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Rice University, 1987
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HF SIDEBANDS IN THE IONOSPHERE

by

ZHONG- HAO HUANG

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

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HOUSTON, TEXAS
APRIL 1967
ABSTRACT

HF Sidebands in the Ionosphere

Sidebands on the high frequency (HF) waves received from two transmitted waves separated by small frequency intervals were first observed by Gordon and Ganguly in January, 1984 at Arecibo, Puerto Rico. Papadopoulos proposed that HF sidebands would arise from the nonlinear interaction of an ELF wave with the two high frequency waves. The nonlinear mechanism for ELF and HF sideband generation is parametric decay of a high frequency radio wave into a low frequency compressional Alfven wave and a high frequency sideband [Papadopoulos et. al., 1982].

In 1984 Fejer proposed another nonlinear mechanism (phase modulation theory) for HF sideband generation. Fejer points out that the effect of the ponderomotive force is a reduction in the electron density and an increase in the phase delay which is greater for more powerful waves. Numerical solutions show that the phase delay would relate to the power density of the HF wave. This modulation index theory using the W.K.B. approximation shows that the amplitude of sidebands is related to: [1] the effective HF power; [2] the HF frequency; [3] the temperature of the electrons; [4] the scale height of the ionosphere; and [5] the difference frequency of the two pump waves. Observations of HF sidebands were again made in January, 1986. The 430 MHz incoherent radar provided the ionospheric background measurements. The modulation index theory provides the most complete interpretation of the observations.
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CHAPTER 1
INTRODUCTION

In 1901 the experiments of Marconi in which radio signals were successfully transmitted across the Atlantic Ocean indicated the presence of a reflecting layer in the upper atmosphere. In 1902 A. E. Kennelly and O. Heaviside postulated that the presence of free electrons produced by solar ultraviolet radiation at less than 1000 A wavelengths could reflect radio waves. In 1925 the frequency change experiments of Appleton and Barnett in England demonstrated the existence of downcoming waves by an interference technique, and the pulse-sounding experiment of Breit and Tuve in 1926 in America finally demonstrated the existence of an ionized layer. In 1931 Chapman gave a basic theory governing the formation of this photoionized layer. In 1957–1958 of the International Geophysical Year; in 1959 of the International Geophysical Cooperation; and in 1964–1965 of the International Quiet Sun Year, observations greatly enhanced our knowledge of the ionosphere.

In 1906 J. J. Thomson calculated the amount of the electromagnetic wave scattered by a single free electron and pointed out that electromagnetic waves are scattered very weakly by free electrons. In 1958 W. E. Gordon proposed that the free electrons of the ionosphere scatter electromagnetic waves and the incoherent scattered waves could be detected by sensitive radars. The proposal of Gordon [1958] was immediately verified by Bowles [1958] by using a powerful radar in Illinois. Incoherent scattering measurements offered a new technique not
only for determining the profile of electron density but also for
determining the profile of electron and ion temperatures and ionic
composition. The capabilities of incoherent scatter radar have progressed
much more than originally envisaged by Gordon. The incoherent scattering
radar can obtain information not only on the dynamics, winds, electric
fields of the ionosphere and the thermosphere [Evans, 1978] but also on
dynamical properties of the troposphere, stratosphere, and mesosphere
[Woodman and Guillen, 1974] using the radio scattering properties of the
irregularities in these media.

In ionospheric communication it is customary to assume that the
response of the ionosphere to an incident wave is linear. In 1933 Tellegen
discovered experimentally that the assumption of linearity breaks down
for relatively modest transmitted powers [Tellegen, 1933], the modulation
of a moderately powerful radio station [220KW, 525.1 kHz] was heard on
the carrier of another station [650 kHz]. Bailey and Martyn pointed out that
this was attributed to ohmic heating by powerful waves modulating the
attenuation of the other signal [Bailey and Martyn, 1934]. This heating
effect has been used for probing the D layer of the ionosphere since 1955.
[Fejer, 1970]. In principle, the nonlinear heating effects exist for any
strength of incident radio wave. In practice, the heating effects become
detectable only for a relatively large strength of the incident wave. When
the strength of the incident wave is increased above a threshold, the
ionospheric plasma becomes unstable and a perturbation wave
spontaneously grows. This process is usually called a parametric
instability. Because the nonlinear effects and instabilities are of great
interest to the physics of plasma and also have applications for ionospheric diagnostics, several high power, ground based, high frequency radio transmitters operating below the vertical incidence penetration frequency of the ionosphere have been constructed and operated in ionospheric heating experiments. Up to now the following manifestations of the nonlinear effects and parametric instabilities have been observed:

[1] Artificial Spread F: One of the most prominent and unexpected effects of ionospheric heating experiments is the artificial generation of the spread F layer which was the first evidence of intentional ionospheric heating by the Platteville high power transmitter facility. With O-mode polarization at frequencies below the F-region critical frequency the Platteville heating facility first detected Spread F in April 1970 [UtIaut et al., 1970]. Artificial heating of the ionospheric F-region produces ionization irregularities of a variety of scales ranging from 3 m to 1 km which is the same range of sizes as natural spread F. It was observed by Herman that naturally occurring spread F frequently accompanies the observation of radar echoes from F region field aligned electron density fluctuations. The artificially created spread F layer caused by large scale field aligned irregularities was observed on ionograms about ten or more seconds after a powerful modifying transmitter was switched on. Thermal focusing was first suggested as an explanation by Georges [Georges, 1970]. Stimulated Brillouin scattering was later suggested as another explanation by Fejer [Fejer, 1973]. The ionospheric irregularities responsible for artificial spread F are aligned with the earth's magnetic field and support backscatter at frequencies well above the local plasma
frequency. It has been possible to demonstrate a potential usefulness of ionospheric modification for telecommunication purposes. Voice, teletype, and facsimile transmissions have been sent by means of the scattering region above the modifier between ground terminals separated by several thousands of km and using frequencies which would not have been useful for those paths.

[2] Airglow: An effect of the ionospheric heating experiments is the artificial enhancement of the airglow red line 6300 A of atomic oxygen \([^1D_2 - ^3P_2]\) and, occasionally, artificial enhancement of the airglow green line 5577 A of metastable oxygen \([^1D_2 - ^1S_0]\) [Biondi et al., 1970; Haslett and Megill, 1974]. The artificial enhancement is about 20 Rayleighs, but much larger values have been observed particularly when the heating transmitter operated near the second harmonic of the electron gyrofrequency. The artificial enhancement of 6300A airglow has also been observed for modifications over Arecibo, but is much less intense than over Platteville. An estimate of the polarization for 6300A is given by Chamberlain [J.W.Chamberlain, 1974]. The artificial enhancement only occurs when the heating transmitter has ordinary polarization. The airglow artificial enhancement is believed to be due to collisional excitation of oxygen atoms by electrons which have a non-Maxwellian distribution because the interaction between the pumping waves and the plasma produces a high energy tail on the electron velocity distribution. Through Landau damping these electrons are believed to have been accelerated to suprathermal energies by parametrically excited Langmuir
waves which are excited by the pumping waves. [Fejer and Kuo, 1973; Perkins et al., 1974].

[3] Anomalous Absorption: Another unanticipated effect of the ionospheric heating experiments is strongly enhanced radio wave absorption. The increase of the electron temperature in excess of 300 K above the ambient value of 1000 K was first observed by Gordon and Showen using the 430 MHz radar in Arecibo. The F-region observations show remarkable heating and cooling curves with time constants of about 40 sec and less than 10 sec respectively [Gordon and Showen, 1971]. The observations indicate that approximately half of the incident HF energy is absorbed in the ionosphere with nearly equal contribution from anomalous absorption and deviative absorption. The energy is imparted to the electron gas through ohmic dissipation as the electron temperature is raised, the field-aligned pressure gradients make the plasma expand along the magnetic field. The predicted changes in the electron temperature are in fair agreement with the measured increases obtained by incoherent backscatter techniques at Arecibo [Gordon and Carlson, 1974] and airglow suppression measurements at Platteville [Blondi et al., 1970]. The time constants for heating and cooling are also in good agreement with theoretical predictions. In the F layer ohmic heating due to deviative absorption and nonlinear heating mechanisms account for the absorption of as much as 60 percent of the incident power. The generation of large plasma wave intensities contributes about 10 to 30 percent of the absorption [Dubois et al., 1973; Maltz et al., 1972; Perkins et al., 1974], while another 10 to 30 percent of the incident HF energy is absorbed through ohmic or joule heating. Only a
small fraction of the energy, about 10 percent, goes into fast electrons [Haslett and Megill, 1974; Weintock, 1974; Fejer and Graham, 1974]. In 1974 Gordon and Carlson observed that the electron temperature during an experiment in which the heater power was cycled from zero to 50 kW and 100 kW the enhancement of the electron temperature at 100 kW was approximately double that for 50 kW. So the electron temperature is approximately proportional to the heating HF power.

[4] HF Sidebands: Another interesting effect of ionospheric heating experiments is the artificial generation of high frequency sidebands and the artificial generation of VLF and ULF waves using two powerful HF waves at closely spaced frequencies. In recent years a method of ionospheric heating using two powerful HF radio waves was proposed by Papadopoulos [K.Papadopoulos et al., 1982, 1983]. The nonlinear interaction sidebands using two powerful waves were first observed by Gordon and Ganguly in January 1984 at Arecibo.
CHAPTER 2
THEORETICAL BACKGROUND

The ionosphere as a natural plasma without walls supports several modes of waves. The interaction between the high power HF waves and the ionospheric plasma is nonlinear. It is possible to transfer power from the strong wave to the weaker natural modes of waves, so that the parametric instabilities will occur. The amplitudes of the excited wave will grow exponentially if an instability threshold is exceeded. The saturation mechanism theory predicts that the growth would be restricted. The saturated plasma wave lose energy through linear damping and the plasma waves establish an equilibrium through nonlinear damping.

2.1 Waves and Instabilities in the Ionosphere

In the absence of the HF pump waves, ionospheric plasma supports several natural modes of waves. With the continuity equation, the momentum equation, Maxwell's equation, and the equation of state the natural modes of waves can be derived [Chen, 1974]:

[1] the low frequency electrostatic ion acoustic wave ($B_0=0$; or $k\parallel B_0$) with the dispersion relation:

$$\omega/K = \nu_S = \left( k_b T_e + 3k_b T_i / M \right)^{1/2}$$

[2-1]
[2] the low frequency electrostatic ion cyclotron wave (K\|B₀) with a frequency:

\[ \omega^2 = \omega_c^2 + K^2 v_s^2 \]  \[2-2\]

[3] the low frequency electromagnetic ion wave (K\|B₀) [Alfvén wave] with a frequency:

\[ \omega^2 = K^2 v_A^2 \]

\[ v_A = \left[ B_0^2 / \mu_0 \sigma_0 \right]^{1/2} \]  \[2-3\]

[4] the low frequency electromagnetic ion wave (K\_|B₀) [magnetosonic wave] with the dispersion relation:

\[ \omega^2 / K^2 = c^2 [ v_s^2 + v_A^2 ] / [ c^2 + v_A^2 ] \]  \[2-4\]

[5] the high frequency electrostatic plasma wave [Langmuir wave] (B₀=0; or K\|B₀) with a frequency:

\[ \omega^2 = \omega_p^2 + 3K^2 v_{th}^2 / 2 \]  \[2-5\]

[6] the high frequency electrostatic electron wave (K\_|B₀) [upper hybrid oscillations] with a frequency:
\[ \omega^2 = \omega_p^2 + \omega_c^2 = \omega_h^2 \]  

[2-6]

[7] the electromagnetic wave with the dispersion relation:

\[ \omega^2 = \omega_p^2 + c^2 k^2 \]  

[2-7]

In the above equations \( \omega_p = [n/e_0 m]^{1/2} \) is the plasma frequency; \( v_s \) is the sound speed in a plasma; \( \omega_c = e B_0 / M \) is the ion cyclotron frequency; \( v_A \) is the speed of Alfvén wave; \( v_{th} = (2k_B T_e / m)^{1/2} \) is the thermal velocity and \( \omega_h \) is the upper hybrid frequency.

The interaction of high power HF waves with the ionospheric plasma is strongly nonlinear. The nonlinear properties of the ionospheric plasma make a transfer of power from the strong pump wave to the weaker waves possible. If the transferred power exceeds the dissipation then the weak waves will grow.

In view of the large number of waves that are possible in the ionospheric plasma, there is a large variety of parametric instabilities that have been theoretically investigated. The most important type of parametric instabilities is the three wave interaction in which the strong pump wave with frequency \( \omega_1 \) and wave number \( K_1 \) causes the growth of a high frequency wave with frequency \( \omega_2 \) and wave number \( K_2 \) and low frequency wave with frequency \( \Delta \omega \) and wave number \( \Delta K \), where at
threshold \( \omega_2 \) and \( \Delta \omega \) are real and the frequency and wave number matching conditions are given by:

\[
\omega_1 = \omega_2 + \Delta \omega \quad K_1 = K_2 + \Delta K
\]  

[2-8]

The two conditions [2-8] can be easily understood by analogy with quantum mechanics. Multiplying these two equations by Planck’s constant \( \hbar \), we have:

\[
\hbar \omega_1 = \hbar \omega_2 + \hbar \Delta \omega \quad \hbar K_1 = \hbar K_2 + \hbar \Delta K
\]  

[2-9]

Therefore, equations [2-9] simply state the conservation of energy and the conservation of momentum.

An HF pump wave can lead to the following four major types of parametric instability in the ionosphere:

1. The parametric decay instability: the upward HF pump wave with frequency near the plasma frequency may decay into a Langmuir wave and an ion acoustic wave. This instability is believed to be an extremely important factor in ionospheric heating experiments.

2. Raman scattering: the upward HF pump wave with frequency \( \omega_1 \geq 2\omega_p \) may decay into a downward scattered electromagnetic wave and an upward Langmuir wave.

3. Brillouin scattering: the upward HF pump wave with frequency
\( \omega_1 > \omega_p \) may decay into a downward electromagnetic wave and an upward ion acoustic wave.

(4) The oscillating two stream instability: the upward HF pump wave with frequency \( \omega_1 - 2\omega_p \) may decay into two oppositely directed Langmuir waves with the same frequency as the pump wave.

