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RADIATION PRESSURE AND THE GEOCORONA

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RADIATION PRESSURE AND THE GEOCORONA

by

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A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE
DOCTOR OF PHILOSOPHY

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ABSTRACT

The theory of planetary exospheres is extended to incorporate solar radiation pressure in a rigorous manner, and an evaporative geocoronal prototype (classical, motionless exobase) is constructed using Liouville's theorem. Calculations for density and kinetic temperature at points along the Earth-Sun axis (solar and anti-solar directions) reveal an extensive satellite component, comprising √2/3 of the total hydrogen density near 10 Earth radii, and a temperature profile suggestive of a near-isotropic quasi-Maxwellian kinetic distribution for the bound component. A geotail is also evident in this model as an enhancement of the local midnight density compared to local noon that increases radially outward from roughly 25% at 10 Earth radii to over 60% at 20 Earth radii. Additional mechanisms acting upon the geocorona alter this evaporative case in notable ways. Solar ionization has been included in a simple fashion; the effect is to deplete the density somewhat without otherwise altering the structure. Interaction with a simple plasmasphere via the Boltzmann equation results in heating the geocorona and enhancing the escape flux at the expense of the density of the bound component, an effect not appreciated in earlier studies; the geotail survives this interaction.
"It is a tale which they narrate in Poictesme, saying:

In the old days lived a pawnbroker named Jurgen; but what his wife called him was very often much worse than that. She was a high-spirited woman, with no especial gift for silence. Her name, they say, was Adelais, but people by ordinary called her Dame Lise.

They tell, also, that in the old days, after putting up the shop-windows for the night, Jurgen was passing the Cistercian Abbey, on his way home; and one of the monks had tripped over a stone in the roadway. He was cursing the devil who had placed it there.

'Fie, brother!' says Jurgen, 'and have not the devils enough to bear as it is?'

'I never held with Origen,' replied the monk; 'and besides, it hurt my great-toe confoundedly.'

'NONE the less,' observes Jurgen, 'it does not behoove God-fearing persons to speak with disrespect of the divinely appointed Prince of Darkness. To your further confusion, consider this monarch's industry! day and night you may detect him toiling at the task Heaven set him. That is a thing can be said of few communicants and of no monks. Think, too, of his fine artistry, as evidenced in all the perilous and lovely snares of this world, which it is your business to combat and mine to lend money upon. Why, but for him we would both be vocationless! Then, too, consider his philanthropy! and deliberate how insufferable would be our case if you and I, and all our fellow parishioners, were to-day hobnobbing with other beasts in the Garden which we pretend to desiderate on Sundays! To arise with swine and lie down with the hyena? - oh, intolerable!'

Thus he ran on, devising reasons for not thinking too harshly of the Devil. Most of it was an abridgement of some verses Jurgen had composed, in the shop when business was slack.

'I consider that to be stuff and nonsense,' was the monk's glose.

'No doubt your notion is sensible,' observed the pawnbroker: 'but mine is the prettier.'"

from Jurgen by James Branch Cabell (McBride & Co., 1919)
Acknowledgments

I would like to express my gratitude towards Joseph W. Chamberlain, for his suggesting the topic of my doctoral research, for his providing of the means by which to undertake this research, and ultimately for his patience while I stumbled along. In studying his work, I have come to appreciate his style (something never spoken of in graduate studies) and to partially realize my own.

While I cannot list or even remember each of the men and women who have aided and inspired my efforts in the past, I must give mention to my mother, whose example has long been a lesson on what to expect in this world. In many ways that can never be apparent, this has been a familial enterprise.
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CHAPTER 1: Introduction

The exosphere, as a distinct layer or region of a planet's (neutral) gaseous envelope, is most often characterized as the region wherein constituent collisions are rare and hence negligible in regards to the basic structure. For some bodies, the associated atmosphere is entirely exospheric in this sense - Mercury and the Moon are prominent examples. Planets that possess thicker envelopes, like the Earth, invariably exhibit a vertical differentiation of that envelope into distinct regions, each such layer being controlled by a distinct set of physical processes. While variations in atmospheric structure are to be expected among the planets, at some altitude all neutral atmospheres cease to be collisionally dominated and so become exospheric, coming into the purview of rarified gas dynamics and celestial mechanics.

Studies pertaining to the Earth's exosphere, also known as the geocorona, can be traced back to the earliest work in the kinetic theory of gases. (This history has been summarized by Chamberlain (1963).) The central question of exospheric research in the past was the phenomenon of atmospheric escape or evaporation, the classic approach to this being the formulation by Jeans (1925). Jeans treated the thermal escape problem as an effusion phenomenon and obtained for the escape flux at the
altitude $r_C$ (called the critical level) where collisions cease to be frequent the expression

$$F_{esc} = \frac{N(r_C)U(T_C)}{2\pi^{1/2}} \left( \frac{v_o^2}{U^2} + 1 \right) e^{-\frac{v_o^2}{2U^2}}, \quad (1.1)$$

where $v_o = \sqrt{2GM/r_C}$ is the minimum escape speed from the planet of mass $M$ and $U = \sqrt{2kT_C/m}$ is the most probable velocity of an atmospheric constituent of mass $m$ belonging to a Maxwellian distribution at temperature $T_C$; $N(r_C)$ is the local density of the gas being considered. Beyond this formula, though, little was known. A hidden belief that the barometric law

$$N(r) = N(r_C) \exp[-(r-r_C)/H] \quad (1.2)$$

(where $H$ is the isothermal scale height) remained valid to all altitudes hindered further efforts until the work of Opik and Singer (1959, 1961) explicitly eliminated this supposition, a development paralleled independently by Chamberlain (1960) and Johnson and Fish (1960). In 1963, Chamberlain published a comprehensive exospheric theory based on Liouville's theorem and Keplerian orbital mechanics that has since served as the standard reference (Chamberlain, 1963). (As the concepts employed therein have gained widespread usage, particularly in the interpretation of satellite observations, the reader may find helpful the synopsis of this theory provided in Appendix I.)
It wasn't until the dawn of the space age that questions about the structure of the exosphere and in particular about the component gravitationally bound to the planet were first asked. Lyman-α photometers sent aloft in the mid-to-late 1950's revealed an unexpected and ubiquitous airglow above 100 kilometers, even in the night sky. A controversy ensued as to the origin (interplanetary or atmospheric) of this airglow. Subsequent investigation based on the work of Thomas (1963), who constructed a technique for handling the transport of resonant radiation in a spherically symmetric exosphere, eventually established geocoronal containment of solar Lyman-α as the responsible mechanism (Meier and Mange, 1970). Attention thus became focused on the exosphere itself. (Thomas (1963) also outlines these early rocket observations and the interpretational difficulties that arose.)

In the last two decades, the techniques and perspectives set forth in the above-mentioned papers of Thomas and Chamberlain have been influential. The common approach since then has been to isolate complications neglected in the basic theory and to determine or estimate the consequences of incorporating such factors. Examples of complications so studied are corrections to the assumed Maxwell-Boltzmann distribution at the exobase (Brinkmann, 1971; Chamberlain and Smith, 1971; Fahr and Nass, 1978), diurnal variations of exobase temperature and density.
(Tinsley, 1973), the constancy of the escape flux and the associated mesospheric bottleneck (Hunten, 1973; Liu and Donahue, 1974a,b,c), and the mechanism of charge exchange with hot plasmaspheric protons to make up the difference between the limiting and thermal escape fluxes (Cole, 1966; Chamberlain, 1977; Maher and Tinsley, 1977; Hodges, et al., 1981).

One long-recognized question has stood out. The Keplerian framework of classical exospheric theory permits the existence of satellite trajectories that never intersect the critical level. The extent to which such trajectories are populated, thus constituting a satellite component to the exosphere, has not been settled by past researchers and, indeed, has replaced atmospheric escape as the central problem in exospheric physics. The main difficulty in this regard lies in constructing a technique for investigating such questions. A notable effort is that of Richter, et al. (1979), based on a Boltzmann equation formulation to account explicitly for rare exospheric collisions and extending the Chamberlain concept of a collision-determined satellite critical radius in a rigorous fashion. Although a worthwhile study, emphasizing the importance of collisional processes, doubt can be cast as to its applicability to the geocoronal situation; in particular, the supposition of Keplerian particle trajectories cannot be justified in the light of deep space observations of the exosphere.
The direct measurement of geocoronal properties has proven to be a formidable though rewarding task, and some significant results have been achieved. Noteworthy in regards to global studies are photometric measurements of the geocoronal Lyman-α intensity carried out on the OGO series of space platforms, particularly OGO-5 (Thomas and Bohlin, 1972; Bertaux and Blamont, 1973; Bertaux, 1978), on the Mariner 5 mission to Venus (Wallace, et al., 1970), and most recently on Dynamics Explorer 1 (Rairden, et al., 1983). As noted by Bertaux (1978), the Chamberlain theory has held up well and remains a straightforward way to estimate global exospheric quantities. Still, several features deduced from observations have not been amenable to the Chamberlain formulation. The most striking of these is the presence of a "geotail" (Thomas and Bohlin, 1972) - i.e., a relative enhancement of hydrogen densities on the nightside in the outer geocorona. This discovery, not entailing the use of intensity inversions (the Thomas and Bohlin study is noteworthy in this regard), prompted the recognition that resonant scattering by geocoronal hydrogen of solar Lyman-α must result in a net acceleration in the anti-solar direction. By perturbing the Keplerian trajectories of exospheric atoms, it was suggested by Thomas and Bohlin that this scattering causes the formation of the geotail. However, a quantitative characterization was not offered.
The same conclusion was reached by Bertaux and Blamont (1973) on the basis of hydrogen density profiles retrieved by inverting photometer intensities and a computer simulation of the effects of resonant photon scattering on the motion of individual atoms traveling in the Earth's gravitational field. This analysis suggested a satellite critical radius $r_{cs}$ of ~2.5 Earth radii and the presence of a mechanism or mechanisms for removing satellite particles above radial distances of 10 Earth radii on the dayside of the planet, a picture in keeping with the analysis by Wallace, et al. (1970) of a density profile of the outer exosphere on the dayside along the Earth-Sun line obtained by Mariner 5 on its way to Venus. The mechanisms mentioned and incorporated in the computer simulation were dynamical modifications of trajectories by solar Lyman-$\alpha$ radiation pressure, solar photoionization, and charge exchange with magnetosheath ions, the latter two acting simply to remove geocoronal hydrogen. The main effects of photon scattering, treated as a sequence of discreet antisolar-directed momentum percussions acting at regular intervals, were reported to be (i) a "decay" of particle perigees, for a large range of initial conditions, leading to a return to the thermosphere, and (ii) a movement of particle apogees to locations far removed from the initial position, exhibiting a preference for nightside orientation. This simulation, conducted for a limited group of initial satellite
trajectories, thus offered an appealing hint of the geotail structure. It cannot, however, be taken as a proper account since the trajectories utilized in the study were arbitrarily chosen, and the manner and extent to which these trajectories ought to be populated was not addressed.

In a related analysis of OGO-5 measurements of the geocoronal Lyman-α line-shape, Bertaux (1978) managed to estimate the extent of the satellite population. By comparing these OGO-5 results to a sequence of Chamberlain models of varying satellite critical radii, an $r_{CS}$-profile was obtained that indicated the satellite critical radius required to bring about agreement in terms of density between measurement and theory. That is, no single satellite critical radius fits the entire geocorona; even for the inner region taken alone such a characterization is not possible. On the face of it, such an $r_{CS}$-profile represents a generalization of the Chamberlain theory, although an ad hoc one. By way of explanation, Bertaux merely cited radiation pressure dynamics as likely necessitating a revision of the satellite critical radius concept and mentioned plasmaspheric charge exchange collisions as perhaps having an influence, declining to suggest a way by which this revision might be carried out.

Thus, the experimentalists have mapped the exosphere rather completely, have demonstrated the importance of radiation pressure, and have highlighted the questionable
status of the satellite component. It ought to be noted, though, that the observations invariably rely on optical
techniques, principally photometric measurement of the
scattered Lyman-α intensity, while the primary quantity
sought is the hydrogen density profile. Herein lies a
dilemma. As the measured quantity is intensity and so
embodies an integration along the line-of-sight, the local
density must be recovered in some fashion by inverting the
data, requiring the use of a model. As a result, the
observational results and their interpretation are often
couched in terms of the Chamberlain theory and so may be
misleading, as when relying on the concept of the satellite
critical radius to indicate the presence and extent of the
satellite component. Indeed, in the common picture of the
satellite component that can be found in the literature,
collisions are thought to be the main mechanism for
populating such trajectories and radiation pressure a major
destruction mechanism via "orbital decay" (Fahr and Shizgal,
1983), a picture clearly reflecting the notions upon which
the satellite critical radius concept is based. Instead, it
must be recognized that photon scattering needs to be
superposed with gravitational attraction in the
determination of particle trajectories and cannot be looked
upon simply as an aeronomic loss mechanism.

An account is here offered of a first-principles
investigation into the confused question of the (radiation
This study incorporates photon scattering in a rigorous (classical) manner, taking Hamiltonian mechanics as the starting point, and determines some of the characteristics of an exosphere acted upon by this mechanism. A central aim has been to delineate the properties of the resulting satellite component. The results take the form of profiles along the Earth-Sun line in both the solar and anti-solar directions of the following quantities: total density, satellite component density, kinetic temperature, and escape flux. In addition, column densities are calculated. An evaporative prototype constitutes the basic case studied. Mechanisms known to act upon geocoronal hydrogen in addition to radiation pressure, namely solar ionization and resonant charge exchange collisions with plasmaspheric ions, are then incorporated singly to assess the relative significance of each. In particular, by comparison of the plasmasphere interaction results with those obtained in the strict evaporative case, the effects of radiation pressure and charge exchange collisions can be unraveled and a truer picture of the satellite component obtained. The techniques and results presented in the following Chapters provide a natural basis for the explanation of most known geocoronal features.
CHAPTER 2: Formal Construction

The purpose of this study has been to resolve and quantify the action of solar Lyman-α radiation pressure on the hydrogen geocorona. The simplest model (henceforth referred to as the evaporative model) which retains the pertinent aspects of the real situation consists of a spherical motionless planet generating a pure atomic hydrogen exosphere by exobase evaporation, the whole bathed by a unidirectional stream of Lyman-α photons. A classical uniform exobase is assumed (Jeans, 1925), pictured as a surface separating the neutral collisionless regime and the upper thermosphere (characterized by a uniform, isothermal Maxwellian velocity distribution). Parcels of gas arising from this surface are followed along the appropriate trajectory, employing Liouville's theorem, and are summed to yield composite quantities like density and kinetic temperature. The basic approach to this problem is that taken by Chamberlain (1977) in his Boltzmann equation study of the effects of the plasmasphere-exosphere interaction (see also Hartle, 1971, and Vidal-Madjar and Bertaux, 1972).

A review of the simplifications inherent in this model serves to establish the geocoronal setting. For example, to incorporate the resonant scattering of solar Lyman-α photons in the trajectory determinations, it is preferable to replace the random impacts by an average acceleration \( \mathbf{a} \) in
the anti-solar direction (see Appendix II). This acceleration has been taken to be constant in the neighborhood of the Earth. Comcomitant simplifications have been to ignore the Earth's geometric shadow and geocoronal shielding. First, the shadow occupies a comparatively small region of space and so may be neglected for this reason alone. Second, the action of radiation pressure compared to gravity is significant only at altitudes where the optical depth happens to be small - i.e., geocoronal hydrogen is comparatively cool, most of it being held near the planet where $a << (GM/r^2)$. Hence, except when directly behind the Earth, a hydrogen atom can be taken to be "bare" to the solar output while the time of transit of the shadow region comprises an ignorable fraction of the total flight time.

