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AN OPTIMAL CONTROL APPROACH TO THE TAXATION OF EXHAUSTIBLE RESOURCES

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RICE UNIVERSITY

AN OPTIMAL CONTROL APPROACH TO THE TAXATION
OF EXHAUSTIBLE RESOURCES

by

MINE KUBAN YÜCEL

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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APPROVED, THESIS COMMITTEE:

Peter Mieszkowski, Professor
of Economics, Chairman

Richard D. Young, Professor
of Economics and Mathematical
Science

Richard J. Stoll, Assistant
Professor of Political Science

HOUSTON, TEXAS

DECEMBER, 1983
To my Mother and Father
ABSTRACT

AN OPTIMAL CONTROL APPROACH TO THE TAXATION OF EXHAUSTIBLE RESOURCES

by

Mine Kuban Yucel

The effects of taxes on the behavior of a profit maximizing firm which explores and produces an exhaustible resource, are analyzed in an optimal control framework. Specifically, a severance tax, a royalty, a profit tax and various partial expensing provisions are considered for competitive and monopolistic market structures. The producer's response to these taxes is studied and the deadweight losses from the taxes are calculated. The case of extraction from a fixed reserve base and a variable reserve base are taken separately but the study concentrates on the simultaneous nature of the extraction and exploration decisions. Time paths of exploratory effort, extraction and prices are computed.

Two formulations are developed. The first is based on a model of Pindyck. The second formulation assumes a Cobb-Douglas production function for exploration and the current extraction decisions.

It is shown that, in this dynamic partial equilibrium process, severance taxes and royalties induce the producer to reduce production and exploration and cause a shift in
the time path for prices, similar to the static results. This shift in price, quantity and exploration paths is more pronounced in the competitive case. The profit tax, with complete expensing of costs, is neutral.

The deadweight losses associated with the taxes under consideration are very low, especially for low tax rates, in competitive market structures. Higher deadweight losses for monopoly are shown to result from sharper reductions in producer profits rather than higher consumer losses.
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NOMENCLATURE

R  Level of reserves
q  Quantity produced
p  Price
P_1  Exogenous price of extractive effort
P_2  Exogenous price of exploratory effort
w  Level of exploratory effort
x  Level of accumulated reserves
tx  Severance tax rate
tr  Royalty rate
tp  Profit tax rate
\xi  Fraction of extraction costs that cannot be expensed
\mu  Fraction of exploration costs that cannot be expensed
\gamma  Fraction of royalties that cannot be expensed
\alpha  Production function parameter
\beta_1, \beta_2  Exploration function parameters
I. INTRODUCTION

An exhaustible resource is one which cannot be regenerated and whose continued use will result in the depletion of the resource. The allocation of exhaustible resources over time, therefore, is an important problem for us today and for future generations. If the resource market is competitive, it will be efficient in the standard sense. However, competitive pricing and production may lead to premature depletion of the resource. A central planner may then try to alter the production path for the resource through taxation. In the case of market imperfections, taxes can again be used to correct these distortions. As Meade (1975) has observed, taxes should be considered as an instrument for controlling the extraction and exploration of exhaustible resources. However, the tax must be levied so as to have a minimum burden on society.

The optimal extraction rate of a natural resource has been analyzed by economists since Hotelling's seminal article "The Economics of Natural Resources" in 1931. In this article he discusses the optimal price and extraction paths of the resource in perfectly competitive and monop-
poly market structures. The optimal price paths maximize the discounted present value of profits for the respective producers. Hotelling shows that the competitive producer will either produce zero quantity or some maximum amount depending on whether the rate of growth of prices is equal to the rate of interest. At the margin, producers will be indifferent between current and future production when the rate of growth of prices is equal to the rate of interest. This is the Hotelling rule.

For monopoly, the rate of growth in marginal revenue will be equal to the rate of interest. The rate of change in prices depends on the market demand curve. When the elasticity of demand increases as prices increase, we get the expected result that the optimal solution for the monopolist has a lower quantity and higher price than the competitive producer.

Pindyck (1978) has extended the Hotelling model to incorporate exploratory effort. He analyzes an optimal control problem where the quantity produced and the level of exploratory effort are control variables and the level of reserves and the level of accumulated reserves from exploration are the state variables. Price is a parameter for the competitive producer but becomes a control variable for the monopolist. Pindyck calculates price,
quantity and exploration paths for both types of producer. Exploration is undertaken to renew the depleted reserve base and to bring extraction costs down, since the extraction cost is an explicit and inverse function of the level of reserves and there is no differentiation between old and new reserves. Pindyck's analysis concludes that the pattern of production and exploratory effort will depend on the value of initial reserves. If the initial reserves are small, the price path will be U-shaped; if they are large, price will increase as the reserve base is drawn down until it becomes no longer profitable to produce. As in the Hotelling study, Pindyck's results reveal that a monopolistic producer extracts and explores at a lower rate than the competitor.

In models with uncertainty, exploration also serves the purpose of providing information about other recoverable deposits. This is an externality which has been discussed by many investigators such as Gaffney (1967), Peterson (1975), Stiglitz (1975) and Gilbert (1979).

The effect of taxes on an extractive firm have been studied as early as 1914 by Gray and also by Hotelling in his 1931 article. Hotelling's basic model has been used to analyze various tax schemes by Dasgupta and Heal (1978) and by Dasgupta, Heal and Stiglitz (1980) in an
intertemporal context. In their models, a producer will extract from a fixed stock of a resource in a way that equalizes the present value of net profits earned in each period of time. The cost of extraction is assumed to be constant. Their primary conclusions are as follows.

First, a constant profit tax will not have distortionary effects on prices and quantities. This is because both the present value of the tax liabilities and net profits are constant. A profit tax that rises over time will lead to production being shifted from the future towards the present, since the present value of the tax now increases with time. Second, a production tax or sales tax, imposed at a constant rate, will be distortionary as it will induce the producers to decrease production, thus raising prices. Hence, this tax will conserve the resource. The explanation of this result is as follows. Since net price grows at the rate of interest, sale price grows at a lower rate. The tax is based on the sale price of the resource. Hence tax liabilities fall through time, leading the firm to postpone production to the future. Royalties are shown to have the same effect as the sales and production taxes.

The results of Conrad and Hool (1980, 1981) are consistent with the findings of Dasgupta, Heal and Stiglitz.
The Conrad and Hool study makes allowances for a per-unit tax on output, an ad valorem severance tax and a per-unit tax on ore. The study also makes allowances for various grades of ore. They conclude that the per-unit taxes will distort production away from the present while the effects of an ad valorem tax are inconclusive. Since the tax liabilities are dependent upon output price, the ad valorem tax will have the same effects as a per-unit tax if price rises by less than the interest rate and vice versa for the opposite case, with no effect if price rises at the rate of interest. The taxes will also have different effects on the grade selection profile and cut-off grades.

Uhler (1979) is one of the studies that analyzes the effects of taxation with a model that includes exploration. His model is developed in two stages. He first examines the optimal path of exploration where the amount of exploration adds to geological information about future reserves, but also reduces the stock of undiscovered reserves since it results in discoveries. Uhler then analyzes extraction from a known reservoir, but integrates the physical behavior of the reservoir into his model. He finds that the optimal extraction path is dependent on pressure conditions in the reservoir. He then considers
the effects of royalty and profit taxes on exploration and extraction. His conclusion is that royalties and profit taxes have no effect on extraction rates as long as the optimal extraction rate is governed by well pressure. If investment in pressure maintenance is allowed, then both types of taxes lead to lower extraction rates because both decrease the marginal value of pressure and the investment pressure maintenance. Royalties and profit taxes will affect exploration negatively, since they lower the price of reserves in the ground. Profit taxes will have no effect on exploration however, if exploration costs are allowed as a deduction.

The review of the literature shows that there is a general consensus on the effects of certain taxes on extraction and exploration decisions. However, quantitative analyses of the effects of such taxes and the excess burden associated with them is not very common. To the best of my knowledge, the only study that deals numerically with the effects of taxes on energy and calculates social benefits or costs associated with the taxes is by Weimar and Blankenship (1980). Specifically, Weimar and Blankenship consider a windfall profits tax that is phased out over time and the deregulation of oil and gas. They consider an oil producing firm within a forty year time horizon which takes the world price of oil as given.
Their measure of social value is the present value of gross profits which includes the windfall profits tax and the corporate profits tax. This measure is based on the assumption that world prices are fixed and a barrel of domestic production displaces a barrel of imported oil. Their basic conclusions are: first, the windfall profits tax has the effect of postponing production to the future, specifically until the tax is fully phased out; and second, the revenue from the tax is not a monotonic function of the tax rate. Windfall profit tax rates of thirty percent and seventy percent yield approximately the same present value of tax revenue. In fact, if the tax is decreased from seventy percent to zero, the increase in the measure of social value is approximately twice the decrease in the present value of tax revenues. These findings signify that the losses associated with such a windfall profits tax are quite high.

Two other studies that investigate numerically the losses associated with taxes are by Shoven and Whalley (1983) and Hausman (1981). Shoven and Whalley analyze a general equilibrium model with two final goods (manufacturing and non-manufacturing), two factors of production (labor and capital), and two classes of consumers. They utilize a CES production function for each good with consumer demands derived from the maximization of a CES
utility function subject to a budget constraint. Their measure of social welfare is the Hicksian Compensating and Equivalent Variations. Shoven and Whalley find that a fifty percent tax on capital will give rise to very large losses. Welfare loss brought about by the tax is .62 percent when expressed as percentage of national income and 25 percent when expressed as percentage of tax revenues. The marginal deadweight loss is 79 cents for each dollar of tax revenue.

The Hausman (1981a) study also reports fairly large deadweight loss figures, in this case from a progressive income tax. Hausman investigates the labor-leisure choice problem in a static, one-period, two-good model. He estimates labor supply functions, using recently developed econometric techniques and allows for variation in net wages, differences among workers and a combination of federal, state and payroll taxes. The deadweight loss measure employed by Hausman is Hicks's consumer's surplus minus the tax revenue raised. He compares the effect of the current tax system with an imaginary system where taxes are raised in a non-distortionary lump sum manner. The deadweight loss figures associated with the progressive income tax increase with increasing income and are as high as 54 percent, expressed as percentage of tax revenue.
In an era of rising prices, there is a tendency to tax energy resources since it is assumed that the tax burden falls mainly on rents and the excess burden associated with these taxes is relatively small. In this dissertation I analyze, in an optimal control framework, the effect of taxes on the behavior of a profit maximizing firm which explores and produces an exhaustible resource. I assume that the resource market is initially in equilibrium and then is distorted away from this equilibrium with the implementation of various taxes. The producer's response to these taxes is studied and the deadweight losses from the taxes are calculated. The case of extraction from a fixed reserve base is studied, but I concentrate on the simultaneous nature of the extraction and exploration decisions and compute time paths of exploratory effort, extraction and prices. Two formulations are developed. The first is based on Pindyck's model. The second formulation is an alternate specification of the exploration and current extraction decisions.

In analyzing the various shocks to the equilibrium state, I consider severance taxes, royalties, profit taxes and certain partial expensing provisions. With full expensing of costs, profit taxes have been shown to be neutral. I study how this neutrality is affected when the producer can only partially expense his extraction and exploration costs. The partial expensing of royalties
is also considered. The effects of the various taxes and expensing provisions are studied for competitive and monopolistic market structures in a partial equilibrium framework. An innovative approach concerns the method of solution which uses the highly effective Modified Quasilinearization Algorithm for optimal control problems developed by Miele and co-workers (1970).

The dissertation is organized as follows. Chapter Two presents the mathematical model. The first formulation based on Pindyck's model is utilized for the analysis of both a fixed and a variable reserve base. In the Cobb-Douglas formulation, only the case of a variable reserve base is analyzed as one of the special cases of the Cobb-Douglas formulation gives rise to a situation with no exploration. Since it is not possible to solve the problem analytically, solutions for only the time rate of change of the control variables are presented for the Pindyck formulation and for the control variables for the Cobb-Douglas formulation (not in closed form). In Chapter Three I introduce the various parameters and functions used in the simulation and discuss the method of solution. I then analyze the results of the base case with no taxes. The optimal time paths for price, quantity and exploration are presented for both competition and monopoly. The effects of the parameters and functions on these paths
are studied. The fourth chapter investigates how the time paths for price, quantity and exploratory effort change with the implementation of the taxes. Deadweight losses are calculated and the incidence and efficiency of these taxes are compared for all of the cases discussed in Chapter Three. Finally, the results of the dissertation are summarized and possible extensions are discussed in Chapter Five.
FOOTNOTES

1Stiglitz (1976) has proven that a monopolist will extract at the same rate as a competitive producer if demand is isoelastic.
II. MODEL

Two different formulations of a basic model of extraction and exploration are employed in this dissertation. The Pindyck formulation is utilized for both a variable and a fixed reserve base. The second formulation assumes a Cobb-Douglas production function and a Cobb-Douglas exploration function for new reserves and is employed to analyze only a variable base case. The general system is solved for the time rate of change of the control variables in the Pindyck case and for the control variables themselves in the alternate formulation.

1. Pindyck Formulation for a Fixed Reserve Base Case

   This model follows Pindyck's construction. The formulation is essentially the same for competition and monopoly, but the method of solution is different. The competitive firm is a price taker, whereas the monopolist can vary both price and quantity. Both types of firms have a fixed reserve base and the same cost functions and face the same demand curve.
A. Competition

I assume a competitive firm which owns a fixed reserve base and tries to maximize the present value of his profits over a time horizon $T$. The firm has an average cost function $C(R,q)$ that is dependent on both the level of reserves and the quantity produced. Special cases of the general cost function can be analyzed. Costs vary inversely with the level of reserves since it becomes more difficult and costly to extract the resource as it is depleted ($C_R(R,q) < 0$). Costs vary directly with the amount produced, $q$ ($C_q(R,q) > 0$). Since the cost function is assumed to be convex, $C_{qq} > 0$. The other partial derivatives of the cost function are taken to be $C_{RR} > 0$ and $C_{qR} < 0$, with $C(R,0) = 0$. Thus the problem facing the producer is, given the price $p$, find $q$ such that the following integral is maximized:

$$
\int_0^T \left[ pq(1-tx) - (1-\xi)C(R,q)q - (1-\gamma)tr \cdot pq \right] e^{-rt}(1-tp)
- \left[ \xi C(R,q)q + \gamma tr \cdot pq \right] e^{-rt}
$$

subject to the differential constraint $\dot{R} = -q$. In the above expression $tx$ is the severance tax rate, $tr$ the royalty rate and $tp$ the profit tax rate. $\xi$ is the fraction of extraction costs that cannot be expensed, and similarly $\gamma$ is the fraction of royalties which cannot be
expensed. The quantity produced is the control variable and level of reserves is the state variable. The Hamiltonian is given by

\[
H = \left[pq(1-tx)-(1-\xi)C(R,q)q-(1-\gamma)tr\cdot pq\right]e^{-rt}(1-tp) \\
-\left[\xi C(R,q)q+\gamma tr\cdot pq\right]e^{-rt}-\lambda(-q).
\] (2.1)

This can be further simplified as

\[
H = \left[pq(1-tx)(1-tp)-\theta C(R,q)q-\psi tr\cdot pq\right]e^{-rt} \\
-\lambda(-q)
\] (2.2)

where

\[
\theta = (1-\xi)(1-tp)+\xi \quad (2.3) \\
\psi = (1-\gamma)(1-tp)+\gamma \quad (2.4)
\]

The multiplier \(\lambda\) can be interpreted as the shadow price of reserves.

