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MONETARY AGGREGATION AND WEAK SEPARABILITY: A TEST OF
HOUSEHOLD PREFERENCES OVER ASSETS AND COMMODITIES

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MONETARY AGGREGATION AND WEAK SEPARABILITY:
A TEST OF HOUSEHOLD PREFERENCES OVER ASSETS AND COMMODITIES

by

KENNETH J. HEAGHNEY

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

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Weak Separability and Monetary Aggregation:
A Test of Household Preferences Over Assets and Commodities

by
Kenneth J. Heaghney

ABSTRACT

Economic theory predicts that money is an important determinant of macroeconomic variables but fails to clearly define which assets should be considered money. Benjamin Friedman (1984) rejected the hypothesis that the major monetary aggregates move closely together over substantial periods of time. Thus, the central bank cannot simply pick one monetary aggregate, target it appropriately, and ignore mixed signals. Separability of preferences provides an explicit criteria for determining which assets should be considered as money.

An aggregate based on separability will satisfy two properties. First, the total amount of the aggregate desired can be determined independent of the size of the aggregate's individual components. Second, changes in the relative size of the aggregate's components which leave the aggregate's total unchanged will not affect the choice of other assets and commodities. These properties indicate that an aggregate based on separability will have a more stable relationship with GNP than any other aggregate. Barnett (1980) first argued this proposition.
Separability imposes structure on the Hessian matrix. Imagine a partition of the n goods of the choice set into m groups. If preferences are separable in this partition, the marginal rate of substitution between any pair of goods in the same group is independent of the level of consumption of any good which is not an element of that group. The result is a two-level Hessian matrix corresponding to the partition.

Theil translates this Hessian matrix into a two-level Rotterdam demand model. This model first allocates total expenditure among groups of goods. Next, each group allocation is allocated among the group's individual components. This two-level demand model permits the testing of alternate hypotheses regarding separable partitions of the choice set.

Three separate partitions of choice set were tested. Each was rejected. Thus, the data do not support using separability to define a monetary aggregate for policy purposes.
ACKNOWLEDGEMENTS

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CHAPTER 1

INTRODUCTION

The correct approach to implementing monetary policy remains controversial. At the risk of over-simplification, two principal approaches to monetary policy can be identified. First, monetary policy should allow the money supply to grow steadily at a rate compatible with long-run economic growth trends and stable prices. This view holds that policymakers lack sufficient information to use monetary policy countercyclically. Information lags and lags in the impact of policy changes on economic variables preclude policymakers from adequately assessing the correct policy choice at any point in time. In this view, policymakers must recognize their limitations by following a monetary growth rate so that they minimize the damage their policy decisions inflict on the economy. The alternative view holds that policymakers have a wide range of information available on economic conditions and trends. A simple monetary growth rule forces policymakers to ignore all information except the money supply in policy decisions. This view holds that the relationship between the money supply and other economic variables are not so predictable that all other information can be ignored. Hence, policymakers must consider all available information.

The contrasting views of the proper conduct of monetary policy have received increased exposure in the popular press. A. Malabre (1983) points out the difficulty of picking the best measure of money for policy purposes when the policymaker lacks complete information.
Moreover, he notes that monetary policy decisions become even more complex as behavioral change occurs. One indicator of behavioral change is shifts in monetary velocity. Velocity has experienced long-term growth since World War II with above average growth during cyclical expansions and decline during recessions. However, velocity declined markedly during the 1981-1982 recession and failed to resume trend growth after the recession ended. If the policymaker does not follow a simple growth rule, he must decide whether to offset this velocity shift. Velocity's deviation from its long-term trend indicates a change in the relationship between the money supply and economic activity. If the policymaker offsets this shift with faster money supply growth and velocity resumes its long-term trend, policy will be overly expansionary. If the policymaker does not offset the velocity shift and velocity remains below trend, monetary policy will be overly restrictive. The policymaker could choose to wait and react as velocity changes. However, the lag between a policy change and its impact on the economy limits the effectiveness of this approach. A policymaker operating under a monetary growth rule would not offset the velocity shift. His presumption is that the shift is temporary and velocity will resume its long-term trend. Further, any attempt to accommodate short-term shifts will tend to destabilize rather than stabilize the economy due to the lags inherent in policy.

In the United States, the Federal Reserve Board conducts monetary policy. In October 1979, the Fed established a policy of setting upper and lower target growth rates for alternative monetary aggregates. At times, the Fed has found its targets incompatible in that one monetary aggregate would be within the bounds defined by its
target growth rates while a second aggregate would fall outside its bounds. Thus, an important issue is the question of the aggregate on which the Fed should focus.

B. Friedman (1984) rejects the proposition that "the major monetary aggregates move together over sustained spans of time so that the central bank can simply pick one aggregate and target it appropriately without having to worry about mixed signals." This indicates that some criteria must be defined to determine which monetary aggregate to target. A general principal is that the policymakers should pick the aggregate which has the most stable relationship with economic activity.

Barnett (1980) proposes using aggregation theory to define the group of assets and the aggregation methodology to determine this monetary aggregate. Aggregation theory requires that preferences satisfy certain underlying structure. Generally, as the conditions placed on the aggregate become more rigorous, the structures placed on preferences become more restrictive. Barnett's approach would incorporate information about preferences into defining a money aggregate. Thus, it could improve the performance of monetary policy under a monetary growth rule or a broad-based policy approach.

The relationship between traditional monetary aggregates and economic activity is far from exact. This may result in part from portfolio shifts due to income and relative price changes. By incorporating such information on preferences into the definition of the monetary aggregate, the relationship between this aggregate and economic activity should be more stable than that between any aggregate which ignores this information and economic activity.
The choice of monetary aggregate for policy purposes cannot be limited to those defined by the Federal Reserve. Economic agents can choose among a wide variety of assets in which to store their wealth. The Fed offers a variety of groupings of these assets as alternative monetary aggregates. These aggregates hide several issues which are important in the choice of monetary aggregate for policy purposes.

The Fed implicitly calculates each monetary aggregate by summing the total holdings of each asset included in the aggregate for every economic agent. The first issue that arises is that the aggregates ignore shifts of assets among different sectors of the economy. Individual economic agents can be grouped into different sectors such as households, various types of businesses, and alternative levels of government. If each sector's behavior is motivated by different factors, shifts in the monetary aggregate among the sectors could have significant implications for the economy. This aggregation issue also exists within sectors.

A second issue is the appropriateness of summing the individual assets to calculate the aggregate. This implies that dollar for dollar shifts between assets in the same aggregate will not affect the economy. Potentially, linear or non-linear aggregation of individual assets may provide more useful information.

A third issue concerns the impact of structural changes and behavioral shifts. In recent years, financial markets have changed rapidly with new assets appearing as government regulation eased. These changes could alter the relationship between the monetary aggregate and other economic variables.
These issues must be considered in defining the optimal
definition of money for policy purposes. The problem of aggregation
across sectors is handled by dealing with household sector data.
Data for individual assets is used so that assumptions regarding
linear or non-linear aggregation are not needed. The data span a
time period of relatively tranquil financial markets so that the
problem of rapid structural change is avoided.

The criteria outlined by Barnett attempt to determine if
preferences regarding monetary assets can be modeled by weakly
separable preference structures. A monetary aggregate which
satisfies these criteria should contain more relevant information
than a simple sum aggregate and bear a more stable relationship to
the level of economic activity than any simple sum aggregate. This
analysis tests the existence of separable preferences through
estimation of the Rotterdam model. This estimation exploits Theil's
(1976) analysis of the impact of weak separability on the model.
Theil derives a two-level demand system which allows for testing of
alternatives hypotheses of weakly separable structures.

These tests will be used to examine the appropriateness of weakly
separable preference structures for household preferences. These
tests will indicate if a monetary aggregate can be defined based on
this structure.
Footnotes

CHAPTER 2
THE PROBLEM OF DEFINING A MONETARY AGGREGATE

In the United States, monetary policy falls in the domain of the Federal Reserve System. The Fed's policy choices are important since money plays a significant role in determining macroeconomic variables. Still, wide disagreement exists regarding the relative efficacy of monetary and fiscal policies and the optimal method of implementing monetary policy so as to achieve the objectives of the central bank.

B. Friedman (1975) developed a simple, stochastic macro-model to examine the monetary control problem. Friedman's model consists of three linear equations: a Hicksian commodities market equilibrium (IS) function, a Hicksian money demand (LM) function, and a money supply function. Respectively, these take the form

\[ Y = a_1 r + a' Z + U_Y \]

\[ M = b_1 Y + b_2 r + b' Z + U_{MD} \]

\[ M = c_1 R + c_2 r + c' Z + U_{MS} \]
where \( Y \) = national income
\( r \) = nominal interest rate
\( M \) = money stock
\( R \) = nonborrowed bank reserves
\( Z \) = vector of variables exogenous to monetary policy

The coefficients of the model include the scalars \( a_1, b_1, b_2, c_1 \), and \( c_2 \) and the vectors \( a, b, \) and \( c \). The scalars \( a_1 \) and \( b_2 \) are expected to be less than zero while \( b_1, c_1, \) and \( c_2 \) are anticipated to be positive. The random disturbance terms \( U_Y, U_{MD}, \) and \( U_{MS} \) are assumed to have expected values of zero, variances equal to \( \sigma_Y^2, \sigma_{MS}^2, \) and \( \sigma_{MD}^2 \), and covariances \( \sigma_{Y,MD} \), \( \sigma_{Y,MS} \), and \( \sigma_{MS,MD} \).

This model contains four variables \((Y, r, M, \) and \( R)\) in three equations. Hence, one variable must be assumed exogenous in order that the model be determined. The central bank's perspective indicates either the interest rate or the stock of nonborrowed bank reserves should be considered exogenous.

Friedman derives the reduced form of the structural model for both cases. Supposing that the central bank exerts direct control over the interest rate, the reduced form of the model is

\[
\begin{bmatrix}
Y \\
M \\
R
\end{bmatrix}
= \begin{bmatrix}
\gamma_{11} & \gamma_{12} \\
\gamma_{21} & \gamma_{22} \\
\gamma_{31} & \gamma_{32}
\end{bmatrix}
\begin{bmatrix}
r \\
Z
\end{bmatrix}
+ \begin{bmatrix}
u_Y \\
u_M \\
u_R
\end{bmatrix}
\]
where

\[ \gamma_{11} = a_1, \quad \gamma_{21} = b_2 + b_1 a_1, \quad \gamma_{31} = c_1^{-1}(b_1 a_1 + b_2 - c_2), \]

\[ \gamma_1 = a, \quad \gamma_2 = b + b_1 a, \quad \gamma_3 = c_1^{-1}(b_1 a + b - c), \]

\[ u_Y = U_Y, \quad u_M = U_{MD} + b_1 U_Y, \quad u_R = c_1^{-1}(b_1 U_Y + U_{MD} - U_{MS}) \]

The model's reduced form under the alternate choice of controlling the nonborrowed reserve stock is

\[
\begin{bmatrix}
Y \\
M \\
r
\end{bmatrix} =
\begin{bmatrix}
g_{11} & g_1 \\
g_{21} & g_2 \\
g_{31} & g_3
\end{bmatrix}
\begin{bmatrix}
R \\
Z
\end{bmatrix} +
\begin{bmatrix}
e_Y \\
e_M \\
e_r
\end{bmatrix}
\]

where

\[ x = (b_1 a_1 + b_2 - c_2)^{-1} \]

\[ g_{11} = X a_1 c_1 \]

\[ g_{21} = c_1 + X c_1 c_2 \]

\[ g_{31} = X c_1 \]

\[ g_1 = a - X a_1 (b_1 a + b - c) \]

\[ g_2 = c - X c_2 (b_1 a + b - c) \]

\[ g_3 = X (b_1 a + b - c) \]

\[ e_Y = U_Y - X a_1 (b_1 U_Y + U_{MD} - U_{MS}) \]

\[ e_M = U_{MS} - X c_2 (b_1 U_Y + U_{MD} - U_{MS}) \]

\[ e_r = -X (b_1 U_Y + U_{MD} - U_{MS}) \]
These reduced form models allow the two monetary policy control variables to be evaluated once a policy objective has been defined. Assume that this objective is minimization of the variance of actual national income about its desired level (e.g. full employment income). Denote this optimal level $Y^*$. Given the structural model, the expected value of $Y - Y^*$, the discrepancy between actual income and optimal income, is zero regardless of the central bank's choice of monetary control variable. However, the variance of actual income about optimal income is non-zero for each choice. Further, the variances are not defined by identical expressions so that the central bank should choose to control the variable which yields the smaller variance of actual income about optimal income.²

This model abstracts greatly from the reality of the monetary policymaking process. A degree of reality can be admitted by considering the impact of information lags on policymaking. Typically, the central bank has information on bank reserves and the interest rate for the most recent time period, but receives information on national income with a lag. B. Friedman demonstrates that, under certain assumptions, using the most recent observation on the money stock to generate an expectation of the discrepancy of actual income from desired income in the present period can be optimal.³ This expectation can then be used to determine the current period's monetary policy.

Another step toward reality can be taken by admitting structural lags in which policy actions taken in past periods affect current and future values of the policy targets and in which imperfect knowledge exists concerning the parameters of the structural model. M.
Friedman (1959) emphasizes the difficulties that the uncertainty surrounding the size and timing of the impact of past monetary policy actions creates for the central bank.

M. Friedman concludes that the problems of implementing countercyclical policy in a world of incomplete information and long and variable lags between a policy action and its ultimate effect on the economy result in larger short-run fluctuations in economic activity then would have occurred in the absence of countercyclical policy. In this view, the problems confronting the Fed are overwhelming and the best policy course is a constant rate of growth in the money supply. The Fed adopted a policy of setting monetary growth rates in October 1979.

