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A model for plasma transport in a corotation-dominated magnetosphere

Pontius, Duane Henry, Jr., Ph.D.

Rice University, 1988
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A MODEL FOR PLASMA TRANSPORT IN A COROTATION-DOMINATED MAGNETOSPHERE

BY

DUANE HENRY PONTIUS, JR.

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

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Abstract

The gross structures of the magnetospheres of the outer planets are decided by processes quite different from those predominant in that of the earth. The terrestrial plasmapause, the boundary beyond which plasma motion is principally determined by magnetospheric interaction with the solar wind, is typically inside geosynchronous orbit. Within the plasmasphere, rotational effects are present, but gravity exceeds the centrifugal force of corotation. In contrast, the Jovian plasmasphere extends to a distance at least twenty times farther than synchronous orbit, affording a large region where rotational effects are expected to be clearly manifest [Brice and Ioannidis, 1970]. The goal of this thesis is to develop an appropriate theoretical model for treating the problem of plasma transport in a corotation dominated plasmasphere.

The model presented here is intended to describe the radial transport of relatively cold plasma having an azimuthally uniform distribution in a dipolar magnetic field. The approach is conceptually similar to that of the radial diffusion model in that small scale motions are examined to infer global consequences, but the physical understanding of those small scale motions is quite different. In particular, discrete flux tubes of small cross section are assumed to move over distances large compared to their widths. The present model also differs from the corotating convection model by introducing a mechanism whereby the conservation of flux tube content along flowlines is violated. However, it is quite possible that a global convection pattern co-exists with the motions described here, leading to longitudinal asymmetries in the plasma distribution.
Acknowledgements

I would like to thank my thesis advisor Dr. Hill for his guidance and patience and for giving me sufficient rope to pull myself along or hang myself as I saw fit. As for my closest friends, Naomi and Juan, there is no way for me to properly express in print the gratitude I feel for their ridiculously unflagging support and faith, so I'll leave that for some drunken evening. Finally, I would like to thank my cat, Malcolm Q. Mouseripper, for inspiring the bulk of the research presented here and for typing the manuscript.

With eternal love, respect, and admiration, I dedicate this work to my father, who taught me how to think and the necessity for doing so.

This research was supported by grant ATM - 8606843.
BEDEVERE
And that, my lord, is how we know the Earth to be banana shaped.

ARTHUR
This new learning amazes me, Sir Bedevere. Explain again how sheep's bladders may be employed to prevent earthquakes.

BEDEVERE
Of course, my Liege...

LAUNCELOT
(he points)
Look, my Liege!

They all stop and look.

ARTHUR
(with thankful reverence)
Camelot!

CUT TO shot of amazing castle in the distance. Illuminated in the rays of the setting sun.

CUT BACK TO ARTHUR and his group. They are all staring with fascination.

Galahad
Camelot!

LAUNCELOT
Camelot!

GAWAIN
(at the back, to PAGE)
It's only a model.

ARTHUR
(turning sharply)
Sh!

Monty Python and the Holy Grail
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Chapter I

Introduction and Background

The magnetospheric plasma associated with a planet having sufficiently high ionospheric conductivity should corotate with the planet to a high degree in accordance with the theory known as Ferraro isorotation. If sufficient plasma exists throughout a magnetosphere, then for many purposes magnetic field lines may be considered to be equipotentials, i.e., $\mathbf{E} \cdot \mathbf{B} \equiv 0$. The perpendicular electric fields $\mathbf{E}_\perp = -\mathbf{v} \times \mathbf{B}$ associated with plasma motion are mapped along magnetic field lines and drive a complementary motion of any plasma on the same field line, such that all the plasma initially associated with a particular field line will move together. To first order, the motion of ionospheric plasma is simply that of the planet’s rotation, and this is in turn imposed on the more distant magnetospheric plasma.

The mapping of potentials between magnetosphere and ionosphere means that any motion across field lines in the magnetosphere will generate currents in the conducting layer of the ionosphere. This is accomplished via Birkeland (field-aligned) currents which close through the Pedersen conducting layer of the ionosphere. If ionospheric conductivity were infinite, plasma in the magnetosphere would be rigidly locked to the ionosphere, and its motion would be strictly governed by that of the ionosphere. However, a finite ionospheric conductivity permits differential plasma motion between the magnetosphere and ionosphere but extracts energy from it. For example, if plasma moves radially outward as viewed from a corotating frame, it gains energy from the centrifugal potential field. Rather than all of this energy going into the kinetic energy of motion, some of it is dissipated through resistive heating in the ionosphere [Dessler, 1980; Eviatar and Siscoe, 1980]. Radial transport also requires a transfer of angular
momentum, and the finite conductivity prevents the ionosphere from enforcing corotation as plasma moves out to large distances [Hill, 1979]. Finally, if new plasma is loaded into the system, a transfer of angular momentum is required to accelerate the new mass up to corotation speed. Because only a finite current density can be generated, the resulting force cannot instantaneously accelerate the newly introduced plasma, and if there is steady mass loading, the result will be a finite deviation from corotation [Pontius and Hill, 1982].

Several models have been proposed to describe the bulk transport of plasma in a corotation dominated magnetosphere. The corotating-convection approach generally assumes that plasma motion is organized into a global pattern, and small scale variations of plasma parameters are not considered explicitly, although a large scale longitudinal dependence of magnetic field and/or boundary conditions is an essential feature [Hill et al., 1983]. Chen [1977] formulated the problem originally in his examination of loss mechanisms for the Io plasma torus. Hill et al. [1981] rederived the governing equations and showed they could be expressed as a pair of coupled, non-linear differential equations that determine the steady state plasma distribution and flow pattern under given boundary conditions. Unfortunately, owing to the complexity of these equations, a general analytic solution has not been found. Summers and Siscoe [1982] analyzed a special case in which these equations reduce to a single third order equation and provided solutions under the assumption that flow velocities are much smaller than the corotation speed.

One consequence of the convection equations is that the mass per unit magnetic flux, or flux tube content, \( \eta = \int \rho ds/B \) is conserved along flowlines in the absence of production and loss processes (\( \rho \) is mass density, B is magnetic field strength, and the
integration is performed along a magnetic field line). As pointed out by Summers and Siscoe [1982], this makes convection analogous to the problem of isentropic flow in hydrodynamics, in which some quantity is conserved everywhere downstream of a source. Unfortunately, this presents a serious obstacle to the interpretation of Voyager observations at Jupiter. The prevailing consensus is that the principal loss mechanism from the Io plasma torus is outward transport. Thus a steady state convection pattern must contain flowlines connecting the Io torus to the outer magnetosphere, where it is proposed that convection breaks down and plasma is lost into the solar wind [Hill et al., 1981]. A spacecraft moving slowly compared to the corotation speed would be expected to encounter each such flowline about once each planetary rotation. Because the particle lifetime outside the Io torus (for process other than transport) is estimated to be much longer than the convection time [Hill et al., 1981], a value of $\eta$ measured in the middle magnetosphere should be observed repeatedly throughout the magnetosphere. Although the problem of time aliasing is very important in considering the limited amount of data available from the Voyager encounters, it is generally agreed that the plasma distribution in the middle magnetosphere of Jupiter does not exhibit this feature. To the contrary, the inferred profile of $\eta$ has an appreciable radial gradient and comparably small variations with longitude [Bagenal and Sullivan, 1981; Siscoe et al., 1981; Bagenal et al., 1986]. The observed density profile does show regular spin-periodic (10 hour) modulations [McNutt et al., 1981]; however these are usually interpreted as being caused by the wobbling of the plasma sheet due to the $\sim 10^\circ$ tilt of Jupiter's magnetic dipole with respect to its spin axis [for a thorough discussion, see Vasyliunas, 1983]. Several recent models [Summers and Siscoe, 1986; Hill and Liu, 1987; Liu and Hill, 1987] have attempted to further elucidate the process by which corotating convection can be driven; however, the observed profile of flux tube content has not as yet been explained.
An alternative theoretical framework that has been put forth to explain radial transport in the magnetosphere at Jupiter is the radial diffusion model. This approach assumes that a net radial transport results from random small scale motions and neglects any large scale longitudinal variation. These models assume that, on the average, all relevant parameters are independent of longitude, and plasma content is usually expressed in terms of the magnetic flux shell density, which is proportional to the longitudinally averaged value of $\eta$ (a flux shell is the surface formed by rotating a magnetic field line about the dipole axis. See Siscoe and Summers [1981] for a discussion of these and related variables). Bulk radial transport is assumed to arise from averaging over the random motions of many individual flux tubes and is treated in terms of violation of the third adiabatic invariant [Fälthammar, 1968]. Adjacent tubes of equal magnetic flux will tend to exchange places if the total potential energy of their configuration decreases upon interchange [Gold, 1959; Sonnerup and Laird, 1963]. Thus, if the dominant contribution to the total energy is the centrifugal potential energy, two radially adjacent flux tubes will arrange themselves such that the outermost will have the greater mass per unit magnetic flux [Melrose, 1967]. Net outward transport occurs stochastically if the flux shell density decreases with distance and if many such interchanges occur randomly throughout the magnetosphere. Siscoe and Summers [1981] derived a radial diffusion coefficient for the Jovian magnetosphere by calculating the centrifugal buoyancy force between two radially adjacent tubes and determining the interchange time. They arrive at an expression for the radial variation of flux shell density that can adequately account for the in situ Voyager observations, although the radial dependence of the size of interchange cells is not determined, without which the exact slope is left unspecified.