The exobase is itself a device by which to avoid subtle questions pertaining to the transition between the collision-dominated (fluid) region below and the adjoining collisionless region above (Fahr and Shizgal, 1983). In reality, this transition occurs in a layer of thickness on the order of a scale height of the major (background) gas. For the case of the Earth, the major thermospheric gas - atomic oxygen - establishes a transition region of thickness $\sim 60$ kilometers at an altitude of roughly 500 kilometers, where atomic hydrogen constitutes a minor gas ($[H]/[O] \sim 4 \times 10^{-3}$) (U.S. Standard Atmosphere, 1976). The true atomic
hydrogen kinetic distribution cannot remain Maxwellian at exobase altitudes when this transition is properly accounted for, though just what the proper distribution is (or even how to calculate it) as yet is not known.

The planet has been held stationary, thus entailing a neglect of both planetary rotation and motion about the Sun. The latter, referred to as the "solar motion" across the sky, presents an angular shift in the direction of the incident photons at a rate $1.992 \times 10^{-7}$ radians/second and can readily be ignored so long as average flight times remain on the order of 10 days or less. The influence of planetary rotation on exospheric structure has in the past been studied in conjunction with the effects of upper thermospheric winds and non-uniformities in exobase temperature and density. While such factors affect the thermosphere-exosphere transition region, the net effects in the outer exosphere have long been known to be minor (Vidal-Madjar and Bertaux, 1972) and consist mainly in enhancing slightly the escape rate (a rotation effect) along with inducing a ballistic flow. In particular, an exosphere generated by evaporation of a realistic exobase (sans radiation pressure), such as that computed by Tinsley, et al. (1975), reveals no geotail.

In keeping with the neglect of the solar motion, the solar gravitational acceleration is not explicitly included in the formulation, even though by coincidence the Lyman-\(\alpha\)-
induced acceleration of hydrogen atoms is almost canceled by solar gravity in an inertial frame (Burns, et al., 1979). By using Earth-centered coordinates, the free-fall of the planet toward the Sun is implicitly accounted for (an approximation valid for the time intervals pertinent to this study) and in this frame of reference the radiation pressure acceleration takes on its full value. Also, lunar gravitational perturbations on the geocorona are quite small. At the Earth the lunar gravitational acceleration has a magnitude $3.32 \times 10^{-3}$ cm/sec$^2$ and a tidal strength $1.73 \times 10^{-13}$ sec$^{-2}$. Across a structure of the size of the geocorona ($10^{10}$ cm), the lunar acceleration gradient can thus be ignored compared to the effective (planet-centered) radiation pressure tidal strength of $2 \times 10^{-10}$ sec$^{-2}$ along the Earth-Sun axis (for a characteristic radiation pressure acceleration of 1 cm/sec$^2$).

In addition to exerting an effective pressure, the solar output can remove geocoronal H in two ways - photoionization by photons of wavelength < 911 A and collisional ionization by solar wind particles. The average lifetime of geocoronal hydrogen for each mechanism is roughly $2.0 \times 10^6$ seconds outside the magnetopause (Wallace, et al., 1970), yielding a net lifetime against solar ionization of ~10 days. As this is comparable to the flight times of hydrogen atoms in the outer exosphere when experiencing radiation pressure perturbations, this
mechanism has been incorporated in a simple way in some of the calculations.

The existence of the terrestrial plasmasphere greatly affects the geocorona. Since the plasmaspheric ion temperature is higher than that of the neutral atoms, charge exchange collisions effectively heat the geocorona at altitudes above the exobase and enhance the escape flux of hydrogen from the Earth at the expense of the bound population. This phenomenon has also been included in some of the calculations; discussion of the appropriate equations and results is deferred until Chapter 4. In the remaining sections of this Chapter the techniques used to construct an exosphere are described, with explicit reference to the evaporative case; the results pertaining to this prototype are then presented in Chapter 3.

TRAJECTORY DETERMINATION

It is straightforward to set up a Hamiltonian formalism to describe the particle motion above the exobase at temperature $T_C$, when the planet's shadow and motion are ignored. The radiation pressure is provided by the potential $m a \cos \chi$, where $m$ is the mass of the hydrogen atom, $r$ is the radial distance to the center of the planet, $\chi$ is the angle between the Earth-Sun axis and the local radial vector, and $|\mathbf{a}|$ is the acceleration provided by Lyman-\(\alpha\)
scattering. The total Hamiltonian $H$ for motion about the Earth (mass $M$) is (superior dots denoting time derivatives)

$$H = \frac{m}{2} [\ddot{r}^2 + (r\dot{\chi})^2 + (r\dot{\phi}\sin\chi)^2] - \frac{GMm}{r} + m\alpha\cos\chi$$

$$= \frac{1}{2m} [p_r^2 + (p_{\chi}/r)^2 + (p_{\phi}/r\sin\chi)^2] - \frac{GMm}{r} + m\alpha\cos\chi,$$

the cyclic position azimuth $\phi$ being taken to be zero in the ecliptic plane on the evening side of the planet (see Figure 2.1). The equations of motion for the momenta are

$$\dot{p}_r = -\frac{\partial H}{\partial r} + \ddot{r} = r\dot{\chi}^2 + r\dot{\phi}^2\sin^2\chi - \frac{GM}{r^2} - \alpha\cos\chi$$

$$\dot{p}_{\chi} = -\frac{\partial H}{\partial \chi} + r\ddot{\chi} + 2r\dot{\chi} = r\dot{\phi}^2\sin\chi \cos\chi + \alpha\sin\chi$$

$$\dot{p}_{\phi} = -\frac{\partial H}{\partial \phi} + r\dot{\phi}\sin\chi + 2r\dot{\phi}\sin\chi + 2r\dot{\chi}\dot{\phi}\cos\chi = 0.$$ 

In handling distribution calculations, dimensionless quantities are convenient. Using a notation similar to that introduced by Chamberlain (1963) -

$$\lambda = \frac{GMm}{(kT^C_r)}$$

$$\xi = \frac{p_r}{\sqrt{(2mkT^C)}}$$

$$\alpha = \frac{(GM/2)(m/kT^C)^2}{a}$$

$$\zeta = \frac{p_{\chi}}{[r/(2mkT^C)]}$$

$$\tau = \frac{(21/2/GM)(kT^C/m)^{3/2}}{t}$$

$$\omega = \frac{p_{\phi}}{[r\sin\chi/(2mkT^C)]}$$

where $k$ is the Boltzmann constant, the Hamiltonian becomes

$$h = \frac{H}{kT^C} = \xi^2 + \zeta^2 + \omega^2 - \lambda + (2\alpha/\lambda)\cos\chi$$
FIGURE 2.1: Geometry employed in the formulation. The positional coordinates are: distance from the center of the planet \( r \), polar angle with respect to (wrt) the solar direction \( \chi \), and azimuthal angle wrt the evening ecliptic \( \phi \). The momentum coordinates are also spherical: (normalized) magnitude of momentum \( \psi \), here pictured as centered on the local point, polar angle wrt local zenith \( \delta \), and azimuthal angle wrt \( \hat{\chi} \) about local zenith \( \varepsilon \). Radiation pressure acts along \(-\hat{\varepsilon}\).
FIGURE 2.1: ILLUSTRATION OF COORDINATES
while the equations of motion translate to

\[ \frac{\partial \lambda}{\partial \tau} = -\lambda^2 \xi \]

\[ \frac{\partial \chi}{\partial \tau} = +\lambda \xi \]  \hspace{1cm} (2.5a)

\[ \frac{\partial \phi}{\partial \tau} = +\lambda \omega / \sin \chi \]

for the positional coordinates and

\[ \frac{\partial \xi}{\partial \tau} = \lambda (\xi^2 + \omega^2) - \lambda^2 / 2 - \alpha \cos \chi \]

\[ \frac{\partial \zeta}{\partial \tau} = \lambda \omega^2 \cot \chi - \lambda \xi \zeta + \alpha \sin \chi \]  \hspace{1cm} (2.5b)

\[ \frac{\partial \omega}{\partial \tau} = -\lambda \xi \omega \cot \chi - \lambda \xi \omega \]

for the momenta. \( \tau \) is a dimensionless time parameter. Transforming to a spherical coordinate representation of the momentum space -

\[ \psi = [\xi^2 + \zeta^2 + \omega^2]^{1/2} \hspace{1cm} \xi = \mu \psi \]

\[ \delta = \cos^{-1}(\xi / \psi) \hspace{1cm} \zeta = \psi(1-\mu^2)^{1/2} \cos \epsilon \]  \hspace{1cm} (2.6)

\[ \epsilon = \tan^{-1}(\omega / \zeta) \hspace{1cm} \omega = \psi(1-\mu^2)^{1/2} \sin \epsilon \]

where \( \mu = \cos \delta \), the equations of motion become

\[ \frac{d\lambda}{d\tau} = -\lambda^2 \mu \psi \]  \hspace{1cm} (2.7a)
\[
\frac{d\chi}{d\tau} = +\lambda \psi (1-\mu^2)^{1/2} \cos \varepsilon \\
\frac{d\phi}{d\tau} = +\lambda \psi (1-\mu^2)^{1/2} \sin \varepsilon / \sin \chi \\
\frac{d\psi}{d\tau} = -\mu \lambda^2 / 2 + \alpha [(1-\mu^2)^{1/2} \sin \chi \cos \varepsilon - \mu \cos \chi] \\
\frac{d\mu}{d\tau} = \lambda \psi (1-\mu^2) - \lambda^2 (1-\mu^2)/2\psi \\
&\quad - \alpha (1/\psi) [(1-\mu^2) \cos \chi + \mu (1-\mu^2)^{1/2} \sin \chi \cos \varepsilon] \\
\frac{d\varepsilon}{d\tau} = -\lambda \psi (1-\mu^2)^{1/2} \cot \chi \sin \varepsilon \\
&\quad - \alpha [\sin \chi \sin \varepsilon / \psi (1-\mu^2)^{1/2}].
\]

\(\varepsilon\) is a momentum azimuth (here referred to as the "steering" angle) and equals zero when the particle velocity lies along \(\hat{\kappa}_\chi\). The Hamiltonian is simply \(h = \psi^2 - \lambda + (2\alpha/\lambda) \cos \chi\).

The equations expressing energy conservation and conservation of the momentum canonical to \(\phi\) are, respectively,

\[
\psi^2(\lambda') = \psi^2(\lambda) - \lambda + \lambda' + 2\alpha \{\cos \chi(\lambda)/\lambda - \cos \chi(\lambda')/\lambda'\} \quad (2.8)
\]
along a trajectory and

\[
A = (1 - \mu(\lambda)^2)^{1/2} \psi(\lambda) \sin \chi(\lambda) \sin \varepsilon(\lambda) / \lambda, \quad (2.9)
\]
where \(A\) is the angular constant. Eqn. (2.8), rather than Eqn. (2.7d), has been used in the actual calculations to follow the evolution of \(\psi\).

The exobase \(\lambda_c\) forms a natural boundary for
terminating trajectory integrations, the intersection point acting as the source point for populating the trajectory in the evaporative case. Those trajectories that escape to infinity necessitate the introduction of an outer boundary beyond which the trajectory need not be followed. An intuitive choice for this boundary is that radial distance beyond which the magnitude of the radiation-induced acceleration exceeds that of the Earth's gravity field. This boundary, here called the exopause in analogy to the various other atmospheric layers, will be denoted by \( \lambda_p \). A trajectory that intersects this radial shell is taken to have escaped even if the total energy \( h \) is negative, since the Hamiltonian permits the continuation of negative energy trajectories to infinity on the nightside of the planet.

LOCAL DISTRIBUTION FUNCTION

The local distribution function is, in the evaporative case, provided by invoking Liouville's theorem. By determining the exobase intersection point and the corresponding momentum coordinates in a specified direction (called the past), the local kinetic distribution function can be written

\[
f(\lambda, \chi, \phi, \psi, \mu, \varepsilon) = f_c(\lambda_c, \chi_c, \phi_c, \psi_c, \mu_c, \varepsilon_c).
\]

The exobase kinetic distribution is taken to be a simple
Maxwellian -

\[ f_C(\lambda_C, \chi_C, \phi_C, \psi_C, \mu_C, \varepsilon_C) = N_C \times \exp[-\psi_C^2] / \pi^{3/2} \quad (2.10) \]

- where the exobase temperature \( T_C \) (employed in the definition of the dimensionless quantities) and the exobase density \( N_C \) are constants.

Having determined the local kinetic distribution function for a specified trajectory in one direction, the function in the opposite direction along the same trajectory can be evaluated by propagating the trajectory into the future, and if the exobase is intersected again, by invoking time reversal, which is equivalent to transforming the future exobase momentum orientation as \( \varepsilon_C + \psi_C - \pi \) and \( \mu_C + \mu_C \). Since in this model the exobase is uniform and stationary, the only quantity necessary for the specification of the kinetic distribution function on the selected trajectory is the exobase value of the kinetic energy \( \psi_C \).

The kinetic distribution can be expressed in a somewhat more explicit form. Note that a solution for the steady-state Liouville's theorem can always be written down when the motion occurs in potential fields - namely

\[ f = K_0 \exp[-H] \]

identically solves the Liouville relation

\[ \frac{df}{dt} = \sum_i \frac{\partial H}{\partial q_i} \frac{\partial f}{\partial p_i} - \frac{\partial H}{\partial p_i} \frac{\partial f}{\partial q_i} = 0 \]
along a trajectory, where $H$ is the Hamiltonian. For the evaporative situation, this expands to (in dimensionless coordinates)

$$f = K_0 \exp[-(\psi^2 - \lambda + (2\alpha/\lambda)\cos\chi)].$$

Comparing to the exobase distribution (Eqn. (2.10)), it is apparent that

$$K_0 = \frac{N(r_c)}{\pi^{3/2}} \exp[-(\lambda_c - (2\alpha/\lambda_c)\cos\chi(\lambda_c))].$$

$f$ can also be written in terms of the no-radiation kinetic distribution $f_{NR}$ (Eqn. (AI.1b)) as

$$f = f_{NR} \exp[-((2\alpha/\lambda)\cos\chi(\lambda) - (2\alpha/\lambda_c)\cos\chi(\lambda_c))]. \quad (2.11)$$

**PHASE SPACE REGIONS**

The set of all trajectories threading a given point in the exosphere can be classified according to the respective past and future histories. (It ought to be noted as a point of clarity that in this time-independent model, the trajectories are strictly geometric entities. When reference is made to the past or future history of a trajectory, what is really meant is the history of a particle that is traveling along the trajectory in question.) This partitioning of trajectories into classes can also be described as a partitioning of phase space into
dynamically distinct regions. In the application, positions in the exosphere have been selected at which quantities like density and kinetic temperature are computed. In what follows, the term "phase space" will be taken to denote the momentum space that attaches to the position in question, although this is only a subspace of the true phase space which by definition is the ensemble of all trajectories. Also, the dimensionless quantities are not all assignable to canonical pairs (i.e., $\xi$ is not canonical to $\lambda$) and hence in another way to call the set of trajectories described by Eqns. (2.7a)-(2.8) containing a given point a "phase space" is erroneous. Still, the use of the term is convenient and should not give rise to any confusion of meaning.

