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Microwave and optical sensor fusion for the shape extraction of 3D space objects

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Rice University, 1988
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MICROWAVE AND OPTICAL SENSOR FUSION FOR THE SHAPE
EXTRACTION OF 3D SPACE OBJECTS

by

SCOTT W. SHAW

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

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Houston, Texas
April, 1988
ABSTRACT

Microwave and Optical Sensor Fusion for the Shape Extraction of 3D Space Objects

Scott W. Shaw

Two sensors that have been proposed for use on a space robot are cameras and radar. Considered individually, neither of these sensors provides enough information for a computer to derive a good surface description of a remote object. Their combination, however, can produce a complete surface model.

The lack of atmosphere in space presents special problems for optical image sensors. Frequently, edges are lost in shadow and surface details are obscured by diffraction effects caused by specularly reflected light. An alternate sensor for space robotic applications is microwave radar. The polarized radar cross-section (RCS) is a simple, well-understood, microwave measurement that contains limited information about a scattering object’s surface shape.

These two data sets are fused through an error minimization procedure. First, an incomplete surface model is derived from the camera image. Next, the unknown characteristics of the surface are represented by some parameter. Finally, the correct value for this parameter is computed by iteratively generating theoretical predictions to the RCS and comparing them to the observed value.
A theoretical RCS may be computed from the surface model in several ways. One such RCS prediction technique is the method of moments. The method of moments can be applied to an unknown surface only if some shape information is available from an independent source. Here, the camera image provides the necessary information. When the method of moments is used to predict the RCS, the error minimization algorithm will converge in most cases.

By combining the microwave and optical information in this way, the shapes of some three-dimensional objects have been accurately recovered. Simulations and experiments were performed on plates, ellipsoids, and an arbitrary curved object. Simulations show that error in the recovered shapes is very small when the RCS measurement error is not too large. Experiments prove that the RCS can be measured within this tolerance.

In general, this investigation has shown the usefulness of sensor fusion applied to the shape reconstruction problem in space. Furthermore, a specific framework has been developed and proved effective for integrating the two types of sensors that are typically found on space vehicles.
ACKNOWLEDGEMENTS

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CHAPTER 1

Introduction

1.1. Motivation

To motivate sensor fusion for space applications, consider the following scenario. A free-flying, autonomous space robot is given the task of retrieving a small, specialized, and expensive wrench that was lost by an astronaut during an extra-vehicular repair activity. A satellite is positioned between the robot’s current location and the wrench. The robot must navigate around the satellite to get to the wrench. The robot is equipped with a video camera, an image processing/understanding system, a laser range scanner, and tactile sensors. The robot first uses its image interpretation system to locate the satellite in the video camera’s field of view and computes the positions of edges. To conserve fuel, the robot must take the shortest path around the satellite. This means navigating close to the edges of the obstacle. As the robot approaches the edge of the satellite as computed from the video image, its tactile sensors report the presence of an obstacle. Directly in front of the robot lies a solid barrier where the imaging system predicted free space. The robot now switches to its laser range scanning system and discovers the edge of the satellite extends two feet beyond that which was computed from the video image. The robot navigates to the new edge and once again encounters an obstacle. By using tactile sensors located on an end-effector, the robot is able to slowly
groped along the satellite until the true edge is found. Unfortunately, by this time, the wrench is out of retrieval range and the robot must give up its mission, abandoning 10,000 dollars of taxpayer’s money. The robot turns around and starts back to the orbiter. However, when the imaging system attempts to match the orbiter shape with the objects in the field of view, no match is found. The only large object is a high intensity disc of light emanating from the general area where the orbiter had been. The robot navigation system now gives up and is guided back to the orbiter by telemetry.

What went wrong with this robot mission? Why did the optical sensing systems fail to guide the robot around the satellite and back to the orbiter? The answer lies in the fundamental limitations of optical methods. There are three such limitations here: First, to form an optical image, the target object must be illuminated; second, the object must reflect light; and third, the reflected light must pass through lenses on its way to the image sensor. The first edge detected by the video camera was not a physical edge, but a shadow caused by an occlusion of the light source. The true satellite edge was not detected by the laser scanner because black paint had been applied along it, inhibiting reflection. On the return trip, the orbiter was obscured in the video image because a parabolic communications antenna caused an anomalously bright point of light. When this single intense point of light passed through the video camera lens, it caused a diffraction fringe, or Airy Disc[1] that completely obscured the rest of the orbiter.
Although this scenario describes a worst case, the problems encountered are all real. Optical techniques may be all that is needed in the earth's atmosphere. But in the vacuum of space, with less than ideal lighting conditions, other types of sensors are required. Tactile sensors are beneficial; both pressure and centroid sensors are available for use on end effectors. These sensors require that the robot be near to the unknown object. They are also limited by the slow speeds of the robot arm, and provide only point information. If one were presented with this problem on earth, the most obvious non-optical sensor type chosen would be one based on ultra-sonic imaging. Unfortunately, the extensive theory and hardware developed for ultra-sonic imaging is useless in space due to the lack of atmosphere in which sound waves may propagate. The only real alternatives to optical imaging in space are electromagnetic sensors.

Electromagnetic sensors with wavelengths longer than infrared are generally known as radar. The acronym RADAR, which originally stood for Radio Detection and Ranging, has come to include a wide variety of measurements. Radar systems now exist for measuring range, range rate, scattered amplitude, and scattered phase in various polarizations. Research is now progressing on systems that will construct images of remote objects using wideband microwave measurements taken over a variety of scattering angles. These radar imaging systems would be an ideal complement to optical imaging in space. They are, however, extremely complex and require much signal processing. In addition, the resulting image is often difficult to interpret, yielding intense blobs at the target's so-called scattering centers. A radar image
would certainly be useful to a space robot, but is difficult to obtain. What kind of shape information is available from the returns of simple, currently available, microwave radar systems?

The Radar Scattering Cross-section (RCS) is the ratio of scattered to incident power normalized for wavelength, range and antenna gain. In this thesis, it is shown that the co- and cross-polarized RCS yield some shape information about the scattering object. Furthermore, by considering the RCS in conjunction with a degraded optical image, an object surface characterization is derived which is more reliable and complete than those derived from either data set taken alone. The process of integrating several sensor measurements into a single object description is known as sensor fusion.

1.2. Comparison of Optical and RCS Sensing

To support the claim that microwave radar can complement optical imaging, we need to establish the relative advantages and disadvantages of these sensors. In situations where microwaves are most useful, light wave frequencies are least effective, and vice versa. The problem areas where one sensor excels over the other are: matte surfaces, metallic surfaces, dielectric blankets, occlusions, ambiguity, and resolution.

The material surfaces of space objects may be roughly divided into two categories, matte and metallic. A matte surface is slightly rough, metallic surfaces are completely smooth. The NASA orbiter is a good example of an object with a matte surface, while many satellites possess metallic surfaces. Matte surfaces are better suited for optical imaging than metallic surfaces due to their uniform scattering
properties. The analysis of optical images for 3D shape is most easily accomplished when light is reflected from a surface equally in all directions. The bidirectional reflectance function for these ideal Lambertian reflectors is constant for any given surface [2]. This makes it easier to invert the optical brightness image to derive surface normals. Such an inversion may be done by photometric stereo, or shape-from-shading.

A metallic surface reflects light primarily in the specular direction. This presents problems for optical image sensors since surfaces whose normals do not point along the viewing direction are difficult to discern. Laser range scanners also have difficulty determining the shape of specular reflectors since a laser beam may not be reflected back from non-specular points with enough power for detection. The simplest models for microwave scattering, however, are smooth surfaces. The task of inverting scattering cross-sections is much easier when the scattering surface is free from irregularities. Furthermore, good electrical conductors usually exhibit metallic surfaces. When the surface material is perfectly conducting, the task of computing microwave scattering cross-sections is greatly simplified.

Another problem with light-wave techniques that microwaves may help to solve is penetration. Optical frequencies are easily attenuated compared to microwaves. This is a problem with space objects covered by dielectric solar blankets. These mylar coverings reflect harmful radiation, but also obscure surface details on space objects. If a robot has to grasp and manipulate such objects, the shape underneath the blankets is critical. Microwaves are capable of penetrating these dielectric films.
Reflected microwave energy is returned with information relating to the underlying object shape. Other types of occlusions can obscure object surface details. For instance, one object may occlude another along the camera’s line of sight, or an object may occlude itself. A big problem resulting from self-occlusion in any imaging situation is that we can never see the back side of an object from a single view. Owing to the long wavelength of microwaves compared to optical frequencies, radar energy may diffract around objects. This diffracted radiation, known as the creeping wave in GTD, can be, partially reflected back to the receiver. If this portion of the return can be separated from the RCS of the illuminated region, we may infer some geometric facts about the back side of a scattering object.

When interpreting narrow-band, single-look RCS returns, a shape extraction system is faced with a high degree of ambiguity. Many object shapes may result in the same polarized RCS. The technique presented in this thesis intends to obviate this problem by using the optical image -- the optical image should provide an initial shape close enough to the true surface so that the numerical procedure will converge to the correct shape.

A problem related to ambiguity is resolution. The RCS is comprised of power reflected from every target within the beamwidth of the radar system. Some range separation is possible through time gating, but this only limits the response to a segment of a ring covering the expected range within the beam. It is through the much higher resolution of the optical image that we are able to localize the scattering object within the microwave beam. These relative advantages and disadvantages are sum-
marized in table 1.1. Advantages are designated by the symbol + and disadvantages by -.

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<td>Slightly Rough Surfaces</td>
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Table 1.1 Relative advantages (+) and disadvantages (-) of low-resolution microwave RCS and optical images.

1.3. An Overview

This thesis describes a new system for extracting the 3D shape of space objects using optical images and the polarized RCS. The system describes the unknown surface parametrically and, using electromagnetic modeling techniques, minimizes the error between the predicted and the observed polarized RCS set. The optical image is
incorporated by providing an initial estimate of the scattering surface shape. This numerical procedure is embedded in a global system intended to make certain heuristic decisions about the scattering object and modeling procedure. Obviously, the success of such a method is highly dependent on the modeling technique used. Many choices are available, but the ones considered here are two most popular: The Geometrical Theory of Diffraction (GTD), and the Method of Moments (MM). These two modeling procedures have been shown to be accurate under proper controlled conditions. The state of the art in electromagnetic modeling is still primitive at best, so we can only hope to achieve good results for simple, perfectly conducting objects. Such simple objects would include plates, ellipsoids, polygonal solids, and perhaps a rudimentary satellite model. However, the future should see the introduction of more powerful computers and microwave equipment, making accurate modeling possible for more complex targets.

1.4. A Detailed Introduction

1.4.1. The Individual Components

The proposed SF application requires the merging of at least three separate disciplines. The first of these is the field of image processing and understanding. The second is microwave scattering theory, and the third is artificial intelligence. All three will be combined in a novel way.

The techniques borrowed from the field of image processing/understanding are three-dimensional (3D) object classification, image registration, and shape from shad-
The object classification is required to identify simple geometric shapes in order that we may predict their cross-sections using analytical techniques. Image registration is a critical step since the size of a scattering object is required before any shape information can be extracted from the polarized RCS. Finally, we need a "shape-from-shading" technique such as photometric stereo, or shape from shading proper, to extract a first estimate of the shape of the scattering object. This thesis will not dwell on these concepts, but accept them as given facts. Of course, these are still ongoing research topics and the existing algorithms are far from reliable. If the reader wishes to explore them further he should consult a standard reference, such as Horn [2].

We are more interested in a branch of computer vision known as sensor fusion. This is a discipline that has emerged in recent years to deal with the vast amounts of conflicting and complementary sensory data now available to functioning robots. The real contribution of this thesis is to that field. The purpose of sensor fusion is to combine the interpreted outputs of various sensors into a consistent world-view that is in some way better than its component interpretations. Often, the sense of confidence in an interpretation is better, or the resulting composite surface or workspace may contain more details than any single sensor observation, or it may be a combination of the two. The research presented here accomplishes this with microwave RCS sensors and camera image sensors. A comprehensive review of sensor fusion techniques is given in an appendix to this thesis.

One particular sensor fusion technique is of interest to us due to its similarity to our work. Wang and Aggarwal [3] have developed a system that combines occluding
contours from a camera image, and surface details from structured light. This technique is diagramed in figure 1.1. The occluding contours from multiple views are projected from the image plane towards the target to generate cylindrical volumes. The surface details are allowed to move along these cylindrical volumes until good matches are found at their edges. This technique resembles ours in that the occluding contours are fused with interior surface details. In the present investigation, the interior details are derived from the RCS rather than structured light, and only one view

![Diagram](image)

**Figure 1.1**

A surface reconstruction sensor fusion scheme due to Wang and Aggarwal [3]. Occluding contours are projected towards the object to form cylindrical volumes, which guide the positioning of surface details derived from structured light.
is needed. Another similarity is that occluding contours from the camera image are used to guide the application of alternate sensor; in our case, the contours aid the inversion of the microwave data.

The second field we shall draw on is the study of microwave scattering. A large amount of effort has gone into discovering accurate procedures for modeling electromagnetic scattering on digital computers. We shall use these forward modeling procedures to determine the shape of unknown objects. This system may be seen as a microwave inversion technique, although we have avoided many of the rigorous details associated with that term. A later chapter will show how scattering models are developed, and discuss similarities and differences between this and the use of the RCS in previous techniques.

1.4.2. The System as a Whole

Figure 1.2 is a conceptual block diagram showing how the various parts of our MW/optical SF system interact. From the optical image we extract occluding contours and a partial surface shape description for the scattering object. The occluding contours are easily derived from a thresholded image, and the partial surface shape from photometric stereo or shape-from-shading. The most important information about the internal surface shape is an estimate of surface normal orientation at sparse locations. These surface normals provide critical knowledge to the EM modeling procedure. From this optical information, an initial surface description is derived. The unknown characteristics of the surface are controlled by a parameter vector \( p \), whose true value is to be determined by an iterative numerical procedure.
Figure 1.2
A block diagram of the RCS/camera sensor fusion system. This framework is required if the numerical reconstruction algorithm is to be used in a working space robot.

From the radar system, we extract an estimate of range, and a set of polarized radar scattering cross-sections. We shall consider all propagating EM fields to be
comprised of the vector sum of two components each polarized along directions that are perpendicular to each other and mutually orthogonal to the direction of propagation. We shall denote these directions of polarization by the letters $h$, and $v$. The polarized RCS observation thus consists of four numbers: The power received polarized in the $h$ direction due to incident radiation polarized in the $h$ direction (denoted by $\sigma^{hh}$), power received polarized in the $v$ direction due to incident $h$ power ($\sigma^{vh}$), and \textit{vice versa} ($\sigma^{vv}$ and $\sigma^{hv}$).

The "intelligent decision module" first attempts to match the shape in the optical image with some simple geometric shapes. This is done so that, if possible, a closed-form expression, derived through the GTD can be used to predict the RCS. If no match is found, the decision module constructs a grid over the unknown surface and passes this information to the EM modeling procedure which will employ the method of moments to compute successive polarized RCS's for the numerical error minimization procedure.

This iterative numerical procedure is the well-known nonlinear least squares technique [4]. Successive approximations to the parameter $p$ are generated by minimizing the difference between the observed and predicted RCS's. Such a procedure requires computation of the Jacobian matrix. Because of the complex relationship between the RCS and $p$, an analytic expression for the Jacobian is difficult to derive. It will be shown that by choosing the parameters $p$ judiciously, the computation of the Jacobian may be simplified.
1.4.3. An Example

As an example, consider an unknown object whose peripheral shape and occluding contours are known from the optical image, but whose central portion is obscured by an intense specular reflection. One possible surface reconstruction procedure would be to represent the unknown portion of the surface by a B-spline patch. B-splines are a technique of interpolating a function of given continuity between a set of fixed points or knots[5]. The knots on the periphery of the B-spline surface patch can be extracted from the optically derived surface. The parameter p here would be the z height of the interior knots. If, for instance, a doubly-quadric B-spline patch represents the unknown surface portion, nine knots are required (see figure 1.3). The eight boundary knots would be taken from the optical data, leaving only a single parameter, the height of the central knot, to be determined through the error

![Figure 1.3](image)

A surface patch surrounding the specular point on an object may be reconstructed with B-splines. The peripheral knots are fixed by the camera image, and the height of the central knot is allowed to vary.
minimization process. If the order of the B-spline patch is increased to doubly-cubic, a four by four array of knots is required. The unknown heights of the internal knots here comprise a parameter vector of length four. It should be noted that this is the most complex situation allowed by a single observation of the polarized RCS set. Since the number of parameters cannot exceed the number of observables in the non-linear least-squares procedure, the maximum length of the parameter vector is four. This limitation can be circumvented by taking RCS measurements at multiple frequencies or aspects, thereby increasing the number of observations. An example of such a B-spline representation of the unknown surface is given in the simulation chapter of this thesis.

1.5. Thesis Outline

Because the problem we wish to solve is the failure of computer vision techniques in the space environment, and because these techniques are used to provide information to our SF method, chapter 2 discusses the demands and limitations of procedures that extract 3D information from 2D camera images in space, showing why additional sensors are needed to correctly define space scenes. The last section in chapter 2 contains an analysis of a typical space camera system, showing how intense specular points can degrade a digital image. Chapter 3 contains brief introduction to EM scattering, explains the EM modeling techniques that will be used, and provides a look at some of the EM inversion techniques that resemble the new approach being presented here. In chapter 4, the mathematical details of the surface extraction procedure are given, and it is shown why knowledge of the occluding
contours of the scattering object is necessary. Finite approximations to the scattering equations are derived, and the numerical minimization procedure is described in detail. Also, this chapter explains the requirements for convergence of this procedure. Chapter 5 provides example simulations and experiments that show the usefulness of the concepts described in the previous chapters. First, simulated scattering data is used, and second, an attempt is made to apply this system to some actual RCS observations acquired in controlled anechoic chamber conditions. Chapter 6 contains a discussion of the experimental results, conclusions about the effectiveness of this method, and suggestions for further research.
CHAPTER 2

The Problem of Optical Imaging In Space

2.1. Introduction

The reason for incorporating the microwave RCS into a space robot’s sensory information, is to overcome some of the difficulties associated with optical imaging in space. In this chapter, the demands on a space robot imaging system will be analyzed, along with the capabilities of optical methods for meeting those demands. The last section contains an analysis of specular glare in a space optical image, and shows that sometimes, an object may be totally obscured the diffraction caused by its specular point.

2.2. Space Sensor Demands

A sensor system for a space robot must be able to gather and interpret information about the spatial location of discrete objects in a scene, their shape and motion. This information must be complete and accurate enough to allow the robot to navigate within its workspace, and manipulate selected objects. Clearly, the demands on a space robot sensor system are dictated by the type of scene that it expects to encounter. Furthermore, the scene type is dictated by the task that the robot must perform.
Recall that the shape extraction algorithm assumes that targets consist of isolated, smooth shapes constructed of some perfectly conducting material. In reality, more than one target may be present, and the shape of these targets may be complex. Certain questions must be answered about the scene to be viewed, and the objects that make up these scenes. Regarding scenes, their expected complexity, range, and motion must be determined.

The attributes of the individual objects that comprise the scene are also of interest. Target properties that affect the accuracy of the scattering model are shape, surface roughness, dielectric constant, and motion. Objects may in turn be decomposed into components that have well-defined properties.

Typical scenes which an automated sensor system might encounter in space vary widely. They may be simple, or complex in terms of numbers of objects, and they may be in motion with respect to several degrees of freedom. In general, the scene characteristics depend on the task at hand. The characteristics that must be determined for each task are complexity, range, and motion.

Jobs that could be automated in space usually fall into the three broad categories of tracking, retrieval, and servicing. Each of these tasks involve different types of scenes which a robot's sensing system must deal with. Over the life of a given robot mission all three tasks could be encountered, but a single vision system should only have to interpret one type of scene at a time. Consider an automated satellite maintenance mission. Such a task requires that the robot first track the target from a distance, then approach the target at close range, and finally dock and execute the neces-
sary servicing operations.

2.2.1. Tracking

Tracking is an important function in space. For instance, a maneuvering vehicle may want to predict the presence of obstacles so that it may avoid them. To plan an efficient path around an obstacle, its location and shape must be determined well before possible collision. Alternatively, a stationary platform like NASA’s Space Station must be aware of approaching vehicles and debris. A tracking scene may consist of many objects, hence, scene complexity is high. Given narrow fields of view, this complexity may be reduced somewhat, but due to the long range characteristic, multiple objects are unavoidable. The sizes of individual objects within a tracking scene will vary from vehicles and satellites several meters across, to minute, high-speed, particles only millimeters in diameter. Since the goal of tracking is early detection, the objects will be distant. This does not mean their range is restricted, however, since objects occurring over a wide variety of distances must be tracked simultaneously. Although the scene may be in complex motion, primarily the range rate of individual objects is of interest to a tracking system.

2.2.2. Retrieval

The EVA retriever is a robot now under development at NASA’s JSC [6]. Its function is to aid astronauts in extra-vehicular activities. The EVA retriever may be asked to venture a short distance from the orbiter and retrieve anything from a small tool to a disabled astronaut. This type of task, where a robot must approach from a
moderate distance and maneuver around some object, is similar to docking or proximity operations. Scenes that a retriever may encounter are of moderate complexity. The target of interest is readily identifiable, although there may be clutter in the periphery of the beam or field of view. The object sizes range from centimeters for tools, to one or two meters for an astronaut. Ranges are limited and vary from moderate to close, typically in the tens of meters. Scene motion is slow relative to the retriever, and may involve some predictable object rotation.

