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THE TWO STATE STRUCTURE OF THE SOLAR WIND
AND ITS INFLUENCE UPON PROTON TEMPERATURES
IN THE INNER HELIOSPHERE:
OBSERVATIONS OF HELIOS 1 AND HELIOS 2

BY
RAMON E. LOPEZ

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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THE TWO STATE STRUCTURE OF THE SOLAR WIND
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ABSTRACT

This thesis examines the two state structure of the solar wind and its influence upon proton temperatures. To examine this question, data from the Helios spacecraft are examined. The results from the data are interpreted in light of several theoretical models of the solar wind. In particular, it is found that the interplanetary heating of the protons observed by Helios is consistent with models that rely on extended deposition of energy and momentum in the form of Alfvénic waves.

Analysis shows that between 4-10% of the time the data are consistent with two fluid models which do not include extended deposition of energy and momentum. For the rest of the data, the magnetic fluctuations are analyzed and it is found that there is dissipation of wave energy. Calculations show that the heating required by the protons can be accounted for by the apparent dissipation of Alfvénic wave energy. The relationships of temperature to velocity, to number density, and to momentum flux are also examined and are found to be consistent with a bifurcation of the solar wind based upon Alfvén waves. A qualitative scenario for
the generation of the two state solar wind wherein all the energy for the solar wind comes from convection in the sun is discussed.
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I dedicate this thesis to my loving parents, through whom all things for me have become possible.
INTRODUCTION

The solar wind can be described as having a two state structure. These two states are best characterized by flow velocity: a low-speed, "cool" wind \( (T \sim 4 \times 10^4 \text{ K at 1 AU}) \) and a high speed wind, with relatively high temperatures at 1 AU \( (T \sim 2 \times 10^5 \text{ K}) \). The fast wind is concentrated in high speed streams that issue from coronal holes [Krieger, et al., 1973; Levine, et al., 1977b] which exhibit a consistent variation over the solar cycle. Numerous authors have discussed this bifurcation of the solar wind [Bame et al., 1977; Feldman, 1977; Rosenbauer et al., 1977; Schwenn, 1983] and have argued that the basic acceleration processes in the solar wind must be at the root of this division. The relationship between the two state structure and the evolution of the solar wind proton temperature in the inner heliosphere will be the central topic of this thesis.

There are important physical differences between high and low speed wind. It will be seen that these differences can be seen in the context of a dominance of certain acceleration processes in one state as contrasted with dominance of alternative processes in the other state. As all of these processes are at work throughout the wide spectrum of solar wind activity, one of the goals of this thesis will be to attempt to relate the observed proton temperatures to both states within a consistent phenomeno-
logical model. Thus the basic philosophical approach will be to use the two state paradigm as a prism to separate the observations and organize the analysis in an attempt to uncover the commonalities, as well as the origin of the differences, between the temperatures in the low and high speed plasma.

To study the solar wind, proton data from the Helios 1 and Helios 2 spacecraft are analyzed. The data span almost six years, from 1974-1980, and cover a heliocentric distance of 0.3 to 1 AU. Thus the Helios data sets offer an extensive survey of the inner heliosphere from which statistical and secular trends in the solar wind may be extracted. In addition, the use of both data sets allows one to investigate times when the satellites were in radial alignment and measured the same plasma. These results will then be interpreted in the light of theoretical models which attempt to resolve the discrepancies between the early models of the solar wind and actual observations.

These models were based on what Hollweg [1978] called the "standard physics." They are notable for their spherically symmetric, thermally driven, radial flows. Some of those assumptions made in early investigations [e.g., Parker, 1958, 1963; Weber and Davis, 1967; Hartle and Sturrock, 1968] have been called into question [Nerney and Barnes, 1977; Hollweg, 1978]. In particular the assumption of radial magnetic topology has been challenged by the fact
that coronal hole geometry is distinctly non-radial [Munro and Jackson, 1977; Levine et al., 1977a]. Several studies have shown that the greater than $1/r^2$ divergence typical of coronal holes can have dramatic dynamical consequences [Durney and Pneuman, 1975; Pneuman, 1976; Kopp and Holzer, 1976; Holzer and Leer, 1980; Habbal and Rosner, 1984].

The inability to produce the high speed wind was one of the major inadequacies of the early models of the solar wind. Another fundamental problem encountered by basic solar wind models was that they predict proton temperatures that are lower by an order of magnitude or more than the observed average temperatures. A favored solution to these problems is to invoke an external energy source, generally in the form of MHD waves, to supply the required heat and acceleration needed. This approach has been taken by several authors [Barnes et al., 1971; Belcher, 1971; Hollweg, 1975; Jacques, 1977; Hollweg, 1978; Leer and Holzer, 1980; Leer et al., 1982] and it is upon this work that much of this thesis builds.

Predictions for electron temperatures in the basic two fluid models are also in disagreement with observations. Whereas the observed temperatures are $1.1 - 1.2 \times 10^5$ K during quiet times, the two fluid model predicts an electron temperature of $3.5 \times 10^5$ K, and the measured electron heat conduction flux is $\sim$1.5 orders of magnitude lower than predicted [Hollweg, 1978]. In addition the electrons are,
on average, hotter in the slow solar wind. While these are questions of undeniable dynamic consequences, they cannot be addressed by this study simply because the data set analyzed contains only proton data. Similarly the role of minor ions (i.e., He+) cannot be examined even though their role is not inconsequential. For example, it is known that the differential ion flow speed is proportional to the local Alfvén speed. As the plasma moves outward the Alfvén speed (which scales as $\sqrt{1 + r^2 / R}$) drops, so this differential streaming could provide a source of free energy [Marsch et al., 1983]. Thus all results must be viewed with some caution given the uncertainties in drawing general conclusions about the solar wind from proton data only.
CHAPTER I. REVIEW OF SOLAR WIND MODELS

It is instructive to review the basic models of the solar wind. We shall begin with the original model of Parker [1958], which will illustrate how a supersonic wind arises from the fluid equations. We will then briefly discuss additions to the one-fluid model (such as $\mathbf{B}$, electrons) which attempted to reconcile theory and observations. We shall see these models were unable to explain the average solar wind. We shall then proceed to discuss the conductive model of Leer et al. [1982], which includes energy and momentum deposition above the coronal base, as well as nonradial flow. It is the results of this model which will form the physical picture for our understanding of the solar wind. As we shall see, the results of Leer et al. [1982] concerning subsonic momentum addition and higher coronal base temperatures, versus supersonic deposition of energy and momentum, are crucial to the analysis.

The modern concept of the solar wind originates with E. N. Parker [1958, 1963]. Parker [1958] assumed a hydrodynamic, frozen in flux, one-fluid flow. In this case the radial one-fluid momentum equation can be written

$$n \mu V \frac{dV}{dr} = -\frac{d}{dr} (2n k T) - \frac{G n u M}{r^2} \quad (1-1)$$

where $n$ is the number density, $V$ is the flow speed, $T$ is the
temperature, and \( r \) is the heliocentric radius. The constants \( M, \mu, K \) and \( G \) are the solar mass, proton mass, Boltzmann constant and the gravitational constant, respectively.

The continuity equation is

\[
\mu n v A = \text{constant} = \mu n_o V_o A_o = P_o \tag{1-2}
\]

where \( A \) is the area of the flux tube and the subscript refer to the coronal base level. For an energy relation Parker [1963] took a polytropic law

\[
P \propto \rho^\gamma \tag{1-3}
\]

where \( \gamma \) is the polytropic index.

Parker [1958] defined the dimensionless quantities

\[
\xi = r/r_o \tag{1-4a}
\]

\[
\tau = T(r)/T_o \tag{1-4b}
\]

\[
\lambda = \frac{G \mu M}{2r_o^2 K T_o} = \frac{V^2 \text{esc}}{U^2} \tag{1-4c}
\]

\[
U = \left( \frac{2 K T_o}{\mu} \right)^{1/2} \tag{1-4d}
\]

\[
\psi = \frac{\mu V^2}{2 K T_o} = \frac{V^2}{U^2} \tag{1-4e}
\]
Using (1-2) we can eliminate n from (1-1), and assuming spherical geometry (A α (r/r_o)^2),

\[
\frac{d\psi}{d\xi} (1 - \tau/\psi) = -2\xi^2 \frac{d}{d\xi} (\tau/\xi^2) - \frac{2\lambda}{\xi^2}
\]  

(1-5)

Inspecting (1-5), we see that the right-hand side can be zero if either \(d\psi/d\xi = 0\) or \(\tau = \psi\). Solutions of the former type have \(d\psi/d\xi\) negative thereafter, and so only \(\tau = \psi\) can give "wind" solutions.

Parker chose an isothermal corona (γ = 1), in which case the left-hand side of (1-5) when \(\tau = \psi\) is

\[
2\xi^2 \frac{d}{d\xi} \left(\frac{1}{\xi^2}\right) + \frac{2\lambda}{\xi^2} = 0
\]

(1-6)

which results in

\[
\xi = \frac{\lambda}{2} = \frac{G \mu M}{4r_0 K T_0}
\]

(1-7)

At this radial distance \(\psi = 1\), or

\[
V = (\frac{2KT}{\mu})^{1/2},
\]

(1-8)

Since this is close to \((\gamma P/R)^{1/2}\), the speed of sound, this point is referred to as the "sonic critical point."

In the isothermal case where \(\psi = 1\) at \(\xi = \lambda\), we can integrate (1-5) to get
\[ \psi - ln\psi = -3-4 \ln \frac{\lambda}{2} + 4 \ln \xi + \frac{2\lambda}{\xi} \quad (1-9) \]

Far from the sun (\(\xi \gg 1\)) the above reduces to

\[ \psi = 4 \ln \xi \quad (1-10) \]

If we assume that the temperature, \(T(r)\), can be written \(T(r) = T_0 f(r)\), such as if there were a simple poltropoic relationship between density and pressure in the heliosphere, then we can rewrite (1-10) as

\[ V = T^{1/2} \phi(r/r_0) \quad (1-11) \]

where \(\phi(r/r_0)\) contains all the radial dependence. So a Parker model wind with an isothermal corona implies the proportionality far from the sun at a given radius

\[ V_{SW} \propto T^{1/2}. \quad (1-12) \]

For \(T_0 = 1 \times 10^6\) K Parker's model gives a velocity of \(\sim 400\) km s\(^{-1}\) at 1 AU. Thus the model could give the observed velocities. Parker's model, however, gives \(n \sim 500\) cm\(^{-3}\) at 1 AU, in contrast to the observed density of \(n \sim 5-10\) cm\(^{-3}\). It was very early [Parker, 1963] that the original model of Parker [1958] was seen as an inadequate description of the solar wind. What followed was a series of
efforts to expand the range of physical considerations and effects considered in the models of the solar wind.

Weber and Davis [1967] were the first to explicitly include the magnetic field in the MHD formalism. Their model included the $\phi$ component of the velocity, as well as the radial velocity. This adds a $\mathbf{J} \times \mathbf{B}$ and a centrifugal force term to the radial momentum equation. For our sun these terms are relatively small, so the radial dynamics of the Parker model are not greatly effected. One difference, however, is the appearance of two more critical points in addition to the sonic point. They are the Alfvénic critical points, where the flow velocity is equal to the radial Alfvén speed ($M_A = 1, r = r_A$), and where $V^2 = |\mathbf{B}|^2/4\pi \rho$. The Weber and Davis model, however, cannot produce the high proton temperatures at 1 AU. Thus while $\mathbf{B}$ was self consistently included in the MHD model, this did nothing to narrow the differences between theory and observation.