Past work of Chamberlain (1979) has indicated that when radiation pressure is treated as a small perturbation to the gravitational acceleration, satellite trajectories have a finite lifetime - i.e., a particle traveling along a trajectory that would be a Keplerian satellite trajectory as defined by the current position and momentum coordinates in the no-radiation (NR) case (i.e., the osculating orbit) actually crashes into the exobase within a finite time interval. In this study, no approximations based on the magnitude of the radiation pressure are made and so the behavior studied by Chamberlain cannot be expected to obtain here for those trajectories that extend well away from the planet. However, in a subsequent paper, Chamberlain (1980)
obtained near-exact solutions for the case of trajectories that lie in the ecliptic plane. The results presented therein indicate that for motion in a plane containing the Earth-Sun line, satellite particles again exhibit finite lifetimes, either colliding with the planet through a systematic lowering of perigee or escaping the planet altogether. Since the exospheric positions that have been selected for this study lie along the Earth-Sun axis, it is then reasonable to expect that a clean partitioning of trajectories threading the points of interest can be accomplished, the resulting phase space regions being identifiable as ballistic, satellite, escaping, and hyperbolic fly-by. These regions, as applicable to the model, are defined in terms of the included trajectories:

BALLISTIC - trajectories that arise directly from, or return to, the exobase without an intervening turning point (i.e., a position along a trajectory at which \( \mu \) changes sign, like at apogee in a Keplerian orbit). These are bound and intersect the exobase in both directions within a finite time interval.

SATELLITE - trajectories that possess a turning point in each direction prior to intersecting the exobase (within a finite time interval).

ESCAPING - trajectories that intersect the exobase only once (in the past) and intersect the exopause once on their way out into the interplanetary environment. At the altitudes of interest, such trajectories do not usually exhibit turning points in the past.

HYPERBOLIC FLY-BY - trajectories that come from infinity and either return to infinity or collide with the planet (thus corresponding to time-reversed escaping trajectories.)
The first three classes are all populated by an evaporating exobase. The last class will be taken to be devoid of hydrogen, except when interaction with the plasmasphere is included. Note that a trajectory that intersects the Earth-Sun axis at least once remains planar.

To dwell a bit on the concept of a satellite component generated by exobase evaporation, it should be realized that at the locations of interest in this study, the action of radiation pressure not only causes a nominal satellite trajectory to intersect the exobase within a finite time interval; by time-reversing the motion an exobase intersection is imposed in the past and hence the trajectory becomes populated as required by Liouville's theorem. Thus, the trajectories governed by the equations of motion Eqns. (2.7a)-(2.8) naturally partition into only two components (ignoring for now the hyperbolic fly-by class) - i.e., a bound and an escape component, the distinction between these in this application being that the former comprises trajectories along which atoms are moving in both directions while on the latter the direction of motion is strictly away from the point of exobase intersection. The distinction between the satellite and ballistic components in the conventional picture of the geocorona, wherein the mechanisms controlling the population of the satellite component (e.g., collisions) differ from that primarily responsible for establishing the ballistic component (i.e.,
evaporation), is no longer valid. Still, it is convenient to isolate the nominal satellite component as it appears in the evaporative model, if only for purposes of comparison with past work. The boundary between the satellite and ballistic components in this work is embodied in the (distorted) cone-of-acceptance \( \mu^0 \), which physically represents for a selected position the family of trajectories of a given kinetic energy that just graze the exobase in a specified direction without an intervening turning point. Those trajectories inside the cone-of-acceptance directly intersect the exobase along the specified direction while those outside pass through a perigee and continue to loop around the planet until intersecting the exobase at some later (or earlier) time. Note that a trajectory that has a satellite osculating orbit at some position would be counted as a ballistic trajectory during the early and late segments of flight and so descending bound trajectories that lie within the distorted cone-of-acceptance are classified as ballistic even if they looped the planet prior to entering the cone - these are the counterparts of the descending ballistics in the no-radiation (NR) case. Thus, the classification of a trajectory as satellite is not absolute and only refers to an evolutionary stage. In a way, the difficulty of the satellite component is one of terminology - namely, there is no satellite component, at least in the Keplerian sense.
COMPOSITE QUANTITIES

Quantities like density and kinetic temperature can be written in terms of the kinetic distribution \( f(\lambda, \chi, \phi, \psi, \mu, \varepsilon) \) (Chapman and Cowling, 1970). Indeed, the exosphere serves as an excellent example of a non-uniform gas. Expressed as a sum of population components, the density can be written

\[
N(\lambda, \chi, \phi) = \left\{ \int_{-1}^{1} \int_{0}^{1} \psi^1 + \int_{0}^{1} \left[ \int_{0}^{\mu^0} + \int_{-\mu^0}^{0} \right] \psi^0 \right. \\
\left. + \int_{0}^{2\pi} \int_{0}^{\mu^0} \int_{-\mu^0}^{\mu^0} \int_{0}^{\infty} \right\} \int_{0}^{2\pi} \int_{0}^{1} \int_{-\psi^1}^{\psi^1} \int_{0}^{\mu^0} \int_{0}^{\psi^0} f(\lambda, \chi, \phi, \psi, \mu, \varepsilon) \psi^2 \, d\psi \, d\mu \, d\varepsilon.
\]

(2.12)

The terms represent, respectively, the isotropic ballistic, restricted ballistic (ascending and descending), satellite, and escaping components. The maximum isotropic speed \( \psi^1 \), the distorted cone-of-acceptance \( \mu^0 \), and the minimum escape speed \( \psi^0 \) all are functions of position and steering angle \( \varepsilon \). In addition, \( \mu^0 \) depends on speed \( \psi \) while \( \psi^0 \) varies with inclination \( \mu \). Hence, prior to performing the integration, \( \psi^0 \) and \( \mu^0 \) must be evaluated simultaneously for each selected position \( (\lambda, \chi, \phi) \) and steering angle \( \varepsilon \). Examples of the integration boundaries \( \mu^0 \) and \( \psi^0 \) are shown in Figures 2.2a and 2.2b, compared to the corresponding NR quantities. The distorted cone-of-acceptance can be seen to be little different from the corresponding quantity as
FIGURES 2.2a AND 2.2b: Comparison of the minimum escape speed $\psi^0$ (diamonds) and the cone-of-acceptance $\mu^0$ (plusses) that result when radiation pressure acts to the no-radiation values, the straight horizontal line showing $\psi^0=\sqrt{\lambda}$ and the solid curve showing the Chamberlain cone-of-acceptance. The phase space regions established by the RP boundaries are also indicated.
Radius: \(11.3552 \, R_E\)

Solar Angle: 0 degrees

\[
\begin{align*}
\psi & \quad \text{hyperbolic fly-by} \\
& \quad + \quad \text{distorted cone-of-acceptance} \\
& \quad \diamond \quad \text{minimum escape speed} \\
\end{align*}
\]

\[
\begin{align*}
\psi^0(NR) & \\
\end{align*}
\]

\[
\begin{align*}
\text{satellite} \\
\text{ballistic} \\
\text{escaping} \\
\end{align*}
\]

FIGURE 2.2a: PHASE SPACE REGIONS AND BOUNDARIES - NOON
Radius: \( \ldots \ldots \) 11.3552 \( R_E \)

Solar Angle: \( \ldots \ldots \) 180 degrees

**Figure 2.2b**: Phase space regions and boundaries - Midnight
defined in the Chamberlain theory. The behaviour of the minimum escape speed is more interesting but not surprising; the expectation that escape requires greater (less) speed for motion toward (away from) the Sun is confirmed in a satisfying manner. These Figures also illustrate the trajectory classes described above.

As elaborated in the third Appendix, the trajectories that are required for the density computation may be specified once the boundaries \(\psi^1, \psi^0, \text{ and } \mu^0\) are known. These trajectories are then populated according to Liouville's theorem and combined as stipulated by the summation scheme invoked. The third Appendix contains a synopsis of the computational steps and techniques employed in this study. Further discussion of the trajectory classes and the associated boundaries in momentum space may be found there; generally, these concepts are closely akin to the parallel concepts of the Chamberlain (1963) theory, an example being the cone-of-acceptance \(\mu^0\) (see Appendix I).

In addition to density, the escape flux, the kinetic temperature, and the column density have been chosen for evaluation. The expressions for these are

**ESCAPE FLUX**

\[
F(\lambda, \chi, \phi) = \int_0^{2\pi} \int_{\mu^0}^1 \int_{\psi^0}^\infty f \mu \psi^3 \, d\psi \, d\mu \, d\phi,
\]  
(2.13)

**KINETIC TEMPERATURE**

\[
T_{\text{kin}} = \frac{2}{3} T_C \left[ <\psi^2> - <\mu \psi>^2 \right],
\]  
(2.14)
angular brackets denoting the average

\[ <g> = \left( \iiint d\psi d\mu d\varepsilon \, \psi^2 \, g \right) / \left( \iiint d\psi d\mu d\varepsilon \, \psi^2 \right) \]

\[ = \left( \frac{1}{N(\lambda, \chi, \phi)} \right) \iiint d\psi d\mu d\varepsilon \, \psi^2 \, g \]

(where the integrals are properly restricted), and

**COLUMN DENSITY**

\[ N_{\text{col}}(\lambda_C, \chi, \phi) = \left( \frac{GMm}{kT_C} \right) \int_{\lambda_p}^{\lambda_C} \left[ N(\lambda, \chi, \phi) / \lambda^2 \right] d\lambda, \quad (2.15) \]

above the exobase (zenith-looking), the exopause being treated as a lid. Each of these quantities provides a piece to the puzzle presented by the observations.
CHAPTER 3: Evaporative Case

The prototype of exospheric theory, constructed by invoking Liouville's theorem along exobase-intersecting trajectories, can be generalized to incorporate the dynamical effects of resonant photon scattering; the scheme for this has been outlined in the last Chapter and in the third Appendix. This Chapter contains results for a model of this type, in the form of radial profiles for total density, satellite density, and kinetic temperature. While one ought not to expect a realistic prediction of exospheric features in so simplistic a model, it exhibits the main traits arising from radiation pressure and sets the stage for incorporating additional mechanisms. One such complication - solar photoionization - can be incorporated within the Liouville theorem formulation; corresponding results are presented after discussion of the simple evaporative case.

To make matters explicit, it is necessary to select the exobase temperature and the acceleration induced by radiation pressure. As both are established by solar conditions in the real situation, it is best to use values conforming to the same level of solar activity. The epoch of the OGO-5 mission (1969-1970) suggests $T_C = 1020K$ and $|a| = 0.75$ cm/sec$^2$ to be suitable (Vidal-Madjar, et al., 1973). This places the exopause $r_p$ at a geocentric distance
of 36.20 Earth radii. The constituent mass (m) has been fixed by the selection of atomic hydrogen as the exospheric gas. The remaining parameters are the exobase radius and density; these are not critical quantities in this application. The exobase has been placed at the conventional altitude of 500 kilometers. Reference to the exobase density $N_C$ can be eliminated by expressing densities in relative terms, either relative to the exobase density or normalized by some standard profile. Lastly, it might be noted that actual geocoronal properties cannot be measured out to the exopause $\lambda_P$ but that the sensible exosphere only extends out to $\sim$16 Earth radii (Thomas and Bohlin, 1972). Due to the very low absolute densities of hydrogen of terrestrial origin beyond this radius, the intensity of geocoronally scattered Lyman-\(\alpha\) cannot be detected against the background intensity of interplanetary (solar-scattered) and galactic origin. The profiles contained in this and the next Chapters do not therefore extend beyond 20 Earth radii. The calculations have been performed at selected altitudes along the Earth-Sun line for solar angles of $\chi = 0$ degrees and $\chi = 180$ degrees - i.e., along the noon and midnight directions in the ecliptic plane. This selection has the distinct benefit of rendering the calculation of exospheric quantities tractable, as discussed in Chapter 2. (Note that most reported observations have centered on day/night differences.)
SIMPLE EVAPORATIVE CASE

Total density is the basic quantity sought for. Denoting the results obtained for the evaporative case when radiation pressure acts by the label "RP", the profile obtained by averaging the noon and midnight RP density profiles as calculated according to Eqn. (2.12) is shown in Figure 3.1, where it is compared to profiles calculated by the Chamberlain theory. Beyond indicating the general trend and the failure of the averaged profile to match the experimentally-suggested \( r_{CS} = 2.50 \) Earth radii (\( R_E \)) profile closely, this manner of presenting results is not informative. However, it is a rather conventional graph and is included here for this reason. The Chamberlain profiles selected for comparison in Figure 3.1 are used repeatedly in this work. The only parameter varied is the satellite critical radius \( r_{CS} \), the exobase temperature \( T_c \) for these models being the same as that used in the RP calculations. The reason for using an \( r_{CS} = 2.50 \) \( R_E \) model has been indicated. Of the other two models, one represents an exosphere completely devoid of satellite atoms (\( r_{CS} = r_c \), the exobase), while the second (\( r_{CS} = r_p \), the exopause) possesses a bound component characterized by a completely isotropic velocity distribution at the radii of interest (\( r < r_p \)). Hence these two outer Chamberlain profiles must envelop the evaporative RP results.
FIGURE 3.1: Comparison of the averaged evaporative RP density profile (plusses) with selected Chamberlain models -

- - $r_{CS} = 2.50 \ \text{R}_E$
- - - $r_{CS} = r_p$ (isotropic bound component)
- - - - $r_{CS} = r_c$ (no satellites).

Densities are normalized to the exobase density.
FIGURE 3.1: RELATIVE DENSITY, SIMPLE EVAPORATIVE CASE
Figure 3.2 is more revealing. Here the RP densities have been normalized by the no-satellite $r_{CS} = r_C$ model, and are plotted against $\lambda = GMm/kT_C r$. The noon ($\chi = 0$ degrees) and midnight ($\chi = 180$ degrees) profiles for total density are separately displayed, providing a first illustration of the geotail. The geotail feature can be isolated in the manner shown in Figure 3.3, by displaying the night-to-day total density ratio; the nightside enhancement becomes notable beyond geosynchronous radius. The extent to which the nominal satellite component as defined in the last Chapter contributes to the total density is indicated in Figure 3.4, where the ratio of the satellite component density to the total density is shown.

A straightforward interpretation of this behaviour is evident. As all populated trajectories arise from the exobase (and return when bound), these constituting the ballistic and escape components in the no-radiation (NR) case, the bending of these trajectories by radiation pressure effectively increases the pathlength traveled by exospheric hydrogen atoms and so increases the total amount of hydrogen contained in the exosphere. This is borne out in Figure 3.2 and by considering column densities (Eqn. (2.15)) as in Table 3.1. The total content enhancement is not large, nor should it be. First, the lengthening of path can only be significant for trajectories reaching greater altitudes than can be attained by the bulk of the rather
FIGURE 3.2: Density normalized by the NR no-satellite model. RP results for noon (plusses) and midnight (stars) are compared to Chamberlain models with $r_{cs} = r_p$ (——) and $r_{cs} = 2.50 R_E$ (— _—).

