EARNING_i + Z\gamma + u_{3i},\]

with \(i = 1...N\) and \(\gamma = g - d.\)

4.5.1 Identification

The structure of system \((4.1')\) is not estimable unless some nonsample, \textit{a priori} information is also available. This sub-section reviews the requirements for identification conditions of two different versions of the estimation model.

According to Judge et al. [1985], a necessary condition for the identification of the \(i\)th equation is that the number of linear restrictions must be greater than or equal to the total number of simultaneous equations minus one.\(^{28}\) Let \(M\) be the total number of dependent variables included in the system. Then, under zero coefficient restrictions, the generalized rank condition is
\[(4.9) \quad \text{Rank}(R_iA) \begin{cases} 
= M-1 \quad \text{then the } i\text{th equation is just identified} \\
> M-1 \quad \text{then the } i\text{th equation is overidentified.}
\end{cases}\]

\(R_i\) is an admissible zero-one restriction matrix resulting from \textit{a priori} information, and the matrix \(A\) contains all the parameters of the simultaneous equation model.\(^{29}\) Table 4.3 presents the \(R_iA\) matrix for the estimation model in \((4.1')\)

---

\(^{28}\) If the normalization rule is excluded.

\(^{29}\) Notice that \((4.9)\) is necessary but not a sufficient condition. A sufficient condition will require that every equation whose error is uncorrelated with the \(i\)th equation is identifiable [see Judge et al. (1985, p.580)]. In addition, the generalized rank condition applies only to simultaneous equation models that are linear in parameters and in variables. Identification conditions under nonlinearities are discussed in Brown [1983].
Table 4.3—Identification Conditions

<table>
<thead>
<tr>
<th>Variable</th>
<th>Conditional Model</th>
<th>Unconditional Model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Wage Eq.</td>
<td>Hours Eq.</td>
</tr>
<tr>
<td>Intercept</td>
<td>d₀</td>
<td>h₀</td>
</tr>
<tr>
<td>logEARNING</td>
<td>-1</td>
<td>h₃</td>
</tr>
<tr>
<td>log(1-HOURS)</td>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>ASSETS</td>
<td>0</td>
<td>h₂</td>
</tr>
<tr>
<td>AGE</td>
<td>0</td>
<td>h₁</td>
</tr>
<tr>
<td>SEX</td>
<td>d₁</td>
<td>h.₁</td>
</tr>
<tr>
<td>MARITAL</td>
<td>d₂</td>
<td>h.₂</td>
</tr>
<tr>
<td>SCHOOL</td>
<td>d₃</td>
<td>h.₃</td>
</tr>
<tr>
<td>CHILDREN</td>
<td>d₄</td>
<td>h.₄</td>
</tr>
<tr>
<td>ENGLISH</td>
<td>d₅</td>
<td>0</td>
</tr>
<tr>
<td>IMMIGR</td>
<td>d₆</td>
<td>h.₆</td>
</tr>
<tr>
<td>POVERTY</td>
<td>d₇</td>
<td>h.₇</td>
</tr>
<tr>
<td>HOUSEHLD</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FAMILY</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>URBAN</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>INDAGR</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>INDCONST</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>INDMANUF</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>INDSERV</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>OCCUTECH</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>OCCUSERV</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>OCCUFARM</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>OCCUPROD</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>PRIVATE</td>
<td>d₁₉</td>
<td>0</td>
</tr>
</tbody>
</table>

An important constraint is given by the exclusion of the ability to speak English as a regressor in the hours equation. The amount of time allocated to market activities is uncorrelated explicitly with language abilities of undocumented workers. Labor supply decisions are related to such a variable solely through the wage rate. In addition, note that the distinction between the conditional and the unconditional models in Table 4.3 is given by the exclusion of all but one of the demographic variables. These restrictions are imposed in order to evaluate the effects of the host environment over the labor supply and the participation decision of undocumented Mexican workers. Moreover, urban characteristics of undocumented workers are included in the immigration decision, but not in the wage and the hours equations. The existence of informal recruitment networks within Mexican settlements supports this restriction. Such networks permit finding jobs regardless of the residence location of workers in the United States. Lastly, based in statistical tests, the wage
equation was found to be more robust if the industry variables were dropped (INDCONST, INDMANUF and INDSERV) which let non-agriculture activities be approximated by occupational variables (OCCUTECH, OCCUSERV and OCCUPROD).

Inspection of Table 4.3 reveals that identification restrictions for the conditional model are given by

\[ \mathbf{R}_{WAGE}^{A} = \begin{bmatrix} -1 & h_2 & h_1 \\ 0 & 0 & y_1 \\ 0 & 0 & 0 \end{bmatrix} : \text{Rank}(WAGE) = 2 = (M-1), \quad \text{iff } y_1 \neq 0; \]

\[ \mathbf{R}_{HOURS}^{A} = \begin{bmatrix} d_5 & d_{19} \\ -d_5 & 0 \end{bmatrix} : \text{Rank}(HOURS) = 2 = (M-1), \quad \text{iff } (d_5, d_{19}) \neq 0; \]

\[ \mathbf{R}_{GAP}^{A} = \begin{bmatrix} 0 & 0 & d_{19} \\ -1 & h_1 & 0 \end{bmatrix} : \text{Rank}(GAP) = 2 = (M-1), \quad \text{iff } d_{19} \neq 0. \]

Likewise, the unconditional model is also identifiable under these conditions. However, here the wage and the hours equations restrictions become sufficient, but not necessary. In particular, the coefficient \( g_{10} \) may be constrained to be non-zero in order to further satisfy the generalized rank condition.

4.5.2 Estimation

The empirical specification described in (4.1) is reproduced here using the basic estimation framework outlined in section 4.3. The structural model may be written as

\[
(4.3) \quad \mathbf{YB} + \mathbf{X}\Gamma = \mathbf{U},
\]

where
\[
\mathbf{Y} = [\log EARNING \quad \log(L-\text{HOURS}) \quad \text{GAP}],
\]
\[
\mathbf{X} = [1 \quad \text{AGE} \quad \text{ASSETS} \quad \mathbf{Z}],
\]
\[
\mathbf{U} = [u_1 \quad u_2 \quad u_3] \sim \mathcal{N}(0, \Sigma),
\]
\[
B = \begin{bmatrix}
1 & -h_3 & -\gamma_2 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\quad \text{and} \quad
\Gamma = \begin{bmatrix}
d_0 & h_0 & \gamma_0 \\
h_1 & 0 & 0 \\
h_2 & \gamma_1 \\
d & -h & \gamma
\end{bmatrix}.
\]

Accordingly, the reduced-form of (4.3) yields

\begin{align*}
(4.10a) & \quad \log EARNING_i = [d_0 + Zd] = v_{1i}, \\
(4.10b) & \quad \log(LHOURS_i) = [h_0 + d_0h_3] + h_1AGE_i + h_2ASSETS_i - Z(h - dh_3) = v_{2i}, \\
(4.10c) & \quad GAP_i = [-[\gamma_0 + d_0\gamma_2] + \gamma_1ASSETS_i + Z(\gamma + d\gamma_2)] = v_{3i},
\end{align*}

where \(L\) is the total amount of leisure available in a given period of time and \(v\) is a vector of random terms such that \([v_{1i}, v_{2i}, v_{3i}] \sim N(0, \Omega)\). Following the description of the stochastic terms in section 4.3.1, the covariance matrix takes the form:

\[
\Omega = \begin{bmatrix}
\omega_{11} & \omega_{12} & \omega_{13} \\
\omega_{12} & \omega_{22} & \omega_{23} \\
\omega_{13} & \omega_{23} & \omega_{33}
\end{bmatrix} = \begin{bmatrix}
\Omega_{11} & \Omega_{12} \\
\Omega_{12} & \Omega_{22}
\end{bmatrix}.
\]

Applying the estimation procedure developed in expression (4.8), the log likelihood function of the simultaneous system (4.10) is governed by

\[(4.11) \quad \log L = -N_u \log(2\pi) - \frac{N_u}{2} \log |\Omega_{11}| - \frac{N_u}{2} \sum_{i=1}^{N_u} \left\{ \frac{pdf_i}{2} - \log[1-CDF_{1i}] + \log[1-CDF_{2i}] \right\} , \]

where the probability and the cumulative distribution functions are defined in terms of the reduced-form stochastic terms \(v_i\) and its respective variance-covariance matrix:

\[
\text{pdf}_i = \frac{1}{|\Omega_{11}|} \left[ v_{1i}^2 \omega_{22} - 2v_{1i}v_{2i} \omega_{12} + v_{2i}^2 \omega_{11} \right].
\]
CDF_{1i} = \Phi \left[ \frac{v_{3i}}{\Omega_{22}^{1/2}} \right],

CDF_{2i} = \Phi \left[ \frac{v_{3i} - \left( \frac{v_{1i} (\omega_{22} + \omega_{12} \omega_{23}) + v_{2i} (\omega_{11} \omega_{23} - \omega_{12} \omega_{13})}{\sqrt{\Omega_{22}^{1/2} \left( \omega_{22}^{2} + 2 \omega_{12} \omega_{13} \omega_{23} + \omega_{11} \omega_{23}^{2} \right)}} \right)^{1/2}}{\sqrt{\Omega_{22}^{1/2}}} \right].

Note that the covariance matrix \( \Omega \) is required to be nonsingular since \( \Omega^{-1} \) must exist. In addition, it is assumed \( \Omega \) to be positive-definite in order to assure global concavity. Drymies [1970] proposes a suitable transformation that guarantees this result.

**Proposition 4.1:** Let \( \Omega \) be a positive definite matrix of order 3. Then there exist an upper triangular matrix \( P \) such that \( \Omega = PP' \).

**Proof:** [See Dhrymes, 1970 p. 579; Proposition 15 and Remark 9]

Accordingly, the variance-covariance matrix may be rewritten as

\[
\Omega = \begin{bmatrix}
P_{11} & P_{12} & P_{13} \\
0 & P_{22} & P_{23} \\
0 & 0 & P_{33}
\end{bmatrix} \begin{bmatrix}
P_{11} & 0 & 0 \\
P_{12} & P_{22} & 0 \\
P_{13} & P_{23} & P_{33}
\end{bmatrix},
\]

where the following definitions applied

\[
\omega_{11} = P_{11}^2 + P_{12}^2 + P_{13}^2
\]
\[
\omega_{22} = P_{22}^2 + P_{23}^2
\]
\[
\omega_{33} = P_{33}^2
\]
\[
\omega_{12} = P_{22} P_{12} + P_{23} P_{13}
\]
\[
\omega_{13} = P_{33} P_{13}
\]
\[
\omega_{23} = P_{33} P_{23}
\]

Furthermore, Maddala [1983] recommends that "[b]ecause \( \gamma \) is estimable up to a scale factor, we shall assume that [Var (\( v_{3i} \))]=1." [Ibid. p.223] This normalization rule implies that
$P_{33}^2 = 1.30$

Estimation of parameters in the simultaneous system (4.10) is undertaken for both conditional and unconditional models. Three econometric procedures are used: iterative three stage least squares (IT3SLS), standard full information maximum likelihood (FIML) and Tobit maximum likelihood (Tobit ML). IT3SLS estimation was applied directly to equations (4.10a) and (4.10b) using the mainframe version of SAS/ETS$^\text{TM}$ simultaneous linear procedures (proc syslin IT3SLS). FIML and Tobit ML estimations required maximization of the log likelihood function (4.11) by iterative numerical procedures. GQOPT4/I subroutines were employed in these cases. In particular, the FIML model was estimated using DFP and GRADX algorithms, while Tobit ML estimation required only DFP optimization.

Convergence is assured by global concavity of the log likelihood function (4.11). Nonetheless, it was appropriate to start the iterative numerical approximation with good initial values in order to improve the speed of convergence. Tobin [1958] uses a simple estimator based on a linear approximation of the reciprocal of Mill's ratio to start his iteration. However, Amemiya [1973] shows that Tobin's initial estimator is inconsistent. In this study, IT3SLS estimators were taken as initial values on the optimization process. The drawback of this approach is that no initial estimates are calculated for the parameters associated with the decision rule, i.e. $\gamma_0, \gamma_1, \gamma_2, g, \omega_{13}$ and $\omega_{23}$. Consequently, these values are initialized at zero. Both set of conversions obtained in this study, conditional and unconditional, report optimum parameter estimates.