The open question remains how to define the money supply or, more precisely, how to determine which assets to include in a monetary aggregate and the method to use to construct the aggregate. The Fed has historically issued data on various measures of the money supply. One traditional measure, denoted M1 or narrow money, consisted of demand deposits and currency in the hands of the public, while a broader measure, M2, included M1 plus time deposits at commercial banks. Still broader measures of the money supply could be obtained by including additional liquid assets. Recently, the Fed altered its definitions of various measures of the money supply.4 Largely, these new definitions reflect the increasing importance of non-commercial bank, interest bearing deposits which function similarly to demand deposits. Redefining the monetary aggregates in this way emphasizes the view that demand deposits plus currency are
just two of many liquid assets which economic agents can choose to hold and that, over time, the distinctions between these assets have become increasingly blurred.

The concept that demand deposits and currency are just two of many assets, each characterized by a unique combination of risk, return, and liquidity, was forcefully presented by Gurley and Shaw (1960). This concept raises the possibility that assets bearing similar characteristics may be close substitutes for each other. For example, the liabilities of financial institutions such as savings and loan deposits might be close substitutes for narrow money. In this case, a change in the relative price of narrow money and savings and loan deposits will induce asset holders to rearrange their portfolios. The extent to which this substitution relationship exists has been shown to have important implications for short-term movements in the level of nominal income and to create problems for the conduct of monetary policy.

This problem can be seen in the context of Patinkin's (1965) four sector general equilibrium model. Patinkin derives excess demand functions for the labor, goods, money, and bond markets. Assume that the Fed maintains a constant money supply and that this rule applies to narrow money. All other liquid assets can be considered to be aggregated into the bond market. Any portfolio shift between narrow money and other liquid assets would shift the market equilibrium curves if the Fed continued to maintain a constant money supply. This shift of the equilibrium curves would alter the interest rate and the price level and the equilibrium process would cause fluctuations in nominal income. Alternatively, assume that the
monetary authority controls a monetary aggregate which includes close portfolio substitutes for narrow money. Here, these liquid assets are aggregated into the money market. Portfolio shifts among these assets would not shift the market equilibrium curves if the Fed continued its policy of a constant money supply. Consequently, the portfolio shift among monetary assets would not affect the interest rate, income, or prices. The crucial factor in this example is that the relationship between nominal income and the broader monetary aggregate is more stable than that between nominal income and narrow money.

This analysis indicates that the group of assets controlled by the Fed is important, at least in the short run. While macroeconomic theory predicts a close relationship between the quantity of money and the level of nominal income, it does not provide clear cut guidelines for determining exactly which assets should be regarded as money. Fisher (1978) identifies two approaches to defining money. First, he proposes a pragmatic approach. Money is that group of assets most useful for attacking the problem at hand. Thus, money is whatever the researcher wants it to be and will vary with the needs of that researcher. Alternatively, the characteristics or functions of money can be identified. Money, in this case, is that group of assets which possesses the postulated characteristics or perform the postulated functions. Traditionally, the functions of money have been identified as medium of exchange, unit of account, and store of value. Demand deposits and currency clearly perform all three functions and these assets comprise the dominant concept of money employed by economists. Other assets exist which do not perform all
three functions, but do perform one or two functions. For example, time deposits do not perform the medium of exchange function, but do act as a store of value. Time deposits, in fact, can be considered superior to narrow money as a store of value since they earn an explicit rate of return.\(^6\) Despite this, time deposits and similar assets have generally been excluded from the definition of money. This emphasizes the role of medium of exchange as the essential function of money.

M. Friedman and A. Schwartz (1970) point out that a monetary economy can be distinguished from a barter economy by the ability of economic agents to separate the act of purchase of a commodity from the act of sale of a commodity. An agent can sell his labor services today for a medium of exchange and purchase food with a medium of exchange tomorrow. Two conditions must exist for this separation of events in time to occur. First, the medium of exchange must exist to complete each transaction. Second, a store of value must exist to bridge the time span between the sale and the purchase. It is not essential that one asset serve both roles nor is it clear that an asset which performs only one role can be ruled out as money. This reasoning opens the way to a definition of money broader than demand deposits plus currency.

The first problem is to determine which assets to include in a monetary aggregate. This issue would be unimportant if the relative growth rates of liquid assets were stable.\(^7\) The data listed in Table 1 and Table 2 indicate substantial looseness in the relationships among liquid assets. Table 1 lists the relative sizes of various measures of liquid assets. For example, M1 accounted for 30% of total liquid assets in 1965, 22% in 1975, and 17.4% in 1981.
In contrast, M2, as a percent of total liquid assets, has declined at a much slower rate than M1. M2 accounts for 78.7% of liquid assets in 1965, 73% in 1975, and 69.9% in 1981. M3 has maintained an essentially constant proportion of total liquid assets during the time period. The trends apparent in Table 1 are largely secular in nature. However, Table 2 demonstrates that significant variations in Quarter-to-quarter growth rates of alternative measures of money have also occurred. Thus, these data suggest secular and cyclical variations in the composition of liquid assets and reinforce the theoretical notion that the choice of a monetary aggregate by the Fed has significant short run implications for movements in nominal income.

A second problem is to determine how individual holdings of each monetary asset should be combined into a single aggregate total. The construction of current monetary aggregates can be viewed as a three-step process. First, holdings of each monetary asset for each individual in every sector of the economy are totaled. Next, the totals for each individual in a sector are summed. Finally, the sector totals are added together. This method assumes these assets are perfect substitutes for each other.

The issue of aggregating across sectors is also an important issue. An aggregate can obscure substantial changes in behavior by the various sectors in the economy. Table 3 compares relative holdings of currency and checkable deposits for five sectors. These data indicate large shifts in relative holdings of these assets. Thus, analysis of the data at an aggregate level could be misleading. We deal with this problem by considering household data only.
Many researchers have studied the problem of defining money. The most common approach is to estimate various cross-elasticities of demand between money and other assets. These elasticities would then determine which assets should be included in the definition of money. A major problem with this approach is determining the critical value of a cross-elasticity of demand which could be used to designate the asset as a component of money. Disagreement on this critical value would lead researchers with similar quantitative results to draw opposite qualitative conclusions.

To overcome this problem, a theoretical basis must be devised to determine which assets should be included in a monetary aggregate. The theoretical basis proposed here comes from microeconomic theory. In particular, the theoretical construct of separable preferences will be employed. This construct will lead to a two stage budgeting procedure. Separable preferences will be examined in Chapter 4.

An aggregate based on separable preferences will satisfy two important properties. First, households will treat the aggregate as if it were a single good. Households can choose the amount of the aggregate they desire without regard to the size of the individual components of the aggregate. Second, changes in the internal composition of the aggregate which do not affect the aggregate total will not affect the household's choice of other assets and commodities. For example, suppose M2 is a consistent aggregate. Then households treat M2 as if it were a single good and can determine the total amount of demand deposits, currency, and time deposits they wish to hold without considering how this total will be broken down among M2's individual components. Also, since M2 is devised by simple summation of the individual components,
transferring one dollar from the holdings of one component to the
holdings of a second component will not alter the aggregate total.
This indicates, via the second property, that the household's choice
of other assets and commodities will not be altered by this transfer.

This second property has important implications for reducing the
variation of nominal income created by the substitution among money
and other liquid assets. These fluctuations arise because of the
monetary authority's inability to predict and react to changes in
velocity, the relationship between the stock of money and the
purchase of commodities. A monetary aggregate based on separable
preferences might internalize some of the substitutions between
narrow money and other liquid assets. The relationship between this
aggregate and purchases of commodities would then be more stable than
that between narrow money and commodities. Therefore, velocity would
be more stable if based on this monetary aggregate than on
alternative definitions of money. Smaller short run fluctuations in
nominal income should result from controlling this aggregate than
from controlling any other aggregate.

The investigation of money and money substitutes has generated a
great deal of empirical evidence without reaching a consensus on how
well other assets substitute for demand deposits and currency in the
aggregate household portfolio and the extent to which the monetary
authority must consider money substitutes in its choice of policy.
The separability properties examined in Chapter 4 provide an explicit
theoretical framework, utility maximization, within which to view the
choice among assets, and an explicit criteria for judging the
consistency of alternative monetary aggregates.
Chetty (1969) was the first to apply a utility maximization approach to the money substitution problem, but his work contains several problems. First, he chose to construct an empirical definition of money based solely on estimates of elasticities of substitution between money and alternative assets. Thus, he ignores the conditions which allow for aggregating individual assets into a composite asset. Second, Chetty derives demand equations which are not true demand equations in the sense that quantity demanded is a function of all prices and total expenditure. This stems from Chetty's failure to include the budget constraint when he solves for the demand equations from the first order conditions. Third, Chetty assumes that the choice of commodities and long-term assets is weakly separable from the choice of short-term assets. Since separability of preferences is the criteria for judging an aggregate, assuming separability is not proper. The implications of this assumption will be examined further in Chapter 4.

The principle goal of this project is to investigate the household sector's choice of narrow money and other liquid assets in terms of separability of preferences. This will generate evidence on which assets should be included in an empirical definition of money and which assets should be of primary concern to the monetary authority in their execution of policy. In order to avoid the implicit assumption of separability of long-term assets and commodities from short-term assets, all will be included. Asset prices will be measured in a way which reflects the stock-flow nature of including both assets and commodities in the analysis. The measure of prices was devised by Clements (1976) in his study of Australian consumer behavior.
The model chosen for empirical work is the Rotterdam model. This model allows for direct testing of the appropriateness of separability conditions. The Rotterdam model has been subjected to much criticism because of certain inherent restrictions. Barnett (1979) demonstrates that these criticisms are misdirected and that the Rotterdam model is a very general tool for modeling household demand.
Footnotes

1. The Fed, the U.S. central bank, implements monetary policy through three main tools: reserve requirements on deposits at member banks, the discount rate charged member banks on their borrowing from the Fed, and open market operations.

2. If the central bank chooses to control the interest rate, the variance equals

\[ E(e_Y)^2 = \sigma_Y^2 \]

If the central bank chooses to control the stock of nonborrowed bank reserves, the variance equals

\[ E(e_Y)^2 = (b_1 a_1 + b_2 - c_2)^2 \left[ (b_2 - c_2)^2 \sigma_Y^2 + \sigma_{MD}^2 + \sigma_{MS}^2 - 2(b_2 - c_2) \sigma_Y \sigma_{MD} + 2(b_2 - c_2) \right] \]

3. Use of the money stock in this way is optimal only if the money demand function is stable (\( \sigma_Y, MD = 0 \)) and insensitive to the interest rate (\( b_2 = 0 \)). Both propositions are generally accepted by those favoring monetary aggregate targeting or a monetary growth rule.

4. The Fed defined M1A to equal demand deposits plus currency in the hands of the public. M1B equals M1A plus negotiable orders of withdrawal, automatic transfer service accounts at banks and thrift institutions, credit share draft accounts, and demand deposits at mutual savings banks. The Fed has now dropped the M1A definition, and M1B has been renamed M1.

5. This assumes differences in reserve requirements among financial institutions and financial assets are neutralized, so that shifts among assets do not alter the total supply.

6. The emergence of savings accounts which earn interest and can be used for transactions purposes has altered this relationship. These accounts may dominate demand deposits. This fact is reflected in the new definitions of money reported by the Fed and in the liberalization of restrictions prohibiting commercial banks from paying interest on demand deposits.

7. If this were so, the problem would be to choose the optimal growth rate for any monetary aggregate.

8. This will be examined further in Chapter 3.
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<thead>
<tr>
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<td>17.4</td>
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</table>

Source: Federal Reserve Bulletin various issues.

1/ M1 = Currency + demand deposits + travelers checks + other checkable deposits.

2/ M2 = M1 + overnight repurchase agreements + eurodollars + money market mutual fund shares + savings and small time deposits.

3/ M3 = M2 + large time deposits and term repurchase agreements.
TABLE 2  
Correlation Matrix of Quarter to Quarter Percent Changes 
In Alternative Measures of the Money Supply

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<th>M3</th>
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Source: Federal Reserve Flow of Funds.

1/ Excludes financial sector and mail float.
CHAPTER 3
REVIEW OF THE LITERATURE

3.1 Introduction

The concept of an empirical monetary aggregate relates to two diverse areas of the economic literature. First, this work follows a long line of investigations into the demand for money and money substitutes. Second, the search for an empirical monetary aggregate based on separability conditions of the utility function places the analysis within the domain of consumer demand modeling. This implies interest in the application of empirical demand models derived from the assumption of constrained utility maximization.

3.2 Money and Money Substitutes

Gurley and Shaw's (1960) seminal work demonstrated the potential effect on the economy of the presence of close substitutes for money narrowly defined. A large quantity of empirical evidence has been generated to test the hypothesis that the liabilities of financial intermediaries are close substitutes for narrow money. Three separate approaches will be examined below.

The first approach takes the viewpoint that the "moneyness" of a financial asset can be evaluated by its ability to explain changes in income relative to the narrow money stock's ability to do so. Timberlake and Forston (1967) take this view and estimate the
parameters of the equation

\[ Y = a + b_1 M1 + b_2 S + e \]

where \( Y \) is national income, \( M1 \) is narrow money, \( S \) is time deposits at commercial banks and savings and loan associations, and \( e \) is a random error term. Their data consists of annual observations for the U.S. for 1913-1965. The equation was estimated in first differences for ten sub-periods. The sub-periods were based on Friedman and Meiselman's reference cycles (1963). The authors claim that "if time deposits \( S \) have some degree of moneyness, the ratio \( b_2/b_1 \) should be greater than zero, but less than one. A ratio of one would indicate that time deposits had "moneyness equal in degree to the items in the narrow money stock." A negative ratio indicates that "time deposits serve more in the nature of investments: that people actively reduce their transactions balances to 'buy' time deposits." In seven of the ten sub-periods, the ratio \( b_2/b_1 \) was negative and varied between \(-4.51\) and \(-.163\). Only during two sub-periods which consist of the Great Depression and the latest sub-period does the ratio become positive. The ratio varied from 7.997 for 1933-1938 to .1521 for 1953 to 1965. The authors attribute the large positive result for the earlier period to loss of confidence in demand deposits by asset holders. They conclude that only the final sub-period estimate lends any credence to the notion of moneyness of time deposits.