With $\lambda = \frac{G M m}{k T_C r}$, some corresponding radial distances in Earth radii are:

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.9006</td>
<td>exobase</td>
</tr>
<tr>
<td>6.0000</td>
<td>1.2404</td>
</tr>
<tr>
<td>5.0000</td>
<td>1.4884</td>
</tr>
<tr>
<td>4.0000</td>
<td>1.8606</td>
</tr>
<tr>
<td>3.7211</td>
<td>2.0000</td>
</tr>
<tr>
<td>3.0000</td>
<td>2.4807</td>
</tr>
<tr>
<td>2.0000</td>
<td>3.7211</td>
</tr>
<tr>
<td>1.1221</td>
<td>6.6323 (gs)</td>
</tr>
<tr>
<td>1.0000</td>
<td>7.4422</td>
</tr>
<tr>
<td>0.7442</td>
<td>10.0000</td>
</tr>
<tr>
<td>0.4651</td>
<td>16.0000</td>
</tr>
<tr>
<td>0.3721</td>
<td>20.0000</td>
</tr>
</tbody>
</table>
FIGURE 3.2: NORMALIZED DENSITY, SIMPLE EVAPORATIVE CASE
FIGURE 3.3: Profile of the midnight ($\chi = 180$ degrees) to noon ($\chi = 0$ degrees) RP density ratio.
FIGURE 3.4: Profiles for the noon (plusses) and midnight (stars) RP satellite fractional density compared to Chamberlain models with $r_{CS} = r_{P}$ (-----) and $r_{CS} = 2.50$ RE (-----).
FIGURE 3.4: SATELLITE FRACTION, SIMPLE EVAPORATIVE CASE
TABLE 3.1: COLUMN DENSITIES ABOVE EXOBASE

<table>
<thead>
<tr>
<th></th>
<th>noon</th>
<th>midnight</th>
</tr>
</thead>
<tbody>
<tr>
<td>RP column density</td>
<td>$1.217 \times 10^{13}$ atoms/cm$^2$</td>
<td>$1.235 \times 10^{13}$ atoms/cm$^2$</td>
</tr>
<tr>
<td>SI column density</td>
<td>$1.183 \times 10^{13}$ atoms/cm$^2$</td>
<td>$1.200 \times 10^{13}$ atoms/cm$^2$</td>
</tr>
<tr>
<td>NR column density</td>
<td>$1.111 \times 10^{13}$ atoms/cm$^2$</td>
<td></td>
</tr>
</tbody>
</table>

(Note: A ceiling has been placed at the exopause.)

cool exobase-generated atoms. Second, the optical depth of the exobase to Lyman-α in the sub-solar region has been reliably measured and is accounted for reasonably well by a no-satellite Chamberlain model, hinting that radiation pressure dynamics ought not to greatly alter this quantity (Vidal-Madjar, et al., 1973).

A corollary is that the resultant RP trajectories go a long way toward establishing an isotropic bound velocity distribution in the exosphere out to beyond 10 Earth radii, akin to the isotropic ($r_{cs} = r_p$) Chamberlain model. This is seen in Figure 3.4, where the satellite component approaches the $r_{cs} = r_p$ profile, especially on the nightside, and is further indicated by comparing the kinetic temperature (Eqn. (2.14)) of this distribution to the NR Chamberlain temperatures, as in Figure 3.5. The averaged RP profile can be seen to mimic the isotropic NR profile in a remarkable fashion. The profiles that arise in the restricted Chamberlain models are noticeably warmer, due to the
FIGURE 3.5: Comparison of the averaged evaporative RP kinetic temperature profile (plusses) with selected Chamberlain models -

- \( r_{cs} = 2.50 \ R_{E} \)
- \( r_{cs} = r_{p} \) (isotropic bound component)
- \( \cdots r_{cs} = r_{c} \) (no satellites).
FIGURE 3.5: KINETIC TEMPERATURE, SIMPLE EVAPORATIVE CASE

- no satellites
- averaged noon & midnight
- isotropic

r_{cs} = 2.50 \text{ RE}
FIGURE 3.6: Noon (plusses) and midnight (stars) kinetic temperature profiles in the evaporative RP case.
FIGURE 3.6: KINETIC TEMPERATURE, SIMPLE EVAPORATIVE CASE
"throttling" of trajectories by the cone-of-acceptance which becomes narrower as energy increases. Figure 3.6 shows the separate noon and midnight RP kinetic temperature profiles.

Each of these facets of the RP exosphere is a direct consequence of the looping behavior of trajectories belonging to the nominal satellite class. Although the minimum escape speed at a given location depends on orientation, this dependence is not a particularly strong one at the locations selected (see Figures 2.2a and 2.2b). The local distribution depends only on speed at the exobase (Liouville's theorem) and so to the extent that the variation of the radiation pressure potential term in the Hamiltonian over the exobase can be ignored, a Maxwellian isotropic velocity distribution can be expected for the bound (ballistic + satellite) component. In other words, atoms reaching a given location along different trajectories yet having equal local speeds will have had roughly the same speeds when leaving the exobase, regardless of the differing points of origin.

A SIMPLE-MINDED MODEL

The above remarks suggest a simple way to calculate exospheric quantities in a phenomenological fashion for points on the Earth-Sun axis. Prior to establishing this model, it will prove convenient to express the density as
\[ N(\lambda, \chi, \phi) = \iiint f \psi^2 \, d\psi d\mu d\varepsilon = \iiint f_C \psi^2 \, d\psi d\mu d\varepsilon \]

\[ = \frac{N_C}{\pi^{3/2}} \iiint e^{-\psi^2} \psi^2 \, d\psi d\mu d\varepsilon \]

\[ = \frac{N_C}{\pi^{3/2}} e^{-(\lambda C - \lambda)} \exp\left(-\frac{1}{2} \frac{\alpha}{\lambda} \cos \chi \right) \times \]

\[ \iiint e^{-\psi^2} \exp\left[\frac{1}{2} \frac{\alpha}{\lambda C} \cos \chi \left(\lambda C\right) \right] \psi^2 \, d\psi d\mu d\varepsilon \]

(3.1)

where Liouville's theorem and energy conservation have been invoked (Eqn. (2.11)) and the integrals restricted to count only those trajectories that intersect the exobase. The \( \alpha \)-potential at the point of exobase intersection - i.e., the radiation pressure potential \((2\alpha/\lambda C)\cos \chi (\lambda C)\) - is an implicit function of local position and momentum. So far, this is an exact formulation for the density. The following simplifications are now applied:

(i) the bound component velocity distribution is taken to be isotropic - i.e., all bound trajectories (eventually) intersect the exobase.

(ii) in describing the escape component, the cone-of-acceptance \( \mu^0 \) as defined in the Chamberlain theory (Eqn. (AI.3)) is taken to closely approximate the distorted cone-of-acceptance. This can be inferred from Figures 2.2a and 2.2b.

(iii) since \((2\alpha/\lambda C)\cos \chi (\lambda C)\) is generally a small correction, it is ignored.

(iv) an "average" minimum escape speed \( \langle \psi^0(\lambda, \chi, \mu) \rangle = \psi^0(\lambda, \chi) \) is assumed. Since the contribution of the escape component to the total density is comparatively small within the sensible exosphere, \( \psi^0(\lambda, \chi) \) can better be looked upon as the ceiling energy for the [ballistic + satellite] component.
The density in this simple-minded model is then given by

\[ N(\lambda, \chi) = \frac{N_c}{\pi^{3/2}} e^{-\frac{(\lambda - \lambda_c)}{2}} \exp\left[-(2\alpha/\lambda)\cos\chi \right] \times \]

\[ \left(4\pi \int_0^{\psi_0} e^{-\psi^2} d\psi + 2\pi \int_{\psi_0}^{\infty} (1 - \mu(\psi)) e^{-\psi^2} d\psi \right) \]

(3.2)

where the first (second) term accounts for the bound (escape) component. (Note that in this application \( \phi \) and \( \epsilon \) are irrelevant quantities.)

The trick is to specify \( \psi^0 \). With escape defined in this study by exopause passage, the most straightforward approach is perhaps to refer directly to the exopause via energy conservation along a trajectory -

\[ \psi_0^2(\lambda, \chi, \mu) = (\lambda - \lambda_p) + \psi_0^2(\lambda_p, \chi_p, \mu_p) + 2\alpha(\cos \chi_p/\lambda_p - \cos \chi/\lambda). \]

Averaging over local momentum orientation (\( \mu \)) yields

\[ \psi_0^2(\lambda, \chi) = (\lambda - \lambda_p) - 2\alpha \cos \chi/\lambda + \langle \psi_0^2 \rangle_p + 2\alpha \langle \cos \chi_p \rangle/\lambda_p \]  

(3.3)

where the averaged quantities are functions of local position. As these averages cannot be explicitly carried out in a rigorous manner, one idea is to replace the averaged terms with a simple function containing adjustable parameters. As an example, one might write

\[ 2\alpha \langle \cos \chi_p \rangle/\lambda_p + \langle \psi_0^2 \rangle_p = (2\alpha/\lambda_p) r(\lambda, \chi) + (2\alpha/\lambda) q(\lambda, \chi). \]  

(3.4)

In particular, \( r(\lambda, \chi) \) and \( q(\lambda, \chi) \) are here taken to be
constants on each side of the planet. The technique is simply to vary $r$ and $q$ until the density profiles as determined by Eqn. (3.2) closely match the RP profiles shown in Figure 3.2.

As shown in Figure 3.7a, this procedure can yield strikingly good results, even though no special effort has been made to find optimal values for $r$ and $q$. Not only is the geotail faithfully reproduced (Figure 3.7b), but the satellite component is also well-fitted (Figure 3.7c), where in this simple-minded model

$$N_{\text{Sat}} = \frac{N_C e^{-(\lambda_C-\lambda)}}{\pi^{3/2}} \exp[(-2\alpha/\lambda)\cos\chi] \int_{\psi^1}^{\psi^0} e^{-\psi^2} \mu^0(\psi)\psi^2 d\psi. \quad (3.5)$$

Lastly, since an isotropic bound component has been assumed, the temperature profile of this model necessarily closely resembles that given by the Chamberlain $r_{cs} = r_p$ model. It is tempting to attach some significance to the quantities $r$ and $q$ - for instance, to identify $r(\lambda, \chi) = \langle \cos x_p \rangle$. This, however, cannot be carried out in a unique fashion without a second, independent relation involving $r$ and $q$, and it seems that such a relation cannot be readily justified on the basis of simple physics. Hence this will not be further pursued here.

The main purpose for this simple-minded model is to provide a handy way to reproduce the RP case results. Its
FIGURE 3.7a: Comparison of the normalized RP density profiles (plusses - noon, stars - midnight) with profiles given by Eqn. (3.2) using the indicated parameter values.
FIGURE 3.7a: NORMALIZED DENSITY, SIMPLE-MINDED MODEL

\[ r_{\text{day}} = -0.59 \quad q_{\text{day}} = +1.53 \]
\[ r_{\text{night}} = -0.14 \quad q_{\text{night}} = -1.00 \]
FIGURE 3.7b: Comparison of the RP geotail ratio (plusses) with that of the simple-minded model.
r_{day} = -0.59 \quad g_{day} = +1.53
r_{night} = -0.14 \quad g_{night} = -1.00

FIGURE 3.7b: GEOTAIL RATIO, SIMPLE-MINDED MODEL
FIGURE 3.7c: Comparison of the RP satellite fraction profiles with those computed using Eqn. (3.5).
\[ r_{\text{day}} = -0.59 \quad q_{\text{day}} = +1.53 \]
\[ r_{\text{night}} = -0.14 \quad q_{\text{night}} = -1.00 \]

**FIGURE 3.7c:** SATELLITE FRACTION, SIMPLE-MINDED MODEL
usefulness extends beyond this, though. The fact that the model works so well in itself supports the interpretation or contention that the RP bound component exhibits a nearly isotropic velocity distribution. It can also serve as a diagnostic tool. For instance, the decrease in normalized density shown in Figures 3.2 and 3.7a above 10 Earth radii, shared by the satellite component in Figures 3.4 and 3.7c, can be attributed to two related factors: (i) an increasingly large portion of phase space corresponds to the hyperbolic fly-by class of trajectories, and (ii) escape resulting from radiation pressure perturbations becomes more likely in the outer geocorona, the escape component thus making up a larger fraction of the total density. The temperature profiles are not affected by these factors, primarily because the effusive flow term in Eqn. (2.14) eliminates any effect on temperature this escaping outward flow may have; the distribution still looks isotropic, just "thinner".

One question cannot be settled by this simple-minded model - namely, what is the principal cause of the geotail? As it is based explicitly on energy conservation (Eqn. (2.8)), the model can be used to study properties involving energy only. However, two separate aspects of radiation pressure dynamics are isolated in Eqn. (3.2) -

(i) the anti-solar orientation is explicit in the α-potential; thus in terms of energetics, the nightside is more accessible.
(ii) the bending/lengthening of trajectories is accounted for in an approximate fashion via the assumed velocity distribution; this distribution is characterized by the bound component ceiling energy $\psi^0$, which measures the extent of this component in phase space.

The latter aspect is primarily an effect involving angular momentum - i.e., radiation pressure, acting as a torque, distorts the trajectory shape. So, although $\psi^0$ is found via energy conservation, the density of trajectories in phase space is not necessarily that conveyed by the assumed velocity distribution. In brief, the geotail can be attributed to energetics and/or to a relatively higher density of occupied trajectories threading the nightside exosphere, but which is dominant cannot yet be ascertained.

SOLAR IONIZATION

As noted in the preceding Chapters, solar ionization (SI) processes are likely to be significant loss mechanisms in the geocoronal situation. These can be combined and incorporated in a simple fashion into the framework of the evaporative RP model via a factor $\exp[-\tau/\tau_{SI}]$, where $\tau$ is the (dimensionless) time of flight between a selected location in the exosphere and the exobase along a specified trajectory and $\tau_{SI}$ is the net lifetime against solar ionization (here corresponding to 10 days). The kinetic distribution when solar ionization acts is thus taken to be
\( f_{SI}(\lambda, \chi, \phi, \psi, \mu, \varepsilon) = f_{RP} \times \exp(-\tau/\tau_{SI}). \) \hfill (3.6)

Figure 3.8 shows the effect on density of incorporating this mechanism. The density depletion at higher altitudes is rather substantial and can be seen to bring the dayside density profile closer to the \( r_{CS} = 2.50 \; R_p \) NR profile. Figure 3.9 demonstrates that the geotail survives this depletion; the weakening of the feature indicates that the hydrogen on the nightside has a somewhat longer mean flight-time than on the dayside. Figure 3.10 illustrates that, as expected, ionization depletes the satellite component to a greater extent than the other components. This mechanism has little effect on the temperature structure; although not shown here, the result of solar ionization is mainly to heat slightly the outer geocorona by preferentially removing slower-moving constituents, these having longer residency times.

Thus, the main effect of solar ionization mechanisms is to deplete the density; the extent of this depletion is further indicated in Table 3.1. An interesting sidelight of this ionization example is a capability to determine an effective or mean "age" of the hydrogen in the exosphere as a function of radius. By comparing the average density profile \( N_{SI}(\lambda) \) in the ionization case to that shown in Figure 3.1 \( (N_{RP}) \), an effective age can be defined by

\[ <\tau> = \tau_{SI} \log \left( \frac{N_{RP}}{N_{SI}} \right). \] \hfill (3.7)
FIGURE 3.8: Comparison of the SI density profiles (plusses - noon, stars - midnight) with the evaporative RP results shown in Figure 3.2 (---, curves fit by cubic splines). The NR $r_{CS} = 2.50$ $R_E$ model results are also shown (--- -).
Figure 3.8: Normalized Density, Solar Ionization Case
FIGURE 3.9: Geotail ratio in the presence of solar ionization (plusses). The solid curve is a cubic spline fit to the points plotted in Figure 3.3.
FIGURE 3.10: Comparison of the SI satellite fraction profiles (plusses - noon, stars - midnight) with the RP profiles of Figure 3.4 (—, curves fit by cubic splines). The NR $r_{CS} = 2.50\ R_E$ profile is also shown (—).
FIGURE 3.11: Effective age of exospheric hydrogen in the evaporative model as a function of radial position, as calculated by Eqn. (3.7).
This is plotted in Figure 3.11. While exospheric theory has not been applied to non-steady state situations as yet, this graph can be used to estimate the response time of the geocorona at a specified altitude to sudden shifts in exobase conditions, such as can happen during a magnetospheric substorm (the geomagnetic effect.) Also note that the ages indicated in this Figure justify the initial neglect of the solar motion across the sky.