Finally, the model (4.10) is estimated using a labor supply defined in terms of the value $\bar{L}$, the maximum amount of leisure in a given period of time. However, the appropriate measure of labor supply is a matter of debate. Hanoch [1980] develops a theoretical model in which nonperfect substitution between different time definitions of leisure is ex-

30 Notice that under this normalization $\omega_{13}$ and $\omega_{23}$ are not unique since $P_{33} = \pm 1$, i.e. $P_{33} = \sqrt{1}$.

31 Recall that under the assumptions of linearity and normality IT3SLS and FIML methods are equivalent. This equivalence is empirically tested in the next section.

32 See Goldfeld and Quant [1986], GQOPT4/I version 4.01. Princeton University: Mineo. The estimation routines were performed in a 80386 IBM compatible running at 25 MHz. with math coprocessor. FORTRAN '77 was used as a programming language.

33 For descriptions of DFP and GRADX algorithms see Powell [1971] and the quadratic hill-climbing explanation in Goldfeld and Quant [1972], respectively. Appendix B presents the Fortran program used in these estimations.

34 Of course, convergence does not guaranteed that a global optimum has been located.
explicitly recognized. He concludes that different types of leisure are distinguished by their corresponding time dimension. Hence, "leisure during working weeks" (L1) and 'leisure during nonworking weeks' (L2), which are separate arguments in the utility function, and thus are not perfect substitutes." [Ibid, p.120] Nonetheless, following suggestions from Ghez and Becker [1975], Smith [1977] and Heckman and MaCurdy [1980], the present model estimates a labor supply function defined for \( \bar{T} = 8760 \) hours, the total number of hours in a year.

### 4.6 Empirical Results

The procedures outlined above were employed in the estimation of two alternative empirical specifications given by systems (4.1) and (4.1'). The first model recommends the use of a wage equation in the labor and participation decisions, while the second model includes an earnings function. Both specifications were estimated using iterative three stages least squares (IT3SLS), full information maximum likelihood (FIML) and the proposed Tobit maximum likelihood procedure (Tobit ML). The evaluation parameters are presented in Table 4.4. These results seem to suggest that sample selections considerations are inconsequential when the wage equation is considered in the estimation model. In contrast, truncation problems and the unobservability of the Mexican regime are extremely important if earnings are included in the empirical specification. The analysis presented below correspond to the latter case.

<table>
<thead>
<tr>
<th>Specification Function</th>
<th>Wage [System (4.1)]</th>
<th>Earnings [System (4.1')]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>R²</td>
<td>Log Likelihood Value</td>
</tr>
<tr>
<td>IT3SLS</td>
<td>0.303</td>
<td>-405.33</td>
</tr>
<tr>
<td>FIML</td>
<td></td>
<td>-405.12</td>
</tr>
<tr>
<td>Tobit ML</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Estimates of all parameters in the structural model (4.1') are reproduced in Tables 4.5-4.8. For comparative purposes, the first column reports IT3SLS estimates. The other

---

35 Recall that direct comparation of these parameters is not appropriate since such specifications are composed by different dependent variables, i.e. wage rate and earnings.
columns provide FIML and Tobit ML estimates for both *conditional* and *unconditional*
models. Inspection of these estimates reveals that IT3SLS and FIML generate statistically
identical results. Consequently, given that the structural model is linear, the assumption of
normal distributed random terms is supported by the empirical results. Evidence of nor-
mality is prerequisite *sine qua non* to avoid misspecification errors under ML procedures.
Amemiya [1985] points out that asymptotic bias is found to be especially great in standard
Tobit models when the true distribution function is nonnormal, e.g. Laplace.

4.6.1 The Variance-Covariance Matrix and Specification Tests

While it is impossible to conclude from the empirical analysis that the proposed Tobit ML
estimation method gives better results, the comparisons show that it does lead to different
empirical estimates. Moreover, it is clear from the theoretical analysis in Section 4.2, that
Tobit ML estimation is efficient and asymptotic unbiased. In contrast with IT3SLS and
FIML methods, the likelihood function proposed in (4.11) introduces specification prob-
lems resulting from the use of a truncated sample (CDF1) and unobserved Mexican regime
(CDF2). As expected, standard procedures (IT3SLS and FIML) are biased and inconsistent
because they fail to correct for the potential self-selection bias. These differences seem to
favor the proposed Tobit ML method. However, whether the source of misspecification is
significant enough to justify the computational burden imposed by Tobit ML techniques in
the wage and hours equation represents an empirical query.36

To address this question two arguments may have to be considered. First, the cor-
relation between the wage and hours functions and the participation rule Ω12, i.e. Ω13 and
Ω23. Large and statistically significant correlation values will imply important misspecification
errors if self-selection bias is omitted from the estimation strategy. Second, the main
reason to compute and report FIML results in addition to Tobit ML estimators is to estab-
lish a general reference model against which likelihood ratio tests can be calculated. This
measure establishes the performance level of Tobit ML estimation with respect to standard
procedures.

Consider the estimates of the transformation matrix P reported in Table 4.5. Ap-
plying the definitions used in (4.11), the upper triangular variance-covariance matrix for the

36 Notice that this controversy does not applies to the participation rule (GAP), since under standard
procedure this function is not estimable, i.e. GAP is not observable. In this context, Tobit ML estimation
is unambiguously superior.
unconditional model is

\[
\Omega = \begin{bmatrix}
0.346 & -0.001 & \pm0.588 \\
& 0.012 & \pm0.067 \\
& & 1.000
\end{bmatrix},
\]

where the terms in parenthesis are asymptotic standard errors and the asterisk denotes a coefficient significantly different than zero at the 95 percent level of confidence.\textsuperscript{37} The statistical inference suggest that the decision rule (GAP) influences wage earnings and labor supply decisions of undocumented workers already participating in the U.S. labor force. The exclusion of the GAP equation will deny such a correlation, with the corresponding asymptotic bias. Furthermore, these results indicate that important efficiency gains may be obtained using full information procedures since the hypothesis of a diagonal covariance matrix is rejected.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Conditional</th>
<th>Unconditional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FIML</td>
<td>Tobit ML</td>
</tr>
<tr>
<td>P₁₁</td>
<td>0.708</td>
<td>0.274</td>
</tr>
<tr>
<td></td>
<td>(0.007)*</td>
<td>(0.007)*</td>
</tr>
<tr>
<td>P₂₂</td>
<td>0.095</td>
<td>0.088</td>
</tr>
<tr>
<td></td>
<td>(0.001)*</td>
<td>(0.001)*</td>
</tr>
<tr>
<td>P₃₃</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>P₁₂</td>
<td>-0.343</td>
<td>-0.081</td>
</tr>
<tr>
<td></td>
<td>(0.011)*</td>
<td>(0.011)*</td>
</tr>
<tr>
<td>P₁₃</td>
<td>-</td>
<td>-2.26</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.053)*</td>
</tr>
<tr>
<td>P₂₃</td>
<td>-</td>
<td>0.107</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.006)*</td>
</tr>
</tbody>
</table>

Log Likelihood Value: \(-647.8\) \(-395.4\) \(-400.8\) \(-144.8\)
Observations (Nu)= 4662

* Denotes a coefficient significantly different than zero at the 95 percent level of confidence.

Aside from generating consistent and efficient parameter estimates, Tobit ML esti-

\textsuperscript{37} On the indeterminate of covariances \(\omega_{13}\) and \(\omega_{23}\), see footnote 30.
mation has another function here. This is to serve as a reference model against which the performance of standard FIML regression models are measured. Recall that the Tobit ML model is equivalent to the FIML specification if \( \omega_{13} \) and \( \omega_{23} \) are equal to zero. Consequently, a likelihood ratio test may be performed under these restrictions.\(^{38}\) Tobit ML estimation constitutes the unrestricted model from which an alternative restricted (FIML) model can compare its empirical performance. According to Table 4.5, the values of the LR test (\(-2\) times the log of the likelihood ratio) for both, conditional and unconditional models are 504.8 and 512.0, respectively. Using a \( \chi^2 \) distribution with \( N_U = 48 \) degrees of freedom allows to reject the null hypothesis (FIML model) in favor to Tobit ML estimation, with 99 percent of confidence in both cases.

Likewise, a similar test may be conducted under the Tobit ML empirical structure comparing the conditional and the unconditional models. Here, the restricted model is the conditional specification, where all the demographic dummies but one (PRIVATE), are set to zero. In this case, the \(-2\)log likelihood ratio is 501.2 which, under a \( \chi^2 \) distribution with \( N_U = 48 \) degrees of freedom, permits reject the null hypothesis in favor of the unconditional models.

To summarize, Tobit ML estimation provides the most efficient and asymptotic unbiased estimators when comparing with standard procedures that do not account for sample selection. Moreover, the inclusion of all available information, in terms of demographic variables, under the unconditional model also improve the performance of the estimation. Lastly, efficiency gains are expected with the use of full information procedures that include the participation equation (GAP), since the hypothesis of a diagonal covariance matrix is rejected.

4.6.2 The Earnings Function

\(^{38}\) In general, the likelihood ratio (LR) test may be stated as:
Consider
\[
H_0: \, r(\theta) = 0 \quad \text{and} \quad H_1: \, r(\theta) \neq 0.
\]
Optimal log likelihood values are given by
\[
L_{U} = L(\hat{\theta} \mid Y) \quad \text{where} \quad \hat{\theta} \quad \text{is the unrestricted ML estimate and}
\]
\[
L_{R} = L(\theta_0 \mid Y) \quad \text{where} \quad \theta_0 \quad \text{is the restricted ML estimate.}
\]
Then, under \( H_0 \)
\[
LR = 2[\log L_{U} - \log L_{R}] \overset{d}{\sim} \chi^2 \quad \text{with} \quad q \quad \text{degrees of freedom.}
\]
Empirical estimates of the logarithm of annual wage earnings of undocumented Mexican workers are presented in Table 4.6. From the set of personal characteristics, only sex and marital status have substantial effects on U.S. earnings. Single male illegal workers are more likely to have higher earnings than females or married males. Although this result is supported by the two Tobit ML models reported in Table 4.6, IT3SLS and FIML contradicts the effect of marital status over earnings. These estimation models suggest that married workers actually have a greater propensity for higher wages. This contradiction may be explained by misspecification errors due to self-selection bias in standard estimation methods.

An additional source of divergence is found with the statistical inference of the SCHOOL’s estimate. In contrast to IT3SLS and FIML coefficients, the unconditional Tobit ML model reports that formal education has a statistically significant positive effect on wages.\textsuperscript{39} Contrary to conventional wisdom, this result implies that undocumented immigrants are actually able to transfer some of their Mexican-specific education to the U.S. labor market. Investment in formal human capital (school education) may be a profitable investment even in the host-country. Unitary increases in the immigrant’s schooling raise the annual earnings by 4.5 percent. Accordingly, Mexican formal education is transferable across regimes.

In contrast, education specific to the U.S. regime, e.g. ability to speak English, is less likely to affect earnings of undocumented workers. The parameter estimate of ENGLISH is not statistically significant at 95 percent of confidence in both IT3SLS and unconditional models. This result supports the tendency observed by illegal workers to show little incentive to learn English. Such a disposition may describe the low skill orientation of their jobs, where none or small dominance of English results in a satisfactory job performance.

The lack of statistical correlation between English speaking ability and earnings may be explained due to cohort or network effects. Here, Mexican workers tend to locate themselves in jobs that reproduce their home-environment by working with fellow-countrymen. In this setting, job coordination takes place inside the cohort without the need of foreign language communication. Only one bilingual person, e.g. the foreman, is required as a bridge between English speaking bosses and undocumented workers. Moreover, the defi-

\textsuperscript{39} Recall that the existence of statistical correlation between two variables does not imply causality.
ciency in English proficiency may provide evidence in favor the transitory characteristics of Mexican undocumented immigration, where low rates of social and cultural assimilation are observed.