Liu [Liu and Kaw, 1976] explained that the dominant nonlinear coupling effects responsible for all these interactions are [1] the nonlinear current density arising due to density fluctuations, and [2] the ponderomotive force \( \propto ve^2 \) produced by the pump wave and other of the excited waves. Liu supposed that the equilibrium consists of electrons oscillating with the electric field of the HF pump wave and with the ions, forming a stationary background. A propagating density perturbation associated with an electrostatic wave disturbs this equilibrium. The electron density fluctuation will be carried by the electric field of the pump wave and will lead to currents at \( \omega_1 - \Delta \omega, K_1 - \Delta K \). These currents will generate mixed electromagnetic electrostatic sideband modes at \( \omega_1 - \Delta \omega, K_1 - \Delta K \). The sideband modes interact with the pump wave field and produce a ponderomotive force. The ponderomotive force amplifies the original density perturbation. This is then a positive feedback process and will lead to parametric instability if growth exceeds damping effects.
2.2 GENERAL FORMALISM FOR TWO COUPLED HARMONIC OSCILLATORS

We consider a specially homogeneous HF pump field, \( E(t) \), which can be written as:

\[
E(t) = 2E_1 \cos \omega_1 t
\]

[2-10]

We also consider two damped oscillations, \( X(t) \) and \( Y(t) \) [in the ionospheric plasma \( X(t) \) and \( Y(t) \) correspond to motions of the ions and electrons], described by two coupling operators \( L_1 \) and \( L_2 \):

\[
L_1[X(t)] = \left[ (d^2/dt^2) + 2\gamma_1 (d/dt) + (\Delta \omega^2 + \gamma_1^2) \right] X(t)
\]

[2-11]

\[
L_2[Y(t)] = \left[ (d^2/dt^2) + 2\gamma_2 (d/dt) + (\omega_2^2 + \gamma_2^2) \right] Y(t)
\]

[2-12]

where \( E_1 \) and \( \omega_1 \) are constants, the frequencies \( \omega_1, \omega_2 \) and damping rates \( \gamma_1, \gamma_2 \) are constant \( (\Delta \omega < \omega_2) \). The coupled mode equations for the oscillator amplitudes \( X(t) \) and \( Y(t) \) can be written as:

\[
L_1[X(t)] = \lambda E_1(t) Y(t)
\]

[2-13]
\[ L_2[Y(t)] = \mu E_1(t) X(t) \]  

where \( \lambda \) and \( \mu \) are real coupling constants. The above equations neglect the products of the small quantities \( X \) and \( Y \). The frequency \( \Delta \omega \) corresponds to the low frequency oscillation and the frequency \( \omega_2 \) corresponds to the high frequency oscillation in the unpumped plasma. The pump field \( E_1(t) \) modulates each oscillation mode, and produces a forced oscillation in the corresponding coupled mode.

The energy transfer occurs only if the matching condition is satisfied \((\omega_1 = \omega_2 + \Delta \omega)\). Using the matching condition and Fourier transform of the equations \(2-13\) \(2-14\) can be written as:

\[ D_1(\omega) X(\omega) = \lambda E_1 \left[ Y(\omega + \omega_1) + Y(\omega - \omega_1) \right] \]  

\[ D_2(\omega) Y(\omega) = \mu E_1 \left[ X(\omega + \omega_1) + X(\omega - \omega_1) \right] \]  

\[ D_2(\omega + \omega_1) Y(\omega + \omega_1) = \mu E_1 \left[ X(\omega) + X(\omega + 2 \omega_1) \right] \]  

where \( D_1(\omega) = -\omega^2 - 2i \gamma_1 \omega + \Delta \omega^2 + \gamma_1^2 \)

\( D_2(\omega) = -\omega^2 - 2i \gamma_2 \omega + \omega_2^2 + \gamma_2^2 \)

and thus
\[ Y(\omega+\omega_1) = \frac{\mu E_1}{D_2(\omega+\omega_1)} X(\omega) \]  
\[ Y(\omega-\omega_1) = \frac{\mu E_1}{D_2(\omega-\omega_1)} X(\omega) \]

Substituting the above equations yields:

\[ D_1(\omega) X(\omega) = \lambda E_1 \left\{ \frac{\mu E_1 X(\omega)}{D_2(\omega+\omega_1)} + \frac{\mu E_1 X(\omega)}{D_2(\omega-\omega_1)} \right\} \]

Considering frequencies \( \omega \approx \omega_2 \) then:

\[ D_2(\omega+\omega_1) = -(\omega+\omega_1)^2 - 2i\omega_2(\omega+\omega_1) + \omega_2^2 + \Phi_2^2 \]

Equation [2-21] yields:

\[ -[\omega^2 + 2i\omega_2 - A\omega^2 - \Phi_2^2] = (k/2\omega_2) \left\{ (\omega-\delta+i\Phi_2)^{-1} - (\omega+\delta+i\Phi_2)^{-1} \right\} \]

where \( k = \lambda \mu E_1^2 \); \( \delta = \omega_1 - \omega_2 \). Consider \( \omega = x + iy \) consisting of real and imaginary parts that can be separated out.

Equating real components yields:

\[ [x^2-(y+\Phi_1)^2 - A\omega^2] = -(x^2-\delta^2) + (y+r_2)^2 / F(x,y) \]

where \( F(x,y) = (\omega_2/\delta k)((x+\delta)^2 + (y+r_2)^2)(x-\delta)^2 + (y+r_2)^2 \)
Equating imaginary components yields:

\[ x[y + r_1 - (y + r_2)/F(x,y)] = 0 \]  \hspace{1cm} [2-24]

Therefore the two possible solutions can be derived:

(a) \( x = 0 \)

(b) \( x \neq 0 \), and \( y + r_1 = (y + r_2)/F(x,y) \)

The solution (a) gives a purely growing mode instability (the oscillating two stream instability) because \( \omega \) is imaginary and the solution (b) gives a decay mode instability because it involves the decay of the pump wave into two lower frequency waves. The solutions (a) and (b) give completely independent excitation modes since the products of the small oscillating quantities \( x \) and \( Y \) are neglected in the formulation of the coupled mode.

Purely Growing Mode:

The purely growing mode solution can be found by setting \( x = 0 \) and requiring \( y > 0 \). For \( r_1^2 \Delta \omega^2 \) the threshold criterion value of \( k \) can be expressed as a function of \( \delta \):
\[ I_{th}(\delta) = -\Delta \omega^2 \omega_2 [\delta + (r_2^2/\delta)] \]  

[2-25]

The minimum threshold value of I occurs at \( \delta = \delta_m \) and is given by:

\[ I(\delta_m) = I_m = 2\Delta \omega^2 \omega_2 r_2 (\sigma \mu) \]  

[2-26]

which yields a threshold value of pump field energy

\[ E_m^2 = 2\Delta \omega^2 \omega_2 r_2 / |\mu| \]  

[2-27]

with \( \omega_1 = \delta + \omega_2 = \omega_2 - (\sigma \mu) r_2 \)

In general, the purely growing mode solution can be excited when \( E_1^2 > E_m^2 \). For this case, the pump frequency and the response frequency of Y are approximately equal and the response frequency of X is coupled through a zero frequency mode.

Decay Mode:

In the decay mode solution for case (b) the threshold criterion value of I can be expressed by a function of \( \delta \) and by putting \( y = 0 \):

\[ I_{th}(\delta) = r_1 r_2 \omega_2 / \delta [4s^2 + (r_2^2 + 2r_1 r_2 + \Delta \omega^2 + r_1^2 - \delta^2)^2 / (r_1 + r_2)^2] \]  

[2-28]
The frequency mismatch value of $x$ also can be expressed as a function of $\delta$ by putting $y=0$:

$$x_{th}(\delta)=\pm[(r_2 \Delta \omega^2 + r_1 \delta^2 + r_1^2 r_2 + r_1 r_2^2)/(r_1 + r_2)]^{1/2}$$  \hspace{1cm} [2-29]

Assuming $\Delta \omega \gg r_1$ the minimum threshold value of $x$ and the frequency shift $\delta_m$ can be found by setting $dE_{th}(\delta)/d\delta=0$:

If $\Delta \omega \gg r_2$, $\delta \to 0$ (imaginary root)

$$E(\delta_m)=E_m=4\Delta \omega r_2 r_1^2 [1+(r_1^2/4\Delta \omega^2)]$$  \hspace{1cm} [2-30]

with $\omega_1=\omega_2+(\sigma \mu \omega_0)\Delta \omega$

If $\Delta \omega \gg r_2$, $\delta \to r_2/[3]^{1/2}$ (imaginary root)

$$E(\delta_m)=E_m=(16[3]^{1/2}/9)\omega_2 r_1 r_2^2$$  \hspace{1cm} [2-31]

with $\omega_1=\omega_2+(\sigma \mu \omega_0) r_2/[3]^{1/2}$

For the case which is appropriate for ionospheric plasma where the ion acoustic frequency is much greater than the Langmuir wave damping rate, the decay mode has a threshold about 2.5 times lower than the
growing mode threshold. In the incoherent backscatter observation two plasma instabilities should be distinguishable in the plasma line power spectrum. The purely growing mode instability excites an electron plasma wave with HF pump frequency \( f_{HF} \); the decay mode instability excites another electron plasma wave with frequency \( f_{HF} - f_{IA} \), in which \( f_{IA} \) is the ion acoustic frequency. This is shown schematically in Figure 2.1. Fig 2.1 also shows that the decay line dominates the plasma line power spectrum. This is because the decay mode generally has the lower instability threshold.
2.3 SATURATION MECHANISMS OF THE IONOSPHERE

In the linear approximation a radio wave propagating in the ionospheric plasma does not influence any of the other radio waves because the plasma oscillations are decoupled. But once an instability threshold is exceeded, the linear theory predicts that the amplitudes of the excited wave will grow exponentially, and the saturation mechanism theory predicts that the plasma restricts this growth. When the growth rate of the excited waves becomes greater than their damping losses a parametric instability is excited. The saturated plasma waves lose energy through linear damping processes, but above instability threshold the pump wave supports more energy than linear damped away, so that the enhanced plasma waves must evolve to an equilibrium through nonlinear damping processes. We consider a number of damping processes:

[1] The linear damping processes: linear damping is an important part of the total wave-plasma energy balance even above instability threshold. Linear damping includes:

(1) Collisional damping: the collisional dissipation of the radio plasma wave is a fundamental linear damping processes. As a plasma wave
passes through the plasma medium the energy of plasma wave couples to
discrete particle motions. The wave energy retained by the discrete
particle is damped away when the particle undergoes collisions. This
process acts through ohmic damping by the electrons. The wave energy
dissipated by ohmic damping goes into the thermal electrons, raising the
local electron temperature.

(2) Linear electron Landau damping: electron Landau damping is a
curious physical phenomenon which is a dissipative process in which wave
energy is transformed into thermal energy without collisions. In the
ionospheric plasma the electron velocities have a Maxwellian distribution.
There are electrons whose velocities are close to the phase velocity of
plasma waves which are called resonance electrons. A resonance electron
might gain or lose a lot of energy from the wave because it feels a
consistent electric field over a long period. The electrons moving slightly
slower than wave gain energy from the wave and the electrons moving
slightly faster then the wave lose energy. A Maxwellian electron velocity
distribution has more slow electrons than fast ones, consequently there
are more electrons taking energy from the wave so that the wave is
damped. Linear electron Landau damping generates a high energy electron
tail in the Maxwellian velocity distribution.

(3) Nonlinear electron trapping: for large amplitudes of radio wave
oscillations, electrons can be trapped in the wave potential. When the
wave potential energy becomes greater than the thermal energy of the
electrons, which are moving at the wave phase velocity, the electrons become trapped.

The potential energy $U_p$ of an electric field per volume can be expressed as:

$$U_p = \frac{E^2}{8\pi}$$  \[2-32\]

The thermal energy $U_t$ of the plasma per volume can be given by:

$$U_t = nK_bT_e/2$$  \[2-33\]

When $U_p > U_t$ the nonlinear electron trapping becomes important part of the saturation process:

$$\eta^2 = \frac{U_p}{U_t} > 1$$  \[3-34\]

where $\eta = E/[4\pi nK_bT_e]^{1/2}$ and $E$ is the pump electric field.

DeGroot and Katz [1973] have shown that electron trapping occurs when $2 < \eta < 7$, and that electron trapping does not significantly affect the plasma saturation process when $\eta < 2$. For ionospheric heating experiments of the F region, if the electric field of the pump wave is 100mV/m then $\eta$ equals 6; therefore electron trapping must be considered as contributing to the plasma wave saturation process.

[2] Nonlinear damping processes: The plasma wave energy is
transferred from an unstable region to a stable region of wave number space through a nonlinear saturation mechanism.

Nonlinear damping includes:

(1) Nonlinear ion Landau damping: The interaction of two high frequency electron plasma waves \( [\omega_1, K_1] \) and \([\omega_2, K_2] \), with \( \Delta \omega = \omega_1 - \omega_2 \), \( \Delta K = K_1 - K_2 \), generates a beat frequency \( \Delta \omega \) to form a low frequency plasma wave response moving with a velocity \( \Delta \omega / \Delta K \). The low frequency wave can exchange energy with the resonant ions when the beat velocity falls within the ion velocity distribution. This low frequency ion acoustic wave beats with the high frequency plasma wave to induce a transfer of energy from the high frequency plasma wave \( [\omega_1, K_1] \) to another high frequency daughter wave \( [\omega_2, K_2] \). This unstable high frequency daughter wave \( [\omega_2, K_2] \) grows to an amplitude which also becomes another pump wave and destabilizes a second high frequency daughter wave \( [\omega_3, K_3] \), and the second high frequency daughter wave \( [\omega_3, K_3] \) stabilizes the first high frequency wave \( [\omega_1, K_1] \), as it simultaneously destabilizes a third high frequency wave \( [\omega_3, K_3] \), etc.

(2) Plasma mode coupling: plasma mode coupling is a three wave interaction process in which an electron plasma wave decays into an ion acoustic wave and another plasma wave, or inversely, an ion acoustic wave and a plasma wave mix to act as a source of a new electron plasma wave. The plasma mode coupling does not dissipate plasma wave energy, but only redistributes this energy in wave number space as nonlinear Landau
damping. The plasma mode coupling is a comparably important saturation mechanism to nonlinear Landau damping for lower pump powers [Godfrey et al., 1973].
2.4 INCOHERENT SCATTER

An ionosonde measures the echo delay time as a function of HF frequency. From the ionogram the electron density profile can be determined under some conditions by solving an integral equation, but no information is obtained above the $F_2$ peak from the ionogram. The incoherent scatter radar can measure not only the electron density profile but also the temperature profile of the ionosphere even above the $F_2$ peak.

H. G. Booker and W. E. Gordon [1950] were first to introduce the basic theory of radio wave incoherent scattering from atmospheric density fluctuations. Later, W. E. Gordon [1958] suggested that although most of the radiated HF power by a transmitter largely penetrates when the HF wave frequency is above the critical frequency of the $F_2$ layer, a minute fraction is scattered back by the free electrons of the ionosphere just as the molecules of the atmosphere scatter sunlight. K. L. Bowles [1958] first observed the incoherent scatter signals from the ionosphere. Since that time, the confirmation of incoherent scatter theory by experiment has made incoherent scatter radar a powerful diagnostic tool for probing the ionosphere.

J. V. Evans and D. T. Farley [1971] excellently reviewed and discussed the incoherent scatter theory and developed its ionospheric applications. In the following brief introduction the physical arguments of the incoherent scatter theory are given which are essential to the
interpretation of the electron density profile, the incoherent scatter spectra, and the electron and ion temperature profile of the ionosphere.

Free electrons scatter a small part of an incident electromagnetic wave with a Thomson scattering cross-section, \( \sigma_e = 10^{-28} \text{m}^2 \).