SUMMARY

Chamberlain (1979) demonstrated the primary effect of solar radiation pressure on geocoronal hydrogen to involve angular momentum. Namely, radiation pressure acts as a torque distorting the shape of exospheric trajectories. His analysis centered on a perturbation technique for handling radiation pressure; while it is reliable when used to evaluate the evolution of tightly bound orbits (i.e., orbits that remain near the planet where \(|a| \ll |g|\)), trajectories that extend well away from the planet are beyond its range of applicability because of the long times spent near apogee. Nevertheless, this work pointed to the existence of a near-isotropic velocity distribution for the hydrogen component bound to the planet, by showing that osculating satellite trajectories eventually crash into the exobase. More importantly, Liouville's theorem necessarily
implies that these trajectories will then be occupied by atoms originating there. The working out of the consequences of this for a simple evaporative prototype has been the subject of this Chapter.

To summarize the basic picture that emerges from the evaporative case, the primary effect of radiation pressure is the lengthening of the paths traveled by exospheric hydrogen, increasing somewhat the total content of the exosphere and generating an extensive nominal satellite component. The looping behaviour of the satellite trajectories establishes a near-isotropic quasi-Maxwellian velocity distribution for the bound geocoronal hydrogen component, at least along the Earth-Sun axis. Since radiation pressure acts strictly in the anti-solar direction, the resultant preferential bending of trajectories in that direction increases the total density on the nightside compared to the dayside, both by making trajectories on the nightside more accessible in terms of energy and by increasing the density of occupied trajectories there; this produces the geotail. Solar ionization acts to partially deplete the total density. The main conclusion to be drawn is this: that solar radiation pressure acting alone, without collisional mechanisms, creates a substantial satellite component with an observable day/night asymmetry.
CHAPTER 4: Plasmasphere Interaction

The collision

\[ H_{\text{cool}} + H^+_{\text{hot}} \rightarrow H^+_{\text{cool}} + H_{\text{hot}}, \]

as a resonant charge exchange reaction, establishes an effective coupling of the exosphere to the plasmasphere, a region of comparatively dense thermal plasma contained by the geomagnetic field exhibiting a rather sharp outer boundary called the plasmapause (Chappell, 1972; Carpenter and Park, 1973). It is a mechanism that has often been cited as supplementing the evaporative escape of neutral hydrogen from the Earth since the incident plasmaspheric proton temperature \( T^* \) is notably higher than that of the neutral hydrogen geocorona. The intention in this Chapter is to isolate and characterize some basic aspects of the exosphere–plasmasphere interaction, by using the Boltzmann equation to incorporate this collision in the evaluation of the geocoronal hydrogen kinetic distribution. Solar ionization mechanisms are ignored in this Chapter.

The plasmaspheric model employed in this treatment is greatly simplified and no attempt is made to describe or follow the ion population in a realistic manner. The actual plasmasphere on Earth is highly dynamic, both in regards to structure and composition. During magnetically quiet periods, it can extend out to geosynchronous radius (6.632
Earth radii) at the magnetic equator and roughly exhibits the dipole shape, terminating at exobase altitudes at mid-latitudes. Plasma temperatures near the exobase are several times that of the local neutral gas and increase outward (along magnetic field lines) by almost an order of magnitude in the vicinity of the equatorial plasmapause (Serbu and Maier, 1970). Protons (H⁺) are generally the primary positive ionic species, although at near-exobase altitudes and during active times singly-ionized helium (He⁺) and oxygen (O⁺) can constitute major components. The distribution of this last species has important implications for geocoronal H due to the near-degenerate ionization energies of O and H, leading to a near-resonant charge exchange cross-section for the reaction

\[ O^+ + H \rightarrow O + H^+. \]

Indeed, the H⁺ density in the topside ionosphere is established by a chemical equilibrium between H, H⁺, O, and O⁺. To keep matters simple, the O-O⁺ exchange paths have been ignored in this study; it is assumed that detailed balancing holds fairly well at near-exobase altitudes, so that no great error arises from this neglect. In the outer plasmasphere, the net effect of oxygen collisions with geocoronal hydrogen is most likely to augment the creation of fast H (via O + H⁺ collisions) at the expense of thermal H (via O⁺ + H).
THEORY OF THE INTERACTION

In the $H + H^+$ charge exchange collision, the momenta of the incident particles are taken to be undeflected; this is justified in view of the large cross-section for the charge exchange process compared to that for momentum-changing (e.g., elastic) collisions. The Boltzmann equation \( df/dt = (df/dt)_{\text{coll}} \) in canonical coordinates then expands to

\[
\begin{align*}
(p_r/m) \frac{\partial f}{\partial r} + (p_x/mr^2) \frac{\partial f}{\partial x} \\
+ \left[ \left( \frac{p_x^2 + p_\phi^2}{\sin^2 \chi} \right)/mr^3 - Gm/r^2 - ma \cos \chi \right] \frac{\partial f}{\partial p_r} \\
+ \left[ \frac{p_\phi^2}{mr^2 \sin^2 \chi} \cot \chi + mar \sin \chi \right] \frac{\partial f}{\partial p_x}
\end{align*}
\]

\( \equiv \iint Q(g)g[f^*(r,p_i')f(r,p_j') - f^*(r,p_j')f(r,p_i')] \, dp_j' \quad (4.1) \)

where \( f \) (\( f^* \)) is the neutral (ion) distribution function and \( g \) is the relative collision speed. \( Q \) is the cross-section for resonant charge-exchange. The primed momenta appearing in the integrand are linear momenta, with \( p_i' \) corresponding to the canonical momenta on the left-hand side of the equation.

As in Chamberlain (1977), a factorization technique is employed, by setting

\[
f(r,p') = f_0(r,p') [1 + \phi(r,p')]; \quad (4.2)
\]

\( f_0 \) is the collisionless distribution function in the evaporative case, found in the last Chapter. The proton
kinetic distribution is simply taken to be that of a spherically symmetric (i.e., the dipole shape is ignored), diffusive equilibrium plasmasphere at a constant temperature \( T^* = \rho T_C \), terminated at the plasmapause \( \lambda_{pp} \):

\[
f^*(r, p_i') = \frac{N_C^* e^{-(\lambda_C - \lambda)/2\rho}}{(2\pi m k T_C \rho)^{3/2}} \exp[-\psi^2/\rho] \quad (\lambda > \lambda_{pp})
\]

\[
= 0 \quad (\lambda < \lambda_{pp}).
\]

(4.3)

No trajectory restrictions are placed on the protons which are trapped by the geomagnetic field. \( N_C^* \) is the proton density at the neutral exobase level and the proton scale height incorporates the gravitational-separation electric field via the factor of two. The factor \( \phi \) is not required to remain small (hence to call this a perturbation technique is somewhat erroneous.) What is expected is that the ballistic component, which comprises the bulk of the total density at lower altitudes, remains relatively unperturbed by charge exchange collisions. It is in this region, within the first few neutral scale heights above the exobase, that the majority of collisions must occur. A hint is taken from the experimental results and a Chamberlain model characterized by a satellite critical radius \( r_{cs} \) of 2.50 Earth radii has been used in evaluating the collision integral.

Since \( f_0 \) solves the homogeneous Boltzmann equation (i.e., Liouville's theorem), Eqn. (4.1) becomes
\[
\frac{\partial \phi}{\partial r} \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial \phi^2} = \left[ \frac{(p_x^2 + p_y^2)}{\sin^2 \chi} \right] / m \rho - G M m / r^2 - m a \cos \chi \frac{\partial \phi}{\partial p_x} \\
+ \left[ \frac{(p_y^2 / \sin^2 \chi)}{\rho} \right] c t n \chi + m a r \sin \chi \frac{\partial \phi}{\partial p_x} \\
= \frac{Q}{f_0} \int \int \int f(i) f_0(j)(1+\Phi(j)) - f(j) f_0(i)(1+\Phi(i)) \, d^3p_j
\]

(collapsing the integrand arguments to the momentum index).

Some comments are in order: first, in the charge exchange collision, the trajectories of the neutrals entering into the collision are not explicitly followed since what matters is the total amount of neutral H at a specific location available for reaction (recall that momentum exchange has been ignored and that no attempt is made to follow the \text{He}^+ population); second, the \( r_{cs} = 2.50 \ \text{R}_\oplus \) Chamberlain model seems to model the dayside geocoronal density profile rather well on the basis of observations and, as seen in the last Chapter (Figure 3.5), has an appropriate kinetic temperature profile at altitudes < 2.50 \( \text{R}_\oplus \) thus mimicking to some extent the effects of radiation pressure on the neutral kinetic distribution. By using the experimentally-suggested Chamberlain model as a zeroth-order approximation, whatever errors are involved may be minimized somewhat. The collision cross-section is slowly varying at thermal velocities (\( \sim 10^6 \) cm/sec) and is here taken to be constant at \( Q = 4.5 \times 10^{-15} \) cm\(^2\) (Newman, et al., 1982).

The collision term must be further simplified to
become tractable. When both neutral and ion kinetic distributions are complete Maxwellians, the mean collision
speeds for the creation and destruction terms are

\[ g_o = \frac{U^2}{2v_i} + v_i \quad \text{and} \quad g^* = \frac{(U^*)^2}{2v_i} + v_i, \quad (4.5) \]

approximately \((U^2 = 2kT/m)\). The approach here is to replace the collision integrals with

\[
\int \int \int g \left[ f^*(i)f_0(j)(1+\phi(j)) - f^*(j)f_0(i)(1+\phi(i)) \right] d^3p_j
\]

\[
= g_o N_c e^{-\left(\lambda_c-\lambda\right)\alpha_0} f^*(i) - g^* N_c e^{-\left(\lambda_c-\lambda\right)/2\rho} f_0(i)(1+\phi). \quad (4.6)
\]

\(\alpha_0(\lambda)\) is the partition function for the neutral component
entering the collision, and as discussed above has been
selected to correspond to the \(r_{CS} = 2.50 \) \(R_E\) Chamberlain
model. The cool plasma produced by charge exchange
collisions is not followed but can be visualized as quickly
heating to \(T^*\) via electron-ion interactions. Within the
forms taken for the neutral and plasma kinetic distributions
and for the collision speeds, the destruction term is exact.
The creation term is also fairly exact when \(\alpha(\lambda) = 1\); for
heights at which \(\alpha(\lambda) < 1\), this approximation tends to
overestimate the effective collision speed since in the
exact integral the high velocity neutrals are restricted to a
narrower range of trajectories - i.e., the high velocity
neutrals are "choked off" by the cone-of-acceptance. Still,
most charge exchange collisions occur at lower altitudes.
where $\Xi(\lambda) \sim 1$ so the approximation ought to be good.

Now to express Boltzmann's equation as given by Eqns. (4.4) and (4.6) in dimensionless quantities:

$$
\frac{d\Phi}{d\tau} = \left(\frac{2GMQN_c^*}{U_c^2}\right) e^{-\frac{(\lambda_c-\lambda)}{2\rho}} x \left(\frac{(\rho-1)\psi^2}{\rho}\right)^{3/2} - \left(\frac{(\rho/2)\psi}{\rho}\right)(1+\phi)
$$

within the plasmasphere ($d\Phi/d\tau = 0$ for $\lambda < \lambda_{pp}$), where the left-hand side has been collapsed back to a total derivative. In the creation term the factor $(1/f_0)$ has been replaced by its NR form (Eqn. (2.11)), which ought to be an admissible approximation since the $\alpha$-potential differences remain small at those altitudes where most collisions occur. This quantity can now be integrated along a dynamical trajectory. The resulting kinetic distribution used to calculate exospheric quantities is then given by Eqn. (4.2).

Boundary values for $\phi$ are not hard to establish. For bound component trajectories, possessing exobase intersections at both endpoints, it is necessary to set $\phi_{\text{initial}}$ to zero for motion in each direction - i.e., $\phi_{\text{initial}} = 0$ for ascent away from the exobase. In general, $\phi$ is nonzero at the end of flight and indeed can be negative (though $\phi > -1$ must always be satisfied). For the escape component as defined in Chapter 2, the same considerations apply - i.e., for motion away from the exobase, $\phi_{\text{initial}} = 0$. The plasmasphere interaction will place atoms on these
trajectories moving downward, though; above the plasmapause these trajectories will remain devoid of atoms moving toward the exobase, implying $\Phi(\lambda<\lambda_{pp}) = -1$ for this class. Likewise, the hyperbolic fly-by trajectories that do not intersect the exobase at all can become populated. In an effort to keep the calculations manageable, this class has been ignored. It can be appreciated that this neglect cannot affect the final results to any great extent. Since atoms on such trajectories do not "stay around" very long, they cannot contribute much to the total density; the primary error arising from their neglect is likely to be a slight underestimate of the kinetic temperature. Also, the "splashback" of downward-moving atoms on escape component trajectories - i.e., the albedo effect (Chamberlain, 1977) - has been ignored, in keeping with the initial assumption of a Maxwellian kinetic distribution at the exobase.

(As a bit of terminology, the results described in this Chapter, obtained by incorporating both radiation pressure and plasmaspheric charge exchange, will be denoted by the label "RP+PS", as opposed to the "RP" epithet used for the results presented in the last Chapter.)

RESULTS OF THE INTERACTION

To carry out the calculations, parameter values must be selected that are in accordance with the geocoronal
FIGURE 4.1: Example of a neutral hydrogen kinetic distribution function calculated for a Chamberlain NR model exosphere ($r_{CS} = 2.50$ $R_E$) interacting with the simple plasmasphere model (Eqn. (4.3)):

A - ascent ($\mu = +1$, $\psi > 0$). Some depletion at slower speeds evident. Considerable enhancement for speeds $\psi > \psi^*$.

B - descent ($\mu = -1$, $0 < \psi < \psi^*$). Significant depletion due to long pathlengths inside plasmasphere (rising + falling). $\psi^*$ is the minimum energy required to pass the plasmapause.

C - descent ($\mu = -1$, $\psi^* < \psi < \psi^0$). Considerably less depletion compared to segment B because slower motion (apogee) occurs above plasmapause.

D - descent ($\mu = -1$, $\psi > \psi^0$). Generated entirely by plasmaspheric charge exchange collisions.

Compared to an evaporative kinetic distribution (Eqn. (A1.1b)), arising from a Maxwellian exobase. Both distributions normalized by respective $\psi = 0$ values.

Altitude - 2.00 Earth radii, vertical motion ($\mu = \pm 1$).
FIGURE 4.1: KINETIC DISTRIBUTION EXAMPLES, NR CASE
example specified in the last Chapter. The plasmaspheric parameters that are admitted in the simple model employed are the proton temperature \( T^* = \rho T_c \), the proton density at the exobase \( N_C^* \), and the position of the plasmapause \( r_{pp} \). Representative values for these quantities are

\[
\begin{align*}
\rho &= 4 \\
N_C^* &\sim 10^4 \text{ cm}^{-3} \\
\end{align*}
\]

\( r_{pp} \sim \) geosynchronous radius.