Laumas (1968) repeats the procedure, but uses quarterly U.S. data for 1947-1966 and three separate measures of savings deposits. The first measure includes only time deposits at commercial banks, the
second measure corresponds to that of Timberlake and Forston, while
the third measure adds mutual savings bank deposits to the second
measure. Laumas finds that the ratio \(b_2/b_1\) is positive for
each measure of savings deposits. This ratio equals .5785 for the
first measure, declines to .4798 for the second measure, and declines
further to .3241 for the third measure. Laumas concludes that each
measure of savings deposits embodies some degree of moneyness as each
estimate of the ratio exceeds zero. Additionally, the broader the
measure of savings deposits, the lower the degree of moneyness
embodied by the measure as the ratio declines as the measure of
savings deposits is broadened.

The second approach taken in investigating the relationship
between narrow money and alternate liquid assets is to estimate ad
hoc demand equations which yield estimates of cross-elasticities of
demand. The ability of financial assets to serve as close
substitutes for narrow money is determined on the basis of these
elasticities. Lee (1966) estimates this model as

\[
\ln(M_1/p^0) = a + b \ln(Y^0/p^0) - c \ln(i_g - i_d) - d \ln(i_{sm} - i_d) + e
\]

where \(M_1\) = narrow money stock in per capita terms
\(p^0\) = permanent price level
\(Y^0\) = permanent per capita income
\(i_{sm}\) = average weighted interest rate paid on savings and loan
and mutual savings bank shares
\[ i_g \] = weighted average interest rate paid on long-term and short-term government securities
\[ i_d \] = yield on demand deposits taken to be the negative of service charges on demand deposits

Lee asserts without providing justification that interest rate differentials rather than levels are the proper explanatory variables in a demand for money equation. Annual U.S. data for 1934-1964 excluding the war years 1942-1945 was used. Lee reports an estimate of \(-0.640\) for \(d\) which is significantly different from zero at the one percent level. This parameter measures the relationship between the demand for per capita money balances and the difference in the rates of return on liquid assets and demand deposits. From this, Lee asserts that the liabilities of financial intermediaries are good substitutes for money narrowly defined. Lee breaks this variable into two components in a second step for two reasons. First, the insurance provisions of the Federal Savings and Loan Insurance Corporation were liberalized in case of default by insured institutions. Second, an increasing percentage of savings deposits were being held in insured institutions. Therefore,

\[
d \ln (i_{sm} - i_d) = d_0 \ln (i_{sm} - i_d)_0 - d_1 \ln (i_{sm} - i_d)_1
\]

where the variable subscripted 1 equals zero before 1950 and the variable subscripted 0 equals zero after 1949. Lee claims that, if \(d_1\) exceeds \(d_0\) in absolute value, liquid assets have become better substitutes for narrow money in the latter period. Lee finds that \(d_1\) equals \(-0.570\) and \(d_0\) equals \(-0.497\). These estimates differ
significantly from each other at the five percent level. Lee interprets this as strong support for the hypothesis that the liabilities of financial intermediaries are close substitutes for narrow money. Lee repeats this procedure but replaces the dependent variable with the narrow money stock plus time deposits at commercial banks in per capita terms. Here, Lee finds that $d_1$ equals -.767 and $d_0$ equals -.276 which are significantly different from each other at the one percent level. Again, Lee interprets this as strong evidence of a close substitution relationship between narrow money and the liabilities of financial intermediaries.

Feige (1964) uses a similar model on U.S. pooled cross-section time series data. The cross-section is for individual states while the time series is annual observations for 1949-1959. Feige estimates the demand for per capita commercial bank demand deposits as a function of per capita permanent income, a vector of per capita holdings of assets, a vector of interest rates on alternative assets, an own rate of interest, and various dummy variables using ordinary least squares. Feige's estimate of the cross-price elasticity of demand between demand deposits and the rate on time deposits at commercial banks is .10 which Feige interprets as evidence of a weak substitution relationship. The cross-price elasticity between demand deposits and the rate paid on savings and loan shares is -.30 which indicates that the assets act as complements. Feige re-estimates his model using a technique, seemingly unrelated regressions, which yields Zellner efficient estimates. However, the quantitative and qualitative results are essentially the same as his OLS results.
Feige, in contrast to Lee, concludes that the liabilities of financial intermediaries are not close substitutes for money narrowly defined.

Cagan and Schwartz (1975) seek to evaluate whether the growth of money substitutes has altered the effectiveness of monetary policy by increasing the interest elasticity of the demand for money. They propose a standard money demand equation

$$\ln (M/P) = k + a \ln (Y/P) + b I + e$$

which is estimated for two periods, 1921-1931 and 1954-1971, using quarterly U.S. observations. M measures the narrow money stock, Y equals gross national product, P equals the consumer price index in the first period and the GNP deflator in the second period, and I equals the interest rate on commercial paper. The coefficient b was estimated as -2.7 in the first period (t-statistic equal to 7.2) and as .2 in the latter period (t-statistic equal to .4). Elasticities were derived by multiplying the coefficient by the average level of the interest rate which was .044 in each period. The interest elasticity of the demand for money was lower in the second period than in the first period which tends to refute the notion that liquid assets are close substitutes for narrow money. Cagan and Schwartz felt that these results were inadequate, and that a variable should be included to account for a time trend. They chose the average rate paid on savings accounts. This relieves the commercial paper rate from acting as a proxy for the upward trend in the savings deposit rate and should allow the coefficient of the commercial paper rate,
b, to reflect the short-run response of the demand for money to changes in market rates. Including this rate on savings accounts has little effect on the estimates of b. For 1921-1931, b equals -2.8 (t = 7.1) and for 1947-1971, b equals .4 (t = 1.4). Cagan and Schwartz conclude that despite the problems they encountered in finding a satisfactory money demand model, the evidence indicates that the interest elasticity of the demand for money has not risen and if anything has fallen in the postwar period. This, in the authors' judgement, suggests that money is no longer held as an asset but only for transactions purposes.

This second approach, the estimation of ad hoc demand equations, relies upon demand theory to suggest the explanatory variables to include in the empirical model. Chetty (1969) builds upon this approach and explicitly assumes that households maximize utility from the holdings of money and assets. This follows the theoretical expositions of Patinkin (1965) and Friedman (1958) and allows for the derivation and estimation of theoretically plausible demand equations. Several problems contained in Chetty's application of this approach have been discussed previously. We now turn to a discussion of his model and results. The utility function specified by Chetty is a generalized constant elasticity of substitution function of the form:

\[ U = U(M^{-p} + \sum_{i=1}^{n} B_iX_i^{-p_i}) \]
where $M$ is holdings of money narrowly defined, $X_i$ is the holdings of the $i$th financial asset, and $p$ and $p_i$ are measures of elasticities of substitution. The budget constraint takes the form:

$$Y = M + \sum_{i=1}^{n} \left(\frac{1}{1+r_i}\right) X_i$$

where $r_i$ is the interest rate on the $i$th asset and $Y$ is total expenditure on the $n+1$ assets. Chetty asserts that maximization of the utility function subject to a budget constraint yields demand equations of the form:

$$\ln X_i = -1/(1+p_i) \ln(Bp_i/B) - 1/(1+p_i) \ln[(1/(1+r_i)) + (p+1)/(1+r_i)] \ln(M)$$

Chetty estimates this equation independently for three assets; time deposits at commercial banks, mutual savings bank shares, and savings and loan shares. Chetty proposes a new definition of money based on the utility function. Normalizing $B$ to equal one, the definition is

$$M_a = (M^{'954}+1.026T^{'975} + .88MC^{'956} + .616SL^{'921})^{1.026}$$

Chetty constructs three measures of velocity based on three definitions of money; $M_1$, $M_2$, and $M_a$. Mean velocity for $M_1$ is 2.99 with a coefficient of variation of 21.77. $M_2$ has a mean velocity of 2.03 with a coefficient of variation of 12.1. $M_a$ has mean velocity of 1.65 with a coefficient of variation of 7.6. Chetty's proposed definition yields a more stable measure of velocity than the narrow definition of money, $M_1$, or the broader definition of money, $M_2$. 
Moroney and Wilbratte (1976) use an approach which they claim is
dual to Chetty's approach and yields demand equations identical to
those of Chetty.\textsuperscript{5} Households are assumed to maximize wealth from
their holdings of assets subject to a transactions technology
constraint. By making further assumptions, Moroney and Wilbratte
arrive at Chetty's demand equations. The equation is estimated for
four assets: commercial bank time deposits, government securities of
less than one year maturity, bonds issued by private corporations,
and an aggregate of savings and loan and mutual savings banks
deposits. Moroney and Wilbratte are unable to reject the hypothesis
that the elasticities of substitution between money narrowly defined
and each of the four assets are all equal. Therefore, "if one
believes a weighted liquidity aggregate is the most appropriate
macroeconomic measure of 'moneyness' in which the weights are
substitution elasticities of various assets for money, this aggregate
must include short-term and long-term bonds (and probably other
assets we have not considered in this paper) on an equal footing with
the limited range of assets included, for example, by Chetty (1969)
in the 'adjusted' money supply."\textsuperscript{6}

Barth, Kraft, and Kraft (1977) assume utility maximizing behavior
and derive theoretically plausible demand equations. The form
specified is the $S_1$-branch utility tree. The $S_1$-branch utility
tree separates goods into groups or branches. The Allen elasticity
of substitution between goods in the same branch is unrestricted but
the elasticity of substitution between goods in different branches is
fixed at one. The authors study five assets; demand deposits plus
currency, time deposits at commercial banks, savings and loan shares,
mutual savings bank shares, and treasury bills suing annual
(1957-1970) data. The authors find that the optimal partition in
terms of maximizing the likelihood function is all assets, but
treasury bills in one branch and treasury bills in the second
branch. This suggests a broader definition of money and supports the
findings of Lee (1966) and Chetty (1969).

The problem inherent in this methodology and implied by Moroney
and Wilbratte is that no criteria exists for choosing which assets
should be included in a definition of money and which should be
excluded. Barnett (1980) resolves this problem by using the
conditions which allow for aggregating individual goods into
composite goods. He accepts the necessity of finding a quantity and
a price aggregate such that their product equals total expenditure on
the individual assets contained in the aggregate. The necessary and
sufficient condition for such aggregation is homothetic separability
of the utility function. Thus, this analysis occurs within the
context of utility maximization.

Barnett assumes a multistage budgeting procedure and the blocking
of the individual assets into homothetically separable groups.
Although the household allocates spending from the top of a utility
tree downward; that is from broad groups to more narrow groups,
estimation can occur from the bottom of the tree up. Barnett posits
a constant elasticity of substitution utility function at each level
of the utility tree. Barnett concentrates on a branch of the utility
tree which is assumed to contain only total transactions balances and
passbook savings deposits at three institution types. This branch is
split into two sub-branches; one contains only transactions balances,
the other contains the three types of savings deposits. We can express this as

\[ U(x_t) = U(x_{1t}, U_2(x_{2t})) \]

where \( x_{1t} \) equals transactions balances,
\( x_{2t} \) equals a vector of savings deposits balances,
\( U_2 \) equals a homothetic utility subfunction, and
\( U(x_t) \) is itself a homothetically weakly separable function nested in an overall utility function.

Barnett first estimates \( U_2 \) with the CES specification. Data is U.S. quarterly averages for the period 1970I-1978I for real per capita holdings of commercial bank passbook accounts, savings and loan passbook accounts, and mutual savings accounts. Barnett concludes that these assets are very close substitutes for each other, and that simple linear aggregation is reasonable although a simple sum index is inadequate.

Barnett then proceeds to the next stage of the utility function and estimates \( U \). Two assets exist at this level, total transactions balances, and the passbook savings account aggregate defined in the first stage. Again the utility function follows the CES specification. Barnett finds that transactions balances and the aggregate savings deposit are not close substitutes and rejects linear aggregation.

Finally, Barnett constructs a sample economic monetary aggregate based on the assets already described, plus negotiable and non-negotiable certificates of deposit at commercial banks. This sample aggregate, computed as a Fisher ideal index, yields a velocity
measure which differs substantially from that yielded by simple summation of these assets. Also, the range of velocity is much smaller for the economic index than for the simple sum index.

Barnett's work represents a tremendous step forward in the search for an empirical monetary aggregate. Several problems still exist, however. First, he acknowledges that an important step would be to search for the optimal blocking of assets rather than to state the blocking as a maintained hypothesis. Second, his recurvise estimation technique does not provide any information on the household's choice among assets and commodities. Finally, the concept of aggregation used here is very rigorous, and the conditions it requires are very restrictive. In particular, the requirement of sectoral homotheticity is very strong. By easing the rigor of the aggregate, homotheticity can be relaxed.

3.3 Consumer Demand Models

Our approach will assume that households maximize utility subject to an expenditure constraint. This results in a system of demand equations which can be estimated. The system of demand equations approach can be defined as follows. Assume that \( x \) is a vector of quantities of \( n \) goods: assets, commodities, or both, and that \( p \) is a vector of prices for the \( n \) goods. The set of \( n \) consumer demand functions can be written as

\[ x = x(y, p) \]
These functions relate the quantity demanded to all prices and to \( y \) which is defined by

\[ y = p'x \]

so that \( y \) is total expenditure on the \( n \) goods.

To estimate such a system of demand equations, a particular functional form must be assigned. Two approaches are possible. First, the demand equations can be assigned a particular form. Alternatively, the demand equations can be derived from a specific utility function. In this latter approach, a common form chosen for the utility function is the Klein-Rubin utility function (1947) which was first applied empirically by Stone (1954). Maximization of the Klein-Rubin utility function

\[ U_t = \sum_{i=1}^{n} B_i \ln(x_{it} - \gamma_i) \]

yields the linear expenditure system

\[ p_{it} x_{it} = p_{it} \gamma_i + B_i (y_t - \sum_{j=1}^{n} p_{jt} \gamma_j) \quad i=1, \ldots, n \]

In this system, \( x_i \) is the consumption of good \( i \), \( B_i \) represents the proportion of an additional dollar of income which will be spent on good \( i \), the \( \gamma_i \) are often interpreted as subsistence quantities or as the "minimum socially acceptable consumption requirement" of good \( x_i \), \( p_i \) is the price of good \( x_i \), and \( y \) is total expenditure on the \( n \) goods.

