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Seismic imaging and wave scattering in zones of random heterogeneity

Gibson, Bruce Sanderson, Ph.D.
Rice University, 1988
SEISMIC IMAGING AND WAVE SCATTERING IN ZONES OF
RANDOM HETEROGENEITY

by

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SEISMIC IMAGING AND WAVE SCATTERING IN ZONES OF RANDOM HETEROGENEITY

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ABSTRACT

Most current interpretations of lower crustal seismic reflectivity suggest the existence of fine-scale (=100 m thick) layered structures at depth. Typical common-midpoint (CMP) stacked images of deep structures are, however, noisy and show discontinuous reflections characterized by numerous subhorizontal segments. Present interpretation techniques cannot definitively resolve whether such a reflection response is attributable to heterogeneity at the target or whether the seismic image is distorted by propagation effects and contaminated by noise. The quantitative assessment of lateral heterogeneity in the deep crust is fundamental to understanding mechanisms of crustal formation and evolution.

Here, crustal heterogeneity is represented by velocity structures that vary randomly in two dimensions, with a correlation distance comparable to the dominant wavelength of the seismic source. Synthetic CMP seismic data are computed for various models using a fourth-order finite-difference solution to the acoustic wave equation. A conventional data processing sequence produces CMP stacked sections with greater lateral continuity than is present at the target and an overall appearance comparable to that of field-recorded data. Lateral coherence is quantified using the spectral coefficient of coherence, applied to trace pairs having various spatial separations. Increased continuity in the CMP stack is attributable to the dip-filtering action of stacking and can be compensated by the application of migration before stack or equivalent processes.

The reflection response observed in common-shot trace gathers shows amplitude and lateral coherence increasing with offset, an effect attributable to the source-receiver geometry. Observed wavefield coherence is related to the correlation properties of the target
through a convolutional expression. A field data example from the Black Forest, Germany shows that enhanced coherence can be expected in wide-angle field experiments and that a model having two-dimensional heterogeneity matches field data previously interpreted in terms of extreme fine-scale layering. A densely-sampled field data set from the Basin and Range Province, Nevada shows increased coherence for $P_{mP}$ at large offsets. Observed lateral coherence values are lower than predicted for scattering purely in the Moho transition zone; coherence levels can be modeled by including scattered energy from inhomogeneities above the target zone.
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Chapter 1
Seismic Imaging and Models of Heterogeneous Crustal Structure

INTERPRETATION OF DEEP SEISMIC DATA

In the past decade, seismic reflection surveys have been notably successful in documenting structural complexity in the deep crust and at the Moho discontinuity. The continental crust in particular has been explored with a variety of seismic techniques employed by numerous researchers (Mooney and Brocher, 1987). Interpretations of crustal seismic data have focused on describing general patterns of reflectivity and in determining gross velocity structure (using large-offset data sets). Several styles of crustal reflectivity have been identified and correlated in a large volume of common-midpoint (CMP) stacked data (Mooney and Brocher, 1987), although a general mechanism for the reflection process at depth is still widely debated (Hurich and Smithson, 1987, Klemperer, 1987). Indeed, an explanation for reflectivity in deep crystalline rocks is a preeminent problem in crustal geology and geophysics today.

The interpretation of deep seismic data is complicated by the paucity of direct evidence for the structure and composition of targets being surveyed. Without well log information or direct lithologic samples (except xenoliths), interpreters have relied on surface geology, non-seismic geophysics, and geochemical arguments to guide the assessment of deep seismic sections (Smithson and Brown, 1977; Kay and Kay, 1981; Fountain and Salisbury, 1981). In some instances, deep structure can be constrained by tracing surface features to depth (Smithson et al., 1979; Oliver, 1982). Most interpretations, however, are based on upper crustal analogs (Lynn et al., 1981; Brown et
al., 1983; Smithson and Iverson, 1983) and supported by numerical seismic modeling studies (Clowes and Kanasewich, 1970; Hale and Thompson, 1982; Jones and Nur, 1984). Modeling studies are usually directed at matching the relative strength and gross reflection signature seen in the field-recorded data. In some recent studies, modeling has been enhanced by more quantitative analyses of frequency content and reflection strength for particular events (Louie and Clayton, 1987; Warner, 1988; Reynaud, 1988).

Although the seismic reflection method provides an efficient means of surveying regional geology, the interpretation of deep structure on CMP stacked sections is often problematic. Deep seismic data typically have a poor signal-to-noise ratio, that in some cases is not improved by the CMP stacking process (Klemperer et al., 1986). The image of deep targets is often characterized by bands of numerous short (<1 km) reflection segments, in contrast to the simple lateral continuity of reflections from many sedimentary sections. A correlatable band of segments may extend more than 1 s in vertical traveltime and can be traced laterally for distances from hundreds of meters to many tens of kilometers (Mooney and Brocher, 1987).

The generally poor signal-to-noise ratio and discontinuous nature of deep reflections poses a fundamental problem in the detailed interpretation of individual targets: Is the lateral variability of the reflection response caused by heterogeneity at the target or is the variability attributable to distortion of the seismic image and noise contamination? Most interpreters (e.g., Reynaud, 1988) have tended to the latter view, arguing 1) that such short segments are artifacts since they lie below the theoretical resolving power of the the CMP method, and 2) that a planar horizon must have a scale length of kilometers in order to reflect a detectable amount of energy. Some investigators do suggest that short events result from interference effects in the wavefield scattered from a heterogeneous target (Hurich and Smithson, 1987; Reston, 1987).
Aside from standard analyses of resolution, most previous studies of crustal reflectivity have paid little attention to the data acquisition and processing choices that can profoundly affect the seismic image presented for interpretation. In regional-scale interpretations of crustal structure, the visibility of reflection events has been the only measure of quality for a seismic section. In evaluating the structure of deep targets and the mechanism of their formation, however, a more detailed understanding of the seismic image is required. As noted below, lateral heterogeneity and scattered noise pose fundamental limitations to the quality of the final processed image; these limitations are related and of central importance in understanding deep reflection patterns.

In this research project, I have studied seismic imaging and wave-scattering effects for velocity structures that have random vertical and horizontal heterogeneity. Random variations in velocity have been used previously to explain scattering effects in earthquake records and such models provide an efficient statistical description of subsurface heterogeneity. Finite-difference synthetic data computed for random targets reveal fundamental biases in the CMP and wide-angle seismic methods: Both techniques inherently tend to produce images with greater lateral continuity than exists at the target. For instance, synthetic CMP stacks for random velocity models show numerous subhorizontal reflection segments where no preferred orientation of structure exists.

The synthetic data suggest that interpretations of CMP and wide-angle field data may be heavily biased toward layered structures. The central goal of this research is to provide means of estimating the bias in a particular experiment and means of correcting for it. A corrected seismic image could then yield a quantitative estimate of lateral variability at the target. As deep seismic interpretation progresses from the general description of reflectivity patterns to detailed analyses of specific targets, quantitative measures such as the scale of lateral continuity should become central to the assessment of geologic structure. As
discussed in the concluding chapter, the utility of this approach to interpretation will require geologic (e.g., outcrop) studies of the expected target.

In the following sections of this introductory chapter, I will first review some general characteristics of models for the lower crust and Moho discontinuity as determined from CMP and wide-angle seismic experiments. I then describe models of random heterogeneity developed by earthquake seismologists and review methods for estimating the parameters of these models from observations of earthquake coda and attenuation. The next section explores the important connection between crustal heterogeneity, seismic wave scattering, and the processing of seismic reflection data. The final section contains an overview of the remaining chapters.

VELOCITY MODELS OF DEEP GEOLOGIC TARGETS

One general feature of lower crustal and Moho models is a fine-scale layering of units with contrasting seismic properties. Individual layers in such models have thicknesses on the order of 100 m and layers are usually characterized by compressional (P-wave) velocity (Clowes and Kanasewich, 1970), although in some cases shear (S-wave) velocity (Sandmeier et al., 1988) or density (Hale and Thompson, 1982) are also specified. Velocity is usually constant within each layer and velocity contrasts between layers typically range up to 12% percent (e.g., Hale and Thompson, 1982; Sandmeier and Wenzel, 1986). In some models, velocity values are chosen from measurements on rock types expected at the target depth and the ordering of layers is based on observed stratigraphic relations (Hale and Thompson, 1982; Collins et al., 1986). Other models use alternating "high" and "low" velocity values to represent generic "mafic" and "silicic" rocks in adjacent layers (Sandmeier and Wenzel, 1986; Roy-Chowdhury et al., 1988).
Most modeling studies conclude that numerous thin layers with large velocity contrasts are required to explain the relative strength and extended time duration of deep reflections. Zones of high-contrast layering up to 7.5 km thick have been proposed to explain the reflection character of the lower crust (Sandmeier and Wenzel, 1986). The average thickness of individual layers within these models is adjusted to match the tuning frequency of reflections observed on stacked sections. Given typical seismic bandwidths and rock velocities, that average is most often near 100 m.

Here I should emphasize that the models cited above are all strictly one-dimensional in nature; that is, they consist of layers with plane horizontal interfaces and no lateral variation. In most cases, normal incidence plane waves are propagated in these models and only primary reflections are considered (Hale and Thompson, 1982; Jones and Nur, 1984). Although simplistic, such a technique is often chosen because it allows seismic traces to be produced from a convolutional model, with little computational effort. Other studies have included multiple reflections (Roy-Chowdhury et al., 1988) and have used point sources and computed seismograms at non-zero source-receiver offsets (Sandmeier and Wenzel, 1986).

Clearly, one-dimensional models can only approximate real geologic structures in a local sense. Beyond their computational efficiency, layered structures are popular because they match to some extent studies of surface geology. Studies of outcrops with structures presumably formed at mid- and lower crustal depths have shown mylonite zones associated with ductile shear (Hurich et al., 1985; Frost and Okaya, 1985), layered zones differentiated by metamorphic grade (Smithson and Brown, 1977), and interlayering of contrasting lithologies (Fountain and Salisbury, 1981). Each of these examples shows well-defined layering vertically, but the lateral scale of correlatable layers is variable, ranging from meters to kilometers.
The fact that deep structures may have significant lateral variability over a range of scale lengths is explicitly recognized in several outcrop and seismic modeling studies. For instance, Figure 1.1, taken from Fountain and Salisbury (1981), shows a set of generalized cross-sections for lower continental crust developed from surface studies in several areas. Intrusive bodies are common to all but one section and such intrusions would presumably have scale lengths from meters to kilometers. A model derived from surface mapping of the Ivrea Zone (Figure 1-2, from Hurich and Smithson, 1987) also shows structural complexity and considerable lateral variation. Surface studies of exhumed oceanic crust (Karson et al., 1984) show similar scales of lateral heterogeneity.

Some recent seismic modeling studies have tested the response of deep structures that are laterally variable. Some of these studies have used layer shapes taken from surface mapping (e.g., Collins et al., 1986), including in one instance velocities for the mapped units determined from laboratory measurements (Hurich and Smithson, 1987). Other studies have used hypothetical models for the target structure and generic values of velocity (Braile and Chiang, 1986; Wenzel et al., 1987).

Synthetic data in these studies were calculated using a variety of techniques, including ray theory (Collins et al., 1986; Braile and Chiang, 1986), diffraction response methods (Hurich and Smithson, 1987), and acoustic-wave finite-difference algorithms (Wenzel et al., 1987). The models in all studies retain a basically layered framework; that is, a discrete selection of constant velocities applied to layers with greater horizontal than vertical extent. In each case, models were constructed so that synthetic data matched the variable reflection response seen in field data. Significantly, in all studies some amount of random noise was added to synthetic sections to help mask lateral continuity and improve the match to field-recorded data. In contrast, the synthetic data I show below have lateral variability that is inherent in the reflecting targets.
Understanding the scale of lateral variability has important implications for studies of crustal reflectivity. First, the large velocity contrasts in many of the plane-layered models above may be attributable in part to their one-dimensional nature. That is, in a model composed only of specular reflectors, large reflection coefficients are required to produce a strong constructive interference. In contrast, lateral heterogeneities may provide comparable scattering potential with smaller velocity variations. For instance, a comparison of 1-D and 2-D models in Chap. 4 suggests that 2-D heterogeneities can produce scattered amplitudes similar to those from a 1-D model from velocity contrasts approximately half as great. Thus, accurate estimates of the magnitude of vertical heterogeneity will depend critically on knowing the extent of lateral heterogeneity also.

Estimates of the scale and magnitude of velocity variations for deep targets will be important to understanding reflection mechanisms in crystalline rocks. Several of the proposed explanations for crustal reflectivity are summarized in the concluding chapter; included among the possibly reflective structures are ductile shear zones (with possible anisotropy, Jones and Nur, 1984; Reston, 1988; Sandmeier et al., 1988), igneous intrusions (Warner, 1988), and fluid-saturated zones (Etheridge, 1988). The various mechanisms produce reflectivity of differing magnitude and spatial scale. Thus, quantitative estimates of crustal heterogeneity could provide important information for assessing the mechanism responsible for a particular structure.

RANDOM HETEROGENEITY AND EARTHQUAKE DATA ANALYSIS

Heterogeneous velocity structures are also an integral part of the analysis of earthquake recordings. In particular, seismic wave scattering from three-dimensional inhomogeneities has been widely accepted as the cause of coda following direct arrivals (Aki and Chouet, 1975) and scattering is likely an important mechanism is the attenuation
of propagating phases as well (Aki, 1982). Heterogeneous velocity structures have also been employed to explain arrival time and amplitude anomalies for teleseismic events recorded at large arrays (Aki, 1973). Analyses of coda decay, arrival time anomalies, and scattering attenuation have been used to estimate the scale length and magnitude of inhomogeneities in the lithosphere (Aki, 1973 and 1980b; Aki and Chouet, 1975). Some of these estimates are likely to characterize crustal heterogeneity, as Wu and Aki (1985a) note that scattering effects for shallow earthquakes (focus < 50 km) are dominated by crustal velocity structure.

In this section I briefly review the form of velocity models used by earthquake seismologists and the techniques of estimating the parameters of heterogeneity. The statistical description of these velocity models is adapted in the primary investigation of this project to synthesize crustal targets. In the analysis of scattered waves on earthquake records, standard velocity models have three-dimensional, isotropic, random fluctuations superimposed on a constant background velocity. The fluctuations are characterized statistically by their variance $\sigma^2$ and a spatial autocorrelation function $N(r)$, where $r$ is a scalar distance in three-dimensional space. Variations are isotropic in that $N$ depends only on distance and not on direction; thus, velocity variations have no preferred spatial orientation. In published studies, the autocorrelation function most often has a Gaussian or exponential form (Aki and Chouet, 1975), although fractal forms have been investigated (Wu and Aki, 1985a; Frankel and Clayton, 1986). Velocity fluctuations with various correlation properties are presented in Frankel and Clayton (1986).

Clearly, an isotropic random medium provides a generalized view of velocity variation at crustal or lithospheric scales, just as layered models simplify the nature of deep crustal targets. Many models of crustal structure, however, do have components with roughly spherical symmetry; note, for instance, the schematic representation of intrusive bodies in the cross-sections of Figure 1-1. Also, structures with complex vertical and
horizontal structure (e.g., Figure 1-2) may best be characterized in a statistical sense. Such a description may be the only practicable approach to the detailed seismic interpretation of such structures at depth. Note that the form of any correlation function $N(r)$ can be adjusted to account for greater continuity in the horizontal direction as compared to the vertical. Interpretations of isotropic random media will be discussed further in the concluding chapter.

In the analysis of seismic scattering, isotropic random media are an important generalization of subsurface structure that makes the calculation of scattered responses a tractable problem. Seismic scattering in random media has been extensively studied for acoustic waves by Chernov (1960) and for elastic waves by Wu and Aki (1985b). Both analyses assume single scattering of incident plane waves from a finite volume of perturbations in an otherwise constant-velocity medium.

Scattering theory has been applied in three main techniques for estimating subsurface heterogeneity. First, Chernov's theory can be used to predict amplitude and arrival time variations for plane waves traveling in random media and these predictions were directly applied by Aki (1973) to estimate scattering parameters for the lithosphere. Second, the power of coda waves can be compared to the power of direct $S$ arrivals to estimate the backscattering coefficient for $S$ waves. Aki and Chouet (1975) used scalar scattering theory (Chernov, 1960) to relate that coefficient to the power spectrum of velocity variations, where the power spectrum is given by the Fourier transform of the autocorrelation function $N(r)$. Wu and Aki (1985b) recently derived the backscattering coefficient for the full elastic case and applied a similar analysis (Wu and Aki, 1985a). Third, Chernov (1960) predictions of the frequency-dependent loss of energy due to scattering have been applied to $S$ arrivals by Aki (1980b). The scale and magnitude estimates of lithospheric heterogeneity based on the various techniques were summarized by Aki (1982). His compilation is reproduced here as Figure 1-3.
While isotropic random media provide a useful description of many wave propagation effects observed in earthquake records, the need to consider other forms of velocity variation has been noted in several studies. In particular, layered structures are known to change the scattered response and both numerical and analytic techniques have been employed to assess such effects (Malin, 1980; Menke and Chen, 1984).

The studies above show that waves scattered from crustal heterogeneities are readily observed in earthquake records. We can therefore expect that such scattering should also influence deep reflection data, since crustal seismic surveys can image to the focal depths of shallow earthquakes. In the following section I consider the connections between crustal heterogeneity and various issues in the processing of seismic reflection data.

CRUSTAL HETEROGENEITY AND SEISMIC IMAGING

For the discussion that follows, it will be useful to focus on spatial resolving power as one quantitative measure of the reliability of a seismic image. To make a comprehensive study of resolving power for deep crustal seismic data, we must consider the following issues: 1) to what extent is the propagating seismic wavelet distorted or attenuated by crustal heterogeneities, 2) to what extent do noises, particularly source- or signal-generated noises, contaminate the data, and 3) to what extent can data processing improve resolving power. As we discuss these issues in more detail below, we will see that they are interrelated.

Standard analyses of vertical and horizontal resolution for reflection data are based on highly idealized models of the seismic experiment. Vertical resolving power is assessed in terms of the bandwidth and phase characteristics of an effective seismic wavelet that propagates as a normal incidence plane wave and interacts with horizontal specular reflectors (e.g., see Neidell and Poggiagliolmi, 1977). Horizontal resolving power is
usually measured in reference to the size of a Fresnel zone, which is defined in terms of a monochromatic spherical wave at the target reflector (Sheriff, 1977). In reflection seismology, the Fresnel zone is taken to be the region that contributes to a specular reflection, and thus defines a basic spatial averaging of the target image. Significantly, the size of a Fresnel zone depends on frequency; therefore, both horizontal and vertical resolving power depend on the characteristics of the incident seismic pulse (the effective wavelet). Berkhout (1984) has discussed in detail the important connections between vertical and horizontal resolution.

*Apparent attenuation and multiple scattering in heterogeneous media.* A well-known effect of propagation through heterogeneous media is the attenuation of seismic waves by multiple scattering. This type of transmission loss is usually referred to as "apparent" attenuation to distinguish the effect from the "intrinsic" attenuation caused by anelastic absorption. Both apparent and intrinsic attenuation act to reduce the bandwidth of the propagating pulse, and thus they similarly degrade resolving power.

The phenomenon of apparent attenuation has been extensively studied by both exploration and earthquake seismologists: For media with only vertical variations of velocity, Wuenschel (1960) and O'Doherty and Anstey (1971) have shown that multiple reflections make important contributions to the transmitted pulse. These studies and others have shown that the phase delays of the multiples cause the effective transmitted pulse to lengthen and exhibit apparent attenuation (Schoenberger and Levin, 1974 & 1978; Richards and Menke, 1983; Menke, 1983; Resnick et al., 1985).

The effects of scattering in two- and three-dimensionally inhomogeneous media have also been studied. Aki and Chouet (1975) discuss two scattering models as sources of coda waves and use these models to estimate crustal heterogeneity from the coda of local earthquakes; the frequencies studied by Aki and Chouet (1-25 Hz) are in the normal passband of seismic exploration techniques. Synthetic seismograms calculated by Frankel
and Clayton (1984, 1986) confirm that the level of apparent attenuation is related to the magnitude of heterogeneity in velocity; their studies show that the coda contains the (generally higher) frequencies scattered from the transmitted pulse. Aki (1982) modeled the observed attenuation of earthquake events as the result of scattering and thereby estimated upper and lower bounds for heterogeneity in the lithosphere. The attenuation and coda associated with scattering are important to resolution studies of reflection data because both effects serve to increase the time duration of the propagating seismic wavelet. These effects of propagation through a heterogeneous medium such as the crust should strongly influence the character of deep reflections.

*Noise and scattering in heterogeneous media.* Anstey (1980) and Widess (1982) have pointed out that the level of recorded noise places a fundamental bound on resolution. That is, the useable bandwidth of a signal includes only those frequencies which have acceptable signal-to-noise ratios. The nature of recorded noise is therefore of primary interest in our assessment of resolution.

Numerous investigations have shown that scattering from lateral heterogeneities is a fundamental source of noise in seismic data. For instance, Geyer (1976) describes an extensive noise study of several continental areas; in his tests, much of the recorded noise is associated with scattering of surface waves by topography and near-surface irregularities such as faults. Larmer et al. (1983a) identified out-of-plane scattering from irregularities near the seafloor as the source of linear patterns of noise observed on stacked marine data. Tsai (1984, 1985) has shown that energy scattered from a rough basaltic layer appears as noise that disrupts deep reflections in a marine survey. Hill and Levander (1984; also Levander and Hill, 1985) have shown for SH and P-SV models that small-scale roughness at the boundary of a low-velocity surface layer can produce noise by scattering upcoming reflections into the modes of the layer.
The literature on theoretical and numerical studies of wave scattering (for both plane wave and point sources) is extensive, and it is beyond the scope of this section to adequately survey these results. Let me emphasize, however, that scattered waves have not been extensively studied in the context of the complicated data processing sequence that is applied to produce a seismic reflection profile.

The heterogeneity of the crust and the variety and prevalence of scattering mechanisms noted above suggest that the level of signal-generated noise will be significant in most deep seismic data. This fact is of particular importance in the acquisition and processing of seismic data; whereas ambient noise can in theory be overcome by increasing the source strength, such an approach only produces more signal-generated noise. Effective attenuation of such noise requires a detailed understanding the mechanisms that produce it (Larner et al., 1983a).

*Data processing, lateral heterogeneity, and noise.* Digital processing of the recorded data clearly affects the resolving power of the final image. For instance, various forms of deconvolution are routinely applied to individual traces to improve vertical (i.e., temporal) resolution. In addition, migration can be formulated in terms of a spatial deconvolution and, properly executed, that process improves horizontal resolution (Berkhout, 1984). Other processes, such as CMP stacking, can improve or degrade resolution, depending on whether the recorded data meet the assumptions of the process.

CMP stacking is particularly important to resolution because it can greatly increase the signal-to-noise ratio. Where velocity varies laterally, however, stacking may degrade data quality because the assumptions of a common reflection point and simple hyperbolic moveout may not be fulfilled. Hale (1984), for example, has discussed the problem of reflection point smearing for CMP gathers collected over a simple dipping interface. Larner et al. (1983b) demonstrate degradation of a stacked image for events below a severe lateral
velocity contrast; in that study, the lateral variation of velocity causes further imaging problems when the data are input to a conventional time migration algorithm.

Noise also limits the ability of processes such as deconvolution to improve resolution. Larner et al. (1983a) show that predictive deconvolution is considerably more effective when scattered noise is removed from a field data set before that process is applied. The tests in the next chapter show that different types of scattered noise can affect the deconvolution process differently. Because data processing has such a marked effect on final data quality, understanding the noise and the magnitude of lateral velocity variation in a data set is of central importance to any analysis of resolving power.

The discussion above has outlined the following interconnections between three of the issues that affect resolution (and hence reliability): signal attenuation and noise have a common connection in lateral heterogeneity. While one major goal of data processing is to compensate for such attenuation, we find that many data processes are seriously compromised by the existence of lateral velocity variation and noise in the data. Understanding signal-generated seismic noise is thus a key element in understanding the reliability of a seismic image.

OVERVIEW OF REMAINING CHAPTERS

In Chapter 2, acoustic-wave finite-difference synthetic data are computed for two velocity models that represent basic forms of crustal heterogeneity. The scattered noise and wavefield distortion produced by an irregular low-velocity surface layer in one model are compared to equivalent phenomena in a similar model that has a deeper zone of random velocity variations. Scattering effects in both models degrade the processed seismic image. Most importantly, however, the dip-filtering action of CMP stacking causes reflections from the random velocity zone to have artificial lateral continuity.
Since the smearing effect of CMP stacking will clearly affect any estimate of lateral heterogeneity, the CMP dip filter is quantitatively assessed in Chapter 3, using a revised set of finite-difference synthetic data. In this chapter, I demonstrate how stacking smear depends on target depth and survey parameters and develop a quantitative measure of lateral continuity in a seismic record. I also demonstrate that migration can only improve the fidelity of the seismic image if the data are stacked correctly or migration before stack is used. The chapter concludes with a field data example showing the effects of stacking smear on quantitative measurements of lateral coherence.

In Chapter 4, I investigate the seismic image of a randomly heterogeneous target as seen in a wide-angle common-shot gather. Shot gathers from the CMP survey in Chapter 3, show increased amplitude and extensive lateral continuity in the reflection response from the random zone at source-receiver offsets greater than about twice the target depth. The continuity and amplitude effects can be explained with traveltime relations and simple wave theory. A simple relation is then developed between the lateral correlation of the wavefield and the correlation function of the velocity variations. Two field data sets are examined here as well: In one, a model with 2-D random velocity variations provides an adequate match to field data previously interpreted in terms of an extensively-layered structure. In the second case study, a field gather with dense receiver spacing shows a progressive increase of lateral coherence at increasing source-receiver offsets. The kinematic analysis, however, predicts greater lateral continuity than is observed; the discrepancy is attributed to energy scattered by heterogeneities above the target and the presence of noise in the data.

Chapter 5 includes a summary of the main conclusions from preceding chapters and a discussion of the lateral heterogeneity of various models proposed for deep crustal reflectors. The chapter concludes with a discussion of approaches to crustal interpretation suggested by these results and suggestions for further work.
FIGURE CAPTIONS

Fig. 1-1. Generalized cross-sections of exposed continental crust in various areas (from Fountain and Salisbury, 1981).

Fig. 1-2. Cross-sectional model of the Ivrea-Verbano Zone based on surface mapping. Velocities and densities in this model were used by Hurich and Smithson (1987) to produce synthetic seismograms.

Fig. 1-3. Scale length and magnitude of heterogeneity in the lithosphere (from Aki, 1982).
Figure 1-1
(from Fountain and Salisbury, 1981)
Fig. 11. Geologic model of a lower crustal terrane derived from mapping in the Ivrea Zone [19]. Although derived from a specific geologic environment, the geometry of the various rock units are probably typical of geometries in many other deformed crystalline terranes. Vp = P-wave velocity (km/s), Rho = density (g/cm³).

Figure 1-2
(from Hurich and Smithson, 1987)
FIG. 5. Inhomogeneity of the earth's lithosphere. Plots of mean square fractional velocity fluctuation versus correlation distance estimated from various measurements. Reproduced from Sato (1982a). Numerals correspond to the following cases. (1) $Q^{-1}$ by the Born approximation neglecting scattered energy into the forward half-space. S waves in the Kanto district, Japan (Wu, 1981). (2) $Q^{-1}$ by Karal and Keller's theory. The explosion data in Nevada (Beaudet, 1970). (3) Teleseismic P-wave analysis by the Chernov theory. LASA in Montana (Aki, 1973). (4) Teleseismic P-wave analysis by the Chernov theory. LASA in Montana (Capon, 1974). (5) Teleseismic P-wave analysis by the three-dimensional inversion method (Aki et al., 1976). (6) Acoustic well log in the Kanto district, Japan, I, Iwatsuki; S, Shimokasa; F, Fuchu (Suzuki et al., 1981). (7) Acoustic well log in New Mexico (Wu, 1981). Alphabets represent parameters of the von Karman autocorrelation function for which $Q^{-1}$ is computed as shown in Figure 6. The order number of the von Karman function is taken as 1/2 for cases A and B, 1/3 for case C, 1/4 for case D, and 1/10 for case E. Case C is circled because it appears the best fit to the observations.