The exobase temperature and the radiation pressure acceleration are taken to have the same values as used in the last Chapter. Figure 4.1 presents an NR example of a kinetic distribution function that results when the plasmaspheric interaction is incorporated via Boltzmann's equation. The perturbed distribution (solid curve) is considerably enhanced in the "wings" and a corresponding depletion at the slowest speeds is evident.

The results of this interaction for the RP+PS model are shown in Figures 4.2 - 4.6.\(^1\) An interesting phenomenon can be seen in the first Figure. The total density in the exosphere is depleted to a considerable extent, and at altitudes inside the plasmapause \( r_{pp} \) the profiles approach

\[\text{--------------------------}\]

\(^1\) Cost restrictions necessitated limiting the RP+PS calculations to points in the outer geocorona; these points are both (i) those at which the effects of the interaction are most evident and (ii) less expensive to evaluate.
FIGURE 4.2: Comparison of the normalized RP+PS density profiles (plusses - noon, stars - midnight) with the RP profiles of Figure 3.2 (___, curves fit by cubic splines.) Also shown is the NR $r_{CS} = 2.50$ $R_E$ profile (___ _)._
FIGURE 4.3: Comparison of the RP+PS geotail ratio (plusses) with the RP results shown in Figure 3.3 (——, curve fit by cubic splines).
FIGURE 4.4: Comparison of the RP+PS satellite fraction profiles (plusses - noon, stars - midnight) with the RP profiles of Figure 3.4 (___, curves fit by cubic splines.) Also shown is the Chamberlain $r_{CS} = 2.50 \, R_E$ profile (_____).
FIGURE 4.4: SATELLITE FRACTION, PLASMASPHERE INTERACTION CASE
FIGURE 4.5: Comparison of the RP+PS kinetic temperature profiles (plusses - noon, stars - midnight) with the mean RP results shown in Figure 3.5 (., curve fit by cubic splines).
FIGURE 4.5: KINETIC TEMPERATURE, PLASMASPHERE INTERACTION CASE
FIGURE 4.6: Normalized escape flux for the RP, RP+PS, and Chamberlain models. (See discussion in text.) The Chamberlain model (described in caption to Figure 4.1) has been evaluated outside the plasmapause only. Note that in this Figure the geometric \((1/r^2)\) spreading is automatically accounted for.
that of the Chamberlain no-satellite \( r_{cs} = r_c \) model. Beyond the plasmapause the normalized density climbs fairly rapidly, on the dayside attaining a level close to that appropriate to the Chamberlain \( r_{cs} = 2.50 \) \( R_E \) model near 10 Earth radii before falling off. This outer geocoronal behaviour is similar to that suggested by the observations of Wallace, et al. (1970) and Bertaux and Blamont (1973). The geotail feature survives the plasmasphere interaction, as demonstrated in Figure 4.3. Figure 4.4 displays the satellite fractional density, indicating that the depletion mechanism acts on the satellite component to a greater degree than on the ballistic population. The kinetic temperatures that result in the RP+PS case are shown in Figure 4.5; as anticipated, the outer geocorona is distinctly warmer.\(^2\)

The density depletion throughout the plasmasphere and the rise of the normalized density near the plasmapause reveal the importance of gravitational "cooling" of the neutral component in this interaction. A parcel of atoms on a bound trajectory effectively cools (the kinetic energy converting to potential energy) as it climbs in altitude

\(^2\) It must be conceded that the total density depletion shown in Figure 4.2 indicates that the use of the \( r_{cs} = r_c \) Chamberlain model in the creation collision term of Eqn. (4.7) would perhaps have been a more self-consistent choice than the \( r_{cs}=2.50 \) \( R_E \) model. The effects described would not be changed, however.
while the plasmaspheric ions remain (in this model) at a constant temperature (in the real plasmasphere $T^*$ actually increases with altitude.) Hence as this parcel slows, any charge exchange collisions that occur are more likely to yield atoms at speeds sufficient to escape and the rate of depletion exceeds to a greater degree the rate at which atoms are scattered onto the bound trajectory in question (illustrated by segment B in Figure 4.1). Satellite component trajectories, due to the longer pathlengths and the relatively slow speeds, will suffer this net depletion to a greater extent. A trajectory of sufficient energy to extend beyond the plasmasphere is less affected for two reasons: (i) the faster-moving parcel has a greater likelihood of gaining atoms via plasmaspheric reactions than a slower-moving parcel, and (ii) a parcel on such a trajectory spends less time within the plasmasphere, with the lowest segment of flight situated outside the plasmapause; the net effect is shown by segment C in Figure 4.1. Also note that the near-isotropy of the RP bound component kinetic distribution no longer holds and, in particular, that along a given trajectory parcels traveling in opposite directions generally no longer carry equal numbers of atoms.

The interpretation of the effect of the plasmasphere interaction on kinetic temperature is likewise straightforward. The warmer RP+PS outer exosphere reflects
the gentler drop-off of the escape population with increasing $\psi$, implying a greater spread of $\psi^2$ about $<\psi>^2$ in Eqn. (2.14) (refer to segment A of Figure 4.1). The concommitant depletion of slower-moving (cooler) atoms clearly contributes to increasing $<\psi^2>$. At altitudes within the plasmasphere the presence of downward-moving atoms on escape trajectories further raises the kinetic temperature (again refer to Figure 4.1, segment D).

The aspect of the plasmasphere interaction that has been most studied in past work (reviewed by Fahn and Shizgal, 1983) is the enhanced escape of hydrogen; the density depletion illustrated in Figure 4.2 complementing this escape process has not been generally appreciated. Profiles for the escape flux (Eqn. (2.13)) that arise in both the RP and RP+PS models studied in this work are shown in Figure 4.6, normalized by the Jeans evaporative flux (Eqn. (A1.4)). It should come as no surprise that radiation pressure increases somewhat the evaporative escape component since atoms need only reach the exopause to effectively take leave of the planet. The increase (decrease) of the evaporative flux on the nightside (dayside) as radial distance increases is partly a manifestation of the bending of trajectories in the anti-solar direction. In the RP+PS case, this day/night behavior is also evident, particularly at altitudes beyond the plasmapause.

It is apparent in Figure 4.6 that the difference
between the RP+PS noon and midnight profiles is essentially the same as for the RP profiles, and again this is expected. The neutral atoms emerging from the charge exchange collisions are on the average moving too fast to be appreciably perturbed by radiation pressure at the altitudes of interest; also, most of the atoms entering into these collisions are on ballistic trajectories, the collisions occurring at lower altitudes. Thus, any day/night asymmetry in the escape flux arising from charge exchange ought to be small. The decrease of the mean enhanced flux inside the plasmapause is also simply accounted for. As in the evaporative case, the escape flux has been assumed to be confined by the cone-of-acceptance - i.e., those trajectories that trespass the plasmasphere and intersect the exopause in both directions, the hyperbolic fly-bys, are neglected here, even though these can become populated in the plasmasphere interaction. The decrease of mean flux is simply due to the transferral via the loss term of Eqn. (4.7) of escaping atoms within the cone-of-acceptance onto hyperbolic fly-by trajectories where they can no longer be counted. (Recall that no trajectory restrictions were placed on the protons entering into the charge exchange collisions.) Beyond the plasmapause the mean flux profile flattens out, the collisions having ceased. In a more rigorous treatment of the plasmasphere interaction, it would be necessary to consider a distorted cone-of-acceptance with
respect to plasmapause intersection (as opposed to exobase intersection) to encompass these non-exobase-intersecting escape trajectories.

SUMMARY AND COMMENTS

The central result of this Chapter on the exosphere-plasmasphere interaction is contained in Figure 4.2 - namely, the strong depletion of total density, interpreted as being due to the conversion of slow-moving atoms into fast escaping atoms. The erosion of the satellite component is especially interesting. In a way, the plasmasphere interaction reinstates a physical cone-of-acceptance in that satellite atoms, by repeated passage of the plasmasphere, are removed to a greater extent than ballistic atoms of the same energy, which on ascent pass only once. Further, the heating effect of the interaction yields kinetic temperature profiles reminiscent of those belonging to the NR models, in which the cone-of-acceptance acts as a "throttle". In any event, the roles assigned to the mechanisms of solar radiation pressure and plasmaspheric charge exchange in the conventional picture of exospheric structure need to be reversed: the former acts to create the satellite component while the latter acts to destroy it.

The handling of charge exchange collisions in this study is patterned on the method introduced by Chamberlain
(1977), to which several major modifications have been made. The most significant improvement has been to couch the Boltzmann equation (Eqn. (4.7)) in terms of time (t), avoiding the confusion which led Chamberlain to integrate the Boltzmann collision integral in a dimensionally-reduced phase space. The proper retention of the loss term proportional to \( \Phi \) in Eqn. (4.7) is also important. After all, it is this term which is responsible for the bound component depletion of Figure 4.2 and the corresponding satellite erosion of Figure 4.4. Its inclusion does not influence the escape flux calculations in any major way since, as previously discussed, this term mainly acts to transfer escaping atoms onto hyperbolic fly-by trajectories. Thus, if the loss term had been neglected, as in Chamberlain (1977, in particular his Eqn. (20)), the total escape flux from the planet would not have been substantially different.

Perhaps the most important outcome of the work described in this Chapter is the technique itself. In a "realistic" Monte-Carlo model of the geocorona, Hodges, et al. (1981) also sought to determine the neutral hydrogen velocity distribution with an eye toward the analysis of Lyman-\( \alpha \) line profiles. Although this model took into account charge exchange collisions with plasmaspheric \( H^+ \) and \( O^+ \), photon momentum impulses, and in contrast to this work, thermospheric rotation, winds and non-uniformities, the Earth's shadow, and a realistic plasma temperature profile,
the statistical nature of the results and the rather large bin sizes precluded the distilling out of any firm conclusions. In particular, the results were statistically reliable only out to about four Earth radii so that the geotail phenomenon went entirely unnoticed, nor could any handle on the satellite component per se be gained. The benefits of employing theoretical techniques (e.g., Gaussian summation) ought to be obvious.
CHAPTER 5: Review and Conclusion

Two global facets of the terrestrial exosphere, as it is known today, have provided the impetus for this study. First, satellite measurements of the Earth's Lyman-α airglow intensity have revealed a nightside geotail - i.e., an asymmetric density distribution of hydrogen aligned along the Earth-Sun axis, noticeable beyond geosynchronous radius. Second, mesospheric studies have indicated a rate of escape of hydrogen from the planet several times larger than the rate arising from simple Jeans evaporation. A simple picture of the exosphere, such as that of Chamberlain (1963), cannot account for these features. Subsequent work has isolated the mechanisms that must come into play, namely the effective radiation pressure resulting from resonant scattering of solar Lyman-α photons (Thomas and Bohlin, 1972; Bertaux and Blamont, 1973; Chamberlain, 1979) and charge exchange collisions between geocoronal hydrogen and plasmaspheric protons. Beyond this, though, it must be conceded that a good deal of confusion has made its way into the literature.

This state of affairs centers on the existence and extent of a satellite component to the exosphere - i.e., the family of hydrogen atoms gravitationally bound to the planet possessing (at least temporarily) perigees above the exobase. It has not always been apparent how to extract the
true influences of the complicating mechanisms of radiation pressure and plasmaspheric charge exchange from the observations, yet these must control the satellite component. On the other hand, theoretical efforts in the past have not been capable of incorporating radiation pressure in an appropriate manner. The work of Maher and Tinsley (1977) and Chamberlain (1977) has managed to estimate, each in a plausible way, the effect of the plasmasphere interaction of the escape rate of hydrogen, but only by ignoring the satellite component.

In the present study, exospheric theory has been re-established along the guidelines of Chamberlain's earlier work, but has included radiation pressure in a rigorous fashion. By generalizing the techniques set forth in Chamberlain (1977), vertical profiles out to 20 Earth radii for the density, kinetic temperature, and escape flux of atomic hydrogen above the subsolar and antisolar points have been calculated. These calculations hinge on the discovery, implicit in Chamberlain (1979, 1980), that satellite trajectories intersecting the Earth-Sun line crash into the exobase within a finite time interval; by Liouville's theorem, such trajectories are then populated according to the velocity distribution at the exobase. Radiation pressure accomplishes this by continuously altering the angular momentum - i.e., the "shape" - of the trajectories threading the exosphere traveled by atomic
hydrogen. One manifestation of this torquing by Lyman-α scattering is the establishment of a near-isotropic quasi-Maxwellian kinetic distribution for the bound (ballistic + satellite) component. A second is an increase in the total hydrogen content of the exosphere, via the generation of this nominal satellite component. The geotail itself results from the bending of trajectories toward the nightside, increasing the density of occupied trajectories there, and from the energy-accessibility of such trajectories, since less energy is needed at the exobase to ascend to great distances on the nightside.

The inclusion of a simple plasmasphere that interacts with the geocorona via the mechanism of resonant charge exchange alters the evaporative situation in several ways; however, the geotail survives. As might be expected, the exosphere is heated to a higher gas-kinetic temperature. The satellite component can become severely depleted by plasmaspheric charge exchange collisions, thus feeding the enhanced escape rate. This depletion also brings the dayside density profile into rather good agreement with the satellite observations of Wallace, et al. (1970). Solar ionization mechanisms have been incorporated in a simple way and act mainly to deplete the total density somewhat while leaving the geotail intact. Unfortunately, it has not been possible with the formalism used to study the solar loss and the plasmasphere interaction in a combined fashion, although
it seems evident that in the real geocorona the plasmaspheric charge exchange mechanism outweighs the solar loss in significance.

For future work, two directions can be suggested by the author. The first involves the elucidation of geocoronal structure for arbitrary solar angle. In particular, the line-shape measurements of geocoronally scattered Lyman-\(\alpha\) reported by Bertaux (1978) and noted in the Introduction are along an orientation quite different from that used here. In the present study, local quantities have been evaluated at positions along the Earth-Sun line in the solar and anti-solar directions, whereas Bertaux's OGO-5 reductions are column integrations, the lines-of-sight scanning roughly 90 degrees away from the Earth-Sun line. Preliminary work in this region has revealed an extensive class of trajectories that do not exhibit systematically-lowering perigees. It may be possible to characterize this behaviour as has been done for tightly-bound orbits (Chamberlain, 1979). As concerns the geocorona proper, this class or component likely remains devoid of atomic hydrogen in exospheres acted upon by photoionization due to the extremely long flight times, perhaps resulting in a density depletion relative to the solar and anti-solar directions. The inclusion of the plasmasphere interaction cannot alter this assessment in any major way, since this mechanism tends to act destructively. The resulting exosphere would indeed
have a non-simple shape. Such a structure can be experimentally investigated via careful studies of the line-shape of resonantly scattered Lyman-α. To this end, theoretical line-shapes illustrating the effects of radiation pressure dynamics could be generated for pertinent orientations, as in Chamberlain (1976).

