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ESSAYS ON THE INFLATION TAX

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ESSAYS ON THE INFLATION TAX

by

James C. Dyer, IV

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

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HOUSTON, TEXAS

APRIL, 1981
This study, "Essays on the Inflation Tax," is three related, but independent papers. They are united by the central theme that inflation is a tax on the purchasing power of money. As a tax, inflation should be analyzed for its efficiency and equity. The former is the sole topic of this study. The contribution to economic theory is essentially one of synthesis. The thesis endeavors to integrate hypotheses and concepts from monetary theory, the economics of information and uncertainty, capital market theory, welfare economics and public finance, especially tax incidence theory, regarding market behavior and inflation. The core issue in this synthesis is whether the inflation tax is a neutral tax.

The inflation tax is efficient, or neutral, if no product substitutions arise from its imposition. It is shown that the inflation tax is not neutral. The tax is inefficient because even in an idealized economy, real resources will be consumed in attempts to avoid or shift the burden of the tax. There are four primary determinants to the magnitude of the inefficiency of the tax: (1) the size of the direct burden of the tax, (2) the accuracy and pervasiveness of expectations, (3) the uncertainty about future inflation, and (4) institutional rigidities and market imperfections associated with economic behavior which is restricted to responses to
nominal prices. The incidence of the tax can be identified only when all market responses caused by an exogenous change in the tax are analyzed and measured simultaneously. The global burden of the inflationary tax is equal to its direct burden if and only if the cost of the tax in any single market is strictly independent of the cost of the tax in all other markets. This condition also describes tax neutrality.

The concept of "neutral money" has a long standing place in economic theory. It has recently received renewed attention because of its close association with the concept of rational expectations. Simply stated, changes in the money supply which are anticipated "rationally" cause no real effects. Now, if inflation is assumed to be caused solely by changes in the money supply, it follows that rationally anticipated inflation has no real (substitution) effects. But the inflation tax is not neutral. Herein lies a serious conceptual conflict. This text is a study and resolution of this conflict.

If the inflation tax is endogenous, the expected inflation rate and real market quantities are jointly and simultaneously determined. An endogenous inflation tax is "neutral" in that it has no causal connection with real behavior. This is the interpretation of the Fisherian Hypothesis, which states that the nominal rate of interest is equal to the real rate of interest plus the expected future
inflation rate. However, an endogenous inflation tax may have caused a loss in social welfare greater than the inflation tax revenues, i.e., excess burden which implies non-neutrality. Hence, the inflation tax can at one moment be non-neutral (exogenous), and yet at a later moment it can be neutral, or more accurately "endogenous."

The three essays which comprise this work examine this issue of the neutrality-endogeneity of the inflation tax by critically evaluating existing theories and hypotheses about the costs and effects of inflation. In general, it is shown that most of these are either seriously flawed or simply erroneous.
Dedicated to the Memory of

Dolores Rosemary Birdsong Dyer,

Nova A. Vernus and Isabel Birdsong
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PREFACE

In the four years since I began this study much has occurred throughout the world to draw attention to the economics of inflation. Perhaps the most significant event has been the double-digit inflation in the United States. More than at any previous time, the economics of inflation is the dominate issue in the economics profession. The scientific literature and public press abound with pieces on the cause or causes of inflation, the costs and effects of inflation, and the remedies for inflation. Yet, today, as four years ago, there is no unified theory of inflation, save for the Classical Quantity Theory which implies inflation causes no disturbances to real choices.

This study is an attempt to integrate existing knowledge about inflation into a unified theory. This is done first by establishing that inflation is a tax on money. As such, the concepts of tax incidence theory can be applied to the study of inflation. Incidence theory provides the overall framework for the analysis of inflationary financing; a framework which dictates a coherent structure to the questions about inflation. The study of tax incidence is the study of the size and distribution of the tax burden. This requires the conceptual distinction between the source of the tax and the use of the tax, either of which may influence real economic choices. If real choices are not impaired by the tax (except for the income effort), the
tax is said to be neutral, a concept similar to the neutrality condition in the Quantity Theory. If real choices are affected, however, the problem becomes one of ascertaining who bears the ultimate burden of the tax. This involves the study of the excess burden and growth effects from the tax, as well as an analysis of any shifting of the tax between sectors. In short, the theory that inflation is a tax organizes and discriminates among the various concepts and folklore about inflation.

This study is three separate but interrelated papers. The first paper, "The General Theory of Inflationary Finance," develops the basic conceptual issues of the inflation tax and integrates core concepts from monetary theory, the economics of uncertainty and information, capital market theory, welfare economics, and public finance. The second paper, "The Question of Inflation Tax Neutrality: A Review of Selected Literature," applies the concepts from the first paper to an analysis of the workings of the financial and labor markets during inflationary periods. The concepts are applied by evaluating the large existing body of literature on inflation in these markets. This approach is adopted to further integrate existing knowledge about the process of inflation. The final paper, "The Inflation Tax and the Theory of the Firm," presents a model of the rate of return to the firm which is conceptual consistent with contemporary corporate finance theory and a non-neutral inflation tax.
The inflation tax is not a neutral tax. The tax is inefficient because even in an idealized economy, real resources will be consumed in attempts to avoid or shift the burden of the tax. The inflation tax may, however, be endogenous. The distinction between inflation tax neutrality and endogeneity is the cornerstone to the unified theory of inflation presented in this thesis. It is, therefore, the theme which ties the three essays together.

This study has benefited from discussions with my colleagues, Michael Maher and Tom Respess, and from the critical appraisal of Dr. Thomas McCaleb of Florida State University. My mentor, Dr. Marian Krzyzaniak, a strict disciplinarian in economics, gave me the freedom to make my own mistakes, and to learn from them the true meaning of scientific inquiry. His insightfulness to the issues was exceeded only by his patience of an often stubborn and frequently mistaken student. The personal debt I owe him goes far beyond this treatise. Needless, to say, only I bear the final responsibility for the errors yet remaining.

The preparation of this study was facilitated by the support provided by Texas Commerce Bank, while I was an Economist and Assistant Vice President, and from Texas Energy Petroleum Inc., where I enjoy the position of Director of Planning and Coordination. I value highly the excellent job performed by Ms. Linda Lagerquist and Mrs. Peggy Romans who typed and structured more versions of this document than any of us care to remember.
I gratefully acknowledge the moral support given by my father and mother to my academic pursuits. Although I frequently doubted whether this point would ever be reached, they never lost faith that it would if only I was dedicated to it. Finally, my greatest debt is to my wife, Karen, who unknowingly on her wedding day married what turned out to be only a part-time husband; and to my baby, Jimmy, who has achieved the advanced age of ten with precious little help from his father. Without them I would have completed years earlier; yet I also know that without them I may never have finished.
THE GENERAL THEORY OF INFLATIONARY FINANCE

A. INTRODUCTION

The basic postulate of this essay is that inflation is a tax on money. This is not a novel concept. Nonetheless, this concept has not been integrated into the traditional public finance curriculum, reflecting in part the fact that the cause of inflation remains an open and debated issue. The objective of this essay is to specify the necessary conditions for inflation to be properly classified as a tax, and to identify and elaborate upon the fundamental concepts of inflationary finance, by which we mean the set of assumptions and economic properties which constitute the inflation tax paradigm.

The contribution to economic theory this essay endeavors to make is essentially synthetical. The model of inflationary finance presented here provides a framework from which both explanatory and predictive judgments can be made about the consequences of exogenous
changes in the inflation tax rate. Much of this framework already exists in the various areas of economic specialty, including monetary theory, the economics of uncertainty and information, capital market theory, welfare economics, and of course public finance. The objective of this paper is to integrate seemingly unrelated principles and concepts from these areas, and thus form the coherent whole we call "the general theory of inflationary finance."

Having established in Section B that a theory of inflationary finance implies that inflation is a monetary phenomenon, we proceed to examine the rudimentary public finance principles of the incidence of the inflation tax. The following three principles are established:

1. Inflation is a tax on the purchasing power of money balances.

2. The inflation tax is an inefficient tax because, even in an idealized economy, real resources will be consumed in attempts to avoid or shift the direct burden of the tax, causing relative price effects.

3. The excess burden of the inflation tax, that is the welfare loss to society not offset by revenues to the government, is greater the less certain the private sector is about what the actual inflation tax rate will be. Uncertainty results in unanticipated inflation taxes which distort real choices. It is reasonable to expect that the greater is the extent to which the inflation tax is unanticipated the larger will be the welfare loss to society and the longer it will take for real choices to adjust to the unanticipated shock.
These three principles form the core of the theory of this study. In essence the theory states that inflation is a tax which has real effects, i.e., is nonneutral. At the moment a change in the inflation tax is announced by the government or the moment before it becomes anticipated by the private sector there is an exogenous shock to the private economy which induces people to change their choices - their behavior - so as to avoid the new tax. Once the new tax is anticipated, however, it becomes endogenous, which is to say it no longer causes adjustments to real choices. At any point in time, anticipated inflation is the "legacy" of a stream of past unanticipated changes in the inflation tax rate. Consequently, to understand the incidence of the inflation tax it is necessary to study the process by which the economy modifies its real behavior and thereby converts an exogenous shock into an endogenous phenomenon.

Assume that the inflation rate had always been 3 percent and that it is expected to continue to be the same 3 percent forever. In an otherwise perfect and stationary economy, the actual inflation tax will have no affect on current real choices. It is fully reflected in current prices, gross-of-tax rates of return, etc. In other words, by this time the inflation tax is perfectly endogenous. Once the tax rate is known, real variables adjust until the net-of-tax returns are equalized. No new adjustments are called for as long as the tax rate remains constant. Suppose however, that new information becomes available which indicates that the future inflation tax rate will be 4
percent instead of 3 percent. There is, in the terminology of this study, an ex-ante unanticipated inflation effect, which is a change in anticipations. There is an exogenous change in the anticipated inflation rate. The second principle above states that this form of unanticipated inflation is a shock to the economic system which causes people to elect alternative investment-consumption plans. In other words there are substitution effects as the private economy adjusts with nonzero duration from the old (pre-new information) equilibrium to the new equilibrium. As we shall demonstrate, however, once the new equilibrium is achieved, the new anticipated 4 percent inflation tax no longer causes any substitution behavior. At that point the tax is once again fully anticipated, fully reflected in prices, and thus endogenous. But what if the actual inflation rate is not equal to the anticipated rate? The answer to this question leads to the third principle. As explained in Section D, there will be ex-post unanticipated inflation effects. The literature refers to this as simply "unanticipated inflation." For the most part we do not distinguish between ex-ante unanticipated inflation and ex-post unanticipated inflation. The distinction is not found in the literature. Furthermore, predictable economic responses are qualitatively equivalent for both. The analysis of inflation expectations does yield two propositions:

1. Perfectly anticipated inflation has no real effect, its effects having occurred in the past before it was first anticipated.
2. Only errors in anticipation of the rate of inflation are exogenous and thus capable of causing substitution effects.

The standard textbook approach to demonstrating the market responses to taxation is ill-suited for the inflation tax. First, as argued in Section D, expectations, especially of the future inflation rate, play a crucial role in determining the costs and effects of inflationary finance. Standard tax evaluation models ignore expectations. Second, public finance models traditionally (and implicitly) treat the tax in question as if it were assessed in a world of certainty. This of course explains the absence of expectational considerations. In Section E we argue that the uncertainty about the future inflation rate is conceivably as significant a determinant of real behavior as is the inflation rate.

The customary approach, as found in the scholarly literature, is to assess the economic consequences of inflation by applying partial equilibrium, single equation models. But by treating inflation as a tax this approach is immediately seen to be unsatisfactory. Today it is generally accepted in the field of public finance that only a general equilibrium model can adequately account for all of the price and quantity relationships disturbed directly or indirectly by a change in a tax rate. This applies with equal force to the inflation tax. Indeed, we contend it is precisely a failure to evaluate
inflation in a simultaneous multi-market context that explains the widely divergent empirical results found in the inflation literature. This issue is explored in Section F.

The rudimentary aspects of inflationary finance are presented in Section B. One of the objectives is to elaborate on the theoretical link between the Monetarists theory of the monetary process and the concept of inflation as a tax. Section C describes some of the social costs and economic effects of inflation which are frequently cited in the literature. The aim is to define certain concepts and to establish the fact that the accuracy of expectations of the future inflation rates, more than any other single factor, determines the economic consequences of inflation.

Finally, a caveat is in order. Throughout this study the words "anticipated" and "expected" are used as synonyms. Technically of course when "expected" refers to the first moment of the conditional probability distribution function of forecasts it is but just one measure of "anticipated."
B. INFLATIONARY FINANCE: THE BASICS

The concept of inflation as a tax is not germane to "normal" public finance theory. This at least is the implication of the fact that public finance textbooks do not, as a general rule, describe the incidence of the inflation tax. Yet it is common knowledge that governments must balance their budgets by either taxing the incomes and wealth of the private sector, by borrowing, or by expanding the (high powered) money supply. The latter alternative, according to the Quantity Theory, ultimately results in a proportional rate of increase in the general level of prices. The rise in prices lowers the purchasing power of existing real cash balance. Thus, in effect, purchasing power is transferred from the private sector to the public sector. But is that not precisely what any conventional tax does.

The concept of inflation as a method of taxation is by no means new. Keynes devoted an entire chapter in *Monetary Reform* [1923] to public finance and changes in the money stock. He stated of inflation taxation:

A Government can live for a long time, even the German Government or the Russian Government, by printing paper money. That is to say, it can by this means secure the command over real resources,—resources just as real as those obtained by taxation. The method is condemned, but its efficacy, up to a point, must be admitted. A government can live by this means when it can live by no other. It is the form of taxation which the public find hardest to evade and even the weakest Government can enforce, when it can enforce nothing else. (p. 41)
Keynes also commented with respect to the incidence of the inflation tax,

On whom has the tax fallen. . . . The burden of the tax is well spread, cannot be evaded, costs nothing to collect, and falls, in a rough sort of way, in proportion to the wealth of the victim. No wonder its superficial advantages have attracted Ministers of Finance. (p. 48)

A. C. Pigou [1929] also identified the inflation tax,

Plainly, the creation of bank credits, whether for the government directly or for private people who have taxes and loan subscriptions to pay to the government, is a mediating, not an ultimate, operation. Its effect is to give the government more purchasing power, and thus to deplete the real value of the purchasing power left to private persons. In this way it enables the government to get possession of more things and services, and so constitutes; as against the public, a concealed form of taxation. This taxation, moreover, is not graduated in any degree and not adjusted in any degree to the size of a man's family. It is simply proportionate to income, without even an allowance to very poor people of a tax-free minimum of subsistence. This kind of taxation is generally acknowledged to be exceedingly oppressive to the poor. (p. 268)

Thompson [1824] presented an early study of the inflation tax in the context of a discussion of the workings of the economy as modelled by the Quantity Theory.

The concept of inflation as a tax can be traced to the earliest usage of coins as money, when the tax took the form of "debasement." Rulers would obtain their revenues by "princely
counterfeiting", including admixture of base metals, seigniorage, or simply by clipping. Indeed, it has even been suggested that the early despots did not introduce coinage for the convenience of the public, but rather as a revenue measure.

Before proceeding to evaluate the mechanics of inflationary finance, it is appropriate that we clarify the connection between the government's budget and inflation, at least as it is defined for this study. In part, this is a question of definition. Inflation is defined as a process of continuously rising prices, or equivalently of continuously declining value of money. But the relationship is also a question of causality. As Laidler and Parkin [1975] note, there is far more agreement that inflation is fundamentally a monetary phenomenon, than there is agreement about the cause of inflation. It is not necessary to repeat here the arguments for and against the various theories of inflation. In any event, it would be impossible to improve on the collective surveys by Bronfenbrenner and Holzman [1963], Conard [1964], Johnson [1967], Morley [1971], Trevithick and Mulvey [1975], Laidler and Parkin [1975], and Frisch [1977], among others. One point that must be emphasized, however, is that this study assumes, as Friedman [1970] has stated that "Inflation is always and everywhere a monetary phenomenon in the sense that it is and can be produced by a more rapid increase in the quantity of money than in output."
For inflation to be classified as a tax it must be the case that government controls at least the tax rate, in this instance the inflation (tax) rate. Taxing is an active event, as distinguished from passive income received from gifts or windfall. The government may be influenced by current economic conditions in its choice of a particular tax policy. However, at some point in the process government finally exercises its monopolistic power to set a given (direct) tax rate. In the case of inflationary financing, government sets the tax rate by electing a given domestic credit policy. Expanding the credit supply at a rate equal to the growth of real income generates revenues to the government which can be used to finance expenditures. More rapid expansion of the credit supply leads to inflation which is as if the government taxed away an equal value in purchasing power by some more conventional means.

This scenario is obviously a Classical Monetarists theory of inflation. In light of such, several ageless issues come to mind; for instance whether public debt is real wealth (i.e., the definition of "credit"), whether the stock of money is endogenously determined (Banking School) or exogenously determined (Currency School), whether in the presence of growth of nonbank intermediaries, the monetary authority controls secular and cyclical monetary policy (i.e., the definition of "money"), etc. In other words, the assumption that inflation is always and everywhere a monetary phenomenon does not resolve many issues related to a tax theory of inflation. However,
adopting a public finance approach to inflationary finance does "resolve" most of these issues. For inflation to be a "tax" it must (1) result in income to the government, and (2) be subject to control as to either the amount of revenue or the rate of taxation. Inflation must be exogenous (to the private sector).

The relationship between the monetarist theory of inflation and public finance theory of the inflation tax is spelled out more completely in the following sections. The first concept to appreciate, however, is that a theoretical link exists between a monetary theory of the price level and the theory of inflation as a tax.

The critical link between the Monetarists theory and the concept of inflation as a tax is the central premise of the monetarists school that, at least in the long-run, money is neutral. Reinterpreted in public finance terminology, this assumption is equivalent to asserting that there is no excess burden from the inflation tax. That is, changes in the money stock, or more accurately in its rate of growth, do not produce changes in the equilibrium value of real variables.

It is judicious to detract from the main stream of development to briefly review the concepts of excess burden and efficient taxation - concepts used repeatedly throughout this essay. In short, any tax has an income effect because resources are transferred to the
public sector. This income effect has no bearing on whether the tax is efficient. An efficient tax is one for which there are no substitution effects. Substitution effects occur when relative prices are altered by a change in the tax. When this occurs the total loss of welfare from the tax as it is imposed exceeds the loss in welfare which would result if the same tax revenue were collected without distorting economic decisions in the private sector. The differential loss is the excess burden of the tax. An efficient tax is one which has zero excess burden, and hence is neutral because there are no substitution effects.

The literature on inflation is complete with countless references to the "cost and effects" of inflation, which generally pertains to income and wealth redistributions beyond the income effect. Clearly such references imply the inflation tax is not neutral; for income or wealth redistribution, apart from the income effect, can only result from relative price changes, a fact developed throughout this essay. Neutrality of the inflation tax implies no substitution effects on the "sources side" of the money supply. What substitution effects result must arise from the "uses side" of the tax. Neutral money, on the other hand, implies no excess burden.

The demonstration of these statements is, in principle, the overall objective of this study. In effect we seek an integration of contemporary monetary theory and public finance, incidence theory. To
this end, we turn to the first model of inflationary finance which we
denote the Bailey Model. Because it is a partial equilibrium
analysis, the Bailey Model is relatively simple, and hence is ideal
for demonstrating the rudimentary concepts of inflationary finance.
Other more elaborate partial and general equilibrium models will then
be presented in this and the following sections which integrate
monetary theory and tax incidence theory.

8.1 The Bailey Model of Inflationary Finance

An inflation tax which is levied in an otherwise perfect economy
and which is fully anticipated causes no redistribution of income in
the private sector. This result, which we evaluate later in this
paper, has been demonstrated by Sargent and Wallace [1975]. However,
inflation may still be an inefficient tax if economic resources are
used to economise on money (i.e., the tax base), which is costless to
produce. This concept was first succinctly demonstrated by Bailey
[1956]. His simple model is well suited for illustrating the first
and second principles of the inflation tax introduced in the previous
Section.

5
Bailey observed that most arguments against "open inflation" as a method of government finance concentrate on the income and wealth redistributive and disruptive aspects of inflation. He suggested another aspect of the inflation tax, namely its welfare cost.

This aspect is a welfare cost of open inflation, which, in effect, is a tax on the holding of cash balances, a cost which is fully analogous to the welfare cost (or "excess burden") of an excise tax on a commodity or productive service. (pp. 93-94)

Consider Figure I which depicts a liquidity preference function. The area under the demand schedule measures the social utility of holding real balances equal to \((M/P)_0\). The welfare cost is associated with a real-balance effect, and should be distinguished from redistributive-disruptive costs. Seven assumptions help to distinguish between these costs.

A.1) Homogenous expectations regarding the future rate of inflation.

A.2) No transactions costs or other market frictions to prevent appropriate adjustments to nominal prices.

A.3) No restrictions on the payment of interest on deposits and there is competition in the creation of deposits.

A.4) All economic agents are free of money illusion.

A.5) The marginal propensity to consume is equal for all economic agents.

A.6) The economy is at full employment output.

A.7) A stationary economy.
Assumptions A.1 through A.6 are sufficient for the neutrality of money. In such a world there is no incidental redistribution of real income from a sustained rise in prices. Assumption A.3 implies that the real value of demand deposits is not altered by inflation. Bailey assumed, "bank deposits are not used as money or are negligible in amount." The purpose of either A.3 or Bailey's assumption is to simplify the analysis by excluding any redistribution to the banking sector as a result of inflationary finance. Assumption A.3 has been chosen because it is sufficient to achieve the purpose, whereas Bailey's assumption was not. As Friedman and Schwartz [1969] have indicated, in the definition of money one must distinguish between the dimensions of "moneyness" and "interest-payingness." Finally, assumption A.7 is introduced merely for simplicity.

Assume the economy has real balances of \( \$1,000 \) at the general price level \( P_0 \) equals 1.0. Further assume that economy obeys the Fisharian Hypothesis, \( i = r + E(\rho) \), where \( r \) is the equilibrium real rate of interest which is assumed to be constant and \( E(\rho) \) is the expected future inflation rate, and assume that initially there is no anticipated inflation.

Now suppose that at time \( t_{0+\epsilon} \) (\( \epsilon \) very small) the government announces its intention to use inflationary financing to balance its budgeted deficit. The money supply will be increased by \( \$200 \). The anticipated one-period ahead inflation rate instantaneously goes from
zero to 20 percent, due to assumptions A.1, A.2, A.4, and A.5. Desired money holdings decline to \( M_0/\left[P_0+P_{0+\varepsilon}\left(_{1}\right)\right] \) as the market rate of interest increases to \( i_{0+\varepsilon} \) in accordance with the Fisherian Hypothesis. By the end of the period, that is by time \( t_1 \), the government has exploited its monopoly powers to create money and has acquired real resources in exchange for new money. The price level increases by exactly the anticipated rate of inflation (expectations are assumed to be errorless). The market interest rate retreats to its beginning of the period level because, by assumption, there is no information which indicates inflationary financing in period \( t_2 \). The private sector now holds \$1,200 in money balances worth the original \$1,000 in purchasing power - and to that extent is no worse off than at time \( t_0 \).

\[
\frac{M_0}{P_0+P_{0+\varepsilon}(\rho)} = \frac{\$1000}{1.20} = 833.33
\]

\[
\frac{\Delta M}{P_1} = \frac{\$200}{1.20} = 166.67
\]

\[
\therefore \frac{M_1}{P_1} = \frac{\$1200}{1.20} = 1000.00
\]

The model just described obscures the facts because it nets the sources and uses sides of the inflation tax. The sources side is depicted in Figure I as the move along the liquidity preference curve from \( \Omega_0 \) to \( \Omega_1 \). In a full employment, stationary economy, the
FIGURE I

Inflation and the Demand for Real Money Balances

\[ i_0 + \varepsilon \]

\[ i_1 = i_0 \]

\[ \frac{M_0}{P_0 + E_0 + \varepsilon (P_1)} \]

\[ \frac{M_0}{P_0} \]
percentage rate of monetary expansion, in equilibrium, equals the rate of inflation, and the flow demand for real balances equals the rate at which the existing equilibrium stock depreciates. The government revenue from inflationary finance is thus measured as $\rho(M_0/P_0)$, or $\$200$. In beginning of the period prices the inflation tax revenue is equal to $\rho(M_0/P_0)/P_1$, or $\$166.67$. This is the direct burden of the inflation tax, the area ($i_0$, $i_{0+\epsilon}$, $\Omega_1$, $\Omega'$). Because of the assumptions above, the direct burden is also equal to the uses side of the inflation tax (i.e., the incremental real money supply). All economic agents have the purchasing power of their original money balances exactly restored by the new money (the uses side of the tax). The important point is that the beginning of the period desired holdings of real cash balances depreciated in purchasing power by a sum equal to the direct burden. The same result would be achieved if the government merely confiscated 20 percent of the original real balances.

Bailey's main contribution to the theory of inflationary finance was the attention he drew to the "social cost" of the tax, that is to its excess burden. By assumption, the area under the liquidity preference schedule is the social value of real balances. Given assumption A.3, the real loss to society from holding cash balances during a period of inflation is the conventional triangle [$\Omega'$, $\Omega_1$, $\Omega_0$], as Bailey noted. However, if A.3 is replaced by the assumption of noninterest bearing demand deposits (and currency), then
the excess burden is the area $\{(M_0/P_0 + E_0 \epsilon (\rho_1)), \Omega_1, \Omega_0, M_0/P_0\}$. The rectangle is included because the initial holding of real balances $M_0/P_0$ is suboptimal from not paying interest on money. The market rate of interest measures the yield foregone on holding one dollar in cash balances (i.e., $i$ measures the marginal productivity of money). Therefore the area under the demand curve, for a given interval of real cash balances, "measures the flow of productivity from the indicated quantity of real balances."\textsuperscript{10} Consequently, because the services of cash balances cannot be provided by any alternative asset forms, the area also measures the welfare costs of the reduction of real balances by inflation.

Several authors have extended Bailey's original analysis. All of those studies reinterpreted Bailey as a study of the optimal rate of monetary expansion.\textsuperscript{11} The general hypothesis is that given the income growth rate, the demand for money function, and a specified inflation expectations mechanism, there should exist a revenue maximizing rate of inflation. This issue has received considerable attention in the literature. It is crucial, however, that it not be mistakenly equated with "optimal taxation." The optimality of a tax pertains to the difference between the total welfare cost of a tax and the loss which would result if the same tax revenue were collected without distorting economic decisions in the private sector. This difference is the excess burden of the inflation tax. It is fitting, nonetheless, that some attention be given to the corollary problem of
the revenue maximizing rate of inflation; both because it is an interesting problem in its own right and because it impinges upon the question of how the excess burden from the inflation tax should be measured. But so as not to detract from the basic concepts which we seek, this issue is addressed in Appendix A of this essay.

Economists familiar with monetary theory will recognize the Bailey model above as a special case of the Pigou effect. The distinction rests in the effects of inflation expectations. The transmission mechanism that produces the direct burden (i.e., the wealth effect) is different from the mechanism that yields the excess burden. The direct burden is obtained by the proportionality and neutrality conditions of the quantity theory. Therefore, as shown below, a Patinkin [1965, especially Ch. XI] type model is sufficient to guarantee the inflationary revenue. Moreover, no assumption regarding expectations formation is required. The increase in the money supply results in an implicit tax, even if the change is perfectly anticipated. The neutrality condition merely implies that, in the aggregate, economic agents understand that the unit of exchange has been changed. The level of real income, the real rate of interest, the real rate of capital formation, and the volume of employment are thus unaffected.
B.2 The Neoclassical Long-Run Demand for Money

The quantity theory of money held that the velocity of money circulation, the velocity of commercial bank deposits in circulation, and the proportion of reserves held by banks against demand deposits were all three independent of the money supply in the long run. Hence, in full employment, the money supply affects only the price level, which is to say the demand for money is a rectangular hyperbola (price elasticity of its demand is equal to unity). Consequently, money is neutral. As is well known to students of monetary theory, the neoclassical system was characterized by what is generally called the Classical dichotomy. The separation of the real and monetary sectors explained why the price level would change equiproportionately to a change in the money supply, but not how. The real sector determined relative prices, whereas the monetary sector determined the general price level. In other words, the demand for goods was assumed to be homogeneous of degree zero. At the same time, an increase in the money supply of x percent initially results in an excess supply of money, and hence an excess demand for goods. It is this excess demand for goods which ultimately forces the price level up by x percent. Clearly, the demand for goods cannot simultaneously be independent of the money supply and the mechanism by which money is neutral.
This brief review of monetary theory sets the stage for a serious attempt to integrate the monetary theory of the price level with a theory of inflation as a tax. It is generally known that Patinkin was largely responsible for solving the Classical dichotomy. In a series of articles, which were later expanded into a book, Patinkin sought to integrate value theory (i.e., price theory) and monetary theory. He did this by distinguishing between an individual experiment and a market experiment, which in turn led him to argue that the neoclassical theory failed to recognize that a change in the general price level would affect the demand for real balances (of money) and thus the demand for goods. It is of significant interest to this study to understand the workings of Patinkin's model. If the money supply is the source of inflation, and if in a monetary model money has real effects but is neutral in the long run, then seemingly a public finance interpretation would conclude that the inflation tax is neutral. Neutral money implies no excess burden to the inflation tax. Yet, incidence theory tells us that if money has real, substitution effects, if only in the short run, there will be welfare costs to the inflation tax, which by definition says money cannot be neutral. Clearly there is yet another "classical" dichotomy to be solved.

In the remainder of this section we present a brief review of a Patinkin-type theory of the neoclassical theory of the long-run (neutral) demand for money. Following that we reinterpret the model
treated inflation as a tax. The inflation tax is clearly shown to be non-neutral. Yet, it is conceivable money could be neutral in the long-run. This apparent contradiction is explained.

In a Patinkin [1965] world consumption, demand is a function of real prices, real goods, and real (money) balances. By assumption, real goods and real balances are both "superior" (positive sloped Engel curve). The homogeneity assumption (A.1) for the Bailey model is strengthened to say there are "no expectations of possible future general price-level changes." In conjunction with the other assumptions set out above, it follows that as the price level rises, the demand for real balances will fall and conversely, cet. par. This relationship is commonly referred to as the negative real balance effect. The negative real balance effect is the partial derivative of money demand with respect to the price level. The money supply is held constant. When both the goods and real balances are superior, the cash balances curve has a demand elasticity of less than unity. In addition to the negative real balance effect there is also the market adjustment which reflects the sum of the initial household responses to an increase in wealth due to an increase in the money supply. This is the positive real balance effect -- the partial derivative of money demand with respect to the nominal money supply.
The distinction between the two real balance effects must be clear. Figures II and III provide alternative representations of the Patinkin neoclassical demand for money. The aggregate demand for money balances is depicted as the $D_0D_0$ curve in Figure II. The $D_0D_0$ curve is the aggregation of individual-experiments. Equilibrium in the money market is at point $\Omega_0$. If the money supply instantaneously doubles, the typical household, having no prior expectations of the increase cash balances, believes its real wealth has increased. Before any price reactions, actual real balances exceed optimal real balances for each household and in the aggregate. With the economy at full employment (assumption A.6), the aggregate excess supply of money is equivalent to the aggregate excess demand for goods. The disequilibrium is represented by the segment $\Omega_0\Omega_1$ in Figure II. Given the increment in money balances each household's demand for real balances increases, which is to say an increase in wealth generates a greater transactions demand for real balances. This causes the aggregate demand curve for real balances to shift out to $D_1D_1$. If this positive real balance effect were sufficiently strong to shift the demand schedule to $\Omega_1$, there would be no excess demand for goods. More likely the demand curve will shift somewhere between $\Omega_0$ and $\Omega_1$, leaving an excess demand for goods equal to $\Omega_2\Omega_1$. 
FIGURE II
Nominal Money Demanded and Supplied

Price of Money

\( \frac{1}{P_0} \)

\( \Omega_0 \)

\( \Omega_1 \)

\( \Omega_2 \)

\( \Omega_3 \)

Money Balances

\( M_0 \)

\( 2M_0 \)

\( D^* \)

\( D_0 \)

\( D_1 \)
By assumption, the doubling of the money supply is achieved by increasing all household money balances in the same proportion - i.e., no redistributional effects. Consequently, all households hold an excess supply of money. With the private sector at full employment and with an excess demand for goods both in the aggregate as well as individually, the disequilibrium can only be resolved by a rise in the general price level. Patinkin assumed no relative price changes. This results from his assumption of a nonredistributional increment to the money supply, provided the economy is free of money illusion (assumption A.4 above) and if marginal propensities to consume are equal (A.5). Under these conditions, a rising price level only causes a negative real balance effect, contracting the demand for real balances. The economy moves down the \(D_1D_1\) curve, and continues to do so as long as the disequilibrium excess demand for goods exists. Equilibrium is restored at \(\Omega_3\) where real balances are equal to their original optimal level.

The movement from \(\Omega_0\) to \(\Omega_3\) is, therefore, the result of two separate forces: the positive real balance effect which shifts the money demand curve, and the negative real balance effect which moves the economy along the new demand curve. These are both short-run adjustments. The long-run market equilibrium demand for money is the locus of points containing \(\Omega_0\) and \(\Omega_3\), which in Figure II is
represented by the D*D* curve. This market equilibrium curve is a rectangular hyperbola. In other words, in the long run money is neutral.

Figure III is an alternative graphic presentation of the Patinkin model in the goods - real balances space. The corresponding points to Figure I are marked with the same symbols, the only difference being the fact that $\Omega_0$ and $\Omega_3$ are the same point in Figure III. The intent here is not to present a comprehensive general equilibrium proof that money is neutral in the neoclassical long-run model. The purpose is to recall the principle features of the economic mechanism which yields this result. These are the real balance effects. The excess supply of real balances generated by an increase in the money supply is fully absorbed by the positive and negative real balance effects. It is the separate but joint operation of the real balance effects that guarantees the price level will change in exact proportion to a nonredistributive change in the money supply.

The Classical dichotomy, which implies money is neutral and that households base their choices on real prices, is a characterization of the long-run equilibrium. The dichotomy is equivalently a separation theorem, and as any separation theorem, is not intended to describe the mechanism which generates the disjointness. In addition to the behavioral forces of the real balance effects, the assumptions that
FIGURE III

Inflation and the Demand for Money and Commodities
money balance changes are nonredistributive and all price changes are
equiproporionate are crucial to the neutrality of money. We now
reconsider the model from a public finance interpretation.

Recall Figure II. The positive real balance effect which shifts
the money demand curve out from \( D_0 \) to \( D_1 \) is, in
conventional tax incidence theory, the uses side associated with the
inflation tax. The nonredistributional assumption implies that the
allocation of the expenditure benefits is proportionate to all
households. What truly determines the equivalence of benefits are the
assumptions of no money illusion (A.4) and equal marginal propensity
to consume (A.5). In other words by assumption, whatever tax revenues
are generated by the inflation tax they are ultimately returned dollar
for dollar to the individual households.

The receipt of the (tax) expenditure in the form of incremental
cash balances induces each household to desire to maintain more real
balances. In the unlikely event that the incremental money demand
just equals the incremental money balances, households will be forced
to raise their consumption demand to achieve their respective
constrained utility maximization. As described above, the collective
behavior of all households attempting to eliminate their excess demand
for goods causes all prices to rise equiproporionately. As this
happens, the value of outstanding money balances declines. At this
point, one simple fact becomes absolutely crucial, namely that money
balances (i.e., monetary base) held in the private sector represent financial obligations of the government. A reduction in the value of these obligations is equivalent to taxing the private sector a like value by some more conventional means. Consequently, the behavioral responses in the private sector aimed at removing a disequilibrium in the goods and money markets brought about by an (unexpected) increase in money balances create a general price-level increase which taxes away some purchasing power (value) nothing more than the negative real balance effect. Hence we have the following equivalences:

\[
\begin{align*}
\text{Uses side of} & \quad \leftrightarrow \quad \text{Positive Real} \\
\text{the inflation tax} & \quad \text{Balance Effect} \\
\text{Sources side of} & \quad \leftrightarrow \quad \text{Negative Real} \\
\text{the inflation tax} & \quad \text{Balance Effect}
\end{align*}
\]

If there is anything peculiar about the inflation tax model just presented, it is the apparent "reverse causality." It is customary to describe the workings of a tax in three parts: (1) the tax rate is established, (2) the tax is applied (sources side), and (3) the tax revenues are expended (uses side). In our model the process is reversed: (1) the tax revenues are expended, (2) the tax rate is determined (in the market place) and (3) the tax revenues are generated (decline in the purchasing power of money balances). This, reverse causality is, however, an artifact of the (Patinkin) assumption of memoryless households. Households do not anticipate future price level changes and cannot profit from their past
experiences. Consequently, the inflation tax rate can only become reality after the injection of new money into the private sector. Yet, even if the prohibition on expections is removed, the inflation tax must still be determined by the expenditure of the tax revenues—the new money. This is the mechanism of the inflation tax. The sources and uses sides cannot be separated. It is the generation of new money, along with its distribution, which produces the inflation tax.

The process by which a new inflation tax is imposed is, in fact, only seemingly reversed as compared to the traditional description of a tax process. The entire process is:

1. the monetary authority determines the desired inflation tax rate (and revenues);
2. the inflation tax revenues are received by creating new money (valued at the existing price-level);
3. the inflation tax revenues are expended; and
4. the general price-level rises, lowering the value of all outstanding money balances.

Moreover, Krzyzaniak [1980] has recently argued it is false logic to treat any tax revenue as independent of the expenditure side. Taxes are, in his words, a minimal binary policy mix. The separation of sources and uses is only imaginary.
Notwithstanding Krzyzaniak's recent contribution, it is instructive to continue with our analytic distinction between the sources and uses side of the inflation tax. Specifically, the distinction is useful in answering two obvious questions: If the inflation tax is neutral, as the neoclassical model suggests, what is the incentive to impose it? Further, does the relaxation of the assumption of no expectations of possible future general price-level changes (A.1) alter the neoclassical monetary neutrality condition?

The answer to the first question is simply: None, absolutely none. In order for the inflation tax to be neutral, all tax revenues must be "collected" and tax expenditures made in just such a manner as to not disturb relative prices and incomes. The tax must not only be a zero-sum transfer between any two subsets of the private sector, it must be a zero-sum transfer for each element (household). A zero-sum game implies no excess burden. Whatever revenues the taxing authority receives it is compelled to spend according to exactly one plan. In addition, the cost of performing the taxing and spending activities must not change relative prices and incomes. Clearly, there is no incentive to engage in such a pointless exercise. The objective of government in matters of taxation is precisely to affect the distribution of resources. The mere fact that governments engage in inflationary finance is prima facie evidence that the inflation tax is not neutral.
The second question raised above pertains to the sensitivity of the neoclassical money neutrality with respect to the "no-expectations" assumption (A.1). Although this issue is examined in greater detail in Sections D and E, an important concept must be introduced at this time. The model above revealed how unanticipated inflation causes short-term real effects (which may happen to be exactly offsetting). The corollary to this is that perfectly anticipated inflation causes no real effects. This principle, well accepted by the contemporary rational expectations school, is nonetheless generally misunderstood. Much of the literature gives the impression that somehow only perfectly anticipated inflation has no real effects. In point of fact, any perfectly anticipated tax causes no real effects. Again, substantial insights can be gained by integrating contemporary monetary theory and public finance theory. The issue is relatively trivial: a perfectly anticipated tax is endogenous, and a endogenous tax does not produce substitution (real) effects. This can be demonstrated by recalling any standard tax incidence model.

Consider, for example, the incidence of the corporate income tax. Assume the textbook case where the supply of capital to the economy is fixed, but it is mobile between the corporate and noncorporate sectors. As displayed by Figure IV, for a given pre-tax rate of return $i_0$, the total capital is divided between the two sectors according to the marginal efficiency of capital schedules ($MEC_0$ and
FIGURE IV
Incidence of Corporate Income Tax
MEC_0^*). Now suppose a tax on capital income in the corporate sector is imposed equal to p^C. The net-of-tax rate of return to investors in the corporate sector falls to (1-p^C)i_0. This is equivalent in Figure IV to a decline in the marginal efficiency of capital to MEC,*. Capital being mobile is reallocated from the corporate sector into the noncorporate sector until investment earns the same net-of-tax rate of return in both sectors. The return in the corporate sector rises to i to earn (1-p^C)i, or i_1. As has been demonstrated most admirably by Harberger [1962], Krzyzaniak [1967], among others, the shift in capital may involve additional adjustments from either the sources or uses side of the tax. For our purposes the process may be assumed to terminate at i_1, K_1^*, K_1.

Until the corporate income tax rate is altered or until new information becomes available to indicate the tax rate will be changed, the corporate and noncorporate sectors remain in equilibrium with K_1^* and K_1 investments respectively, cet. par. The tax rate is fully anticipated. The tax revenues are thus determined each period jointly and simultaneously by those influences (variables) determined from outside the system (the business sector). In short, the corporate income tax (revenue) is endogenous. No further substitution effects occur once the tax is perfectly anticipated. The substitution effects are caused by errors in expectations, change in the tax rate. What is an endogenous tax at one point in time must
have been an exogenous shock to the system at some earlier date. It was at the earlier date that the adjustments in the economic system took place.\textsuperscript{17}

Returning to the question raised before, it should be clear that the relaxation of the assumption of no expectations of possible future general price-level changes does not necessarily alter the neoclassical monetary neutrality condition. The role played by expectations in the neutrality of the inflation tax depends on the type of expectations formed. Expectations inconsistent with the real processes will cause distortions and real effects. We have demonstrated it is not necessary for the private sector to have any price-level expectations in order for there to be an effective inflation tax. The tax is the consequence of a transfer of purchasing power from the private sector to the public sector. In the Bailey model, the sources-side excess burden is the result of homogenous inflation expectations. All households agree which parameters of the inflation process are important to consumption decisions, and furthermore they agree as to the values of each parameter. It is then precisely the fact that the public comes to expect a state of continuous price increase which induces the rise in interest rates and the social cost or excess burden. In the neoclassical monetary model, there are no price-level expectations. Yet, there is a sources-side excess burden. In both cases, however, the change in the money supply was neutral. Clearly this neutrality is a stronger condition than the
usual neutrality condition in tax incidence analysis. In the former, the sources and uses side must be jointly neutral, i.e., precisely offsetting. In the latter case only the sources side is neutral. Although there are general-price expectations in the Bailey model, the homogeneity assumption implies that when one household forms an imperfect inflation expectation, all households suffer the same condition. Hence, in both the Bailey model and the neoclassical monetary model, households collectively as well as individually encounter the same disequilibrium in the demand for goods caused by an increase in the money supply. Driven by an absence of money illusion they restore equilibrium without distorting relative prices.

What is the incidence of the inflation tax when heterogenous price-level expectations are allowed? What is the incidence of the inflation tax in a world of market imperfections, such as income taxes and fixed payment obligations? Is there a tendency for society to alter market rules and to strive for conditions more consistent with the neoclassical monetary model, the greater the expected future inflation rate? Do subsets of the private sector repeatedly form differentially more accurate inflation forecasts? Do they earn economic profits from their forecasts, and if so how are they able to prevent the other elements in the private sector from capturing this profit? These questions and many more naturally come to mind. The remainder of this essay is devoted to providing a conceptual framework to evaluate these issues. In certain instances, specific responses
are given. In general, it is shown inflation expectations can play a crucial role in not achieving a neutral inflation tax. Furthermore, it becomes apparent that, except for the case when the inflation tax is neutral, the incidence of the tax can only be determined in a general equilibrium context. This important point is ignored in the literature on inflation. Yet, since the publication of Harberger's [1962] now famous study of the corporate income tax, general equilibrium analysis has been the standard model of incidence analysis.[18]

To summarize some of what has been discussed, there is an inherent and fundamental connection between the Monetarists theory and the concept of inflation as a tax. This conceptual link has many important implications, not the least of which is the principle of neutral money - neutral inflation tax. Even under the most ideal conditions, however, there is almost certainly an excess burden or welfare cost associated with the inflation tax. In an inflationary economy, economic resources are used to economise on money which is costless to produce. Additional social costs may be attributable to inflationary financing if there are pre-existing market and behavioral imperfections (e.g., money illusion). Similarly, inflation expectations determine the incidence of the inflation tax. More specifically, both the incidence and the excess burden of the
inflation tax are determined in time and magnitude by the degree of unanticipated inflation - a principle developed more thoroughly in the subsequent sections.