Figure 1-3
(from Aki, 1982)
Chapter 2
Modeling and Processing Scattered Waves in Seismic Reflection Surveys

SUMMARY

Various wave-scattering mechanisms are known to degrade signal and produce noise in seismic reflection data. Synthetic 2-D acoustic-wave finite-difference data sets illustrate the effects of two such mechanisms. Twenty-five shot gathers were generated for each of two models and the data were processed as a standard CMP survey. In one model, an irregular low-velocity surface layer produced multiply-scattered surface waves that appear as linear noise trains in common-shot gathers and stacked sections. The scattering of upcoming reflections at the lower interface of the layer also produced a significant amount of noise. When predictive deconvolution is applied before stack to reduce reverberations, the spectral character of the scattered surface waves seriously inhibits the action of that process.

In the second model, a zone of smooth random velocity variation was imposed between two reflectors deeper in the model. The heterogeneous zone (±5% rms velocity variation) substantially degraded signal reflected from below it; events produced by body-wave scattering are characterized by higher phase velocities than seen in the first model. Conventional CMP stacking produced discontinuous subhorizontal events from the disturbed zone. The limited bandwidth of the propagating signal and spatial filtering attributable to CMP stacking cause these events to bear no simple relation to the velocity anomalies of the model, even after migration.

20
SYNTHETIC DATA

As a first step in analyzing the effects of lateral heterogeneity, we have generated and processed synthetic reflection data for two models. These models have been designed to represent two basic types of scattering mechanisms: the scattering of surface waves and body waves by an irregular surface layer and the scattering of body waves by velocity inhomogeneity at depth.

The synthetic data were generated with a fourth-order finite-difference scheme for the constant-density two-dimensional (2-D) acoustic wave equation. In both models studied below, velocities were specified on a computational grid that had 6 m spacing in both the vertical and horizontal directions. The sides and bottom of the finite-difference grid satisfied the Clayton and Engquist (1977) A2 absorbing boundary condition. The top of the grid satisfied a vanishing pressure condition (corresponding to the surface of the Earth). A band-limited acoustic line source was inserted into the grid in the manner of Alterman and Karal (1968). The line source is the 2-D analog to the 3-D point source.

To minimize grid dispersion in a fourth-order finite-difference scheme, the shortest wavelength propagated on the grid must be a least five times the grid dimension (see Levander, 1988). Considering the minimum velocity in the models below, this criterion corresponds to a maximum frequency of 42 Hz. The source wavelet for the modeling program is antisymmetric (a differentiated Gaussian pulse) with a dominant frequency of 18 Hz. Relative to the peak frequency, the amplitude spectrum of the source wavelet is down 70 percent at 42 Hz. The stability criterion for the finite-difference solution is based on the maximum velocity in the two models; this criterion set the time step for the calculations at 1.225 ms.

The two velocity models studied are shown in Figures 2-1a and 2-1b. In Figure 2-1a, the low-velocity surface layer has a rough lower interface (±12 m variation about an
average depth of 72 m, thickness is laterally constant over 36-m intervals). The maximum two-way traveltime anomaly introduced by the rough interface is ±6.5 ms. The irregular surface layer can produce noise by two mechanisms: multiple scattering of surface waves trapped in the layer and resonant coupling of upcoming reflections into the layer modes (Hill and Levander, 1984; Levander and Hill, 1985). At 800 and 2000 m, velocity changes produce horizontal specular reflectors with reflection coefficients of 0.1.

The second velocity model (Figure 2-1b) has a smooth surface layer with the same average thickness as that in Figure 2-1a (72 m) and primary reflectors at the same depths. Velocity in the zone between the two reflectors, however, varies randomly. Velocity at each grid point in the model was perturbed by a uniformly-distributed random amount; variations were then smoothed over 72 m vertically and horizontally and scaled to have a standard deviation that is 5% of the mean layer velocity (2400 m/s). This second model will produce noise by the multiple scattering of body waves in the random zone.

The finite-difference algorithm was used to generate 25 synthetic shot records for each model. Shots were spaced at 96-m intervals and groups of seismograms from the 6-m grid were summed to simulate 48-m long receiver arrays (48-m group interval, 2304-m maximum offset, 2.5 s maximum record length). The sample interval of the data was also increased to 4 ms. Representative shot records for both models are shown in Figures 2-2a and 2-2b.

The records in Figures 2-2a and 2-2b exhibit distinctly different noise characteristics. In Figure 2-2a, trapped waves scattered in the irregular surface layer produce strong linear noise trains that start at the first breaks. These scattered waves are themselves dispersive and have low phase velocities corresponding to the waveguide modes of the surface layer and the deeper medium. In contrast, the noise scattered by the deeper inhomogeneities (Figure 2-2b) starts only after the first reflection and these noise events exhibit much higher
phase velocities. Note also that the lateral velocity variation in the deep scatterer model (Figure 2-1b) causes substantial disruption of the second reflection event at about 2 s.

Differences in noise character are equally apparent when the data are sorted into CMP gathers (Figures 2-3a and 2-3b). In Figure 2-3a, most of the scattered noise appears as strong horizontal events (normal moveout correction not applied). This linear pattern results from the horizontal travel paths these waves take in the 2-D model. In Figure 2-3b, the scattered waves appear as hyperbolas having various moveout velocities according to their lateral position relative to the common midpoint. Note also that the moveout and reflection character of all events below the first reflection are affected to some extent by the lateral velocity variation.

DATA PROCESSING

The CMP gathers were muted as shown in Figures 2-3a and 2-3b, corrected for normal moveout (NMO) using a single velocity function (the rms velocity of the basic layered model), and then stacked. The CMP gathers vary from single-fold at the ends of each model to 25-fold in the center. Figures 2-4a and 2-4b show the center portions of each stack, where the fold is above 18. In both sections, CMP stacking has substantially enhanced the reflected events, and again the noise characteristics of the two models are distinct. Noise scattered in the surface layer shows up as strong linear patterns (an effect described in detail by Lerner et al., 1983a), but the noise does not seriously disrupt either reflection.

In contrast, the scattered body waves from the random zone of Figure 2-1b appear as numerous short subhorizontal segments. Multiple scattering in the random zone causes noise to persist beyond the second reflector, where it decays at 8 db/s. While the deeper
reflector clearly stands out from the background noise, that event shows variations in amplitude and timing attributable to the overlying lateral velocity variation.

The stacked sections in Figures 2-4a and 2-4b also show that reverberation in the surface layer of both models has substantially extended the source pulse. It is interesting to compare reverberation effects in the two cases. While the surface layer of Figure 2-1a efficiently scatters surface waves, the vertical layer roughness (±12 m) is relatively small compared to even the shortest wavelengths in the source. Thus the reverberation pattern in the irregular surface layer is essentially identical to that produced when the layer is smooth. Since the reverberations obviously affect any measure of resolution, we have studied how successfully this problem can be treated with the standard process of predictive deconvolution.

Figures 2-5a and 2-5b show the results of applying predictive deconvolution to the stacked data of Figures 2-4a and 2-4b. The deconvolution operators had a prediction distance of 40 ms (240 ms maximum length, 0.1% white noise added), and were designed and applied to each stacked trace separately. A 90° phase-shift filter was included in the deconvolution process to convert the source pulse to a symmetric (zero-phase) wavelet.

In both Figures 2-5a and 2-5b, the deconvolution process has substantially compressed the reflected wavelet by removing most of the reverberation effect. A close examination of the stacked data, however, reveals that the deconvolution process was somewhat more successful in the case of the data generated for the deep scatterer model. The differences between the deconvolution results are most evident in the character of the deeper reflection.

In conventional data processing, predictive deconvolution is most often applied before stacking in order to condition the data for processes such as velocity and residual statics estimation. In Figures 2-6a and 2-6b, 40-msec gapped deconvolution operators were designed on and applied to individual unstacked traces, which were then stacked with
the previous muting and NMO correction. The results here are quite different from those shown in Figures 2-5a and 2-5b; predictive deconvolution has had practically no effect on the character of the reflection events for the surface scatterer model (compare Figures 2-4a and 2-6a). In contrast, the results for the deep scatterer data are essentially the same as before (compare Figures 2-5b and 2-6b).

The marked difference in the results of Figures 2-5a and 2-6a (and the similarity of Figures 2-5b and 2-6b) can be explained by examining the spectral character of the two sets of data. Figures 2-7a and 2-7b show amplitude spectra for the "signal" in the surface and deep scatterer data, respectively. Figure 2-7a is the average spectrum computed for a 300-msec window covering the upper reflection event of Figure 2-4a; the average was computed over eleven traces from the center of that section. Figure 2-7b is similarly computed for the data in Figure 2-4b.

Figures 2-7a and 2-7b both show the general trend of the source spectrum, with deep notches that are attributable to the effects of the source and receiver ghosts and reverberation in the surface layer. The goal of the deconvolution process is to remove the spectral notches (and their associated phase shifts). Noise in the data presents a serious obstacle to achieving that goal.

Figures 2-7c and 2-7d show amplitude spectra for the noise in Figures 2-4a and 2-4b. These average spectra were computed for a window between the two reflection events (1.1 - 1.8 s), over the same traces used before. Noise in the surface scatterer model has an average spectrum (Figure 2-7c) that is close to that of the source wavelet, but does not show any prominent notches. Scattered waves from the random velocity zone, however, have an average amplitude spectrum (Figure 2-7d) that shows the same reverberation notches seen in the upper reflection (Figure 2-7b).

The differences between Figures 2-7c and 2-7d are attributable to the different noise mechanisms of the two models. The scattered noise seen in the stack of Figure 2-4a results
from waves that have never left the surface layer. These trapped waves have therefore not been modified by the (source and receiver) reverberation filters that are imposed on energy scattered from the deeper random zone (Figure 2-4b).

The results of predictive deconvolution can then be explained by the fact that the action of that process depends on the spectral characteristics of the trace (signal plus noise) to which it is applied (Berkhout, 1977). Combining the corresponding signal and noise spectra of Figure 2-7, we see that the scattered surface-wave noise will tend to fill in the reverberation notches. When deconvolution is applied after stack, the signal-to-noise ratio (SNR) is large enough (about 3) that the noise has little effect. On the unstacked data, however, the noise is strong enough (SNR about 1.7) to obscure the notches; the resulting deconvolution operator does not compensate for the reverberation effect.

IMAGING RANDOM VELOCITY VARIATION

Another important issue in this study is the image of the deep scattering zone as seen in the CMP stacked sections (Figures 2-4b, 2-5b, and 2-6b). While we have been considering events from that zone as scattered "noise", these returns are also the reflection response of that part of the medium. Interestingly, that reflection response shows numerous subhorizontal event segments typical of much field-recorded reflection data at late traveltimes. The typical length for these segments ranges from 200 to 400 m (about 25 to 50 percent the size of a Fresnel zone for the dominant frequency of 18 Hz).

While the reflection response seen in Figure 2-4b suggests a layered velocity structure between the two primary reflectors, recall that the velocity variation in this zone is smoothly random, both vertically and horizontally, with a scale length much less than the typical segment size. In an effort to sharpen the reflection image, finite-difference migration was
applied to the stacked data in Figure 2-4b. The result (shown in Figure 2-8) did not improve, and probably degraded, the horizontal resolution of the image.

The artificial horizontal continuity seen in Figures 2-4b and 2-8 is primarily attributable to the failure of CMP stacking for this inhomogeneous medium. These data were stacked with velocities appropriate for the horizontal interfaces; the stack then acted as a dip filter for non-vertical arrivals from the random zone. Migration further smears the dip-filtered stack because steeply-dipping diffraction energy was not preserved.

To assess the results of the migration process, consider a series of 1-D seismograms for the deep scatterer model; Figure 2-9a was produced by computing vertical reflection coefficient sequences at each horizontal position in the model of Figure 2-1b. Adjacent sequences were summed to produce one sequence every 24 m. The average sequences were then convolved with a 5-40 Hz zero-phase wavelet to produce the 1-D primaries-only synthetic seismograms shown. This section represents an ideal migrated section, where all multiple reflections have been completely suppressed.

For comparison, Figure 2-9b shows the result of migrating the stacked section that had deconvolution applied after stack (Figure 2-5b). The deconvolved stack was migrated so that the effective wavelets in the two sections would be similar. In comparing the two images of the random zone, however, note that only the strongest events can be reasonably correlated. The fully processed section in Figure 2-9b shows substantially greater lateral continuity than exists in the velocity model. Several events in Figure 2-9b that have considerable lateral extent (>400 m) are not at all evident in Figure 2-9a. We attribute this result to the complex patterns of multiple scattering in the random zone.

The lateral continuity seen just below the first reflector in Figure 2-9b is attributable to two effects other than those discussed above: 1) the finite-length deconvolution operator produces some residual ringing on output because it cannot remove the free surface ghost
completely (free surface reflection coefficient = -1.0), and 2) the top of the random zone is located 100 m below the first specular reflector and acts as a (non-specular) reflector itself.

DISCUSSION

In the examples presented above, a particular type of scattered noise can obviously degrade the results of predictive deconvolution. An important issue for the interpreter is whether data processing can suppress such noise and improve the final image. For instance, the result of Figure 2-6a could be considerably improved by suppressing the scattered noise with a frequency-wavenumber filter before deconvolution. Larner et al. (1983a) demonstrate the effectiveness of such filtering on field-recorded data; their work emphasizes the fact that effective noise suppression depends on several factors but in all cases demands a detailed understanding of the mechanisms by which noise is generated.

Figures 2-2, 2-3, and 2-4 show that noises from the basic mechanisms we have chosen to study can be distinguished at virtually any stage of the data processing sequence. In field-recorded data, a number of complicating factors will make this task considerably more difficult. First, the scattered noise problem is fundamentally a three-dimensional problem. Larner et al. (1983a) and Tsai (1984) have discussed in greater detail the nature of noise scattered in three dimensions from both shallow and deep roughness. Second, for land data in particular we must consider the propagation of both elastic and acoustic waves rather than the acoustic-only models we have studied here. We could expect that, even in two-dimensions, conversions between P and S waves would produce noise events with a range of phase velocities much greater than that observed here.

Other aspects of the real earth will also play a substantial part in determining the noise character of a given set of data. Most obviously, anelastic attenuation will be important in determining the strength of scattered events at a given point in the record. Aki (1982) has
noted that the coda of earthquakes owes it particular characteristics to the relative strengths of scattering and attenuation. The relative roles of apparent and intrinsic attenuation is a subject not well understood.

One mechanism that may be of substantial importance to the character of the noise field is the resonant coupling of upcoming reflections to surface-layer trapped waves, described by Hill and Levander (1984; see also Levander and Hill, 1985). As noted earlier, this mechanism should be active in the surface-scatterer model of Figure 2-1a, but the effect is not visible in the data because of the strong scattering of direct waves (see Figure 2-2a).

To assess the interaction of reflections with the irregular lower interface of the surface layer, we modified the model in Figure 2-1a and generated a shot record at the same position as that shown in Figure 2-2a. The surface layer of the modified model was the same as before, but the specular reflectors at depth were removed by keeping the velocity below the surface layer constant at 1900 m/s. The shot record for this model is shown in Figure 2-10a. It contains only those direct and scattered waves from the source that have never left the surface layer.

The record in Figure 2-10a was subtracted from that in Figure 2-2a to produce the section shown in Figure 2-10b. This resulting gather shows only the two primary reflections and events that are generated by the interaction of the reflections with the rough interface. The scattered waves have significant amplitudes, in many instances comparable to those of the reflections themselves. The scattered events in this case are predominantly scattered body waves; the existence of trapped modes is not evident in this model because of its limited extent in time and space. The beginning of dispersive mode behavior can be seen at the longest offsets (which in this case are outside the mute pattern).

The importance of this noise mechanism is that it is continually excited with each upcoming reflection. Such signal-generated noise may be an important contributor to the
background field, especially at late reflection times. We should also note that the spectral character of this noise will be different from that seen in Figure 2-7c, because the upcoming reflections will have at least the reverberation pattern associated with the source position imposed upon them. In three dimensions, the resonant coupling mechanism will produce a field that crosses a 2-D linear array from all azimuths. The distinct $\omega-k$ dispersion relation of the scattered field observed in 2-D simulations (Levander and Hill, 1985) will thus not be readily observable, as the noise field will appear at all apparent phase velocities.

The other main issue in this study is how accurately the zone of random velocity variation (Figure 2-1b) can be imaged. Our synthetic data clearly show that the image of the random zone has more lateral continuity than does the underlying velocity structure. This point is of considerable importance to field-data interpretation, as groups of short segments such as those seen in Figures 2-4b or 2-8 are often interpreted in terms of layered structures (e.g., Cook et al., 1981).

Improved data processing could possibly refine the image of the random zone. We have noted that the horizontal continuity is in part attributable to dip-filtering by the stack and migration processes. Spatial resolution might therefore be improved by either 1) using a more sophisticated stacking technique to produce a section that retains a wider range of dips (i.e., includes the steeply-dipping diffraction energy), or 2) employing some form of migration before stack. The first approach is generally more efficient and easier to implement than the second.

One approach to improving the stack is the dip-moveout correction (DMO) developed by Hale (1984). The main effect of that process is to remove the variation of stacking velocity with dip so that the CMP stack can preserve a full range of dips. The sequence of DMO correction, stack, and migration after stack amounts to an efficient application of time migration before stack.
When dip-moveout correction was applied to the unstacked data of the deep scatterer model, however, the resulting stacked section (not shown) was not significantly different from that shown in Figure 2-4b. Migration of the DMO-corrected stack thus produced a section virtually identical to Figure 2-8. The failure of the dip-moveout process here is most likely caused by the lateral variation of velocity in the random zone. As noted above, the enhanced processing sequence implements a form of time migration before stack; time migration algorithms (before or after stack), however, cannot properly treat data where velocity varies laterally (Larner et al., 1983b). Thus some form of prestack depth migration may be necessary in this case.

Even the more-costly process of depth migration before stack, however, may not provide a clear image of the random zone, because much of the energy recorded from that zone has been multiply-scattered. The effects of multiple scattering can be seen in the stacked and unstacked data at traveltimes below the second reflector. At these late times we see a considerable amount of energy is still being returned from the random zone. We note also the uncorrelatable events seen in the comparison of Figures 2-9a and 2-9b. An accurate depth migration would thus have to be based on a two-way wave equation (that can accommodate multiple scattering) and a detailed description of velocity variation in the zone that we seek to image.

The effects of multiple scattering raise another issue of importance to the interpreter. That is, the residual noise (or coda) generated by scattering in the random zone can be strong enough to obscure weak reflections from below the zone (independent of any lateral velocity variation effects). This issue has been studied for one-dimensional layered media by Hosken and Mayo (1984) and is inherent in the formulation presented by Resnick et al. (1985). The 2-D and 3-D cases of this problem are important topics of future research for both conventional and deep reflection data.
CONCLUSIONS

These modeling studies indicate that different types of scattered noise can have substantial effects on the appearance of the final processed section. While different noise types can be easily distinguished in these 2-D acoustic models, we realize that similar investigations of field-recorded data will be more difficult. The reality of three-dimensional wave propagation certainly complicates the scattered noise field. A more complete description of such noise must also include the effects of shear waves and anelastic attenuation. Relatively simple models such as these, however, provide insights into the nature of scattered noise and the effects of the conventional seismic data processing sequence.

Since the specific nature of heterogeneity at all levels of the crust has important geologic implications, we find that a better understanding of the resolving power of seismic surveys is an important goal. Most previous studies have not taken adequate account of the effects of noise, inhomogeneity, and data processing on the section that is finally interpreted. Another important direction for future work then is the characterization of apparently random zones and the subsequent analysis of their effects on resolution.
FIGURE CAPTIONS

Fig. 2-1. (a) 2-D acoustic model of a layered structure with an irregular surface layer. The surface layer boundary fluctuates ±12 m about a mean thickness of 72 m; its depth changes randomly every 36 m horizontally. Velocity is specified on a 6-m square grid and is constant in each layer. Reflection coefficients of the interfaces at 800 m and 2000 m are approximately 0.1.

(b) 2-D acoustic model of a layered structure with a deep zone of random velocity fluctuations (constructed by varying velocity randomly at each grid point and then smoothing vertically and horizontally with a 72-m running average). The velocity fluctuation is 5% about the mean (again 2400 m/s). Velocity in the random zone is plotted in a seismic trace format. Velocity is constant in other layers of the model and the surface layer has a smooth lower interface at 72 m.

Fig. 2-2. (a) Common shot gather generated by finite-difference acoustic-wave modeling using the velocity structure in Fig. 2-1a. The source was located in the surface layer at a horizontal position of 648 m. The group interval is 48 m.

(b) Common shot gather generated for the velocity model in Fig. 2-1b. Source position and group interval are the same as for Fig. 2-2a. The bar provides scale for the source-receiver offset axis.

Fig. 2-3. (a) Two adjacent CMP gathers for the velocity model in Fig. 2-1a. First arrivals have been muted but normal moveout corrections have not been applied. These gathers, with midpoints near the middle of the model, have source-receiver offsets that range from 0 to ±2304 m.

(b) Adjacent CMP gathers for the velocity model in Fig. 2-1b. The mute pattern, midpoint locations, and offset ranges are the same as for Fig. 2-3a.

Fig. 2-4. (a) CMP stacked section produced from 25 synthetic shots computed for the rough-surface model (Fig. 2-1a). The stacked traces shown span the center of the model where CMP fold is 18 or greater.

(b) The CMP stacked section of the 25-shot experiment for the deep-scatterer model (Fig. 2-1b). The bar on this and subsequent stacked sections indicates a horizontal distance of 500 m.

Fig. 2-5. (a) CMP stack of Fig. 2-4a with predictive deconvolution applied after stack (DAS).

(b) CMP stack of Fig. 2-4b with DAS. Note that the reflection events in this section are somewhat more compressed than those in Fig. 2-5a.
Fig. 2-6. (a) CMP stack of the data used in Fig. 2-4a, with predictive deconvolution applied before stack (DBS). For these data, DBS fails to compress reflection events (compare with Figs. 2-4a and 2-5a).

(b) CMP stack of the data used in Fig. 2-4b with DBS. For these data, the results of predictive deconvolution applied before stack are comparable to those obtained when the process is applied after stack (see Fig. 2-5b).

Fig. 2-7. Amplitude spectra for signal and noise in the stacked data generated from Figs. 2-1a and 2-1b. Spectra labeled "signal" (a and b) were calculated over a window surrounding the first reflection event in Figs. 2-4a and 2-4b, respectively. Spectra labeled "noise" (c and d) were calculated over a window between the two reflections on the same sections.

Fig. 2-8. Finite-difference migration of the data in Fig. 2-4b. The migration was performed using the unperturbed interval velocities and layer thicknesses common to both models.

Fig. 2-9. (a) Vertical incidence seismic section constructed by convolving a zero-phase wavelet with the primary reflection series obtained from the deep-scatterer model. The convolved wavelet has a passband similar to that of the finite-difference source, and the velocity structure has been laterally averaged over intervals equal to the stacked trace spacing (24 m). This section represents the CMP stack for which deconvolution and migration were performed perfectly and therefore represents the highest possible vertical and horizontal resolution obtainable from the CMP experiment.

(b) Migration of the data in Fig. 2-6b (predictive deconvolution applied after stack). Note that horizontal resolution for this section appears poorer than that for the primary reflection series in Fig. 2-9a.

Fig. 2-10. (a) Shot record generated for a modified version of the model in Fig. 2-1a; the two specular reflectors at 800 and 2000 m have been removed.

(b) Shot record produced by subtracting Fig. 2-10a from Fig. 2-2a. This difference section reveals the two reflections and scattered waves produced by the interaction of the reflections with the irregular lower interface of the surface layer. The dashed line indicates the mute pattern applied before stacking.
Figure 2-1
Chapter 3
Analysis of Lateral Coherence in CMP-Stacked Images

SUMMARY

The synthetic data in the previous chapter suggest that interpretations of CMP stacks can be heavily biased toward layered structures because the seismic image is dip filtered in the stacking process. In this chapter, I quantitatively assess the lateral correlation of reflection events and how correlation is affected by the stacking filter. The first section describes a measure of lateral correlation based on the spectral coefficient of coherence. Next I describe the parameters of a second set of finite-difference synthetic data produced specifically to evaluate the effects of CMP acquisition and processing for a target with random heterogeneity. The new velocity model is simplified relative to that described in Chapter 2 and has spatial dimensions more appropriate to crustal exploration; reducing the magnitude of velocity fluctuations and removing the complications of a low-velocity surface layer allow the CMP stacking effects to be examined more clearly.

The third section investigates the primary reflectivity of the heterogeneous target and the zero-offset section of the synthetic seismic survey. Primary reflectivity represents an ideal image of the subsurface; conventional seismic imaging techniques (i.e., post-stack migration) are designed to recover primary reflectivity from zero-offset input data using the idea of an exploding reflectors experiment. The next section reviews a quantitative measure of the stacking dip filter and shows to what extent stacking fails to produce the desired zero-offset result. While migration should accurately image lateral heterogeneity, the dip-filtering effects of stacking cannot be reversed in the post-stack migration process; the
synthetic data are used to demonstrate the necessity of migration before stack or equivalent processing (advanced stacking techniques such as dip moveout correction). The final section presents a field data example that illustrates the effects of CMP stack filtering using lateral coherence functions measured on stacked and unstacked sections.

MEASUREMENT OF LATERAL CORRELATION

I will quantify the lateral correlation of a seismic wavefield with a statistic known as the coefficient of coherence (Foster and Guinzy, 1967). This statistic measures the likeness of two traces as a function of frequency; in this study, the average coherence for pairs of traces a fixed distance apart will be computed. The lateral correlation of the 2-D velocity model and various wavefields will be characterized by the manner in which coherence diminishes with increasing trace separation. As shown later, for a given frequency the variation of coherence with spatial separation (lag) amounts to a spatial correlation function. The spatial correlation function of a recorded wavefield can then be related to that of the velocity structure through the geometry and wave propagation effects of a surface seismic experiment.

For two signals $x(t)$ and $y(t)$, the coefficient of coherence at a frequency $\omega$ is defined as,

$$\gamma_{xy}(\omega) = \frac{|\Phi_{xy}(\omega)|}{\sqrt{\Phi_{xx}(\omega) \Phi_{yy}(\omega)}}, \quad (3.1)$$

where, $\Phi_{xy}(\omega)$ is the cross-power spectrum of $x$ and $y$, $\Phi_{xx}(\omega)$, $\Phi_{yy}(\omega)$ are the standard (auto-) power spectra of the individual signals. The power spectra are Fourier transforms of the appropriate cross- or autocorrelation functions. Thus,
\[ \Phi_{xy}(\omega) = \int_{-\infty}^{\infty} \phi_{xy}(\tau) \exp(-i\omega \tau) \, d\tau , \]

\[ = \int_{-\infty}^{\infty} \left[ \int_{-\infty}^{\infty} x(t) \, y(t+\tau) \, dt \right] \exp(-i\omega \tau) \, d\tau . \]  \hspace{1cm} (3.2)

Foster and Guinzy (1967) show that \( \gamma(\omega) \) will range from 0 to 1, where a constant value of 1 at all frequencies means the traces are identical and a constant value of 0 means the traces are completely uncorrelated. When coherence is not constant, the value of \( \gamma(\omega) \) is interpreted as the fraction of power on one trace that is predictable from the other, at the given frequency. Foster and Guinzy (1967) also give expressions relating \( \gamma \) to signal-to-noise ratio, assuming contamination by random noise.