A second direction for future research deals with the strong interaction between the exosphere and the plasmasphere; to date, this has only been studied with regard to the effects on the exosphere. This interaction must have some influence on the plasmasphere, particularly on the energy balance within the plasmasphere. A study of this would entail the construction of a more realistic plasmasphere than has been used here. Further, the question of the constancy of the total escape flux of hydrogen from the Earth over a solar cycle can now be addressed. By sequencing calculations of varying solar conditions, using appropriate topside ionospheric parameters, the escape flux can be computed for the various levels of solar activity (including the escape flux arising from the loss of satellite atoms) and compared to mesospheric model predictions. In this way the evolution of the exobase hydrogen density could be analysed. Thus, these studies would have considerable bearing on unraveling the aeronomy of hydrogen in the thermosphere and on reconstructing the evolution of the Earth's hydrogen inventory.
As a concluding remark, it is hoped that the simplicity of the exosphere has been noted by the reader, though at times this may have been clouded by the formalism and/or techniques employed, or by the style of the author.
APPENDIX I: Chamberlain Formulation

When restricted to situations of spherical symmetry, Liouville's theorem\(^1\) can be directly integrated to obtain the kinetic distribution at radius \(r\) above the critical level \(r_c\), which expressed in terms of canonical momenta is

\[
f(r,p_r,p_\chi) = \frac{N_c}{(2\pi mk T_c)^{3/2}} e^{-\left(\lambda_c - \lambda\right)} e^{-\frac{p_r^2}{2mkT_c}} e^{-\frac{p_\chi^2}{2mkT_c r^2}}, \quad (\text{AI.1a})
\]

a Maxwellian distribution for the gas of constituent mass \(m\) having been assumed at the exobase \((N_c, T_c\) constant). The presence of a gravitational field is accounted for by the barometric factor \(\exp[-(\lambda_c - \lambda)]\), where \(\lambda = \frac{GMm}{rk T_c} = \frac{v_o^2}{U^2}\). When integrating this distribution to obtain the local density, care must be taken to ensure that only particles emanating from the critical level or gravitationally bound to the planet are counted, taking interplanetary space to be otherwise empty. By rewriting the distribution in terms of spherical momentum coordinates -

\[
\psi^2 = \left(\frac{p_r^2}{2mk T_c}\right) + \left(\frac{p_\chi^2}{2mk T_c r^2}\right)
\]

for the (normalized) kinetic energy and

\[\text{--------------------------}\]

\(^1\) Note that in this work Liouville's theorem is taken to be equivalent to the homogeneous Boltzmann equation, in accordance with common usage.
\[ \delta = \tan^{-1} \left( \frac{p_x}{rp_x} \right) \]

for the polar momentum angle with respect to local zenith - the kinetic distribution becomes

\[ f(\lambda, \psi, \mu) = \frac{N_C}{\pi^{3/2}} e^{-(\lambda_C - \lambda)} \exp[-\psi^2]. \]  \tag{AI.1b}

One can now write an expression for the total density -

\[ N(\lambda) = N_C e^{-(\lambda_C - \lambda)} \Xi(\lambda), \]  \tag{AI.2}

\[ \Xi(\lambda) = \left( 2/\pi^{1/2} \right) \int e^{-\psi^2} \psi^2 \, d\psi d\mu, \]

where \( \mu = \cos \delta \). The integration limits for the partition function \( \Xi(\lambda) \) are restricted by dynamical considerations; when no restrictions are imposed \( \Xi = 1 \) and the barometric law is obtained. Introducing the concept of the cone-of-acceptance \( \mu^0 \) which separates those trajectories that intersect the critical level or exobase from those that possess angular momentum large enough to have perigee above the exobase -

\[ \mu^0 = \left[ 1 - \left( \frac{\lambda}{\lambda_C} \right)^2 \left( 1 + \frac{(\lambda_C - \lambda)}{\psi^2} \right)^{1/2} \right], \quad \psi > \psi^t \]  \tag{AI.3}

\[ = 0, \quad \psi < \psi^t, \]

where \( \psi^t = \lambda/\sqrt{(\lambda_C + \lambda)} \) is the maximum speed that still intersects the exobase irrespective of direction of motion - one can define component partition functions that provide
the densities of the ballistic, bound, and escape components in terms of the incomplete gamma function \( \gamma(a,b) \). (Expressions for these quantities are not needed here and may be found in Chamberlain (1978).) The ballistic and escape portions are straightforward as they both arise from the exobase and are bounded by the cone-of-acceptance \( \mu^0 \) and by the minimum escape speed \( \psi^0 = \sqrt{\lambda} \). The main uncertainty is the extent to which the satellite component is populated. In the pure gravitational theory the only mechanism for populating this component is trajectory transferral via rare elastic collisions. In the absence of perturbing factors, those satellite trajectories that are so populated continue to gain particles until a balance with collisional depopulation is reached. Such a satellite component thus exhibits the same Maxwellian property as exhibited by Eqns. (AI.1) and together with the ballistic component comprises the bound component mentioned above.

As the geocorona is subject to perturbing influences (e.g., photoionization acting as a loss mechanism), such long-lived satellite atoms must be depleted to some extent. To account for this, Chamberlain introduced the concept of a satellite critical radius: those satellite trajectories possessing perigees below this radius are occupied according to the same kinetic distribution as characterizes the ballistic and escape components while the remaining trajectories are empty, thus mimicking in a simple way the
increased significance of the loss mechanisms with altitude. The Chamberlain model has five parameters:

\[ \begin{align*}
    m & \quad \text{exospheric constituent mass} \\
    N_C & \quad \text{density at the exobase} \\
    T_C & \quad \text{temperature at the exobase} \\
    r_C & \quad \text{exobase radius} \\
    r_{CS} & \quad \text{satellite critical radius}
\end{align*} \]

The first four are established by thermospheric processes and can be determined by \textit{in situ} measurements or by theoretical considerations. The last has often been handled as an empirical parameter, its value being fixed through the analysis of photometric observations.

The Jeans flux in this notation is

\[ F(\lambda) = \frac{N_C e^{-\lambda C}}{2\pi^{3/2}} U(\lambda C + 1)(\lambda^2/\lambda C^2), \quad \text{(AI.4)} \]

accounting for geometric spreading. Two other quantities needed for the present study are the kinetic temperature

\[ T_{\text{kin}} = (2/3) T_C [\langle \psi^2 \rangle - \sum_i (\langle v_i^2 \rangle / U^2)], \quad \text{(AI.5)} \]

the angular brackets denoting averages and the summation extending over the linear velocity components, and the column density above the exobase (zenith-looking)

\[ N_{\text{COL}}(\lambda_C) = \int_{r_C}^\infty N(r) \, dr - (GMm/kT_C) \int_{\lambda_C}^{\lambda_P} N(\lambda) / \lambda^2 \, d\lambda, \quad \text{(AI.6)} \]

placing a lid on the exososphere at a distance \( r_P \).
APPENDIX II: Radiation Pressure Acceleration

The phase function for Lyman-α scattering by atomic hydrogen is of the form \( I(\theta) = A - B \sin^2 \theta \), where \( \theta \) is the angle between the directions traveled by the incident and scattered photons. Since this phase function is an even function in \( \theta \), the effect of a scattering event (averaged over many events) will be to transfer the photon momentum to the hydrogen atom, the net velocity of which will then increase in the antisolar direction by an amount \( \Delta v \):

\[
\Delta v = h\nu/cm_H = h/\lambda m_H = 3.257 \times 10^2 \text{ cm/sec.}
\]

The average number of photons scattered per atom per second, called the \( g \)-factor, must be evaluated. The solar Lyman-α profile is taken to be flat at line center and broad enough to envelop the geocoronal absorption core. Then, for each geocoronal atom (neglecting attenuation),

\[
g = \int dv \pi F_0 \sigma_v = \pi F_0 \int dv \sigma_v = \pi F_0 \sigma_{12},
\]

where \( \pi F_0 \) is the Lyman-α flux at the solar emission line center (in photons/cm\(^2\)-sec-Hz). The absorption cross-section \( \sigma_{12} \), integrated over the natural line profile, is here taken to be \( 1.103 \times 10^{-2} \) cm\(^2\)-Hz. \( \pi F_0 \) is commonly quoted in units of photons/cm\(^2\)-sec-A so one must convert

---

1 Refer to Bertaux and Blamont (1973) and Vidal-Madjar, et al. (1973).
between Angstroms (Å) and frequency: \( \Delta \nu = (c/\lambda^2) \Delta \lambda \), implying 1 Å = 2.028 \( \times \) 10^{12} Hz at Lyman-\( \alpha \) (\( \lambda = 1215.67 \) Å). Combining, the net average acceleration is

\[
a = \Delta \nu g = 1.774 \times 10^{-12} \pi F_0 \text{ cm/sec}^2,
\]

for \( \pi F_0 \) expressed in photons/cm²-sec-Å. For the epoch selected in this study (1969-1970), \( \pi F_0 \) lies in the range of 4 to 5 \( \times \) 10^{11} photons/cm²-sec-Å, so the value of the radiation pressure acceleration has been set equal to 0.75 cm/sec².
APPENDIX III: Numerical Procedure

This Appendix contains a brief but fairly explicit description of the numerical techniques employed in the calculations of this study. The order of exposition parallels the discussion of Chapter Two: the technique of integrating individual trajectories is outlined first, then the construction of the dynamically appropriate phase space regions, and lastly the calculation of densities. This Appendix concludes with some comments on calculating the plasmaspheric enhancement factor $\phi$ and the column densities.

TRAJECTORY INTEGRATION

The backbone of the computational framework is the evaluation of trajectories as represented by the set of coupled partial differential equations Eqns. (2.7)-(2.8). To maintain a specified degree of precision, a fourth-order Runge-Kutta (RK) integrator has proven adequate, when refined to permit self-correction. The correction scheme is quite simple. After completing an integration step, the routine returns to the step initial point, halves the stepsize and via a linked two-step RK computation recrosses the original increment; then the routine compares the results for the trajectory variables of the two trials to ascertain whether the required precision has been attained.
Specifically, the logic of the step routine requires of the results

\[ | x_1(\tau + \Delta \tau) - x_2(\tau + \Delta \tau) | < 10^{-6} \]
\[ | u_1(\tau + \Delta \tau) - u_2(\tau + \Delta \tau) | < 10^{-6} \] (AIII.1)

where the subscript indicates the number of steps used to span the trajectory increment. If both conditions are satisfied, the routine proceeds to the next increment. If not, the first half-step is itself treated as the basic step and is halved for comparison, this procedure repeating until a stepsize guaranteeing satisfaction of conditions (AIII.1) is found. It is this capability of self-correction which lead to the selection of the RK technique over the more common choice of a predictor-corrector method. The quantities \( x \) and \( u \) were selected as being the most significant for the precision tests by reason of the rapid variations in these quantities that occur near trajectory turning points (e.g., apogee) in the latter case and near the coordinate axis (Earth-Sun line) in the former.

The endpoints of a trajectory, when propagated backward and forward in time from the specified initial point in the exosphere, are the points of intersection with either the exobase \( \lambda_c \) or the exopause \( \lambda_p \). Since such boundaries are described by a simple radial position, it is somewhat difficult to obtain the intersection points precisely when time \( \tau \) is taken as the independent
parameter. To remain consistent with the idealization of a distinct exobase, which serves as the source for populating the trajectories in the evaporative model, an artifact was spliced onto the RK integrator—namely, the introduction of a spherical shell lying just above the exobase within which radiation pressure is ignored. This shell will here be called the Kepler shell. In the model, it extends from the exobase to an altitude of √800 kilometers. When a trajectory passes into the Kepler shell, it is continued on to the exobase via the Keplerian algebraic relations. Considering the relative strength of the gravitational acceleration to the unattenuated radiational acceleration in the shell (√0.090%) and the mean occupancy time in this shell (√100 seconds), the neglect is certainly justifiable. The exopause crossing point need not be determined so precisely and it has been quite simple to estimate the intersection using constant force formulae, since both the curvature and strength of the gravitational field vary negligibly over a stepsize at the exopause radius (36.20 Earth radii).

The viability of the method described above was tested in a couple of ways. First, since the component of angular momentum canonical to azimuthal position is a constant of the motion and is expressable as a function of all the dynamically important variables of the motion, as seen in Eqn. (2.9), it can serve as a monitor of the accumulation of
integration errors. The deviation of the angular constant tends to increase as flight-times increase but in all cases the variation remained at or below the $5 \times 10^{-6}$ level. Second, an obvious test is step-by-step comparison with the Kepler theory in the no-radiation (NR) case along an entire trajectory. Such comparisons were amazingly good — amazing in that when a deviation between the trajectory parameters as computed by the two techniques arose, as near turning points where angular parameter values varied most rapidly, the deviation would dampen out quickly and the values would again become equal (to the six significant digit level of monitoring).

The accumulation of error along a trajectory integration can be estimated in a simple way. By having invoked conditions (AI.1.1) in the RK span of a trajectory step, the disagreement between $\chi_1$ and $\chi_2$ must be roughly equal to the step error, since $\chi_2$ has an intrinsic error smaller than that accruing to $\chi_1$ by a factor of $\sqrt{16}$ (error per step scales as $(\text{stepsize})^5$). Since the step error can be either positive or negative and apparently has about equal tendency in either direction (as is perhaps indicated by the NR comparison described above), the error accumulation is somewhat like a random walk; the net accumulated deviation or error is thus roughly

$$\Delta(\text{error}) = \text{RMS}(\text{step errors})$$
$$= (\text{step error}) \sqrt{\text{(number of steps)}}.$$
For example, for a trajectory requiring $10^4$ steps, the accumulated error is likely to be on the order of $10^{-4}$, assuming a step error of $\sqrt[4]{10^{-6}}$. The actual error accumulation must, of course, increase in some manner as stepsize increases. The stepsize $\Delta \tau$ utilized in the initial RK span of an increment had the form

$$\Delta \tau = K \times \psi / \lambda^2$$

so as to expand or contract along the trajectory in an efficient fashion. The selection of the coefficient $K$ was based on the desire to balance the two basic conflicting criteria of numerical integration - i.e., efficiency (to take as few steps as possible, which in this application means the avoidance of multiple increment crossings) and accuracy (to take small steps). Tests indicated $K = 0.10$ to be a good choice. As an example, for marginal escape in the NR case where $\psi^0 = \sqrt[4]{\lambda}$, the stepsize $\Delta \tau$ as given by the above expression has the value $0.0055$ at the exobase, corresponding to a step pathlength of roughly 600 km while at 10 Earth radii the stepsize has increased to $0.156$ and the step pathlength to $\sqrt[4]{6000}$ km.

PHASE SPACE BOUNDARIES

In the application, it is necessary to isolate the several distinct trajectory classes. This is both to allow
study of the corresponding component densities and to help ensure numerical accuracy when integrating the non-smooth distribution function over momentum space. The specification of the appropriate boundaries is not as straightforward as in the no-radiation case, particularly in regard to the minimum escape velocity $\psi^0$. Also, the cone-of-acceptance $\mu^0$ which contains the occupied trajectories in the NR case is not independent of the direction of motion and must be determined numerically.

The cone-of-acceptance $\mu^0$ serves to separate those trajectories that directly connect to the exobase in at least one direction (i.e., without an intervening turning point, at which $\mu$ changes sign), which are nominally "ballistic" trajectories when bound, from those trajectories which possess turning points in both directions, which are the nominal "satellite" trajectories. In the presence of radiation pressure, the cone-of-acceptance is distorted and depends in a complicated way on both position ($\chi$) and orientation ($\varepsilon$), as well as on energy ($\lambda, \psi$). The way used to determine $\mu^0(\lambda, \chi, \psi, \varepsilon)$ in the application was via an iterative search, taking the no-radiation value $\mu^0(NR)$ as the initial guess. Propagating the trajectory back toward the exobase, having taken $\mu^0(initial) > 0$, upon reaching the Kepler shell the resulting $\mu^0(shell)$ was compared to the $\mu^0(NR, shell)$ value, which is known from the Chamberlain theory. Unless the value $\mu^0(shell)$ satisfied the condition
\[ | \mu^0(\text{shell}) - \mu^0(\text{NR,shell}) | < 10^{-6}, \quad (AIII.2) \]

A correction was computed, to increase (decrease) \( \mu^0(\text{initial}) \) according as \( \mu^0(\text{shell}) \) was too small (large). In the event that a turning point was encountered prior to reaching the Kepler shell, the \( \mu^0(\text{initial}) \) value was increased until intersection occurred. In practice, the iteration was quick.