A next step was to include electrons in the model and see if proton-electron coupling could dramatically alter the predicted solar wind values at 1 AU. Various two fluid models have been published [e.g., Hartle and Sturrock, 1968; Wolff et al., 1971; Durney, 1973]. These models are solved by numerical integration of the two-fluid equations. It is the predictions of these models which are of interest, in particular the predictions for proton temperature and velocity, and which will be briefly discussed.
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The basic assumption of these models is a spherically symmetric, thermally driven wind. The collision frequencies are proportional to the coulomb logarithm which some authors [Hartle and Sturrock, 1968] hold constant. Others allow it to vary with radial distance [Durney, 1973]. The coulomb logarithm, defined as \( \ln \Lambda = \ln \left( \frac{T^{3/2}}{\eta^{1/2}} \right) + \) constant, only varies by roughly a factor of two over the radial range in question, therefore, the results of the two models are not very different. Table 1, from Nerney and Barnes [1977], illustrates the differences between the two as well as showing the basic shortcomings of the thermally driven, two-fluid models. Whereas the observed average "quiet" solar wind has \( V \approx 350 \text{ km s}^{-1} \) and \( T \approx 40,000 \text{ K} \), the models predict \( V \approx 250 \text{ km s}^{-1} \) and \( T < 10^4 \text{ K} \). The number density problem, however, was greatly improved. Hartle and Sturrock [1968] found a density of \( 15 \text{ cm}^{-3} \) at 1 AU; still somewhat high.

Nerney and Barnes [1977] have carried out two-fluid calculations that give \( V = 450 \text{ km s}^{-1} \) at 1 AU for very low base densities \( N_e(1R_s) = 2-4 \times 10^6 \text{ cm}^{-3} \) and high coronal temperature \( T_p(1R_s) = 2.5 - 4 \times 10^6 \text{ K} \), but they are unable to duplicate velocities seen in high speed streams. They argue additional energy and momentum or non-radial flow are required. In addition, coronal conditions derived from using observed solar wind parameters at 1 A.U. indicate that \( T_{po} \approx 2T_{eo} \). Even in the case of strong coupling (the colli-
sion time is larger than the solar wind expansion time), the base temperatures are 30% different, the protons being hotter. Therefore, the two fluid models, to be consistent with even the quiet solar wind, imply extended coronal heating of protons.

The effects of nonradial flow, as well as of energy and momentum deposition, in conductive one-fluid models have been examined in a series of papers [Durney and Pneuman, 1975; Kopp and Holzer, 1976; Holzer and Leer, 1980; Leer and Holzer, 1980; Leer et al., 1982]. The thermally driven model, which we will discuss first, employs equations (1-1) and (1-2) (momentum and continuity), as well as an energy equation

\[ 3 V n k \frac{dT}{dr} = 2 V k T \frac{dn}{dr} - \frac{1}{A} \frac{d}{dr} (qA) \]  

(1-13)

which replaces the polytropic law. This equation can be integrated using (1-1) to give \( \phi \), the energy flux

\[ \phi = F \left( \frac{1}{2} V^2 + \frac{5}{m} \frac{K T}{r} - \frac{G M}{r} + \frac{A}{F} q \right) = \text{constant}. \]  

(1-14)

At 1 AU, bulk flow dominates, therefore

\[ \frac{\phi}{F} = \frac{V^2}{2} \]  

(1-15)

The radial heat flux, \( q \), is given by
\[ q = q_a = K_0 \frac{T^5}{2} \frac{dT}{dT} \]  

(\( K_0 = 7.8 \times 10^{-7} \text{ erg cm}^{-1} \text{ s}^{-1} \text{ K}^{-7/2} \)) when in the collisional regime [Hollweg, 1978]. If one takes account of the collisional inhibition of the field, then

\[ q = q_a \cos^2 \theta \]  

(1-17)

where \( \theta \) is the angle between \( \mathbf{B} \) and the radial direction. In the collisionless regime, \( q \) is given by

\[ q = \frac{3}{2} a nKTV \]  

(1-18)

where \( a \) is an arbitrary parameter [Hollweg, 1976].

For an isothermal corona (1-1) can be integrated to yield the flow speed at the critical point. One finds [Leer et al., 1982] that

\[ V_C \propto \exp \left( -\frac{GM}{2KT/T_C} \right). \]  

(1-19)

Because \( nV(1 \text{ AU}) \propto V_C \), for strong gravitational confinement (\( GM/r_C \gg 2KT/\mu \)), the number flux is very dependent on the coronal temperature. Another way of viewing the situation is to realize that a hotter corona raises the scale height of the plasma, and so the number flux at the critical point increases. However, since \( q \propto T^{7/2} \), it does not increase as
with temperature as rapidly as does the mass flux invariant, \( F \). Therefore the conductive energy per unit mass, \( Aq/F \), decreases more rapidly than the enthalpy per mass, \( 5 \, \text{KT/m} \), increases (as long as \( n_o \) is \( \geq 10^7 \, \text{cm}^{-3} \)). Therefore the total energy per mass and \( V \) at 1 AU decrease as the coronal temperature increases [Leer et al., 1982].

This is the most important result of the conductive models. This cannot be seen in a polytropic model because the mass flux scales out and would seem to be unimportant as far as flow speed at 1 AU is concerned. The effect of low altitude heating on the velocity at 1 AU is crucial to the consideration of momentum and energy addition to the solar wind. Such an addition might increase the number flux and lower the energy/particle by the mechanism described above.

In this framework, the addition of momentum and energy can simply be included by modifying (1-1) and (1-13) which become [Leer and Holzer, 1980; Leer at al., 1982]

\[
\rho \, V \, \frac{dV}{dr} = -\frac{dP}{dr} - \frac{GM \rho}{r^2} + \rho D \quad (1-20)
\]

\[
3nVK \, \frac{dT}{dr} = 2VKT \, \frac{dn}{dr} - \frac{1}{A} \frac{d}{dr}(qA) + Q \quad (1-21)
\]

where \( \rho D \) and \( Q \) are the rates of momentum and energy density deposited in the fluid. The energy invariant (equation (1-14) is modified by the addition of

\[
\Delta \phi = \int dr(DF + AQ) \quad (1-22)
\]
There are some general results, independent of flow geometries, which will now be discussed. It was found [Leer and Holzer, 1980; Leer et al, 1982] that heat added below the sonic point raised the mass flux (for reasons discussed above), but that $\Delta \phi$ compensated enough so that the flow speed at 1 AU, was almost unchanged. A somewhat different result was found for the case of subsonic momentum addition. The added momentum increased the mass flux invariant more than heat (for equal energy inputs) and so caused a drop in the energy per unit particle and the bulk flow speed at 1 AU. Heat and momentum added after the sonic critical point had no effect on the mass flux. Thus all the additional energy and momentum deposited above the critical point went into increasing the downwind, or terminal, flow speed.

The effects of nonradial divergence are important in determining flow parameters at 1 AU [Durney and Pneuman, 1975; Holzer and Leer, 1980], but they do not fundamentally alter the results quoted above [Leer and Holzer, 1980]. A greater than $r^2$ divergence tends to move the critical point closer to the sun, and so increase $F$. Thus there is a positive correlation between higher terminal flow speeds and less rapid divergence [Levine, 1978]. In coronal holes the divergence of the magnetic field is more rapid than $r^2$, but the center of the hole has a less rapid divergence [Hollweg, 1978]. Accordingly there should be a velocity gradient across a coronal hole.
The effects of nonradial flow, however, are probably not sufficient to account for high speed streams, and so some kind of extended acceleration is implied [Leer et al., 1982]. This energy, in the form of MHD waves could then heat the protons in interplanetary space. We shall show that this is the origin of the correlation between high proton temperature and high velocity.

Thus we see that thermally driven models cannot account for the observed average solar wind or high speed streams. One might ask, however, if there is a component of the solar wind which does agree with these models, and if so, what distinguishes it from the rest of the solar wind. This, in turn, might shed some light on the differences between the low and high speed states of the solar wind and how this influences proton temperature.

From Helios data we shall attempt to glean such information, as well as some understanding of the dynamics of the formation of these two states. Before proceeding, however, it is necessary to discuss the Helios data and the methods of analysis.
II. HELIOS OBSERVATIONS AND METHODS OF DATA ANALYSIS

Helios 1 was launched December 10, 1974 into an elliptic heliocentric orbit with perihelion at 0.309 AU. Helios 2 was launched January 15, 1976, with a perihelion of 0.29 AU. The data from these spacecraft that we shall examine run until 1980 and so they cover a solar cycle from solar minimum (~1976) almost to solar maximum (~1981). Thus the data form a large and consistent set, with which large scale features of the solar wind may be investigated.

The details of the instrumentation for the plasma experiment are given in Rosenbauer et al., [1977]. Similarly, the instrumentation for the magnetic experiment is discussed in Musmann et al., [1977]. The data consist of hourly averages of plasma and magnetic quantities, along with their standard deviations.

Specifically, for the plasma we have hourly averages of the bulk flow speed, the proton number density, and the radial proton temperature. This temperature is obtained by integrating the proton distribution along the radial direction. Schwenn [1981b] states that this temperature shows little difference from the normal temperature, defined as the trace of the temperature tensor, in this case \( (2T' + T) / 3 \). The magnetic data consists of the magnitude of the x, y, and z field components (where x is sunward, y is azimuthal and z is out of the equatorial plane), as well
as the hourly average total field. The standard deviations of the hourly averages of these quantities are also provided. Each hour of data has plasma data or magnetic data or both, as well as the time and position of the spacecraft.

With such a large data set we are able to conduct a reasonably good statistical study of radial gradients. Since the most immediately noticeable feature of the two state solar wind is the dichotomy of speed, bulk flow velocity is a natural organizing parameter. For example, the "average" solar wind was studied by Schwenn [1983] and it was found that the radial temperature gradient is dependent on the velocity. Thus it is natural to use a separation into velocity bins to organize the data.

We use the methods of least squares to fit

\[ \ln y = A \ln r + B \]  

(2-1)

where \( r \) is the radial position in AU, \( y \) is the dependent variable, and \( A \) and \( B \) are constants. Even quantities which are not expected to be power laws are fit with (2-1) for convenience. For example, the magnitude of the interplanetary magnetic field (IMF) given by the Parker spiral model [Parker, 1958] has a functional dependence of \( r^{-2}(1 + r^2)^{1/2} \). However, for the purposes for which we will have need of the radial variation of a given quantity, a power
law fit will be sufficient.

With these fits we are able to normalize the data to 1 AU and construct a Carrington plot of the solar wind. Such a plot has been made of the bulk flow velocity (Fig. 1a) with the velocity color coded in four divisions. The velocity data are unnormalized since it is assumed that \( V \) is essentially constant over the radial range in question. At the bottom of the plot is the Carrington longitude of the spacecraft at the time of observation so that each line of data is an apparent rotation of the sun. On the right vertical and left vertical sides of the plot, are the year and Carrington rotation number, respectively. Thus time runs from right to left and from the top down, and we are able to display 6 years of data (over 48,000 data points in Figure 1a) on a single plot.

This type of display, developed by John W. Freeman [Freeman, 1983], easily allows one to see large scale features such as the high speed stream in Figure 1a at \( \sim 210^\circ \) in 1975. Also, secular changes in the solar wind become visibly accessible, such as the disappearance of long-lived high speed streams as one goes towards solar maximum. At the "center" of the coronal hole pointed out above in Figure 1a, one can see a high speed peak. This most likely is due to the weaker divergence at the center of the hole compared to the edges discussed in the last chapter.
In Figure 2 we see the same type of plot in which the radial proton temperature is color coded. The temperature has been normalized to 1 AU using power law fits derived from the data. Immediately apparent is the correlation between high temperatures and high speed streams. Figure 3 plots the x (sunward) component of the IMF. One thing that can be seen is an abrupt change in the sector structure of the IMF from a two sector structure to a four sector structure around Carrington rotation number 1640. The utility of this type of display is obvious and such plots of numerous solar wind quantities as well as descriptions of their variations over the solar cycle are given in Freeman and Lopez [1985b].

Figure 1b is a Carrington plot of velocity from Helios 2 data. Except for the fact that this is only four years of data instead of approximately five years for Helios 1, the two plots are essentially the same. In general the analysis and the quoted results will be for Helios 1, with some results from Helios 2 also presented to show the high level of consistency between the data gathered by the two spacecraft. However, since the Helios 2 data set does not extend to the period immediately before solar minimum, calculations of the changes over the solar cycle in specific fits to the Helios 1 data cannot be checked completely by Helios 2.