In the coherence analysis of a 2-D wavefield \( p(x,t) \), we will compute the average coefficient of coherence for all pairs of traces separated by a distance \( \delta \). Repeating this process for various trace separations will produce a lateral correlation function for any frequency of interest. For two traces, \( p(x,t) \) and \( p(x+\delta,t) \), let

\[ \gamma_{x,x+\delta}(\omega) = \frac{|\Phi_{x,x+\delta}(\omega)|}{\sqrt{\Phi_{x,x}(\omega) \Phi_{x+\delta,x+\delta}(\omega)}} . \]  \hspace{1cm} (3.3)

Assuming that the statistics of the target zone (and thus the reflected wavefield) do not vary spatially, all trace pairs of constant separation should have the same cross-power spectrum, and it follows that the average power spectrum of a single trace is also constant. We can define the expected cross-spectrum for trace separation \( \delta \) as
\[ \Phi(\delta, \omega) = \frac{1}{\Delta x} \int \Phi_{x,x+\delta}(\omega) \, dx, \quad (3.4) \]

where the integration is over the range \( \Delta x \). We then define the spatial coherence function as

\[ \gamma(\delta, \omega) = \frac{|\Phi(\delta, \omega)|}{|\Phi_0(\omega)|}, \quad (3.5) \]

where \( \Phi_0(\omega) = \Phi(0, \omega) \) is the average single-trace power spectrum.

We can usefully explore (3.5) a little further. First rewrite (3.4), using an operator notation \( \text{FT}\{\} \) to indicate a Fourier transform over \( \tau \). Recalling the definition of the cross-power spectrum in (3.2),

\[ \Phi(\delta, \omega) = \int \text{FT}\{ \phi_{x,x+\delta}(\tau) \} \, dx \]

\[ = \int \text{FT}\left\{ \int p(x,t) \, p(x+\delta, t+\tau) \, dt \right\} \, dx, \]

where all integration limits are \( \pm \infty \). The normalizing factor \( \Delta x \) in (3.4) has been dropped since it is cancelled by the denominator and the integration limits for \( x \) have been extended. Now commute the Fourier transform with the integral over \( x \) and write

\[ \Phi(\delta, \omega) = \text{FT}\left\{ \int \int p(x,t) \, p(x+\delta, t+\tau) \, dt \, dx \right\}. \quad (3.6) \]

We can see from (3.5) and (3.6) that the coherence \( \gamma(\delta, \omega) \) represents a normalized spatial correlation function at a given frequency. We will use the coherence function to
quantify lateral correlation in the target and the reflected wavefield. Coherence measures of the random velocity zone, the primary reflectivity, and the zero-offset section are described below.

SYNTHETIC DATA

The synthetic data in this chapter and the next were calculated for a velocity model that extends from 0 to 40 km horizontally and from 0 to 16 km in depth. For clarity, Figure 3-1 shows only the center portion of the model (10-30 km horizontally to 12 km depth), with velocity plotted in a seismic trace format. The average velocity in the model (constant shaded areas) is 6 km/s; between depths of 4 and 12 km across the entire horizontal extent of the model, velocity has been perturbed with smooth random variations. For reference, velocity all across the model increases to 6.5 km/s at a depth of 14 km to produce a specular reflector; this reflector will not appear in the CMP data presented in this chapter. Velocities are specified on a 25-m square grid for finite-difference modeling as described below.

The velocity perturbations were produced by filtering a grid of uniformly-distributed random numbers in the 2-D Fourier domain. The filter applied had a power spectrum of the form

$$P_{vel}(k) = \frac{a^2}{(1 + k^2 a^2)^2}$$

where $k_r^2 = k_x^2 + k_z^2$ is the radial wavenumber and the correlation distance $a = 100$ m. This distribution of velocity perturbations is similar to the exponential and self-similar distributions studied by Frankel and Clayton (1986); at high wavenumber, the power of
the filter used here rolls off as $k_r^{-4}$, whereas the self-similar and exponential distributions roll off as $k_r^{-2}$ and $k_r^{-3}$, respectively, making this distribution somewhat smoother than exponential. The important property of the velocity variations for this study, however, is that they are radially symmetric — that is they have the same correlation properties in all directions.

Velocity fluctuations for various power distributions are shown in Figure 3-2, where a single grid of uniform random numbers has been filtered according to four different power distributions (self-similar, exponential, Gaussian, and the filter described in eqn 3.7). Note that the fine-scale variation of the fluctuations decreases as the slope of the distribution increases. The anomalies produced by the Gaussian filter are generally smaller than those from (3.7) because the Gaussian distribution has more energy at normalized wavenumbers between 1-5 (see Figure 3-2).

The smoothed velocity perturbations in Figure 3-1 were scaled to have a standard deviation $\sigma$ equal to 0.5% of the 6 km/s average. For the particular realization used here, velocity in the random zone has extreme values of 5866 and 6122 km/s ($\pm 4\sigma$). The extremely low magnitude of velocity variation was chosen to eliminate effects caused by wave propagation through a laterally heterogeneous medium and the effects of multiple scattering. The correlation distance $a=100$ m was chosen so that the velocity variations fulfilled the condition $ka=1$ for a frequency of 10 Hz.

Synthetic seismograms were calculated with the same fourth-order acoustic-wave finite-difference algorithm used in Chapter 2. Again Clayton and Engquist (1977) A2 absorbing boundary conditions were specified on all edges of the grid. To produce CMP data gathers, source points were located at intervals of 400 m from positions of 15 to 25 km on the model. For each shot point, seismograms were calculated to a maximum time of 7 s at a time step of 1.731 ms (75% of the maximum possible time step dictated by the stability limit of the differencing calculations). Traces recorded at each grid point were
resampled to an interval of 4 ms. Calculations for a single shot required about 4 min total CPU time on an NEC SX-2 supercomputer located at the Houston Area Research Center in The Woodlands, Texas. Selected traces from the 26 shots were sorted into CMP gathers and processed as described in later sections. In the CMP study, traces were selected from the grid at 100-m intervals to produce a midpoint interval of 50 m.

The source wavelet has the form of the derivative of a Gaussian pulse and an amplitude spectrum constructed so that frequencies above 40 Hz were attenuated by 20 dB relative to the peak frequency of 14 Hz. At 40 Hz, the grid size of 25 m samples at least 5 points per wavelength at the slowest velocity in the random zone.

Before discussing the CMP-stacked section, we will first consider the primary reflectivity of the random zone and its relation to the zero-offset section.

PRIMARY REFLECTIVITY AND THE ZERO-OFFSET SECTION

In relating the correlation properties of a reflected wavefield to the target structure, we must first consider how the velocity structure of the target relates to seismic reflectivity. In this section I demonstrate that a simple, locally 1-D model adequately describes the reflectivity measured by a zero-offset experiment. We can further show that the lateral correlation properties of reflectivity are the same as those of the underlying velocity variations, if the variations are small.

The reflectivity model of the subsurface will be composed of a series of 1-D, primaries-only, vertical-incidence, synthetic seismograms computed separately for each column of the finite-difference velocity model. I will refer to this collection of seismograms as the "primary reflectivity section" or PRS. The PRS is a standard conceptual model of subsurface reflectivity, and is often regarded as an ideal seismic image
(Claerbout, 1985). The PRS for the velocity model in Figure 3-1 is shown in Figure 3-3, with traces displayed at 50-m intervals.

Each trace in Figure 3-3 is computed from velocity values in the corresponding column of Figure 3-1 as follows: The normal-incidence reflection coefficients are calculated for each interface in the column (layer thicknesses 25 m) and assigned the integrated two-way traveltime to the interface depth. The irregularly-spaced reflection coefficient sequence is interpolated to a constant sampling interval of 4 ms by distributing each coefficient linearly to the nearest sample points. The sequence is then convolved with the source wavelet, which was measured in a separate finite-difference run (not shown).

The PRS can be related to the zero-offset (and thus the CMP-stacked) section through the idea of an exploding reflectors experiment (Claerbout, 1985). In such an experiment, the surface-recorded wavefield is calculated by taking each scattering element to be a point (or line) source with strength proportional to the local reflection coefficient. All sources are activated at time zero and the composite wavefield is recorded at desired surface locations. To adjust for two-way travel, medium velocities are halved or the time axis stretched by a factor of two.

Note that in a laterally heterogeneous medium, the reflectivity seen by seismic waves will certainly be more complex than that described by the locally-layered structure above. For instance, the PRS does not characterize the angular dependence of reflection coefficients. Such effects are usually neglected in simulations of CMP experiments, however, and a test with the synthetic data (below) shows the exploding reflectors experiment to be an adequate representation of subsurface reflectivity for these small-magnitude velocity variations.

The exploding reflectors experiment was simulated for this velocity model by processing the PRS with the modeling technique of inverse Stolt migration. Stolt migration recovers subsurface reflectivity from zero-offset input data by a stretching and scaling
operation in the 2-D frequency domain (Stolt, 1978). Using the PRS as input data, and reversing the frequency domain operations, produces a synthetic unmigrated zero-offset section, shown here in Figure 3-4a. The trace spacing in Figure 3-4 is 50 m, and for clarity, only a limited portion of the section is shown. To avoid stretching artifacts, the broadband PRS was inverse migrated and then the source wavelet used in Figure 3-3 was convolved with each output trace.

The zero-offset section produced by finite-difference modeling is shown for comparison in Figure 3-4b (again, 50-m trace spacing). Since shot points were located at 400-m intervals, traces in between those with true zero offset were produced by applying normal moveout (NMO) corrections to near-offset traces (maximum source-receiver offset 400 m). A few finite-difference test shots (not shown) located at 50- and 100-m intervals confirm that the NMO-corrected traces show no detectable differences with those produced at true zero offset.

The sections in Figures 3-4a and 3-4b match each other closely, particularly at times above 2.5 s. As expected, differences are more obvious at late times, but at all levels, events as short as 3-4 traces can be correlated between the two. The exploding reflectors concept and the PRS provide an acceptable representation of the seismic experiment because this model has small velocity variations. Thus, single scattering interactions are likely to dominate, and ray-bending effects are small. Some differences are noticeable at depth, however, since late arrivals in the finite-difference experiment come from a wavefield more complex than that modeled by exploding reflectors.

Accepting the PRS as a reasonable measure of subsurface reflectivity, let us now compare its lateral correlation properties with those of the velocity variations in the random zone. For reference, detailed views of the PRS and the random velocity zone are shown in Figure 3-5; these sections correspond to the same spatial window displayed in Figure 3-4.
For small-magnitude velocity fluctuations, the process of computing the broadband PRS can be approximated by taking a vertical derivative of the velocity field and rescaling $k_z$ to frequency $\omega$ (Sheriff, 1980). Assuming a constant background velocity $V$, the conversion from time to depth can be done first, and the derivative can be represented as multiplication by $i\omega$. Thus, the $\omega \cdot k$ power spectrum of the PRS can be written as,

$$P_{prs}(k_x,\omega) \propto \omega \cdot P_{vel}(k_x,\omega), \quad (3.8)$$

where $\omega = 2k_z/V$ and $P_{vel}$ is given in (3.7). Convolution with the source wavelet can also be represented by multiplication in the frequency domain. Given a source amplitude spectrum $W(\omega)$, the bandlimited PRS has the power spectrum

$$\tilde{P}_{prs}(k_x,\omega) = \omega \cdot W(\omega) \cdot P_{vel}(k_x,\omega), \quad (3.9)$$

where the constant of proportionality is absorbed in $W$.

The discussion above reveals that the conversion of the velocity field to primary reflectivity amounts to a filtering operation along the time axis, once the vertical axis is converted from depth. The assumption of constant background velocity means that the same filter is applied to each column of the model; operating on each column separately means no filtering is applied in the horizontal direction. Foster and Guinzy (1967) show that the same filter applied to both signals in a coherency measurement leaves the coherency values (3.1) unchanged. Thus we expect the PRS to have the same lateral correlation properties as the velocity model when we compute the lateral coherence described in (3.5).

Figure 3-6 shows coherence values as a function of trace separation for the velocity model and the PRS, calculated for a frequency of 10 Hz. These estimates are made for a set of 80 traces from the middle of the model (trace spacing 50 m) and the values were
calculated over a vertical window that spans the random zone (1.333–4.000 s vertical traveltime). Also shown in Figure 3-6 are theoretical 10-Hz coherence values for the velocity model, computed from the power spectrum in (3.7). Coherence values for velocity were calculated by fixing $k_z$ in (3.7) at the equivalent of 10 Hz and then performing an inverse Fourier transform over $k_x$.

First note that the coherence values for the velocity model match the theoretical values almost exactly, as expected. Coherence values for the PRS closely track those of the velocity model, but are slightly lower. This difference can be attributed to the fact that reflection times in the PRS were calculated according to the true velocity variations, rather than with a constant velocity assumption. Thus, the reflection time to a particular depth varies slightly from trace to trace in the PRS and this causes the coherence to drop. For small variations in velocity, however, we will assume that the coherence of the velocity model and the PRS are the same.

Before describing the CMP stacked data, let us return briefly to the data in Figure 3-5. On closer inspection, these plots of the PRS and velocity model seem to contradict the results of Figure 3-6 and the analysis above. While the velocity anomalies in Figure 3-5b do not appear to have any preferred spatial orientation, the reflection response in the PRS is characterized by numerous subhorizontal event segments. In some cases (e.g., just above 2 s), segments are aligned to suggest an event that can be correlated laterally over about 4 km. The PRS appears to have greater horizontal continuity than the velocity model, although the coherence measurements (Figure 3-6) show that the PRS actually has slightly less.

This apparent contradiction can be resolved by examining the power spectra of (3.7), (3.8), and (3.9), which are plotted in Figure 3-7. The $\omega$-$k$ spectrum of the velocity variations (Figure 3-7a) shows the circular symmetry that is necessary for the fluctuations
to be isotropic. Note here that Figure 3-7a shows the spectrum of the smoothing filter used to produce the random zone (3.7) and not a sample spectrum of the actual velocities.

The spectrum of the broadband PRS (Figure 3-7b) lacks circular symmetry, however, because of the derivative with respect to time (see equation 3.8). The spectrum in Figure 3-7b shows a progressive attenuation of power as spatial direction departs from the horizontal. That is, the process of converting velocity to primary reflectivity involves a filtering that suppresses steeply-dipping components of the fluctuations. This dip filtering results from the asymmetric treatment of differentiating the vertical direction but not the horizontal.

The attenuation of steeply-dipping components can be seen again after the source wavelet is convolved (Equation 3.9, Figure 3-7c). The consequence of the dip filtering evident in Figures 3-7b and 3-7c is an apparent increase in the continuity of sub-horizontal events in Figure 3-5a. The results of Figure 3-6 emphasize that the increase is only apparent: The time derivative cannot increase lateral continuity (i.e., decrease bandwidth in $k_x$), since it has exactly the same effect at all values of $k_x$. The derivative has instead decreased vertical correlation by increasing vertical bandwidth (through the multiplication by $\omega$). The relative change of bandwidths causes the visual impression of lateral continuity.

In fairness, I should note that the differences between Figures 3-5a and 3-5b are accentuated by the limited dynamic range of a black and white seismic display. Thus, segments in Figure 3-5a are dip-filter enhanced consequences of real velocity variations that are simply difficult to see in Figure 3-5b. To confirm that the variable area display format was not seriously biasing the plotted images, the section was also transposed and replotted (results not shown). In the transposition, rows of samples on the section were converted to traces (columns), so that after plotting at the same 1:1 scale, the plot had to be rotated 90° for comparison. Horizontal continuity in the transposed section was the same as seen in
Figure 3-5a, and so the shading of positive peaks can be rejected as a significant source of bias.

The dip-filtering effect described above has important consequences for the interpretation of any surface-recorded seismic section. To the extent that a conventional reflection survey measures vertical reflectivity, scattered events from isotropic variations will have a bias toward subhorizontal alignment. This effect, which is not accounted for in qualitative interpretations, will slightly enhance the impression of layering in structures at all depths. This bias in conventional reflection images is a result of surveying geologic targets only from above; this result is well-known in seismic tomography, where coverage of a target from all directions is required to produce an optimally-resolved image (Wu and Toksöz, 1987).

The bias of surface recording can be corrected with an integration along the time axis. Such an integration has been widely implemented in the exploration technique of forming a pseudo-velocity log from processed seismic data (Lindseth, 1979). The pseudo-log technique makes the same assumptions about reflectivity used above and simply reverses the differentiation in (3.8) to estimate reflection coefficients. Another approach to estimating velocity structure is iterative forward modeling as discussed by Gelfand and Larner (1984); this method has the advantage of explicitly including the effects of a source wavelet (see Equation 3.9).

As shown in the next section, the bias of surface recording is small relative to the smearing effects introduced by the CMP stacking dip filter.

QUANTITATIVE ANALYSIS OF DIP FILTERING IN THE CMP STACK

The first three shots in the finite-difference CMP survey (source locations of 15.0, 15.4, and 15.8 km) are shown in Figure 3-8. Traces in these gathers have a group interval
of 100 m and the maximum source-receiver offset has been restricted to ±5 km; the direct wave has been muted and the data are plotted with no time-variant gain. Events reflected from the random zone appear as a complex pattern of short segments with the steeply-dipping limbs of hyperbolic scattering (aka, diffraction) patterns evident below 2 s. At zero-offset, primary reflections from the random zone arrive between 1.333 and 4.000 s; the termination of primary reflections from the random zone is obscured by energy scattered from the side.

Traces from the 26 synthetic shots were sorted to produce CMP gathers with a midpoint spacing of 50 m and a nominal fold of 6 (7 where a zero-offset trace is present). In all processing, the direct wave was muted and the traces were filtered at 5-40 Hz to suppress any residual spurious energy. CMP gathers were NMO-corrected using the average medium velocity of 6 km/s and then stacked.

The CMP stack corresponding to Figures 3-1 and 3-3 is shown in Figure 3-9. In this plot, and all other seismic plots in this chapter, a time-variant gain proportional to \( \sqrt{t} \) has been applied to compensate for geometric spreading. In Figure 3-9, the reflection response of the random zone is again characterized by numerous short segments, but in this stack, the segments have a preferred orientation near horizontal (as seen in many stacks of field data and in the synthetic data of Chapter 2). Note that the reflection response in Figure 3-9 is somewhat more smeared near the top of the random zone than at the bottom, and again the lower limit of the zone is masked by scattered arrivals. The progressive change in reflection character with depth is discussed below.

As noted in Chapter 2, the appearance of sub-horizontal segments in the stack results from the attenuation of steeply-dipping energy. This effect is evident when the stack is compared to the zero-offset section. The more random appearance of the zero-offset section (Figure 3-10a), relative to the stack (Figure 3-10b), is clearly attributable to the presence of dipping scattered events. In these plots, and similar ones in this section, the spatial window
shown is the same as in Figures 3-4 and 3-5, and time samples have been plotted so that the sections have a 1:1 scale.

Conventional CMP stacking is generally applied to improve the signal-to-noise ratio of a data set and to produce a section that corresponds to a zero-offset experiment. The attenuation of dipping features is the most obvious way in which a stack fails to reach the desired zero-offset result and such attenuation limits the resolving power of CMP-stacked images (Berkhout, 1984). Interestingly, the loss of dipping scattered events in Figure 3-10b reduces the random appearance of the section and thus stacking appears to have accomplished half its goal. In this data set, however, all random events are signal, since they are the reflection response of the target being surveyed.

The filtering effect in Figure 3-10 occurs because the optimum stacking velocity for a given event depends on its dip (i.e., on the relative positions of the reflection point and the CMP). This dip dependence results in a well-known problem when a single velocity function must be chosen for NMO corrections: NMO corrections that properly align horizontal events will over-correct dipping ones. Subsequent stacking of the misaligned features effects the dip filter.

The over-correction problem is illustrated in Figure 3-11, which shows an unstacked data volume characterized by midpoint position, source-receiver offset, and reflection time. The larger face of the volume is the zero-offset section (in isometric perspective) and the smaller face is an NMO-corrected CMP trace gather. The CMP reflection response of individual events on the zero-offset section can be examined by correlating features at the corner of the volume. Such correlation confirms that events misaligned in the corrected gather correspond to dipping events in the zero-offset section. (Note that a maximum offset of 10 km has been plotted for this gather in order to make the misalignment obvious.) CMP stacking is accomplished by summation over the shorter horizontal direction.
Analysis of the moveout equation shows that the attenuation of steep dips increases with the ratio of maximum offset to target depth (Hale, 1984). The stacking dip filter is thus most severe for early reflection times and when large offsets are included in the stack. As expected, Figure 3-11 shows that the misalignment problem decreases with increasing reflection time, which explains the change of reflection response noted on the stacked section in Figure 3-9.

The time interval in Figure 3-10 represents depths of 4.5-10.5 km, with corresponding ratios of maximum offset to target depth being 1.11 and 0.48. Survey geometries and trace muting in typical exploration data usually result in maximum ratios near 1. Stacking data with such ratios is known to attenuate specular reflections with extremely steep dips (e.g., fault-plane reflections; Hale, 1984), and Figure 3-10 suggests that the equivalent effects on randomly heterogeneous targets will be moderate.

In crustal reflection and refraction surveys, long source-receiver offsets are acquired to provide deep velocity control. In some wide-angle surveys with dense receiver spacing, traces from long (>10 km) offsets have been stacked to image deep targets (Levander et al., 1987; Flueh and Okaya, 1987). To illustrate the influence of long offset traces, the synthetic CMP gathers were extended to include maximum offsets of ±10 km. Including the longer offsets increased the nominal multiplicity to 12. Gathers were processed as before and the resulting stack is shown in Figure 3-12.

The offset-to-depth ratios for Figure 3-12 range from 2.22 at the top to 0.95 at the bottom and the change in lateral continuity relative to Figure 3-10b is dramatic. In particular, compare the event at 1.9 s on the left of Figure 3-12 to equivalent features in Figures 3-10a and 3-10b. With 10-km offsets stacked, this event can be followed over 2 km laterally; the zero-offset section in the same area shows 10 separate segments, with the original stack having intermediate continuity. While an offset-to-depth ratio of 2 is large for most CMP reflection surveys of the deep crust, recent wide-angle surveys such as those
cited above can produce CMP-stacked data with that characteristic. Thus, the effects of stacking smear will be an important issue as the distinction between CMP reflection and wide-angle refraction surveys is reduced.

The stacking dip filter has been quantified by Bolondi et al. (1982). Their analysis uses geometrical optics and acoustic wave theory to produce a second-order partial differential equation that relates a constant-offset section (i.e., any plane parallel to the large face of Figure 3-11) to the zero-offset section. Under the assumptions of geometric ray theory, the mapping from one offset to another can be separated into standard NMO correction followed by a second timing correction that accounts for dip-dependent moveout. This analysis leads to an efficient pre-stack method of "offset continuation" that allows subsequent stacking to enhance events with a wide range of dips.

In their analysis, Bolondi et al. (1982) also derive a 2-D frequency domain filter that relates a conventional stacked section to a stack after processing with offset continuation. Within the assumptions of their technique, the stack with offset continuation is equivalent to a zero-offset section, and thus their filter will be used here to relate the results of Figure 3-10a to those in 3-10b.

The development of Bolondi et al. (1982) explicitly recognizes the time-variant aspect of the filtering effect; denoting the dip filter for reflection time T as $S(\omega, k_x; T)$, the relation between stacked and zero-offset sections is written

$$P_{stk}(\omega, k_x) = P_{zo}(\omega, k_x) \cdot S(\omega, k_x; T) \quad \text{(3.10)}$$

where, $P_{stk}$ and $P_{zo}$ are the 2-D Fourier transforms of the stacked and zero-offset sections. $S$ is computed from the integral expression
\[ S(\omega, k_x; T) = \frac{1}{\phi_{\text{max}}} \int_0^{\phi_{\text{max}}} \exp(-i\phi^2) \, d\phi \]  

(3.11)

with

\[ \phi = \frac{k_x h}{\sqrt{2\omega T}} \]  

(3.12)

If the maximum source-receiver offset is \( 2h_{\text{max}} \), the upper limit of integration in (3.11) is given by (3.12) with \( h=h_{\text{max}} \). The integral in (3.11) amounts to a summation over offset (i.e., stacking), with the integrand representing dip-dependent phase factors (i.e., dip moveout time shifts).

The integral (3.11) was evaluated numerically for a reflection time of 3 s and the maximum offset of 5 km, and the results plotted in Figure 3-13. This plot provides a quantitative measure of how energy in the stack is progressively attenuated with increasing dip. Another description of the dip-filtering effect is that, at any given frequency, the loss of steep dips decreases spatial bandwidth. In the next section I show the corresponding increase in lateral correlation. I further explore the quantitative relationship between spatial bandwidth and lateral correlation in the next chapter, for both zero-offset and common-shot geometries.

To demonstrate the stack filter and to test the assumptions made in the development of Bolondi et al. (1982), \( S(\omega, k_x; T) \) shown in Figure 3-13 was applied to the zero-offset data of Figure 3-10a. The filtered section (Figure 3-14b) closely approximates the results of stacking the actual CMP gathers (Figure 3-10b). Events in Figures 3-10b and 3-14b are easily matched, especially within 500 ms of the design time (i.e., from 2.5-3.5 s). At earlier times, the detailed character of equivalent events differs because the dip filter applied
in Figure 3-14b is not severe enough. Overall, Figures 3-10b and 3-14b show that $S(\omega,k_z;T)$ accurately characterizes the filtering effects of the CMP stack.

MIGRATION AND LATERAL CORRELATION IN THE SEISMIC IMAGE

Let us now consider how migration affects the lateral correlation observed in the wavefield reflected from the random zone. As noted in Chapter 2, migration increases lateral resolution (Berkhout, 1984) and is an essential process for imaging zones of lateral heterogeneity. The effect of migration on the data presented here can be quantified by computing lateral coherence functions.

Lateral coherence values for the unmigrated zero-offset and stacked sections (Figure 3-10) are plotted in Figure 3-15 (frequency of 10 Hz). In estimating these values, the data windows shown in Figure 3-10 (101 traces) were tapered over 300 ms at each end and correlations were computed to lags of ±150 ms. For reference, the coherence function for the velocity variations (shown in Figure 3-6) is plotted as a dashed line. Note that coherence in the zero-offset section (closed circles) is similar to that of the the velocity model, except for a prominent sidelobe near trace separations of about 5. Coherence in the zero-offset data would be greater if the target were deeper; this effect will be discussed in detail in the next chapter. Smearing in the CMP stack is manifested by coherence values (open circles) that are higher than those of the zero-offset data. We generally expect migration to improve the quantitative estimates of lateral coherence at the target by focusing the surface-recorded wavefield and increasing lateral resolution. In this example, the zero-offset section is only slightly more coherent than the target, but migration will still improve the match, as shown below.

The migration process can be couched in terms of a deconvolution that increases spatial bandwidth (Berkhout, 1984). As noted in Chapter 2, however, migration cannot
restore the bandwidth lost to smearing in the stacking process. This fact is illustrated by the results of migrating the zero-offset and stacked sections. The results of Stolt (1978) migration of these data sets are shown in Figures 3-16a and 3-16b.

Before examining quantitative measurement of coherence in these data, note that the reflection character of the migrated zero-offset section (Figure 3-16a) closely matches that of the PRS (Figure 3-5a); individual features as small as 3-5 traces in length can be matched between the two sections. Some high frequencies have been lost in the migration as a result of the steepening of dipping energy (Stolt, 1978), but the result in Figure 3-16a confirms that migration of zero-offset data recovers the primary reflectivity of the medium. In contrast, the migrated stack (Figure 3-16b) has greater lateral continuity attributable to uncompensated stack smear. The same results are obtained when migration is performed with a $45^\circ$ finite-difference algorithm (Figure 3-17). The results in Figure 3-17 show that for this scale of lateral heterogeneity, finite-difference migration produces a sufficiently accurate image of the random zone.

Coherence functions for the Stolt migrations (Figure 3-16) are shown in Figure 3-18, with coherence of the velocity model again plotted for reference. Note that values for the migrated zero-offset section follow the trend of coherence for the model even more closely than in Figure 3-15. In contrast, values for the migrated stack remain higher than the model by intervals of 0.1 to 0.4. Note there is a slight overall increase in coherence levels of the migrated data at trace separations greater than 5 relative to the unmigrated results; this effect may be attributed to mild edge effects in the limited aperture of data processed.