The importance of price expectations to the study of inflation, unemployment, the business cycle, and other economic problems has been recently rediscovered. Indeed, some contended that the basic contribution of Friedman and other contemporary monetarists has been to reassert the role of expectations. In their recent survey on inflation, Laidler and Parkin [1975] made the following comment concerning the classification of developments in inflation theory,

We have found the distinction between equilibrium, or perfectly anticipated inflation, and disequilibrium or imperfectly anticipated inflation, more useful. Much recent work, particularly of a theoretical nature, on monetary explanations of inflation concentrates on the former, but clearly all the interesting and important practical questions concern macro-economic disequilibrium in which inflation is imperfectly anticipated: inflation can only be perfectly anticipated in any actual economy if all people and organizations (e.g. Trade Unions, Trade Associations) hold the same expectations, since otherwise some expectations are bound to be wrong. The interaction of prices and wages with excess demand, unemployment, inventory changes and the like, when expectations and realisations are different, is an important element in the subject-matter of modern short-run macro-economics. Analysis of such problems goes back through the work of Keynes and Hayek to Fisher and Wicksell. When expectations are not fulfilled the burden of adjustment falls partly on quantities of employment, output, and inventories, and partly on prices. Study of the behavior of wage and price setters and their role in the transmission process of monetary and fiscal impulses to price and quantity decisions provides a necessary micro-foundation for macro-analysis and has attracted much attention recently. (p. 743)
C. THE SOCIAL COSTS AND ECONOMIC CONSEQUENCES OF INFLATION

It has been argued that inflation, interpreted as a monetary phenomenon, is equivalent to a tax on cash balances. Moreover, there may be an excess burden to the inflation tax because economic resources are consumed to avoid the tax which is levied on an asset that is costless to produce. The actual dollar value of this cost is disputed. Turvey [1961] contends inflation has constituted a very important tax. H. G. Johnson [1972], however, maintains that for mild inflations, unlike those considered by Bailey [1956] the social cost of the excess burden is likely to be trivial. But the Bailey type excess burden is not the cost most frequently discussed in the scholarly journals or the public press. As the statement by Laidler and Parkin repeated in the previous section suggests, the costs most frequently discussed are those that are attributable to what we defined in Section A as "ex-post unanticipated inflation," i.e., the difference between the actual inflation rate and the anticipated rate. Yet, Johnson argues such costs "may or may not be a social cost." He claims that the effects,

actually do not involve economic costs in the sense of waste of resources and sacrifice of potential consumption or investment - that is, redistribution of income from creditors to debtors, or from old age pensioners, social security recipients, and public sector employees to private sector employees and entrepreneurs, or from wives to husbands. (p. 23)
How is it that Johnson can maintain the potential economic consequences of inflation he describes do not involve "economic costs"? The answer, we assume, is in the distinction between individual household and firm effects on the one hand, and the aggregate economic effects on the other. Redistribution of income from one wage earner to an entrepreneur is viewed as a zero-sum money transfer. Likewise, the redistribution of wealth from a creditor to his debtor, caused by a decline in the purchasing power of money, is also viewed as a zero-sum money transfer. As described, the zero-sum transfer between individuals is a pure income effect. However, as previously discussed, the redistribution of income or wealth may induce relative price changes or substitution effects for the economy as a whole. The outcome will depend upon such factors as the supply of labor and capital, the elasticity of substitution of capital for labor in different sectors, and the elasticity of substitution of product consumption. In addition, "economic costs" or excess burden may be attributable to a psychological reaction, as Ackley [1978] recently noted,

All income redistributions, whether among classes or individuals, increase personal insecurity and lessen personal satisfactions (even on the part of the beneficiaries), and heighten interpersonal and institutional tensions. Thus they are destructive of the social and political fabric, and ultimately of economic efficiency. Even those who are not really hurt by inflation often think they are. They regard their gains in money incomes as the result of their own cleverness, the effectiveness of their union or trade association or friends in political power — or just good luck. These benefits would have occurred whatever happened. The rise
in the prices they pay therefore is regarded as unjustly robbing them of what is rightfully theirs. Thus they think they are hurt; and if they think they are, they are. All seek someone to blame - those greedy employers or nasty trade unions; the bankers, landlords, farmers, or foreigners; our economic system, the government, society. A significant real cost of inflation is thus what it does to morale, to social coherence, and to people's attitudes toward each other. (p. 151)

Irrespective of the validity of Ackley's concept of the sociological cost of inflation, this study assumes all costs of the inflation tax are observable and measurable in economic values. We concentrate on the hypothesis that inflation distorts relative prices, with respect to the Pareto optimal solution. This may occur because of changes in relative factor rewards which determine the redistribution or incidence on the sources side of income, or because of changes in relative product prices which determine the redistribution or incidence on the uses side of income. In general, whether inflationary finance induces changes in real values is a question of the efficiency of expectations - most importantly of inflation rate expectations. Equivalently, it is a question, as Scitovsky and Scitovsky [1964] state, of whether the pricing system fulfills the function of transmitting important economic information to economic decision-makers so that in the aggregate the allocation of resources and the pattern of output conform by and large to society's wishes. 2
Countless authors have addressed the question of the economic consequences of inflation. J. W. Conard [1964], however, amply summarized the consensus:

The major objections that have been raised to inflation are based on the view that it has these effects: (1) It arbitrarily and regressively changes the distribution of income and of real wealth among individuals and groups. (2) Inflation, once it starts, may feed on itself and thus become self-perpetuating or even accelerating. (3) It may weaken the desire to save, and thus reduce the rate of growth of the economy; similarly, uncertainty about future prices discourages long-term contracts. (4) It may arbitrarily affect the profitability of various kinds of economic activity, thus distorting the allocation of resources. (5) It may discourage both the amount and the efficiency of production, partly because it encourages speculation in contrast to productive activity, partly because under inflation swelling costs may be passed on readily to buyers through higher prices, partly because inflation permits inefficient producers to stay in business and partly because work opportunities are usually so readily available during inflationary periods that efficient work on the part of labor is not a requisite of holding a job. (6) Price and wage decisions become increasingly determined by power struggles, with the prize going to the strong rather than to the productive, thus increasing social strife and reducing efficiency. (7) Under a system of fixed exchange rates, such as the present one or the international gold standard, inflation in this country, if greater than that abroad, can price us out of world markets, thereby causing unemployment and imposing deficits in our balances of payments. Under systems of flexible rates, inflation here greater than that abroad would bring falling exchange rates, causing uncertainty in foreign trade and investment, and weakening confidence in the dollar. (pp. 76-77)

It can be argued that all of the effects of inflation described by Conard may be classified into: (i) those affecting the distribution of income and wealth, and (ii) those affecting economic efficiency. The
basic question, of course, is whether the first causes the second. At this point, however, we wish merely to spell out what economists and even the general public have traditionally meant by type (i) inflationary effects.

The redistribution effects associated with inflationary finance which are of concern to this study are those that pertain to market determined prices. Furthermore, we are concerned with the effects of imperfect expectations of future inflation rates. To be sure, redistributions can occur even if expectations are errorless. If, for example, the government uses inflationary financing but holds nominal transfer payments constant, then the real value of the payments declines and a redistribution of income occurs. This describes the popular impression of the plight of social security and other fixed income recipients. We exclude such redistributions from consideration here, not because they are insignificant, but because they are the consequence of nonmarket processes.

In general, redistribution of income and wealth is the result of imperfect relative price adjustments, vis-a-vis the inflation tax rate. The relative prices most frequently discussed are "the" interest rate, "the" wage rate, and "the" rental rate. The issue is whether the relative price moves contemporaneously with changes in the expected future inflation rate. When a change in the interest rate lags a change in the expected future rate of inflation, debtors gain
at the expense of their creditors because the interest rate fails to fully compensate the creditors for the expected time value of money, since the dollars borrowed are more valuable in purchasing power than the dollars repaid. We shall refer to this wealth transfer as the Debtor-Creditor Hypothesis, the title it has acquired since the Alchian-Kessel studies of the 1960s. Indeed, as Kessel and Alchian [1962] commented,

The expectation that the current price level will persist implies that the equilibrium money rates of interest observed in capital markets are unaffected by unanticipated inflation. No changes in the relative demand and supply of bonds are implied by the observed depreciation in the purchasing power of money. Consequently, interest rates fail to rise enough (real yields on bonds fall below that required) to maintain preinflation economic relations between debtors and creditors. The nominal rate of interest fails to reflect rising prices because estimates of the course of future prices are biased, not because of market imperfections. As a result there are transfers of wealth from net monetary creditors to debtors. What is true of debtors and creditors linked by indebtedness in the form of bonds is equally true of all debtors and creditors regardless of the specific security creating this relationship, which may be bonds, mortgages, notes, bills, acceptances, or contingent sales contracts; relationships between debtors and creditors are systematically affected. No income effects, other than those flowing from the wealth transfer, are implied. (p. 524)

The debtor-creditor hypothesis applies to the effects of unanticipated inflation on wealth. Income redistribution is customarily evaluated in terms of the Wage Rate Hypothesis. In addition some studies have also examined the question of the Rent Lag Hypothesis. Again, as Kessel and Alchian [1960] stated,
What, then, is the wage-lag hypothesis. To answer this question, we have turned to the works of those economists who have used this idea. The most important "explanation," importance being measured by either the extent to which it has been used or its deviation from the way economists explain behavior in nonlabor markets, is the belief that wages have more "inertia" or "sluggishness" than other prices because of custom, weak bargaining power of labor, or lack of foresight of workers. (p. 44)

Scitovsky and Scitovsky [1964] elaborated,

The distribution of income is determined by the prices the market sets on products and the services of different kinds of labor; and at a time when all prices are rising, income distribution may be affected by the fact that some prices, being more flexible, rise faster or sooner than others. In particular, economists have for long regarded as axiomatic the fact that, since the prices of products are more flexible than the prices of labor, inflation lowers the share of labor and increases the share of capital (profits) in society's total income; and that such fixed income earners as salaried employees, civil servants, and the aged living on pensions and accumulated savings are affected even worse by inflation than wage earners are. The theory on which these expectations are based is reasonable and simple enough. (p. 450)

A crucial question arises regarding inflation induced zero-sum money income and wealth transfers: Do such redistributions necessarily generate additional excess burden in addition to the sources side burden from anticipated inflation? One can imagine that inflation induced private sector redistributions do not interfere with private choices, being zero-sum. In such an event, a distinction is must be made between direct and excess burden on the one hand, and redistribution "effects" on the other.
The literature is silent on this question. In point of fact, however, such redistributions do imply excess burden. Put another way, the debtor-creditor wealth transfer, the wages-lag effect, and other inflation tax induced redistributions are the consequence of distortions of private choice, and hence imply excess burden. This point is so fundamental to an understanding of the cost and effects of the inflation tax that it warrants demonstration and elaboration. The principle involved here may be established by proving that any inflation caused redistribution yields excess burden. Accordingly, the debtor-creditor transfer is chosen for illustrative purposes.

To begin, as explained above, excess burden is the difference between the total loss of welfare of a tax as it is actually imposed and the loss which would result if identical tax revenues were collected without distorting economic decisions in the private sector. What must be established, therefore, in regard to the efficiency of any tax is the extent to which the first-order conditions for pre-tax equilibrium differ from the first-order conditions for post-tax equilibrium.

Consider a two period world with one homogeneous good c and money, which in this economy is a "consumed" commodity, meaning that people derive utility from holding money in the same way they derive utility from the nonmonetary good, c. The demand for money is derived from its liquidity benefits, which translates to "real balances". The
utility function for the representative household is given by EQ(1), where a differentiation is made between contemporaneous consumption commodity \( c_0 \) and money \( M_0/P_0 \), and one period-ahead commodity \( c_1 \) and money \( M_1/P_1 \).

\[
EQ(2) \quad U_h = U(c_0, c_1, \frac{M_0}{P_0}, \frac{M_1}{P_1})
\]

The time \( t_0 \) wealth constraint is represented by EQ(2-a), where \( y_0 \) is time \( t_0 \) real income. The term \( B_0 \) denotes current period bonds, which may be either positive or negative. If \( B_0 \) is positive then household \( h \) is a lender or creditor. Alternatively, household \( h \) is a borrower or debtor when \( B_0 \) is negative. Endowment money balances are denoted by \( M_0 \). The time \( t_1 \) wealth constraint is given by EQ(2-b). Money balances maintained at time \( t_0 \), that is

\[
EQ(2-a) \quad P_0c_0 + M_0 + B_0 = P_0y_0 + M_0
\]

\[
EQ(2-b) \quad P_1c_1 + M_1 - B_1 = P_1y_1 + M_0 + E_0(\Delta M_1)
\]

\( M_0 \), are available for use at time \( t_1 \) and hence appear on the sources of income side of the constraint (i.e., right hand side). The term \( E(\Delta M_1) \) represents the expected autonomous increment to money endowment between \( t_0 \) and \( t_1 \). The bond value \( B_1 \) is the repayment
of the bond transaction of the previous period such that \( B_1 = B_0(1 + \hat{i}) \), where \( \hat{i} \) denotes the contract or coupon rate of interest.

The two wealth constraints must be aggregated before the utility function can be evaluated. This is accomplished by discounting the period ahead budget constraint EQ(2-b) by "the" market interest rate \( i \). The overall wealth constraint is EQ(3). The bond holding \( B_0 \) can be dropped out of the equation if the coupon rate \( \hat{i} \) equals

\[
\text{EQ}(3) \quad P_0 c_0 + M_0 + B_0 + fP_1 c_1 + fM_1 - fB_1 = P_0 y_0 + \tilde{M}_0 + fP_1 y_1 + fM_0 + fE(\Delta M_1),
\]

where \( f = (1+i)^{-1} \)

the market rate \( i \). However, for reasons which will soon be apparent, it is preferable not to cancel out the bond variables.

To close the model, four conservation equations are introduced.

\[
\text{EQ}(4-a) \quad \sum c_0 = \sum y_0 \\
\text{EQ}(4-b) \quad \sum c_1 = \sum y_1 \\
\text{EQ}(4-c) \quad \sum M_0 = \sum \tilde{M}_0 \\
\text{EQ}(4-d) \quad \sum M_1 = \sum M_0 + \sum E(\Delta M_1)
\]
The summations are over all the economic decision-making bodies in the economy (only households in this case).

The first order optimum conditions for the households constrained utility maximization problem are as follows:

\[ \text{EQ}(5-a) \quad \frac{\partial c_1}{\partial c_0} \bigg|_{U} = -\frac{P_0}{P_1} (1+i) \]

\[ \text{EQ}(5-b) \quad \frac{\partial (M_0/P_0')}{\partial c_0} \bigg|_{U} = -(1+i) \]

\[ \text{EQ}(5-c) \quad \frac{\partial (B_1/P_1')}{\partial (B_0/P_0')} \bigg|_{U} = -\frac{P_0}{P_1} (1+i) \]

\[ \text{EQ}(5-d) \quad \frac{\partial (M_1/P_1')}{\partial c_1} \bigg|_{U} = -1 \]

The marginal rate of substitution of periods \( t_0 \) and \( t_1 \) bond is achieved by treating bonds as a substitute for both the nonmonetary commodity \( c \) and money. The fact that the marginal rate of substitution of period \( t_1 \) money and commodity \( c_1 \) is equal to unity is a perversion of the two-period model. In a multiperiod model, EQ(5-d) would be exactly the same form as EQ(5-b). Only the terminal period has the property of unitary substitution.
Having derived the standard first order conditions, we may explore the question of inflation tax distortions. The first requirement is to establish the relationships for a non-inflationary economy. Such an economy is characterized by EQ(6), where $\rho$ is the inflation rate.

EQ(6-a) \hspace{1cm} P_1 = P_0

EQ(6-b) \hspace{1cm} E_0(\rho_1) - \rho = 0

The most obvious adjustment in this case is to equate the real interest rate and the market interest rate, $r = i$. Moreover, the coupon rate $\hat{i}$ is equal to the real coupon rate $\hat{r}$, and both are equal to "the" interest rate. Under these conditions, the bond holdings $B_0$ may be properly omitted from the wealth constraint EQ(3). Utility is not a function of bond holdings. This is because the ex-ante bond valuation relationship EQ(7-a) is identical to the ex-post valuation relationship EQ(7-b). The marginal rates of

EQ(7-a) \hspace{1cm} B_1 = B_0(1+i)

EQ(7-b) \hspace{1cm} \frac{B_1}{P_1} =\frac{B_0(1+i)}{[1+E_0(\rho_1)]}

EQ(8-a) \hspace{1cm} \frac{\partial c_1}{\partial c_0} \bigg|_U = -(1+r)
\[
\text{EQ}(8-b) \quad \left. \frac{\partial (M_0/P_0)}{\partial c_0} \right|_U = \frac{-(1+r)}{r}
\]

\[
\text{EQ}(8-c) \quad \left. \frac{\partial (B_1/P_1)}{\partial (B_0/P_0)} \right|_U = -(1+r)
\]

Substitution derived above are reduced to functions solely of the real interest rate. Equations EQ(8-a), E(8-b), and EQ(8-c) are substituted for EQ(5-a), EQ(5-b), and EQ(5-c) respectively.

Now assume the equality symbol in the price equation EQ(6-a) is replaced by the "greater than" conjunction. Equation EQ(6-b) remains unaltered. This is the Bailey case of perfectly anticipated inflation. Inflation is caused by an anticipated increase in money balances. But when the actual increase in the money supply (i.e., the actual inflation tax imposed) equals the anticipated increase there is no distortion of the expected future money balances relative to the current balances so choices are not disturbed. In short the inflation tax is \textit{endogenous}. The economic system does not receive any new shock (i.e., exogenous information) which could cause an adjustment in the individual's demand for commodities or money, either in substitution for one another or in substitution over time (e.g., \( c_0 \) for \( c_1 \)). The actual incremental money endowment \( \Delta M_1 \) can be substituted in the appropriate equations above for the expected increment \( E_0(\Delta M_1) \) and utility will still be independent of the incremental money supply. This is what is meant by the "rational
expectations neutrality of money", a concept of immense interest in the academic literature -- and a concept about which we shall have a great deal to say.

The essential feature of perfectly anticipated inflation is that the inflation tax does not affect real choices. This is clearly demonstrated with the model at hand by recognizing that \( (P_1/P_0) \) in EQ(5-a) and implicitly in EQ(5-b) and EQ(5-c) is equal to the expected price ratio written \([1 + E_0(\rho_1)]\). Hence, these equations collapse to their respective "no inflation" counterparts: EQ(8-a), (8-b), and (8-c). Clearly, real choices are not influenced by perfectly anticipated inflation. However, the very fact the inflation tax is endogenous implies the shock to the private economy caused by a change in the tax rate or a change in the expectation of the tax rate must have occurred in some previous period. For the tax to be endogenous at time \( t \), there must be some exogenous force at time \( t+\varepsilon, \varepsilon < 0 \).

A minor digression is in order. The model of choice presented above is free from institutional rigidities, the size of the government, and market imperfections such as income taxes. When such imperfections do exist, the inflation tax, although perfectly anticipated, will generally magnify the distortions of choice caused by these imperfections. Feldstein and Summers [1978], among others, have shown that even if the market rate of interest "fully adjusts" for expected inflation, there will still be effects on real variables
in the presence of progressive income tax rates. For example, an investor in the 50 percent marginal tax rate will find that a net-of-tax return of 4 percent in the absence of inflation vanishes when the inflation rate is 8 percent. In order to fully understand the social costs and economic consequences of inflation, it is necessary to distinguish between distortions in the efficient use of resources caused jointly by market imperfections and inflation, and the costs and effects of inflationary finance in an otherwise perfect economy. The distinction is crucial because the policy decisions to correct for the distortions are different. Where efficient resource allocation and use are disturbed by inflation magnifying existing market imperfections, the preferred solution could well be to correct the existing market imperfections instead of eliminating the inflation tax.

Consider now the case of unexpected inflation. There are two ways unexpected inflation may occur in the model above. The first is if there is an autonomous increase in time $t_0$ money endowment $\bar{M}_0$. The second source of unexpected inflation is when the actual increment to time $t_1$ money endowment exceeds the expected increment, that is when $\Delta M_1 > E(\Delta M_1)$. This is what was defined in Section B as ex-post unanticipated inflation. The case of ex-ante unanticipated inflation requires a three-period model: the first period when
\( E_0(\rho_2) \) is determined; the second period when the expectation is altered to \( E_1(\rho_2) \neq E_0(\rho_2) \); and the third period when the incremental money supply occurs.

The first case of unexpected money causes distortions in private choice only if the expected \( t_1 \) increment to money endowment is nonzero. In this event the increased money endowment at time \( t_0 \) enlarges the aggregate of current money relative to future money; and since interest is a premium on current money as against future money, the interest rate must change. This follows from the conservation equation EQ(4-d). With \( E_0(\Delta M_1) \) nonzero for at least one household, the aggregate of future money is larger than the aggregate of current money endowments. Hence, an unexpected increase in current total money endowments leads to a smaller proportionate change in \( \Sigma M_1 \). In this case the inflation will tend to lower the real interest rate. Should the anticipated future increment to money endowment be zero for all households, any unexpected increase in current aggregate money endowment will be neutral in terms of real choices. In this situation, the aggregate proportionate change in \( \Sigma M_0 \) necessarily yields the same proportionate change in \( \Sigma M_0 \) and \( \Sigma M_1 \). With prices initially held constant, the increase in wealth leads to an additional demand for consumption goods \( c_0 \) and \( c_1 \) and a desire to divest of the excess real money balances. But according
to conservation equations EQ(4-a) and EQ(4-b), the endowment of goods is fixed. Consequently, equilibrium is restored by a proportionate increase in the price levels $P_0$ and $P_1$.

The case of unexpected inflation just described is perfectly analogous to the Patinkin [1965] money neutrality discussed above. The non-neutrality of an unexpected increase in the current money endowment, when $\Sigma E(\Delta M)$ is nonzero, while interesting in terms of comparative statics analysis, is not the type of inflation tax distortion generally discussed in the literature. The social costs and economic consequences of inflation discussed in the literature pertain more to unanticipated (inflation) shocks to the economic system after contracts for goods and services have been negotiated. This is analogous to our second type of unexpected inflation, namely when $\Delta M_1$ exceeds $E_0(\Delta M_1)$. In this event the actual rise in prices $(P_1/P_0)$ is greater than the beginning of the period expected rise $[1 + E_0(p_1)]$. As a result, the EQ(5) marginal rates of substitution do not collapse to their respective EQ(8) "no inflation" equivalents. The marginal rate of substitution in consumption, ex-ante, is independent of inflation, as EQ(9-a-1) indicates. However, the ex-post marginal rate of substitution is less in absolute value than the efficient solution. Thus, the error in

\[
\text{EQ}(9\text{-}a\text{-}1) \quad \left. \frac{ac_1}{ac_0} \right|_{U} = \frac{-(1+i)}{[1+E_0(p_1)]} = -(1+r)
\]
EQ(9-a-2) \[ \frac{\frac{\partial c_1}{\partial c_0}}{U} = \frac{-(1+i)}{1+r_1} > -(1+r) \]

expectations forces a "wedge" between the beginning of period efficient consumption and optimal consumption as measured at the end of the period. Hence there is a **substitution effect**. Households would prefer to have had relatively less future consumption \((c_1)\). Since there is a substitution effect, there is also excess burden.

Figure V is a partial equilibrium demand and supply diagram for consumption good \(c\). Initial equilibrium in the no inflation (or perfectly anticipated inflation) world is denoted by "a". The budget equation passes through the point \((y_0, y_1)\) with slope \(-(1+r)\). Now suppose the government, needing revenues to finance its budget, imposes a "lump-sum" tax equal in terms of current income to \(y_0 - y_0'\). The budget line shifts down to include the income point \((y_0', y_1)\). Optimal consumption declines in both periods to \((c_0', c_1')\), as utility falls from \(U_0\) to \(U_1\). However, there is no substitution of current consumption for future consumption. Consequently, the tax causes only income effects and no excess burden.

The government may also balance its budget by unexpectedly increasing the money supply. As shown above, this lowers the (ex-post) marginal rate of substitution. The budget line correspondingly rotates down such that equilibrium consumption is at
FIGURE V
Inflation and Consumption Decisions
"b" or for \((\hat{c}_0, \hat{c}_1)\). Period \(t_0\) consumption is substituted for period \(t_1\) consumption -- even more so than would be optimal from the decline in income to point "x". This substitution of current consumption \(\hat{c}_0 - c'_0\) for future consumption \(c'_1 - \hat{c}_1\) is the substitution effect. The result is the uncompensated loss in welfare equal to the area \((x,b,d)\).

Figure V also depicts the plight of a net creditor. With income at point "z" and desired consumption of "a", the representative household is a net lender of income \(y_0 - c_0\). The choice to be a creditor in this amount is conditional upon an expected price rise of \(E_0(\rho_1)\). However, the actual inflation is \(1 - (P_1/P_0)\). As

\[
\text{EQ}(9-\text{c}-1) \quad \left. \frac{\partial(B_1/P_1)}{\partial(B_0/P_0)} \right|_U = \frac{-(1+i)}{[1+E_0(\rho_1)]} = -(1+r)
\]

\[
\text{EQ}(9-\text{c}-2) \quad \left. \frac{\partial(B_1/P_1)}{\partial(B_0/P_0)} \right|_U = \frac{-(1+i)}{(1+\rho_1)} > -(1+r)
\]

before, the ex-ante optimal bond relationship is given by \text{EQ}(9-\text{c}-1)\) or \text{EQ}(8-c). The ex-post optimal marginal rate of bond substitution is less in absolute value than the ex-ante rate of substitution, which says that, had the representative creditor known at the beginning of the period what the actual inflation rate for the period would be, he would not have lent the same resources in the market without the contract rate \(\hat{i}\) bid appropriately higher to compensate for the loss in
purchasing power. In fact, in the diagrammatic scheme of Figure V, the representative household would have chosen to be a net debtor, borrowing current period resources of \( \hat{c}_0 - y_0 \). The loss to the creditor from the unexpected inflation is the debtor's gain. But the welfare loss, area \((x,b,d)\), is an uncompensated loss.

The proposition that a zero-sum money income or wealth transfer implies excess burden from the inflation tax has thus been proven. Inflation causes welfare loss because real resources are used to avoid the tax. But this cost is generally thought to be immaterial compared to the welfare loss from the induce income and wealth redistributions associated with imperfectly anticipated inflation. We have introduced several specific ways in which factor incomes are believed to be distorted by inflation expectation errors, such as the Debtor-Creditor Hypothesis. It is now clear such distortions necessarily imply that inflationary finance is inefficient — compared to a tax which would obtain the same tax revenues without altering relative choices. But there is a far more serious consequence to this conclusion. All of value theory is subject to attack. If it is true that one subsector, such as debtors, systematically benefits from inflation expectation errors, then we must openly question the very nature of the market place and the function of prices as the means to efficient resource allocation. The principle that prices represent value surrogates
presupposes an unbiased mechanism for price determination. This, however, is prima-facie inconsistent with a systematic income or wealth transfer due to imperfect inflation anticipations.
D. THE ROLE OF EXPECTATIONS

The previous section examined the process referred to as "inflationary finance" from a public finance perspective, that is as a tax. It was demonstrated that both the type and extent of the social costs and economic consequences of inflationary finance are largely determined by the accuracy of expectations of future inflation rates. This section looks more closely at the role of expectations, particularly those pertaining to money creation and the future course of prices in general. The basic issue is the same as in the previous section; but the approach is different. The issue, of course, is the neutrality of the inflation tax. In the terminology of this section, however, the issue is rephrased to one of the rationality of expectations and the neutrality of monetary policy.

A precise taxonomy of expectations concepts is required to model the role expectations play in economic choice. The first distinction is between an "expectation" per se, and the "use of the expectation," as Tait [1967] stated,

The impact of price changes will depend not only on their original asset structure, but first, on their [individual's] ability to anticipate price changes correctly; secondly and separately, their ability to adjust their earnings, assets, liabilities, and consumption to the anticipated change. There is a double criterion consisting first of the correct anticipation, and secondly the ability to use that forecast. (p. 651)
We distinguish between fully anticipated or perfect forecasts, and partially anticipated or imperfect forecasts. Perfect forecasts are errorless: the expected value and the subsequently observed value are always identical. Imperfect forecasts involve error, i.e., have a nonzero error variance. These forecasts may be either unbiased (in which case the expected value of the error is zero), or they may be biased. Efficient forecasts are defined as those which, subject to cost constraints, exhaust all available information pertinent to the variable in question. Efficient forecasts are either perfect or unbiased. The concept of efficient forecasts originated independently by Muth [1961] and Fama [1970].

Imperfect inflation forecasts always impose an unanticipated inflation tax. However, the form of the tax may be of two types. An unanticipated increase in the money supply in the hands of the Federal Reserve (assumed to be part of the consolidated public sector) is imperfectly anticipated inflation tax revenue. An unanticipated shock to the real sector, resulting in a decline in aggregate product, also imposes an imperfectly anticipated inflation tax, because with the velocity of money constant, the corresponding rise in prices lowers the value of outstanding money balances which reduces the value of aggregate claims on the public sector. The difference between the two forms of inflationary financing is that the government adopts an active role in the case of inflationary financing from money supply manipulation, but it may be either active or passive in receiving the
benefits of a decline in the purchasing power of outstanding money balances. This is an important distinction. As indicated earlier in this study, taxing is an active event. Inflationary financing which is unanticipated by the private sector but nevertheless planned by the public sector is taxation. But when inflation is unanticipated by both the private and the public sectors, the financing is a windfall to the public sector.

Insofar as forecasts affect market behavior, we may introduce the concept of an efficient market, defined as a market where prices instantaneously and unbiasedly reflect all available information. For our purposes the relevant aspect of the efficient market theory is the expected return or "fair game" model. The market is said to be a fair game with respect to the information sequence $\emptyset$ when the following condition holds,

$$\text{EQ}(10) \quad E_t(z_j,t+1 | \emptyset_t) = 0$$

$$z_j,t+1 = P_j,t+1 - E_t(P_j,t+1 | \emptyset_t)$$

The excess market value for commodity $j$ ($z_j$) at time $t+1$ has an expected value at time $t$ of zero, based on the information available and used by the market to determine the price of commodity $j$ ($P_j$). This condition holds if all available information has been exhausted by the market in arriving at the expected one period ahead price ($P_j,t+1$). The fair game condition also implies that the joint
probability density function for commodity prices at time $t+k$ ($k \geq 0$), assessed at time $t$ on the basis of the information set used by the market, equals the "true" joint probability density function for the commodity prices at time $t+k$ implied by the set of information available at time $t$ which is relevant for determining commodity prices. The equality of the joint distribution functions is implied by the efficient market condition that the information used by the market to determine prices at time $t$ includes all available information. This includes knowledge of the mapping of states of the world into probabilistic outcomes.

Muth [1961] had previously defined an equivalent concept to the efficient market theory referred to as rational expectations. He emphasized that economic agents, that is consumers, workers, investors, have price incentives to gather information relevant to the variable of concern in order to form more accurate forecasts of the future values of the variable. Furthermore, he argued, that expectations, which are simply informed predictions of future events, are essentially the same as the predictions of the relevant economic theory. An important corollary was first noted by Walters [1971], which we shall title the Axiom of Consistency. This axiom holds that if the expectations of the market are inconsistent with the observed data, the market will alter the expectations formation mechanism so that the predictions agree with the observations. The Consistency Axiom is clearly an error learning hypothesis. It also suggests
something of a Bayesian decision theory. Econometrically, the Axiom implies the possibility of respecification. Inflation forecast errors can induce the market to correct its expectations mechanism. The correction may involve a respecification of the structural model, for example including additional independent variables; or the correction may be to the prior probabilities about the variables. The latter is certainly Bayesian. However, adjustments to specification are "Bayesian" in nature if the adjustments are predicated on the informational significance of the model forecast errors. In the strict classical case, the estimated structural model is theoretically correct or it is not, but the population parameters are fixed values corresponding to stable distributions of the variables. Consequently, no new information can be expected about the distributions and, therefore, the market cannot learn from its predictions. The Axiom of Consistency states the market will respecify its inflation expectations model to incorporate the informational content of unanticipated inflation (errors). The process continues until the expectations are efficient and unbiased, i.e., rational.

The relevance of the concept of efficient market - rational expectations to the study of the effects of inflationary finance is best appreciated by noting the fundamental implication of a fair game market, namely the impossibility of trading systems based only on information $\phi_t$ that have expected profits or returns in excess of equilibrium expected profits or returns. Consequently, it is said
that in an efficient market, prices "fully reflect" all available information. Prices are unbiased, rational indicators of value. This leads to the following proposition:

P(1) Market prices fully reflect all available information (at the beginning of the period) regarding the process generating inflation, if and only if the inflation (tax) rate expectations are rational.

A market, such as the capital market, is not "efficient" if prices do not "fully reflect" the expected future inflation rate. In such an economy, the observed inflation rate forecast error has no ex-ante economic value. If there is no systematic behavior to the forecast error it has no ex-post predictive value either. On the other hand, if there is a systematic pattern to the errors then not all information has been exhausted. According to the Axiom of Consistency, however, the market will, in time, modify its expectations formation mechanism to incorporate the excluded information.

When expected profits in excess of equilibrium expected profits are earned from trading systems based on publicly available information related to the period-ahead inflation rate, the unanticipated inflation has economic significance; which is to say it causes real effects. These effects may be of two different types. First, as just stated, by the Axiom of Consistency, the forecast
errors may induce a respecification of the expectations formation mechanism. This will alter behavior and the values of all economic variables which are derived conditional on the expected inflation rate. The consequences may be numerous as laborers seek to renegotiate their contracts, creditors seek means to adjust the effective (real) rate of interest on their loans, etc. The second way unanticipated inflation causes real effects is through institutional pricing systems denominated in nominal prices. An example is the additional tax liability imposed by progressive individual income tax rates. Such effects can be monetarily significant. But as argued in the previous section, they must be distinguished from the "expectational effects." In all likelihood, the incidences of the imposed costs of the two are not the same. Furthermore, the appropriate measures to correct for the effects of inflation depend on the source of the inefficiencies.

In deterministic models, rational expectations are equivalent to perfect expectations. Several authors have explored this fact in the context of money growth models. Sargent and Wallace [1975] and Barro [1976] [1977] have demonstrated what is generally considered the core of rational expectations; namely a purely deterministic money supply rule can have no effect on the probability distribution of real income. In the language of the previous sections, only exogenous changes in the money supply cause real effects. Lucas [April, 1972] was one of the first to reformulate this classic Quantity Theory
result. He presented a no-money-illusion general equilibrium model which demonstrated a systematic relation between the rate of change in nominal prices and the level of real output (i.e., a variant of the Phillips curve). In the limiting case where the only factor disturbing the economy is monetary, the classical neutrality of money holds even in the short run. Real cash balances, employment, and consumption remain unchanged even in the face of unanticipated monetary changes. This result is achieved for precisely the same reasons as in Section B. The sources side effects are exactly offset by the uses side effects.

The relevance of the Sargent, Wallace, et. al. rational expectations of the money supply theory to the issue of inflationary financing is twofold. Given the assumption inflation is always and everywhere a monetary phenomenon, there is a correspondence between an unanticipated change in the money supply and an unanticipated change in the general price-level. Accordingly, money is neutral if the inflation tax is rationally anticipated (i.e., endogenous). Similarly, expectation studies which evaluate the real effects, such as a Phillips Curve, associated with monetary disturbances have a direct bearing on the neutrality of the inflation tax. In a less direct way, without resorting to the monetary inflation assumption, the rational expectations concepts are just as pertinent to an understanding of the costs and effects of inflation. All economic choices are based on expectations of future events, in as much as the
budget constraint is equivalent to the value of expected future services. Therefore, the process by which expectations of the future covariance of prices are formed is paramount to the allocation of current resources.

Rational expectations models, at the macroeconomic level, tend to be concerned with establishing the conditions for the existence of a Phillips Curve. As Barro and Fisher [1976] point out there are three key issues of concern:

1. Does the model generate a short-run Phillips Curve, which in this context can be viewed as an effect of unanticipated movements in money on the levels of output and employment.

2. If the Phillips Curve exists, is it exploitable, so that monetary policy can systematically affect the economy's time path of output and employment., and

3. If an active policy role for money is feasible, is it also desirable?

Uncertainty, that is the existence of forecast errors, is a necessary condition for a Phillips Curve. However, uncertainty is not a sufficient condition. As Barro [1976] demonstrated, some limitation to current information which causes individuals to be unable to distinguish between monetary and real shocks is also required. In Barro's model, market participants are assumed to be unable to discriminate between relative price shifts and absolute price shifts. Such a situation occurs when the money supply is generated by one
stochastic process, say a random walk, and net excess (real) demand is generated by another process, perhaps a mean regressive process. Individuals then form expectations about the one-period ahead price level by guessing how much of the (unanticipated) price movements are attributable to the monetary and real sources. To the extent individuals misjudge the sources of price rises monetary disturbances will have some (usually positive) effect on output. Equivalently, a Phillips Curve relation will exist between unanticipated money-price level movements and output. Lucas [1973] and Barro [1976] have both shown that the output response to money supply shocks varies inversely with the responsiveness of price expectations to current price observations. The greater the variance of money, the more sensitive expectations will be, and the less sensitive output (i.e., lower magnitude of the Phillips Curve).

Models in which there are real effects as a result of expectational errors, with expectations formed rationally based on less than complete information, have profound implications for monetary policy. An increase in the money supply variance increases the variance of output above the level that would have been obtained under conditions of complete information, _cet. par._ The reason, as Barro and Fischer [1976] states, is that the "increased variance of money reduces the informational content of observed prices and therefore makes it more difficult for individuals to respond 'appropriately' to changing patterns of relative supplies and
In other words, real effects may be observed as a consequence of the variance in the money supply and hence the inflation rate, a point elaborated in the next section.

The question of monetary stabilization or the ability of the authorities to exploit the inflation tax and the Phillips Curve has been shown to depend on whether the authorities have superior information about the economy. Nevertheless, the mere existence of such real effects as a Phillips Curve does not imply that the policymakers can manipulate the economy. The crucial factor is what differential information about future events exists as between the policymakers and the rest of the economy. When information is equally and costlessly available to all agents the Phillips Curve cannot be exploited. When the monetary rule is general knowledge but the authorities have superior information about the economy, such as in the case of the national income figures, a countercyclical monetary policy can reduce the discrepancy between the nonintervention output and the output that would exist for complete information. Taylor [1975] contends if the monetary authorities have superior information about the consequences of their own actions, then a deceptive monetary policy can have systematic output effects during the "transition periods" in which the public learns the policy. This point is contested by Sargent and Wallace [1976].
The instances just described of real effects caused by unanticipated inflation can be reduced to a single class of events: a failure of the perfect market condition that information be costlessly available to all market participants. The joint probability density function for prices at time t+k (k≥0), assessed by the market at time t on the bases of the information set used by the market does not equal the true joint probability density function for prices at time t+k, because the information set is heterogenous across market participants. Information regarding the future course of the general price-level is not equally available to all. Consequently, the observed unanticipated inflation causes redistributions of real incomes and wealth within the private sector. Unequal access to information is not necessary, however, for real unanticipated inflation effects. All information may be equally available to all participants in the market. Nonetheless, some or all of the participants may be formulating their price forecasts on the basis of an invalid market equilibrium model. This best describes the concept of money illusion. As traditionally portrayed, a segment of society is said to suffer from money illusion when each member of the segment seeks its optimum consumption - investment portfolio according to absolute (or nominal) prices, when in fact equilibrium is driven by relative prices. In this case it is not the prices which fail to incorporate all available information, but rather the perception of prices.
To summarize, we have concluded if prices do not incorporate all available information regarding the expected future inflation rate, then unanticipated inflation will produce real effects, such as described above and in Section C. We are, therefore, now able to derive the second expectations proposition. Recall that "endogenous" is the negation of "exogenous." Then by applying the modus tollendo tollens law of logic we obtain P(2).

P(2) If the inflation tax is endogenous, then the joint probability density function for commodity prices at time t+k (k>0), assessed at time t on the basis of the information set used by the market with respect to the future time-series pattern of the general price-level, equals the "true" joint probability density function for the commodity prices at time t+k, as implied by the set of information available at time t.

The tautology we wish to formally establish is now eminently obvious. A formal derivation of P(3) is presented in Appendix B of this essay.

P(3) Inflation rate expectations are rational if and only if the inflation tax is endogenous.

It is absolutely essential to an understanding of the inflation tax that P(3) be properly interpreted. The proposition implies it is not sufficient for prices to reflect all available information pertaining solely to expected future general price-level movements in order for the inflation tax to be endogenous. This describes what
might be called, "weak form" endogeneity. For a "strong form" endogenous tax, prices must necessarily fully reflect all expected future economic effects caused by the anticipated future inflation. This includes all expected inescapable market distortions caused by the expected future inflation tax. Money illusion is escapable, according to the Axiom of Consistency.

Most criticisms of the rational expectations models relate to either information problems or to convergence problems. The same can be said of the "rationality" of inflation expectations. This is equally true for the criticisms of the efficient market theory and its applications. Shiller [1978] is a case in point. As he stated,

First, it is often asserted that individuals do not actually have the information which would enable them to behave in accordance with these models. A great deal of information may actually be required in order to behave in accordance with these models, and this information may be either costly or impossible for them to obtain. Even when the information is readily available, economic phenomena may be affected by the behavior of people who are unable to make proper use of it. Second, assuming that individuals are capable of obtaining the information required, we are yet faced with the problem: how does the economy converge on a rational expectations equilibrium. If a model should change, so that an expectations mechanisms which had formerly been rational is no longer rational, will a new expectations mechanism be found by individuals which will, after its impact on the economy is felt, turn out to be rational. If we do know that such a mechanism will be found do we know this will happen in a reasonably short interval of time. (pp. 33-34)
It is not our intention to defend rational expectations or the efficient market theory. Indeed, both the information problem and the convergence problem (i.e., the dynamics of the adjustment period) do warrant further attention in the literature. Yet, in spite of these criticisms, we are reminded of the voluminous literature which reports empirical results consistent with both theories, especially with regards to the capital markets. But, there is also evidence that the capital markets are not completely efficient. Corporate insiders and specialists on the exchanges apparently have access to information which is not fully reflected in prices. Realistically, the body evidence suggests that if investors only have access to publicly available information then the proposition that the markets are efficient is a good first approximation of reality. Clearly, for our purposes, the significance of this depends on what information about the future general price-level is publicly available and when does it become so.

Judging from the literature, one might be inclined to believe that "rational expectations" refers solely to a monetary phenomenon. We maintain this is erroneous. Economic agents are assumed to compete, subject to cost constraints, for all available information about the future price, quality, timing, and quantity of all economic variables, as well as for information about the processes, economic and other, which determine these factors. The inflation rate is just one variable whose probable future values are of concern to decision
makers. In the aggregate, it may not even be an important variable. The crucial point is that logical consistency dictates any model which assumes decision makers form expectations about the future course of any single variable must also assume that expectations are formed about all other factors that affect the decision problem at hand. This concept of consistency of expectations is sorely missing in the literature, as demonstrated in the second essay of this volume.

The accuracy of expectations, but not the pervasiveness of expectations, has been interpreted as the determining factor in the main schools of macroeconomic thought, at least this is our reading of Leijonhufvud [1967] [1968] and others.\textsuperscript{15} The workings of the classical model hinged on the Walrasian auctioneer providing costless and errorless information to the markets. Such information permitted the classical model to achieve complete price adjustments and maintain equilibrium in all relevant markets. Keynes, according to Leijonhufvud, challenged this perfect information - perfect expectations assumption. Leijonhufvud's Keynes emphasized the role of incomplete information, which for an exogenous decline in aggregate demand was responsible for output and employment reductions leading price adjustments. This deflationary process could be averted if the market rate of interest would fall sufficiently so to stimulate investment and consumption. But according to Leijonhufvud's interpretation of Keynes this does not occur because of "inelastic expectations," that is because the Axiom of Consistency fails.
Whatever the case, this clearly reveals how entire theories of economic behavior inherently depend on the assumptions of how markets form expectations about future events.
E. INFLATION UNCERTAINTY

As revealed in the previous section, unanticipated inflation can produce changes in relative prices together with changes in the allocation of resources. There are two separate dimensions to unanticipated inflation: the expected rate of inflation, and second the attendant inflation uncertainty. The latter implies risk. In this section we propose the argument that inflation risk or uncertainty will alter investment decisions, wage demands, consumption decisions, or other economic phenomena which are subject to "the possibility of suffering harm or loss" due to imperfect but presumable unbiased inflation expectations. In short, the uncertainty about future inflation can itself be a determinant of real behavior even if the inflation tax is otherwise neutral (i.e., neutral with respect to the expected rate of inflation).