The first order conditions for a maximum are

\[
Hq = [p(1-tx)(1-tp)-\theta[C_q+e^{-rt}]\psi tr\cdot pq]e^{-rt}+\lambda = 0 
\] (2.5)

\[
\dot{\lambda} = H_R = -\theta C_R q e^{-rt} 
\] (2.6)

Assuming all the firms in the industry face the same
cost functions, they will produce the same quantity \( q \). The demand function for the industry can be put in the form \( p = f(Q) \) where \( Q = nq \) with \( n \) being the number of firms in the industry. Hence \( p = f(nq) \). The analysis for competition will be made as if there exists one large firm which behaves in a competitive manner and produces the joint output of all the \( n \) competitive firms. The first order conditions can be solved together with the demand function to obtain optimal trajectories for quantity and price. It is not possible to solve the problem analytically: one must resort to the computer to get numerical results. However, the analysis can be carried one step further to yield the rates of change for the quantity and/or price.

Differentiating Eq. (2.5) with respect to time, we obtain

\[
\begin{align*}
(\dot{p}(1-\tau)(1-t)p) - & \theta \{ C_{qq} \dot{q} + 2C_{qq}q + C_{qR} \dot{R}q + C_{R} \dot{R} \} e^{-rt} \\
- & r[p(1-tx)(1-tp) - \theta (C_{q}q + C_{p}p)e^{-rt} + \lambda] = 0. \quad (2.8)
\end{align*}
\]

Substituting \(-\theta C_{R}q e^{-rt}\) for \( \lambda \) and \( p = f'q \), we have

\[
\begin{align*}
(f'(1-tp)(1-tx) - & \theta (C_{qq}q + C_{qR} \dot{R}q - C_{R} \dot{R}q^2) \\
- & r[p(1-tx)(1-tp) - \theta (C_{q}q + C_{p}p)e^{-rt} + \psi tr \cdot p] = 0 \quad (2.9)
\end{align*}
\]

then one obtains for \( \dot{q} \)
\[ q = \frac{r(p(1-tx)(1-tp)-\Theta_{q+C})-\psi tr'p-\Theta Cq^2}{f'[(1-tx)(1-tp)-\psi tr]-\Theta [Cqq + 2Cq]} \] (2.10)

B. Monopoly

I analyze the same maximization problem for the monopolist, facing the same demand and cost functions. However, price is now a control variable instead of a parameter and the demand function must enter the optimization problem.

Maximize \[ \int_0^t [f(q)q(1-tx)(1-tp)-\Theta C(R,q)q-\psi tr\cdot f(q)q]e^{-rt} \]

subject to the constraint
\[ \dot{R} = -q \]

All the variables are defined as in the competitive case and \( f(q) \) is the demand function.

The Hamiltonian is

\[ H = f(q)q(1-tx)(1-tp)-\Theta [C_{qq} + C] - \psi tr\cdot f(q)q]e^{-rt} \]
\[ -\lambda (-q) \]

with first order conditions

\[ Hq = \{f'q + f\} (1-tx)(1-tp)-\Theta [C_{qq} + C] - \psi tr\cdot [f'q + f] \} e^{-rt} \]
\[ + \lambda = 0 \]

(2.12)

(2.13)
\[ \dot{\lambda} = H_R = -\theta C_R q e^{-rt} \quad (2.14) \]

Proceeding in the same way as in competition, we obtain

\[ \dot{q} = \frac{r (f'q+f)((1-tx)(1-tp)-tr\psi)-\theta (C_q q+C ) -\theta C_R q^2}{2f'[(1-tx)(1-tp)-tr\psi]-\theta [C_{qq} q+2C_q]} \quad (2.15) \]

This is assuming a linear demand function. The full derivation can be found in the appendix.

C. Taxes

To see the effects of a severance tax on a competitive producer, assume all other tax and expensing variables to be zero. We get for \( \dot{q} \)

\[ \dot{q} = \frac{r[p(1-tx)-(C_q q+C) q-C_{qR} q^2]}{f'(1-tx)-C_{qq} q-2C_q} \quad (2.16a) \]

or

\[ \dot{q} = \frac{rp-(rC_q q+rC+C_{qR} q^2)/(1-tx)}{f'(C_{qq} q+2C_q)/(1-tx)} \quad (2.16b) \]

Recall that all variables in the denominator are negative and that \( C_{qq} > 0 \). One can see that the rate of change
in quantity produced will become smaller with the tax. Ceteris paribus, this means that the producer will produce less and prices will be higher with the implementation of a severance tax.

The effect of a royalty will be the same. With only a royalty, the equation for \( \dot{q} \) will become,

\[
\dot{q} = \frac{r[p(l-tr)-(C_q+R)-C_R q^2]}{f'(1-tr)-C_{qq} q - 2C_q}
\]

(2.17)

This equation for \( \dot{q} \) is the same as that for a severance tax with only a change in the names of the tax variables. Hence royalties and severance taxes will have the producer reacting in exactly the same manner. Qualitatively, the effects of a severance tax or royalty on a monopolistic producer are the same as in the case of the competitor. In the absence of all other taxes, the rate of change in quantity with a severance tax or royalty will be:

\[
\dot{q} = \frac{r[f'q+f(1-t)-C_q q + C(R, q) - C_R q^2]}{2f'(1-t)-C_{qq} q - 2C_q}
\]

(2.18)

The tax decreases the numerator and increases the denominator of \( \dot{q} \), hence overall the quantity produced will
be decreased with the tax.

As is well known, the profit tax is neutral when all other taxes and costs are full expensed. \( \psi = \theta = (1-\tau_p) \).

This can be seen in the model as follows: \( \psi \) and \( \theta \) both reduce to \( (1-\tau_p) \) when \( \xi^{-} = 0 \) and \( \gamma = 0 \). Equation (2.10) takes the form

\[
q = \frac{r[p(1-\tau_x)(1-\tau_p)-(1-\tau_p)(C_q+\gamma)-(1-\tau_p)\tau r \cdot p]-C_{qq}q^2(1-\tau_p)}{f'[1-(1-\tau_x)(1-\tau_p)-(1-\tau_p)\tau r]-(1-\tau_p)[C_{qq}q+2C_q]}
\]

(2.19)

The \( (1-\tau_p) \) terms cancel out and the result is identical to the equation for \( q \) as without a profit tax. The only effect of the profit tax is to decrease the producer's profits, as can be observed from the integral to be maximized:

\[
\text{Max} \quad \int_0^T [pq-C(R,q)q]e^{-rt}(1-\tau_p)
\]

or

\[
\text{Max}(1-\tau_p) \quad \int_0^T [pq-C(r,q)q]e^{-rt}
\]

The optimal solution will not change, but the maximized profit will be decreased by a factor of \( (1-\tau_p) \).
To see the effects of partial expensing, assume, first, a royalty of which $\gamma$ cannot be expensed. We get for $\dot{q}$

$$\dot{q} = \frac{r[p(1-t_p)-(1-t_p)[C_q+C\psi tr\cdot p]-(1-t_p)C_rq^2]}{f'[r-(1-t_p)-(1-t_p)[C_qq+2C_q]}$$

(2.20)

$$\dot{q} = \frac{r[p-C_q-C\cdot[1-\gamma\cdot\psi tr\cdot p]C_rq^2]}{f'[r-\psi t_p-C_qq+2C_q]}$$

(2.21)

call $(1-\gamma)+\gamma/(1-t_p)=\psi'$

then

$$\dot{q} = \frac{r[p(1-\psi tr)-C_qq-C^2]}{f'[1-\psi tr-C_qq-2C_q]}$$

(2.22)

The joint effect of the profit tax and partial expensing of royalties comes through the $\psi'$ variable. Since $\psi'>1$, it has the effect of increasing both the denominator and the numerator. The magnitude of the distortion of the tax will be determined by how divergent $\psi'$ is from unity and how large in absolute value the denominator and numerator of $\dot{q}$ are.

If extraction costs are partially expensed,
\[
\dot{q} = \frac{r[p - \frac{(1-\xi)}{1-\tau_p}(C_q + C)] - [(1-\xi)+\frac{\xi}{1-\tau_p}]C_R q^2}{f' - [(1-\xi)+\frac{\xi}{1-\tau_p}] (C_{qq} q + 2C_q)}
\] (2.23)

let \(\theta' = (1-\xi) + \frac{\xi}{(1-\tau_p)}\)

\[
\dot{q} = \frac{r[p-\theta'(C_q + C) - \theta' C_{qq} q^2]}{f' - \theta (C_{qq} q + 2C_q)}
\] (2.24)

In this case \(\theta'\), which is \(>1\) has the effect of decreasing the numerator and increasing the denominator of the expression for \(\dot{q}\). Hence the effect is unambiguous: the quantity produced will decrease and prices will increase.

For the monopolist, the effect of partial expensing and profit taxes are the same as for the competitive firm. With partial expensing of royalties,

\[
\dot{q} = \frac{r[1-trH'](f'q + f) - C_{qq} q - C_{q} q^2]}{2f'[1-trH'] - C_{qq} q + 2C_q}
\] (2.25)

and with partial expensing of costs,

\[
\dot{q} = \frac{r[f'q + f - \theta'(C_q + C) - \theta' C_{qq} q^2]}{2f' - \theta' (C_{qq} q + 2C_q)}
\] (2.26)
2. Pindyck Formulation for a Variable Reserve Base Case

When firms can add to their reserve base by exploration, the optimal control problem becomes more complex with two control variables and two state variables. The level of exploratory effort and the amount of accumulated reserves are respectively the additional control and state variables. There are two factors that reinforce each other and that drive exploration in this model. First, exploration adds to the reserve base. Since extraction costs are inversely proportional to the level of reserves, exploration brings extraction costs down. Second, exploration augments revenues directly: reserves are increased and more is produced and sold in each period.

A. Competition

A competitive firm undertaking exploration has essentially the same maximization problem as in the case without exploration. The cost of exploration enters the integral to be maximized:

\[
\int_0^T \left[ pq(t, x) - (1 - \xi) C(R, q)(1 - \mu) CP(w) - (1 - \gamma) tr \cdot pq \right] e^{-rt} (1 - tp) \\
- \left( \xi C(R, q)q + \mu CP(w) + \gamma tr \cdot pq \right) e^{-rt}
\]

subject to the differential constraints
\[ \dot{R} = G(x,w) - q \quad (2.27) \]
\[ \dot{X} = G(x,w) \quad (2.28) \]

where \( G(x,w) \) is the exploration function, \( CP(w) \) the cost of exploration, \( x \) the amount of accumulated reserves and \( w \) the level of exploratory effort. All other variables and parameters are defined as before. The exploration function depends on both the amount of exploratory effort and the level of accumulated reserves. It is assumed that exploration is decreased as more reserves are found, hence \( G_x < 0 \), but \( G_w > 0 \). The cost of exploration is directly proportional to the level of exploratory activity with \( CP_w > 0 \). \( CP(w) \) is the total cost of exploration, whereas the extraction cost is an average cost. The Hamiltonian is

\[
H = [pq(1-tx) - (1-\xi)C(R,q)q - (1-\mu)CP(w) - (1-\gamma)tr \cdot pq] e^{-rt}(1-tp)
- [\xi C(R,q)q + \mu CP(w) + \gamma tr \cdot pq] e^{-rt} - \lambda_1(G-q) - \lambda_2 G. \quad (2.29)
\]

This can be simplified as

\[
H = [pq(1-tx)(1-tp) - \Theta C(R,q)q - \phi CP(w) - \psi tr \cdot pq] e^{-rt}
- \lambda_1(g-q) - \lambda_2 G, \quad (2.30)
\]

where \( \phi = (1-\mu)(1-tp) + \) and \( \Theta \) and \( \psi \) are defined as in Equations
2.3 and 2.4. Keeping $p$ constant, the first order conditions are

$$H_q = [p(1-tx)(1-tp) - \theta(C_q + C) - \psi \text{tr} \cdot p] e^{-rt} + \lambda_1 = 0$$  \hspace{1cm} (2.31)

$$H_w = - \phi C_R q e^{-rt}$$  \hspace{1cm} (2.32)

$$\dot{\lambda}_1 = H_w = - \theta C_R q e^{-rt}$$  \hspace{1cm} (2.33)

$$\dot{\lambda}_2 = H_w = - G_x (\lambda_1 + \lambda_2)$$  \hspace{1cm} (2.34)

Proceeding in the same manner as in case 1 and substituting $f(q)=p$ in the first order conditions, we obtain the following for the rate of change in quantity produced:

$$q = \frac{r[P(1-tx)(1-tp) - \theta[C_q + C] - \psi \text{tr} \cdot p] + \theta[C_{qR} q(G-q) + C_R G]}{f'[1-tx)(1-tp) - \psi \text{tr}] - \theta[C_{qq} q + 2C_q]}$$  \hspace{1cm} (2.35)

To find the rate of change in exploratory effort, we get from $H_w$

$$H_w = - G_w (\lambda_1 + \lambda_2) = \phi C_P e^{-rt} - (\lambda_1 + \lambda_2) = \frac{\phi C_P e^{-rt}}{G_w}$$  \hspace{1cm} (2.36)

taking the time derivative of $H_w$, we have
\[ - (\dot{\lambda}_1 + \dot{\lambda}_2) = \frac{G_w \{ \phi C P e^{-rt} \} - r \phi C P e^{-rt}}{G_w^2} - \phi \frac{C P e^{-rt}}{G_w} \quad (2.37) \]

Substituting the expression for \( \dot{\lambda}_1 \) and \( \dot{\lambda}_2 \) and arranging terms,

\[ \cdot \frac{G_x G \phi C P - G \phi \phi C P - \theta C R Q G^2}{\phi C P C G^2} \quad (2.38) \]

B. Monopoly

As in the competitive case, the monopolist faces the problem of maximizing

\[ \int_0^T \left[ f(q)q(1-tx) - (1-\xi)C(R,q)q - (1-\mu)CP(w) - (1-\gamma)tr.pq \right] e^{-rt} \]

\[ (1-tp) - [\xi C(R,q)q + \mu CP(w) + \gamma tr.pq] e^{-rt} \]

subject to

\[ \dot{R} = G(x,w) - q \quad (2.39) \]

\[ \dot{X} = G(x,w) \quad (2.40) \]
The price is again a control variable instead of a parameter. The rates of change for quantity and exploratory effort are:

\[ \dot{r} = \frac{r[(f'q+f)(1-tx)(1-tp)-\theta(C_q q+C)-\psi tr]+\theta[C_q q^2(G-q)+C_R G]}{(f'' q+2f')[(1-tx)(1-tp)-\psi tr]-\theta[C_q q^2+2C_q]} \]  \hspace{1cm} (2.41)

\[ \dot{w} = \frac{\phi CP_{w} - G_w - \phi r CP_{w} - \theta C_R q G}{\phi CP_{w} G_{ww}} \]  \hspace{1cm} (2.42)

C. Taxes

For both competition and monopoly the severance tax and royalties have the same effect in this case as in the case with a fixed reserve base. With the introduction of a severance tax, the price will rise, the quantity will decrease and, as would be expected, the amount of exploration will also decrease.