2.2.3. Servicing

Servicing refers to the tasks a robot performs at close range. Such a task might be to replace a battery on an orbiting telecommunication satellite. The robot performs this operation while docked with the satellite. Although individual object detail may be great, the overall scene is uncomplicated. The typical scene consists of a single, large object. In addition, the object is stationary with respect to the robot. It is at this stage that we must consider object shape and complexity in detail.

2.3. Space Objects

Scenes are comprised of one or many objects. These objects are themselves complex, and are made up of components. A single object may be made of several materials. Often, space objects are covered by solar blankets that obscure the surface details. The complexity of space objects is determined by the number of components required to describe them. Some objects, such as satellites, are simple, perhaps nothing more than a cylindrical body attached to a parabolic nozzle. Other objects are
complex, for example, the Space Station will include various sizes of grid-work, cylinders, and flat plates. Most space objects are rigid, but some, such as astronauts, are highly deformable, and others, such as the Space Station, will deform slightly in predictable ways.

The components that make up space objects are the smallest level of resolution we will consider. Components are simple geometric objects made up of a single material. This is the only level where our radar scattering model corresponds closely to reality. Component shapes may be polyhedral, with sharp discontinuities and broad flat areas, or they may be curved, continuous surfaces, such as ellipsoids or cylinders. Surfaces of space object components may, in general, be divided into two classes: metallic, and matte. Metallic surfaces reflect light only in the specular direction. Satellites and objects covered with solar blankets often exhibit metallic surfaces. Matte surfaces are rough and reflect light in all directions. Targets such as the orbiter and the space station have close to matte surfaces. A matte surface in one portion of the electromagnetic spectrum may be considered smooth at longer wavelengths. Component materials are seldom perfectly conducting, although some can be approximated this way with good results.

2.4. Optical Sensor Capabilities

An important aspect of the shape reconstruction algorithm being proposed in this dissertation is the use of optical sensing to derive a first approximation to the target shape. A computer vision system that is able to even partially determine the shape of three-dimensional objects will consist of many independent sub-systems.
Each of these subsystems may be the result of years of research and testing, so the literature on this subject is vast. The following discussion will cover only those topics that are of immediate concern at this point. For a more detailed look at optical sensing and image processing, the reader is referred to Horn [2], Rosenfeld [7], or Castleman [8].

Computer vision system components are usually broken into two categories: those that deal with low-level processes, and those that perform high-level functions. Low-level vision includes image acquisition, digitalization, and simple linear filtering. High-level vision involves the more complex tasks of matching and shape determination. The first approximation surfaces that are required by the numerical shape reconstruction procedure described in chapter 4 result mostly from low-level processes. However, to construct the most accurate first approximation possible, these low-level routines may have to be supplemented by some degree of high-level reasoning.

2.4.1. Low-Level Vision Subsystems

Image acquisition is the first step in the optical imaging sequence. In this step, the light that comprises the image enters the camera through a series of lenses, and is focused onto a sensing element. The sensing element converts the light into an electrical signal. Important characteristics of this stage in the image formation process are the aspect ratio of the camera, the focal length and diameter of the optical lenses, and the type and characteristics of the light sensor.
The closed circuit television (CCTV) system developed by NASA for its Space Shuttle routes images of the payload and payload bays, and crew compartments to various monitors, transmitters, and recorders on the orbiter [9]. This system makes use of monochrome and color vidicon TV cameras. The monochrome cameras have a zoom lens with a focal length adjustable from 18 mm to 108 mm, irises adjustable from f1.6 to f16, the field of view for close-up and medium range activities varies from 40.9° to 6.6°. The light sensor for these cameras is a silicon-intensified target vidicon, that has anti-blooming capabilities [10].

The analog video image must then be converted into digital form so that the optical information may be manipulated by computer. This process, known as digitalization, also affects the usefulness of the final image product. Parameters such as the size, number, and spatial distribution of the pixels influence the amount of information from the analog image which survives into the digital image. Other relevant characteristics at this point are word length, and the accuracy of image registration.

Low-level vision also includes some simple filtering techniques. These traditional image processing routines are important the the resulting shape interpretation and must be given careful consideration. The two low-level processing steps that have most relevance to the present research are thresholding and edge detection. A vision system must be able to separate objects from background to approximate their shape for the initial parameter determination. This is most easily accomplished through thresholding, where a bright object is assigned one brightness value, and the dark background another. The numerical shape reconstruction system described in
chapter 4 depends heavily on knowledge of the scattering target's occluding contours. To determine an object's occluding contours, edge detection is required.

2.4.2. High-Level Vision Subsystems

High-level vision subsystems are those that not only process the optical image, but attempt to extract some information regarding the location, shape, motion, etc. of the objects in the image. There are two approaches to this problem. The first approach is to compare the optical image to a library of stored images, or images generated from a library of stored object models. The second approach is to generate a three-dimensional shape directly from the information in the optical image. The matching approach is much more reliable, but requires a large library of stored models, and a vast amount of knowledge about the various images these models may generate.

The second approach is manifested in such techniques as shape-from-shading and photometric stereo. These shape-from techniques are limited by the quality of illumination available. They have only proven successful in controlled experiments with simple shapes. Also, the shape-from techniques only generate surface normals. Reconstructing a unique surface shape from these normals is not always possible.

Another method for generating shape directly from the optical image is stereo vision. In stereo vision, simultaneous images from cameras separated by a small distance are correlated so that points in the two images that correspond to the same physical location are matched. From knowledge of camera placement and the disparity between two matched points, the 3D location of the corresponding physical point can
be computed. The most difficult aspect of this procedure is the correlation between images. Seldom is a perfect match found throughout the image, and specular point present particularly troublesome matches. Stereo is, however, the most promising 3D vision technique and most of the space robotic vision systems under design make use of it. Perhaps an incomplete correlation resulting in a partial 3D surface description can be augmented by the shape from RCS method described in this research.

2.4.3. Alternative Optical Sensors

Other 3D optical sensing techniques exist that are not necessarily imaging systems. Laser range scanners are popular as 3D shape determination devices. These devices scan across objects and construct range pictures by measuring phase delays in the reflected light. They are currently the most reliable remote shape determination devices, however they can fall prey to the same problems as optical imaging, i.e. poor reflectors, specular surfaces, etc.. Another alternative to traditional optical imaging is the use of structured light. In these systems, stripes of light composing a regular grid are projected onto an unknown surface. Shape is inferred from the manner in which straight lines are distorted in the resulting image. Since optical techniques are used, this method exhibits the same problems that have already been discussed. In addition, there are sometimes problems associated with identifying stripes in the final image.

Even the best of these 3D shape extraction systems do not always generate a reliable shape under normal conditions. More sensory information is required if a good shape extraction system is to be constructed. Some workers have proposed the
use of multiple images to circumvent the problems encountered with single images. While this is effective in some cases, limited time, space, and illumination may prevent the acquisition of multiple views of an object.

2.5. Examples of Space Robotic Vision

Various sensing and processing systems have been proposed for space robotic applications. Some of this research addresses the global architecture for a complete sensing system [11, 12], while other papers deal with narrow aspects of such a system, such as low-level sensors [13], or high-level object identification and attitude estimation [14].

Bronze et al. have described the plans for developing a free-flying space robot intended for use as a transfer device [12]. This robot, the "ROBIN", would be required to ferry satellite orbital replacement units from the space station to disabled satellites. The designers envision a robot equipped with stereo video cameras capable of high-resolution grey-scale imaging. The cameras will have fully automated pan, tilt focus and zoom capabilities, as well as automatic tracking of end-effectors. The authors of this paper stress that special care must be given to the remote lighting system in order to remove glare and illuminate shadows.

Wilcox et al. have studied the sensing system for a space robot in great detail [11]. They plan to use a pipelined image processing system to do feature extraction, matching and object tracking. The use of stereo is also important to their system. The camera outputs are fed into a motion stereo analyzer that in turn sends information to a stereo matcher. At this point the 3D object descriptions are input to a model
matcher that provides the input to the tracking system. The tracking output is then fed back to the motion analyzer for improved motion stereo matching. Communication is provided between each of these processes. These authors note the importance of camera calibration, and describe a procedure for doing this. Calibration is performed for each camera by measuring the space between dots on a specially prepared image.

A team of Rockwell International engineers have developed a testbed for space robotic sensors [13]. This testbed is a mobile platform equipped with stereo video cameras, a laser rangefinder, and an acoustic mapping sensor. The camera has a variable focal length lens so that the same camera can be used for wide-angle searching, and telephoto target tracking. The laser range device supplies only range information at single points, it is not a range scanner. Depth maps are provided by the ultra-sonic mapping system which scans over the full range and azimuth of the workspace. This acoustic device is intended to simulate future radar imaging systems. The investigators found that the laser range and acoustic imaging sensors were hampered by the lack of an effective data representation. Since this was a telerobotic application, the end-user of the system was a human operator. The data representation used here was a display device, but when the sensor analyzer is a computer, the same issue applies. These researchers might benefit from a review of Harmon’s work on sensor fusion [15]. As stated in chapter 2, Harmon observed that efficient integration of multiple sensors is best accomplished when the sensory data is contained in a uniform format.
deFigueiredo, Markandey, and Kehtarnavaz, have investigated the high-level function of object identification and attitude determination in space [14, 16, 17]. Their technique is to extract the edges of an object, build a wireframe representation, and use attributed graph matching to compare the unknown object to a group of stored models. This technique, known as the MIAG algorithm, appears to work well on good quality images of simple shapes, but suffers when edges in the image are indistinct. This technique may be useful to the present research as a simple 3D shape recognizer. Recall from chapter 1 that in our shape extraction system, the unknown object is first examined to see if a match to some simple geometry can be made so that a closed-form GTD solution for the RCS may be used. The MIAG algorithm could be used to identify targets that are candidates for GTD generated cross-sections.

2.6. Analysis of Specular Glare in Space Images

Consider the image acquisition system shown in figure 2.1. The light enters a lens with magnification $M$, passes through an aperture of diameter $D$, and is focused on an image sensor placed a distance $f$ from the lens. The image sensor may consist of a charge-coupled-device (CCD) array, a vidicon tube, or light sensitive film.

When light passes through an aperture, the resulting image displays diffraction effects to varying degrees. When the light source is a point, and the aperture is circular, the resulting diffraction pattern is well-known. The radial diffraction pattern due to a circular aperture is shown in figure 2.2. The intensity of such a pattern drops off away from the image point as a function of
A camera imaging geometry with a lens magnification $M$, focal length $F$, and aperture diameter $D$.

\[ \left[ \frac{J_1(\rho)}{\rho} \right]^2, \]

where $\rho$ is a function of the radial distance away from the image point, and $J_1(\rho)$ is a Bessel function of the first kind. If $\theta$ is the angular radial distance from the image point, $\rho$ may be expressed as

\[ \rho = kR\sin\theta, \]

where $k$ is the wave number of incident light, and $R$ is the radius of the aperture. For small angles,
\[ \theta = \frac{\rho \lambda}{\pi D} \]

The central lobe of this diffraction pattern is known as the Airy disc [1]. Ordinarily, with incoherent, white light, the side lobes of the diffraction pattern cancel out with neighboring points and are invisible. The remaining central lobe, the Airy disc, represents the resolving power of the lens. A typical camera image is the sum of the Airy discs of each point in the image. What if a single point of light is several orders of magnitude more intense than its surrounding points? The side lobes of the diffraction pattern no longer cancel out, and a disc may be visible out to the second or third

![Graph](image_url)

**Figure 2.2**
The radial diffraction pattern due to a circular aperture.
sidelobes of the Bessel function.

What image effect might an intense specular reflection cause in the worst case? Such a case would be a telephoto image of a small bright object. The long range would require a high magnification lens, a bright scene would require a small aperture, and a small object is more likely to be obscured by diffraction effects. Let the object being imaged be a sphere 10 cm in diameter, and the longest wavelength entering the lens be \( \lambda = 1 \times 10^{-3} \, mm \). Consider a lens with a magnification \( M = 1000 \), a focal length \( f = 100 \, mm \), and a lens speed of f16, indicating an aperture diameter of \( D = 6.25 \, mm \). If the specular point has an intensity 80 \( \text{dB} \) higher than that of its surrounding points, at the third lobe of the diffraction pattern, the intensity would still be about 30 \( \text{dB} \) above the surrounding intensity. The edge of the third sidelobe occurs at the third zero of \( J_1(\rho) \), which is \( \rho = 10.173 \). The angular spread of the diffraction pattern at this point is

\[
\theta = \frac{10.173 \times 10^{-3}}{6.25\pi} = 5.181 \times 10^{-4}.
\]

Thus, the width of the diffraction pattern out to the third sidelobe on the image sensor would be about 0.05 \( mm \). This compares in the image to the total sphere diameter of \( \frac{100}{M} = 0.1 \, mm \). Thus, the diffraction pattern out to the third sidelobe obscures about half the image of the sphere. Additionally, if the sphere appears against a black background, the diffraction fringe due to the specular point will be visible far beyond the third sidelobe, making the sphere appear larger than is really is. While the size of this diffraction pattern seems small, it is entirely resolvable by the light sensors on
modern video cameras. Typical CCD cameras have sensor areas of tens of millimeters, and as many as 3000 pixels per square millimeter. High-resolution vidicon-tube cameras may have on the order of 100 scan lines per millimeter [18].

Of course, if the diffraction pattern only covers a portion of the image, a stereo camera arrangement can be used to image the entire sphere. However, the specular point will cause a false correlation in shape-from-stereo algorithms. How far apart must two cameras be spaced in the above situation so that the central portion of the sphere that is lost in one image is visible in another? This situation is depicted in figure 2.3. If the specular disc extends to one half the radius of the sphere, the two cameras must be separated by an angle $\phi$ of at least $60^\circ$. If the sphere being imaged is $10\, m$ from the cameras the cameras must be separated by at least $S = 11.55\, m$. This is a much larger $S$ than customary for stereo vision. A separation of $11\, m$ would be unwieldy for a small space robot, thus stereo does not easily solve this problem encountered in 3D imaging.
Figure 2.3
A stereo imaging system showing the camera separation necessary to resolve diffraction effects out to the third sidelobe.
CHAPTER 3

Electromagnetic Scattering Preliminaries

3.1. Introduction

A key component of the SF technique described in this dissertation is the computer generation of synthetic scattering data. Minimizing the error between the observed and predicted RCS is only useful if the RCS can be predicted with a great degree of accuracy. This is not easily accomplished for complex scattering targets. In fact, the question of how to accurately model the diffracted field from even moderately complex targets is still a subject of ongoing research. The goal of this thesis is not to develop a shape reconstruction device that can immediately be put to useful work, but to show that, with the appropriate tools, such a system is possible. By demonstrating the method on simple shapes for which the scattering characteristics can be reliably modeled, the possibility, or impossibility of extracting shape from RCS will be shown. With the development of more powerful computers and modeling algorithms, and the advent of microwave systems capable of making more accurate measurements over a wider range of frequencies than is currently practical, a usable shape reconstruction device may very well be realized. In this chapter, a few of the electromagnetic modeling techniques that will be used here are described. In addition, the related topic of electromagnetic inversion and similarities between our technique and previous research will be discussed.
3.2. The Radar Scattering Cross-Section

For this research, the measurable quantity of interest is the Radar cross-section \( \sigma \). The theoretical definition of \( \sigma \) is \( 4\pi \) times the limit of the ratio of incident power to received power as the range becomes infinite \([19]\).

\[
\sigma = 4\pi \lim_{R \to \infty} R^2 \frac{|\vec{E}^{sc}|^2}{|\vec{E}^{inc}|^2} = 4\pi \lim_{R \to \infty} R^2 \frac{|\vec{H}^{sc}|^2}{|\vec{H}^{inc}|^2},
\]

(3.1)

where \( \vec{E}^{sc} \) is the magnitude of the vector electric field measured at the receiver, and \( \vec{E}^{inc} \) is the incident magnitude. Correspondingly, \( \vec{H} \) represents the magnetic field vector. The RCS is measured in units of squared area. When the transmitter and receiver are located at the same point, the RCS is said to be monostatic, and when they are separated, the RCS is bistatic. The term backscatter may be substituted for monostatic. This formal definition of the RCS is not very useful for quantifying actual radar measurements. For this reason, a second definition, related to the basic radar equation is sometimes used \([20]\)

\[
\sigma = \frac{P_r R_r^2 (4\pi) R_t^2 (4\pi)^2}{G_r \lambda^2 G_t},
\]

(3.2)

where \( P_r \) and \( P_t \) are the received and transmitted powers, \( G_r \) and \( G_t \) are the receiver and transmitter antenna gains, \( R_r \) is the distance from target to the receiver, and \( R_t \) is the distance from the transmitter to the target. This definition ignores any losses in the system or transmitting medium \([21]\).

The above definitions make no mention of polarization. The RCS is, in fact, a function of incident and received polarization \([19]\). This polarization dependence of
the scattered field is commonly expressed in a form known as the scattering matrix. If the incident and received electric fields are decomposed into orthogonally polarized complex components denoted by $E_1$ and $E_2$, they are related by the matrix equation [21]:

$$
\begin{bmatrix}
E_1^{sc} \\
E_2^{sc}
\end{bmatrix} =
\begin{bmatrix}
S
\end{bmatrix}
\begin{bmatrix}
E_1^{inc} \\
E_2^{inc}
\end{bmatrix},
$$

(3.3)

where $[S]$ is the 2x2 scattering matrix. The components of the scattering matrix are related to the polarized RCS by the expression:

$$
S_{ij} = (4\pi r^2)^{-1/2}\sqrt{\sigma_{ij}},
$$

(3.4)

where $\sqrt{\sigma_{ij}}$ is a complex number which may contain phase information [19]. An alternate means for representing a field's polarization information in terms of strictly real numbers is the Stokes' vector $\mathbf{g}$ [22].

$$
\mathbf{g} =
\begin{bmatrix}
|E_1|^2 + |E_2|^2 \\
2\text{Im}\{E_1^*E_2\} \\
|E_1|^2 - |E_2|^2 \\
2\text{Re}\{E_1^*E_2\}
\end{bmatrix}
$$

(3.5)

The Stokes' parameters are irreducible [23]. The Stokes' parameters are useful, but computing them requires a radar system that measures the phase of an incoming signal. This requirement significantly boosts the complexity of the radar system and introduces another source of measurement error. Henceforth, only the polarized RCS will be considered. Although the polarized RCS contains less information than the Stokes' vector, it requires only knowledge of the magnitude of the received radia-
tion, and not the phase. Deducing shape information from the Stokes parameters may be a good subject for further research.

3.3. Electromagnetic Modeling Techniques

3.3.1. Overview of Electromagnetic Modeling

A large body of literature exists pertaining to the prediction of diffraction patterns due to isolated objects. The majority of these techniques can be broken down into two categories: those which model the surface currents induced on a body by the incident radiation, and those which attempt to model the diffracted field directly. The techniques of the first kind make use of exact theories involving the solution of integral equations, whereas the second kind depend on asymptotic techniques. The integral equations resulting from the exact theories must be solved by computer. This solution can be very complex and may also introduce many inaccuracies. The approximate solutions, however, result in closed form expressions for the scattered field which may be solved very quickly. The exact solutions are valid over a wide range of frequencies and object shapes, although this range is limited by the feasibility of numerical solutions. The approximate theories are valid only in the case where the scattering object is much larger than a wavelength.

3.3.2. Exact RCS Prediction Techniques

The most basic exact prediction technique is a direct solution of the vector Helmholtz equation which results from combining Maxwell’s equations. This well-known wave equation is
\[ \nabla^2 \vec{F} + k^2 \vec{F} = 0, \]  

(3.6)

where \( \vec{F} \) represents either the electric or magnetic field in free space [24]. An analytic solution to this equation is usually found by substituting a series appropriate to the coordinate system into 3.6 and solving the resulting expression for the unknown coefficients. Such a solution is only practical when the geometry of the scattered field contains a great deal of symmetry and conforms closely to one of the standard coordinate systems, such as rectangular, cylindrical, spherical, etc. Recently, supercomputers capable of solving massive systems of linear equations have made possible finite element solutions to the vector wave equation. Impressive results have been achieved using this technique, which may prove useful in the future. A more practical exact solution approach is to numerically solve an integral equation which results from applying Green’s theorem to the vector wave equation.