The existence of two independent, yet experimentally consistent, satellites allows us to search the data for
Fig. 1a. This figure plots the (color coded) bulk flow velocity vs. Carrington longitude and Carrington rotation number. Each pixel represents one hourly average of the bulk flow velocity from Helios 1. Each horizontal line is one Carrington rotation. The Carrington rotation number corresponds to the line of data nearest or just below the top edge of the number. The year is indicated on the right axis.
Fig. 1b. Same as la, except for Helios 2.
Fig. 2. This figure plots Helios 1 solar wind proton radial temperatures in the same format as Figure 1. Note the high speed stream activity in 1974 and 1975 and transient events closer to solar maximum. Temperatures are in kilodegrees Kelvin. The times when $T < 15000 \, ^\circ K$ are in red, the temperatures have been normalized to 1 AU using the data in Table 2.
Fig. 3. This figure plots the unnormalized radial component of the interplanetary magnetic field seen by Helios 1 in the same format as Figure 1.
radial alignments such that both satellites were in the vicinity of the same solar wind plasma. This type of analysis has been done by Schwenn et al. [1981a,b] and as well as Schwartz and Marsch [1983]. The latter authors looked at one particular two and a half hour lineup between the Helios spacecraft and did an analysis of the distribution functions of the protons and of the alphas. Schwenn et al., [1981a,b] did a more statistically oriented analysis involving several lineup constellations. Our approach is also a statistical one, since what we want is an adjunct to the overall statistics.

We define the radial error time to be the actual time difference between two data hours minus the (radial) plasma travel time between the two spacecraft. The longitudinal error is given by the longitudinal difference between the two satellites minus the net angular motion in the rotating sun centered frame. The net angular motion is the rotation rate of the sun times the satellite time difference minus the orbital angular motion of the reference spacecraft (Helios 2).

The angular velocity of an object in orbit is given by [Goldstein, 1980].

\[ \mu r^2 \frac{d\theta}{dt} = \ell \quad \text{(2-3)} \]

where \( \mu \) is the reduced mass (= \( M_{\text{Helios}} \)) and \( \ell \) is the
(constant) angular momentum. For an elliptical orbit

\[ \frac{L^2}{\mu K} = a(1 - \varepsilon^2) \]  

(2-4)

where \( a \) is the semimajor axis of the ellipse, \( \varepsilon \) the eccentricity and \( K \) is the force constant in the equation \( F = \frac{K}{r^2} \). For such an ellipse

perihelion = \( a(1 - \varepsilon) \)  

(2-5a)

aphelion = \( a(1 + \varepsilon) \)  

(2-5b)

so we can easily calculate \( a \) and \( \varepsilon \) for Helios. From these equations we find that

\[ \frac{d\theta}{dt} = \frac{1}{r^2} \left( G M a(1 - \varepsilon^2) \right)^{1/2} \]  

(2-6)

which gives us the angular velocity in terms of known quantities.

Helios 1 and 2 data were sorted to look for alignments where the radial error is less than 12 hrs. Schwenn [1981b] has pointed out that over a longitude of 1° there can be substantial changes in solar wind properties, therefore we have further restricted our data to that with a longitudinal errors less than 0.8°. Furthermore we have selected three lineups in particular: one with \( V > 600 \) km s\(^{-1}\), one with
Fig. 4. This figure shows the velocity data (in km s⁻¹) from the three radial alignments used. As can be seen, there are three lineups: \( V < 400 \), \( 400 < V < 600 \) and \( V > 600 \).
$400 \text{ km s}^{-1} < V < 600 \text{ km s}^{-1}$, and one with $V < 400 \text{ km s}^{-1}$ (Fig. 4). These were selected because they have relatively large radial separations. These three lineups also give a reasonable range of velocities.

One of the lineups ($V < 400 \text{ km s}^{-1}$) is of limited utility. The data are at a time when there are large local fluctuations of the density, and it is also near a sector boundary. Thus the radial gradients derived from the lineup are not entirely valid for comparison with average values derived from statistics. One point of interest is that the slow speed flux tube seems to inflate $\sim 30\%$ more than for radial expansion over the distance of 0.4 to 0.6 AU. This effect was also mentioned by Schwenn et al. [1981a], who gave a result of $\sim 25\%$ for the expansion over and above radial.

In general, we will primarily rely on the statistics of the Helios 1 data set. These results will be compared to Helios 2 results, as well as the the lineup derived gradients, where applicable. Aside from radial gradients, we shall analyze the relationship between proton temperature, velocity and number density. From these analyses of the protons and magnetic data we will attempt to draw conclusions about influence of the two state structure of the solar wind upon proton temperature.
CHAPTER III. THE COLD SOLAR WIND: 
AN EXTREMUM OF SOLAR WIND BEHAVIOR

As pointed out in Chapter 1 the standard multi-fluid models of the solar wind have been considered inadequate in a number of respects. Most notably, they predict proton temperatures of about 5000 °K and flow velocities of about 250 km s\(^{-1}\) at 1 AU. These predictions are in contrast to the observed average temperatures of 1 AU which are an order of magnitude higher and average velocities of about 350 km s\(^{-1}\) for the slow solar wind, and still higher values in the high speed solar wind. Moreover, it is also known that, in general, the solar wind protons cool much more slowly than expected from pure adiabatic expansion [e.g., Schwenn, 1983]. While the average solar wind doesn't resemble the thermally driven two-fluid model winds, the range of temperatures and velocities in the solar wind is great. Therefore there might be a component of the solar wind which does resemble the two-fluid models.

While examining hourly averages of the plasma data from the Helios 1 and 2 spacecraft we were surprised by the large percentage of time the radial component of the proton temperature showed extremely low temperatures (T < 15000 K or 1.3 eV). In order to study this low temperature or cold component further we normalized Helios 1 average proton temperatures to AU using values for \(\gamma\) (the power of r)
obtained from a least squares linear fit to the heliocentric distance following a velocity sort (see Table 2 and Schwenn [1983]). We then sorted the full normalized data set for times when the temperature was less than 15000 K and analyzed the resulting data subset.

The Helios 1, T < 15000 K data subset was first sorted by year and the percentage of hours in each year were calculated. The results, shown in Figure 5 indicate some solar cycle variation in the occurrence frequency of the cold solar wind with the highest percentage, about 10%, occurring toward solar maximum in 1980. Feldman et al., [1977] have tabulated average solar wind properties as calculated from IMP 6, 7 and 8 data between March 1971 and July 1974. They found that 5% of the data had Tp below 10,000 K. Inspecting Figure 5 we see that a figure of ~5% around 1972-73 is a reasonable number assuming an oscillation of the cold solar wind component keyed to the solar cycle. A closer examination of earlier solar wind data would allow a quantitative study of this variation and allow an investigation of its relationship to other solar variations.

Figure 2, which shows the Helios 1 color coded hourly averages of solar wind proton temperature, emphasizes the intervals of cold solar (T < 15000 K) by plotting them in red. It can be seen that the intervals of cold solar wind are often sustained for substantial periods, up to several days. Careful inspection of Figure 2 also shows, again, the
Fig. 5. This shows the percentage of the time that $T < 15000$ K in a given year.
higher incidence of cold solar wind approaching solar maximum as opposed to solar minimum.

Comparing Figures 1a and 2, note that the cold solar wind avoids times of transient events and high speed streams. A cursory examination of solar activity charts at the longitudes of cold solar wind events indicates no significant solar features that can be identified with the source regions of cold solar wind. However, a proper study with adequate mapping to the sun remains to be done.

Next we examine the relationship between the cold solar wind temperature and the simultaneous solar wind velocity. Figure 6 is a scatter plot showing that the velocities associated with the cold solar wind are very low and lie in the range from about 200 to 350 km s\(^{-1}\). The relationship that is evident in Figure 6 is continuous with the temperature-velocity relationship for the full data set as is shown in Figure 7. In Figure 7 we see that there is a distinct change in the character of the temperature-velocity relationship at about 500 km s\(^{-1}\). Data above the knee at 500 km s\(^{-1}\) is probably characteristic mostly of high speed streams plus a small percentage of points from transient events. The discussion of the bend in the T-V data will be taken up in Chapter 5. We present it here in order to show that the cold solar wind is not a separate or physically distinct component of the solar wind but rather part of the continuum of the solar wind below 500 km s\(^{-1}\).
Fig. 6. This figure shows a scatterplot of velocity and low temperature data (normalized to 1 AU).
Fig. 7. This figure shows velocity and temperature data (normalized to 1 AU) from the full Helios 1 dataset.
The proton density for the cold solar wind tends to be higher than for the full Helios 1 dataset. From a power law fit to the radial distance, we obtained a mean value of 10.8 ± .9 protons cm⁻³ at 1 AU for those hourly averages with T < 15000 K.

**Cooling Rate**

Schwenn [1983] using Helios data has calculated the best fit exponents of r, which he called γ, following a velocity sort at 100 km s⁻¹ intervals. We have repeated Schwenn's analysis and the results are shown in Table 2. The value for γ for the 200 to 300 km s⁻¹ interval is about 1.3 which is consistent with the pure adiabatic cooling for the solar wind in that velocity interval. γ decreases dramatically at higher velocities indicating heating processes are taking place. Results of gradients derived from radial alignment data are generally consistent with the results in Table 2. Any theory of a heating source, e.g., Alfvén wave, heat flux, etc., must explain this strong velocity dependence on the heating. The question of the heating and its relation to velocity will be the subject of the next chapter.

It is important to note that the very low velocity wind (which we have shown is essentially the cold solar wind) cools nearly adiabatically as it expands outward in the region of 0.3 to 1 AU. We assume that inside 0.3 AU the
expansion is adiabatic in to $\sim 3 R_\odot$. At this point it is reasonable to expect a decoupling of the protons and electrons [T. E. Holzer, personal communication, 1985]. This radius also is roughly the (sonic) critical point. The observed lower limit of the cold solar wind is about 3500 K. For adiabatic expansion this extrapolates to about $1 \times 10^6$ K at $r \sim 3 R_\odot$.

Apparently the cold solar wind protons escape the influence of the heat source, (waves, particle heat flux, etc.) that is responsible for the extended interplanetary heating in higher velocity streams. It is likely that the cold solar wind is emitted from source regions on the sun where this heat source is diminished. The cold solar wind is therefore a good candidate for comparison with two-fluid solar wind models which do not attempt to incorporate an external heat source.

**In-Transit Effects**

We have considered the possibility that these periods of cold solar wind arise from in-transit effects such as stream-stream interactions, or that they may be associated with magnetic clouds [Burlaga et al., 1982]. To evaluate the former we scanned the data for a correlation with decreasing values of the IMF, or bulk velocity gradients, using the method of superimposed epochs. No significant correlation was found. To investigate the possibility of a
connection with magnetic clouds we plotted $\delta$, the angle the magnetic vector makes with the equatorial plane. Again no significant correlation was seen except in a few cases. Moreover, the extended duration of the cold solar wind phenomena over continuous periods as long as about 4 days (e.g., Carrington Rotation No. 1687, Figure 2) argues against these possibilities. Further support for this comes from the fact that the intervals of cold solar wind avoid times of high speed streams and transient events as can be seen in the velocity data (Figure 1a). It is, however, likely that some small fraction of our cold solar wind events are associated with coronal mass ejections and the resulting magnetic clouds, particularly near solar maximum.

Comparison to Two-Fluid Models and Interpretation

The two-fluid solar wind model of Hartle and Sturrock [1968] predicts a proton temperature at 1 AU of $4.4 \times 10^3$ K, an electron density of 15 cm$^{-3}$, and a velocity of 250 km s$^{-1}$. Figure 6 shows that when the temperature is $4.4 \times 10^3$ K, 250 km s$^{-1}$ is a reasonable value for the observed solar wind velocity. Moreover, we note again that the observed lower limit to the solar wind temperature is about 3500 K which corresponds to reasonable coronal temperatures for the observed adiabatic cooling. The mean temperature for the cold solar wind data set is $\sim 9000$ K. This is higher than either the Hartle and Sturrock [1968] value or the
result of Durney [1973] (see Table 1). The small amount of heat required could probably be supplied by the degradation of the particle heat flux. The observed mean proton density of 10.8 ± .9 cm⁻³ is in satisfactory agreement with the Hartle and Sturrock predicted value of 15 cm⁻³. In view of the fact that Hartle and Sturrock overestimated the coronal base density [Athay, 1976], the similarity between the observed and predicted densities might indicate relatively higher coronal base and/or critical point number fluxes in the source regions for the cold solar wind.