<table>
<thead>
<tr>
<th>Variable</th>
<th>IT3SLS</th>
<th>Conditional</th>
<th>Unconditional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>FIML</td>
<td>Tobit ML</td>
</tr>
<tr>
<td>Intercept (d0)</td>
<td>7.61</td>
<td>7.50</td>
<td>11.45</td>
</tr>
<tr>
<td></td>
<td>(0.093)*</td>
<td>(0.079)*</td>
<td>(0.438)*</td>
</tr>
<tr>
<td>SEX (d1)</td>
<td>-0.379</td>
<td>-0.606</td>
<td>-2.93</td>
</tr>
<tr>
<td></td>
<td>(0.039)*</td>
<td>(0.038)*</td>
<td>(0.405)*</td>
</tr>
<tr>
<td>MARITAL (d2)</td>
<td>-0.127</td>
<td>-0.177</td>
<td>0.685</td>
</tr>
<tr>
<td></td>
<td>(0.025)*</td>
<td>(0.025)*</td>
<td>(0.294)*</td>
</tr>
<tr>
<td>SCHOOL (d3)</td>
<td>0.003</td>
<td>0.002</td>
<td>-0.047</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>CHILDREN (d4)</td>
<td>0.014</td>
<td>0.003</td>
<td>-0.019</td>
</tr>
<tr>
<td>ENGLISH (d5)</td>
<td>0.033</td>
<td>0.073</td>
<td>0.517</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.008)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>IMMIGR (d6)</td>
<td>0.026</td>
<td>0.052</td>
<td>0.306</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
<td>(0.022)*</td>
<td>(0.085)*</td>
</tr>
<tr>
<td>POVERTY (d7)</td>
<td>0.215</td>
<td>0.196</td>
<td>0.875</td>
</tr>
<tr>
<td></td>
<td>(0.005)*</td>
<td>(0.005)*</td>
<td>(0.049)*</td>
</tr>
<tr>
<td>HOUSEHLD (d8)</td>
<td>0.121</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.040)*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FAMILY (d9)</td>
<td>-0.406</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.041)*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>INOAG (d11)</td>
<td>-0.234</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.058)*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>OCCUTECH (d15)</td>
<td>-0.115</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.077)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>OCCUSERV (d16)</td>
<td>-0.218</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.063)*</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>OCCUPROD (d18)</td>
<td>-0.052</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>PRIVATE (d19)</td>
<td>0.168</td>
<td>0.186</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(0.056)*</td>
<td>(0.094)*</td>
<td>(0.018)</td>
</tr>
<tr>
<td>Weighted R^2</td>
<td>0.317</td>
<td>-647.8</td>
<td>-395.4</td>
</tr>
<tr>
<td>Log Likelihood Value</td>
<td>-647.8</td>
<td>-395.4</td>
<td>-400.8</td>
</tr>
<tr>
<td>Observations (N)</td>
<td>4662</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Denotes a coefficient significantly different than zero at the 95 percent level of confidence.

A robust result is obtained in the estimate of the IMMIGR variable. In particular, under the unconditional Tobit LM model an additional five years of immigration are more likely to increase wage profits an average of 3.5 percent a year. This effect is understated.
when using standard estimation procedures, e.g. 2.4 percent under the comparable FIML model. In any case, longer immigration periods may suggest higher assimilation rates of undocumented workers into the labor force. In fact, U.S.-specific market and nonmarket experience, measured by immigration years, constitutes a source of human capital investment. Unlike others forms of U.S.-specific human capital investment (e.g., English speaking ability), immigration time has a significant direct effect on earnings.\(^{40}\)

In sum, human capital investment of Mexican settlers may take the following forms: formal domestic education (SCHOOL), U.S.-specific abilities (ENGLISH) and U.S. experience (IMMIGR). It is shown that formal education in Mexico may be transferred across regimes positively affecting U.S. earnings. In addition, ability to speak English does not constitute a profitable investment, since undocumented workers tend to locate in low skill jobs where cohort organization eliminates the need of English-speaking interaction. Lastly, time of immigration constitutes a proxy for on-the-job training, where the degree of assimilation into the U.S. labor market is proportional to the migration experience.

4.6.3 The Labor Supply

Estimates of the labor supply are presented in Table 4.7. Here empirical results are generally consistent with theoretical predictions described in Section 3.4.\(^{41}\) According to the life cycle model of undocumented immigration, time-preferences and interest rates are captured by the estimate coefficient of AGE. Although older immigrants may have a greater tendency to work more, as expected in the theoretical model, this parameter is not statistically significant under both unconditional models (FIML and Tobit ML). The rationality of this behavior may be given by the equivalence between the time-preference and the real interest rate, i.e., \(r=r\).\(^{42}\) The estimate coefficient of ASSETS, in contrast, is entirely conventional. Increase in the U.S. property income reduces the amount of hours allocated to market activities. A consistent result that is found elsewhere and confirmed by the estimate of \(h_2\) in Table 4.7.

\(^{40}\) Note that immigration time is not necessarily positive correlated with the ability to speak English.
\(^{41}\) Notice that the labor supply in the U.S. regime is reciprocal to leisure demand, i.e. \(H_u = 1/L_u\).
\(^{42}\) According to expression (3.28) \(h_1 = -\log R\), where \(R = (1+\rho)/(1+r)\). Then \(h_1 = 0\) if \(\rho = r\).
Table 4.7—Regression Estimates of the Labor Supply: Log(E - HOURS)
(Assumptive Standard Errors in Parenthesis)

<table>
<thead>
<tr>
<th>Variable</th>
<th>IT3SLS</th>
<th>Conditional</th>
<th>Unconditional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>FIML</td>
<td>Tobit ML</td>
<td>FIML</td>
</tr>
<tr>
<td>Intercept (h_{0})</td>
<td>9.72</td>
<td>9.78</td>
<td>9.41</td>
</tr>
<tr>
<td></td>
<td>(0.338)*</td>
<td>(0.226)*</td>
<td>(0.064)*</td>
</tr>
<tr>
<td>AGE (h_{1})</td>
<td>-0.0002</td>
<td>-0.0004</td>
<td>-0.0004</td>
</tr>
<tr>
<td></td>
<td>(0.00014)</td>
<td>(0.0001)*</td>
<td>(0.0001)*</td>
</tr>
<tr>
<td>ASSETS (h_{2})</td>
<td>3.3x10^{-6}</td>
<td>5.7x10^{-6}</td>
<td>6.1x10^{-6}</td>
</tr>
<tr>
<td></td>
<td>(1.4x10^{-5})*</td>
<td>(1.1x10^{-5})*</td>
<td>(1.1x10^{-5})*</td>
</tr>
<tr>
<td>logEARNING (h_{3})</td>
<td>-0.102</td>
<td>-0.114</td>
<td>-0.060</td>
</tr>
<tr>
<td></td>
<td>(0.042)*</td>
<td>(0.029)*</td>
<td>(0.006)*</td>
</tr>
<tr>
<td>SEX (h_{4})</td>
<td>0.010</td>
<td>0.024</td>
<td>0.020</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.018)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>MARITAL (h_{5})</td>
<td>0.012</td>
<td>0.016</td>
<td>-0.004</td>
</tr>
<tr>
<td></td>
<td>(0.006)*</td>
<td>(0.006)*</td>
<td>(0.005)</td>
</tr>
<tr>
<td>SCHOOL (h_{6})</td>
<td>-0.001</td>
<td>-0.001</td>
<td>-0.0005</td>
</tr>
<tr>
<td></td>
<td>(0.0004)*</td>
<td>(0.0003)*</td>
<td>(0.0005)</td>
</tr>
<tr>
<td>CHILDREN (h_{7})</td>
<td>-0.002</td>
<td>-0.002</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>IMMIGR (h_{8})</td>
<td>-0.005</td>
<td>-0.007</td>
<td>-0.007</td>
</tr>
<tr>
<td></td>
<td>(0.002)*</td>
<td>(0.002)*</td>
<td>(0.002)*</td>
</tr>
<tr>
<td>POVERTY (h_{9})</td>
<td>-0.010</td>
<td>-0.0115</td>
<td>-0.009</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.006)*</td>
<td>(0.005)</td>
</tr>
<tr>
<td>HOUSEHL (h_{10})</td>
<td>0.003</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>FAMILY (h_{11})</td>
<td>0.007</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>INDA (h_{12})</td>
<td>0.027</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.024)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>INDCOST (h_{13})</td>
<td>-0.001</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>INDMIN (h_{14})</td>
<td>0.023</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>INDS (h_{15})</td>
<td>0.017</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>OCCUTECH (h_{16})</td>
<td>0.018</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>OCCUSERV (h_{17})</td>
<td>0.024</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>OCCUAGR (h_{18})</td>
<td>0.002</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>OCCUFPROD (h_{19})</td>
<td>0.009</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Weighted R^2 0.317
Log Likelihood Value
-647.8   -395.4   -400.8   -144.8
Observations (N)= 4662

* Demotes a coefficient significantly different than zero at the 95 percent level of confidence.
The labor supply of undocumented Mexican workers appears to be positive and relatively inelastic to changes in the U.S. wage rate. Under the Tobit ML model, estimated earnings elasticities of the demand for leisure are $-0.052$ and $-0.060$ for the unconditional and conditional models, respectively.\(^{43}\) However, the earnings elasticity is substantially overestimated, in absolute terms, when using standard estimation procedures. IT3SLS and FIML report estimates of earnings elasticities ranging from $-0.102$ to $-0.114$. All estimates reject the null hypothesis at 95 percent of confidence in favor of non-zero coefficients.

The theoretical specification of the labor supply in (3.28) allows the identification of intertemporal effects.\(^{44}\) Under the unconditional Tobit ML model, the life cycle component of the labor supply $l_{43}$ is likely to be $-0.947$. Hence, leisure decisions are affected by permanent changes in earnings. Transitory effects perceived as such by undocumented workers will have little effect over their lifetime labor supply. Relatively inelastic labor supplies are expected if intertemporal effects are included in the regression model.

Similar studies of life cycle behavior by Smith [1977], MaCurdy [1985] and Altonji [1986], among others appear to confirm this result.\(^{45}\) Smith uses synthetic cohort data from a 1967 sample of white and black men, finding leisure demand elasticities of $-0.06$ and $-0.10$, respectively. Cross-section data on prime-age white married men is used by MaCurdy, obtaining labor supply estimates ranging from $-0.08$ to $-0.15$. Lastly, Altonji calculates lifetime sensibility of labor supply decisions to a longitudinal sample of married white men, estimating elasticities in the order of $0.09$-$0.17$. Unfortunately, there are no similar studies on undocumented immigration in the economic literature.

According to the unconditional Tobit ML model, single Mexican workers are more likely to work longer periods than married settlers; this seems to be consistent with their expected earnings reported in the wage equation. Undocumented workers with higher education and with longer immigration periods will tend to allocate more time to market activities than low educated and recent immigrants. Although these estimates are statistically sig-

\(^{43}\) Notice that according to system (4.1') the actual labor supply elasticity is $0.0548$.
\(^{44}\) Recall that the uncompensated wage elasticity $h_2$ is composed of a current and lifetime components, i.e. $h_2=(1+l_{43})$.
\(^{45}\) See Killingsworth [1983] for a comprehensive summary of estimates for lifetime elasticities of labor supply, p.298.
nificant, their overall impact on the labor supply may be rather small. Finally, the estimate parameter for the number of children is statistically significant only in the unconditional Tobit ML model, where an additional child may imply a decrease of 0.2 percent in the number of hours worked. This issue is specially important in the case of female undocumented workers.

4.6.4 The Participation Decision

The most important contribution of this analysis constitutes the empirical estimation of determinants of the undocumented participation rule. Given that the Mexican regime is not observable, this objective may be achieved only by the use of estimation methods based on limited dependent variables models. Table 4.8 shown the empirical results of the decision rule under both conditional and unconditional Tobit ML models.

The first step in the interpretation of estimates for the participation rule is to define the economic meaning of GAP. It is well known that if positive quantities of a market good are purchased, a necessary equilibrium condition is that its price equals its marginal value; while if the price exceeds the marginal valuation of a market good, no purchases will be observed. For Mexican undocumented participation, a similar condition applies. Immigrants will switch back and forth across regimes according to the relative composition of net benefits in each country. In this context, undocumented immigration is observed since the marginal valuation or shadow price of U.S. participation exceeds the opportunity costs of an alternative activity in the Mexican regime.