There are some conceptual difficulties involved in the use of a diffusion formalism to describe centrifugally driven transport, and its strict applicability is open to question
[Hill, 1983]. For the centrifugal buoyancy force to drive the interchange motion, the flux shell density must decrease with distance. Consider an azimuthally symmetric magnetosphere filled with plasma having such a gradient, and allow a tube to move outward by exchanging places with a less dense neighbor. The density of this newly introduced tube is greater than that of the ambient plasma at that distance, and the buoyancy force inducing it to overturn with its next radial neighbor is even stronger. As long as a flux tube conserves its mass and remains distinct, this process will continue. It will move into flux shells where the difference between its own value of $\eta$ and the background value is increasingly large, thereby making the centrifugal buoyancy force greater. But the diffusion model calculates the strength of interchange by assuming that the parameters determining that motion are given by longitudinal averages. Here, the system very quickly moves away from the initial distribution, and the disturbance grows much faster than as calculated under this assumption. Alternately, to reduce the density to the background value, it is necessary to postulate some other diffusive process, which must operate on a time scale short compared to the interchange time scale, else the flux tube will simply overturn again. But such a process would be at least as significant in determining the radial density profile as the centrifugally driven mechanism under consideration and would need to be explicitly included in a self-consistent treatment.

In this thesis, we present an alternative model to those mentioned above. This model is an extension of ideas presented originally in Pontius et al. [1986], with the goal of updating those ideas with respect to a finer examination of the data as provided by Richardson et al. [1987]. As in the radial diffusion model, we examine bulk plasma transport resulting from the motion of small, discrete flux tubes. In contrast, however, we assume that each such tube remains identifiable over a distance much larger than its equatorial dimensions. In particular, a discrete flux tube is allowed to move through an
azimuthally uniform background, which in turn circulates around it, rather than to simply exchange positions with another tube. We therefore make a distinction between these small, discrete flux tubes, whose plasma content differs significantly from the longitudinally averaged value, and the more predominant flux tubes that make up this average background. The former, which will be called transient tubes, move steadily under the influence of the centrifugal force, while the latter only move in response to the passage of a transient.

This dichotomy is not meant to imply that a flux tube of one kind is forever distinct from those of the other. We propose that transient flux tubes arise from the background and are eventually reabsorbed into it. In our scenario, discrete flux tubes are created when some instability displaces a tube of plasma radially. Before this displacement, the plasma content of the flux tube is the same as that of the background, and though the system is in a state of unstable equilibrium, the centrifugal force is unable to drive radial motion without some longitudinal variation in density. After displacement, the tube has a net surplus or deficit of \( \gamma \) compared to that of the background and feels a radial force proportional to this difference. Flux tubes having a greater density than the background will move outward, while those that are more tenuous will move inward, and both types will be referred to as transient flux tubes. Eventually such a flux tube will lose its identity, either by moving into the source or sink regions, or by other mechanisms to be discussed which return the flux tube content to the background value.

The rapid transport made possible by these discrete flux tubes, along with the relatively slow redistribution of the background, provides a mechanism for moving plasma through the magnetosphere. To account for the observed radial decrease in flux shell content, we assume that a relatively small amount of plasma can pass between an
individual flux tube and the background by single particle microdiffusion. This process is generally judged to be unimportant for transporting plasma on a global scale, owing to the small magnitude of the density gradient observed at Jupiter. However, discrete flux tubes introduce a locally sharp gradient across which microdiffusion is significant. The plasma content of a heavy individual tube is lowered by this diffusion as it moves outward, but in steady state the process is not fast enough to reduce it to the background density. The result is that plasma is transported through the magnetosphere by the motion of transient flux tubes which in turn distribute plasma by microdiffusion. Additionally, this provides a mechanism for converting the transport energy of the transient tubes into thermal energy, as will be shown. It is this combination of global transport and local microdiffusion that distinguishes the present model.

Chapter II presents calculations of the velocity and distortion of an isolated flux tube after some initial perturbation has displaced it radially outward. The velocity expressions are similar to those found by Pontius et al. [1986], apart from the introduction of a factor to account for the geometry of the flux tube cross section. However, the assumptions and subsequent derivation differ in important respects and offer some insights into the physical processes taking place. In Chapter III, a simple form is adopted to describe the diffusion of plasma from a flux tube into the surrounding medium. It is found that the radial dependence of the coefficient parameterizing microdiffusion is vital to determining the radial slope of plasma content. A discussion of measurements and observations of the Jovian magnetosphere in relation to the predictions of this model is presented in Chapter IV. Finally, Chapter V contains a summary of this model and its implications.
Chapter II
The Dynamics of Transient Flux Tubes

We now derive the equations governing the motion of a small, isolated flux tube in a corotation-dominated magnetosphere. For simplicity, the magnetic field is represented by a static, spin aligned dipole with surface equatorial field strength \( B_0 \), and no attempt is made to model the contribution of the ring current to the magnetic field. Calculations are carried out in coordinates which corotate with the neutral atmosphere, such that the centrifugal and Coriolis forces appear explicitly. The pressure gradient force is assumed negligible, and in consequence, magnetospheric plasma can be treated as lying in a thin equatorial sheet [Hill and Michel, 1976; Siscoe, 1978; Vasyliunas, 1983]. Variations in plasma parameters through the sheet are assumed to be unimportant, and magnetospheric drift currents will be integrated along \( \mathbf{B} \) to give the corresponding sheet current \( \mathbf{J}_\perp \) [see Hill, 1983a, and references therein, for derivations of the various sheet currents]. Likewise, ionospheric currents will be integrated through the Pedersen conducting layer of the ionosphere. Finally, we assume that production and loss processes are negligible in the region of interest.

Consider a flux tube having a circular or elliptical footprint in the ionosphere. We treat this class of flux tubes because analytic solutions can be obtained, and the results found are taken to be indicative of the general behavior for arbitrary cross section. For all points in the flux tube, we want to know the electric field that arises from the surface charge density on the wall of the tube due to the divergence of various drift currents. We will adopt the ordering identified by Hill [1983a] for the currents associated with a rotating coordinate system appropriate to the condition \( |v| \ll \Omega r \). Take \( \mathbf{J}_\perp \) to be the centrifugal force sheet current in the plasma sheet:
\[ J_\perp = \eta \Omega^2 r \hat{\phi} \] (1)

The field aligned current density \( j_{\parallel} \) is determined by the divergence of \( J_\perp \):

\[ \nabla \cdot J_\perp = \frac{1}{r} \frac{\partial}{\partial \phi} \eta \Omega^2 r = 2 j_{\parallel} \] (2)

The factor of two arises to take into account both hemispheres. The field-aligned current density in the ionosphere \( j_{\parallel}' \) is proportional to that in the magnetosphere by the inverse ratio of the differential areas

\[ -j_{\parallel}' = j_{\parallel} \frac{dA}{dA'} = \frac{L}{\sin \theta} \frac{dL}{d\theta} j_{\parallel} = 2L^3 j_{\parallel} = L^3 \Omega^2 \frac{\partial \eta}{\partial \phi} \] (3)

where \( L = r / R_0 \) is the (dimensionless) radial distance from the spin axis to a point in the equatorial plasma sheet, and primes are used to indicate quantities in the ionosphere. The equations that map a dipole field line from its intersection with the plasma sheet to a conjugate point in the ionosphere have been used implicitly in equation (3), and these are described in the appendix. Using Ohm's law and Poisson's equation in the ionosphere:

\[ \nabla \cdot J_\perp = \nabla \cdot \Sigma \mathbf{E}' = \frac{\Sigma \rho'}{\varepsilon_0} \] (4)

where the height-integrated Pedersen conductivity \( \Sigma \) is assumed constant. The charge density in the ionosphere is therefore
\[ \rho' = \frac{\varepsilon_0 L^3 \Omega^2}{\Sigma} \frac{\partial \eta (L, \phi)}{\partial \phi} \] (5)

So \( \rho' \) is determined explicitly by the dipole field geometry and the azimuthal gradient of flux tube content at the conjugate point in the plasma sheet. Unlike the ionospheric charge density, the charge density in the plasma sheet \( \rho \) cannot be determined directly without knowledge of the electric field. It is interesting to note that while \( \rho' \) is non-zero only where \( J_\perp \) diverges, the same is not in general true of \( \rho \). Thus it is necessary to calculate the electric field in the ionosphere and map it into the plasma sheet.