As in the previous case, in both monopoly and competition a profit tax has no effect on price and exploration paths when all costs, royalties and severance taxes are fully expensed. With partial expensing however, one obtains different results. If royalties are partially expensed
\[ q = \frac{r[p(1- \psi' tr) - C_q q - C] + C_q R q(G-q) + C_R G}{f'(1- \psi' tr)} \] (2.43)

where \( \psi' \) is defined as before. For values of \( \psi' \) close to unity, the effect on \( q \) is minimal. As the profit tax rate rises, or as \( \gamma \) increases, the profit tax will have distortionary effects. Since the effect of partial expensing on exploratory effort manifests itself through \( q \) only, there is no change in \( w \) until the tax rate becomes very high. The monopoly case will show similar behavior.

In extraction costs are partially expensed,

\[ q_c = \frac{-\theta'[r(C_q q+C) - C_q R q(G-q) - C_R G] + rp}{f' - \theta'(C_q q + 2C_q)} \] (2.44)

for competition, and

\[ q_m = \frac{r(f'q + f) - \theta[r(C_q q+C) - C_q R q(G-q) - C_R G]}{2f' - \theta'(C_q q + 2C_q)} \] (2.45)

for monopoly. If there is no profit tax, \( \theta' = 1 \). Given the profit tax, if extraction costs are partially expensed, \( \theta' > 1 \). All the terms in the coefficient of 2.44 are positive, with the exception of \(-C_q R (G-q)q\). This term will generally be negative unless the level of exploration is
higher than production. If that is the case, then the
effect of partial expensing (through $\theta'$) will be to increase
the negative part of the numerator. If $G-q<0$, then the
size of the coefficient of $q$ will depend on whether
$-C_{qR}(G-q)q<rC_q+qC_C-C_RG$. In our case however, the cost
function used in the simulation is only a function of $R$ and
the $C_{qR}$ term drops out. Hence the quantity produced should
be lower than with complete expensing, ceteris paribus. A
decrease in extraction costs however, could lead to higher
levels of production.

To see how the exploratory effort is affected, we look
at $w$, which has the same expression for competition and
monopoly. The actual level of exploration is different
because production varies between the two market structures.

$$
\frac{G^2C_{qR}(1-\theta')+\frac{G}{1-\theta'}}{C_{qR}} - C_G + rC^2
\quad (2.46)
$$

There are two interacting effects here. The effect of
$\theta'(>1)$ will be to increase the negative part of the expres-
sion. The quantity produced is decreasing, and this has
the effect of decreasing the negative component of the
expression. With production reduced, the level of reserves
in the ground is larger, which implies that $C_R$ is smaller.
This further decreases the negative component of the above expression. Hence if the decreases in quantity and $C_R$ are relatively small, $\theta'$ will be large enough to offset this negative effect. If the decrease in quantity is large enough, it will offset $\theta'$ and the amount of exploration will decrease (ceteris paribus). As the tax rate goes up, $\theta'$ increases. As the percentage of costs that cannot be expensed goes up, (i.e., $\xi$ increases) $\theta'$ also increases.

In monopoly, the decrease in quantity due to partial expensing ranges from .1% to 2%. In competition the range is .8% to 20%. However, in both cases $\theta'$ dominates and exploratory effort increases. The higher $\theta'$ is, the higher is exploratory effort.

If exploration costs are partially expensed, the expensing factor does not enter the $q$ equation explicitly and prices and quantities are changed indirectly through exploration costs. With partial expensing,

$$w = \left[ C_x G_w - r G_w^2 - \frac{C_R q G_w^2}{\phi' CP_w} \right] / G_{ww}$$

(2.47)

where $\phi' = (1-\mu) + \mu/(1-\tau_p)$. With no profit tax or full expensing, $\phi'=1$. With partial expensing, $\phi' > 1$. The only positive term in the numerator of the above expression for $w$ is that involving $\phi'$. Therefore $\phi' > 1$ implies a decrease in the positive part of the $w$ expression and
exploration will decrease.

3. Alternate Cobb-Douglas Formulation for Exploration Case

In this second formulation, production is a function of the level of reserves and the amount of extractive effort which is made up of various factors of production. The cost of production is a function of the level of extractive effort and the cost of effort. Exploration is undertaken to renew depleted reserves and the exploration function is a diminishing returns to scale production function for new reserves. The cost of exploration is a function of the level of exploratory effort and its cost.

Assume a Cobb-Douglas production function for q

$$q = K_1 \lambda^\alpha R^{1-\alpha}$$  \hspace{1cm} (2.48)

where $\lambda$ is the extractive effort and $R$ the level of reserves and a Cobb-Douglas exploration function $G$ where

$$G = K_2 w^{\beta_1} x^{\beta_2}, \beta_1 + \beta_2 < 1$$  \hspace{1cm} (2.49)

$w$ is exploratory effort and $x$ is accumulated reserves as before. The cost functions are $C_1 = P_1 \cdot \lambda$ and $C_2 = P_2 \cdot w$ where $P_1$ and $P_2$ are exogeneous prices for extractive and explora-
tory effort, respectively.

A. Competition

The competitive producer maximizes

\[ \int_0^T [pq(\xi, R)(1-tx)-(1-\xi)C_1(\xi)-(1-\mu)C_2(w)-(1-\gamma)tr\cdot p\cdot q(\xi, R)]e^{-rt} \]

\[ -[\xi C_1(\xi)+ \mu C_2(w)+ \gamma trpq]e^{-rt} \]

subject to the differential constraints,

\[ \dot{R} = G(x, w)-q \quad (2.50) \]

\[ \dot{X} = G(x, w) \quad (2.51) \]

In this problem, the control variables are \( \xi \) and \( w \), the extractive and exploratory effort respectively. The state variables are the level of reserves and accumulated reserves from exploration (x). The Hamiltonian is

\[ H = (pq(\xi, R)(1-tx)(1-tp)-\theta P_1 \xi - \phi P_2 w- \psi trpq(\xi, R)]e^{-rt} \]

\[ -\lambda_1 (G-q)-\lambda_2 G \quad (2.52) \]
where $\theta, \phi$ and $\psi$ have been substituted for the tax and expensing parameters as defined in section 2A. 

First order conditions are:

$$H_L = \left[ pq_L (1-tx)(1-tp) - \theta_P - \psi trpq_L e^{-rt} + \lambda_1 q_L \right] = 0 \quad (2.53)$$

$$H_w = -\phi P_2 e^{-rt} - G_w (\lambda_1 + \lambda_2) = 0 \quad (2.54)$$

$$\lambda_1 = H_R = \left[ pq_R (1-tx)(1-tp) - \psi trpq_R e^{-rt} + \lambda_1 q_R \right] \quad (2.55)$$

$$\lambda_2 = H_x = -G_x (\lambda_1 + \lambda_2) \quad (2.56)$$

Since $q_L = \alpha x^{\lambda_1} l^{-1} = \alpha q/l$, substituting into $H_L$ we obtain

$$\left[ pq_L (1-tx)(1-tp) - \theta P + \psi trpq_L e^{-rt} \right] + \lambda_1 q_L = 0 \quad (2.57)$$

$$\theta P_1 = \alpha pq[(1-tx)(1-tp) - \psi tr] + \lambda_1 \alpha q e^{rt} \quad (2.58)$$

$$\lambda = \frac{\alpha pq[(1-tx)(1-tp) - \psi tr] + \lambda_1 \alpha q e^{rt}}{\theta P_1} \quad (2.59)$$

To obtain exploratory effort, we have

$$H_w = -\phi P_2 e^{-rt} - G_w (\lambda_1 + \lambda_2) = 0 \quad (2.54)$$
from \( H \) we have

\[
\lambda_1 = \frac{\theta P_1 \lambda - p \alpha q[(1-tx)(1-tp) - \psi tr]}{\alpha q e^{rt}}
\]

(2.60)

Substituting \( \lambda_1 \) and \( G = K_2 w^{\beta_1} x^{\beta_2} \) into \( H_w \),

\[
-\phi P_2 e^{-rt} - K_2 \beta_1 w^{\beta_1-1} x^{\beta_2} - \lambda_2 - K_2 \beta_1 w^{\beta_1-1} x^{\beta_2} = 0
\]

(2.61)

\[
w = \left[ \frac{K_2 \beta_1 x^2 (p[(1-tx)(1-tp) - \psi tr] - \lambda_2 e^{rt} - \frac{\theta P_1 \lambda}{\alpha q})}{-\phi P_2} \right]^{1-\beta_1}
\]

(2.62)

In general, the above stated system is not solvable analytically, the Cobb-Douglas production functions further complicate the problem in that they are differentiable in \( q \) of order \( n \). Hence the above expressions for \( \lambda \) and \( w \) are for demonstrative purposes only.

B. Monopoly

The maximization problem facing the monopolistic producer is the same as the competitor's. As in the preceding formulation, the monopolistic producer must take the market demand function into consideration. The
Max $\int_T^0 [f(q)q(\ell,R)(1-tx)-(l-\ell)C_1(\ell)-(l-\mu)C_2(w)-(l-\gamma)tgpq)e^{-rt}(l-tp)-(\xi C_1(\ell)+\mu C_2(w)+\gamma tgpq)e^{-rt}$

subject to the constraints

\dot{R} = G(x,w) - q(\ell,R) \quad (2.63)
\dot{X} = G(x,w) \quad (2.64)

The Hamiltonian is

\[ H = [f(q)q(\ell,R)(1-tx)(1-tp)-\psi tr]-\phi P_1 \ell - \phi P_2 w e^{-rt}-\lambda_1 (G-q)-\lambda_2 G \] \quad (2.65)

First order conditions for a maximum are:

\[ H_{\ell} = (f_{q}q+f_{\ell}q)[(1-tx)(1-tp)-\psi tr]-\phi P_1 \ell e^{-rt}+\lambda_1 q = 0 \] \quad (2.66)

\[ H_w = -\phi P_2 e^{-rt} - G_w (\lambda_1 + \lambda_2) = 0 \] \quad (2.67)

\[ \lambda_1 = H_R = (f_{R}q+f_{R}q)[(1-tx)(1-tp)-\psi tr]e^{-rt}+\lambda_1 q_R \] \quad (2.68)

\[ \lambda_2 = H_x = -G_x (\lambda_1 + \lambda_2) \] \quad (2.69)

Noting that $f = A+Bq+Cy$ and

\[ f_\ell = Bq_\ell \quad \text{and} \quad f_R = Bq_R \]
we get for extractive effort

\[ \lambda = \frac{apq \chi + \alpha \lambda q e^{rt} \alpha Bq^2 \chi}{\theta P_1} \quad (2.70) \]

where \( \chi = (1-tx)(1-tp) - \psi tr \) and for exploratory effort

\[ w = \frac{K_2 \beta_1 \chi^2 \left[ (p-Bq) \chi - \lambda_2 e^{rt} \right]}{\phi P_2} \quad (2.80) \]

Note that, ceteris paribus, extractive effort and exploratory effort for the monopolistic producer are less than those for the competitive producer. Details of the calculations are in the Appendix.

C. Taxes

The competitive and monopolistic producers behave qualitatively in the same manner for all the different taxes and expensing provisions. As in the previous formulation, partial expensing of extraction costs stimulates both producers to increase exploratory effort but, as will be seen in the numerical analysis, this increase is very slight. This result is because the cost of extraction is not explicitly dependent upon the level of reserves. Only the competitive case will be analyzed here, since both types of producer behave similarly.
As in previous cases, severance taxes and royalties have identical effects on the system. With a severance tax or royalty, the amount of extractive effort is decreased, leading to lower quantities produced, consequently to higher prices. To observe the effects of a severance tax, assume all other tax and expensing variables to be zero. We get for $\lambda$

$$\lambda = \frac{apq(1-tx)+\lambda_1aqe^{rt}}{\theta p_1}$$

(2.81)

All the terms in the above expression with the exception of $\lambda_1$ are $x$ positive (for an explanation, see Appendix). A severance tax decreases the positive portion of the expression and thus reduces the overall amount of extractive effort. A royalty payment, fully expensed, has the exact same result since the expression for $\lambda$ becomes

$$\lambda = \frac{apq(1-tr)+\lambda_1aqe^{rt}}{\theta p_1}$$

(2.82)

The expression for exploratory effort, with a severance tax in effect is

$$\frac{K_2\beta_1 x^2 [p(1-tx)-\lambda_2 e^{rt} - \frac{P_1\lambda}{aq} - \frac{1}{1-\beta_1}]}{\phi p_2}$$

(2.83)
All variables in the expression for \( w \) are positive. The severance tax decreases the numerator, thus reducing exploratory effort.