We conclude that the cold solar wind data are in good agreement with at least one solar wind model that does not incorporate nonthermal pressures and extended heating. As such, it represents an extremum in solar wind behavior. Furthermore, the incidence of cold solar wind is anticorrelated with high speed stream activity, the other extremum of (non-impulsive) solar wind behavior.

This implies that the remainder of the solar wind reaches its higher velocities (and temperatures) because of nonthermal processes such as the deposition of wave energy and momentum. The high speed wind might then be a primarily wave-driven wind. To investigate this we shall next examine why the proton temperatures are so large at 1 AU and see what we might conclude about the dynamics of the formation of the two state solar wind.
CHAPTER IV.

WAVE HEATING OF SOLAR WIND PROTONS

It has been long known that coronal temperatures are on the order of $10^6$ K and it is the high coronal temperatures that, in the early models, drives the solar wind [Parker, 1958, 1963; Weber and Davis, 1967; Hartle and Sturrock, 1968]. As we say in Chapter 1, one major problem with these early models is that they predict temperatures of $\sim 5000$ K at 1 AU, whereas the observed average temperature is $\sim 20$ times that. In the last chapter it was shown that there is a cold component of the solar wind which does have the expected temperatures and it was argued that this cold solar wind escaped any heating beyond $\sim 3$ $R_\odot$. This component of the solar wind only accounts for 3% to 10% of the Helios data (Figure 5), and so the rest of the solar wind must be undergoing some kind of additional heating.

To investigate the radial variation of the temperature, following the development of Barnes and Hollweg [1974], we write the heat conduction equation

$$(\gamma P - 1)^{-1} \left[ \frac{\partial P}{\partial r} + \nabla \cdot (P \mathbf{v}) \right] + P \mathbf{v} \cdot \nabla + q \cdot \mathbf{v} \cdot \hat{q} = 0$$

(4-1)

where $P$ is the pressure, $\rho$ is the mass density, $q$ is the
heat flux and $\gamma_p$ is the ratio of specific heats. If we multiply by $(\gamma_p - 1) \rho^{-\gamma} \text{ equation (4-1) becomes}$

$$\rho^{-\gamma_p} \left( \frac{\partial}{\partial x} + \nabla \cdot \mathbf{v} \right) p + \gamma_p \rho^{-\gamma_p} \rho \mathbf{v} \cdot \mathbf{v} = -\rho^{-\gamma_p} (\gamma_p - 1) \mathbf{v} \cdot \hat{\mathbf{q}}$$

(4-2)

The continuity equation is

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

(4-3)

Expanding the gradient and multiplying by $-\gamma_p \rho^{-1} p + 1 + \gamma_p$ yields

$$-\gamma_p \rho^{-1} p + 1 + \gamma_p \left( \frac{\partial}{\partial x} + \nabla \cdot \mathbf{v} \right) \rho = \gamma_p \rho^{-\gamma_p} \rho \mathbf{v} \cdot \mathbf{v}$$

(4-4)

which when substituted into (2) immediately results in

$$\left( \frac{\partial}{\partial x} + \nabla \cdot \mathbf{v} \right) \left( \frac{p}{\rho \gamma_p} \right) = - (\gamma_p - 1) \rho^{-\gamma_p} \mathbf{v} \cdot \hat{\mathbf{q}}$$

(4-5)

In the absence of a heat flux gradient, (4-5) becomes the familiar adiabatic law [Barnes and Hollweg, 1974; Gazis 1984]

$$\left( \frac{\partial}{\partial x} + \nabla \cdot \mathbf{v} \right) \left( \frac{p}{\rho \gamma_p} \right) = 0$$

(4-6)

For $\gamma_p = 5/3$ and radial expansion, the temperature should be
proportional to $r^{-4/3}$ and the quantity $T/n^2/3$ should be constant. Observations have been made of radial temperature gradients both inside and outside 1 AU [Rosenbauer et al., 1977; Schwenn, 1983; Smith and Barnes, 1983; Mihalov, 1983; Gazis, 1984]. One common feature of these investigations is that they show radial temperature profiles flatter than the expected $r^{-4/3}$ law. This means that the RHS of (4-6) is not zero, and that there is either an appreciable heat flux gradient or a heating term not explicitly included in (4-1).

The question of whether the particle heat flux could be responsible for the deviation from adiabaticity has been examined by Marsch et al. [1983] and Marsch and Richter [1984]. In Marsch et al. [1983] the rise in temperature due to the observed radial variation in the heat flux was calculated and compared to the deviation from adiabaticity in the observed temperature. It was found that the heat flux gradient was insufficient by a factor of 3-5 in the low speed solar wind ($V < 400 \text{ km s}^{-1}$), and by a factor of $>10$ in the high speed wind ($V > 600 \text{ km s}^{-1}$), to account for the observed temperatures at 1 AU.

Marsch and Richter [1984], in a parallel approach, calculated a polytropic index dependent upon the heat flux. They found that the index was $\sim 5/3$ (adiabatic) and did not vary significantly with velocity or radial distance. Thus, in both calculations it was shown that the heat flux gradient was insufficient to account for the observed tempera-
Burlaga and Ogilvie [1970] showed that the high solar wind temperatures at 1 AU could not be due to turbulent heating from stream-stream interactions. They also showed that shocks could not raise protons temperatures on a large scale. Pizzo et al. [1973] also determined that $T_p$ is more strongly correlated with $V$ than $dV/dt$. This is consistent with our results from Helios data which show little correlation between bulk velocity gradients and average proton temperatures. Correlation coefficients between temperature and velocity are $\sim 0.7$ while correlation coefficients between temperature and bulk velocity gradients are $\sim 0.3$. In addition, radial fits to the temperature separated by velocity show a distinct velocity dependence, again with correlation coefficients 0.7 to 0.9. In contrast, separation of the data by velocity gradients results in little correlation between radial position and temperature, and the fits show no particular dependence on $dV/dt$.

Magneto-hydrodynamic waves have been invoked by many authors to reconcile theoretical models with observed solar wind properties [e.g., Belcher, 1971; Barnes and Hollweg, 1974; Hollweg, 1975, 1978; Leer et al., 1982]. It seems clear that some form of extended energy and momentum deposition is needed to produce the high speed streams [Leer et al., 1982]. The remnants of that energy flux could then heat the protons between 0.3 and 1 AU.
There is considerable evidence for wave heating of solar wind protons. This evidence is primarily in the form of observations of the proton distribution in velocity space. For a collisionless plasma where the scale length is much greater than the gyro radii or the debye length (as is the case in the solar wind), the variation of $T_\parallel$ and $T_\perp$ are given by the double adiabatic invariants [Chew et al., 1956]

$$\left( \frac{3}{8} \varepsilon + \nabla \cdot \mathbf{v} \right) \left( \frac{T_\perp}{B} \right) = \left( \frac{3}{8} \varepsilon + \nabla \cdot \mathbf{v} \right) \left( \frac{T_\parallel B^2}{n^2} \right) = 0 \quad (4-7)$$

The variation of $B$ between 0.3 and 1 AU can be given by a power law fit [Musmann et al., 1977; Mariani et al., 1978], which we have verified, in which case

$$B = B_0 (r/r_0)^{-1.6} \quad (4-8)$$

Therefore we have $T_\perp \propto r^{-1.6}$ while $T_\parallel \propto r^{-0.8}$. Closer to the sun $B \sim B_\parallel \propto r^{-2}$ so that $T_\perp \propto r^{-2}$ while $T_\parallel$ is constant. In both cases we have that $T_\perp$ should decrease faster than $T_\parallel$, resulting in a "cigar-shaped" distribution at 1 AU.

This is not what is seen in the fast solar wind. Bame et al. [1975] and Marsch et al. [1982b] show that high speed streams have $T_\perp > T_\parallel$, the latter finding that this anisotropy is most pronounced at perihelion (0.3 AU). By contrast $T_\parallel > T_\perp$ is generally observed in the low speed solar wind. However, even in some data (near perihelion)
with bulk speeds less than 400 \text{ km s}^{-1} there is evidence of $T_1 > T_2$, as well as simultaneous Alfvénic wave activity [Marsch et al., 1981]. In addition the observed $T_a/T_p \sim 4$ in high speed streams [Marsch et al., 1982a] is consistent with heating by hydromagnetic waves. Therefore it appears that wave heating is the most likely candidate to cause the non-adiabatic behavior of the protons, which is predominantly found in the high speed wind.

As discussed in Chapter 1, the most plausible addition of momentum and energy to produce high speed streams is a magneto-hydrodynamic wave flux of solar origin. Since the energy/particle is larger for high speed streams, if the energy were supplied by waves, one would expect a positive correlation between proton temperature and velocity if some fraction of the wave flux was dissipated into heat. This would explain the dependence of the deviation from adiabaticity upon velocity. A brief inspection of Figures 1a and 2 lends support to this supposition.

In fact, average magnetic fluctuations are quite large in the Helios dataset with $0.267 \lesssim \Delta B/B \lesssim 0.494$, therefore one would expect nonlinear effects that could lead to damping of the fluctuations. Because the fluctuation in the field components is much larger than the fluctuation in the field magnitude, we interpret these fluctuations as being primarily Alfvénic in nature. Alfvén waves have been observed by many authors and are the predominant, though not the only,
wave mode [Belcher and Davis, 1971; Burlaga and Turner, 1976; Mariani et al., 1978; Denskat and Neubauer, 1981].

Following Schwenn [1983], we separated our data by velocity and, using a least squares routine, calculated power law fits as a function of radius to the temperature (see Schwenn [1983], Table 1), and to the double adiabatic invariant (Fig. 8a,b). What we see is a clear dependence of the deviation from adiabaticity upon the velocity. Low speed data ($V < 400$ km s$^{-1}$) show almost adiabatic behavior, whereas high speed data ($V > 600$ km s$^{-1}$) show the greatest deviation. The cold component of the solar wind that exhibits adiabatic behavior has a mean velocity of 250 km s$^{-1}$, in agreement with the two-fluid models which do not include waves as discussed in Chapter III [Freeman and Lopez, 1985a]. This suggests that the dynamics which form the high speed streams are also responsible for the extended heating of the solar wind.

To investigate the role of waves in the proton temperature, we need to know the radial dependences of the Alfvénic wave amplitudes. We use the magnetic variances of the hourly averages and assume that these are the average wave amplitudes for fluctuations with periods less than an hour. We thus define

$$\sigma_W \equiv (\sum \sigma^2_i)^{1/2}$$  \hspace{1cm} (4-9)
Fig. 8a. This figure plots the negative power of \( r \) from the power law fit to \( T/n^{2/3} \) (the adiabatic invariant) for Helios 1. The figure indicates an increase in \( T/n^{2/3} \) as \( r \) increases which is velocity dependent.
Fig. 8b. Same as Figure 8a, but for Helios 2.
where \( \sigma_i \) is the variance of the \( i \)th component of the magnetic field.

Alfvén waves are notable in that they are non-compressive, so the presence in the data of fluctuations in the magnitude of \( \mathbf{B} \), \( \sigma_B \), is indicative of other wave modes. These might be fast modes propagating nearly parallel to \( \mathbf{B} \) or they might be Alfvén waves coupled to fast modes, as suggested by Barnes and Hollweg [1974] and Burlaga and Turner [1976]. Fits to the entire data set give \( \sigma_B/\sigma_W \sim 0.2 \), so compressive modes account for \( \sim 20\% \) of the fluctuations.

The average characteristics of \( \sigma_B \) merit some mention. The compressive fluctuations have flatter radial gradients than \( \sigma_W \), which seems surprising since compressive modes should damp faster than noncompressive Alfvénic modes [Barnes, 1979]. The flatter gradients might then imply local generation of compressive modes, perhaps through the nonlinear decay of the Alfvén mode. The average value of \( \sigma_B \) rises with rising velocity, but the ratio \( \sigma_B/\sigma_W \) drops with increasing velocity. This last point is consistent with results from both Helios and other observations which found that the largest predominance of Alfvénic fluctuations were to be found in high speed streams [Belcher and Davis, 1971; Denskat and Neubauer, 1983].