We represent the flux tube content as being constant in azimuth except for a discontinuous step at the flux tube boundary. The integration of \( \rho' \) across this step gives \( \sigma' \), the surface charge density on the flux tube within the ionosphere. Letting \( \mathbf{n} \) be the inward normal to the boundary:

\[ \sigma' = \int \rho' \sin \theta R_o \, d\phi \, \hat{\phi} \cdot \mathbf{n} = \frac{\varepsilon_0 L^{5/2} \Omega^2}{\Sigma} \Delta \eta R_o \, \hat{\phi} \cdot \mathbf{n} \] (6)

The quantity \( \Delta \eta \) represents the difference between \( \eta \), the flux content in the tube, and \( \eta_o \), the local background value. As will be discussed in more detail later, both of these may depend on radial distance, but at a given position, \( \Delta \eta \) is assumed to not vary across the flux tube. To find the internal electric field, we set up a local coordinate system in the ionosphere normal to the magnetic field and having its origin at the center of the flux tube. The configuration is assumed to have translational symmetry along \( \mathbf{B} \), and the boundary of the flux tube is expressed as:
\( R \) is the position of a point on the boundary, and \( X \) is a point at which \( E(x,y) \)' will be evaluated. The vector components are expressed in local rectilinear coordinates as shown with \( x, y \) as unit vectors. In terms of the global spherical coordinates of the ionosphere, \( y \) is in the direction of increasing \( \theta \) and \( x \) is in the negative azimuthal direction.

If the change in \( L \) across the flux tube is negligible, the only variations in equation (6) are due to \( n \cdot x \). Then \( \sigma'(x,y) \) depends only on the \( y \) component of the flux tube boundary, and we can write the differential charge per length along \( B \) as \( \sigma' \, dR = \sigma_0' \, dR \cdot y \), where \( dR \) is the arclength along the boundary and \( \sigma_0' \) is the surface charge density when \( dR \) is parallel to \( y \). Then

\[
\sigma' \, dR = \sigma_0' \, g \cos y \, dy
\]  

(7)

The general solution for points inside the tube is:

\[
E(x, y)' = \frac{-g}{g + f} \frac{\sigma_0}{\varepsilon_0} \, x
\]  

(8)
Using \( \mathbf{v} = \mathbf{E} \times \mathbf{B} / B^2 \) and the approximation that the ionospheric magnetic field has a constant magnitude \( 2B_0 \), the resulting interior velocity field is

\[
\mathbf{v}(x, y)' = \frac{g}{g + f} \frac{\sigma_0'}{2B_0 \varepsilon_0} y
\]  

Note that as \( g \) increases the solution for \( \mathbf{E}' \) approaches that of a pair of infinite parallel plates. The external solution, responsible for moving plasma around the flux tube, is expressible in local elliptical cylindrical coordinates as

\[
\mathbf{E}(u, w) = \frac{\sigma_0}{\varepsilon_0} \frac{gf}{g^2 - f^2} \frac{e^{-u}}{(\sinh^2 u + \cos^2 w)^{1/2}} (\cos w, \sin w)
\]  

where

\[
x = a \sinh u \cos w \\
y = a \cosh u \sin w \\
a^2 = g^2 - f^2
\]

Because of the unfamiliar coordinates, the solution represented by equation (10) may be difficult to visualize, so the corresponding systems of equipotentials have been graphed in Figures (1a) and (1b) for \( g = f \) and \( g = 2f \), respectively. These equipotentials, also flowlines, are defined by the curves parametrized by
Figure 1a. The equipotential distribution around a flux tube due to the centrifugal force current, from equations (12) in the text for $g = f$. The arrows indicate the direction of plasma flow. Because this figure is for the ionospheric foot of the flux tube, the flow inside the tube is towards the pole below the page. The corresponding equipotentials in the plasma sheet are geometrically similar to these but inverted and expanded vertically by a factor of 2.
Figure 1b. Same as Figure 1a, except for $g = 2f$. 
\[ x = \frac{g + f}{2} \frac{\cos w}{\cos w_0} \cos w \left\{ 1 - \frac{g - f}{g + f} \frac{\cos^2 w_0}{\cos^2 w} \right\} \quad (12a) \]

\[ y = \frac{g + f}{2} \frac{\cos w}{\cos w_0} \sin w \left\{ 1 + \frac{g - f}{g + f} \frac{\cos^2 w_0}{\cos^2 w} \right\} \quad (12b) \]

where \( w_0 \) defines the intersection of a given equipotential with the flux tube boundary. For \( f = g \), the bracketed terms in (12) reduce to unity, the parameter \( w \) becomes the local polar angle, and the solution is that of a dipole in two dimensions. For an elliptical boundary, the equipotentials are stretched out parallel to the major axis and compressed along the minor axis. We note that, although the coordinate system defined in equation (11) is strictly applicable only for \( g > f \), there is a similar coordinate system for \( f > g \) in which the same calculations can be carried out. The solutions inside the flux tube are identical, while those outside have minor changes in form but are essentially the same. In particular, the expressions for the flowlines, equations (12), are identical.

We map the internal solution into the plasma sheet and take \( \sigma_0' \) from equation (6) to find the electric field that drives an isolated flux tube.

\[ E (L, \phi) = \frac{-G}{G+2F} \frac{L \Omega^2 R_o \Delta \eta}{\Sigma} \hat{\phi} \quad (13) \]

where \( G \) and \( F \) are the radial and azimuthal dimensions of the flux tube in the plasma sheet. Finally
\[ v = \frac{G}{G + 2F} \left( \frac{\Omega^2 R_o \Delta \eta}{\sum B_o} \right) L^4 \hat{r} \]  

(14)

If the magnitude of \( v \) is much less than the local corotation speed, the Coriolis force will be negligible, and this will be the first order motion of the flux tube [Hill, 1983a, and Pontius et al., 1986]. For representative values of the parameters in equation (14) (Table 1) and taking \( \Delta \eta \) to be some fraction of the observed flux tube content at the orbit of Io (see Chapter IV below), this condition should be satisfied throughout the middle magnetosphere of Jupiter.

Equation (14) agrees with the previous result of Pontius et al. [1986] except for the correction factor preceding the parenthesis, which represents the fraction of the diverging current that closes inside the flux tube in the ionosphere. The distinction between the two results arises from the implicit assumption of Pontius et al. [1986] that all the current in the ionosphere is confined to the flux tube and thus can be simply related to the plasma sheet current, whereas here we have equated the divergences of the two currents. While the exact form of the correction term is probably different for a flux tube of arbitrary cross section, we expect it to exhibit the general characteristics found here, i.e., the amount of current closing within the flux tube increases as the ratio of radial to azimuthal dimension increases. As that ratio becomes very large, the correction factor approaches unity and the result of Pontius et al. [1986] is achieved.

The above procedure can be repeated for the Coriolis force sheet current, and the resulting electric field can be treated as a perturbation to that arising from the centrifugal force drift current. From Hill [1983a], the Coriolis force sheet current is
**Table 1**

Jovian Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Planetary Radius</td>
<td>$R_o$</td>
<td>$7.14 \times 10^7$ m</td>
</tr>
<tr>
<td>Rotation Frequency</td>
<td>$\Omega$</td>
<td>$1.74 \times 10^{-4}$ rad-s$^{-1}$</td>
</tr>
<tr>
<td>Surface Equatorial Magnetic Field Strength</td>
<td>$B_o$</td>
<td>4.2 G</td>
</tr>
<tr>
<td>Ionospheric Pedersen Conductance</td>
<td>$\Sigma$</td>
<td>0.1 - 10 mho</td>
</tr>
</tbody>
</table>
\[ J_{\text{Cor}} = 2 \eta \Omega v \] (15)

Using the first order \( v \) found above, the only divergence of \( J_{\text{Cor}} \) is due to the discontinuity in \( \eta \) at the flux tube boundary. This leads to a surface charge density

\[ \sigma_{\text{Cor}} = -\varepsilon_0 \frac{L^{3/2} \Omega \Delta \eta}{\Sigma} \left[ v_\phi \hat{\phi} \cdot n - v_r \hat{\theta} \cdot n \right] \] (16)