When the condition \( \alpha = \frac{K \lambda_d}{4 \pi \lambda_d / \lambda} \ll 1 \) is satisfied, the radar wavelength \( \lambda \) is much smaller than the plasma Debye length \( \lambda_d = \left[ \frac{\pi k T_e}{n_e e^2} \right]^{1/2} \) and the ions can be neglected in comparison with the electrons. Thomson scattering is purely incoherent and the total signal is simply the sum of the individually scattered power.

When the condition \( \alpha \ll 1 \) is satisfied, the radar wavelength is much greater than the plasma Debye length, the electron motion correlates with the ions, and the scattering by the electrons can not be considered as purely incoherent. The electrons have an effective scattering cross-section \( \sigma_T = \sigma_e / (1 + T_e / T_i) \), for thermal equilibrium \( T_e = T_i \), \( \sigma_T = \sigma_e / 2 \), which is half the cross-section found for the purely incoherent scattering case [Buneman, 1962].

For the Arecibo incoherent scatter radar, the operating frequency is 430 MHz, the radar wavelength is 70 cm, and the typical Debye length is 3 cm \( [T_e = 1500 \text{K}, n_e = 8 \times 10^{11} \text{m}^{-3}] \) for the F region. Consequently, \( \alpha = 0.05 \), and the ions must be considered important in the backscatter power
spectrum.

The echo power $P_r$ from the incoherent radar transmitter scattered from electrons in the ionosphere is a function of electron density, and the average scattering cross-section of the electron, and can be written as:

$$P_r \propto N[h] \sigma_T[h] / h^2 \quad [2.35]$$

From this expression, which is known as the radar equation for a beam filled with targets, the electron density profile can be obtained.

For $\alpha \ll 1$ the power spectrum has two principal components. Most of the backscattered signal comes in the ion component, or ion line, which scatters from collisionless acoustic waves, and has a frequency width of the order of Doppler shifts for the ion thermal velocity $[2kT_i/m]^{1/2}$. The ion temperature $T_i$ can be measured from the width of the ion component at half power points. The shape of the ion component also depends on different ratios of electron temperature and ion temperature, so that the ratios $T_e/T_i$ can be obtained by determining the shape of the ion component. Fig 2.2 shows that theoretical spectra of the ion component for various values of electron to ion temperature ratio. The curves calculated by Fejer (1961) assume that the radar wavelength is much greater than the plasma Debye length; only singly charged atoms are present; only one ion species is present; and the earth’s magnetic field and collisions can be neglected.
In Fig 2.3 the observation of the electron and ion temperature in January 16, 1986 is shown [Dr. Craig Tepley, private communication 1986] using 430 MHz incoherent radar. Fig 2.3a shows the electron temperature, Fig 2.3b shows the ion temperature, Fig 2.3c shows the ratio of the electron temperature to the ion temperature, Fig 2.3d shows the ratio of the signal to noise.

The other component of the backscattered signal is the plasma line, which scatters from weakly damped electrostatic plasma oscillations near the plasma frequency [Serpeter, 1960].

The center frequency for the plasma line is the electron resonance frequency \( f_r \). When \( f_p^2 \gg f_c^2 \) the electron resonance frequency can be written by [Yngvesson and Perkins, 1968]:

\[
f_r^2 = f_p^2 + 12kT_e/v^2m_e * f_c^2 * \sin^2 \theta \quad [2-36]
\]

Where \( f_p \) is the plasma frequency, \( f_c \) is the electron gyrofrequency, \( \theta \) is the angle between the radar vector and the magnetic field. The unenhanced intensity of the plasma line is less than the intensity of ion component by a factor of approximately \( \alpha^2 \), but the parametrically enhanced plasma line can be 3 or 4 orders of magnitude more intense than the unenhanced lines [R. L. Showen and D. M. Kim, 1978].
Figure 2.1 Schematic backscatter spectrum showing the enhanced plasma line. The structural terminology is adopted from Kantor (1972).
Fig 2.2 Theoretical spectra of the ion component
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(A) TE  Fig 2.3a

(B) TI  Fig 2.3b

(C) TE/TI  Fig 2.3c

(D) S/N  Fig 2.3d

FIGURE 2.3  TEMPERATURE VERSUS TIME
CHAPTER 3

TWO HF HEATING THEORY

In 1982 Papadopoulos presented a method of ionospheric heating using two powerful HF waves for ELF generation [Papadopoulos et al., 1982]. Papadopoulos pointed out that a HF wave can decay into an ELF whistler wave and a second HF wave. The sidebands would arise from the nonlinear interaction of the ELF wave with the two high frequency waves because of an instability. In 1984 Gordon and Ganguly first observed the HF sidebands using two powerful waves in Arecibo. In 1984 Fejer proposed a phase modulation theory to explain the HF sidebands. In 1986 Gordon-Huang introduced modulation index theory (wave-particle interaction theory) using the W.K.B. approximation to describe the HF sidebands.

3.1 PAPADOPoulos Theory

The interaction of high-power HF waves with the ionospheric plasma is strongly nonlinear. Electrostatic waves or electromagnetic waves with frequencies different from the pump frequencies can be generated by the ponderomotive force or the thermal force. The generation of VLF and ULF waves, besides its scientific merit, has a practice of applications for Navy communications due to its low propagation losses in the earth-ionosphere waveguide and its low absorption properties in seawater.
The first nonlinear mechanism for ELF generation is parametric decay of a high-frequency radio wave into a low-frequency compressional Alfvén wave or magnetosonic wave and a high-frequency sideband [Papadopoulos et al., 1982]. The process is a conventional stimulated forward Brillouin scattering [Liu, 1976] and is considered as collisionless in the region above 150 Km [F region]. The nonlinear force is dominated by the ponderomotive force rather than thermal nonlinearity. The maximum downconversion efficiency is limited by the Manley-Rowe relation [Sagdeev and Galeev, 1969] to $\omega_0/\omega_1$. The power threshold for complete decay of HF waves into Alfvén waves is very large and can be found by the inverse scattering transform method [Zakharov and Manakov, 1975]. Papadopoulos pointed out that if the thermal nonlinearity caused by collisional electron dominates over the ponderomotive force, the power threshold for complete decay is reduced by a comparable $\omega_d/\Delta\omega$ factor and the downconversion efficiency is increased by a factor $\omega_d/\Delta\omega$ where $\omega_d$ is the electron collision frequency [Papadopoulos et al., 1983]. This is the case for ELF generation in the lower ionosphere [E region].

The second nonlinear mechanism for ELF generation is local modulation of existing ionospheric current, such as the auroral and equatorial electrojets. It is called nonlinear demodulation or ionospheric detection because the HF waves produce modulated electron heating with modulation of the local conductivity. When the natural ionospheric currents pass through the illuminated region the currents are modulated by the conductivity changes and produce a radiating dipole current pattern. The
low-frequency wave couples to the earth-ionosphere waveguide and propagates to long distances with low propagation losses [Cheng et al., 1981].

VLF signals were observed with amplitude $10^{-3}$ with maximum frequency for 2kHz by the Max Planck group [Stubbe et al., 1982,a,b] using the Tromso Norway ionospheric heater to modulate the polar electrojet. VLF signals in the 500Hz to 5 kHz range were observed by Ferraro [Ferraro et al., 1982] using the Arecibo HF heater to modulate the equatorial electrojet.

The third nonlinear mechanism for ELF generation is a spontaneously generated magnetic field with time variation when $\nabla n = 0; \nabla T_e = 0$. where $\nabla n$ is the ambient ionospheric density gradient and $\nabla T_e$ is the electron temperature gradient in the heated region [Papadopoulos and Chang, 1985]. For frequencies below 40 Hz, the spontaneous field generation dominates over the current modulation.

For the nonlinear mechanism of parametric decay in the collisionless region [F region] Papadopoulos derived the growth rate for the parametric instability in a homogeneous plasma. Papadopoulos considered a HF wave $e_y E_0 \exp[-i(\omega_d t - K_0 x)]$ propagating in the x-z plane in the ionosphere; the z axis is in the direction of the earth's magnetic field, while the x axis lies in the vertical plane. For pump frequency $\omega_d > \omega_B$, where $\omega_B$ is the electron cyclotron frequency, the electron high-frequency response is unmagnetized. The high-frequency waves produce a low-frequency ($\Delta \omega$, $\Delta k$) ponderomotive force on the electrons, given by:
\[ F = -ie^2 \Delta K(E_0 E_- + E_0 E_+) / 2m \omega_0^2 \]  

where \( \Delta \omega, \Delta K \) are the Alfvén wave frequency and the Alfvén wave number respectively; and \( \omega_+ = \omega_0 + \Delta \omega; \omega_- = \omega_0 - \Delta \omega; K_+ = K_0 + \Delta K; K_- = K_0 - \Delta K \) are the frequency and wave number of the two high frequency sidebands respectively.

The low frequency ponderomotive force drives a low frequency current and a density fluctuation, given by:

\[ J = \sigma [E \times E_p] \]

\[ n = -K \sigma [E \times E_p] / \omega \]

where \( \sigma \) is the electron conductivity tensor and \( E \) is the self-consistent electric field.

The nonlinear current at the sideband is given by:

\[ J_+^{NL} = -neV_0 / 2 \]

\[ J_-^{NL} = -neV_0^* / 2 \]

Using the above equations the wave equation of the sidebands, the
nonlinear dispersion relation, and the growth rate for the parametric instability can be obtained. In the two limits of $\Delta \omega \approx \omega_1$ or $\Delta \omega \ll \omega_1$ the growth rate reduces to:

$$\gamma_0^2 = K^2 \gamma_0^2 \frac{\omega_1^2}{16 \Delta \omega \omega_0}$$ [3-6]

where $\omega_1$ is the ion cyclotron frequency.

An order of magnitude estimate of the growth rate is given by Papadopolous: $\gamma = 10^{-2} \text{s}^{-1}$, at $\Delta \omega = 10^3 \text{ rad s}^{-1}$ for a 10MW, 5MHz transmitter with $g = 10^3$; $P = 5 \times 10^{-2} \text{W/m}^2$, $E_0 = 6.6 \text{V/m}$, $V_0 = 4 \times 10^6 \text{cm/s}$. A limit on ELF power of the order 200 kW also is given for a 20MW transmitter because of the Monley-Rowe theoretical upper limit of the efficiency $n = \Delta \omega c / V_A \omega_0 = 10^{-2}$. 200KW is large enough for long-range waveguide excitation.
3.2 FEJER-HUANG PHASE MODULATION THEORY

The amplitude of the electric field of an ionospherically reflected radio wave is described by the Airy function in the reflection region in the absence of an external magnetic field. But the amplitude of the electric field of high power radio waves is no longer described by the Airy function because of an appreciable artificial horizontal stratification due to the ponderomotive force.

A powerful radio wave normally incident on the ionosphere modifies the ionospheric electron density by several mechanisms. Over longer time scales of tens of seconds the enhanced electron temperature plays a dominant role in the F region, resulting in an electron density enhancement due to a decrease of the NO\textsuperscript+ and O\textsubscript{2}\textsuperscript+ recombination rates with increasing electron temperature [Stubbe et al., 1982; Figure 10]. Over shorter time scales [about 0.2 s or less] the effect of the ponderomotive force dominates; some of its effects were discussed in Fejer et al. [1983], showing computed density profiles. The effect of the ponderomotive force is a reduction in the electron density which is spatially quasiperiodic due to the standing wave nature [closely resembling the Airy pattern] of the radio wave. The reduction in density is particularly pronounced near the height of reflection. Figure 3.1b (Fig 6b of Fejer et al., 1983) shows the resemble Airy pattern of the electric field intensity for two different field intensities of the incident wave. From Figure 3.1b it is clearly seen that at any given height the wavelength of the standing wave is shorter for
the more powerful wave and, in addition, the more powerful wave
"reaches" a greater height. Both have the effect of increasing the phase
delay which is thus greater for the more powerful wave. Since the
intensity of the resultant of two powerful incident radio waves at closely
spaced frequencies varies at the beat frequency, this must result in the
variation of the phase delay at the beat frequency.

The density profiles of Figure 3.1a (Fig 6a of Fejer et al. [1983]) can
not be established instantly after switching on the heating transmitter
because the ion thermal velocity is relatively slow. The spatial
quasiperiodicity is established relatively rapidly; the observations of
Belikovich [1975, 1977, 1978] show a time constant of about 20 ms. The
reduction in electron density implied by Figure 3.1a will be established
relatively rapidly close to the reflection region; in the example of Figure
3.1a the first dip in density extending over about 100 m would be
established in roughly 0.1 seconds, the time taken by an ion to travel that
distance at thermal velocity.

The propagation of a radio wave in a cold unmagnetized plasma is
governed by the differential equations given by Budden [Budden,
1961;1985]:

\[ \frac{d^2E}{dz^2} + k^2 n^2 E = 0 \]  \[3-7\]

where \( k = \omega/c; \) \( c = (\epsilon \mu)^{-1/2} \)

But for the propagation of powerful HF waves, the density change due
to the ponderomotive force must be considered. The differential equations
for the normalized fields $E^*$, $B^*$ are given by Fejer [Fejer, 1963]:

$$\frac{\partial E^*}{\partial z^*} = -B^* \quad \quad [3-8]$$

$$\frac{\partial B^*}{\partial z^*} = [1 - N^* + N^*E^*/4]E^* \quad \quad [3-9]$$

where the dimensionless variables are defined by $z^* = z\omega/c$, $E^* = [\epsilon_0/(NK_bT)]^{1/2}E$, $B^* = [\mu_0 NK_bT]^{-1/2}B$, and $N^* = N\omega_0^2/\epsilon_0 m \omega^2$; $K_b$ is Boltzmann's constant, $T$ is the temperature, $z$ is the depth below the height where $N^* = 1$ ($\omega = \omega_p$), $N$ is the unperturbed number density of electrons, and $\epsilon_0$ is the electric polarizability of free space; $\mu_0$ is the magnetic permittivity of free space; $e$ and $m$ are the charge and mass of electron respectively.

The third term in the parentheses on the right of [3-9] represents the change in electron density caused by the ponderomotive force of the electromagnetic wave alone [neglecting the effect of parametrically excited electrostatic waves]. In the computations of phase delay to be described here a linear density profile will be used and equations [3-8] and [3-9] will be numerically integrated downward, starting from above the reflection point and ending at the bottom of the ionosphere.