We shall examine the "Alfvénic" component of the fluctuations and investigate any dissipation of these "Alfvén
waves." It is assumed that $\sigma_W$ represents a simple superposition of compressive and noncompressive modes. Let the background field be $\hat{B}_0$ and the average Alfvénic wave fluctuation be $\delta \hat{B}$. For Alfvén waves the fluctuation is noncompressive, so

$$(\hat{B}_0 + \delta \hat{B})^2 = \hat{B}_0^2 \quad (4-10)$$

therefore

$$2\delta \hat{B} \cdot \hat{B}_0 = -\delta \hat{B}^2. \quad (4-11)$$

If we add a compressive fluctuation, $\sigma_B$, parallel to $\hat{B}_0$, and take a time average

$$\langle \cos \theta \rangle = \frac{-\langle \delta B \rangle}{2\langle \hat{B}_0 + \sigma_B \rangle} = -\frac{\langle \delta B \rangle}{2\langle \hat{B}_0 \rangle} \quad (4-12)$$

where $\theta$ is the angle between $\hat{B}_0$ and $\delta \hat{B}$. The $\sigma_W$ of equation (7) has both a compressive and noncompressive component. To find the noncompressive part we write

$$\sigma_W^2 = (\delta \hat{B} + \delta \sigma_B)^2 = \delta \hat{B}^2 + 2\delta \hat{B} \sigma_B \cos \theta + \sigma_B^2 \quad (4-13)$$

then

$$\langle \delta B^2 \rangle = \frac{\langle \sigma_W^2 \rangle - \langle \sigma_B^2 \rangle}{(1 - \langle |\sigma_B| / \langle \hat{B}_0 \rangle \rangle^2)}. \quad (4-14)$$
For the purpose of this calculation, the radial power law fits to the data using equations (4-14) and (4-9) do not differ significantly. That is to say that $\sigma_B(r)$ and $\sigma_W(r)$ are not very different, but we will use (4-14) to calculate the wave amplitudes from the data.

Using a least squares fitting program we have calculated power law fits to $\delta B(r)$ obtained from the data, separated by velocity. Using these fits we have constructed a Carrington plot of $\delta B$ (Figure 9) similar to the velocity and temperature plots. What we see is a rough coincidence between high temperature (velocity) and $\delta B$. This relationship can be seen in a clearer, if more statistical, way in Figures (10a, b) and (11a, b). In these we see the logarithm of the value of the power law fit to $\delta B(r)$ at 1 AU vs. velocity and temperature. There is a clear rise in $\delta B$ vs. $V$ and $T$. This is consistent with a higher (wave driven) deviation from adiabaticity for higher velocities.

For dissipationless propagation of Alfvén waves in the WKB limit

$$\frac{\delta B^2}{n^{3/2}} (1 + V_A/V)^2 = \text{const.} \equiv \text{WKB invariant} \quad (4-15)$$

where $V_A$ is the Alfvén speed and $V$ is the solar wind speed [Hollweg, 1974; Jacques, 1977]. Power law fits to the WKB invariant were calculated from the data and the relevant result is plotted in Figures (12) and (13). What can be
Fig. 9. This Carrington plot plots the value of $\delta B$ (in gammas) normalized to 0.3 AU in the same format as Figure 1.
Fig. 10a. This figure plots the value of the power law fit to $\delta B(r)$ (in gammas) at 1 AU versus velocity. The data is from Helios 1.
Fig. 10b. Same as Figure 10a, but for Helios 2.
Fig. 11a. This figure plots $\delta B(r)$ at 1 AU versus temperature for Helios 1.
Fig. 11b. Same as Figure 11a, but for Helios 2.
Fig. 12. This figure plots the negative power of r from the radial fit to the WRB invariant (see equations (4-15)) versus velocity. The figure indicates a decrease in the invariant as r increases.
Fig. 13: This figure plots the negative power of \( r \) from the radial fit to the WKB invariant versus temperature. Note that the lowest temperature group (\( T < 15000 \) K) appears to have no dissipation of waves.
seen is that the WKB invariant, instead of being constant, decreases as r increases except, perhaps, in the cold solar wind (T < 15000 K). We interpret this decrease as a function of r in the WKB invariant as dissipation of wave energy into the plasma. Data from the radial alignments indicate a decrease in the WKB invariant in the high and middle speed lineups. The slow speed lineup shows little or no decrease in the WKB invariant, which is not what the statistical gradients indicate.

Further support for some dissipation of wave energy is the fact that \( \delta B(r)/B_0(r) \) is roughly constant. This is shown in Figure (14), where we have plotted the exponent of the power of r derived from a fit to \( \delta B(r)/B_0(r) \) versus solar wind speed. What one sees is the power of r is very nearly zero for all solar wind velocities so \( \delta B/B_0 \) is approximately constant over a radial distance of 0.7 AU. This has been interpreted as evidence for the saturated propagation of waves [Hollweg, 1978; Villante, 1980]. The saturation level of wave energy density might then be a threshold for wave-particle heating and damping of the waves [Mariani et al., 1978].

It is interesting to note that the average value of \( \delta B/B \) increases with velocity and temperature (Figures 15 and 16). Least square fits to the number density (Figures 17 and 18) show the well known decrease of density with velocity and temperature. The number flux invariant, \( F_0 \),
Fig. 14. This figure plots the negative power of r from the power law fit to $\delta B/B$ versus velocity.
Fig. 15. This figure plots the value of $\delta B/B$ from the power law fit versus velocity.
Fig. 16. This figure plots the value of $\delta B/B$ from the power law fit versus temperature.
Fig. 17. This figure plots the value of $n(r)$ at 1 AU versus velocity.
Fig. 18. This figure plots the value of $n(r)$ at 1 AU versus temperature.
also decreases with increasing velocity. Thus, if the average power dissipation for Alfvénic heating is proportional to the density (or inversely proportional to $V_A$), it would help explain why the threshold for heating in terms of $\delta B/B$ is, on average, higher in that part of the solar wind where the influence of waves seems to be more pronounced. In fact the value of fits to the data at 1 AU of the function $nV_0\delta B/B$ is roughly constant as a function of velocity.

Let us assume the dissipation of waves implied by Figures (12) and (13) directly heats the protons. Hollweg [1974] self-consistently included wave dissipation in the energy equation for the protons. This adds a wave related term to the left-hand side of equation (4-1)

$$\quad - v \frac{d}{dt} \frac{\delta B^2}{8\pi} + \frac{1}{A} \frac{d}{dt} \left[ A\left( \frac{\delta B^2}{4\pi} (V_A + \frac{3}{2} V) \right) \right]$$

(4-16)

where $A$ is the flux tube area. The two terms in (4-16) can be understood on physical grounds by observing that the force per unit volume exerted by the waves is

$$\quad - v \frac{\delta B^2}{8\pi}$$

(4-17)

and the Alfvénic energy flux is

$$\quad \frac{\delta B^2}{4\pi} \left( \frac{3}{2} V + V_A \right)$$

(4-18)
Thus (4-16) is the divergence of the (radial) Alfvén wave flux plus the work done on the plasma.

Using equation (4-16) and assuming \( \mathbf{v} \cdot \mathbf{\hat{q}} \sim 0 \), we can rewrite equation (4-5) as

\[
\frac{PV}{(\gamma_p - 1) \frac{d}{dr} \ln (\rho/\rho_p)} = V \frac{d}{dr} \frac{\delta B^2}{8\pi} - \\
\frac{1}{r^2} \frac{d}{dr} \left[ r^2 \left( \frac{\delta B^2}{4\pi} \left( V_A + \frac{3}{2} V \right) \right) \right]
\tag{4-19}
\]

where spherical symmetry is assumed. We now substitute into the above equation \( \gamma_p = 5/3 \) and the velocity binned power law fits derived from Helios data and integrate (4-19) from .3 to 1 AU. Specifically let

\[
\frac{\delta B(r)}{\delta B(r_0, V)} = (\frac{r}{r_0})^{-n(V)} \tag{4-20a}
\]

\[
\frac{P}{\rho \gamma} (r) = \frac{P}{\rho \gamma} (r_0, V) (\frac{r}{r_0})^{\theta(V)} \tag{4-20b}
\]

\[
T(r) = T(r_0, V) (\frac{r}{r_0})^{-\gamma(V)} \tag{4-20c}
\]

\[
n(r) = n(r_0, V) (\frac{r_0}{r})^2 \tag{4-20d}
\]

where \( \gamma(V) \) is given in Table 2 and we select \( r_0 = 1 \) AU. It is also assumed that \( V \) is constant over the radial range in question.
We know, however, that $V$ is not constant from examining radial alignments between Helios 1 and Helios 2. Schwenn [1981] found a velocity gradient in the solar wind of $50$ km s$^{-1}$ AU$^{-1}$ in streams with $V$ less than $400$ km s$^{-1}$. Velocity gradients decreased in higher speed streams. Our examination of Helios lineups results in somewhat higher, but generally consistent, velocity gradients. Thus for an average slow solar wind stream, max $(\Delta V/V) \sim 10\%$. Since most of the data is taken closer to aphelion the error is $\lesssim 5\%$, which we shall see is small compared to other uncertainties involved.

The integrals of the RHS and LHS of (4-19) are tabulated in Table 3. What one sees is that the ratio of thermal energy gained to wave energy loss is on the order (roughly) of unity, particularly in high speed streams. The extremums given in Table II also show that the range of this ratio can be quite great. This is because a relatively small change in the exponent of $r$ can produce large changes in the result. Gradients derived from radial alignments in high and middle speed streams give similar results. For slow speed streams we do not have a completely reliable lineup, but if we use the expansion geometry derived from the lineups then we obtain the figures on Table 3 that are in parentheses. While this is a rather rudimentary calculation, it appears that the observed average temperatures may be accounted for by the apparent dissipation of
wave energy.

In fact, we see that there seems to be an excess in wave energy dissipated compared to heat added. Some of the Alfvénic wave flux might be degenerating into lower frequency modes with periods greater than an hour. So the fact that the dissipated wave energy is larger than the heat required by the protons is not very troublesome.
CHAPTER V

THE RELATIONSHIP OF TEMPERATURE
TO OTHER DYNAMICAL QUANTITIES

We saw in Chapter IV how the dissipation of magnetic fluctuations could produce the high temperatures at 1 AU. Implicit in that analyses was a relationship between temperature and velocity. In addition, the T-V scatter plot (Fig. 7) introduced in Chapter III illustrated a definite correlation between proton temperature and bulk flow speed. Furthermore, this relationship changed at $V \sim 500$ km s$^{-1}$. We now proceed to analyze this relationship, as well as the relationship of T to other quantities.

Early investigations of the solar wind noted a qualitative relationship between proton temperature and velocity [Neugebauer and Snyder, 1966; Hundhausen et al., 1970]. In Chapter I we saw that the simplest solar wind model, a one-fluid, isothermal corona, hydrodynamic flow, predicted a rough proportionality ($V \propto T^{1/2}$) between temperature and velocity. Thus, in looking for turbulent heating in the solar wind, Burlaga and Ogilvie [1970] fit Explorer data to the relation

$$T^{1/2} = AV + B \quad \text{(5-1)}$$

This was extended by Burlaga and Ogilvie [1973] using
additional data that, together with the earlier data, covered more than five years. Their result is shown in Figure (19).

To compare Helios findings to these earlier results we must first make our data consistent with that in Burlaga and Ogilvie [1973]. Inspecting Figure 19, one immediately sees that the data lies essentially below 500 km s\(^{-1}\). Yet it is also at 500 km s\(^{-1}\) that the T-V relationship qualitatively enters a new region. This will turn out to be of some importance. Thus we must exclude data with \(V > 500\) km s\(^{-1}\)

In addition, all of the data analyzed in Burlaga and Ogilvie [1973] were taken at 1 AU, whereas Helios data is distributed over radial distance. We remove this discrepancy by normalizing the temperatures to 1 AU using the power law fits to the data given in Table 2. These statistical gradients should give us the average temperatures at 1 AU associated with those inner data points. As in the last chapter we assume that the bulk flow velocity is a constant over .7 AU, however, in the case of the T-V relationship, a \(\Delta V/V \sim 5\%\) is not a completely insignificant number compared to the other uncertainties. However, 5% more error in the fits will not affect the basic results. And so, with velocity binned and properly normalized data we calculate a least squares fit to equation (5-1).
Fig. 19. Temperature versus velocity data (from Burlaga and Ogilvie [1973]). The line on the graph is the fit derived by Burlaga and Ogilvie [1973] for the T-V relationship.
The results are given in Table 4, along with results from Burlaga and Ogilvie [1973]. The similarity of the results extending from 1966 to 1980 lends strong support to the conclusion of Burlaga and Ogilvie [1973] that the fit to (5-1) does not substantially change over the solar cycle. This means that the relationship of proton temperature to flow speed of 1 AU, or alternatively total energy/unit particle, is roughly a solar constant.