The first term in the brackets results in a field proportional to equation (8), while the field from the second term can be determined in an analogous way. The internal field in the ionosphere is

\[ E_{\text{Cor}}(x, y') = \frac{1}{g + f} \frac{L^{3/2} \Omega \Delta \eta}{\Sigma} (g v_\phi', f v_r) \] (17)

and the resulting second order velocity in the plasma sheet within the flux tube is

\[ v_{\text{Cor}}(r, \phi) = \frac{1}{G + 2F} \frac{L^3 \Omega \Delta \eta}{\Sigma B_o} (G v_\phi', -F v_r) \] (18)

Because the flux tube motion is radial to first order, the main effect of the Coriolis current will be due to the second term in (18), contributing an azimuthal component to the flux tube velocity. The potential distribution in the ionosphere that results from the combined effects of the centrifugal and Coriolis currents is drawn in Figure (2) for \( g = 2f \) and \( v_r \) equal to half the local corotation speed.
Figure 2. The equipotential distribution appropriate to the combined centrifugal and Coriolis currents for \( v_r = \Omega r / 2 \) and \( g = 2f \).
We next examine the consequences of variations in $\sigma'$ across the flux tube. Because the main contribution to $\sigma'$ is from equation (8), it is reasonable to neglect the effects of the Coriolis force for now. Continuing in the $x, y$ coordinate system in the ionosphere, we adopt a general form linear in $y$:

$$
\sigma_1 (L)' = \sigma_0 (L_c)' \zeta \frac{y}{g}
$$

(19)

where $\zeta$ is a dimensionless scaling factor and $L_c$ is the center of the flux tube. The factor $\sigma_0(L_c)$ is the first order charge density on the boundary at $y = 0$. From the expression defining the elliptical boundary, $y = g \sin \gamma$. The general solution for points inside the tube is:

$$
E_1 (x, y)' = -\zeta \frac{g}{(g + f)^2} \frac{\sigma_0}{\varepsilon_0} (y, x)
$$

(20)

while the exterior solution, again in elliptical cylindrical coordinates, is

$$
E_1 (u, w) = \frac{f g^2}{(g^2 - f^2)^{3/2}} \frac{\sigma_0}{\varepsilon_0} \zeta \frac{e^{-2u}}{\left(\sinh^2 u + \cos^2 w\right)^{1/2}} (\sin 2w, -\cos 2w)
$$

(21)

leading to the system of equipotentials drawn in Figures (3a) and (3b). These curves are parametrized by
Figure 3a. The perturbation potential distribution due to the variation in the centrifugal force current, as described by equations (22) in the text for $g = f$. 
Figure 3b. Same as Figure 3a, except for $g = 2f$. 
\[ x = \frac{g + f}{2} \left( \frac{\sin 2w}{\sin 2w_0} \right)^{1/2} \cos w \left\{ 1 - \frac{g - f}{g + f} \frac{\sin 2w_0}{\sin 2w} \right\} \]  
\[ (22a) \]

\[ y = \frac{g + f}{2} \left( \frac{\sin 2w}{\sin 2w_0} \right)^{1/2} \sin w \left\{ 1 + \frac{g - f}{g + f} \frac{\sin 2w_0}{\sin 2w} \right\} \]  
\[ (22b) \]

For \( f = g \), this solution corresponds to that of a two dimensional quadrupole. The motion induced within the flux tube by equation (20) is

\[ v_1(x, y) = \frac{\zeta g}{(g + f)^2} \frac{\sigma_0}{2B_0 \varepsilon_0} ( -x, y ) \]  
\[ (23) \]

For positive \( \zeta \), the flux tube simply expands uniformly in the \( y \) direction while contracting along \( x \) such that an ellipse having its semi-major axis aligned with \( y \) will be distended into an increasingly eccentric ellipse. In the plasma sheet this becomes:

\[ v_1(L, \phi) = \frac{\zeta G}{(G + 2F)^2} \left( \frac{\Omega^2 R_o \Delta \eta}{\Sigma B_o} \right) L^4 (\Delta L, -L \Delta \phi) \]  
\[ (24) \]

As was shown in equations (6) and (16), the surface charge density in the ionosphere is proportional to the component of the sheet current normal to the flux tube boundary. By the procedure outlined here, any current that diverges along an elliptical boundary can be separated into radial and azimuthal components, and the subsequent electric fields will be proportional to those solutions found above.
To review, the first order motion of the flux tube is driven by the centrifugal force and is as given by equation (14). Equation (18) describes the second order velocity driven by the Coriolis force, which would make a flux tube moving outward lag from corotation in general agreement with the ideas proposed by Hill [1979]. Note that for a flux tube having $\Delta \eta < 0$, i.e. a local plasma depletion or cavity, the centrifugal buoyancy force drives the tube inward, and the azimuthal acceleration is such that the angular frequency is increased, even into superrotation. Finally, equation (24) gives the relative motion of a point inside the flux tube a distance $(\Delta L, L \Delta \phi)$ away from the center.

These results can be combined to find the behavior of the flux tube cross section as a function of radial distance. In equation (6), assume $\Delta \eta$ to be constant across the flux tube and expand $L$ about $L_c = (\sin \theta_c)^{-2}$, the center of the flux tube.

$$L^{5/2} = L_c^{5/2} \left( 1 + \frac{5}{2} \frac{\Delta L}{L_c} \right)$$  \hspace{1cm} (25)

Comparison with equation (19) and some algebraic manipulation yields $\zeta = -5g L_c^{1/2} / R_o$ (The minus sign arises from equation (A5) in the appendix). If equation (6) is substituted into equation (23) then

$$v_1(x, y)' = v_y' = -\frac{5}{2} \frac{g L_c^{1/2}}{R_o (g + f)} (-x, y)$$  \hspace{1cm} (26)

The distance from the spin axis to the center of the flux tube in the ionosphere is $R_o \sin \theta_c$, which means that $v_y'$ equals the time derivative of $R_o \sin \theta_c$. Likewise, $v_{1y}'$ is the rate of change of $y$. With this in mind we can write the y component of equation (26) as
\[ \frac{dy}{dt} = \left( \frac{5g}{g + f} \right) \frac{-y}{R_o \sin \theta_c} \frac{d}{dt} \left( R_o \sin \theta_c \right) \]  

(27)

\[ \frac{1}{y} \frac{dy}{dt} = \left( \frac{g}{g + f} \right) \frac{d \ln \sin^{-5} \theta_c}{dt} \]  

(28)

If we choose to solve for \( y = g \), then

\[ \left( \frac{g + f}{g^2} \right) \frac{dg}{dt} = \frac{d \ln \sin^{-5} \theta_c}{dt} \]  

(29)

As shown above, the cross section evolves by a linear distortion along \( x \) and \( y \). Thus the product of \( g \) and \( f \) is proportional to the area of the flux tube. This in turn remains approximately constant with time owing to conservation of magnetic flux within the tube and the approximate constancy of the high-latitude ionospheric magnetic field. The left-hand side of equation (29) can now be written

\[ \left\{ \frac{1}{g} + \frac{f g_o}{g^3} \right\} \frac{dg}{dt} = \frac{d}{dt} \left\{ \ln g - \frac{f_o g_o}{2 g^2} \right\} \]  

(30)

where \( fg = f_o g_o \) is constant. Upon integrating, the solution is therefore

\[ g e^{\Delta \psi} \left\{ -\frac{f_o g_o}{2 g^2} \right\} = \text{(constant)} \sin^{-5} \theta_c \]  

(31)
If we map this into the plasma sheet and evaluate boundary conditions then

\[
\frac{G}{G_0} \exp \left\{ \frac{F_0}{G_0} - \frac{F_0 G_0}{G^2} \left( \frac{L}{L_0} \right)^3 \right\} = \left( \frac{L}{L_0} \right)^4
\]  

(32)

where \( L_0 \) is an arbitrary reference distance where \( G = G_0 \) and \( F = F_0 \). The azimuthal width \( F \) can be found from the relation \( FG \propto |B|^{-1} \propto L^3 \).