3.3.3. The Electric and Magnetic Field Integral Equations

The starting point for integral equation solutions to scattering problems is the Stratton-Chu integral [25]. The relationship between the time harmonic electric and magnetic fields and the electric and magnetic currents in a homogeneous, isotropic medium inside a closed surface is given by

\[ \int_V \{ j \omega \mu \vec{E} \times \vec{H} + \nabla \times \vec{G} - \frac{1}{\varepsilon} \nabla \nabla G \} \, dv = \int_\Sigma \{ j \omega \mu \vec{E} \times \nabla \vec{H} - (\hat{n} \cdot \vec{E}) \nabla \vec{G} - \hat{n} \cdot \vec{E} \nabla G \} \, \xi(s', \gamma) \]  

(3.7)

In the above equation,
$V$ is a closed volume,
\(\Sigma\) is the surface enclosing \(V\), composed of \(S\) and \(S_1\) as shown in figure 3.1,
\(\omega\) is the radial frequency of the time-harmonic fields \(E\) and \(H\),
\(\mu\) is the magnetic permeability of the medium,
\(\varepsilon\) is the electric permittivity of the medium,
\(\rho\) is the charge density in the medium,
\(G\) is the free-space Green’s function
\[ G = \frac{e^{-j\kappa |\mathbf{r} - \mathbf{r}'|}}{|\mathbf{r} - \mathbf{r}'|}, \]
\(\mathbf{J}\) is the electric current density,
\(\mathbf{K}\) is the magnetic current density,

The integrations in equation 3.7 are performed over the source variable \(\mathbf{r}'\). For the purposes of predicting the RCS of a target, equation 3.6 will be used to compute the electric currents \(\mathbf{J}_s\) on the surface of the target. In this case, both the observation point \(\mathbf{r}\) and the source point \(\mathbf{r}'\) lie on the surface. For this reason, the singularity in

![Figure 3.1](image)

*Figure 3.1*

The surface of integration for the Stratton-Chu integral, showing the inner surface, \(S\), and the outer surface, \(S_1\).
the function $G(\hat{r}, \hat{r}')$ presents problems. If the surface of integration, $S$, is broken into two portions as shown in figure 3.2, and limit of the integral over $S$ is taken as $r$ becomes very small, a principal value integral denoted by $\int [\cdot] \, ds'$ results. According to a derivation given in Poggio [24], if the surface $S_1$ is taken at infinity, the resulting expressions for the electric and magnetic fields on $S$ with no sources in $V$ are

$$
\vec{E}(\vec{r}) = \frac{1}{2} \vec{E}^{inc}(\vec{r}) - \frac{1}{2\pi} \int_{S_1 + S} \left[ j\omega \mu (\hat{n}' \times \vec{H}) \times \vec{V}' G - (\hat{n}' \times \vec{E}) \times \vec{V}' G - (\hat{n}' \cdot \vec{E}) \vec{V}' G \right] \, ds',
$$

(3.8)

and

$$
\vec{H}(\vec{r}) = \frac{1}{2} \vec{H}^{inc}(\vec{r}) - \frac{1}{2\pi} \int_{S_1 + S} \left[ j\omega \varepsilon (\hat{n}' \times \vec{E}) \times \vec{V}' G - (\hat{n}' \times \vec{H}) \times \vec{V}' G - (\hat{n}' \cdot \vec{H}) \vec{V}' G \right] \, ds',
$$

(3.9)

with the restriction that the fields $\vec{E}(\vec{r})$ and $\vec{H}(\vec{r})$ go to zero faster than $\frac{1}{|\vec{r}|}$ as $|\vec{r}|$ increases.

---

*Figure 3.2*

The scattering object's surface used to compute the principal value integral.
goes to infinity. This is known as the radiation condition [26]. Equations 3.8 and 3.9 are known as the electric field integral equation (EFIE) and the magnetic field integral equation (MFIE) respectively.

The remainder of this dissertation will be concerned only with perfectly conducting objects in free space. In order to find the currents induced on these perfectly conducting objects, take the vector cross product between the surface normal vector at the observation point, and equations 3.8 and 3.9. Noting that \( \mathbf{h} \times \mathbf{H} = \mathbf{J}_S \), for the perfectly conducting scatterer, the EFIE and MFIE reduce to

\[
\mathbf{h} \times \mathbf{E}^{inc} (\mathbf{r}) = \frac{1}{4\pi j\omega \epsilon} \mathbf{h} \times \int_S \left[ -\omega^2 \mu \epsilon \mathbf{J}_S \mathbf{G} + (\nabla' \times \mathbf{J}_S) \nabla' \mathbf{G} \right] ds', \quad \mathbf{r} \in S, \tag{3.10}
\]

and

\[
\mathbf{J}_S (\mathbf{r}) = 2\mathbf{h} \times \mathbf{H}^{inc} (\mathbf{r}) + \frac{1}{2\pi} \mathbf{h} \times \int_S \left[ \mathbf{J}_S \times \nabla' \mathbf{G} \right] ds', \quad \mathbf{r} \in S. \tag{3.11}
\]

These forms of the EFIE, and particularly the MFIE will be used for the remainder of this dissertation.

3.3.4. Integral Equation Solutions

The above equations are Fredholm Integral Equations of the First and Second kinds, respectively. These equations may not be solved directly for the surface currents, \( \mathbf{J}_S \), but series approximations have been developed which converge to a solution. The resulting electric and magnetic fields can be shown to satisfy Maxwell’s equations and the radiation condition [26]. These integral equations may not be applied to points on a surface where the outward normal vector is undefined,
such as edges and conical points.

3.3.5. Uniqueness

Although a solution $\mathcal{J}_S$ to 3.10 or 3.11 is indeed valid for any closed surface $S$ that satisfies the continuity condition, such a solution is not guaranteed to be unique in all situations. Since the integrals in 3.10, and 3.11 are linear operators on $\mathcal{J}_S$, if more than one solution exists, their difference must satisfy

$$\frac{1}{4\pi j_{\omega e}} \hat{\mathbf{a}} \times \int_S \left[ \omega^2 \varepsilon_0 \mathcal{J}_S \mathbf{G} + (\nabla' \cdot \mathcal{J}_S') \nabla' \mathbf{G} \right] ds' = 0,$$

(3.12)

and

$$\frac{1}{2\pi} \hat{\mathbf{a}} \times \int_S \left[ \mathcal{J}_S \times \nabla' \mathbf{G} \right] ds' - \mathcal{J}_S (\mathbf{r}) = 0.$$

(3.13)

Jones [26] has shown that solutions to equations 3.12 and 3.13 exist only when the electric or magnetic fields are zero everywhere on the surface $S$. This can only occur when the volume enclosed by $S$ contains electric or magnetic modes of resonance. In other words, the solutions to 3.10 and 3.11 are non-unique when the scattering target is a cavity resonator at the frequency of the incident radiation. Jones suggests that numerical solutions to equations 3.10 and 3.11 must be viewed with suspicion when the target dimensions exceed half a wavelength.

In practice, numerical solutions to 3.10 and 3.11 are computed. The most common technique for solving these equations in known as the Method of Moments (MM) This is a computational procedure derived through the calculus of variations [27]. When given a space of functions in which the approximate solution will lie, the MM
finds the "best" solution in that space. This requires the solution of a large set of linear equations. The success of the MM in finding an accurate solution to 3.10 and 3.11 depends on the choice of basis functions. This is a subject of ongoing research, and although this paper may, at times, assume that these functions can be automatically chosen, in practice, they must be carefully and individually crafted for each different scattering geometry. By compiling the experience of experts in this field, it is hoped that an artificially intelligent system may be constructed which assigns basis functions for arbitrary scatterers. This system should be a key element in the intelligent decision module shown in the block diagram of figure 1.2. More details on the MM are presented in chapter 5.

3.3.6. The Scattered Field

The field scattered by an object which has surface currents $\mathbf{J}_s$ induced on it may be derived from the MFIE. In this case, the source point and observation point are non-coincident, so no principal value integral is necessary. In a source-free region of free space containing a perfectly conducting object with currents known at all points on the surface $S$, the magnetic field at a point $\mathbf{r}$ is given by the equation

$$\mathbf{H}^{sc}(\mathbf{r}) = \frac{1}{2\pi} \int_S \left[ \mathbf{J}_s(\mathbf{r'}) \times \nabla' G(\mathbf{r}, \mathbf{r'}) \right] d\mathbf{r'}.$$  

(3.14)

Thus, predicting the RCS of a perfectly conducting target is a two-step process. The surface currents are computed from equation 3.10, or equation 3.11 and the resulting scattered magnetic field can be computed from equation 3.14. Finally, the RCS is found by taking the magnitude of the scattered field and inserting it into equation 3.2.
3.3.7. Physical Optics Approximation

Another technique for approximating a solution to 3.10 or 3.11 when the size of the scattering object is large compared to a wavelength is the Physical Optics (PO) approximation [24]. The PO solution to the diffraction problem ignores any interaction between fields on the scattering surface. It also assumes that the field on the shadow side of a scattering object is zero. A PO solution essentially ignores the integral term in equation 3.11. The field on the surface is assumed to be the result of the incident field at that point, and nothing else.

This solution is also known as Kirchoff diffraction, or the tangent plane model. The term "tangent plane model" results from the observation that the PO reflection from a point on an object is the same as that which would result from an infinite plane tangent to the object at that point. The term Kirchoff diffraction comes from the similar assumptions used to compute the far-field diffraction from a slit in an opaque screen. The field transmitted by the screen is assumed to be equal to the incident field on the slit and zero everywhere else [1]. This assumption holds true everywhere except near the edges of the slit. The problem with this approximation is that when the slit is very narrow, edge diffraction becomes the dominant process, and the Kirchoff model is no longer valid. The same is true in scattering from an object. The edges, or shadow boundaries, exhibit diffraction effects which are not explained by the PO model. When the object is large compared to a wavelength, scattering from the interior of the object is much more intense than the scattering from the edges. On the other hand, when the object is small, the edge effects are more notice-
able and must be taken into account to produce an accurate model.

If the normalized RCS of a scattering object is plotted against its longest linear dimension, the resulting response curve may be broken into three regions [20]. Such a plot for a metallic sphere is shown in figure 3.3. The first region, where the diameter of the sphere is much smaller than a wavelength is called the Rayleigh region. In this region, the normalized RCS is approximately linear with respect to diameter. The second region, where the diameter is on the order of a wavelength, is known as the resonance, or Mie region. The response of a target in this region fluctuates with

![Figure 3.3](image)

*Figure 3.3*

A plot of the RCS of a sphere versus the wavelength normalized by the sphere's diameter.
respect to linear dimension. In third region, known as the physical optics region, the
diameter is much larger than a wavelength. The normalized RCS of a sphere is
approximately constant in the PO region.

Some researchers are now proposing a compromise between the MM solutions
and the PO approximation. These so-called hybrid methods use the PO approxima-
tion to compute scattering from the interior of the object, and moment methods to
compute scattering from the edges and shadow side. One such technique proposed
by Kim and Thiele [28] splits a conducting body into two surfaces, the shadow side,
and the lit side. These surface are further decomposed into MM regions and asymptotic regions. The currents on these surfaces are also decomposed into MM currents,
and PO currents. The integration in the MFIE (equation 3.11) is then broken up into
eight parts, one for each of these currents on each of these surfaces. The MM solu-
tion only needs to be applied to the integrals concerning the MM currents. The solution is further simplified by observing that in certain regions, some currents are negli-
gible compared to others. This type of solution has the accuracy of a full moment
method solution, while retaining some of the computational simplicity of the PO
solution. Hybrid solutions may represent an avenue for bringing RCS modeling into
common usage, thereby yielding the sensor fusion system described in this thesis
practical for everyday use.

3.3.8. Asymptotic Techniques

The accuracy of the PO approximation degrades significantly as the receiving
angle moves away from specular [19]. This phenomenon was observed by J. B.
Keller [29], who found that the inaccuracies were due to edge diffraction. Keller’s theory, known as the Geometrical Theory of Diffraction, (GTD) provides a powerful tool for predicting the RCS of radar targets. The GTD extends the theory of geometrical optics to account for diffraction from edges and shadow boundaries. Three principles govern the basic application of the GTD as it was developed by Keller [30]:

1. Diffracted fields propagate along rays which are determined by a generalization of Fermat’s principle [1].

2. Diffraction is a localized phenomenon at high frequencies. It depends only on the surface and incident field in the immediate neighborhood of the point of diffraction.

3. The normal laws of Geometrical Optics apply.

In the GTD, a ray incident on an edge is said to produce a cone of diffracted rays as shown in figure 3.4. Keller derived expressions for diffraction coefficients which predict the intensity of rays in this cone as a function of the incident and scattering angles $\psi^i$ and $\psi^s$. The GTD has been successfully used to predict the RCS of many geometric objects such as plates, wedges, and ellipsoids. The effectiveness of the GTD compared to PO in computing the RCS of a flat plate is demonstrated in a paper by Ross [31]. In this figure, the solid line represents the GTD prediction, and the Dashed line PO. Notice how the PO approximation diverges from the experimentally measured RCS at about 30 degrees, while the GTD remains accurate up to about 80 degrees angle of incidence.
Figure 3.4
The cone of diffraction used to compute an approximation of the field scattered by an edge using the geometrical theory of diffraction.

There are some drawbacks to the GTD. In deriving expressions for the diffraction coefficients, Keller employed a wide-angle approximation to a surface integral. This approximation leads to diffraction coefficients which are singular at shadow or reflection boundaries. This effect occurs at the 90 degree point for a flat plate. Kouyoumjian and Pathak [32] developed the Uniform Theory of Diffraction (UTD) to circumvent this difficulty. They attempted to formulate an exact theory of diffraction which would account for scattering at edges and shadow boundaries and also apply to curved boundaries. The UTD employs a Fresnel integral expression to eliminate the singularity in the GTD diffraction coefficients. Other, more accurate asymptotic techniques have been developed, e.g. the method of equivalent currents, the physical theory of diffraction, and the incremental length diffraction coefficient
[19], but they are not relevant to this particular research. Certainly, however, in a working shape-from-microwave SF system, where complex and non-ideal targets are encountered, the most accurate and efficient RCS prediction techniques available must be employed.

3.4. Microwave Inversion Techniques

Predicting the diffraction pattern due to a given scattering target geometry is known as the forward problem. The task of recovering a scatterer’s size, shape, material composition, or other physical properties from knowledge of the diffracted wave is the inversion problem. The later is much more difficult. The term inverse scattering generally denotes a large body of work which has been developed to cope with the mathematical difficulties involved. The most prominent being the ill-posed nature of the inverse problem due to the lack sufficient information to perform a complete inversion. At the present time, the technology for realizing these techniques lags behind the mathematical theory [33]. The present research does not pretend to deal with these complex mathematical issues. However, since we are concerned with extracting the shape of an object from scattered field information, certain aspects of the inverse problem must be discussed. There exists a small amount of inversion research which resembles our shape extraction technique. Particularly, the work done by Roger [34, 35], and Chaudhuri [36-38] is of interest.
3.4.1. Inverting the RCS of Ellipsoids

Chaudhuri and Boerner [36, 38] developed a technique for reconstructing the profile of a perfectly conducting ellipsoid from the monostatic polarized RCS. They claim that "It is well known that the differential scattering cross-section of an object is related to the Gaussian Curvature at the point of reflection.". This inverse problem is therefore related to the Minkowski problem of differential geometry, where the shape of an object must be recovered from knowledge of the Gaussian Curvature. They produced good results with their method, however the inversion formulae are derived from expressions for the shape of the ellipsoid and so only apply to that particular target geometry. Also, monostatic scattered field observations are required over a wide range of viewing angles. This technique is relevant to the present research since the authors did manage to extract shape information from the polarized RCS.

Chaudhuri is also responsible for a low-frequency inverse scattering technique used to recover the semi-axes of a prolate spheroid [37]. This technique is based on the premise that the RCS in the Rayleigh region is directly related to the shape of the scattering target. In particular, the ratio of RCS's due to horizontally and vertically polarized incident radiation is related to the ratio of semi-major to semi-minor axes of the ellipsoid, i.e. the degree of eccentricity. The scattered field was described by a dipole expansion and the coefficients of the first few terms were found via a non-linear optimization technique. The numerical optimization aspect of the method closely resembles the shape extraction technique presented in this paper. Chaudhuri
tested his technique by adding noise to the observations and observing the convergence characteristics. As will be shown in chapter 4, his results were comparable to those presented here.

### 3.4.2. Inversion by Functional Derivatives

The E-M inversion technique which most closely resembles our method, and which reinforces the likelihood of its success, is due to Roger [34]. Roger makes use of a technique for solving non-linear equations based on the contractive mapping theorem [39]. He refers this technique as the Newton-Kantorovich algorithm and uses it to solve for the profile of cylindrical scatterers from the bistatic scalar RCS. The starting point for Roger's technique is an expression for the scattered scalar field $B(\theta)$ in terms of the cylinder's profile $F(\theta)$, and the layer potential $\phi(\theta)$ on the cylinder's surface.

$$B(\theta) = \int_{0}^{2\pi} e^{-jkF(\theta')cos(\theta-\theta')}\phi(\theta')d\theta',$$

(3.15)

where $\phi(\theta)$ is found by solving another integral equation. This procedure is similar to computing the scattered field by 3.14 and computing $\tilde{\mathcal{J}}(\vec{r})$ via the MFIE 3.11. Roger used the layer potential $\phi(\theta)$ in the same way that the surface currents $\mathcal{J}(\vec{r})$ are used in this paper. The RCS is computed from $B(\theta)$ by $\sigma = \frac{1}{4k}|B(\theta)|^2$. The inverse problem reduces to solving the operator equation

$$\sigma = O\left[ F \right].$$

(3.16)

where $O\left[ \cdot \right]$ is the integral operator in 3.15. Roger used the Newton algorithm to
solve for the cylinder profile's $F_0$ given the RCS $\sigma_0$. The Newton algorithm as implemented by Roger is an iterative procedure which, if started close enough to $F_0$, will converge to that solution. Given an initial estimate $F_1$, the procedure computes a better estimate $F_2$ by solving the linear equation

$$\sigma_0 - \sigma_1 = \partial_F \left[ F_2 - F_1 \right], \quad (3.17)$$

where $\partial_F$ is the functional derivative, or Frechet differential of the operator $O \left[ \cdot \right]$. In the inverse problem of the perfectly conducting cylinder, $\partial_F$ is an Fredholm integral operator of the first kind which relates a small variation $\delta F(\theta)$ of the profile to a small variation $\delta \sigma$ of the RCS. Roger's approach was to express $\partial_F$ analytically and numerically solve the resulting integral equation. Since Fredholm integral equations of the first kind are inherently unstable, the solution requires the use of regularization. Using Tikhonov regularization [40], the resulting integral equation to be solved for an update of $F_i$ is

$$\left[ \partial_F^* \partial_F + r R^* R \right] \delta F = \partial_F^* \delta \sigma, \quad (3.18)$$

where $r$ is the regularization parameter, $R$ is the regularization operator, and $^*$ denotes the adjoint operator, $\delta F = F_i - F_{i+1}$, and $\delta \sigma = \sigma_0 - \sigma_i$. Roger produced acceptable results from equation 3.18 by expanding $F$ in a Fourier basis and solving for the coefficients. This approach to inversion is very similar to the one which has been adopted for our research, however there are some important differences.

A) Roger used a linear Newton technique, referred to as the Gauss-Newton method by Dennis [4] This technique constructs a locally linear model of the function to
be solved at each iterate of the variable. The research presented in this paper will employ a variation of the Newton technique, which constructs a trust region at each step in the procedure and constrains further estimates to lie within this region. This is similar to using a full Newton technique and estimating the second derivative matrix by secant methods [4].

B) Roger computed an analytic expression for the functional derivative of the equation to be solved. The technique presented in chapter 5 describes the RCS parametrically, and uses the Jacobian matrix in place of the functional derivative. The Jacobian matrix is composed of ordinary partial derivatives of the RCS with respect to the parameters. In a later paper [35], Roger addresses this difference. He claims that the functional derivative is superior to parametric techniques since it provides an exact measure of the variation in the RCS function with respect to a variation in the target shape. The functional derivative is independent of the way in which the scattering surface is parameterized. Indeed, our technique is highly sensitive to the choice of surface parameters. The advantage of the parametric approach is that when the parameters are chosen correctly, the numerical complexity of a solution may be reduced. Also, the RCS may be related to the target shape in such a way that an analytic expression for the functional derivative is difficult to derive. In this case, the only option may be to approximate the Jacobian matrix with finite differences. It must be noted that at some point in Roger’s technique a numerical solution must be computed. If using computers to solve for continuous functions, one
cannot escape using a discrete representation of those functions. Therefore, even when using the functional derivative, the scattering surface must still be given a parametric representation. As will be shown in chapter 5, if the choice of basis functions used to solve the integral equations are related to the parameters of the surface for which the equations are being solved, the computation of the Jacobian may be greatly simplified.

C) Roger did not address the issue of deriving a first approximation to the cylinder's profile. He, somewhat arbitrarily, used a circular profile which produced the same observed RCS as a first approximation. The success of the Newton method in converging to a correct solution is highly dependent on the choice of an initial approximation. The hallmark of the present research is the use of the optical image to derive a first approximation to the target shape. This assures that the iterative solution procedure will start close to the true solution, and also helps resolve ambiguities from which the need for regularization arises.
CHAPTER 4

The Numerical Reconstruction Algorithm

4.1. Introduction to the Derivations

This chapter demonstrates how knowledge of the scattering target’s occluding
contours can facilitate extraction of the interior shape by non-linear minimization and
the method of moments. This demonstration will lead to a finite approximation of the
scattering cross-sections for an arbitrary surface. From this finite approximation, an
expression for the components of the Jacobian matrix will be derived. This matrix is
required for the iterative error minimization procedure. The Jacobian will be derived
for a general collocation solution to the scattering equations, and it will be shown
how its computation by finite differences may be accomplished in an efficient
manner. Finally, the convergence of the non-linear least squares (NLLS) algorithm
applied to the surface reconstruction problem will be analyzed.