Other Functional Fits to the Data

As previously stated, the Helios data show a distinct change in the T-V relationship at \( V \sim 500 \text{ km s}^{-1} \). To investigate the behavior of these two regions above and below 500 km s\(^{-1}\) we can calculate a least squares fit to (5-1) as well as the following functions:

\[
\frac{T_1}{3} = AV + b \tag{5-2a}
\]

\[
\frac{T_2}{3} = AV + B \tag{5-2b}
\]

\[
T = AV + B \tag{5-2c}
\]

\[
T = AV^B \tag{5-2d}
\]

\[
T = Ae^{BV} \tag{5-2e}
\]
These are not the only ones imaginable, but they give a reasonably wide range of functional dependence, while being forms that can be conveniently fit using the method of least squares. Some of these forms like $T_1/2 = AV + B$ (or for that matter $V^2 = AT + B$) are implied by polytropic one-fluid models, while the function $Ae^{BV}$ is not related to any solar wind model. However, the linearity of the data on a semi-log plot (Fig. 7) makes such a fit seem a reasonable thing to calculate. In addition, if the argument, BV, is small, then $Ae^{BV}$ is roughly a low order polynomial. As we shall see, it can be a convenient form to work with.

To determine which function provides the best fit, we calculate $\chi^2_v$ as follows:

$$\chi^2_v = \frac{1}{N-1} \sum_{i=1}^{N} \frac{(T_i - T(V_i))^2}{\sigma_i^2}$$  \hspace{1cm} (5-5)$$

The total standard deviation of each temperature point is

$$\sigma_i = \sigma_{T_i} + \frac{dT(v)}{dv} \sigma_{vi}$$  \hspace{1cm} (5-6)$$

where $\sigma_{T_i}, \sigma_{vi}$ are the standard deviations of the hourly temperature and velocity averages, respectively. The best fit is the function that gives a minimum in $\chi^2_v$. The results for the full Helios 1 data set (28532 points with $V < 500$ km s$^{-1}$, 8080 points with $V > 500$ km s$^{-1}$) are given in Table 5. Although the $\chi^2_v$ are not much different from each other
it appears that for $V < 500$ km $s^{-1}$, $T^{1/3} \sim (or \ T^{1/2} \sim V)$ and for $V > 500$ km $s^{-1}$, $T \sim V$ are the best fits. We do not present Helios 2 results since they are quite similar to those values in Table 5.

To examine the variation of the temperature velocity relationship over the solar cycle, we separated the Helios 1 data into two subsets. The first subset contains data from 1974-1976, and the other has data from 1978-1980. These data sets were further subdivided by velocity and then fits to the equations given in (5-2) were calculated. The results are given in Table 6. Again, similar results from Helios 2 are not presented.

What can be seen by inspecting Table 6 is that the fits do not change greatly over a solar cycle. Furthermore, the values in the functional part of the fits become more like each other. Thus the differences between the two states appear to diminish near solar maximum. This is a reasonable result, since the long-lived high speed streams disappear as one approaches solar maximum and the solar wind becomes more "blended." Still the basic functional difference between the two states is preserved over the solar cycle. This result, taken with the results of Burlaga and Ogilvie [1973], is a strong indication that the $T-V$ relationship is rooted in the acceleration and heating of the solar wind.
The T-V relationships imply an interesting fact about the solar wind at 1 AU. Namely, if we define $\chi$ to be

$$\chi = \frac{d \ln T}{d \ln V}$$

(5-7)

then it represents the fractional change in temperatures ($dT/T$) for a given fractional change in velocity ($dV/V$). Since $2dV/V = dE/E$, where $E$ is the total energy per unit particle (a flow constant), and $kT$ is the thermal energy per unit particle, then $\chi$ is a measure of the efficiency of the mechanism by which an increase in total energy per particle results in an increase in thermal energy per particle.

For an exponential fit to the data ($T \propto \exp (\alpha V)$), $\chi$ is easily found to be

$$\chi = \alpha V$$

(5-8)

Since the fits are averages over their respective velocity ranges, we should use the average velocities (380 km s$^{-1}$ and 650 km s$^{-1}$). If we define $\chi_S = \chi (V < 500 \text{ km s}^{-1})$ and $\chi_F = \chi (V > 500 \text{ km s}^{-1})$, then $\chi_S = 3.28$ and $\chi_F = 1.93$. The result that $\chi$ is larger for slower speeds holds in all the fits to the data and so is stastically significant. Thus it seems that the slow speed wind is more effectively heated than the fast wind. This seems contradictory, since we know the high speed wind to be hotter, but the result is not
contradictory, since $\chi$ only refers to relative increases, not absolute numbers.

Let us now examine the radial variation of $\chi$. We differentiate $\chi$ to get

$$\frac{d\chi}{dr} = \frac{d}{dr} \left( \frac{d \ln T}{d \ln V} \right) = \frac{d}{d \ln V} \frac{d \ln T}{dr}$$  \hspace{1cm} (5-9)

The last step in (5-9) is possible since $V$ at 1 AU is independent of $r$. Since we know the radial variation of $T$, $(T \propto r^{-\gamma})$ equation (5-9) becomes

$$\frac{d\chi}{dr} = -\frac{1}{r} \frac{d \gamma(V)}{d \ln V} = -\frac{V}{r} \frac{d \gamma(V)}{d V}$$ \hspace{1cm} (5-10)

Inspecting Table 2 we see that, in general, $\gamma(V)$ decreases with $V$, therefore $d\chi/dr > 0$. Thus $\chi$ should decrease as we move inward towards the sun.

We can calculate in two parallel ways what the $T-V$ relationship should be at a point closer to the sun, say 0.3 AU. The most straightforward method is simply to normalize the temperatures to 0.3 AU and then calculate the $T-V$ fits as before. The second is to write the temperature as

$$T = f(r,V) \tau(V)$$  \hspace{1cm} (5-11)

where $\tau(V) = \tau_0 e^{aV}$ is the temperature given by the exponential $T-V$ fit at 1 AU and $f(r,V)$ contains all the
radial dependence. Then, if we use our radial fits to the data

\[ T = r^{-\gamma(V)} \tau_0 \exp(aV) = \tau_0 \exp(aV - \gamma(V) \ln r) \] (5-12)

We parameterize \( \gamma(V) \) to be (excluding data with \( V > 800 \) km s\(^{-1}\))

\[ \gamma(V) = \begin{cases} 1.75 - V/600 & V < 500 \text{ km s}^{-1} \\ 1.145 - V/1800 & V > 500 \text{ km s}^{-1} \end{cases} \] (5-13)

These expressions are necessarily crude, given the small number of points in Table 2, but they give reasonable values for \( \gamma \). With equations (5-12) and (5-13) we can calculate an "effective" \( a \) and \( \tau_0 \)

\[ a' = a'(r) = a - \frac{d\gamma}{dV} \ln r \] (5-14)

\[ \tau_0' = \tau_0'(r) = \tau_0 \exp \left[ (V \frac{dV}{dr} - \gamma) \ln r \right] \] (5-15)

in which case

\[ T(V, r) = \tau_0'(r) \exp(a'(r) V) \] (5-16)
There is one other way to get a T-V fit close to the sun, and that is simply to use only data with 0.3 AU < r < 0.4 AU. This reduction in the data set still leaves 4319 low speed and 1006 high speed data points in the Helios 1 data set alone. Thus we still have a good statistical sampling. The results of these three methods of calculating an exponential T-V fit at .3 AU, as well as the 1 AU values, are given in Table 7. What we can see is that the normalized data results and the parameterized calculation are in close agreement, which is not surprising. What is of interest is that the empirically derived fits differ greatly from the other fit derived from near perihelion data.

We see that the empirical prediction for the low speed wind is a decrease in $\alpha$ of $\sim .0019$ from its 1 AU value, but the data in that region shows a decrease of .00076 in $\alpha$ from its 1 AU value. Since $\chi = \alpha V$, it is clear that $\frac{d\chi}{dr}$ is not as large as expected from the values of $T$ and $V$ at 1 AU. If we consider $\gamma$ to be a function of $r$, as well as of $V$, then (5-10) becomes

$$\frac{d\chi}{dr} = -(V \ln r \frac{d}{dr} \frac{d\gamma}{dV} + \frac{V}{r} \frac{d\chi}{dV})$$ (5-17)

A smaller than expected $\frac{d\chi}{dr}$ means that

$$\ln r \frac{d}{dr} \frac{d\gamma(V, r)}{dV} > 0.$$ (5-18)
Since inside 1 AU \( \mathcal{L} n < 1 \), \( \frac{d\gamma}{dV} \) increases as \( r \) decreases. Since \( \frac{d\gamma}{dV} < 0 \), this means that the difference in heating between higher and lower speeds within the low speed wind diminishes as one approaches the sun.

In the high speed wind we see that \( \alpha \), and thus \( \chi \), increase as we go to \( .35 \) AU. But for \( \frac{dx}{dr} \) to be negative implies that

\[
\ln r \frac{d}{dr} \frac{d\gamma}{dV} > -\frac{1}{r} \frac{dx}{dV}
\]  

(5-19)

When \( \frac{dx}{dr} = 0 \) we find that

\[
-\frac{d}{\ln r} (\ln r) = \frac{d}{dV} (\frac{d\gamma}{dV}) \frac{dx}{dV}
\]

(5-20)

so

\[
\frac{d\gamma}{dV} = \frac{d}{dV} \frac{dx}{dV} = \frac{K(V)}{\ln r}
\]

(5-21)

where \( K \) is positive and depends only on \( V \). Again \( \frac{d\gamma}{dV} < 0 \), and we see that \( \left| \frac{d\gamma}{dV} \right| \) decreases as \( r \) decreases, so the differences in heating between velocities also decreases.

Thus another difference between the slow and fast winds is that \( (\frac{dx}{dr})_{\text{AVE}} \) is less than zero for the fast wind and greater than zero for the slow wind. For the slow wind we found a lessening of relative heating difference as one approaches the sun. We found the same thing for high speed
wind, except that to have $\frac{d\gamma}{dr}$ less than zero, $|\frac{d\gamma}{dV}|$ must
decrease faster than $1/n r$ as one goes towards the sun. Thus, $|\frac{d\gamma}{dV}|$ must decrease faster in the fast wind than in the slow wind as $r$ decreases.

**Other Temperature Relationships**

In this section we will examine the relationship of temperature to number density and momentum flux. The inverse relationship between velocity and number density has been noted by various authors [Steinitz and Eyni, 1980; Schwenn, 1983, Fig. 17]. Thus there should be an inverse relationship between number density and temperature. In Figure 18 we see that this is as expected. The T-V-N relationship is depicted pictorially in Figure 20 and the two states can clearly be discerned. Inspecting Figure 20, we can see that in the low speed wind there is a much greater variation of the density when compared to the high speed wind. Fits to the data show that $n \propto V^{-2}$ in the slow wind, while $n \propto V^{-1}$ in the fast wind. This fact gives a qualitative explanation for why $\chi_S > \chi_F$, as noted in the previous section. The last chapter showed that the heating of protons increases monotonically with $V$. A faster drop in $n$ for a given rise in $V$ (and the heat deposited) would give a correspondingly greater rise in $T$, thus $\chi_S > \chi_F$. 
Fig. 20. This figure plots velocity, temperature and (color-coded) density data.
The relationship between proton temperature and momentum flux density shows a clear bifurcation into two fundamentally different states. This can be seen in Figures 21 and 22 which plot the temperature versus momentum flux density per unit mass for data with $V < 400 \text{ km s}^{-1}$ and $V > 600 \text{ km s}^{-1}$, respectively. These scatterplots show that for low speeds, $T$ decreases, or remains constant, with increasing $nV^2$, while for high speeds, $T$ increases with increasing $nV^2$.