The issues of concern for this study pertaining to inflation uncertainty are twofold. The first is the question of the static equilibrium properties of inflation uncertainty. The second pertains to the stability of the probability distribution of inflation expectation errors. Several recent theoretical and empirical works have suggested that the second moment of the forecast errors is not constant and furthermore is covariant with the expected value of the inflation rates.
E.1 Inflation Uncertainty and Portfolio Choice

The majority of the work on the static equilibrium effects of inflation uncertainty has, not surprisingly, dealt with the investment portfolio implications of this source of risk. Sarnat [1973] and Fischer [1975] examined the demand for purchasing power indexed bonds under uncertain inflation. The empirical evidence offered by Sarnat did not reveal any material change in the composition of efficient portfolios. Long [1974] and Brealey and Schaefer [1977] incorporated inflation uncertainty into models of the term structure of interest rates. As Brealey and Schaefer observed, in the absence of inflation uncertainty, the distinguishing feature of bonds is that they have a known value at maturity, and hence the returns are negatively correlated. This gives bonds a unique hedging property. However, when inflation uncertainty is the only source of uncertainty, and the required real return is independent of the expected inflation rate, bonds cease to have any value as hedges. In short, inflation uncertainty can cause real effects.

Two seminal papers appeared in 1975 on the effects of inflation uncertainty on investment portfolio decisions. Jointly these papers, by Biger [1975] and Chen and Boness [1975], clearly established that inflation uncertainty "matters" in investment decisions. Gordon and Halpern [1976], in a working paper, reaffirmed this result. Biger applied the mean-variance selection criterion to the problem of
generating efficient portfolios; first under the assumption the inflation rate is perceived as certain, and second that investors properly account for the inflation rate as a random variable. The variance and covariance of real returns were the basic variables in Biger's analysis. Applying elementary probability theorems, Biger

\[ \sigma^2(\tilde{r}_j) = \sigma^2(\tilde{\rho}) - 2 \text{COV} (\tilde{i}_j, \tilde{\rho}) + \sigma^2(\tilde{i}_j) \]

\begin{align*}
  i_j &= \text{nominal rate of return for security } j \\
  r_j &= \text{real rate of return for security } j \\
  \rho &= \text{rate of inflation} \\
  \sigma^2 &= \text{variance (of)} \\
  \text{COV} &= \text{covariance} \\
  \sim &= \text{denotes random variable}
\end{align*}

demonstrated the real variance of any security will be less the greater its covariance with the rate of inflation. This is clear from EQ (11). He also observed that the mean of real returns is just a scaling of the mean of nominal returns. However, no clear monotonic transformation was evident for the variances of nominal and real returns. Analysis of the composition of optimal portfolios revealed that portfolio decisions which ignore purchasing power risk are clearly risk-return dominated by real optimal portfolios.
Chen and Boness generalized the traditional Capital Asset Pricing Model (CAPM) to account for inflation uncertainty. They showed that the traditional form of the CAPM overstates the market price of risk when an uncertain inflation is expected, and that CAPM overstates the firm's relative risk if its returns are positively correlated with the inflation rate. In general, inflation uncertainty affects the cost of capital through the market price of risk and the systematic risk of particular projects. The optimal investment-production decision is, therefore, dependent on the uncertainty about future inflation rates. However, the theoretical separation of the investment decision and the financing decision, in the absence of corporate income taxes, is preserved.

Friend, Landskroner, and Long [1976] disputed Chen and Boness' results. They concluded:

1. The traditional CAPM (measured in nominal terms) understates the market price of risk if an uncertain inflation is expected (i.e., \( \sigma^2(\bar{E}(\rho)) \neq 0 \)), and if there is a positive covariance between the rate of return to the market and the rate of inflation.

2. The CAPM overstates the risk of an asset if \( \sigma^2(\bar{E}(\rho)) \neq 0 \) and if there is a positive covariance between the rate of return to the specific asset and the inflation rate (same as Chen and Boness).

3. The CAPM overstates the earned rate of return on a risky asset if the correlation between the rate of return on a risky asset and the rate of inflation is greater than product of the correlation between the rate of inflation and the rate of return on the market and the correlation between the rate
of return on the risky asset and the rate of return on the market (given that \( \sigma^2(E(\rho)) \neq 0 \) and all correlations are positive).

Friend, et al., argued Chen and Boness reached quite different and incorrect conclusions because the latter assumed a quadratic utility function for terminal wealth, in which the coefficients were independent of the inflation rate. The coefficients of such a model are the same irrespective of whether wealth is measured in nominal or real values.

Subsequent works by Chen [1979] and Bodie [1979] have demonstrated still further that, although the exact form of the effects of inflation uncertainty on optimal portfolio decisions may be contestable, there is little doubt inflation uncertainty affects the market price of risk.

E.2 Inflation Uncertainty and Macroeconomic Effects

In the previous section we demonstrated how only unanticipated inflation causes real variables to deviate from their ex-ante values. In models by Lucas [1973] and Barro [1976], the structural coefficients for unanticipated inflation were shown to vary inversely with the variance of the actual price level in both the unemployment (labor market) equation and the aggregate supply equation. Barro
[1976] illustrated how an increase in the variance of the money supply, holding fixed the variance of real shocks (e.g., changes in tastes, productivity, etc.), increases the variance of output above the level that would be realized with perfect information. The basic question is does the incremental variance of money induce substitution effect?

Assume the representative household's utility is a function of consumption, which is itself a function of expected income (a permanent income type model). Further assume that in the aggregate individuals are risk-averse. Indirect utility (of income) thus declines as a result of a mean-preserving spread in the probabilities of possible future income levels. Given $E_t(y_{t+1})$, suppose $\sigma^2(y)$ increases to $\hat{\sigma}^2(y)$. Then, provided utility functions are quasiconcave and continuous, it follows:

$$EQ(12) \quad \exists E_t(\hat{y}_{t+1}) < E_t(y_{t+1})$$

such that

$$U[E_t(\hat{y}_{t+1}), \hat{\sigma}^2(y)]$$

\[ \Theta \]

$$U[E_t(y_{t+1}), \sigma^2(y)]$$

In other words, there exist a lower level of expected future income which, for the original variance in income, is equivalent (indifferent) under U (utility) to the original expected income with
higher uncertainty. When translated into consumption demand, this implies the higher the uncertainty of income, the lower the consumption demand, \textit{cet. par.}\footnote{2} If the increase in the variance of income is mean preserving, the decline in consumption demand is matched by a rise in desired savings, i.e., in demand for financial assets. Assuming a Tobin [1958] type liquidity preference, the increase in demand for financial assets will be manifested as higher money demand but lower bond demand for a given equilibrium interest rate.

Evidence supporting the hypothesis that higher inflation will induce a substitution of savings (investment) for consumption was documented by Mueller [1959] and Juster and Wachtel [1972]. The latter found their evidence supported the position that the demand for durables and services declines with a rise in unanticipated inflation, while the demand for consumables remains essentially constant. In the aggregate, therefore, expenditures decline and savings advance. However, just the opposite was found to be true for a rise in anticipated inflation.

Klein [1977] presented a rigorous analytic and empirical analysis of the role of inflation uncertainty for the long-run money demand function. He claimed the demand for money ($M_1$ and $M_2$) is positively related to the uncertainty of future inflation if the stream of monetary services is proportional to real balances and the
demand for money is interest inelastic. Previously, Klein [1975]
[1976], had argued the evidence for the U. S. suggests that long-term
inflation uncertainty has risen relative to short-term inflation
uncertainty, and thus one would expect a corresponding decrease in the
demand for (and supply of) long-term debt relative to short-term
debt.3

Mullineaux [1980] tested two hypotheses: that increased
uncertainty about the near-term inflation causes an increase in the
unemployment rate, and that increased inflation uncertainty reduces
the level of industrial production. Empirical evidence which
Mulleneaux reported for the period 1958-1975 was consistent with both
hypotheses. However, the evidence was less conclusive for tests over
the longer period, 1950-1975. Parenthetically, Mullineaux also
reported unanticipated inflation apparently lowers the unemployment
rate, but does not significantly increase industrial production.
Blejer and Leiderman [1980] reported results consistent with
Mullineaux, assuming one accepts their concept of relative price
variability as a measure of inflation uncertainty.

Fama [June, 1976] explored the impact of uncertainty about the
rate of inflation on Treasury Bill returns as a re-examination of the
hypothesis proposed in [1975] that expected real returns on bills are
constant through time. Fama's [June, 1976] study is quite involved
and need not be outlined here. The important result is that virtually
all the uncertainty regarding the one-period ahead \textit{real} return on Treasury Bills is attributable to uncertainties about expected future inflation rates.\textsuperscript{4}

Notice that the inflation effect just described is inconsistent with the commonly held belief that inflation induces a substitution of consumption for savings; the "purchase before prices rise" syndrome. The argument is usually couched in terms of unanticipated changes in the \textit{rate} of inflation. No consideration is given to the uncertainty (e.g., variance) about the future inflation rate. It is this second dimension to the inflation tax which apparently has a negative impact on real consumption. Conceptually, therefore, there can be both a stimulative and hence destabilizing effect on consumption from accelerated inflation, and a retarding or stabilizing influence from additional inflation uncertainty. Which of these dominates is a question subject to empirical test. Clearly, however, any theoretical or empirical study of inflation which does not differentiate between these two effects will yield biased and inconsistent estimates of the neutrality of the inflation tax, at least as regards to the degree to which markets reflect available information relevant to predicting future inflation rates. In point of fact, the economic literature has generally not differentiated between the expected inflation \textit{rate} and the \textit{variance} (uncertainty) of the expectations, and hence is inconclusive on this issue.
In summary, there is both analytical and some empirical support for the hypothesis that real choices are affected by the variability of the inflation rate. At the same time, the notion of inflation uncertainty has just recently received any serious attention by the economics profession. Accordingly, hypotheses about the cost and effects inflation uncertainty are especially tentative.

E.3 Stability of the Probability Distribution Functions

The second question raised above is whether the probability distribution of inflation expectation errors is constant. Empirical evidence implies an association exists between the actual or expected inflation rate and the variability of inflation rates. Ackley [1978], in his recent review of the costs of inflation, stated, "there is clear evidence that the variability of inflation increases with its average extent." He cited evidence from Okun [1971], Logue and Willet [1976], and Klein [1976]. Ackley concluded that the variability of the inflation rate involves costs which are more serious the higher the inflation rate. Unfortunately, he provided neither analytic nor empirical evidence to substantiate this claim. Instead he alleged, "any textbook in macroeconomics accumulates a considerable list of probable effects of inflation," which would include the fact that, "by increasing consumer fear and feelings of insecurity, it [inflation] increases the propensity to save." The
principle here is fundamentally important: namely, fear-insecurity are heightened by accelerating inflation, and this psychological response manifests itself in real (economic) behavior.

If in the aggregate individuals are risk averse, then variance of the inflation rate is a surrogate measure of inflation uncertainty provided the utility functions are of the appropriate form (e.g., quadratic). Reinterpreting Ackley's "fear and feelings of insecurity" as "uncertainty," and assuming that a positive relation exists between the variance of the inflation rate and its average rate, it follows that an increase in the inflation tax rate will cause substitution effects. Moreover, such real effects will occur even if the tax is otherwise neutral, that is neutral with respect solely to the expected inflation rate (weak form).

Another example of the significant (short-run) role of inflation uncertainty is found in Friedman [1977]. The author observed that the theory of the relation between inflation and unemployment has progressed through three stages: (i) the original negatively sloped Phillips Curve, (ii) the natural rate hypothesis, and (iii) the positively sloped Phillips Curve; the third stage being an empirical reality for many countries. As an explanation for this phenomenon, Friedman hypothesized that a positive association between the variability of the inflation rate and its average level raised the natural rate of unemployment in two ways. First, increased volatility
shortens the optimum length of unindexed commitments and makes indexing more advantageous. But since actual practice adjusts slowly, existing arrangements introduce rigidities that reduce the effectiveness of markets. Second, increased volatility of inflation lowers the efficiency of market prices as signals for the coordination of economic activity. Friedman suggested that,

Again, the effect on economic efficiency is clear, on unemployment less so. But, again, it seems plausible that the average level of unemployment would be raised by the increased amount of noise in market signals, at least during the period when institutional arrangements are not yet adapted to the new situation. (p. 467)

Okun [1971] presented evidence that countries with higher rates of inflation experience greater variability in inflation rates. Logue and Willett [1976] verified Okun’s findings except for the highly industrialized countries. Foster [1978], however, recently confirmed the positive association even for the highly industrialized countries. Gordon and Halpern [1976] reported the correlation between the mean percentage price change and their standard deviations was .91 for 33 selected countries.

Klein [1976], on the other hand, found that with respect to U. S. inflation over the period 1880 to 1972, although there had been periods of demonstrably diverse inflation rate variability, these same periods had not been characterized by significantly different mean rates of price change. As we commented above, Klein did, nonetheless,
maintain inflation uncertainty differs depending on the time horizon, and that the evidence supports the proposition that from the mid-1950's there had been an increase in the long-term price unpredictability relative to short-term unpredictability of inflation. Again, there would be real substitution effects; as one would expect upward pressure on the "long-term" segment of the term-structure of interest rates. This would induce a shift in the demand for shorter lived assets (e.g., equipment) at the expense of the demand for longer lived assets (e.g., structures).

Jaffee and Kleinman [1977] argued the case for low expected inflation rates is strengthened if it is true that the variance of inflation expectations is a positive function of the expected inflation rate. However, after critically reviewing the works of Okun [1971], Gordon [Brookings 2, 1971] and others, the authors concluded that only a weak relationship exists between the average inflation rate (of a country) and the fluctuations around the rate. The authors did, nevertheless, emphasize the significance of the welfare costs of uneven or what they called "sporadic" inflation.9

It is possible for the level of inflation, inflation risk, and risk premiums to be identical in their time series representations and yet not causally connected. Brito [1979], using Brazilian data, has shown both the level of inflation and inflation risk do follow the
same autoregressive structure. This would be expected if inflation uncertainty was a univariate function of the inflation rate. It is not, however, sufficient to insure a casual connection.

Two recent studies which applied different techniques to test for the independence of the first and second moments of the probability distribution of inflation forecast errors provide the strongest evidence to date of the lack of independence. Parks [1978] examined relative price variability assuming homogeneous expectations. In addition to observing that the rate of inflation is positively associated with its variance, he also found fully anticipated inflation has no affect on the changes in relative prices which occur as a result of real factors influencing supply and demand. At the same time, Parks reported unanticipated inflation does increase relative price variance. In another form of his model, Parks did observe some evidence consistent with a separate real effect for the rate of inflation, i.e., non-neutrality.

Adopting yet another technique, Cukierman and Wachtel [1979] used the variance across survey respondents of the twelve-month expected inflation rate in the CPI as published by Survey Research Center of the University of Michigan. The authors calculated a time-series for the variance of expected inflation rates across survey respondents for the period 1947-1975 as a measure of heterogeneous expectations. Their results are summarized as follows:
\begin{align*}
\text{COV}[\sigma^2(\hat{E}(\hat{\rho})), \sigma^2(Y)] &> 0 \\
\text{COV}[\sigma^2(\hat{E}(\hat{\rho})), \sigma^2(\rho)] &> 0,
\end{align*}

where \( \hat{E}(\hat{\rho}) \) = expected rate of inflation across survey respondents.

\( \hat{Y} \) = rate of change of nominal GNP.

In words, periods with large variances in the rate of inflation and large variances in the rate of change of nominal income also tend to be periods with large variances of inflationary expectations. As the authors observed, when these results are considered in conjunction with Wachtel's [1977] finds that the level and variances of expected inflation are positively correlated, the indication is clearly,

Greater variability of inflationary expectations also leads to greater incidence of unanticipated inflation and consequent unanticipated redistributions in nominally denominated asset markets. This may well be an important cost of higher inflation. (p. 606)

E.4 Conclusions

Unquestionably, the ultimate issue as pertains to this study is whether the variability of the inflation rate is a contributing source of excess burden associated with the inflation tax. We have shown inflation uncertainty affects relative prices and hence will induce
substitution effects. Accordingly, the inflation tax distorts choices even if the anticipated tax rate is an unbiased estimate of the actual future inflation rate.

In a world where at least some price adjustments are less than instantaneous, there is risk as to the real value of future monetary claims, including future income. This risk is more significant the greater is the variability of the future inflation rates about their expected value. The empirical evidence suggests a rise in the uncertainty about the future rates of inflation will be associated with a decline in consumption demand and industrial production, and an increase in savings, money demand, and the unemployment rate. Some or all of the increase in savings may be attributable to faulty investment decisions based on market equilibrium models which do not fully incorporate all measures of asset risk including inflation risk, such as the traditional specification of the Capital Asset Pricing Model. As documented in Sections B and C, such substitutions, whether voluntary or not, are inherently inconsistent with an efficient tax, as the concept is understood in the field of public finance. Studies of tax incidence have traditionally (tacitly) assumed a world of certainty. As a result, no body of evidence has been amassed which might be called upon to suggest the relative significance of inflation tax uncertainty vis-à-vis the uncertainty of other taxes.
The ability to conceptually as well as empirically differentiate between the costs and effects of unanticipated inflation on the one hand, and the costs and effects attributable to uncertainty about future inflation on the other, is inversely related to the degree of dependence of one to the other. The evidence to date tends to support the hypothesis of positive covariance. This being the case, any failure to account for the variance in inflation will distort the estimate of the efficiency of the inflation tax. This point was raised earlier in this section, but it warrants repetition. In effect, estimates of the income and substitution effects for the anticipated inflation rate will be distorted by their covariance with the omitted inflation uncertainty substitution effect.

Let $X_t$ be some variable measured in nominal values which is determined by the expected inflation rate, $E(\rho)$, and the period $t$ variance of the expected inflation rate, $\sigma_t^2(E(\rho_{t+1}))$, as:

$$EQ(13) \quad X_t = \beta_0 + \beta_1 E(\rho_t) + \beta_2 \sigma_t^2(E(\rho_{t+1})) + \varepsilon_t$$

Suppose equation EQ(14) is estimated instead of EQ(13). Solving for the estimated coefficient obtains EQ(15), which says the estimated

$$EQ(14) \quad X_t = \hat{\beta}_0 + \hat{\beta}_1 E(\rho_{t+1}) + \hat{\varepsilon}_t$$

$$EQ(15) \quad \hat{\beta}_1 = \beta_1 + \beta_2 \text{COV}[E(\rho_{t+1}), \sigma_t^2(E(\rho_{t+1}))] \sigma^2$$
coefficient is an unbiased estimate of the true coefficient for anticipated inflation if and only if there is no covariability between the anticipated inflation rate and the variance of the inflation rate. But by assumption the covariance is positive. Consequently, the omission of a measure of the inflation rate variability results in an upward biased estimate of the effect of the anticipated inflation rate.\footnote{11} Alternatively, if $\beta_2$ is negative, as the empirical evidence implies when $X$ represents "consumption," then $\hat{\beta}_1$, will be biased downward. In either case, statements about the anticipatory efficiency of the economy will be distorted. This will contribute to further distortions in the private section if public policy is responsive to such statements.\footnote{12}

In this section we have endeavored to establish the concept that the variability of the inflation rate is an important determinant of the real effects from inflationary finance. This concept has been systematically omitted from empirical estimations. Attempts to derive a coherent theory of the costs and effects of inflationary finance are jeopardized to the extent that such omissions distort estimates of the extent to which risk-averse markets anticipate future inflation.
F. GENERAL EQUILIBRIUM AND SIMULTANEITY BIAS

We have argued the effects of inflationary finance should be measured within the context of a general equilibrium model of the economy. Partial equilibrium analysis is unacceptable because it fails to take into account the interdependence among markets and what occurs in product and factor markets simultaneously when the inflation tax is imposed. Inflationary finance imposes a tax on one form of capital, namely money. Since the tax is a general tax on all money it would be borne by the owners of the money regardless of the demand for money or mobility of money, were it not for the fact that the supply of financial capital is not perfectly inelastic.\(^1\) Capital substitution matters. The substitution effects cannot be captured in a partial equilibrium analysis.\(^2\) Likewise, differential inflation expectations will cause resource misallocations, the costs of which will be borne disproportionately throughout the economy depending on individual abilities to both anticipate the tax and to shift the burden or retain any benefits.

It is accepted that general equilibrium is the appropriate framework for assessing the impact of changes in the inflation rate. This is attributable in no small way to the recognition that inflationary finance is equivalent to an excise tax on money. However, the application of general equilibrium analysis has been limited almost exclusively to theoretical treatises. Works by Barro
and Grossman [1976], Turnovsky [1977], and on a smaller scale by Drazen [1979] and Levi and Makin [1978] [1979], are exemplary. Drazen's paper is particularly relevant to this study because he demonstrates how the commonly used partial equilibrium approach seriously diminishes the generality of the models of the optimal rate of inflation (cf. Section B above). Rational expectations models are inherently general equilibrium systems of equations. The very concept of the rational expectations approach is to construct expectations on the model itself. Consequently, the expectations are covariant with the stochastic errors of the structural equations.

The customary methodology, in the study of inflation, is to model a single dependent variable, such as the (nominal) interest rate, as a function of a set of behavioral variables which includes and may even be restricted solely to the anticipated rate of inflation. The model is then estimated by ordinary least squares (OLS), or perhaps ordinary least squares with an autocorrelation correction technique. The OLS procedure is of course consistent with the assumption of a partial equilibrium model. Numerous illustrations of this methodology are reviewed in the second essay in this volume. At this point, what is required is an evaluation of the broader methodological issue of the reliability of the estimates of inflation effects using single equation or partial equilibrium models.
Suppose we are interested in testing the Fisherian Hypothesis. The model is written as:

\[ E(16) \quad i_t = \beta_0 + \beta_1 E_t(\rho_{t+1}) + \beta_2 X_t + \epsilon_t \]

where, \( i_t = \) market interest rate,
\( X_t = \) a determinant of the real rate of interest, e.g. the money supply,
\( \epsilon_t = \) stochastic disturbance, such that,
\[ E(\epsilon_t) = 0, \forall t \]
\[ E(\epsilon_t, \epsilon_{t+j}) = 0, \forall j \neq 0 \]

For now assume the expected inflation rate is independent of \( \epsilon \), either because \( E_t(\rho_{t+1}) \) forms a set of fixed numbers or because \( E_t(\rho_{t+1}) \), the market or sample expected inflation rate, equals \( E_t(\rho^*_{t+1}) \), the true expected inflation rate plus a stochastic error \( \hat{\epsilon} \), i.e., \( E_t(\rho_{t+1}) = E_t(\rho^*_{t+1}) + \hat{\epsilon} \). However, let \( E(\hat{\epsilon}) \) equal zero with \( \epsilon \) independent of \( \hat{\epsilon} \). This eliminates the errors in variables bias.

Consider first the case where \( X \) is near exogenous. Assuming the model is correctly specified, the OLS estimates of the \( \beta \)'s will be unbiased. The problem in this case is not statistical, but rather theoretical. Equation \( E(16) \) is merely a specific price equation. For the estimates of the coefficients to be unbiased, the model as given must also satisfy an implicit unidirectional causality constraint. The variables on the right-hand-side must be independent
(exogenous) of the price \( i \). The variable \( X \) explains the changes in the real interest rate. Candidates for \( X \) are the real money supply and real income. Yet, under virtually any comprehensive economic theory, both of these variables are functions of the interest rate. Therefore, theory must be modified or an alternative determinant of the real interest rate found. This points to the paradoxical nature of the problem. In order to guarantee simultaneity bias-free estimates for a single product or factor model either, the behavioral variables must be near exogenous, a requirement which invariably will conflict with economic theory if not observed behavior; or the equation must be a reduced form of some unspecified structural system, in which case it is impossible to conclude whether the underlying theory has been confirmed or rejected by the model estimates.\(^3\)

Assume now, in addition to \( i \) a function of \( X \) in EQ(16), that \( X \) is a function of \( i \). Ordinary least squares coefficients will be biased and inconsistent. In short, a partial equilibrium model is a biased representation if a phenomenon is truly general equilibrium in nature.

One way to avoid this problem is to assume that although \( X \) may be a function of \( i \), the reverse is not the case. In our particular example this could be justified by assuming the expected value of the real interest rate is constant, as in Fama [1975] [June, 1976]. However, unless the model is estimated over a relatively short period, such an assumption is unlikely to be a good first approximation of
reality. Without evidence to justify the exclusion of X, the model will be misspecified and OLS estimated coefficients will suffer from an omitted variable bias. This is the problem addressed in the previous section. Such a model takes the form:

\[
EQ(17) \quad z_t = \alpha_0 + \alpha_1 E_t(\rho_{t+1}) + \mu_t
\]

where \( z_t \) is a factor rate of return. Implicitly EQ(17) states the equilibrium expected real value of \( z \) is constant. Yet, such equations are commonly estimated with several years of data, suggesting the equilibrium expected real \( z \) is independent of the stochastic shocks which befall the economic system. One such shock is unanticipated changes in the inflation tax. Hence, the estimate of the coefficient \( \alpha_1 \) is consistent and unbiased if and only if the inflation tax is neutral:

Consider now the case where the assumption \( E_t(\rho_{t+1}) \) is exogenous is relaxed. If the expected inflation rate is a function of a series of strictly historical inflation rates, then the model may be estimated (block) recursively. However, if the anticipated inflation rate cannot be reduced to a function of its own lagged values, then it must be determined simultaneously along with other endogenous variables. Such interdependence is even more likely when the factor and product markets are influenced by the variability of the inflation rate.
The possible cases we have considered may be reduced to the following:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Partial Equilibrium Analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( E_t(\rho t+1) ) is near exogenous.</td>
<td></td>
</tr>
<tr>
<td>a. ( X ) is near exogenous.</td>
<td>Valid, cet. par. However, will be difficult to theoretically support the assumption of unidirectional casualty from ( X ) to ( i ).</td>
</tr>
<tr>
<td>b. ( X = f(i) )</td>
<td>Invalid. Estimates will be biased and inconsistent.</td>
</tr>
</tbody>
</table>
|   c. \( X = f(i) \)  
   \( i \neq f(X) \) | Valid, cet. par. However, will be difficult to theoretically support the implication that the real rate \( (r) \) is constant, \( E_t(r t+1) = \bar{r}, \forall t. \) |
| 2. \( E_t(\rho t+1) \) is endogenous. |  |
|   a. \( E_t(\rho t+1) = f(\rho t, \rho t-1...) \) | Valid, cet. par. |
|   b. \( E_t(\rho t+1) = f(e_t) \) | Invalid. Estimates will be biased and inconsistent. |

In summary, if the inflation tax is in fact neutral, a general equilibrium model is not required, cet. par. If inflation is non-neutral, however, only a general equilibrium, simultaneous equation model will provide consistent estimates of the full extent of the costs and consequences of the tax. Whether or not the inflation tax is neutral is an issue subject to empirical determination. What
is certain is that any model of economic equilibrium which simply assumes or implies inflation tax neutrality cannot be rejected solely because empirical tests fail to confirm conditions consistent with the model. This all important property of "falsification" is possible only if the tests do not jointly depend upon the neutrality of the inflation tax and the correctness of the equilibrium model.
G. SUMMARY

This essay has examined the theory of inflation as a tax on the purchasing power of money. The contribution to economic theory is essentially one of synthesis. We have endeavored to integrate many of the hypotheses and concepts from monetary theory, the economics of information and uncertainty, capital market theory, welfare economics, and public finance, especially tax incidence theory, regarding market behavior and inflation.

The inflation tax is an efficient tax if no product substitutions arise from its imposition. Substitution, caused by changes in relative prices, leads to the condition where the total loss of welfare from the tax as it is imposed exceeds the loss in welfare which would result if the same tax revenue were collected without distorting economic decisions in the private sector. The differential loss is the excess burden of the inflation tax. If this excess burden is zero for all values of the tax, the tax is said to be neutral.

There are four principle determinates of the neutrality of the inflation tax: (1) the size of the direct burden of the tax, (2) the accuracy and pervasiveness of expectations, (3) the uncertainty about future inflation, and (4) institutional rigidities and market imperfections associated with economic behavior which is restricted to responses to nominal prices (e.g., U.S. income tax laws). Moreover,
the proposition was advanced that the incidence of the inflation tax can be estimated only when all market responses caused by an exogenous change in the tax are identified and measured simultaneously. The costs and effects of the inflation tax, apart from the direct burden, are the consequence of the restoration of general equilibrium. The global burden of the inflation tax is equal to the direct burden if and only if the cost of the tax in any single market is strictly independent of the cost of the tax in all other markets.

The inflation tax is not neutral. The tax is inefficient (in the sense defined above) because even in an idealized economy, real resources will be consumed in attempts to avoid or shift the direct burden of the tax.

The excess burden of the inflation tax, that is the welfare loss to society not offset by revenues to the government, is greater the less certain the private sector is about what the future inflation tax rate will be. Uncertainty about future inflation tax rates alters relative market prices, depending on which market segment bears the risk of uncertainty. It is reasonable to assume the greater in absolute value is the unanticipated inflation tax, the longer it will take for real choices to adjust to the unanticipated shock, and thus the larger will be the welfare loss to society.
The hypothesis has been advanced that perfectly anticipated inflation has no real effects. Only the uncertainty about future inflation rates and the errors in anticipation of the inflation rate are capable of causing substitution effects. Such errors occur in one of two ways. A change in inflation rate expectations causes what this paper titles, "ex-ante unanticipated effects." Alternatively, if the actual inflation rate is not equal to the expected inflation rate then product substitution may occur, producing "ex-post unanticipated inflation effects." What is characteristic of both is the exogeneity of the unanticipated inflation.

If the inflation tax is endogenous, then the expected inflation rate, market prices, and real quantities are jointly and simultaneously determined. An endogenous inflation tax is neutral. A neutral inflation tax has no excess burden. However, an endogenous inflation tax may have caused a loss in social welfare greater than the tax revenue, i.e., excess burden. This would appear to be a logical contradiction. How can an endogenous, neutral tax produce an excess burden? The reconciliation of the apparent impossibility is the key to a unified and general theory of inflation.

We have established that in point of fact there is no contradiction. The inflation tax can be both neutral, as the rational expectations literature maintains, and yet non-neutral, as measured by public finance, incidence theory standards. This is no simple game of
words. It is the reflection of what has been until now two distinct theories of inflation. The expected rate of inflation embodied in the market equilibrium pricing mechanism is endogenous. As such, it has no causal influence. In this sense the expected inflation tax is said to be "neutral." The inflation tax becomes endogenous when equilibrium is restored following a market distortion from a change in the expected inflation rate or from an inflation rate forecast error. Both of these are exogenous. If on the occurrence of either exogenous disturbance relative prices do not deviate, there will be no excess burden from the tax, and the tax will be optimal because it is "neutral" (in the strong form). Clearly, therefore, an endogenous tax at time t does not imply an absence of excess burden at some previous time t-\(j\) (\(j>0\)). Market equilibrium conditions, such as the Fisherian Hypothesis, imply endogeneity, but not neutrality, i.e., the possibility of excess burden remains. On the other hand, neutrality (no excess burden) implies the Fisherian Hypothesis.

We are reminded of the statement by the British mathematician and philosopher Alfred North Whitehead, "When you have a new concept, and you consider by what name you want to call it, you can do either of two things: you can use an old word which you redefine, but if you do, you will always be misunderstood; or you can invent a new word, but if you do, you will never be understood." We have endeavored to circumvent this paradox as it would apply to the theory of inflation by not introducing new concepts, but rather by demonstrating that the
myriad of concepts already existing can be reduced to a relatively simple set of principles. These principles constitute the theory of the inflation tax - a theory about the neutrality of inflation and the incidence of its burden. This theory must ultimately be subject to empirical verification. But empirical testing is conditional upon the existence of a well defined and coherent theory.
FOOTNOTES

B. INFLATIONARY FINANCE: THE BASICS

1 The topic of inflation, if covered at all in public finance textbooks, has generally been limited to what Rolph and Break [1961] call "compensatory finance." This is the study of the activities of government which compensate for variations in private expenditures. No reference to the incidence of the inflation tax was found in: Bastable [1922], Dalton [1932], Allen and Brownlee [1947], Shultz and Harriss [1949], Hicks [1948] [1968], Musgrave [1959], Cauley [1960], Prest [1960], Rolph and Break [1961], Sharp and Sliger [1964], Herber [1967], and Musgrave and Musgrave [1976]. Lutz [1947] explicitly acknowledges inflation as a "kind of taxation" (p. 137), but did not consider the question of the incidence of the tax. Pigou [1929] appears to have been one of the few to comment on the burden of the tax (see pp. 268-270). Buchanan and Flowers [1975] are typical of the growing literature which examines the distortions in the income taxes caused by inflation. They assume resource misallocation caused by inflation. Pesek [1960] is perhaps the best example of a study of inflation as a tax. He compared the distributional burden of inflation with the distributional burden of an income tax, a sales tax with food not taxable, and a sales tax with food taxed. His calculations showed that over 70 percent of U.S. families have a "clear incentive to support the use of an alternative equilibrating tool, namely, the use of income taxation." Pesek simply computed the direct burden (i.e., tax liability) for the alternative taxes. He implicitly assumed that real factors, such as real family income before taxes, are perfectly inelastic. His methodology does not permit conclusions about the differential incidence of the inflation tax. This same criticism applies to three other well known empirical studies of the "incidence" of the inflation tax, namely, Bach and Ando [1957], Brownlee and Conard [1961], and Brimmer [1971].

2 In a review of the literature on inflation it quickly becomes apparent that no other single topic could possibly have received more attention by economists than has inflation. It is impossible to provide a definitive bibliography on the subject. However, certain works warrant acknowledgment. In addition to the works cited above, Branson [1975] provides a comparison of the Keynesian and the Monetarist models of the transmission of inflation. Fischer's [1975] review of the recent developments in monetary theory is particularly helpful in highlighting the changes in the conceptual
framework that have taken place. Fischer contends that the greater significance attached to inflation, or more accurately to inflation expectations, is the result of a shift from comparative static analysis to dynamic analysis. Fand [1969] made a similar argument a few years earlier in his review of the Keynesian theories of the 1960's. Hagger [1964] modelled and then criticized the monetarists' theory of inflation, raising the objection that inflation tax need not result in a change in consumption demand. He asks, "To put the question in another way, may he [the consumer] not continue to determine real consumption spending in the light of real income before the implicit tax and react by rearranging the disposition of real saving - by devoting less to building up his real bond-holding and more to building up his real cash-holding - than he would have done had prices been stable?" (p. 145) A positive relationship between cash balances and inflation expectations has in fact been observed, for example in Schmidt [1978] and Harberger [1978]. Such evidence is inconsistent with generally accepted theory, but confirms Mueller's [1959] earlier discovery that consumer spending has responded inversely to price expectations. In Section E we suggest such results may be due to omitted variable misspecification error.

3 Friedman [1971] was among the first to argue that if real income is growing over time, then the demand for cash balances will likewise be increasing. Consequently, the government can obtain revenue simply by supplying this incremental demand, without resorting to inflationary financing.

4 In incidence study, the question of who bears the burden of the tax (sources side) is customarily distinguished from the question of who benefits from the expenditures financed by the tax (uses side).

5 Bailey drew upon Friedman's [1953] analysis of inflation as a tax on real (money) balances.

6 The first three assumptions are generally equivalent to those employed by Bailey, [1956, p. 94].

7 Cf. Patinkin [1965].

8 Bailey [1956], p. 94.


10 Bailey [1956], p. 95.
As a general rule, the demand for money equation used in these studies, including Bailey's is a variant of Cagan's [1956] model from his classic hyperinflation study. An exception is Barro [1970] [September/October, 1972]. Using a sophisticated version of the Baumol-Tobin inventory theoretic approach to the (transactions) demand for money, with an endogeneous payment period, Barro concluded that inflationary finance is unacceptable on social welfare grounds. The net welfare cost of inflation was derived from the transaction costs which individuals incur in order to avoid the private interest cost associated with holding cash balances. The "marginal collection cost" of inflationary finance, that is the change in the welfare cost as a function of the change in government revenue, was reported to exceed one-half for all positive rates of inflation.

For simplicity we assume the private sector consists solely of households.

Patinkin [1965], cf. pp. 45, 50.
Patinkin [1965], p. 74.
Cf. Patinkin [1965], Chapter III.
Cf. Patinkin [1965], Hansen [1970]

A tax may be endogenous at time t and yet the economic system can still be adjusting from the exogenous shock caused by a change in the tax rate at time t-n. Endogeneity describes the state of the process where the value of the variable, in
this case the tax revenues, is determined within the system. The econometric equivalent is the case of lagged endogenous variables.

Laidler and Parkin [1975] concluded their comprehensive review of inflation with much the same view.

In the light of this work the question as to whether monetary expansion is a unique "cause" of inflation seems to us to be one mainly of semantics, and one that distracts attention from another, more important theme to emerge from our survey, namely that analysis of the inflationary process must involve the study of the whole economic system and not just of one or two markets in isolation. The quantity theory of money might provide one with hypotheses about the behaviour of the money market in the inflationary process, and the Phillips curve with hypotheses about the labour market, but precisely because inflation involves changes in the value of money, its analysis must deal with all markets in which money serves as means of exchange and a unit of account. That, at least in a developed economy, must mean virtually all markets. (p. 796)
C. THE SOCIAL COSTS AND ECONOMIC CONSEQUENCES OF INFLATION


According to economic theory, the ultimate social cost of anticipated inflation is the wasteful use of resources to economize holdings of currency and other noninterest-bearing means of payment. I suspect that intelligent laymen would be utterly astounded if they realized that this is the great evil economists are talking about. They have imagined a much more devastating cataclysm, with Vesuvius vengefully punishing the sinners below. Extra trips between savings banks and commercial banks? What an anti-climax! (p. 15)


3. Cf. Jaffee and Kleiman [1977] who maintain that welfare costs also arise because inflation is uneven (i.e., the variance is nonzero). See Section E for additional comments.

4. The analogue between "fully anticipated" and endogenous, and "unanticipated" and exogenous characterizes the link between the theoretical and econometric properties of a neutral inflation tax. A variable which is "truly" endogenous has no effect, in a causal sense, on other variables. Two endogenous variables will at most exhibit contemporaneous covariance. Hence, if one variable behaves in a well defined manner with respect to the value of another variable at some previous date, then a "causal connection" may be inferred. Our use of the terms "exogenous" and endogenous" is in accordance with their meaning as found in Ando, Fisher, and Simon [1963], especially as used by F. M. Fisher [1961] reprinted therein.
D. THE ROLE OF EXPECTATIONS


2. See Fama [1970] for details. We have elsewhere elaborated on the efficient market hypothesis, cf., Dyer [1976].

3. The concept of rational expectations has virtually become synonymous with "distributed lag forecasts." In part this confusion is attributable to Muth [1961], who illustrated rational expectations by a case which turned out to be extrapolative. Rational expectations is a theory of information demand. Distributed lag forecasts are a class of expectations formation mechanisms which may or may not be the "true" model of market equilibrium. Rutledge [1974] and Nelson [1975] have elaborated on these distinctions.

4. Fama [1970], p. 385. Poole [1976] and more recently Mishkin [1978] have explored at length the implications of the efficient market theory and rational expectations to macro models and monetary policy.

5. The progressivity of the rates (i.e., convexity) implies that even if the expectation errors are mean zero, the net additional tax will not be zero. Let, $Y$ be anticipated income, $Y(\varepsilon)$ the unanticipated nominal income due to unanticipated inflation, and $T(Y)$ the tax revenues. Then for $Y(\varepsilon_0) = -Y(\varepsilon_1); T(Y) \leq T(0.5(Y+Y(\varepsilon_0)) + 0.5(Y+Y(\varepsilon_1)))$.


7. A "purely deterministic money supply rule" is one where the growth rate of money is not a random variable.


8. Lucas [April, 1972], p. 114. Sargent and Wallace [1976] summarize Lucas [April, 1972] but in a less rigorous manner and with more emphasis on economic policy. Their conclusion warrants repeating,
The conundrum facing the economist can be put as follows. In order for a model to have normative implications, it must contain some parameters whose values can be chosen by the policymaker. But if these can be chosen, rational agents will not view them as fixed and will make use of schemes for predicting their value. If the economist models the economy taking these schemes into account, then those parameters become endogenous variables and no longer appear in the reduced-form equations for the other endogenous variables. If he models the economy without taking the schemes into account, he is not imposing rationality. (p. 183)

9 Barro and Fischer [1976], p. 156.

10 Ibid., pp. 161-162.


12 Cf. Barro [1976].

13 Both the information problem and the convergence problem have been discussed in many works, including the literature generally referred to as the "new microeconomics of money, prices, and unemployment." See Gordon [April, 1976], Barro and Fischer [1976], and Frisch [1977] for limited summaries. Brock [1972] has considered the related problem of the microeconomics of expectations.

In defense of the efficient market - rational expectations theory, Barro and Fischer [1976] comment,

Although the rational expectations hypothesis provides a consistent explanation of some cyclical phenomena, it arouses a number of strong reactions. One reaction is that it is absurd to assume individuals can make the necessary calculations since most economists still cannot do so; this is reminiscent of criticisms of microeconomic theory on the grounds that most consumers have never seen a Lagrangean, and it is equally beside the point. (p.163)
Poole [1976] maintains that a distinction must be made between auction-markets, such as the stock market, and markets where spatial and temporal price differentials in excess of those "reflecting equalizing differences" cannot be arbitraged away, such as the labor market (according to Poole). Rational expectations is said to pertain more to the former markets than the latter.

Keynes assumed inelastic expectations in all markets, although he concentrated on the bond market. See Leijonhufvud [1968], especially Chapter V.
E. INFLATION UNCERTAINTY

1. For more extensive evaluation of the Bigger and Chen and Boness papers, see the discussants' comments pp. 503-507 same issue (Journal of Finance).

2. This same analysis may be applied to any number of economic variables. An interesting application is the case of wage demand. Suppose that at time $t$ a one-period wage contract is being negotiated. The expected one-period ahead inflation rate is $E_t(\rho_{t+1})$ and is known with certainty. Therefore, the wage demand $w^d_{t+1}$, for a desired real wage increment, $g^d_{t+1}$, is determined simply as:

$$\left(w^d_{t+1} \mid \sigma^2_t(\rho_{t+1})=0\right) = (w_t+g^d_t)[1+E_t(\rho_{t+1})]$$

Now suppose that $E_t(\rho_{t+1})$ is not known at time $t$ with certainty, but is subject to a probability distribution of outcomes $[\sigma_t(E_t(\rho_{t+1}))>0]$. It follows from what was said above that there exists a wage demand $w^d_{t+1}$ such that:

$$[\hat{w}^d_{t+1} \mid \sigma^2_t(\rho_{t+1})>0] > [\hat{w}^d_{t+1} \mid \sigma^2_t(\rho_{t+1}) = 0]$$

s.t. $U[\hat{w}^d_{t+1} \mid \sigma^2_t(\rho_{t+1})>0] + U[\hat{w}^d_{t+1} \mid \sigma^2_t(\rho_{t+1}) = 0]$

$$g_{t+1} = \bar{g}$$

$$E_t(\rho_{t+1}) = \bar{\rho}$$

The important result is that empirically one will not substantiate the Fisherian Hypothesis for the simple model,

$$w^d_{t+1} = \beta_0 + \beta_1 E_t(\rho_{t+1}) + \epsilon_t$$

In fact, $\beta_1$ will be greater than unity provided $w^s$ is equal to $w^d$. Of course, management has the incentive to offer:
\[(w_{t+1}^S | \sigma_t^2(\rho_{t+1}) > 0) < (w_{t+1}^S | \sigma_t^2(\rho_{t+1}) = 0)\]

A corollary is that the moving standard deviation of the annual rate of price change does not properly measure the unpredictability of prices.

Cf. Brealey and Schaefer [1977] who extended Fama's [June, 1976] analysis to the term structure question. They concluded, by analytic inquiry, that the equilibrium rate of interest on a forward contract is equal to the expected spot rate plus a premium. The premium, however, is strictly a risk premium and is unrelated to maturity preferences as is frequently assumed in the term structure literature. The premium depends solely on the degree of inflation uncertainty.