The ponderomotive potential can be derived from the electron density perturbation [Fejer, 1979] [Belikovich, 1976]:

$$-eE = F = -\nabla \phi \quad \quad [3-10]$$
\[ \gamma = \frac{e^2 E^2}{4 m \omega_0^2} \]  
\[ \gamma / k_b [T_e + T_i] = \frac{e^2 E^2}{4 m \omega_0^2} k_b [T_e + T_i] \]

\[ \text{E is peak value of the electric field. The fractional electron density} \] 
\[ \text{perturbation can be given by [Fejer, 1983]:} \]
\[ \Delta N = -\frac{e^2 E^2 N}{4 m \omega_0^2} k_b [T_e + T_i] \]

\[ \text{Equation [3-13] can be rewritten as:} \]
\[ \Delta N / N = (e_0 E^2 / 4 NK_b T_e) (N e^2 / e_0 m \omega_0^2) \approx -E x^2 N^x / 4 \]

The effective normalized number density \(N^x\) would then be reduced from \(N^x\) to \(N^x - N x^2 E^2 / 4\). It agrees with equation [3-9]. Equations [3-6] and [3-9] can not be solved exactly but can be numerically integrated using the following technique:

\[ \frac{dE}{dz} = f_1(z, E, B) \]
\[ \frac{dB}{dz} = f_2(z, E, B) \]
\[ K_{11} = f_1(z_0, E_0, B_0) \hbar \]
\[ K_{12} = f_2(z_0, E_0, B_0) \hbar \]
\[ K_{21} = f_1(z_0 + h/2, E_0 + K_{11}/2, B_0 + K_{12}/2)h \]
\[ K_{22} = f_2(z_0 + h/2, E_0 + K_{11}/2, B_0 + K_{12}/2)h \]
\[ K_{31} = f_1(z_0 + h/2, E_0 + K_{21}/2, B_0 + K_{22}/2)h \]
\[ K_{32} = f_2(z_0 + h/2, E_0 + K_{21}/2, B_0 + K_{22}/2)h \]
\[ K_{41} = f_1(z_0 + h, E_0 + K_{31}, B_0 + K_{32})h \]
\[ K_{42} = f_2(z_0 + h, E_0 + K_{31}, B_0 + K_{32})h \]
\[ \Delta E = (K_{11} + 2K_{21} + 2K_{31} + K_{41})/6 \]
\[ \Delta B = (K_{12} + 2K_{22} + 2K_{32} + K_{42})/6 \]

Numerical solutions of equations [3-8] and [3-9] were obtained for \( N^* = 1 - z^*/H^* \). This corresponds to a linear density profile with a normalized scale height of \( H^* = 4000 \); for a frequency of 3.175 MHz this corresponds to an actual scale height of about 50 km. The downward integration was started at \( z^* = -40 \) and was carried to \( z^* = 4000 \). For \( z^* = -40 \) the wave is evanescent and the third term on the right of [3-9] may be neglected. It may then be shown that the solutions for \( B^* \) and \( E^* \) are simple exponential function of \( [N^* - 1]^{1/2}z^* \) and that their ratio is given by:

\[ B^*/E^* = [N^* - 1]^{1/2} \]

[3-15]

which for \( N^* - 1 = z^*/4000 = 0.01 \) becomes -0.1.

The integration was then carried out for different starting values of
E* * . The final value \( E_1^* \) and \( B_1^* \) and the phase delay \( \psi = 2 \tan^{-1} \left( B_1^*/E_1^* \right) \) were calculated. It was expected that the phase delay would be proportional to the power density of the incident wave which is given by

\[ P = cN_0kT(E_1^*E_1^* + B_1^*B_1^*)/4. \]

Several values of the ratio:

\[ r = [\psi_1 - \psi_2]/[P_1 - P_2] \]  

were calculated and were found to yield the same result for incident power densities ranging from \( 2 \times 10^{-6} \) W/m to \( 3 \times 10^{-4} \) W/m, \( r = 6529 \) radians m²/Watt with \( N = 1.25 \times 10^{11} \) m⁻³, \( T = 1000 \) K. For an incident power density of \( 10^{-4} \) W/m this leads to a phase delay of 6529 radians. An incident power density of \( 10^{-4} \) W/m could be produced for a radiated power of 400 kW with an antenna gain of 23 db at a height of 250 km, parameters typical of the Arecibo experiments to be described in the next section. No computations were carried out for other values of \( H^* \).

The next step in the theory is the calculation of the power in the sidebands. For this purpose it is assumed that two frequencies are transmitted from the ground with equal power. Then the resultant field incident on the ionosphere at some height just below it will have the form:

\[ E_0[\sin \omega_1 t + \sin \omega_2 t]/2 = E_0 \cos(\omega_1 - \omega_2) \sin(\omega_1 + \omega_2) t/2 \]  

\[ \omega_1 > \omega_2 \]

It is further assumed that \( \omega_1 + \omega_2 \gg |\omega_1 - \omega_2| \) so that the field has an
"instantaneous amplitude" \( E_0 \cos\left(\left(\omega_1 - \omega_2\right)/2\right) \). In terms of linear circuit theory the field has no Fourier component at the frequency \([\omega_1 + \omega_2]/2\) but in order to calculate the effect of the nonlinear amplitude-dependent phase change it is correct to use the concepts of instantaneous frequency and amplitude. Note that power density incident on the ionosphere is variable and its maximum is \( c_0 r E_0^2/2 \), where \( c \) is the speed of light. The nonlinear phase change of the reflected wave is therefore given in terms of time. The field of the reflected wave therefore has the form:

\[
E_0 \cos\left(\left(\omega_1 - \omega_2\right)t/2\right) \sin\left(\left(\omega_1 + \omega_2\right)t/2 + c_0 \tau E_0^2 \cos\left(\left(\omega_1 - \omega_2\right)t/2\right)\right) \]

\[
E_0 \cos\left(\left(\omega_1 - \omega_2\right)t/2\right) \sin\left(\left(\omega_1 + \omega_2\right)t/2 + c_0 \tau E_0^2 \left(1 - \cos\left(\left(\omega_1 - \omega_2\right)t\right)/4\right)\right) \tag{3-19}
\]

where \( c_0 \tau E_0^2/2 \ll 1 \) was assumed and where \( \tau = t - t_D \), \( t_D \) is the linear time delay involved in the reflection process. We now use the identity [Morse and Feshbach, p. 1322]:

\[
\exp(iz\cos\beta) = \sum_{m=-\infty}^{\infty} \hat{e}_m i^m \cos(m\beta) J_m(z) \tag{3-20}
\]

where \( e_m = 2 \) for \( m > 0 \) and \( e_m = 1 \) for \( m = 0 \). The first factor on the right of [3-19] then becomes:

\[
\sin\left(\left(\omega_1 + \omega_2\right)t/2 + r c_0 E_0^2/4\right) J_0\left(-r c_0 E_0^2/4\right) + \]
\[ +2\cos[(\omega_1+\omega_2)r/2+c_0E_0^2/4]\cos[(\omega_1-\omega_2)r/2-c_0E_0^2/4]J_1(-c_0E_0^2/4) - \\
-2\sin[(\omega_1+\omega_2)r/2+c_0E_0^2/4]\cos[2(\omega_1-\omega_2)r]J_2(-c_0E_0^2/4) \]

\[ [3-21] \]

With the help of [3-21] expression [3-19] becomes

\[ E_0J_0(-c_0E_0^2/4)(\sin\omega_1 r+\sin\omega_2 r)/2 + \\
+ E_0J_1(-c_0E_0^2/4)(\cos[2(\omega_1+\omega_2)r] - \cos[2(\omega_1-\omega_2)r])/2 - \\
- E_0J_2(-c_0E_0^2/4)(\sin[2(\omega_1+\omega_2)r] - \sin[2(\omega_1-\omega_2)r])/2 \]

\[ [3-22] \]

This expression for the field of the reflected wave clearly shows the presence of the nonlinear sidebands.
3.3 GORDON-HUANG MODULATION INDEX THEORY

The modulation index theory of HF sidebands using the W.K.B. approximation are described. When the ionosphere is illuminated with two strong HF signals of slightly different frequencies the nonlinear wave-particle interaction process arises. This wave-particle interaction causes an electron density fluctuation [Fig 3.1]. The propagating electron density fluctuation will lead to currents at \( \omega_1 + \Delta \omega \), \( \omega_2 - \Delta \omega \). These currents will generate mixed electromagnetic electrostatic sideband modes at \( \omega_1 + \Delta \omega \), \( \omega_2 - \Delta \omega \). The sideband modes interact with two pump waves and generate HF sidebands at \( \omega_1 + \Delta \omega \), \( \omega_1 + 2\Delta \omega \), \( \omega_2 - \Delta \omega \), \( \omega_2 - 2\Delta \omega \). So that the HF signals reflect from the ionosphere and contain sidebands which are multiples of the difference frequency between the two pump waves. It is shown that the amplitudes of the sidebands are related to: [1] the effective HF power; [2] the HF frequency; [3] the temperature of the electrons; [4] the scale height of the ionosphere; and [5] the difference frequency of the two HF pump waves.

A linear theory of parametric excitation of ionospheric heating using two powerful HF waves was proposed by Arnush et al. [1973]. The theory was developed by Fejer et al. [1978]. Fejer predicted that a standing Langmuir wave at exactly the arithmetic mean of the two pump frequencies and a standing low-frequency wave at exactly half the difference between the two pump frequencies are excited which is called the arithmetic mean instability. Showen demonstrated the presence of the
standing Langmuir waves by experiments [Shawen et al., 1978]. The experiments of Sulzer demonstrated the presence of the standing low frequency wave by the technique using multiple-pulse and pulse-to-pulse correlation [Sulzer et al., 1984]. Wong also observed the presence of the standing Langmuir waves and their discrete lower frequency sidebands and demonstrated the strongest coupling when the frequency separation of two pumps equals two times the ion acoustic frequency [Wong et al., 1979]. Fejer points out that a rather different nonlinear process operates when two powerful HF waves are used with a separation less than a few hundred Hz.

This modulation index theory using W.K.B. approximation can be described as follows:

Two strong HF waves at the bottom of the ionosphere can be represented by:

\[ E_0 \sin \omega_1 t + E_0 \sin \omega_2 t = 2E_0 \cos(\Delta \omega t/2) \sin \omega_0 t = E \sin \omega_0 t \quad [3-23] \]

\[ \omega_0 = (\omega_1 + \omega_2)/2 \quad \Delta \omega = \omega_1 - \omega_2 \quad E = 2E_0 \cos(\Delta \omega t/2) \]

The propagation of powerful electromagnetic waves in a cold unmagnetized ionosphere is governed by the differential equations [3-8] and [3-9] given by Fejer [1983].

Equations [3-8] and [3-9] have been numerically integrated by Fejer and Huang in the direction of increasing \( z^* \) assuming a linear profile for
the unperturbed number density $N$. The field distributions still closely resemble the Airy function. In the equation (3-23) there are four waves, therefore there is a standing wave of fluctuating amplitude so that the fields vary spatially on the relatively short scale of the Airy pattern.

The nonlinear effects of the HF waves can be explained in terms of the ponderomotive force and ponderomotive potential which are given by equations (3-10) and (3-11). If $\omega_0 \gg \Delta \omega$ the fields of the HF waves can be thought to have an "instantaneous amplitude" $E = 2E_0 \cos(\Delta \omega t/2)$ and an "instantaneous angular frequency" $\omega_0 = (\omega_1 + \omega_2)/2$. It is very useful to calculate the effects of the nonlinear modulation. The concept of instantaneous amplitude and instantaneous angular frequency was given by Fejer.

From the equations (3-8) [3-9] and (3-13) we can get the following differential equation which is the same equation given by Budden [1961] except for $n_1^2$:

$$\frac{d^2E}{dz^2} + k^2[n_0^2 + n_1^2]E = 0$$  \hspace{1cm} [3-24]

$$n^2 = n_0^2 + n_1^2$$

$$n_0^2 = 1 - \frac{Ne^2}{\varepsilon_0 m \omega_0^2}, \hspace{0.5cm} n_1^2 = \frac{e^4E^2N}{4\varepsilon_0 m^2 \omega_0^6 k_0 [T_0 + T_1]}$$  \hspace{1cm} [3-25]

where, $K$ is the wave number; $E$, the electric field; $\varepsilon_0$, the electric polarizability of free space; $\mu_0$, the magnetic permeability of free space;
and $n_1^2$ represents the change in refractive index caused by the ponderomotive force and corresponds to the nonlinear interactive term.

Using the W.K.B. approximation the propagation of HF radio waves in a slowly varying horizontally stratified ionosphere has the solution:

\[
E = A \exp[2iK_0 z ndz] \\
= A \exp[2iK_0 z (n_0^2 + n_1^2)^{1/2} dz] \quad [3-26]
\]

This W.K.B. approximation for the propagation of high-power HF waves is better than the W.K.B. approximation for the propagation of low-power HF waves in which the approximation condition is certain to fail near the value of $z$ where $n_0^2$ passes through a zero. But for high-power waves in the reflection region the refractive index $n^2 = n_0^2 + n_1^2$ doesn't pass through a zero.

Equation [3-25] shows that the refractive index $n_0^2$ of the unperturbed electron density is much larger than the refractive index $n_1^2$ caused by the ponderomotive force except within a fraction of one free space wavelength of the level of the reflection. Equation [3-26] becomes approximately:

\[
E = A \exp[2iK_0 z n_0 dz] \exp[iK_0 z n_1^2/n_0 dz] \\
= A' \exp[iK_0 z n_1^2/n_0 dz] \quad [3-27]
\]

where,

\[
A' = A \exp[2iK_0 z n_0 dz]
\]
From equation (3-26) \( 2E_0\cos[\Delta \omega t/2] \) is the instantaneous amplitude. If the difference frequency \( \Delta \omega \) is much smaller than \( \omega_0 = (\omega_1 + \omega_2)/2 \) then equation (3-27) becomes:

\[
E = A''\exp[iK\{(\omega_0^2 n''d_0)\cos\Delta \omega t\}]
\]

where,

\[
A'' = A'\exp[iK\{N\varepsilon_0^2E_0^2/2\varepsilon_0 m^2k_b[Te+T_i]n_0\}dz],
\]

and

\[
n'' = \int_0^z\varepsilon_0^2N\varepsilon_0^2E_0^2/2\varepsilon_0 m^2\omega_0^2k_b[Te+T_i]n_0dz
\]

From equations (3-23) and (3-28), the electric field which reflects from the F layer can be written:

\[
E = B\sin(\omega_0 t + K\{\omega_0 n'dz\}\cos\Delta \omega t)
\]

\[
= B\sin(\omega_0 t + m_p\cos\Delta \omega t)
\]

where,

\[
m_p = \{\varepsilon_0^2/2\varepsilon_0 m^2\omega_0^3c_kb[Te+T_i]\}\int_0^z(E_0^2/dn_0)dz
\]

The distribution of the electric field closely resembles the Airy function, so that the amplitude of the electric field \( E_0 \) is not a constant. In the reflection region the electric field has swelled by a factor of approximately 10, because the group velocity of the HF waves has slowed down. The amplitude of the electric field \( E_0 \) in the inside of the integral of the equation (3-30) can be approximated by the value at the reflection region, moved outside the integral and replaced by \( B|E_0|^2 \) for calculation.
simplicity. Then equation [3-30] can be given as:

\[ m_p = \left( e^{\frac{e^2 B E_0^2}{2 \sigma_0 m^2 \omega_0^3 c k_b (T_e + T_l)}} \right)_0^z \frac{(N/n)}{dz} \]  

[3-31]

where \( \beta \) is related to the absorption of the HF waves, and the value of \( \beta \) ranges from 0.5 to 1.