The linear expenditure system incorporates several important restrictions in addition to the restrictions imposed by classical consumer demand theory. First, the B's are restricted to sum to one. Also, if nonsatiation is assumed, each B must exceed zero. This restriction would exclude the possibility of inferior goods.
Nonsatiation implies that marginal utility is positive. Thus

$$\frac{\partial u}{\partial x_i} = B_i (x_i - \gamma_i)^{-1}$$

Since the utility function is defined only for \( x_i > \gamma_i \), the term \((x_i - \gamma_i)^{-1}\) must be positive. Therefore, nonsatiation requires that \( B_i \) exceed zero. Since \( B_i \) represents the marginal budget share, the possibility of inferior goods is eliminated. Second, the LES permits no complementary goods. The compensated cross-price elasticity of demand of the consumption of good \( i \) with respect to the price of good \( j \) is

$$\gamma_{ij} = \frac{B_i (x_j - \gamma_j)}{p_j} \quad i \neq j$$

Since each term \( B_i, p_i, \) and \( (x_j - \gamma_j) \) is positive, the entire term is positive and all pairs of goods are substitutes. Third, the own price elasticities of demand are forced to lie between zero and negative one. The own price elasticity of demand is given by

$$\eta_{ii} = \frac{(1-B_i)\gamma_i}{x_i} - 1$$

\( B_i \) is restricted to be less than one and \( \gamma_i \) is less than \( x_i \), so that the first half of the expression lies between zero and one. Thus, the entire term has a value less than zero and greater than negative one.

Pollack and Wales (1969) added dynamics to the LES by modifying the subsistence parameters. These different modifications allow the subsistence parameters to vary over time. One assumption is that \( \gamma_i \) is a linear function of time, thus

$$\gamma_{it} = \gamma^0_i + \delta_i t$$
Second, they propose a proportional habit formation model in which the current level of subsistence of good $i$ depends upon the previous period's consumption of good $i$.

$$\gamma_{it} = \delta_i x_{i,t-1}$$

Finally, they propose a linear lagged subsistence model.

$$\gamma_{it} = \gamma_{oi} + \delta_i x_{i,t-1}$$

Pollak and Wales obtain estimates for four goods using annual U.S. price and per capita consumption figures for good, clothing, shelter, and miscellaneous goods for the years 1948-1965. Several important results emerge from this study. First, the choice of a dynamic specification of $\gamma_{it}$ is very important. In some cases, the subsistence parameters exceeded the quantities consumed. In this case, the utility function is undefined. Also, the estimates of the $B_i$'s varied with the specification of the subsistence parameters. Pollak and Wales also find that results vary with the estimation technique. One technique is to minimize the sum of squared residuals over all expenditure equations and time periods. This requires no a priori specification of the error structure, but also leads to no knowledge of the properties of the estimators. A second technique is to drop one equation and find the parameter estimates which maximize the likelihood function. This assumes that the sum of the error terms over the four expenditure equations is zero. This yields knowledge of the asymptotic properties of the parameter estimates. Pollak and Wales conclude that maximum likelihood estimation is necessary.
Clements (1976) extends the LES to analyze household behavior in consuming commodities and holding financial assets (1976). Clements chooses to call this system the linear allocation of spending power system (LASS). Households are assumed to maximize utility from the consumption of commodities and the holdings of assets, both real and financial. As Clements notes, this is an extension of the Friedman-Patinkin concept that real cash balances generate a nonpecuniary yield.

Clements estimates this system using seasonally unadjusted per capita data for the period 1968III-1973II drawn from Australia. Goods were aggregated into nine subsets consisting of three commodities and six assets. These are:

1. Food, cigarettes, tobacco, and alcohol
2. Clothing, footwear, and drapery
3. Other consumption
4. Liquid assets
5. Government securities and debentures
6. Equities
7. Dwellings
8. Durable goods
9. Debt

Among his major results, Clements finds that the marginal propensity to consume out of spending power (not income) is .083. This is the sum of the marginal budget shares of the first three groups. Liquid assets and dwellings have the highest marginal budget shares accounting for nearly seventy percent of any additional spending.
The marginal budget share for liquid assets is .409; for dwellings, it is .286. Additionally, Clements finds that consumption is responsive to interest rate changes. The interest elasticity of demand for all categories of consumption with respect to the interest rate on each asset except liquid assets is significantly negative, but small. The elasticity of demand with respect to the interest rate on liquid assets for the three consumption categories is significant and positive, but very small. Clements cautions that any interpretations of these results must be tempered because of the seeming presence of autocorrelation.

Schmidt (1978) uses the LASS to investigate the effect of expected inflation on the behavior of households. Schmidt initially demonstrates that expectations of inflation have a significant and negative effect on the demand for money for the period 1968I-1976IV. Expected inflation was found not to be a significant determinant of money demand during an earlier time period 1954II-1967IV. The early time period corresponds to a period of relatively low inflation, while the later time period corresponds to a time of relatively high inflation. In Schmidt's view, this provides evidence in support of the existence of a threshold effect with regard to the effect of expected inflation. This can be justified in the following manner. At low levels of inflation, the cost of inflation are not sufficient to induce households to alter their behavior; that is, to cause adjustments in their holdings of assets or purchases of commodities. This could be due to the costs of gathering information to generate inflationary expectations exceeding the costs imposed by inflation on the consumer or that the transactions costs of adjusting to inflation
exceeds the costs imposed by inflation. In either scenario, as the expected rate of inflation increases, the costs imposed upon consumers increases. When the expected rate of inflation reaches a high enough level, the costs associated with inflation will exceed the information costs of generating expectations. At that point, households will choose to alter their behavior in response to their expectations.

While Schmidt's principal concern is the effect of expected inflation on money demand, he recognizes that shifts in the demand for money must imply shifts in the demand for other goods; either assets or commodities. Thus, Schmidt alters the LASS by assuming that expected inflation alters only the marginal spending power shares of the LASS, and that this effect occurs in a linear manner. Thus, the marginal budget share, $B_i$, becomes

$$B_i + (B_i^0 \cdot PE_t)$$

where $PE_t$ equals the expected rate of inflation in period $t$ and the sum of the entire expression over the $n$ goods is one. Schmidt uses the following normalization conventions in estimating his version of the LASS.

$$\sum_{i=1}^{n} B_i = 1 \quad \quad \sum_{i=1}^{n} B_i^0 = 0$$

The modified LASS allows the marginal spending power shares to differ in each period, while the normalization conventions require that the "decline in commodity and asset allocations which result from inverse relationships with expected inflation will be exactly matched by increases in the allocation of spending to the commodities and assets that are positively related to expected inflation." Schmidt estimates his version of the LASS for eight goods.
1. Nondurables
2. Services
3. Savings accounts
4. Durables
5. Equities
6. Bonds
7. Demand deposits and currency
8. Debt

The system was estimated for the same period that indicated a negative relationship between expected inflation and the demand for money, namely 1968I-1976II. The main result is that expected inflation causes households to substitute away from equities and services and towards money balances, nondurables, durables, and savings accounts, but primarily toward money and savings accounts.

An alternative to specifying the form of the utility function is to employ the differential properties of consumer demand theory to arrive at demand equations. The Rotterdam model (Theil (1965), Barten (1964)) represents such a model. For our purposes, this model has two advantages over specifying the form of the utility function. First, the empirical results are not tied to a particular specification of the utility function. Second, Theil (1975) has shown that the Rotterdam model is capable of modeling weakly separable preferences.

Constrained utility maximization over a choice set of n goods implies n + 1 first order conditions necessary for a maximum. Differentiating these first order conditions with respect to total
expenditure and price yields, after rearranging, the Fundamental
Matrix Equation of consumer demand theory. This equation has the form

\[
\begin{bmatrix}
u & p \\
p' & 0
\end{bmatrix}
\begin{bmatrix}
aq/\partial y & aq/p' \\
-\lambda/\partial y & -\lambda/\partial p'
\end{bmatrix}
= 
\begin{bmatrix}
0 & \lambda I \\
1 & -q'
\end{bmatrix}
\]

where \( q \) is an \( n \)-element sector of quantities of the \( n \) good
\( p \) is an \( n \)-element sector of prices of the \( n \) goods
\( y \) is a scaler equal to total expenditure
\( \lambda \) is the Lagrangean multiplier
\( I \) is the \( n \times n \) unit matrix
\( U \) is the \( n \times n \) Hessian matrix of second order derivatives of the
utility function

This equation can be solved for price and income derivatives which
Theil uses to develop expressions for changes in budget shares.
Define \( w_i \) to equal \( p_i q_i/y_i \). Then
\[
\begin{align*}
dw_i &= \left( \frac{3i}{y} \frac{dp_i}{dq_i} \right) + \left( \frac{p_i}{y} \frac{dq_i}{dy} \right) - \left( \frac{piq_i}{y^2} \frac{dy}{dy} \right) \\
&= w_i d(\log p_i) + w_i d(\log q_i) - w_i d(\log y)
\end{align*}
\]
Theil notes that demand theory holds total expenditure and prices
constant so that the quantity component of \( dw_i \) is endogenous for the
consumer. Hence,
\[
w_i d(\log q_i) = \frac{p_i}{y} d(q_i)
\]
This component becomes the dependent variable of the Rotterdam
model. This component can be stated in terms of the price and total
expenditure elasticities which follow from solving the fundamental
matrix equation. This component becomes

$$d\frac{q_i}{y} = \frac{\partial q_i}{\partial y} \frac{dy}{y} + \sum_{j=1}^{n} \left( u_{ij} - \frac{\lambda}{\partial \lambda / \partial y} \frac{\partial q_i}{\partial y} \frac{\partial q_j}{\partial y} - \frac{\partial q_i}{\partial y} q_j \right) dp_j$$

where $U_{ij}$ is the $i,j$ element of the inverse of the Hessian.

Multiplying by $\frac{p_j}{y}$ yields

$$w_i \frac{d(\log q_i)}{y} = \frac{p_i}{y} \frac{\partial q_i}{\partial y} \frac{dy}{y} + \frac{p_j}{y} \sum_{j=1}^{n} \left( \lambda U_{ij} - \frac{\lambda}{\partial \lambda / \partial y} \frac{\partial q_i}{\partial y} \frac{\partial q_j}{\partial y} - \frac{\partial q_i}{\partial y} q_j \right) dp_j$$

Theil manipulates this equation to arrive at the Rotterdam demand
equation

$$w_i \frac{d(\log q_i)}{y} = \mu_i(y,p) \left[ d(\log y) - \sum_{k=1}^{n} w_k(y,p) d(\log p_k) \right]$$

$$+ \sum_{j=1}^{n} v_{ij}(y,p) \left[ d(\log p_j) - \sum_{k=1}^{n} v_{jk}(y,p) d(\log p_k) \right]$$

where $\mu_i$ and $v_{ij}$ are parameters. The first term in brackets
represent the change in real income. The second term in brackets
measures the total substitution effect of relative price changes.

Note that the parameters of this model vary with total
expenditures and prices, and that the independent and dependent
variables reflect infinitesimal changes in prices, quantity, and total
expenditures. For empirical purposes, Theil ignores the dependency
of the parameters on prices and total expenditure to develop a finite
change version of the Rotterdam model. With the addition of the time subscript, \( t \), the model becomes

\[
\tilde{w}_{it} Dq_{it} = u_i DQ_t + \sum_{j=1}^{n} v_{ij} (D_{pj} - \sum_{k=1}^{n} \mu_k D_{pk}) + e_{it}
\]

where \( \tilde{w}_t = (w_{it} + w_{it-1})/2 \)

\( D = \log \) change operator: \( D a_t = \log a_t - \log a_{t-1} \)

\( DQ_t = \sum_{j=1}^{n} \tilde{w}_{it} Dq_{it} \)

\( e_{it} = \) random error term

Theory imposes constraints upon the parameters of the model. First, the sum of the \( u_i \), the marginal budget shares, equals 1. Second, the matrix of the price coefficients, the \( v_{ij} \)'s, is symmetric negative definite. Third, the sum of the \( n \) price coefficients for good \( i \) is proportional to the marginal budget share of good \( i \). The proportionality constant, \( \phi \), is the income flexibility parameter. This parameter is the "reciprocal of the income elasticity of the marginal utility of income." \(^9\) Theil also demonstrates that the price coefficient matrix is proportional to the inverse of the Hessian matrix in expenditure terms. \(^10\)

3.4 Conclusion

Much work has been done in examining the relationship between the demand for money and the demand for liquid financial assets. Barnett's application of aggregation theory has been a most important
advance in this area. This work will extend Barnett's work in several ways. First the assumption of sectoral homotheticity will be relaxed. Thus, this also extends the work of Schmidt and Clements, since the linear expenditure system imposes homotheticity with respect to a point defined by the subsistence parameters. Second, the demand for financial assets will be examined within the context of the household sectors demand for both assets and commodities. Finally, we will attempt a search for the optimal partition. On the basis of this partition, the assets to include in an empirical monetary aggregate will be determined.
Footnotes

1. For a more extensive survey, see Feige and Pearce (1977).


4. Donovan (1975) demonstrates that Chetty improperly derived the demand equations. The first order conditions for maximization of the utility function include the budget constraint. Chetty fails to include this condition in deriving his demand equations. Proper derivation would yield quantity demanded as a function of all prices and total expenditure.

5. Moroney and Wilbratte also fail to include the last first order condition in arriving at their demand equations. The quantity demanded should be a function of all prices and the number of transactions T. Although the form of the equation is identical to Chetty's, the parameter estimates will differ with differences in the data on transactions compared to the data on total expenditure.