4.2. The Scattering Equations

The problem is to reconstruct a surface from its co- and cross-polarized scattering
cross-sections, and the occluding contours from the optical image. This will be
done by characterizing the surface by a parameter vector \( \mathbf{p} \in \mathbb{R}^n \) and finding the value
of \( \mathbf{p} \) that minimizes the quantity
\[
\frac{1}{2} \| \mathbf{\sigma}(p) - \sigma_o \|_2^2,
\]  
(4.1)

where \(\sigma_o\) is the vector of observed cross-sections;

\[
\sigma_o = \begin{bmatrix} 
\sigma_{ovv} \\
\sigma_{ohh} \\
\sigma_{ohv} \\
\sigma_{ovh} 
\end{bmatrix},
\]  
(4.2)

and \(\sigma(p)\) is the vector of cross-sections generated by a computer model.

\[
\sigma(p) = \begin{bmatrix} 
\sigma_{vv}(p) \\
\sigma_{vh}(p) \\
\sigma_{hv}(p) \\
\sigma_{hh}(p) 
\end{bmatrix}
\]  
(4.3)

Since \(\sigma(p)\) is a highly non-linear function of \(p\), this is a non-linear least squares problem [4]. Equation 4.1 will be minimized by generating successive models \(\sigma(p_i)\) of the observations until a desired tolerance is reached. The occluding contours are used to generate models of \(\sigma(p)\), particularly when the modeling procedure requires solution of an integral equation. Since the scattering cross-sections are measures of backscattered power, the model for generating \(\sigma(p)\) is;

\[
\sigma_{vv}(p) = (H_{vv}^{sc}(p))(H_{vv}^{sc}(p))^*,
\]  
(4.4)

where * denotes complex conjugate. Notice that this definition of \(\sigma(p)\) differs from the definition of the RCS given in chapter 3. The proper definition of the RCS contains factors for normalization with respect to range, antenna gain, and incident
power. These factors remain constant for a given target regardless of its shape, so for simplicity in the following discussion, these constants will be ignored. In the application of this theory to actual scattering data, the power measurements need to be calibrated, usually to a sphere with the approximate dimension of the target. This calibration procedure will account for the constant terms in the definition of the RCS from chapter 3. The scalar quantity $H_{\eta V}^{sc}$ is the back-scattered magnetic field polarized in the \( V \) direction caused by an incident field polarized in the \( \eta \) direction. All polarizations will be expressed as the linear combination of the orthogonal directions \( h \), or \( \nu \). The total scattered vector field \( \vec{H}^{sc} \) is given by the integral;

$$\vec{H}^{sc}(\vec{r}, p) = \frac{1}{4\pi} \int_{S(p)} [\vec{J}(\vec{r}', p) \times \nabla' G(\vec{r}, \vec{r}', p)] d^2 r'$$

(4.5)

where \( \nabla' \) indicates the gradient with respect to the variable of integration. \( \vec{J}(\vec{r}, p) \) must be computed by solving the Magnetic Field Integral Equation (MFIE) that relates the surface currents to the incident magnetic field;

$$\vec{J}(\vec{r}, p) = 2\hat{n}(p) \times \vec{H}^{inc}(\vec{r}, p) + \frac{1}{2\pi} \int_{S(p)} \hat{n}(\vec{r}', p) \times [\vec{J}(\vec{r}', p) \times \nabla' G(\vec{r}, \vec{r}', p)] dS'$$

(4.6)

where the bar denotes a principal value integral. The Electric Field Integral Equation (EFIE) could also be used, but presents computational hazards that we will discuss in a later section. The problem, then, is to simultaneously estimate the surface and the currents on it by 4.6 so that the resulting cross-sections generated by 4.4 and 4.5 are the best least squares approximation to the observed cross-sections.
4.3. Integral Equation Solution

Now consider a numerical solution to 4.6. The remainder of this section will be devoted to finding an algebraic form for this solution, but first we will show in general how the method of moments can be used to solve an integral equation. The following derivation is based on the developments given by Harrington [27].

4.3.1. The Method of Moments Applied to a Scalar Problem

Let $V$ be a linear space of complex-valued, square integrable functions of a real variable defined on the interval $[a,b]$ with the inner product

$$
\langle f,g \rangle = \int_a^b f(t) g^*(t) \, dt, \quad f, g \in V.
$$

We are given a bounded linear operator $I$ that maps functions from $D(I) \subset V$ into $R(I) \subset V$, where $D(\cdot)$ and $R(\cdot)$ denote the domain and range of the operator. The method of moments attempts to approximate a function $A(x) \in D$ that satisfies

$$
I \left[ A \right](x) = B(x)
$$

for a given $B(\cdot) \in R(I)$. Note that in the present section, $A(x)$ and $B(x)$ are scalar-valued functions of a scalar-valued position variable $x$. In the following section, when applying the method of moments to our problem, they will become vectors in three-space. $A(\cdot)$ can be approximated by expanding it in a set a basis functions and solving a set of linear equations for the coefficients. The procedure detailed below is derived through the calculus of variations [26]. Let
\[ A(x) = \sum_{n=1}^{N} a_n f_n(x), \quad f_n(x) \in D(I), \quad (4.8) \]

and substitute this expression into 4.7 to yield

\[ \sum_{n=1}^{N} a_n I \left[ f_n(\cdot) \right](x) = B(x). \quad (4.9) \]

Next, choose a set of linearly independent weighting functions, \(q_1(x), \ldots, q_M(x)\), from the domain of the adjoint operator \(I^A\). The equations used to solve for the \(a_n\)'s are derived by forming the inner product between each \(q_m\) and 4.9.

\[ \sum_{n=1}^{N} a_n \langle q_m, I \left[ f_n \right] \rangle = \langle q_m, B \rangle, \quad m=1, \ldots, M. \]

Geometrically, this is equivalent to projecting the residual error

\[ \varepsilon = I \left[ \sum_{n=1}^{N} a_n f_n \right] - B \]

into a subspace spanned by the weighting functions. In matrix form, we solve the equation

\[ [Z]A = E, \quad (4.10) \]

where \([Z]\) is an \(N \times M\) matrix, \(A\) is length \(N\) and \(B\) is a length \(M\) vector defined by

\[ (Z)_{ij} = \langle q_i, I \left[ f_j \right] \rangle, \]

and,

\[ A_j = a_j, \quad B_i = \langle q_i, B \rangle, \quad i = 1, \ldots, N, \quad j = 1, \ldots, M. \]

When \(f_n = q_n\), the solution procedure is known as Galerkin's method, and is
equivalent to the Rayleigh-Ritz variational method used in finite element analysis [41]. Many moment method solutions use delta functions for the \( q_i \)'s, which greatly simplifies the inner product computation. The \( f_j(x) \)'s should be chosen to suit the geometry of the problem.

Solutions using pulse weighting functions are known as point-matching when the basis functions used are also impulses. If bases are chosen to be some kind of piecewise functions, the solution is known as collocation. Because of their simplicity, point-matching solutions will be used in these simulations. It can be shown that if \( I^{-1} \) is a bounded linear operator, and the above conditions on \( D \) hold, then a point matching solution converges to the solution of 4.7 as the number of points goes to infinity [26]. It should be noted, however, that the rate of this convergence is not necessarily rapid, and a great many points may be required for a correct solution. Because accurate point matching solutions result in an unwieldy linear system, other types of basis functions are chosen when great accuracy is required and computational capacity is limited. Some of these are described in the next section. For the sake of simplicity and rapidity of computation in the simulations of chapter 5, point matching solutions to the MFIE have been computed. It should be kept in mind that these solutions are for demonstration purposes only, and may not necessarily correspond to actual measured values of the polarized RCS.

4.3.2. Method of Moments Applied to the MFIE

We shall now show how a specific application of the method of moments produces a finite approximation to the surface currents by numerical solution of 4.6
which is a Fredholm integral equation of the second kind [26]. The operator in question is an integral over the surface $S'(p)$. Figure 4.1 shows that $S'(p)$ may be represented as a mapping from a domain $[u_0, u_1] \times [w_0, w_1]$, to a surface in three-space. If this mapping is diffeomorphic with metric $g_{uw}(p)$, the 3D surface position vector $\vec{r}'$ may be expressed as a function of $u$, and $w$, and the integral in 4.6 may be taken over the above domain [42]. Consider, therefore, a linear space $V$ of complex, vector-valued, square integrable functions of two real variables $u$, and $w$ defined on the interval $[u_0, u_1] \times [w_0, w_1]$, with the inner product

\[ M(u', w', p) \]

\[ u' \]
\[ w' \]
\[ w_0(u) \]
\[ w_1(u) \]
\[ u_0 \]
\[ u_1 \]
\[ x \]
\[ y \]
\[ z \]

**Figure 4.1**

*The Surface $S(p)$ will be represented as a mapping from a two-dimensional parameter domain to three-space. All of the $p$ dependence is placed in the mapping.*
\[
\langle f, g \rangle = \int_{w_0}^{u_1} \int_{w_0}^{u_0} \left[ \mathcal{F}(u, w) \cdot \mathcal{G}^*(u, w) \right] du dw.
\]

An element \( \mathcal{F}(u, w) \) of CV has three component functions: \( f_x(u, w), f_y(u, w), \) and \( f_z(u, w) \).

Rewriting 4.6 as

\[
2\hat{h}(p) \times \overline{\mathcal{F}}^{inc}(p) = \overline{J}(p, p) - \frac{1}{2\pi} \oint_{S^*(p)} \hat{h}(p') \times [\mathcal{F}(p', p) \times \nabla G(p', p', p)] \, ds', \tag{4.11}
\]

the operator \( \mathcal{I} \) is defined by

\[
\mathcal{I}(p) \left[ \overline{J}(p, p) \right] = 2\hat{h}(p) \times \overline{\mathcal{F}}^{inc}(p, p), \tag{4.12}
\]

where \( \overline{J}(p, p) \) is the unknown vector function corresponding to \( A(x) \) in the scalar case (A.1), and \( 2\hat{h}(p) \times \overline{\mathcal{F}}^{inc}(p, p) \) corresponds to \( B(x) \), where \( p \) plays the role of \( x \). \( \mathcal{I}(p) \) is a vector integral operator whose kernel depends on \( p \). Let \( D(\mathcal{I}) \) be a subspace of \( V \) for which the integral in equation 4.11 exists. If

\[
\mathcal{L}(u, w) = \int_{w_0}^{u_1} \int_{w_0}^{u_0} \left[ \mathcal{F}(u', w') \times \nabla' G(u', w', u, w, p) \right] \sqrt{g_{ww}(p)} du' dw', \tag{4.13}
\]

then

\[
D(\mathcal{I}) = \{ \mathcal{F}(u, w) \in CV \mid L_\alpha(u, w) < \infty \ \forall \ (u, w) \in [u_0, u_1] \times [w_0, w_1], \ \forall \alpha \in \{x, y, z\} \}.
\]

The operator defined by 4.12, then, is a linear mapping \( \mathcal{I}(p) \) from \( D(\mathcal{I}) \) to \( R(\mathcal{I}) \subset V \).

Notice that in 4.13 the \( p \) dependence of \( S'(p) \) appearing in the limits of the surface integral in 4.6 has been eliminated. This was done through knowledge of the occluding contours of the scattering surface seen in the optical image. This is an important simplification, and a key step in fusing the optical and microwave data.
The solution of 4.6 for the surface currents proceeds exactly as for the scalar problem described in the previous section. As in the scalar case, we need a set of basis functions, \( \vec{J}_i(\vec{r}) \in D V \), \( i = 1, 2, \cdots, N \). The solution, \( \vec{J}(\vec{r}, \vec{p}) \) is expanded in this basis analogously to 4.8, so that the surface current vector field may now be written:

\[
\vec{J}(\vec{r}, \vec{p}) = \sum_{i=1}^{N} \vec{J}_i(\vec{p}) \vec{f}_i(\vec{r}).
\]

Thus we have placed all the \( \vec{p} \) dependence in the coefficients of \( \vec{J}_i(\vec{p}) \), allowing the same set of basis functions to represent a variety of surface currents. Of course, there is no one set of basis functions that will suffice for all the current distributions that might be encountered on arbitrary surfaces. The best success with the method of moments has been achieved by choosing a set of basis functions that closely resembles the form of the final solution. Wire grid basis functions are a popular means for representing currents on arbitrary scatterers, but thin wire approximations do not necessarily converge to a continuous surface solution. For this reason, surface patch basis functions are a better choice for representing arbitrary current distributions. In their Electromagnetic Surface Patch Code, Newman, et. al [43], used quadrilateral basis functions to model currents on polygonal plates. Rao, et. al. [44], have suggested the use of plane triangular surface elements for solving the EFIE on arbitrary scatterers. In a working MW/optical surface reconstruction system, a set of basis functions that serve for a broad class of scatterers will be specified by the heuristic decision module (figure 1.2) at the outset of the numerical procedure. Each basis function will have support over a finite subsection of the surface parameter domain.
The parametric boundaries of each basis function will be determined beforehand and remain fixed throughout the minimization procedure. Once again, this is only possible through knowledge of the scattering object's occluding contours. After choosing a set of weighting functions, \( \mathbf{\tilde{w}}^j(\mathbf{\tilde{r}}) \), \( j = 1, \ldots, M \) from the domain of \( \mathbf{\tilde{T}}^N(\mathbf{p}) \), the coefficients \( \mathbf{J}(\mathbf{p}) \) are found by solving the linear vector equations corresponding to equation 4.15.

\[
[K(\mathbf{p})] \mathbf{J}(\mathbf{p}) = \mathbf{H}(\mathbf{p}),
\]

where,

\[
[K(\mathbf{p})]_{ij} = \langle \mathbf{\tilde{w}}_i, \mathbf{\tilde{T}}(\mathbf{p}) \mid \mathbf{\tilde{T}}_j \rangle,
\]

\[
\mathbf{H}_i(\mathbf{p}) = \langle \mathbf{\tilde{w}}_i(\mathbf{\tilde{r}}), 2 \hat{\mathbf{n}}(\mathbf{\tilde{r}}_p) \times \mathbf{\tilde{N}}^{inc}(\mathbf{\tilde{r}}_p, \mathbf{p}) \rangle,
\]

and, \( \mathbf{J}(\mathbf{p}) \) is the vector of coefficients in the expansion of \( \mathbf{\tilde{T}}(\mathbf{\tilde{r}}_p, \mathbf{p}) \). For the rest of this section, \( \mathbf{p} \) dependence of all factors is assumed.

The specific application of the moment method that we shall employ is subsec-
tional collocation. In this method, the surface is covered by a two-dimensional grid of \( N \) nodes at locations \( \mathbf{\tilde{r}}_i, i = 1, 2, \ldots, N \). Each basis function \( \mathbf{\tilde{T}}_i(\mathbf{\tilde{r}}) \) has constant value at the \( i \)th node, 0 at all other nodes, and is non-zero only in a finite subsection surrounding node \( i \). As previously mentioned, the weighting functions \( \mathbf{\tilde{q}}_i(\mathbf{\tilde{r}}) \) are impulses at \( \mathbf{\tilde{r}}_i \), however, here there must be three vector weighting functions for each of the \( M \) grid points. For grid point \( i \) they are:
\[ q_i(\vec{r}) = \begin{bmatrix} \delta(\vec{r} - \vec{r}_i) \\ 0 \\ 0 \end{bmatrix}, \quad q_{i+M}(\vec{r}) = \begin{bmatrix} 0 \\ \delta(\vec{r} - \vec{r}_i) \\ 0 \end{bmatrix}, \quad q_{i+2M}(\vec{r}) = \begin{bmatrix} 0 \\ 0 \\ \delta(\vec{r} - \vec{r}_i) \end{bmatrix} \]

Vector integral equation 4.6 may be thought of as a system of three scalar valued integral equations. With this in mind, an element of \( J \) is a scalar coefficient of a basis function \( \varphi_i(\vec{r}) \) that is non-zero in only one component, and \( J_p, J_{i+N}, \) and \( J_{i+2N} \) all correspond to the same surface location. The elements \( H_i, H_{i+M}, \) and \( H_{i+2M} \) represent the \( x, y, \) and \( z \) components of \( 2\hat{A}(\vec{r}_i) \times \vec{H}(\vec{r}_i) \). If there are \( N \) grid points for both basis functions and weighting functions then \( M=N \). Furthermore, the vectors \( J \) and \( H \) will have length \( 3N \), and \( [K] \) becomes a \( 3N \times 3N \) matrix.

\[
J = \begin{bmatrix}
J_s(\vec{r}_1) \\
\vdots \\
J_s(\vec{r}_N) \\
J_p(\vec{r}_1) \\
\vdots \\
J_p(\vec{r}_N) \\
J_{i+N}(\vec{r}_1) \\
\vdots \\
J_{i+N}(\vec{r}_N) \\
\vdots \\
J_{i+2N}(\vec{r}_1) \\
\vdots \\
J_{i+2N}(\vec{r}_N)
\end{bmatrix}
\]
\[ H = \begin{bmatrix}
-2n_s(\rho_1)H_s(\rho_1) & \cdots & \cdots & \cdots \\
-2n_s(\rho_0)H_s(\rho_0) & 2n_s(\rho_1)H_s(\rho_1) & \cdots & \cdots \\
\vdots & \ddots & \ddots & \cdots \\
2n_s(\rho_N)H_s(\rho_N) & \cdots & 2n_s(\rho_1)H_s(\rho_1) - 2n_s(\rho_0)H_s(\rho_0) & \cdots \\
2n_s(\rho_N)H_s(\rho_N) - 2n_s(\rho_0)H_s(\rho_0) & \cdots & \cdots & \cdots 
\end{bmatrix} \]

The matrix \([K]\) is composed of nine \(N \times N\) blocks:

\[
[K] = \begin{bmatrix}
[K_{xx}] & [K_{xy}] & [K_{xz}] \\
[K_{yx}] & [K_{yy}] & [K_{yz}] \\
[K_{zx}] & [K_{zy}] & [K_{zz}] 
\end{bmatrix}
\]

where each element of \([K]\) is found by numerically computing a principal value integral:

\[
[K_{\alpha\beta}]_{ij} = \left[ \delta_{ij}\alpha\beta f_j(\rho_i) - \int_{\rho'} \Gamma_{\alpha\beta}(\rho_i, \rho', \rho_j(\rho')) dS \right], \quad \alpha, \beta \in \{x, y, z\}, \quad i, j = 1, 2, \ldots, N. \tag{4.17}
\]

Here, \(\Gamma_{\alpha\beta}(\rho_i, \rho', \rho_j)\) is the kernel component in the scalar version of equation 4.6 relating the \(\beta\) component of \(\vec{J}(\rho')\) to the \(\alpha\) component of \(\vec{J}(\rho)\). Also, the integral is taken over the parameter domain as in equation 4.13. If \(A_{\alpha\beta}\) is a 3\(\times\)3 matrix consisting of \(\pm 1\)’s and 0’s, a convenient expression for \(\Gamma_{\alpha\beta}\) is:

\[
\Gamma_{\alpha\beta} = \delta(\rho_i, \rho_j) A_{\alpha\beta} \nabla G(\rho_i, \rho_j, \rho) \tag{4.18}
\]

The appropriate \(A_{\alpha\beta}\)’s are:
\[
\Lambda_{xx} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
\Lambda_{xy} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
\Lambda_{xz} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{bmatrix}
\]

\[
\Lambda_{yx} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
\Lambda_{yy} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \\
\Lambda_{yz} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}
\]

\[
\Lambda_{zx} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} \\
\Lambda_{zx} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
\Lambda_{zz} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}
\]

In expression 4.17, the merits of the MFIE over the EFIE become apparent. As the matrix \([K]\) becomes larger, the integral portion of each element becomes smaller. However, the identity term produced by \(\delta_{ij}\alpha\beta\) maintains the conditioning of the matrix by keeping the diagonal elements close to one. The corresponding \([K]\) for the EFIE contains no identity term, so it becomes progressively ill-conditioned as the sampling grid becomes larger.

4.4. Approximating the Scattering Cross-Sections

We shall now derive finite approximations to the integrals that define the vector \(\sigma(p)\). This expression is a quadratic form on the hermitian matrix \(H_{\nu}H_{\nu}'\) and will be used in the minimization of 4.1.