Because it is possible to compare the relative magnitudes of utility differences, the participation index GAP arranges migration preferences in an ordinal fashion. GAP, however, does not provide a quantity measure since utility preferences do not observe cardinal properties. Intrapersonal comparisons of utility are not feasible. Indeed, the decision rule indicates the direction of migration flows, but it does not quantify the magnitude of such flows. Nonetheless, a cardinal measure of undocumented Mexican immigration may be obtained by a suitable transformation of the participation rule into a probability statement.46 Here, the likelihood of participation is quantified in terms of their impact multipliers which are a function of all exogenous variables in the decision rule and its density function. Consequently, an ordinal criteria like the participation index GAP can be proportionally mapped

46 Such a transformation is given by \( \Pr(GAP>0)=\Pr(T13X>v3)=CDF(-T13X) \). A comparative static exercise results in \( \frac{\partial \Pr(GAP>0)}{\partial \gamma_i}=\frac{\partial CDF(\cdot)}{\partial \gamma_i}=pdf(\cdot)\times \partial T13/\partial \gamma_i \), where by definition pdf(\cdot)>0.
into a cardinal measure given by the probability of switching regimes.

Table 4.8—Regression Estimates of the Participation Rule: GAP

<table>
<thead>
<tr>
<th>Variable</th>
<th>Conditional</th>
<th>Tobit ML</th>
<th>Unconditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept ((\gamma_0))</td>
<td>14.61</td>
<td>3.66</td>
<td></td>
</tr>
<tr>
<td>ASSETS ((\gamma_1))</td>
<td>(-4.6\times10^{-6})</td>
<td>(-3.2\times10^{-4})</td>
<td>(4.7\times10^{-8})*</td>
</tr>
<tr>
<td>logEARNING ((\gamma_2))</td>
<td>(-1.38)</td>
<td>(-0.500)</td>
<td>(0.007)*</td>
</tr>
<tr>
<td>SEX ((g_{1:d_1}))</td>
<td>0.017</td>
<td>2.93</td>
<td>(-0.198)</td>
</tr>
<tr>
<td>MARITAL ((g_{2:d_2}))</td>
<td>(-0.149)</td>
<td>(-0.834)</td>
<td>(-0.679)</td>
</tr>
<tr>
<td>SCHOOL ((g_{3:d_3}))</td>
<td>0.006</td>
<td>0.0</td>
<td>(-0.059)</td>
</tr>
<tr>
<td>CHILDREN ((g_{4:d_4}))</td>
<td>0.004</td>
<td>0.0</td>
<td>0.038</td>
</tr>
<tr>
<td>ENGLISH ((d_5))</td>
<td>(-0.517)</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>IMMIGR ((d_6))</td>
<td>(-0.306)</td>
<td>(-0.035)</td>
<td></td>
</tr>
<tr>
<td>POVERTY ((d_7))</td>
<td>(-0.875)</td>
<td>(-0.167)</td>
<td></td>
</tr>
<tr>
<td>HOUSELD ((d_8))</td>
<td>(-0.146)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>FAMILY ((d_9))</td>
<td>(-0.303)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>URBAN ((g_{10}))</td>
<td>(-0.033)</td>
<td>0.0</td>
<td>(0.028)</td>
</tr>
<tr>
<td>INDAGR ((d_{11}))</td>
<td>(-0.133)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OCCUTECH ((d_{15}))</td>
<td>(-0.0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OCCUSERV ((d_{16}))</td>
<td>(-0.146)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>OCCUPROD ((d_{18}))</td>
<td>(-0.0)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Demotes a coefficient significantly different than zero at the 95 percent level of confidence. The characters in bold refer to estimates of the vector \(g\). Recall that demographic characteristics in the GAP equation are composed by \(g_\text{ed}d\). See Table 4.5 for estimates of vector \(d\).

Taking these considerations into account, inspection of estimates in Table 4.8 reveals that married males with a low level of formal education are more likely to engage in migration activities. In particular, human capital in terms of Mexican-specific education appears to have a stronger effect in the participation decision than in the U.S. undocumented labor supply. Not surprisingly, it is shown that improvement and expansion of the Mexican educational system may significantly reduce immigration flows to the United States. Moreover, such a contraction may be viewed by domestic residents as a lifetime decision. U.S. immigration policy aimed at the promotion of Mexican-specific human capital investment
appears to be a more efficient strategy to control undocumented flows, than conventional non-economic policies, e.g. border interdiction and employment sanctions.\footnote{These policy instruments do not affect the life cycle component of the decision process of undocumented workers. It has been shown, therefore, that such transitory distortions may have little impact on the participation rule.}

Although the number of children (CHILDREN) and years of immigration (IMMIGR) show relatively small effects on the estimate of hours worked in the U.S., these variables appear to play an important role in the determination of the immigration decision. In general, increasing the number of children in the household may encourage participation in the U.S. labor market. In contrast, larger periods of immigration (IMMIGR) are more likely to promote repatriation of undocumented workers to the Mexican regime.

Empirical estimates of these two variables (CHILDREN and IMMIGR) seem to provide evidence of the household productivity hypothesis developed in Chapter 2. There, transitory immigration is an optimal solution even with the existence of continuously large wage differentials in favor of the U.S. regime. Undocumented workers switch back to their home-communities because the marginal productivity of household activities offsets possible U.S. market gains. Mexican residents immigrate in periods of low productivity and return to their home-communities in periods of high productivity.

In this context, larger families will tend to distribute household activities among their members in a more comprehensive way than smaller families. Higher number of children may liberate human resources that can be directed to participation in the United States. The positive correlation observed in Table 4.8 between number of children and incentives to participate in the U.S. appears to support the former hypothesis.

Because the marginal productivity of household activities appreciates exponentially over the immigration period, further participation in the U.S. labor market diminishes relative benefits of undocumented workers in terms of alternative activities in their home-communities. This creates a progressive deterioration of the family wealth that will result in Mexican repatriation. Such an argument seems to be supported by the empirical evidence presented here, where the estimate of the immigration parameter (IMMIGR) is negatively correlated with the decision rule.

The two economic variables included in the participation equation, ASSETS and
logEARNING, are statistically significant at 99 percent level of confidence. As expected, the estimate parameter of the net worth is negative and larger, in absolute terms, than the similar estimate in the labor supply (3.2x10^{-4} v.s. 3.7x10^{-6}). This result indicates that relative to the labor supply, the determination of the immigration decision may be affected in a larger proportion by changes in the property income. Although the empirical estimate of such a variable is rather small, it is shown that the changes in the household financial wealth may significantly impact a behavior characterized by switching regimes.48

In addition, the estimate coefficient of the logarithm of earnings may be interpreted as the uncompensated elasticity of the GAP index. Relative to the labor supply, the participation decision appears to be more sensitive (less inelastic), in absolute terms, to earnings, i.e. −0.5 v.s. 0.052 [see Figure 4.5]. Accordingly, changes in the low-skill U.S. wage rate may have a small effect over the undocumented labor supply, but a relative large impact in terms of divert Mexican participation.

\[ \text{U.S. Earnings} \]
\[ \text{U.S. Wage} \]
\[ \text{GAP} \]

\[ \text{Participation Rule} \]
\[ \text{U.S. Hours} \]

Figure 4.5—Mexican Undocumented Labor Supply and GAP Wage Elasticities

Indeed, contrary to conventional wisdom, the empirical evidence presented here suggests that an increase in U.S. earnings may actually discourage Mexican participation. Recall from section 3.4.2a and in particular equation (3.27), that the wage elasticity of the participation rule is defined as a linear combination of three major elements: A lifetime (permanent) component [(a+b)u_{3}<0], a direct (current) multiplier [b>0] and a job-searching cost parameter [bf>0], i.e. \( \gamma_2 = [(a+b)u_{3}+b(1+f)] \). Consequently, given that the empirical estimate of \( \gamma_2 \) is negative, current and searching costs effects seem to dominate the

48 Undocumented participation in the U.S. will be promoted when household wealth is deteriorating, and it will be discouraged when net worth appreciates.
life cycle component of the wage elasticity \([a+b]l_{u3} < b(1+f)\). In this context, Pioe [1986] suggest that

"[t]he best way to limit immigration is by direct control over employment conditions, by raising wages and improving working conditions of jobs to which immigrants are attached in the hope that this will eventually attract national workers in their place." [Ibid. p.37]

For instance, an increase in the legal minimum wage will actually hurt new immigration flows, while undocumented workers already participating in the U.S. labor force will benefit from this policy.

Further conclusions may be derived from the analysis of different elements of the wage elasticity. According to the discussion on the labor supply in 4.6.3, the life cycle component \(l_{u3}\) is identifiable if the wage elasticity of the hours equation is known, i.e. \(h_3 = (1+l_{u3})\). Consistent with the theoretical specification, it was suggested that the estimate of \(l_{u3}\) may be equal to \(-0.947\). Assuming constant returns to scale in the utility function, i.e. \((a+b)=1\), the coefficient of both current and searching costs effects in the participation rule is \(1.447.49\) Unfortunately, the effect of searching cost over the participation rule cannot be explicit deduce without further assumptions.

The negative correlation between the immigration decision rule and U.S. earnings reported above is likely to be driven by the consideration of searching costs. Notice that in the absence of searching costs \((f=0)\), life cycle information outweighs current effects if \(l_{u3}<1\), i.e. \((a+b)l_{u3}>b\). In this case, the wage elasticity of the participation decision is unambiguously positive. Consequently, given that \(\gamma_3<0\) it may be concluded that searching costs play a significant role in the determination of the participation rule: Large values of \(f\) will result in less inelastic negative functions of GAP with respect to wage rates. Any attempt to neglect such an effect may led to erroneous conclusions on undocumented Mexican participation in the United States.

Nonetheless, the former results have to be taken with caution since both variables GAP and U.S. earnings \(E_{u3}\) are assumed to be endogenous. It is shown that increasing wage rate in the U.S. labor market may discourage Mexican participation i.e.

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49 In fact, notice that by construction \(0<b<1\) and \(0<f<1\). Then, current and searching costs coefficients are restricted to the interval \([1,2]\), i.e. \(1\leq b(1+f)<2\).
\( \partial \text{GAP}/\partial \text{E}_{\text{US}} < 0 \). However, an inverse relationship may also applied: The influx of Mexican workers could depress U.S. wage rates, i.e. \( \partial \text{E}_{\text{US}}/\partial \text{GAP} < 0 \). This hypothesis can not be tested since GAP does not appear as a regressor in the wage equation.\(^{50}\) In this context, the empirical exercise has established a statistically significant correlation between the immigration decision and U.S. wages, but their causality direction is ambiguous.\(^{51}\)

To summarize, estimation of the participation rule constitutes a major contribution of this empirical exercise. It is found that undocumented immigration is more likely to occur among married household members. Mexican-specific formal education is expected to have a statistically significant negative effect over the probability of undocumented participation. In particular, promotion of further investment in human capital specific to the Mexican regime may contribute to significantly reduce undocumented population in the United States. This may be a more efficient immigration policy than traditional instruments that fails to promote permanent involvement in home-communities. The empirical estimates of the parameters for number of children and years of immigration appear to confirms the hypothesis that relates the marginal productivity of household activities with switching back and forth across regimes. Moreover, the deterioration of the household net worth seems to promote undocumented participation, while its appreciation may result in repatriation of Mexican illegal workers. Finally, the empirical evidence suggests undocumented immigration is discourage with further expansion of U.S. earnings. This result may be driven by the impact of job-searching costs over the elasticity of GAP, where large searching costs yield relative elastic negative participation functions.

\(^{50}\) Since participation rule is missing, such a specification will considerably complicate the empirical model.

\(^{51}\) The author acknowledges Peter Harley for bring to his attention this point.
Empirical conclusions regarding the economic impact of undocumented workers on the U.S. economy have traditionally been based on circumstantial rather than on direct evidence. Such results are frequently based on simulated effects of illegal immigration, where the relevant elasticities underlying the simulations are merely approximations. In fact, little is known about the elements driving Mexican immigration and its characteristics of the labor market in which undocumented workers operate.