The visual result is easily verified by calculating $T(nV^2)$ for the same kinds of functions listed in equation (5-2). For the sake of brevity, we present only the functional dependences for three functional forms to the $T(nV^2)$ relationship, with the results given in Table 7. One sees that for $V > 500 \text{ km s}^{-1}$, $T(nV^2)$ has a positive slope, and that for $V < 500 \text{ km s}^{-1}$ the opposite is true.

This result can be interpreted in the framework of Leer et al. [1982]. In that model, momentum added below the sonic point decreased the flow speed at 1 AU, while momentum added above the critical point simply raises the flow speed at 1 AU. In addition, it has been shown that the average momentum flux is not very sensitive to velocity [Stienitz and Eyni, 1980; Schwenn, 1983]. Thus it would appear that, on average, both states of the solar wind have the same amount of momentum to distribute in a "budget."

If the slow solar wind has a momentum addition term
Fig. 21. This figure plots temperature versus momentum flux density per unit mass (data normalized to 1 AU) for data with $V < 400$ km s$^{-1}$. $T$ appears to decrease as $nV^2$ increases.
Fig. 22. Same as Figure 21, except data is that with $V > 600$ km s$^{-1}$. $T$ clearly increases as $nV^2$ increases.
If the slow solar wind has a momentum addition term characterized by D in equation (1-24), then the data results would indicate that the momentum is deposited below the sonic point, relatively low in the corona. This would increase the mass flux and so lower the flow energy/particle at 1 AU as well as the total amount of thermal energy per particle. This would generate the cool, slow wind. On the other hand, this implies that momentum addition in the high speed wind occurs beyond the sonic point. Thus any added momentum will lead to higher flow speeds at 1 AU and so the average thermal energy also increases. This would generate the hot, fast wind.

This point becomes a key by which we may advance our understanding of the differences between the two states of the solar wind and how this influences temperatures at 1 AU. The next chapter will tie these observations together into a phenomenological model for the solar wind.
CHAPTER VI. CONCLUSION

In this chapter we will review several of the major points discussed in the preceding chapters. These points are consistent with the view that the high temperatures in high speed streams, as well as the streams themselves, are generated by the extended deposition of MHD wave momentum and energy. Finally, a phenomenological model of solar wind generation which fits our results, as well as other important results (also, incidentally, from Helios data) is discussed. We now proceed to reiterate the major results of the analysis of Helios data.

1. Analysis of the hourly averaged magnetic fluctuations show that the WKB invariant (equation 4-15) is not constant. In fact, it decreases with increasing radial distance. Under the assumption that the fluctuations are predominantly Alfvén waves, the nonconservation of equation (4-15) implies a dissipation of waves into heat.

2. The magnetic fluctuations show, on average, a positive correlation with velocity and proton temperature; higher $T_p$ and $V$ are associated with higher $\delta B$.

3. The dissipation of wave energy implied by point (1) is calculated and found to be of adequate magnitude to account for the bulk of the observed heating of the protons between 0.3 and 1 AU. This calculated heating is greater in high speed streams due to point (2) and is consistent with
the observed dependence of the deviation from adiabaticity upon bulk flow speed.

4. We have examined an extremum of solar wind behavior, the cold solar wind. The cold solar wind is notable in that it is consistent with two-fluid models of the solar wind that do not include extended heating and acceleration by MHD waves of solar origin. There is some evidence that there is little or no Alfvénic dissipation in the cold solar wind (Figure 13).

5. The relationship of temperature to velocity (or temperature to total energy per particle), while being different for slow and fast streams, is essentially constant at least from 1965 to 1980. Extrapolations of the T-V relationship to 0.3 AU differs from the T-V relationship derived from data between 0.3 and 0.4 AU. This indicates that the heating changes over the radial range in question and that the logarithmic gradients of T and δB are themselves radially dependent.

6. As we discussed in Chapter V, the relationship of temperature to momentum flux density implies that momentum/energy addition in the low speed wind is "localized" below the sonic point and vice versa for the high speed state.

If we examine points (1), (2) and (3) we see that there is reasonable evidence for the view that waves in the solar wind provide the dominant source of heat for the protons in
the inner heliosphere. Points (2) and (3) can then provide a qualitative explanation for the positive T-V correlation, while the constancy of the T-V relationship (point (5)) implies that the heating mechanism is the same over the solar cycle.

Point (4) lends additional support to the importance of MHD waves in the solar wind. The cold solar wind is clearly part of the solar wind continuum. Since the rest of the solar wind appears to reach its higher temperatures due to wave heating, the link between the Alfvénic wave flux and temperature provides strong, though indirect, evidence for the link between MHD waves and the higher velocities in the rest of the solar wind. Furthermore, point (6), taken with points (2) and (3), is consistent with MHD wave effects as predicted by the model of Leer et al. [1982]. We shall now discuss a scenario which is consistent with the above ideas as well as observations of the solar wind energy flux.

**Other Considerations and a Possible Scenario for Solar Wind Generation**

Hollweg [1978] derived an expression for R, the ratio of Alfvénic wave flux to total energy flux at the sun.

\[ R = 2 \left( \frac{\|B\}}{V} \right) \]  \( \frac{V_A}{V} \) 1 AU

(6-1)

This was derived assuming that bulk flow dominates at 1 AU,
that $V >> V_A$ at 1 AU and $V_A >> V$ near the sun, and that between the sun and the earth, $\delta B$ is given by equation (4-15). With Helios data, equation (6-1) gives average ratios of between ~2% for slow wind to ~12% for fast wind. However, we know that $\delta B$ has a greater radial gradient than that implied by equation (4-15), so these percentages are underestimates and a figure of ~1/3 might be reasonable for the high speed wind. In addition, Hollweg [1982] has presented evidence of sufficient Alfvénic wave flux in the corona to produce high speed streams.

While the percentage of energy flux in waves seems to be a relevant distinguishing characteristic between the two states of the solar wind, the total energy flux (equation 1-14) is constant for all velocities [Schwenn, 1983; Marsch and Richter, 1984]. In addition, the momentum flux is also roughly the same for all velocities [Stienitz and Eyni, 1980; Schwenn, 1983]. What varies greatly, and is responsible for the large energy per particle in high speed streams, is the anticorrelation between velocity and number flux. This probably comes about because of the different results from subsonic and supersonic energy and momentum deposition. As we saw in Chapter I, the model of Leer et al. [1982] demonstrated how higher coronal temperatures or pre-sonic momentum addition can lead to larger particle fluxes. Thus the energy-momentum budget seems to be a global constant, and so what is important is where and how
energy and momentum are deposited along the trajectory of the flow.

A phenomenological model of solar sources of the solar wind should be consistent with our conclusions concerning the origin of the high proton temperatures at 1 AU, as well as the observations of a global energy flux. There is a general picture which can fit these constraints that is that the ultimate source of solar wind energy is convection within the sun.

This is not a new idea. Parker [1983] has proposed a mechanism by which mechanical energy of convection, via the coronal magnetic field, is directly dissipated and that energy heats the corona. What Parker [1983] proposed is that closely packed, twisted, closed flux tubes could not form an equilibrium configuration. At the boundary between two of any three flux tubes, reconnection would occur, and the dissipated energy could heat the corona. It is the twists in the flux tube that produce the appropriate magnetic topology for reconnection. The energy for twisting the flux tubes around each other would come from the convective motion of the plasma in which the foot of the flux tube is anchored.

However, if the flux tube is open, like in a corona hole, some of the twists could propagate out as MHD waves. The fast and slow modes would rapidly damp out, heating the corona, while the Alfvén mode could propagate out until \( \delta B/B \)
becomes large enough and the waves nonlinearly damp. This has been investigated by Hollweg [1981], who found that it is likely that sufficient energy flux could be generated and propagated out of the open region to account for high speed streams. This wave flux could then deposit the post-sonic energy and momentum. The net power in either waves or direct dissipation should be roughly the same since the driver in both cases is convection in the photosphere. This would be consistent with the observation of a global energy flux.

The phenomenological model brings these two complementary ideas together within the solar topology for the sources of the solar wind discussed by Hundhausen [1979]. Figure 23 illustrates this concept for a single open region. The slow state is pictured to come from the edges of open regions, close to the areas where almost all the convective energy supplied by the sun is transformed into heat by mechanical dissipation of magnetic field. This heats the nearby edge of the open region. Thus the slow wind, in this model, is primarily driven by thermal pressures. On the other hand, the high speed wind has a very large Alfvénic energy flux propagating out along open flux tubes, and so can be described as being a wave driven state of the solar wind.
Fig. 23. This figure illustrates a phenomenological model for the generation of the solar wind. Convective motions in the sun provides the energy source. Thermally driven, low speed wind comes from the edges of the open region. In the center of the open region a significant fraction of the energy is convected by MHD waves which then can provide momentum and energy in the supersonic regime, giving rise to the fast solar wind.
This picture is consistent with both the observations of Helios and the model of Leer et al. [1982]. This scenario, however, is conjecture based upon interpretations of Helios data, and not a solid conclusion. What can be said is that the observed evolution of the proton temperature between 0.3 and 1 AU strongly suggests a scenario very much like one presented above, in which MHD waves of solar origin are at the root of the observed proton temperatures and the two state structure of the solar wind in the inner heliosphere.
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TABLE 1

A comparison between a variable coulomb logarithm case and a constant coulomb logarithm case for two-fluid solar wind models with the same base conditions ($T_{eo} = T_{po} = 2 \times 10^6$ K, $N_o = 3 \times 10^7$ cm$^{-3}$). The values given are at 1 AU. (From Nerney and Barnes [1977].)

<table>
<thead>
<tr>
<th></th>
<th>$T_p$ (10$^6$ K)</th>
<th>n (cm$^{-3}$)</th>
<th>V (km s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variable $L^*$</td>
<td>4.4</td>
<td>15</td>
<td>250</td>
</tr>
<tr>
<td>Constant $L^+$</td>
<td>6.1</td>
<td>16</td>
<td>260</td>
</tr>
</tbody>
</table>

$^*$Hartle and Sturrock [1968]

$^{+}$Durney [1973]
TABLE 2

$\gamma(V) -$ The Power of $r$ in the Fit to the Temperature

<table>
<thead>
<tr>
<th>$V$ (Km s$^{-1}$)</th>
<th>Helios 1</th>
<th>Helios 2</th>
<th>Both</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 300</td>
<td>1.324</td>
<td>1.272</td>
<td>1.305</td>
</tr>
<tr>
<td>300 - 400</td>
<td>1.217</td>
<td>1.13</td>
<td>1.183</td>
</tr>
<tr>
<td>400 - 500</td>
<td>1.02</td>
<td>1.036</td>
<td>1.035</td>
</tr>
<tr>
<td>500 - 600</td>
<td>0.82</td>
<td>0.921</td>
<td>0.866</td>
</tr>
<tr>
<td>600 - 700</td>
<td>0.767</td>
<td>0.836</td>
<td>0.797</td>
</tr>
<tr>
<td>700 - 800</td>
<td>0.827</td>
<td>0.76</td>
<td>0.777</td>
</tr>
<tr>
<td>&gt; 800</td>
<td>-0.781</td>
<td>1.775</td>
<td>0.636</td>
</tr>
</tbody>
</table>
Table 3

Right Hand and Left Hand Sides of the Integral of Equation (4-19) From 0.3 to 1 AU, as Calculated From Helios 1 Data ($\times 10^{-3}$ ergs cm$^{-2}$ s$^{-1}$)

<table>
<thead>
<tr>
<th>Solar Wind Velocity (km s$^{-1}$)</th>
<th>LHS</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 300</td>
<td>$-0.08 \pm 0.84$</td>
<td>$3.23 \pm 1.47$</td>
</tr>
<tr>
<td></td>
<td>($-0.19 \pm 2.0$)</td>
<td></td>
</tr>
<tr>
<td>300-400</td>
<td>$1.12 \pm 1.26$</td>
<td>$6.65 \pm 1.96$</td>
</tr>
<tr>
<td></td>
<td>(2.7 $\pm 3.0$)</td>
<td></td>
</tr>
<tr>
<td>400-500</td>
<td>$3.96 \pm 2.29$</td>
<td>$17.5 \pm 4.9$</td>
</tr>
<tr>
<td></td>
<td>(9.4 $\pm 5.7$)</td>
<td></td>
</tr>
<tr>
<td>500-600</td>
<td>$8.2 \pm 3.7$</td>
<td>$11.0 \pm 3.3$</td>
</tr>
<tr>
<td>600-700</td>
<td>$12.3 \pm 4.4$</td>
<td>$25.3 \pm 7.1$</td>
</tr>
<tr>
<td>700-800</td>
<td>$15.0 \pm 9.7$</td>
<td>$37.5 \pm 15.7$</td>
</tr>
</tbody>
</table>