If the observation point lies in the far field with respect to the entire surface, we can make the approximation that \(|\vec{r} - \vec{r}'| = R_0 - \frac{\vec{r} \cdot \vec{r}'}{R_0}\), where \(R_0\) is the distance along the direction of look from the origin to the receiver point. The resulting approximation to \(\nabla^2 G\) is:
\[ \nabla' G(\vec{r}, \vec{r}', p) = jk e^{-jkR_0} \frac{e^{-jk\vec{r}'(p)}}{R_0} \] (4.19)

Substituting this into 4.5 yields

\[ H^{sc}(\vec{r}, p) = \frac{jk e^{-jkR_0}}{4\pi R_0} \int_{S} \left( \hat{J}(\vec{r}', p) \times \hat{k} \right) e^{-jk\vec{r}'(p)} d^2 \vec{r}' . \] (4.20)

In terms of scalar polarized components, the scattered field may be written

\[ \begin{bmatrix} H_h^{sc} \\ H_v^{sc} \end{bmatrix} = \frac{jk e^{-jkR_0}}{4\pi R_0} \int_{S} \begin{bmatrix} J_v(\vec{r}') \\ J_h(\vec{r}') \end{bmatrix} e^{-jk\vec{r}'(p)} d^2 \vec{r}' . \] (4.21)

By inserting the expansion 4.14 into the above expression, the \( \eta \) component of \( H^{sc} \) may be written

\[ H_{\eta}^{sc} = \frac{jk e^{-jkR_0}}{4\pi R_0} \sum_{i=1}^{3N} q_i^{\eta} f_i(\vec{r}') e^{-jk\vec{r}'(p)} d^2 \vec{r}' . \] (4.22)

where \( q_i^{\eta} \) is a vector that transforms the \( x, y, \) and \( z \) components of \( J(\vec{p}) \) into the \( h \) or \( v \) components required to compute \( H_{\eta}^{sc} \). Notice that if \( h \) and \( v \) are associated with \( x \) and \( y \) directions, the \( q_i^{\eta} \)'s are composed solely of \( 1 \)'s and \( 0 \)'s. If we let

\[ \alpha_i^{\eta}(\vec{p}) = q_i^{\eta} jk e^{-jkR_0} \int_{S} f_i(\vec{r}') e^{-jk\vec{r}'(p)} d^2 \vec{r}' , \] (4.23)

the scattered field polarized in the \( \eta \) direction caused by an incident field in the \( \nu \) direction may be expressed as

\[ H_{\eta\nu}^{sc}(\vec{p}) = \sum_{i=1}^{3N} \alpha_i^{\eta}(\vec{p}) J_{\nu i}(\vec{p}) , \] (4.24)

where \( J_{\nu} \) is the \( J \) vector due to incident radiation with \( \nu \) polarization. If we arrange
the $\alpha^\eta$s in a vector $A_\eta$, we get

$$H^S_\eta = A^T_\eta J_\eta.$$  

Substituting this expression into 4.4 yields;

$$\sigma^{\eta \nu} = (A^T_\eta J_\eta)(A^T_\eta J_\eta)^* = A^T_\eta J_\eta J'_\nu A^*_\eta, \quad (4.25)$$

where $J'$ is the hermitian transpose of $J$. Inverting 4.15 and substituting into the previous expression gives us;

$$\sigma^{\eta \nu} = A^T_\eta (K^{-1} H_\nu)(K^{-1} H_\nu)' A^*_\eta \quad (4.26)$$

$$= A^T_\eta K^{-1} H_\nu H'_\nu (K^{-1})' A^*_\eta. \quad (4.27)$$

Now, letting

$$L_\eta = (K^{-1})' A^*_\eta, \quad (4.28)$$

The cross-section can be expressed in the form

$$\sigma^{\eta \nu} = L'_\eta H_\nu H'_\nu L_\eta. \quad (4.29)$$

Thus, the vector of computed cross-sections is given by;

$$\sigma(p) = \begin{bmatrix} L'_h H_h H'_h L_h \\ L'_h H_h H'_h L_h \\ L'_\nu H_\nu H'_\nu L_\nu \\ L'_\nu H_\nu H'_\nu L_\nu \end{bmatrix} \quad (4.30)$$
4.5. Non-Linear Minimization

The Gauss-Newton technique for minimizing equation 4.1 is to build a locally quadratic model of the residual vector \( R = (\sigma(p) - \sigma_c) \) around the current point \( p_c \), and to update \( p \) by the expression

\[
p_+ = p_c - (\partial^T(p_c)\partial(p_c))^{-1}\partial^T(p_c)R(p_c)
\]

(4.31)

where \( \partial(p) \) represents the Jacobian matrix of \( R(p) \), and the Hessian, or second derivative matrix has been approximated by \( \partial^T(p)\partial(p) \). The complexity of such a procedure evidently hinges not only on the computation of \( R(p) \), but also on the computation of the Jacobian \( \partial(p) \). The following section derives in detail an expression for the Jacobian, and shows how its computation by finite differences can be simplified.

Taking the gradient of equation 4.30 with respect to \( p \) yields

\[
[\partial(p)]_{ij} = \sum_{k=1}^{4} \frac{\partial\sigma_i}{\partial p_j}
\]

or,

\[
\frac{\partial\sigma_{\nu\nu}}{\partial p_k} = \left[ \frac{\partial L_{\nu}}{\partial p_k} \right]^T (H_{\eta}H_{\eta})L_{\nu} + L_{\nu}(H_{\eta}H_{\eta}) \left[ \frac{\partial L_{\nu}}{\partial p_k} \right] + L_{\nu} \frac{\partial (H_{\eta}H_{\eta})}{\partial p_k} L_{\nu}
\]

\[
= 2 \text{Re} \left\{ \left[ \frac{\partial L_{\nu}}{\partial p_k} \right]^T (H_{\eta}H_{\eta})L_{\nu} + L_{\nu}(H_{\eta}H_{\eta}) \frac{\partial H_{\eta}^T}{\partial p_k} L_{\nu} \right\}.
\]

(4.32)

This expression is complex and requires the solution of another \( 3N \times 3N \) system of equations for each \( \frac{\partial L_{\nu}}{\partial p_k} \). Since the quantities in 4.32 are functions of the surface...
height and normals that are in turn nonlinear functions of the parameter \( p \), these derivatives are difficult to express analytically for a general surface. If the Jacobian is to be approximated by finite differences, two extra \( 3N \times 3N \) linear system solutions are required at each step in the minimization procedure. This amount of computation time is unacceptable. On the other hand, we can express the derivatives in 4.32 analytically as functions of \( \frac{\partial z}{\partial p_k} \), and \( \frac{\partial h}{\partial p_k} \). In this way, these surface derivatives can be computed quickly by finite differences, and the computation of each \( \frac{\partial \sigma^v}{\partial p_k} \) will require only one \( 3N \times 3N \) linear system solution per update. To compute the Jacobian as given in 4.32, we need expressions for the derivatives of \( H_1 \) and \( L_\nu \). When the proper reference geometry is chosen, these expressions may be written explicitly.

The geometry chosen for this case is shown in figure 4.2. The incident and received plane waves propagate along the \( z \) axis and the receiver is a distance \( R_0 \) from the origin. The \( h \) direction will be along the \( x \) axis, and \( \nu \) along the \( y \) axis. Let the unknown portion of the surface be described by the surface patch surrounding the specular point \( (x_s, y_s) \), where the normal points along the \( z \) axis. All references are to a right-handed coordinate system with the \( z \)-axis extending towards the viewer. Furthermore, the surface \( S(p) \) will be considered a Monge patch [42], i.e. the parameters \( u \) and \( w \) used in the evaluation of the surface integrals are simply the \( x \) and \( y \) axes. For this type of surface,

\[
\sqrt{g_{uw}(p)} = \frac{1}{n_z(u, w, p)}.
\]
Figure 4.2
The geometry for deriving a finite approximation to the RCS.

Using this geometry, the $\mathbf{H}$ vectors are given by

$$
\begin{bmatrix}
-2n_{x1}e^{jkz_1} \\
\vdots \\
-2n_{xN}e^{jkz_N} \\
0 \\
\vdots \\
2n_{x1}e^{jkz_1} \\
\vdots \\
2n_{yN}e^{jkz_N}
\end{bmatrix}
$$

$$
\begin{bmatrix}
0 \\
\vdots \\
0 \\
2n_{x1}e^{jkz_1} \\
\vdots \\
2n_{xN}e^{jkz_N} \\
-2n_{y1}esup_{jkz_1} \\
\vdots \\
-2n_{yN}esup_{jkz_N}
\end{bmatrix}
$$

The derivatives of these vectors are
\[
\frac{\partial H_v}{\partial p_k} = e^{-jkR_0} \begin{bmatrix}
-2e^{jkz_1} \left( \frac{\partial n_{11}}{\partial p_k} + jkn_{11} \frac{\partial z_1}{\partial p_k} \right) \\
\vdots \\
-2e^{jkz_N} \left( \frac{\partial n_{1N}}{\partial p_k} + jkn_{1N} \frac{\partial z_N}{\partial p_k} \right) \\
0 \\
\vdots \\
0 \\
2e^{jkz_1} \left( \frac{\partial n_{21}}{\partial p_k} + jkn_{21} \frac{\partial z_1}{\partial p_k} \right) \\
\vdots \\
2e^{jkz_N} \left( \frac{\partial n_{2N}}{\partial p_k} + jkn_{2N} \frac{\partial z_N}{\partial p_k} \right)
\end{bmatrix}.
\]

\[
\frac{\partial H_k}{\partial p_k} = e^{-jkR_0} \begin{bmatrix}
0 \\
\vdots \\
0 \\
2e^{jkz_1} \left( \frac{\partial n_{11}}{\partial p_k} + jkn_{11} \frac{\partial z_1}{\partial p_k} \right) \\
\vdots \\
2e^{jkz_N} \left( \frac{\partial n_{1N}}{\partial p_k} + jkn_{1N} \frac{\partial z_N}{\partial p_k} \right) \\
-2e^{jkz_1} \left( \frac{\partial n_{21}}{\partial p_k} + jkn_{21} \frac{\partial z_1}{\partial p_k} \right) \\
\vdots \\
-2e^{jkz_N} \left( \frac{\partial n_{2N}}{\partial p_k} + jkn_{2N} \frac{\partial z_N}{\partial p_k} \right)
\end{bmatrix}.
\]

Th derivative of \( L_\eta \) is found by solving the equation

\[
\left[ K'(p) \right] \frac{\partial L_\eta}{\partial p_k} = \left[ \frac{\partial A_\eta(p)}{\partial p} \right]^* - \left[ \frac{\partial K'(p)}{\partial p} \right] L_\eta(p),
\]

(4.35)
which results from taking the derivative of 4.28. The solution to 4.35 depends on the quantities $\frac{\partial A_n}{\partial p_k}$ and $\frac{\partial [K]}{\partial p_k}$. The $A_n$ vectors may be expressed in terms of integrals of the basis functions over the surface parameter domain,

$$
A_n = \frac{jk e^{-jk R_0}}{4\pi R_0} \left[ \begin{array}{c}
\int f_i(x', y') \frac{e^{j k (x' - p)}}{n_i(x', y')} \, dx' \, dy' \\
\cdots \\
\int f_i(x', y') \frac{e^{j k (x' - p)}}{n_i(x', y')} \, dx' \, dy' \\
0 \\
\cdots \\
0 \\
0 \\
0
\end{array} \right]
$$

$$
A_h = \frac{jk e^{-jk R_0}}{4\pi R_0} \left[ \begin{array}{c}
0 \\
\cdots \\
0 \\
\int f_i(x', y') \frac{e^{j k (x' - p)}}{n_i(x', y')} \, dx' \, dy' \\
\cdots \\
0 \\
0
\end{array} \right]
$$

Because the limits of the integrals in the above equation are known from the occluding contours, the derivatives of $A_n$ may be expressed as
\[
\frac{\partial A_{\eta i}}{\partial p_k} = \frac{jke^{-jkR_0}}{4\pi R_0} b_{\eta i} \int_{S^\prime} f_{i,mod}(x,y)e^{jx\phi} \left[ jk\frac{\partial^z(p)}{\partial p_k}(n'_i(p))^2 - \frac{\partial n'_i(p)}{\partial p_k}(n'_i(p))^2 \right] dx'd\phi, \tag{4.37}
\]

where \( b_{\eta i} \) is either 1 or 0 to select the proper components given in equation 4.36.

Finally, since the principal value integral does not include the singularity in \( \Gamma_{\alpha\beta} \), and the bounds of \( S^\prime \) are not dependent on \( p \), from equation 4.17 the derivative of the \( K \) matrix is

\[
\frac{\partial [K_{\alpha\beta}]_{ij}}{\partial p_k} = -\int_{S^\prime} \left[ \frac{1}{n'_z} \frac{\partial \Gamma_{\alpha\beta}(\vec{r},\vec{r}')}{\partial p_k} - \frac{\partial n'_z}{\partial p_k} \Gamma_{\alpha\beta}(\vec{r},\vec{r}') \right] f_{i}(\vec{r}')dx'dy', \tag{4.38}
\]

\[\alpha, \beta \in \{x, y, z\}, \quad i,j = 1,2,\ldots,N,\]

where

\[
\frac{\partial \Gamma_{\alpha\beta}(\vec{r},\vec{r}')}{\partial p_k} = \frac{\partial \Lambda_{\alpha\beta}(\vec{r})}{\partial p_k} \varphi G(\vec{r},\vec{r}',p) + \Lambda_{\alpha\beta} \frac{\partial \varphi'}{\partial p} G(\vec{r},\vec{r}',p), \tag{4.39}
\]

and

\[
\frac{\partial \varphi'}{\partial p} = \left[ \frac{\partial z_i}{\partial p} - \frac{\partial z'_i}{\partial p} \right] e^{-jk|\vec{R}'_i|} \left[ \frac{\vec{R}'_i - R_{1'i}}{|\vec{R}'_i|} - \varepsilon' R_{2'i} \right], \tag{4.40}
\]

where

\[
\vec{R}'_i = \vec{r}_i - \vec{r},
\]

\[R_{1'i} = -jk|\vec{R}'_i|^{-3} - k^2 |\vec{R}'_i|^{-2} - 3|\vec{R}'_i|^{-4},\]

and
\[ R2_i' = |\vec{R}_i'|^{-3} + jk|\vec{R}_i'|^{-2}. \]

Combining these expressions, we see that the only quantities that have not been expressed analytically are the derivatives of the surface normals and \( z \) heights. These remaining unknowns may be easily and accurately computed by finite difference approximation on the surface.

4.6. Convergence of the NLLS Algorithm

The variation of the NLLS algorithm that shall be applied to the surface reconstruction problem is the Levenberg-Marquardt (L-M) method. This technique establishes a trust region around the current estimate of \( p \), and constrains updates to fall within that region. The convergence of this procedure depends on convergence of the basic Gauss-Newton technique, therefore we need only shown that the G-N algorithm converges, and the L-M algorithm will converge at least as well.

It can be shown [4] if

1) \( \mathbf{R} : \mathbb{R}^n \to \mathbb{R}^m \),

2) \( f(p) = \frac{1}{2} \mathbf{R}^T(p) \mathbf{R}(p) \) is \( C^2 \) in \( D \subset \mathbb{R}^n \),

3) \( \exists \ p_* \in D \), such that \( \partial f(p_*)^T \mathbf{R}(p_*) = 0 \),

4) \( \exists s \geq 0 \) such that \( \| (\partial f(p) - \partial f(p_*)^T \mathbf{R}(p_*) \|_2 \leq s\| p - p_* \|_2, \forall p \in D \),

5) \( \exists \lambda \geq 0 \) such that \( \lambda \) is the smallest eigenvalue of \( (\partial f(p_*)^T \partial f(p_*)) \),

6) \( s < \lambda \).
then, for all initial approximations $p_0$ in an open neighborhood of radius $\varepsilon$ around $p_*$, the Gauss-Newton method converges to $p_*$.

From the above theorem, criteria can be established for the convergence of the numerical surface reconstruction algorithm. Items 1), 2), and 3) can be assumed true for most applications. From 5), a $\lambda$ can be established by noting that if $e$ is an eigenvalue of $A$, then $e \geq \frac{1}{\|A^{-1}\|}$. All that remains is to show that $s < \lambda$, where

$$s = \sup_{p \in D} \frac{\left\| \partial^T(p) - \partial^T(p_*) \right\|_2}{\|p - p_*\|_2}, \quad (4.41)$$

Since $s$ is an approximation to the second derivative of the residual, the quantity $\frac{s}{\lambda}$, which should not exceed 1, indicates the relative non-linearity of the problem. If the problem being solved has zero residual, i.e. the model matches the observation exactly at $p_*$, $s$ becomes zero, and the algorithm is guaranteed to converge for $p_0$ inside $N(p_*, \varepsilon)$. Small residuals are unlikely in the present application because of errors in the observed RCS, and in the model RCS.

The particular implementation of the NLLS algorithm that shall be used is the L-M algorithm found in the MINPACK collection of programs. This algorithm has even stronger convergence properties than the Gauss-Newton technique. The L-M algorithm compares the G-N step to a threshold and if it is too large, constrains the step to lie within some trust region. This allows convergence even when the matrix $\partial(p)^T\partial(p)$ is singular. According to a theorem by More [45], when the gradients of $R(p)$ are continuously differentiable, the steps generated by the L-M algorithm will
converge to zero.

4.6.1. A Simplified Convergence Example

To analyze convergence for the G-N method in the moment method solution case, it must be shown that $\partial^T(\mathbf{p})\partial(\mathbf{p})$ is invertible, when the components of $\partial(\mathbf{p})$ are given by equation 5.27. Because of the complexity of this expression, it is impossible to make any general comments concerning $\partial^T(\mathbf{p})\partial(\mathbf{p})$. However, when the PO approximation is made, and the single parameter, point matching solution is considered, non-convergent situations can be predicted.

Consider the case where $\mathbf{p}$ consists of only one variable, and only the physical optics scattering solution is required. When the physical optics approximation holds, there is no cross-polarized scattered field, and the co-polarized fields are equal. Thus, the matrix $\partial^T(\mathbf{p})\partial(\mathbf{p})$ has only one component, $\left[ \frac{\partial \sigma_{PO}}{\partial p} \right]^2$. We are interested in identifying situations where this quantity goes to zero. The physical optics approximation to the matrix $K$ is the identity matrix $I$. Substituting $I$ for $K$ in the expression for $L_v$, produces $L_v^{PO} = A_v^*$. Substituting this into equation 4.32 yields

$$
\frac{\partial \sigma_{PO}}{\partial p} = 2 \text{Re} \left\{ \left[ \frac{\partial A}{\partial p} \right]^T (\mathbf{HH}^*) A^* + A^T \mathbf{H} \frac{\partial \mathbf{H}^*}{\partial p} A^* \right\},
$$

(4.42)

where the subscripts have been dropped since polarization has no relevance in the PO case. From the components of $\mathbf{H}$, $A$ and their derivatives given in expressions 4.33, 4.34, 4.36, and 4.37, it can be seen that the vector inner products $A^T \mathbf{H}$, $\frac{\partial A^T}{\partial p} \mathbf{H}$, and
$A^T \frac{\partial H}{\partial p}$ consist of only $N$ terms each due to multiplication by 0's. Thus, these vectors can now be thought of as length $N$ vectors. The derivatives of the components of these length $N$ vectors may be written

$$\frac{\partial H_i}{\partial p} = \alpha_i H_i, \quad \frac{\partial A_i}{\partial p} = -\alpha_i^* A_i,$$  \hspace{1cm} (4.43)

where,

$$\alpha_i = jk \frac{\partial z_i}{\partial p} + \frac{1}{n_{zi}} \frac{\partial n_{zi}}{\partial p}.$$

Substituting expressions 4.43 into 4.42, and writing the inner products in terms of summation over the components, it can be shown that

$$\frac{\partial \sigma_{PO}}{\partial p} = 8 \left[ \sum_{m=1}^{N} \sum_{n=1}^{N} \left( \text{Im}\{\alpha_m^*\} - \text{Im}\{\alpha_n^*\} \right) \text{Re}\{A_m H_m\} \text{Im}\{A_n H_n\} \right].$$  \hspace{1cm} (4.44)

By substituting the expressions for the real and imaginary parts of these terms into expression 4.44, the explicit form of $\frac{\partial \sigma_{PO}}{\partial p}$ is

$$\frac{\partial \sigma_{PO}}{\partial p} = \frac{8k^3}{(2\pi R_0)^2} \sum_{m=1}^{N} \sum_{n=1}^{N} \left( \frac{\partial z_m}{\partial p} - \frac{\partial z_n}{\partial p} \right) \sin 2k(z_m - R_0) \cos 2k(z_n - R_0).$$  \hspace{1cm} (4.45)

This quantity must not be zero if the G-N method is to converge. According to expression 4.45, the situation to avoid would be one in which all $z$ values change at the same rate with respect to $p$. For instance, a cylinder with fixed attitude whose length is $p$. Also, sampling grids should be avoided where all differences between $z$ values are multiples of $\frac{\lambda}{8}$. This is unlikely to occur, but can be avoided by sampling.
the surface at a rate higher than $\frac{\lambda}{8}$ in the $z$ direction.
CHAPTER 5

Simulations and Experiments

5.1. Introduction

Simulations have been performed to demonstrate the principles detailed in the previous chapters. In addition, experimental RCS measurements have been made on a simple scattering target in an effort to prove the usefulness of the technique when applied to real data. When selecting the situations and scattering targets for these experiments, the requirements of simplicity in problem formulation were balanced with difficulty in surface reconstruction. To demonstrate effectively the concepts being tested, the scattering geometry and surface parameters must be simple enough that reasonable conclusions may be inferred from the results of the simulations. On the other hand, the problem should not be so simple that the solution is trivial, or may be more easily solved by other means.

In this chapter, three basic applications are examined. Two of these involve simple, constrained target shapes and RCS modeling techniques, while the third illustrates the application to arbitrary surfaces. The first two applications are demonstrated both on simulated data and experimental measurements.

The first application considered is that of reconstructing the edge of a flat plate. Three edges of the plate and its approximate orientation are assumed known from the camera image. The co-polarized cross-sections at three frequencies are used to solve
for the width and exact orientation of the plate. The expressions for the RCS of a flat plate are derived by GTD in a paper by Ross [31]. Experimentally measured cross-sections are also reported in Ross' paper, and these will be used to show the success of the edge reconstruction technique when applied to real data.

The second application is to determine the size and orientation of a perfectly conducting ellipsoid. The information taken from the optical image is the width and height of the ellipsoid projected into the image plane. The remaining two axes of the ellipsoid, as well as the azimuthal orientation are determined by the error minimization process. The observation vector consists of the apparent ellipsoid width in the image plane, the co-polarized RCS and the cross-polarized RCS. The predicted co-polarized RCS is computed by a geometric optics approximation, and the cross-polarized RCS is computed by an expression derived by Chaudhuri and Boerner [38]. Chaudhuri used this expression to invert the profile of an ellipsoid from monostatic scattering data.