As proven in the preceding section, a profit tax has no effect on the optimal solution of the system aside from reducing the producer's profits by a factor of \((1-tp)\). In this formulation one can see how the costate variables change with the profit tax by analyzing the control variables. With a profit tax,

\[
\xi = \frac{\alpha q(1-tp) = \lambda_1 qe^{rt}}{P_1(1-tp)} \tag{2.84a}
\]

\[
\xi = \frac{\lambda_1}{\frac{P_1}{(1-tp)}} q e^{rt} \tag{2.84b}
\]

Since the optimal trajectory does not change with the tax, the new costate variable is reduced by a factor of \((1-tp)\), i.e.,

\[
\lambda_1 = \frac{\lambda_1}{1-tp} \tag{2.85}
\]

The same result can be seen for \( \lambda_2 \) in the expression for \( w \). Exploratory effort with a profit tax becomes
\[
\begin{align*}
\lambda_2 &= \frac{\lambda_2}{1-tp} \quad (2.87)
\end{align*}
\]

Hence shadow prices for reserves fall with a profit tax. The effects of partial expensing on this system are qualitatively the same for all types of partial expensing. Both extractive effort and exploratory effort decrease when costs or royalties cannot be fully expensed. Unlike the previous formulation, there are no ambiguities. To see the effects of partial expensing, assume first a royalty of which \(\gamma\) percent cannot be written off in the current period from the producer's profit tax liabilities. Then,

\[
\ell = \frac{\alpha p q (1-tp-tr\psi) + \alpha_1 q e^{rt}}{(1-tp)P_1} \quad (2.88)
\]

Let \(\psi' = \psi/(1-tp) = 1 - \gamma + \gamma/(1-tp)\) as before \(\psi' = \psi = 1\) for full expensing and \(\psi' > 1\) for partial expensing.

\[
\ell = \frac{\alpha p q (1-tr\psi') + \alpha_1 q e^{rt}}{P_1} \quad (2.89)
\]
decreases the positive part of the expression, as the negative part is increased by a factor of \((1-tp)\). Thus extractive effort is decreased with the partial expensing of royalties. The magnitude of the decrease will depend upon the profit tax rate and the higher the percentage of royalties one cannot expense, the greater will be the decrease in extractive effort. For the change in exploratory effort:

\[
\frac{K_2 \beta_1 x^2 \rho (1-\psi' tr)}{p_2} \left[ \frac{\lambda_2 r t}{1-tp} - \frac{\lambda_1 q e^{rt}}{aq} \right] \frac{1}{1-\beta_1} = \frac{2.90}{p_2}
\]

The analysis is the same as for changes in extractive effort. The numerator of \(w\) is decreased which leads to a reduction in exploratory effort.

With partial expensing of extraction costs, we have for \(\lambda\) and \(w\)

\[
\lambda = \frac{apq(1-tp)+a\lambda_1 q e^{rt}}{\theta p_1} = \frac{apq+\alpha \lambda_1 q e^{rt}}{\theta' p_1} = 2.91
\]

where \(\theta' = \theta/(1-tp) = 1-\xi + \xi/(1-tp) > 1\).

The numerator of the expression for \(\lambda\) is decreased while the denominator is increased, which reduces \(\lambda\) as a whole. Similarly,
\[
K_{2} \beta_{1} \beta_{2} (p - \frac{\lambda_{2} e^{rt}}{1-tp} - \frac{\theta'P_{1}l}{\alpha q}) \frac{1}{1-\beta_{1}} \]
\[
w = \left[ \frac{p - \frac{\lambda_{2} e^{rt}}{1-tp} - \frac{P_{1}l}{\alpha q}}{\phi' P_{2}} \right] \frac{1}{1-\beta_{1}}
\]
(2.92)

As in the previous case, we again have two interacting effects. \( \theta' \) serves to decrease the numerator of \( w \). From the above analysis, though, we know that \( l \) is decreased with partial expensing. Hence whether exploratory effort decreases or increases would depend upon \( \theta' \) and the magnitude of the decrease in \( l \).

If exploration costs are partially expensed,

\[
w = \frac{K_{2} \beta_{1} \beta_{2} (p - \frac{\lambda_{2} e^{rt}}{1-tp} - \frac{P_{1}l}{\alpha q})}{\phi' P_{2}} \frac{1}{1-\beta_{1}}
\]
(2.93)

\( \phi' = \phi/(1-tp) = 1 - \mu + \mu/(1-tp) > 1 \). The numerator of the expression for \( w \) is decreased and the denominator increased. Hence, exploratory effort is less when exploration costs are partially expensed. There is no direct effect on extractive effort.

4. Deadweight Losses

It has been established in economic theory that the deadweight loss from taxing a commodity is equal to the
sum of the loss in consumer's surplus plus the loss in producer's surplus. This is the difference in revenue received by the government from the tax and the loss suffered by consumers and producers. Producer's surplus is profits plus rent, so that a loss in producer's surplus becomes just the change in profits due to the tax. Consumer's surplus can be defined as "the amount the consumer would pay or would need to be paid to be just as well off after the price change as he was before the price change" due to the tax. Although this definition corresponds to the Hicksian measure of compensating variation, there is no consensus on the exact measurement of this consumer's surplus because the Hicksian uncompensated demand functions are not observable. Marshallian surplus, or the area to the left of the market demand curve has been used as a proxy for compensating variation, but for the two measures to be equal, there must be no income effects. With no income effects, Harberger's measure of $1/2 2E.t.p.q$ becomes a correct measure. Robert Willig (1976) derives exact upper and lower bounds on the percentage errors approximating the compensating and equivalent variations with Marshallian consumer's surplus. He shows that the error in approximation will be very small if the income elasticities of demand and the proportion of income spent
on the good are not very large. I will use an exact measure of consumer's surplus and deadweight loss developed recently by Jerry Hausman. Hausman (1981) utilizes the dual approach to consumer behavior and using certain properties of the expenditure function and indirect utility function, derives an exact measure of Hick's compensating variation for two types of market demand curves, linear and CES.

Compensating variation is defined as

\[ CV(p_0, p_1, y_0) = e(p_1, u_0) - e(p_0, u_0) = e(p_1, u_0) - y_0 \]  \hspace{1cm} (2.94)

in terms of the expenditure function where \( p, u, y \) are the price, utility and level of income before the tax and \( p \) is the after tax price of the good. Using Roy's identity, he obtains,

\[ \frac{dy}{dp_1} = \alpha p_1 + \beta y + \gamma y - q \]  \hspace{1cm} (2.95)

where \( q \) is the market demand function. Integrating (2.95), solving for the indirect utility function and substituting expenditure for income, he obtains the expenditure function

\[ e(p_1, \bar{u}) = e^{\frac{\delta}{\delta} p_1 \bar{u} - \frac{1}{\delta}(\alpha p + \frac{\alpha}{\delta} + \gamma y)} \]  \hspace{1cm} (2.96)

From this, the compensated variation is calculated to be
\[ CV = \frac{1}{\delta} e^{\delta(p_1-p_0)} \left[ q_o(p_0,y_0) + \frac{\alpha}{\gamma} \right] - \frac{1}{\delta} \left[ q_1(p_1,y_0) + \frac{\alpha}{\delta} \right]. \]

Using this measure of compensating variation, I calculate the deadweight loss to society from the tax as

\[ DWL = \Delta \text{Producer's surplus} + \Delta \text{Consumer's surplus} \]

\[ DWL = \Delta PR + CV - TXR \]

where \( \Delta PR \) is the change in producer's profits and PR is the present value of total profits over the entire time horizon. TXR is the present value of total tax revenues and CV is the sum of the present values of compensated variation at each time \( t \). The value of \( CV - TXR \) corresponds to the "welfare triangle".
III. SIMULATION AND DISCUSSION OF BASE CASE WITH NO TAXES

In this chapter, I report simulation results for the models discussed above. The optimal time paths for the price, current quantity produced and the amount exploratory effort are calculated in a situation without taxes. These base case simulations, calculated for competitive and monopoly market structures are used as the bases of the deadweight loss calculations for various regimes. I also calculate the time period in which exploitation of the resource stops. However, to facilitate comparisons between the two market structures, production paths for a fixed, common production period is analyzed.

In analyzing the effects of various tax provisions, each provision is analyzed separately by assuming other taxes and related provisions are equal to zero. Once the optimal price and quantity paths are calculated, the deadweight losses associated with different taxes are calculated.

A single, predetermined demand function is utilized throughout the analysis. In order to test the robustness of the results the demand side is modified in some of the simulations. Also, certain parameters, the interest rate,
the time horizon, the form of the exploration function and the level of initial reserves are varied in a variety of sensitivity experiments.

I. Parameter Values and Functions Used in the Simulation

In the formulation modeled after Pindyck, the cost and exploration functions are those estimated by Pindyck for the Permian Basin Region of Texas, with data for oil production for the years 1965-1974. Pindyck reports the average cost of producing a barrel of oil in 1966 was $1.25 and reserves were 7170 barrels. The per-unit cost of extraction is assumed to be a function of reserves alone, and is computed by Pindyck to be

\[ C(R) = \frac{8960}{R} \]  \hspace{1cm} (3.1)

The cost of exploration estimated by Pindyck is

\[ CP(w) = 103.2 + .067w \]  \hspace{1cm} (3.2)

and the exploration function is

\[ G(x,w) = 10.9w^{.599} e^{-0.0002258x} \]  \hspace{1cm} (3.3)

In the alternate formulation, I use a constant returns to scale Cobb-Douglas function, with equal weights on the factors:
q = λ.5 R.5 \quad (3.4)

with q being the quantity produced and λ the level of extractive effort. The exploration function is a decreasing returns to scale production function for new reserves:

\[ G(x, w) = 2w^{\beta_1} x^{\beta_2}. \quad (3.5) \]

w and x are defined as in the Pindyck case. Initially, the parameters \( \beta_1 \) and \( \beta_2 \) are chosen so as to "mimic" the Pindyck function, with \( \beta_1 = .6 \) and \( \beta_2 = -.05 \). Later, values of .3 and .8 for \( \beta_1 \) and -.1 for \( \beta_2 \) are used to investigate the effects of changing the exploration function on price and quantity paths. The exogenous price of extractive effort is chosen so the total costs at initial time period \( t_0 \) are equal in the Pindyck and Cobb-Douglas formulations as follows. Total cost in the Pindyck case is

\[ TC = C(R)q = 1.25q \quad (3.6) \]

Total cost in the Cobb-Douglas case is

\[ TC = p_1 \lambda \]
where \( p_1 \) is the exogenous price of extractive effort. Solving for extractive effort, from the production function,

\[ \lambda = \frac{q^2}{R} \ . \]

Substituting,

\[ TC = p_1 \frac{q^2}{R} \ . \]

Equating total costs, we obtain

\[ p_1 = \frac{1.25 \ R}{q} \ . \]

For the Pindyck formulation under competition the initial quantity produced is approximately 600 units and \( p_1 \) is equal to $14.94. For monopoly initial quantity is approximately 300 units and \( p_1 \), the cost of extractive effort is equal to $29.87. I take the average of these two values for extractive effort, $22.41 in all of the simulations. The price of exploratory effort, \( p_2 \), calculated from drilling and production data for Texas for 1964, is equal to $0.32 per barrel of reserves.

The principal demand function is a variation of the one used by Pindyck. In order to calculate the welfare
effects of taxes, I have modified this function by adding an income variable. The demand function is

\[ q = 460 - 20p + .0109y \]  \hspace{1cm} (3.7)

where \( q \), \( p \) and \( y \) are quantity, price and income respectively.

For this linear demand function the price elasticity of demand \( \epsilon_d \), is relatively small at low prices, \( \epsilon_d = -.1 \) for \( p = 3 \) and rises as price increases. The price elasticities vary between 0 and -2.0, the income elasticities vary between .3 and 1. These elasticities are consistent with the price and income elasticities estimated for U.S. energy demand, as reported by Taylor (1976).

The Permian Basin Region in Texas represents only a portion of overall U.S. production and consumption. The absolute level of income for the market under consideration is not important provided that the coefficient of income is changed to keep the income elasticity unchanged. I have used both Texas personal income and U.S. income, both of which are imperfect. The estimates of deadweight loss are not sensitive to the income measure used. Though deadweight losses expressed as a percentage of income are very sensitive to the choice of income variable.
To allow for growth in demand over time, I have considered cases where income grows at .5 percent a year and 2.5 percent a year. In these cases, I use the initial demand function with

\[ y = y_o + 100t \quad \text{and} \quad y = y_o + 500t, \quad (3.8) \]

respectively. (Note that the price elasticity decreases with increasing income).

The following taxes have been analyzed. For the cases with exploration, the effects of a production tax imposed at rates of 5%, 10%, 15% and 20% are studied. As discussed in Chapter II, this is equivalent to a production royalty imposed at the same rate. A profit tax is varied in the range 5% to 40%. Costs and royalties are initially assumed to be fully expensed under the profits tax. Cases of partial expensing of costs and royalties to the extent of 20, 50 and 80 percent are also analyzed. The royalty rate is set at 5% for the partial expensing experiment.

The optimal time horizons for production and exploration are finite for the two market structures, competition and monopoly. For the case of fixed reserves, the finite horizon is explained by the finite level of reserves. With exploration, the increasing cost nature of the production function influences the time horizon and explains its finite nature. As will be demonstrated, the relaxation of the assumption of the rising cost of exploration gives rise to
a steady state solution where the amount of new reserves
developed through exploration is approximately equal to
production.

The optimal time horizon is the period of production
which yields the maximum present value of profits for the
producers given that all production and exploration de-
cisions are made under conditions of perfect certainty for
all periods into the future. At any time horizon greater
than the optimum, the present value of total profits de-
clines and marginal net profits turn negative.

The optimal time horizons for the competitive and
monopoly market structures are different as the monopolist
extracts and explores at a slower rate than the competitive
industry. In order to measure the tax effects and to make
comparisons between the two market structures, I assume the
same fixed production periods for the two market structures.
These are 15 and 50 years respectively for the fixed and
variable reserve base cases.