The results in Figures 3-10, 3-15, 3-16 and 3-18 argue that the accurate quantitative assessment of lateral heterogeneity will require some form of migration before stack (MBS). In true MBS techniques, energy is migrated to its proper spatial position first; subsequent stacking can be done with a velocity function that does not depend on dip. Thus the dip-filtering effects of conventional stacking are avoided and the full spatial bandwidth
is preserved. While true MBS techniques provide superior accuracy over conventional post-stack migration, they are also expensive and require detailed velocity information at an early stage of processing (Yilmaz, 1987).

Numerous alternative approaches have been developed in order to address the practical problems of migration before stack (Yilmaz, 1987). Instead of migrating the large volume of unstacked data, alternative techniques improve the stacked section by passing a wider range of dips. With full spatial bandwidth preserved, the stack is closer to the desired zero-offset section, and accurate post-stack migration can be applied to improve lateral resolution. Such approaches have the advantage of allowing refined velocity estimation before migration and they produce an unmigrated stack in cases where that section aids interpretation (Yilmaz, 1987).

The offset continuation technique of Bolondi et al. (1982) is one example of these techniques, known generically as "pre-stack partial migration". Most current implementations of pre-stack partial migration are variants of the "dip moveout" (DMO) correction described by Hale (1984). DMO processing amounts to a more accurate version of offset continuation: NMO-corrected common-offset sections are transformed to zero offset by a frequency-domain transformation that accounts for dip-variable moveout. After residual velocity estimation, the DMO-corrected data are be stacked and then migrated. Migration is typically done with techniques such as the Stolt (1978) algorithm, so that a maximum range of dips can be preserved.

To demonstrate the importance of maintaining full spatial bandwidth in the stack, the stacking filter $S(\omega,k_x)$ can be removed with a post-stack deconvolution. Figure 3-14a shows the results of applying the inverse filter $S^{-1}(\omega,k_x;T)$ to the stacked data (Figure 3-9). Recall in this example that the filter $S$ is calculated for a reflection time of 3 s and a maximum offset of 5 km. In Figure 3-14a, steeply dipping energy has been restored and the data have a reflection character similar to that of the zero-offset section in Figure 3-10a.
Events in Figures 3-10a and 3-14a can be correlated near the design time of 3 s; at earlier times, the post-stack filtered section shows greater continuity because the inverse filter has not restored the full spatial bandwidth.

The processing in Figure 3-14a is similar to a post-stack technique described by Bale and Jakubowicz (1987). Their approach treats the time-variant nature of the filter with a transformation of the time axis and they apply a filter based on an analysis similar to that of Bolondi et al. (1982). The effect of noise on post-stack deconvolutions of this kind remains unanswered. In producing the results in Figure 3-14a, 0.01% white noise was added to the filter S before performing the frequency-domain division. Adding white noise protects against instability in the filter (division by small values) but it also limits the output spatial bandwidth. The small amount of white noise used here (and by Bale and Jakubowicz, 1987) has no significant effect on the final bandwidth. Noise in field-recorded data, however, may seriously compromise the ability of post-stack deconvolution to restore steep dips. Thus, the pre-stack processing of DMO appears to be preferable, since subsequent stacking can help mitigate noise problems. The results in Figure 3-14a suggest that DMO processing will provide a close approximation to the zero-offset section, at least for small random variations in velocity.

To complete the example, the data filtered post stack with $S^{-1}(\omega,k_x,T)$ were migrated with the Stolt algorithm used above and the results plotted in Figure 3-19. This section has a general reflection character similar to the migrated zero-offset result in Figure 3-16a; since migration has moved scattered events generally higher in time, the detailed comparison of events is best between 2-3 s. Figure 3-20 shows lateral coherence values for the filtered stack before and after migration. Note that both the unmigrated and migrated results closely track coherence values for the velocity function, except for a sidelobe in the migrated results above separations of 5. The agreement between the processed results and the model
is particularly good considering the assumption of constant traveltime made in this inherently time-variant problem.

FIELD DATA EXAMPLE: CENTRAL VALLEY, CALIFORNIA

The following field data example is presented to demonstrate the effects of dip filtering in the CMP stack and to show the use of the lateral coherence function in assessing the lateral continuity of reflection events. Since this data set was recorded over a thick sedimentary section, it provides examples of both laterally continuous and discontinuous reflectors. Comparing coherence measurements for different reflections allows us to address the basic interpretation question raised in the introductory chapter: we find that the coherence function for a laterally-constant reflector contaminated by noise is fundamentally different from that of a laterally-variable reflector.

These field data are part of a contractor's speculative survey that was purchased by the U. S. Geological Survey for use by the Central California Deep Crustal Studies Group. The data were acquired in the Central Valley, California, using seismic vibrators with sweep frequencies of 10-58 Hz and a maximum record time of 6 s. The original field records were subsequently recorrelated with a truncated version of the pilot signal to produce records having a maximum time of 12 s and an effective sweep of 10-44 Hz. For each vibrator point, 96 channels were recorded at a group interval of 110 ft (33.54 m). A vibrator point interval of 220 ft results in CMP trace gathers with a nominal multiplicity of 24 spaced at intervals of 55 ft. The entire data set was reprocessed at Rice by Anne Meltzer and is described in Meltzer (1988); a standard land data processing sequence including field and residual static corrections and automatic gain control was applied to the data before stack. Zone A corresponds to an approximate depth interval of 4800-7400 ft (1460-2250 m) and Zone B to an interval of 7400-10600 ft (2250-3230 m).
A portion of the stacked section is shown in Figure 3-21. In this analysis, we will consider the reflection response in two time windows, hereafter referred to as Zones A and B. In Zone A (1450 - 2000 ms) reflection events are laterally variable, with typical segment lengths of 600 - 1200 ft. In Zone B, the reflection response is dominated by an event between 2300 - 2400 ms; this strong event and the other reflections in Zone B are laterally consistent, exhibiting only gradual phase changes across the window shown.

Geologically, events in Zone A have been interpreted as reflections from members of the Etchegoin Group and events in Zone B as reflections from within the Monterrey Fm. (Meltzer, 1988). The Etchegoin Group consists of siltstones, sandstones, and conglomerates, deposited in bay and esturine (tidal to sub-tidal) settings (Loomis, 1988). In outcrop, the Etchegoin shows sand- and conglomerate-filled channels; interpreted as tidal creeks and channels, these features have vertical scales of feet to tens of feet and lateral scales on the order of hundreds of feet (Loomis, 1988). In contrast, sand and shale members of the Monterrey Fm. were deposited in deeper marine settings (Williams, 1988), and therefore can be expected to have much greater horizontal continuity. Thus, geologic studies of these formations in outcrop are consistent with the reflection character seen in Figure 3-21. The magnitude of velocity variations in the two zones was estimated from a sonic log taken in a well near the line (Bass-Westhaven 54, provided by Anne Meltzer; see Meltzer, 1988). In both zones, the standard deviation of velocities measured at 10-ft intervals was about 7-8% of the average.

In the sections below, I will characterize events in Zone B as laterally invariant and will refer to Zone B as a region of "specular" reflectors. Here, the term "specular" is used in an informal sense to describe a reflecting horizon that is essentially planar and has a reflection character that is approximately constant along its lateral extent.

In the synthetic data examples shown before, stacked data have been compared to zero-offset sections. Since zero-offset traces are not recorded in this field data set, we will
use NMO-corrected common-shot gathers to study the unstacked reflection response of these two targets; after NMO correction, a common-shot gather can be viewed as an approximate zero-offset section. Six adjacent shot records were selected so that their source-receiver midpoints cover the CMP range of interest (2750-2850).

An NMO-corrected shot record (source position at CMP 2762) is shown in Figure 3-22, along with a comparable portion of the CMP stack from Figure 3-21. All data have had automatic gain control (AGC) applied, using a 500 ms gate. The shot gather has been muted to retain the widest possible spatial coverage for each zone; the standard mute applied before stacking allows a maximum source-receiver offset of 5400 ft for Zone A and 8000 ft for Zone B. Maximum source-receiver offsets in this data set are small enough that the coherence effects described in the next chapter can be ignored. Reflection amplitude does not vary consistently with offset in either zone; the maximum angles of incidence for Zones A and B are estimated to be 36° and 32°, respectively, based on an interval velocity model determined from stacking velocities.

In comparing the shot record to the CMP stack, the effects of the stacking process are clear. The stacked data are more laterally continuous and show less evidence of random variation than the unstacked data. Lateral coherence functions for the unstacked data in Zones A and B (frequency, 20 Hz) are shown as closed triangles in Figures 3-23a and 3-23b. The coherence functions shown are averages of individual functions calculated for the six shot records. Individual functions in each zone were calculated for the indicated time intervals, using a 50 ms taper on each end of the window and computing correlation functions to a maximum lag of ±150 ms. The frequency of 20 Hz was chosen because the signal-to-noise ratio there was better than at the 10-Hz frequency used in previous plots (10 Hz is at the edge of the swept band).

For Zone A (Figure 3-23a), the coherence function decreases rapidly at midpoint separations of 0 - 5, and then maintains a more gradual decrease to the maximum calculated
separation of 16. The function for Zone B (Figure 3-23b) also decreases markedly at midpoint separations less than 5, and then maintains a roughly constant level near 0.5 at larger separations.

Lateral coherence functions for equivalent portions of the stacked sections are plotted as closed circles in Figure 3-23a and 3-23b. As expected from a visual examination of Figure 3-22, stacked coherence values for both zones are uniformly higher than equivalent values for the unstacked data. The stacked coherence function for Zone A (Figure 3-23a) shows the same general decrease with separation observed in the unstacked case. For Zone B, the stacked coherence function (Figure 3-23b) is almost constant at values of 0.95-1.00.

Let us now consider the lateral coherence functions in light of the basic interpretation issue raised earlier. The stacked coherence function for Zone B (Figure 3-23b) confirms the visual evidence on the stacked section; the value near 1, essentially constant with midpoint separation, is the result expected for a specular reflector. For Zone A, one might question whether the discontinuous reflection response results from noise contamination or from heterogeneity at the target. The steady decrease of coherence observed in Figure 3-23a, argues that the reflector is laterally variable over the midpoint range shown. If the response in Zone A were that of a specular reflector contaminated by noise we would expect the coherence function to reach some constant level as seen in the results of Zone B (Figure 3-23b). This simple interpretation suggests that the lateral coherence function provides a useful discriminant between noise contamination and target heterogeneity.

*Stacking Smear and Lateral Coherence*

The interpretation of lateral coherence values is, however, complicated by the effects of stacking smear. While stacking improves the signal-to-noise ratio (SNR), the synthetic data studies in this chapter show that stacking smear will alter the apparent correlation
properties of the wavefield. Conversely, the coherence values measured on unstacked data are not affected by smear, but these data are contaminated by higher levels of noise. In this section, I will demonstrate how stacking smear affects stacked coherence values in Figure 3-23a. In the next section, we then consider how to correct unstacked coherence values for the presence of noise and compare the results to those measured on the stack.

The rise in coherence values after stack is conventionally attributed to noise reduction in the stacking process. The standard theory of the CMP stack (Yilmaz, 1987) predicts that stacking N traces will improve the SNR by a factor of \( \sqrt{N} \). As discussed in Appendix D, SNR can be determined from coherence values under certain assumptions, and thus the theoretical effects of noise reduction can be calculated. In Zone B the average CMP multiplicity is 22 (reduced from the nominal 24 by the mute pattern); in Figure 3-23b, the unstacked coherence function for Zone B was adjusted for the effects of a 22-fold stack, using expressions developed in Appendix D. The resulting coherence values match those measured from the stacked data almost exactly. Thus for the specular reflector (the standard assumption of the stacking process) the process of stacking works as expected.

In assessing the relation between stacked and unstacked data in Zone A, however, the effects of stacking smear become evident. When the unstacked coherence function for Zone A is adjusted for the (13-fold) stacking process, the resulting coherence values are significantly higher than those measured on the stacked section (see Figure 3-23a). For reference, adjusted values for 2-fold and 4-fold stacking are also plotted in Figure 3-23a, indicating that the trend of the stacked result does not follow the general predictions of stacking theory.

The discrepancy in Figure 3-23a can be resolved by the application of the stack smear operator described in (3.11). Figure 3-24 shows the results of applying \( S(\omega, k_x, T) \) to the shot record in Figure 3-22, for values of T corresponding to the center of Zones A and B (\( T = 1.7 \) and 2.3 s, respectively). With the resulting increase in lateral continuity and
decrease in random noise, the filtered shot records look more like the stacked section (Figure 3-22) than before. The increased continuity caused by the stacking filter was measured by computing lateral coherence functions for each zone on the six filtered shot records and averaging the results.

The average coherence functions for the filtered shots are shown as open triangles in Figures 3-25a and 3-25b; for reference, the coherence functions for the stack and unfiltered shots (Figure 3-23) are repeated in Figure 3-25. Application of $S(\omega,k_x)$ has increased the coherence values somewhat, but the values in both cases are still lower than those measured from the stacked data. The difference in overall level of the coherence values is attributable to the fact that $S(\omega,k_x)$ has not provided as much reduction of the random noise level as does the stacking process.

Since application of $S(\omega,k_x)$ at a fixed frequency amounts to a spatial convolution, the noise reduction attributable to this process can be calculated from the spatial width of the filter. Recall that spatial convolution amounts to a spatial mixing of traces and is thus similar to the process of stacking. A standard measure of filter length (equivalent to the second moment of the function; Berkhout, 1984) shows that $S(\omega,k_x)$ is effectively 4.2 traces wide for the $T = 1.7$ s filter, and it is 5.8 traces wide at $T = 2.3$ s. Thus, the expected SNR improvements attributable to $S(\omega,k_x)$ are $\sqrt{4.2}$ and $\sqrt{5.8}$ for Zones A and B, respectively. To match the SNR improvement of the actual stacking process, then, we need to adjust the filtered coherence results for a further SNR improvement. The parameters for that improvement amount to a further 3-fold stack for Zone A and a 4-fold stack for Zone B.

After corrections for the reduction of random noise, the coherence values predicted for the stacked section match the measured values almost exactly. Particularly for Zone A (Figure 3-25a), the predicted values track the measured ones at all midpoint separations. This result and the poor match of the conventional stacking theory in Figure 3-23a confirm
the importance of stacking smear in the interpretation of lateral coherence. For Zone B (Figure 3-25b), note that the predictions that include stack smear match the measured values as well, although in this case the standard model of a specular reflector is met and the smearing operator is not necessary to explain the observations.

**Interpretation of Unstacked Coherence Values**

Let us now consider how to assess the lateral coherence of the reflected wavefield using coherence values measured on the unstacked data. While not affected by stacking smear, the unstacked values are clearly affected by noise. In this section, I show how to correct the unstacked values for the presence of noise and compare the corrected results to those observed on the stack.

The unstacked coherence values in Figures 3-23a and 3-23b reveal that the noise field is composed of both uncorrelated and spatially-correlated components. Spatial correlation in the noise is inferred from Figure 3-23b, where the specular reflector should have a constant value of coherence at all trace separations other than zero. Since we observe that the reflections in Zone B appear specular (Figure 3-21), and the relationship between the coherence of unstacked and stacked data confirms this fact, we will assume that the rise in coherence between trace separations of 1 and 5 (Figure 3-23b) is attributable to spatial correlation in the noise. This change of trend is also observed at the same trace separations in the coherence values for Zone A (Figure 3-23a). Based on the model of near-surface scattering discussed in Chapter 2 we expect spatial correlation in seismic noise that is generated by the scattering of upcoming reflections. Coherence measurements at late reflections times on the shot records (not shown) confirm existence of spatial correlation in the noise; the measured coherence functions have the same general form as the estimated function discussed below.
In this section we will estimate the noise coherence by assuming that the reflectors in Zone B are specular and that deviations in that lateral coherence function are due to noise correlation. With these assumptions, relation (D9), developed in Appendix D, is used to estimate the lateral coherence function of the noise. The estimated coherence is shown in Figure D-1 (Appendix D).

Given lateral coherence values for the noise, the effect of noise correlation can be removed from the unstacked coherence values in Figure 3-23a using expressions (D11-D14). The correction is made assuming an overall SNR of 1 for these data (as discussed in Appendix D), and the results are plotted in Figure 3-26 (along with the stacked and unstacked values from Figure 3-23a). Note that coherence values at trace separations of 1-5 now follow the trend defined at larger trace separations (compare with the original unstacked values).

Note also that the lateral coherence function can be extrapolated to an intercept of 0.65 at zero trace separation, and this intercept value defines the basic SNR (=1.36, using D4) for these data. At zero trace separation, reflected signal will be perfectly correlated and the intercept value represents the ratio of correlated power to total power on the trace (See Appendix D; measured values at zero separation will always be 1, since only one trace is available for analysis).

This interpretation of the intercept value can be made only after correcting for correlated noise. Note that the correction does not change the total noise power; correlated noise is simply replaced by uncorrelated noise. After this adjustment, the lateral coherence function should represent only the correlation properties of the reflected signal. The intent of the correction is to remove the trend caused by noise correlation, so that trend is not attributed to the correlation properties of reflected signal.

Given the SNR defined by the intercept coherence, the effects of uncorrelated noise can be removed using (D19). The resulting lateral coherence function (shown in Figure 3-
estimates the lateral correlation of reflected signal if that signal could be observed in a noise-free zero-offset section. Note that the estimated noise-free values are consistently lower than those measured on the stack, due to the effects of stacking smear.

Overall, coherence values are consistent with the scales of lateral heterogeneity reported in the geologic studies mentioned earlier (Loomis, 1988). Differences between the adjusted unstacked values and those for the stack (Figure 3-26) are not, however, as dramatic as seen in the previous synthetic example. This result can be explained by the fact that the heterogeneities in Zone A are not isotropic; as described by Loomis (1988), reflecting structures within the Etchegoin Group are generally lenticular, having much greater continuity laterally than vertically. Thus, the effects of stacking smear will be less apparent for these targets. As discussed in the concluding chapter, the geologic interpretation of lateral coherence values requires further study. That is, the lateral coherence of various geologic targets needs to be measured in outcrop (and possibly borehole) studies in order, 1) to determine to what extent lateral coherence is diagnostic of particular geologic settings, and 2) to assess the significance of differences such as those observed here between stacked and unstacked coherence values.

CONCLUSIONS

The examples above demonstrate a quantitative approach to understanding the effects of CMP stack smear. The evaluation of the stack filter S allows the loss of steep dips to be considered in terms of reduced spatial bandwidth that corresponds to increased lateral correlation in the stack relative to the desired zero-offset section. The maximum source-receiver offset in the CMP trace gather is a primary factor controlling the amount of stack smear, and should be carefully considered in stacks of wide-angle seismic records.
For CMP data, true lateral heterogeneity can be observed directly on migrated sections if, 1) stacking smear can be eliminated, 2) the migration process preserves the full spatial bandwidth available in the data, and 3) stacking adequately suppresses noise. In other words, lateral coherence in a properly stacked and migrated section can be analyzed with the WYSIWYG method (what you see is what you get). The field data example from California demonstrates the effects of stacking smear and shows how noise affects the interpretation of lateral coherence measurements.

In the next chapter, I consider the lateral coherence of reflected events in common-shot gathers and develop an alternative approach that does not require migration. In either approach, the stack smear effect can be estimated and compensated for through the analysis of spatial bandwidth. The relation of spatial bandwidth to lateral correlation is also explored in the next chapter.
FIGURE CAPTIONS

Fig. 3-1. Velocity model for finite-difference synthetic seismogram calculations, with velocity represented in seismic trace format. Entire model extends 0-40 km horizontally and 0-16 km in depth. Grid interval is 25 m.

Fig. 3-2. Top: Logarithmic power spectra for self-similar, exponential, and Gaussian distributions and the distribution defined by (3.7). Distributions plotted in closed symbols have the form \((1+k^{2}a^2)^{-n}\), where \(n=1\) and 1.5 for the self-similar and exponential distributions, respectively, and \(n=2\) for the distribution in (3.7). The Gaussian distribution has the form \(\exp(-k^2a^2/4)\). Below: Examples of velocity perturbations produced by filtering the same grid of random numbers with the four different filters. Each panel shows 10 km horizontally and 5 km vertically, at a trace spacing of 50 m. The correlation scale length \(a=100\) m for all filters.

Fig. 3-3. Primary reflectivity section for velocity structure shown in Fig. 3-1. Trace spacing is 50 m.

Fig. 3-4. a) Inverse Stolt migration of PRS in Fig. 3-3 (detailed view).

b) Zero-offset section formed from 26 shots taken across Fig. 3-1.

Fig. 3-5. a) Detailed view of PRS shown in Fig. 3-3.

b) Detailed view of velocity structure shown in Fig 3-1.

Fig. 3-6. Lateral coherence measurements for the PRS and the velocity model. Also shown is theoretical coherence calculated from power spectrum of velocity smoothing filter. Values shown for a frequency of 10 Hz.

Fig. 3-7. a) Frequency-wavenumber power spectrum of the velocity smoothing filter (Eqn. 3.7)

b) Power spectrum for broadband primary reflectivity based on constant background velocity (Eqn. 3.8).

c) Power spectrum for primary reflectivity including convolution with source wavelet (Eqn. 3.9).

Fig. 3-8. Shot gathers at horizontal positions of 15.0, 15.4, and 15.8 km. Group interval 100 m.

Fig. 3-9. CMP stack of synthetic data gathers. Nominal fold is 6, maximum offset 5 km, midpoint spacing 50 m.

Fig. 3-10. a) Zero-offset section (same as Fig. 3-4b).

b) Detailed view of stacked section in Fig. 3-9.
Fig. 3-11. Unstacked data volume, with axes of midpoint, offset and time. Larger face is the zero-offset section; narrow face is a CMP trace gather (maximum offset 10 km) with NMO correction at 6 km/s.

Fig. 3-12. Detailed view of CMP stack data with maximum offset of 10 km. Compare to conventional stack in Fig. 3-10b.

Fig. 3-13. Stacking filter for reflection time of 3 s and maximum offset 5 km. This $\omega$-$k$ filter is developed in the analysis of Bolondi et al. (1982). Note progressive attenuation of steep-dip components.

Fig. 3-14. a) Simulated zero-offset section produced by filtering stacked data with the inverse of the filter shown in Fig. 3-13.

b) Simulated stacked section produced by filtering zero-offset data with the stack filter shown in Fig. 3-13.

Fig. 3-15. Lateral coherence values measured on zero-offset and stack sections, with theoretical coherence of the velocity model shown for reference. Values shown for a frequency of 10 Hz.

Fig. 3-16. a) Stolt migration of the zero-offset section.

b) Stolt migration of the stacked section.

Fig. 3-17. a) Finite-difference migration ($45^\circ$ algorithm) of the zero-offset section.

b) $45^\circ$ finite-difference migration of the stacked section.

Fig. 3-18. Lateral coherence values measured on migrated zero-offset and stacked sections, with theoretical coherence of the velocity model shown for reference. Values shown for a frequency of 10 Hz.

Fig. 3-19. Stolt migration of the post-stack filtered data shown in Fig. 3-14a.

Fig. 3-20. Lateral coherence values for the migrated and unmigrated section with post-stack filtering as described in text. Theoretical values for velocity model shown for reference. Values shown for a frequency of 10 Hz.

Fig. 3-21. CMP stack of data collected in the Central Valley, California.

Fig. 3-22. Left: NMO-corrected shot gather for vibrators located at the position of CMP 2762). Muting has been applied to show the analysis windows for Zones A and B. Maximum source-receiver offsets are 5400 ft for Zone A and 8000 ft for Zone B. Right: Equivalent portion of CMP stack from Fig. 3-21.
Fig. 3-23. a) Lateral coherence functions computed in Zone A for average unstacked gathers (triangles) and stacked section (circles). Coherence values predicted from the theory of CMP stacking are shown as X's (13-fold stack) and dashed lines (2- and 4-fold stacks). Note that the trend of coherence measured on the stacked section does not follow that predicted by theory. All coherence values are shown for a frequency of 20 Hz.

b) Lateral coherence functions computed in Zone B, as described above. Predicted stack values, based on 22-fold stacking, closely match the measured values.

Fig. 3-24. Left: NMO-corrected shot gather from Fig. 3-22 after application of the stacking smear filter $S(\omega, k_x; T)$ for the reflection time of $T = 1.7$ s. Application of this filter approximates the effects of stacking smear for Zone A (1450-2000 ms). Right: Same as left panel, with stacking filter computed for $T = 2.3$ s. This version of the stacking filter approximates the smearing effects in Zone B (2000-2600 ms).

Fig. 3-25. a) Open triangles: Average unstacked lateral coherence function computed for Zone A on the NMO-corrected shot gathers after application of the stacking smear filter. Closed triangles and circles show unstacked and stacked coherence values from Fig. 3-23a. After stack smear and adjustment for additional noise reduction are considered (see text), predicted stack values (X's) match the measured values closely. Values again shown for 20 Hz.

b) Lateral coherence functions computed for Zone B as described in (a).

Fig. 3-26. Asterisks: Lateral coherence function for unstacked data in Zone A after adjustment for noise correlation. Closed triangles and circles show unstacked and stacked coherence values from Fig. 3-23a. Predicted values of coherence for reflected signal without noise are shown as X's.
Figure 3-2
Figure 3-6

- ▲ Velocity model
- △ Velocity meas.
- ◇PRS measured

Coherence vs. Trace separation
(1 unit = 50 m)
Figure 3-7
Figure 3-11

$V_{nmo} = 6 \text{ km/s}$

$H_{max} = 10 \text{ km}$
Stack filter: $S(\omega, k)$

$T_0 = 3\text{ s}$

$H_{\text{max}} = 5\text{ km}$

Figure 3-13
Figure 3-15

- ZO measured
- Slant measure
- Velocity model

Coherence

Trace separation (1 unit = 1 m, depth = 50 m)
Figure 3-18

- ZO measured
- Stack meas.
- Velocity model

Trace separation (1 unit = 1 midpt = 50 m)

Coherence
Figure 3-23a: Zone A

Figure 3-23b: Zone B
Fig 3-26: Zone A

Coherence

Midpoint separation
(1 unit = 1 midpt = 55 ft)

- Unstacked
- Uncorrelated
- Noise free
- Stack
Chapter 4
Analysis of Lateral Coherence and Amplitude in
Common-Shot Records

SUMMARY

In crustal-scale surveys, seismic recordings made at large source-receiver offsets provide necessary information for the interpretation of CMP reflection data (Mooney and Brocher, 1987). The primary goal of most wide-angle experiments is to provide velocity control at depth, and the standard interpretation procedure for wide-angle data is traveltime analysis of reflected and refracted phases. Recent wide-angle surveys have used receiver spacings dense enough that the detailed character of reflected events can also be observed (e.g., Valasek et al., 1987; Mooney et al., 1988; Goodwin et al., 1988) and in some cases CMP stacking of traces could be undertaken (Levander et al., 1987; Flueh and Okay, 1987). The reflection character seen in most densely-sampled wide-angle experiments (Valasek et al., 1987; Mooney et al., 1988; Goodwin et al., 1988) tends to support the interpretation of layered structures at depth.

In the following sections, I analyze the reflection response of the isotropically random target as seen in a common-shot trace gather. In the first section, a shot gather from the data set analyzed in Chapter 3 is used to show that the wavefield scattered from the random zone has significantly greater lateral continuity and amplitude when observed at long offsets than it does at the offsets typical of reflection surveys. The increase of amplitude is qualitatively the same as that expected for specular reflectors, and thus reflection character in wide-angle experiments may bias interpretations toward layered structures.
In this chapter, ray theory and the theory of scattering in random media are combined to explain the coherence and amplitude effects. The results show that wavefield coherence is dominated by the kinematics of scattering, which are fundamentally different for the random target than they are for specular reflectors. The increased coherence in particular can be explained in terms of a dip filter that results from the common-shot geometry.