The third application demonstrates the use of Moment Method solutions to arbitrary surface scattering. This application implements the surface parameterization described in section 5.3.2. The target shape to be determined is an ellipsoid with an indentation at the specular point. The area around the specular point is assumed to be obscured in the optical image, and is approximated by various spline surface patches. This application is a simulation of the example described in the first chapter of this dissertation, section 1.4.3.
The absence of real data to compare to the arbitrary surface simulation results from two factors. First, no experimental measurements are available for the target shape considered. Second, due to its computational speed and ease of programming, a point-matching solution to the MFIE was used. This type of solution, while useful for demonstration purposes, does not always closely correspond to experimental measurements. A better approach to demonstrating the arbitrary surface application would be to use a professionally written, commercially available scattering code to approximate the scattering cross-sections. When such codes are available, they correspond more closely to experimental results than point-matching solutions.

5.2. Flat Plate Edge Reconstruction

The geometry for this example is shown in figures 5.1 and 5.2. A thin,

![Diagram of a perfectly conducting flat plate](image)

Figure 5.1
Dimensions of the perfectly conducting, rectangular flat plate.
perfectly-conducting, rectangular, flat plate with width $2a$ and height $2b$ is rotated an angle $\phi$ with respect to a viewer in the horizontal plane. A viewer with a camera, microwave transmitter, and microwave receiver is located at point a distance $r$ from the center of the plate. The light source for the camera image is separated from the viewer, and a shadowing object lies between this light source and the plate. Assume that the upper, lower, and left-hand edges of the plate can be determined from a camera image. The object lying between the light source and the plate casts a shadow on

Figure 5.2
Plan view diagram of the flat plate edge reconstruction problem. The right-hand edge of the plate is lost in shadow and will be determined from the RCS data.
the plate, obscuring the right-hand edge against the dark space background. This is a simplified version of the situation shown in the space image of figure 5.3, where the edge of a satellite is lost against the dark background. The goal of the sensor fusion system here is to determine the position of the unknown plate edge given the approximate orientation of the plate, the three edges extracted from the optical image, and the polarized RCS at various frequencies. The expressions used to predict the cross-sections are taken from a paper by Ross [31]. For an incident plane wave with wavenumber $k$ transmitted and received with vertical polarization, the RCS is

![Figure 5.3](image)

*Figure 5.3*

*A space photograph of a satellite showing edges lost in deep shadow and diffraction effects due to intense specular reflections.*
\[
\sigma_{VV} = \frac{4b^2}{\pi} \left[ \cos 2k \sin \phi - \frac{j \sin 2k \sin \phi}{\sin \phi} \right] \\
- \frac{e^{-j2ka - j(\frac{\pi}{4})}}{\sqrt{2\pi}(2ka)^2} \left[ \frac{1}{\cos \phi} + \frac{e^{j2ka - j(\frac{\pi}{4})}}{4\sqrt{2\pi}(2ka)^2} \left( \frac{(1 + \sin \phi)e^{-j2k \sin \phi}}{(1 - \sin \phi)^2} + \frac{(1 - \sin \phi)e^{j2k \sin \phi}}{(1 + \sin \phi)^2} \right) \right] \\
\times \left[ 1 - \frac{e^{-jka - j(\frac{\pi}{2})}}{8\pi(2ka)^3} \right]^{-1/2}.
\]

(5.1)

For horizontally transmitted and horizontal received polarization, the expression is

\[
\sigma_{HH} = \frac{4b^2}{\pi} \left[ \cos 2k \sin \phi + \frac{j \sin 2k \sin \phi}{\sin \phi} \right] \\
- \frac{4e^{-j2ka + j(\frac{\pi}{4})}}{\sqrt{2\pi}(2ka)^2} \left[ \frac{1}{\cos \phi} - \frac{e^{-j2ka + j(\frac{\pi}{4})}}{2\sqrt{2\pi}(2ka)^2} \left( \frac{e^{-j2k \sin \phi}}{1 - \sin \phi} + \frac{e^{j2k \sin \phi}}{1 + \sin \phi} \right) \right] \\
\times \left[ 1 - \frac{e^{-jka + j(\frac{\pi}{2})}}{\pi(2ka)^2} \right]^{-1/2}.
\]

(5.2)

These expressions are derived through the geometrical theory of diffraction, and are accurate for angles of orientation less than about 80 degrees. Expressions are available for co-polarized cross-sections only. Vertical and horizontal polarizations are considered separately, however. Accurate expressions for larger angles, including edge-on incidence are derivable through UTD, but will not be considered here.
In these simulations, the RCS observations were modeled by adding uncorrelated, zero-mean, Gaussian random noise to the theoretical RCS predictions. The same variance was used for all observations within a given trial. This variance was adjusted from trial to trial to show the effect of RCS observation signal-to-noise ratio on convergence of the algorithm. This same basic simulation technique is used for all simulations in this chapter. This noise simulation may not resemble "real-world" conditions in the sense that all observations are not necessarily equally reliable. For instance, cross-polarized returns are likely to have greater error than co-polarized observations.

Presumably, the estimate of orientation, $\phi$, is derived through stereo, shape-from-shading, or photometric stereo. Because these are unreliable techniques, this orientation is treated as an initial approximation only. The exact orientation is an unknown parameter to be determined through the error-minimization process. The initial estimate of orientation extracted from the optical image is simulated as a uniform random variable distributed in the range $\pm \delta \phi$ about the exact value of $\phi$. Various values for $\delta \phi$ were used in an attempt to find the tolerance of the algorithm to orientation error.

A plot of $\sigma_{VV}$ versus $a$ for a plate with height 3.25 cm oriented at 45° to an incident plane wave with a wavelength of 1.28 cm (Ku band) is shown in figure 5.4. Note the presence of many local minima in this expression. These can and do cause problems for the non-linear-least-squares algorithm; the algorithm tends to converge to these minima if they lie between the initial approximation and the true solution.
Figure 5.4
A plot of $\sigma^{ww}$ versus plate width $a$ for a perfectly conducting flat plate rotated 30° in the horizontal plane.

To reduce the effect these minima have on the residual error function, the observation vector was chosen to consist of cross-sections measured at three different frequencies. So that the local minima do not coincide, the observation frequencies should not be harmonics of one another.

From the preceding discussion, the parameter vector for this simulation is of length two, and the observation vector is length six.

$$p_{plate} = \begin{bmatrix} \alpha \\ \phi \end{bmatrix},$$

(5.3)
\[ \sigma_{plate} = \begin{bmatrix} \lambda_1 \\ \sigma_{VV} \\ \sigma_{HH} \\ \lambda_2 \\ \sigma_{H\ell} \\ \lambda_3 \\ \sigma_{\ell\ell} \end{bmatrix}, \]  

(5.4)

where \( \lambda_1 = 1.28 \, \text{cm} \), \( \lambda_2 = \frac{\lambda_1}{2^{\frac{2}{3}}} = 0.76 \, \text{cm} \), and \( \lambda_3 = \frac{\lambda_1}{2^{\frac{1}{3}}} = 0.55 \, \text{cm} \).

5.2.1. Flat Plate Simulation Results

Simulations were performed for plates oriented at 30°, 45°, and 60°. The true width of the plate is assumed to be 6.5 cm yielding a correct value of 3.25 cm for the variable \( a \). The initial estimate of \( a \) was arbitrarily chosen as 2.0 cm in each case. Results of these simulations are shown in figures 5.5-5.7. These figures contain plots of signal-to-noise ratio versus percent error in plate width. The signal-to-noise ratio for this case is the ratio of squared noise variance to squared magnitude of the RCS vector.

\[ SNR = \frac{||\sigma||^2}{\text{var}^2}. \]  

(5.5)

Even with multiple frequencies, the numerical error-minimization algorithm still exhibited a tendency to converge to local minima. This is because the MINPACK Levenburg-Marquardt version of the error minimization algorithm terminates if the improvement between successive approximations to \( \sigma \) is below a certain tolerance. The algorithm tends to descend into a "valley" and creep slowly along the floor.
% Error in 30° Plate edge

Figure 5.5
Percent error in flat plate edge reconstruction for the case $\phi = 30^\circ$. 
% Error in 45° Plate Edge

Figure 5.6
Percent error in flat plate edge reconstruction for the case $\phi = 45^\circ$. 
To overcome this difficulty, a supervisory step was added to the algorithm. At each termination of the numerical algorithm, the magnitude of the residual error vector, \( \| R(p) \|_2 \), is compared to a threshold, \( t \). If \( \| R(p) \|_2 > t \), the initial estimate of \( a \) is perturbed by a small amount and the numerical algorithm restarted. Since the size of \( \| R(p_*) \|_2 \) is related to the variance of the additive noise, \( t \) was chosen to be proportional to the noise variance. A proportionality constant of 5 was found to be effective. Lowering this constant may improve global convergence slightly at low SNR's.
The effects of this supervisory procedure can be seen in the error plots. Note that in each case, the errors tend to form two groups; one group of large errors at low SNR's, and another group of small errors at the high SNR's. The break point between these groups represents the point at that the variance of the additive noise became as large as the residual at the incorrect termination. Beyond this point, the algorithm loses the ability to distinguish between termination at the wrong $p$, and termination at the correct, noisy $p$.

![Graph of $\lambda$ vs. $a$](image)

**Figure 5.8**
The quantity $\lambda$, that is the smallest eigenvalue of $\partial_e^T \partial_e$ for reconstructing the edge of a rectangular flat plate.
5.2.2. **Flat Plate Convergence**

The number of function evaluations required for convergence in these simulations averaged around 100. The value of the convergence parameter $s$ for a 45° plate with noise variance 0.1, and $a_\ast = 3.25209$ was found to be 6.42. This small value of $s$ relative to $\lambda$ indicates that the problem is not strongly non-linear. The matrix $\partial^T(p)\partial(p)$ was always invertible. A plot of $\lambda$ vs. $p$, where $\lambda$ is the smallest eigenvalue of $\partial^T(p)\partial(p)$, is shown in figure 5.8. In no place does this quantity become zero, indicating $\partial^T(p)\partial(p)$ has full column rank throughout $D$. The ratio $\frac{s}{\lambda}$ is plotted in figure 5.9. Note that this quantity stays less than 1, that guarantees convergence according to the requirements stated in the previous chapter 4. The largest value this ratio attains is 0.00613, at $a = 2.265$.

5.2.3. **Flat Plate Example with Measured Data**

Experimentally measured cross-sections for a perfectly conducting flat plate are given in Ross' paper [31]. Ross reports the cross-section of three sizes of plates at the same RCS, but these have been scaled to represent measurements on the same size plate at three different frequencies. The plate size is chosen as $6.5cm \times 6.5cm$, and the three wavelengths are 1.28 cm, 1.66 cm, and 2.08 cm. The deviations of the experimental data from theoretical predictions are shown in Table 6.1 for the three orientations 30°, 45°, and 60°. If an estimate of SNR for this data is $\frac{||\sigma_0||^2}{||\varepsilon||^2}$, where $\varepsilon$ is the vector of deviations from table 5.1, it is seen that this data lies just inside the group of poorly converging SNR's shown in figures 5.5-5.7.
Figure 5.9

The ratio $\frac{s}{\lambda}$ for the flat plate edge reconstruction problem. The fact that this quantity is much less than one indicates the problem is not too non-linear.
<table>
<thead>
<tr>
<th>$\phi$</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
</tr>
</thead>
<tbody>
<tr>
<td>error in $\sigma_{VV}^{\lambda_1}$</td>
<td>$-1.26 cm^2$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>error in $\sigma_{HH}^{\lambda_1}$</td>
<td>$-1.41 cm^2$</td>
<td>$-1.58 cm^2$</td>
<td>$-2 cm^2$</td>
</tr>
<tr>
<td>error in $\sigma_{VV}^{\lambda_2}$</td>
<td>$+1.78 cm^2$</td>
<td>$+1.12 cm^2$</td>
<td>$+1.26 cm^2$</td>
</tr>
<tr>
<td>error in $\sigma_{HH}^{\lambda_2}$</td>
<td>$-3.16 cm^2$</td>
<td>$-1.41 cm^2$</td>
<td>$-1.26 cm^2$</td>
</tr>
<tr>
<td>error in $\sigma_{VV}^{\lambda_3}$</td>
<td>$+1.12 cm^2$</td>
<td>0</td>
<td>$-1.25 cm^2$</td>
</tr>
<tr>
<td>error in $\sigma_{HH}^{\lambda_3}$</td>
<td>0</td>
<td>$-1.58 cm^2$</td>
<td>0</td>
</tr>
<tr>
<td>SNR</td>
<td>11.1 $dB$</td>
<td>16.35 $dB$</td>
<td>16.89 $dB$</td>
</tr>
</tbody>
</table>

*Table 5.1 Deviations of experimentally measured cross-sections of a perfectly conducting flat plate from theoretical predictions.*

When the algorithm is applied exactly as in the simulations section, the updates of $\phi$ wander far from the correct value, even when $\phi_0$ lies close to $\phi_*$. To prevent incorrect estimates of $\phi$ from misguiding the algorithm in this low SNR case, $\phi$ is assumed known exactly and not allowed to vary. Another approach to solving this difficulty when $\phi$ is not known exactly would be to limit the variation of $\phi_*$ from $\phi_0$, and consider the equivalent constrained optimization problem. Table 5.2 lists the results of applying the edge reconstruction algorithm to the experimental data with $a_0 = 2.0 cm$. 
<table>
<thead>
<tr>
<th>$\phi$</th>
<th>$a_*$</th>
<th>% error in $a_*$</th>
<th>residual</th>
<th>no. f.e.v</th>
</tr>
</thead>
<tbody>
<tr>
<td>30°</td>
<td>3.24938</td>
<td>0.02</td>
<td>4.241</td>
<td>48</td>
</tr>
<tr>
<td>45°</td>
<td>3.23325</td>
<td>0.52</td>
<td>2.781</td>
<td>22</td>
</tr>
<tr>
<td>60°</td>
<td>3.26761</td>
<td>0.54</td>
<td>2.940</td>
<td>48</td>
</tr>
</tbody>
</table>

Table 5.2 Results of applying the edge reconstruction algorithm to experimental data of a perfectly conducting 6.5 cm X 6.5 cm flat plate.

Satisfactory results were obtained and produced residuals of the same order as the SNR.

5.3. Rotated Ellipsoid Example

The second application used to test the sensor fusion system is to determine the size and attitude of an ellipsoid rotated an angle phi in the horizontal plane. Figures 5.10 and 5.11 depict the geometry of this example. An ellipsoid centered at the origin has axes of length $a$ along the $x$ axis, $b$ along the $y$ axis, and $c$ along the $z$ axis. A plane wave propagates along a line lying in the $x$–$y$ plane making an angle $\phi$ with the $y$ axis. The angle of polarization, $\alpha$, is measured with respect to the $x$–$y$ plane. The backscattered microwave power is measured by a microwave receiver placed at the same point as the microwave transmitter and camera. The camera detects the outline of the ellipsoid projected into the image plane. Thus, the information available
Figure 5.10
The axes of the ellipsoid to be reconstructed. Axis $c$ will be determined from the camera image, axes $a$ and $b$ from the RCS.

from the optical image is the exact width of the ellipsoid, equal to $2c$, and the projected width of the ellipsoid along the $x$ axis, denoted by $w$. Assuming a parallel image projection, $w$ can be written in terms of the axes of the ellipsoid and the viewing angle $\phi$.

$$w = 2\sqrt{\frac{2A}{B^2 + 2AC}},$$

(5.6)

where,
\[ A = \frac{\sin^2 \phi}{b^2} + \frac{\cos^2 \phi}{a^2}, \]

Figure 5.11
Plan view of the rotated ellipsoid problem. The apparent width, \( w \), can be determined from the optical image.

\[ B = \frac{(a^2 - b^2)}{a^2 b^2} \cos \phi \sin \phi, \]

and,

\[ C = \frac{\sin^2 \phi}{a^2} + \frac{\cos^2 \phi}{b^2}. \]
5.3.1. Derivation of the Scattering Equations

Expressions for the co- and cross-polarized RCS of an ellipsoid are not well-known. No exact expression has yet been found for a general ellipsoid, although scattering from the prolate spheroid (an ellipsoid with two equal axes) is fairly well understood. The models used here are taken from separate sources. The expression for predicting the co-polarized RCS is a geometric optics (GO) solution that approximates the scalar RCS as proportional to $\frac{1}{K}$, where $K$ is the Gaussian curvature of the scattering target at the specular point. In the experimental portion of this section it will be seen that the GO expression is accurate if only the specular portion of the reflected energy is considered, i.e. the leading edge of the time domain response. The GO expression for the RCS of an ellipsoid rotated $\phi$ in the $x-y$ plane is [21]

$$
\sigma_{GO} = \frac{\pi a^2 b^2 c^2}{[a^2 \cos^2 \phi + b^2 \sin^2 \phi]^2}.
$$

(5.7)

The model used for the cross-polarized RCS is taken from a paper by Chaudhuri and Boerner on inverting the profiles of ellipsoids [38]. In this paper, the authors use the co- and cross-polarized RCS measured over a variety of viewing angles to solve for the profile of an ellipsoid. Their technique requires knowledge of the relative phase between the co-pol fields at orthogonal polarizations. These measurements are used in an iterative procedure to determine curvatures and hence the shape of a scattering ellipsoid. The technique presented in the present investigation represents an improvement over Chaudhuri's technique in the sense that only one viewing angle is required if the camera image is available. Also, our technique does not require
knowledge of phase, although if an additional axis of rotation were introduced, as in the Chaudhuri technique, phase would be useful. The cross-polarized RCS is derived by a first order correction to the physical optics approximation. Instead of substituting the PO surface current into the scattered field surface integral, a first order correction surface current is found by substituted the PO surface currents into the MFiE surface integral. This corrected PO surface current is then used to compute the scattered field. The expression Chaudhuri and Boerner derived for the magnitude of the scattered field with $-\alpha$ polarization due to a transmitted field with $\alpha$ polarization depends on the difference between the target's maximum curvature, $K_1$, and minimum curvature $K_2$ at the specular point.

$$|\vec{H}^{sc}| = \cos \alpha \sin \alpha |K_1 - K_2| \left( \frac{G}{\omega} \right)^2 + \left( \frac{2\Gamma G}{\omega} \right)^2 \Gamma^{\frac{1}{2}},$$

(5.8)

where,

$$\Gamma = \sqrt{\frac{a^2b^2B_1}{C_1^2 - B_1E_1}},$$

$$G = \frac{a^2b^2}{B_1} \left| \frac{C_1^2}{B_1^2} - \frac{E_1}{B_1} \right| \frac{1}{2} \Gamma^3$$

and,

$$B_1 = b^2 \sin^2 \phi + a^2 \cos^2 \phi,$$

$$C_1 = (a^2 - b^2) \sin \phi \cos \phi,$$
\[ E_1 = a^2 \sin^2 \phi + b^2 \cos^2 \phi. \]

Note that the cross-polarized response goes to zero when the polarization of the incident or received fields is aligned with the \( x \)-\( y \) plane or the \( z \) axis. For this reason, \( \alpha \) must lie between 0° and 90°, and preferably not to close to the limits of this range, so that the cross-polarized return is not too small.

The parameters to be determined for the ellipsoid are the axes \( a \) and \( b \), and the orientation \( \phi \). The observations are the co-polarized GO RCS, the cross-polarized RCS, and the apparent width \( w \). For this application,

\[
P_{\text{ellip}} = \begin{bmatrix} a \\ b \\ \phi \end{bmatrix}
\] (5.9)

and

\[
\sigma_{\text{ellip}} = \begin{bmatrix} \sigma_{\text{GO}} \\ \sigma_{\text{spot}} \\ w \end{bmatrix}
\] (5.10)

Noise was added to the observation vector in the same manner as the flat plate application. The observations were determined by the theoretical expressions, and zero mean Gaussian noise with progressively larger variance was added to them.

5.3.2. Ellipsoid Simulation Results

The ellipsoid used for this simulation has dimensions \( a = 16cm \), \( b = 12cm \), and \( c = 3cm \). The error results for simulations at \( \phi = 30^\circ \), and \( 45^\circ \) are shown in figures 5.12 and 5.13. The signal-to-noise ratio was defined the same as for the flat plate example, and the error is given by
Figure 5.12

Percent error in reconstructing the parameters of an ellipsoid rotated by an angle $\phi = 30^\circ$. 
Figure 5.13

Percent error in reconstructing the parameters of an ellipsoid rotated by an angle $\phi = 45^\circ$. 
\[ \| e \| = \frac{1}{M} \sum_{i=1}^{M} \| p_i - p_{exact} \|_2, \]

where \( M \) is the number of samples, \( p_\star \) is the parameter found by the NLLS algorithm, and \( p_{exact} \) is the exact parameter used to compute the noiseless observations. The initial estimate of \( a \) in these simulations was 3.0 cm, and the initial \( b \) was chosen to be equal to the observed \( w \), 8.44 cm for the 30° case, and 6.99 cm for the 45° case. Some ambiguity exists in that \( w \) does not depend on the sign of \( \phi \). Because of this, the success of the method in these two examples depended on the sign of the initial estimate of phi. Solutions were restricted to positive values of \( \phi \), and a small positive angle was chosen as the initial approximation to \( \phi \). Occasionally, at lower signal-to-noise ratios, the algorithm produced correct \( a \)'s and \( b \)'s, but chose \( \phi \) that was \( 2\pi + \phi_\star \). This was not considered an error, and the extra period was subtracted from the final answer. Figures 5.12 and 5.13 show that errors become large at signal to noise ratios of -10 dB for the 30° case, and 20 dB for the 45° case.

\[ \| e \| = \frac{1}{M} \sum_{i=1}^{M} \| p_\star - p_{exact} \|_2, \]

where \( M \) is the number of samples, \( p_\star \) is the parameter found by the NLLS algorithm, and \( p_{exact} \) is the exact parameter used to compute the noiseless observations.