2. Method of Solution

The method of solution follows Miele's (1974) general
algorithm for optimal control problems with differential
and nondifferential constraints. In the case without ex-
ploration, the differential system is of order two with one
initial condition which is the level of reserves, and one
final condition which is that the multiplier $\lambda_1$ is zero. This assures that production ends when the present value of reserves falls to zero. In the exploration case, the system is of order four with two initial and two final conditions. The additional initial condition is on $x$, the level of accumulated reserves, which is taken as zero. The second final condition is on the second multiplier which must be zero at final time $T$. Nominal functions for the state, control and costate variables are chosen, satisfying the initial conditions. The performance index $S$ is defined as

$$S = P + Q$$  \hspace{1cm} \text{(3.9)}$$

where

$$P = \int_0^T (\dot{R} - \phi)^2 + (\dot{X} - \psi)^2 \, dt$$  \hspace{1cm} \text{(3.10)}$$

is the error in the constraining equations and

$$Q = \int_0^T (\dot{\lambda_1} - H_R)^2 + (\dot{\lambda_2} - H_x)^2 + H_q^2 + H_w^2 + [\lambda_1(T)]^2 + [\lambda_2(T)]^2$$  \hspace{1cm} \text{(3.11)}$$

is the error in the optimality conditions. Here $\phi$ and $\psi$ are the differential constraints on the state variables $R$ and $x$, respectively. The first variation of $P$ and $Q$ are
\[ \delta P = 2(\ddot{R}-\ddot{\phi})\delta (\ddot{R}-\ddot{\phi}) + 2(\ddot{\chi}-\ddot{\psi})\delta (\ddot{\chi}-\ddot{\psi}) \]  
(3.12)

\[ \delta Q = 2(\ddot{\lambda}_1-H_R)\delta (\ddot{\lambda}_1-H_R) + 2(\ddot{\lambda}_2-H_\chi)\delta (\ddot{\lambda}_2-H_\chi) + 2 H_q\delta H_q + 2 H_w\delta H_w \]  
(3.13)

The constraint equations and optimality conditions are linearized to obtain the following system of variations:

\[ \delta (\ddot{R}-\ddot{\phi}) = -\alpha (\ddot{R}-\ddot{\phi}) \]  
(3.14a)

\[ \delta (\ddot{\chi}-\ddot{\psi}) = -\alpha (\ddot{\chi}-\ddot{\psi}) \]  
(3.14b)

\[ \delta (\ddot{\lambda}_1-H_R) = -\alpha (\ddot{\lambda}_1-H_R) \]  
(3.14c)

\[ \delta (\ddot{\lambda}_2-H_\chi) = -\alpha (\ddot{\lambda}_2-H_\chi) \]  
(3.14d)

\[ \delta H_q = -\alpha H_q \]  
(3.14e)

\[ \delta H_w = -\alpha H_w \]  
(3.14f)

\[ \delta [\lambda_1(T)] = -\alpha \lambda_1(T) \]  
(3.14g)

\[ \delta [\lambda_2(T)] = -\alpha \lambda_2(T). \]  
(3.14h)

This yields

\[ \delta S = \delta P + \delta Q = -2\alpha S \text{ where } \alpha > 0 \]  
(3.15)

which represents the descent property of the algorithm for small variations.
Using the transformations

\[ \Delta R = \alpha A_1(t) \]  
\[ \Delta \chi = \alpha A_2(t) \]  
\[ \Delta \lambda_1 = \alpha A_3(t) \]  
\[ \Delta \lambda_2 = \alpha A_4(t) \]  
\[ \Delta q = \alpha B(t) \]  
\[ \Delta w = \alpha C(t) \]  

one obtains from equation 3.14

\[ \dot{A}_1 = \phi_R A_1 + \phi_\chi A_2 + \phi_q \Delta q + \phi_w \Delta w - (R - q) \]  
\[ \dot{A}_2 = \psi_R A_1 + \psi_\chi A_2 + \psi_q \Delta q + \psi_w \Delta w - (\chi - \psi) \]  
\[ \dot{A}_3 = H_{RR} A_1 + H_R A_2 + H_{Rq} \Delta q + H_{Rw} \Delta w + H_R \lambda_1 \lambda_2 A_3 + H_R \lambda_2 A_4 - (\lambda_1 - H_R) \]  
\[ \dot{A}_4 = H_\chi R A_1 + H_\chi A_2 + H_\chi q \Delta q + H_\chi w \Delta w + H_\chi \lambda_1 \lambda_2 A_3 + H_\chi \lambda_2 A_4 - (\lambda_2 - H) \]  
\[ H \omega R A_1 - A + H_q q B + H q = 0 \]  
\[ H_w A_2 + H \lambda_1 A_3 + H \lambda_2 A_4 + H_{ww} C + H_w = 0 \]

which are independent of \( \alpha \). This linear system is solved by the method of particular solutions to find the perturbations \( A(t) \), \( P(t) \), and \( C(t) \). The functions \( \Delta R(t) \), \( \Delta \chi(t) \), \( \Delta \lambda_1(t) \) and \( \Delta \lambda_2(t) \) are updated according to
\[
\begin{align*}
\hat{\chi} &= R(t) + \alpha A_1(t) & (3.18a) \\
\hat{\chi} &= (t) + \alpha A_2(t) & (3.18b) \\
\hat{\chi}_1 &= \lambda_1(t) + \alpha A_3(t) & (3.18c) \\
\hat{\chi}_2 &= \lambda_2(t) + \alpha A_4(t) & (3.18d) \\
\hat{q} &= q + \alpha B(t) & (3.18e) \\
\hat{w} &= w + \alpha C(t) & (3.18f)
\end{align*}
\]

The scaling factor \( \alpha \) is determined such that \( \hat{\chi} < S \). A bisection process starting from the reference value \( \alpha = 1 \) is employed for this purpose. The algorithm is stopped when \( S < \varepsilon \). In this study \( \varepsilon = 10E-4 \).

3. Pindyck Formulation for a Fixed Reserve Base

In this case the resource stock is fixed, and producers vary the rate of production of a given stock of resource. For this case the optimal time horizons are relatively short. The optimal time horizon for a competitive producer is approximately 27 years. However, as convergence to the desired level of accuracy for the algorithm could not be achieved for production periods of 17 years and over, the results for the optimal horizon are not reported here.

The optimal time horizon for the monopolist is about
36 years. The monopolist producer extracts at a low rate, decreasing production as the reserve base is depleted. Initial price is $19.89 and rises to a price of $33.00 at the end of the production period. Ninety-three percent of the resource is extracted during this time. The present value of profits for the monopolist is $67968 million.

For the common comparison period of 15 years both the competitive and monopolistic industry extract at higher rates than in their respective optimal time horizons. This is because the period of extraction and maximization is shorter. For competition 91 percent of the reserve base is depleted in 15 years. The time paths for quantity produced and the reserve base are presented in Figures 1a-1b.

For monopoly the production path is very flat, almost horizontal and prices increase only 2.3 percent in 15 years. This is explained by the cost structure and the short production period. The monopolist extracts at a slow rate and extraction costs are small, especially in the early years. The effects of time discounting are also modest as the production horizon is relatively short. Without discounting and constant extraction costs, the producer would extract the same quantity in each time period and prices would remain constant. As demonstrated
in figures 1c and 1d, this is approximately the behavior of the monopolist. The decline in the reserve path is approximately constant and the monopolist extracts 63 percent of the resource in 15 years.

4. Pindyck Formulation for a Variable Reserve Base Case

As expected the optimal production periods are considerably longer when reserves are augmented through exploration. The optimal production periods for competition and monopoly are 57 and 130 years respectively.

Since extraction costs are inversely related to the level of reserves, extraction costs will be small and production will begin at a high rate. When the reserve base is relatively small, exploratory activity will be substantial resulting in a large increase in reserves in the early years of the production period. The level of exploratory activity will then decline for a period and then increase once again to replenish the depleting reserve base. As cumulative discoveries increase over time, the diminishing returns to scale character of the exploration function comes into play and the exploratory activity has to increase to offset this effect. As reserves are depleted, extraction costs increase and production declines. At the end of the time horizon, reserves fall to a level
Figure 1. Pindyck Formulation for a Fixed Reserve Base. Base Case
a. Competitive Reserve Path; b. Competitive Quantity; c. Monopoly Reserve Path;
d. Monopoly Quantity
where addition to profits from an extra unit of reserves becomes zero. That is, the shadow price of reserves $\lambda_1$ becomes zero.

For competition initial price is $5.00 and the production rate is 558 million barrels. Price rises steadily to $25.00 and production declines to 157 million barrels. The total amount of new discoveries through exploration is 15,745 million barrels.

For monopoly, there is also a high level of exploratory effort at the start of the production period and an increase in the initial reserve base. Costs are lowered by this increase in reserves and production increases in the first 6-7 years, leading to a drop in price. As the reserve base is depleted, production declines and price increases monotonically until the end of the time horizon. However, most extractive activity takes place in the first half of the time horizon. Hence, the price path is fairly flat in the first 50 to 55 years, becoming steeper as production drops. The general path of exploratory effort is the same as for competition. Exploration decreases after the initial surge of exploratory activity but increases later to keep the reserve base from being depleted. Exploratory activity drops to zero at the end of the time horizon. Optimal trajectories for price and
quantity were found to vary little for time horizons of 95 years or more. Initial price and quantity remain the same in this range. Also, the change in profits is found to vary little with the length of the time horizon. For example, the percentage difference between profits in 97 years and in 130 years is .06%, increasing from $92962 million to $93017 million. On the other hand, prices in the final years and total discoveries increase with the time horizon. Total discoveries are 17250 million barrels and price reaches $30.20 in 130 years.

For a 50 year time horizon in the competitive case, initial and final prices are lower than those for the "optimal" 57 year time horizon. Since maximization takes place over a shorter length of time, initial production is higher, resulting in lower initial prices even though initial extraction costs are the same as in the "optimal" case. Initial price and quantity are $4.88 and 560 units with prices increasing to $18.90 in the 50 years. As before, initial exploratory activity is high and leads to an increase in reserves. The reserve base, quantity, price and exploratory activity paths are shown in Figure 2. The lower final prices are explained by the shorter time horizon. There are more reserves left in the ground at the end of the production period and extraction costs are
lower. Hence, marginal extraction costs and prices are lower. Total maximized discounted profit is $45823 million in the 50 years.

The optimal paths for the monopolistic producer for a production plan of 50 years are qualitatively the same as for the optimal time horizon of 130 years. High exploratory activity in the first five year period increases reserves and production. Prices fall from an initial value of $17.57 to $17.46. Although the path for exploratory activity is also sinusoidal in this case, it is rather flat after the initial period. As shown in Figure 3c, the paths for quantity and price are also very flat: prices increase only 5% in 50 years. Again this is explained by the cost structure. Marginal costs are very low because the monopolist depletes the reserve base slowly. Hence, production and prices remain stable for long periods. Sixty-eight percent of the reserve base is depleted in 50 years with total discoveries of 10145 million barrels. Total discounted profit is $88467 million.

5. The Alternate Formulation (The Cobb-Douglas Formulation)

As noted in Chapter II, extraction costs are not an explicit function of the level of reserves in this formu-
Figure 2. Pindyck Formulation for a Variable Reserve Base. Competition Base Case

a. Reserve Path; b. Quantity; c. Exploratory Effort; d. Price
Figure 3. Pindyck Formulation for a Variable Reserve Base. Monopoly Base Case

a. Reserve Path; b. Quantity; c. Exploratory Effort d. Price
lation. The major differences in the optimal solution of this system and Pindyck's formulation can be explained by differences in cost structure. The optimal time horizon is between 90 and 95 years under competition and between 105 and 110 years for the monopolist.

For a 90 year time horizon, the competitive producer starts with a low level of exploration activity since initial reserves are high enough to support high extraction rates. The depletion of the reserve base increases extractive costs (effort) and exploratory effort is increased. The resource is extracted rapidly in the initial years until a quasi steady-state is reached at around 30 years. In this initial phase, price increases from $9.56 to about $20.00. The system remains stable for 50 years with only slight changes in price, extraction rate and exploration levels. Undiscounted profits are relatively stable in these 50 years. However, as discounted profits approach zero, price increases to $23.89, exploratory effort becomes zero and the remaining resource is rapidly extracted. At the end of the time horizon, the competitive producer leaves 339 units (million barrels) of resource in the ground. Total discoveries are 17612 units and total profits are $63044 million. This quasi steady-state behavior is explained by the exploration function. As discussed in Chapter II,
accumulated discoveries have a negative effect on future discoveries, i.e., \( G \prec C \). However, \( G \prec C \) is not a strictly decreasing function; it levels off after a certain level of accumulated discoveries (\( G \prec C \) at some point). This implies that a slightly increasing level of exploratory activity will keep the amount of exploration and hence the reserve base constant. This particular exploration function will be utilized in most of the analysis. Three other exploration functions with different parameters are also used to investigate the differences in price, extraction and exploration paths. It should be noted that such a "plateau" is not necessarily observed in the 50 year base case because the level of accumulated reserves do not increase far beyond the critical level where \( G \prec C \) function becomes flat.

As would be expected, the optimal time horizon for the monopolistic producer is longer than for competition. The rate of extraction is lower, prices are higher and the overall level of exploration is lower. As in the competitive case, initial exploratory activity is very low and the existing stock is rapidly depleted. Since the monopolist extracts at a slower rate, prices do not increase as steeply as under competition. Price increases from an initial value of $18.87 to about $24 in 40 years and continues to rise slowly to $25 during the next 50 years.
Extractive effort and exploratory effort also remain fairly constant in the middle period. As discounted profits tend to zero, exploratory activity stops, the reserve level drops to 295 units and final price jumps to $26.36. Total discounted profit for the monopolist is $84371 million.

In the base case of 50 years, the price and quantity paths are not very different from those observed in the "optimal" time horizon. For competition, initial quantity and price are almost identical to their "optimal" levels. Because of the shorter time horizon, the level of exploratory activity is much higher in this case. Following a sharp initial increase, exploratory activity rises steadily until 30-35 years and declines later as production falls and discounted profits approach zero. The final price of $23.37 is very close to that of the "optimal" case ($23.88). The level of reserves left in the ground is slightly higher: 366 million barrels compared to 339 million barrels. Accumulated discoveries in this case are about one-half of those of the "optimal" case. Total profits are $60350 million.

The 50 year time horizon case for the monopolist is also very similar to the monopolist's behavior in the optimal time horizon. In this case, the monopolist extracts at a slightly higher rate resulting with a lower
price path. Due to the shorter time horizon, initial exploratory activity is higher than in the "optimal" case and rises steadily. Relative to the competitive case, the bulk of exploratory activity is undertaken later in the time horizon. From Figure 4c, we see that exploratory effort peaks around 40 years. For both market structures, but especially for monopoly, the producer does not invest in new capital (i.e., he does not explore), until production becomes significantly more expensive. For monopoly, total discoveries are 1/3 of their "optimal" values; total discounted profits are $80903 million.