The scattering development is then extended to show the quantitative relationship between lateral coherence in the recorded wavefield and the lateral coherence of velocity variations in the target zone. A simple convolutional model allows the lateral heterogeneity of the random zone to be estimated from the surface-recorded data without doing a migration.

The ideas developed above are tested with studies of two field data examples. In the first study, synthetic data are generated for a crustal velocity model developed for the Black Forest, Southwest Germany. The velocity model used had random heterogeneities in the lower crust where previously published results had interpreted extensive horizontal layering. A synthetic shot gather shows coherency effects similar to those observed in the constant-velocity model used before. At long offset, the finite-difference synthetic matches the crustal response observed in the field-recorded wide-angle data.

The second set of field data was recorded on a dense receiver array deployed as part of a larger wide-angle experiment in the Basin and Range Province, Nevada. Analysis of various shots taken into a fixed set of receivers shows a progressive increase in continuity for the Moho reflection (PmP) as offset increases. The kinematic analysis above, however, predicts greater lateral continuity than is observed; the discrepancy can be attributed to energy scattered by heterogeneities above the target and to the presence of random noise.
SYNTHETIC COMMON-SHOT GATHER

A common-shot gather of traces from the synthetic data set described in Chapter 3 is shown in Figure 4-1. This shot was the first in the CMP survey (located at 15 km on the model) and traces are displayed for receivers at locations of 5-40 km. The group interval is again 100 m and the data have been muted, filtered and scaled for plotting as before (see Chapter 3). The strong hyperbolic event with an apex time of 4.333 s is the specular reflection from an interface at 14 km depth where velocity jumps to 6.5 km/s. The reflection has strong amplitude relative to the scattered waves and is essentially unaffected by propagation through the random zone (recall $\sigma=0.5\%$).

Figure 4-1 provides an extended view of the data in the leftmost panel of Figure 3-7, and in general shows a complex pattern of roughly hyperbolic scattering responses. Note, however, that the character of the scattering pattern is dramatically different in different parts of this extended record. Near zero offset, the gather shows events with a wide range of apparent slownesses; this range of slowness (i.e., spatial bandwidth) gives the wavefield its characteristic random and discontinuous appearance. For offsets beyond about 10 km, however, the scattered field has a much less random appearance and gains considerable lateral continuity. This effect is particularly pronounced at times near the first arriving energy and can be seen at early times near zero offset as well.

Another noticeable effect in these data is the increase in amplitude of reflections from the random zone. For instance, the rms amplitude of events at early times near 20 km offset is about twice that of equivalent events near zero offset. This increase in amplitude looks like a critical-angle effect, but recall that this model has minimal velocity variations and there are no specular reflectors in the random zone. To emphasize this point, suppose that the extreme velocities in the random zone (5.866 and 6.122 km/s) were juxtaposed across a specular boundary at 4 km depth: The critical distance in that case would be more
than 45 km — clearly beyond the offset range shown. Measurements shown below reveal that the amplitude increase is independent of $\sigma$. For random zones with standard deviations up to 3%, the increase of amplitude remains twofold from 0 to 20 km offset. Both the lateral coherence and amplitude effects can be explained in terms of the kinematics of scattering in the random zone, as described in the next section.

TRAVELTIME RELATIONS FOR SCATTERED EVENTS

In this section I analyze the scattering response of the random zone in terms of its traveltime behavior and find that the geometry of a wide-angle source-receiver pair causes scattered events to appear at the receiver with a narrow range of apparent slownesses. The resulting distribution of energy amounts to a dip filter that increases spatial correlation in the scattered wavefield. A similar analysis can explain the change in amplitude from near to far offsets.

The geometry of scattering from the random zone is shown in Figure 4-2 for the two-dimensional case. This figure is drawn to scale for the offset (7.5 km) and traveltime (2.4 s) in the center of Box C, Figure 4-1. In Figure 4-2, the source location is the origin ($x=0$), the receiver is located at offset $H$ (with $h=H/2$), and the medium has constant velocity $V$. For a fixed traveltime $t$, scatterers that contribute energy to the seismogram recorded at $H$ lie along the arc of an ellipse that cuts through the random zone. We assume the random zone has a uniform density of scatterers, with the coordinates of a scattering point being $(x,z)$. From the standard expression for an ellipse centered at $(h,0)$, the scattering curve is defined by

$$\frac{(x-h)^2}{a^2} + \frac{z^2}{b^2} = 1 \quad ,$$

(4.1)
where,

\[ a = \frac{V t}{2} \quad , \]

\[ b = \frac{V t_0}{2} \quad , \text{and} \]

\[ t_0^2 = t^2 - H^2/V^2 \quad . \quad (4.2) \]

The distance \( b \) is the maximum depth of the scattering curve and the quantity \( t_0 \) is the zero-offset (vertical) traveltime to that depth. Equation (4.1) can be solved to determine the horizontal range over which \( z \geq D \), where \( D \) is the depth at the top of the random zone.

Now the traveltime from the source to a scatterer and back to the receiver is

\[ t = \frac{1}{V} (r_1 + r_2) \quad , \quad (4.3) \]

where, \( r_1 = \sqrt{x^2 + z^2} = \text{distance from source to scatterer}, \) and

\[ r_2 = \sqrt{(x-H)^2 + z^2} = \text{distance from scatterer to receiver}. \]

In the analysis of coherence we will need the apparent slowness (or time dip) of energy contributed by each scatterer on the curve defined by (4.1). In a common-shot gather, the slowness \( p \) is the derivative of \( t \) with respect to receiver position \( H \),

\[ p = \frac{dt}{dH} = \frac{1}{V} \frac{d r_2}{dH} = \frac{-(x-H)}{r_2 V} \quad (4.4) \]

The slowness \( p \) is plotted versus \( x \) in the lower part of Figure 4-2 for the geometry shown there. From equation (4.4), note that \( p \) depends only on the receiver position.
relative to the scatterer and not the source location. For a scatterer directly below the receiver, energy arrives vertically and thus has zero apparent slowness. The asymmetric distribution of slowness values along the scattering curve shown in Figure 4-2, clearly indicates that energy arriving at the receiver will be concentrated in a narrow range of positive values.

DISTRIBUTION OF ENERGY VERSUS SLOWNESS

The energy of scattered waves arriving at the receiver can now be calculated as a function of apparent slowness \( p \). In this analysis, the energy of scattered waves received at offset \( H \) and traveltime \( t \) is determined by the product of three factors. That is,

\[
E(p) = G(p) S(p) D(p)
\]

where \( G(p) \) depends on the geometry of the scattering curve; \( S(p) \) describes the geometric spreading of the scattered waves; and \( D(p) \) describes the angular dependence of scattering. The factor \( G \) can be evaluated purely in terms of the kinematics of scattering in the random zone, while spreading effects are given by standard 2-D wave theory. The directivity factor \( D \) was derived by Frankel and Clayton (1986) in their 2-D adaptation of Chernov's (1960) scattering theory.

For the geometric factor \( G(p) \), we assess the energy of events that arrive with slownesses in an infinitesimal range about some value of \( p \). Assuming a uniform distribution of scatterers along the curve described by (4.2), energy in the range \( (p, p+dp) \) will be proportional to the arc length over which scatterers contribute at those slownesses. Thus,
\[ G(p) \propto \left| \frac{ds}{dp} \right| \text{, or} \]
\[ G(p) = \left| \frac{ds}{dx} \frac{dx}{dp} \right| \text{,} \quad (4.6) \]

where we assume a unit constant of proportionality, and the variable \( s \) represents arc length. An element of arc \( ds \) and the equivalent element \( dx \) corresponding to the slowness range \( dp \) are shown in Figure 4-2. The separate factors in equation (4.6) can be evaluated by differentiating equations (4.1) and (4.4). Given

\[ \frac{ds}{dx} = \sqrt{1 + \left( \frac{dz}{dx} \right)^2} \]

we find from (4.1) that

\[ \frac{ds}{dx} = \sqrt{1 + \frac{x^2b^4}{z^2a^4}} \quad . \quad (4.7) \]

Similarly, from (4.4)

\[ \frac{dp}{dx} = \frac{1}{r_2} \left( \frac{d}{dx} \frac{(x-H)}{r_2} \right) \]

Note that in evaluating this derivative, we are seeking the change in slowness with horizontal position for points on the scattering curve. Thus the value of \( z \) in the definition of \( r_2 \) depends on \( x \) according to the shape of the scattering curve (see Equations 4.1 and 4.4). Taking that into account,

\[ \frac{dp}{dx} = \frac{1}{r_2^3V} (r_2^2 - (x-H)(x'-h')) \quad , \quad (4.8) \]
where \( x' = x \left(1 - \frac{b^2}{a^2}\right) \), and
\[ h' = h \left(\frac{b^2}{a^2} - 2\right) . \]

The factor \( G(p) \) is plotted in Figure 4-3 for the geometry shown in Figure 4-2 (dB scale, normalized to the maximum value). Values of \( G(p) \) were determined using (4.1) and (4.6-4.8), for values of \( p \) determined at equally-spaced increments of \( x \) using (4.4). The distribution \( G(p) \) peaks at the maximum slowness of 15.3 ms/trace; this concentration of amplitude is the result of the asymmetric position of the receiver relative to the scattering curve (Figure 4-2). We shall see below that the other factors (\( S \) and \( D \)) have less effect on \( E(p) \) than the basic geometry of the experiment.

For a two-dimensional (line) scatterer, geometric spreading causes an energy decay of
\[
S(p) = \frac{1}{r_1 r_2} , \tag{4.9}
\]
where \( r_1 \) and \( r_2 \) vary along the scattering curve in such a way that
\[ r_1 + r_2 = vt = \text{constant} . \]

While the complete traveltime for any point on the scattering curve is the same, the geometric spreading will differ slightly, with the spreading effect most severe when \( r_1 r_2 \) is a maximum. Differentiating \( r_1 r_2 \) with respect to \( r_1 \) produces the result that
\[ r_1 = \frac{V_t}{2} = r_2 \]
at the minimum value of \( S(p) \). Knowing \( r_1 \) and \( r_2 \) for any scatterer location and using (4.4) to find \( p \), the factor \( S(p) \) is plotted in Figure 4-3, also normalized to its maximum values which occur at the extreme values of \( x \).

The final factor \( D(p) \) characterizes the directivity of scattering from a random zone. In this study we will use the angular dependence predicted by the standard theory of plane-wave single scattering in a random medium (Chernov, 1960). Frankel and Clayton (1986) adapted the 3-D Chernov theory to the 2-D case and give the following expression for the power \( \Phi^2 \) of waves scattered from an inhomogeneous zone of area \( A \) (their equation C7):

\[
\left( \frac{\Phi}{\Phi_0} \right)^2 = \frac{k^3 \sigma^2 A}{|x|} P[2k \sin(\theta/2)] ,
\]

where, \( \Phi_0^2 \) and \( k \) are the power and wavenumber of the incident plane wave, \( \sigma^2 \) and \( P(k) \) are the variance and power spectrum of velocity perturbations in the random zone, \( |x| \) is the distance of the receiver from the scattering region, and \( \theta \) is the angle between \( x \) and the propagation direction of the plane wave.

For the spectrum of velocity variations used in this experiment (Equation 3.7),

\[
D(p) \propto (1 + 4k^2a^2 \sin^2(\theta/2))^{-2} .
\]

Having chosen the correlation distance so that \( ka=1 \) for 10 Hz, we will define the directivity factor as

\[
D(p) = (1 + 4\sin^2(\theta/2))^{-2} ,
\]
where for fixed source and receiver, \( \theta \) is a function of \( x \) and thus ultimately of \( p \) (again through Equation 4.4). Like the spreading term \( S(p) \), \( D \) has maxima at the extreme values of \( x \) allowed for a particular scattering curve (Figure 4-3).

While our interest lies solely with the angular dependence of scattered power, we should note that our experiment does not completely meet the assumptions underlying (4.10). For instance, we illuminate the random zone with a point source rather than a plane wave and our receiver can be as close as about 10 wavelengths from the random zone for low frequencies and near offsets. The extra geometric spreading attributable to the point source was accounted for in the spreading factor \( S(p) \) discussed. The validity of the angular dependence described in (4.11) was tested by measuring the amplitude variation of a single isolated anomaly at 5 km depth (not shown). The isolated anomaly was generated by filtering a single-point velocity perturbation with the smoothing filter described in (3.7). Finite-difference seismograms with the same parameters used above were produced for sources at various horizontal positions relative to the scatterer. The observed angular dependence in the synthetics matched that in (4.11) within 5% out to source-receiver offsets of 20 km.

The final curve plotted in Figure 4-3 shows the product of the three factors that comprise the energy distribution function. At the offset and traveltime in Figure 4-2, note that energy arrives between -6.6 and +15.3 ms/trace and that most energy is concentrated at slownesses above 12 ms/trace. This uneven distribution of energy can be thought of as a dip filter: The most obvious events seen in the common-shot gather will fall in a narrow range of slowness and will thus be relatively coherent (note the arrivals in Box C, Figure 4-1). In this model, energy can propagate at any slowness between \( \pm 16.7 \) ms/trace (as dictated by the medium velocity of 6 km/s and the trace spacing of 100 m). The dip filter attributable to this surface observation point restricts the observed field and thus introduces
spatial correlation in the same way that a bandpass filter produces temporal correlation when it is applied to a random time series.

The dip filtering effect at various offsets is shown in Figure 4-4. The top curve shows the energy distribution for zero source-receiver offset at a traveltime of 1.70 s (corresponding to the center of Box A, Figure 4-1). In this case, our analysis suggests that scattered events will arrive with about equal amplitude over a range of slownesses between ± 10.3 ms/trace. The dip range is symmetric because of the zero offset but restricted (relative to ±16.7 ms/trace possible maximum). The other curves in Figure 4-4 show energy distributions for offsets of 5-20 km and times that fall along a hyperbolic moveout curve from the zero-offset time above. In comparison to the zero-offset result, the non-zero offset curves show a progressive shift of range toward positive slowness and a progressive peaking of amplitude at large slowness values. The effects of this dip filtering are apparent in an examination of Figure 4-1; along the corresponding moveout path, scattered events have increasing lateral continuity.

Energy distribution curves for all three boxes in Figure 4-1 are shown in Figure 4-5 (curves for Boxes A and C come from Figures 4-4 and 4-3). Note here that the distribution function for Box B (also at zero offset) is symmetric and occupies a larger range of slowness (±15.2 ms/trace) than that for Box A (±10.3 ms/trace). In Figure 4-1, the data within Box B look somewhat less coherent than the shallower data in Box A. The difference is attributable to the larger arc of the scattering curve at later time. The distribution of amplitude over the larger slowness range corresponds to a more gentle dip filter in Box B.

For comparison with Figure 4-5, frequency-wavenumber power spectra (Figure 4-6) were calculated for the data within Boxes A, B, and C. The resulting spectra are shown with the slowness ranges predicted by the kinematic theory superimposed as bold lines. For Box A, energy is distributed about uniformly between the symmetric dip limits
predicted by the theory. In Box B, the maximum amplitude appears concentrated near the extreme slowness values, a result consistent with visual inspection of Figure 4-1. The theoretical energy distribution for Box B predicts a difference of about 4 dB between the extreme slowness values and those at zero dip, which is consistent with the f-k spectrum in Figure 4-6. In Box C, energy is again distributed between the predicted slowness values, but here the distribution is distinctly asymmetric and energy is concentrated near large slowness values. The light line in Figure 4-6 corresponds to a dip of 11 ms/trace, where in Figure 4-5 the energy distribution falls to -9 dB.

VARIATION OF SCATTERED AMPLITUDE WITH OFFSET

In addition to increased lateral coherency, the other obvious attribute of the scattered field in Figure 4-1 is the increase of amplitude at long offsets. Again, in horizontally-layered media, this effect would be attributed to angular changes in reflection coefficients (especially near the critical angle). Such an explanation is not readily applicable in the case of the random target: Besides there being no specular reflectors or significant velocity contrasts, we have found that the relative change in amplitude does not vary when the standard deviation of velocity variation in the zone is increased substantially (by a factor of 6). The amplitude variation can be explained by simply integrating the energy distribution discussed above; before presenting that development, we first quantify the amplitude changes seen.

Figure 4-7 displays the rms amplitude of events scattered from the random zone discussed here and values for two other random zones with different magnitudes of velocity variation. The other velocity models had the identical spatial fluctuations shown in Figure 3-1, with the standard deviation of the perturbations scaled to 1% and 3% of the 6
km/s mean value. In each case, amplitude was measured on a common-shot gather having a source location in the center of the model (20 km position).

The rms amplitudes are computed for time windows that correspond to specular reflection depths between 4 and 8 km at each offset. That is, the time interval at zero offset is 1.333 - 2.667 s and the window at 20-km offset was 3.590 - 4.269 s. The analysis window used for all three shots is shown in Figure 4-8 (data for the 0.5% random zone plotted). Amplitude values in Figure 4-7 have been normalized to that at zero offset on each shot. From 0 - 20 km, the three shots produce nearly identical values and show a twofold increase in amplitude over that range.

The scattering model developed above can be used to explain the amplitude increase. At fixed offset, the amplitude expected at any time t is proportional to the square root of the power contributed by scatterers along the curve defined in (4.1). The total power at time t will be proportional to the integral of the spreading and angular factors (S and D) along the scattering curve, namely,

$$A^2(t) = \int_{x_1(t)}^{x_2(t)} S(x) \ D(x) \ \frac{ds}{dx} \ dx$$

(4.12)

where $x_1$ and $x_2$ are the horizontal limits of the scattering curve, and they depend on offset $H$, depth to the random zone $D$, and the traveltime $t$. Individual factors of the integrand can be evaluated using expressions (4.1), (4.3), (4.7), (4.9) and (4.11).

Equation (4.12) was evaluated numerically for each time sample within the analysis windows used in Figure 4-7. The rms amplitude predicted for each window is plotted in Figure 4-7 as a solid circle, again normalized to amplitude at zero offset. Note that the predictions of (4.12) closely follow the values observed in the finite-difference synthetics.
At the far offset (20 km), the slight departure in amplitude is consistent with the 5% deviation noted in applying the angular correction $D$ to the scattered response of the isolated anomaly.

**DISCUSSION**

Figures 4-2 and 4-3 show that changes in both coherence and amplitude are largely controlled by the kinematics of scattering in a random zone. The kinematic dependence is explicit in the coherence analysis, where the geometric term $G(p)$ dominates. We should note here that the kinematics of random-zone scattering are significantly different from those of specular reflection. For a specular reflection, kinematics predicts that the energy recorded at a given offset and traveltime would all arrive at one value of slowness. This single value corresponds to the one point on the target, midway between source and receiver, where reflection (i.e., scattering) takes place. Beyond kinematics, Kirchhoff wave theory predicts that the specular reflection is actually formed over the Fresnel zone (Berkhout, 1985) centered on the reflection point; contributions from the frequency-dependent Fresnel zone would result in a narrow range of observed slowness values, controlled by the spectral content of the source.

Even with Fresnel-zone effects, the slowness values observed for a specular reflection would be centered on the value corresponding to the midpoint reflection. In contrast, most of the scattered energy received from a random zone is scattered between the midpoint and the source; this effect can be seen in the geometry of Figure 4-2 and the energy curves of Figures 4-3 and 4-4. Thus, scattering from an isotropically random target implies a fundamental spatial bias in addition to the coherence and amplitude effects described. The bias might be observed in field data as a shift of the expected slowness values recorded at some offset. Observing such a shift will likely be difficult, however, as
it would require a detailed and accurate velocity model and a dense and evenly-spaced receiver array.

In the next section we present a quantitative analysis of lateral coherence and show how the dip-filter effect described here can be assessed in field-recorded data. Before undertaking that analysis, let us review the dip-filter and amplitude effects inherent in the common-shot survey geometry. The coherence of a wavefield recorded near a given receiver depends on the range of angles at which energy arrives. For a specular reflector, energy arrives within a narrow range of angles and the wavefield is highly correlated from trace to trace. For a random target, the range of arrival angles (slownesses) depends on the offset of the receiver relative to the target depth. At short offsets and late times, energy arrives from many directions and the resulting wavefield is less coherent than at long offset, where the range of arrival angles is much narrower. The effect of this restricted range and its action as a smoothing filter will be quantified in the next section.

The amplitude effects shown above can also be interpreted in terms of the scattering kinematics. First, note that the total energy received over a given time window at fixed offset will be proportional to the area of the random zone lying above the deepest scattering curve. Thus the variation of total power with offset will depend on two components: 1) the change in shape of the scattering curve as offset changes, and 2) the change in time interval. In the amplitude analysis, the time window was defined by the specular reflection times from 4 and 8 km depth. Using the previously quoted reflection times, we find that the time window at zero offset was 1.96 times longer than that at 20 km. Ignoring any changes in shape for the scattering curves, this difference almost exactly explains the twofold increase in power observed at 20 km. That is, relative to zero offset, at 20 km the same amount of energy arrived in an interval only half as long.
LATERAL COHERENCE OF VELOCITY STRUCTURE AND RECORDED WAVEFIELDS

In this section, I develop the quantitative relationship between the lateral coherence of the random zone and that of the reflected wavefield recorded at the surface. Coherence will be characterized by the function described in (3.5) and the connection between target and wavefield coherence will depend on the energy distribution function described above.

I will first explore this relation for the zero-offset experiment: The exploding reflectors model of the zero-offset section provides a convenient framework for relating the observed wavefield to its coherence function and that of the target. I show that event correlations in a zero-offset section can be explained by an energy distribution similar to that developed above. After examining the zero-offset case, the analysis is extended to the common-shot geometry discussed above.

Scattering Curves and the Exploding Reflectors Model

In the previous chapter, I showed how the exploding reflectors model can be used to relate subsurface structure to the zero-offset section. Below, the zero-offset scattering curve will be used to cast this relation in a convolutional form. From that convolution, the connection between lateral coherence in the wavefield and that in the target can be developed directly.

Assuming the exploding reflectors model for a medium of constant average velocity, the wavefield recorded at a given travelt ime on a zero-offset trace will be the superposition of responses for scatterers lying along a semi-circle as shown in Figure 4-9a. The reflectivity of each subsurface point will be taken from the PRS, and we have assumed that the statistics of reflectivity are stationary along the curve.
Note that for any source-receiver point, the shape of the scattering curve at a fixed traveltime $T$ remains the same. Thus the response for any horizontal location has the form of a convolution over the lateral coordinate. For a limited range of times, we will assume that the scattering curve changes slowly enough that the average effects can be described by the shape at the central time. With this assumption, the zero-offset traces in a time window around $T$ can be represented by a 2-D convolution,

$$p_{zo}(x,t) = p_{prs}(x,t) * f_{zo}(x,t;T)$$  \hspace{1cm} \text{(4.13)}$$

where $p_{zo}(x,t)$ is the zero-offset response, $p_{prs}(x,t)$ is the PRS, and $f_{zo}(x,t;T)$ represents the effects of a zero-offset scattering curve at traveltime $T$. The shape of $f_{zo}$ (the arc of a circle) is sketched in Figure 4-9b to indicate the range of scattering elements contributing to a given zero-offset trace. As we will see below, $f_{zo}$ embodies the kinematics of the scattering process and includes the effects of geometric spreading. At zero offset, the angular dependence of scattering is not an issue, since all scattering angles are $180^\circ$.

Note that $f_{zo}$ characterizes the dip-filtering implicit in a zero-offset experiment over a random target. As in the common-shot experiment analyzed earlier, correlation in the zero-offset wavefield will depend on the range of slownesses recorded at the surface. The surface-recorded data will have an energy-slowness distribution (discussed later) that is constrained by the zero-offset raypath geometry.

The lateral coherence function for $p_{zo}$ is found by inserting (4.13) into (3.5) and (3.6). After some manipulation (described in Appendix A), the resulting expression relates correlation in the zero-offset section to that in the target:
\[ \gamma_{zo}(\delta, \omega) = \frac{|\Phi_{prs}(\delta; \omega) \ast \Phi_{fzo}(\delta; \omega)|}{|\Phi_0(\omega)|} \]

\[ = \frac{|\Phi_{vel}(\delta; \omega) \ast \Phi_{fzo}(\delta; \omega)|}{|\Phi_0(\omega)|}, \quad (4.14) \]

where \( \Phi_{prs}(\delta; \omega) \), \( \Phi_{fzo}(\delta; \omega) \), and \( \Phi_{prs}(\delta; \omega) \) are correlation functions defined by (3.6): they are the temporal Fourier transforms of the 2-D autocorrelation functions for \( p_{prs} \), \( f_{zo} \), and the velocity structure respectively. The function \( \Phi_{vel}(\delta; \omega) \) replaces \( \Phi_{prs}(\delta; \omega) \) on the basis of the discussion in Chapter 3 and the results in Figure 3-5. In (4.14), \( \omega \) is indicated as a parameter to emphasize that the convolution is over the spatial dimension only. The denominator represents the average power spectrum of the traces being analyzed, and has the sole function of normalizing \( \gamma(0, \omega) = 1 \). The lateral coherence function \( \gamma_{zo}(\delta, \omega) \) and the function \( \Phi_{fzo}(\delta; \omega) \) are calculated for a window centered at the traveltime \( T \) (see Figure 4-9c).

Equation (4.14) represents an important (and relatively simple) result: The lateral coherence measured on the zero-offset section is related to that of the velocity model (and the PRS) by a simple convolution. Given the coherency at zero offset, one might attempt to deconvolve the autocorrelation \( \Phi_{fzo}(\delta; \omega) \) and find the correlation properties of the velocity structure. Limited spatial bandwidth, however, will likely make the approach of forward modeling more stable. To do the forward modeling, one would calculate \( \Phi_{fzo}(\delta; \omega) \) as described in the next section, and convolve it with progressively refined estimates of \( \Phi_{vel}(\delta; \omega) \).
Lateral Coherence and the Zero-Offset Energy Distribution

Recall in Figure 3-14 that the coherence of the zero-offset section was slightly greater than that of velocity variations at the target. This increase can be explained in terms of the range of slownesses dictated by the scattering curve. The effect of the scattering curve is described below.

To compute the coherence filter $\Phi_{fz0}(\delta;\omega)$, it will be convenient to transform (4.13) into the $\omega$-$k$ domain. The power spectrum of the zero-offset section is then

$$P_{z0}(k_x,\omega) = P_{prs}(k_x,\omega) F_{z0}(k_x,\omega)$$  \hspace{1cm} (4.15)

where here and in the following we will implicitly assume a data window near traveltime $T$. The quantity $F_{z0}(k_x,\omega)$ is the Fourier transform of $\Phi_{fz0}$ and thus represents the 2-D spectral distribution of scattered energy.

Note that an energy distribution in $\omega$-$k$ is naturally interpreted in terms of 2-D dip (i.e., slowness) components. In fact, $F_{z0}(k_x,\omega)$ is directly related to the energy-slowness distribution $E(p)$ discussed earlier. $E(p)$ had previously been calculated for a common-shot geometry; for a zero-offset section, the energy distribution will differ slightly, because each detector location has a different source position as well. As detailed in Appendix B, deriving the zero-offset distribution $E_{z0}(p)$ involves only a slight modification of the previous geometric factor; the spreading term remains the same (although spreading loss is constant along a given scattering curve) and the directivity factor can be dropped.

Since $k_x = p\omega$, we can relate $E_{z0}$ to $F_{z0}$ according to

$$F_{z0}(k_x,\omega) = E_{z0}(p = k_x/\omega)$$  \hspace{1cm} (4.16)
$F_{ZO}(k_x, \omega)$ represents the spectrum of the smoothing filter, and we can calculate the autocorrelation function $\Phi_{fZO}(\delta; \omega)$ by performing an inverse Fourier transform. Since each frequency component is treated separately, the transform only needs to be done over $k_x$; for fixed $\omega$ we find

$$\Phi_{fZO}(\delta; \omega) = \text{FT}^{-1}\{ E_{ZO}(k_x/\omega) \}.$$  \hspace{1cm} (4.17)

The procedure for calculating the filter $\Phi_{fZO}(\delta; \omega)$ then simply amounts to the inverse transformation of a stretched version of the energy distribution function. The stretching function is specified by the frequency of interest.