For the case \( \phi = 60^\circ \), results were not so favorable. It was found that \( p_0 \) had to be close to \( p_{exact} \) for \( p_\star \) to be close to a correct answer. The reason for this is shown in figures 5.14 and 5.15, which depict the magnitude of the residual function, \( R(p) \), as a function of \( b \) and \( \phi \). Figure 5.14 is
Figure 5.14
A plot of $\| R(p) \|$ as a function of $b$ and $\phi$, in the region of $\phi = 30^\circ$, with $a$ fixed at 6.0 cm.
Figure 5.15
A plot of $\| R(p) \|$ as a function of $b$ and $\phi$, in the region of $\phi = 60^\circ$, with $a$ fixed at 6.0 cm.
for the case $\phi = 30^\circ$, and figure 5.15 is for the case $\phi = 60^\circ$. Notice the presence of two minima in the $60^\circ$ case. If $\phi_0$ is chosen to be less than approximately $45^\circ$, the NLLS algorithm will converge to an incorrect $p$ with residuals as small as can be expected for correct convergence. This problem can not be remedied with supervision as in the flat plate example. There is no way to distinguish between correct and incorrect convergence. The corresponding plot of $\|R(p)\|_2$ for the $30^\circ$ case exhibits only one minimum, assuring correct convergence for $\phi$'s less than $45^\circ$. To compare errors, simulations were performed on an ellipsoid oriented at $60^\circ$ with $a_0 = 5.0\ cm$, $b_0 = 9.0\ cm$, and $\phi_0 = 57^\circ$. The results of these simulations are shown in figure 5.16. Notice the broad flat area at $b = 9.0\ cm$ in figure 5.15. This is why $\phi_0$ needs to be so close to $\phi_*$ for this simulation. If the initial value of $b$ was closer to $b_*$, the restrictions on $\phi_0$ could be relaxed, although even with an accurate $b_0$, the algorithm will not converge properly if $\phi_0$ is less than $45^\circ$.

5.3.3. Convergence for the Ellipsoid Application

Convergence in this example was faster than the flat plate example, primarily because no search procedure was required. The number of function evaluations required for the ellipsoid example was typically greater than 50 and less than 60. The value of the convergence parameter $s$ for a $45^\circ$ ellipsoid with initial parameters given in the previous section, and noise variance 0.1, was $8.97661 \times 10^{-12}$.

A plot of $\lambda$ vs. $b$ and $\phi$, when $a = 6.0\ cm$, is shown in figure 5.17. In no place does $\lambda$ reach zero. However, at small values of $\phi$, and when $b$ approaches $a$, $\lambda$ becomes small, indicating the problem is poorly posed at these points. This is due to
two phenomena. When $\phi = 0^\circ$, the ellipsoid is presented broadside, and small changes in $\phi$ do not significantly effect the RCS's or $\nu$. This makes the problem nearly independent of $\phi$, hence $\partial^T(p)\partial(p)$ is poorly conditioned. When the value of $b$ is close to $a$, the ellipsoid becomes a prolate spheroid with its axis parallel to the $z$ axis, and is therefore over-parameterized as an ellipsoid. A change in $\phi$ does not affect any of the observations, so again, $\partial(p)$ does not have full column rank, render-
ing $\partial^T(p)\partial(p)$ completely ill-conditioned. Fortunately, although the Gauss-Newton technique would have trouble converging under these circumstances, the successful

*Figure 5.17*

A plot of $\lambda$ vs. $b$ and $\phi$ for $a = 6.0 \, \text{cm}$, and $\phi_\ast = 30^\circ$. 
convergence of the Levenburg-Marquardt least-squares minimization algorithm does not depend on the existence of \( \left( \partial^T(r) \partial(r) \right)^{-1} \). The L-M algorithm requires only that the second derivatives of \( R(p) \) be continuously differentiable [45]. This is another benefit of this algorithm over Roger's functional derivative technique. While Roger was forced to use regularization to prevent ill-conditioning, the parameterization technique employed in this investigation allows the use of numerical minimization codes that can deal with poorly conditioned matrices.

5.3.4. Ellipsoid Example with Experimental Data

Co- and cross-polarized RCS's were measured for a perfectly conducting ellipsoid at a variety of aspect angles. These measurements were performed in the small anechoic chamber at NASA's Lyndon B. Johnson Manned Space Flight Center. A CW/FM RCS measurement system was employed. In this technique, the transmitted frequency was swept linearly from 7 GHz to 12 GHz, and the reflected signal was detected by matched filtering. Room and target support reflections as well as antenna leakage were subtracted out, leaving only the target response [19]. The frequency domain response was transformed via an FFT to yield a band-pass time domain signal. These procedures were all performed by a Hewlett Packard 8510 network analyzer, and are described in detail in their application brochure. A diagram of the RCS measurement system is shown in figure 5.18. Note that the antennas are not exactly monostatic, but are separated by about 5°. Also, the far field criterion was not achieved over the entire angular range of measurement. This does not significantly affect the co-polarized RCS, but may have introduced error in the cross-polarized.
Figure 5.18

Plan view of the device used to measure polarized RCSs of an ellipsoid.

measurements.

Photographs of the experimental setup are shown in figures 5.19 and 5.20. Polarization isolation was achieved by rectangular horn antennas that have a cross-pol rejection capability of about 20 dB. This was the best that could be achieved under the circumstances, but better polarization isolation is possible and desirable. Power measurements obtained in this manner need to be scaled according to some calibration target. To assure close correspondence between theoretical and
Figure 5.19
A photograph of the target ellipsoid in the anechoic chamber used to measure the RCS.
Figure 5.20
A photograph of the instrumentation used to measure the RCS of an ellipsoid. The horn antennas in this figure are set up to measure the cross-polarized RCS.
Figure 5.21
The calibrated geometrical optics RCS of a perfectly conducting, 1:2:4 ellipsoid as a function of azimuth. The theoretically predicted GO RCS is shown for comparison.
Figure 5.22
The raw, calibrated, cross-polarized RCS of a perfectly conducting 1:2:4 ellipsoid as a function of azimuth. Shown for comparison is the cross-polarized RCS computed by a first-order correction to the PO approximation.
experimental measurements, an ellipsoid at $\phi = 0^\circ$ was used for calibration. Of course this would be impractical in a working system with an unknown target. Some well-understood scattering target such a cylinder or sphere would be used as a calibration target. The GO RCS was picked from the leading impulse of the time domain response, whereas the cross-polarized response was taken from the frequency domain signal at 11.5 GHz.

The resulting data compared to the theoretical responses are shown in figures 5.21 and 5.22. Note the poor correspondence between the experimental and theoretical cross-polarized RCS. This may be largely due to the fact that the impulse frequency response was inadvertently subtracted from the frequency domain traces. In an effort to compensate for the uneven frequency response of the calibration target (a cylinder has an RCS that increases with frequency), an average value was subtracted from the frequency domain traces at the time of measurement and not recorded. Observe that when bulk shifts were applied to three angular segments of the cross-polarized data, a closer correspondence is seen between theoretical and experimental measurements. A plot showing the three segments before and after correction is shown in figure 5.23, and a plot showing the composite corrected cross-pol data is shown in figure 5.24. The overly shallow null in the corrected experimental data could be attributed to the fact that far-field requirement was not completely satisfied [19]. The corrected cross-pol data was used for these experiments.

The discrepancies between theoretical and experimental data are listed in table 5.3.
Figure 5.23
A plot of the experimentally measured cross-polarized RCS data calibrated to 0°, and the two segments using alternate calibrations. A plot of the theoretically predicted RCS is shown for comparison.
<table>
<thead>
<tr>
<th>$\phi$</th>
<th>error in $\sigma_{go} (dB)$</th>
<th>error in $\sigma_{pol} (dB)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>-0.415</td>
<td>-0.01</td>
</tr>
<tr>
<td>$10^\circ$</td>
<td>+0.031</td>
<td>-0.106</td>
</tr>
<tr>
<td>$30^\circ$</td>
<td>-0.7</td>
<td>-0.273</td>
</tr>
<tr>
<td>$40^\circ$</td>
<td>-0.332</td>
<td>-1.89</td>
</tr>
<tr>
<td>$50^\circ$</td>
<td>-0.413</td>
<td>+9.461</td>
</tr>
<tr>
<td>$60^\circ$</td>
<td>-0.7</td>
<td>-1.44</td>
</tr>
<tr>
<td>$70^\circ$</td>
<td>-0.175</td>
<td>+0.432</td>
</tr>
<tr>
<td>$80^\circ$</td>
<td>+0.219</td>
<td>+2.542</td>
</tr>
<tr>
<td>$90^\circ$</td>
<td>-0.101</td>
<td>-7.86</td>
</tr>
</tbody>
</table>

*Table 5.3 Discrepancies between theoretical predictions and experimental measurements of radar cross-sections for an ellipsoid at various aspect angles.*

The error minimization algorithm was applied to the experimental data, and the resulting errors are given in figure 5.25. As in the simulations, for azimuths greater than $45^\circ$, the initial parameters were placed closer to the true solution. The error in the parameters at $\phi = 0^\circ$ is considerably lower than at other azimuth angles. This does not include the fact that the value found for $\phi$ was wrapped around many times. As could be predicted from the plot of $\lambda$ in figure 5.17, the iterative algorithm took large steps in the $\phi$ direction due to the poor conditioning of $\partial^T \partial$ at $\phi = 0^\circ$. Since
Figure 5.24
The composite, corrected, cross-polarized RCS with the theoretical predictions shown for comparison.
Figure 5.25
The percent error in reconstructing ellipsoid parameters from experimentally measured RCS data.
the residual function is periodic in $\phi$, this was not counted as an error. This problem could be eliminated by adjusting the Marquardt parameter that limits the step size in the L-M algorithm. The remaining errors averaged around 15%, with the largest errors occurring at 40°, where the cross-polarized RCS diverged greatly from the theoretical predictions. These experimental errors were as good as or better than those predicted by the simulations.

Figure 5.26
The top half of the arbitrary surface whose specular region is to be reconstructed. The peripheral details are assumed known from the optical image, and the central portion is recovered using the RCS data.
5.4. An Arbitrary Surface Example

The flat plate and ellipsoid examples demonstrated the application of the shape extraction technique to targets with simple, constrained geometries. An application will now be described that allows unknown targets to possess a wide variety of shapes, not necessarily related to simple geometric objects. The method of moments has been employed to predict the polarized RCS's of an arbitrary scattering surface by the technique explained in the previous chapter. The shape used is an electrically small prolate spheroid with an indentation at the specular point. The top half of this target is shown in figure 5.26, and the scattering geometry is shown in figure 4.2. The minor axis of this target measures $\frac{1}{2}\lambda$, and the major axis is $\frac{3}{4}\lambda$, where $\lambda$ is the wavelength of the incident radiation. The horizontally polarized component of the incident radiation has its magnetic field aligned with the major axis. The reason for choosing a small target is to reduce the number of sampling nodes that are required.

It is assumed that the specular region of this target is obscured in the camera image, but the peripheral shape and occluding contours have been reconstructed. This may have been done through shape-from-shading, or stereo imagery. Both of these 3D vision techniques fail in the presence of intense specular points as may be found in space images. Figure 5.27 is an image of such a target showing the intense region around the specular point. Note the vertical diffraction line emanating from the specular point in figure 5.27. This behavior was predicted in chapter 2. For this reason, we want to determine the shape of the specular area from the co- and cross-polarized RCS's.
Figure 5.27
A photograph of the ellipsoid taken under space-like lighting conditions. Loss of detail surrounding the specular point and some diffraction effects are visible.

To implement the point-matching procedure, a grid of sampling points consisting of 9 radial lines of 7 points each encloses the target. The central point and first ring lie within the unknown specular region as shown in figure 5.28. B-spline surface patches were used to approximate the surface within this region. The peripheral knots of the spline patch are completely fixed by the camera-derived surface. The interior knots are fixed in the \(x-y\) plane, and their \(z\) heights are left as parameters in the error minimization scheme. In the first example of this section, a bi-quadratic B-spline patch was used, resulting in one free parameter. In the second example, the specular region is represented by a B-spline patch that is cubic in the \(y\) direction and quadratic in the \(x\) direction. This leaves two interior knots to be determined by the non-linear least squares algorithm. These configurations are diagramed in figure
5.29. For the first example, the parameter vector was

\[ p_{arb} = z_1, \]  \hspace{1cm} (5.11)

where \( z_1 \) is the \( z \) height of the central interior knot. For the second example,

\[ p_{arb} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \]  \hspace{1cm} (5.12)

where \( z_1 \) is the \( z \) height of the left interior knot, and \( z_2 \) the height of the right-hand knot. For both examples, the observation vector consisted of the co-polarized RCS in both polarizations, and the cross-polarized RCS.
Figure 5.29a
The knots used in a bi-quadratic B-spline reconstruction of the specular portion of an arbitrary surface. The eight peripheral knots are fixed by the optical image, and the z height of the central knot is allowed to vary.

Figure 5.29b
The grid of knots used for a B-spline reconstruction that is cubic in the x direction and quadratic in the y direction. The ten peripheral knots are fixed by the optical image, and the z heights of the central knots are allowed to vary.
The quantity $\sigma_{vv}$ for an arbitrary surface whose specular region has been approximated using a bi-quadratic spline patch with a variable central knot.

\[
\sigma_{arb} = \begin{bmatrix}
\sigma_{hh} \\
\sigma_{hv} \\
\sigma_{vv}
\end{bmatrix}.
\]  

(5.13)

Only three observations are available at each frequency since the principle of reciprocity predicts $\sigma_{vh} = \sigma_{hv}$. Noise was added to the observations in the same manner as the previous two applications. In addition, scaling was performed to lend equal weight to the various RCS’s that differ from each other by orders of magnitude; $\sigma_{vv}$ was scaled by 10, and $\sigma_{vh}$ by 100.

5.4.1. Arbitrary Surface Simulation Results

A plot of $\sigma_{vv}$ for the 2x2 example is shown in figure 5.30. From this figure it can be seen that local minima may occur as in the flat plate example. The same
supervisory system was used to overcome this problem. The resulting errors are plotted against signal-to-noise ratio in figures 5.31 and 5.32. In figure 5.31, an initial \( z_1 \) was chosen close to the correct solution so that no search was necessary. In figure 5.32, the initial estimate of \( z_1 \) was chosen far from the correct solution so that the supervision was required. Error and SNR are defined as in the previous examples, and similar results can be observed. Figure 5.31 shows a smooth relationship between SNR and error. As in the flat plate application, figure 5.32 shows two groups of errors. The error groups are divided at approximately 35 dB, where noise variance becomes as large as the residual at false convergences.
Figure 5.32
Percent error in spline parameter recovery using a bi-quadratic patch. A supervisory procedure was used. Note the sharp break in errors at about 30 dB.

The magnitude of $R(p)$ for the mixed degree spline is plotted in figure 5.33 as a function of $z_1$ and $z_2$. Notice that there is a shallow valley that runs from left to right across this figure with its lowest point in the middle at $p_*$. When the minimization procedure is started with $p_0$ in the leftmost quadrant where $z_2$ is large and $z_1$ is small, the algorithm converges nicely. When started in the rightmost quadrant with large $z_1$ and small $z_2$, the algorithm descends into the valley and make little progress, terminating before it reaches $p_*$. The solution to this problem is not so simple as in the single parameter case. A perturbation from the initial estimate of $p$ requires direction as well as magnitude; it is not clear how to choose this direction. One possibility is to perturb $p_0$ in random directions with increasing radius until a sufficiently small
residual is obtained. The errors resulting from applying the algorithm with an initial $z_1$ of 0.42, and an initial $z_1$ of 0.37 are shown in figure 5.34. Errors and signal-to-noise ratios are defined as in the previous examples.

### 5.4.2. Arbitrary Surface Convergence

The number of function evaluations here typically ran about 100, although this number could be reduced substantially if the procedure for calculating

![Graph showing % error vs SNR](image)

**Figure 5.34**

*Percent error in spline parameter recovery using a mixed degree B-spline patch.*
Figure 5.33
A plot of $||R(p)||$ versus $z_1$ and $z_2$ for the mixed degree B-spline reconstruction. $p_*$ occurs at (0.4,0.4). Note the shallow "valley" that runs from left to right across the figure.
the Jacobian described in the previous chapter was used. A plot of \( \lambda \) for the mixed
degree spline case is shown in figure 5.35. Although these values appear small, this
is partially due to the scaling of the cross-sections that have small numerical values.
The minimization program scales the Jacobian so this is not a problem. Convergence
is rapid in the first part of the procedure, and slows down when approaching a solu-
tion in the valley of figure 5.33. A value of \( 1.72188 \times 10^{-7} \) for \( s \) indicates that the
problem is not strongly non-linear in this region. The quantity \( \frac{s}{\lambda} \) is plotted in figure
5.36, and remains well below 1.0 throughout the domain of \( p \).
Figure 5.35
A plot of $\lambda$, the smallest eigenvalue of $\partial_c^T \partial_c$. Since this quantity does not become 0 anywhere, one requirement for convergence is satisfied.
Figure 5.36
A plot of $\frac{S}{\lambda}$ for the mixed degree spline reconstruction. Since this quantity is always less than one, convergence is guaranteed.
CHAPTER 6

Conclusions

6.1. Summary

A free-flying space robot needs detailed information about the location of objects in its workspace and the shape of individual objects. The most common sensors for acquiring this information are electronic cameras sensitive to optical wavelengths. The space environment, however, presents unique problems to an optical 3D shape sensing system. The lack of atmosphere creates deep shadows and intense illumination. Specular points become very bright, and shadowed edges are indistinguishable against dark backgrounds. These problems cannot always be easily overcome by simple image processing or enhanced vision algorithms. Sensory data from some independent physical regime must be used to augment a purely optical robot sensor system.

The process of integrating diverse physical observations to produce a single consistent world-view is known as sensor fusion. The goal of a sensor fusion system may be to localize objects within a robot workspace, or to determine the shape of individual objects. The application considered here is extracting the shape of 3D objects. Various sensors which have been studied by other researchers for robotic applications include multiple camera views, multiple processes on a single view, tactile sensors, laser range images, and ultrasonic imaging. This dissertation has focused
on the fusion of camera images with electromagnetic scattering data of conventional bandwidths (not high resolution) and frequencies, i.e. X band, Ku band, etc., to determine the shape of remote 3D objects in space.

Ideally, a robot working in space would use high-resolution microwave imaging to augment its tactile and optical sensors. This type of system requires large bandwidth, multiple viewing angles, and sophisticated signal processing, and produces an image consisting of intense regions at points of specularity. The technology and equipment required for such a system is, however, prohibitively cumbersome and expensive for present space applications, and interpretation techniques lag behind the imaging technology. An alternative to microwave imaging is the use of polarized radar scattering cross-sections (RCSs). The RCSs do not yield high-resolution images, but the analysis of depolarization can provide important shape information about the scattering surface; especially at the specular point, where the optical image often fails. To circumvent the ambiguity of the RCS, the incomplete optical image can be used to guide the conversion of RCS into shape. The resulting scattering surface shape description is more complete than can be derived from the RCS or the camera image alone.

The main contribution of this investigation has been the development of numerical algorithms that determine some characteristics of a surface from an incomplete optical image and the observed RCS. This is done by modeling the unknown characteristic of a surface by some parameter vector and applying a non-linear-least-squares algorithm to minimize the difference between the observed RCS and a theoretical
RCS that results from the parameterized surface model. The optical image is used to provide an initial estimate of the parameterized surface. The success of such an algorithm depends to a large extent on the accuracy of the theoretical RCS model used. Models using the geometrical theory of diffraction, moment methods, and physical optics have been considered. Another limitation on the success of the algorithm is the type and quality of information fed to the numerical algorithm. However, in a typical application the algorithm may be used in a configuration such as the one shown in figure 1.2.

An application of particular interest is one in which the method of moments is used to model the RCS. Although this involves the numerical solution of a complex integral equation, it is significant because it allows the computation of microwave scattering from arbitrary surfaces. Simpler modeling techniques such as the geometrical theory of diffraction or physical optics constrain the scattering target to simple geometric shapes such as cylinders and plates. The method of moments requires that the currents on the surface of the scattering object be expanded in some orthonormal basis. Without some independent shape information, no constraints exist on the shape and configuration of these basis functions. This information must be derived from the optical image. In chapter 5 it was shown that a non-linear error minimization employing the method of moments is simplified if the shape and configuration of the basis functions remain fixed throughout the procedure. This is only possible if the occluding contours of the scattering object are known at the outset. These observations indicate that the optical image is absolutely essential to the feasibility of this
technique. Throughout this work, we have assumed that the occluding contours mentioned above can be extracted from the camera data using standard image processing techniques.

Several simulations and experiments were performed demonstrating the application of the RCS/image sensor fusion system. Three recognition problems were considered. The first problem was to recover the edge of a rectangular plate which had become lost in shadow. The second was to determine the size and attitude of an ellipsoid rotated in the horizontal plane. The third application was to recover the shape of the specular portion of an arbitrary surface. The first two applications included experimental data as well as simulations, whereas the third was simulated only. The results show that surfaces can be recovered with very little error when the polarized RCS and certain parameters from the camera image can be measured accurately.