This study addressed the impact of Mexican illegal immigration on the U.S. labor market. It constitutes a first step towards developing rigorous structural econometric models which empirically analyze undocumented labor force dynamics. Rigorous structural estimation requires the solution of intricate theoretical problems that have not been addressed in previous literature. The analysis developed here identifies those problems and proposes innovative solutions. In particular, the participation decision and the labor supply function of illegal Mexican workers were studied in the context of life cycle theory and stochastic behavior. The empirical part of the analysis addressed the problems of sample selection and missing observations that characterize the available data on Mexican migration. The major conclusions derived from this study are enumerated below.

*Human Capital and Earnings:*

- Mexican undocumented workers appeared to have little economic incentives to invest in human capital specific to the U.S., such as the ability to speak English. This behavior may be the result of partial transferability of home skills (e.g. formal education) to
the secondary labor market in the United States. Contrary to conventional wisdom, the empirical evidence seemed to be consistent with the transferability hypothesis. Undocumented immigrants may be actually able to transfer some of their Mexican-specific education to the U.S. labor market, positively affecting earnings of undocumented workers. Unitary increases in the immigrant’s schooling may boost their annual income in the U.S. by 4.5 percent. Accordingly, investment in formal human capital (school education) appeared to be a profitable investment even in the host-country.

- Education specific to the U.S. regime, e.g. ability to speak English, is less likely to affect earnings of undocumented workers. The ability to speak English does not constitute a profitable investment, since undocumented workers are employed in low-skill jobs where cohort organization eliminates the need for English-speaking interaction. Mexican workers will tend to locate themselves in jobs that reproduce their home-environment by working with fellow countrymen. In this setting, job coordination takes place inside the cohort without the need of foreign language communication. Only one bilingual person, e.g. the foreman, is required to bridge English speaking bosses and undocumented workers. The coordinator is likely to be an Hispanic native worker or a long time permanent immigrant. This result is consistent with the tendency observed by illegal workers to show little incentives to learn English.

- Mexican illegal workers are likely to be characterized by individuals with transitory immigration patterns. The deficiency in English proficiency and the transferability of Mexican-specific human capital may provide evidence in favor of this hypothesis. Mexican undocumented immigration may be viewed as a temporary phenomenon, distinguished by low rates of social and cultural integration to the U.S. economy.

- The time elapsed since immigration may represent a proxy for on-the-job training, where the degree of assimilation into the U.S. labor market is proportional to the migration experience. Unlike others forms of U.S.-specific human capital investment (e.g. English speaking ability), immigration time may have a significant direct effect over earnings.

Labor Supply and Marginal Displacement Effects:

- The labor supply of undocumented Mexican workers is likely to be positive and very inelastic to changes in the U.S. earnings. Under the proposed econometric model, the uncompensated labor supply elasticity was likely to be in a 0.055–0.064 range. However,
wage elasticities are substantially overestimated if standard linear estimation procedures are used, i.e. 0.114–0.129.

- The life cycle specification developed in the theoretical model permitted the identification of current and lifetime effects. It was shown that labor supply decisions may be affected mainly by permanent changes in earnings. Transitory effects perceived as such by undocumented workers will have little effect over their lifetime labor supply. This result appears to be consistent with Friedman’s permanent income hypothesis.

- Low marginal displacement rates of native workers were observed as result of Mexican undocumented immigration. Following Johnson [1980], the labor supply elasticity of domestic workers in the low-skill market is plausibly to be around 0.2, which is relatively more elastic than the labor supply of illegal workers estimated in this study. Accordingly, given an expansion of the wage rate in the secondary market, e.g. a hike in the legal minimum wage, the number of native born workers in the labor force will increase proportionally more than the increase in undocumented workers. The rather inelastic labor supply of illegal settlers may reflect little marginal displacement effects in the low-skill market.

The Participation Decision and Household Productivity:

- The dichotomy between home-production possibilities in each country suggested that leisure in Mexico is a poor substitute for leisure in the United States. This constituted a major element in explaining the coexistence between temporary migration and continuously large wage differentials favoring the U.S. labor market. Mexican residents are productive in household activities and in the labor market, while undocumented workers are productive only in the U.S. labor force.

- Mexican workers switch back and forth across countries as a result of fluctuations in the marginal productivity of home activities either in the market or in the household. The life cycle specification predicted that Mexican immigrants respond to social, cultural and particularly economic attachments with their home-communities. Illegal settlers, however, will immigrate permanently if these anchors are broken, for instance by family immigration.
• The empirical evidence obtained here appears to be consistent with the previous hypothesis. Indeed, longer immigration periods are more likely to discourage Mexican participation in the U.S. labor market. Extended immigration periods diminishes relative benefits of undocumented workers in terms of alternative activities in their home-communities. In fact, the marginal productivity of household work appreciates exponentially over the immigration period. This creates a progressive deterioration of the family wealth that will result in Mexican repatriation.

The Participation Decision and Formal Education:

• Formal education in Mexico seemed to have strong negative effects on the probability of undocumented laborers participating in the U.S. labor force. It was shown that improvement and expansion of the Mexican educational system may significantly reduce immigration flows to the United States. Moreover, such a contraction may be viewed by domestic residents as a lifetime decision. Immigration policy aimed at the promotion of Mexican-specific human capital investment appears to be a more efficient and permanent strategy to released immigration pressures than conventional non-economic policies, e.g. border interdiction and employment sanctions.

The Participation Decision and U.S. Earnings:

• The theoretical model predicted that under determined conditions expansion in U.S. wages may, in fact, dissuade participation of further Mexican settlers. Because of the consideration of search costs, undocumented labor force participation may be discouraged as result of increases in domestic wages. The basic hypothesis suggests that search costs are negatively correlated with the domestic excess-demand in the U.S. labor market. When the labor market shows large net domestic demand, jobs are easy to locate because employment competition is relatively low across participants. Conversely, requirements of undocumented workers are reduced when local residents are able to clear the domestic labor market. In this context, illegal immigration acts as a “reserve army” that responds to contractions and expansions of domestic labor supply and demand.

• According to the empirical results, Mexican undocumented workers were likely to operate in a labor market characterized by conditions of excess demand. The uncompensated earnings elasticity of the participation index was likely to be near −0.50. This implied that, contrary to conventional wisdom, the empirical evidence suggested that an exogenous
increases in U.S. wages may actually discourage Mexican participation. For instance, an increase in the legal minimum wage will in fact hurt new immigration flows, while undocumented workers already participating in the U.S. labor force will benefit from this policy. Consequently, search costs play a significant role in the determination of the participation rule. Any attempt to neglect such an effect may lead to erroneous conclusions on undocumented Mexican participation in the U.S. labor market.

- Notice that the empirical result described above established a statistically significant correlation between the immigration decision and exogenous changes in U.S. earnings. However, their causality direction remained ambiguous when earnings are considered endogenous. In this case, the impact of Mexican undocumented participation on the determination of domestic wage rates (\( \text{GAP} \rightarrow E_u \)) cannot be assessed within this framework.

Finally, recent immigration policy has introduced costly and inefficient distortions to the U.S. economy, in general, and to the low-skill labor market, in particular. Policy makers may learn from this experience and, hopefully, recognize the need for a different approach. In this context, some policy recommendations are proposed below.

First, establish a formal network of policy coordination between Mexican and U.S. authorities. Coordination may lead the U.S. to abandon unilatera*l immigration measures, and Mexico may be willing to recognize the migration phenomenon as a national priority and thus a legitimate topic for negotiation. The fundamental labor dependency between these nations cannot be neglected. Yet, dialogue and cooperation in a context of mutual respect and understanding constitutes a unique channel in which both countries may certainly benefit from further Mexican migration to the United States.

Second, restore a comprehensive guest-worker program that could provide temporary Mexican workers for the secondary market in the United States. The domestic economy would benefit in terms of efficiency gains, development of new markets and tax revenues of transitory workers. Legalization of the *de facto* undocumented immigration process could provide a transparent mechanism for equal redistribution of efficiency gains across the economy.
Although the proposed contract-labor program may be regulated and overseen by immigration authorities, management would rely on private immigration agencies. Akin to the role played by import-export agencies in international trade, immigration agents would act as intermediaries between industries with excess labor demand and Mexican workers willing to participate in the U.S. labor market. The proposed program may be self-financing. Immigration authorities would charge license fees to authorized agents and employers would pay commissions for intermediary services. Resources to monitor the program could come from reallocation of funds already used in extensive border control and inefficient employer sanctions enforcement. Under these circumstances Mexican immigrants would not have economic incentives to settle permanently in the United States. They will switch back and forth according to the economic forces in both countries and not due to artificial distortions in the market.

Third, continued support to educational and regional development programs that promote attachment to home-communities. Following the empirical evidence presented here, these policies may substantially increase domestic productivity while reducing the likelihood of Mexican immigration. Moreover, economic involvement with local communities may be promoted by attracting immigrant workers’ remittances to investments in regional agro-industrial projects.
APPENDIX A

COMPOSITION OF THE IMMIGRATION CYCLE

A.1 The Non-Stationary Mexican Regime (V2)

This process assumes that consumers allocate time to only non-market activities in their home-community ($H_m=0$). Using model (3.1) and setting the depreciation rate $\delta$ to unity, the Hamiltonian function of a non-stationary process is

$$H(t,C,I,L_m,A,S,\lambda_m,\mu_m,\gamma) = \log C + \log(SL_m) + \lambda_m(rA + SI - C) + \mu_m(S - I - L_m) + \gamma(1 - I - L_m).$$

Under an interior solution the assumptions of and non-satiation, the necessary and sufficient conditions required for a local optimum are

(i) State Equations: 

$$\dot{A} - rA = SI - C$$  \hspace{1cm} (A.2)

$$\dot{S} - \sigma S = -1 \text{ since } \gamma > 0;$$  \hspace{1cm} (A.3)

(ii) Costate Equations: 

$$\dot{\lambda}_m + r\lambda_m = 0$$  \hspace{1cm} (A.4)

$$\dot{\mu}_m + \sigma \mu_m = -\left(\lambda_m I + \frac{1}{\delta}\right);$$  \hspace{1cm} (A.5)

(iii) Optimality Conditions:

$$C = \frac{1}{\lambda_m}$$  \hspace{1cm} (A.6)

$$L_m = \frac{1}{\mu_m + \gamma}$$  \hspace{1cm} (A.7)

$$\lambda_m S = \mu_m + \gamma \text{ since } I > 0.$$  \hspace{1cm} (A.8)
Notice that $V_2$ is the last value functions in the overall optimization problem and under the assumption no bequest [$A(T) = 0$], $V_2$ does not show a salvage term. Following Kamien and Schwartz [1983] a straightforward solution can be found for the costate equation (A.4), given the initial condition $\lambda_m(t_2) > 0$,

(A.9) \[ \lambda_m^*(t) = \lambda_m(t_2)e^{-\tau(t-t_2)}. \]

where a balanced budget set is implied by the assumptions of non-satiation and finite resources.

In addition, the difference equation (A.3) may be integrated under the initial condition $S(t_2) > 0$,

(A.10) \[ S^*(t) = S(t_2)e^{\sigma(t-t_2)} - \frac{1}{\sigma} [e^{\sigma(t-t_2)} - 1]. \]

For stability purposes, the non-stationary case requires $\dot{S}^* > 0$; then condition (A.10) satisfies $\sigma S(t_2) < 0$.