We can verify the convolutional relationship (4.14) by considering coherence measurements made on the zero-offset section. Coherence values for 10 Hz are shown as closed circles in Figure 4-10; these values were estimated on 80 zero-offset traces from the middle of the section, with a time window of 1.333–2.667 s. As before, the window was tapered over 300 ms at each end and correlations computed to lags of ±150 ms. For reference, coherence in the velocity model $\gamma_{vel}(\delta, \omega=10 \text{ Hz})$ is shown as a dotted line; the increased coherence attributable to dip-filtering is evident, although a stronger effect will be seen at far offsets in a common-shot gather.

The open circles in Figure 4-10 are predicted coherence values based on (4.14) and (4.17). The autocorrelation $\Phi_{fZO}(\delta; \omega)$ was computed from $E_{ZO}(p)$ for the center time of 2.0 s. $\Phi_{fZO}(\delta; \omega)$ was convolved with the correlation function $\Phi_{vel}(\delta; \omega=10 \text{ Hz})$ shown. The predicted coherence values closely track those measured on the zero-offset section out to the maximum midpoint separation of 10. For the zero-offset case, then, the application of (4.14) adequately predicts the degree of lateral correlation.
Lateral Coherence in Common-Shot Trace Gathers

The ideas developed above for the zero-offset case can now be extended to the analysis of coherence in common-shot gathers. In summary, the results for this case will be analogous to those in (4.14); namely, the lateral correlation seen near a given offset in a common-shot gather will be the convolution of a correlation function for the target and a smoothing filter that depends on the source-receiver geometry. The smoothing filter will be directly related to the common-shot energy distribution function $E(p)$ discussed previously.

To begin, consider two traces recorded in a common-shot experiment as shown in Figure 4-11a. Here receivers $R_1$ and $R_2$ are positioned at offsets $H_1$ and $H_2$. Also shown in Figure 4-11a are the scattering curves associated with the response at each receiver and raypaths for scattering at the source-receiver midpoints. The scattering curves in Figure 4-11a are drawn (with some exaggeration) to indicate that the traveltimes $T_1$ and $T_2$ are the same.

The situation sketched in Fig 4-11a is similar to the zero-offset case (Figure 4-9), in that the response at a given receiver is the superposition of events from the associated scattering curve. Since traveltimes is fixed, however, a significant difference is obvious; as the receiver moves to the right in Figure 4-11a, the scattering curve becomes shallower. At sufficiently long offset, the scattering curve cannot intersect the random zone for the specified traveltimes. This change in the nature of the scattering curve seriously complicates an analysis similar to that for the zero-offset case.

Not surprisingly, coherence analyses of common-shot data are more naturally accomplished when moveout is taken into account. In Figure 4-11b, raypaths and scattering curves are drawn for the case where traveltimes $T_1$ and $T_2$ fall on a moveout curve (i.e., they have the same normal incidence time $T_0$),
\[ T_1 = \sqrt{T_0^2 + H_1^2/V^2} \quad \text{and} \quad T_2 = \sqrt{T_0^2 + H_2^2/V^2}. \] (4.18)

Note here that all scattering curves with the same \( T_0 \) have the same maximum depth (\( b = VT_0/2 \)), and so as the observation point changes, the scattering curve shifts laterally but does not change its vertical extent. Although vertically the curve is constant, its width changes with offset. Using the finite-difference synthetic data, I will show that for a reasonable range of offsets, the scattering curve changes slowly enough that it can be considered constant with some average shape.

Having accounted for moveout in the common-shot geometry, scattering from the random zone looks much the same as for the zero-offset case. For fixed \( T_0 \), the response at receivers over some offset range can be modeled as the lateral convolution of the scattering curve with the reflectivity structure of the medium. We can expect, then, that lateral correlation in the common-shot record will depend on the energy-slowness distribution, as controlled by the shape of the scattering curve. Again, if energy observed in some window of offset and traveltime arrives with a narrow range of apparent slownesses, the wavefield will be highly correlated from trace to trace. Conversely, if scattered energy arrives from a wide range of directions, the wavefield will have lower coherency.

Before writing the convolution formula, we should slightly revise the description of the common-shot geometry. Note that receiver separation \( \Delta H \) results in the center of the scattering curves being displaced by the midpoint separation \( \Delta x = \Delta H/2 \). Since we assume that the average properties of the target are laterally invariant, it will be convenient to measure the separation of traces by their midpoint coordinates. Thus the common-shot record \( p_{cs}(H,t) \) can be denoted as \( p_{cs}(x,t) \), where \( x = H/2 \). This change will allow an easy comparison with the results of the zero-offset case.

For fixed \( T_0 \), the convolutional model of the common-shot record near some midpoint position \( X \) can be written in a form analogous to (4.13),
\[ p_{cs}(x,t) = p_{prs}(x,t) * f_{cs}(x,t;T) \quad , \tag{4.19} \]

where \( T^2 = T_0^2 + 4X^2/V^2 \). In (4.19), \( p_{prs} \) is the same reflectivity section (PRS) used in the zero-offset case and \( f_{cs} \) again has the shape of the scattering curve. As noted above, in this geometry, \( f_{cs} \) changes over midpoint as well as traveltime, but for a window of midpoints near \( X \), we assume \( f_{cs} \) is invariant and represents effects for the average scattering curve.

Given (4.19), we can proceed with the same development used in the zero-offset case, and write the relation between coherence of a common-shot wavefield and that of the target as

\[ \gamma_{cs}(\delta,\omega) = \frac{|\Phi_{vel}(\delta;\omega) * \Phi_{fcs}(\delta;\omega)|}{|\Phi_0(\omega)|} \quad , \tag{4.20} \]

where \( \delta \) represents spatial (midpoint) lag near the reference midpoint \( X \) and

\[ \Phi_{fcs}(\delta;\omega) = FT^{-1}\{ E(k_x/\omega) \} \quad . \tag{4.21} \]

The validity of (4.20) can also be tested using the finite-difference synthetic data. Before presenting those results, let us note that in the development above coherence is measured on individual traces at times that vary according to the moveout equation (4.18). In practice, this measurement is most easily accomplished by first applying a normal moveout (NMO) correction to the data and then doing the coherence analysis over a window with fixed center time.

If NMO were applied to correct all traces to zero-offset, however, the coherence results could be degraded by the phenomenon of NMO stretch. NMO stretch will be a
particular problem for traces with long offsets relative to target depth. Thus, in the analyses that follow, only partial NMO corrections are applied to adjust each trace in the spatial window to the offset at the center (i.e., the reference offset). Partial NMO corrections limit stretching to less than 10% in all analyses.

If we again consider Figure 4-11b, note that even partial NMO corrections will affect the traveltime relations that are central to the development of the distribution $E(p)$. These effects (and the effects of frequency shifting) are included in the calculation of $\Phi_{\text{FCS}}$ used below and are detailed in Appendix C.

The finite-difference data analyzed in this section come from the same shot used in the previous discussion of lateral continuity and amplitude effects. For review, this shot was located in the middle of the model ($x=20$ km) and traces will be analyzed at a group interval of 100 m (every fourth finite-difference grid point). The same window used in the amplitude analysis (shown in Figure 4-8) will be considered here. Coherence functions were calculated for reference offsets $H = 0$, 8, and 16 km. The spatial window for each analysis consisted of traces within ±2 km of the reference trace (41 traces total), with traces NMO corrected to the reference offset. The time window was linearly tapered over 300 ms at each end (100 ms for the short window at 16 km), and correlation functions were computed for lags of ±150 ms. The NMO-corrected and tapered windows are shown in Figure 4-12.

Results of analyses at the three reference offsets are displayed in Figure 4-13. In each of these figures (a–c), the measured coherence functions are shown as closed symbols, with values predicted by (4.20) shown as open symbols for comparison. Each figure also shows the correlation function of the velocity model as a dotted line for reference. Uncertainty in the coherence estimates ranges from about ±0.05 for values of $\gamma_{\text{FCS}}$ above 0.7 to about ±0.1 for values below 0.2.
In comparing the measured results in Figure 4-13, note the expected increase in lateral correlation. The data near zero offset show a slight increase in lateral coherence over that of the target, as seen for the zero-offset results (Figure 4-10). The results from 8 and 16 km show progressively more coherence, especially at midpoint separations beyond about 5. Overall, the measured values agree well with the predicted values at all trace separations.

Some discrepancy between measured and predicted values is evident, however, in Figure 4-13b. Here, values measured for the traces in Figure 4-12 are generally higher than the predicted values for separations of 2-5, while at other lags values match well. This deviation has two sources: First, the data window for the analysis is relatively small and thus the results can be biased by the limited selection of events. To increase the number of events sampled, a coherence function was computed for traces in the same offset range on the other side of the shot (offsets -6 to -10 km, not shown). These results are shown as closed triangles in Figure 4-13b, and they match the predicted values more closely at separations of 2-5 and depart slightly at greater lags. Average coherence estimates over a wider range of positive offsets (+4 to +12 km, not shown) produced a similar improvement.

The residual mismatch in Figure 4-13b can also be attributed to a rapid change in the energy-slowness distribution in this offset range. Curves plotted in Figure 4-4 show a quick narrowing of the energy distribution with increasing offset, at ranges near twice the target depth. At shorter offsets, the energy distribution is relatively flat over the slowness range; at longer offsets, all energy curves are sharply peaked near the maximum possible slowness for the medium. Thus, the assumption of an average scattering effect for the data window is most severely tested for offsets near 8 km in this model. The results in Figure 4-13b, however, suggest that the maximum deviations are acceptable.

Overall, the trends of predicted and measured values match closely and systematic differences between them are comparable to the basic uncertainty in the coherence analysis.
Based on the results in Figure 4-13, then, the convolutional relation in (4.20) provides an adequate approximation for the analysis of lateral coherence in field-recorded common-shot data.

FIELD DATA EXAMPLE: BLACK FOREST, SOUTHWEST GERMANY

A particularly striking model of layered crustal structure was published recently by Sandmeier and Wenzel (1986). From an interpretation of wide-angle data collected in the Black Forest, Southwest Germany, they propose a laminated model for the lower crust that includes high and low velocity layers (compressional velocity 6.0-7.2 km/s) of 120 ± 30 m thickness (Figure 4-14a). The P-wave velocity variation in the lower crust ranges from ±4% to ±8% of the mean.

The velocity contrast and thickness of the lamellae were determined by extensive 1-D elastic-wave modeling of long-offset data using a reflectivity algorithm. Portions of the field data and the corresponding synthetic section are shown in Figure 4-15. Shear-wave velocities in the model were calculated for a fixed Poisson's ratio of 0.25 and densities were calculated from velocities with the Nafe-Drake relation. Details of the layered structure were constrained by matching the scattered response and relative amplitude of the lower crustal phase marked P₁ in Figure 4-15.

While the synthetic seismograms from the 1-D model (Figure 4-15b) adequately match the wide-angle field data (Figure 4-15a), near-vertical incidence field records showed a laterally discontinuous lower crustal response (Wenzel et al., 1987); one such record from Wenzel et al. (1987) is reproduced in Figure 4-16. To match the near-offset data, Wenzel et al. calculated seismograms (also shown in Figure 4-16) using a 2-D acoustic-wave pseudo-spectral algorithm and a laterally-variable model based on the velocity structure in Figure 4-14a. In their 2-D model, the lower crust consists of rectangular, high-
velocity lamellae with horizontal lengths of 400 - 1200 m. The lamellae had the same distribution of thickness as in Figure 4-14a (giving aspect ratios of 3:1 to 10:1) and had the same degree of velocity contrast at each depth.

In this section, I present 2-D acoustic-wave finite-difference synthetic data for an alternative velocity model that matches the lower crustal response seen in the Sandmeier and Wenzel (1986) field data (Figure 4-15a). This model, with smooth random velocity variations in two dimensions, can explain the lower crustal response at both near and far offsets.

The basic vertical velocity structure used here (shown in Figure 4-14b) is based on that used by Sandmeier and Wenzel et al. (1986) with the following changes: 1) The uppermost velocity structure was replaced with a single layer of velocity 6 km/s to eliminate the lowest velocities and allow higher frequencies to be modeled with the finite-difference algorithm; 2) the velocity from 14 to 22.5 km increases linearly with depth from 5.5 to 6.8 km/s; and 3) the Moho transition was modeled as a series of simple steps rather than as alternating layers; this change has negligible effect on the wide-angle Moho reflection. With these modifications, the vertical velocity model is simplified relative to that of Sandmeier and Wenzel (1986) but still retains the essential aspects of a velocity reversal in the middle crust, a velocity gradient throughout the lower crust, and a transitional zone at the Moho.

For the velocity model in this study, variations in the lower crustal zone (14 - 22.5 km) were isotropic 2-D perturbations having the same spatial power spectrum used in previous synthetics (see 3.7), with the correlation distance $a = 200$ m. The resulting field of smooth random perturbations was scaled to have a standard deviation of 3% of the local velocity (which increases because of the gradient). Figure 4-17 shows a sketch of the complete model, with perturbations in a portion of the random zone expanded and plotted in a seismic trace format. For the entire model, velocities are specified on a grid that
extends 100 km horizontally and 32 km in depth at an interval of 50 m in both directions. The vertical velocity functions at horizontal positions of 25 and 50 km on the grid are shown in Figures 4-14c and 4-14d.

Synthetic seismograms were computed with the usual finite-difference algorithm for a source at the horizontal position of 10 km and a depth of 250 m. Absorbing boundaries were used on all sides of the model so there is no free surface to produce ghosts or surface multiples. The maximum frequency of the source wavelet was 20 Hz for 5-point spatial sampling of the shortest wavelength. A total of 18 s of data were calculated at a time step of 2.813 ms (again 75% of the stability limit; final traces resampled to 8 ms). The calculations for this model required about 7 min of CPU time on the NEC SX-2.

Figure 4-18 shows a portion of the resulting common-shot gather. Traces are displayed for every fourth grid point (offset increment of 200 m) and are plotted at a level adjusted to best show returns from the random zone (no time-variant gain applied). The strong linear event at the top is the direct wave traveling in the surface layer. The reflection response of the random zone is a complex collection of scattered events that begin just before 5 s at zero source-receiver offset. Reflections from the Moho transition (23.5 - 25.0 km depth) are too weak to be seen at near offset (8.0 - 8.5 s vertical traveltime) but brighten enough to be seen beyond about 45 km. At times later than the Moho arrival the response from the random zone has significant amplitude and decays relatively slowly.

The reflection response of the random zone displays the same changes in coherence and amplitude seen in previous synthetic studies. At offsets beyond about 50 km, events from the random zone have substantial lateral continuity — individual peaks or troughs can be followed over distances of almost 30 km. In addition, scattered events at 65 km offset have rms amplitudes roughly 12 dB greater than those near zero offset (16 dB after correction for geometric spreading). If viewed only at long offset, the lower crustal response suggests a distinctly layered structure.
The scale of this model suggests that increased amplitude and coherence should be observable in field-recorded data. Lateral coherency of returns from the lower crustal random zone is not evident for the 2-km seismogram spacing of the wide-angle field record shown in Figure 4-15a, where the wavefield is spatially aliased. Figure 4-19 shows traces from Figure 4-18 displayed as a reduced-time record section, with trace-normalized seismograms spaced at 2 km intervals as in the field survey. The reducing velocity is 8 km/s, and for clarity, the direct wave has been muted.

In Figure 4-19, the dominant event at offsets of 60 - 70 km is the Moho reflection ($P_{mP}$). At longer offsets, the Moho event is complicated by the emerging head wave. The complex events preceding $P_{mP}$ are returns from the lower crustal regions of the model ($P_l$); at the 2-km trace spacing, these events have no obvious spatial coherence. The $P_{mP}$ phase grows weaker relative to the scattered waves at shorter offsets until it is not clearly visible inside 60 km. Traveltime curves for the reflections $P_l$ and $P_{mP}$, and the mantle refraction $P_n$ are plotted at the top of Figure 4-19.

The results in Figures 4-18 and 4-19 can be directly compared to the field and synthetic data of Sandmeier and Wenzel (1986) in Figure 4-15. The comparison reveals obvious differences because their synthetic data were calculated with an elastic-wave reflectivity algorithm for frequencies up to 30 Hz, while the data presented here include only acoustic waves and have a maximum frequency of 20 Hz. The finite-difference data in Figure 4-19, however, show the same general behavior of $P_{mP}$ and $P_l$ as the reflectivity synthetic data calculated by Sandmeier and Wenzel using the 1-D velocity model shown in Figure 4-14a. In both synthetic data sets, $P_l$ is a complex reverberatory wave train, while $P_{mP}$ is most clearly visible at 65-80 km and is complicated by the head wave at longer offsets.

The synthetic wide-angle data from the random lower crust model match the Black Forest field data in the appearance of $P_l$ and in two other respects as well. First, $P_{mP}$
becomes difficult to identify at offsets less than 60 km in both the field data and the finite-difference synthetic records in Figure 4-19. Second, the field data and the finite-difference synthetics show a similar amplitude decay for events following $P_mP$: an incoherent "coda" following $P_mP$ decays rapidly at offsets of 75-85 km but is much more persistent for records at 45-60 km. Sandmeier and Wenzel's reflectivity synthetics also show significant energy arriving after $P_mP$. This coda is composed of converted shear phases that have lateral coherence because of the layered structure. The incoherence and persistence of coda after $P_mP$ in Figure 4-19 is attributable to the scattering of waves in a random 2-D medium, an effect not possible in 1-D models. A comprehensive description of events following $P_mP$ in the field record requires considering both elastic effects and randomness.

Finally, note that the near-offset finite-difference data collected over the random lower crust in Figure 4-17 show discontinuous reflection events similar to those in the field and synthetic data of Wenzel et al. (1987). Figure 4-20 shows traces from every surface point on the finite-difference grid (50-m spacing) to a maximum offset of $\pm 5$ km. Note that this near-vertical reflection response is a collection of discontinuous segments, similar in extent to those seen in the field data (Figure 4-16). The sub-horizontal alignment of events near zero offset is enhanced by the bias of vertical reflectivity described for the zero-offset section in Chapter 3. In Wenzel et al.'s (1987) near-vertical synthetics (not shown), random noise was added to mask lateral continuity and improve the match to the field data. In Figure 4-20, the random appearance and the variable length of event segments are interference effects. In the complex wavefield reflected from a random target, "noise" is built in.

The high-contrast layering in Sandmeier and Wenzel's (1986) velocity model were introduced to explain the amplitude and reverberatory nature of the $P_l$ events. CMP reflection data recorded in the same area and interpreted by Lüschen et al. (1987) support the layered model, although those data have not been compensated for the stack-filter
effects described in Chapter 3. Note that if \( P_l \) had been densely sampled at large offsets, its 
extensive continuity would also tend to support a layered interpretation of the lower crust.

The random model in Figure 4-17 efficiently explains many features of both the near-
vertical and wide-angle field data presented here, including amplitude and coda effects. In 
this study, the lower crust has a more general form of velocity variation than that used in 
layered models; with 2-D inhomogeneities, local velocity contrasts can be much less than 
those required across specular reflectors (compare velocities in Figure 4-14).

The synthetic data in this study are not intended to suggest that the isotropically 
random heterogeneities of Figure 4-17 provide a complete description of the lower crust in 
this area. Instead, random heterogeneities and conventional layering provide useful end-
member models for the structure of such targets. The field and synthetic data presented here 
show that considerable lateral heterogeneity can be accommodated in models derived from 
standard wide-angle experiments and emphasize the need for dense receiver spacing in 
future surveys.

FIELD DATA EXAMPLE: BASIN AND RANGE PROVINCE, NEVADA

The field data considered in this section are taken from a large experiment conducted 
during 1986 in the northwestern Basin and Range Province of Nevada by PASSCAL 
(Program for Array Seismic Studies of the Lithosphere). In this experiment, a combination 
of reflection, refraction/wide-angle reflection, and earthquake seismology were used to 
investigate crustal thickness and other aspects of lithospheric structure (Mooney et al., 
1988). The data analyzed in this section were recorded on multichannel exploration 
seismographs deployed during the shooting of a conventional wide-angle refraction survey.

These so-called "piggyback" reflection data are of particular interest here because this 
experiment was able to record large chemical explosions on dense receiver arrays deployed
at offsets from near zero to over 100 km. During the complete experiment, various reflection spreads were deployed along three different profiles. In this section, I analyze data from selected receivers in the second deployment.

Figure 4-21 shows a plan view of receiver locations for Deployment 2; receivers connected to five different recording instruments were positioned along the profile, with group intervals of 100 m for three of the instruments and 67 m for the other two (Jarchow et al., 1987). Figure 4-21 also shows locations of the five shots used in the coherency analysis below. Charge sizes ranged from 227 kg (500 lbs) at Shotpoint 4B (SP 4B) to 1818 kg (4000 lbs) at SP 5.

In this section, I consider the lateral coherence of reflections from the Moho, $P_{mP}$. The piggyback data set provides an unusual opportunity to study $P_{mP}$ at a wide range of source-receiver offsets: First, $P_{mP}$ is clearly visible on near-offset traces (i.e., on the record from SP 4B; Jarchow et al., 1987). Second, the shot-receiver geometry allows one to trace $P_{mP}$ almost continuously from zero offset to beyond 80 km at a nominal trace spacing no greater than 100 m. In coherence analyses one can thus be confident of measuring the properties on the same phase from one offset to another.

$P_{mP}$ appears as a composite reflection event having a time duration of about 1.0 - 1.5 s. Near zero offset, $P_{mP}$ occurs between 8.5 and 10.0 s as a complex and laterally variable event (Jarchow et al., 1987). As shown below, the Moho reflection becomes stronger relative to background noise and has shorter duration at large offsets. Near-offset seismic records also show a prominent reflection response from the upper crust; reflections have an abrupt onset near 4 s (roughly 10 km depth) and persist for up to 3 s two-way traveltime (Jarchow et al., 1987). The lower crust appears relatively non-reflective, with reduced reflection amplitude within 1-2 s of the onset of the Moho reflection complex.

$P_{mP}$ can be traced to larger offsets by combining traces recorded from different shots, and treating the result as one large common-shot trace gather. Although some lateral
variation of structure is present across the survey area, relative time shifts can be compensated by eye and $P_mP$ matched between data sets with different source points. The reflection time of $P_mP$ is sketched in the inset of Figure 4-21, along with the shots used at different offsets. Continuous offset coverage is available over 0-80 km, except for a gap at 40-45 km.

While the composite spread provides a wide range of offsets, traces for the coherence analysis must be carefully selected to avoid problems caused by near-surface variations. For instance, it would be unwise to compare coherence values measured for one shot over different parts of the spread, because near-surface statics, instrument type, and the linearity of receiver positions can differ substantially.

Coherence in this study will thus be measured on a fixed subset of receivers (Stations 438-478) shown in Figure 4-21. This sub-spread is approximately 2.6 km long and the five shots studied provide offsets from 3.178 to 75.570 km. Group interval on the spread is nominally 67 m and data were recorded on an MDS-10 system (deployed by the University of Wyoming) using receiver groups of twelve MD81 geophones.

This subset of receivers was selected because data recorded there were of consistently good quality for all shots. Furthermore, these receivers were located in an area with apparently little near-surface variation. Near-surface problems were assessed by examining first arrivals from the shot at SP 1, which is located approximately 50 km northeast of SP 2 (off the map in Figure 4-21). First arrivals from SP 1 recorded at Stations 400-480 are shown in Figure 4-22, where individual seismograms have been trace-normalized and the time scale is reduced by 10 km/s. Offsets to SP 1 range from 120.9 to 126.1 km.

The time of first arrival on the selected subspread ranges from 9.35 s at Stn 438 to 9.60 s at Stn 478, and the observed event has a duration of about 400 ms at all stations. This event is the refracted crustal wave (apparent velocity 6.4 km/s), which can be approximated as a plane-wave arrival at this distance. The crustal wave has excellent
coherence from trace to trace, indicating that near-receiver scattering is relatively small. The arrival time of this event is also nearly linear, with only minor static time shifts near Stn 450. Since the coherence is measured for separate frequency components, static shifts of this magnitude will not significantly affect the resulting values.

Having established that near-surface effects are small and receiver response across the array is consistent, let us now consider the measurement of coherence on \( P_{mP} \) as observed for the five shots. The top row of Figure 4-23 shows the \( P_{mP} \) event recorded on the subspread for each shot. The onset and termination of \( P_{mP} \) was picked by hand from the composite gather of all traces from the five shots. Arrival times across the subspread were shifted to align the onset at time zero.

Coherence values were measured separately for each shot, with traces linearly tapered over 50 ms at each end and correlation functions computed to lags of ±300 ms. Since receiver spacing is slightly irregular, average source-receiver offsets were computed from the traces analyzed, for each value of receiver separation. Coherence values were computed for receiver separations out to about 1 km (15 traces @ 67 m), and are shown for each shot in Figure 4-24 (frequency 10 Hz).

Figure 4-24 shows a general increase of lateral coherence with increasing offset. SP 4B clearly has the lowest coherence at all offsets, falling to zero near 200 m separation. SP's 3 and 4 are comparable and somewhat above SP 4B, with SP's 2 and 5 having significantly greater values. Note, however, that SP 5 (central offset 57.3 km) has somewhat greater coherence than SP 2 (74.3 km).

Irregularities in the coherence trend are most likely attributable to the relatively small spatial sample provided by the 2.6-km subspread. Individual analyses may therefore be contaminated with anomalous events, such as the one seen at the lower left side of the data for SP 3. In addition, systematic variation in the character of the Moho target could occur over the scale of this survey (i.e., the statistics of lateral heterogeneity may not be
stationary). Note, for instance, that SP 2 and SP 5 are on opposite sides of the subspread, with source-receiver midpoints separated by about 65 km.

A visual examination of Figure 4-23 confirms the quantitative results of Figure 4-24. The correspondence is particularly clear when the seismic passband is restricted to a narrow range around 10 Hz: The bottom row of Figure 4-23 is the result of applying a trapezoidal filter (5-7-13-15 Hz) to the data in the top row. The reflection response for SP 4B has a number of short discrete segments, and lateral coherence is markedly increased for SP’s 2 and 5.

At this point, note that the results in Figure 4-24 suggest that the Moho transition zone has significant lateral variations at scale lengths near 1 km. In the absence of noise, a specular (horizontally invariant) reflector would have a lateral coherence of 1 at all receiver separations. If the target were essentially specular with some random variation, we would expect lateral coherence to decay to some base level above zero, as seen in the field data example of Chapter 3. The model of a noise-contaminated specular reflector clearly does not fit the coherence values measured for these data, which drop to zero at receiver separations less than about 1 km.

The convolutional relation developed earlier in this chapter can now be applied to investigate the lateral coherency of the Moho target. The first step in assessing lateral coherency for these data is to compute the common-shot smoothing filter \( \Phi_{cfs}(\delta; \omega) \); normally, we would then try to estimate a velocity correlation function \( \Phi_{vc}(\delta; \omega) \) and model the observed lateral coherence through expression \( (4.20) \). Since this data set covers the entire crust, the scattering curve that underlies the computation of \( \Phi_{cfs} \) must be calculated for a velocity model that varies with depth. This extension of the constant-velocity theory is necessitated by the effects of Snell’s Law in a medium having a wide range of velocity.

An approximate velocity model for this area, provided by the U. S. Geological Survey (R. Catchings, personal communication), is shown in Figure 4-25, along with
scattering curves that will be discussed below. In this model, the Moho is at 30 km depth with a mantle velocity of 8.0 km/s. For a fixed offset and travelt ime, the shape of each scattering curve is determined by an iterative raytracing technique: First, the ray parameter for a specular reflection to some depth is determined by manual trial and error. This raypath corresponds to scattering at the deepest point on the scattering curve and the ray parameter gives the slowness value associated with that point (refer to Figure 4-2). Hereafter, we consider the ray connecting source and scatterer separately from that connecting receiver and scatterer.