6.2. Observations

To assess the results of this investigation, the original goal and motivation for undertaking the project should be reviewed. It was observed that optical sensing in the space environment does not always provide enough information to reconstruct 3D surfaces. The fusion of radar scattering data with the camera images was proposed as a solution to this problem. Specifically, the use of polarized radar scattering cross-sections was suggested. Successful sensor fusion results in an workspace interpretation which is better than the component interpretations in the sense of either confidence or completeness. The goal of this thesis has been to show that such a
fusion is possible. Note that the central goal was to prove the possibility, although for the technique to be of use, the issues of practicality and feasibility must also be addressed. Certainly, in the simulations and experiments chapter, fusion was achieved. The complete surfaces were indeed recovered from the component interpretations. Also, it could be said that the confidence in the final surface has increased over the individual components. This is due to the fact that the component interpretations cooperated to achieve a correct solution. Thus, the independent sensors appeared to have perceived the same physical object.

The overall favorable results of the applications chapter should be interpreted in light of the simplifications that were made. First of all, the problems were designed with the application in mind. These were not the most difficult situations imaginable. In particular, the plate and ellipsoid were scattering geometries which would probably seldom occur in a real space robotic situation. On the other hand, many complex scattering geometries can be approximated as collections of smaller simple objects. The cylinder, in particular, has a well-known and easily predictable RCS; it is one of the most thoroughly studied scattering geometries in existence since exact solutions to the wave equation are available for this case. The RCS of spacecraft, satellites, etc. are often computed by the NEC scattering code which models complex objects as connected cylinders of various shapes. Thus, recovering the shape of simple scattering geometries is a first step to developing solutions for more complex geometries. The plate was chosen for its well-known and accurate closed-form RCS expression, but the ellipsoid has no such expression. The ellipsoid remains one of the
most difficult scattering problems, and the prediction of diffraction patterns for general ellipsoids is still a research problem. It is encouraging, therefore, that good results were obtained for the ellipsoid, even though gross simplifications were made in computing the RCS. The time-domain scattering results were essential here, since the specular return, for which the simplifications hold, could be separated from the creeping wave reflection.

The hidden simplification in the design of the arbitrary surface experiment is that the surface used to generate the observations was constructed in the same way as the approximation surface. Splines of the same degree were used to define both surfaces, so an exact solution was always possible. A better test of this method would be to construct the surface used to generate the observations from splines of a higher degree than those used to model the scattering response. Alternately, the observation-generating surface could be defined arbitrarily, with no reference to splines, perhaps containing sharp corners or other features that cannot be exactly represented by continuous functions. The results of a spline approximation in such a case would reveal insights about the usefulness of the shape reconstruction technique applied to truly arbitrary surfaces.

Another simplification that affected the outcome of the experiments is the assumption that the scattering targets are perfectly conducting. In practice, this condition is seldom met, although it may be approximated in certain space objects. If the target is not perfectly conducting, its dielectric constant must be determined along with the target shape. In addition, the scattered field expressions increase in com-
plexity. While this difficulty is not insurmountable, its addition increases the number of surface parameters that must be determined, and therefore places more demands on the number of observations.

One last assumption that was made and that is seldom achieved in a real space situation is target stationarity. Usually, objects are rotating about some axis and moving translationally with respect to the sensor. This creates Doppler shifts in received radar frequency, and target scintillation. While Doppler shift is desirable in target tracking situations, it creates problems for RCS measuring systems. One way in which this might be overcome is to use a laser range sensor to determine target velocity and predict Doppler shift.

In the flat plate and arbitrary surface simulations it was seen that the purely numerical error minimization procedure alone did not always converge to the correct solution. This was overcome by comparing the residual error at convergence to a threshold related to the noise variance of the data. If the residual was greater than the threshold, the initial parameter was perturbed slightly and the numerical procedure started again. This need for supervision in order to ensure global convergence even in the simplest case points out a limitation in the purely numerical algorithm. In order to implement such adjustment a supervisory system would have to be incorporated in a given application as represented by the intelligent module of figure 1.2.

Overall, the results of this investigation suggest that there is merit in using information from some independent source to guide the inversion of electromagnetic data. Previous work in shape determination from scattering data has concentrated on the
scattered electromagnetic field as the sole source of information. Much of this previous work consisted of attempts to regularize the ill-posed inversion problem. The ill-posed nature of the inversion problem stems from the lack of complete information about the scattered field. If information from some other physical regime is available about the scattering target it can be used to complement the limited scattered field description. In our case, this has been accomplished by using the camera information to reduce the number of unknown surface parameters. In each reconstruction problem, the number of possible surfaces which must be considered has been reduced by the constraints imposed by the camera image. If this constraint was not available, the number parameters to be determined would exceed the number of observations, requiring the use of a pseudo-inverse or regularization to solve the ambiguity. The problem as it has been set up in this research avoids all issues of ill-posedness; in each example in chapter 5, the Jacobian matrix was invertible over the entire parameter space. In fact, the inverse problem has been regularized in this investigation by choosing a scattering surface that is closest to the approximate surface defined by the optical image.

6.3. Further Investigations

Several areas of investigation are left open in this research. These include using advanced E-M modeling techniques to predict cross-sections, testing the system on a wider variety of surface shapes, examining the surface reconstruction problem in terms of inverse theory, addressing sensor fusion issues, and adding a model-matching step to the surface reconstruction procedure.
The simulations involving arbitrary surfaces were hampered by the lack of good moment-method code with which to generate cross-sections. The point matching technique that was used performed well as a demonstration technique, and allowed fast computation of the RCS, but did not correspond well to experimental data. Professionally developed scattering codes using wire-grid or quadrilateral planar surfaces as basis functions are available in the Numerical Electromagnetic Code package. The next step in testing the optical/RCS SF technique should be to work these programs into the error minimization procedure.

One possible simplification of the surface extraction problem has not been mentioned. In space, the number of possible objects that must be considered is very few. Usually, a robot working on a satellite has relatively complete knowledge of what types of objects it may encounter. This knowledge can be incorporated into the conceptual framework of the shape reconstruction procedure. Once a surface has been computed it can be compared to models of surfaces stored in the robot's memory. This will increase confidence in a given interpretation and help to resolve any ambiguities that remain.

A related topic of investigation that might be considered is the problem of object localization. The radar range and radial velocity of unknown objects can easily be incorporated into a robot's sensory system. This information places constraints on the 3D location of these objects. Consecutive lists of range and range rate pairs must be correlated and resolved with the camera images. This information should be considered in a dynamic sense; sequences of images and sequences of radar measure-
ments can be combined in solving tracking problems. This type of system fits into the framework of previous sensor fusion research, and should not be difficult to derive.
APPENDIX A

Review of Previous Sensor Fusion Research

A.1 Introduction to Sensor Fusion

Sensor fusion (SF) techniques have been developed to combine information from such diverse sources as optical imagery, laser ranging, structured light, and tactile feedback. Other inputs may come from applying different processing algorithms to the same data, such as combining occluding contours with shading details, both derived from a gray-level image, to compute a complete surface description. A practical SF system must also take into account successive time snapshots of the same sensory data. Fusing microwave data with the other available sensory inputs is a slightly different problem than any of these because the microwave data does not come in a form that relates directly to spatial characteristics of the robot environment.

Currently available hardware allows the range and range rate of an object to be accurately determined from micro- or millimeter-wave data. Although it is not the focus of this thesis, combining the range information with the optical data is a valid topic of future investigation that will briefly be discussed. These range measurements and their error probabilities must be taken into account along with other available information and their inconsistencies resolved. Such a system falls within the scope of previous SF research, since range and range rate are direct measurements of workspace geography.
A more relevant subject is the fusion of a set of polarized RCS measurements with optical imagery. The polarized RCS yields detailed information about an object's surface only if some other independent surface information is available. An interpreter needs information from an alternate source such as an optical image to convert the RCS into spatial information that a general SF system can use. Once this conversion is completed, the traditional concepts of SF can take over. It may be worthwhile to examine the state-of-the-art in SF and determine what techniques are of use to us, and where our proposed SF applications fit in.

A.2 Sensor Fusion in General

The purpose of SF is to combine the interpreted outputs of various sensors into a consistent world-view that is in some way better than its component interpretations. Often, the sense of confidence in an interpretation is better, or the resulting composite surface or workspace may be more complete than any single sensor observation, or it may be a combination of the two. Although all SF systems produce some kind of spatial, or geographic information, they may be divided into two groups. The first type of fusion system sees the world as a collection of discrete objects and tries to localize these objects [46-48]. The second type attempts to describe the details of continuously connected surfaces [3, 49, 50]. A common thread that runs through all these methods is the ability to deal with uncertain, conflicting, and incomplete data.

The essential function of most SF systems is to sift through a collection of tokens representing spatial primitives and when possible, merge two or more tokens into one. Example instances of these tokens may be frames describing objects [15],
or individual surface patch and contour descriptions [50]. Harmon [15] stresses that
the design complexity of a system for fusing information from different sources may
be reduced greatly by employing a data representation that is uniform for all sensors.
For example, a robot may obtain information about surface normals from optical
techniques or tactile sensors. The SF system does not need to know which of these
sensors a particular normal estimate came from. Information about a particular phy-
sical property should be placed in a uniform data structure containing the value, prob-
ability of error, and perhaps a time stamp or other pertinent data. This data structure
should be the same, regardless of the source of the information.

We have stated that the goal of a SF system is to fuse all pieces of information
from different sensors that describe the same concept. Various techniques have been
developed to perform this fusion step. Harmon [15] divides the approaches into three
categories: averaging, deciding, and guiding. In averaging techniques, confidence
measures are used to weight various estimates of the same property to compute an
average value that may not be exactly equivalent to any individual estimate. When
deciding, one particular estimate is picked from many others to represent the entire
data set. Again, this choice is based on confidence measures. The third technique,
guiding, uses one estimate to direct the acquisition of further estimates.

Our proposed shape extraction SF method is primarily a guiding technique. The
partial surface model acquired from the image data guides the conversion of RCS
data into a complete surface model.
A.3. Object Localization Techniques

Researchers have investigated a variety of statistical methods for integrating uncertain and redundant geometric information about discrete objects. These methods include weighted estimates [48], confidence measures [51], and Bayesian estimation [46]. Other researchers have concentrated on the overall architecture of the resulting information-processing system [15, 47, 52]. Both aspects of the problem must be investigated so that the most efficient knowledge and information handling mechanisms can use the most accurate statistical model.

A.3.1. Statistical Models

Shekhar, et.al. have developed a system for determining the six degrees of freedom of an object by combining multiple estimates [48]. The sensory data considered was from tactile sensors placed on the robot end-effector. Error in the positioning of the manipulator leads to uncertain estimates of attitude and position. These measurement errors are assumed to be known a priori and are used to weight final estimates. Orientation and position estimates are carried out independently. Position estimates are computed by a simple weighted average. Orientation estimates are derived by representing the rotation matrix as a quaternion and solving the resulting linear system of equations by a weighted left inverse. This method assumes much information is provided to the system. This information includes complete stored object descriptions and correspondence between sensed points and model points. This technique is clearly of the averaging variety.
A complete sensor fusion system is described by Luo, et. al. [51]. The decision process and the statistical models are considered together. A group of sensor observations is first selected based on the task at hand. Observations of the same physical property are fused by a two-step process. The first step selects observations to be fused, and the second step combines them into one estimate. Each observation is characterized by a normal p.d.f., and a distance is defined between p.d.f.'s. These distances are then thresholded and the largest connected group of observations is chosen to be fused. The optimal estimate is then derived by maximizing the sum of conditional probabilities for the estimate weighted by the probability of each observation. Finally, an attempt is made to compensate those observations discarded in the first step of the process. This technique constitutes a hybrid between deciding and averaging.

A complex method for integrating disparate sensor observations is presented by Durrant-Whyte [46]. Uncertain measurements are combined in such a way that geometric consistency is maintained in the world-model. To insure robustness, the p.d.f.'s of each sensor observation are characterized as the sum of two normal distributions, of which, only one covariance is known. Observations that fall outside an ellipse enclosing the mean of the known distribution are discarded. The remaining samples are considered jointly normal with known covariance. A Bayesian estimate of the sensor value is found by minimizing a loss function under geometric constraints. Durrant-Whyte emphasizes that he is not solving a recognition problem. The SF system deals only with abstract geometric information. This system is also a
combination of averaging and deciding.

A.3.2. Computing Architectures

Various architectures have been proposed for handling multiple sensory information. Harmon, et. al. describe a system based on a distributed blackboard [15]. Each sensor subsystem has its own copy of the up-to-date world model, made up of tokens, each of which represents an object, a selected object property and its value, along with an error range, a confidence factor and a time stamp. Communication between subsystems is accomplished through a local area network. Subsystems share only high-level, abstract information, leaving the recognition task to the individual sensors.

A system using two types of knowledge representation is described by Kent, et. al [47]. The purpose of this system is to fuse multiple views of the same workspace taken over time. A world model is built up and compared to internally stored information. The system is also capable of handling objects that have no stored model. Spatial information about the workspace is represented by an octree. Knowledge about discrete objects and their features is maintained separately in a list. The system operates by generating predictions about the world and then matching these predictions with the observed data. The system deals with dynamic information in real time, and is updated continuously.

Chiu, et. al. claim that sensor fusion should be represented hierarchically as a data-flow process [52]. They propose that this process be implemented on a parallel processor.
A.3.3. A Radar/Image Object Localization System

Since this thesis concerns the fusion of microwave and optical sensors in space, a brief parenthetical discussion of an object localization technique is appropriate. An application suggests itself for combining the outputs of range and range rate measuring radar and binary optical images to locate discrete objects. This is an immediately useful SF technique due to the simplicity of the measurements involved. This system would be of use to the EVA MMU retriever under development at the Johnson Space Center [6]. This robot, designed to aid astronauts in extra-vehicular activities, will carry a video camera and a 100 Ghz radar system. Successive time-snapshots from these sensors can be used to localize objects in the robot's 3D workspace. As the previous review has shown, a substantial body of work exists relating to the combination of disparate sensor observations for object localization and attitude determination. Range measurements from doppler radar would fit neatly into these well-known SF schemes.

When combining images and radar range or range rate measurements, we are not fusing disparate measurements of the same physical property, but matching a set of different physical properties to a set of objects. For example, consider combining the output over time of a CW Doppler radar with successive thresholded gray-level images. From two sequential images, we can extract two sets of 2D object positions by analyzing the shape of connected areas. The radar output produces two sets of range rates corresponding to the image times. We wish to find the subset of the four-way Cartesian product of these measurement sets that satisfies the physical con-
straints of the problem. The number of combinations that must be considered can be reduced by considering the constraints on motion imposed by the range rates. This type of problem is similar to the constraint pruning technique of Grimson that is described below. We are able to infer new physical properties from directly observed quantities and compare these inferences to other measurements. For instance, when an object is correlated between two images, its average velocity may be calculated from knowledge of camera motion over the elapsed time between images. The resulting range rate estimate for each pair of objects must then be reconciled with the set of range rates obtained from the radar data. If we can compute the change in projected area between correlated objects, another estimate of average range rate may be obtained. This is a topic of further investigation and will not be considered in any detail in the present thesis.

A.4. Surface Reconstruction Techniques

The RCS-optical system that is the central focus of this thesis falls into the category of SF techniques intended to reconstruct continuous surfaces rather than collections of discrete objects. Previous work in this area has been done by Allen [53], Grimson [49], Crowley [50], and Wang [3].

A.4.1. Review of Surface Reconstruction SF Techniques

Allen and Bajcsy demonstrated that the combination of multiple sensors can produce a 3D object description that is better than those derived from individual sensors. They have used the paradigm of computational stereo [54] to build occluding
contour descriptions that contain gaps and inaccuracies. The interior of the surface and the uncertain points on the contours are filled in with an active tactile sensor. Coons patches (composite bicubic surfaces) [55] are used to interpolate between critical points. The resulting object description is a 2 1/2 D sketch of the unknown object. This is a good example of a guiding technique since the optical information controls the movement of the tactile sensor.

By restricting unknown objects to be composed of polyhedral faces, Grimson [56] was able to generate some constraints that allowed sparse isolated surface points and normal estimates to be matched with stored models. The surface data could, in principle, be derived from any source, the specific cases being range and tactile observations. Using heuristics derived from the geometry of polyhedral objects, such as maximum and minimum distances between pairs of points on different facets, consistent normal angles, amounts of rotation, etc. the investigators were able to prune the search tree of point-facet pairings. With the pruning done beforehand, most point labelings need not be considered when determining a proper model match. Grimson found that even a poor estimate of surface normals greatly simplifies the attitude determination problem. in a later publication, they expand their method to include recognition of partially occluded objects. [49] Also, the efficiency of the algorithm was improved by estimating possible configurations by Hough transforms.

Crowley [50] saw the need, when combining information from different sources, for a standard knowledge representation. He used primitives representing planar surface elements and contours joining elements. Each attribute of a primitive is
represented as an uncertainty range in which the exact number is believed to lie. The result is surface elements and contours that are not two- and one-dimensional manifolds in three-space, but represent some volume where such manifolds may exist. Both stored knowledge and acquired data may be represented in the same form. A model that is being constructed need not be complete, or represent any physically realistic surface to exist. A surface patch need not have bounding contours on each side, and a contour does not need to point to two surface patches. These missing facts may be inferred, however, in the fusion process, and incomplete models matched to stored library models. The form of each piece of data remains intact as the surface model is constructed and primitives are added and merged. Each primitive has a confidence factor associated with it, and elements are combined or removed based on this confidence factor. This SF technique is of the deciding variety.

Wang and Aggarwal also dealt with modeling 3D surfaces using diverse sources of information [3]. Their technique is similar to ours in that they used one source of information to determine occluding contours, and another source to fill interiors of 3D objects. The occluding contours are derived from a thresholded optical image, and partial surface structures are inferred from structured light. Multiple views of an object are considered. This scheme is depicted in figure 1.1. The partial surface structures are allowed to move along an axis defined by the occluding contours observed at the same time. The actual position of any one surface along its axis is determined by matching it with the cylindrical volume derived from occluding contours in and other view. Pairs of surfaces and bounding volumes that are most nearly
orthogonal to each other are combined. This allows one surface structure to be positioned to the greatest accuracy without relating to other surface structures.

Wang and Aggarwal mention that efficient data structures such as octrees are used to store the surface data, but the issue of uniform data representation was not directly addressed in this paper. It is possible that an octree structure may have to be altered as new surface data is incorporated at different levels of resolution [47]. This may result in some wasted time in a general SF system. Most SF systems use lists of tokens of equal significance that may be linked bidirectionally in various ways as opposed to an hierarchical data structure such as an octree. As new information is added in this type of system, relationships may be noted without disturbing the links that have already been established. The final smooth surface representation is derived by spline approximation. Therefore, the spline basis functions and their knots may be thought of as the surface knowledge representation primitives. If conflicting surface points arise, their weighted average is computed. Thus, this SF technique falls into the averaging category. It might also be classified as a guiding technique, since the occluding contours from one view are used to guide the placement of partial surface structures in another.

A.4.2. RCS/Image Surface Reconstruction

Examining these previous attempts at sensor fusion, we are able to make comparisons and draw suggestions from the work that has gone before. Even though our system uses a new combination of information, there remain strong similarities with other techniques. In addition there are shared pitfalls. Although the microwave and
video SF system may at first appear to be a simple guiding technique, the problem of how to handle conflicting and uncertain information needs to be addressed. If a partial surface may be reconstructed from the RCS, how can it be resolved with the optical image surface? Are there points of contradiction? Which interpretation should be given more weight, and how are such weights assigned? Also, what is the most efficient and uniform data structure for storing, inferring, and reconstructing surface knowledge derived from the optical image, RCS and library models?

In one particular application of our SF technique, interior surface patches are to be reconstructed from the RCS by an iterative minimum error technique. This surface patch will cover a portion of the surface whose optical image has been degraded in some fashion. The surface patches to be reconstructed will have some predetermined finite support, such as a rectangle, in the parameter domain. It is unlikely that the degraded portion of the image will have this exact shape, so the smallest support of predetermined shape that completely covers the degraded portion must be used. This results in overlapping areas between the optically derived surface and the RCS reconstructed surface. Although the two surfaces may agree at the initial estimate, the minimization procedure could easily produce a surface patch that does not match the optical surface exactly at its edges. Since both surfaces are uncertain to some degree, some convention must be adopted to resolve these conflicts. If the surface is represented by a continuous manifold, an averaging technique must be used to preserve continuity. If, however, the surfaces are represented by a collection of discrete points, a decision process may be applied to choose an appropriate z value at
a given location. The averaging step that preserves continuity could then be taken care of by spline interpolation. The weight or confidence assigned to each point depends on a variety of possible sources of error. For the optically derived surface, uncertainty arises from errors in camera positioning, light source location, noise, model inaccuracy, and image registration. For the RCS surface, factors such as detection error, ambient reflections, and range uncertainty can contribute to RCS measurement error, and inappropriate modeling assumptions, numerical inaccuracies and insufficient grid density can produce error in the conversion from RCS to surface.

In the simulations of chapter 5, discrete surface point samples have been adopted as a means for representing 3D geometric knowledge. This type of representation lends itself well to microwave scattering computations. In addition to a location in three-space, each surface point should carry with it an estimate of surface normal at that point. If no surface normal estimate is available, a coarse one may be inferred from the surrounding points. A token that represents a surface point must also carry a confidence factor appropriate to the decision process to be used. Also, a range of possible values may be specified for each numerical attribute in the manner adopted by Crowley [50]. This additional information is necessary in order to resolve inconsistencies between the optical surface and the final surface derived through the error minimization process.
Bibliography