The "$\lambda_m(t_2)$-demand" functions are obtained by plugging the equilibrium values (A.8) and (A.9) into the optimality conditions (A.6) and (A.7),

(A.11) \[ C^*(t) = \frac{1}{\lambda_m(t_2)} e^{r(t-t_2)}, \]

(A.12) \[ L^*_m(t) = \frac{1}{S^*(t)\lambda_m(t_2)} e^{r(t-t_2)}. \]

$S(t)^*$ is given by (A.10). Substitution of equations (A.11) and (A.12) into the budget constraint (A.2) yields

(A.13) \[ \dot{A} - rA(t) = S(t_2)e^{\sigma(t-t_2)} - \frac{1}{\sigma} [e^{\sigma(t-t_2)} - 1] - \frac{2}{\lambda_m(t_2)} e^{r(t-t_2)}. \]
Proposition A.1: The utility function (2.1') and a balanced budget set imply that the marginal utility of income is a decreasing function of the state variables evaluated at age \( t_2 \),

\[
\frac{\partial \lambda_m(t_2)}{\partial A(t_2)} < 0 \quad \text{and} \quad \frac{\partial \lambda_m(t_2)}{\partial S(t_2)} < 0.
\]

Proof: Under boundary conditions \( A(t_2) > 0 \) and \( A(T) = 0 \), the solution for the first order difference equation (A.13) is

\[
(A.14) \quad A(t_2) = \frac{1}{r \sigma} \left[ e^{\frac{r}{\sigma} (T-t_2)} - 1 \right] - \frac{\sigma S(t_2) - 1}{\sigma (\sigma - r)} \left[ e^{(\sigma - r)(T-t_2)} - 1 \right] + \frac{2}{\lambda_m(t_2)} (T-t_2).
\]

Differentiating (A.14) with respect to the endowment \( A(t_2) \) yields

\[
1 = -\frac{2}{\lambda_m(t_2)^2} (T-t_2) \frac{\partial \lambda_m(t_2)}{\partial A(t_2)}.
\]

Accordingly, \( \frac{\partial \lambda_m(t_2)}{\partial A(t_2)} < 0 \) in order to maintain the equality. Likewise, differentiating (A.14) with respect to the marginal productivity of household work \( S(t_2) \) implies

\[
0 = -\frac{\sigma}{\sigma (\sigma - r)} \left[ e^{(\sigma - r)(T-t_2)} - 1 \right] - \frac{2}{\lambda_m(t_2)^2} (T-t_2) \frac{\partial \lambda_m(t_2)}{\partial S(t_2)},
\]

where \( \frac{\partial \lambda_m(t_2)}{\partial S(t_2)} \) must be negative, since the first term in this expression is also less than zero.

[Q.E.D.]

The marginal utility of wealth in period \( t_2 \) may be obtained by solving equation (A.14) for \( \lambda_m(t_2) \). Notice that \( \lambda_m(t_2) \) is a function of elements outside the current period \( t \). In order to obtain an explicit equilibrium solution for \( V_2 \), the "\( \lambda_m(t_2) \)-demand" equations may be plugged into the non-stationary objective function:
\[ V_2^* = \int_{t_2}^{T} \left[ \log \frac{1}{\lambda_m(t)} e^{r(t-t_2)} \right] dt. \]

Taking natural logarithms, this expression may be rewritten as

\[ V_2^* = \int_{t_2}^{T} 2[r(t - t_2) - \log \lambda_m(t_2)] dt. \]

Notice that \( \lambda_m(t_2) \) does not depend of the current period \( t \). Straightforward integration yields

(A.15) \[ V_2^* = (T - t_2)[r(T - t_2) - 2\log \lambda_m(t_2)] \quad \text{with} \quad \lambda_m(t_2) \text{ given by (A.14)}. \]

Expression (A.15) constitutes an explicit solution for the original maximization problem since current events in period \( t \) have been integrated into the limit points \( t_2 \) and \( T \).

A.2 The U.S. Regime (\( V_1 \))

The consumer migrates to the U.S. labor market at time \( t_1 \) and switches back to his home-community at time \( t_2 \). The Hamiltonian function in this case may be written as

\[ H(t, C, L_u, A, s, \lambda_u, \mu_u, r) = \log C + \log L_u + \lambda_u (rA + W_u H_u - C) + \mu_u \sigma S + r(1 - H_u - L_u). \]

Here the boundary conditions are set free and greater than zero. The necessary and sufficient conditions are:

(i) State Equations: (A.16) \[ \dot{A} - rA = W_u H_u - C \]

(A.17) \[ \dot{S} - \sigma S = 0; \]

(ii) Costate Equations: (A.18) \[ \dot{\lambda}_u + r\lambda_u = 0 \]

(A.19) \[ \dot{\mu}_u + \sigma \mu_u = 0; \]
(iii) Optimality Conditions:

\begin{align*}
\text{(A.20)} & \quad C = \frac{1}{\lambda_u} \\
\text{(A.21)} & \quad L_u = \frac{1}{\gamma} \\
\text{(A.22)} & \quad \lambda_u W_u = \gamma \quad \text{since } H_u = 0.
\end{align*}

In this case, the motion equation associated with the marginal utility of wealth (A.16) has a non-zero terminal value, i.e. \( \lambda_u(t_2) > 0 \), since assets at the end of the period are positive [\( A(t_2) > 0 \)]. This case contrasts with the previous model, where the boundary condition required \( A(T) = 0 \). Accordingly, the solution for costate equation (A.18) is given by

\begin{equation}
\text{(A.23)} \quad \lambda_u^*(t) = \lambda_u(t_2) e^{\sigma(t_2-t)}.
\end{equation}

In addition, the state associated with non-market productivity, equation (A.19), is independent of the maximization process in the host-regime. Then (A.19) yields a solution

\begin{equation}
\text{(A.24)} \quad S(t_2) = S(t_1) e^{\sigma(t_1-t_2)}.
\end{equation}

Notice that (A.24) is not a function of current time \( t \) in the receiving labor market. Moreover, the household productivity of the immigration period, \( S(t_1) \), is given by the discounted value of \( S(t_2) \) which is determined by an exponential factor \( \sigma \) and the duration of the undocumented status, \( t_1-t_2 \).

The "\( \lambda_u(t_2) \)-demand" functions are obtained by plugging the equilibrium equations (A.22) and (A.23) into the optimality conditions (A.20) and (A.21),

\begin{align*}
\text{(A.25)} & \quad C^*(t) = \frac{1}{\lambda_u(t_2)} e^{-\tau(t_2-t)} , \\
\text{(A.26)} & \quad L_u^*(t) = \frac{1}{W_u \lambda_u(t_2)} e^{-\tau(t_2-t)} .
\end{align*}

Using functions (A.25) and (A.26), the state condition associated with the budget constraint (A.16) can be integrated under the boundary conditions \( A(t_1), A(t_2) > 0 \),

\begin{equation}
\text{(A.27)} \quad A(t_1) = A(t_2) e^{-\tau(t_2-t_1)} + \frac{W_u}{\tau} [e^{-\tau(t_2-t_1)} - 1] + \frac{2(t_2-t_1)}{\lambda_u(t_2)} e^{-\tau(t_2-t_1)} ,
\end{equation}
where $A(t_2)$ is given by equation (3.14). Following the same procedure outlined in Proposition A.1, $\partial A_0(t_2)/\partial A(t_1) < 0$ as well as $\partial A_0(t_2)/\partial W_u < 0$. Then the equilibrium value function, $V_1^*$, may be calculated by the substitution of equations (A.25) and (A.26) into the objective function in $V_1$,

$$
V_1^* = \int_{t_1}^{t_2} \left[ \log \frac{1}{\lambda_u(t_2)} e^{-r(t_2-t)} \right] dt + \int_{t_1}^{t_2} \left[ \log \frac{1}{W_u} e^{-r(t_2-t)} \right] dt.
$$

Taking logarithms, $V_1^*$ may be rewritten as

$$
V_1^* = \int_{t_1}^{t_2} \left[ 2r(t_2-t) + \log W_u + \log \lambda_u(t_2) \right] dt.
$$

Lastly, integration leaves an explicit solution in terms of end point conditions

(A.28) \quad V_1^* = -(t_2 - t_1)[r(t_2 - t_1) + \log W_u + 2\log \lambda_u(t_2)]

with $\lambda_u(t_2)$ given by (A.27).

Unlike the home-country model, the benefit function (A.28) is affected by current ($W_u$) and life-cycle [$\lambda_u(t_2)$] events. In general, it is expected that lifetime elements dominate current events. In this context, notice that an increase in the U.S. wage rate will encourage undocumented migration if

$$\frac{\partial V_1^*}{\partial W_u} = \left[ \frac{1}{W_u} + \frac{2}{\lambda_u(t_2)} \frac{\partial \lambda_u(t_2)}{\partial W_u} \right] > 0;$$

this implies that lifetime effects dominate current events.
A.3 The Steady-State Mexican Regime ($V_0$)

A stationary process is defined by home-residents allocating their time to market and non-market activities in every period [H_m(t), I(t)>0]. The Hamiltonian function is

$$\mathcal{H}(t,C,I,L_m,A,S,\lambda_m,\mu_m,y) = \log C + \log(SL_m) + \lambda_m(rA+W_mH_m+S-I-C) + \mu_m(\sigma S-I-L_m) + \gamma(1-I-H_m-L_m).$$

The first order conditions are given by:

(i) State Equations: \hspace{1cm} (A.29) \hspace{1cm} \dot{A} - rA = W_mH_m - S - C

(A.30) \hspace{1cm} \dot{S} - \sigma S = - (I + L_m);

(ii) Costate Equations: \hspace{1cm} (A.31) \hspace{1cm} \dot{\lambda}_m + r\lambda_m = 0

(A.32) \hspace{1cm} \dot{\mu}_m + \sigma \mu_m = - (\lambda_m I + \frac{1}{S});

(iii) Optimality Conditions

(A.33) \hspace{1cm} C = \frac{1}{\lambda_m}

(A.34) \hspace{1cm} L_m = \frac{1}{\mu_m + \gamma}

(A.35) \hspace{1cm} \lambda_m S = \mu_m + \gamma \text{ since } I \geq 0

(A.36) \hspace{1cm} \gamma \lambda_m W_m = \gamma \text{ since } H_m \approx 0.

Under the end point condition $\lambda_m(t_1)>0$, costate equation (A.31) yields

(A.37) \hspace{1cm} \lambda^*_m(t) = \lambda_m(t_1)e^{r(t_1-t)}.

From section 2.2.1, it follows that expression (A.30) has an equilibrium solution given by

(A.38) \hspace{1cm} S(t)^* = \frac{W_m}{R} \quad \text{where} \quad R = \frac{2\sigma - r}{\sigma - r} > 1, \text{ for } \sigma > r.

Note that (A.38) implies that $S(t)^*=S(0)=S(t_1)$. Then "$\lambda_m(t_1)$-demand" equations are

(A.39) \hspace{1cm} C^*(t) = \frac{1}{\lambda_m(t_1)} e^{-r(t_1-t)},
\[ L_m^*(t) = \frac{R}{W_m \lambda_m(t_1)} e^{-\tau(t_1-t)}, \]

\[ I^*(t) = \frac{\sigma W_m}{R} - \frac{R}{W_m \lambda_m(t_1)} e^{-\tau(t_1-t)}. \]

Using functions (A.39) to (A.41), the budget constraint (A.29) may be rewritten as

\[ \dot{A} - rA(t) = W_m \left[ 1 - \frac{\sigma^2 W_m}{R(2\sigma-r)} \right] - \frac{2}{\lambda_m(t_1)} e^{-\tau(t_1-t)}. \]

Under the boundary conditions \( A(0) = A_0 \) and \( A(t_1) \) free, the above expression yields

\[ A(0) = A(t_1) e^{-\tau t_1} + \frac{W_m}{r} \left[ 1 - \frac{\sigma^2 W_m}{R(2\sigma-r)} \right] [e^{-\tau t_1} - 1] + \frac{2t_1}{\lambda_m(t_1)} e^{-\tau t_1}. \]

Notice that Proposition A.1 applies to equation (A.42); therefore, \( \partial \lambda_m(t_1)/\partial A_0 \) and \( \partial \lambda_m(t_1)/\partial W_m \) are less than zero. In this case, however, the latter condition requires \( R(2\sigma-r) > \sigma^2 W_m \).

Finally, the equilibrium benefit function \( V_0^* \) may be obtained using the "\( \lambda_m(t_1)\)-demand" equations (A.39) to (A.41), where some algebraic manipulation yields

\[ V_0^* = \int_0^{t_1} \left[ -2 \left[ r(t_1-t) + \log \lambda_m(t_1) \right] \right] dt. \]

Straightforward integration gives

\[ V_0^* = -t_1[r t_1 + 2 \log \lambda_m(t_1)] \quad \text{where } \lambda_m(t_1) \text{ is obtained from (A.42)}. \]

Notice that direct current events are not observed in this state of the world. Consumers' initial wealth and domestic wage rates affect the home-country benefit function solely through indirect life-cycle changes.
Appendix B

GQOPT4/I Programming Code for the Optimization of the Likelihood Function

The database is available from the author upon request.