The 50 year cases are quite similar to the "optimal" time horizon cases for both market structures. The major difference is that total level of exploration is much higher in the longer time horizon in order to keep the reserve level from falling to zero, from prolonged extraction. The 50 year case will be considered the "base case" for both market structures. A comparison of the monopoly and competition base case paths for reserves, quantity, exploratory effort and price are presented in Figures 4a through 4d.

I have also calculated optimal solutions for a 30 year time horizon. Here initial extraction and exploration rates are higher for both producers. Overall, the price
Figure 4. Cobb-Douglas Formulation. Base Case

a. Reserve Path; b. Quantity; c. Exploratory Effort; d. Price

Legend: ———— Competition; ———— Monopoly
path is shifted down. Total discoveries and profits are also lower. The trend that becomes evident is that as the time horizon increases, producers decrease initial exploratory activity, postponing it to the future when costs will be discounted more heavily and prices will be higher. Extraction rates are also lower since there is a longer period to exploit the resource.

Optimal solutions of the Cobb-Douglas formulation with different reserve bases were also explored. The effects of doubling and halving the initial reserve base on the time paths of quantity and exploratory effort can be seen in Figures 5a through 5d. When the initial reserve base is doubled to 14340 units, the competitive producer extracts at a higher rate for a longer initial period. Hence prices are lower than in the base case. For example, initial price is down by 45% ($5.29) and final price by 37% ($22.50). Initial exploratory activity is very low in the first 15 years, but increases to keep the level of reserves from falling too sharply. Total discoveries are 30% lower than in the base case. Although the production rate is high in the beginning, extractive effort and extractive costs are low due to the high level of reserves. However, since exploratory activity is less and extraction rate are higher, extractive effort in the later years is much higher than in
the base case. Due to lower prices and higher extraction costs in the labor years, total discounted profit is 6.3% less at $56530 million.

Doubling the reserve base in the monopoly case increases the initial production rate. The high level of reserves results in low extraction costs and low exploratory activity. Total new discoveries are only 22% of those in the base case. Since there is very little exploration, the time paths of quantity and reserves are similar to the solutions for the Pindyck formulation for a fixed reserve base. The quantity path is shown in Figure 5c. The reserve path is almost linear, leading to an approximately constant production profile. Since marginal extraction and exploration costs are nearly constant, undiscounted profits are also fairly equal in each time period. The small increase in price, together with low extraction and exploration costs lead to a total discounted profit of $93616 million which is 16% higher than the base case. Hence, a large reserve base implies increased profits for the monopolist and reduced prices for the consumer but virtually no exploration.

If the initial reserve base is halved, production levels decrease and prices increase in both market structures. Extractive effort and exploratory effort are
higher in each period. Profits decrease by 10% in competition and by 18% in monopoly. The smaller decrease in competitive profits can be explained as follows: halving the reserve base leads to an increase of 50% in initial prices for competition. While extraction and exploration costs are high, instantaneous profits are very high in the initial period. In monopoly however, prices increase by only 10% initially and profits are lower in each period than the base case.

I have varied the parameters in the exploration function to study the effects of different exploration functions. The weights \( \beta_1 \) and \( \beta_2 \) were initially set as .6 and -.05. I first keep \( \beta_1 \) constant and let \( \beta_2 = -.1 \). The exploration function is still a diminishing returns to scale function, but more so: an extra unit of cumulative reserves has a more negative effect for future discoveries. Hence, overall exploratory effort is less with this new exploration function. Production is also lower to make up for the decrease in exploratory activity and to keep the reserve base from being depleted too quickly. Despite lower extraction rates, 82% of the resource is depleted by the competitor and 70% by the monopolist in the first half of the 50 year time horizon. This increases price for both producers. In fact, price
Figure 6. Different Exploration Functions. Cobb-Douglas Formulation. Competition.

a. Reserve Path; b. Quantity; c. Exploratory Effort; d. Price

Legend: —— Base Case; ——— $\beta_1 = .6, \beta_2 = -.1$ ——— $\beta_1 = .8, \beta_2 = -.05$ ——— $\beta_1 = .3, \beta_2 = -.05$
Figure 7. Different Exploration Functions. Cobb-Douglas Formulation. Monopoly.

a. Reserve Path; Quantity; c. Exploratory Effort; d. Price

Legend: Base Case; $\beta_1 = .6, \beta_2 = .1$; $\beta_1 = .8, \beta_2 = .05$; $\beta_1 = .3, \beta_2 = .05$
paths for the monopolist and the competitor are very similar in the final years. Final prices for the competitive producer and monopolist are $27.89 and $28.18 respectively. Due to lower extraction and exploration costs and higher prices, total discounted profit for the competitor is $64585 million, 7% higher than in the base case. Although costs are lower for the monopolistic producer, the percentage decrease is less. Since the monopolist is also operating at a very elastic portion of the demand curve, instantaneous profits are lower at each period relative to the base case. This results in a total discounted profit of $78692 million, which is 98% of the base case.

The second variation in the exploration function consists of letting $\beta_1 = .3$ and $\beta_2 = -.05$. This dampens exploration since new discoveries due to an extra unit of exploratory activity decreases sharply as $\beta_1$ is decreased. With this exploration function, exploratory activity and total discoveries are so low that price and quantity paths simulate the case with no exploration. Firms start with a lower production rate since additions to the reserve base will be very small. The competitive producer depletes 95% of the initial reserve base in the first half of the time horizon. This yields a price increase of 150% in 25 years. Extraction rates are very
low in the second half of the time horizon with prices reaching the cut-off level and reserves being completely depleted. The time path for exploratory activity is similar to the path observed when the initial reserve stock is doubled. However, we obtain very different price and quantity paths in these two cases as is shown in Figures 5, 6 and 7. Price and quantity paths are not constant in this case because the decrease in exploratory effort is not motivated by a high resource stock. In the doubling of the initial resource stock, extraction costs were very low due to high reserve levels. In this case marginal extraction costs are higher and increasing with time, leading to increasing prices. Total discoveries for competition and monopoly are 3.5% and 10% of those in their respective base cases. Due to low exploration costs and high prices, maximized profit is $65652 million, 8.8% higher than in the base case. Similar behavior is observed in the monopoly case, but profits drop by 7.3% to $74987 million, as the monopolist operates in the more elastic portion of the demand curve relative to the base case.

The third exploration function considered is one where $\beta_1 = .8$ and $\beta_2 = -.05$. This has the effect of increasing the marginal productivity of exploratory
activity and leads to increased levels of exploration. However, the paths for exploratory effort are quite different.

In competition, exploratory activity is very high in the first years of production, raising the reserve level above the initial value. In all other cases, exploration serves the purpose of maintaining the reserve base. In the present case, exploratory activity is high, adding to the initial reserve base for the first 20 years. Due to increased reserves, production also increases in this period and level of extractive effort decreases. Hence, prices drop further to $4.14 from $5.16. As cumulative discoveries increase however, the diminishing returns to scale character of the exploration function becomes dominant and the depletion of the resource begins. Production starts to fall and prices begin to increase. At the end of the production period, reserves are 62% of the initial reserve base. The extraction rate is high and prices increase to only $5.69. Total profits are only $16349 million as a result of low prices and high costs from high levels of extraction and exploration. Total discoveries are 194% higher than in the base case.

In the monopoly case, although total discoveries are higher, the total level of exploration is much lower
since less exploratory effort is needed to discover a unit of new reserves. The reserve path is very flat and decreases very slowly. Due to increased discoveries, production is higher in each time period than in the base case. If the marginal productivity of exploratory effort is unity without diminishing returns to scale effects from the accumulated reserves, one obtains a steady state with a constant level of exploratory activity which is just enough to offset production in each period. Although the diminishing returns to scale structure of the exploration function is evident here, the effects are slight. Time paths for all variables are very flat and undiscounted profits in each time period are approximately equal. At the end of the time horizon, 37% of initial reserves are left in the ground and total discoveries are 139% more than in the base case. Profits are 10% higher in this case as a result of low extraction and exploration costs and production in a very elastic portion of the demand curve. The time paths for price, quantity, reserves and exploratory effort are shown in Figures 7a through 7d.

Finally, an exploration function with $\beta_1 = .95$ and $\beta_2 = 0$ is considered. The assumption of diminishing returns has been relaxed to search for a long run steady
state for the system. In this case, both producers settle at a steady state where marginal revenue (MR) equals the sum of the marginal extraction and exploration costs as shown in Chapter 2.

Numerically, the steady state is characterized by both types of market structures producing at the maximum point of their total revenue curves, where MR is equal to zero. For the monopolist, \( MR = p(1+1/\varepsilon) = 0 \) implies that the monopolist produces where price elasticity of demand equals 1. This translates to a price of $16.50 with the constant demand function employed in this study. However, due to discounting effects, the monopolistic steady state price is $17.00. For the competitive producer, \( MR = 0 \) implies that price equals zero. Hence the competitive producer tries to bring both her marginal cost and price down to zero. The steady state price for the competitor was found to be $1.60.

I experimented with two other discount rates, \( r = 0.05 \) to see how the system responds to different levels of discounting. It is well known that a rise in interest rates will reduce the level of exploration.\(^3\) If the discount rate is increased to 0.08 both producers shift extraction toward the present and exploration toward the future. Hence, they produce more when the reserve level
is relatively high and extraction costs low. Exploration is undertaken in later years when the exploration costs are more heavily discounted. Although total discoveries are slightly less with the higher interest rate for both market structures, exploratory effort in the later years is higher than in the base case, as shown in Figures 8 and 9. Final prices are higher in this case. Maximized profits decrease for both types of producers, by 35% in competition and by 30% in monopoly. If the discount rate is decreased to .01 the system responds in the opposite manner. As the discount rate is close to zero, production and exploration in each period are relatively equal. Production is postponed to the future leading to higher initial prices and lower final prices. As indicated by Figures 8c and 9c, exploratory activity is very high initially and stays constant for about 30 to 35 years and then falls below the base case level in the final years. Hence exploratory activity is shifted to the present since future costs weigh heavier than in the base case. Total discoveries are about 13% higher for both monopoly and competition with the lower discount rate. Since profits in each period are discounted less, total maximized profits increase by 112% in competition and by 101% in monopoly.
Figure 8. Different Interest Rates. Cobb-Douglas Formulation. Competition.

a. Reserve Path; b. Quantity; c. Exploratory Effort; d. Price

Legend: —— Base Case; —— r = .01; ---- r = .08
Figure 9. Different Interest Rates. Cobb-Douglas Formulation. Monopoly.

a. Reserve Path; b. Quantity; c. Exploratory Effort; d. Price

Legend: — Base Case; —— r=.01; ——— r=.08
The last two experiments in the alternate formulation were with shifting demand functions resulting from increases in income. Income grows at .5% a year in the first case and at 2.5% in the second. Shifting demand has the same qualitative effects on both types of producers. As demand increases, all variables of the system increase. There is more production and exploratory activity in each period. The price schedule is shifted up. The greater the increase in demand, the higher are maximized profits.
FOOTNOTES

1. As defined in Chapter II, $R$ is the level of reserves, $W$ the level of exploratory effort and $X$ the level of accumulated reserves from exploration.


3. The exploration decision is an investment decision and is similar to a decision for investment in reproducible capital. In the long run, as the interest rate rises, exploration and extraction will be reduced. (See Church (1981)).
IV. SEVERANCE AND PROFIT TAXES

The simulation results for taxation and partial expensing are presented in this chapter. The numerical results are consistent with the theoretical analysis of these taxes. A severance tax shifts the price schedule upward for both market structures and decreases the rates of extraction and exploration, thus enhancing the conservation of the resource. A production royalty has the same effects as the severance tax.

The literature has shown profit taxes to be nondistortionary. This study confirms that profit taxes which allow for complete expensing of extraction and exploration costs are nondistortionary. Simulation experiments show that with incomplete expensing, profit taxes are essentially nondistortionary as long as the tax rate is low and the percentage of costs and royalties that cannot be expensed is low.

The deadweight losses obtained in this study are surprising low, especially in the Pindyck formulations. Contrary to expectations, the marginal deadweight losses are also not very different from average deadweight loss figures for low tax rates. The Cobb-Douglas and the Pindyck formulations have been utilized to analyze the effects of severance taxes on competitive and monopoly market struc-
tures. The competitive case, with a Cobb-Douglas formulations is analyzed first.

1. The Cobb-Douglas Formulation

In the Cobb-Douglas formulation, the effect of severance taxes in a competitive market is small. The total effect, as measured by the deadweight losses, varies with different parametrizations of the model. In general, though, the deadweight losses are quite low. One important reason underlying this result is that the initial reserves are a fixed factor of production and the competitive producer has positive economic rents. The impact of taxes on prices is buffered to the extent that the tax burden is absorbed by producers in the form of lower rents. Thus, the severance tax is not passed on fully to the consumers, e.g., a 5% tax increases initial price by 25% and a 20% tax by 10%. The tax becomes more distortionary as the tax rate increases. A 50 percent tax increases prices by 34 percent. The effects of severance taxes on price and quantity paths are shown in Figures 10-13. As is evident from the graphs, price increases due to the tax becomes more prominent in the later years. This results partially from the fact that the resource is depleted toward the end of the time horizon and the shadow price of reserves are driven to zero. The producer then passes a higher percent-
age of the tax burden on to the consumers. In the base case, the reduction in producer's rents by taxes of 5, 20 and 50 percent is slightly lower than the corresponding tax rate. Deadweight losses, expressed as percentage of tax revenue (hereafter referred to as percentage deadweight losses) are 1.1, 5.2 and 17.5 percent for tax rates of 5, 20 and 50 percent, respectively.\textsuperscript{1}

The relative magnitude of deadweight losses is altered by extraction and exploration costs, as increases in these quantities reduce rents. Increased levels of exploration and extraction due to different extraction and exploration functions increase the respective total costs. However, the net effect on deadweight losses is complicated by the fact that there are also demand side effects. Increased exploration may lead to production in a very inelastic portion of the demand curve. For this case, severance taxes will lead to low compensating variation figures, and this coupled with inelastic supply will result in lower deadweight losses than those in the base case.