Ackley [1978], p. 149.

Ibid., p. 152.

Friedman [1977], p. 466.

Foster [1978] commented,

Since growth in the rate of inflation also tends to raise the average rate for the country, using \(\sigma\) as a measure of variability could bias the results in some cases, producing from the systematic components a positive correlation between \(m\) and \(\sigma\). (p. 347)

Where \(m\) = average rate of inflation and \(\sigma\) = standard deviation of the annual rates of inflation about \(m\). Foster proposed the average absolute change in the inflation rate as a suitable substitute for \(\sigma\).

The Jaffee and Kleiman [1977] distinction between ex-ante and ex-post costs of inflation parallels the same distinction described in Section C above. As Jaffee and Kleiman commented,

Ex ante costs occur whenever inflation is expected and individuals are affected or change their behaviour on this account. Thus, as long as inflation is expected, ex ante costs will occur regardless of whether the inflation actually takes place. Similarly, when inflation is a surprise, then
there will be no ex ante costs. The excess burden of the inflation tax on cash balances is a clear case of ex ante costs. Ex post costs of inflation, in contrast, arise when the realised inflation rate differs from the anticipated rate. Thus, ex post costs can arise either because the prior probability distribution was wrong, or, even if the distribution were right, because individuals did not take actions to anticipate its effects. An undesired income redistribution resulting from an unanticipated inflation is a clear example of ex post costs. (p. 288)

10 The authors also used the semiannual survey of forecasters and business economists conducted by Joseph A. Livingston.

11 The actual variance of the anticipated inflation rate is less than the true variance, raising the true probability of a Type I error.

12 Cf. Levi and Makin [1979] who contend that it is precisely the failure to control for changes in the real rate of interest that are associated with changes in anticipated inflation, including changes in the variability of inflation, that explains the contradictory evidence to the Fisherian Hypothesis (cf., "The Question of Inflation Tax Neutrality: A Review of Selected Literature," Section B, this volume).
F. GENERAL EQUILIBRIUM AND SIMULTANEITY BIAS

1 This is a general result for a tax on all uses of a product or factor. Cf. McLure and Thirsk [1975].


3 An exception is if the model is exactly identified.

4 Strictly historical excludes the contemporaneous inflation rate.

APPENDIX A

Measuring the Social Welfare Cost of Inflation

Bailey [1956] and Marty [1967] were the first to estimate the social welfare cost of inflation. Bailey simulated the tax consequences for "Sylvania," a country "with the most favorable situation likely to occur anywhere." He found that the Sylvania government could obtain up to 18.4 percent of the national income by inflation taxation. If the government took approximately 10 percent of income, the excess burden would be about 15 percent of the amount collected. Bailey concluded by commenting,

The idea discussed here of a deliberate, openly announced inflation has little to do, of course, with inflation as it is customarily experienced . . . Governments typically do not admit that they have in any way contributed to an existing inflation, whether rapid or moderate; in their view they are helpless pawns forced to issue increasing quantities of money in response to price rises generated by forces beyond their control. In practice this has meant, on the one hand, that it was possible for governments to get far more resources, by increasingly rapid money-creation constantly outrunning the public's expectation, than is implied by our results and, on the other hand, that the redistributive and disruptive effects of inflation were considerable. (p. 110)

Comparative static analyses of the revenue maximizing rate of inflation include those by Marty [1967] and Friedman [1971]. Mundell [1965] examined the hypothesis that inflationary financing contributes to economic development. In addition to challenging the validity of
this hypothesis, Mundell demonstrated that comparative static analysis overstates the growth-maximizing rate of inflation. He argued that even rapid inflations are likely to add less than 1.5 percent to the growth rate of the economy, assuming no resource misallocation from inflation. Tower [1971] took issue with Marty [1967] and Bailey regarding their definition of the "efficiency" of the inflation tax. Both had used the ratio of the excess burden of inflation to the revenues from inflation and similar measures to determine the maximum justifiable inflation tax rate. Tower correctly argued such "efficiency" concepts are average cost measures, whereas the optimal inflation tax pertains to the ratio of marginal cost to marginal revenue. In a growing economy, allowance must be made for the fact that the denominator in such measures naturally increases. Hence the appropriate measure of the "efficiency" of the inflation tax is, according to Tower, the marginal cost of the tax as a ratio of the marginal cost of collecting the same revenue by alternative taxes. The marginal cost of inflationary finance was defined by Tower as the incremental excess burden divided by the incremental revenue collected from an infinitesimally small increase in the rate of inflation. Tower demonstrated the efficiency of the inflation tax declines as the real income growth rate increases because growth reduces the marginal revenue from inflation. This can be seen by referring to Figure I from Section B, which is reproduced here. An increase in the expected inflation rate from $E(\rho_1)$ to $E(\rho_1) + E(\varepsilon)$, raising market
FIGURE I

Inflation and the Demand for Real Money Balances

Interest Rate

\( i_0 = i_1 \)

\( i_{0+\epsilon} \)

\( i_2 \)

\( \frac{M_0}{P_0 + E_0 + (\rho_1)} \)

\( \frac{M_0}{P_0} \)

Real Money Balances

\( \Omega_0 \)

\( \Omega_1 \)

\( \Omega' \)
interest rates to \( i_2 \), generates incremental revenues of area "a" minus area c. The excess burden, however, increases by the sum of the areas b + c + d.

Cathcart [1974], Auernheimer [1974], and Frenkel [1976] addressed the issue of the magnitude of the excess burden and the inflation tax revenues conditional on dynamic adjustments of cash balances and the rate of inflation in the transition from one equilibrium to another. Cathcart concluded the marginal cost of the inflation tax is higher than Bailey's analysis had implied, but not as high as Tower reported. More important, he found that, in addition to lowering the efficiency of inflationary finance, the growth in real income probably decreases the excess burden. Lags in inflation expectations were shown to yield more revenue for the government.

Frenkel [1976] stressed the importance of controlling for the time factor in comparisons of the inflation tax excess burden and government earned revenue stream. This is done by comparing the present discounted values of the burden and revenue streams. Frenkel claimed to demonstrate that for a "lagging economy," (i.e., an economy with less than instantaneous restoration of long-run equilibrium) the government revenue initially exceeds the excess burden. As time passes and "expectations start catching up," the demand for cash balances declines and the excess burden increases. However, since the benefits accrue during the early phase of the adjustment period while
the costs accrue later, there necessarily exists some positive discount rate for which the net present value of the entire series of costs and benefits is positive. If this rate is less than the social discount rate then positive (net) social welfare gains are possible.

More recently, Aghevli [1977] reconsidered Mundell's [1965] analysis which had also claimed the possibility of growth benefits outweighing welfare costs. Like Frenkel, Aghevli concentrated on the present discounted value of the incremental consumption derived from a higher growth rate of output. He showed that for "plausible values" of various parameters of his model, a case could be made for "moderate rates of inflation." But Aghevli's results were quite sensitive to both the social discount rate and the coefficient for inflation expectations in the demand for money function. All of these studies, however, explicitly assumed that inflationary finance does not distort choices between consumption and savings, work and leisure, etc.*

*Resource reallocations induced by inflation would be observed as a consequence of money illusion. Interestingly, Goldman [1972] has demonstrated how money illusion can be stabilizing. Money illusion mitigates against the conversion of excess demand for goods into unexpected inflation, in a world in which inflation is self-generating (i.e., expectations systematically underestimate the inflation rate, thus inducing a decline in the demand for cash and further excess demand for goods).
APPENDIX B

Proof That The Inflation Tax Is Endogenous If And Only If Inflation Rate Expectations Are Rational

Using the symbol ¬ to denote negation, \( \rightarrow \) to denote “implies,” and \( \iff \) to represent "biconditional," let each of the following be defined.

\[ P(A): \quad \alpha \iff \beta \]

\[ \alpha = \text{Unanticipated inflation causes real effects.} \]

\[ \beta = \text{Inflation tax is exogenous.} \]

\[ P(B): \quad \neg \lambda \iff \alpha \]

\[ \neg \lambda = \text{Prices do not incorporate all available information about expected future inflation rates.} \]

Definition \( \neg \beta \iff \eta \)

\[ \eta = \text{Inflation tax is endogenous} \]

Propositions:

\[ P(1) \quad \lambda \iff \theta \]

\[ \theta = \text{Inflation rate expectations are rational.} \]

\[ P(2) \quad \eta \rightarrow \neg (\neg \lambda) \]

Propositions P(1) and P(2) are those derived in Section D.
Now prove (1), "If the inflation tax is endogenous, the inflation rate expectations are rational."

(1) \( \eta \rightarrow \theta \)
1. \( \eta \rightarrow \sim \beta \) by definition.
2. \( \sim \beta \rightarrow \sim \alpha \) by P(A) and modus tollendo tollens.
3. \( \sim \alpha \rightarrow \sim (\sim \lambda) \) by P(B) and Law of Double Negation.
4. \( \lambda \rightarrow \theta \) by P(1)

:\( \eta \rightarrow \theta \)

Now prove (2), "If inflation rate expectations are rational, then the inflation tax is endogenous."

(2) \( \theta \rightarrow \eta \)
1. \( \theta \rightarrow \lambda \) by P(1).
2. \( \lambda \rightarrow \sim \alpha \) by P(B), law for biconditional sentences, and modus tollendo tollens.
3. \( \sim \alpha \rightarrow \sim \beta \) by P(A), law for biconditional sentences, and modus tollendo tollens.
4. \( \sim \beta \rightarrow \eta \) by definition.

:\( \theta \rightarrow \eta \)

By the Law for Biconditional Sentences, we obtain,

\( (\eta \leftrightarrow \theta) \leftrightarrow (\theta \rightarrow \eta) \) and \( (\eta \rightarrow \theta) \).
Thus, we achieve Proposition P(3) of Section D which states, the inflation tax is endogenous if and only if inflation rate expectations are rational.
THE QUESTION OF INFLATION TAX NEUTRALITY:
A REVIEW OF SELECTED LITERATURE

A. INTRODUCTION

The neutrality of any tax stems from the degree of relative price change caused by a change in the tax. A neutral tax has only income effects. An unanticipated change in the inflation rate is a change in the tax on the purchasing power of outstanding money balances. This tax induces consumers, laborers, and firms to adjust their behavior. These adjustments not only affect the distribution of the direct tax burden, but they also influence the efficiency of private sector resource use and, hence, the overall burden of the tax. Choices among products are affected, as is the choice between goods and leisure, and the choice between present and future consumption - investment. The degree to which these choices are disturbed depends on the accuracy and pervasiveness of expectations regarding the future inflation
rate. Consequently, to achieve any conclusion about the effects of inflationary finance one must begin with an understanding of how markets incorporate expectations into prices.

This paper studies the workings of the financial and labor markets with respect to inflationary financing. The analysis is conducted within the context of a critical review of the pertinent literature. This approach is adopted because a new hypothesis or theory about inflation is not needed at this time. The concept of inflation as a tax is proposed, not because it is relatively new and original, but because it is a unifying theme. What is required is not another empirical study of the effects of inflation, but rather a consolidation of the vast literature from so many fields of economics - monetary theory, welfare economics, information and uncertainty economics, capital market theory, and public finance. The process of unification - of integration of theory - begins with a constructive yet critical review of the existing knowledge about the process of inflation. To our knowledge, no such attempt has been made at such a broad, conceptual level. It is a further step in the recent movement to produce the microeconomic foundations to macroeconomic behavior.

The analysis is limited to the workings of the financial markets and of the labor market because they have been well studied, and because two of the more commonly cited sources of inflation cost and market distortion pertain to these markets, namely the debtor-creditor
hypothesis and the wages-lag hypothesis. The central question is whether the Fisherian Hypothesis is a defensible approximation of the equilibrium condition for financial rates of return and wage rates, as judged by the empirical evidence.

The analysis proceeds as follows. First, in Section B, we take a closer look at the Fisherian Hypothesis. The discussion commences with a consideration of the precise interpretation of the hypothesis—an issue of utmost significance to a proper assessment of inflation tax neutrality (or lack thereof). This issue is addressed again in Section D after certain basic concepts have been introduced in the preceding sections. In Section B, we also review Irving Fisher's empirical studies of the behavior of interest rates in an inflationary environment. These works had a profound influence on subsequent empirical investigations.

Section C reviews the theoretical and empirical literature pertaining to inflation and the real rate of interest. We first generalize the concept of Bailey's welfare cost of inflation and confirm the proposition reported in the previous essay that even fully anticipated inflation has real (excess burden) effects. The commonly held opinion that the real interest rate falls with accelerating inflation is queried by examining selected scholarly works on the issue.
Section D opens by re-examining the meaning of the Fisherian Hypothesis, with emphasis on the logical relation of the hypothesis and the concept of inflation tax neutrality. The majority of Section D is a review of the empirical literature on interest rates and inflation. The review concludes by observing that most authors have rejected the Fisherian Hypothesis, especially as to the short-run. It is argued, however, that the models were generally misspecified, and apart from the specification problem the models are frequently interpreted in a manner inherently inconsistent with an expectational theory of interest rate determination. The misspecification is the consequence of failing to incorporate the four basic determinants of inflation effects: the effect of the direct burden, expectations, inflation uncertainty, and market separability.

Section E takes up the issue of whether common stocks are inflation hedges. This question is merely another way of expressing the Fisherian Hypothesis and of testing the behavior of real rates of return during inflationary periods. The review of the literature affords the opportunity to define several key concepts and to emphasize the necessity to model market responses to inflation in a general equilibrium structure. The crucial role of uncertainty is demonstrated.
Section F reviews the literature pertaining to capital market responses to money supply changes. We assume throughout that inflation is always and everywhere a monetary phenomenon. Section F consolidates this assumption with the rate of return - inflation analyses, by evaluating the behavior of interest rates to changes in the money supply. Traditional economic theory assumes changes in the money supply affect desired portfolio balances and hence cause interest rates to adjust as portfolio equilibrium is re-established, i.e., interest rates lag money supply. However, if the capital markets are efficient, then changes in the money supply will be anticipated and interest rates will fully reflect all available information regarding the future and of course the current money stock, i.e., interest rates will lead money. An efficient market also fully reflects all available information about the expected future general price-level.

Section G examines the debtor-creditor hypothesis, i.e., the proposition that wealth, in the form of purchasing power, is transferred from creditors to debtors during inflationary periods. The first consideration is to ascertain what may properly be concluded from the tests of the Fisherian Hypothesis regarding the existence of a debtor-creditor wealth transfer. The role of differential expectations is investigated and shown to be a sufficient condition for a failure of the Fisherian Hypothesis and a cause of the debtor-creditor effect. An illustration of asset valuation with
imperfect inflation expectations is presented to help clarify the issues. The literature review reveals that the debtor-creditor hypothesis is by no means an indisputable proposition of fact.

Section H examines the effects of inflation on the labor market. Two basic issues are addressed: (1) Do wages lag behind price changes, and (2) What is the implication of the wages-lag hypothesis to the possibility of an inflation-unemployment trade-off. The answer to both questions depends on homogeneity and accuracy of inflation forecasts, which is to say the answer is determined by whether or not the Fisherian Hypothesis (in wages) is an appropriate characterization of the labor market. The empirical evidence on both questions is shown to be faulty because of the same misspecification problems which plague the rate of return tests.

Finally, in Section I, we summarize the principle conceptual issues addressed in the study.
B. THE FISHERIAN HYPOTHESIS

The theory of portfolio balance is in the end analysis a transmission mechanism theory in which some exogenous disturbance is traced through a set of interrelated responses producing adjustments in the stock of assets or liabilities, whichever constitutes the portfolio of interest, until an equilibrium is established that obeys the first principle of price theory, namely that the ratios of the marginal utilities of the assets to their own price are equal for all assets in the portfolio. Today the portfolio balance theory is most commonly associated with an approach to money demand popularized by Tobin, Hicks, Markowitz, and of course Friedman. However, one of the earliest applications of the theory was by Irving Fisher in his study of interest rates. Fisher argued that in a world of perfect markets and errorless anticipations, an aggregate portfolio comprised of a monetary asset and a real asset is in equilibrium when the money rate (i.e., the nominal rate) of interest is equal to the real rate plus the expected future (annuity equivalent) rate of change in prices. This famous equation, which we write as EQ(1), is referred to throughout this study as the Fisherian Hypothesis.
EQ(1) \[ i_t = r_t + E_t(\rho_{t+j}) \]

where, \( i_t \) = nominal rate of interest,
\( r_t \) = real rate of interest,
\( E_t(\rho_{t+j}) \) = the 't' th period expectation of the inflation rate over time t to t+j

The interpretation of EQ(1) is not uncontested in the literature. We interpret Fisher to have described an idealized or normative condition represented more accurately as EQ(1-a). The condition (the differential) implies that the real rate of interest is independent of the expected inflation rate, although not necessarily constant over time. This interpretation could also be referred to as the strong form

EQ(1-a) \[ \frac{d_i_t}{dE_t(\rho_{t+j})} = 1 \]

EQ(1-b) \[ \frac{d_i_t}{dE_t(\rho_{t+j})} \bigg|_r = 1 \]

Fisherian Hypothesis. An alternative interpretation of Fisher's theory, represented by EQ(1-b), is that, ceteris paribus, the market rate of interest is unitary elastic with respect to the expected future inflation rate. This would permit EQ(1) to be definitionally true, but would place no restrictions on the behavior of the real rate of interest with respect to inflation expectations. We call this the weak form Fisherian Hypothesis. This proposition is especially
relevant to this study because, when coupled with an expectation condition described in Section D, it asserts that the inflation tax is expectation neutral. In other words, if the weak form Fisherian Hypothesis holds then any inefficiencies caused by inflationary financing are attributable to institutional factors, to attempts to avoid or shift the tax (cf. Section C, Mundell Effect), or to inflation uncertainty, but not to money illusion regarding the future level of prices.

Irving Fisher's work on capital theory spanned several decades. His first major work was Appreciation and Interest, published in 1896. This was followed in 1906 by The Nature of Capital and Income, which served as a foundation for The Rate of Interest, published in 1907. Fisher elaborated on his monetary theory of the business cycle in his 1911 publication, The Purchasing Power of Money. The series culminated in 1930 with The Theory of Interest. All of these works originated from a request by the American Economic Association to Fisher for a study of the findings by Thomas Tooke and A. H. Gibson that a positive correlation is observed between commodity price levels and the money rate of interest, an issue which we address shortly.

In an idealized world of no money illusion, Fisher [1911] maintained that EQ(1) must hold. As he stated,
It is plain that the man who lends $100 at the beginning of the year must, in order to get 5 per cent interest in purchasing power, receive back both $103 (then the equivalent of the $100 lent) plus 5 per cent of this, or a total of $108.15. That is, in order to get 5 per cent interest in actual purchasing power, he must receive a little more than 8 per cent interest in money. The 3 per cent rise of prices thus ought to add approximately 3 per cent to the rate of interest. Rising prices, therefore, in order that the relations between creditor and debtor shall be the same during the rise as before and after, require higher money interest than stationary prices require. (p. 57)

The competitive equilibrium mechanism suggested by the above was articulated in The Rate of Interest, particularly Chapters V and XIV. Yet, while Fisher's name is attached to the competitive equilibrium condition given by EQ(1), he in fact did not believe that EQ(1) explained the behavior of short-run prices and interest rates. In the very next paragraph following the one quoted above Fisher [1911] stated,

Yet we are so accustomed in our business dealings to consider money as the one thing stable,—to think of a "dollar as a dollar" regardless of the passage of time, that we reluctantly yield to this process of readjustment, thus rendering it very slow and imperfect. When prices are rising at the rate of 3 per cent a year, and the normal rate of interest--i.e. the rate which would exist were prices stationary--is 5 per cent, the actual rate, though it ought (in order to make up for the rising prices) to be 8.15 per cent, will not ordinarily reach that figure; but it may reach, say, 6 per cent, and later, 7 per cent. This inadequacy and tardiness of adjustment are fostered, moreover, by law and custom, which arbitrarily tend to keep down the rate of interest. (pp. 57-58)
As this statement reveals, Fisher held that the economy was subject to money illusion. Indeed, he wrote a book on the subject by that title. In it Fisher presented the argument that during inflation debtors gain at the expense of creditors - what this study refers to as the Debtor-Creditor Hypothesis. Fisher [1928] lectured on the costs of the debtor-creditor wealth transfer with furor, as when he stated,

Think of the disturbance of loan contracts. When, for instance, there is inflation, and the price level rises, the creditors lose and the debtors gain.

It might seem at first that this is as broad as it is long since the debtor gains exactly as much as the creditor loses. It might be argued that no harm can be done to society as a whole either by inflation or deflation since the average wealth would not be changed.

But one might as well reason that when a bank vault is robbed or when your house is burglarized, society is none the poorer. If you, the victim of the robbery, should be told, "What you have lost the burglar has gained, and therefore society as a whole is just as well off!" that would be cold comfort to you.

In somewhat the same sense this burglarizing dollar is defrauding people, even if it does so impersonally. Something is taken away from its rightful owner. The evil is not (primarily at least) general impoverishment; it is social injustice. Unlike burglary or personal fraud, there is no violation of the letter of the law as to debts, but there is a defeat of its spirit and intent. (pp. 60-61)

The wealth transfer from creditors to debtors, according to Fisher, results in an abnormal return to equity capital owners. Business profits rise because expenses, which include interest
charges, do not increase proportionately to the increase in (nominal) revenues. Therefore, profits rise faster than prices, resulting in a capital gain to equity owners. The validity of this argument rests on the assumption that the (implicit) change in relative factor incomes does not distort product demand. This is assured if all consumers have the same propensity to consume out of income and wealth.

Fisher's empirical studies of the behavior of interest rates with regard to inflation are among the more ingenious, especially for his day. Contrary to the idealized relationship, Fisher found that, interest rates do not adjust completely to changes in expected inflation, at least in the short-run. To establish that expected inflation is a determinant of interest rates, Fisher compared the rates of return on U. S. currency denominated bonds to the rates of return on coin denominated bonds (i.e., payable in gold or silver). The semiannual rates of return data from 1870 to 1896 revealed a difference in the rates of return which changed in sign over time. In 1870, for example, investors realized 6.4 percent for gold bonds but only 5.4 percent for currency bonds. Fisher explained the inferiority of the gold bonds as a reflection of the anticipated resumption of the gold standard and the consequent rise in the value of currency (i.e., deflation). From 1879 to 1894 paper money was roughly on par with gold and the rates of return differed only slightly. Fears of inflation from Greenbackism and Free-Silverism produced a divergence.
in the late 1890s. The currency bond rates rose above the gold bond rates, indicating that investors adjusted interest rate demands for expected future inflation rate changes.

Fisher also compared the interest rates for gold and rupee (i.e., silver) bonds. The interest on rupee bonds was paid in draft on India, so that the sum received in English pounds was a function of the exchange rate. Since both bonds were issued by the same government and possessed the same degree of security, the only important difference between them was the standard in which they were expressed. Fisher's tests revealed that the interest rates did not differ significantly until about 1875 when the Indian exchange rate started to decline rather rapidly. The rupee bonds then commanded a premium. The results, Fisher argued, "afford evidence that the fall of exchange (after it once began) was, to some extent, discounted in advance and affected the rates of interest in those standards."

As further evidence that the market interest rate is affected by price level changes, Fisher compared the pattern of signs for interest rate and price level changes. The international data revealed that in 38 of 53 trials, price level changes and market interest rate changes occurred contemporaneously and with the same sign. For the industrial countries (England, Germany, and the United States), the correlation coefficient for interest rates on price level changes was
reported at about .70. Yet, Fisher concluded that "there is little or no apparent relationship between price changes and interest rates." The empirical evidence would seem to contradict Fisher's conclusion. It is only when Fisher's works are interpreted collectively that this apparent self-contradiction is resolved.

The currency and coin bond interest rate data, and the frequency of consistent sign changes in interest rates and price changes confirmed the hypothesis that interest rates are a function of inflation rates. This hypothesis stood in direct conflict with the Gibson Paradox and Wicksell's [1935] business cycle theory. But while interest rates are determined by inflation rates, the relationship is not, according to Fisher, as perfect (i.e., complete and instantaneous) as theory would have it. The reason - money illusion. As Fisher [1930] argued:

Most people are subject to what may be called "the money illusion," and think instinctively of money as constant and incapable of appreciation or depreciation. Yet it may be true that they do take account, to some extent at least, even if unconsciously, of a change in the buying power of money, under guise of a change in the level of prices in general. (pp. 399-400)
Fisher hypothesized that under EQ(1) the money interest rate should be more volatile than the real rate. What he discovered was just the opposite - the real rate of interest was from seven to thirteen times as variable as the market rate. This reinforced his opinion that, "men are unable or unwilling to adjust at all accurately and promptly the money interest rates to changed price levels." Another "symptom of the same imperfection of adjustment" is the fact that the adjustment is very slow. Using correlation statistics, Fisher concluded that over long periods the interest rate follows price movements, which "corroborate convincingly the theory that a direct relation exists between P' [inflation rate] and i." 

One of the more remarkable aspects of Fisher's study of money, interest and prices, was his application of distributed lag techniques, which he invented. Using the correlation coefficient as a significance statistic, Fisher found that changes in inflation, which are distributed over time with declining weight as defined by EQ(2), affect the interest rate with considerable lag, even as long as 28 years. This conclusion both substantiated his interest rate - inflation rate theory, and simultaneously refuted the neutrality hypothesis.

EQ(2) \[ \alpha_t = 2(n-t+1) [n(n+1)]^{-1} \]

where \( \alpha_t \) = \( t \)'th period weight
\( n \) = number of lag periods
\( t \) = time: 1, 2, ..., \( n \)
Fisher summarized his empirical evidence in four propositions, the first three of which are as follows; where \( P \) is the price level, \( P' \) the rate of change in prices, and \( \bar{P}' \) the sum of the distributed lag of \( P' \).

1. The rate of interest tends generally to be high during a rising price level and low during a falling price level;

2. The rate of interest lags behind \( P' \) so that often the relationship is obscured when direct comparison is made;

3. The rate of interest correlates very markedly with \( \bar{P}' \), representing the distributed effect of lag. For recent years in Great Britain, the close relationship is indicated by \( r = +0.98 \) when \( i \) is lagged and the effects of \( P' \) are distributed over 28 years. (p. 430)

An important question is whether Fisher ever actually tested EQ(1). The relevant variable in EQ(1) is the expected future inflation rate. While there are statements throughout Fisher's works, but especially in *The Theory of Interest*, which emphasize the clear distinction he made between ex-ante and ex-post measurements, there is no explicit statement that the distributed lag is the expectation generating mechanism, as it is now so frequently assumed. Moreover, Fisher constructed the real rate of interest series as the nominal rate of interest minus the period's actual rate of inflation. This is inherently inconsistent with a distributed lag expectations
formation. The actual rate of inflation $\rho_t$ is the appropriate measure for $E_t(\rho_{t+j})$ in EQ(1) if and only if $\rho_t$ follows a random walk, i.e.,

$$\text{EQ(3)} \quad \rho_{t+1} = \rho_t + \varepsilon_{t+1}$$

where $E_t(\varepsilon_{t+1}) = 0$

$$\text{COV}(\varepsilon_t, \varepsilon_{t+j}) = 0, \forall j \neq 0$$

On balance, the evidence, some of it implicit, tends to support the view that Fisher assumed economic agents determine their inflation forecasts according to the distributed lag law of motion. The issue is not all that critical for this study since this is not a treatise on Fisher's contribution to monetary theory. What is important, however, is the fact that Fisher believed interest rate changes lag changes in the (expected) inflation rate. Given the market equilibrium mechanism represented by EQ(1), a lag in interest rates implies the capital markets are not efficient in the sense of instantaneously and completely reflecting all available information in capital asset prices.

Fisher thus provided a theory of the idealized behavior of money, interest, and prices; one which explains how inflation rate changes could, theoretically at least, be neutral. But his empirical analyses
led him to conclude that changes in the inflation rate are not neutral. This non-neutrality was the foundation of his business cycle theory.

Two corollary conclusions of Fisher warrant mention. The first is the "fourth principle" which states:

4. The rate of interest tends definitely to be high with a high price level and low with a low price level.

This of course is the Tooke-Gibson phenomenon, (i.e., Gibson Paradox) which Roll [1972] argues is, in conjunction with the stochastic process in the level of commodity prices, what causes the current interest rate and a distributed lag of past commodity price changes to be positively correlated. To Fisher, however, the process is just the reverse. Given EQ(1) and a lag in interest rates relative to price changes, the interest rate will be raised cumulatively during a period of generally rising prices, so that by the end of the inflationary period both interest rates and the price level are high. In other words, the Gibson Paradox is an artifact of Fisher's first three principles.13

The second ancillary point relates to a comment by Fisher, which he did not elaborate upon, but is especially relevant to the more contemporary empirical studies of inflation and interest rates. He
stated, "It is quite definitely demonstrated that, in times of marked
price changes, as in the World War period, the effects of price
movements are felt rather quickly upon the rates of interest, even in
the case of long-term bond yields." This implies a variable
coefficients model of distributed lags. Several authors have since
suggested that the empirical validity of EQ(1) increases when
inflation is high.
C. INFLATION AND THE REAL RATE OF INTEREST

Fisher observed that, contrary to the idealized theory, the real rate of interest is more volatile in the short run than is the market interest rate. Considering the manner in which Fisher derived his real interest rate series, this was equivalent to postulating that the inflation tax is non-neutral in that the market interest rate does not move proportionately to changes in inflation. One possible explanation is that the real rate of interest itself may be influenced by the (anticipated) inflation rate. In this section we examine the literature which deals explicitly with the apparent covariability between the real rate of interest and the inflation rate.

In a brief paper published in 1960, Charles Kennedy took issue with the Radcliffe Report which had concluded that during a period of inflation investors would substitute equities for bonds. Kennedy argued the Radcliffe Committee had erred by essentially limiting their analysis to portfolios of bonds and equities. Kennedy suggested that a correct Keynesian analysis would not have bond prices falling, but equity prices rising, reflecting the higher marginal efficiency of capital. Ball [1965] objected to this hypothesis. He maintained some decline in equity yields and some rise in bond yields would follow an upward revision in expected future inflation rates. Keynes [1936] of course had argued that a rise in expected future inflation rates increases the (nominal) yield on real capital investment. This in
turn leads to an increased demand for capital, which raises the capital supply price and ultimately raises the level of money income. The real rate of interest remains constant provided the money supply is increased. A fixed money supply, on the other hand, forces portfolio adjustments which terminates with a lower real rate of interest.\footnote{3}

It was in terms of this debate that Mundell [1963] came to write one of the most frequently cited papers on inflation and interest rates. Mundell maintained that Keynes had been misinterpreted. He argued that an increase in the anticipated rate of inflation will be accompanied by a decline in the equilibrium real rate of interest. He adopted a model proposed by Metzler, which may be summarized as follows:

1. wages and prices are flexible,
2. economy is at full employment,
3. the share of profits in full employment income is constant,
4. wealth is held in money and shares, the real value of shares is equal to the capitalized rate profits at going interest rate,
5. real investment is a function of the real rate of interest,
6. real savings are a function of real (cash) balances, and
7. the ratio of money to equities is a function of the money rate of interest.
The initial equilibrium in IS-LM schedules is characterized by equality between the real rate and the money or nominal rate of interest. This is depicted in Figure I as point \( \Omega_0 \). The LM schedule is derived on the basis of a money rate of interest, "as that measures the true cost of holding money." With anticipated inflation the LM curve, in real interest rate space, shifts down by a distance equal to the change in rate of inflation, \( E(\rho) \). This shift is produced by the fall in real balances from \((M/P)_0\) to \((M/P)_1\). The IS schedule, in real interest space, remains fixed by assumptions (i) through (vii). The result is a fall in the real rate.

The equivalent result is achieved when the analysis is conducted in money interest rate space. This was Bailey's [1956] contribution. The LM schedule remains fixed, but the IS schedule shifts upward by the rate of inflation. With a less than perfectly inelastic LM schedule, the increase in the money interest rate is less than the increase in the expected future inflation rate. According to the Fisherian Hypothesis this implies a fall in the real rate.

Mundell thus demonstrated that changes in the market interest rate can be interpreted as a consequence of a shift in the marginal efficiency schedule (as a function of money interest), or from a shift in liquidity preference (as a function of real interest). He interpreted Keynes as a case for the former.
FIGURE I

Mundell - Bailey Effect

Interest Rate

\( i_0 = r_0 \)

\( r_1 \)

\( i_1 \)

Real Balances

\( \left( \frac{M}{P} \right)_1 \)

\( \left( \frac{M}{P} \right)_0 \)
The key concept here is that the market interest rate rises by less than the rate of inflation, and consequently the real rate and the inflation rate are inversely related. It is paramount, however, that the cause of the decline in the real interest rate be clearly identified. It is the consequence of a theory of "dichotomous markets." Investment-savings is modelled as a function of the real interest rate, but the demand for real balances is assumed to be a function of the market interest rate. This dichotomy is based on the noninterest bearing property of money. It is this property which makes money balances the basis for the inflation tax. Therefore, the decline in the real rate from anticipated inflation, what has come to be called the Mundell Effect, is the result of the direct burden of the inflation tax.

Inflationary finance is a tax on the purchasing power of money balances. This tax reduces the demand for real balances from \((M/P)_0\) to \((M/P)_1\) in Figure I above. The tax on money raises the demand for investment-savings. Equilibrium is restored, for a fixed anticipated (permanent) rate of inflation, when the net of inflation tax rate of return to money and investment are equal. The real rate of return to investment declines because investment is substituted for money balances. This is qualitatively analogous to the behavior of the return to capital in the corporate and noncorporate sectors from a change in the corporate income tax rate.
The Mundell Effect reflects the direct burden of a tax on one form of capital, specifically money. There are substitution effects as with any tax in which the factors of production or products are mobile and not perfectly inelastic in supply. We suggest, however, that the Mundell Effect is not the "cost of inflation" people tend to be concerned about. The "troublesome" costs are those from resource misallocation, including the excess burden identified by Bailey.

The Mundell Effect is simply the "Bailey Effect" with an upward sloping IS curve. This point appears to be lost to the literature. If the supply of capital is fixed the burden of the inflation tax falls solely on desired money holdings. The real rate of interest remains unaffected. This is depicted graphically in Figure I as a shift in the vertical IS schedule, raising the market interest rate from \(i_0\) to \(i_1\), which is the full value of the anticipated inflation. When capital is not perfectly inelastic the Mundell Effect is obtained. Technically, Bailey's model, and not Mundell's, is consistent with the no-growth assumption adopted by both.

Gibson [May-June, 1970] offered what he claimed was an alternative, albeit more rigorous, explanation to Mundell's analysis of the behavior of the real interest rate to changes in inflation expectations. We present his analysis in a full IS-LM diagram, Figure II, with nominal money balances, but real aggregate product.
FIGURE II

Economic Adjustment to Distributed Lag Inflation Expectation Formation
An increase in the money supply from $M^S_0$ to $M^S_1$ causes the LM schedule to shift out from $LM_0$ to $LM_1$, assuming income effects lag and no immediate change in expectations regarding future inflation rates. The fall in the real interest rate to $r_1$ represents the liquidity effect. At the lower interest rate both consumption and investment are stimulated. The corresponding excess demand for goods forces prices upward. At the original market clearing interest rate, planned investment equals planned savings. But at the lower interest ($r_1$), planned investment exceeds planned savings. Prices continue to rise until the imbalances are corrected. Gibson refers to this price induced responses as the income effect, which is depicted as step 2 in Figure II. The rise in commodity prices, Gibson claimed, affects peoples expectations about future prices. Specifically, people come to expect higher inflation and thus attempt to shift out of money balances and into real assets. The price expectations causes the liquidity preference schedule to shift inward and the LM curve outward along the IS schedule, reflecting the decline in the yield to real assets as capital prices are bid up. The Gibson equilibrium is finally achieved at $\Omega_2$, with a lower real rate of interest.

The complete IS-LM presentation of Gibson's version of the Mundell Effect reveals that precisely nothing is added to our understanding of the phenomenon, and in fact the principle concepts are distorted by assuming a Keynesian underemployment initial output level ($y$). Clearly $\Omega_2$ is the long-run equilibrium only if the economy was
originally at less than full employment. If \( y \) is the full employment income, then excess demand at \( y_2 \) will force prices up until the real interest rate again equals \( r_0 \). We interpret Gibson's observation, that continuing increases in money and prices are necessary to sustain the price expectations and \( \Omega_2 \), as an acknowledgment of this fact.\(^5\)

Once \( y \) is interpreted as full employment income, it is clear that Gibson's model does not capture the inflation tax induced decline in the real interest rate referred to as the Mundell Effect. Gibson's model describes the workings of the economy with respect to inflation expectations derived from a (implied) distributed lag in inflation rates.

The first essay in this volume introduced the Sargent and Wallace et. al. theory of the neutrality of perfectly anticipated inflation (and money). At first consideration this theory seemingly conflicts with the non-neutrality of money in growth models and with the Mundell Effect. This apparent paradox, however, is readily explained. The neutrality of Sargent and Wallace type models follows from their assumptions, especially the endogeneity of inflation, and the exclusion of a wealth term in the behavioral equations. Including a wealth parameter gives rise to the Mundell non-neutrality. The proof of this statement is left as an exercise for the reader, but we note that the issue is precisely the same as that addressed by Patinkin [1965] several years prior to the application of rational expectations in macroeconomic analysis. Patinkin argued that neoclassical theory
failed to account for the fact that a change in the general price produces a divergence between an individual's actual and his desired real balances and therefore affects his demand for real goods (i.e., the negative real balance effect).

The commonly held opinion is that the real interest rate declines with inflation, as for example with Kessel and Alchian [1962] who contend that an increase in anticipated inflation lowers the yield to real assets as individuals substitute away from money. This position was also taken by Tobin [1965]. However, Darby [1975] and Feldstein [1976] have argued that the existence of corporate and personal income taxes may cause market interest rates to rise by substantially more than the rate of inflation. By comparison, however, Feldstein and Summers [1978] contend for the same reason the real rate of interest net of tax available to investors is reduced dramatically by inflation. The outcome depends on whether the marginal personal tax rate exceeds the marginal corporate tax rate, and on the sign of the change in the real capital stock with respect to a change in inflation. The possibility of the real rate of interest increasing with changes in the anticipated rate of inflation arises solely from the structure of the tax system which permits the effective tax rate on capital income to change with the inflation rate. We should also mention that Steindl [1973], using the Patinkin model with price expectations, argued the derivative of the real rate
with respect to inflation expectations is a priori indeterminate. Obst and Rasche [1976] clarified some of the assumptions necessary for Steindl's results.

Lintner [1975], observed that several empirical studies have refuted the classical contention that owners of levered equity benefit from a real capital gain whenever there is an unanticipated increase in the expected future rate of inflation. (This topic is reviewed in depth in Section E.) Lintner maintained most of the evidence must be discounted because the authors failed to account for institutional factors, such as the use of historical rather than current replacement costs as the basis for depreciation for tax purposes. He suggested that even if these factors were accounted for, however, the real rate of interest will still decline as inflation expectations are revised upward. Lintner's theory warrants a critical appraisal.

Lintner's theory can be reduced to a model where the total demand for external funds to maintain real growth is a linear function in the change in current dollar sales and the level of sales. The ratio of external to internal financing for such a model increases with inflation. Consequently "the relative dependence on external financing necessarily varies directly with realized inflation rates." This reduces the value of outstanding equity because it involves a "deadweight dilution" of the real returns on owning equities over the period. The reduction in the current period's real
returns on equity is even larger in the case of an increase in anticipated future inflation. According to contemporary capital market theory, if the real rate of interest is constant and the risks of equity ownership in real terms are unchanged, then the expected real rates of return on equities, net of dilution, must be maintained. Consequently the firm's real cost of capital must rise, thereby reducing the optimal growth rate of the firm, which produces a capital loss to equity.

A close examination of Lintner's model raises doubts about the validity of his conclusion. With no loss of generality, the model can be further simplified by assuming firms maintain neither trade debt nor trade credit, and that cash flows are perfectly synchronized, eliminating the (transactions) demand for money. Lintner assumed that depreciation is proportional to the capital stock at replacement cost, and that the corporate profit tax rate is constant over all levels of income. Qualitatively this is equivalent to ignoring both depreciation and the corporate tax.

In the modified Lintner model the demand for external funds, \( \Delta F_t \), is a fixed percentage of current period's sales.

\[
\text{EQ(4)} \quad \Delta F_t = I_t - R_t = b S_t
\]

where \( I_t \) = investment at current costs, 
\( R_t \) = retentions, 
\( \Delta F_t \) = demand for external funds, and 
\( S_t \) = period \( t \) sales
The balance sheet equation is simply: capital equals debt plus equity ($K_t = B_t + V^e_t$). Now suppose that during the period $t$ to $t+1$ there is no expected or actual inflation. Let $I_t$ exceed $bS_t$. The debt to equity ratio at time $t+1$ is equal to the ratio at $t$ only in a very special case:

$$
\text{EQ}(5) \quad \frac{B_{t+1}}{V^e_{t+1}} \leftrightarrow \frac{B_t}{V^e_t} \iff \frac{R_t}{I_t} \leftrightarrow (1 - \frac{B_t}{V^e_t}) = \frac{V^e_t}{K_t}
$$

The post-investment debt to equity ratio in inflationary conditions depends on both the proportion in which real debt and real equity are used to finance the acquisition, and the inflation rate. We state without proof:

$$
\text{EQ}(6) \quad \frac{B_{t+1}}{V^e_{t+1}} = \frac{B_t}{V^e_t} \iff \rho \leftrightarrow \frac{\Delta F_{t+1}(V^e_{t+1})}{K_t} B_{t+1} - \frac{R_{t+1}}{K_t}
$$

As EQ(6) reveals, the debt to equity ratio may decline, contrary to Lintner's conclusion. In Lintner's model the real rate of return on equity is inversely related to changes in the anticipated and unanticipated inflation rate because he fails to account for the appreciation in the capital stock which is captured as a capital gain (part of $V^e_t$). In other words, Lintner's analysis is correct with respect to the cash flow determination of the debt to equity ratio, but is incomplete for lack of a wealth effect.
This brief review of the behavior of the real rate of interest with respect to inflation points to one basic conclusion: the Fisherian Hypothesis, as it was originally stated, is misspecified. A correct specification must incorporate the Mundell Effect - the direct burden of the inflation tax. Likewise, the hypothesis must be corrected for inflation uncertainty and for relevant institutional distortions, such as the personal and corporate income taxes. All of these factors influence the equilibrium real interest rate. Therefore, Marshallian partial equilibrium analysis is inappropriate because the inflation tax induced changes in the equilibrium real rate of interest will alter the demand and supply of all goods, services, and capital assets. A change in the expected rate of inflation causes real substitution effects. Consequently, the inflation tax is not a neutral tax.
D. INTEREST RATE TESTS

Interest: What factor or commodity earns it as its reward? What determines the rate? Such questions have intrigued students of economy as analytic issues since at least the ninth century; although not seriously dealt with until the classic period of scholasticism (thirteenth century). As reported in the previous section, it has only recently, in this time perspective, been determined how changes in the general level of prices might alter interest rates. The question addressed above was how changes in the inflation rate affect the real interest rate. In this section we turn our attention to the complementary question of how the market or nominal interest rate responds to changes in inflation, actual or anticipated. The objective is to critically review the literature so as to ascertain what general conclusions, if any, are substantiated by a coherent body of empirical evidence. Moreover, we seek to determine if what appears to be contradictory or anomalous evidence can be explained as the consequence of a failure to control for one or more of the principles of inflationary finance set out in the first essay.

This section is subdivided into five parts. The first section (D.1) is a chronological review of the more noteworthy studies of the Fisherian Hypothesis. The sampled literature is restricted for the most part to studies which tested a simple single equation model of the hypothesis. Section D.2 is devoted entirely to a review of the
tests of short-term interest rates as predictors of inflation. This is a key section in this thesis as it brings together the conceptual issues of market efficiency, the assumptions about the time series properties of the real interest rate, and the Fisherian Hypothesis. In Section D.3, attention is directed to the specification problem traditionally referred to as the Gibson Paradox: Is the market interest rate a function of the price level or the rate of change in prices. Section D.4 examines the sparse literature which has tested the Fisherian Hypothesis with models that accounted for the variability in real interest rates. Finally, conclusions are presented in Section D.5.