The radial dependence of \( F \) and \( G \) represented by equation (32) is graphed in Figure (4), along with \( L^4 \), which would represent the behavior of \( G \) if the solution of Pontius et al. [1986] were exact. In that case, all points within the flux tube would have the same functional dependence of distance on time as that of the center, except for displacement by some constant \( \Delta t \). Thus, the time required for a given flux tube to pass any particular radial distance would be constant, and its radial width \( G \) would be proportional to \( v_r \). The resulting solution \( G \sim L^4 \) can be interpreted as a first approximation to the problem of flux tube distortion, in which the front of the tube outruns the back simply due to the increase of the centrifugal force current with distance. As can be seen from equation (32), this is a fairly accurate description of what happens for large \( L \). However, for a nearly circular tube the behavior is quite different, and in particular, the azimuthal width does not decrease as \( L^{-1} \) as would be predicted. In Figure (5), the ratio of \( F \) to \( G \) is graphed, along with the correction factor \( G/(G + 2F) \) found in equation (14). In Chapter III, we will discuss other processes which can also affect the cross section, so the last results should be viewed as illustrative of the effect of flux tube distortion, and not definitive.
Figure 4. The variation of flux tube dimensions with radial distance, as given by equation (32) in the text. The parameter $L_0$ has been chosen such that $F = F_0$ and $G = G_0$ at that distance.
Figure 5. The variation of the ratio of flux tube dimensions with radial distance, as given by equation (32) in the text. Also shown is the corresponding correction term to the first order velocity which takes into account the fraction of the ionospheric current that closes outside the flux tube.
Chapter III
The Interaction of Transient Flux Tubes with the Background

We now turn to the problem of the interaction of these discrete flux tubes with a background wherein the flux shell content decreases with increasing radius. If no process existed to limit the life of a heavy isolated flux tube, then it would travel from the point of formation until it encountered a sink, presumably near the magnetopause. Likewise, a plasma cavity would move inward to the source. However, in this scenario, there is no process to refill the region between the source and sink. A magnetosphere loaded with plasma would fragment into discrete tubes which would deplete the initial distribution, and in a short time, the only plasma in this region would be that in transient flux tubes. This was the model proposed by Pontius et al. [1986] wherein observed variations in flux shell content were proposed to result from an averaging by spacecraft instruments over full flux tubes and the empty background. An examination of high resolution PLS data from Voyager by Richardson et al. [1987] has made this hypothesis untenable by setting an upper limit on small scale density variations. They conclude that the flux tube content in adjacent flux tubes can differ by no more than ten percent in the range $L = 6$ to 10. The purpose of this chapter is to examine mechanisms by which this observational constraint may be satisfied.

One process that will tend to limit the life of an outward moving flux tube is the diffusion of individual particles into the background. We will refer to this as microdiffusion to distinguish it from the radial diffusion model as discussed in Chapter I. Unfortunately, the details of microdiffusion from an isolated moving flux tube are not well understood. Some recent results of interest have been obtained by Borovsky and Hansen [1987] in their treatment of the effect of the polarization current on plasma
clouds. Under the assumption that $\beta$, the ratio of thermal to magnetic energy density, is much less than unity, they numerically trace the evolution of an $\mathbf{E} \times \mathbf{B}$ drifting magnetized plasma cloud in an externally imposed perpendicular electric field $\mathbf{E}_o$. Their treatment is somewhat complementary to that of Chapter II, the principal difference being that the particle density of the plasma cloud is assumed to vary gradually from that of the background across some distance, in contrast with the discontinuous step-function distribution adopted for the flux tubes of the present model. Another, less fundamental, difference is that Borovsky and Hansen treat the motion arising from the polarization current as a deviation from the externally imposed $\mathbf{E}_o \times \mathbf{B}$ motion. In this thesis, the flux tube polarization is caused by the rotational force drift currents, and the velocity expressions found here are deviations from the corotation velocity field. However, all these are inertial currents proportional to mass density, and if the plasma clouds treated by Borovsky and Hansen are viewed in the reference frame moving at $\mathbf{E}_o \times \mathbf{B}/\mathbf{B}^2$, then the results may be qualitatively applied to the present model, although the distinction between "frontside" and "backside" is reversed. Also, Borovsky and Hansen do not treat the problem of variations in $\mathbf{E}_o$ across the plasma cloud, so the solution we found for the distortion of flux tube shape does not apply to their model.

The qualitative result of Borovsky and Hansen [1987] that is of principal interest here is that, for a gradual transition in particle density across some distance, the edges of the flux tube are stripped off to form a long string of low density plasma behind the cloud. To see how this occurs, consider a flux tube similar to those we have been considering, but having a pair of discontinuous density steps at two concentric circles of nearly the same radius. Thus, there is a central circular region of maximum density surrounded by a thin annulus of density intermediate between that in the center and in the ambient medium. A surface charge density will arise on each of the circular boundaries
in proportion to equation (6) by the magnitude of the respective density steps, and the net electric field will be the superposition of two solutions, each proportional to the field described by equations (8) and (10) and pictured in Figure (1). Recall that motion driven by that field is radial inside the charge boundary and circulates the surrounding plasma around the boundary. In the center region, the net field is proportional to the interior solution, so that the velocity is purely radial. However, in the annulus the contribution from the inner boundary is the exterior solution, such that the plasma motion is partially radial, although slower than that of the center plasma, and partially that of the external, circulating plasma. Thus, there is a velocity shear between the two circular boundaries, and plasma in the annulus will fall behind that in the center. However, in their numerical investigations Borovsky and Hansen also found that the inclusion of microdiffusion caused the stripped-off plasma to be convected back into the plasma cloud rather than falling off behind it. Another finding was that small ripples on the frontside of a flux tube are unstable to a fluting instability which would break up the boundary into a series of long, thin fingers of plasma, although microdiffusion tends to suppress the formation of such structures. However, the flux tubes under consideration here are presumed to have scale sizes on the order of several ion gyroradii, which would tend to stabilize against such instabilities [see Schmidt, 1979].

Depending upon the rate of microdiffusion versus that of transport, the interaction between a transient flux tube and the ambient plasma can be manifested in a variety of complicated ways. Rather than attempting to analytically model such a process, we will adopt a few simplifying assumptions concerning its nature and examine their consequences. The first assumption is that the results of Chapter II are applicable, which implies the following: that the diffusive process does not directly alter the geometry of the flux tube boundary, and that $\Delta \eta$ is constant across a flux tube at a given instant, though it
may change with the tube's position or from one flux tube to another. While this may not accurately depict the physical situation, such a description will allow further analysis within the framework of the present model. We adopt a simple form to describe the loss rate from a discrete tube

$$\frac{d\eta}{dt} = - \alpha \Delta \eta$$

(33)

The parameter $\alpha$ characterizes the rate of diffusion, and while it may have a functional dependence on $L$, we assume that it is independent of either $\eta$ or $\Delta \eta$. The rate of change of the radial position is determined by equation (14), so we can change variables and express the loss rate of $\eta$ as a function of $L$.

$$\frac{d\eta}{dL} \frac{dL}{dt} = \frac{d\eta}{dL} \frac{v_r}{R_o} = - \alpha \Delta \eta$$

(34)

Upon substituting $v_r$ from equation (14), $\Delta \eta$ is eliminated to arrive at

$$\frac{d\eta}{dL} = - \alpha \frac{\Sigma B_o}{\Omega^2 L^4} \left\{ \frac{G + 2F}{G} \right\}$$

(35)

This is a striking result. The decrease in plasma content of a transient flux tube per unit radial distance is independent of the plasma content of either the flux tube or the background or their difference. The reason is essentially as follows: both the rate of diffusion and the transport speed depend linearly on $\Delta \eta$. Therefore, a flux tube that diffuses rapidly will move quickly through a flux shell such that it deposits the same
mass per unit flux as a tube that diffuses slowly but stays in the region longer. There is a small dependence on the geometry of the flux tube cross section, as represented by the bracketed term, but, for an outward moving tube, the ratio of $G$ to $F$ increases rapidly and this factor approaches unity. Additionally, equation (14) ignores the effect of the Coriolis force, which tends to reduce a positive $v_r$ as shown in equation (18). However, the correction to $v_r$ is proportional to $v_0$ and is therefore of third order in $\nu/\Omega r$. Thus, to a good approximation, the amount of mass per unit magnetic flux deposited into a flux shell by a transient tube is determined by $\alpha(L)/L^4$.

As described above, a transient flux tube having $\Delta \eta > 0$ loses plasma to the background by microdiffusion, and in consequence, the average plasma content of the background $\eta_0$ must increase. Likewise, as was depicted by the flowlines in Figures (1) - (3), the passage of a flux tube is associated with a redistribution of the background plasma. This is necessary to maintain the requirement that there be no net longitudinal drop in electric potential, i.e., $\int_{2\pi} E_\phi \ d\phi = 0$. However, the assumption that the average background distribution is in steady state, along with the general idea that the discrete flux tubes act independently of each another, means that each transient flux tube must correct the effects of its passage and leave the background as it found it. Thus these two effects must compensate for each other in order for the background to maintain a steady state distribution. We now calculate the magnitudes of these two effects and equate them.

The amount of plasma diffused from a transient tube per unit $L$ is given by the product of the loss rate of $\eta$ times the amount of magnetic flux in the tube $\Delta \Phi$ (recall that $\eta$ is mass per unit magnetic flux). The resulting increase in $\eta_0$ is equal to this value divided by the magnetic flux per unit $L$ of the background.
\[ \Delta \eta_o = \frac{d\eta}{dL} \left( \frac{dL}{d\Phi_o} \right) \Delta \Phi \quad (36) \]

If \( \Delta \eta < 0 \), then this term is negative, and plasma diffuses into a cavity as it moves inward. Note that if \( d\eta/dL \) is approximately the same for all flux tubes, as suggested by equation (35), then equation (36) depends only on the amount of magnetic flux a given transient flux tube contains, and not upon the particular plasma content.