7. This will be discussed in greater detail in Chapter 4.

8. Schmidt (1978)


10. op. cit. p. 29.
CHAPTER 4
AGGREGATION: SOME THEORY AND IMPLICATIONS

4.1 Introduction

Green (1964) has defined an elementary good as one where any unit of that good is a perfect substitute for any other unit of the same good. Many economists have investigated the theoretical conditions which allow the aggregation of two or more elementary goods into a single composite good. One purpose of this project is to apply these aggregation conditions to the empirical investigation of the household sector's demand for assets and commodities. From this analysis, it may be possible to suggest which assets can be aggregated into a single composite asset called money.

The general result of this research into aggregation is that aggregation is possible only if underlying preferences follow a particular structure. This structure varies with the requirements placed on the aggregate. One possibility which will be examined in section 4.3 requires the existence of an aggregator function which combines several goods into a single composite variable within the household sector's utility function. A second possibility, considered in section 4.4, demands the existence of a price aggregate and a quantity aggregate such that their product equals total expenditure on the individual goods within the aggregate.¹

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4.2 The Basic Household Problem

The basic household problem is to maximize utility:

\[(4.1) \quad u(q) \text{ such that } p \cdot q \leq y\]

where \( q \) is an \( n \)-element vector of elementary goods with \( q_i \) denoting the quantity of the \( i \)th element of that vector. The \( n \)-element vector of prices is denoted \( p \). Again, \( p_i \) indicates the price of the \( i \)th good. The scalar, \( y \), equals total expenditure which can be allocated by the household. Generally, the solution to (4.1) can be expressed as

\[(4.2) \quad q^*(q_1, q_2, \ldots, q_n) = (\phi^1(p, y), \phi^2(p, y), \ldots, \phi^n(p, y))\]

where \( \phi^i(p, y) \), \( i=1\ldots n \), is the demand function for good \( i \).

The utility function, \( u \), satisfies two properties by assumption. First, the marginal utility of each of the \( n \) goods is strictly positive. Mathematically, this requires

\[(4.3) \quad \frac{\partial u}{\partial q_i} > 0, \quad i=1\ldots n\]

Second, the principle minors of order \( k \) \((k \geq 3)\) of the determinant of the bordered Hessian, \( H \),

\[(4.4) \quad H = \begin{bmatrix} 0 & -p_1 & \cdots & -p_n \\ -p_1 & u_{11} & \cdots & u_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ -p_n & u_{n1} & \cdots & u_{nn} \end{bmatrix}\]
have the sign \((-1)^{k+1}\) where \(u_i = a_i u / a q_i\) and \(u_{ij} = a^2 u / a q_i a q_j\), \(i, j = 1 \ldots n\). This property implies that satisfaction of the first order conditions for maximizing \(u\) subject to a total expenditure constraint yields a maximum.²

Now, define the vector \(I\) to be

\[
I = (1, 2, \ldots, n)
\]

This represents the set of integers which identify the variables over which preferences are defined. Imagine a partition of the vector \(I\) into \(m\) subsets, \(m < n\).

\[
I = (I^1, I^2, \ldots, I^m)
\]

This partition satisfies three properties. These are that each element of \(I\) is an element of a subset, no element is a member of more than one subset, and no subset is empty. Define \(n_g\) equal to the number of goods which are elements of \(I^g\), so that \(n_1 + n_2 + \ldots + n_m = n\) and arrange the vector \(q\) so that those of \(I^1\) come first followed by those of \(I^2\) and so on.

Imposing this partition on the solution to (4.1), we can rewrite³ (4.2) as

\[
q = (q^1, q^2, \ldots, q^m) = (\phi^1(p, y), \phi^2(p, y), \ldots, \phi^m(p, y))
\]
where $q^g$, $g=1...m$, is the subvector of quantities of elementary goods in group $g$. The subvector of demand functions for the goods in group $g$ is represented by $\varphi^g(p, y)$.

4.3 The Existence of Aggregator Functions

We seek an aggregator function over a subset of the $n$ choice variables, the $q_i$. Imposing the partition $I$ on $q$, we express the general utility function $u = u(q)$ as

$$u = u(q^1, q^2, ... q^m).$$

Now, hypothesize the existence of $m + 1$ functions $u^g$, $g = 1...m$ and $u$ such that

$$\bar{u} = u(u^1(q^1), u^2(q^2), ... , u^m(q^m)).$$

(4.6)

The functions $u^g$, $g = 1...m$, are aggregator functions. Each $u^g$ aggregates the individual components of $q^g$, the elementary goods $q_i$, into a single composite variable within the utility function.

Equation (4.6) validly represents preferences only if preferences are weakly separable in the partition $I$. Assuming $u$ satisfies (4.3) and (4.4), preferences are weakly separable if and only if

$$\frac{\partial u}{\partial q_k}(\frac{\partial u}{\partial q_i}) = 0, j \in I^g, k \in I^h, g \neq h.$$
This condition states that the marginal rate of substitution between any pair of goods in a group is independent of the level of consumption of all goods which are members of the complement of that group. Blackorby, Primont, and Russell have shown that the aggregator functions can be defined to inherit the properties of the macro-function $u$. Thus, $u^g, g = 1...m$, can be defined to satisfy all the properties of a utility function and can be termed a subutility function.

The restrictions imposed by separability on preferences translates into a structure imposed on the Hessian matrix of the utility function. We consider the impact on the Hessian expressed in expenditure terms. Differentiating equation (4.6) with respect to $q_i$ yields

$$\frac{\partial u}{\partial q_i} = \frac{\partial u}{\partial q_j} \frac{\partial u^g}{\partial q_i} \quad i \in Ig$$

which when differentiated with respect to $q_j, j \in I^h$, results in

$$\frac{\partial^2 u}{\partial q_i \partial q_j} = \frac{\partial^2 u}{\partial q_j \partial q_i} \quad \frac{\partial u^g}{\partial q_i} = \frac{\partial^2 u^g}{\partial q_j \partial q_i} \quad \frac{\partial u}{\partial q_i} = \frac{\partial u}{\partial q_j}$$

In expenditure terms, this becomes

$$(4.7) \quad \frac{\partial^2 u}{\partial (p, q_i) \partial (p, q_j)} = \lambda^2 \frac{\partial^2 u}{\partial q_i \partial q_j} \frac{\partial u^g}{\partial q_i} \frac{\partial u}{\partial q_j}$$

This relies upon the first order condition $\partial u / \partial q_i = \lambda p_i$. 
This term (4.7) summarizes the preference relationship between pairs of goods which are elements of different separable groups. Note that the right hand side of 4.7 is independent of the subscripts i and j. Thus, if two goods are from different groups, utility interactions between the two goods occur at the group level. If goods i and j are elements of the same group, the Hessian term in expenditure form becomes

\[
(4.8) \quad \frac{\partial^2 u}{\partial (p_{ij} q_i) \partial (p_{ij} q_j)} = 2 \frac{\partial^2 u}{\partial u_i \partial u_j} + \frac{\partial^2 u}{\partial u_i \partial u_j} \frac{\partial^2 u}{\partial (p_{ij} q_i) \partial (p_{ij} q_j)}
\]

This expression does depend upon i and j and interactions specific to the two goods occur.

Equations (4.7) and (4.8) can be combined to yield the Hessian in expenditure terms. This matrix summarizes the utility interactions arising from a weakly separable utility function.

\[
(4.9) \quad P^{-1} U P^{-1} = K \begin{bmatrix} 
V_1 & C_{12} & i_1 i_2' & \ldots & C_{1m} i_1 i_m' \\
C_{21} & i_2 & i_1 & \ldots & i_2 \\
& \ddots & \ddots & \ddots & \ddots \\
& & & \ddots & \ddots \\
C_{m1} & & & \ldots & V_m 
\end{bmatrix}
\]

where \( P \) = the diagonal matrix of the price vector \( p \).

\( i, g \) = the unit vector of rank \( n_g \)

\[
V_g = \phi \chi y \quad \frac{\partial^2 u}{\partial u_i \partial u_j} \quad g \neq h
\]

\[
C_{gh} = \phi \chi y \quad \frac{\partial^2 u}{\partial u_i \partial u_j} \quad g \neq h
\]

\[
K = \frac{\lambda}{\phi y}
\]
\[ \phi = \text{reciprocal of the elasticity of the marginal utility of income.} \]

4.4 Decentralizability

Typically, empirical studies of household behavior select a subset of the household's choice set for inclusion in the study. The quantities consumed of the goods included in that subset are expressed as a function of total expenditure on the goods in the subset and the prices of all goods in that subset. Recall that the solution to the basic household allocation problem (4.2) expressed quantity demanded as a function of total expenditure on all goods in the choice set and the prices of all goods in the choice set. Thus, in the typical empirical demand study, the general solution is replaced by

\[ x^r = \phi^r(p^r, y^r) \quad r = 1 \ldots m \]

Estimation of such a system of demand equations for a subset of assets and commodities in the household's total choice set implicitly assumes that the goods included in the estimation are weakly separable from the remaining goods in the choice set.

Consumers face the problem of maximizing utility from all goods in the choice set. The general solution expresses the demand for a single good as a function of all prices and total expenditure. The empirical approach represented in Equation 4.10 simplifies this by assuming an allocation of total expenditure to broad categories of goods. Demand for any individual good is then a function of prices of goods in that same category and total category expenditure.
The existence of demand functions such as (4.10) has been termed weak decentralizability of the household budgeting procedure. The necessary and sufficient conditions for decentralizability is that the utility function be weakly separable in the partition I. Recall that weak separability allows the utility function to be expressed as

\[ u = u(u^1(x^1), u^2(x^2), \ldots, u^m(x^m)) \]

To see the validity of the necessary and sufficient conditions, note that maximization of utility requires maximization of the \( u^r, r = 1, \ldots, m \). Consequently, the demand for the goods in group \( r \) results from utility maximization subject to a budget constraint, \( \sum x_i = y^r \), which yields demand equations of the form (4.10).

The preceding analysis demonstrates the implicit assumption of separability inherent in demand analysis which concentrates on a subset of the choice set. If the assumption of separability is not appropriate, the econometric results will suffer from omitted variables bias. The implication of this analysis for our study is to suggest the necessity of including a wide range of assets and commodities in the empirical study. We have seen that separability is a crucial property in the investigation of aggregates. Therefore, separability assumptions should be avoided wherever possible. Consequently, assets of varying characteristics and durable and nondurable commodities will all be included in the empirical analysis.
4.5 Additive Price Aggregation

Green has argued that the aggregator functions derived from a weakly separable utility function are inadequate. He reasons that the aggregation of two or more elementary goods yields a single composite good. This good should have a single price, and the product of the price and quantity of this composite good should equal total expenditure on this good. The aggregator function defined in equation (4.6) based solely on weak separability of the utility function fails to satisfy Green's criteria. A much more restrictive structure than weak separability must be imposed upon the utility function in order to meet Green's condition.

The necessary and sufficient condition for the existence of a price aggregate and a quantity aggregate for some subset of the household's choice set such that their product equals total expenditure on the goods in that subset are that the utility function is continuous, positively monotonic, and strictly quasiconcave, that U is separable in the partition I, and that each aggregator function \( u^r, r = 1, \ldots, m \), is homothetic.\(^9\) These conditions are termed homothetic separability. The existence of such aggregates has been termed both additive price aggregation and additive quantity aggregation. Formally, additive price aggregation or, alternatively, additive quantity aggregation is defined as the existence of positive linear homogenous functions \( \pi^r \) and \( \pi^r, \pi^r, r = 1, \ldots, m \), such that

\[
\pi^r(p^r) \cdot \pi^r(x^r) = y^r
\]
where $x^r$ and $y^r$ are optimal quantities of $x^r$ and optimal group expenditures respectively.

While this type of aggregate has great appeal, it also places severe restrictions on preference structures. The demand function for each good which is an element of the aggregate must satisfy the same properties as any demand function derived from a homothetic utility function. Foremost among these properties is the requirement that group expenditure elasticities equal one for all price vectors and total expenditure levels. Consequently, all goods partitioned into a homothetically separable group must have identical total expenditure elasticities. This indicates that such intuitive groupings as food or transportation are inappropriate, since luxuries and necessities cannot be grouped together.

4.6 The Strotz-Gorman Two-Stage Budgeting Procedure

Barnett (1980) applied the necessary and sufficient conditions for additive price aggregation to the problem of a consistent monetary aggregate based on the Strotz-Gorman (1959) two-stage budgeting procedure. Additive price aggregation and the Strotz-Gorman two-stage budgeting procedure are closely related but are not identical. Formally, homothetic separability is sufficient but is not necessary for the consistency of the two-stage budgeting procedure. The necessary and sufficient conditions for the consistency of this procedure is that the utility function be expressible as

$$U = \sum_{r=1}^{d} v^r + u(u^{d+1}) , \ldots , U^m(x^m)$$

(4.12)
where $U^r_\cdot$, $r = d+1, \ldots, m$, is linear homogenous in $x^r$ and $V^r$
takes the Generalized Gorman Polar form

$$V^r = G_r(y^r/\pi^r) + H_r \quad r = 1, \ldots, d$$

where $G_r$ is an increasing function of $(y^r/\pi^r)$, $\pi^r$ is a
positively homogenous function and $H_r$ is homogeneous of degree
zero.\textsuperscript{10} Thus, the conditions for two-stage budgeting are more
general than those for additive price aggregation. However, the
strong separability condition inherent in $V^r$ is not innocent.

Houthaker (1960) has shown that if the utility function is strongly
separable, the marginal cross price response must be proportional to
the marginal total expenditure response. Further, strong
separability results in the own price elasticity being approximately
proportional to the total expenditure elasticity. Barnett's model
assigns a particular functional form to the $u^r$ of equation (4.12)
in that it imposes homothetic seperability.\textsuperscript{11} He uses this model
to construct a new monetary aggregate. The problem here is the
theoretical restrictiveness of the maintained hypothesis of
homothetic separability.\textsuperscript{12} Moreover, homotheticity or linear Engel
curves have generally been rejected in empirical tests.