1. The main program:

```plaintext
IMPLICIT REAL*8 (A-H,O-Z)
INTEGER*2 NDATA
EXTERNAL FUNC1,DFP,GRADX
DIMENSION X(48)
CHARACTER*8 ALABEL(48)
```

```
Next are data arrays

DIMENSION NDATA(4662,24)
COMMON/USER1/NDATA

For next 6 cards see GQOPT handbook

COMMON/BOPT/IVER,LT,IFP,ISP,NLOOP,IST,ILOOP
COMMON/BPRINT/IPT,NFILE,NDIG,NPUNCH,JPT,MFILE
COMMON/BSTACK/AINT(1200)
COMMON/BSTAK/NQ,NTOP
COMMON/BSTOP/NVAR1,ISTOP(3)
COMMON/BTRAT/ITRFLG

Open input/output files

OPEN(8,FILE='MEXICO.DAT')
OPEN(9,FILE='THEESIS.OUT',STATUS='OLD')
```

Set parameters for optimization procedure
CALL DFLT
ACC=1.D-10
ITERL=200
ITRFLG=1
ISP=1
MAX=1
NFIL=9
NP=48
NO=12000
CALL LABEL(ALABEL,NP)

Read in the data

DO 200 I=1,4662
   200 READ(8,*) (NDATA(I,J), J=1,24)

Initial values from IT3SLS estimation

Economic Parameters: X(1)-X(8)
X(1)= 7.61201463
X(2)= 9.71857777
X(3)= -0.000179396
X(4)= 0.00000330348
X(5)= -0.10240450
X(6)= 0.
X(7)= 0.
X(8)= 0.

Variance-covariance parameters: X(9)-X(13)
X(9)= 0.67366
X(10)= 0.1047
X(11)= -0.3454
X(12)= 0.1
X(13)= 0.2

Demographic parameters of the Earning Eq.: X(14)-X(27)
X(14)= -0.37910637
X(15)= -0.12688934
X(16)= 0.002913175
X(17)= 0.01390780
X(18)= 0.03319968
X(19)= 0.02458901
X(20)= 0.21526888
X(21)= 0.12139216
X(22)= -0.40601473
X(23)= -0.23384645
X(24)= -0.11507345
C Demographic parameters of the Labor Supply: X(28)-X(43)
X(28)=-0.01007414
X(29)=-0.01224382
X(30)= 0.001210684
X(31)= 0.001575521
X(32)= 0.005159367
X(33)= 0.009609170
X(34)= 0.003516572
X(35)= 0.006982463
X(36)= 0.02685115
X(37)= 0.001474021
X(38)= 0.02316441
X(39)=-0.01713336
X(40)= 0.015797218
X(41)=-0.02409924
X(42)=-0.001562307
X(43)=-0.0087644196

C Demographic parameters of the Participation Rule: X(45)-X(48)
X(45)= 0.
X(46)= 0.
X(47)= 0.
X(48)= 0.
ISTOP(1)=0
ISTOP(2)=0
ISTOP(3)=0
CALL OPT(X,NP,F,DFP,ITERL,MAX,IER,ACC,FUNC1,ALABEL)
CALL OPT(X,NP,F,GRADX,ITERL,MAX,IER,ACC,FUNC1,ALABEL)

C Finish up
CLOSE(9)
STOP
END

2. The likelihood function:

SUBROUTINE FUNC1(X,NP,FUO,*)
IMPLICIT REAL*8 (A-H,O-Z)
INTEGER*2 NDATA(4662,24)
DIMENSION X(NP)

C Next card shows how to communicate data to the function subroutine

COMMON/USER1/NDATA
PI=3.1415926535897988
NOB=4662
Variance-covariance definitions

\[ W_{11} = X(9) * X(9) + X(11) * X(11) + X(12) * X(12) \]
\[ W_{22} = X(10) * X(10) + X(13) * X(13) \]
\[ W_{33} = 1.0D0 \]
\[ W_{12} = X(10) * X(11) + X(12) * X(13) \]
\[ W_{13} = X(12) \]
\[ W_{23} = X(13) \]
\[ DETW = W_{11} * W_{22} - W_{12} * W_{12} \]
\[ FUO = -NOB * (DLOG(2*PI) + DLOG(DETW)/2) \]
\[ DO 20 \ l=1, NOB \]

Demographic definitions

The Earning Equation

\[ \text{DEM1} = X(14) * \text{NDATA(I,2)} + \]
\[ X(15) * \text{NDATA(I,4)} + \]
\[ X(16) * \text{NDATA(I,6)} + \]
\[ X(17) * \text{NDATA(I,8)} + \]
\[ X(18) * \text{NDATA(I,10)} + \]
\[ X(19) * \text{NDATA(I,20)} + \]
\[ X(20) * \text{NDATA(I,9)} \]
\[ \text{DEM1A} = X(21) * \text{NDATA(I,10)} + \]
\[ X(22) * \text{NDATA(I,11)} + \]
\[ X(23) * \text{NDATA(I,13)} + \]
\[ X(24) * \text{NDATA(I,17)} + \]
\[ X(25) * \text{NDATA(I,18)} + \]
\[ X(26) * \text{NDATA(I,20)} + \]
\[ X(27) * \text{NDATA(I,21)} \]

The Labor Supply

\[ \text{DEM2} = X(28) * \text{NDATA(I,2)} + \]
\[ X(29) * \text{NDATA(I,4)} + \]
\[ X(30) * \text{NDATA(I,6)} + \]
\[ X(31) * \text{NDATA(I,8)} + \]
\[ X(32) * \text{NDATA(I,10)} + \]
\[ X(33) * \text{NDATA(I,9)} \]
\[ \text{DEM2A} = X(34) * \text{NDATA(I,10)} + \]
\[ X(35) * \text{NDATA(I,11)} + \]
\[ X(36) * \text{NDATA(I,13)} + \]
\[ X(37) * \text{NDATA(I,14)} + \]
\[ X(38) * \text{NDATA(I,15)} + \]
\[ X(39) * \text{NDATA(I,16)} + \]
\[ X(40) * \text{NDATA(I,17)} + \]
\[ X(41) * \text{NDATA(I,18)} + \]
\[ X(42) * \text{NDATA(I,19)} + \]
\[ X(43) * \text{NDATA(I,20)} \]

The Participation Rule

\[ \text{DEM3} = (X(14) - X(44)) * \text{NDATA(I,2)} + \]
\[ (X(15) - X(45)) * \text{NDATA(I,4)} + \]
\[ (X(16) - X(46)) * \text{NDATA(I,6)} + \]
3 \( (X(17) - X(47)) \times NDATA(I,8) + \)
4 \( X(18) \times NDATA(I,7) + \)
5 \( X(19) \times NDATA(I,5) + \)
6 \( X(20) \times NDATA(I,9) \)
7 \( DEM3A = X(21) \times NDATA(I,10) + \)
1 \( X(22) \times NDATA(I,11) + \)
2 \( X(48) \times NDATA(I,12) + \)
3 \( X(23) \times NDATA(I,13) + \)
4 \( X(24) \times NDATA(I,17) + \)
5 \( X(25) \times NDATA(I,18) + \)
7 \( X(26) \times NDATA(I,20) \)

Reduced-form components: \( V_1, V_2, V_3 \)

\[
V1 = DLOG(\text{EARNINGS}) - (X(1) + DEM1 + DEM1A) \\
V2 = DLOG(\text{HOURS}) - (X(2) + X(3) \times \text{AGE} + X(4) \times \text{ASSETS}) \\
V3 = -(X(6) + X(7) \times \text{ASSETS}) \\
(\text{DLOG}(\text{EARNINGS}) - V1) + \text{DEM2} + \text{DEM2A}) \\
(\text{DLOG}(\text{EARNINGS}) - V1) + \text{DEM3} + \text{DEM3A}) \\
\]

Elements of the likelihood function:

PDF (normal likelihood estimation),
CDF1 (due to sample truncation),
CDF2 (due to unobservability of GAP)

\[
\begin{align*}
\text{PDF} &= (W22 \times V1 \times V1 - 2 \times W12 \times V1 \times V2 + W11 \times V2 \times V2) / \text{DET}\text{W} \\
\text{CDF1} &= \text{PHI}(V3 / \text{DSQRT}(W33)) \\
\text{CDF2} &= \text{PHI}((V3 - (V1 / \text{DET}\text{W}) \times (W22 \times W13 - W12 \times W23)) \\
&\quad - (V2 / \text{DET}\text{W}) \times (W11 \times W23 - W12 \times W13)) / \\
&\quad \text{DSQRT}(W33 - (W22 \times W13 \times W13 - \text{DET}\text{W})) \\
\end{align*}
\]

Checking for errors on the cumulative distribution functions since CFS<1

IF (CDF1.GE.1.0D0) THEN
CDFONE=1.0D0-1.0D-15
ELSE
CDFONE=CDF1
ENDIF
WRITE(*,100) CDF1
C 100 FORMAT(1X,'CDF1 is equal to one',D20.10)
END IF
IF (CDF2.GE.1.0D0) THEN
CDFTWO = 1.0D0 - 1.0D-15
ELSE
CDFTWO=CDF2
ENDIF
C WRITE(9,101) CDF2
C 101 FORMAT(1X,'CDF2 is equal to one',D20.10)
   END IF
C
C The likelihood function
C
FUG= FUG–PDF/2–DLOG(1.0D0–CDFONE)+DLOG(1.0D0–CDFTWO)
C
20 CONTINUE
   RETURN
   END

3. Specification of the cumulative distribution function:

REAL*8 FUNCTION PHI(X)
EXTERNAL ERF
REAL*8 X
PI=3.1415926535897988
PHI = .5D0*(1.0D0+ERF(X/DSQRT(2.0D0))
IF (PHI.LT.1.0D0) RETURN
C
C WRITE(9,100) X
C 100 FORMAT(1X,'PHI is greater than one',D20.10)
   PHI = 1.0D0 – 1.0D–7
   RETURN
   END
BIBLIOGRAPHY

Altonji, J. [1986]
Intertemporal Substitution in Labor Supply: Evidence from Micro Data

Amemiya, T. [1974]
Multivariate Regression and Simultaneous Equation Models When the Dependent Variables are Truncated Normal
Econometrica 42: 999-1012.

——— [1984]
Tobit Models: A Survey
Journal of Econometrics 24: 3-61.

——— [1985]
Advanced Econometrics
Cambridge: Harvard University Press.

Arrow, K. and M. Kurz [1970]
Public Investment, The Rate of Return and Optimal Fiscal Policy
Baltimore: John Hopkins Press.

Bean, F., A. King and J. Passel [1983]
The Number of Illegal Migrants of Mexican Origin in the United States: Sex Ratio-Based Estimates for 1980

Bean, F., H. Browning and P. Frisbie [1984]
The Sociodemographic Characteristics of Mexican Immigrants Status Groups: Implication for Studying Undocumented Mexicans
International Migration Review 18(Fall): 672-91.

Becker, G. [1965]
A Theory of the Allocation of Time
Economic Journal 299(Sep): 496-517.
Bellman, R. [1957]  
*Dynamic Programming*  
Pricenton: Pricenton University Press.

Ben-Porath, Y. [1967]  
The Production of Human Capital and the Life Cycle of Earnings  

Bernard, W. [1953]  
Economic Effect of Immigration  
In B. Ziegler ed., *Immigration: An American Dilemma*  
D. C. Health and Co. (Boston): 50-70

Bertsekas, D. [1987]  
*Dynamic Programming: Deterministic and Stochastic Models*  
London: Pretince-Hall.

Blau, F. [1986]  
Immigration and the U.S. Taxpayer  
In S. Pozo ed., *Essays on Legal and Illegal Immigration*  

Blundell, R. and I. Walker [1982]  
Modelling the Joint Determination of Household Labour Supplies and Commodity Demand  
*Economic Journal* 93(Jun): 351-64.