If we suppose that the electron density profile is:

\[ N = N_0 \left[ 1 - \left( \frac{x}{H} \right)^j \right] \]  

[3-32]

where \( H \) is the scale height of the ionosphere, \( j \) ranging from 1 to 2, then equation [3-31] can be written:

\[ m_p = B \left( \frac{(2-j) / (2j)}{2j} \right) e^{2 \frac{e B E_0^2}{j m \omega_0 c k_b (T_e + T_l)}} \]  

[3-33]

where \( B \left( \frac{(2-j) / (2j)}{2j} \right) \) is Beta Function. If the electron density profile is linear \( (j=1) \), then the equation [3-33] can be written:

\[ m_p = 2 e^{2 \frac{e B E_0^2}{3 m \omega_0 c k_b (T_e + T_l)}} \]  

[3-34]

The electric field returned from the ionosphere contains the sidebands and can be calculated by the following identity [Terman, 1943]:
\[
\sin(\omega_0 t + m_p \cos \Delta \omega t) = J_0[m_p] \sin \omega_0 t \\
+ J_1[m_p] \{ \sin(\omega_0 + \Delta \omega) t - \sin(\omega_0 - \Delta \omega) t \} \\
+ J_2[m_p] \{ \sin(\omega_0 + 2\Delta \omega) t + \sin(\omega_0 - 2\Delta \omega) t \} \\
+ J_3[m_p] \{ \sin(\omega_0 + 3\Delta \omega) t - \sin(\omega_0 - 3\Delta \omega) t \} \\
+ \ldots \]  

\[3-35\]

Here \(J_n[m_p]\) is the Bessel function of the first kind and nth order with modulation index \(m_p\). The amplitude of the different sidebands depends upon the modulation index and can either be calculated with the aid of a table of Bessel functions or be obtained from Figure 3.2. Figure 3.2 shows the relationship of modulation index \(m_p\) with the sideband amplitudes where the nth sideband amplitude is \(\text{db}\) with respect to the transmitter power \((J_n[m_p]/J_0[m_p])\):

The relationship between the modulation index \(m_p\) and the difference frequency of the two HF pump waves can be derived:

The nonlinear modulation using two HF waves is caused by the ponderomotive force. If only term of the ponderomotive force is considered the electric field can be represented by:

\[
E = A \exp[2iK/n_1 dz] \]  

\[3-36\]
where \( n_1 = e^2 E(N)^{1/2} / 2m\omega_0^2 (\pi e K_B T_e + T_i)^{1/2} \)

\[ E = 2E_0 \cos(\omega t / 2) \]

This case is approximately true in the reflection region because the refractive index \( n_1 \) caused by the ponderomotive force is larger than the refractive index \( n_0 \) of the unperturbed electron density in the reflection region.

Fig 3.3 shows that HF waves with the angular frequencies \( \omega_1 \) and \( \omega_2 \) are reflected at different heights 1 and 2 of the ionosphere. The difference height \( \Delta x \) is increasing if the difference frequency \( \Delta f \) is increasing. We assume the following profile for the unperturbed number density \( N \) in the reflection region:

\[ N = N_0 [1 - (\Delta x / H)] \]

where \( H \) is scale height of the ionosphere. This profile for the unperturbed number density is very close to practical profile. The difference height \( \Delta x \) caused by the difference frequency is given by:

\[ \Delta x = H (2\Delta f / f_0)^{1/2} \]

The electric field return from the ionosphere can represented by:
\[ E = B \sin(\omega_0 t + 2K J n_1 \, dz) \]
\[ = B \sin(\omega_0 t + 2K \{e^2(N)^{1/2}E_0/m \omega_0^2(e_0K_B[T_e + T_I])^{1/2}\}) dz \cos(\Delta \omega t/2) \]  

[3-39]

The modulation index caused by the difference frequency of the two HF pump waves can be written as:

\[ m_{pf} = m_p - \Delta m_p \]  

[3-40]

From the equation [3-39] the modulation index \( \Delta m_p \) can be given by:

\[ \Delta m_p = 2K \{e^2(N)^{1/2}E_0/m \omega_0^2(e_0K_B[T_e + T_I])^{1/2}\} \, dz \]

\[ \Delta m_p = \{2He^2E_0(\beta N_0)^{1/2}/m \omega_0 C(e_0K_B[T_e + T_I])^{1/2}\} [2\Delta f/f_0]^{1/2} \]  

[3-41]

where \( m_p \) is given by the equation [3-34]; \( E_0 \) is the electric field amplitude at the reflection region and \( \beta \) is a coefficient which is concerned in the absorption of the electric field.

Fig. (a) The normalized unperturbed number density (straight line) and the perturbed number densities (curves) as functions of the normalized depth \( z^* \) below the \( \omega = \omega_p \) point for two incident waves of different amplitudes (solid and dashed lines). The normalized electric field for the same incident waves is shown by Figure (b) The normalized electric field \( E^* \) as a function of the normalized depth \( z^* \) below the \( \omega = \omega_p \) point for two incident waves of different amplitude (solid and dashed lines). (After Fejer, 1983)
FIGURE 3.2 FIRST SIDEBAND AMPLITUDE VERSUS MODULATION INDEX
Figure 3.3 Two HF waves reflect at different heights.
CHAPTER 4
EXPERIMENT

4.1 HIGH FREQUENCY HEATING FACILITY

The HF heating facility is located near Islote, Puerto Rico at 18°26'35" N and 66°40'00" W geographic latitude and longitude respectively, which is 17.1 km northeast of the Arecibo Observatory. This facility consists of an array of 32 log periodic inverted pyramid antennas arranged in an 8 by 4 configuration for operation in the frequency range 3-12 MHz. This array is orientated as shown in Figure 4.1. The faces of the pyramid are at an angle of 45° with the ground and contain two nonplanar log-periodic antennas (NLPA). One NLPA is contained in the north and south faces, and the other is contained in the east and west faces. The apex of the pyramid is at the ground. Figure 4.2 is sketch of an array element.

This facility consists of two separate halves, each half comprised of 4 by 4 pyramids. The individual pyramids are separated by 65 m. Power is supplied by four 200KW CW transmitters. Each transmitter feeds one linear polarization to one half of the array.

In the sideband experiments the transmitter \( T_1 \) with HF frequency \( \omega_1 \) feeds E-W dipoles in the array 1 of 4 by 4 pyramids; the transmitter \( T_2 \) with HF frequency \( \omega_1 \) feeds N-S dipoles in the array 1 of 4 by 4 pyramids; \( T_3 \) with HF frequency \( \omega_2 \) feeds E-W dipoles in the array 2 of 4 by 4 pyramids;
pyramids; \( T_4 \) with HF frequency \( \omega_2 \) feeds N-S dipoles in the array 2 of 4 by 4 pyramids.

In order to produce circular polarization in each array the electric field of the HF wave with transmitter \( T_1 \) has 90\(^\circ\) phase delay to the electric field of the HF wave with transmitter \( T_2 \) and the electric field of the HF wave with the transmitter \( T_3 \) has 90\(^\circ\) phase delay to the electric field of the HF wave with the transmitter \( T_4 \). It is shown in the Fig 4.1. In the sideband experiments we just consider the far zone in which the fields are transverse to the radius vector \( R \) and fall as \( R \), typical of radiation fields.

Wave Polarization: The phases of electric field of each of the four transmitters is switchable to produce various polarizations in following cases:

(A) Four Transmitters: The electric field of the HF wave from transmitter \( T_1; T_2; T_3 \); and \( T_4 \) can be expressed by \( E_x; E_y; E_x3; \) and \( E_y4 \) respectively. The total electric field can be expressed by:

\[
E(z,t) = i E_x(z,t) + j E_y(z,t)
\]  \hspace{1cm} [4-1]

\[
E_x(z,t) = E_x1 + E_x3
\]

\[
= E_0 \sin(K_1 z - \omega_1 t) + E_0 \sin(K_2 z - \omega_2 t)
\]

\[
= 2E_0 \sin(K_0 z - \omega_0 t) \cos((\Delta K z - \Delta \omega t)/2)
\]  \hspace{1cm} [4-2]
\[ E_y(z,t) = E_{y2} + E_{y4} \]
\[ = E_0 \sin[K_1 z - \omega_1 t + 90^0] + E_0 \sin[K_2 z - \omega_2 t + 90^0] \]
\[ = 2E_0 \sin[K_0 z - \omega_0 t + 90^0] \cos[(\Delta K z - \Delta \omega t)/2] \]

\[ \omega_0 = (\omega_1 + \omega_2)/2 \quad \Delta \omega = \omega_1 - \omega_2 \quad K_0 = (K_1 + K_2)/2 \quad \Delta K = K_1 - K_2 \]  \[ [4-3] \]

When \( z = 0 \):

\[ E_x(t) = -2E_0 \sin[\omega_0 t] \cos[\Delta \omega t/2] \]

\[ E_y(t) = -2E_0 \sin[\omega_0 t - 90^0] \cos[\Delta \omega t/2] \]

The equations [4-4] and [4-5] show the HF wave has circular polarization, but the amplitude of the electric field is changing by the factor of \( \cos[\Delta \omega t/2] \). When the HF wave enters the ionosphere the wave with right circular polarization is called the \( O \)-wave in which the wave vector changes monotonically to \( \epsilon_p = 90^0 \) and its electric field is parallel to the earth's magnetic field as the wave approaches the reflection height where \( X = 1 \); the wave with left circular polarization is called the \( X \)-wave and is reflected in the region \( X = 1 - Y \). (Here \( \epsilon_p \) is the angle between the magnetic field and wave vector; and \( X = \omega_p^2 / \omega^2 \), \( Y = \omega_H / \omega \). \( \omega_p \) is the angular plasma frequency, \( \omega_H \) is the angular gyro-frequency.

(B) Three Transmitters: If only three transmitters are operating, (for
example the transmitter $T_4$ is not operating), the polarization of HF wave changes with time:

When $z=0$:

$$E_x = -2E_0 \sin(\omega_1 t - \Delta \omega t/2) \cos(\Delta \omega t/2) \quad [4-6]$$

$$E_y = -E_0 \sin(\omega_1 t - 90^0) \quad [4-7]$$

The equations [4-6] and [4-7] show that the polarization of the HF wave changes with time, the amplitude of $E_x$ is changing by the factor of $\cos(\Delta \omega t/2)$.

(C) Two Transmitters: If only two transmitters are operating there are different cases: (1) The transmitters $T_1$ and $T_3$ or $T_2$ and $T_4$ are operating, the HF wave has linear polarization but the amplitude of the electric field varies with time at the frequency $\Delta \omega$; (2) The transmitters $T_1$ and $T_2$ or $T_3$ and $T_4$ are operating, the HF wave is the circular polarization with constant amplitude. (3) The transmitters $T_1$ and $T_4$ or $T_2$ and $T_3$ are operating, the polarization of the HF wave is changing with time: from circular polarization to elliptical polarization; to linear polarization; to elliptical polarization; to circular polarization and so on.
if $z=0$: $[T_1$ and $T_4$]

$$E_x = -E_0 \sin \omega_1 t$$ \hspace{1cm} [4-8]

$$E_y = -E_0 \sin[\omega_1 t - \Delta \omega t - 90^\circ]$$ \hspace{1cm} [4-9]

When the HF wave enters the ionosphere the HF wave splits into two waves, one wave is called the O-wave with the right circular polarization the wave vector $K$ is changing monotonically to $\theta_p = 90^\circ$ as the wave approaches the reflection height where $X=1$, and the electric field is parallel to the magnetic field in the reflection region; the other wave is called the X-wave with the left circular polarization and is reflected in the region $X=1-Y$. Fig 4.3 shows the path of vertically incident wave packets.

Pattern Multiplication: The pattern multiplication technique is based upon the calculation of the total pattern of an array by taking the product of an array factor $[AF]$ with the element pattern.

Suppose there are "N" of colinear isotropic radiators separated by a distance $d$. The far field, at a point "P", due to the $N^{th}$ radiator has a phase factor $\exp[\text{imKdcos} \gamma]$:

$$E \propto \exp[\text{imKdcos} \gamma]$$ \hspace{1cm} [4-10]

The total field at "P" can be expressed:
\[ E \propto \Sigma \exp[imK \cos \phi] = \left\{ \frac{1 - \exp[iNK \cos \phi]}{1 - \exp[iK \cos \phi]} \right\} \]

\[ |E|^2 \propto AF = \sin^2 \left[ NK \cos \frac{\phi}{2} \right] / \sin^2 \left[ K \cos \frac{\phi}{2} \right] \quad [4-11] \]

The expression of equation [4-11] for the 4- element and 8- element arrays can be given by equation [4-12] and [4-13]:

\[ AF_1 = \sin^2 \left[ 2K \cos \frac{\phi_1}{2} \right] / \sin^2 \left[ K \cos \frac{\phi_1}{2} \right] \quad [4-12] \]

\[ AF_2 = \sin^2 \left[ 4K \cos \frac{\phi_2}{2} \right] / \sin^2 \left[ K \cos \frac{\phi_2}{2} \right] \quad [4-13] \]

The Figure 4.1 shows the orientation of the 4- and 8- element array on X,Y coordinate system. The Y - axis is taken to be north and the X- axis as east.

In the sideband experiments if only transmitters \( T_1 \) and \( T_3 \) are operating the HF wave has a linear polarization. In the X-Z plane the electric field can be expressed by:

\[ E \propto \sin[2K_1 \cos \phi] \sin[K_1 R - \omega_1 t] / R \sin[K_1 \cos \phi / 2] \]

\[ + \sin[2K_2 \cos \phi] \sin[K_2 R - \omega_2 t] / R \sin[K_2 \cos \phi / 2] \quad [4-14] \]
In these sideband experiments the factor of \( \frac{\sin[2K_1\cos\phi]}{\sin[K_1\cos\phi/2]} \) is very close to the factor of \( \frac{\sin[2K_2\cos\phi]}{\sin[K_2\cos\phi/2]} \) and can be replaced by the factor of \( \frac{\sin[2K_0\cos\phi]}{\sin[K_0\cos\phi/2]} \). The electric field can be given by:

\[
E \propto 2\sin[2K_0\cos\phi] \cos[(\Delta K - \Delta \omega t)/2]/R\sin[K_0\cos\phi/2]*\sin[K_0(R - \omega_0 t)]
\]

[4-15]

The last factor on the right hand side of equation [4-15] represents the progressive HF plane wave. We now just consider the distribution of the electric field influenced by the difference frequency of the two HF pump waves. The electric field can be expressed by:

\[
E \propto 2\sin[2K_0\cos\phi]/R\sin[K_0\cos\phi]*\cos[(\Delta K - \Delta \omega t)/2]
\]

[4-16]

The last factor on the right hand side of equation [4-16] also represents a progressive low frequency plane wave. Fig 4.4 shows the distribution of the electric field and the variation with time due to the difference frequency of the two HF pump waves. The amplitude of the electric field of the HF waves is dominated by the factor of the total pattern of the antennas and is sliding in X-Z plane gently toward X direction with the low frequency \( \Delta \omega \).
Strength of the Electric Field: An estimate of the electric field at an altitude, (neglecting absorption and swelling factor) can be given as follows:

The effective radiated power (ERP) is given by the product of the antenna gain, G, and transmitter power P:

\[ ERP = G \cdot P \]  \hspace{1cm} [4-17]

The power density, \( P(h) \), at an altitude \( h \) of an HF wave is given by:

\[ P(h) = \frac{ERP}{4\pi h^2} \]  \hspace{1cm} [4-18]

The power density in term of the electric field is:

\[ P(h) = \frac{E^2}{377} \]  \hspace{1cm} [4-19]

From equations [4-17] to [4-19] the electric field is:

\[ E^2 = 377 \frac{GP}{4\pi h^2} \]  \hspace{1cm} [4-20]
4.2 ARECIBO OBSERVATORY

The Arecibo Observatory, the world’s largest radio-radar telescope, is located in the mountains of northern Puerto Rico about 15 km south of the city of Arecibo and at \( 18^\circ \, 20' \, 46.2'' \) N and \( 66^\circ \, 45' \, 10.5'' \) W geodetic latitude and longitude respectively (\( 18^\circ \, 21' \, 13.7'' \) N and \( 66^\circ \, 45' \, 18.6'' \) W astronomic latitude and longitude respectively). Magnetic dip angle at Arecibo is \( 50^\circ \) and the magnetic declination is \( 6^\circ \, 30' \) west.