The redistribution of the background can be modeled by simply displacing the radial profile of flux shell content a distance sufficient to compensate for the passage of a transient tube. The magnitude of this displacement is given by the net magnetic flux \( \Delta \Phi_o \) that must be shifted divided by the magnetic flux per unit \( L \) of the background. The resulting change in \( \eta_o \) is therefore

\[ \Delta \eta_o = \frac{d\eta_o}{dL} \left( \frac{dL}{d\Phi_o} \right) \Delta \Phi_o \quad (37) \]

This term is negative [positive] for an outward [inward] moving flux tube when the radial gradient of flux shell content is negative. At this point the result is clear. The magnitudes of equations (36) and (37) are required to be equal if \( \eta_o \) is to remain constant, and because the net amount of magnetic flux transported must be zero, \( \Delta \Phi_o = \Delta \Phi \). This means that

\[ \frac{d\eta}{dL} = \frac{d\eta_o}{dL} \quad (38) \]
i.e., the loss rate of plasma in the transient tube equals the radial slope of the background flux shell content. This also means that $\Delta \eta$ remains constant for a given flux tube. If this requirement is satisfied, discrete flux tubes can form and move radially without altering the background profile, although a net amount of plasma will be transported outward regardless of whether the transient is heavier or more tenuous than the background.

The distance that a flux shell must be shifted is determined only by the magnetic flux of a passing transient tube, such that the same redistribution would occur for any two transient tubes having the same $\Delta \Phi$, regardless of their respective mass contents. The assumption that the two processes which alter $\eta_0$ cancel one another pivots on the result that the amount of mass transferred through microdiffusion likewise depends only on $\Delta \Phi$. If the dependence on cross section geometry is neglected, then the response of the background to both microdiffusion and redistribution is proportional to $\Delta \Phi$ and nothing else, and our assumption is reasonable.

Equation (38) implies the existence of a canonical slope for the background distribution such that the ambient plasma content is unchanged by the passage of a transient flux tube. Because the left hand side of equation (38) is determined by equation (35), this result implies that, if $\alpha$ is indeed independent of $\eta$ and $\eta_0$, then the radial slope of the background plasma is determined by the loss rate of the transient tubes and not the converse. To understand how local microdiffusion from transient tubes can determine the distribution of background plasma, we now perform a gedanken experiment in a magnetosphere which is analogous to the one we have been considering, but wherein the effects of dipole geometry are removed to elucidate the phenomenon of interest (see Figure 6). In particular, consider a one dimensional "magnetosphere" having a constant
Figure 6. Configuration of the one-dimensional analog magnetosphere described in the text. The two thin planes represent the conducting layers of the ionosphere, and the thick plane defines the location of the plasma sheet.
B, in which the ionosphere is represented by parallel conducting planes normal to B separated by some large distance. The centrifugal force is replaced by a constant external force F in the direction r, and the system has a uniform width parallel to F \times B. If we were to repeat the earlier calculations for a transient flux tube in this system, we would find that \nu_r has no direct dependence on position and that there is no distortion of an elliptical cross section. Also, d\Phi_o/dr is constant, which, from equation (37), means that the passage of a given transient tube will shift the background density profile by a constant distance \Delta r over the entire distance it travels. Finally, for simplicity, we choose \alpha to be constant, such that the increase in the background content \Delta \eta_o due to microdiffusion, as determined by equation (36), is likewise constant.

It is now simple to evaluate the effects of flux tube passage. If the profile of \eta_o(r) is initially described by a curve \eta_o(r) = f(r), where f is some function of r, then the passage of a transient will change the profile in the region affected to \eta_o(r) = f(r + \Delta r) + \Delta \eta_o, the relationship between \Delta r and \Delta \eta_o being determined by equations (36) and (37). Thus we can take any hypothetical background distribution and find the result of the passage of a heavy [tenuous] flux tube by merely moving the background distribution inward [outward] a distance \Delta r and up [down] a distance \Delta \eta_o. If the condition described by equation (38) holds, then the result is that the background is unchanged.

Figures (7) – (12) present sketches representing various scenarios, where different initial background distributions are chosen while the slope of transient content remains the same. In these graphs of plasma content versus distance, the wide, dark line represents the initial background profile, while the dotted curve shows the resulting background distribution after transient passage. The dashed line represents the flux content of the transient tube, with open arrows indicating the direction of transport.
When there are two sketches in a single figure, the top and bottom sketches will be for $\Delta \eta$ positive and negative, respectively. It should also be recognized that, for the sake of demonstration, the degree to which the background is affected by each flux tube has been greatly exaggerated compared to that anticipated in the Jovian magnetosphere.

The steady state distribution is depicted in Figure (7), where the initial and final curves lie on top of one another. This result is independent of the particular value of $\Delta \eta$ adopted, in both magnitude and sign. In Figure (8), the magnitude of the background slope is greater than the canonical value, and a flux tube that traverses the entire region merely changes the local flux shell content but does not alter the shape of the background profile. However, in Figure (9) $|d_L \eta| > |d_L \eta_0|$ such that a transient tube reaches a point where its density equals that of the background, and it is consequently reabsorbed into the ambient medium. As can be seen, the result is an overall increase of the magnitude of the average slope. A variation of this effect is shown in Figure (10), where the formation of transient flux tubes at the edge of a sharp drop serves to smooth out the background distribution. Note that if microdiffusion is strong enough to make the profile of transient content steeper than that of the background flux shell content across the entire magnetosphere, then this mode of plasma transport becomes very inefficient. In the absence of other transport processes, the flux tube content in the source region would then increase and eventually produce flux tubes sufficiently heavy to alter the background distribution and increase its average slope.

Consider this analog magnetosphere to be empty except for an isolated plasma population near a localized source. The evolution of the background distribution will be determined by the formation and motion of heavy transient tubes of varying densities, each of which moves outward until its density is reduced to the local background value. As can be seen from the series of consecutive sketches in Figure (11), the result is that
Figure 7. Hypothetical profile of the background flux shell content in the magnetosphere of Figure (6) before (heavy dark line) and after (dotted line) the passage of a transient flux tube. In this figure, the slope of the background is the same of that of the transient content (dashed line), and the background is undisturbed. The top and bottom sketches are for local plasma enhancements and depletions, respectively, and open arrows indicate direction of transient motion.
Figure 8. Same as Figure (7), except that the initial profile of background flux shell content is steeper than the profile of transient flux tube content. The result is a decrease [increase] in the background content for a heavy [tenuous] transient flux tube. The effect has been greatly exaggerated for clarity.
Figure 9. Same as Figure (7), except that the initial profile of background flux shell content is not as steep as the profile of transient flux tube content. A transient tube will reach a point where its density matches the ambient value, and it is then reabsorbed into the background. The effect has been exaggerated for clarity.
Figure 10. The distribution here has the same initial slope locally as that in Figure (9), but a sharp drop has been added. The result is that the finite step is spread out, and if further transients flux tubes are formed in a similar manner, the distribution will eventually exhibit a smooth transition across this region.
Figure 11. Hypothetical time evolution of the profile of flux shell content in an initially empty system. The source lies to the left in the region not depicted, and various initial values of transient content have been chosen for demonstration purposes. The steps in the distribution are smoothed out as suggested by Figure (10).
the background is gradually filled with plasma over the region traversed by transient flux tubes, and the flux shell content approaches a distribution defined by the slope of transient plasma content and by the maximum density in the source region. Slightly different initial values of \( \eta \) have been used to illustrate how the buildup of the background might proceed, but no attempt is made to depict the evolution of the source region, which has been omitted from the sketches. The finite steps evident in the newly formed profile are artifacts of the above mentioned exaggeration adopted for clarity. More likely, the assimilation of a discrete flux tube into the background would result in a background that gradually changes from the altered to the unaltered distribution over the scale size of the flux tube. Of course, the magnitude of the difference would be very small for any individual flux tube. However, the particulars of this phenomenon are unimportant, as any such step will be smoothed out by additional flux tube motion as shown in Figure (10). This has been suggested in the sketches of Figure (11) by gradually decreasing the slope at these steps in successive frames.