Successive points on the scattering curve are determined at constant depth intervals (200 m in the cases presented here). To find the scattering point at a given depth, a set of rays is first traced from the receiver to that depth level; ray parameters are chosen so that the rays cover a horizontal interval that includes the scattering point being sought. A similar set of rays is then traced from the shot. Raytracing is accomplished with the parametric ray equations for a layered medium with linear velocity gradients in each layer (Gibson et al., 1979).

Linear interpolation is then used to find shot rays that intersect the desired depth level at the same positions as the traced receiver rays. Raytracing and interpolation produces traveltimes and ray parameters for each of the source-to-receiver raypaths. The scattering point is taken as that position where the total travelt ime is closest to the specified value for the curve. In the model considered here, rays were traced at intervals of 0.0001 s/km, which resulted in horizontal increments of 20-50 m at the various depth levels. Traveltimes for all points on the scattering curves shown were within 5 ms of the desired time.

For the model in Figure 4-25, the maximum depth of the scattering curve was taken as 35 km (10.95 s zero-offset travelt ime). The top of the scattering layer was taken as the Moho depth (30 km) and scattering curves were computed for source-receiver offsets corresponding to each shotpoint. In Figure 4-25, scattering curves are shown for
Shotpoints 4B and 5, with the curves shifted to align their source-receiver midpoints. Once the scattering curves were found, energy distribution functions \( E(p) \) (Equation 4.5) were determined by numerical calculation, but only the geometric factor \( G \) (Equation 4.6) was included. As noted in the development earlier, the spreading and directivity terms are second order effects relative to \( G \). Given the preliminary nature of the velocity information available, the refinement provided by the other two terms is not justified. Given the functions \( E(p) \), smoothing filters \( \Phi_{fc} \) were computed through (4.17).

Observed and predicted values of lateral coherence for Shotpoints 4B and 5 are shown in Figure 4-26. The observed values were estimated from the data in Figure 4-23 as described previously, but with time windows truncated to correspond to the 35-km maximum scattering depth; that is, traces from SP 4B were truncated at 1.200 s after the onset of \( P_{m}P \), and traces from SP 5 were truncated at 0.750 s. The decreased time window at long offset is a well-known effect caused by coalescing of reflection traveltine branches.

The predicted values of coherence in Figure 4-26 are computed for an uncorrelated velocity structure (i.e., \( \Phi_{vel}(\delta;\omega) \) has the form of a delta function in spatial lag). The values shown in Figure 4-26 thus represent minimum estimates of lateral coherence; any positive spatial correlation of velocity, such as that used in previous synthetic models, would tend to increase the coherence values predicted.

Although the predicted values in Figure 4-26 are conservative estimates of coherence, these values are significantly greater than those observed at both the near and far offset. As noted below, other studies (e.g., Hurich and Smithson, 1987; Reston, 1987) have found that observed coherence is generally less than that expected from theory. The mismatch evident in Figure 4-26 can be explained by considering two effects of the real earth not included in the synthetic models presented previously.

First, in previous models, the randomly inhomogeneous target was considered as the only source of scattering in the model; in fact, heterogeneity exists at all subsurface levels.
In this data set, the strong reflection response of the upper crust was emphasized by Jarchow et al. (1987). Heterogeneity throughout the section means that scatterers above the target can contribute energy at the offsets and traveltimes of interest. Importantly, the energy contributed from scatterers above the target will arrive at slownesses that will increase the spatial bandwidth and thus decrease the predicted lateral coherence.

Scatterers above the Moho can be accounted for in the model considered here by allowing the scattering curve to extend above a depth of 30 km. Extended curves are shown in Figure 4-27. The curves terminate where a ray from source or receiver reaches its turning point; thus points on these curves represent scattering for pre-critical wave propagation only. For the calculation of the energy distribution functions, scatterers above the Moho are assigned a strength one-half that of scatterers below.

Second, we must also consider that the field-recorded Moho reflection is contaminated by background noise; as detailed in Appendix D, noise will tend to lower measured coherence values. Results in Chapter 2 suggest that a likely source of background noise is the scattering of upcoming reflections by irregularities in near-surface layers. The data in Figure 4-22 show this mechanism can produce significant levels of noise without extensively disrupting the lateral coherence of reflections. For the data analyzed here, I will adjust predicted values of coherence for the presence of random noise, using expression (D19) from Appendix D. While results in Chapter 3 show that background noise may have some spatial correlation, spatially-uncorrelated noise was used here because no reliable estimate of noise correlation was available for these data.

Lateral coherence predictions for the extended scattering curves of Figure 4-27 are shown in Figure 4-28. For both Shotpoints 4B and 5, noise-free predictions are shown as open circles. While extending the scattering curves has produced noticeably lower coherence estimates (particularly for SP 4B), the predicted values are still significantly higher than those observed. The other curves plotted in Figure 4-28 show predicted
coherence values adjusted for various levels of random noise (amplitude signal-to-noise ratios, SNR = 0.5, 1, and 2). Predicted values are now comparable to the observed values for both shotpoints. For SP 4B, observed coherence values follow a trend between SNR values of 0.5 and 1; for SP 5, observed values fall on a trend between SNR's of 1 and 2. These values of SNR are consistent with the appearance of the data (Figure 4-23). The increase of SNR at longer offsets can be attributed to the relative arrival time of PmP: Near zero offset, PmP follows reflections from all levels of the crust and can be contaminated by direct and scattered waves trapped near the surface. At far offsets, PmP is closer to a first arrival; thus near-surface scattering of direct source waves is not possible and there are fewer preceding reflections to produce background noise.

While predicted and observed coherence values in Figure 4-28 now match more closely, I should emphasize that the predicted values represent only one combination of velocity structure, lateral heterogeneity and noise level that can explain the observations. Clearly, an uncorrelated velocity structure for heterogeneities in the mantle is not reasonable on geologic grounds. The uncorrelated model is used here to demonstrate that the change in lateral coherence from near to far offset is consistent with the theory of lateral coherence due to shot-receiver geometry. We should also note that some loss of coherence for PmP may in general be attributable to propagation through laterally variable material in the crust. Understanding the influence of inhomogeneous overburden will require further study, but this effect does not appear significant in these data; the extremely coherent first arrival in Figure 4-22 corresponds to a diving ray that has turned near 20 km depth, thus traversing most of the crust.

A model with positive spatial coherence for mantle scatterers would produce noise-free coherence estimates greater than those shown in Figure 4-26. Reconciliation of those curves with observed values would require a different distribution of scattering above the Moho and different noise conditions than those used here. Uncorrelated noise was used
here because it required a minimal assumption about the noise field; the inclusion of spatially-correlated noise may have further improved the match. Most importantly, I should emphasize that the modeling undertaken demonstrates a framework for analyzing scattering phenomena; the results are non-unique because more information is required about scattering in the upper crust and near-surface than is available from the analysis of this data set.

CONCLUSIONS

The synthetic data presented in this chapter show that the geometry of a wide-angle common-shot experiment tends to produce seismic images with enhanced lateral continuity. Thus, the interpretation of large-offset data sets, especially those collected with dense receiver spacings, will tend to support the layered models developed from CMP-stack sections. The field data example from the Black Forest shows that lateral heterogeneity and elastic wave propagation effects are both important aspects of models developed to explain complex scattered events. Relative to the previous layered model, the 2-D random structure modeled in that study shows that a more general form of velocity variation can adequately explain the observed seismic response. Although the 2-D random model does not correspond directly to proposed geologic models of the lower crust, it has the attractive feature of having less severe small-scale variations of velocity than its layered counterpart.

The theory developed to explain amplitude and lateral coherence effects in the synthetic data provides a useful framework for analyzing issues of wave scattering in heterogeneous media. The field data example from the Basin and Range Province, however, reveals the complex nature of scattering in the crust. The fact that lower crustal reflection segments have less coherency than expected has been noted in other published studies (Hurich and Smithson, 1987; Reston, 1987). These researchers, however, have
attributed the numerous short segments to interference effects in the wavefield reflected from a heterogeneous target. The correlation analysis here shows that such interference cannot be the explanation in this case; even an uncorrelated random velocity structure should have a reflection response with substantial lateral correlation. Accurate estimates of lateral heterogeneity for deep crustal targets will thus require understanding of both the biases in seismic imaging techniques and a detailed knowledge of heterogeneity at all crustal levels.
FIGURE CAPTIONS

Fig. 4-1. Common-shot trace gather for source position of 15 km on model in Fig. 3-1. Group interval is 100 m and the direct wave has been muted. The strong hyperbolic event below 4.5 s is a reflection from the specular interface at 14 km depth (see text).

Fig. 4-2. Scattering geometry for offset and traveltime in the center of Box C, Fig. 4-1 (7.5 km, 2.4 s). The scattering curve is drawn to scale (depth to the random zone 4 km). Differential arc length ds is shown displaced from the scattering curve for clarity.

Fig. 4-3. Energy distribution as a function of slowness (in dB). Open symbols are values for individual factors comprising E(p). Closed symbols indicate values for E(p). All curves normalized to their maximum value.

Fig. 4-4. Energy distribution function for various source-receiver offsets. The traveltime at zero offset is 1.7 s (center of Box A, Fig. 4-1), and times at other offsets are determined according to hyperbolic moveout with a velocity of 6 km/s. Each curve normalized to its maximum value.

Fig. 4-5. Energy distribution functions for offsets and traveltimes in the center of Boxes A, B, and C, Fig. 4-1. Each curve normalized to its maximum value. The range of slowness observed in Box B is greater than that for Box A because a wider range of arrival angles is possible at the greater depth.

Fig. 4-6. Frequency-wavenumber power spectra for the data in Boxes A, B, and C, Fig. 4-1. Bold lines indicate slowness range as shown in the energy distribution functions of Fig. 4-5. Light line on plot for Box C indicates a dip of 11 ms/trace, where the energy function in Fig. 4-5 is down 9 dB.

Fig. 4-7. RMS amplitude values for various source-receiver offsets in Fig. 4-1. Time window for estimate is shown in Fig. 4-8.

Fig. 4-8. Time window for estimation of rms amplitudes in Fig. 4-7.

Fig. 4-9. a) Schematic view of random velocity zone. Seismic response on a given zero-offset trace (vertical bold line) is the superposition of scattered events from points along the semi-circular scattering curve shown.

b) Schematic view of PRS, with scattering curve superimposed. Strength of scattering is determined by local reflectivity at each point on the curve.

c) Schematic view of zero-offset section. Over a limited window in space and time, the zero-offset response can be cast as a convolution between the scattering curve and the reflectivity structure of the medium. This formulation assumes the shape of the curve is constant over the window modeled.
Fig. 4-10. Lateral coherence values for zero-offset traces (solid), predictions from convolutional expression (open), and the velocity structure of the model (dashed).

Fig. 4-11. a) Sketch of ray paths and scattering curve for equal traveltimes at two different offsets. Note the scattering curve becomes shallower as offset increases.

b) Same as (a) for different times having the same normal incidence time (see text). Maximum depth of the scattering curve remains the same, although horizontal extent changes with offset.

Fig. 4-12. Partially NMO-corrected window of data plotted in Fig. 4-8. NMO corrections are made to central offset in each window (0, 8, and 16 km).

Fig. 4-13. a) Lateral coherence values measured for data at 0-km offset in Fig. 4-12 (solid) and values predicted from theory (open). Coherence of velocity structure shown as dashed line; all values for 10 Hz.

b) Same as (a) for data at 8-km offset. Solid triangles are values for trace at similar offsets on the opposite side of the shot (see text).

c) Same as (a) for data at 16-km offset.

Fig. 4-14. a) P-wave velocity model of Sandmeier and Wenzel (1986).

b) Unperturbed velocity model for this study.

c) and d) Vertical velocity functions at 25 and 50 km on the velocity grid used to calculate finite-difference synthetic seismograms. See Fig. 4-17.

Fig. 4-15. Field and synthetic wide-angle data from Sandmeier and Wenzel (1986). Synthetic data calculated with a reflectivity algorithm as described in the text.

Fig. 4-16. Near-vertical common-shot trace gathers from Wenzel et al. (1987).

Fig. 4-17. Sketch of velocity model used to produce finite-difference synthetic data, with a portion of the lower crustal random zone expanded. Velocity in random zone presented in seismic trace format (higher velocities dark). High-velocity areas visible have typical sizes of 1–2 km.

Fig. 4-18. Acoustic-wave finite-difference common-shot trace gather. Group interval 200 m (every fourth grid point). Moho reflection PmP is marked where it is plainly visible at long offsets.

Fig. 4-19. Trace-normalized reduced-time record section of seismograms from Figure 4-18. The reducing velocity is 8 km/s and the direct wave has been muted. Traveltimes for the reflections from the lower crust and mantle (solid) and the mantle refraction (dashed) are shown at the top.
Fig. 4-20. Detailed view of traces within ±5 km offset at the finite-difference grid spacing of 50 m. Reflections from the Moho transition arrive at 7.95 to 8.35 s (data shown to 8.5 s).

Fig. 4-21. Plan view of receiver locations and shot points for data used in coherence study of P_mP. Location of Stn 438-478 shown as short line. Inset: Sketch of P_mP traveltime and duration, with indication of shot contributing at various offset ranges.

Fig. 4-22. First arrivals from shot at SP 1, as recorded at Stn 400-480. Consistency of moveout and waveform across the subspread Stn 438-478 indicates a relatively simple near-surface.

Fig. 4-23. Top row: Broadband (5-40 Hz) plots of P_mP as recorded on the subspread from 5 different shots. First arrival of P_mP is shifted to time zero. Bottom row: Narrow bandpass filter (7-13 Hz) applied to data above.

Fig. 4-24. Lateral coherence functions for data in top row of Fig. 4-23. Values computed for a frequency of 10 Hz.

Fig. 4-25. Velocity model for calculation of coherence values. Scattering curves for SP 4B and SP 5 are computed for depths between the Moho and 35 km and are shifted to align their source-receiver midpoints.

Fig. 4-26. Observed and predicted lateral coherence functions for SP 4B and SP 5. Predicted values are computed from energy distribution functions for the scattering curves in Fig. 4-25. Observed values are measured for a time window corresponding to the maximum depth of 35 km.

Fig. 4-27. Velocity model from Fig. 4-25 with scattering curves computed for all pre-critical raypaths. In the calculation of energy distribution functions, scatterers above the Moho are assumed to produce half the power per unit distance as those below.

Fig. 4-28. Observed and predicted lateral coherence values for SP 4B and SP 5. Noise-free predicted values are computed for the extended scattering curves shown in Fig. 4-27 and are adjusted for the presence of random noise as discussed in the text. SNR is measured in terms of amplitude.
Figure 4.8

Offset (km)

Time (s)
Figure 4-10

- Measured
- Predicted
- Velocity model

Coherence vs. Trace separation (1 unit = 1 midpt = 50 m)
Figure 4-12
Figure 4-13a: 0±2 km offset

Figure 4-13b: 8±2 km offset

Figure 4-13c: 16±2 km offset
Fig. 3. (Top) Reflection seismograms from the central Black Forest in southwest Germany and (bottom) numerical solutions of the two-dimensional wave equation for a random lamellae model. Traces are spread along 16 km; the source is located in the center. The lower crustal response appears between 5 and 8.5 to 9 s TWT. The sections have the same frequency content. The synthetics are superimposed by noise adopted from the 10- to 12-s TWT range of the data. Note the similar appearance of data and the numerical solution.

Figure 4-16
(from Wenzel et al., 1987)
Deployment 2
PASSCAL
Nevada, 1986

Figure 4.21
Figure 4.22

Station

Offset (km)

(\textsuperscript{11}/X) 01
Figure 4-26a: SP 4B

Figure 4-26b: SP 5
Figure 4-28a: SP4B

Figure 4-28b: SP5
Chapter 5
Summary and Conclusions

SUMMARY

The main conclusions from preceding chapters are summarized below:

- In the 2-D case, near-surface scattering mechanisms can be differentiated from those at depth by the moveout of scattered events. In 3-D, this discrimination will be more difficult because scattered waves will appear with a wide range of phase velocities corresponding to arrivals from all azimuths.

- Scattered waves trapped in the near-surface will appear as noise over the entire length of reflection seismic records. This noise can seriously degrade the results of data-dependent deconvolution, because the scattered energy masks the spectral character of reverberation effects that deconvolution seeks to correct.

- Irregular near-surface layers can also produce a significant amount of noise by scattering energy from upcoming reflections into the trapped modes of the layer. This mechanism is likely a large contributor to background noise levels, since it is active at all reflection times.

- CMP stacks of targets with isotropic random velocity variations show numerous short subhorizontal events similar to those seen in field-recorded data. This reflection character results from the loss of steeply-dipping energy through a filtering process inherent in the conventional NMO-stack process. Other steps in the data processing sequence (e.g., frequency-wavenumber filtering) can further suppress dipping energy and
thus enhance lateral continuity. The effects of the stacking dip-filter can be quantified through the formulation of Bolondi et al. (1982).

- Post-stack migration focuses the surface-recorded wavefield and thus improves lateral resolution, but cannot correct the effects of the stacking filter. In order to make accurate estimates of lateral coherence, the unmigrated stack must approximate a zero-offset section and post-stack migration must preserve the full range of stacked dips. The stacking filter is most efficiently treated with techniques such as dip-moveout (DMO) correction.

- The effects of stacking smear can be seen in field data from the Central Valley, California by analyzing lateral coherence functions for stacked and unstacked sections. While coherence values for unstacked data are not affected by stacking smear, interpretation of these values requires adjustments for the presence of correlated and uncorrelated noise.

- Surface-recorded images of laterally heterogeneous targets are slightly biased toward lateral continuity by the geometry of the reflection experiment. This bias occurs because the reflected field depends vertically on the derivative of material properties, but horizontally it depends on the properties directly. This asymmetric treatment can be compensated by integration along the time axis, which is a standard exploration technique for converting reflectivity to velocity information.

- When viewed at large source-receiver offsets in common-shot trace gathers, the reflection response of isotropic random zones has increased lateral continuity and amplitude relative to observations near zero offset. Both effects are particularly noticeable at offsets beyond about twice the target depth and can be explained with a combination of geometric ray theory and simple wave theory.

- Using the model of an exploding reflectors experiment, the correlation properties of a random velocity structure can be related to the lateral coherence of a reflected wavefield. Thus observations of lateral coherence in a restricted data window can be used to estimate lateral heterogeneity at the target. Note that this approach does not require migration of the
data; field-recorded wide-angle shot gathers usually lack sufficient velocity information and sampling density for successful migration processing (e.g., McMechan and Fuis, 1987).

- A 2-D isotropic random velocity model for the lower crust produces acoustic-wave synthetics which match the character of field-recorded data from the Black Forest at both near and far offsets. Although the far-offset field data had receiver spacings too coarse to see increased lateral continuity, the relative amplitudes of lower crustal scattering and the Moho reflection were matched closely. Also, the random lower-crustal model produced a complex coda following the main arrivals that was similar to energy observed in the field records.

- Observations of $P_{m}P$ on a dense receiver array deployed in the Basin and Range Province show an marked increase of lateral coherence with increasing offset. In general, however, lateral coherence was much less than that predicted by theory. The discrepancy can be attributed to energy scattered from inhomogeneities above the target.

GEOLOGIC MODELS OF DEEP REFLECTORS

As noted in the first chapter, the physical nature of deep reflecting structures in the crust and at the Moho is the subject of much current research and debate. Understanding the reflection mechanism for a specific target will require a shift in current interpretation technique: Qualitative descriptions of reflection patterns will have to be replaced with quantitative measures of reflection character, such as absolute reflection strength, variation of amplitude and frequency content with offset, and measures of the scale of lateral heterogeneity.

Currently, the structures and physical conditions that give rise to deep reflections can be grossly divided into three categories: 1) Ductile shear zones or structures produced by ductile strain banding; 2) zones of compositional variation; and 3) the presence of free
fluids in various types of structures. In this section, I briefly review models of each of the three, emphasizing the scales of lateral variation and the likely contrasts in material properties at the reflector. In the next section, I consider how models of random heterogeneity can be applied to the interpretation of these types of structures.

**Ductile Shear Zones**

The reflectivity of ductile shear zones is inferred from the correlation of surface faulting with reflection events at great depth in the crust. Reflections marking major faults such as the Wind River Thrust and the Sevier Desert Detachment can be traced from surface outcrop into the middle crust (Smithson et al., 1979; Allmendinger et al., 1983). Below the brittle-ductile transition, brittle faulting is thought to give way to zones of ductile shear (Sibson, 1977). Shear zones have also been invoked to explain dipping reflectors in extensional regimes where correlative surface geology is not available (Reston and Blundell, 1987).

The structure of ductile shear zones has been examined where such features outcrop in metamorphic core complexes (e.g., Hurich et al., 1985). From the geometry of the core complexes, these zones are taken to be exhumed deep fault zones and are characterized by the presence of mylonitic rocks. The mylonite zones tend to be extensive planar features that separate bodies of unmylonitized crystalline rocks (Fountain et al., 1984). Within the shear zones, mylonites can be interbedded with unmylonitized rocks in a laminar or anastomosing fashion (Fountain et al., 1984).

Typical thicknesses for mylonite zones are 1-2 km, with the scale of interbedding mentioned above ranging from less than a meter to hundreds of meters (Hurich et al., 1985). At lower crustal depths, anastomosing shear zones may define lens-shaped bodies
with scale in the range of kilometers (Sibson, 1977; Frost and Okaya, 1985; Reston, 1987).

Fountain et al. (1984) attribute the reflectivity of mylonite zones to contrasts in mineralogy relative to the surrounding rocks and to the preferred orientation of mineral grains within individual mylonitized units. Their data, and that of Jones and Nur (1984), show that strongly oriented grains of quartz and phyllosilicates can produce anisotropy of 2-20%, with slower velocities for propagation normal to the mylonitic fabric.

Fountain et al. (1984) conclude that both contrasting mineralogy and anisotropy will reduce the velocity in mylonitized rocks slightly relative to surrounding units. Their model of such shear zones includes multiple interfaces with low (0.03) reflection coefficients and a 2-D geometry taken from a mapped zone; synthetic seismograms show multi-cycle reflections with lateral continuity over kilometers.

*Compositional Variation*

Contrasting composition is a feature found in several outcrop studies of presumed lower crustal rocks (Smithson and Brown, 1977; Fountain and Salisbury, 1981; Percival, 1986; Hurich and Smithson, 1987). One aspect of compositional change is a regional increase in metamorphic grade, which is a primary identifier of such zones as exhumed crustal sections (Fountain and Salisbury, 1981). While metamorphic layering represents compositional change on the scale of kilometers, the sections summarized by Fountain and Salisbury (1981) also show compositional variation both vertically and horizontally on the scale of meters to hundreds of meters. Mechanisms for these variations include the tectonic emplacement of metamorphosed supracrustal rocks and igneous intrusion caused by partial melting of lower crustal material (Smithson and Brown, 1977; Fountain and Salisbury, 1981). Note that igneous intrusion into higher regions of the crust can produce isolated sills
with vertical scales of hundreds of meters over lateral distances of kilometers (Goodwin et al., 1988).

In the model of Smithson and Brown (1977), the composition of the lower crust can range from that of granite to that of gabbro. From the model of Hurich and Smithson (1987) shown in Figure 1-2 is based on mapping of the Ivrea Zone and shows typical reflection coefficients in the range 0.05–0.10. Note that the smallest reflection coefficients in this model (0.03) are the same as those used for individual interfaces in the mylonite zone model of Fountain et al. (1984).

*Fluids in the Deep Crust*

The presence of fluids (other than magmas) as the cause of reflections at mid- and lower crustal depths is the most controversial of the three models discussed here. Standard petrologic arguments suggest that the existence of anhydrous metamorphic rocks (granulites) in the lower crust requires that water is not present at those depths (Smithson and Brown, 1977; Fyfe, 1986). Also, estimates of permeability by Jones and Nur (1984) suggest that any water in the lower crust would diffuse out in geologically short time periods.

Other investigators suggest that water is widely present based on the relatively high electrical conductivity at lower crustal depths (Klemperer, 1987; Lüschen et al., 1987). Fyfe (1986) discusses how processes of subduction, continental collision, and magmatic underplating could contribute large volumes of water to the lower crust on a roughly continuous basis. Etheridge (1988) has also discussed how water might be trapped at intergrain boundaries in units of limited porosity; he also suggests that structural weakening due to elevated pore pressure could cause local hydro-fracturing and a subsequent increase in porosity.
Since the presence of free fluids is a secondary geologic phenomenon not visible in outcrop, it is difficult to predict the spatial distribution and relative velocity anomalies they might produce. Klemperer (1987) reviews models that suggest fluids would preferentially, but not necessarily, accumulate along the lines of pre-existing composition. The migration of fluids is also likely controlled by patterns of ductile shear, moving more easily along zones of relative weakness. Thus, Klemperer (1987) suggests that fluids could act to enhance the reflectivity of shear zones and zones of variable composition. The effect of fluid in any environment will depend on local porosity, but fluids are thought capable of producing extremely large reflection coefficients. For instance, a strong reflection (estimated coefficient, 0.18-0.36) from 15-18 km depth has been speculatively interpreted as caused by fluids in local porosity (Wille et al., 1985).

FURTHER GEOLOGIC AND SEISMIC STUDIES

The preceding discussion of models for deep-crustal reflectivity can be summarized in a table shown below, which lists the ranges of lateral variability and the strength of velocity contrasts.

<table>
<thead>
<tr>
<th></th>
<th>Shear zones</th>
<th>Composition</th>
<th>Free fluids</th>
</tr>
</thead>
<tbody>
<tr>
<td>Horizontal scale</td>
<td>≥ km's</td>
<td>10's m - km's</td>
<td>10's m - 10's km</td>
</tr>
<tr>
<td>Velocity anomalies</td>
<td>low</td>
<td>low - high</td>
<td>med. - v. high</td>
</tr>
<tr>
<td>Reflection coefficients</td>
<td>&lt; 0.05</td>
<td>0.01 - 0.10</td>
<td>0.05 - 0.10+</td>
</tr>
</tbody>
</table>

Clearly, lateral scale and the degree of velocity contrast are not, by themselves, sufficient to differentiate the three models in all cases. The ambiguity evident here is similar
to that present in the conventional interpretation rock composition based on velocity values. Thus, other information, such as that provided by shear wave reflection surveys, is necessary for a more complete delineation of subsurface properties (Wenzel et al., 1986).

As with surveys of the upper crust, the interpretation of seismic characteristics must be conducted within a larger geologic framework. The fact that the table above is largely qualitative suggests that much remains to be known about the physical properties of deep structures. Quantitative estimates of these properties derived from detailed seismic studies may have considerable diagnostic power; the success of such studies will require the synthesis of knowledge provided by seismologists, rock physicists, and surface geologists.

The application of lateral coherency analysis as presented here will first require geologic (e.g., outcrop) studies to determine the statistics of lateral heterogeneity for targets such as those expected in the lower crust. With detailed velocity measurements in two (or three) dimensions, we could then attempt to characterize various geologic features with a combination of layered and random structures. Let me emphasize again that layered and isotropically-random structures should be considered together as end-members in the description of geologic targets. Outcrop-based models, such as Figure 1-2, show that one-dimensional fine-scale layering alone is not an acceptable model of lower-crustal heterogeneity; as shown in the Black Forest field data example (Chapter 4), seismic interpretations conducted within the framework of such models result in extreme estimates of lower-crustal properties.

The goal of outcrop studies would then be to characterize the magnitude and spatial scale of variations in material properties through laboratory measurements, and to correlate these data with geologic descriptions of composition and structure. To characterize the complete heterogeneity of a particular target, multiple measurements of physical properties should be made within apparently homogeneous rock units. Preliminary studies of such
redundant measurements suggest that rocks of nominally constant composition may have random velocity variations of 1-2% (Fountain, 1988, personal communication); this degree of heterogeneity is a significant fraction of the 3% standard deviation used here to model the lower crust in the Black Forest (Chapter 4).