An important step would be to seek a monetary aggregate based on
the weaker restrictions of separable preferences. This would
eliminate the dependence of the results on the highly restrictive
conditions of homothetic separability or strong separability. The
cost of relaxing these restrictions is to lose the rigor of additive
price aggregation.
4.7 Conclusion

The analysis presented here has important implications for this study. First, we have seen that aggregates can be constructed on different levels of rigor; each level requiring different necessary and sufficient conditions. Barnett constructs a monetary aggregate based on the most rigorous of these conditions, homothetic separability. Homotheticity is, however, a very strong condition to impose on consumer behavior. The approach taken here relaxes the assumption of homotheticity but maintains the framework of separable preferences. If separability were rejected, homothetic separability would also be rejected. We assume that demand deposits plus currency are included in the monetary aggregate. Other assets will also be included in the monetary aggregate if they are grouped with demand deposits plus currency in the optimal partition of all assets and commodities included in the study.

A second insight we have gained is the implied separability assumption of empirical demand analysis which includes only a subset of the entire choice set. Failure of this assumption to hold true indicates the empirical results suffer from omitted variables bias. Separability is also an important part of the necessary and sufficient conditions for aggregation. Therefore, we prefer to avoid assuming separability unnecessarily. As a result, we include a wide spectrum of assets, durables, and non-durables in the analysis.

Two methods exist for choosing a monetary aggregate based on the properties explicated here. First, a functional form can be chosen which allows for direct testing of various separability properties.
Unfortunately, the current functional forms, including the flexible functional forms such as the trans-log, do not allow for testing of weak separability or of homothetic separability. The second approach, followed here is to choose a functional form which imposes the desired property with respect to a partition chosen by the researcher. The problem is to find the optimal partition of the goods. The model will be described in detail as will the estimation procedure in Chapter 6.

First, we turn to an examination of the data in Chapter 5.
Footnotes

1. In general, we require that a quantity aggregate be defined solely as a function of quantities and price aggregates be defined solely as a function of the prices of individual goods.

2. The first order conditions for utility maximization are:
   
i. \( \frac{\partial u}{\partial q_i} = p_i \quad i = 1, \ldots, n \)
   where \( p_i \) is the Lagrangean multiplier
   ii. \( y - \sum p_i q_i = 0 \)

3. This is strictly a change in notation. Equation (4.5) imposes no additional properties or constraints on the general solution, equation (4.2).

4. Again, no additional structure is imposed on preferences by this step.

5. For proof see Blackorby, Primont, and Russell (1978) p. 108.

6. Definition of separability due to Leontief (1947). Blackorby, Primont, and Russell (1978) offer an alternative definition of separability which does not require differentiability of the utility function. Both definitions are identical if the utility function is differentiable.


8. This discussion follows from Theil (1975) Vol. 2.


11. Barnett's model imposes a particular form of homotheticity. The use of subsistence parameters indicates that the Engel curves are linear but emanate from a point defined by the subsistence parameters. This has been termed quasi-homotheticity.

12. This assumption is even stronger since this particular functional form imposes strong separability rather than weak. Thus, Barnett ignores the tradeoff between homotheticity and separability in equation (4.12).

13. Blackorby, Primont, and Russel (1978), p. 308-310, show that the trans-log is incapable of modeling weak separability and can model only a certain form of homothetic separability. Denny and Fuss (1977) show if the trans-log is interpreted as an approximation to the true utility function, it is capable of modeling weak separability, but the statistical properties of the estimators are valid only at the point of approximation.
CHAPTER 5

DATA

5.1 Introduction

The consumer demand model which will be estimated includes both nondurables and durables along with assets. Estimation of the model requires data for each time period on the price of each good, total expenditure for each good, and total expenditure on all goods included in the system. The inclusion of both assets and commodities in the analysis poses a problem since observed data measures the flow of nondurables consumed and the end of period stocks of financial assets and durables. An adequate measure of the price of an asset or durable is not obvious in this case. Clements (1976) attacks this problem by stating the wealth constraint facing households in each time period and making an assumption on the timing of changes in the household portfolio. Thereby, Clements derives a measure of the price of an income earning asset or depreciating asset in a stock/flow context. This chapter summarizes Clements' price measure and details the construction of the data employed in this study.

5.2 Asset Prices

Clements develops this price measure by noting that, in any period t, the constraint on the maximization of utility from the flow of nondurables and the holding of real and financial assets is that
the value of consumption plus the value of the portfolio in period $t$ must equal income in period $t$ plus the value in period $t-1$ of the portfolio held at the end of period $t-1$, minus the depreciation of the stock of durables.

This wealth constraint together with timing assumption on changes in the portfolio yield a measure of the prices of the assets, the $\pi_t$. For each asset, two terms are required to construct its price: $r$, the rate of return, and $\pi^*$, the pure asset price.

$$\pi_t = [I_n - 1/2 r_t] \pi^*.$$

5.3 Data Sources

The data on the nine goods used in this study were taken from three main U.S. sources: Federal Reserve System's Flow of Funds, the Federal Reserve Bulletin, and the Department of Commerce's Survey of Current Business. Data is quarterly for the period 1960I-1978II and seasonally unadjusted except where noted. All prices were scaled to equal one in the first time period.

1. **Currency Plus Demand Deposits**

   End of period holdings of the household sector were taken from the Flow of Funds. The price was measured by the consumer price index taken from the Survey of Current Business. The rate of return on currency plus demand deposits is assumed to equal zero.

2. **Time Deposits at Commercial Banks**

   End of period holdings are taken from the Flow of Funds for the household sector. Price is measured by the CPI published in the Survey of Current Business. Interest rates were provided by the Flow of Funds Section, Federal Reserve System.
3. **Other Savings Accounts**

   End of period holdings by the household sector of all savings accounts at institutions other than commercial banks reported by the *Flow of Funds*. Prices are measured by the CPI reported in the *Survey of Current Business* and interest rates were provided by the *Flow of Funds* Section, Federal Reserve System.

4. **Government Securities**

   End of period holdings of all treasury issues by the household sector excluding savings bonds taken from the *Flow of Funds*. Rate of return is measured by the average of the three-month treasury bill rate and the ten-year bond rate as reported in the Federal Reserve *Bulletin*. The pure price is measured by the inverse of one plus the rate of return.

5. **State and Local Government Securities**

   Household sector's end of period holdings reported in the *Flow of Funds*. Interest rate reported in the Federal Reserve *Bulletin* and the pure price is measured by the inverse of one plus the interest rate.

6. **Corporate and Foreign Bonds**

   The household sector's end of period holdings are from the *Flow of Funds*. The rate of return is measured by the rate on corporate bonds reported in the Federal Reserve *Bulletin*. The pure price is measured by the inverse of one plus the rate of return.

7. **Equities**

   End of period holdings for the household sector reported in the *Flow of Funds*. The pure price of equities was measured by
Standard and Poors' index of 500 leading stocks as reported in the Federal Reserve Bulletin. The rate of return was measured as the dividend price ratio for common stocks reported in the Federal Reserve Bulletin.

8. Durables

No seasonally unadjusted stock series was available for consumer durables. Seasonally unadjusted series were available for purchases of durables and depreciation of durables. The end of period stock series was created by taking the seasonally adjusted stock of durables for 1951IV and adding net changes for each period. Starting the procedure prior to the start of the sample period is designed to negate the influence of using a seasonally adjusted stock as the base. The pure price of durables is measured by the durables component of the CPI as reported in the Survey of Current Business. The rate of return on durables is computed as

\[ r_t = \frac{D_t}{\frac{1}{2} \alpha_t V_t + \frac{1}{2} \tilde{V}_{t-1}} \]

where

- \( D_t \) = Depreciation charge
- \( \alpha_t \) = Pure price of durables
- \( V_t \) = Quantity of durables
- \( \tilde{Z}_t \) = Realization price of durables
- \( \tilde{\alpha}_t \) =
9. Consumer Goods

Expenditures on consumer goods taken from the Federal Reserve Bulletin. The pure price is measured by the nondurables component of the CPI reported in the Survey of Current Business. No rate of return is required. Population statistics from the U.S. Bureau of the Census were used to compute expenditures to per capita terms.

5.4 Data Considerations

The data contain three goods, demand deposits plus currency, time deposits at commercial banks, and other savings deposits whose rates of return were capped by federal government regulations. Regulation Q prohibited commercial banks from paying interest on demand deposits and capped interest rates for time and savings deposit. Regulation Q's impact should be reflected in changes in relative prices of regulated assets compared to unregulated assets and commodities.

The data span a period of relatively stable financial markets. The period predates the widespread emergence of negotiable order of withdrawal accounts, money market mutual funds, and small certificates of deposits resulting from the gradual deregulation of the banking and financial services industry. The emergence of these assets could alter the preferences which underlie the data.

An important assumption implicit in the analysis is that the household sector's housing stock is weakly separable from the nine goods included in the analysis. Decisions regarding housing may be
the most important decisions that the average household makes. Thus, this assumption could have important consequences for the estimation results.

Housing decisions seem compatible with the decentralization implied by weak separability. At the time the home is purchased, the household typically draws from its accumulated wealth sufficiently to meet the required down payment. The assumption that this decision can be made independently of current prices of all other goods appears plausible. The major determinants of housing purchases most likely are expectations of future income and relative housing prices. After the home is purchased, the assumption of weak separability becomes more reasonable. Here, the mortgage agreement fixes housing expenditures. These expenditures are independent of current spending power and prices, including housing prices. These factors suggest that the assumption of weak separability is appropriate and that housing may be excluded from the analysis without biasing the results.

An additional reason for excluding housing is that housing decisions do not fit well with the model used. The Rotterdam model assumes equilibrium and continuous choice. High transaction costs limit the homeowner's ability to rapidly adjust housing to price change. Note that some transaction costs depend upon the price of the home, not the homeowner's equity. Also, housing choices are better characterized as a discrete rather than a continuous variable. The purchase of a home requires a large minimum equity payment. Thus, households must accumulate sufficient wealth to purchase the home. After purchase, homeowners are unable to freely adjust their equity portion in response to price changes.
If the exclusion of housing is improper, the parameter estimate for durables would be the estimates most likely affected. Home purchase and durable goods purchases are linked as the new homeowner often must purchase major appliances. Also, as housing prices rise, homeowners that convert this rise in wealth into liquid form through second mortgages are likely to reallocate this wealth to durables.
CHAPTER 6
SPECIFICATION AND ESTIMATION OF THE ROTTERDAM MODEL
UNDER SEPARABLE PREFERENCES

6.1 Introduction

Examination of B. Friedman's macro model has demonstrated
conditions under which optimal monetary policy consists of
controlling the stock of nonborrowed bank reserves. M. Friedman has
suggested that the uncertainties of implementing monetary policy
under real world conditions require that the monetary authority
follow a simple growth rule regarding the money supply. However,
many assets exist which can be considered as money or near-money and
the central bank's choice of definition of the money supply under
such a policy can have important short-run implications for the
national economy.

The theoretical construct of separable preferences defines an
explicit criteria for determining the existence of an aggregate.
Thus, this criteria may determine which assets should be treated as
money. The task then is to model preferences so as to determine the
appropriateness of applying separability conditions to determine
which assets to include in a monetary aggregate.

As previously noted, we narrow the problem by considering only
household behavior. Thus, this methodology abstracts from the
problem of aggregating across sectors of economic agents. We assume
that households maximize utility from their holdings of assets and

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from consumption of commodities subject to a total spending power constraint. This places the analysis within the framework developed by M. Friedman (1955) and Patinkin (1965).

The empirical analysis will employ extensions of the Rotterdam model examined in Chapter 3. These extensions allow the impact of weak separability assumptions to be incorporated into the analysis. We now turn to the Rotterdam model and its weakly separable extensions.

6.2 The Rotterdam Model

Recall from Chapter 3 that the finite change version of the Rotterdam model takes the form:

\[ \tilde{W}_{it}Dq_{it} = \mu_i Dq_t + \sum_{j=1}^{n} V_{ij} (Dp_{jt} - \sum_{k=1}^{n} \mu_k Dp_{kt}) + \varepsilon_i \]

\[ i = 1 \ldots n \]

Theil denotes this the relative price version of the Rotterdam model. The origin of this name lies in the term in parantheses which compares the log change in the price of the jth good to a price index constructed by weighting the log changes of the prices of the n goods by their marginal budget shares. The relative price version is nonlinear in its parameters and, therefore, may be quite difficult to estimate. Theil, however, developed a linear version of the Rotterdam model known as the absolute price version. This takes the form:
\[ (6.2) \quad \tilde{w}_{it} \Delta q_{it} = \mu_i \Delta q_t + \sum_{j=1}^{n} \pi_{ij} \Delta p_{jt} + \varepsilon_{it} \]

i = 1, \ldots, n

Here, absolute prices appear rather than relative prices as in (6.1).

The price coefficient of (6.2), the \( \pi_{ij} \)'s, are related to the price coefficients of (6.1) by:

\[ \pi_{ij} = v_{ij} - \delta \mu_i \mu_j. \]

Economic theory imposes three constraints on the absolute price model. First, the marginal budget shares, the \( \mu_i \), sum to one. Second, the matrix of price coefficients is symmetric, negative semi-definite. Third, the sum of the price coefficients for each good \( i \) totals zero.

The Rotterdam model yields easily calculated elasticities. The income elasticity of demand for good \( i \) is:

\[ \eta_i = \frac{\mu_i}{w_i} \]

While the price elasticity of demand for good \( i \) with respect to price \( j \) is:

\[ k_{ij} = \frac{\pi_{ij}}{w_i} \]

The Rotterdam model is derived from the differential properties of consumer demand theory. At this theoretical level, the model includes infinitesimal changes in exogenous variables and specifies coefficients as dependent upon prices and income. The infinitesimal version imposes no specific form on the utility function. However,
empirical implementations of the model requires that coefficients be
held invariant and that the exogenous variables be measured as finite
changes. This finite change model corresponds to (6.2).