One interesting phenomenon of this model is that, in a magnetosphere where the slope of background flux shell content is the same as or steeper than that of the transient flux tube content, a local increase in slope will steepen into a step as shown in Figure (12). It is reasonable to suppose that such a region would favor the formation of discrete flux tubes, and thus the process depicted in Figure (12) should continue until a sharp ledge is formed. If the flux shell content is otherwise in the canonical distribution defined by equation (38), then transients which cross such a step will simply move it without altering its shape, and the step will remain a feature of the profile until it is moved into the source or sink region. If the respective amounts of magnetic flux in heavy and tenuous transients are nearly equal, such that the background motion is very small, then a sharp drop could persist for a long time.
Figure 12. The initial distribution has the slope required for steady state, except over a small region of increased slope. Transient flux tubes formed in the steeper region will reduce its radial extent and increase its average slope. The background will eventually steepen to a sharp drop which will persist until it is moved to the source or sink.
Finally, we turn to the question of plasma heating. It was shown by Gold [1959] that if a dipole flux shell is displaced adiabatically while conserving magnetic flux, then the resulting change in volume requires that the plasma temperature vary as $L^{-8/3}$. In contrast, if transport takes place by some mechanism which conserves the first adiabatic invariant, then $v_{\perp}^2 \sim B$ and it would be expected that $T_{\perp} \sim L^{-3}$. Under either assumption, an outward moving flux tube is expected to lose thermal energy, while magnetic flux tubes that move inward should be heated. In the present model, this applies to both the transient flux tubes and the background distribution, which will be compressed and expanded in response to transient passage. There is, however, another mechanism in this model that can provide additional heating. When plasma moves from a transient flux tube to the background by microdiffusion, or vice-versa, each particle will gain an amount of thermal energy on the order of the transport energy associated with the relative motion. This can be seen from the following simple argument. The speed of the background immediately adjacent to the azimuthal axis of a transient, i.e. at points corresponding to $\gamma = 0$ or $\pi$, is identical to equation (14) except that the correction factor is $2F/(G + 2F)$. Therefore, the difference in velocity $\Delta v_{\text{max}}$ across the boundary at these points is given by equation (14) without the geometric correction factor, and the magnitude of the velocity shear varies from this maximum to zero at the front and back of the flux tube. Consider a small population of particles in a transient tube having thermal velocities of common magnitude $v_{\text{th}}$ and random directions in the plane perpendicular to $B$, and allow them to be moved across this maximum velocity shear into the background. Seen from the rest frame of the plasma in the background, this newly introduced plasma will have velocities given by the sum of $\Delta v_{\text{max}}$ and the thermal velocities. Because the initial thermal velocities by definition have random direction, the mean square speed of the diffused plasma in the background frame will be $(\Delta v_{\text{max}})^2 + (v_{\text{th}})^2$. It should be noted that this result also applies to plasma diffused the other way, i.e., from the
background into a transient flux tube. This calculation yields an upper limit on the amount of thermal energy that can be gained in such a process and is only intended to suffice for order of magnitude purposes. But it demonstrates how transport energy, which is derived from centrifugal potential energy, may be converted to thermal energy. The consequences of this will be discussed further in the comparison of theory with observation in the next chapter.
Chapter IV
Discussion

In the previous chapter, we proposed that there are certain background distributions that are not changed by the passage of transient flux tubes. Furthermore, we presented qualitative arguments as to why a magnetosphere might be expected to assume such a distribution. The principal assumptions of physical interest that led to these findings were (1) that discrete flux tubes move and remain identifiable over distances much larger than their radial extent, and (2) that the rate of diffusion from a transient tube into the surrounding background (or vice-versa) depends only on the difference in flux-tube content and on a parameter $\alpha(L)$ which is a function of radial distance in the plasma sheet only. If these assumptions are valid, then this model represents a viable alternative to other models of plasma transport. To address the question of how well it explains the morphology of the Jovian magnetosphere, we now turn to a comparison of this model with empirical data from the Voyager spacecraft. The limited amount of data available makes it difficult to test directly the central hypothesis of this thesis, i.e., the existence of discrete, long lived flux tubes. It is, however, possible to test certain conclusions derived from this hypothesis, such as the predicted slope of the ambient plasma content. There are also a number of parameters that are as yet unspecified, e.g. the size of transients, the magnitude of $\Delta\Omega$, and the relative fractions of magnetic flux contained in outward and inward moving transients versus that in the background. In this section, we will attempt to provide plausible estimates for these parameters. In consideration of the uncertainties involved in much of the data, geometric correction factors will be ignored.

As was discussed earlier, this thesis departs from the model of Pontius et al. [1986] in that the observed distribution is not assumed to represent an averaging over flux tubes
of vastly different densities. The present model postulates a steady state background which is longitudinally uniform except for local inhomogeneities due to the passage of transient flux tubes. The conclusion of Richardson et al. [1987] is that transient tubes can differ in flux-tube content from the ambient medium by no more than ten percent over the range L = 6 to 11. If we fix Δ η using this constraint, this means that an observation of plasma density made totally within a transient tube would not differ greatly from one sampling only the ambient medium at the same radial distance. Therefore, transient flux tubes are no longer required to be small enough to produce time aliasing within the sampling time of the Voyager instruments. However, in order for the results of the previous analysis to remain valid, we must assume that their radial and azimuthal dimensions are on the order of several tens to hundreds of ion gyroradii, or equivalently, from one to ten percent of the Jovian radius near the Io torus.

We therefore take the observed density profile to be that of the background distribution; i.e., we assume, in accordance with the findings of Richardson et al. [1987] that minor fluctuations due to transient sampling are negligible. Because our model predicts the profile of plasma content rather than density, it is necessary to take into account the distribution of plasma along a field line. This has been done in an extensive analysis by Bagenal et al. [1986], who find that outside the Io torus the number of unit charges per flux shell, denoted Y(L), falls off roughly as L^{-2.2}. It should be noted that this is a refinement of previously reported results [Bagenal and Sullivan, 1981] which takes into account certain particle spectra that were considered unresolvable in earlier analyses. In consequence, some densities and flux shell contents are higher than previously accepted values. The parameter Y is dimensionless and is related to η₀ by
\[ \eta_o = \frac{A^*}{2\pi B_o R_o^2} Y \]  

(39)

where \(A^*\) is the average mass per charge of the plasma ions, which is taken to have a constant value of 21 amu [Siscoe et al., 1981], such that \(\eta_o\) and \(Y\) have the same radial dependence. The value of \(Y\) at \(L = 6\) was reported by Bagenal et al. [1986] to be approximately \(1.7 \times 10^{36}\) which leads to \(\eta_o(L = 6) = 4.5 \times 10^{-3} \text{ kg/W}\), and the background distribution outside the Io torus is thus taken to be described by

\[ \eta_o(L) = (4.5 \times 10^{-3} \text{ kg/W}) \left( \frac{L}{6} \right)^{-2.2} \]  

(40)

from which we determine \(d_L \eta_o = -2.2 \eta_o / L\).

Equations (35) and (39) predict that the gradient of the background distribution will be determined by \(\alpha(L) \propto L^{-4}\). Thus, to account for the observed slope requires \(\alpha(L) \sim L^{0.8}\). Unfortunately, to determine the exact radial dependence of \(\alpha(L)\) would require an analysis of the microdiffusive processes which is beyond the scope of the present work. However, for purposes of comparison, we can assume that \(\alpha\) is proportional to the Bohm diffusion coefficient and estimate the resulting slope at a point where our model is expected to be applicable. Because \(\alpha\) is defined to have units of inverse time, we write

\[ \alpha(L) = \frac{\epsilon}{16} \frac{kT}{e|B|} \frac{1}{\lambda^2} \]  

(41)

where \(\epsilon\) is a positive dimensionless factor less than unity, and \(\lambda\) is the characteristic scale
size of the density gradient at the boundary. We set this parameter equal to the local
gyroradius, about 15 kilometers for singly ionized oxygen at \( L = 8 \). In order to be a well
defined object, a flux tube must have a width of at least several gyroradii at its narrowest
part, so this represents only a lower limit for \( \lambda \), but the ratio of the gyroradius to the
actual gradient scale length can be included in the definition of \( \varepsilon \). We evaluate equation
(36) at \( L = 8 \) using \( T = 50 \text{ eV} \) [Belcher, 1983] and the dipole value for \( B \), which yields
the result \( \alpha(L = 8) = \varepsilon (1.7 \times 10^{-2} \text{ s}^{-1}) \). From equations (35) and (39), this means that
the slope of flux shell content at \( L = 8 \) should be \( d_L \eta_0 = -\varepsilon (5.8 \times 10^{-2} \text{ kg/W}) \). This
compares favorably with the empirical value of \( -6.6 \times 10^{-4} \text{ kg/W} \) if \( \varepsilon \) is on the order of
\( 10^{-2} \), which is reasonable in consideration of the fact that Bohm diffusion represents an
empirical upper limit on the rate of collisionless, cross-field microdiffusion. As shown
by Hill [1983b], the actual coefficient in equation (41) is determined by the ratio of the
effective collision frequency to the local gyrofrequency.