Deadweight losses are lowest in cases with little or no exploration. Rents are relatively high when the producer extracts from a fixed reserve base. Hence the producer absorbs a larger portion of the tax burden and prices increase by a smaller percentage than in cases with exploration. The exploration function with $\beta_1 = .3^2$ results
in a case with almost no exploration. Severance taxes in this case have very little effect on prices, with a 5% tax increasing initial competitive prices by .9% and a 20% tax by 4.5%. Although production takes place in an elastic portion of the demand curve, the presence of high rents effectively prevents the producer from passing the tax burden on to the consumer. Percentage deadweight losses associated with taxes of 5 and 20 percent in this case, are .1 and .6% respectively. As will be seen later, the same type of behavior is also observed in the Pindyck formulation with a fixed reserve base.

The exploration function with \( \beta_1 = .8 \) also leads to lower deadweight losses than the base case (\( \beta_1 = .6 \)). From the preceding analysis, one would expect just the opposite to occur since this exploration function leads to very high levels of exploration. However, the high levels of exploration result in a higher level of reserves and high production rates at a very inelastic portion of the demand curve.

Supply is also relatively inelastic due to higher level of reserves and lower extraction costs. And the decrease in profits due to severance taxes is much less than in the base case. Hence, deadweight losses in this case are lower than in the base case and range from .3 to 2.2 percent for taxes of 5 and 20 percent.
A similar case to the base case arises with the exploration function with $\beta_2 = -1$. This exploration function leads to similar type of behavior as in the base case, although the levels of extraction and exploration are smaller. Rents are higher in this case due to lower extraction and exploration costs. Therefore deadweight losses are slightly lower than in the base case. They are 0.9 and 4.0 percent for taxes of 5 and 20 percent respectively.

As with different levels of exploratory activity, different levels of initial reserves also alter the manner in which severance taxes effect the system. A higher reserve base yields lower extraction costs and lower levels of exploratory activity, dampening the effects of the tax. When the initial reserve base is doubled, the effects of a severance tax are less pronounced than in the base case. Production is in a less elastic portion of the demand curve due to higher reserve levels and lower costs. A similar result occurs in the case of the exploration function with $\beta_1 = .8$ and deadweight losses in this case are also very low. The deadweight losses range from .7 to 3 percent for tax rates of 5 and 20 percent.

When the initial reserve base is halved, deadweight losses are greater. Due to lower reserve levels, costs
are higher and increase sharply as the resource is depleted. Higher costs result in lower rents and coupled with low production levels at more elastic portions of the demand curve, they lead to high deadweight losses.

To check the sensitivity of the deadweight losses due to changing demand conditions, a demand function which shifts with time due to increases in income, is also analyzed. The deadweight losses are not altered greatly. The changes in deadweight losses with the two shifting demand functions specified earlier are shown in Table 3. The percentage changes in all the variables except for total exploration are the same as the base case with severance taxes. Quantitatively, deadweight losses are higher because of higher actual magnitudes for the control variables.

Marginal deadweight losses have also been calculated for the base case, for tax rates varying from 5 to 50 percent. As can be seen in Figure 15, they are higher than the average deadweight loss figures. Both the average and marginal deadweight losses are of the same level of magnitude for low tax rates. However, marginal deadweight losses increase sharply as the tax rate is increased.

As the tax rate increases from 5 to 50 percent, total tax revenues also increase fairly linearly, closely following the percentage increases in the tax rate. When the
Figure 10. Severance Taxes. Cobb-Douglas Formulation. Competition.

a. Reserve Path; b. Quantity; c. Exploratory Effort; d. Price

Legend: ——— Base Case; ——— tx=.5; ——— tx=.2; ———— tx=.05
Figure 11. Severance Taxes. Cobb-Douglas Formulation. Monopoly.

a. Reserve Path; b. Quantity; c. Exploratory Effort; d. Price

Legend: ------ Base Case; --- tx=.5; --- tx=.2; ------ tx=.05
tax rate is increased ten-fold from 5 to 50 percent, tax revenues increase 9.59 times. Both the compensating variation and the reduction in producer's profits increase at a higher rate than the percentage increase in the tax. The rate of reduction in profits is higher than the rate of increase in compensating variation however, and this discrepancy increases as the tax rate increases. This implies that the tax burden is shifted to the producers as the tax rate is increased.

2. The Pindyck Formulation

The effects of severance taxes in the Pindyck formulation have also been analyzed for fixed and variable reserve bases. Deadweight losses, in competition with a fixed reserve base are the lowest encountered in this study, due to high rents. As would be expected, the imposition of a severance tax does not lead to sharp price increases. A severance tax of 5 percent increases initial and final prices by 1.2 and .8 percent respectively. For a tax rate of 20 percent, the respective increases are 5.6 and 2.8 percent. The effect on future prices is less because discounted tax liabilities are reduced in the future.

Tax induced price increases are higher in the Pindyck formulation with exploration. Severance taxes of 5 to
Figure 12. Severance Taxes. Pindyck Formulation for a Fixed Reserve Base.

a. Competitive Reserve Path; b. Competitive Quantity; c. Monopoly Quantity

Legend: —— Base Case; —— tx=.2; ——— tx=.05
20 percent increase prices by 3 to 13 percent. A larger percentage of the tax burden is shifted to the consumer in this case because rents are lower due to increased costs from exploration. Moreover, supply is more elastic than the fixed reserve base case because of exploration. Hence deadweight losses are higher in this case. The percentage deadweight losses associated with taxes of 5 and 20 percent are .3 and 1.6 percent respectively.

When compared to the base Cobb-Douglas case, deadweight losses are lower in the Pindyck formulation with a variable reserve base. The higher deadweight losses of the Cobb-Douglas formulation may be explained in the following manner. In the Pindyck case, exploratory activity in the beginning years increases the initial reserve base, thus reducing extraction costs. Therefore price increases due to the tax are small in the early years. Whereas in the Cobb-Douglas formulation, exploratory activity barely replenishes the diminishing reserve base, resulting in higher extraction and exploration costs and thus higher prices.

The effect of severance taxes on a monopoly market is qualitatively similar to competition. The imposition of the tax leads to increases in price, reduction in extraction and exploration and decreases in profits. Rents in this market are higher than in a competitive market, as
they are made up of monopoly rents plus rents due to the fixed factor of production, the resource base. Hence the monopolist absorbs a higher percentage of the tax burden than the competitive producer through decreased rents and profits. Moreover, monopoly production is always in an elastic portion of the demand curve. This results in lower price increases induced by the tax. In the base case for the Cobb-Douglas formulation, severance taxes of 5, 20 and 50 percent lead to increases in initial prices of .4, 1.9 and 6 percent, respectively. The reduction in monopoly profits with the same taxes are 5.7, 22.5 and 55 percent.

As expected, percentage deadweight losses in monopoly are greater than in a competitive market. The deadweight losses associated with taxes of 5, 20 and 50 percent are 12.0, 14.1 and 20.8 percent respectively. The difference in the deadweight losses for the two market structures are lessened as the tax rate increases and becomes more distortionary. However, the deadweight loss calculations do not cast any light on the differential tax incidence problem in the two market structures. Table 1 shows the deadweight losses, the compensating and reduction in profits for severance taxes of 5 and 20 percent. Although deadweight losses are lower in competition, the compensating variation figures are much higher. Hence the consumer bears
a larger part of the burden of the tax in competition. The higher deadweight losses in monopoly reflect the larger reductions in producer's profits.

3. Profit Taxes and Partial Expensing

As was proven in Chapter 2, the profit tax is nondistortionary in that it does not alter price, extraction or exploration paths. Deadweight losses are zero with a profit tax because the tax revenue is exactly equal to the loss in producer's profits.

Profit taxes are essentially nondistortionary when royalties are partially expensed. The slight distortion due to partial expensing increases as the profit tax rate and the percentage of royalties that cannot be expensed increases. Assume first that the producer can expense from her current profit tax liabilities only 80% of the royalties paid, given that she is paying a production royalty of 5 percent. The change in prices due to partial expensing is nil for a 5% profit tax and at most one-third of a percentage point for a 40% profit tax in all three formulations. The largest distortion is when the producer can only expense 20% of royalties and when the profit tax rate is 40%. Even in this case the most change in prices was 1.6 percent. Consequently, the additional deadweight losses associated with the partial expensing of royalties are not
Figure 13. Severance Taxes. Pindyck Formulation for a Variable Reserve Base.

a, b. Competitive Quantity and Exploratory Effort
c, d. Monopoly Quantity and Exploratory Effort

Legend: —— Base Case; —— tx=.2; ——— tx=.05
very high. In fact, percentage deadweight losses fall as
the tax rate increases due to increasing tax receipts and
nearly constant deadweight losses. Similar to the severance
tax cases, monopoly profits and competitive are affected
more by partial expensing. Percentage deadweight losses
are higher for a monopolistic producer. These results are
summarized in Table 4.

The partial expensing of extraction costs yields dif-
f erent results for certain cases of the different formu-
lations. In the case with no exploration, the partial ex-
pending of extraction costs leads both the competitor and
the monopolist to produce less since it has become more
costly to produce. This affect on quantity and price in-
creases with increasing tax rates and expensing parameters.
Due to lower production rates, the reserve path is shifted
upward with partial expensing.

In the Pindyck formulation for a variable reserve base
and in the alternate formulation, the partial expensing
of extraction costs yields interesting results. As dis-
cussed in Chapter 2, the effect of partial expensing of
extraction costs on exploratory effort was ambiguous and
depended on quantity produced in the Pindyck case and
changes in price and extractive effort in the alternate
formulation. Since there is a single pool of the resource,
the exploratory activity adds to the level of reserves
and serves to bring down extraction costs. This effect is more pronounced in the Pindyck case where extraction costs are an explicit and inversely varying function of the level of reserves. With partial expensing, both the monopolist and the competitor reduce quantities produced and raise prices. Exploratory effort is increased in the first half of the time horizon to keep extraction costs down. This increase is greater in monopoly. In fact, the monopolist's total discoveries is higher than in the base case and increases with the tax rate and expensing parameters. As shown in Figure 14a and b, both producers cut back on exploratory activity in the later years as the discounted present value of the tax liabilities decrease. The production path is shifted down with partial expensing for the monopolist. Although the same is true for the competitive producer for a majority of the time horizon, she starts producing more in the last ten years. For both producers, the partial expensing of extraction costs leads to higher reserve levels over time due to low extraction and high exploration rates.

In the Cobb-Douglas formulation, the partial expensing of extraction costs leads to lower production levels and higher prices. Exploratory effort is higher at the beginning of the time horizon when the present value of tax
liabilities is highest, but later falls below the base case level. The paths for exploratory effort are presented in Figure 14c and d. Total discoveries are lower with the partial expensing of extraction costs for both producers.

In the fixed and variable reserve base cases for competition, partial expensing of costs is less distortionary than severance taxes for low profit tax levels and low expensing parameters, but becomes comparable as these parameters increase. For monopoly in the alternate formulation however, severance taxes cause more distortion. In fact, the highest percentage deadweight loss associated with the partial expensing of extraction costs is less than the loss associated with the lowest severance tax rate.

The partial expensing of exploration costs has similar effects on both types of producer in the two formulations with exploration. With partial expensing, exploration becomes more costly and leads to increases in prices and cuts in exploratory activity. As in the partial expensing of extraction costs, the distortion in optimal trajectories due to partial expensing increases as the profit tax rate and the percentage of costs one cannot expense increases. These results are summarized in Table 4.
Figure 14. Partial Expensing of Extraction Costs.

a, b. Pindyck Formulation for a Variable Reserve Base. Exploratory Effort for Competition and Monopoly; c, d. Cobb-Douglas Formulation. Exploratory Effort for Competition and Monopoly

Legend: ——— Base Case; ——— ξ = .8; ——— ξ = .5; ——— ξ = .2
Figure 15. Deadweight Losses (as percentage of tax revenue)

Legend:
- Average DWL:
- Marginal DWL.
## EFFECTS OF SEVERANCE TAXES

<table>
<thead>
<tr>
<th></th>
<th>TX</th>
<th>PR</th>
<th>%APR</th>
<th>CV</th>
<th>DWL</th>
<th>%DWL</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Competition</td>
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<td>34272.27</td>
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<td>-.223</td>
<td>2194.15</td>
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<td>216.61</td>
<td>.077</td>
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<td>.0906</td>
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<td>PVRB</td>
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<td></td>
<td></td>
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<tr>
<td>Competition</td>
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<td>-.05</td>
<td>1249.51</td>
<td>12.01</td>
<td>.003</td>
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<td></td>
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<td>5598.91</td>
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<td>.016</td>
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<td>Monopoly</td>
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<td>-.05</td>
<td>310.75</td>
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<td>-.223</td>
<td>1438.31</td>
<td>1467.54</td>
<td>.074</td>
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<td>Alternative Formulation</td>
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<tr>
<td>Competition</td>
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<td>-.043</td>
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<td>51.87</td>
<td>.011</td>
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<td>-.179</td>
<td>8850.49</td>
<td>977.86</td>
<td>.052</td>
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<td>Monopoly</td>
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<td>-.057</td>
<td>539.93</td>
<td>551.09</td>
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<td>.20</td>
<td>62699.24</td>
<td>-.225</td>
<td>2323.50</td>
<td>2539.66</td>
<td>.141</td>
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</table>

PFRB - Pindyck Formulation for a Fixed reserve base
PEVB - Pindyck Formulation for a Variable Reserve Base
PR  - Total discounted profits
CV  - Total discounted compensating variation
DWL - Total discounted deadweight losses.