The finite change model has been criticized because it is
compatible with theory only if each consumer's preferences are
Cobb-Douglas. ¹ This permits integrability of the model to a
community utility function, but imposes overly restrictive properties
on preferences. These properties have generally been ignored in
applications. However, Barnett has derived the model in a new and
more general way which does not tie the model's theoretical basis to
a community utility function. Barnett first derives the individual
consumer's allocation decision. At this level, what Barnett terms
the micro level, coefficients vary. Next, he aggregates across
consumers using very weak assumptions. The result is a "limiting
stochastic transformation of aggregate economic theory."² At this
macro level, coefficients are fixed. The essence of Barnett's method
is that constant coefficients at the macro level do not imply
constant coefficients at the micro level. Barnett's parameterization
of the transformation results in a first order Taylor series
approximation to the transformation. This necessitates that a
remainder term be dropped which Barnett argues will be uniformly
small.

6.3 The Rotterdam Model Under Separable Preferences

The primary goal of the project is to evaluate the applicability
of separability conditions for determining an empirical definition of
money. Theil developed the impact of weak separability on the Rotterdam model. The basis for this development is the structure of the \( n \times n \) Hessian matrix in expenditure terms under weakly separable preferences.\(^3\) This matrix was described in Chapter 4.

Note again that weak separability allows the utility function to be expressed as:

\[
    u(q) = \overline{u}(u_1(q^1), \ldots, u_m(q^m))
\]

Theil demonstrates that this utility structure yields a two-level demand system. At the first level, the consumer allocates total spending among the \( m \) groups of goods. At the second level, the consumer spends each group's allocation of total spending power on the individual goods of that group. Hence, the first level is characterized by a system of \( m \) equations, one equation for each group. The second level consists of \( m \) sets of equation systems; each system has the same number of equations as the group does goods.

We begin with Theil's specifications of demand at the first level.\(^4\) We term this the composite demand for group \( g \). These equations take the form:

\[
(6.3) \quad \tilde{w}_{gt} \cdot D_{0t} = M_{gt} DQ_t + \sum_{h=1}^{m} \gamma_{gh} \cdot D\tilde{P}_{ht} + E_{gt} \quad g = 1, \ldots, m
\]

where

\[
\tilde{w}_{gt} = \sum_{i \in I_g} \tilde{w}_{it} \cdot \tilde{D}_{it}
\]

\[
D_{0t} = \sum_{i \in I} \tilde{w}_{it} \cdot \tilde{w}_{gt} \cdot D_{qt}
\]
\[ M_g = \sum_{i \in I_g} \mu_i \]
\[ D_{pht} = \sum_{i \in I_h} \mu_i M_h D_{pit} \]

Thus, \( \bar{w}_{gt} \) equals the average budget share of the group in periods \( t \) and \( t-1 \), \( DQ_{gt} \) measure the log change in the quantity of the composite group good, \( M_g \) is the marginal budget share of the group and equals the sum of the marginal budget shares of the goods which are elements of the group, and \( D_{pht} \) is a price index for group \( h \) constructed by weighting the individual prices of the goods by the ratio of each good's marginal budget share to the group's marginal budget share.

Theoretical restrictions on the parameters of (6.3) follow directly from those placed on (6.1). Again, the marginal budget shares sum to one. Also, the \( M \times M \) price coefficient matrix is symmetric negative semi-definite. Finally, each row of this matrix sums to zero.

The system of equations defined by (6.3) allocates total spending among the groups. Once this allocation has been made, \( m \) second stage allocations must be made. These allocations are defined by:

\[
(6.4) \quad w_{it} D_{qit} = \mu_i \bar{w}_{it} D_{qit} + \sum_{j=1}^{m} \pi_{ij} \bar{w}_{jt} D_{pjt} + \epsilon_{it} \]

The \( m \) sets of demand systems defined by (6.4) are termed conditional demand systems. This follows from the fact that each is conditional upon the initial allocation of (6.3). Once more, the restrictions on
parameters follow directly from (6.1). The marginal budget shares, $\mu_i^g$, sum to one, the sum of price coefficients for each good i totals zero, and the matrix of price coefficients is symmetric negative semi-definite.

Together, (6.3) and (6.4) define a consistent two-level allocation procedure. Further, Theil demonstrates that the disturbance term of the composite demand system are independent of the disturbance term of the m conditional demand systems. Also, the disturbance terms of each conditional demand system are independent of those of the m-1 remaining conditional demand systems. Consequently, each conditional demand system can be empirically evaluated independently.

The coefficient estimates obtained from (6.4) and (6.3) can be consolidated into a demand system of the form (6.1). This consolidation is based on the following relationships.

$$\pi_{ij} = \pi_{ij}^g + \pi_{ij}^* \mu_i^g \mu_j^g \quad i, j \in I_g$$

(6.5) $$\pi_{ij} = \pi_{gh}^* \mu_i^g \mu_j^h \quad i \in I_g, j \in I_h, h \neq g$$

$$\mu_i = M_g \mu_i^g \quad i \in I_g$$

6.4 Estimation Strategy

The two-level demand system defined by (6.3) and (6.4) can be used to investigate the appropriateness of separable structures for household preferences. The starting point for the analysis is to assume no separability. Thus, (6.2) is the applicable model for this case. Note that (6.2) is identical to (6.3) if each group consists of exactly one good.
The second step of the analysis requires that assumptions regarding separable structures be imposed on the utility function and empirically investigated through the two-level Rotterdam model. One goal of these assumptions is to minimize the structure imposed on household preferences at the same time information is generated on the appropriateness of aggregating monetary assets. Consequently, the existence of just one separable group is hypothesized. Mathematically, this assumption becomes:

\[ U(q_1, q_2, \ldots, q_n) = \bar{u}(u^1(q^1), q_j, \ldots q_n) \]

and the individual elements of \( q^1 \) would be defined as components of the money supply. This has three advantages. First, it reduces the number of possible groupings of the n goods. Second, it removes the possibility that an appropriate separable structure imposed on monetary variables would be rejected because an inappropriate structure had been imposed on the non-monetary variables. Third, it minimizes the additional structure beyond rational behavior placed on preferences.

In choosing partitions, currency plus demand deposits will be accepted as money. The partitions will widen the group of assets treated as money by including additional assets in the group with demand deposits plus currency.

Given a partition, the estimation process begins at the second level, equation system (6.4). This system is estimated only for those assets partitioned into the monetary aggregate, since the allocation which occurs at the first level completely defines the demand for the remaining goods. Estimation of (6.4) yields estimates
of the conditional marginal budget shares which can be used to calculate the aggregate price index necessary to estimate equation system (6.3).

Once (6.4) and (6.3) have been estimated, these results can be consolidated using the relationships of (6.5) to derive a nine equation system of the form (6.2). The likelihood function of the consolidated model can be compared with that of the nonseparable version of (6.2) in a likelihood ratio test. This test will determine the appropriateness of the separability assumption.

6.5 Estimation Technique

Two factors complicate the estimation of the parameters of (6.2), (6.3), and (6.4). First, the Rotterdam models are allocation models and, consequently, they form singular equation systems. This necessitates that one equation be deleted during estimation. Barten (1969) has proven that maximum likelihood estimates are invariant to the choice of equations deleted. Second, the individual equations of the system are disturbance related.

Assume the equation system

\[ Y = XB + e \]

where \( Y \) is the \( NT \times 1 \) vector of observation on the endogenous variables, 
\( X \) is the \( NT \times K \) matrix of observations of predetermined variables, 
\( B \) is the \( K \times 1 \) vector of parameters, and 
\( e \) is the \( T \times 1 \) vector of random error terms.
Let $e_{it}$ be the error for the $i$th equation in the $t$th time period.

Using ordinary least squares (OLS) to estimate the system, assume that

1. $E(e_{it}) = 0$  
   $i = 1 \ldots n$  
   $t = 1 \ldots T$

2. $E(e_{it}e_{js}) = \sigma_{ij}$  
   if $i = j$  
   $s = t$

   $= 0$  
   if $i \neq j$

   or $s \neq t$

This assumes that the individual equations are not disturbance related. OLS estimates are unbiased, but not minimum variance if, in fact, the equations are disturbance-related.

Zellner's Seemingly Unrelated Regressions (SUR) estimation relaxes the OLS assumptions on error terms and admits contemporary correlation between equations. The SUR estimation assumes:

1. $E(e_{it}) = 0$  
   $i = 1 \ldots n$  
   $t = 1 \ldots T$

2. $E(e_{it}e_{jt}) = \sigma_{ij}$  
   $t=s$

The covariance matrix for the entire vector of error term is:

$$
E(e'e) = \begin{bmatrix}
\sigma_{11} I_t & \sigma_{1n} I_t \\
\sigma_{21} I_t & \sigma_{nn} I_t
\end{bmatrix} = \phi
$$

In Kronecker product notation this becomes:

$$\phi = \otimes \sigma I_t$$
The basis of the SUR estimation is that $\phi$ contains information which OLS estimates ignore. By considering this information, SUR estimates become more precise than OLS estimates. The SUR estimator is:

$$B = [X'(X'X\sigma^{-1})^{-1}X' \Sigma^{-1} \sigma I] Y$$

This estimator uses $\Sigma$ as an estimate of $\sigma$. The estimate of $\Sigma$ is derived from the residuals resulting from OLS estimation of the system. Thus,

$$\hat{\sigma}_{ij} = \tau^{-1} \hat{e}_i \hat{e}_j$$

Equations (6.7) and (6.8) can be combined in an iterative process. First, OLS results are used to compute (6.8) which, in turn, are used to estimate (6.7). The residuals from applying (6.5) to the data can then be used to recompute (6.6). Again, (6.5) can be estimated. This iterative procedure can be repeated until convergence is obtained.

The iterative estimator yields estimates which are asymptotically consistent, unbiased, and fully efficient.

6.6 Autocorrelation

Autocorrelated error terms often trouble economic time series models. Autocorrelation results in unbiased parameter estimates, but not minimum variance estimates. Consequently, a correction for autocorrelation was included in the estimation process. Berndt and Savin (1975) demonstrate that the restrictions of a singular equation system impose restrictions on the autocorrelation process. For the n
equation system

\[ Y = XB + e \]

assume the error terms follow a first-order autoregressive process. Thus,

\[ e_t = R e_{t-1} + v_t \quad t = 2 \ldots T \]

where \( R \) is an \( n \times n \) matrix of unknown autocorrelation parameters and the \( v_t \) are independent and normally distributed error terms with zero mean.

Berndt and Savin demonstrate that singularity of \( R \) requires that each column sum of \( R \) equal an unknown constant. If this restriction is ignored, maximum likelihood estimates will vary with the choice of equation deleted. For simplicity, we assume \( R \) to be diagonal. Berndt and Savin's restriction necessitates equality of the diagonal elements. This implies that all cross-equation impacts arising from the error terms occur contemporaneously and are captured by the seemingly unrelated regressions technique.

Autocorrelation can be analyzed through an iterative process. First, the model was estimated assuming no autocorrelation. This yields maximum likelihood estimates conditional on the assumption that \( R \) is a zero matrix. Second, the error terms from the first step were regressed on lagged error terms with \( R \) specified as a diagonal matrix. This yields estimates of \( R \)'s elements conditional on the parameter estimates of step one. Third, the original data were transformed using the autocorrelation parameter estimated in step two and Rotterdam model parameters were estimated. This process yields consistent and efficient estimates of the Rotterdam model parameters and the autocorrelation parameter.
This process was applied to the nine-good Rotterdam model with no separability imposed. A likelihood ratio test was used to test the hypothesis that the autocorrelation matrix, \( R \), was a zero matrix. This test statistic is

\[
2(L_u - L_R)
\]

when \( L_u \) is the log of the likelihood formation of the unrestricted model and \( L_R \) is the log of the likelihood function of the restricted model. \( L_u \) equals 3132.8, while \( L_R \) equals 3122.0. The test statistic equals 21.6 and exceeds the critical value of the chi-square distribution at 5\( \alpha \) probability and one degree of freedom. Hence, we can reject the hypothesis that \( R \) is a zero matrix. Consequently, a correction for autocorrelation was included in all subsequent estimation.

The presence of autocorrelated error terms suggests two potential problems. First, omitted variables bias may exist. Implicit in this analysis is the assumption that any good not included in the data set is weakly separable from those goods included. If the assumption is invalid, the impact of omitted variables may be reflected in autocorrelated error terms. Second, autocorrelated errors may reflect incomplete adjustment by households to change in exogeneous variables. Thus, autocorrelation parameters capture dynamic changes in household behavior.

6.7 Estimation Results

The first step in investigating the separability properties of household preferences is to estimate the Rotterdam model assuming no separability. The coefficient estimates and their asymptotic
t-ratios are reported in Table 4.8 Table 5 details the spending power and price elasticities. The spending power elasticities vary over a small range with the exception of the elasticities for durables and equities. Equities' elasticity is very high, 2.97, and is highly asymptotically significant.9 The coefficient's t-ratio exceeds 26. The elasticity for durables equals .032 and its underlying coefficient is not significant. The seven remaining spending power elasticities lie between .12 and .30.

The extremely low spending power elasticity on durables indicates that as spending power increases households do not immediately increase purchases of long-lived goods. Note the tight grouping of the spending power elasticities of currency plus demand deposits, time deposits at commercial banks, and other time deposits. This suggests that, over the long-term, the relative proportions of these assets held by the household sector will change slowly if relative prices remain stable. The marginal spending power shares of state and local bonds and corporate bonds are at the low end of the range.

Several interesting patterns exist among the price elasticities. The absolute level of the own- and cross-price elasticities among the three money and savings deposits goods tend to be high. Exceptions include those between time deposits at commercial banks and other savings deposits. However, none of the underlying coefficients is significant. In contrast elasticity estimates indicate that changes in the prices of bonds, equities, durables, and non-durables have little impact on the demand for the money and savings deposit goods. Again, the underlying coefficient estimates are insignificant. Two of the nine own-price coefficients are positive, but neither is significant. The remaining own-price coefficients are negative,
three are significant, and the remainder, while not significant, have
t-ratios higher than those of the positive own-price coefficients.
The log of the model's likelihood function equals 3132.8.