The antenna reflecting surface is a huge \( 70^\circ \) spherical dish with an opening diameter of \( 304.8 \) m, \( 50.9 \) m deep, \( 265 \) m radius of curvature and covers an area of \( 73,000 \) \( m^2 \). The surface is made up of almost \( 38,778 \) perforated aluminium panels. The 600 ton platform is suspended \( 130 \) m above the reflector. The platform hangs on twelve cables, which are strung four each from three reinforced concrete towers with appropriate backstays. Below the triangular frame of the platform is a circular track, on which the azimuth arm turns. The azimuth arm is a bowshaped structure of \( 100 \) m long. The curved part of the arm is another track, on which two carriage houses can be positioned anywhere up to \( 20^\circ \) from the vertical. The various feed antennas are hanging below the carriage houses, each feed tuned to a narrow band of frequencies. The feed is aimed at a certain point on the reflector. The radio waves originating from a very
small area of the sky will be focused on the feed by the reflector. Arecibo's main antenna is the world's most sensitive radio telescope just because of the giant size of the reflector.

A 430 MHz line-feed system in carriage house 1 utilizes a 2.5 MW pulsed peak power output, 150 kW pulsed mode average power with normally right circular polarization. When the zenith angle is zero this feed illuminates the entire 304.8 m reflector aperture, but when the zenith angle is increased toward 20° some of the illumination spills off the reflector. The receiver, also located in carriage house 1, normally translates the echo to a center frequency of 30 MHz before sending it by cable to the control building.

The Arecibo Observatory is part of the National Astronomy and Ionosphere Center, a national research center operated by Cornell University with the support of the National Science Foundation. Use of the telescope and related facilities is available to all scientists from all over the world for observational research in Radio Astronomy, Radar Astronomy and Atmospheric Physics.

The 430 MHz incoherent backscatter radar was used during the second two HF sideband experiments to make background measurements of the ionosphere, such as the electron density and the electron and ion temperature.
4.3 Two Frequency Experiment

The two frequency experiments were made during two different ionospheric modification experiments, one in January 20 - February 3, 1984, the other in February 3-8, 1986. Two HF waves were transmitted near vertically, mixed and reflected from the ionosphere, and received at the Arecibo Observatory. The experimental set-up is illustrated in Figure 4.5. The received sideband spectra were obtained using a spectrum analyzer. Throughout the observation an ionosonde was used to monitor the ionosphere.
Fig. 4.3 The paths of vertically incident wave-packets in the northern hemisphere for a frequency greater than the gyro-frequency. The observer is looking towards the (magnetic) west. (After Budden, 1961)
Figure 4.4 Electric Field Distribution
CHAPTER 5
EXPERIMENTAL RESULTS

In this chapter the salient features of sideband observations made during the 1986 ionospheric modulation experiments are presented. Particular attention is focused upon the properties of the nonlinear modulation process which are basic to the wave-plasma nonlinear interaction caused by the ponderomotive force. The observations show that the amplitude of the sidebands is related to the HF power and the difference frequency. Phase modulation theory and modulation index theory, show agreement with the observations and comparisons of the theories and the observations are made in chapter 6.

The terms sideband SB(+) and sideband SB(-) denote the upper sideband and lower sideband respectively in the received sideband spectrum. The points + and the points - indicate the upper sideband and lower sideband respectively in the Figures. Finally, when times are specified, they refer to the local time at the Arecibo Observatory, that is, Atlantic Standard Time (AST).
5.1 Brief Summaries of the Ionospheric Background Measurements

As discussed in the section 4.3 the 1986 observations are treated in the present study. Electron density profile information is available.

Observations were made on February 3, 4, 5, 7, and 8, 1986. The observations of the $f_0F_2$ values are indicated in Figure 5.1 and Figure 5.2.

5.2 Extended Observations

5.2 (A) FEBRUARY 3, 1986

During an evening observation period extending from 1525 to 2224 AST on February 3, 1986, the HF transmitters were tuned to frequency of 5.1 MHz from 1620 AST to 1812 AST and 4.45 MHz from 1829 AST to 2230 AST. The transmitter spectrum is shown in Figure 5.3, the received sideband spectrum is illustrated in Figure 5.4 and Figure 5.5, the amplitudes of the sidebands are listed in the table 5-1.
TABLE 5-1 THE SIDEBAND AMPLITUDE
(in decibels below the carrier)
February 3, 1986. HF frequency 5.1 MHz, Δf=5Hz

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>the first</th>
<th>the second</th>
<th>the third</th>
<th>total</th>
<th>number of sidebands</th>
</tr>
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<td>AST</td>
<td>HF</td>
<td>sideband</td>
<td>sideband</td>
<td>sideband</td>
<td></td>
<td></td>
</tr>
<tr>
<td>power</td>
<td></td>
<td>B(+)  B(-)</td>
<td>B(+)  B(-)</td>
<td>B(+)  B(-)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1750</td>
<td>200</td>
<td>33db 30db</td>
<td>44db 45db</td>
<td>56db 58db</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>1757</td>
<td>400</td>
<td>29db 29db</td>
<td>45db 51db</td>
<td>57db 56db</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>1805</td>
<td>100</td>
<td>42db 32db</td>
<td>45db 47db</td>
<td>57db 58db</td>
<td>16</td>
<td></td>
</tr>
</tbody>
</table>

In Fig 5.6 the electron density profile and an ion line enhancement is displayed at 1750 AST on February 3, 1986. Fig 5.6 indicates that the reflection height of the 5.1 MHz wave is 219 Km and the scale height is 72 km. Assuming that the ionosphere is not changing too rapidly Fig 5.6 indicates approximately the electron density profile and the HF reflection height from 1740 AST to 1812 AST. From equation [4-20] the amplitude of electric field E₀ can be calculated to be 0.10 v/m when the HF power is 4×25 kw; 0.14 v/m when the HF power is 4×50 kw; 0.20 v/m when the HF power is 4×100 kw. According to a ionogram assuming β=0.8 from equation
[3–34] the modulation index can be calculated to be $m_p = 0.025 \ (p=4\times25$ kw); $m_p = 0.05 \ (p=4\times50$ kw); $m_p = 0.10 \ (p=4\times100$ kw).

Fig 5.7 shows the observations which indicate the relationship between the HF power and modulation index based on the data of the first sidebands, data recorded between 1740 AST and 1812 AST on February 3, 1986. The HF transmitters were operated at the power of $4\times25$ kW; $4\times50$ kW and $4\times100$ kW and tuned to the HF frequency of 5.1 MHz. The difference frequency of two pump HF waves was chosen to be 5 Hz. During this evening observations Arecibo ionograms and electron density profiles given by the 430 MHz radar show $f_{0}F_2$ maintained a value of 6.3 MHz. The curve of Fig 5.7 is calculated according to equation [3–34] and shows the comparison of the data with the theory.
FIGURE 5.1  $f_o F_2$ VALUE  February 3, 1986

TIME (AST)

$f_o F_2$ (MHz)
TRANSMITTER SPECTRUM

FIGURE 5.3a  1758 AST February 3, 1986

f = 5.1 MHz  Δf = 5 Hz  W = 400 KW

FIGURE 5.3b  1804 AST February 3, 1986

f = 5.1 MHz  Δf = 5 Hz  W = 100 KW
FIGURE 5.6 The electron density profile

1750 AST
\( f = 5.1 \text{ MHz} \)
\( L = 0.219 \text{ km} \)
\( H = 72 \text{ km} \)
February 3, 1986

ELECTRON DENSITY (cm\(^{-3}\))

ALTITUDE (km)
Figure 5.7: Modulation Index versus Total HF Power

1740 AST - 1812 AST
f = 5.1 MHz, Δf = 5 Hz
February 1986
5.2 (B) FEBRUARY 4, 1986

In an evening observation period extending from 1530 AST to 2159 AST on February 4, 1986, the HF transmitters were tuned to the frequency of 5.1 MHz from 1633 AST to 2029 AST; and 3.175 MHz from 2039 AST to 2101 AST. The received sideband spectrum is illustrated in Figure 5.8, 5.11, 5.12. The amplitudes of the sidebands are listed in the table 5-2 and 5-3.


<table>
<thead>
<tr>
<th>Total</th>
<th>the first</th>
<th>the second</th>
<th>the third</th>
<th>total number of sidebands</th>
</tr>
</thead>
<tbody>
<tr>
<td>AST</td>
<td>HF</td>
<td>sideband</td>
<td>sideband</td>
<td>sideband</td>
</tr>
<tr>
<td>power</td>
<td>B(+)</td>
<td>B(-)</td>
<td>B(+)</td>
<td>B(-)</td>
</tr>
<tr>
<td>1629</td>
<td>200</td>
<td>30db</td>
<td>37db</td>
<td>50db</td>
</tr>
<tr>
<td>1643</td>
<td>400</td>
<td>32db</td>
<td>35db</td>
<td>55db</td>
</tr>
<tr>
<td>1648</td>
<td>100</td>
<td>41db</td>
<td>40db</td>
<td>52db</td>
</tr>
<tr>
<td>1655</td>
<td>40</td>
<td>54db</td>
<td>38db</td>
<td>76db</td>
</tr>
</tbody>
</table>

TABLE 5-2 THE SIDEBAND AMPLITUDE
(in decibels below the carrier)
February 4, 1986. HF frequency 5.1 MHz, Δf=5Hz
In Fig 5.9a the electron density profile and ion line enhancement is displayed at 1648 AST on February 4, 1986. In Fig 5.9b the plasma line enhancement is displayed at 1650 AST on February 4, 1986. Fig 5.9a and Fig 5.9b show that the reflection height of the 5.1 MHz wave is 208 km and the scale height is 100 km. Assuming that the ionosphere is not changing too rapidly Fig 5.9a shows approximately the electron density profile and reflection height of the HF waves from 1629 AST to 1703 AST. From equation (4-20) the amplitude of the electric field $E_0$ can be calculated to be 0.026 v/m when the HF power is $4*1.5$ kw; 0.066 v/m when the HF power is $4*10$ kw; 0.10 v/m when the HF power is $4*25$ kw; 0.15 v/m when the HF power is $4*50$ kw; 0.21 v/m when the HF power is $4*100$ kw. According to an ionogram assuming $\beta=0.5$ from the equation (3-34) the modulation index can be calculated to be $m_p=0.01$ (p= $4*10$ kw); $m_p=0.025$ (p=$4*25$ kw); $m_p=0.05$ (p=$4*50$ kw); $m_p=0.10$ (p=$4*100$ kw).

Fig 5.10 shows the observations which relate the HF power to the modulation index based on the data of the first sideband observations. Data are recorded between 1629 AST and 1703 AST on February 4, 1986. The HF transmitters were operated at the power of $4*10$ kW; $4*25$ kW; $4*50$ kW.
and 4*100 kW and tuned to the frequency of 5.1 MHz. The difference frequency of the two pumps is chosen to be 5 Hz. In this period the electron density profiles given by the 430 MHz radar and Arecibo ionograms show the $f_0F_2$ in the range from 5.2 MHz to 5.5 MHz. The curve of Fig 5.10 is calculated from equation (3-34) and shows the comparison of the data with the experiments.
TABLE 5-3 THE SIDEBAND AMPLITUDE

(in decibels below the carrier)

February 4, 1986. HF frequency 5.1 MHz, total HF power 200 kw

<table>
<thead>
<tr>
<th>AST</th>
<th>Δf</th>
<th>the first sideband</th>
<th>the second sideband</th>
<th>the third sideband</th>
<th>total number of sidebands</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B(+)  B(-)</td>
<td>B(+)    B(-)</td>
<td>B(+)   B(-)</td>
<td></td>
</tr>
<tr>
<td>1711</td>
<td>5</td>
<td>38db   37db</td>
<td>61db     52db</td>
<td>66db   62db</td>
<td>9</td>
</tr>
<tr>
<td>1718</td>
<td>23</td>
<td>41db   39db</td>
<td>57db     54db</td>
<td>59db   57db</td>
<td>15</td>
</tr>
<tr>
<td>1723</td>
<td>53</td>
<td>46db   40db</td>
<td>56db     54db</td>
<td>62db   7</td>
<td></td>
</tr>
<tr>
<td>1726</td>
<td>97</td>
<td>45db   46db</td>
<td>60db     60db</td>
<td>60db   58db</td>
<td>11</td>
</tr>
<tr>
<td>1730</td>
<td>37</td>
<td>44db   38db</td>
<td>59db     55db</td>
<td>60db   58db</td>
<td>11</td>
</tr>
<tr>
<td>1735</td>
<td>11</td>
<td>41db   41db</td>
<td>61db     59db</td>
<td>68db   7</td>
<td></td>
</tr>
<tr>
<td>1739</td>
<td>5</td>
<td>40db   34db</td>
<td>56db     49db</td>
<td>65db   59db</td>
<td>16</td>
</tr>
<tr>
<td>1743</td>
<td>73</td>
<td>47db   44db</td>
<td>56db     72db</td>
<td>72db   5</td>
<td></td>
</tr>
</tbody>
</table>

In Fig 5.13 the electron density profile is displayed at 1737 AST on February 4, 1986. Fig 5.13 indicates that reflection height of the 5.1 MHz wave is 236 km and the scale height is 70 km. Assuming that the ionosphere is not changing too rapidly the Fig 5.13 displays approximately
the electron density profile; the scale height and HF reflection height from 1711 AST to 1751 AST. According to an ionogram assuming that $\beta=0.6$ and $j=1.4$ from the equations [3-34], [3-40] and [3-41] the modulation index can be calculated to be $m_{pf}=0.028 \ (\Delta f=5\text{Hz})$; $m_{pf}=0.025 \ (\Delta f=11\text{Hz})$; $m_{pf}=0.020 \ (\Delta f=23\text{Hz})$; $m_{pf}=0.015 \ (\Delta f=37\text{Hz})$; $m_{pf}=0.011 \ (\Delta f=53\text{Hz})$; $m_{pf}=0.006 \ (\Delta f=73\text{Hz})$.