D.1 Historical Development

As was noted in Section B, Fisher [1930] computed the correlation coefficient for a distributed lag in actual inflation rates and various interest rates, such as the interest yield on British consols, and the 4-6 month U. S. commercial paper rates. This was equivalent to estimating the following linear model:

\[ EQ(7) \quad i_t = \beta_0 + \beta_1 \sum_{j=1}^{n} \alpha_j \pi_{t-j+1} + \epsilon_t \]
Fisher reported high correlation coefficients for lags of 20 years for U. S. data and 28 years for British.

The simplicity of Fisher's tests is fortuitous in as much as it forces us to address the question of the relationship between inflation expectations based on sequences of past inflation rates and the neutrality of the inflation tax. We interpret Fisher as intending to test the model described below by Eq(8), which is structurally consistent with the Fisherian Hypothesis. Because the expected future inflation rates generated by Eq(8-b) are specified for predetermined coefficients there is no stochastic disturbance term and hence Eq(8) and Eq(7) are econometrically identical. Alternatively, Fisher can be read as having intended to test the proposition that the current interest rate is a convex combination of the real interest rate and a sequence of past inflation rates. This proposition holds that the interest rate is a function of realized inflation with the capital market making no attempt to anticipate future general price level movements. This interpretation of Eq(7) finds support in Fisher's claims that the markets suffer from money illusion.

We believe Fisher's works, interpreted collectively, are most consistent with the Eq(8) interpretation of Eq(7). At the same time, the way Fisher conducted his empirical analysis prevented him from testing either Eq(8) or even the Fisherian Hypothesis. What Fisher inadvertently determined was the optimal period length n* for Eq(7) or
EQ(8-b) which maximized the fit between the market interest rate and the sequence of past inflation rates. Knowledge of \( n^* \) does not answer the fundamental question: Is \( \beta_1 \) in EQ(1) or EQ(8-a) equal to unity?

\[
\text{EQ}(8-a) \quad i_t = \beta_0 + \beta_1 E_t(\rho_{t+1}) + \epsilon_t
\]

\[
\text{EQ}(8-b) \quad E_t(\rho_{t+1}) = \sum_{j=1}^{n} \alpha_j \rho_{t-j+1}
\]

\[
\alpha_j = \frac{2(n-j+1)}{n(n+1)}
\]

\[
\sum_{j=1}^{n} \alpha_j = 1
\]

It is not necessary that we resolve what Fisher's "true" theory of interest rate behavior was. It is, however, necessary to spell out the implications of the various interpretations attached to EQ(7). The theory that interest rates respond to a history of realized inflation rates is inherently inconsistent the concept of market efficiency and the neutrality of the inflation tax. This is true even if \( \beta_1 \) in EQ(7) should exactly equal unity. At the same time, \( \beta_1 \) equal unity in EQ(8-a) does not imply inefficient capital markets, and it is consistent with an endogenous inflation tax (i.e., weak form Fisherian Hypothesis). The fact that inflation expectations are generated by historical price level changes is irrelevant to the validity of the Fisherian Hypothesis. It may imply that the commodity
markets are inefficient if the informational content of past inflation rates is not fully and instantaneously incorporated into current period prices (see discussion below of Fama and Schwert [1979] for exception).\(^2\) The rational expectations - efficient market theory says that the markets will use whatever information is available to improve its forecasts. If movements in the general level of prices tend to persist over time then the history of inflation rates is valuable information to the assessment of the expected future inflation rate. The neutrality of the inflation tax is independent of the type of information used by the market. It is determined by whether the expected future inflation rate is an unbiased estimate of the subsequently observed actual rate, written as:

\[
EQ(9) \quad E_t(\rho_{t+1}) - \rho_{t+1} = u_{t+1}
\]

where \(E_t(u_{t+1}) = 0\)

\(\text{Cov}_t(u_t, u_{t+j}) = 0, \forall j \neq 0\)

If EQ(9) fails with respect to EQ(8), then there is no meaningful economic difference between EQ(8) and the "non-expectations" interpretation of EQ(7) - both imply information inefficient capital markets and distortions from inflationary finance, such as the debtor-creditor wealth transfer.
It is clear, therefore, that the Fisherian Hypothesis is a necessary but not sufficient condition for neutrality of the inflation tax. How inflation forecasts are generated matters. The vast majority of the empirical tests of the Fisherian Hypothesis have been designed in such a manner as to permit conclusions only about the necessary condition.

As we shall shortly reveal, many of the empirical tests of the impact of inflation on interest rates have estimated equations similar to EQ(7), except that $\beta_1$ is predetermined to equal unity and only the $\alpha$'s are estimated. However, the manner in which the models are frequently interpreted is not consistent with a test of the Fisherian Hypothesis. The problem parallels the conflicting theoretical meanings attributable to EQ(7). The common practice has been to evaluate the characteristics of the estimated $\alpha$'s, and to estimate the long-term elasticity of interest rates to a one percent change in the current period actual inflation rate. This is done by computing the change in $i_t$ as the change in the inflation rate passes through the distributed lag coefficients. This is not a test of the Fisherian Hypothesis. It is conceptually equivalent to the "non-expectations" interpretation of EQ(7), i.e., the economy responds to past price level changes - not to expected future price level changes. The characteristics of the $\alpha$'s in EQ(7) are relevant only as to the determination of whether assumption EQ(9) holds with respect to EQ(8-b). In other words, in so far as tests of the Fisherian
Hypothesis are concerned the question is not, "What is the response of the interest rate to a one percent change in the actual inflation rate.", but rather, "What is the response of the interest rate to a one percent change in the expected future inflation rate." The latter question is answered by either multiplying the sum of the $\alpha$'s by one percent, or by estimating $\beta_1$ in EQ(8-a) (assuming $i$ and $\rho$ are measured on the same time scale). Neutrality of the inflation tax requires both that $\beta_1$ equal unity and assumption EQ(9) be satisfied.

Fisher's distributed lags necessarily sum to unity for any value of $n$ in EQ(2), EQ(7), or EQ(8-b). As we shall soon see, it has been customary in subsequent tests to impose the constraint that the weights sum to unity, even for distributed lag models of more complex form. This restriction may lead to a serious underestimation of $\beta$ in the equations above. This point, which was originally made by Sargent [1971], is raised at this time as a caveat to the results to be described. It is crucial when interpreted in its broader sense. If EQ(8-b) describes a rational generator of expectations, then one-period forward expectations formed by EQ(8-b) will be minimum-mean-squared-error forecasts if the actual rate of inflation is generated according to an $(n+1)$th order autoregressive process. However, as Sargent demonstrated, so long as the inflation rate can be approximated as a covariance-stationary stochastic process (and the $\alpha$'s are nonnegative), the $\alpha$'s must sum to less than unity.
Accordingly, it is not possible for EQ(8-b) to simultaneously characterize the process by which rational expectations are generated and to satisfy the unity summation constraint.

Apart from Fisher's statistical tests, little work had been undertaken on the behavior of interest rates to price expectations for the United States until the late 1960s. In 1960, Sargent published what has become a classic paper in the field. The significance of Sargent's study is that it involved a test of the Fisherian Hypothesis for a theoretical model of the demand and supply of loanable funds which specified the determinants of the real rate of interest. This was the first serious attempt to adjust for changes in the real rate of interest. Two typical models are presented in Appendix A. The model is nonlinear in the parameters for a Koyck distributed inflation rate. The Hildreth-Lu procedure was applied to search for the value of $\lambda$ which minimized the ordinary least squares (OLS) sum of squared residuals for values of $\beta$ and $\eta$.

\[
EQ(10) \quad E_t(\rho_{t+1}) = \beta \sum_{j=1}^{t-1} \lambda^{j-1} \rho_{t-j} + \lambda^t \eta
\]

The signs of the parameters agreed with Sargent's loanable funds theory, namely that the nominal interest rate and no other variable clears the market for loanable funds. The expected inflation rate was a highly significant determinate of the nominal interest rate.
Sargent, like Fisher before him, found the estimated distributed lag in inflation to be surprisingly long, peaking at a lag of about 8 years, but having a mean of 20 years or more. In one test, Sargent estimated a second order lag model of price expectations, having two expectation coefficients (\( \beta \) 's) and two separate sets of lag weights (\( \lambda \) 's). This more flexible form had one positive \( \beta \) coefficient and one negative coefficient. Sargent rationalized this as an indication of a regressive effect of price changes on short-term interest rates, that is price changes temporarily generate (partial) expectations of changes in the opposite direction, or back to "normal."

Neither the savings nor the investment equations in Sargent's model included a wealth variable. Therefore, there was no tax base for the inflation tax (i.e., no Mundell Effect). The rate of change in the real money stock variable, which Sargent included in his model, was a proxy for the market interest rate. Sargent recognized the misspecification in a footnote, and suggested that the omitted wealth effect could account for the negative coefficient on inflation expectations in the two decay coefficient model.

Shortly after Sargent's paper was published, Yohe and Karnosky [1969] reported the results of a similar investigation. Their study was an examination of Fisher's second and third propositions, namely that (ii) interest rate movements lag price-level changes, and (iii) there is a marked correlation between interest rates and a weighted
average of past price level changes. These propositions have important implications to monetary policy, which Fisher had not explored. When the lags in interest rate adjustments to price-level changes are as long as indicated by Fisher and Sargent, then recent price behavior can be ignored in evaluating changes in market interest rates. The validity of the Keynesian textbook IS-LM model likewise rests on the length of the lag.

Yohe and Karnosky suggested several areas where previous interest rate studies could be improved. For one, they noted that interest rate data prior to 1952 was of dubious value because of the Federal Reserve's "par-pegging" of Government securities prices. They also argued that the frequently used geometrically decaying lag structure may be too restrictive. They estimated a variety of distributed lags in order to investigate the effect of lag form on the length of the lags. As indicated by Appendix A, Yohe and Karnosky found the lags to be much shorter than in previous studies. Most of the effect of price-level changes on both long and short-term interest rates occurred within two years. In several estimations, the mean lag was less than one year - a marked difference from Fisher's original results.

Yohe and Karnosky suggested that the discrepancy between their results and those of previous studies could be attributable to: (1) an aggregation bias (monthly to quarterly data), (2) the estimation
procedure, or (3) institutional changes. Empirical tests refuted the aggregation bias hypothesis. Alternative estimation procedures did yield different results, from which they concluded that geometrically declining lag structures which require an exponential decay are inappropriate for interest rate and price level data.

Yohe and Karnosky were also among the first to observe what we shall refer to as a "Threshold Effect." They found that the coefficients of their model were more significant when estimated over the period 1961-1969. They attributed the differences to institutional changes, and suggested these changes could account for the much shorter lags in interest rate adjustments.

In one of the simpler studies of price expectations effects on interest rates, Gibson [March, 1970] applied OLS to various interest rate series on lagged inflation rates and changes in inflation rates. He noted that his model was, "in no sense a complete model of interest rate determination: it seeks only to discover the relationship between nominal rates of price change." We interpret this as an admission of misspecification. The empirical results led Gibson to conclude the Fisherian Hypothesis is valid, but that nominal rates do not rise by the full amount of the percentage point increase in the expected inflation rate. Consequently, the real rate must decline. These
results formed the basis for Gibson's [May-June 1970] subsequent and somewhat less than satisfactory analysis of the transmission mechanism of monetary policy on interest rates, reviewed in Section C.

Interestingly, Gibson's empirical results did not support his conclusion of Fisherian Hypothesis verification. For annual data, only the coefficient for the current inflation rate on two long-term yields was significant. Neither the current inflation rate nor any of the lagged inflation rates were significant for any of the tested short-term interest rates. For quarterly data, none of the reported coefficients were significant. Only marginally better results were achieved for changes in interest rates regressed on changes in inflation rates. No explanation was offered for why changes in the rates were even examined. The insignificance of the inflation rate coefficients may have been the consequence of multi-collinearity. Rates of price change will be serially dependent if the commodities market is not information efficient. Therefore, when a sequence of lagged inflation rates is estimated by OLS, the coefficients will suffer from the usual inefficient standard errors.

Of the three studies reviewed thus far, one concluded the inflation expectations effect is distributed over a very long period; another claimed the effect was relatively rapid; and yet another, contrary to its author's conclusion, demonstrated a contemporaneous relationship. Perhaps even more perplexing are the differences in the
estimates of the elasticity of the market rate of interest to inflation. Sargent [1969] reported a one percent increase in the inflation rate raises the interest rate anywhere from 1.29 percent to 6.21 percent, depending on the period and lag structure. Yohe and Karnosky [1969], on the other hand, indicated a sustained one percent increase in the inflation rate raises the interest rate by only 66 basis points. Interpreted solely in terms of the Fisherian Hypothesis, Sargent has the real rate increasing, while Yohe and Karnosky have it falling with an increase in expected future inflation.

Feldstein and Eckstein [1970] sought to reconcile such conflicting evidence. They provided a synthesis of Keynes' theory of liquidity preference and the Fisherian Hypothesis, which formed the basis for their empirical tests. A series of equations of the long-term interest rate on liquidity, inflation, government debt, and short-run expectations (i.e., change in the interest rate) were tested. A third-degree Almon polynomial was applied to past inflation rates. As reported in Appendix A, the explanatory variables explained nearly all of the quarterly variance in long-term interest rates. A decomposition of the interest rate variation revealed that liquidity was more significant than inflation over the period 1954-1965, but that from 1965 to 1969-II, inflation was the dominant force. These tests confirmed Monetarists' interpretations of the Fisherian Hypothesis, namely that in the long-run the real rate of interest is unaffected by the rate of inflation, but that in the short-run the
real rate falls as the inflation rate increases. A one percent permanent increase in inflation ultimately causes a 92 basis point increase in the market interest rate, which is virtually the unit elasticity implied by the Fisherian Hypothesis.

Certain salient aspects of the Feldstein and Eckstein study warrant special attention. They reported a 24 quarter distributed lag structure for inflation rates which was chosen because it gave the "best results." The mean lag was just over two years, which is roughly equivalent to Yohe and Karnosky's results. Feldstein and Eckstein also criticized Sargent for failing to explicitly treat liquidity balances (i.e., Mundell Effect), and suggested that his estimates were biased because of the omission. The question of biased estimation is, however, even more relevant to the Feldstein and Eckstein model. The presence of serial correlation in their equations biased the estimates because of the inclusion of a lagged dependent variable. The reported Durbin-Watson statistics were either significant or indeterminate. Moreover, the explanatory variables are, theoretically at least, correlated with the disturbance terms. The authors even acknowledged this system bias when they stated,

In interpreting the model, it should of course be remembered that this is in effect a single equation in a complete macro-economic system. The coefficients therefore measure only the direct effect of each variable on the interest rate. The monetary base, for example has not only this direct effect but also an indirect effect through its impact on prices and private GNP. (p. 374)
Consequently, the Feldstein-Eckstein results do not appear to be very reliable.

Weil [1970] investigated the returns to bondholders as an indication of the efficiency of price level change expectations. He distinguished among what he called the instantaneous real rate of interest, the anticipated real rate, and the actual (or realized) rate. He defined these measures as EQ(11-a, b, c). Implicitly, he assumed a random walk in inflation expectations, i.e.,

\[ E_t(\rho_{t+1}) = \rho_t. \]

\[
\text{EQ(11-a)} \quad \hat{r}_t = \left( i_t - \frac{1}{P_t} \frac{dP_t}{dt} \right) / \left( 1 + \frac{1}{P_t} \frac{dP_t}{dt} \right)
\]

\[
\text{EQ(11-b)} \quad r^a_t = (i_t - \rho_t) / (1 + \rho_t)
\]

\[
\text{EQ(11-c)} \quad r_t = (i_t - \rho_{t+1}) / (1 + \rho_{t+1})
\]

In addition, Weil estimated the bond holding period return as,

\[
\text{EQ(12)} \quad H_t = \frac{(P_t + V_t - P_{t-1})}{P_{t-1}}
\]

where \( V_t = C_t(1+i_t) + C_t \)
\( C_t \) = coupon payment,
\( i_t \) = reinvestment rate.
Weil conducted a series of single equation OLS tests using a constructed series of holding period returns [from EQ(12), the Standard and Poor's Aaa yield to maturity] and the Durand corporate bond yields. He concluded the period inflation rates for 1900-1968 implied forecasts of annual price changes that were substantially incorrect, or that the real rate was not constant, or both.

By the time Weil published his study it had become standard practice in inflation tests to also evaluate the model using only recent inflation and interest rate observations - the hypothesis being that a threshold inflation rate had been surpassed in the 1960s which was more apt to produce an observable Fisherian Hypothesis. Weil reported the null hypothesis that the inflation expectations coefficient equals unity could not be rejected for the subperiod 1961-1968, thus confirming a threshold effect.

Anderson and Carlson [1970], in a continuation of the St. Louis Federal Reserve study of the Monetarist theory of economic stabilization, developed a multiequation model of the influence of monetary expansion on total spending. The model consisted of five behavioral equations, including an interest rate equation, and three identities. All equations were estimated by OLS. Lag structures, with one exception, were estimated by a second-degree Almon polynomial.
The interest rate equation in the Anderson and Carlson model included an inflation rate expectation, where anticipated price changes were assumed to be generated from past price experience, but "corrected" for current economic conditions. An index of the unemployment rate was used as a leading indicator of future price movements. The period inflation rate was divided by the index to adjust for the assumed lag in price movements to changes in total spending (see EQ(13) below). As reported in Appendix A, the sum of the OLS coefficients on the adjusted lagged inflation rate was significant for both short-term and long-term interest rates. Anderson and Carlson concluded that 95 percent of the price response to a change in total spending is achieved in the first five quarters.

\[
E_t(\Delta P_{t+1}) = Y_{t-1}\left\{[1 - 4.01(\sum_{j=1}^{17} \alpha_{i(t-j)}^u)]^{1/4} - 1.0\right\}
\]

where, \( U = \) index of unemployment, a leading indicator of future price movements.

\( Y = \) total spending (GNP in current prices).

The Anderson-Carlson model is a block-recursive equation system. None of the structural equations included the equilibrium market rate of interest, in keeping with the "strong-form" of the Monetarist theory. Had the interest rate been included in the total spending equation, then the OLS estimates would, of course, be biased. If the
change in the market rate of interest with respect to the inflation rate is not unity, then the real rate must be a function of inflation and there are substitution effects. Yet, according to the design of the Anderson and Carlson model, total spending, aggregate demand, and unemployment are independent of unanticipated inflation effects. Clearly, such separable structural equations are justifiable only if the inflation tax is neutral. It is significant to note, therefore, that Anderson and Carlson reported results consistent with (near) neutrality, i.e., a one percent increase in the expected inflation rate causes a .96 percent increase in the market interest rate.

One of the more innovative inquiries into the affects of expected price changes on interest rates was by Suraj Gupta [1970]. As Gupta observed, if in an appropriately specified demand for money function the market rate of interest is significant, then if the portfolio balance theory of expected future inflation effects is valid, one should observe equivalent coefficients when the cost of holding money is measured by the real interest rate and the expected future inflation rate, as when the cost is measured by the market interest rate alone. This hypothesis merely exploits the Fisherian Hypothesis equation.

Gupta took the yield to maturity on the highest grade long-term bonds as the market interest rate. He constructed a "real" interest rate equal to the depreciation and tax adjusted expected "real"
earnings to corporate equity divided by the real market price of the equity. The expected "real" earnings to corporate equity was estimated by searching for a coefficient in a nonlinear equation which gave the "best" results. The expected future inflation rate was constructed as the difference between the market interest rate and the real rate. Finding significant serial correlation, Gupta applied Johnston's two-step correction. The results, a sample of which are reported in Appendix A, revealed the null hypothesis $b_2 = b_3$ could not be rejected in four out of six subperiods tested.

\[ \text{EQ(14-a)} \quad \log M_t = \log a_0 + a_1 \log Y_t + a_2 \epsilon_t + \epsilon_t \]

\[ \text{EQ(14-b)} \quad \log M_t = \log b_0 + b_1 \log Y_t + b_2 r_t + b_3 \epsilon_t (\rho_{t+1}) + u_t \]

Gupta concluded that asset-holders consider the expected rate of inflation as much a cost of holding money as the real rate of interest foregone. The implication is that decision makers demand information about both the future real rate of interest and the expected future rate of inflation, and that they will bid equally for the information to the point where the economic value of incremental information equals the cost to obtain it.
As Gupta noted, confirmation of the portfolio balance theory requires \( a_2 = b_2 = b_3 \) above. For the overall period 1871-1960 only \( b_2 = b_3 \) could not be rejected. For the most recent subperiod, 1941-1960, either the significance could not be determined or the equality hypotheses were rejected. Gupta's evidence was mixed at best. Indisputable support of the Fisherian Hypothesis was not provided, despite claims to the contrary.\(^{11}\)

Turnovsky [1970] was among the first to go behind the Fisherian Hypothesis and examine the inflation expectations mechanism. Another noteworthy feature of Turnovsky's study was the use of independently derived inflation forecasts. Data on price expectations were provided by Joseph A. Livingston of the Philadelphia Bulletin, who conducts a semi-annual poll of "informed" business economists regarding their predictions for a number of economic series. Turnovsky examined three expectations hypotheses, which for inflation expectations are written as equation EQ(15).

\[
\text{EQ(15-a) Extrapolative expectations,} \\
E_t(\rho_{t+1}) = \beta_1 \rho_t + \beta_2 (\rho_t - \rho_{t-1}) + \epsilon_t
\]

\[
\text{EQ(15-b) Adaptive expectations,} \\
E_t(\rho_{t+1}) = \beta_1 \rho_t + \beta_2 E_{t-1}(\rho_t) + \epsilon_t
\]
EQ(15-c) Weighted expectations,

$$E_t(p_{t+1}) = \sum_{j=1}^{N} \beta_j p_{t-j+1} + \epsilon_t$$

Both the extrapolative and adaptive expectations are special cases of distributed lags. Turnovsky concluded that:

1. Extrapolative expectations are probably marginally best in the short-term predictions, while their margin of superiority is substantial for the 12-month forecasts.

2. In all models, the adjusted R² for short-term and longer-term predictions are similar.

3. Quarterly data do not improve on semi-annual data in terms of adjusted R².

4. There was a significant change in the formation, rationality, and accuracy of businessmen's price expectations during the early 1960s [Threshold Effect].

5. Tests conducted for the period 1962-1969 suggested that expectations tend to be regressive, i.e., businessmen expect past trends to reverse themselves.

A particularly important methodological feature of Turnovsky's study was the test of the rationality of expectations. The test was conducted by regressing actual inflation rates on expected future inflation rates. According to Muth [1961], and the efficient market theory, efficient use of information requires the following:
\[
\begin{align*}
\text{EQ(16)} \quad \rho_t &= \beta_0 + \beta_1 E_{t-j}(\rho_t) + \epsilon_t \\
E(\beta_0) &= 0 \\
E(\beta_1) &= 1
\end{align*}
\]

This is simply the "fair game" condition discussed at length in the first essay in this volume (Section D). Turnovsky found the constant term was significantly greater than zero, implying businessmen's inflation expectations tend to be systematically understated. However, the coefficient \( \beta_1 \) was not statistically different from unity. This favorable evidence did not override the fact that only one of the numerous equations tested was consistent with rationality when the model was re-estimated with actual inflation substituted for expected future inflation. In summary, Turnovsky concluded the expectations hypotheses tend to be inconsistent with the rationality assumption.

Turnovsky did not claim that the capital markets were necessarily inefficient. The failure of a model to conform to the conditions for rational expectations does not necessarily imply inefficient markets with respect to information. In the specific instance of inflation expectations, such tests are not sufficient to refute expectations neutrality of the inflation tax hypothesis. Any test is simultaneously a test of the efficiency of the markets and of the properties of the expectations generating mechanism. Failure to confirm rationality may be due to the fact that the markets are indeed
not efficient in the use of available information. It may also be
because the hypothesized expectations model falsely represents the
true expectations generating process.

Several authors have used Turnovsky's study as a basis for further
investigations of the Fisherian Hypothesis. Among them were Gibson

Gibson concluded market interest rates have been strongly affected
by expectations, particularly since 1959. More important, however, he
demonstrated that the elasticity of the market rate of interest to the
anticipated inflation rate is unity, which implies the real interest
rate is independent of price expectations (see Appendix A). Pyle
compared several models with alternative inflation expectations
structures. His conclusion, apparently based on the goodness of fit
of the model, was that observed price expectations are at least as
powerful as distributed lag proxies in explaining nominal interest
rates. Exemplar results are reported in Appendix A. Carlson [June,
1977] used the Livingston survey data of inflation expectations to
test the hypothesis proposed by Fama [1975] that the expected real
rate is constant. Both studies are examined closely below.

Tests of the Fisherian Hypothesis have reported the entire
spectrum of results: from virtual neutrality of the inflation tax, to
long periods of capital market adjustment, i.e., non-neutrality. Such
tests are being repeated with increasing frequency as the economics profession redisCOVERS the role of expectations in economic behavior (see for example, Feldstein and Summers [1978], Tanzi [1980], and B. Friedman [1980]). Not surprisingly there tends to be a positive correlation between the rate of inflation and the quantity of literature which addresses the issue of capital market price adjustment to anticipated inflation. Needless to say, a substantial body of literature has appeared since about 1970, especially in the U.S. Only a small segment of this post-1970 literature was identified above because as a rule the more contemporary works treat the Fisherian Hypothesis as a component of broader issues, such as inflation hedging. These issues are dealt with separately in this study, which brings us to the issue addressed by Fama of whether short-term interest rates are predictors of inflation.

D.2 Short-Term Interest Rates and Inflation Predictors

Eugene Fama tested the proposition that the market for 1 to 6 month U.S. Treasury Bills is efficient. A substantial empirical literature exists on the efficiency of the Treasury Bill market in terms of the expectations hypothesis about the term structure of interest rates (for example, Roll [1970]; Hamburger and Platt [1975]; Burger, Lang, and Rasche [1977]; and Cargill and Meyer [1980]). Nonetheless, Sargent [February 1972] and others have provided evidence
which contradicts the joint proposition of efficiency, and that expectations determine the yield curve, with or without a liquidity premium.

Fama observed, as had Roll [1972], that Fisher's empirical evidence and most subsequent evidence was inconsistent with a "well-functioning" or "efficient" market. Fama proposed if the inflation rate is predictable, and if the one-period equilibrium expected real interest rate does not change so as to exactly offset changes in the expected future inflation rate, then a relationship between the one-period market interest rate observed at a point in time and the subsequently observed one-period rate of inflation is indicative of an (information) efficient market. Otherwise the market is inefficient because it overlooks relevant and available information about future inflation. Fama's empirical tests led him to conclude:

1. At least during the 1953-1971 period, there were definite relationships between nominal interest rates and subsequently observed rates of inflation.

2. During the 1953-1971 period the bill market was efficient in the sense that nominal interest rates summarized all the information about future inflation rates that is in time-series of past inflation rates.

3. The substantial variation in nominal bill rates seems to be due entirely to variation in expected inflation rates, i.e., the expected real returns on bills seem to be constant.\(^\text{13}\)
The basis for these conclusions is found in Fama's theoretical derivation of the following equation:

\[ \Delta_t = \beta_0 + \beta_1 i_t + \epsilon_t \]

where \( \Delta_t \) = rate of change in purchasing power from \( t-1 \) to \( t \)

\[ \Delta_t = -1 + \frac{P_{t-1}}{P_t} \]

\[ \lim \Delta_t \]

\[ dP \to 0 \]

Because of the significance of Fama's study, it is appropriate to derive EQ(17) in some detail.

Fama began by rewriting the Fisherian Hypothesis as:

\[ \tilde{r}_t = i_t + \Delta_t \]

The tildes (\(^\sim\)) denote random variables. The real rate is random because \( \Delta_t \) is random. The nominal interest rate is not random because, by the very nature of Treasury Bills, once the market (issuance) price is set, the nominal return to the holder is fixed. If the Fisherian Hypothesis is correct, then as a first approximation,
EQ(19) $E_{t-1}^m(\Delta_t | \phi_{t-1}^m) = E_{t-1}^m(\hat{r} | \phi_{t-1}^m) - i_t$

where, $m = \text{"market"}$

$\phi_{t-1}^m =$ the set of information available at $t-1$ and used by the market.

By definition, an efficient market is one where the market's assessed density functions for $\Delta$ and $r$ equal the true density functions implied by the information sets, in which case we may rewrite EQ(19) as,

EQ(20) $E_{t-1}(\Delta_t | \phi_{t-1}) = E_{t-1}(\hat{r} | \phi_{t-1}) - i_t$

When the market sets the equilibrium price of a one-month Treasury Bill at $t-1$, it also determines $i_t$. Given EQ(18), the market's assessed distribution for $r_t$ is implied by $i_t$ and the assessed distribution of $\Delta_t$. If in addition one assumes that the equilibrium expected real return on bills is constant, then with market efficiency it follows that,

EQ(21) $E_{t-1}(\Delta_t | \phi_{t-1}) = E(\hat{r}) - i_t$

where, $E_{t-1}(r_t | \phi_{t-1}, i_t) = E(\hat{r})$
Equation EQ(21) implies that all variation through time in the market rate of interest \( i_t \) is a direct reflection of the variation in the market's assessment of the expected value of \( \tilde{\Delta}_t \). More important, however, when the market fully utilizes all information the variation in \( i_t \) "mirrors variation in the best possible assessment of the expected value of \( \tilde{\Delta}_t \). Once \( i_t \) is set at time \( t-1 \), the details of \( \phi_{t-1} \) become irrelevant because the information in \( \phi_{t-1} \) is fully reflected in the value of \( i_t \). In this sense, the market rate \( i_t \), observed at time \( t-1 \), is the best possible predictor of \( \tilde{\Delta}_t \). This exhaustion of information is the cornerstone of Fama's theory.

To test his theory, Fama introduced another class of models of market equilibrium which includes the constant expected rate of return as a special case. He assumed EQ(22) to be the equilibrium relationship, which with market efficiency and EQ(18) implies EQ(23).

\[
\text{EQ(22)} \quad E^m_t (r_t | \phi^m_{t-1}, i_t) = \alpha_0 + \alpha_1 i_t
\]

\[
\text{EQ(23)} \quad E_t (\tilde{\Delta}_t | \phi_{t-1}) = \beta_0 + \beta_1 i_t
\]

In the special case where the expected real return is constant through time \( \alpha_1 = 0 \) and \( \beta_1 = -1 \), with all the variation in \( i_t \) mirroring variation in \( E(\tilde{\Delta}_t | \phi_{t-1}) \). Estimates of \( \beta_0 \) and \( \beta_1 \) were
obtained by applying OLS to EQ(23), with the hypothesis that, \( E(\tilde{r}) = \beta_0 \) and \( \beta_1 = -1 \). Efficiency requires that the disturbance term in the OLS model be of no additional help in assessing the expected value of \( \tilde{\Delta} \). Consequently, autocorrelations of the disturbance should be zero for all lags. In another test Fama noted if inflation or deflation tend to persist, then \( \Delta_{t-1} \) is relevant information for assessing the expected value of \( \tilde{\Delta}_t \). If the information is not correctly used by the market in setting \( i_t \), then \( \beta_2 \) in EQ(24) will be nonzero. Finally, Fama argued if the equilibrium expected real return is constant through time, it follows by EQ(25-a) and EQ(25-b), that in efficient markets the autocorrelations of real returns are zero for all lags.

\[
\text{EQ}(24) \quad \tilde{\Delta}_t = \beta_0 + \beta_1 i_t + \beta_2 \Delta_{t-1} + \tilde{\epsilon}_t
\]

\[
\text{EQ}(25-a) \quad \tilde{r}_t - E(\tilde{r}) = \tilde{\Delta}_t + i_t - E(\tilde{r})
\]

\[
\text{EQ}(25-b) \quad \tilde{r}_t - E(\tilde{r}) = \tilde{\Delta}_t - E_t(\Delta_t | \phi_{t-1})
\]

Fama found that \( \beta_1 \) in EQ(23) was not statistically different from unity. Autocorrelations of the disturbance terms were close to zero (see Appendix A). He noted EQ(24) is an independent test only because \( \Delta_{t-1} \) contains information about \( \Delta_t \), as exemplified by the nonzero autocorrelations. Yet, when \( \Delta_t \) was regressed against both \( i_t \) and \( \Delta_{t-1} \) only the coefficient for \( i_t \) was significantly different from zero, which is consistent with the hypothesis that \( i_t \)
summarizes all information in $\Delta t_{-1}$ about the expected value of $\Delta_t$. Finally, sample autocorrelations of $r_t$, constructed as per EQ(18), were approximately zero.

In short, the empirical evidence was overwhelmingly consistent with the hypothesis that the bill market is efficient in setting one to six-month nominal rates of interest. The market correctly uses all the information about future inflation rates contained in time-series of past inflation rates. The evidence also substantiated the assumption of a constant expected equilibrium real interest rate.

Fama's study, in one respect, could be considered just a repetition of Gibson [1972]. What makes the work so important is that it provides theoretical and empirical support for the contention the market's inflation expectations are rational in the Muthian sense, and fully incorporated into equilibrium expected market interest rates. As we have previously argued, the Fisherian Hypothesis is a necessary condition for inflation tax neutrality, but it is not sufficient. Neutrality requires unbiased expectations, which are suggested by Fama's evidence.

Fama and MacBeth [1974] was the prelude to Fama [1975]. The 1974 paper tested the multiperiod two-parameter Capital Asset Pricing Model. The Fisherian Hypothesis arose because in order to test the behavior of returns and investment opportunities one must consider
real returns rather than nominal returns. The tests in [1974] were similar to those reported above, and the conclusions were generally identical. There was, however, one very important finding in [1974], with particular significance to this study, which was not duplicated in [1975]. Fama and MacBeth asserted the expected nominal rates of return to the equilibrium market portfolio, $E(\tilde{i}_{mt}^*)$, should reflect changes in the expected value of purchasing power, $E(\tilde{\Delta}_t)$. Their hypothesis was not verified, however. They explained this, in effect, by suggesting their model was misspecified, and that a fully specified structural equation of $E(\tilde{i}_{mt}^*)$ would reveal the Fisherian effect. Their results, including an evaluation of short-term bonds and common stocks as hedges against inflation, are best summarized in Fama and MacBeth's own words.

We think that a reasonable way to interpret the combined results is as follows. The regressions ... indicate that $i_{mt}^*$, the one-month nominal interest rate observed at $t-1$, contains a rather good estimate of $E(\tilde{\Delta}_t)$, the expected value of the percent change in purchasing power from $t-1$ to $t$. Thus the market shows some competence in predicting inflation. Moreover, it seems unreasonable that this competence is only exercised in the market for one-month bonds. More likely, whatever assessment of $E(\tilde{\Delta}_t)$ is included in $i_{mt}^*$ is also included when the market sets the prices of all other assets at $t-1$. Thus the fact that we cannot identify changes in $E(i_{mt}^*)$ in response to changes in $E(\tilde{\Delta}_t)$ does not mean that the response mechanism does not exist. Rather, other sources of variation in $i_{mt}^*$ are so large relative to changes in $E(i_{mt}^*)$ in response to $E(\tilde{\Delta}_t)$ that such changes are not statistically observable.

But even if one-month expected nominal returns on common stocks contain the same assessment of the expected percent change in purchasing power as the one-month
nominal interest rate, the one-month bond is still better as a specific hedge against the one-month change in purchasing power. Once the nominal interest rate is set, the only uncertainty in the real return on the one-month bond is in the deviation of the percent change in purchasing power from its expected value, whereas this is a trivial fraction of the uncertainty in the real return on common stock. (pp. 64-65)

In 1977 a series of papers appeared which challenged the findings of Fama and MacBeth [1974], and Fama [1975] [June, 1976]. These papers, by Carlson [June, 1977], Joines [1977], and Nelson and Schwert [1977], were preceded by Hess and Bicksler [1975]. Because of the significance of the Fama studies it is appropriate to consider the challenges in detail.

Hess and Bicksler compared the robustness of nominal interest rates as predictors of inflation to forecasts derived from a Box-Jenkins [1976] integrated autoregressive moving average process (ARIMA). They rewrote EQ(23) above as EQ(26) and suggested EQ(27) as the best ARIMA. See Appendix A for details.

\[
\text{EQ(26)} \quad E_{t-1}(\rho_t | \phi_{t-1}^*) = (1.0 + \alpha_0 - i_t)^{-1} - 1.0
\]

\[
\text{EQ(27)} \quad E_{t-1}(\rho_t | \phi_{t-1}') = \rho_{t-1} + \rho_{t-12} + \rho_{t-13} + \\
\quad \theta_1 u_{t-1} - \lambda_1 u_{t-12} - \lambda_1 \theta_1 u_{t-13}
\]

where, $\theta = \text{moving average parameter}$

$\lambda = \text{seasonal moving average parameter}$
If the capital markets are efficient, then the information in EQ(27) must be spanned by $\phi_t^*$ in EQ(26). That is, efficiency requires that the forecasts derived from $\phi_t^*$ cannot be improved upon by incorporating $\phi_t'$ into the model. This immediately implies the forecasting errors of the capital markets should be independent of the contemporaneous difference between capital market forecasts and forecasts based upon $\phi'$. The minimum squared errors in EQ(28) will occur for $\beta$ equal zero. Equation EQ(29) is the general form of the OLS estimation of EQ(28). The specific OLS estimated equation substituted EQ(26) and EQ(27) into EQ(29).

\[
\text{EQ(28)} \quad \Delta_t = \beta [E(\Delta_t | \phi_{t-1}')] = (1-\beta)[E(\Delta_t | \phi_{t-1}^*)] + \varepsilon_t
\]

\[
\text{EQ(29)} \quad \Delta_t - E(\Delta_t | \phi_{t-1}^*) = \beta [E(\Delta_t | \phi_{t-1}') - E(\Delta_t | \phi_{t-1}^*)] + \varepsilon_t
\]

The hypothesis that $\beta = 0$ was rejected. Hess and Bicksler therefore concluded the capital markets are not efficient. Not all available information about expected future inflation is embodied in $i_t$, or the equilibrium expected real Treasury Bill rate is not constant, or both.
In order to be more precise about the implications of their findings, Hess and Bicksler retested EQ(29), with the hypothesis that the expected real rate is not constant but is generated by past rates. Equation EQ(30) is the general form of the identified real rate ARIMA.

\[
\text{EQ(30)} \quad E(\tilde{r} | \phi_{t-1}^u) = r_{t-12} - \theta_1 u_{t-1} - \theta_2 u_{t-2} - \theta_3 u_{t-3} \\
- \lambda_1 u_{t-12} + \theta_1 \lambda_1 u_{t-13} + \theta_2 \lambda_1 u_{t-14} \\
+ \theta_3 \lambda_1 u_{t-15}
\]

Again, Hess and Bicksler were able to reject the hypothesis that \( \beta \) in EQ(28) equals zero for the time period 1958-1971. However, for all subintervals of that time period the hypothesis could not be rejected. They concluded that improving the approximation of the expected real rate leads to empirical results much more consistent with the efficient markets model.

Hess and Bicksler summarized their empirical results as inconsistent with Fama's hypothesis of a constant expected real interest rate, but as not inconsistent with the efficient markets hypothesis. They also provided evidence (i.e., distribution of the forecasting errors) which confirmed the existence of a Mundell Effect - an effect which we have shown is inherently inconsistent with a constant expected real rate of interest.
Nelson and Schwert [1977] also applied the ARIMA technique in their review of Fama [1975]. They revealed the autocorrelation function of the observed real rate of interest may be approximately zero for all lags, even if the ex-ante real rates vary substantially and are highly autocorrelated. This is due to the relatively large variance of the errors in inflation expectations. In addition, they re-examined Eq(24) above by replacing $\Delta_{t-1}$ with the "optimal" (i.e., ARIMA) predictions of inflation based on the past history of inflation rates. Efficiency requires $\beta_2$ equal zero.

\[
\text{EQ(31)} \quad \rho_t = \beta_0 + \beta_1 i_t + \beta_2 \rho_t
\]

\[
\text{EQ(32)} \quad \rho_t = \sum_{j=0}^{\infty} \theta^j (1-\theta) \rho_{t-j-1}
\]

Yet, the empirical results revealed $\beta_2$ was large and usually significant. As with Hess and Bicksler, the conflicting evidence implied either the expected real return is not constant, the capital markets are not efficient, or both. Nelson and Schwert emphasized that individual past inflation rates contain very little information about rates of inflation. Accordingly Fama's equation, Eq(24) above, is inefficient because it wastes information.
Carlson [June 1977] was more conclusive in his review of Fama [1975]. He stated the expected real interest rate is not constant, and in addition, the capital markets are not efficient, in the sense that all available information about subsequent inflation rates are incorporated in interest rates. Carlson arrived at these conclusions by retesting EQ(24). He first demonstrated the expected real rate of interest is not constant over time. Using the Livingston inflation expectations data, Carlson constructed a series of expected real yields on six-month bills. He then examined the plot of the expected real returns and suggested the variance was too large to be consistent with a constant expected rate. The variable $\Delta_{t-1}$ in EQ(24) was replaced by the ratio of employment to population, apparently as a surrogate for the "health" of the economy (to capture recessions). Carlson then demonstrated the coefficient on the employment ratio was statistically different from zero.

Joines [1977] asked whether Fama would have achieved similar results based on an information set somewhat broader than the past history of consumer price inflation. He also questioned whether Fama's data and model actually support his hypothesis. To answer the first question, Joines substituted lagged rates of change of wholesale prices for $\Delta_{t-1}$ in EQ(24), under the assumption that wholesale price changes lead consumer prices changes. As with the previous tests of that type, Fama's hypotheses imply the coefficient on the market interest rate will be unity while all other coefficients, save the
constant, will not be significantly different from zero. The test results were inconsistent with these expectations. In addition, Joines also concluded that even Fama's data and model failed to substantiate efficiency because of statistically significant twelfth-order (i.e., yearly) autocorrelation of the residuals from EQ(23).

Fama [1977] responded to his critics in a detailed discussion of their individual methodologies and empirical results, the specifics of which need not be addressed here. In general, Fama argued the marginal contribution to the description of the measured inflation rate by such variables as Carlson's employment to population ratio might represent a spurious component of the inflation rate which is properly ignored by the market in setting the interest rate. Fama concluded by claiming,

Taking the data at face value, the interest rate remains the best single predictor of the inflation rate; and nobody has uncovered variables that make substantial contributions to the prediction of inflation beyond that provided by the interest rate alone. Moreover, one of the more interesting propositions of the model, that the largest part of the variation in nominal interest rates reflects variation in expected inflation rates, seems intact. Finally although the model is not an exact description of the world, the specific deviations discovered so far are to some extent manifestations of measurement errors in the estimates of inflation rates and interest rates. (p. 496)
Fama also responded to his critics in a paper published jointly with Schwert [1979]. In this paper, the authors explored the composition of the consumer price index (CPI). The idea proposed was that the interest rate will respond to the expected future inflation rate only if inflation rates are unambiguous measures, such as when relative prices are constant. But relative prices have varied substantially (during 1953-1977) which, in conjunction with measurement errors, induces spurious short-term autocorrelations in measured inflation rates. This information is properly ignored in the capital markets but appears in econometric models as contradictory evidence to the hypothesis that interest rates contain market forecasts of future inflation rates. This basis for the Fama and Schwert paper was presented in Fama [1977].

According to Fama and Schwert, the Fisherian Hypothesis is a relationship between interest rates and the component of the expected inflation rate which is common to all goods. They assumed differentials in the expected inflation rates of the various components of the CPI exist primarily because of annual seasonals in the components. The authors argued that price seasonals will be observed when the real costs of providing different goods to the market varies seasonally, as for example when the seasonal price movement offsets the costs of storage that arise from nonsynchronized supply and demand (e.g., fruits and vegetables). The interest rate, on the other hand, will not respond to these differential price
changes, but only to the common part of the expected inflation rate, provided the differentials exist to offset differing costs of providing goods to the market at different points in production-storage cycles. Empirically one should observe the Fisherian Hypothesis for the interest rate and the inflation rate for any commodity $j$ in time $t$ ($\rho_{jt}$), and the residuals of the estimated equation should correspond to the time series behavior of the differential expected inflation rate for good $j$ (assuming a constant expected equilibrium real rate of interest). This is precisely what Fama and Schwert did observe. A limited sample of their results is reproduced in Appendix A. Fama and Schwert, therefore, gave credence to the proposition that when price-level data are adjusted for differential (i.e., relative) price movements, the Fisherian Hypothesis is valid.