This unrestricted model can be compared to models with
separability imposed. The validity of alternative separability
assumptions can be tested through a likelihood ratio test. The test
statistic is

$$2(\log L_u - \log L_R)$$

where $L_u$ is the unrestricted model's likelihood function and $L_R$
is the restricted model's likelihood function. This statistic is
distributed chi-square with $q$ degrees of freedom. Theil shows that
the number of free price coefficients equals

$$(6.) \quad \frac{1}{2} M (M-1) + \frac{1}{2} \sum_{g=1}^{M} n_g (n_g-1)$$

Thus, the unrestricted model has 36 free price coefficients.

The first separability assumption considered is that currency
plus demand deposits and time deposits at commercial banks are weakly
separable from the seven remaining goods in the choice set. The
model (6.3) is estimated for eight goods; the hypothesized monetary
aggregate and the seven remaining goods. This model will be referred
to as the stage one model. Table 6 presents the coefficient
estimates and t-ratios and Table 7 details the stage one
elasticities. The spending power shares and elasticities are similar
to those of the non-separable model. The spending power share of the
aggregate nearly equals the sum of the marginal spending power shares
of the individual goods of the aggregate estimated in the non-
separable model. Also, the absolute level of own- and cross-price
elasticities between aggregated money and other savings are higher,
although the underlying coefficient estimates are insignificant. All
eight own-price coefficients are negative and three, those pertaining
to U.S. bonds, corporate bonds, and non-durables, are significant.
The five remaining own-price coefficients are not significant, but
generally have t-ratios exceeding the corresponding t-ratios of the
unrestricted model.

The lack of precision of the price coefficient estimates partly
results from multi-collinearity. Separability assumptions permit
aggregation of goods and reduces the range of the price coefficient
matrix. This may reduce multi-collinearity and increase the
precision of the estimates. This effect can be seen by comparing the
t-ratios of the stage one model to those of the unrestricted model.

Given the stage one allocation, households must allocate group
spending power among the group's components. Table 8 details
coefficient estimates and t-ratio's for this stage two decision;
Table 9 lists the conditional elasticities. Note the low estimates
of the own- and cross-price conditional elasticities compared to
those of the unrestricted model and the stage one model. Coefficient
estimates from stages one and two were consolidated using (6.5).
Table 10 reports these coefficients and Table 11 reports the
elasticities. The log of the consolidated model's likelihood
function equals 3122.5. Thus, the test statistic between this model
and the unrestricted model equals 18.6. Using (6.9), seven degrees
of freedom exist. The critical value of the chi-square distribution
with seven degrees of freedom and 5% probability equals 14.0671. The
test statistic exceeds the critical value and we reject the
hypothesis that demand deposits plus currency and time deposits at
commercial banks are weakly separable from the remaining goods.
Next, we broaden the monetary aggregate to include other savings deposits with currency plus demand deposits and time deposits at commercial banks. This yields seven goods at the stage one level. Tables 12 and 13 detail stage one results. Patterns established in the previous stage one and unrestricted models appear here. The marginal spending power shares of the monetary aggregate approximate the sum of the individual marginal shares of the aggregate's components estimated in the unrestricted model. All seven own-price elasticities are negative, four of these are significant. Generally, the precision of the estimates improved compared to the previous models.

The stage two decision involves an allocation among the three components of the monetary aggregate. Tables 14 and 15 detail the results. Tables 16 and 17 report results for the consolidated model. The consolidated model's likelihood function has a log value of 3115.7 and the model has 12 degrees of freedom. The test statistic of 34.2 exceeds the critical value of 21.026 at 5% probability. Hence we reject the hypothesis that currency plus demand deposits, time deposits at commercial banks, and other savings deposits are weakly separable from the remaining six goods.

The third separability assumption tested is that currency plus demand deposits, time deposits at commercial banks, other savings deposits, and U.S. government bonds are weakly separable from the five remaining goods. This hypothesis results in six goods at the stage one level. Tables 18 and 19 summarize the results. The marginal spending power share for the monetary aggregate does not approximate the sum of the marginal shares of its components from the unrestricted model. This may indicate that aggregation of U.S. bonds
with the currency and deposit accounts poses serious estimation problems. The own-price elasticities are negative and four of them are significant. Again, coefficient estimates seem to have greater precision as the number of price coefficients decreases.

The stage two model has four goods; results are reported in Tables 20 and 21. Consolidated results are reported in Tables 22 and 23. The unique result in the model is the negative marginal spending power share on U.S. bonds. However, the estimate is not significant. The log of the likelihood function for the consolidated model equals 2951.5 and the model has 15 degrees of freedom. The test statistic equals 362.6 compared to a critical value of 24.996. Hence, the hypothesized partition is rejected.

The empirical tests rejected all three hypotheses. However, the proposed aggregates of currency plus demand deposits and time deposits at commercial banks and of these two goods plus other savings deposits were narrowly rejected compared to the aggregate of these three deposit goods plus U.S. government bonds. This may result from the rather uniform marginal spending power elasticities of the three deposit goods and insignificant own- and cross-price responses among these goods. Hence, separability, which restricts price response behavior did not drastically alter the results as long as the hypothesized partition was restricted to the deposit accounts. In contrast, U.S. bonds had a significant own-price coefficient and cross-price coefficients among the deposit accounts and the other goods. Hence, the separability assumption was more restrictive if U.S. Bonds were included in the aggregate.
Footnotes


3. Theil (1975-1976 Vol. 2) details this matrix and derives the resulting demand systems.

4. We follow Theil's development of the absolute price version under weakly separable preference. Theil also develops relative price versions of each of these equations.

5. This two-level procedure must be distinguished from the Strotz-Gorman two-stage budgeting procedure. Strotz-Gorman requires that the first level allocation occur on the basis of total spending power and a single price index for each group. Theil's two-level system uses total spending power and two price indices for each group. The Strotz-Gorman procedure requires more restrictive necessary and sufficient conditions than Theil's procedure. Deaton and Muellbauer (1982) survey other two-stage budgeting procedures.

6. The OLS estimator is:
\[ B = (X'X)^{-1} X'Y \]

7. If \( \Sigma \) is known, the estimator is:
\[ B = [X'(\Sigma^{-1}aI)X]^{-1} X'(\Sigma^{-1}aI)Y \]

\( B \) is unbiased and minimum variance of all unbiased estimators. Also, if the error terms follow a multi-variate normal distribution, \( B \) is a maximum likelihood estimator.

8. The restrictions imposed on these estimates are:
   1) The \( u_i \)'s sum to one.
   2) The price coefficient matrix is symmetric.
   3) Each row of the price coefficient meeting runs to zero. Negative semi-definiteness of the price coefficient matrix is not imposed. These restrictions are not tested. The very large number of parameters involved necessitate some structure on preference. Classical demand theory provides that structure.

9. Henceforth, the term asymptotic is omitted. All references to statistical significance are at the 5% level.
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Asymptotic t-ratios in parentheses  
Log of Likelihood Function = 386.39

### TABLE 9
Stage 2 Spending Power and Price Elasticities  
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TABLE 15
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TABLE 20
Stage 2 Rotterdam Model
Coefficient Estimates
Goods 1-4

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Rho = .02359
Asymptotic t-ratios in parentheses
Lof of Likelihood Function = 1155.3

TABLE 21
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Spending Power and Price Elasticities
Goods 1-4

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Log of likelihood function = 2951.5
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CHAPTER 7

CONCLUSIONS

Monetary economists have proposed policies ranging from simple monetary growth rules to countercyclical policy which incorporate all pertinent information available. Barnett proposed using separability of preferences to define a monetary aggregate to replace current measures of money. Barnett's aggregation method incorporates information about preferences into the construction of the aggregate. The higher information content of this aggregate should permit more efficient monetary policymaking under any policy regime.

This dissertation tested the appropriateness of applying separability conditions to construct a monetary aggregate. The problem was simplified by examining per capita household data on asset holdings and consumption. This abstracts from the problem of aggregation across sectors of economic agents. Also, the data span a time period prior to the relaxation of government regulation of the financial industry. Decreased regulation has permitted the emergence of financial assets such as money market mutual funds and widespread use of negotiable order of withdrawal accounts. Thus, the study also abstracts from the problem of rapid financial innovation.

The Rotterdam model was used to test alternative partitions of the data set. In each test, the assets partitioned with demand deposits plus currency were hypothesized to be elements of the monetary aggregate. Three tests were conducted with the Rotterdam model. First, demand deposits plus currency and time deposits at
commercial banks were assumed to be elements of the monetary aggregate. Second, this partition was broadened to include other savings deposits. Finally, U.S. government bonds were added to the second partition. All three hypotheses were rejected. Further, the broader the monetary aggregate hypothesized, the more decisively the hypothesis was rejected. The hypothesized monetary aggregates which contained only narrow money and deposit accounts were narrowly rejected at the 5% probability level. However, the aggregate which also contained U.S. government bonds was decisively rejected. This indicates that preferences over narrow money and deposit accounts have some degree of independence from the remainder of the choice set but not the degree of independence implied by weak separability. However, the behavior implied by inclusion of U.S. government bonds in the aggregate seriously violates the behavior measured by the unconstrained model. This indicates that the choice among the elements of this aggregate are critically dependent on the prices of the elements of the choice set outside the aggregate. This is not surprising since U.S. government bonds are likely held for quite different reasons than narrow money and deposit accounts. United States government bonds are relatively illiquid investments. A priori, one would expect closer relationships between U.S. bonds and other bond categories and equities than between these bonds and narrow money and deposit accounts. The decisive rejection of this hypothesis lends credence to this expectation.

These tests indicate that Barnett's proposal to use separability to define a monetary aggregate for policy purposes is not supported by the data. This result does not resolve the conflict between those
who support a monetary growth rule and those who favor active countercyclical policy. Those who support a monetary growth rule could interpret this result as confirming their belief that the Fed lacks sufficient information to successfully implement countercyclical policy. Those who support active policymaking might interpret this result as indicating that underlying preferences are not as regular and predictable as many monetary growth rule supporters suggest. Hence, the relationship between money and economic activity is loose, and it is essential that the Fed not rely on a simple growth rule.

While these tests reject separability, Barnett's sample monetary aggregate proved to have a more stable relationship with economic activity than simple sum aggregates. Thus, one might argue that the Fed should control this aggregate even though the theoretical basis for the aggregate was empirically rejected. The problem here is that no evidence exists of causation from the sample aggregate to economic activity. The closer correlation between the sample aggregates and GNP than that between simple sum aggregates and GNP may reflect smoothing of observed data rather than any fundamental relationships. Consequently, the relationship between the sample aggregate and economic activity may quickly break down outside the sample period.

Barnett's proposal relies on microeconomic theory. Thus, one explanation for rejecting separability is that the data set consists of aggregate household data. Hence, aggregation across consumers may pose a major problem for the analysis. While Barnett reformulated the Rotterdam model to explicitly aggregate across consumers, this
aggregation may have important implications for the Rotterdam model results. The data included a wide variety of assets and households of widely different levels of wealth are unlikely to hold similar assets. For example, the lower class may be unlikely to hold equities or bonds, while the upper class are likely to be sufficiently sophisticated investors to minimize their holdings of deposit accounts. This suggests fundamental differences in response to changes in exogenous variables. Thus, a low cross-price elasticity between time deposits and equities may indicate that those who hold time deposits and those who hold equities are largely distinct groups. Consequently, little substitution behavior between these assets would be observed.

The dominance of the marginal spending power share of equities may reflect in part shifts in the distribution of wealth among households. The data span a time period of generally increasing real per capita wealth. One would expect equity holdings to increase by a greater percentage than per capita wealth. Also, increases in the value of equities which happened during the period raised the value of existing equity portfolios and total spending power. This effect might be reflected in the marginal budget share of equities.

The principal conclusion emerging from this analysis is that weak separability of preferences is not an appropriate theoretical restriction to use to define an empirical monetary aggregate. The analysis also indicate that the choice of alternative separable assumptions can significantly alter results. Here, price elasticity estimates proved sensitive to alternative assumptions.
This research suggests several areas which warrant further analysis for monetary policymaking purposes. The rapid development of new financial market instruments has created new investment opportunities, especially for small savers. This may have further eroded the distinction between money and non-money financial assets. Clearly these developments have complicated monetary policymaking and their implications for policy need to be evaluated. Because they complicate policymaking, these developments may support a monetary growth rule. Conversely, an active countercyclical policy may be more essential if these developments loosen the money-economic activity relationship.

While this research was aimed at questions regarding aggregation of assets, the results indicate that aggregation across households poses problems for measuring behavior. These problems may arise, in part, from changes in the distribution of total spending power. Analysis which captures behavioral changes resulting from such shifts would provide important insight into the long-term relationship between money and economic activity. Similarly, aggregation across sectors may create problems for empirical analysis. Shifts in asset holdings among sectors have occurred over time. Understanding these shifts would contribute to understanding monetary policy impacts. Recent financial market dynamics could have important implications for understanding these aggregation problems.

In addition to its implications for monetary policy, this research yielded information on household preferences for assets and commodities. Several extensions of this analysis are apparent. First, incorporation of housing into the analysis would be
beneficial. Clearly, housing decisions are extremely important to households, particularly with regard to wealth formation. However, these decisions are extremely complex and analysis of these decisions within the context of the demand for all goods would require a more sophisticated modeling tool than the Rotterdam model. Second, specification and estimation of a dynamic demand model which retains the Rotterdam model's ability to investigate separability properties would be an important contribution. This analysis assumed that households completely adjust within the period to changes in exogenous variable. A dynamic model would extend the adjustment process. Adding dynamics to a model, however, tends to introduce non-linearity. This creates estimation problems particularly for systems of equations with a large number of goods.

Finally, the analysis assumed that the demand for assets and commodities was independent of the source of spending power. An interesting study would be to differentiate sources of spending power such as labor income, interest income, capital gains, and various forms of debt and analyze their impact on demand. For example, consumer installment credit should have little effect on equity holdings but a large impact on durables and non-durables. An integrated analysis of the sources and uses of total spending power should yield powerful insight into household behavior and the relationship between money and economic activity.