Figure 5.14 summarizes the first sideband observations, the data are recorded between 1711 AST and 1751 AST on February 4, 1986. The HF transmitters were operated at 4x50 Kw power and were tuned to the frequency of 5.1 MHz. The difference frequencies of the two pump waves were chosen as prime numbers of 5 Hz; 11 Hz; 23 Hz; 37 Hz; 53 Hz; 73 Hz and 97 Hz. There are not Schumann resonance frequencies. During this period Arecibo ionograms and electron density profiles show $f_{o}F_2$ maintained a value of 5.4 MHz. Fig 5.15 gives the relation between the difference frequency and the modulation index based on the data of the first sideband observations. The curve of the Fig 5.15 is calculated from the equation [3-40]. The curve shows the agreement with the data of the experiments. This Fig 5.16 shows the modulation index is decreasing with 0.7 power of the difference frequency $\Delta m_{o} \propto (\Delta f)^{0.7}$. 
FIGURE 5.8 RECEIVED SIDEBAND SPECTRUM

(a) 1629 AST  
Feb 4  
1986  
f = 5.1 MHz  
$\Delta f = 5$ Hz  
W = 4*50 KW

(b) 1643 AST  
Feb 4  
1986  
f = 5.1 MHz  
$\Delta f = 5$ Hz  
W = 4*100 KW

(c) 1648 AST  
Feb 4  
1986  
f = 5.1 MHz  
$\Delta f = 5$ Hz  
W = 4*25 KW

(d) 1655 AST  
Feb 4  
1986  
f = 5.1 MHz  
$\Delta f = 5$ Hz  
W = 4*10 KW
FIGURE 5.9a THE ELECTRON DENSITY PROFILE

FIGURE 5.9b ENHANCED PLASMA LINE
Figure 5.11  Received Sideband Spectrum

(a) 1711 AST
Feb 4 1986
\( f = 5.1 \text{ MHz} \)
\( \Delta f = 5 \text{ Hz} \)
\( W = 4 \times 50 \text{ KW} \)

(b) RELATIVE FREQUENCY (Hz)

-40 0 40
-80

1718 AST
Feb 4 1986
\( f = 5.1 \text{ MHz} \)
\( \Delta f = 23 \text{ Hz} \)
\( W = 4 \times 50 \text{ KW} \)

(c) 1723 AST
Feb 4 1986
\( f = 5.1 \text{ MHz} \)
\( \Delta f = 53 \text{ Hz} \)
\( W = 4 \times 50 \text{ KW} \)

(d) RELATIVE FREQUENCY (Hz)

-53 0 53
-80

1726 AST
Feb 4 1986
\( f = 5.1 \text{ MHz} \)
\( \Delta f = 97 \text{ Hz} \)
\( W = 4 \times 50 \text{ KW} \)

RELATIVE AMPLITUDE (dB)
Fig 5.13 The electron density profile
FIGURE 5.14  THE FIRST SIDEBAND AMPLITUDE VERSUS THE DIFFERENCE FREQUENCY

THE FIRST SIDEBAND AMPLITUDE (dB)

THE DIFFERENCE FREQUENCY (Hz)
Figure 5.15 The Modulation Index versus the Difference Frequency

1711 AST - 1751 AST
1 MHz
W=4.50 kW
February 4, 1986

The Modulation Index
5.2 (C) FEBRUARY 5, 1986

During the evening observation period extending from 1609 AST to 2157 AST on February 5, 1986, the HF transmitters were tuned to the frequency of 5.1 MHz from 1609 AST to 2117 AST; 3.175 MHz from 2127 AST to 2157 AST. The received sideband spectrum is illustrated in Figure 5.16, 5.17, 5.18 and 5.19. The amplitudes of the sidebands are listed in the table 5-4.
TABLE 5-4 THE SIDEBAND AMPLITUDE
(in decibels below the carrier)

February 5, 1986. HF frequency 3.175 MHz, total HF power=400kw

<table>
<thead>
<tr>
<th>AST</th>
<th>Δf</th>
<th>the first sideband</th>
<th>the second sideband</th>
<th>the third sideband</th>
<th>total sideband</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>B(+)   B(-)</td>
<td>B(+)   B(-)</td>
<td>B(+)   B(-)</td>
<td></td>
</tr>
<tr>
<td>2120</td>
<td>5</td>
<td>22db 27db</td>
<td>42db 48db</td>
<td>57db 58db</td>
<td>10</td>
</tr>
<tr>
<td>2123</td>
<td>11</td>
<td>21db 20db</td>
<td>45db 44db</td>
<td>60db</td>
<td>7</td>
</tr>
<tr>
<td>2125</td>
<td>37</td>
<td>25db 21db</td>
<td>47db</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>2129</td>
<td>97</td>
<td>24db 23db</td>
<td>52db</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>2132</td>
<td>179</td>
<td>24db 29db</td>
<td>56db 61db</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>2136</td>
<td>271</td>
<td>32db 30db</td>
<td>67db 60db</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>2139</td>
<td>367</td>
<td>31db 36db</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>2141</td>
<td>457</td>
<td>34db 43db</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>2143</td>
<td>761</td>
<td>37db</td>
<td></td>
<td></td>
<td>3</td>
</tr>
<tr>
<td>2145</td>
<td>971</td>
<td>39db 36db</td>
<td>68db</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>2147</td>
<td>853</td>
<td>34db 43db</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>2149</td>
<td>457</td>
<td>36db 39db</td>
<td>69db 55db</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>2151</td>
<td>53</td>
<td>22db 26db</td>
<td>50db 56db</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>2154</td>
<td>5</td>
<td>19db 27db</td>
<td>45db 52db</td>
<td>59db 58db</td>
<td>8</td>
</tr>
<tr>
<td>2156</td>
<td>3</td>
<td>20db 26db</td>
<td>43db 54db</td>
<td>59db 61db</td>
<td>8</td>
</tr>
<tr>
<td>2159</td>
<td>1</td>
<td>22db 19db</td>
<td>35db</td>
<td>50db</td>
<td>6</td>
</tr>
</tbody>
</table>
In Fig 5.20a and Fig 5.20b the electron density profiles are displayed in 212723 AST and in 215729 AST on February 5, 1986 respectively. Fig 5.20a and Fig 5.20b indicate that the ionosphere is in the stable state and the reflection height of 3.175 MHz wave is 265 km and the scale height is 90 km. According to a ionogram assuming that \( \beta=0.5 \) and \( j=1.3 \) from equations [3-34], [3-40] and [3-41] the modulation index can be calculated to be \( m_{pf}=0.18 \) (\( \Delta f=5 \)); \( m_{pf}=0.15 \) (\( \Delta f=37 \)); \( m_{pf}=0.12 \) (\( \Delta f=97 \)); \( m_{pf}=0.09 \) (\( \Delta f=179 \)); \( m_{pf}=0.06 \) (\( \Delta f=271 \)); \( m_{pf}=0.03 \) (\( \Delta f=361 \)).

Fig 5.21 summarizes the first sideband observations. The data are recorded between 2120 AST and 2159 AST on February 5, 1986. The HF transmitters were operated at 4*100 KW power and were tuned to the frequency of 3.175 MHz. The difference frequencies of the two pumps were varied from 1 Hz to 971 Hz, chosen as prime numbers and not Schumann resonance frequencies. During this time the Arecibo ionograms and the electron density profiles indicated \( f_{0}F_2 \) maintained a value of 4.3 MHz. Fig 5.22 shows the relation between the difference frequencies and the modulation index based on the data of the first sideband. The curve of Fig 5.22 is calculated according to the equation [3-40] The calculation shows agreement with the experiments.
FIGURE 5.17  RECEIVED SIDEBAND SPECTRUM

**a**
2132 AST  
Feb 5  
1986  
f = 3.175 MHz  
Δf = 179 Hz  
W = 4 x 100 kW

**b**
2136 AST  
Feb 5  
1986  
f = 3.175 MHz  
Δf = 271 Hz  
W = 4 x 100 kW

**c**
2139 AST  
Feb 5  
1986  
f = 3.175 MHz  
Δf = 367 Hz  
W = 4 x 100 kW

**d**
2141 AST  
Feb 5  
1986  
f = 3.175 MHz  
Δf = 457 Hz  
W = 4 x 100 kW

**RELATIVE FREQUENCY (Hz)**
FIGURE 5.18

RECEIVED SIDEBAND SPECTRUM

(a)

P = 3.175 MHz
ΔF = 761 Hz
F = 4,100 kW

RELATIVE AMPLITUDE (dB)

0 2143 AST
Feb 5
1986

-40
-80

0 2145 AST
Feb 5
1986

-40
-80

(b)

P = 3.175 MHz
ΔF = 761 Hz
F = 4,100 kW

RELATIVE AMPLITUDE (dB)

0 2143 AST
Feb 5
1986

-40
-80

0 2145 AST
Feb 5
1986

-40
-80

(c)

P = 3.175 MHz
ΔF = 853 Hz
F = 4,100 kW

RELATIVE AMPLITUDE (dB)

0 2147 AST
Feb 5
1986

-40
-80

0 2149 AST
Feb 5
1986

-40
-80

(d)

P = 3.175 MHz
ΔF = 457 Hz
F = 4,100 kW

RELATIVE AMPLITUDE (dB)

0 2147 AST
Feb 5
1986

-40
-80

0 2149 AST
Feb 5
1986

-40
-80

(e)

P = 3.175 MHz
ΔF = 457 Hz
F = 4,100 kW

RELATIVE AMPLITUDE (dB)

0 2147 AST
Feb 5
1986

-40
-80

0 2149 AST
Feb 5
1986

-40
-80
FIGURE 5.20a THE ELECTRON DENSITY PROFILE

FIGURE 5.20b THE ELECTRON DENSITY PROFILE
5.2 (d) FEBRUARY 7, 1986

In a morning observation period extending from 0546 AST to 1010 AST on February 7, 1986, the transmitters were tuned to the frequency of 4.45 MHz from 0546 AST to 0611 AST and 5.1 MHz from 0644 AST to 1010 AST. The received sideband spectrum is illustrated in Figure 5.23, 5.24 and 5.25. The amplitudes of the sidebands are listed in the table 5-5.
TABLE 5-5  THE SIDEBAND AMPLITUDE

(in decibels below the carrier)

February 7, 1986. HF frequency 5.1 MHz, total HF power=300 kw

<table>
<thead>
<tr>
<th>AST</th>
<th>Δf</th>
<th>the first sideband</th>
<th>the second sideband</th>
<th>the third sideband</th>
<th>total number of sidebands</th>
</tr>
</thead>
<tbody>
<tr>
<td>0902</td>
<td>37</td>
<td>27db 36db</td>
<td>42db 49db</td>
<td>49db 56db</td>
<td>11</td>
</tr>
<tr>
<td>0904</td>
<td>179</td>
<td>43db 37db</td>
<td>54db 52db</td>
<td>63db</td>
<td>7</td>
</tr>
<tr>
<td>0906</td>
<td>367</td>
<td>42db 35db</td>
<td>60db 61db</td>
<td></td>
<td>7</td>
</tr>
<tr>
<td>0908</td>
<td>761</td>
<td>46db 36db</td>
<td>60db 61db</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>0910</td>
<td>971</td>
<td>53db 41db</td>
<td>51db</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>0913</td>
<td>457</td>
<td>42db 37db</td>
<td>63db 58db</td>
<td>69db</td>
<td>7</td>
</tr>
<tr>
<td>0914</td>
<td>271</td>
<td>44db 37db</td>
<td>56db 57db</td>
<td>65db 67db</td>
<td>12</td>
</tr>
<tr>
<td>0917</td>
<td>97</td>
<td>41db 37db</td>
<td>47db 51db</td>
<td>52db 53db</td>
<td>9</td>
</tr>
<tr>
<td>0919</td>
<td>11</td>
<td>29db 30db</td>
<td>43db 39db</td>
<td>52db</td>
<td>7</td>
</tr>
<tr>
<td>0922</td>
<td>5</td>
<td>26db 32db</td>
<td>49db 43db</td>
<td>53db 53db</td>
<td>16</td>
</tr>
<tr>
<td>0925</td>
<td>3</td>
<td>22db 34db</td>
<td>42db 46db</td>
<td>53db 54db</td>
<td>13</td>
</tr>
<tr>
<td>0929</td>
<td>2</td>
<td>23db 30db</td>
<td>41db 47db</td>
<td>53db 57db</td>
<td>17</td>
</tr>
</tbody>
</table>
In Fig 5.26a the electron density profile is displayed at 0926 AST on February 7, 1986. In Fig 5.26b the electron density profile and ion line enhancement is displayed at 0951 AST on February 7, 1986. Both Figures indicate that the ionosphere is in stable state, the reflection height of the 5.1 MHz wave is approximately 200 km and the scale height is 90 km. According to a ionogram assuming that $\beta=0.7$ and $j=1.2$ from equation [3-34], [3-40] and [3-41] the modulation index is calculated to be $m_{pf} = 0.1$ ($\Delta f = 11$ Hz); $m_{pf} = 0.09$ ($\Delta f = 37$ Hz); $m_{pf} = 0.08$ ($\Delta f = 97$ Hz); $m_{pf} = 0.06$ ($\Delta f = 179$ Hz); $m_{pf} = 0.04$ ($\Delta f = 271$ Hz); $m_{pf} = 0.03$ ($\Delta f = 367$ Hz).