*The values in parenthesis are for the case of inflated flux tubes (see text).
Table 4

\( T^{1/2} = AV + B \) (\( V \) in Km \( s^{-1} \), \( T \) in \( 10^3 \) K)

<table>
<thead>
<tr>
<th>Satellite</th>
<th>Date</th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>*Explorer 43</td>
<td>1971</td>
<td>0.033 ± .001</td>
<td>-4.8 ± .4</td>
</tr>
<tr>
<td>*Explorer 34</td>
<td>1967</td>
<td>0.036 ± .004</td>
<td>-5.6 ± 1.6</td>
</tr>
<tr>
<td>*Pioneer 6</td>
<td>1966</td>
<td>0.032 ± .001</td>
<td>-3.3 ± .4</td>
</tr>
<tr>
<td>Helios 1</td>
<td>1974-80</td>
<td>0.031 ± .002</td>
<td>-4.4 ± .1</td>
</tr>
<tr>
<td>Helios 2</td>
<td>1976-80</td>
<td>0.032 ± .002</td>
<td>-4.7 ± .1</td>
</tr>
</tbody>
</table>

*Burlaga and Ogilvie [1973]*
Table 5

Results of Fits to Helios 1 Data

<table>
<thead>
<tr>
<th>Velocity in Kms⁻¹</th>
<th>Function</th>
<th>$T$ in $10^3$ K</th>
<th>A</th>
<th>B</th>
<th>$\chi^2_{0}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 500</td>
<td>$T^{1/3} = AV + B$</td>
<td>0.0106 ± 0.001</td>
<td>-0.278 ± 0.03</td>
<td>276.9</td>
<td></td>
</tr>
<tr>
<td>&lt; 500</td>
<td>$T^{1/2} = AV + B$</td>
<td>0.0311 ± 0.002</td>
<td>-4.39 ± 0.08</td>
<td>288.5</td>
<td></td>
</tr>
<tr>
<td>&lt; 500</td>
<td>$T^{2/3} = AV + B$</td>
<td>0.0823 ± 0.006</td>
<td>-16.32 ± 1.6</td>
<td>321.7</td>
<td></td>
</tr>
<tr>
<td>&lt; 500</td>
<td>$T = AV + B$</td>
<td>0.504 ± 0.004</td>
<td>-128.4 ± 1.6</td>
<td>504.2</td>
<td></td>
</tr>
<tr>
<td>&lt; 500</td>
<td>$T = AV^B$</td>
<td>$2.46 \times 10^{-7}$ ± $0.3 \times 10^{-7}$</td>
<td>3.22 ± 0.02</td>
<td>319.16</td>
<td></td>
</tr>
<tr>
<td>&lt; 500</td>
<td>$T = Ae^{BV}$</td>
<td>1.8 ± 0.06</td>
<td>0.0086 ± 0.0001</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>&gt; 500</td>
<td>$T^{1/3} = AV + B$</td>
<td>0.0058 ± 0.0001</td>
<td>2.13 ± 0.08</td>
<td>317.6</td>
<td></td>
</tr>
<tr>
<td>&gt; 500</td>
<td>$T^{1/2} = AV + B$</td>
<td>0.0218 ± 0.0005</td>
<td>0.44 ± 0.299</td>
<td>285</td>
<td></td>
</tr>
<tr>
<td>&gt; 500</td>
<td>$T^{2/3} = AV + B$</td>
<td>0.074 ± 0.0017</td>
<td>-11.64 ± 1.03</td>
<td>255.6</td>
<td></td>
</tr>
<tr>
<td>&gt; 500</td>
<td>$T = AV + B$</td>
<td>0.77 ± 0.021</td>
<td>-265.2 ± 12.5</td>
<td>204.8</td>
<td></td>
</tr>
<tr>
<td>&gt; 500</td>
<td>$T = AV^B$</td>
<td>0.0012 ± 0.0004</td>
<td>1.85 ± 0.05</td>
<td>373.1</td>
<td></td>
</tr>
<tr>
<td>&gt; 500</td>
<td>$T = Ae^{BV}$</td>
<td>28.9 ± 1.3</td>
<td>0.003 ± 0.0001</td>
<td>394.9</td>
<td></td>
</tr>
<tr>
<td>Velocity in $\text{Kms}^{-1}$</td>
<td>Function</td>
<td>$T$ in $10^3 \text{K}$</td>
<td>$A$</td>
<td>$B$</td>
<td>$\chi^2_{uv}$</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>----------</td>
<td>------------------------</td>
<td>-----</td>
<td>-----</td>
<td>--------------</td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T = A e^{BV}$</td>
<td>$1.57 \pm 0.06$</td>
<td>$0.0089 \pm 0.0001$</td>
<td>84.3</td>
<td></td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T = A V^B$</td>
<td>$7 \times 10^{-8} \pm 1.8 \times 10^{-8}$</td>
<td>$3.42 \pm 0.04$</td>
<td>88.9</td>
<td></td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T = A V + B$</td>
<td>$0.511 \pm 0.006$</td>
<td>$-135.6 \pm 2.6$</td>
<td>137.1</td>
<td></td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T^{2/3} = A V + B$</td>
<td>$0.085 \pm -0.001$</td>
<td>$-17.9 \pm 0.4$</td>
<td>111.8</td>
<td></td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T^{1/2} = A V + B$</td>
<td>$0.0323 \pm 0.0003$</td>
<td>$-5.02 \pm 0.14$</td>
<td>102.1</td>
<td></td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T^{1/3} = A V + B$</td>
<td>$0.011 \pm 0.0001$</td>
<td>$-0.489 \pm 0.048$</td>
<td>94.4</td>
<td></td>
</tr>
</tbody>
</table>

| $> 500$                     | $T = A e^{BV}$ | $30.2 \pm 1.3$        | $0.0029 \pm 0.0001$ | 508.8 |
| $> 500$                     | $T = A V^B$   | $1.96 \times 10^{-3} \pm 0.54 \times 10^{-3}$ | $1.78 \pm 0.04$ | 470.1 |
| $> 500$                     | $T = A V + B$ | $0.511 \pm 0.014$     | $-118.4 \pm 8.6$ | 262.3 |
| $> 500$                     | $T^{2/3} = A V + B$ | $0.06 \pm 0.001$ | $-3.8 \pm 0.97$ | 335.1 |
| $> 500$                     | $T^{1/2} = A V + B$ | $0.0191 \pm 0.0005$ | $2.0 \pm 0.3$ | 374.9 |
| $> 500$                     | $T^{1/3} = A V + B$ | $0.005 \pm 0.0001$ | $2.4 \pm 0.8$ | 417.2 |

**Table 6b**

Helios 1, 1978–1980, $T$ in $10^3 \text{K}$, $V$ in $\text{Kms}^{-2}$

<table>
<thead>
<tr>
<th>Velocity in $\text{Kms}^{-1}$</th>
<th>Function</th>
<th>$T$ in $10^3 \text{K}$</th>
<th>$A$</th>
<th>$B$</th>
<th>$\chi^2_{uv}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$&lt; 500$</td>
<td>$T = A e^{BV}$</td>
<td>$2.06 \pm 0.07$</td>
<td>$0.0083 \pm 0.0001$</td>
<td>444.6</td>
<td></td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T = A V^B$</td>
<td>$4.8 \times 10^{-7} \pm 1 \times 10^{-7}$</td>
<td>$3.11 \pm 0.03$</td>
<td>474.9</td>
<td></td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T = A V + B$</td>
<td>$0.5 \pm 0.01$</td>
<td>$-125 \pm 2.38$</td>
<td>730.5</td>
<td></td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T^{1/3} = A V + B$</td>
<td>$0.08 \pm 0.001$</td>
<td>$-15.3 \pm 0.33$</td>
<td>437.7</td>
<td></td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T^{1/2} = A V + B$</td>
<td>$0.0304 \pm 0.0003$</td>
<td>$-3.97 \pm 0.12$</td>
<td>387.8</td>
<td></td>
</tr>
<tr>
<td>$&lt; 500$</td>
<td>$T^{2/3} = A V + B$</td>
<td>$0.0103 \pm 0.0001$</td>
<td>$-0.122 \pm 0.039$</td>
<td>377.36</td>
<td></td>
</tr>
</tbody>
</table>

| $> 500$                     | $T = A e^{BV}$ | $25.4 \pm 2.3$        | $0.003 \pm 0.0016$ | 170.6 |
| $> 500$                     | $T = A V^B$   | $5.1 \times 10^{-4} \pm 4.5 \times 10^{-4}$ | $1.98 \pm 0.1$ | 160.7 |
| $> 500$                     | $T = A V + B$ | $1.09 \pm 0.05$     | $-437 \pm 28.5$ | 119.4 |
| $> 500$                     | $T^{2/3} = A V + B$ | $0.09 \pm 0.004$ | $-22 \pm 2.2$ | 151.6 |
| $> 500$                     | $T^{1/2} = A V + B$ | $0.026 \pm 0.001$ | $-1.95 \pm 0.64$ | 160.29 |
| $> 500$                     | $T^{1/3} = A V + B$ | $0.006 \pm 0.0003$ | $1.65 \pm 0.17$ | 165.6 |
TABLE 7

Exponential ($T = A e^{BV}$) $T-V$ Fits at .3 AU

<table>
<thead>
<tr>
<th>V</th>
<th>ln A</th>
<th>B</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 500</td>
<td>2.697</td>
<td>.006753</td>
<td>Normalized data</td>
</tr>
<tr>
<td>&lt; 500</td>
<td>2.696</td>
<td>.006354</td>
<td>Equations (5-14), (5-15)</td>
</tr>
<tr>
<td>&lt; 500</td>
<td>.892</td>
<td>.007882</td>
<td>Data from .3 - .4 AU</td>
</tr>
<tr>
<td>&lt; 500</td>
<td>3.94</td>
<td>.008642</td>
<td>Data normalized to 1 AU</td>
</tr>
<tr>
<td>&gt; 500</td>
<td>5.01</td>
<td>.001788</td>
<td>Normalized Data</td>
</tr>
<tr>
<td>&gt; 500</td>
<td>4.75</td>
<td>.002277</td>
<td>Equations (5-14), (5-15)</td>
</tr>
<tr>
<td>&gt; 500</td>
<td>3.28</td>
<td>.00311</td>
<td>Data from .3 - .4 AU</td>
</tr>
<tr>
<td>&gt; 500</td>
<td>5.07</td>
<td>.00296</td>
<td>Data normalized to 1 AU</td>
</tr>
</tbody>
</table>
**TABLE 8**

Selected Fits to $T(nV^2)$ for Helios 1
for $T$ in $10^3$ K, $n$ in cm$^{-3}$ and $V$ in km s$^{-1}$
$n$ and $T$ are at 1 AU

<table>
<thead>
<tr>
<th>Function</th>
<th>$V$</th>
<th>$A$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T = Ae^{BnV^2}$</td>
<td>&lt; 500</td>
<td>51.5</td>
<td>$-6 \times 10^{-8}$</td>
</tr>
<tr>
<td>$T = A(nV^2)^B$</td>
<td>&lt; 500</td>
<td>186.8</td>
<td>-.098</td>
</tr>
<tr>
<td>$T = BnV^2 + A$</td>
<td>&lt; 500</td>
<td>63.9</td>
<td>$-7.4 \times 10^{-7}$</td>
</tr>
<tr>
<td>$T = Ae^{BnV^2}$</td>
<td>&gt; 500</td>
<td>159.17</td>
<td>$3 \times 10^{-8}$</td>
</tr>
<tr>
<td>$T = A(nV^2)^B$</td>
<td>&gt; 500</td>
<td>9.2</td>
<td>.205</td>
</tr>
<tr>
<td>$T = BnV^2 + A$</td>
<td>&gt; 500</td>
<td>$2.33 \times 10^{-5}$</td>
<td>158.7</td>
</tr>
</tbody>
</table>