### TABLE 1
# ALTERNATE FORMULATION - SEVERANCE TAXES

## BASE CASE

<table>
<thead>
<tr>
<th>$\beta_1 = .6$, $\beta_2 = -.05$</th>
<th>COMPETITION DWL</th>
<th>MONOPOLY DWL</th>
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<tr>
<td>$tx = .05$</td>
<td>.011</td>
<td>.12</td>
</tr>
<tr>
<td>$tx = .20$</td>
<td>.052</td>
<td>.141</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$\beta_1 = .3$, $\beta_2 = -.05$</th>
<th>COMPETITION DWL</th>
<th>MONOPOLY DWL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$tx = .05$</td>
<td>.001</td>
<td>.035</td>
</tr>
<tr>
<td>$tx = .20$</td>
<td>.006</td>
<td>.042</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>$\beta_1 = .8$, $\beta_2 = -.05$</th>
<th>COMPETITION DWL</th>
<th>MONOPOLY DWL</th>
</tr>
</thead>
<tbody>
<tr>
<td>$tx = .05$</td>
<td>.003</td>
<td>.09</td>
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<td>$tx = .20$</td>
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<td>.108</td>
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<table>
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<tr>
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<th>COMPETITION DWL</th>
<th>MONOPOLY DWL</th>
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</thead>
<tbody>
<tr>
<td>$tx = .05$</td>
<td>.009</td>
<td>.098</td>
</tr>
<tr>
<td>$tx = .20$</td>
<td>.04</td>
<td>.115</td>
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<table>
<thead>
<tr>
<th>$r = .01$</th>
<th>COMPETITION DWL</th>
<th>MONOPOLY DWL</th>
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<tbody>
<tr>
<td>$tx = .05$</td>
<td>.013</td>
<td>.147</td>
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<tr>
<td>$tx = .20$</td>
<td>.027</td>
<td>.156</td>
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<table>
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<th>$r = .08$</th>
<th>COMPETITION DWL</th>
<th>MONOPOLY DWL</th>
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<td>.009</td>
<td>.101</td>
</tr>
<tr>
<td>$tx = .20$</td>
<td>.02</td>
<td>.107</td>
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</table>

DWL = Deadweight Loss  
$\beta_1 = \beta_2 =$ exploration function parameters  
$r =$ interest rate  

**TABLE 2**
**ALTERNATE FORMULATION - SEVERANCE TAXES**

**DEADWEIGHT LOSSES**

<table>
<thead>
<tr>
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<th>COMPETITION</th>
<th>MONOPOLY</th>
</tr>
</thead>
<tbody>
<tr>
<td>RØ = 14340 bbm</td>
<td></td>
<td></td>
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<tr>
<td>tx = .05</td>
<td>.007</td>
<td>.058</td>
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<td>tx = .20</td>
<td>.03</td>
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<td>tx = .20</td>
<td>.074</td>
<td>.204</td>
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<tr>
<td>Increasing Demand</td>
<td></td>
<td></td>
</tr>
<tr>
<td>y = y_o + 100t</td>
<td></td>
<td></td>
</tr>
<tr>
<td>tx = .05</td>
<td>.011</td>
<td>.12</td>
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<td>.146</td>
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<tr>
<td>Increasing Demand</td>
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<td>y = y_o + 500t</td>
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<tr>
<td>tx = .05</td>
<td>.013</td>
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<td>.162</td>
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<tr>
<td>Tl = 30 yrs</td>
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<td></td>
</tr>
<tr>
<td>tx = .05</td>
<td>.009</td>
<td>.099</td>
</tr>
<tr>
<td>tx = .20</td>
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<td>.114</td>
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</table>

RØ - initial resource stock

Tl - time horizon

**TABLE 3**
**ALTERNATE FORMULATION - PARTIAL EXPENSING**

**DEADWEIGHT LOSSES**

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<tr>
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<th><strong>COMPETITION</strong></th>
<th><strong>MONOPOLY</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>(tp = 0.05, tr = 0.05, \gamma = 0.2)</td>
<td>0.0069</td>
<td>0.067</td>
</tr>
<tr>
<td>(tp = 0.40)</td>
<td>0.0023</td>
<td>0.018</td>
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<tr>
<td>(tp = 0.05, tr = 0.05, \gamma = 0.5)</td>
<td>0.0072</td>
<td>0.067</td>
</tr>
<tr>
<td>(tp = 0.40)</td>
<td>0.0033</td>
<td>0.021</td>
</tr>
<tr>
<td>(tp = 0.05, tr = 0.05, \gamma = 0.8)</td>
<td>0.0072</td>
<td>0.067</td>
</tr>
<tr>
<td>(tp = 0.40)</td>
<td>0.0042</td>
<td>0.024</td>
</tr>
<tr>
<td>(tp = 0.05, \xi = 0.2)</td>
<td>0.0001</td>
<td>0.021</td>
</tr>
<tr>
<td>(tp = 0.40)</td>
<td>0.003</td>
<td>0.02</td>
</tr>
<tr>
<td>(tp = 0.05, \xi = 0.5)</td>
<td>0.0006</td>
<td>0.03</td>
</tr>
<tr>
<td>(tp = 0.40)</td>
<td>0.013</td>
<td>0.048</td>
</tr>
<tr>
<td>(tp = 0.05, \xi = 0.8)</td>
<td>0.002</td>
<td>0.047</td>
</tr>
<tr>
<td>(tp = 0.40)</td>
<td>0.027</td>
<td>0.075</td>
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<tr>
<td>(tp = 0.05, \mu = 0.2)</td>
<td>0.0003</td>
<td>0.014</td>
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<tr>
<td>(tp = 0.40)</td>
<td>0.0066</td>
<td>0.022</td>
</tr>
<tr>
<td>(tp = 0.05, \mu = 0.5)</td>
<td>0.0014</td>
<td>0.035</td>
</tr>
<tr>
<td>(tp = 0.40)</td>
<td>0.032</td>
<td>not converged</td>
</tr>
<tr>
<td>(tp = 0.05, \mu = 0.8)</td>
<td>0.004</td>
<td>0.056</td>
</tr>
<tr>
<td>(tp = 0.40)</td>
<td>0.06</td>
<td>0.082</td>
</tr>
</tbody>
</table>

\(tp=\) profit tax rate  
\(tr=\) royalty rate  
\(\gamma = \)% of royalties not expensed  
\(\xi = \)% of extraction costs not expensed  
\(\mu = \)% of exploration costs not expensed  

**TABLE 4**
FOOTNOTES

1. The deadweight loss can also be expressed as a percentage of 1975 GNP to facilitate comparisons with monopoly. In this case, the deadweight loss associated with a severance tax of 5% will fall to .004 percent.

2. $\beta_1$ and $\beta_2$ are the exploration function parameters, as defined in Chapter III. $\beta_1$ is the weight on the level of exploratory effort and $\beta_2$ the weight on the level of accumulated reserves.

3. The corresponding deadweight losses in competition were 1.1, 5.6 and 17.5 percent, for the Cobb-Douglas base case.
V. Concluding Remarks

The dynamic analysis of this dissertation confirms the static results regarding exhaustible resources and taxes on a number of accounts. It has been shown that severance taxes and royalties lead the producer to postpone production to the future and cause a shift in the time path for prices. This shift in price, quantity and exploration paths is much more pronounced for the competitive case. Again consistent with the static results, profit taxes are shown to have no effect on optimal paths for quantity, price or exploratory effort, but they reduce the firm's profits by a factor of the tax rate. Lower profits imply a decrease in the rate of return to capital in the extractive industry. This would normally have an effect on the level of investment (i.e., exploration in our case) in the long run. However, since the models employed here do not allow for capitalization, such a reduction in exploratory activity is not observed.

This simulation has also confirmed the fact that in a perfectly competitive market, production and exploratory effort are always higher and prices lower when compared to a monopoly, given a linear demand curve. Monopoly leads to the conservation of both the known resource stock and the undiscovered reserves. Although this may seem attractive
from a conservation point of view, it should be remembered that the excess burden associated with the monopolistic producer is 3 percent, expressed as percentage of GNP.

The deadweight losses calculated in this study are very low, especially for low severance tax rates. For tax rates up to 20 percent, the deadweight losses expressed as percentage of tax revenue are all less than 6 percent for a competitive market. Although deadweight losses from taxing a monopolistic industry are relatively higher, the tax burden falls mainly on the monopolistic producer and not the consumer. The deadweight losses associated with the partial expensing of royalties are also very low. Consequently, if royalties were perceived as a state or local tax on gross value of production, the partial expensing of these taxes would not lead to large excess burdens and would bring higher tax revenues. These findings seem to confirm the point of view that taxes on energy are relatively efficient.

The approach used in this study employs a closed economy model. Although most of the parameters and functions are derived from oil production and exploration data, the international aspects of oil pricing and production are not incorporated into this model. The model is more applicable to the natural gas market since the United States is more
nearly self sufficient in natural gas.

In both the formulations analyzed, there exists only one pool of resource and there is no differentiation between the initial resource stock and the newly discovered reserves. A possible extension of this study would be to generalize the solution method to incorporate a number of reserve bases. This can be done with additional computational effort. The model can then be applied to different markets, specifically to natural gas taxation and deregulation.
VI. APPENDIX

This appendix will present certain derivations that were omitted in the text due to their lengthiness and similarity to previous derivations.

1. Derivation of the time rate of change of quantity for a monopolistic producer with a fixed reserve base:

The first order conditions derived in chapter 2 for the monopolist are given by

\[ H_q = [(f'q+f)(1-tp) - \tau r] - \theta q^2 e^{-rt} + \lambda = 0 \]

\[ \lambda = H_R = -\theta C_R q e^{-rt} \]

taking the time derivative of \( H_q \), we obtain

\[ [(f''q+2f'q)(1-tp) - \tau r] - \theta q^2 q + \theta C_R q + \theta C_R q \]

\[ + \lambda - re^{-rt} [(f'q+f)(1-tp) - \tau r] - \theta (C_q + C) = 0 \]

Substituting for \( \lambda \) and arranging terms,

\[ q^2 f'[(1-tp) - \tau r] - \theta (C_q + 2C_q) = \]

\[ r[(f'q+f)(1-tp) - \tau r] - \theta (C_q + C) + \theta C_R q e^{-rt} \]

\[ q = \frac{r[(f'q+f)(1-tp) - \tau r] - \theta (C_q + C) + \theta (C_R q^2 + C_R q)}{(f''+2f')(1-tp) - \tau r] - \theta (C_q + 2C_q)} \]
2. Derivation of the time rate of change of quantity for a monopolistic producer with a variable reserve base:

The first order conditions obtained from the Hamiltonian are

\[ H_q[(f'q+f)\chi-\theta(C_qq+C)]e^{-rt} + \lambda_1 = 0 \]

where \( \chi = (1-tx)(1-tp)-\psi tr \)

\[ H_w = -C_p\theta e^{-rt} - C_w(\lambda_1 + \lambda_2) = 0 \]

\[ \lambda_1 = H_R = -C_R\theta e^{-rt} \]

\[ \lambda_2 = H_\chi = -C_\chi(\lambda_1 + \lambda_2) \]

Differentiating \( H_q \) with respect to time

Differentiating \( H_q \) with respect to time

\[ [(f''qq+2f'q)\chi-\theta(C_qq+2C_q+qR+qR+R)]e^{-rt} \]

\[ -re^{-rt}[(f'q+f)\chi-\theta(C_qq+C)] - C_R\theta e^{-rt} = 0 \]

\[ q[(f''qq+2f')\chi-\theta(C_qq+2C_q)] = r[(f'q+f)\chi-\theta(C_qq+C)] \]

\[ +\theta(C_qR-C_q^2+C_RG) \]

\[ q = \frac{r[(f'q+f)\chi-\theta(C_qq+C)] + \theta(C_qR-C_q^2+C_RG)}{(f''qq+2f')\chi-\theta(C_qq+2C_q)} \]
The derivation of \( w \) in the monopoly case is exactly the same as in competition and leads to the same expression as one can see by comparing equations (2.38) and (2.42).

3. Derivation of extractive effort and exploratory effort for a monopolistic producer, in the alternate formulation.

The first order conditions for profit maximization yield:

\[
H_\lambda = [(f_{g} + f_{q}) (1-t x)(1-t p) - \psi t r] - \theta p_1 e^{-r_t + \lambda_1 q_x} = 0
\]

substituting \( \chi = (1-t x)(1-t p) - \psi t r \), and \( f_{g} = -\beta q_x \)

\[
(-\beta q_a q + f q_{x}) \chi - \theta p_1 + \lambda_1 q_x e^{r t} = 0
\]

\[
(-\beta a q / \lambda q + f a / \lambda) \equiv - \theta p_1 + a \lambda_1 q / \lambda e^{r t} = 0
\]

\[
(-\beta a q + a f) \equiv \theta p_1 / q + a \lambda_1 e^{r t} = 0
\]

\[
\lambda = \frac{\alpha \lambda_1 q e^{r t} + a p q \chi - a \beta q^2 \chi}{\theta p_1}
\]

one can also solve for \( \lambda_1 \) from \( H_\lambda \),

\[
\lambda_1 = \frac{\theta p_1 \lambda + \beta a q^2 \chi - a p q \chi}{a e^{r t} q}
\]
Substituting $\lambda_1$ into $H_w$,

$$-\theta p_2 - \beta_1 w^{\beta_1-1} \lambda_2^{\beta_2} \beta_2^{\beta_1-1} \lambda_1^{\beta_2} \left[ \frac{\theta p_1 + \beta a q^2 - \alpha p q x}{\alpha e^{rt} q} \right] = 0$$

$$w^{\beta_1-1} \left[ -\beta_1 \lambda_2 e^{rt} - \beta_1 \lambda_2^{\beta_2} \left( \frac{\theta}{\alpha} p_1 \lambda_2 + \beta q x - p x \right) \right] = \theta p_2$$

$$w = \left[ \frac{\beta_1 \lambda_2 (p x - \beta q x - \lambda_2 e^{rt} - \theta p_1 \lambda_2) \theta p_2}{1 - \beta_1} \right]$$

4. Derivation of the negativity of $\lambda_1$, in the alternate formulation:

From the first order conditions for the alternate formulation,

$$H_\lambda = p q x - \theta p_1 + \lambda_1 q_\lambda e^{rt} = 0$$

with $\lambda$ and $\theta$ defined as in Chapter 2. We obtain for $\lambda_1$,

$$\lambda_1 = \frac{\theta p_1 - p q_\lambda}{q_\lambda e^{rt}} = (\theta p_1 / q_\lambda - p)e^{-rt}.$$ 

$q_\lambda$ is the marginal product of extractive effort, and is positive. Since $p$ is the price of the good, it is greater than the factor price, and also greater than $p_1 / q_\lambda$. Thus $\lambda_1$ is negative.
5. Derivation of the exogeneous price of extractive effort.

The price of extractive effort is calculated such that initial total costs are equal in the Pindyck Case and in the alternate formulation.

Total Cost in Pindyck Case:
\[ TC = C(R)q = 1.25q \]

Total Cost in Alternate Formulation:
\[ TC = p_1 \ell \]

Solving for \( \ell \) from the production function,
\[ \ell = q^2/R \]

Substituting
\[ TC = p_1q^2/R . \]

Equation total costs,
\[ 1.25q = \frac{p_1q^2}{R} \]

we obtain
\[ p_1 = \frac{1.25R}{q} . \]

For competition, initial \( q \) in the Pindyck case is approximately 600.
\[ p_1 = \$14.94 \]

For monopoly, initial \( q \) is approximately 300,
\[ p_1 = \$29.87 . \]

Hence \( p_1 = 1/2(p_1^C + p_1^M) = \$22.406 \)
VII. BIBLIOGRAPHY