The average amount of magnetic flux contained in transients can be estimated from
the requirement that the outward mass flux equal the mass loading rate in the source
region. For order-of-magnitude purposes, we will take \( \Delta \eta \) to be constant and equal to
one tenth the ambient value \( \eta_0 \). Recall that the passage of a transient results in a net
outward displacement of \( \Delta \eta \) mass per unit magnetic flux, regardless of whether the
transient is heavier or lighter than the surrounding medium. We adopt the notation of
Pontius et al. [1987] and define \( \xi \) to be the fraction of the magnetic flux per unit \( L \) which
resides in transient flux tubes at a given distance \( L \). If \( dM/dt \) is the mass loading rate in
the Io torus and if loss mechanisms other than radial transport are ignored, then outside
the source the following relation must hold:
\[
\frac{dM}{dt} = \xi \ 2\pi \ L \ R_o \ v_r \ \Delta\eta \ |B| \ (42)
\]

From equation (40), the value of \( \eta_0 \) at \( L = 8 \) is \( 2 \times 10^{-3} \) kg/s, and we use one tenth this value for \( \Delta\eta \). It is therefore convenient to rewrite (42) as

\[
\frac{dM}{dt} = \xi \ \frac{v_r}{\Omega r} \ 2\pi \ R_o^2 \ \Omega \ \Delta\eta \ B_o \ L^{-1} \ \ (43)
\]

\[
= \xi \ \frac{v_r}{\Omega r} \ (4.2 \times 10^{31} \ \text{amu/s}) \quad \text{at} \quad L = 8
\]

The radial velocity \( v_r \) is determined by equation (14), but it should be remembered that the requirement \( v_r < \Omega r \) is necessary to the analysis. If \( \Sigma \) is 1 mho, then the radial velocity will be 5 km/s, which is well below the rigid corotation speed of \( \sim 100 \) km/s. However, the precise value of \( \Sigma \) is not well determined, and plausible estimates have been made ranging from 0.1 to 10 mho [see Hill et al., 1981 and references therein]. In light of this, the radial speed of transient tubes may be expected to differ from this estimate by an order of magnitude. The mass loading rate is generally assumed to be about \( 10^{30} \) amu/s, so if \( v_r \) is a few tenths of the corotation speed, a value \( \xi \sim 0.1 \) will suffice to provide the necessary outflow of plasma.

The magnitude of \( v_r \) has an additional consequence, in that it affects the heating of the ambient plasma. As was discussed in the last chapter, microdiffusion between transient tubes and the background provides a mechanism for converting the transport energy of diffusing ions into thermal energy. If our estimate of \( v_r \) is correct, then diffusing plasma will be heated to a fraction \( < 10^{-2} \) of the local corotation energy. Further heating and/or cooling may take place depending upon the average motion of the
background, but if that is sufficiently slow, it is reasonable to suppose that this temperature might be reflected in the ambient plasma. The results from various examinations of Voyager data indicate that, contrary to what would be expected if transport were purely adiabatic, the plasma temperature increases with radial distance. The temperatures available from direct fits to ion spectra [Bagenal and Sullivan, 1981; McNutt et al., 1981] show a gradual increase outward from about 50 eV just outside the orbit of Io [Belcher, 1983]. This is an order of magnitude lower than the local corotation energy. Several other models (see Vasyliunas [1983] for a thorough review) quantitatively infer the plasma temperature from in situ magnetic field measurements. Their general finding is that the ratio of the thermal speed to the local azimuthal (sub-corotation) speed varies between roughly one tenth and some factor less than unity. These results are in general agreement with our hypothesis, although a more thorough analysis would be necessary to properly include the possibility of adiabatic heating and cooling due to the motion of the ambient plasma.
Chapter V
Summary and Conclusions

In this thesis, we have presented a model to describe the process by which plasma is transported through a corotation-dominated magnetosphere. One consequence of this investigation is that the radial gradient of flux shell content is determined by the rate of single particle diffusion, a surprising result in light of the generally held view (with the exception of Abe and Nishida, 1986) that microdiffusion can be neglected when treating radial transport. The motivation for this work was to explain the observed plasma distribution in the Jovian magnetosphere outward of the Io plasma torus but within the region where interaction with the planet dominates over the solar wind interaction. In the comparison of data with theory in Chapter IV, the model was found to adequately account for the plasma measurements at $L = 8$, a point outside the source but well within the region where the model is proposed to be valid. Unfortunately, the factors controlling microdiffusion are not at this time sufficiently well understood to allow a thorough test of the model.

Although a dipole field was used in these calculations, it should be recognized that the physical processes described herein do not depend upon that assumption, and a analogous treatment could be carried out for any axisymmetric magnetic field configuration. The expressions for ionospheric charge density and transient velocities would be altered owing to the use of different field mapping equations, but the essential idea of this model would remain, i.e. that plasma can be transported by discrete flux tubes in a way such that the background distribution is unchanged. In particular, equations (36) - (38) would still hold.
This thesis has not dealt with the processes by which discrete flux tubes are created from the background and reabsorbed into it. One process that will tend to limit the life of an outward moving flux tube is the azimuthal narrowing described in the previous section. As the width of a flux tube approaches the gyroradius of its constituent ions, it ceases to be a well defined object, and it is reasonable to hypothesize that such a flux tube would be broken up by instabilities. Similarly, a flux tube having $\Delta \eta < 0$ will move inward and will be compressed radially. In this case, the correction term in equation (14) will become increasingly important and decrease the flux tube speed, which means that microdiffusion will have a greater effect. However, it should be kept in mind that the results derived in Chapter II are intended to demonstrate the effect of the cross section on the electric field and are not meant to definitively answer the question of flux tube evolution. In particular, those results should be considered together with those described by Borovski and Hansen [1987] which may also affect the shape of the flux tube cross section.

We have estimated the amount of magnetic flux contained in transients, versus that in the background, with a simple argument based on flow continuity. A more meaningful estimate can be made only when more is known about the rate of discrete flux tube formation and the distance a transient flux tube can move before losing its identity. As an added benefit, this information would also determine the respective amounts of magnetic flux in heavy and tenuous transients at a given distance, which in turn determines the average motion of the background. At any point where these two quantities are equal, the net background motion is zero, and as we showed in Chapter III, a sharp ledge in the radial profile of an otherwise stable background would then persist indefinitely. If, for some reason, outward moving transients were to persist over greater distances than those moving inward, then the average motion of the ambient plasma would be inward.
However, if the converse holds, then the background will gradually move outward as transient holes bubble inward to the source.

We have attempted to develop a model that avoids some shortfalls of the commonly accepted theories. At this time, the available data are inconclusive as to what process is responsible for transporting plasma radially from the Io plasma torus to the outer magnetosphere. Additional data, and/or further analysis of existing data, will eventually allow the question to be answered.
References


Appendix

Two points, one in the ionosphere, the other in the equatorial plane, that lie on the same magnetic field line are described as being conjugate to one another. The equations that define conjugate points for a static dipole field are

\[ \sin^2 \theta = \frac{1}{L} \]

\[ dL = -2L^{3/2} \Lambda d\theta \] (A1)

where \( \theta \) is the colatitude in the ionosphere and \( L \) is the equatorial crossing distance of the field line in units of planetary radii. The factor \( \Lambda = (1 - 1/L)^{1/2} = \cos \theta \) will be approximated by unity, as is appropriate to the polar regions of the ionosphere. From these equations, and the assumption that field lines are equipotentials, we can map quantities between conjugate point as follows:

\[ v_r = -2L^{3/2}v_\theta' \]

\[ v_\phi = L^{3/2}v_\phi' \] (A2)

\[ E_r = -\frac{1}{2L^{3/2}}E_\theta' \]

\[ E_\phi = \frac{1}{L^{3/2}}E_\phi' \] (A3)

where primed quantities are evaluated in the ionosphere and unprimed quantities are evaluated in the magnetic equatorial plane. These are general results for a dipole field, but similar expressions can be derived for any static azimuthymmetric magnetic field. The local (x,y) ionospheric coordinates introduced in the text are related to global coordinates by
\[
\begin{align*}
\text{dy} &= R_0 \, d\theta \\
\text{dx} &= -R_0 \, \sin \theta \, d\phi
\end{align*}
\] (A4)

\[
\begin{align*}
y &= -\frac{R_0}{2 \, L^{3/2}} \, \Delta L \\
x &= -\frac{R_0}{L^{3/2}} \, L \, \Delta \phi
\end{align*}
\] (A5)

Finally, the dimensions of the flux tube in the ionosphere are related to those in the magnetosphere by

\[
\begin{align*}
g &= \frac{1}{2 \, L^{3/2}} \, G \\
f &= \frac{1}{L^{3/2}} \, F
\end{align*}
\] (A6)