Two recent studies of the Argentina and Brazil Treasury Bill markets have also concluded by rejecting Fama's assumption of constant expected real interest rates. Leiderman [1979] reported, for the case of Argentina, an increase in expected inflation is not fully transmitted to the nominal interest rate. The real interest rate (or its determinants) was omitted from the estimation so the estimated coefficients were not unbiased. Brito [1979] went further in his conclusion regarding the Brazilian capital market and stated that the nominal interest rate is not an efficient predictor of inflation nor are real rates constant. In both cases, however, the authors stressed
the evidence is consistent with the hypothesis that the respective bill markets have efficient expectations of inflation in the sense that the capital markets use all the information about the subsequent inflation rate contained in the time-series of past inflation rates. This implies the variance in the real interest rate "matters." However, neither Fama nor his critics addressed the fundamental question of whether the real rate is a function of the expected inflation rate; their methodology, of course, precluded any such an analysis.

D.3 The Gibson Paradox: A Further Question of Specification

Roll [1972], in his critical review of the empirical studies of the Fisherian Hypothesis, argued that inflation expectations based on a sequence of historical inflation rates implies inefficient commodities markets. Roll equated Eq(7) above with the "Fisher Effect" (our Fisherian Hypothesis). He argued the Fisher Effect was an artifact of the Gibson Paradox and the stochastic process in the level of commodity prices. Fisher, recall, viewed the Gibson Paradox as the artifact. Furthermore, we maintain, as stated supra, that Eq(7) is not the "Fisherian Hypothesis."
Roll re-estimated the Feldstein and Eckstein [1970] model adjusted for the Gibson Paradox. The inclusion of the current level of the wholesale price index did materially alter the original results. Anticipated inflation as measured by the Almon lag, was no longer statistically significant, thus lending support to Gibson Paradox interpretation. Selections of Roll's results are presented in Appendix A.

Apart from the excellent but limited literature review, Roll's study is seriously lacking because no theoretical rationale is provided to justify why nominal interest rates should be determined by the level of prices, apart from the attendant relation described by Fisher. Fama [1975] observed if inflations and deflations tend to persist, then there may appear to be a relationship between the level of interest rates and the measured level of prices. Roll's empirical evidence is also subject to question. The estimated coefficients varied both in sign and significance with the inclusion of the price level in the Feldstein and Eckstein model. In most instances the signs were contrary to "normal" economic theory (see Appendix A for examples). Roll did recognize the problem might be attributable to "a bald fitting of a single equation to a multiple equation system."  

The Gibson Paradox was exhaustively studied by Sargent [February, 1973], as part of a review of the long history of estimates of Fisher's equation. He correctly noted economists in general retain
strong reservations about Fisher's work (and that of others) which purport "implausibly" long lags in the formation of inflation expectations. Yet, it was precisely the long lags which, according to Fisher, cause the Gibson Paradox. Using ARIMA models, Sargent concluded the distributed lag weights which would characterize optimal forecasts are much shorter than those estimated by Fisher, et al. The model Sargent adopted is written as:

\[
\begin{align*}
\text{EQ(33-a)} & \quad i_t = a(L)i_t + b(L)\rho_t + \epsilon_t \\
\text{EQ(33-b)} & \quad \rho_t = d(L)\rho_t + u_t
\end{align*}
\]

where, \( \epsilon_t \) and \( u_t \) are mutually independent white noise; \( a(L) \), \( b(L) \), and \( d(L) \) are one-sided polynomials in the lag operator \( L \).

A model such as EQ(33) is distinguished by the fact it admits only unidirectional influence, namely from inflation rates to interest rates. A large value of \( u_t \) increases the inflation rate, and consequently the market interest rate. However, the model asserts no feedback occurs from the interest rate to either the current or subsequent rates of inflation. In this sense, the rate of inflation is implicitly exogenous. Sargent applied cross-spectral analysis to determine whether such a model is a valid characterization of actual economic behavior. His results, which are both too complex and too lengthy to repeat here, revealed that the unidirectional causality is not as efficient a hypothesis as is the system of equations hypothesis which allows for feedback from current rates of interest to
subsequent rates of inflation. Such a model may be written in general form as EQ(34), of which EQ(33) is merely a special case. One implication of this finding is that Fisher’s explanation of the Gibson

\[
\begin{bmatrix}
1-a(L) \\
-c(L)
\end{bmatrix}
\begin{bmatrix}
-i_t \\
-\rho_t
\end{bmatrix}
= \begin{bmatrix}
\epsilon_t \\
\upsilon_t
\end{bmatrix}
\]

Paradox, which is unidirectional from inflation to interest rates, is inadequate. However, as Sargent warned, what appears to be feedback between \( i \) and \( \rho \) within the context of EQ(34) may exist because some omitted series influences both terms.

Sargent also specified a model which included a price expectations mechanism. He then held anticipated inflation to zero and simulated the model for predetermined parameters. Using the artificial data generated by the model, Sargent examined the cross-spectral density functions for the interest rate and the inflation rate and for the interest rate and the lag of the price level. A close contemporaneous relationship between \( i_t \) and the (log of) price level was detected, but not for \( i_t \) and \( \rho_t \). The relationship between inflation and interest in the artificial data was roughly similar to that characterizing the historical data, but since anticipated future inflation was constrained to zero in the simulation the relationship could not be attributed to expectations. Long lags in the formation of expectations of inflation existed in the constructed data which
were also observed in the actual historical data. Summarizing his results, Sargent stated that, "in general there is no reason to expect a regression of interest against current and lagged rates of inflation to reveal very much about the expectations of inflation held by the public."[18]

In summary, what did Sargent demonstrate. First, he showed that interest rates are correlated with the price level, i.e., the Gibson Paradox. Second, if taken at face value, the evidence conflicts with Fisher's explanation of the Gibson Paradox, but it is indirectly supportive of the Wicksell-Keynes theory of capital market behavior in disequilibrium, where the price level tends to rise when the natural rate exceeds the market rate. However, as Sargent's critics noted, the evidence for the first conclusion was much stronger than for the second.[19] Likewise Sargent's claim the simulation experiment produces a Gibson Paradox without inflationary expectations is questionable since there are notable differences in the cross-spectral phase series for the historical and the hypothetical data. Consequently, Sargent did not establish that interest rates are determined by the level of prices instead of anticipated inflation rates. He did, nevertheless, contribute to the controversy about the descriptive accuracy of the competing hypothesis.
D.4 Modelled Real Rate Behavior

Empirical tests of the Fisherian Hypothesis can be categorized into those which assume (or tacitly imply) a fixed real rate of interest, and those in which the real rate is variable but specifiable. Elliott [1977] belongs to the latter group. He is to our knowledge the only author to have evaluated the Fisherian Hypothesis for competing theories about the determination of the real interest rate.

Elliott adopted Sargent's [Brookings, 1973] neo-Keynesian model, in which $y$, $P$, $i$, $r$, and $p$ are endogenous. A Neoclassical-loanable funds model was developed as an alternative explanation of the real interest rate. The model assumes that real rates are determined by the supply of and the demand for financial assets. The endogenous variables are $i$, $B$, $M^S$, and $p$. Elliott also included the Fisherian autoregressive model, which was simply the Fisherian Hypothesis with autoregressive inflation expectations. The models are reproduced in Appendix A.

Elliott estimated two inflation expectations models, in addition to the autoregressive. The first he called the "monetary model," which regressed inflation rates on lagged money supply. The alternative inflation expectations model exploited the assumption that adjustments in labor market variables can be assumed to reveal the
prevalent inflationary expectations. He assumed money wages were exogenous over the sampled period. This inflation expectations model was titled the "labor market model."

Elliott tested the models by first estimating the equations over the period 1960-1974 and computing the residual variation for each model. The results are reproduced as the first column of Table I.

**TABLE I**

**Elliott [1977]**

**Real Interest Rate Tests**

<table>
<thead>
<tr>
<th>Model</th>
<th>Root Mean Square Error</th>
<th>One Period Ahead Predictions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neo-Keynesian r, Labor Market</td>
<td>1.16</td>
<td>1.95</td>
</tr>
<tr>
<td>Neo-Keynesian r, Monetary</td>
<td>1.09</td>
<td>1.69</td>
</tr>
<tr>
<td>Loanable Funds r, Labor Market</td>
<td>1.12</td>
<td>1.90</td>
</tr>
<tr>
<td>Loanable Funds r, Monetary</td>
<td>1.12</td>
<td>1.91</td>
</tr>
<tr>
<td>Fisherian Autoregressive</td>
<td>4.10</td>
<td>3.37</td>
</tr>
</tbody>
</table>

*twice its standard error.

The Neo-Keynesian--Monetary model was the most efficient characterization of expectations. All four macroeconomic models were clearly superior to the Fisherian autoregressive in terms of residual variation. To determine the post-sample predictive power of each
model, Elliott re-estimated the models over the period 1960-1967, and then produced conditional predictions over the four quarters of 1968. He then re-estimated over 1960-1968 for conditional predictions of 1969, and so forth. The one period (quarter) ahead predicted real interest rates were compared to the "actual" real rates. The neo-Keynesian—Monetary model was the most efficient, both in terms of lowest residual standard error and average absolute prediction error. The Fisherian autoregressive model was the least efficient. Elliott concluded,

The neo-Keynesian explanation of the real rate of interest completed by the monetary explanation of inflationary expectations stands out as both the best fitting model over the 1960-74 period and as the most efficient for conditional predictions over the 1968-74 period. It accordingly makes the most effective use of available theory and data. Accordingly, the real interest rate measured by this model is the best representation of Muthian rational expectations among the alternatives considered. At the other extreme, the autoregressive formulation makes the least effective use of current data and is the poorest representation of expectations we have considered. This comparatively poor showing raises questions about the continued use of the autoregressive model in empirical work, when more efficient alternatives are apparently available. (p. 440)

Elliott's study is important for several reasons. First, it demonstrated that macroeconomic explanations of expected inflation are considerably more efficient than models limited solely to the information in past inflation rates. Rutledge [1974] and Nelson [1975] had previously made the same point. This raises serious doubts about the validity of Fama's hypothesis of constant expected
equilibrium real interest rates. Second, but related to the first point, complete systems models were shown to outperform single equations (based on standard errors). Finally, since the expected real rates were unbiased predictions and comparatively accurate for some models, there was no evidence of an inefficient capital market.

Elliott is one of the few who have explicitly acknowledged and adjusted for the multi-market, general equilibrium effects of the inflation tax and the simultaneity bias that otherwise would accompany the estimation of coefficients in a single equation method. Unfortunately, Elliott did not report his estimated equations so we are unable to draw any inferences about the neutrality of the inflation tax.

Feldstein and Chamberlain [1973] estimated what they called a "partial reduced form of the financial sector" in a study of the long-term interest rate. In fact, they estimated a single structural equation containing explanatory variables which are (theoretically) correlated with the disturbance term. Although Keynes [1936] stressed the interdependence between the rate of interest and the state of expectations in the stock market, subsequent empirical developments of the "Keynesian" model tended to ignore the role of equities and focused on only two assets, bonds and money. Their paper sought to reassert the joint dependence of the financial markets. They did this by developing a model of long-term interest rates based on
multi-market expectations. They also considered a broad class of inflation expectations models. Their study was primarily an update and extension of Feldstein and Eckstein [1970].

Feldstein and Chamberlain estimated numerous equations - far too many to review here. (Appendix A presents some typical results.) Using a second-order Pascal lag on past inflation rates as the expected inflation rate mechanism, they first re-estimated the Feldstein and Eckstein [1970] equations. The results were consistent with the earlier estimation, except for the expected inflation coefficient which was not significantly different from unity. They extended the basic specification by including a distributed lag measure of the expected rate of capital gains on equity shares. The authors interpreted the significant positive coefficient on this variable as an indication investors believed common stock prices would continue to rise and thence provide substantial capital gains. However, an alternative interpretation is possible. The past percentage changes in the Standard and Poor's Combined Index of 500 Common Stock Prices was used to generate the expected "capital gains." The S-P index is also a proxy for the return to the market portfolio. Consequently, what Feldstein and Chamberlain probably measured was the systematic risk between their dependent variable (yield to maturity on newly issued Moody's Aaa corporate bonds) and the market as a whole. We also have difficulty accepting a modification of the dynamic specification of their model. The
modification was the incorporation of a lag in the response of the long-term interest rate to changes in liquidity. Such a lag is inconsistent with the efficient market theory and the empirical evidence indicating that expected interest rates reflect pending liquidity changes. This topic is reviewed in Section F.

Hendershott and Van Horne [1973] added yet another dimension to the study of the Fisherian Hypothesis. They contend the two principal economic theories used to explain the behavior of the real interest rate, namely the Keynesian liquidity preference theory and the loanable funds theory, were inadequate. Consequently, tests of the Fisherian Hypothesis which built the real rate specification upon either of these theories were not valid. The liquidity preference theory "appears to have tenuous theoretical foundations;" while the loanable funds theory, "has little empirical promise." The Hendershott and Van Horne approach was to "concentrate on the capital-market equilibrium." The weakness of their criticisms of the traditional theories of the determination of the real interest rate should not detract from their theory.

In the capital market equilibrium approach, rates of expected inflation are inferred from differences in expected returns on an asset whose return tends to be expressed in real terms and one whose
return tends to be expressed in nominal terms.\textsuperscript{22} Assuming a perpetual growth dividend valuation model, Hendershott and Van Horne derived the following equation:

\[
\text{EQ}(35) \quad i_t - \left( \frac{D}{Ve} \right)_t = E_t(\sigma_{t+j}) + g - \sigma
\]

where, \( i_t \) = nominal rate of return on bonds,
\( \frac{D}{Ve} \) = economy wide dividends to market value of equities,
\( g \) = compound annual growth rate in real dividends,
\( \sigma \) = risk premium.

They estimated EQ(35) by substituting various proxies for the unobservable \( g \). The distributed lag coefficients were constrained to sum to unity. The implied expected inflation rates were computed and compared to the actual inflation rates and the Livingston series. Hendershott and Van Horne concluded the estimates of expected inflation were roughly consistent with actual changes in inflation, and definitely superior to the Livingston expectations. The observed corporate bond rates and the rates calculated on the basis of the model were also compared and shown to be nearly identical.

Three salient features of the Hendershott and Van Horne study warrant mention. First, their model of the bond rates produced a series of returns which were consistent with the marked acceleration
in the formation of price expectations in the late 1960s. This established that a well specified model could capture what this study has referred to as the "Threshold Effect."

Second, Hendershott and Van Horne explicitly introduced a risk premium variable, equal to the difference between the expected real return on bonds and the expected real return on stocks. This risk premium, while not exactly the concept discussed in our first essay, does conceptually include the inflation rate uncertainty. Unfortunately, Hendershott and Van Horne elected to treat the risk premium as a constant.

Finally, Hendershott and Van Horne plotted the difference between the observed bond rate and their estimate of the anticipated rate of inflation. The difference, which is an estimate of the expected real yield on bonds, exhibited little variance and no discernible trend. This substantiated Fama's hypothesis that the equilibrium expected real interest rate is constant.

D.5 Conclusion

Most certainly this review of the empirical tests of the behavior of the interest rate to changes in the anticipated inflation rate has failed to consider interesting and important papers. We believe,
however, that no significant conceptual issue has been omitted from consideration. The general impression gained from this review is that virtually any position regarding the impact of anticipated inflation on interest rates can be supported by appeal to empirical evidence. Interest rates either fully reflect anticipated future inflation; adjust contemporaneously to changes in inflation rates; adjust with a short lag, or adjust only after several years - all depending on which studies one chooses to believe. Accordingly, monetary policy is at one extreme anticipated and has an immediate and significant impact on market (and perhaps real) interest rates, or at the other extreme is generally insignificant in the short-run and properly disregarded in arriving at stabilization policy.

The spectrum of empirical evidence naturally suggests the question: Why the discrepancy? Throughout this section we have endeavored to indicate specific methodological weaknesses which detract from the reliability of the particular tests. Beyond these specific problems three general methodological deficiencies have been identified.

First, the interest rate equations have traditionally been partial equilibrium models incapable of capturing the general equilibrium adjustments to the inflation tax. Econometrically, the estimates of the coefficients are subject to simultaneous equation bias. The conditions are very stringent under which a regression of the market
interest rate on current and lagged rates of inflation can be expected to yield a consistent estimate of the distributed lag on inflation model of inflation expectations formation. Least-squares estimation gives consistent estimates if the disturbances are mutually independent and the system is block recursive in a special way. Moreover, the conditions can fail in such a way that interest rates appear to be formed by a long distributed lag of actual inflation, even when inflation expectations are actually formed by a short distributed lag of actual inflation.

Second, the estimates of the elasticity of market interest rates to expected inflation rates are biased because the models are misspecified. The Mundell Effect, that is the inflation tax base, is frequently omitted in the model specification. In addition, the uncertainty about the future inflation rate has been universally ignored, save Fama [June, 1976]. The consistent discovery of a threshold effect may merely reflect the failure to properly control for these factors or institutional considerations, such as the personal and corporate taxes. The simultaneity and possible omitted variable biases are particularly crucial because the assessment of the neutrality of inflationary finance is contingent on whether the estimated coefficient for anticipated inflation is, not just significant, but whether it is significantly different from unity.
Finally, the spectrum of empirical findings regarding the Fisherian Hypothesis is perhaps more than anything just the image of the variety of inflation forecast mechanisms which have been applied. In many instances the expectations generating mechanism appears to have been chosen on the basis of convenience. But since the true inflation expectations generating function is unknown there is no way to establish which model has the correct specification. Furthermore, we must question whether the tacit assumption of "singular expectations" does not further bias the tests of the Fisherian Hypothesis. In virtually all the empirical inquiries, the economy is implicitly assumed to have foresight, however imperfect, only as to one variable -- the inflation rate. Anticipations of future monetary policy, levels of economic output, profit rates, and other economic variables are ignored. Naturally this makes for simpler models. How the results would vary for a broader application of expectations is as yet untested.
E. COMMON STOCKS AS INFLATION HEDGES

The Fisherian Hypothesis, according to Sargent [May 1972] is, "based on the assumption that investors consider assets with yields that are fixed in real terms, such as equities and physical capital, as very good substitutes for bonds, a class of assets whose returns are fixed in nominal terms." This point was made in Section B. As has been demonstrated, there are reasons why the real price of equities and physical capital will not increase proportionately to changes in the general level of prices. In this section we continue the investigation of the Fisherian Hypothesis by concentrating on the generally accepted notion that equities, that is common stocks, are hedges against inflation. Recent experience in the U. S. capital markets may be weakening the previously unquestioned assumption that common stocks do indeed keep pace with inflation. However, the principle is still widely supported. As Sargent stated,

This much of Fisher's doctrine, which asserts that the spread between the nominal rate of return on bonds and the rate of return on real assets fully adjusts to reflect changes in the anticipated rate of inflation, is widely accepted, both by "monetarists" and "Keynesians." (pp. 212-213)

Before reviewing selected studies it is first necessary to define certain concepts. The first of these is a monetary asset,
A monetary asset is a claim to a fixed number of dollars. An asset whose nominal amount is independent of the price level.

Similarly, a monetary liability is defined as,

A monetary liability is an obligation to pay a fixed number of dollars. A liability whose nominal amount is independent of the price level.

A nonmonetary asset is any asset which is not a monetary asset, which is to say it is an asset whose nominal amount is not independent of the price level. Similarly for a nonmonetary liability. A nonparticipating corporate bond is a monetary liability of the issuing body, and a monetary asset to the holders. Common stock is a nonmonetary liability of the issuing corporation, and a nonmonetary asset to the stockholder.

The definitions of "monetary" and "nonmonetary" imply nothing about the behavior of the real rate of return with respect to inflation. This is a question of the extent to which inflation expectations are incorporated into the contracted return for monetary instruments, and of how nonmonetary prices adjust over time to changes in anticipated inflation rates. The latter issue is the topic of this section.
Unless otherwise stated, we adopt the definitions presented by Fama and Schwert [1977] of an inflation hedge. Following on Fama [1975], they wrote the Fisherian Hypothesis as EQ(37). Unexpected inflation was just defined as \( \varepsilon_t \) in EQ(38), so that the complete model could be described as EQ(39). Assuming the equilibrium expected real rate of return is uncorrelated with the expected loss in purchasing power, Fama and Schwert derived EQ(40) as a test form of EQ(39).

\[
\text{EQ(37)} \quad E_{t-1}(i_{j,t} | \phi_{t-1}) = E_{t-1}(\tilde{r}_{j,t} | \phi_{t-1}) + E_{t-1}(\Delta_t | \phi_{t-1})
\]

\[
\text{EQ(38)} \quad \varepsilon_t = \Delta_t - E_{t-1}(\Delta_t | \phi_{t-1})
\]

\[
\text{EQ(39)} \quad E_{t-1}(i_{j,t} | \phi_{t-1}, \Delta_t) = E_{t-1}(\tilde{r}_{j,t} | \phi_{t-1}) + E_{t-1}(\Delta_t | \phi_{t-1}) + \gamma_j \varepsilon_t
\]

\[
\text{EQ(40)} \quad i_{j,t} = \beta_{0,j} + 1 \beta_{1,j} E_{t-1}(\Delta_t | \phi_{t-1}) + \gamma_j \varepsilon_t + \eta_{j,t}
\]

An "inflation hedge" is defined by the parameters \( \beta_{1,j} \) and \( \gamma_j \).

1. Complete Hedge Against Inflation

\[ \beta_{1,j} = 1.0 \quad \gamma_j = 1.0 \]

The nominal return on the asset varies in one-to-one correspondence with both the expected and unexpected components of the inflation rate, and the ex-post real return on the asset is uncorrelated with the ex-post inflation rate. (p. 117)
2. **Complete Hedge Against Expected Inflation**

\[ \beta_{1,j} = 1.0 \]

The expected nominal return on the asset varies in one-to-one correspondence with the expected inflation rate, and the expected real rate of return on the asset is uncorrelated with the expected inflation rate. (p. 117)

3. **Complete Hedge Against Unexpected Inflation**

\[ \gamma_j = 1.0 \]

The ex-post nominal return on the asset varies in one-to-one correspondence with the unexpected inflation rate.

The differential between the market rate of return and the real rate of return to common stocks will not equal that required to just compensate for the expected future inflation rate for three reasons:

1. differential inflation forecast ability by income earning segments,
2. taxes and other market imperfections, and
3. inflation uncertainty.

The first category includes the debtor-creditor hypothesis, and the wages lag hypothesis (see Section H). Briefly, the latter holds that wage rates lag rising prices thus raising the operating income of the firm. The increase in income is captured by common stockholders in the form of abnormal rates of return. The debtor-creditor argument maintains that common stock returns will exceed, equal, or fall short of the rate of inflation as the firm's monetary assets are less than,
equal, or greater than its monetary liabilities (i.e., its net debtor-creditor position). Both arguments rest on the assumption that one income earning segment (bondholders, labor) are disadvantaged in their ability to forecast inflation rates compared to other segments (e.g., equity holders). The disadvantage may reflect unequal understanding of the economic relationships or unequal access to information (which technically is a market imperfection).

Whereas the common stocks are assumed to be more than complete inflation hedges according to the differential forecast hypothesis, the literature is far less conclusive regarding the impact of tax and other market imperfections. Most of the literature has dealt with the specifics of the tax code which prevent common stock prices from rising by the full value of expected inflation. As noted in Section C, Darby [1975] and Feldstein [1976] contend common stocks are more than compensated for expected inflation. However, Nichols [1968], Motley [1969], and Tideman and Tucker [1976] maintain just the opposite.

Finally, the influence of inflation uncertainty has been recognized on a limited scale. The more important works are discussed below. In addition to these papers, we note that Amihud and Barnea [1977], using a Taylor expansion of the inflation rate, find the derivative of \( i \) with respect to a mean preserving spread in the
price-level to be negative, as it is generally imagined. The variance of price-level changes depresses the market rate of return, thereby giving the appearance of a less than complete inflation hedge.

Reilly, Johnson, and Smith published a series of papers in which they empirically evaluated the inflation hedge property of common stocks. They adopted an alternative definition of "inflation hedge" to the one just presented. They defined an asset to be a complete inflation hedge if the real rate of return in inflationary periods is at least as great as the real rate of return in non-inflationary periods. A partial inflation hedge, on the other hand, was defined as an asset whose nominal rate of return in inflationary periods is greater than the nominal rate in non-inflationary periods.

In Reilly, Johnson, and Smith [1970] the period studied was from September 30, 1937 to December 31, 1968. The period was divided into eleven (unequal) subperiods depending upon whether the quarterly percentage changes in the Consumer Price Index indicated inflation, "relative non-inflation," or deflation. Using several indexes of stock prices, such as the Standard and Poor's 500 Composite Index, the authors derived implicit nominal rates of return series which they compared to the type of price change in each period. They found that the "net return," that is the real rate of return minus the nominal rate of return, was generally negative for all price indexes and all
subperiods, thus implying that common stocks are not complete inflation hedges. Subsequent analysis revealed common stocks had been partial hedges at various times.

In Reilly, Smith, and Johnson [1975], the authors corrected for an error in the computation of weighted average real returns in their previous study. After re-estimating they concluded,

In conclusion, the evidence continues to support the contention that common stocks in general are not complete or consistent inflation hedges during periods of significant inflation. As before, there may be some individual stocks that are good inflation hedges, and it is certainly possible that during time intervals within the overall period of inflation common stock returns may be very adequate. Such adequate returns on common stock occur most likely during periods of stability or decline in the rate of inflation. (p. 876)

Bodie [1976] criticized the Reilly, Johnson, and Smith definition of inflation hedge. A security is an inflation hedge according to that definition if it offers "protection" against inflation, which interpreted meant the elimination or at least the reduction of the possibility that the real rate of return on the security will fall below some limit (e.g. zero). But as Bodie noted, in order for common stocks to qualify as an inflation hedge under this definition stocks must be free of "downside" risk stemming from all sources - not just from inflation. Given the large variance of equity returns, it was not surprising the Reilly, Johnson, and Smith studies found the nominal rate of return on common stocks was frequently less than the
rate of inflation. Reilly, Johnson, and Smith failed to account for the independent changes in the real rate of return, which is to say their model suffered from misspecification error.

Van Horne and Glassmire [1972] analyzed the process by which common stock values are affected by unanticipated changes in inflation. They defined the market value of a share of stock as the present value of all expected future dividends for the share, discounted at a required rate of return. Dividends were disaggregated into four components: the expected future earnings before interest, taxes, and depreciation; expected future interest costs on net financial liabilities; expected future depreciation; and the discount rate. All four components are affected by the unanticipated change in inflation. Given expressions for each of these components, Van Horne and Glassmire derived a complex equation for the value per share expected to unanticipated changes in the rate of inflation. The authors were able to show how the sensitivity of prices, wages, and other costs to unanticipated inflation dominate the determination of the response of share prices to inflation. If product prices, wages, and other costs change exactly with unanticipated changes in inflation, then share values are unaffected by operating earnings. In this case, any change in common stock prices accompanying an unanticipated change in inflation will be determined by whether the monetary assets of the firm exceed its monetary liabilities (i.e., net creditor), and by the tax impact of expected depreciation charges.
The Van Horne and Glassmire study is particularly significant. It demonstrates that the question of whether common stocks are a hedge against inflation is equivalently a question about the microeconomics of the firm. In particular, whether common stocks are a hedge depends on the sources and uses of capital (i.e., on the relative share of net monetary assets to net nonmonetary assets), and the lead or lag of product prices over factor payments. Since these factors affect the real rate of return to the firm, the Van Horne and Glassmire study proves the inflation hedge question cannot be answered by simple OLS regressions of common stock returns on inflation rates - anticipated, unanticipated, or actual.

Another very important analytic study of common stocks as hedges against inflation is Roll [1973]. Roll's paper was a study of how assets and commodities acquire equilibrium prices in competitive markets. He constructed a constrained consumer utility (of consumption) maximization problem in a two period world with uncertainty regarding period two prices. Inventory storage costs, transaction - liquidity costs, and depreciation costs were incorporated into the model. The intercommodity conditions were similar to those in the classical certainty case. The intertemporal conditions drove the model. The uncertainty about future prices introduced an important complication, namely a risk premia demanded as part of the anticipated inflation rate. These premia depend on the comovements of commodity j's price with its (uncertain) deterioration
and storage costs and its marginal utility in period two. If in addition it is held investors are risk-averse, then the marginal utility of a particular commodity will depend on the variation in all other commodity prices and in all asset prices. Roll thus indirectly established that for a world of uncertainty, the general equilibrium analysis of the affects of inflation is the only defensible option.

Roll clearly revealed how the expected nominal rate of return to any asset varies with the anticipated rate of inflation entirely as a function of risk. It follows there is no a priori reason why common stocks should be complete inflation hedges. Roll derived the following fundamental equation for the expected rate of return:

\[ \text{EQ(41)} \quad E(i_j) > \frac{1}{\sigma_j E(U_{2k} z_j)} / E(U_{2k}) \]

where, \( i_j = E(i_j) + \sigma_j z_j \)

\( z_j = \) a random variable with mean zero and unit dispersion

\( U_{tj} = \) the marginal utility of consumption of good \( j \) in period \( t \).

Since EQ(41) does not contain a term involving inflation, the co-movement of expected nominal rates of return and expected inflation rates will have the same sign irrespective of the expected inflation rate. The uncertainty about inflation can be attributed to the inflation rate deviating about either a positive or a negative mean,
but the market price of such uncertainty will be the same. This result can be made more specific by assuming a quadratic utility function. In this case the expected nominal return is described as:

\[ E(i_j) = \bar{r} + \left[ E(i_m) - \bar{r} \right] \gamma(r_j) \]

\[ - [ \text{cov}(i_j, \rho) - \text{cov}(i_m, \rho) \gamma(r_j) ] / \rho \]

where, \( \bar{r} \) = nominal return to the risk free asset,
\( i_m \) = a value-weighted average of returns to all assets,
\( r_j \) = an "excess return" in real terms,
\( r_j = \rho (1+i_j) - E(\rho)(1+i_\bar{r}) \)

\( \gamma(r_j) = \text{cov}(r_j, r_m) / \sigma(r_m) \)

Equation EQ(42) clearly indicates additional risk premia, to compensate for inflation uncertainty, contribute to the equilibrium market rate of return.

The possibility of inflation risk leads to the conclusion that the Fisherian Hypothesis cannot be reduced to statements merely about the expected rate of inflation. In the model presented by Roll the elasticity of the expected rate of return to the expected inflation rate could be positive, zero, or negative.
Roll summarized the implications of his model by observing that when economic agents must make simultaneous choices of assets and consumption in a money economy, the (strong form) Fisherian Hypothesis will fail. Siegel and Warner [1977] derived the equivalent result -- the Fisherian Hypothesis depends on the sign of the covariance between the real market rate of return and the expected rate of inflation. Of course, as observed in Section B, Fisher derived the identical conclusion, perhaps less elegantly, nearly half a century earlier.

In a related study, Jaffee [1978] explored the role of taxes on the rates of returns to stocks and bonds in an inflationary environment. He showed that because the personal tax rate on stocks is lower than the personal tax rate on bonds (due to preferential capital gains rates), the net of tax yield on a bond equals the net of tax yield on a stock only if the bond's pretax yield rises more rapidly than the stock's pretax yield. From this fact he concluded that the differential between the yield on a stock and the yield on a bond is inversely related to the inflation rate. Obviously if this is the case, when the Fisherian Hypothesis is valid for bond interest rates, common stock returns will be negatively related to the inflation rate, cet. par.
Oudet [1973] reported a correlation coefficient of -.42 for rates of return on stocks and inflation rates for quarterly observations from 1953 to 1970. However, he acknowledged such a test provides limited information because it does not shed light on the adjustment of the financial sector in periods of inflation.\footnote{To overcome this problem Oudet presented a simple two-equation model of the financial sector (see Appendix B). One equation was a structural bond equation, the other a structural stock (return) equation. These equations were estimated by the two stage least squares technique. Almon lags were used to produce the anticipated inflation rate and the expected end-of-period stock return. Oudet reported estimations of only the structural stock equation and its reduced form. The total sum of the coefficients associated with the past series of stock price ratios was negative and highly significant in both equations. Oudet interpreted this as indicating the "stock market overreacts to new information and then readjusts to a more normal level."\footnote{More important, the coefficients associated with rates of past inflation were significantly negative in the reduced form estimation, leading Oudet to conclude stock returns do not adjust completely to anticipated inflation.}}

The Oudet study was a general equilibrium estimation of the impact of anticipated inflation on stock returns, and therefore was an improvement over the Reilly, Johnson, and Smith studies. But the Oudet paper is subject to several criticisms. First, the model
implicitly assumes the financial markets are separate from all other markets. This justifies treating aggregate income as an exogenous variable. It is an assumption difficult to defend (it may also explain the "wrong sign" for the income coefficient). Second, a savings variable appears in the reduced form stock equation but not in either structural equation or in the algebraic expression of the reduced-form stock equation. What affect it has on the model specification and estimation is unknown. Third, the exogenous expected end-of-period stock return variable is poorly defined, making it impossible to evaluate the meaning of the negative coefficients for lagged values of the variable. The coefficients were significant for the first six quarterly lagged values. If the variable was in fact a stock return measure, then significant lagged values are anomalous evidence regarding the efficiency of the capital markets.

Jaffee and Mandelker [1976] investigated the effectiveness of stocks as inflation hedges by regressing the nominal or the real return on the market portfolio \((i_m, r_m)\) on the actual inflation rate. As should now be clear such models are not robust tests of the behavior of returns on nonmonetary assets. Nevertheless we note that they reported a "split decision." For the period 1875 - 1970, yearly returns on stocks were apparently independent of rates of inflation. However, for the period 1953-1971, the returns were significant and negatively related to the anticipated rate of inflation (see Appendix B).
Bodie [1976] tested the response of the returns to equity to both anticipated and unanticipated inflation. His model was essentially identical to EQ(40) presented above. Numerous OLS regressions were attempted, which revealed the real return on equity to be negatively related to both anticipated and unanticipated inflation, at least in the short run (see Appendix B).

Keran [1971], Hamburger and Kochin [1972], Lintner [1973] [1975], Nelson [May, 1976], and Fama and Schwert [1977], all reported an inverse relationship between the rate of return to common stocks and the anticipated inflation rate (see Appendix B). Branch [1974] regressed the returns to industrial stocks for 22 developed nations against the respective inflation rates and industrial production rates of growth. On the basis of this cross-country analysis he concluded stocks appear to be a partial but not complete long-run hedge against inflation. Stock returns tended to rise in a year by about 45 percent of the inflation rate. Fama and Schwert [1977] disclosed that for the period tested returns to equity were also negatively correlated with unanticipated inflation. They claimed the negative relationship between expected stock returns and the expected inflation rate, "remains an economic enigma, but we cannot as yet reliably conclude that it is evidence of a market inefficiency." Nelson, on the other hand, went further when he concluded,
What is puzzling about the results reported in this paper is the implication that ex ante rates of return on stock may at certain times have been below risk-free yields or, even worse, negative. The first possibility is strongly at variance with modern capital market theory, the second with economic theory generally to say nothing of common sense. If these possibilities are ruled out, then some at least temporary departures from efficiency would seem to be implied. While change, however small, can never be ruled out as the explanation for these results, a really satisfactory explanation is not at all apparent. (p. 482)

Unquestionably, the empirical evidence, if taken at face value, is inconsistent with the hypothesis that common stocks are a complete hedge against inflation. This would not be as surprising if it was the consequence of common stock prices not fully and instantaneously adjusting to unanticipated inflation. But since most of the empirical evidence suggests market rates of return on common stocks do not fully adjust to anticipated inflation, and indeed actually decline, the evidence constitutes a serious challenge to neoclassical economic theory. Yet, as we have previously argued, there are sound reasons why most of the evidence to date should be suspected. In one form or another all the tests were either misspecified or suffered from limited explanatory power (i.e., low $R^2$'s). The criticisms raised in the previous section seem to be even more applicable to the empirical studies just reviewed.

We have argued there are certain reasons why we should not expect a nonmonetary asset, such as common stocks, to be a complete inflation hedge. Differential inflation forecast on the part of various income
earning segments will affect the return to equity holders. Both the debtor-creditor hypothesis and the wages-lag hypothesis imply an elastic response in market rates of return to common stocks from inflation. The income tax laws with regard to the maintenance of capital (i.e., depreciation regulations) and the taxation of capital gains will depress returns to common stocks in inflationary periods. Finally, the uncertainty about the future inflation rate influences the real rate of return, and hence the ability of common stocks to "keep up" with inflation. The single greatest fallacy of the Fisherian Hypothesis - and the hedging issue is simply a rephrasing of this classic hypothesis - is the implicit assumption of risk-neutral investors. Only in a world of certainty or a world of risk-neutral inflation would the nominal price of a capital asset adjust exactly by the amount of the expected rate of inflation.
F. CHANGES IN THE MONEY SUPPLY, RATES OF RETURN, AND THE EFFICIENT MARKET HYPOTHESIS

The Monetarists theory in its "purest" (simplest) form is the Classical Quantity Theory or Cambridge Cash-Balance Theory, in which a change in the stock of money outstanding is transmitted into a proportionate change in the general price-level. Modern Monetarism - the Neoclassical monetary school - says very little which is original compared to its predecessors. However, the modus operandi by which their conclusions are reached is far more robust, and is in fact a contribution to monetary theory in its own right. The key concepts of this structure are: expectations about price changes; the cost of information search and gathering; and the theory of portfolio optimization. With these concepts, Monetarists have been able to construct theoretical explanations of the link between short-run monetary disturbances and long-run equilibrium. The transmission mechanism is the capital market, or more specifically, interest rates. Deviations from the long-term growth rate of the money supply cause relative prices (rates of return) to deviate, thus creating portfolio imbalances. The reconciliation of these imbalances brings about a return to long-run equilibrium.

This section looks at contemporary Monetarist theory as a hypothesis about the behavior of capital markets. Our interest in this is drawn from the dictum of Monetarism, that "inflation is always
and everywhere a monetary phenomenon." This dictum implies "inflation is always and everywhere determined (transmitted) by the operations and efficiency of the capital markets."

We do not concern ourselves with the many issues of debate between contemporary Monetarists and Keynesians regarding the effects of money on the economy and especially on interest rates. We elect to concentrate on the question of the efficiency of the capital markets. That is, do rates of return fully reflect all available information regarding anticipated changes in the money supply? This is a more pertinent question for this study because it concerns the ability of the capital market to properly arrive at ex-ante equilibrium prices, which is another way of saying it concerns the likelihood that the inflation tax is neutral in the Fisherian Hypothesis weak form sense. The degree of capital market efficiency determines the extent to which resources are misallocated as a consequence of inflationary financing.

Wicksell, Keynes, Fisher, and other economists all contributed to the theory of how stock prices respond to changes in the money supply. In more recent times, Sprinkel [1964] [1971] reasserted that changes in the money supply lead changes in stock market prices. Both monetary change and stock prices lead business cycle turning points, according to Sprinkel. However, since monetary changes have a longer lead over business turning points than stock prices, "it follows that monetary change leads stock prices."
Sprinkel arrived at this conclusion by visual examination of money and stock price data. He explained the phenomenon as the product of portfolio adjustments brought about by a disequilibrium producing changes in the money supply, à la Friedman's monetary theory. Palmer [1970] updated Sprinkel's [1964] graphical analyses to 1960-1969 and reaffirmed that money is a leading indicator.

Sprinkel's results are anomalous with the efficient capital markets theory. According to the efficient market theory, the informational content of past money supply changes will be exploited in trading strategies to the point where equilibrium returns fully reflect the information (i.e., fair game). If Sprinkel is correct, the Fisherian Hypothesis is also false, given the monetary theory of the price level assumed above.

As noted in the previous section, Keran [1971] estimated a stock price equation with lagged money growth rates which were found to be statistically significant. This suggests a general principle: Any study which purports that a market rate of return is a function of a lagged money variable is inherently inconsistent with the efficient market (rational expectations) theory.

Reilly and Lewis [1971] updated Sprinkel's [1964] study. They employed both a graphical analysis and a simple OLS regression of the Standard and Poor's 425 Industrial Index on lagged three and six month
moving averages of either the rate of growth in the money supply or the rate of growth in the monetary base. A trend variable was also included as a regressor. Based on the graphical analysis, they concluded, "major and sustained declines in the growth rate of the money supply are followed by stock price declines but false signals are possible." The regression results tended to substantiate the lag in stock price response to money growth rate changes. However, most of the stock price behavior was attributed to the trend variable. Tests of changes in the Standard and Poor's Index confirm the significance of lagged money, although the models explained only about 4 percent of the variation in the index rate of change.

Although Hamburger and Kochin [1972] concluded, "it seems unlikely that the results presented will help one to earn excess profits in the stock market (which is one reason why they are being published)," their empirical results verified a lag in stock price movements to money growth rate changes. In the absence of transactions costs, such a lag is prima facia evidence that excess profits are possible. The Hamburger and Kochin paper was essentially a critical analysis of Keran's [1971] extension of the St. Louis model. They re-estimated Keran's "semi-reduced form" equation after including the long-term bond rate and the price level (Gibson Paradox). The estimated coefficient for lagged money growth changes was even more significant than Keran originally reported. It is unclear, however, how the Hamburger and Kochin equation should be interpreted. Whereas Keran's
"semi-reduced form" merely suffered from misspecification due to excluded predetermined variables, the Hamburger and Kochin equation was subject to simultaneous equation bias because they included the endogenous long-term interest rate.

Homa and Jaffee also observed a significant relationship between stock prices and lagged money supply changes. They, however, attempted to explicitly test what Hamburger and Kochin had surmised, namely that knowledge of money supply changes cannot be exploited so as to earn excess profits. Because their structural stock price equation contained the contemporaneous money supply it was necessary to derive one period ahead money supply forecasts before stock price predictions could be made. Homa and Jaffee tried three money supply forecast models: perfect forecast, naive extrapolation (i.e., extrapolation of the current money supply growth), and regression predictions. Comparisons were made with and without margin trading. The investment yields from these six models were compared to the yields for a simple buy and hold strategy. Only the compounded yields for the naive extrapolation models failed to exceed the buy and hold return. When risk differentials were taken into consideration only the perfect foresight without margin strategy and the regression prediction without margin strategy dominated buy and hold. Malkiel and Quandt [1972] verified these results, but showed that an alternative model based on fiscal rather than monetary variables produced superior forecasts. They also revealed that the forecasts
did very poorly in predicting the percentage change in the actual Standard and Poor's Index. This raises the question of what variable should be forecasted - stock prices or stock returns. This question is best answered after completing the literature review.

While the studies discussed above all revealed a lag in stock price movements to changes in the money supply, implying inefficient capital markets, there is evidence that in fact stock prices anticipate money supply changes; for example, Cooper [1974], Rozeff [1974], Rogalski and Vinso [1977], and Ciccolo [1978].

Cooper was among the first to recognize the implications of lagging stock prices to monetary changes. He combined the simple Quantity Theory (SQ) model with the efficient markets (EM) model. The SQ-EM model states that the money supply is an important determinant of market rates of return but that returns actually lead money changes, since all available information, such as anticipations about future money changes, are incorporated into current returns. Cooper estimated the coherence between money and returns. The spectral results were strongest in the low end of the frequency domain (i.e., longer periods). Cooper interpreted this as consistent with the SQ-EM model, because it implied that the stock prices respond more to the predictable long-run movements in money changes and less to shorter-run fluctuations. Cooper also examined the lead/lag spectrum. The evidence, he concluded, was most consistent with the
SQ-EM hypothesis, with rates of return leading money supply by about one to three months. Cooper did acknowledge the evidence was also consistent with money leading returns by 80 months; a somewhat unlikely event.

Rozeff [1974] raised attention to the fact that the traditional monetary portfolio theory of monetary transmission mechanism conflicts with the efficient market theory. Rozeff conducted a number of tests, including regressions of stock returns on the change in past growth rate of the money supply. The multiple coefficients of determination ($R^2$) tended to be low, consistent with the efficient market theory which requires that the explanatory power of past available data be nil. The $R^2$'s for regressions which included the contemporaneous money supply growth rate ranged from .040 to .104 for the period August 1948 to March 1970. The relationship between stock returns and lagged money growth rates was stronger from 1948 through 1960 than from 1960 to 1970, contrary to what might have been anticipated judging from the numerous studies reporting a threshold effect in the 1960s. The relationship was weakest for weekly data and strongest for quarterly data.