[1986]  
A Life-Cycle Consistent Empirical Model of Family Labour Supply Using Cross-Section Data  

Borjas, G. [1983]  
The Labor Supply of Male Hispanic Immigrants in the U.S.  

[1986]  
Immigrants and the U.S. Labor Market  
In S. Pozo ed., *Essays on Legal and Illegal Immigration*  

[1987]  
Self-Selection and the Earnings of Immigrants  
*American Economic Review* 77(Sept): 531-53

Borjas, J. and J. Heckman [1979]  
Labor Supply Estimates for Public Policy Evaluation  
*IRRA 31st Annual Proceedings*: 320-35.

Bowden, R. [1978]  
Specification, Estimation and Inference for Models of Markets in Disequilibrium  
Brown, B. [1983]
The Identification Problem in Systems Nonlinear in the Variables
Econometrica 51: 175-96.

Browning, H. and Rodriguez, N. [1985]
The Migration of Mexican Indocumentados as a Settlement Process:
Implications for Work
in G. Borjas and M. Tienda eds., Hispanics in the U.S. Acrimony
Academic Press, Inc.

Burciaga-Valdez, R., et al. [1987]
An Annotated Bibliography of Sources on Mexican Immigration
A Rand Note. Prepared for the California Round Table, Rand Foundation.

Burdett, K. and T. Mortensen [1978]
Labor Supply Under Uncertainty
In R. Ehrenberg ed., Research in Labor Economics vol. 2
JAI Press (Greenwich):109-158.

Chiswick, B. [1977]
An Analysis of Earnings Among Mexican Origin Men
American Statistics Association

—— [1978]
The Effect of Americanization on Earnings of Foreign-Born Men

—— [1984]
Illegal Aliens in the United States Labor Market: Analysis of Occupational
Attainment and Earnings
International Migration Review 18(Fall): 714-32.

—— [1986]
The Illegal Alien Policy Dilemma
In S. Pozo ed., Essays on Legal and Illegal Immigration

—— [1988]
Illegal Immigration and Immigration Control

Chow, G. [1983]
Econometrics

Cogan, J. [1980]
Labor Supply with Costs of Labor Market Entry

—— [1980a]
Married Woman's Labor Supply:
A Comparison of Alternative Estimation Procedures

Cornelius, W. [1978]
Mexican Migration to the United States:
Causes, Consequences and U.S. Responses

Corwin, A. [1978]
A Story of Ad Hoc Exemptions: American Immigration Policy Toward México
In A. Corwin ed., Immigrants and Immigrants Perspectives on Mexican Labor Migration to the United States

_______ [1982]
The Numbers Game: Estimates of Illegal Aliens in the United States, 1970-81
Law and Contemporary Problems 45(Spring): 223-97.

De Maria, C. [1983]
Impact of Undocumented Workers in Houston Construction Industry
MA Thesis, Department of Economics, Rice University.

Dhrymes, P. [1970]
Econometrics: Statistical Foundations and Applications

_______ [1986]
Limited Dependent Variables
In Z. Griliches and M. Intriligator eds., Handbook of Econometrics vol. III
North-Holland (Amsterdam): 1568-1626.

Díez-Canedo, J. [1980]
A New View of Mexican Migration to the United States
Ph.D. Dissertation, Department of Economics, M.I.T.

Ethier, W. [1986]
Illegal Immigration: The Host-Country Problem

Friedman, M. [1956]
A Theory of the Consumption Function

Flinn, C. and J. Heckman [1982]
New Methods for Analyzing Structural Models of Labor Force Dynamics
Journal of Econometrics 18(Jan): 115-68.

Ghez, G. and G. Becker [1975]
The Allocation of Time and Goods Over the Life Cycle
New York: NBER.

Goldfeld, S. and R. Quant [1972]
Nonlinear Methods in Econometrics
Amsterdam: North Holland Press.
_______ [1976]
Studies in Nonlinear Estimation
Ballinger Publishing Co.

Greene, W. [1983]
Estimation of Limited Dependent Variable Models by Ordinary Least Squares and
the Method of Moments

The Factor Market Consequences of U.S. Immigration

Grossman, J. [1984]
Illegal Immigrants and Domestic Employment

Hall, R. [1978]
Stochastic Implications of the Life Cycle-permanent Income Hypothesis:
Theory and Evidence

Ham, J. and Ch. Hsiao [1984]
Two-Stage Estimation of Structural Labor Supply Parameters Using Interval Data
from the 1971 Canadian Census

Hamermesh, D. [1976]
Econometric Studies of Labor Demand and Their Application to Policy Analysis
Journal of Human Resources 11(Fall): 507-25.

Hausman J. and D. Wise [1979]
Attrition Bias in Experimental and Panel Data:
The Gary Income Maintenance Experiment
Econometrica 47: 455-73.

Heckman, J. [1974]
Shadow Prices, Market Wages, and Labor Supply
Econometrica (Jul).

_______ [1976]
A Life Cycle Model of Earnings, Learning and Consumption

_______ [1978]
A Partial Survey of Recent Research on the Labor Supply of Women

_______ [1979]
Sample Selection Bias as a Specification Error

_______ [1979a]
New Evidence on the Dynamics of Female Labor Supply
Women in the Labor Market
New York: Columbia University Press.

[1980]
Sample Selection Bias as Specification Error

Heckman, J. and R. Willis [1977]
A Beta Logistic Model for Analysis of Sequential Labor Force
Participation by Married Women

Heckman, J. and T. MacCurdy [1980]
Female Labour Supply

[1981]
Recent Developments in Labor Supply: A Survey with Special Reference to
Statistical and Dynamic Issues
in Z. Griliches, J. Grice and A. Webb eds.,
Econometrics of Labour Market
London: Her Majesty's Stationary Office.

[1986]
Labor Economics
in Z. Griliches and M. Intriligator eds.,
Handbook of Econometrics Vol. III
Amsterdam: North Holland.

Heckman, J., M. Killingsworth and T. MacCurdy [1981]
Empirical Evidence on Static Labour Supply Models:
A Survey of Recent Developments
in Z. Griliches, J. Grice and A. Webb eds.,
The Econometrics of Labour Market
London: Her Majesty's Stationary Office.

Hill, J. [1985]
The Economic Impact of Tighter U.S. Border Security

Hill, J. and J. Pearce [1987]
The Incidence of Sanctions Against U.S. Employers of Illegal Aliens

Hirschman, C. [1978]
Prior U.S. Residence Among Mexican Immigrants

Hotz, J., et. al. [1987]
Intertemporal Preferences and Labor Supply
Mimeo (Forthcoming in Econometrica).
Huddle, D., et. al. [1985]
Illegal Immigration: Job Displacement and Social Costs
The American Immigration Control Foundation.

Johnson, G. [1980]
The Labor Market Effects of Immigration

Johnson, N. and S. Kotz [1972]
Distribution in Statistics: Continuous Multivariate Distributions
New York: John Wiley & Sons.

Judge, G., et. al. [1985]
The Theory and Practice of Econometrics
Second Ed., Wiley Series.

Kamien, M. and Schwartz, N. [1981]
Dynamic Optimization: The Calculus of Variations
and Optimal Control in Economics and Management
New York: North Holland.

Kendall, M. and A. Stuart [1973]
The Advanced Theory of Statistics, vol. 2
New York: Hafner.

Kennedy, P. [1979]
A Guide to Econometrics
Cambridge: The MIT Press (Second ed.)

Killingsworth, M. [1983]
Labor Supply
New York: Cambridge University Press.

King, A. [1986]
The Effects of Hispanic Immigrants on the Earnings
of Native Hispanic Americans
Social Science Quarterly 67(Dec): 673-89.

Lancaster, K. [1971]
Consumer Demand: a New Approach
New York: Columbia University Press.

Lee, L. [1978]
Unionism and Wage Rates: A Simultaneous Equations Model with Qualitative and
Limited-Dependent Variables

[1981]
Simultaneous Equations Models With Discrete and Censored Dependent Variables
In C. Manski and D. McFadden, eds.,
Structural Analysis of Discrete Data with Econometric Applications
Lee, L. and R. Porter [1984]
Switching Regression Models with Imperfect Sample Separation Information with an Application on Cartel Stability

Asymptotic Covariance Matrices of Two-Stage Probit and Two-Stage Tobit
Method for Simultaneous Equation Models with Selectivity

MaCurdy, T. [1981]
An Empirical Model of Labor Supply in a Life-Cycle Setting

[1985]
Interpreting Empirical Models of Labor Supply in an Intertemporal Framework with Uncertainty
in J. Heckman and B. Singer eds., Longitudinal Analysis of Labor Market Data
New York: Cambridge University Press.

Maddala, G. [1983]
Limited-Dependent and Quantitative Variables in Econometrics
New York: Cambridge University Press.

Marshall, A. [1952]
Principles of Economics
London: MacMillan and Co. (Eighth ed.)

Mincer, J. [1962]
in H. Lewis ed., Aspects of Labor Economics
Princeton University Press.

Moffitt, R. [1984]
Profiles of Fertility, Labor Supply and Wages of Married Woman: A Complete Life-Cycle Model

Nelson, F. and L. Olson [1978]
Specification and Estimation of a Simultaneous Equation Model with Limited Dependent Variables

North, D. and M. Houstoun [1976]
The Characteristics and the Role of Illegal Aliens in the U.S. Labor Market

Olsen, R. [1978]
Note on the Uniqueness of the Maximum Likelihood Estimator for the Tobit Model

Passel, J. and K. Woodrow [1984]
Geographic Distribution of Undocumented Immigrants: Estimates of Undocumented Aliens Counted in the 1980 Census by State
*International Migration Review* 18(Fall): 642-71.

Pearce, J. and J. Gunther [1985]
*Immigration from Mexico: Effects on the Texas Economy*

Perloff, J. and R. Sickles [1987]
*Union Wage, Hours and Earnings Differentials in the Construction Industry*
Mimeo. Rice University.

Piore, M. [1978]
*Birds of Passage: Migrant Labor and Industrial Societies*
Cambridge: Cambridge University Press.

——— [1986]
*Can International Migration be Controlled?*
In S. Pozo ed., *Essays on Legal and Illegal Immigration*

Polacheck, S. and E. Horvath [1977]
*A Life-Cycle Approach of Migration: Analysis of the Perspicacious Peregrinator*
In R. Ehrenberg ed., *Research in Labor Economics* vol. 1
JAI Press (Greenwich):103-150.

Pollak, R. and Wachter, M. [1975]
*The Relevance of the Household Production Function and Its Implications for the Allocation of Time*

Portes, A. and R. Bach [1978]
*Dual Labor Markets and Immigration: A Test of Competing Theories and Income Inequality. Comparative Studies of Immigration and Ethnicity. Occasional Paper Series*

Powell, M. [1971]
*Recent Advances in Unconstrained Optimization*
*Mathematical Programming* 1: 26-57.

Reynolds, C. and R. McCleery [1988]
*The Political Economy of Immigration Law: Impact of Simpson-Rodino on the United States and México*
*Journal of Economic Perspectives* 2(Summer): 117-32.

*Determinants of the Requested Rate of Return and the Rate of Return Granted in a Formal Regulatory Process*

Sickles, R. and P. Schmidt [1978]
*Simultaneous Equations Models with Truncated Dependent Variables: A Simultaneous Tobit Model*

Sjaastad, L. [1962]
The Costs of Returns of Human Migration

Smith, J. [1977]
Family Labor Supply Over the Life Cycle
Explorations in Economic Research NBER 2(Spring): 205-76.

Tienda, M and L. Neidert [1980]
Segmented Markets and Earnings Inequality of Native and Immigrant Hispanics in the United States

Tobin, J. [1958]
Estimation of Relationships for Limited Dependent Variables
Econometrica 26: 24-36.

U.S. Department of Commerce, Bureau of the Census [1980]
Census of Population and Housing: 1980
Public-Use Microdata Samples
Technical Documentation.

Varian, H. [1978]
Microeconomic Analysis
New York: Norton & Co. (Second ed.)

A Count of the Uncountable:
Estimates of Undocumented Aliens Counted in the 1980 United States Census

Young, L. [1969]
Lectures on the Calculus of Variations and Optimal Control Theory
Philadelphia: W.B. Saunders Company.