The seismic responses of detailed velocity models could then be tested and compared with appropriate numerical modeling techniques. One standard goal of seismic modeling is to assess what level of detail can be interpreted by conventional means. For instance, the seismic response computed for Figure 1-2 (produced by Hurich and Smithson, 1987; shown here as Figure 5-1), reveals that structural elements having scales less than 2 or 3 km cannot be directly identified; the fine scale structure in Figure 1-2 obviously lies below the conventional resolution limit of the seismic method. The ambiguity of current lower crustal interpretations is thus partly attributable to the fact that salient structural details cannot be resolved.

One goal of computing synthetic data for detailed, geologically-based models would be to provide more realistic examples of expected seismic responses; as noted below, previous modeling studies have various shortcomings. The other important goal would be to assess whether statistical analyses of the fine-scale structure of the seismic response provide useful information for the interpretation of fine-scale structure of the target. Since geologic targets have not generally been described in statistical terms, a body of information on various types of geologic features needs to be assembled before the utility of statistical descriptions can be assessed.

Further work in seismic data acquisition and processing techniques will also be important in studies of heterogeneous targets throughout the crust. The field data examples in Chapters 3 and 4 have shown that seismic scattering at all levels of the subsurface complicates the interpretation of coherency measurements. Accounting for these
complications in field-recorded data thus requires understanding the heterogeneity of all structures above the target of primary interest.

Parameters for current field experiments are most often chosen to optimize the image of features at a particular depth. In future experiments, a combined survey strategy should be adopted to image the entire crustal section. A combined survey over a given target might consist of high-resolution profiling to define irregularities in the near-surface, conventional CMP reflection profiling to image the upper and middle crust, and large explosive reflection/refraction shooting to image deeper structures. Coincident CMP profiling and reflection/refraction shooting have been used in studies such as that described by Goodwin et al. (1988) and Mooney et al. (1988). High-resolution studies of the near-surface would be an important addition when lateral coherence is analyzed: near-surface irregularities can disrupt the coherence of reflected wavefield and generate noise in the process.

Since the cost per distance of combined surveys can be considerably more than that of conventional experiments, the location of such surveys is particularly important. Detailed seismic work is most efficiently applied where reconnaissance surveys have identified an interesting target and have given some indication of the required seismic parameters. Also, detailed seismic experiments could be most profitably employed in areas where studied outcrops can be traced into the subsurface; thus seismic models developed from outcrop data could be tested in situ at various depths of burial. This type of integrated study has been undertaken by the LITHOPROBE group in Canada, with CMP profiling over areas of the Kapuskasing Uplift (Fountain, 1988; Green et al., 1988).

Numerical modeling should be an important step in many stages of future studies, in that synthetic data can be used to plan acquisition parameters, guide processing choices, and confirm interpretations. While many modeling studies of crustal targets have been undertaken (see Chapter 1), all have shortcomings in terms of assessing crustal heterogeneity: In some studies (e.g., Wenzel et al., 1987), velocity models are not
constrained by geologic observations. In others (e.g., Hurich and Smithson, 1987), geologically-based structures are modeled with techniques that require constant layer velocities and cannot account properly for heterogeneity within the target. Realistic modeling of crustal targets requires a sophisticated technique such as finite-difference method; finite-difference methods can properly model the seismic response of targets with heterogeneity at all reasonable scales.

As noted above, advanced seismic experiments will cost more than those currently conducted. Seismic modeling will add to these costs, since the calculation of synthetic data for realistic models requires supercomputer power. The results of this research project suggest that such costs will be inevitable if lower crustal structure and composition are to be understood: Conventional acquisition, processing, and interpretation practice cannot distinguish between alternative models. Increased cost will likely necessitate careful surveys of restricted extent, rather than the extensive reconnaissance coverage currently employed.
Figure 5-1. Synthetic seismic section computed for the velocity model shown in Figure 1-2. Reproduced from Hurich and Smithson (1987).
References


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Appendix A

Convolution Operators in Wavefields and Coherence Functions

In this appendix, I detail the steps that lead from (4.13) to (4.14) in the text. Assuming an average zero-offset smoothing filter for some window, let us rewrite (4.13) and drop the explicit notation of traveltimes $T$,

$$p_{z0}(x,t) = p_{prs}(x,t) * f_{z0}(x,t) \quad .$$

(A1)

Using the definition of the coherence function $\gamma(\delta,\omega)$ in (3.5), we wish to show that

$$\gamma_{z0}(\delta,\omega) = \frac{|\Phi_{prs}(\delta;\omega) * \Phi_{fz0}(\delta;\omega)|}{|\Phi_{0}(\omega)|} \quad ,$$

(4.14)

where $\Phi_{prs}$ and $\Phi_{fz0}$ are correlation functions for $p_{prs}$ and $f_{z0}$ at fixed $\omega$, as given by (3.6).

Derivation of (4.14) requires a general theorem for 2-D correlation functions, namely that if a function $a(x,t)$ can be represented by the 2-D convolution of two other functions,

$$a(x,t) = b(x,t) * c(x,t) \quad ,$$

(A2)

then,

$$\phi_{a}(\delta,\tau) = \int \int a(x,t) a(x+\delta,t+\tau) \, dt \, dx$$

(A3)

$$= \phi_{b}(\delta,\tau) * \phi_{c}(\delta,\tau) \quad .$$
To simplify notation let us represent the definite integrals (with limits \( \pm \infty \)) with an operator notation

\[
\langle \rangle_x = \int_{-\infty}^{\infty} d \, x
\]

and rewrite (A2) as

\[a(x,t) = \langle \langle b(x',t') c(x-x',t-t') \rangle_x \rangle_{x'} > t' . \tag{A4}\]

Substituting (A4) into (A3),

\[
\phi_a(\delta, \tau) = \langle \langle \langle b(x',t') c(x-x',t-t') \rangle_x \rangle_{x'} > t' . \\
\langle \langle b(x'',t'') c(x+\delta-x'',t+t-t'') \rangle_x \rangle_{x''} > t'' \rangle_{x'} > t .
\]

Now define \( \delta' = x''-x' \) and \( \tau'' = t'' - t' \) and rearrange terms,

\[
\phi_a(\delta, \tau) = \langle \langle \langle b(x',t') b(x'+\delta',t'+\tau') c(x,t) c(x+\delta-\delta',t+\tau-\tau') \rangle_x \rangle_{x'} > t' > \delta > \tau > x > t .
\]

then execute the \( x' \) and \( t' \) integrals to get

\[
\phi_a(\delta, \tau) = \langle \langle \phi_b(\delta', \tau') c(x,t) c(x+\delta-\delta',t+\tau-\tau') \rangle_x > t > \delta > \tau > x > t ,
\]

Continuing with the \( x \) and \( t \) integrals produces,
\[
\phi_a(\delta, \tau) = \langle \phi_b(\delta', \tau') \phi_c(\delta-\delta', \tau-\tau') \rangle \delta \tau',
\]

which clearly has the form of the desired convolution in (A3).

Now apply a Fourier transform over \( \tau \) to (A5). The transform can be represented by

\[
\langle \exp(-i\omega \tau) \rangle \tau,
\]

and after doing the \( \tau \) and \( \tau' \) integrals,

\[
\Phi_a(\delta, \omega) = \langle \Phi_b(\delta', \omega) \Phi_c(\delta-\delta', \omega) \rangle \delta,
\]

\[
= \Phi_b(\delta; \omega) \ast \Phi_c(\delta; \omega),
\]

where \( \omega \) is shown as a parameter to emphasize that the convolution is one-dimensional in space. The result in (A6) can be applied in the definition of spatial coherence for the zero-offset section, replacing \( a, b, \) and \( c \) with \( p_{zo}, p_{prs}, \) and \( f_{zo}, \) respectively,

\[
\gamma_{zo}(\delta, \omega) = \frac{|\Phi_{zo}(\delta; \omega)|}{|\Phi_0(\omega)|}
\]

\[
= \frac{|\Phi_{prs}(\delta; \omega) \ast \Phi_{fzo}(\delta; \omega)|}{|\Phi_0(\omega)|}.
\]

As noted in the text, the denominator serves only to normalize the definition of \( \gamma \), and thus any variation in lateral coherence caused by the filter \( f_{zo} \) will be reflected in the numerator. For any coherence analysis, the denominator is equal to the numerator with \( \delta = 0 \).
Appendix B

Energy Distribution Function for a Zero-Offset Section

The energy distribution function for zero-offset recording can be developed in the same manner as for the common-shot geometry discussed in the text. The only significant difference in the two cases results from the fact that in a zero-offset section, the source position is not fixed, but moves with the receiver. The zero-offset energy distribution $E_{zo}$ will be represented as the product of a geometric term $G_{zo}$ and a spreading term $S$,

$$E_{zo} = G_{zo}(p) \ S(p) \ . \tag{B1}$$

The common-shot directivity term $D(p)$ has been dropped here because it is constant for all scatterers in zero-offset recording (scattering angle is always $180^\circ$).

The geometric term is derived from the zero-offset traveltime relations. Using the geometry in Figure B-1 traveltime is given by

$$t = \frac{2r}{V} \ ,$$

and the energy from a fixed scatterer appears on the zero-offset section with slowness

$$p = \frac{dt}{dx} = \frac{2x}{rV} = \frac{2 \ \sin \ \theta}{V} \ . \tag{B2}$$

For a fixed traveltime $T$, the scattering curve will be the arc of a circle given by the expression
\[ r^2 = x^2 + z^2 = \frac{V^2 T^2}{4} = \text{constant} \quad , \quad \text{for } z > D. \tag{B3} \]

As before, the geometric term will be written in two parts,

\[ G_{z0}(p) = \left| \frac{ds}{dx} \frac{dx}{dp} \right| , \]

where from (B2) and (B3)

\[ \frac{ds}{dx} = \sqrt{1 + \left( \frac{dz}{dx} \right)^2} = \sqrt{1 + \frac{x^2}{z^2}} , \tag{B4} \]

and

\[ \frac{dp}{dx} = 2 \frac{d}{dx} \left( \frac{x}{r} \right) = 2 \frac{1}{rV} \quad . \tag{B5} \]

Finally, the spreading term for a line source and line scatterer is \( S(p) = r^2 \); this spreading is constant over \( p \) but varies with traveltime. The complete zero-offset energy distribution (B1) can then be evaluated for a given target depth using the numerical procedure outlined in the text for the common-shot case.
Figure B-1. Geometry of the scattering curve for the zero-offset case. The top of the random zone is located at a depth of $z=D$. 

Appendix C

Effects of NMO Corrections on the Common-Shot Energy Distribution Function

The application of NMO corrections in a common-shot trace gather amounts to a variable trace-by-trace stretching of the time axis. This stretching can be expressed by the change of variable

\[ t' = \sqrt{t^2 - \frac{H^2}{V^2}} \quad , \quad \text{(C1)} \]

where \( t' \) represents NMO-corrected time, \( t \) is traveltime before NMO correction, \( H \) is source-receiver offset, and \( V \) is again velocity. Distortion of the time axis will change the distribution of slowness values observed and thus will change the energy-slowness distribution function. In this appendix, I show how the NMO-corrected energy distribution is related to its uncorrected counterpart.

To derive the relationship, two effects need to be accounted for: First, NMO corrections will change the value of slowness observed for a particular scatterer from \( p \) to \( p' \), where

\[ p' = \frac{dt'}{dH} = \frac{1}{t'} \left( \frac{dt}{dH} - \frac{H}{V^2} \right) = \frac{1}{t} \left( tp - \frac{H}{V^2} \right) \quad . \quad \text{(C2)} \]

This transformation amounts to a shifting and stretching of the slowness axis according to the magnitude of the correction made.
Second, stretching the p axis will affect the magnitude of $G(p)$ in the definition of $E(p)$ in (4.5). After NMO corrections, the new geometric factor will be

$$G'(p') = \left| \frac{ds}{dx} \frac{dx}{dp} \right|,$$

(C3)

where $\frac{ds}{dx}$ is the same as before, but

$$\frac{dx}{dp'} = \frac{t'}{dp},$$

from (C1) and (C2). All other factors in the definition of $E(p)$ are changed only by the mapping of $p$ to $p'$.

If partial NMO corrections are made to some reference offset $H_0$, equation (C1) becomes

$$t' = \sqrt{t^2 - \frac{H^2 - H_0^2}{V^2}}.$$

(C5)

The expressions for $p'$ and $G'(p')$ remain as shown in (C2) and (C3), since $t'$ now represents a partially corrected traveltime.

Finally, we should note that even partial NMO corrections produce some amount of NMO stretch (or compression) that will affect measured coherence values. Distortion of the time axis will cause the frequency of a trace to shift upwards for offsets less than the reference offset, and conversely, frequencies at greater offsets will be shifted lower. In computing predicted coherence values for comparison with those measured on the finite-difference data, energy distribution functions were computed for a range of offset and traveltime points within the analysis window. The distribution functions were then
transformed according to (4.21) and averaged. To account for NMO stretch, the frequency used in (4.21) was modified according to the expression below.

From equation (C5) the NMO distortion at a given offset and travelt ime is

\[ \frac{dt'}{dt} = \frac{t}{t'} , \]  

(C6)

where \( t' < t \) implies stretching and \( t' > t \) implies compression. From (C6), we can see that the frequencies will be shifted according to

\[ \omega' = \frac{t'}{t} \omega . \]  

(C7)

The transformations of individual energy distribution functions with (4.21) were thus performed so that the observed frequency \( \omega' \) remained fixed at 10 Hz. That is, if a trace is stretched during NMO correction \( (t' < t) \), energy observed at \( \omega' = 10 \) Hz will have the correlation properties of a higher frequency \( \omega \), given by rearranging (C7). The amount of distortion caused by the stretching was less than 10% in all windows, as indicated in the text.
Appendix D

Signal-to-Noise Ratios and the Coefficient of Coherence

Foster and Guinzy (1967) discuss the standard interpretation of the coefficient of coherence in terms of signal-to-noise ratio (SNR); in their model, the signal component is considered to be identical on all traces and noise is assumed to be completely uncorrelated from trace to trace. In this appendix, I review the standard model and use it to show the expected effects of CMP stacking on measured values of lateral coherence (as discussed in Chapter 3). I then extend the model to the more general case where signal (i.e., the reflection response) and noise are both partially correlated. The extended model can then be used, 1) to correct observed coherence values for the effects of correlated and uncorrelated noise (Chapter 3), and 2) to adjust predicted values of lateral coherence for the presence of noise in cases where the reflected signal is known to be only partially correlated (Chapter 4).

In the standard model (Foster and Guinzy, 1967), the coefficient of coherence provides a simple measure of the correlatable and uncorrelatable power in two time series. Consider two traces $x(t)$ and $y(t)$ that contain an identical signal $s(t)$ and are each contaminated with noise,

$$
\begin{align*}
x(t) &= s(t) + \alpha n_1(t) \\
y(t) &= s(t) + \alpha n_2(t)
\end{align*}
$$

where the signal power is unity, the noise power is $\alpha^2$, and noises $n_1$ and $n_2$ are uncorrelated. At a given frequency, the coefficient of coherence for $x$ and $y$ is
\[ \gamma = \frac{1}{1 + \alpha^2}, \]

which is the ratio of correlatable power to total power for each trace. This result is based on the assumption of Gaussian signal and noise, but Foster and Guinzy (1967) show that the Gaussian assumption is not critical and that (D2) can be generally applied to seismic data.

Since the signal in (D1) has unit power, the quantity

\[ \alpha^2 = \frac{1 - \gamma}{\gamma}, \]

also represents the ratio of uncorrelatable power to correlatable power for a trace. In the text and this appendix, SNR is quoted in terms of amplitudes and in the following it will be denoted by the variable R. For the model (D1), \( \alpha^2 \) is the inverse of the SNR expressed in terms of power, and thus

\[ R = \frac{1}{\alpha} = \sqrt{\frac{\gamma}{1 - \gamma}}. \]

In Chapter 3, measured lateral coherence values (Figure 3-23a) for unstacked and stacked data were compared to study the effects of dip-filtering and noise reduction in the stacking process. Standard theory predicts that stacking \( N \) traces will improve the SNR by a factor of \( \sqrt{N} \) (Yilmaz, 1987). Thus any coherence value for unstacked data can be adjusted for the theoretical effects of stacking by, 1) converting the coherence value to \( R \) according to (D4), 2) multiplying \( R \) by \( \sqrt{N} \), and then 3) converting the modified SNR back to coherence. This approach was used to predict the coherence values shown in Figures 3-23a and 3-23b for various stack multiplicities.
In other aspects of this research project, the model (D1) has obvious limitations, since the signal reflected from a heterogeneous target will not correlate perfectly from trace to trace, even in the noise-free case. Furthermore, as discussed previously, seismic noise caused by near-surface scattering will be spatially correlated to some extent. To account for signal and noise that may be partially correlated, the basic model (D1) will be extended and rewritten as

\[ x(t) = s(t) + \alpha_r n_1(t) + \alpha_c n_c(t) + \alpha_u n_2(t) \]
\[ y(t) = s(t) + \alpha_r n_3(t) + \alpha_c n_c(t) + \alpha_u n_4(t) \]

where the first two terms on the right-hand side represent the correlatable and uncorrelatable components of the reflected signal and the second two terms represent the same components of the noise. Note again that \( s(t) \) and \( n_c(t) \) are identical on the two traces, \( s(t) \) has unit power, and none of the six component time series on the right-hand side cross-correlate with one another. Relations derived from the extended model will be used to address two issues in the interpretation of lateral coherence values in Chapters 3 and 4.

In Chapter 3 (Figure 3-23), coherence values measured on unstacked traces are not affected by stacking smear but they are contaminated with correlated and uncorrelated noise. The noise effects must therefore be compensated before assessing the heterogeneity of reflectors in Zone A. As shown in the next section, the correlation properties of the noise field can be estimated from the data for Zone B (Figure 3-23b), by assuming that that reflector is specular (i.e., laterally constant). In the following section, I demonstrate how the effects of correlated noise can be removed from the the observed coherence values, so that adjusted coherence values can be interpreted in terms of an uncorrelated noise field. In the final section, I develop corrections for the presence of uncorrelated noise.
The results of the last section are also applied in Chapter 4 (Figure 4-28), where the reverse process is required: Noise-free predicted values for reflected signal are modified for the presence of various amounts of uncorrelated random noise.

ESTIMATION OF NOISE COHERENCE

The lateral coherence function for Zone B shown in Figure 3-23b is reproduced in Figure D-1. As indicated by the dotted lines in Figure D-1, coherence values at midpoint separations greater than about 5 are roughly constant in the range 0.50 - 0.55. Given this result, and the stacked results discussed in the text, let us assume that the reflectors in Zone B are specular and contaminated with noise at an SNR of 1.0 - 1.1 (γ = 0.50 implies SNR = 1). We will further assume that any coherence above the level of 0.55 is caused by correlation in the noise field and that the total noise power is the same on all traces. The lateral coherence function of the noise can then be calculated using the relations developed below.

At the base-level coherence γ₀ = 0.55, all noise is uncorrelated and the noise power α₀² is given by (D3) with γ = γ₀. At midpoint separations in Figure D-1 where γ > γ₀, the noise is spatially correlated but recall that the total noise power (correlated plus uncorrelated) remains the same. Since for a specular reflector, α_r = 0, the model in (D5) can then be rewritten as

\[ x(t) = s(t) + \alpha_c n_c(t) + \sqrt{\alpha_0^2 - \alpha_c^2} n_1(t) \]
\[ y(t) = s(t) + \alpha_c n_c(t) + \sqrt{\alpha_0^2 - \alpha_c^2} n_2(t) \]  \hspace{1cm} (D6)
As noted earlier, for any observed coherence $\gamma$, the quantity $\alpha^2$ given by (D3) can be interpreted as the ratio of uncorrelated to correlated power between two traces. Thus, for (D6),

$$\alpha^2 = \frac{\alpha_0^2 - \alpha_c^2}{1 + \alpha_c^2} ,$$  \hspace{1cm} (D7)

so that

$$\alpha_c^2 = \frac{\alpha_0^2 - \alpha^2}{1 + \alpha^2} .$$  \hspace{1cm} (D8)

The coefficient of coherence for the noise (i.e., ratio of correlated noise power to total noise power) is then

$$\gamma_N = \frac{\alpha_c^2}{\alpha_0^2} .$$  \hspace{1cm} (D9)

The lateral coherence function for the noise is then computed by applying (D8) and (D9) to all Zone B coherence values in Figure D-1 where $\gamma > \gamma_0$. At other midpoint separations, the lateral coherence of the noise is assumed to be zero.

The computed coherence function for the noise (shown as closed triangles, Figure D-1) decreases smoothly to zero at a midpoint separation of 6. Values of the base-level coherence lower than 0.55 would produce small, non-zero coherence at greater lags; the 0.55 value was chosen to address the most significant deviation of the coherence curve in Figure D-1.
ADJUSTMENT FOR CORRELATED NOISE

Given the estimated coherence of the noise $\gamma_N$, we can now develop an adjustment for the observed coherence of reflections $\gamma$. This adjustment will remove the variations of lateral coherence caused by noise correlation, so that noise correlation is not interpreted as variability at the target. The resulting coherence values can be interpreted in terms of a partially-correlated reflection signal contaminated by uncorrelated noise.

For both the reflections and the noise, coherence can again be expressed in terms of the quantities $\alpha$ and $\alpha_N$ according to (D3). As noted before, these quantities each represent the ratio of uncorrelated power to correlated power. Thus from the model (D5),

$$\alpha^2 = \frac{\alpha_r^2 + \alpha_u^2}{1 + \alpha_c^2}.$$  \hspace{1cm} (D10)

Similarly, for the noise,

$$\alpha_N^2 = \frac{\alpha_u^2}{\alpha_c^2}.$$  \hspace{1cm} (D11)

Making the correction for noise correlation requires the determination of $\alpha_r$, $\alpha_u$, and $\alpha_c$. To provide the required third relation for this solution, let us specify the overall signal-to-noise ratio,

$$R^2 = \frac{1 + \alpha_r^2}{\alpha_c^2 + \alpha_u^2}.$$  \hspace{1cm} (D12)

From (D10), (D11), and (D12), we can then find the power of the correlated noise,
\[ \alpha_c^2 = \frac{1 + \alpha^2}{(R^2+1)\alpha_N^2 + R^2 - \alpha^2} . \quad \text{(D13)} \]

Given \( \alpha_c^2 \), \( \alpha_u^2 \) can be determined from (D11) and \( \alpha_r^2 \) can then be determined from (D12). With these three quantities, the modified coherence value is given by

\[ \gamma' = \frac{1}{1 + \alpha_r^2 + \alpha_c^2 + \alpha_u^2} . \quad \text{(D14)} \]

Expression (D14) now represents the ratio of correlatable signal power to total power (signal plus noise). Since the total noise power \( (\alpha_c^2 + \alpha_u^2) \) is constant, variations of \( \gamma' \) with midpoint separation now depend only on the correlation properties of reflected signal (through the quantity \( \alpha_r \)). Note that the original coherence value \( \gamma \) has the form of (D14) with an extra term \( \alpha_c^2 \) added in the numerator.

Modified coherence values for the specular reflectors of Zone B are shown in Figure D-1 as open circles. The modified values were determined from (D14) using a total SNR of \( R = 1.0 \). Note that the adjustment only affects coherence values at separations less than 6 midpoints (where \( \gamma_N \neq 0 \)). Over all midpoint separations, the adjusted curve is consistent with the interpretation of a specular reflector contaminated with noise at an SNR near 1. The choice of \( R = 1.0 \) for the correction was based on the overall level of coherence at large separations; a value of \( R = 1.1 \) would have produced adjusted values that were constant at \( \gamma = 0.55 \) for separations of 1 - 5, but the difference of 0.1 in \( R \) cannot be considered significant.

In the text, the adjustment was applied to coherence values calculated for Zone A (Figure 3-26), also using \( R = 1.0 \). The choice of \( R = 1.0 \) was justified on the grounds that, 1) the rms amplitudes in Zones A and B are within 20% of each other; 2) the scattering of
upcoming reflections is a likely mechanism for generating seismic noise, so that noise amplitude would be directly related to reflection amplitude; and 3) the value $R = 1.0$ results in adjusted values that best follow the trend of other values.

The sensitivity of the adjustment to the assumed SNR is shown in Figure D-2. Here, the observed lateral coherence values are shown as closed circles, with adjusted values shown for SNR's of 2, 1, 0.5 and 0. All possible adjustments fall between the original curve and that defined for SNR $= 0$. The curve for SNR $= 1$ best matches the trend defined at larger midpoint separations; note that corrections produced by values of SNR in the range 0.8 - 1.2 would also follow the trend reasonably well. I should emphasize that the adjusted values for Zone A are also dependent on the assumption that Zone B has laterally-constant structure. Any assessment of lateral correlation for Zone A thus depends on assumptions about Zone B in the same way that paleo-structure determinations depend on the flattening of key horizons.

**ADJUSTMENT FOR UNCORRELATED NOISE**

Having adjusted for noise correlation, the seismic trace model (D5) can now be simplified. The model now includes partially-correlated reflection signal contaminated by uncorrelated noise,

\[
\begin{align*}
    x(t) &= s(t) + \alpha_r n_1(t) + \alpha_u n_2(t) \\
    y(t) &= s(t) + \alpha_r n_3(t) + \alpha_u n_4(t) 
\end{align*}
\]

(D15)

In terms of this model, an observed coherence value would be defined as
\[ \gamma = \frac{1}{1 + \alpha_t^2 + \alpha_u^2}. \]  

(D16)

From the standard definition of the signal-to-noise ratio,

\[ \alpha_u^2 = \frac{1 + \alpha_t^2}{R^2}. \]  

(D17)

so that (D16) can be rewritten as

\[ \gamma = \frac{1}{1 + \alpha_t^2} \frac{R^2}{R^2 + 1}. \]  

(D18)

Now the noise-free coherence value \( \gamma_{nf} \) is given by (D16) with \( \alpha_u = 0 \), so that

\[ \gamma_{nf} = \frac{1}{1 + \alpha_t^2} = \gamma \frac{R^2 + 1}{R^2}. \]  

(D19)

The complete adjustment for correlated and uncorrelated noise is thus accomplished by first applying (D11 - D14) followed by (D19) with \( \gamma = \gamma \). In order to apply (D19), a value of \( R \) must again be provided. When reflectors appear to be specular, as in Figure D-1, the necessary value can be computed from the (nearly-) constant coherence using (D4). For specular reflectors, however, the correction is unnecessary, since, we know that a noise-free specular reflector would have a constant coherence of 1.

For the laterally-variable reflections in Zone A (Figure 3-26), the value of \( R \) is determined by extrapolating the trend of lateral coherence to a midpoint separation of zero. Extrapolation to zero provides an estimate of coherence when reflected signal is perfectly correlated \( (\alpha_t = 0) \) and noise is completely uncorrelated; thus \( R \) can be determined from the
standard model (D1) and relation (D4). The extrapolation is necessary because at zero midpoint separation, only one trace is available for analysis and thus measured coherence will always be 1.

In Figure 3-26, the intercept is 0.65 gives R = 1.36; this value differs somewhat from the R = 1.00 used to adjust for noise correlation in the previous section. The discrepancy may be related to errors in characterizing the noise correlation for Zone A. The value R = 1.36 was used to estimate the noise-free values in Figure 3-26, since using R = 1.00 in (D19) produces values of $\gamma_{nf}$ greater than 1.

Expression (D19) was also used in Chapter 4 to predict the effects of contamination by uncorrelated noise (Figure 4-28). In that case, noise-free coherence values were computed for a common-shot geometry. The noise-free values were then modified for specified values of R, using (D19) to calculate $\gamma$. 
Figure D-1. Closed circles: Lateral coherence values measured on the unstacked data in Zone B. Closed triangles: Estimated lateral coherence for noise, assuming that coherence levels above 0.55 result from noise correlation. Open circles: Adjustment of the unstacked values for the presence of correlated noise, assuming an SNR of 1.
Figure D-2. Coherence values for the unstacked data in Zone B (closed circles) adjusted for the presence of correlated noise using the noise coherence function in Fig. D-1 and various choices for the total SNR. The correction using SNR = 1 (open circles) best fits the trend of values defined at larger trace separations.