The criterion for escape is the more difficult boundary to establish. In the NR case, one has simply \( \psi^0 = \sqrt{\lambda} \), where \( \psi^0 \) is the minimum escape speed. This criterion must be replaced, as some negative energy (\( h < 0 \)) trajectories escape to infinity on the nightside of the planet. Exopause (\( \lambda_p \)) passage is an intuitively appealing alternative and in practice proved to be effective.

The minimum escape energy, as it is so greatly influenced by radiation pressure, is inherently dependent on location and position - \( \psi^0 = \psi^0(\lambda,\chi,\mu,\epsilon) \). As when computing the distorted cone-of-acceptance \( \mu^0 \), the minimum escape energy \( \psi^0 \) for specified position and orientation can be found by iteration. The scheme employed was not as straightforward as that used for finding \( \mu^0 \) for the simple reason that a comparison quantity analogous to \( \mu^0(\text{NR}) \) is not available. Also, the integration paths can be much longer. In the scheme, an initial guess (\( \psi^0 = \sqrt{\lambda} \) is convenient) along with the specified variables (\( \lambda,\chi,\mu,\epsilon \)) defined a trajectory that was propagated both backward and
forward in time until (a) exosbase intersection or (b) exopause intersection. If possibility (a) occurred in both the past and future, the initial guess was increased, otherwise the initial guess was decreased; in each case the correction was large enough to ensure over-correction, thus bracketing the true value between upper and lower bounds. The initial correction was subsequently halved (for example), and reapplied in the next iteration in such a way as to continually refine the upper and lower bounds. (In the actual calculations, a slightly more elaborate scheme for determining the correction was employed.) When the correction satisfied the condition

$$| \Delta \psi | < 10^{-4}, \quad (AIII.3)$$

the iteration stopped and the current minimum energy was taken as the true $$\psi^0(\lambda, \chi, \mu, \epsilon)$$. In practice, the evaluation of $$\psi^0$$, according to the schedule described below, was a CPU-expensive step.

The point of intersection of the $$\psi^0$$ and $$\mu^0$$ curves, called the partition point $$(\mu^{00}, \psi^{00})$$, serves as a sort of critical point between the dynamical regions and must be found at the outset. It is that trajectory (for specified $$(\lambda, \chi, \phi, \epsilon)$$) which barely reaches the exopause in one direction and grazes the exobase in the opposite direction - i.e.,

$$(\mu^{00}, \psi^{00}) = (\mu^0(\psi^0), \psi^0(\mu^0)).$$
Once known, one can then, for example, span the range \([\mu^0,1]\) to find \(\psi^0(\mu)\) as appropriate for the ballistic-escape demarcation, or span \([\psi^1,\psi^0]\) to obtain \(\mu^0(\psi)\) separating the bound satellite and ballistic regimes. The partition point \((\mu^0,\psi^0)\) has been located for each selected position by nested iteration for \(\psi^0\) and \(\mu^0\) using the schemes described above (\(\mu^0\) being found at the start of each iteration for \(\psi^0\)).

Below a certain energy all trajectories arise from the exobase. Due to the action of radiation pressure, this energy \(\psi^1\) depends on location and orientation - \(\psi^1(\lambda,\chi,\varepsilon)\) - and can be characterised as the maximum energy for which \(\mu^0 = 0\). This quantity has also been found for each position of interest in the exosphere, using an iterative search patterned after that used for finding \(\mu^0\). In the NR case, all trajectories are ballistic below this energy \(\psi^1\), which then has the value \(\lambda/\sqrt{(\lambda_C+\lambda)}\), and by analogy all such trajectories in the present model are grouped with the nominal ballistic class. Thus \(\psi^1\) serves as a lower energy bound for the satellite class.

A final energy boundary is the maximum energy considered in the model. As the integrals for density, kinetic temperature, and escape flux formally include trajectories with energies approaching infinity, it is necessary to select a maximum energy \(\psi^2\) that effectively includes these high energy contributions. Testing to be
described below indicated \( \psi^2 = \sqrt{16 + \lambda} \) to be a valid choice.

Ignored in this study are those escaping trajectories that exhibit an intervening turning point between the exobase source and the selected local position – i.e., escaping trajectories are assumed to remain within the cone-of-acceptance. As the escaping trajectories are characterized by relatively fast speeds, such bends are rarely if ever present in the altitude range considered (and even then only at the lowest escaping speeds).

In summary, the phase space regions that are taken to be populated in the evaporative model are as follows:

\[
\begin{align*}
0 & < \psi < \psi^1 \quad -1 < \mu < 1 \quad \text{ISOTROPIC BALLISTIC} \\
\psi^1 & < \psi < \psi^0 \quad \mu^0 < \mu < 1 \quad \text{RESTRICTED BALLISTIC} \\
\quad & < \mu < -\mu^0 \\
\psi^1 & < \psi < \psi^0 \quad -\mu^0 < \mu < \mu^0 \quad \text{SATELLITE} \\
\psi^0 & < \psi < \psi^2 \quad \mu^0 < \mu < 1 \quad \text{ESCAPE}
\end{align*}
\]

THE GAUSSIAN NET

The trajectories actually integrated in these calculations are those specified by the summation schemes employed in the evaluation of quantities like density (Eqn.(2.12)), once the point of evaluation in the exosphere has been chosen. It will be assumed in the remainder of this section that such a point \((\lambda, \chi, \phi)\) along the Earth-Sun line has been selected. Also, for simplicity attention is
here restricted to the evaporative case. (It ought to be noted that the trajectories needed cannot be numerically integrated at points exactly on the coordinate axis since in this formalism the azimuthal variables $\phi$ and $\epsilon$ become undefined. The trajectory calculations were actually carried out at $\chi$-orientations of $0.01\pi$ and $0.99\pi$ on the evening side of the planet ($\phi=0$). It was in the interest of maintaining an ability to study trajectories of arbitrary initial orientation and to evaluate exospheric quantities at points away from the Earth-Sun line that the general spherical formalism has been developed.)

In the application, the sources of hydrogen by which the trajectories are populated are symmetric about the coordinate axis, as is the action of solar radiation pressure. Hence the $\epsilon$-integral simply reduces to a factor of $2\pi$, and a convenient value for $\epsilon$ may be assigned; in the computations the choice for $\epsilon$ was $\pi/2$.

In expressions like Eqn. (2.12), the $\mu$-integral and $\psi$-integral are each partitioned according to regions as already described. For each dynamical range, the corresponding $\mu$-integral was evaluated by an eight-point Gauss-Legendre summation. Likewise, the $\psi$-integrals were converted to eight-point Gaussian summations with the exception of the isotropic ballistic region which only required four-point summation. Since the integration limits on the $\mu$- and $\psi$-integrals are so closely interdependent,
separate (nested) summations are not feasible, and a coupled summation scheme has been formulated. Trajectories were selected for evaluation by a procedure here called a Gaussian net:

Having determined the partition point \((\mu^0, \psi^0)\) for the position of interest \((\lambda, \chi)\) along with the corresponding \(\psi^1\), the quantities \(\mu^0\) and \(\psi^0\) were determined by the iteration schemes described earlier. \(\mu^0(\psi)\) was found for the Gaussian-selected \(\psi\)-values in the intervals \([\psi^1, \psi^0]\) and \([\psi^0, \psi^2]\); the resulting values are denoted \(\mu^0(i)\), for the \(i\)-th Gaussian \(\psi\)-value \(\psi(i)\). Similarly, \(\psi^0(\mu)\) was found at the Gaussian points in the intervals \([0, \mu^0]\) and \([\mu^0, 1]\); the results are written \(\psi^0(j)\) for the \(j\)-th \(\mu\)-value \(\mu(j)\). These coordinate pairs \((\mu^0(i), \psi(i))\) and \((\mu(j), \psi^0(j))\) constitute the boundaries illustrated in Figures 2.2a and 2.2b. Once the boundaries are established, the coupled \((\mu, \psi)\) summation scheme defines the summation trajectories, which comprise or "weave" the "net". An explicit example of net construction is here provided for the restricted ballistic, locally ascending evaporative component. With the boundaries

\[
\mu^0(i) = \mu^0(\psi(i)), \quad (AIII.4a)
\]

\[
\psi(i) = (1/2)[(\psi^0 - \psi^1) \times r(i) + \psi^0 + \psi^1], \quad i = 1, 8
\]

and

\[
\psi^0(j) = \psi^0(\mu(j)), \quad (AIII.4b)
\]

\[
\mu(j) = (1/2)[(1 - \mu^0) \times r(j) + 1 + \mu^0], \quad j = 1, 8,
\]
the coupled double integral

\[ \int_{\mu_0}^{1} \int_{\psi_1}^{\psi_0} \psi^2 \ f(\lambda, \chi, \psi, \mu, \varepsilon) \]

becomes

\[ \sum_j \sum_i \Delta(\mu(ij)) \Delta(\psi(ij)) \ psi(ij)^2 \ f(\lambda, \chi, \psi(ij), \mu(ij), \varepsilon) \] (AIII.5)

where the net points and weights are

\[ \psi(ij) = \frac{1}{2} [(\psi^0(j) - \psi^1) \times r(i) + \psi^0(j) + \psi^1], \quad i,j = 1,8 \]

\[ \mu(ij) = \frac{1}{2} [(1 - \mu^0(i)) \times r(j) + 1 + \mu^0(i)], \quad i,j = 1,8 \]

\[ \Delta(\psi(ij)) = \left( \frac{1}{2} \right) \times w(i) \times (\psi^0(j) - \psi^1) \]

\[ \Delta(\mu(ij)) = \left( \frac{1}{2} \right) \times w(j) \times (1 - \mu^0(i)). \]

The \( r(i) \), \( w(i) \), \( i = 1,8 \), are the Gaussian points and weights, respectively, for the interval \([-1,+1]\). The locally descending component \((\mu < 0)\) is best handled by following the ascending trajectory to the post-history endpoint and there invoking time-reversal symmetry - i.e., \( \varepsilon \rightarrow -\varepsilon - \pi \) and \( \mu \rightarrow -\mu \). Similar summations can be written for the other occupied phase space regions; in all cases, the ascent component was explicitly evaluated and the descent portion accounted for by following the corresponding ascent trajectories to their future conclusions, where the reversed bound component trajectories are populated according to the
FIGURE AIII.1: A simplified (4 x 4) Gaussian net for the NR (Keplerian) satellite component. The indicated points (u,ψ) specify the trajectories needed for performing the double summation in Eqn. (AIII.5) appropriate for this component. (Altitude - 10.00 Earth radii)
exobase Maxwellian kinetic distribution. Thus the locally descending trajectories were not spanned separately, and the entire \( \mu \)-range explicitly spanned was \([0,1]\). Figure (AIII.1) illustrates the satellite component net for a simplified case.

Once the net has been constructed, it can be called out of computer memory to be consumed in the summations for density (Eqn. (2.12)). Note that the quantities so stored are the current position \((\lambda, \chi, \phi)\), the attaching net points \((\psi(ij), \mu(ij), \varepsilon)\), and the past- and post-history boundary intersection points. The net computed for the density calculations was also used in evaluating the integrals for escape flux (Eqn. (2.13)) and kinetic temperature (Eqn. (2.14)).

The test which can be applied to the Gaussian net construction, to ascertain the reliability of the convergence, again relates to the NR case. By comparison to the Chamberlain theory (uniform exobase conditions), the number of Gaussian points needed to adequately span the various kinetic energy \((\psi)\) intervals were established. This comparison also provided the value of the maximum energy \(\psi^2\) which needed to be considered. The accuracy of the summation approximations, according to this test, is roughly six significant digits. The \(\mu\)-integral is much harder to nail down. In the Chamberlain theory, this integral is performed automatically and so a more elaborate comparison
model must be used. An appropriate choice (and indeed, the only one the author could think of) is an exosphere which is generated by evaporation from a non-uniform exobase - i.e., an exobase with spatially varying temperature and density. Such a model can be constructed in a fashion quite similar to that used in the radiation pressure model, with the simplification that trajectories are immediately evaluated via the Keplerian relations. The phase space boundaries, being independent of the manner by which the trajectories are populated, are simply those appropriate for the NR case. Since there are no symmetries left to the problem (even the $\epsilon$-integral must be numerically evaluated - a Simpson arc summation (six steps per hemisphere) worked quite well) - attaining convergence in this model is a stringent test for the ballistic and escape density components. This is particularly so in that the exobase conditions can be varied arbitrarily. The minimum number of Gaussian points that provided convergence to five significant figures were subsequently used and have been given above. Unfortunately, this model comparison cannot indicate the best way to evaluate the satellite component quantities. For simplicity (and cost considerations) the satellite coverage was the same as the restricted ballistic coverage - i.e., using eight-point Gaussian spans for both $\psi$ and $\mu$. Some confidence is provided by the observation that at low altitudes, where the ballistic component is dominant ($\mu^0 =$
0), the ballistic summation worked as well as at high altitudes. Also, the fact that for the ballistic and satellite components descending parcels (on a specified trajectory) are nearly mirror images of the ascending counterparts in the simple evaporative model, while this is not the case in the NR non-uniform exobase model (or when solar ionization or the plasmasphere interaction are included), points to the theoretical simplicity of the evaporative satellite component - i.e., no neurotic behavior is expected. This simplicity is also apparent in Eqn. (2.11) and in the simple-minded model results of Chapter 3. However, it would be best to re-evaluate the satellite component with a sixteen-point Gaussian summation over \( \mu \) for direct comparison with the present results.

MISCELLANEOUS

Two matters are discussed here: the evaluation of column densities and the incorporation of the charge-exchange enhancement factor \( \phi \). The desire to compute column densities above the sub-solar and anti-solar exobase points dictated the selection of the altitudes at which densities were calculated. In the Chamberlain model, with a ceiling placed on the exosphere at 36.20 Earth radii, proper convergence required a sixteen point Gaussian summation for evaluation of the column density integral Eqn. (AI.6).
Hence, the radii of evaluation were these Gaussian-selected altitudes. (More properly, the calculations for density, etc., were carried out at the 16 Gaussian points for $\lambda$ in the range $[\lambda_p, \lambda_C]$.) In addition, several standard radii such as geosynchronous radius ($g_s$) were selected for evaluation.

The inclusion of a plasmasphere interaction via the Boltzmann equation (Eqn. (4.1)) necessitated a re-integration of the summation trajectories taking the previously evaluated endpoints, now placed in the past, as the initial points. The quantity $d\phi$ was evaluated for each RK step with a Runge-Kutta technique incorporating the loss term. Since the accumulated quantity $\phi$ increased dramatically (became negative) for fast (slow) trajectories in the plasmasphere, the stepsize coefficient $K$ was decreased to 0.01 to ensure convergence. It was also necessary to include in the summations a fast escape component, since high velocity trajectories that were negligibly populated in the evaporative case could now become appreciably populated (Figure 4.1). As commented upon in the text, these fast escape trajectories were assumed to lie within the distorted cone-of-acceptance. The maximum energy ceiling for this component was $\psi_{\text{max}} = \sqrt{(36+\lambda)}$, established by comparison to the NR plasmasphere interaction case of Figure 4.1.
REFERENCES