Fig 5.27 summarizes the first sideband observations. The data are recorded in the morning between 0902 AST and 0956 AST on February 7, 1986. The HF transmitters were operated at 4875 kW power and were tuned to the frequency of 5.1 MHz. The difference frequencies of the two HF pump waves were chosen as prime numbers between 5 Hz and 971 Hz and not Schumann resonances. In this morning period the electron density profiles given by the 430 MHz radar and Arecibo ionograms show that $f_0F_2$ had a value of 7 MHz. Fig 5.28 shows the relationship of the difference frequencies with the modulation index based on the data of the first sidebands. The curve of the Fig 5.28 is calculated according to equation [3-40]. The curve shows the agreement with the data.
FIGURE 5.23

(a) 0906 AST, Feb 7, 1986
$f_0 = 5.1$ MHz
$\Delta f = 367$ Hz
$W = 300$ kW

(b) 0908 AST, Feb 7, 1986
$f_0 = 5.1$ MHz
$\Delta f = 761$ Hz
$W = 300$ kW

(c) RECEIVED SIDE-BAND SPECTRUM

(d) RELATIVE AMPLITUDE
FIGURE 5.26a THE ELECTRON DENSITY PROFILE

150   300   450

ELECTRON DENSITY (cm\(^{-3}\))

0.14E+07

0.79E+06

0.12E+06

ALTITUDE (km)

FIGURE 5.26b THE ELECTRON DENSITY PROFILE
THE FIRST SIDEBAND AMPLITUDE VERSUS THE DIFFERENCE FREQUENCY

THE FIRST SIDEBAND AMPLITUDE (dB)
Figure 5.28 The Modulation Index Versus the Difference Frequency

0902 AST - 0956 AST

f = 5.1 MHz  W = 4*75 kW

February 7, 1986
5.2 (E) FEBRUARY 8, 1986

During another morning observation period extending from 0535 AST to 1010 AST on February 8, 1986, the transmitters were tuned to the frequency of 3.175 MHz from 0535 AST to 0643 AST; 5.1 MHz from 0700 AST to 0809 AST and 3.175 MHz from 0930 AST to 1010 AST. The received sideband spectrum is illustrated in Figure 5.29, 5.30 and 5.31. The amplitudes of the sidebands are listed in Table 5-6.
TABLE 5-6  THE SIDEBAND AMPLITUDE
(in decibels below the carrier)

February 8, 1986.  HF frequency 5.1 MHz, total HF power=250kw

<table>
<thead>
<tr>
<th>AST</th>
<th>Δf</th>
<th>the first sideband B(+)  B(-)</th>
<th>the second sideband B(+)  B(-)</th>
<th>the third sideband B(+)  B(-)</th>
<th>total number of sidebands</th>
</tr>
</thead>
<tbody>
<tr>
<td>0804</td>
<td>5</td>
<td>34db 35db</td>
<td>57db 53db</td>
<td>63db 63db</td>
<td>9</td>
</tr>
<tr>
<td>0808</td>
<td>179</td>
<td>48db 44db</td>
<td>67db 60db</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>0809</td>
<td>369</td>
<td>47db 47db</td>
<td>65db 62db</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>0811</td>
<td>761</td>
<td>50db 53db</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>0812</td>
<td>971</td>
<td>55db 53db</td>
<td></td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>0813</td>
<td>457</td>
<td>51db 49db</td>
<td>63db 60db</td>
<td>65db</td>
<td>5</td>
</tr>
<tr>
<td>0814</td>
<td>271</td>
<td>46db 42db</td>
<td>58db 55db</td>
<td>60db</td>
<td>7</td>
</tr>
<tr>
<td>0815</td>
<td>97</td>
<td>43db 45db</td>
<td>55db 55db</td>
<td>60db 59db</td>
<td>8</td>
</tr>
<tr>
<td>0816</td>
<td>37</td>
<td>41db 43db</td>
<td>57db 56db</td>
<td>63db 60db</td>
<td>9</td>
</tr>
<tr>
<td>0818</td>
<td>11</td>
<td>36db 40db</td>
<td>55db 56db</td>
<td>61db 64db</td>
<td>9</td>
</tr>
<tr>
<td>0823</td>
<td>3</td>
<td>33db 28db</td>
<td>54db 53db</td>
<td>60db</td>
<td>8</td>
</tr>
<tr>
<td>0829</td>
<td>1</td>
<td>29db 29db</td>
<td>50db 49db</td>
<td></td>
<td>8</td>
</tr>
</tbody>
</table>
In Fig 5.32a the electron density profile is displayed for 0809 AST on February 8, 1986. Fig 5.32b shows the plasma line for 0812 AST on February 8, 1986. Fig 5.32a indicates that the reflection height is 200 km and the scale height is 80 km. Assuming that the ionosphere is not changing too rapidly Fig 5.32a displays approximately the electron density profile from 0804 AST to 0841 AST. According to a ionogram assuming that $\beta = 0.6$ and $j = 1.1$ from equations [3-34], [3-40] and [3-41] the modulation index is calculated to be $m_{pf}=0.03$ ($\Delta f=5$ Hz); $m_{pf}=0.029$ ($\Delta f=11$ Hz); $m_{pf}=0.028$ ($\Delta f=37$ Hz); $m_{pf}=0.025$ ($\Delta f=97$ Hz); $m_{pf}=0.022$ ($\Delta f=179$ Hz); $m_{pf}=0.018$ ($\Delta f=271$ Hz); $m_{pf}=0.014$ ($\Delta f=369$ Hz); $m_{pf}=0.011$ ($\Delta f=457$ Hz).

Fig 5.33 summarizes the first sideband observations. The data are recorded in the morning between 0804 AST and 0841 AST on February 8, 1986. The HF transmitters were operated at 3075 KW and 1425 KW powers and tuned to the frequency of 5.1 MHz. The difference frequencies of two pump waves were chosen as prime numbers between 5 Hz and 971 Hz not in the Schumann resonance frequency. During the morning observations the Arecibo ionograms and the electron density profiles indicated $f_{0}F_{2}$ had the value of 5.5 MHz. Fig 5.34 gives the relation between the difference frequencies and the modulation index based on the data of the first sidebands. The curve of the Fig 5.34 is calculated from equation [3-40]. The curve indicates the agreement with the data.
TABLE 5-7 THE SIDEBAND AMPLITUDE

(in decibels below the carrier)

February 8, 1986. HF frequency 3.175 MHz. Δf=5Hz.

<table>
<thead>
<tr>
<th>AST</th>
<th>HF power</th>
<th>Sideband</th>
<th>Sideband</th>
<th>Sideband</th>
<th>Sidebands</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B(+)</td>
<td>B(-)</td>
<td>B(+)</td>
<td>B(-)</td>
<td>number</td>
</tr>
<tr>
<td>0952</td>
<td>200kw</td>
<td>20db</td>
<td>32db</td>
<td>39db</td>
<td>36db 53db</td>
</tr>
<tr>
<td>0953</td>
<td>100kw</td>
<td>30db</td>
<td>44db</td>
<td>46db</td>
<td>7</td>
</tr>
<tr>
<td>0957</td>
<td>40kw</td>
<td>41db</td>
<td>40db</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>1001</td>
<td>20kw</td>
<td>50db</td>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

In Fig 5.36 the electron density profile and ion line enhancement is displayed for 1010 AST on February 8, 1986. Fig 5.36 indicates the reflection height of HF waves is 248 km and the scale height is 45 km. Assuming that the ionosphere is not changing too rapidly the Fig 5.36 represents approximately the electron profile from 0951 AST to 1011 AST. From equation [3-20] the amplitude of the electric field $E_0$ can be calculated to be 0.056 v/m when power is 4*5 kw; 0.079 v/m when power is 4*10 kw; 0.12 v/m when power is 4*25 kw; 0.17 v/m when power is 4
by 50 kw. According to a ionogram assuming $\theta=1$ from equation [3-34] the modulation index can be given to be $m_p=0.015$ (p=4*5 kw); $m_p=0.03$ (p=4*10 kw); $m_p=0.08$ (p=4*25 kw); $m_p=0.16$ (p=4*50 kw).

Fig 5.37 shows a morning observations which indicate the relationship between the HF power and the modulation index based on the date of the first sideband observations. The data are recorded between 0951 AST and 1011 AST on February 8, 1986. The HF transmitters were operated at the power of 4*50 kW; 4*25 kW; 4*10 kW; and 4*5 kW and tuned to the frequency of 3.175 MHz. The difference frequency of the two HF pump waves is chosen to be 5 Hz. During this morning observations Arecibo ionograms and the electron density profiles given by the 430 MHz radar indicate that the $f_0F_2$ is at a value of 7.5 MHz. The curve of Fig 5.38 is calculated according to equation [3-34] and shows the agreement with the data.
Figure 5.34 The modulation index versus the difference frequency

0804 AST - 0841 AST
f = 5.1 MHz
W = 3 * 75 + 25 kw
February 8, 1986
FIGURE 5.37 MODULATION INDEX VERSUS TOTAL HF POWER

0951 AST - 1011 AST
f=3.175 MHz Δf=5 Hz
February 8, 1986
CHAPTER 6
ANALYSIS AND CONCLUSION

The intent of this dissertation has been to investigate the mechanisms in the ionospheric plasma that are excited by powerful HF waves and to explore the relationships of the HF sidebands with the ionospheric parameters. The investigation has been aided by using the 430 MHz incoherent radar to measure the ionospheric background such as the electron density profile and temperature of the electrons. From the observation of HF sideband at Arecibo in 1984 the relationships of the sidebands with some ionospheric parameters have been developed qualitatively. Observations of the sidebands of the HF waves introduced by mixing in the ionosphere were made in February, 1986. Our analysis of the 1986 observations show the quantitative relationships of the sideband amplitude with the HF waves and ionospheric parameters.

In 1983 Papadopoulos first proposed parametric instabilities (Briillouin scattering) to explain the HF sideband generation but did not provide any predictions of the relation of the HF sideband with HF waves and ionospheric parameter.

In 1984 Fejer proposed the phase modulation theory to explain the HF sideband generation. Fejer proposed that the powerful radio waves incident on the ionosphere modifying the ionospheric electron density. The effect of the ponderomotive force is a reduction of the electron density which is spatially quasiperiodic due to the standing wave nature of the radio wave (Airy pattern). The numerical solutions show the phase delay of the HF wave is proportional to the HF power density. The phase delay is
\( r = 6 \times 10^3 \) radians per 1 watt/m\(^2\) power density. For an incident power density \( 10^{-4} \) W/m\(^2\) this leads to a phase delay of 0.6 radians. An incident power density of \( 10^{-4} \) W/m\(^2\) would exist at 250 km for a radiated power of 400 kW with an antenna gain of 23 db and this results in \( c r e_0 E_0^2 / 4 = 0.34 \) for the argument of the Bessel functions in [3-22]. This would result in the first order sideband that is only about 15 db below the level of the carrier. For a radiated power of 200 kW the results in \( c r e_0 E_0^2 / 4 = 0.17 \) for the argument of the Bessel functions in [3-22] and would lead to the first order sideband about 21 db below the level of the carrier. In comparison with the observations the phase modulation theory would overestimate the phase delay and therefore the sideband levels. In 1985 we proposed the nonlinear wave–particle interaction theory (the modulation index theory) to predicted quantitatively the sideband amplitude related to [1] the HF power; [2] the HF frequency; [3] the scale height of the ionosphere; [4] the electron temperature. [5] the difference frequency. The theoretical prediction of the phase modulation theory and the modulation index theory have some agreement with the observation as noted in the following:

Analysis of the Experimental Results:

(1) The relationship between the sidebands and HF power: The modulation index theory predicts that the modulation index is proportional to the HF power. The observations also indicate that the modulation index is proportional to the HF power, if the total HF transmitter power is less than about 200kW [Fig 5.7, Fig 5.10, and Fig 5.37]. But the modulation index
of the observation is smaller than the modulation index theory prediction, if the total HF transmitter power is larger than about 200 kw. The phase modulation theory predicts a similar dependence of sideband amplitude on HF power. Both theories do not agree with observations above about 200 kW. This disagreement has two possible explanations:

(1) The HF waves would be strongly absorbed when the HF power is increased above some level. The observations [Gordon and Showen, 1971; Gordon and Carlson, 1974] show that approximately half of the incident HF energy is absorbed from anomalous absorption and deviative absorption. The energy goes into the electron gas through ohmic dissipation (the electron temperature is raised). The generation of large plasma wave contributes about 10 to 30 percent of the absorption. Another 10 to 30 percent of incident HF energy is absorbed through ohmic or joule heating; only 10 percent goes into fast electrons [Haslett and Megill, 1974; Fejer and Graham, 1974]. The HF waves would excite the decay mode instability and enhance the plasma line, absorbing some of the energy of the HF waves when decay mode threshold is exceeded.

(2) The two HF waves would generate an ELF wave: Papadopoulos proposed that a HF wave would decay into an ELF whistler wave and another second HF wave. The sidebands would arise from the nonlinear interaction of the ELF wave with the two high frequency waves. This process is a conventional stimulated forward Brillouin scattering and more energy of the HF waves goes into the ELF wave.

This model suggests for future HF sideband experiments to find some relationship between the HF sideband and the plasma line.

[2] The relationship between the amplitude of the sidebands and the
difference frequency of two pump: The modulation index theory predicts that the modulation index is inversely proportional to the difference frequency with $1/j$ power [$\Delta m_0 \propto (\Delta f)^{1/j}$]. The parameter $j$ which is related to the gradient of the electron density range from 1.1 to 1.4. The observations show agreement with modulation index predictions if the difference frequency is smaller than 200 Hz. But a disagreement occurs if the difference frequency is larger than 200Hz [Fig 5.22; Fig 5.28 and Fig 5.34], because the condition of modulation index theory [$n_1 > n_0$] is broken down. In the deriving of the relation of the modulation index and the difference frequencies of the two HF pump waves only the part of the refractive index $n_1$ caused by the ponderomotive force is considered, the part of the refractive index $n_0$ caused by the unperturbed electron density is neglected. This condition is approximately true in the reflection region. Since the refractive index $n_1$ caused by ponderomotive force is larger than the refractive index $n_0$ caused by unperturbed electron density, the refractive index $n_0$ goes to zero at the reflection point.

No predictions have been made by either the phase modulation theory or by Papadopoulos theory about the relationship between the sidebands and the difference frequency.

[3] The relationship between the amplitude of the sideband and the HF frequency: The modulation index theory predicted the modulation index is inversely proportional to the HF frequency. The observation show that sideband amplitude in the 3.175 MHz HF waves is larger than the sideband amplitude in the 5.1 MHz HF waves. In the observation of February 6, 1906,
0952 AST the first sidebands are B(+) 20 db and B(-) 20 db (3.175 MHz) which are larger than the first sidebands which are B(+) 33 db and B(-) 30 db (5.1 MHz) in the observations of February 3, 1986, 1750 AST. Both cases have the same HF transmitter power of 200 kW and the same difference frequencies of 5 Hz. The phase modulation theory shows the phase delay is related to HF frequency.

[4] The relationship between the amplitude and the scale height of the ionosphere: The modulation index theory predicted that the modulation index is proportional to the scale height. There is a physical explanation: if the scale height is larger the nonlinear interaction region is larger so that the nonlinear mixing effect is also larger. It causes larger modulation index and larger phase delay. The observations show agreement with the theories.

[5] The relationship between the sideband amplitude and the electron temperature: The modulation index theory predicted that the modulation index is inversely proportional to the electron temperature. There is a physical explanation: if the electron temperature is high, the electrons would carry away the HF energy through collisions with ions, so that the nonlinear mixing effect becomes weaker. The observations also show agreement with the theory. The phase modulation theory also shows that the phase delay is related to the electron temperature.

Since the HF sidebands were first observed by Gordon and Ganguly in 1984, three sideband theories have been developed to explain them. Papadopoulos first introduced the parametric instability to explain the HF sideband generation, but no predictions were made about the relation of the HF sidebands to the ionospheric parameters. Later Fejer introduced the
phase modulation theory to explain the sideband generation. Fejer's numerical solution overestimates the sideband amplitude in the relation of the sideband with the HF power. We introduced modulation index theory to describe two HF wave modulation using the W.K.B. approximation due to the wave-particle nonlinear interaction. These wave-particle interactions cause a propagating electron density fluctuation which will lead to currents at the frequencies \( \omega_1 + \Delta \omega \) and \( \omega_2 - \Delta \omega \), these currents will generate mixed electromagnetic electrostatic sideband modes at the same frequencies. The sideband modes interact with two pump waves and generate HF sideband. The modulation index theory provides the most complete interpretation of the observations.
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