Rozeff also evaluated the profitability of trading strategies based on historical money supply data, paying particular attention to the date when the data became available. The buy-and-hold rule outperformed the trading strategies whether original or revised
available data (at the time of trade) were used, and whether the independent variable was growth rates or changes in growth rates of money. On the other hand, the same type of trading strategies given unavailable data (at the time of trade) outperformed buy and hold by a wide margin - about 12 percent. Rozell thus concluded that the capital market appears to be a "fair game" with respect to the informational content of past money supply members, and that evidence to the contrary was in part attributable to a failure to control for the announcement lag in published monetary statistics. Rozell concluded by stating,

In conclusion, the evidence is inconsistent with the predictive form of the monetary portfolio model and consistent with the efficient market model. For the stock market, the lag in effect of monetary policy is essentially zero. Stock returns do not lag behind growth rates of the money supply, nor would we expect them to do so in an efficient market. Still, changes in stock prices are related to monetary variables as the non-predictive form of the monetary portfolio model suggests. A substantial fraction of current stock price change can be linked to current monetary policy. In addition, an important part of current stock price change appears to reflect stock market anticipations of future monetary growth, a result we would expect in an efficient market. (p. 301)

Rogalski and Vinso [1977] investigated the causal relationship between stock returns and money supply by applying the Granger-Haugh causality test. Granger [1969] advanced a concept of "causality" between variables which roughly stated says that variable X "causes" another variable Y if Y can be predicted with less error from the past
values of X and Y together than from the past values of Y alone, other relevant information also being exploited in the predictions. Haugh [1976] developed a procedure for testing this concept of causality. The series X and Y are first transformed by a filtering rule (e.g., Box and Jenkins [1976]) into series of white noise. The resulting residuals are then cross correlated, with statistically significant cross correlation coefficients implying a causal relationship defined by the particular lag. When Rogalski and Vinso estimated the cross correlations for various measures of stock returns and percentage changes in $M_1$, they discovered that the null hypothesis of stochastic independence between the series could be rejected. Moreover, the results of the tests of the hypothesis that causality runs from the money supply to stock returns was also rejected, whereas the reverse causality hypothesis could not be rejected. Contemporaneous correlation was also reported.

Finally, we note that Ciccolo [1978] also applied the Granger [1969] causality concept to a test of the mechanism by which monetary policy actions and financial disturbances are transmitted to the goods market. Briefly, Ciccolo evaluated the importance of the money supply and "q" as independent sources of variation in economic activity, where "q" was Tobin's ratio of the market price to the reproduction price of real assets. He concluded the evidence supported the simple quantity theory of money supply changes inducing changes in economic activity. Stock prices were found to influence economic activity, but
without feedback. No intertemporal relationship between money and stock prices was detected. Consequently, Ciccolo argued, in order for Friedman to be correct about the transmission of monetary policy actions - through changes in stock prices to economic aggregates - the link between the money supply and stock prices must be very fast (less than one quarter, as quarterly data were tested).

Taken at face value the evidence seems to support whichever opinion one may desire: either stock prices lag money supply changes, or stock prices lead (i.e., anticipate) money supply changes. What has been established is that money and stock prices are interdependent.

In our opinion, the evidence is generally consistent with the efficient market theory. This opinion is based on the criticisms raised above of those studies purporting to have demonstrated a lag response in stock prices. It is also based on criticisms enumerated by others (e.g., Rozeff [1974], pp. 287-299). Pesando [1974] expressed several criticisms against the Keran, Hamburger-Kochin, and Homa-Jaffee models which are worth noting.

1. The risk-return model of capital asset pricing is widely accepted, yet these studies made no attempt to incorporate explicit measures of risk into their analysis.

2. Keran and Hamburger-Kochin relied heavily upon the Almon technique. The Almon variables exhausted several degrees of freedom, such that the explanatory power of the models may reflect the ability of the
extraordinarily flexible lag structures to track the observed pattern of stock prices rather than to capture a stable structural relationship.

3. The explanatory power of the Homa-Jaffee model was dependent upon the incorporation of the very pronounced serial correlation into the estimated equation, indicative of the systematic influence of omitted variables in stock price determination.

4. The models were not stable to alternative structural specifications. Including a distributed lag on current and past rates of growth of real corporate earnings, rather than on the actual earnings levels, resulted in significant changes in the magnitude and sign of the coefficients for the other structural variables.

5. Forecasts of stock price indexes based on post-estimation-sample data proved to be inferior to a naive no-change extrapolation for both U. S. and Canadian data.

There is yet one final criticism which must be noted. This refers to the second and third points above and the question introduced previously in this section regarding the appropriate capital asset value measure - price or rate of return. If the empirical studies differ in no other way, they are clearly divided on this question. Every study which reported results consistent with the Quantity Theory (SQ) used capital asset prices. Those studies supporting the efficient market theory employed the rates of return. The dichotomy suggests the choice of price or rate of return matters. Table II indicates why.
The first column in Table II gives the correlation coefficients for the Standard and Poor's Common Stock Index (price) and lagged rates of change of the money supply. The persistently high coefficients are consistent with the hypothesis that money is a leading indicator of stock price movements. Note, however, that correlations of the rate of change of the money supply on lagged SP index values (column two) support just the opposite hypothesis — stock prices are a leading indicator of money. Clearly, both hypotheses cannot both be valid. The problem is spurious correlation. The correlation coefficients are unreliable because the price series is nonstationary, as column three clearly reveals. This explains why Reilly and Lewis and Pesando found that a dummy trend variable explained most of the variance of stock prices. The rate of change in the SP index yields a homogenous stochastic stationary series, which is generally uncorrelated with money for various leads and lags.

To reiterate, we find the evidence most consistent with the propositions that, (i) the Monetarist theory is correct in principle, that changes in the money supply are a determinant of stock prices (or market rates of return), but (ii) stock prices fully reflect all available information regarding the money supply, as the efficient market theory implies. These two propositions, in conjunction with the Monetarist theory of inflation, jointly imply that stock prices should fully reflect all available information concerning expected
future inflation. This proposition stands in stark contrast to the econometric evidence reviewed in the previous sections. Clearly a reconciliation is demanded.

TABLE II

CORRELATIONS OF STOCK PRICES AND MONEY SUPPLY

MONTHLY: 1952 - 1978
(N = 324)

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<th>( \text{m}_{t-j} )</th>
<th>( \text{SP}_{t} )</th>
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<th>( r_{t-j} )</th>
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\( \text{SP} \) = Standard and Poor's Common Stock Index
\( r \) = Rate of change in \( \text{SP} \)
\( \text{m} \) = Rate of change of the money supply, \( M_1 \)
ESSAYS ON THE INFLATION TAX

by

James C. Dyer, IV

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE

DOCTOR OF PHILOSOPHY

ESSAY II. THE QUESTION OF INFLATION TAX NEUTRALITY: A REVIEW OF SELECTED LITERATURE...(Continued)

ESSAY III. THE INFLATION TAX AND THE THEORY OF THE FIRM

HOUSTON, TEXAS

APRIL, 1981
G. THE DEBTOR-CREDITOR HYPOTHESIS

The previous sections of this essay are in some sense a preliminary to the question of whether, as is commonly believed, debtors gain at the expense of creditors during a period of inflation. The concept of a wealth transfer between debtors and creditors, is referred to as the "Debtor-Creditor Hypothesis." The concept of inflation as a tax is, of course, just a special case of the debtor (i.e., government) - creditor (i.e., private sector) hypothesis.

The debtor-creditor wealth transfer is frequently regarded as one of the principal sources of excess burden from inflation. The objective of this section is to clarify the connection between interest rate behavior with respect to changes in the general price-level and the debtor-creditor hypothesis. In addition, we examine the literature which has dealt explicitly with the issue of whether the debtor-creditor hypothesis can be empirically verified.

Although we have challenged the methodological basis for the majority of the studies of the behavior of interest rates (or rates of return generally) to inflation, it must be acknowledged that in the aggregate the evidence contradicts the Fisherian Hypothesis. In other words, $\beta_1$ in EQ(8-a) has been shown to be significantly different from unity.
EQ(8-a) \[ i_t = \beta_0 + \beta_1 E_t(\rho_{t+1}) + \epsilon_t \]

The question is, What implication does this have to the debtor-creditor hypothesis? The answer to this question ultimately depends on the accuracy of the inflation forecasts - ignoring the arguments raised above as to why EQ(8-a) is misspecified. If the inflation forecasts are rational expectations, which is to say all available data which are relevant to assessing the level of future prices are incorporated into the forecast, then a failure of the Fisherian Hypothesis implies the capital market fails to correctly incorporate information into the equilibrium interest rate. If \( \beta_1 \) is less than unity, the market interest rate understates the "true" cost of money. Assuming subsequent realized inflation exactly equals the expected inflation rate, a wealth transfer transpires from creditors to debtors.

The problem with this scenario, however, is that it implies a paradox. On the one hand the market generates rational and accurate forecasts of the future inflation rate, but on the other hand it is incapable of translating this forecast into the equilibrium interest rate. This paradox is the consequence of attempting to deduce market behavior from a "reduced form" equation such as EQ(8-a) which treats the expected inflation rate as if it is determined in a homogeneous "market." This obscures the fact that the equilibrium market interest rate embodies heterogeneous inflation rate expectations.\(^1\)
A debtor-creditor wealth transfer will be observed if the two
groups form differential inflation forecasts with one group making
biased forecasts. This is best explained with the help of Figure III
which presents demand and supply schedules for bonds. Suppose at time
one neither debtors nor creditors anticipate any future inflation.
The equilibrium quantity of bonds is $B_0$ with market interest rate
$i_0$. Subsequently, debtors, who by assumption have superior
information about future inflation, revise their inflation
expectations upward. The equilibrium price of bonds should fall to
$1/i^*$. However, since creditors do not anticipate any inflation there
is a differential in expectations which can be exploited for gain by
debtors. Bonds sold at any price above $1/i^*$ are overpriced, given the
expected rate of inflation (i.e., the real rate is below the
equilibrium rate). Debtors are, therefore, induced to increase the
supply of bonds. The incremental supply will depend on the price
elasticity of bond demand, but is certainly bounded by $B_1^-$. Any
supply in excess of $B_1^-$ would raise the real cost of funds to the
debtors above the equilibrium rate. With transactions costs, $B_1^-$ is
inefficient because when creditors subsequently adjust their bond
demand (to $D_1$) debtors will be forced to enter the market and
repurchase the excess bond supply ($B_1^- - B_0$), moving from $\Omega$ to $\Omega^*$. 
Supply will not exceed $B_1^-$, so that the market rate of interest will
be less than or possibly equal to the Fisherian Hypothesis rate.
Consequently, $\beta_1$ in EQ(8-a) will be less than or equal to one.
FIGURE III
Debtor-Creditor Hypothesis
Suppose bond supply expands to $B_1$. In the subsequent period creditors observe prices rising and adjust their inflation forecasts to equal that of debtors. The higher expected inflation rate causes creditors to adjust their portfolios with the demand curve shifting down to $D_1$. The Fisherian Hypothesis now holds, assuming no new information causes an inflation expectation revision. There has, however, been a wealth transfer from creditors to debtors equal to the shaded area in Figure III. We assume bond contracts can be renegotiated each period so there is no compound effect.

A similar story holds if creditors are assumed to have superior knowledge about future prices. In this event when inflation expectations are revised upward, the demand schedule shifts down as creditors require a lower bond price (higher yield) to induce them to buy the same quantity of bonds. The demand schedule will shift down where $\tilde{B}_1$ bonds clear the market. At any point such as $\tilde{\Omega}_1$ creditors would pay too much for bonds. Creditors have every incentive to exploit their inside information by forcing bond prices down below $1/i^*$. Consequently, $B_1$ in EQ(8-a) will be greater than or equal to one.

While both cases assumed complete ignorance by either creditors or debtors of a change in the expected future inflation rate, it should be clear that precisely the same qualitative results are achieved for any case of heterogeneous anticipated inflation rates. The
differential in inflation expectations causes the Fisherian Hypothesis to fail. Abstracting from measurement errors, $\beta_1$ less than unity implies a wealth transfer from creditors to debtors. For the same assumptions a $\beta_1$ greater than unity implies a wealth transfer from debtors to creditors. In either case, the market (i.e., collective) anticipated inflation rate is biased. This is what Kessel [1956] had in mind when he stated,

Clearly the heart of this particular explanation of wealth redistribution is the assumption that interest rates fail to reflect completely price level changes during inflation. The debtor-creditor hypothesis is based on the postulate that interest rates reflect an implicit biased estimate of the future course of prices. It is because this estimate is assumed to be low that the conclusion - debtors gain and creditors lose during inflation - follows. (p. 128)

While a $\beta_1$ unequal to unity implies a debtor-creditor wealth effect, $\beta_1$ equal to unity does not necessarily imply neutrality. The Fisherian Hypothesis is a necessary but not sufficient condition for inflation neutrality. This principle has already been demonstrated in this essay. In the case at hand, the Fisherian Hypothesis can hold at each point in time, but contracts for streams of future payments fixed in nominal values change in real value whenever inflation forecasts are imperfect or inflation forecasts are revised during the contract period. In either instance, a debtor-creditor wealth transfer occurs. This point is perhaps best explained by numerical example.
Suppose at time 0 a bond is offered which will pay $100 at the end of each year for the next 10 years and a principal payment of $1,000 in ten years. The coupon interest rate is 10 percent. Assume repayment is guaranteed, and that the bond market is separate from all other markets. At the time of issuance the market rate of interest for all similar bonds is 10.25 percent and the expected inflation rate for each of the next 10 years is 5 percent. It follows, therefore, that the equilibrium real rate of return is also 5 percent. Since the required rate of return is slightly greater than the coupon rate the bond sells at a discount for $984.80. Table III summarizes these facts. In all cases the expected future inflation rate $E_t(\rho_{t+j})$ is variable but equal for all future periods.

At the end of the first period the market does not revise its inflation forecast. The actual first year inflation equals the beginning of the period forecasts. The yield to maturity remains 10.25 percent. The period market rate of return ($i^*$) and the market yield since maturity ($i$) are both 10.25 percent. The equivalent real returns are the equilibrium 5 percent. These return concepts are defined by Eq(43).

Now suppose as expected the inflation rate in the second period is 5 percent, but that for whatever reason the market comes to anticipate a 6 percent annual inflation rate forever. In accordance with the Fisherian Hypothesis, the market reassesses the equilibrium market
EQ(43-a) \[ i_t^* = \frac{(B_t + C_t - B_{t-1})}{B_{t-1}} \]

EQ(43-b) \[ r_t = i, \text{ such that: } B_0 = \sum_{j=1}^{t} \frac{c_j}{(1+i)^j} + \frac{B_j}{(1+i)^t} \]

EQ(43-c) \[ r_t^* = \frac{[B_t + C_t - (1+p_t) B_{t-1}]}{(1+p_t) B_{t-1}} \]

EQ(43-d) \[ r_t = r, \text{ such that: } 0 = -B_0 + \sum_{j=1}^{t} \frac{p_0}{p_j} C_j (1+r)^{-j} + \frac{p_0}{p_t} B_t (1+r)^{-t} \]

where, \[ B_t = \text{value of one bond at time } t \]
\[ C_t = \text{coupon payment at time } t \]
\[ p_t = \text{general price level at time } t. \]
### TABLE III
Rates of Return and the Debtor-Creditor Hypothesis:
An Illustration

<table>
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<tr>
<th>Time</th>
<th>$B_t$</th>
<th>$E_t(p_{t+1})$</th>
<th>$\rho_t$</th>
<th>Yield To Maturity</th>
<th>Period Rates of Return</th>
<th>Average $\rho$</th>
<th>Yield Since Issuance</th>
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<td>.05</td>
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</tbody>
</table>

255
rate of interest at 11.3 percent. Since the bond sold at time 0 only earns 10.25 percent the demand for it declines driving the price down to $933.81, at which point the old bond also has a yield to maturity of 11.3 percent. The nominal rate of return to the bondholder during period two is only 4.88 percent and the real return is actually negative. There has been a wealth transfer from the creditor to the debtor. The change in expectations imposes a differential exogenous inflation tax on the capital market. The debtor can now repurchase the bond at its current market price with the real cost since issuance of only 2.53 percent (per year) — nearly half the original contracted real rate.

In the third period inflation expectations are unchanged and the actual inflation rate equals the anticipated rate. The period nominal return equals the yield to maturity so there is no further debtor-creditor wealth effect. The Fisherian Hypothesis holds.

In the fourth year the actual inflation rate is 7 percent whereas only a 6 percent increase in prices was expected. The unanticipated inflation does not (by assumption) induce an inflation forecast revision. The demand and supply for the bond are unchanged so the price is unaffected. Yet there is an additional wealth transfer from the creditor to the debtor. This occurs because the debtor can repurchase the bond with dollars which are less valuable in purchasing power than either the debtor or creditor anticipated at the beginning
of the period. An expected decline in the purchasing power of money was embodied in the value of the bond and hence the yield to maturity. The unanticipated inflation cannot be offset by higher future coupon payments since the coupon rate is fixed. The result is a windfall gain to the debtor which must simply be absorbed by the creditor. Yet, the Fisherian Hypothesis continues to be observed.

In reality the bond market is not separate from other markets, hence it is quite conceivable there would be product substitution caused by the inflation forecast error. The Fisherian Hypothesis would not be verified.

In period six inflation expectations are lowered so there is a wealth transfer back to creditor from the debtor, as witnessed by the period real return which is greater than the assumed equilibrium rate of 5 percent. Although at this point the yield to maturity equals that originally expected, the real yield since issuance is still less than 5 percent so there remains a net creditor to debtor wealth transfer.

In period nine both the expected future inflation rate is lowered and there is an unanticipated decline in the actual inflation rate. The consequence is the same as in periods two and four, but with the wealth effect in favor of the creditor.
Finally, the illustration has been designed so that at the end of the tenth year the average actual inflation rate ($\bar{p}$) just equals the original inflation forecast. Accordingly, the nominal yield since issuance is 10.25 percent and the real yield is 5 percent. There is of course no net debtor-creditor hypothesis. In no way does it alter the fact that there were debtor-creditor wealth transfers.

We are now in a position to draw certain valuable conclusions regarding the debtor-creditor hypothesis.

1. The Fisherian Hypothesis is a necessary but not sufficient condition for the impossibility of the debtor-creditor hypothesis (i.e., neutrality). Debtor-creditor type wealth transfers occur as a result of unanticipated inflation or unanticipated changes in the expected rate of inflation.

2. A change in the expectations of future inflation induces lenders and borrowers to adjust their portfolios thereby changing the value of claims to streams of future payments fixed in nominal value. This wealth effect is a transfer from lenders to borrowers if future inflation rate expectations are raised. The opposite is true for a downward adjustment of expectations.

3. Unexpected inflation results in a change in the real return since issuance of claims to streams of future payments fixed in nominal values, and hence to a debtor-creditor wealth effect. Unexpected inflation does not, however, alter the current market price of such claims.

4. Assuming the anticipated one period ahead inflation rate always equals the anticipated inflation rate for any future period, and that the claim to a stream of future payments fixed in nominal values is multiperiod, then a change of one percent in the expected future rate results in a greater change in the period real rate of return ($r^*$) and the real
yield since issuance ($\bar{r}$) than a one percent error in
the expected inflation rate (expectations unaffected).

These conclusions and the example upon which they are based
critically depend on the inflation forecast generating mechanism. If,
for example inflation rate forecasts were generated by some adaptive
expectations structure, then the expectations error (i.e.,
unanticipated inflation) would enter into the forecast and the current
price of a claim to a stream of future payments fixed in nominal
values would be altered. In this event, conclusion four would not
necessarily hold.

The debtor-creditor hypothesis is inconsistent with the efficient
market theory. A systematic misassessment of the process generating
price-level changes is inconsistent with the efficient use of
information by the capital market. This point was made by Fama
[1975], Hess and Bicksler [1975], and Rozeff [1977]. If the capital
markets are segmented then it is possible for each submarket to fully
utilize all available information it possesses, and yet a
debtor-creditor effect still be observed. What is required is
monopolistic control of information in at least one submarket. A
similar concept of segmented capital markets is found in the
literature on the term structure of interest rates. In that theory,
lenders and borrowers are said to confine themselves to certain
segments of the yield curve for a variety of reasons, including the
cost of information. This causes investors to specialize in market "segments." In the case of the debtor-creditor hypothesis, capital markets are assumed to systematically misassess the process generating price level changes, either because all information is not costlessly available to all individuals, or because the information is available but not incorporated into models of equilibrium price determination.

Finally, we note as did De Alessi [1963], that the debtor-creditor hypothesis does not deny that factors other than inflation may affect wealth positions of firms and individuals. What the hypothesis does assert is: in any period in which such factors are randomly distributed with respect to the net monetary position, a correlation will be observed during an inflationary period between the net monetary position of the economic entities and their relative changes in wealth (assuming imperfect inflation forecasts). The traditional interpretation of the debtor-creditor hypothesis is that the correlation will be positive. We now examine selected studies which have attempted to test such propositions by, so to speak, going "behind the Fisherian Hypothesis."

Several examples have been cited in this study of authors claiming a wealth transfer takes place between debtors and creditors during a period of inflation. Other instances of authors merely assuming a debtor-creditor hypothesis can be found in Willis and Chapman [1935], Meyer [1954], Harris [1959], and Scitovsky and Scitovsky [1964]. The
latter relied heavily on the Bach and Ando [1957] study discussed below. The Willis and Chapman thesis is particularly interesting. They held that inflation is a wealth transfer process which has nothing to do with money and banking, and that even the idea of a price-level is artificial. As they concluded,

Perhaps the most general statement which may thus be offered as a result of the entire study, is the fact that what is properly termed inflation is a problem not, as commonly supposed, of money and credit, but of production and distribution of wealth, is a process by which the distribution and use of purchasing power on the part of the community is altered. Inflation occurs when artificial alteration is brought about in the incomes of the several members of the community, or when they are induced to apply these incomes in a fashion radically different from that in which they were previously employed. (pp. 206-207)

Harris [1959] is a good example of merely assuming the popular beliefs about "who gets hurt" by inflation. Other studies, such as those by Pesek [1960], Brownlee and Conard [1961], and Nichols [1968] assume the debtor-creditor hypothesis in the design of their models.

The accounting profession, especially the American branch of this profession, has for several years considered the problem of restating financial statements for inflation. The early studies were primarily concerned with the measurement issues associated with a change in the purchasing power of the dollar - the unit of measurement. See, for example, Sweeney [1936], American Accounting Association [1951], R. Jones [1955] [1956], Mason [1956], American Institute of Certified
Public Accountants [1963], Gynther [1966], and Accounting Principles Board (A.I.C.P.A.) [1969]. Traditionally the accounting profession has assumed the debtor-creditor hypothesis in its approach to the measurement of general price-level gains and losses. No consideration has been given to the possibility of interest rates or other prices incorporating expected future inflation rates (the Fisherian Hypothesis). Reluctantly, I must include myself in this criticism; cf., Dyer [1973].

More recently, several studies have questioned the decision-making relevance of price-level restated financial statements; cf., Dyckman [1969], Rosenfield [1969], Peterson [1973] [1978], Morris [1975], Heintz [1973], Davidson and Weil [Jan.-Feb., 1975] [May-June, 1975] [March-April, 1976] [1976], Kaplan [1977], and Basu and Hanna [1978]. As might expected, the evidence is mixed. Test results have been reported indicating the capital markets do not respond to the information provided by inflation adjusted earnings numbers. Such evidence could be consistent with the efficient market theory. However, it may also indicate the inability to define investment decision models, or a lack of understanding of restated data.

Kessel [1956] offered one of the first attempts to explicitly test the debtor-creditor hypothesis. His study was followed by Alchian and Kessel [1959] which corrected Kessel's original paper for alleged measurement errors. The first study focused on the rate of return
to the common stocks of banks. Kessel showed that banks were typically net monetary creditors (i.e., monetary assets exceed monetary liabilities), and that the real value of their shares declined during the World War II inflation. Together both facts support the contention that a wealth transfer from creditors to debtors transpires during an inflationary period. No theory was offered by way of explanation for why banks would rationally elect to construct their balance sheets so as to suffer a decline in real value of stockholders' equity. Logic would suggest banks have a significant loss function associated with inflation forecast errors.11

Alchian and Kessel [1959] studied the stock prices of all the industrials with common stock traded on the New York Exchange at any time between 1914 and 1952 and similar firms from the American Stock Exchange for 1933-1952. They found that while individual firms apparently do not frequently change their net monetary status, the distribution of firms by net monetary debtor (or creditor) had "changed spectacularly" since 1914. During the inflationary subperiods net monetary debtors earned a higher equity rate of return than net monetary creditors. During deflations the net monetary creditors did better than the net monetary debtors. Alchian and Kessel concluded the evidence is consistent with the hypothesis that inflation is basically a tax on creditors in favor of debtors.
Rozeff [1977] convincingly demonstrated the Kessel-Alchian studies are unreliable because they failed to account for risk. Rozeff maintained that the differential returns earned by debtor equities are the result of systematic risk differentials. This point, also noted by Hong [1977], rests on studies which had demonstrated a positive covariance between, (1) a firm's systematic risk and its financial leverage, and (2) a positive covariance between a firm's systematic risk and its operating leverages. The net monetary position should be related to the operating and financial leverages, hence the beta coefficient of a stock (systematic risk) will impound the firm's net monetary position. It follows that debtor firms will have higher betas than creditor firms, which explains the rate of return differentials.

Perhaps the best known and most frequently cited study of the debtor-creditor hypothesis was by Bach and Ando [1957]. They are also remembered for their contribution to the study of the wages lag hypothesis, along with Alchian and Kessel, which we explore in the next section. Bach and Ando tested four propositions related to the redistributional effects of inflation. The third proposition was the debtor-creditor hypothesis (we review the other three in later sections). They claimed the total dollar value of creditors' claims wiped out by inflation between 1939 and 1952 was around $500 billion (in 1952 prices). However, their calculations took no account of inflation expectations built into the interest rate. Hence they
overstated the loss to creditors. This is the same error accountants commit in their computation of the gains and losses from general price-level changes.

Bach and Ando examined the distribution of the gain from the inflation induced loss to creditors' real wealth. They computed the net monetary position at various dates for households, unincorporated businesses, nonfinancial corporations, financial corporations, and the government. Households were found to be the major net creditor sector, while the government was the principal net debtor; from which they concluded that inflation had transferred real purchasing power primarily from the household sector to the government. After adjusting for the fact that taxpayers, conceived as responsible for the public debt (as indirect debtors), benefit from the decline in purchasing power associated with interest and principal payments on government debt, Bach and Ando concluded inflation does not drastically redistribute purchasing power among income groups via its effect on the government debt (if the debt is refunded).

Bach and Ando paid special attention to the possibility of a debtor-creditor effect for nonfinancial corporations. They applied tests similar to Alchian and Kessel's on a sample of 52 companies. Surprisingly, however, Bach and Ando found that the debtor-creditor status was a weak factor in the determination of the return on investment and only marginally more significant for common stock price
changes. They noted several possible factors which could account for the discrepancy between the two studies, none of which need be repeated here.

Bach and Stephenson [1974] re-evaluated the Bach and Ando [1957] study. Their main concern was with the redistributitional effects of inflation on the ownership of real and monetary assets. They asserted that over the quarter century between 1946 and 1971 the total dollar value of creditors' claims wiped out by inflation could have reached $1,200 billion (assuming no interest adjustment for anticipated inflation). Like Bach and Ando before them, Bach and Stephenson were interested in determining who gained from the wealth transfer. They verified the previous findings and concluded,

Clearly, households have consistently been massive net creditors. The main offsetting debtor was governments, with different types of businesses accounting for the rest of net debt. Thus, since World War II inflation has apparently caused a massive transfer of wealth from households, as the major net creditors, to the federal government, as the major net debtor, and more recently to business firms. (p. 4)

To assess the debtor-creditor hypothesis more accurately, Bach and Stephenson compared the behavior of a sample of nonfinancial corporations over the period 1952-1970. In addition to the basic debtor-creditor hypothesis, they also tested a modified version which classified firms according to their net monetary position plus depreciation. The objective was to correct for the loss in real
income to the firm because corporate taxes are assessed on accounting income which does not correct for replacement cost depreciation. Bach and Stephenson discovered the increase in stock values and rates of return on book investment were generally consistent with the debtor-creditor hypothesis. Not surprisingly, the results were strongest when the sample sets were restricted to firms which did not change debtor-creditor status over the sampled period. However, when they corrected for systematic risk, Bach and Stephenson could not substantiate this result. They computed the systematic risk (beta) coefficient for each stock and tested the residuals or "excess returns." No significant differences in returns was detected.

Once the beta coefficients are removed, there are no appreciable differences between positive- and negative-exposure companies. This suggests that the effects of inflation are being picked up completely by the beta coefficient, as is not surprising. It would normally be expected that companies with large net debtor positions and large depreciation exposure would be more susceptible to cyclical swings than those with smaller or negative exposure to inflation. (p. 10)

In spite of this evidence and other tests which suggested that the differences in returns were neither statistically significant nor consistent in sign with the theory as between debtors and creditors, Bach and Stephenson concluded that inflation apparently benefitted nonfinancial corporations as net debtors. By way of explanation, they suggested that inflation effects are reflected on income statements and balance sheets by accepted accounting practice only with
substantial lag so that stock prices could either fail to reflect the effects or capture the effects only over a very long period - "security markets may be inefficient." This explanation reflects a gross disregard for the voluminous literature available even in 1974 which indicated the capital markets do not respond to mere accounting numbers, but rather derive equilibrium rates of return on the basis of expected future economic income.

Other authors have concluded that debtor firms tend to earn superior returns compared to creditor firms, for example De Alissi [1963], Conard [1964], and Gonzalez [1973]. Still others, notably Hong [1977], and to a lesser extent Bradford [April 1974], argue that inflation is anticipated in dealings between borrowers and lenders. Two of these studies, specifically De Alessi's and Hong's warrant our attention.

De Alessi derived the following equation (in our notation):

\[ \frac{V_t}{V_0} = \frac{P_t}{P_0} = \beta_0 + \beta_1 \left[ p_t \frac{\text{NMA}_t}{V_t} \right] + \epsilon_t \]

where, \( V \) = market price of the firm's common stock adjusted for stock splits and dividends,

\( P \) = general price level,

\( \text{NMA} \) = estimate of the net monetary asset position of the firms.
The left hand side of EQ(44) is a measure of the real rate of return to common equity. Consequently, $\beta_1$ should be zero if inflation is properly anticipated. The alternative hypothesis assumed the greater the degree of net monetary position (NMA/V) the more nominal wealth the firm will gain as a result of inflation. De Alessi applied five different (but not independent) statistical tests to British companies for the period 1948 to 1957. The results, he concluded, were consistent with the debtor-creditor hypothesis, and thus corroborated Alchian and Kessel's findings.

Apart from the fact that De Alessi failed to account for risk differentials, his conclusion is simply not supported by his empirical findings. Indeed, it is hard to imagine how the paper ever passed an editorial review. De Alessi applied his statistical tests to two different industries and to each of the nine years in the sample period. In all, 144 tests were conducted. Only the significance levels for the cases which, according to De Alessi, rejected the null hypothesis ($\beta_1=0$) were reported. A total of only six cases were significant at the standard 5 percent level. Another seven tests were significant at the 10 percent level. The average reported significance level was 24 percent. De Alessi accepted a rejection level as high as 49 percent. The disturbing aspect of this is that De Alessi is frequently cited as an authoritative work proving the debtor-creditor hypothesis.
Bradford [January 1974] examined the effect of the U. S. corporate tax law on firm earnings during a period of inflation. He restricted the analysis to depreciation effects. The hypothesis was the value of the firm declines the larger the depreciation charge (i.e., the more capital intensive the firm the greater the depreciation-inflation burden). The hypothesis was empirically verified. Bradford concluded that the Kessel-Alchian analysis on monetary assets can be "strengthened" by the consideration of income tax laws in relation to depreciation. Whether the word "strengthened" is appropriate is questionable, since Bradford failed (repeatedly) to establish a statistically constant differential between the rates of returns to debtor and creditor firms.

Hong [1977] estimated cross-sectional regression equations of average monthly rates of return on the systematic risk of the firm (beta), net debtor position, fixed assets, and inventory cost understatement per unit of inflation (the last three as ratios to the book value of the firm). He found no evidence to support the debtor-creditor hypothesis. Instead, a wealth transfer from firms to the government was detected. This transfer occurs because of U. S. tax law. When the fixed asset (i.e., depreciation) and inventory cost variables were omitted from the estimation, the net debtor position coefficient became negative and statistically significant.
This clearly reveals the sensitivity of debtor-creditor hypothesis tests to misspecification error. The systematic risk coefficient was systemically significant.

What can we conclude then about the debtor-creditor hypothesis. The literature does not have a consensus opinion. The popular belief that debtors gain at the expense of creditors during inflationary periods is by no means as supported by empirical fact as the popularity of the belief would imply.15 No study has satisfactorily accounted for both the general equilibrium effects of inflation and the economic (e.g., Mundell Effect), institutional (e.g., U. S. corporate tax law), and psychological (i.e., risk) aspects of inflationary finance. Consequently, notwithstanding the vast literature on interest rate behavior to inflationary conditions, it can be said that the debtor-creditor hypothesis is yet to be properly tested.

The debtor-creditor hypothesis is a statement about differential inflation expectations. It is inconsistent with a neutral inflation tax. The hypothesis is also inconsistent with the efficient market theory. Moreover, the debtor-creditor hypothesis is contradictory to the spirit of the Fisherian Hypothesis. However, the Fisherian Hypothesis can be observed even in the event the debtor-creditor hypothesis is valid. This is because the Fisherian Hypothesis is a necessary condition for neutrality of the inflation tax, but it is not
a sufficient condition. On the other hand, the debtor-creditor hypothesis is sufficient for non-neutrality. Consequently, tests of the debtor-creditor hypothesis, while not entirely independent of the tests of interest rate behavior in inflationary periods, do provide additional evidence of the inflation tax issue.
H. THE LABOR MARKET AND THE WAGES LAG HYPOTHESIS

In the introduction to this essay we remarked that the theoretical principle embodied in the Fisherian Hypothesis is not limited to interest rates. We elected to develop the conceptual issues by examining the interest rate literature simply because the study of the association between interest rates and the price level has had a long and controversial history. We have endeavored to clarify the meaning of the Fisherian Hypothesis. It has been argued that the diversity of results regarding the neutrality of the inflation tax is attributable to the failure to properly account for the four primary determinants of the incidence of inflationary finance: the direct burden (i.e., Mundell Effect), expectations, inflation uncertainty, and general equilibrium relations. In addition, a fifth factor was identified, namely institutional factors, such as the U. S. corporate tax law regarding depreciation. We now turn to the labor market. The question of concern: Is the Fisherian Hypothesis a valid statement regarding the relationship between the rate of change in wages and the expected inflation rate. Just as in the case of interest rates, the answer to this question depends on the endogeneity of the inflation tax, which in turn stems from the efficiency of the markets.

In this section we first explore the literature which purports to demonstrate that wages lag prices. We show how, properly interpreted, this evidence is inconsistent with the Fisherian Hypothesis. Although
this literature no doubt was chiefly responsible for the popular belief that inflation robs the wage earner, the empirical techniques adopted seriously limit the reliability of the results. Again it is a failure to adequately control for the five determinants of inflation tax effects that undermines the generality of the wages lag studies. Next we examine a sample of studies which have refuted the wages lag prices hypothesis. Having established the commonly accepted opinion (that wages do not keep up with inflation) is by no means indisputable, we turn to a brief review of the role the wages lag hypothesis has played in labor market theory. The issue of course is the Phillips curve. Our purpose is to show how the existence of a trade-off between inflation and unemployment arises out of non-neutrality of inflationary finance - but the source of the non-neutrality is crucial. Of primary concern is whether the labor market is characterized by differential expectations in which one element, say employees, form systematically biased inflation expectations.

H.1 A History of the Wages Lag Hypothesis

The wages lag hypothesis, expressed symbolically as EQ(45), has a long history, but it is most commonly associated with two contemporary economists, Hamilton and Mitchell.
EQ(45) \[ \dot{w}_t - \dot{\eta}_t < E_t(\rho t+1) \]

where, \( \dot{w}_t \) = rate of change in the market wage rate,

\( \dot{\eta}_t \) = rate of change in the real wage rate.

Hansen [1925] stated the wages lag hypothesis most distinctly when he concluded,

It is evident that price fluctuations have a powerful influence on the distribution of the national income. The marginal product of each factor is no doubt the normal rate of returns to each. But this normal distribution is constantly being upset not only by changes in the relative supply and efficiency of the various factors, but also by the redistribution of income accompanying price changes. If prices rise, the benefit accrues immediately to the entrepreneurial class. In time, however, competition among entrepreneurs compels them to bid up the prices paid for labor, land and capital until the surplus profits are absorbed. In the meantime, however, a fresh rise in prices has created a new margin of profits above costs. Entrepreneurs, naturally, do not pass on these gains to the other factors until compelled to do so by the pressure of competition. It is therefore inevitable that in rising-price periods wages and other costs should lag behind prices. Rising prices amount in fact to a redistribution of the national income in favor of the entrepreneurial class. It amounts to an enforced taxation of wage-earners, salaried persons, investors and landlords with long-term rent contracts. (p. 40)

Hamilton was certainly the most prolific and adamant proponent of the wages lag hypothesis. His principal studies, [1928], [May, 1929], [August, 1929], [November, 1929], [1935], [November, 1936], [1936], [1937], [1942], and [1952], constitute a life's work in the
analysis of European wage and price behavior from the middle fourteenth century to 1800. All the quantitative price and wage data were taken from the accounting books and financial records in the private, ecclesiastic and public archives of the countries studied. The Spanish data were from the kingdoms of Valencia, Aragon, Navarre, and New Castile. The period spanned the recovery of American treasure, and hence a period of autonomous increases in the money supply, through several decades of war. The French and English data covered a shorter period and were less detailed.

In [May, 1929] Hamilton outlined the process by which American gold and silver were imported into Spain from 1503 to 1660. The influx of precious metals into Europe, Hamilton maintained, "precipitated one of the greatest price revolutions occurring on a specie basis in modern times, if not in all history." This "price revolution" was the primary focus of most of Hamilton's empirical analysis.

In [August, 1929] Hamilton published what was perhaps the most rigorous econometric study of his career. The issue was the wages on Spanish treasure ships. He compared the time series behavior of index numbers of prices in the Andalusia region and of the wages of landmen and seamen. He devoted considerable attention to the issue of the seamen's food subsistence, attempting to demonstrate that the subsistence remained a uniform quality and quantity. Homogenous and
constant subsistence was interpreted by Hamilton as an indication seamen's real wages were fixed, especially in that no trade-off between real wages and incremental employment was possible at sea. Nominal wage rates were fixed for the periods at sea, which lasted from seven months to more than a year. The index numbers of wages and prices clearly revealed a decline in the real wages of seamen, and a decline in wages relative to the same craftsmen on land.

Hamilton concluded his study of seamen wages with the observation that the general lag in wages behind prices augmented profits and was probably the greatest single factor in the rise of modern capitalism. This theme was elaborated in subsequent works, especially in [November, 1929]. Based on a time series comparison of prices and wages in England and France, Hamilton concluded that,

In England and France the vast discrepancy between prices and wages, born of the price revolution, deprived labourers of a large part of the incomes they had hitherto enjoyed, and diverted this wealth to the recipients of other distributive shares. As has been shown, rents, as well as wages, lagged behind prices; so landlords gained nothing from labour's loss. For a period of almost two hundred years English and French capitalists - and presumably those of other economically advanced countries - must have enjoyed incomes analogous to those American profiteers reaped from a similar divergence between prices and wages from 1916 to 1919. (p. 355)
Hamilton explored the "profit inflation" interpretation of the wages lag hypothesis in further detail in [1942]. The data were commodity prices and money wages in London (1729-1800) and New Castile, Spain (1729-1800). Hamilton again demonstrated that prices were higher than wages leading him to conclude,

By keeping the normal rate of profit far above the prevailing rate of interest, the lag of wages behind prices stimulated the investment of savings as they took place. According to one estimate, in 1740-1800 the supply of capital in Great Britain exclusive of land, more than quintupled. (p. 263)

Felix [1956] argued Hamilton misread the evidence, and that contrary to Hamilton's opinion, profit inflation generated by lagging wages and rising prices was not the stimulus to capitalist development. Felix interpreted history in nonmonetary terms. Capital accumulation, he said, resulted from reinvested profits which arose from increases in productivity. The observed decline in real wages was attributable to rising agricultural prices and major increases in labor supply. The most obvious criticism of the profit inflation thesis, according to Felix, was the fact that there is no correlation either between inflation and profit inflation or between the rates of profit inflation and the apparent rates of industrial growth.²

Demographic and institutional influences on the supply of wage labor, which occurred largely independent of inflation, seem to account for much of the wide discrepancies between price and profit
inflation. Regarding the profit inflation in France, Hamilton relied on studies by Sabrousse, who according to Felix did not accept the wages lag hypothesis, but rather attributed the decline in real wages to population growth and relatively mediocre production increases. In the case of Spain, Hamilton did not clearly describe any economic rationale for why profit inflation should be given credit for the economic gains. Indeed, Hamilton [1947] even acknowledged the lag in wages behind commodity prices apparently resulted from the near doubling of the Spanish population in the eighteenth century and from mass migration into urban districts. 3 Felix explained the English profit inflation as a statistical artifact of Hamilton's price index, which when decomposed showed little or no profit inflation in industry.

If, as appears to be the case, capital in the technically advancing industries came predominately from reinvested profits, it was more likely the great increases in productivity rather than rising prices which accounted for the expanded profits. If real wages fell during the Industrial Revolution, it was because of rising agricultural prices, conjoined with a major increase in wage labor supply from population growth and enclosures. Hamilton's price index, heavily loaded with agricultural items, has misled him as to the chief beneficiaries of the rising prices. The classical economists who made favorable agricultural terms of trade the focus of their concern were not wrong in their assessment of the facts. (p. 457)

Hamilton's empirical studies are easily subjected to the host of criticisms raised in the previous sections regarding the methodology of testing interest rate behavior with respect to inflation. Apart from the fact that, as Kessel and Alchian [1960] noted, Hamilton
generally did not explicitly control for factors affecting wage rates other than inflation, his evidence itself was far from conclusive. In the case of the Valencia, Aragon, and Navarre data, wages lagged prices only for specific and sometimes relatively short periods. Moreover, Hamilton frequently drew his conclusions from analyses of turning points in charts of prices and wages. Such casual empiricism simply cannot detect complex lead/lag relationships, as is demonstrated later in this section. Yet if we ignore the effects of real factors, such as employment, labor productivity, technology, and capital intensity, Hamilton's evidence in the aggregate does point to the inability of labor to maintain its real income during inflationary periods.

Mitchell [1903] [1908] evaluated wage and price series in the North for the period of the U. S. Civil War. He used the same casual empirical analysis as did Hamilton. He concluded the rise in wages was subsequent, not prior, to the rise of prices, thus refuting a wages cost-push theory of inflation. Mitchell [1908] outlined the response pattern as follows,

The process of change was attended by serious friction, but the final chart shows that despite this friction the price-system displayed remarkable flexibility. Wholesale prices lagged behind gold, to be sure; retail prices lagged behind wholesale prices, cost of living behind retail prices, and wages behind cost of living. (p. 282)
Mitchell interpreted his data as revealing an inefficient labor market in the 1860s and 1870s. Lerner [1955], in his study of the Confederacy, likewise concluded real wages fell as a consequence of inflation. However, Kessel and Alchian [1959] offered an alternative interpretation of the evidence. The observed fall in real wages can be explained, they said, as the consequence of three nonmonetary phenomena: (1) the outbreak of war between the North and South which curtailed or eliminated previous trading relationships; (2) special economic characteristics of the base year (1860); and (3) the tax system (excise and import duties) used to divert resources from the private to the public sector. The first factor was thought to be the most important. The outbreak of the Civil War essentially destroyed the triangular trading relationship among England, the North, and the South. The foreign exchange earned by the sale of cotton in the South provided the basis for North-South trade, and in turn the means of settlement for imports to the North. After the war began, the North abandoned the gold standard in favor of an inconvertible paper standard and a floating exchange rate, thus solving the balance of payments problem which had arisen. But the election to finance the war by printing greenbacks resulted in a fall in the exchange value and a rise in the relative price of imports. Hence, when Mitchell computed the real wage rate by a price index (such as the wholesale price index) which included foreign purchased goods, the real wage was observed to decline. Therefore, Mitchell's results did not necessarily imply an inefficient labor market - one characterized by
money illusion. The imputed decline in real wages was greater with respect to internationally traded goods than it was with respect to domestic goods. The wage lag hypothesis, however, makes no allowance for such differentials. Moreover, the fall in real wages was a statistical artifact of the finance system chosen by the government to balance its war budget. As Kessel and Alchian point out, if the inflation tax had been replaced by a tax on income or wealth, Mitchell's real wage measure would have been unaffected. In short, real wages did decline, but as the consequence of the war.

The U. S. Bureau of Labor Statistics published a series of studies at the turn of the century dealing with the standard of living. Rubinow [1914] raised serious objections to the Bureau's technique for constructing price indexes. The majority of Rubinow's study dealt with the construction of wage and price indexes. He identified several factors which altered real wages other than inflation. He also emphasized the distinction between wages and income. The income of the wage-worker's family can increase, Rubinow argued, with no increase in return for the labor of the individual wage-worker. Smaller families, rapid development of woman labor, and increases in employment of married women all contribute to income. These factors, all of which are as relevant today as they were in Rubinow's time, alter both family incomes (or expenditures) and the characteristics of a labor market equilibrium.
Rubinow's empirical evidence revealed wages did lag general price-level changes, for the period 1890-1912, but real wages did not necessarily decline.

In years of falling or even slowly rising prices, the American wage-worker was able to hold his own or to improve his condition to a slight extent. But when confronted with a rapidly rising price movement (accomplished as it was by a violent growth of profits), the American wage-worker, notwithstanding his strenuous efforts to adjust wages to these new price conditions, notwithstanding all his strikes, boycotts, and riots, notwithstanding all the picturesque I. W. W.-ism, new unionism, and the modish sabotage, has been losing surely and not even slowly, so that the sum total of economic progress of this country for the last quarter of a century appears to be a loss of from 10 to 15 per cent in his earning power. (p. 813)

We suspect the fact that real wages were observed to fall during accelerating inflation could be attributable to inflation uncertainty. If the uncertainty about the future inflation rate is positively covariant with the rate of inflation, then labor may seek to offset this uncertainty by trading inflation equivalent wage increases for job security. This job security could manifest itself in several ways, such as in a rise in unionism and a larger share of women in the work force. We elaborate on this possibility later in this section (also see Section E of the first essay).

Fairchild [1916] also examined the question of how to measure the standard of living. He suggested the early studies of the wages lag hypothesis were in fact studies of the standard of living (his
definition of "standard of living" was essentially what today would be called "discretionary income"). Fairchild constructed a budget of necessities, as distinct from "culture wants" (i.e., luxuries). He then determined the cost of this "standard budget" at two points in time and deducted the costs from income at the two dates. The remainder, that is the "expenditure for culture wants," as a ratio of income constituted a relative measure of the "standard of living." This was done for the "average" family (father, mother, three children, and $600 income). The standard of living index was 15.4 percent in 1890, but declined to 14 percent by 1908. Fairchild concluded that, although the data do not prove the common laborer's family was better off in 1890 than 1908, the evidence was consistent with that hypothesis.

It is intuitively appealing to think that a decline in the standard of living must imply a failure of wages to keep up with prices. We have, however, already noted ways in which the two are dissimilar. Still it is enlightening to set out the conditions under which a decline in a measure of the standard of living, such as Fairchild's, implies wages lag. The necessary conditions are quite stringent, as the following proof reveals. The first assumption is that all income is received as wages. The second is a nonsatiation assumption. Savings can be treated as a special consumption good without loss of generality. The third assumption precludes relative price changes. Given a Fairchild type standard of living definition,
it is entirely possible for the first assumption to hold at all times and yet the standard of living index change solely as a result of the implicit consumption weights applied to relative price changes. Assumption four implies there is no employment change. Assumptions five and six fix real expenditures and the standard budget. Given these assumptions, a change in the standard of living is determined completely by the difference between the rate of inflation and the rate of change in wage rates.

Assumptions, all with respect to the representative household (h).

(1) $Y_t = W_t$

where, $Y_t = \text{income}$
$W_t = \text{wages, } w_tL_t$
$L_t = \text{labor quantity (i.e., man hours)}$

(2) $Y_t = U_t + N_t$

where, $N_t = \text{total expenditures on necessaries}$

$$N_t = \sum_{j=1}^{m} p_{j,t} \check{c}_{j,t}$$

$U_t = \text{total expenditures on luxury goods and services, (unnecessaries)}$

$$U_t = \sum_{j=1}^{m} \left[ p_{j,t} (c_{j,t} - \check{c}_{j,t}) \right]$$

$\check{c}_j = \text{necessary good } j.$

(3) $\frac{dp_{j,t}}{dt} \frac{1}{p_j} = \rho_t \forall j$
(4) $L_{t+1} = L_t$

(5) $c_{t+1,j} = c_{t,j} \forall j$

(6) $\bar{c}_{t+1,j} = \bar{c}_{t,j} \forall j$

Definition

$$\alpha_t = 1 - \frac{N_t}{Y_t}, \text{ standard of living.}$$

Proposition

$$\alpha_{t+1} - \alpha_t \quad \frac{\Delta w_t}{w_t} \quad \frac{\Delta Y_t}{Y_t} \quad \rho_{t+1}$$

Proof

By assumptions (2) and (3):

$$Y_{t+1} = \sum_{j=1}^{m} (1 + \rho_{t+1}) p_{t,j} \bar{c}_{t+1,j}$$

$$+ \sum_{j=1}^{m} (1 + \rho_{t+1}) p_{t,j} (c_{t+1,j} - \bar{c}_{t+1,j})$$

By assumptions (5) and (6):

$$Y_{t+1} = (1 + \rho_{t+1}) Y_t$$

Substituting into $\alpha_t$:

$$\alpha_{t+1} = 1 - \frac{(1 + \rho_{t+1}) \sum_{j=1}^{m} p_{j,t} \bar{c}_{j,t}}{(1 + \rho_{t+1}) Y_t}$$
By assumption (1):

\[ a_{t+1} = \frac{\left(1 + \rho_{t+1}\right) \sum_{j=1}^{m} p_{j,t} c_{j,t}}{w_{t+1} L_{t+1}} \]

Let \( \beta_{t+1} \) be defined as:

\[ \beta_{t+1} = \frac{w_{t+1} L_{t+1}}{w_{t} L_{t}} - 1 \]

By assumption (4): \( 1 + \beta_{t+1} = \frac{w_{t+1}}{w_{t}} \)

Therefore:

\[ a_{t+1} = 1 - \frac{(1 + \rho_{t+1}) \sum_{j=1}^{m} p_{j,t} c_{j,t}}{(1 + \rho_{t+1}) Y_{t}} \]

Therefore:

\[ a_{t+1} - a_{t} \leq 0 \iff \beta_{t+1} > \rho_{t+1} \]

Jones [1917] criticised Rubinow's relative price indexes. He revised both the wage and price series and presented evidence confirming the decline in purchasing power over the sampled period (1890-1912). The real wage index was computed as the nominal wage index divided by the concurrent actual retail price index, in a fashion similar to that of all the wage studies reviewed thus far. The Fisherian Hypothesis as applied to wages, however, would have the nominal wage index divided by an index of expected future prices. Expectations were ignored in the early empirical literature on the
wages lag hypothesis. But this does not destroy the validity of those
tests. Wage rates were rarely fixed for more than a year during the
periods studied. As unionism expanded, the length of contracts did as
well, but never to the point where the time horizon for expectations
of the future inflation rates would exceed three years. Therefore, a
one year ahead expected future inflation rate forecast would not
seriously misstate the actual forecast period. In this event, a
significant decline in the average real wage rate computed from the
concurrent actual inflation rate is consistent with the failure of the
Fisherman Hypothesis analogue for wages.\footnote{5}

Douglas and Lamberson [1921] also raised objections to the
Rubinow-Fairchild-Jones price indexes, but their revision of the
series did not alter the general conclusion reached by the previous
authors. Douglas and Lamberson were interested in the effect of World
War I on real wages. They used an adjusted Rubinow price index to
evaluate the behavior of wages from 1890 to 1918. They concluded the
purchasing power of an hour's wages was 20.7 percent less in 1918 than
it had been during the years 1890-1899. The decrease, however, was
consentrated almost wholly in the two periods 1908-1912, and
1916-1917. From 1912 to 1916 money wages not only "held their own"
but actually gained slightly on prices. Real wages again rose in 1918.
The Douglas and Lamberson study played an important part in the progression from the "standard of living" era to the contemporary problem of identifying and estimating the specific proposition that wages lag prices. A close examination of their data adds still further evidence of a wage lag.

If wages consistently lag prices, then the real wage should increase when the price level declines. The Douglas and Lamberson price index fell in 1892, 1894-1896, 1911, and 1915. In each case the purchasing power of wages per hour increased. In addition, over the twenty-nine year period evaluated, the average annual increase in wages per hour was 2.8 percent, whereas prices rose by the statistically significant higher average rate of 7.3 percent. Together these facts suggest the decline in real income during and shortly after World War I was the product of a systematic bias in the labor market. Again the caveat - forces in the labor market other than inflation are assumed constant.

Another important name in the wages lag hypothesis literature is Rufus Tucker [1933] [1934] [1936]. Tucker [1936] took up the issue of the proper indexation of the cost of living in his study of the real wages of artisans in London for the period 1729 to 1935. Tucker explicitly identified a declining real wage as the consequence of lagging wage rates (cf. p. 82). His time series revealed that real wages were constant (1756-1793) or increasing (1847-1897) during
periods when the price level was constant or declining - behavior which one expects if wages consistently lag prices. However, from approximately 1895 through 1935 money wages increased nearly proportionately to the change in the price level, as the Fisherian Hypothesis dictates they should. Moreover, as Kessel and Alchian [1960] have observed, every time Tucker observed a fall in real wages he was able to explain it by real factors such as poor crops, and wars. Yet he ignored these explanations when he drew his conclusions.

The assumption that wages lag prices can be found in numerous other works, for example Fisher [1911, cf. p. 190] [1928, cf. p. 95], Willis and Chapman [1935, cf. pp. 85, 183, 189], Bresciani-Turroni [1937], Holzman [1950], Meyer [1954], Schwartz [1973], and R. Bhatia [1962] who tested Kaldor's [1959] assertion that the rise in money wages depends on the increase in profits of the previous year, which tacitly assumes price increases in the previous period exceeded wage increases. For U. S. data (1935-1959) Bhatia concluded the response of wages to the level of profits and the changes in profits had been very quick. The computed wage adjustment period in the post war period was only about two months. As we remarked in a previous section, Bresciani-Turroni [1937] proposed a special version of the wages lag hypothesis based on the observation that employees are generally creditors of their employers. As creditors, employees lose to their employers during an inflationary period in the same manner as financial creditors allegedly lose to debtors. We do not expect this
debtor-creditor version of the wages lag hypothesis to be a significant economic factor, except perhaps for long-term, nonindexed contracts.

Certainly there is an abundance of evidence supporting the proposition that real wages have not always and everywhere risen — for this is technically all that can be legitimately concluded with respect to most of the studies just reviewed. We have indicated ways in which such data can also be interpreted as being consistent with the hypothesis that wages lag behind prices. This necessitates that all determinants of the market wage rate, other than inflation, be constant. If inflation uncertainty is ignored, this is equivalent to assuming the marginal product of labor is inelastic. It is not a question of whether the marginal product of labor is itself a function of the (expected) inflation rate. The simple manner in which the real wage rate is computed insures that if for whatever reason the marginal product of labor declines coincidently with a rise in the (expected future) inflation rate, then the computed real wage rate will be observed to decline in purchasing power. The problem is one of ascertaining causality. The fall in the real wage rate so computed cannot logically be ascribed to inflation. Clearly, the only defensible test of the wages lag hypothesis is one that estimates the elasticity of the market wage rate with respect to the expected future inflation rate, having fully adjusted for changes in the determinants of the marginal product of labor.
H.2 Endogeneity of Inflation and Wages

Having examined a sample of the literature which argues wages lag prices, we now turn to a brief review of those works which have refuted the wages lag hypothesis. Our purpose is to put into perspective a hypothesis generally accepted as valid. In addition, we seek to discover possible explanations for this dichotomy in the empirical literature, and by so doing achieve a better understanding of how to express the structural labor market relationships.

Kessel [1956] raised questions about the validity of the wages lag hypothesis, questions which were put to the test in subsequent studies published jointly with Alchian. In "Redistribution of Wealth through Inflation," they reported the correlation coefficient between the ratio of wages expense to stockholders' equity and relative stock price changes was not significant for a sample of 113 firms listed on the New York Stock Exchange (1940-1952). A more exhaustive analysis of the same data was presented in Kessel and Alchian [1960]. According to the wages lag hypothesis, they argued, the greater the wage to stockholders' equity the larger should be the rise in equity values as a result of inflation. They found, however, that the average equity rise was greater the lower the wage to equity ratio. After accounting for the positive correlation between net monetary status and increases in equity values, Kessel and Alchian reported the partial correlation coefficient was only -.09, thereby rejecting the
wages lag hypothesis. Yet in their review of the effects of inflation, Kessel and Alcian [1962] argued that anticipated inflation does lead to a decline in wages (and other factor prices). This is not a manifestation of market imperfections (e.g., money illusion), but a consequence of the direct burden of the inflation tax (i.e., what we have called the Mundell Effect, cf., Section C). Their explanation was essentially the same as what we described above as the general equilibrium adjustment to the inflation tax with mobile and elastic factors of production.

A. Conard [1959] applied ordinary least squares multiple regression to the annual wage and salary increase as a function of the change in real output, the change in real output per unit of labor, the change in unemployment, and the average wholesale price index. Because he used the price index (level) his model was more nearly a test of the Gibson Paradox (cf., Section II.D.3) as applied to wages instead of the Fisherian Hypothesis. Nonetheless, Conard signifies one of the first attempts to rigorously treat productivity and employment influences. His model only explained about 14 percent of the variance of wage changes, so it is of little value to us. Conard did, however, speak to the question of wage and profit shares of income, commenting that the evidence supports a cyclical pattern, with wages and profits alternating in capturing short run gains, but moving together secularly.
Bodkin [1966] studied the effects of changes in prices (of finished goods) and changes in output per man-hour on changes in money wages in manufacturing (U. S. 1900-1957). He examined monthly and quarterly time series and concluded the lag of wage changes behind price level changes is very short - "so short in fact that quarterly data conceal it." Using annual data Bodkin estimated the coefficient for the current period inflation rate at 1.193. This is consistent with the Fisherian Hypothesis for wages, assuming further that inflation rates are generated by a random walk. Rees and Hamilton [1967] critiqued Bodkin's study. They applied different functional forms to Bodkin's data and in the process verified a significant and nearly unitary elastic relation between the percent change in wages and prices.

Cargill [1969] applied spectral analysis to U. S. and British wage and price data. Almost all the low frequencies (long run) exhibited significant coherence values for U. S. data, but the majority of the phase estimates were not significantly different from zero (i.e., no lead or lag patterns between the wage rate and inflation rate series). For the high frequencies there was virtually no significant phase lag. The evidence from the 1849-1935 period London time series had several significant coherence values and phase shifts in the low frequencies. The same was true of the building trades of southern England time series. For both sets of data the high frequencies were absent of significant phase shifts. This is an intriguing result, for
it is consistent with the existence of a long run Phillips curve but no short run trade-off between inflation and unemployment - just the opposite of the natural rate theory. We will shortly clarify the relationship between the wages lag and Phillips curve theory.

Whereas Cargill estimated the lead/lag relationship between wages and prices in the frequency domain, Mehra [1977] applied a similar technique to the time domain. He first computed (ad hoc) distributed lag models of quarterly U. S. inflation rates and industry wage rates (1954-1970). He then applied the Granger causality test (see Section F). This is equivalent to regressing the residuals from one distributed lag equation on a distributed lag of the other variable. The null hypothesis of no causality is identified by zero coefficients in the second equation. Mehra's empirical results were mixed, but he nevertheless concluded the tests of causal patterns supported a bidirectional feedback structure between money wages and consumer prices. This is prima facie inconsistent with the unidirectional pattern implied by the wages lag hypothesis.

A close examination of Mehra's results, however, leads to a different interpretation. The bidirectional feedback was strongest at the aggregate manufacturing level. At the individual industry level the F-values and the size of the distributed lag coefficients on the future values of money wages clearly implied, as Mehra acknowledged, that the interperiod lag distributed coefficients of the industry
money wages were dominated by a consistent influence pattern from consumer prices. In other words, knowledge of the consumer price index improves the forecast of the industry money wages by more than could be made on the basis of the lagged industry wages alone - and that by definition is causality.

The income share studies, as we shall call them, which refute the wages lag theory were an outgrowth of the standard of living studies of the early 1900s. The most important of these were Radice [1935], Bach and Ando [1957], Brimmer [1971], Budd and Seiders [1971], Bach and Stephenson [1974]. The Hollister and Palmer [1969] study of the impact of inflation on the poor can be added to this list, although the income variable in their analysis was broader than just wages (see for example pp. 20-24).

In general, the income studies report the wages and salaries share of income did not decline over long periods of time which included inflationary subperiods. In addition, most report the profits share of income declined over the same period. As we remarked above regarding the early standard of living inquiries, income share numbers are not sufficient to either establish or reject the wages lag hypothesis, at least not without stringent and for the most part unrealistic assumptions about the multitude of factors determining the labor and product markets. Some would even argue that shares are meaningless because of output effects. Nevertheless, it is
instructive to review the income share studies so to give a balanced view of the empirical evidence regarding the fate of labor's purchasing power during inflation.

As we noted in Section G, Bach and Ando summarized four basic propositions regarding the redistributional effects of inflation, the first of which was that inflation redistributes real purchasing power from those whose incomes rise more slowly as a result of inflation (e.g., wage earners) to those whose incomes rise more rapidly (dividend and profit earners). The data for the period 1939 to 1952 revealed the labor share of total personal income rose by 6 percent. Corporate profits before taxes, however, also increased, although by only 3 percent. Interest income was the overall loser. A curious feature of their figures, which Bach and Ando apparently failed to notice, is the negative covariance between labor share of income and corporate profits (before taxes and after inventory valuation adjustments). Negative covariance is expected if profits lead wages during the onset of higher expected inflation, with wages subsequently adjusting to both the higher realized inflation rates and the lost real wages.10

Bach and Stephenson [1974] updated the earlier study. They found wages and salaries as a percentage of national income rose by 6.6 percent over the period 1950 to 1971, as did interest income. Corporate profits, on the other hand, declined by 6.2 percent.
Moreover, the shift from profits to wages and salaries was concentrated during the strongly inflationary subperiods: 1950-52, 1955-57, and 1965-71. Their conclusion warrants repeating,

Of course, the fact that income shares changed as described during inflation periods does not demonstrate that the changes were caused by inflation. Perhaps they would have occurred anyhow in a noninflationary world, or perhaps the shifts would have been in opposite directions. But if inflation tended to shift income distribution in accord with the conventional wisdom, these inflationary pushes were so weak as to be overwhelmed by other forces. (p. 2)

Contrary to what Radice [1935] wanted to demonstrate, judging from his numerous normative remarks regarding the equity of inflation induced shifts in wage and salary incomes, Radice's data failed to establish that the wage and salary share of income systematically declines with inflation. The proportion of income from profits and dividends did gain at the expense of the proportions earned by labor and by the holders of long-term contracts between 1914 and 1917, but those were war years. From 1914 to 1920 there was no significant decline in wages and salaries, although Radice did argue salaried employees lost to wage earners. Between 1923 and 1928 this pattern was reversed, which led Radice to conclude,

changes in the distribution of labor income, as between wages and salaries, are due to general causes affecting the supply and demand of different types of labor rather than to monetary causes such as the various types of inflation here considered. (p. 252)
Brimmer [1971] reported U. S. wages and salary incomes as a percentage of personal income rose between 1947 and 1968, dividend share remained roughly constant, but proprietors' income share declined - all seemingly contrary to the wages lag hypothesis. Less aggregated figures did reveal income shifts consistent with the hypothesis. The relative real wages of a factory worker failed to grow between 1961 and 1965 when real disposable income grew by an annual rate of 4.6 percent. Brimmer attributed this to two nonmonetary sources: a rapid increase in the number of multi-earner families and a more rapid occupational upgrading of the employed labor force (due to excess demand for labor).

Finally, Budd and Seiders [1971] simulated the impact of inflation on the value and share in real income, for a model based on predetermined price adjustment coefficients. The wage and salary coefficient was set at 1.00 based on their regressions of the share of wages and salaries in total income (details were not reported). Budd and Seiders claimed they ran the regression for the total economy and for different sectors with varying hypothesized lags. The results, they said, failed to support either the wages lag hypothesis or the alternative wages lead hypothesis.
H.3 Conclusions from the Literature Review

What can we conclude as between those expositions supporting the proposition that wages lag price changes, and those studies just examined along with others (e.g., Douglas [1926] and J. Conard [1964]) which have not disclosed a consistent bias in market wages? Certainly nothing definite regarding the validity of the Fisherian Hypothesis as it relates to the labor market. In this respect we are no more able to make empirically defensible statements about the endogeneity of the inflation tax than we are based on interest rate tests. Laidler and Parkin's [1975] conclusion is most appropriate, "the only safe conclusion about the wages lag hypothesis must be that it postulates a phenomenon which is certainly not universal, but which may from time to time have happened."\(^{11}\) But this does not imply the literature review was without product. First, we have established that further tests of the wage lag hypothesis are needed. The outcome of such tests cannot be predicted at this moment. Second, we have established a very probable reason for the conflicting empirical evidence, namely the failure to account for the five fundamental factors which determine the effects of inflation: the direct burden (i.e., Mundell Effect), expectations, inflation uncertainty, general equilibrium relations, and institutional factors such as the progressivity of the personal income tax. As we remarked above, failure to control for these factors distorts the estimate of the degree of inflation tax neutrality in the labor market, except in very special and unlikely
cases. Causality as between prices and wages can be deduced from the
time series behavior of computed real wages, as the difference between
the market wage rate and the price level, if and only if all the
determinants of market wage rates (save inflation) are both
economically and statistically independent of inflation. In other
words, failure to control for the sources of inflation effects
distorts the estimate of the weak form Fisherian Hypothesis for the
labor market except in the unlikely case where the strong form
Fisherian Hypothesis holds in all markets.

H.4 Inflation Tax and the Phillips Curve

As is well known, Phillips [1958] presented empirical evidence to
support of his theory that the rate of change of money wage rates can
be explained by the level of unemployment and the rate of change of
unemployment. Phillips explicitly assumed cost of living adjustments
have little or no effect on the rate of change of money wages, except
when the rise in import prices is sufficient to offset the tendency
for increasing productivity to reduce the cost of living. What
Phillips meant by this assumption, we believe, is that the inflation
rate does not enter the wage rate equation separately, except in the
case of a rapid rise in import prices. In Phillips' model inflation
was reflected in the money wage rate, with employment adjusting to
compensate for situations of excess labor demand or supply. Lipsey
[1960] repeated Phillips' analysis but applied more standard statistical techniques to eliminate the econometric problems associated with Phillips' estimations. Subsequently, the original wage change - unemployment relationship was transformed into an inflation rate - unemployment relationship by generalizing Phillips' exception, i.e., prices change whenever wages rise more rapidly than labor productivity.

We do not intend to enter into an exhaustive review of the developments in the theory of inflation and unemployment since the Phillips - Lipsey studies. This has been admirably performed for us by R. Gordon [1976], Laidler and Parkin [1975], and especially Santomero and Seater [1978]. Solow [1975] has a particularly succinct review of the alternative theoretical models of the inflation - unemployment trade-off. The important point for our purposes is the role of inflation expectations as the microeconomic foundation for the Phillips curve. The natural unemployment rate theory contends that the very possibility of keeping unemployment below its natural rate depends on a process of deception, i.e., of deriving higher employment and productivity out of labor than would otherwise be achieved. Labor is assumed to systematically underestimate the future rate of inflation, thus expecting to earn a higher return than subsequently realized. This is depicted in Figure IV.
FIGURE IV

Inflation and the Labor Market
Initially employers fix labor demand according to their expected inflation rate \( E(\rho') \). Labor supply is determined by the market wage rate less labor's expected inflation rate \( E(\hat{\rho}) \). Suppose individuals are risk neutral, maintain no real balances, and that there are no market imperfections (including institutionally caused distortions from inflation). The equilibrium level of employment \( L_0 \) is the natural level of employment, provided \( E(\rho') \) equals \( E(\hat{\rho}) \), and either one is an unbiased maximum likelihood estimate of the true future inflation rate.

Now suppose new information is obtained which calls for an upward revision of inflation expectations. The labor demand schedule (in market wage growth rates) moves out as firms compete for labor, being stimulated by potential excess profit (i.e., higher product prices but unchanged wage rates). If labor has the same information set and the same capacity to take the incremental information and derive rational wage and price expectations, then the labor supply schedule will shift inward to \( L_s \). Higher market wage increases will be required to induce the same labor supply. But assume labor underestimates the future inflation rate. The labor supply price will not increase sufficiently, causing the ex-ante real wage rate to decline. This induces an increase in employment (from \( L_0 \) to \( L_1 \)). Subsequently, when the actual inflation rate occurs labor realizes the "folly of its ways" and demands the growth rate of money wages be raised to \( \dot{w} \). Clearly if the incremental employment is to be maintained...
permanently, the differential in expectations must also be maintained. This implies inflation will become increasingly rapid in any state in which unemployment lies below its "natural" rate; hence the title, accelerationism.

The process described by Figure IV should look familiar by now. It is essentially the same as that described by Figure I and III for interest rates, with the axes simply relabeled. More important, however, it should be clear that the steps from $a_0$ to $a_1$ to $a_2$ in Figure IV map a lagging wage response to the expected future inflation rate. The wages lag hypothesis is a sufficient, although not necessary, condition for a Phillips curve.

We have argued that a distinction must be made between this expectations neutrality of the inflation tax (i.e., endogeneity or weak form Fisherian Hypothesis) and the general neutrality of the tax (i.e., strong form Fisherian Hypothesis). The latter cannot be observed - inflation has real effects. Consequently, a Phillips curve trade-off may appear even if expectations are perfect. As Phelps [1970] has stated ($u^*$ is the steady-state equilibrium unemployment rate),

It is standard doctrine to acknowledge that an "anticipated inflation" does have real effects, especially through their effects upon fiscal and monetary efficiency. If the real rate of interest does not fall by a compensating amount, an anticipated inflation impairs the function of money as a medium of exchange:
Nominal interest rates will be higher under anticipated inflation, and "liquidity" will thereby be reduced. This might affect $u^*$, though it is hard to say in what way. (p. 160)

M. Friedman [1977] among others has suggested that inflation uncertainty will have real effects on the labor market, with the possibility that the Phillips curve could be positively sloped (negatively in Figure IV). It is possible, nevertheless, to simultaneously have both the accelerationist's long run vertical Phillips curve and greater inflation uncertainty. Again consider Figure IV. Suppose both labor and management form the same future inflation rate expectations. In a risk neutral world, the growth rate of market wages rises from $\dot{w}_0$ to $\dot{w}'$, as previously described. But with risk aversion the equilibrium growth rate may rise to say $\dot{w}^*$. Labor may prefer to insure its future income and consequently trade the incremental inflation uncertainty to management. In order to induce management to accept the differential risk, labor will lower its supply price (rate of increase). Assuming inflation expectations are subsequently realized, and abstracting from all other labor market effects, a higher inflation rate is observed but the rate of unemployment remains unchanged. The imputed real wage rate will decline. Wages will behave "as if" they lag prices in general.

This state does not describe a long run equilibrium. Eventually, with prices systematically increasing at a rate faster than wages, real income declines, and hence in turn so does aggregate demand. In
response, management cuts its labor demand (to $L_D^*$). If the inflation uncertainty (insurance) premium remains unchanged (i.e., $w' - w^*$) a new equilibrium will be achieved at $\alpha_4$ with higher employment, as Friedman suggested.

This analysis is by no means as rigorous and complete as it could (as perhaps should) be. The point is merely that the effect of inflation uncertainty on unemployment cannot be determined a priori. For one, we have omitted any consideration of differential relative risk aversion as between management and labor. We have also omitted any consideration of the effects of unemployment insurance. Since businesses do not receive unemployment compensation, it is conceivable that they may attempt to induce labor to share the inflation uncertainty risks, for example by trading risk for no strike clauses. Assume in Figure IV that the utility of unemployment insurance just compensates for the disutility of incremental inflation uncertainty. The labor supply curve moves to $L_s^-$. However, labor demand shifts out to a point less than $L_D^-$, say to $L_D^-$. Equilibrium is established at $\alpha_5$ with $L_3$ labor clearing the market. Hence, the differential employment risk created by unemployment insurance can result in an increase in unemployment during an inflationary period.

The Phillips curve models which have been subjected to empirical estimation are distinguished from most of the wages lag hypothesis tests in that the former tends more so to control for factors
affecting labor market equilibrium. The usual form of the Phillips curve is given by EQ(46), where \( X_t \) is a vector of variables which influence labor supply and demand prices. Recall that unemployment, \( U_t \), is endogenous and hence EQ(46) is a semi-reduced form. No conclusions about causality can be drawn from it.

\[
\text{EQ(46)} \quad \dot{w}_t = f(U_t, X_t) + \beta E_t(\rho_{t+1})
\]

The estimated coefficient \( \beta \) is the crucial term. The natural rate (accelerationist) - Fisherian Hypothesis implies \( \beta \) equals one. For \( \beta \) less than one, a long run trade-off between inflation and unemployment is said to exist. We return to this interpretation shortly.

The majority of tests of EQ(46) have rejected the null hypothesis that \( \beta \) equals unity. Reuber [1968] concluded the empirical estimates which have been made are less in disagreement than is sometimes suggested - in the long run prices and wages may not adjust fully to expectations, and in any event the adjustment process is very slow (perhaps on the order of thirty years). R. Gordon [1970] estimated \( \beta \) at .453. In subsequent works Gordon obtained various values for \( \beta \) depending on the structural form of EQ(46): R. Gordon [1971], .354 to .466; R. Gordon [1972], .183 to .705. Perry [1964] described the range for \( \beta \) as between .288 and .532; with \( \beta \) for 1953-1960 equal to .680, but \( \beta \) for 1948-1953 negative (-.025). Perry [1970] confirmed
the overall relation \( \beta = 0.385 \). Brechling [1968] derived an EQ(46) type Phillips curve from a political decision model and achieved 0.386 for \( \beta \).

Eckstein and Brinner's [1972] estimated \( \beta \) coefficient was 0.244. However, they included a threshold inflation (dummy) variable which was not significantly different from unity. This term tends to distort the meaning of the expected inflation variable. Turnovsky and Wachter [1972], used the Livingston inflation expectations data and several models of the inflation expectations mechanism. The range of \( \beta \) over all these models was from a low of 0.427 (simple distributed lag) to a high of 0.720 (error adjusted expectations). De Menil and Bhalla [1975] re-estimated the R. Gordon [1972], Perry [1970], and Eckstein and Brinner models with a constructed inflation expectations series derived from the household survey data collected by the Survey Research Center of the University of Michigan. The tests were too numerous to describe here, but in general the hypothesis that \( \beta \) equals one was rejected.

Treynor [1975] discovered a negative relationship between the annual change in the inflation rate and various unemployment measures, which conflicts with the (long-run) natural unemployment rate neutrality theory. However, the model did not control for any other variables which affect unemployment and prices. Finally, Lucas and Rapping [1969] estimated an alternative form to EQ(46). In their
model the unemployment rate was the dependent variable, while inflation and the wage rate were independent variables. According to the natural rate theory, the coefficient on the (expected) inflation rate should be zero. Instead, they found a significant negative relationship. A detailed analysis of their model led Lucas and Rapping to conclude the Phillips curve (for the U. S.) has not been stable, that it may exist in the short run, but probably does not hold over the long run. (Recall Cargill's [1969] suggested just the opposite.)

A few studies have reported $\beta$ equal to unity. For example, see R. Gordon [1977], Vanderkamp [1972], Turnovsky [1972], and Wachter [1976]. Then there are the "mixed results" studies such as Sargent [Brookings 2, 1973] and Lucas [1973] (see Santomero and Seater [1978] for others). Lucas found the behavior $\beta$ varied across countries - the more volatile the price in the country the more likely nominal income changes will be associated with equal and contemporaneous price movements with little discernible effect on real output. Sargent's [Brookings 2, 1973] evidence, in the aggregate, pointed to $\beta$ less than unity, although some evidence was consistent with the vertical Phillips curve theory (see pp. 460-461). In his conclusion Sargent raised the very important point that the failure to confirm the natural unemployment (accelerationist) hypothesis in estimation does not imply policy makers should attempt to stabilize or stimulate the economy through inflation.
On the one hand, one test of the natural unemployment rate hypothesis above - which is the model's centerpiece - points to rejection of that hypothesis and seems to imply some scope for policy makers to influence the mean of the unemployment rate via a suitable policy rule. On the other hand, I am aware of no evidence that shows that any particular existing structural model embodying a specific alternative to the natural rate hypothesis can outperform it in predicting the course of the unemployment rate. Such evidence ought to be in hand before it is reasonable to believe that economists know enough to design policies that can affect the expected value of the unemployment rate. (p. 464)

There are two points we wish to raise regarding the empirical tests of the Phillips curve. The first is that most of the models have been single, semi-reduced form equations. Therefore, they are incapable of accurately capturing the multi-market adjustments to a change in the expected future inflation, adjustments which arise in response to the direct burden of the inflation tax. This point has been amply defended elsewhere in this and the previous essay, so it need not be repeated here.

The second point concerns the bias in most tests of EQ(46) in favor of $\beta$ less than one. The bias is the result of the expectations scheme used to generate $E(\rho)$. The most common scheme is a distributed lag in past inflation rates, with constrained coefficients (e.g., all greater than or equal to zero). As a general rule, the authors do not report whether the structure leads to systematically incorrect expectations, thus conflicting with the Axiom of Consistency. The problem is that a once and for all shift in the prices (with fixed
distributed lag coefficients) **always** implies permanent change in the unemployment rate. This is incompatible with the natural rate hypothesis which postulates that anticipated once and for all jumps in any order difference of the price-level have no permanent effect on the unemployment rate.\(^{15}\) This is equivalent to a linear restriction that the sum of the coefficients equal zero. Rational expectations implies no such restriction. Sargent [Brookings 2, 1973] has addressed this problem. The solution he offered was the method adopted in most of the efficient market literature. He supposed the unemployment rate is generated by a general mixed autoregressive, moving average representation,

\[
E(47) \quad u_t = \sum_{j=1}^{k} \lambda_j u_{t-j} + \sum_{j=0}^{n} \alpha_j \epsilon_{t-j}
\]

where \(\epsilon\) is a serially uncorrelated normally distributed random variable. The natural rate hypothesis, Sargent argued, permits \(\epsilon_t\) to be correlated with values of endogenous variables dated \(t\) and later, but requires \(\epsilon_t\) to be uncorrelated with past endogenous and exogenous variables,

\[
E(48) \quad E(t-1(\epsilon_t | \phi_{t-1}) = 0
\]

where \(\phi_{t-1} = \) set of variables upon which the expectation is conditional.
Consequently, the value of past information is fully reflected in the expected unemployment rate.

This specification captures the heart of the natural unemployment rate hypothesis, and implies that there is no better way to predict subsequent rates of unemployment than fitting and extrapolating a mixed autoregressive, moving-average process in the unemployment rate itself. This suggests that the natural unemployment rate hypothesis can be tested against specific competing hypotheses by setting up statistical prediction "horse races." (p. 451)

The "horse race" Sargent adopted was to estimate an autoregressive form of EQ(47), then re-estimate the model with lagged (log of) prices and lagged (log of) wage rates, and compute the pertinent F-statistic. The natural unemployment rate hypothesis says the statistic should not be distinguished from zero. As we remarked above, the results were mixed, and depended on the set of elements in \( \varphi_{t-1} \).

H.5 Conclusions

This section has examined the literature on labor market behavior in periods of inflation. We derived no new basic propositions regarding the welfare costs of inflationary finance; but then none were expected. What we have done is to integrate the commonly held belief that wages do not keep up with inflation with the Phillips
curve inflation - unemployment trade-off. Certainly the wages lag hypothesis is regarded as one of the principle social welfare costs of the inflation tax, as we noted in Section C. This opinion is reinforced by the interpretation that the link between the wages lag hypothesis and the Phillips curve characterizes an inefficient labor market, where the market wage rate does not fully and instantaneously reflect all available information.

We have shown the empirical evidence is mixed as to whether or not the inflation tax causes: (1) a systematic depression in the standard of living, or (2) wages to lag prices, or (3) a trade-off between employment and wage changes. Moreover, the tests of the labor market hypotheses are not reliable. A serious deficiency has been the failure to adequately control for changes in the real wage rate caused by social, political, and economic events other than the inflation tax. But the fundamental problem is attributable to the manner in which inflation has been incorporated into the economic theory of labor. Inflation has been introduced into labor market models, but a theory of inflation has not. We have argued that the appropriate theory of inflation is that it is a tax (ignoring pure regulatory features of Federal Reserve policy). The key conceptual components of this theory, apart from the issues of incidence, are: the direct burden of the inflation tax; inflation expectations and market efficiency; inflation uncertainty; and institutional factors which restrict real behavior to nominal measures (e.g., the U.S. personal
income tax laws). Furthermore, it has been suggested that the very nature of an inflation tax necessitates a general equilibrium approach to the identification and estimation of labor market equilibrium.

We do not wish the critical and somewhat negative conclusions in this section to be misinterpreted. The science of economics is generally progressing. It is in fact in the very nature of things that a study such as this critically evaluate prior works in order to draw out those analytical concepts which may prevail over time. What we have demonstrated is that there is nothing conceptually peculiar about the labor market with respect to the costs and effects of inflation. The Fisherian Hypothesis, and all the attendant hypotheses, concepts, and theories, as applied to the capital markets are fully applicable in a general theory of inflationary finance, including a theory of labor market equilibrium.
I. SUMMARY

The classical Quantity Theory, modern monetary theory, the efficient market hypothesis, rational expectations, capital market (investment-production) theory, and uncertainty theory - what do these theories have in common? We have shown the commonality is the assumptions about the processing of information related to future political, sociological, and natural phenomena as they pertain to the supply and demand for goods and services. This is especially true of information about the future course of the general price-level.

The Quantity Theory is a statement about macroeconomic equilibrium. It assumes well behaved and perfect markets. Although faulted for its lack of a coherent microeconomic foundation, it is safe to say the classical Quantity Theory presumes households and firms act "as if" they have complete information about relative prices. As a first approximation, changes in the supply of monetary balances are assumed to be instantaneously translated into equiproporionate changes in the level of prices in general. This neutrality condition implies a competitive equilibrium and is, at the same time, a necessary condition for Pareto efficiency.

The efficient market theory, and its equivalent the theory of rational expectations, are statements about values. If prices fully reflect all available information, then they are unbiased
representations of value. In that case, the information is neutral because all decisions can be made from knowledge of relative prices. Equivalently, the efficiency of the markets implies a competitive equilibrium.

Clearly, if the Quantity Theory is valid, then markets must be efficient. Not that the Quantity Theory is especially important to contemporary monetary theory; but the theoretical interconnection of two seemingly unrelated theories about market equilibrium is very important. Couple this with the fact that the Quantity Theory implies neutrality of inflation (money) in the long-run, and we have an apparent contradiction in logic. The efficient market - rational expectations theory of information neutrality has never been conceptually or empirically limited in its relevant time domain. Assuming information about any future price-level is relevant to the determination of (real value) equilibrium, there will be profit incentives to collect and exploit all information about future inflation rates, consistent with the cost of searching and analysis. Trade decisions based on such information will transpire until all expected abnormal profits to be made from further trades have been bid away. If markets are efficient, then in setting the nominal price of commodities and investments, the markets correctly use all available information to assess the distribution of future (random) inflation rates. But, the neutrality of the information does not translate to neutrality of inflation, except in the long-run, according to the
Quantity Theory. Consequently there is a dichotomy: the Quantity Theory requires efficient markets, and markets which are efficient are so in the short-run as well as the long-run; but the neutrality of inflation is only a long-run effect.

It has been argued in this essay that the inflation tax is not a neutral tax. The neutrality concept promulgated by the Quantity Theory is possible only if the excess burden from the sources side of the inflation tax is exactly equal to the excess benefit (consumer surplus) from the user side of the tax (cf., Essay I, Section B). Put in these terms for the first time, it becomes readily apparent that the Quantity Theory is theoretically incomplete. It follows, therefore, that the neutrality paradox between the Quantity Theory and the efficient markets theory is an artificial paradox, which is to say the premise (inflation is neutral) is false. The legitimate question is when do net-of-tax prices fully reflect changes in the inflation tax rate? As has been demonstrated, this is a question of the dynamics of adjustment. The issue is not the neutrality of the inflation tax, but rather the endogeneity of the tax.

It is not our intention to defend the classical Quantity Theory. Our objective rather is to point out the conceptual link between it and the efficient market - rational expectations theory, a link virtually ignored in the economic literature. The Quantity Theory is a model of market equilibrium. It is the link between the joint
probability density function for prices assessed by the market on the basis of available information, and the equilibrium prices. We do not suggest the Quantity Theory is a valid model of market equilibrium. Its predictive power may be unacceptably poor. Conceptually, however, it is rich, particularly with respect to the concept of inflation neutrality. This essay has explored this concept and these relations.

The Fisherian Hypothesis is a surrogate for the Quantity Theory. The hypothesis asserts that market interest rates change equiproportionately to changes in the expected inflation rates. This "neutrality" condition is identical to the neutrality condition of the Quantity Theory, assuming as we have that inflation is always and everywhere a monetary phenomenon (ignoring pure regulatory features of Federal Reserve policy).

The Fisherian Hypothesis has received the serious attention of the economics profession for decades. In the last several years it has been implicitly the subject of attention in the finance and capital market literature in relation to studies of stock price behavior. This essay has critically, but hopefully constructively, examined this vast literature. We have argued that an analysis of the process which gives rise to the market condition characterized by the Fisherian Hypothesis is essential to an understanding of the costs and effects of the inflation tax.
Strictly interpreted, the Fisherian Hypothesis does not imply the inflation tax is neutral. The possibility of excess burden from the tax remains even if market prices are unitary elastic with respect to the expected inflation rate. On the other hand, a neutral inflation tax does imply the Fisherian Hypothesis. The Fisherian Hypothesis is a necessary condition for neutrality, it is not a sufficient condition. What the Hypothesis does imply is endogeneity of the inflation tax. An endogenous inflation tax has no causal connection; it is determined simultaneously and cojointly with all market prices. The inflation tax is endogenous if and only if the Fisherian Hypothesis is a valid model of market equilibrium.

The works of Irving Fisher were examined above to expose the full depth of his contribution to our understanding of market processes. Clearly Fisher believed his model was simply an idealization. His empirical evidence and observations led him to conclude markets suffer from money illusion.

The Fisherian Hypothesis implies the real rate of return is independent of changes in the expected inflation rate. This proposition was shown to be false. The inflation tax is a tax on the purchasing power of money balances, a non-interest bearing asset. Therefore, since money is mobile and less than perfectly inelastic in supply, substitution effects will be caused by the tax. We referred
to this as the Mundell or Bailey Effect. Judging from the literature to date, one cannot specify a priori the sign of the derivative of the real rate of return to the expected inflation rate.

Empirical tests of the Fisherian Hypothesis must include a model of inflation expectations formation. The most commonly accepted form of this stochastic process is the distributed lag in past inflation rates. This model originated with Fisher himself. We argued the fact that inflation expectations are possibly determined by past inflation rates says nothing about the neutrality of the inflation tax. The neutrality, or more accurately the endogeneity of the inflation tax, is measured by whether the expected future inflation rate is an unbiased estimate of the actual rate. This critical point has been lost all too often in the literature.

The empirical and analytic studies of the Fisherian Hypothesis in its simplest form do not support any strong conclusion about the validity of the hypothesis. Interest rates either fully reflect anticipated future inflation; adjust contemporaneously to changes in inflation rates; adjust with a short lag, or adjust only after several years - all depending on which studies one chooses to believe. The myriad of results may be attributed to three general methodological deficiencies: (1) single equation estimation of inherently a system of equation phenomenon; (2) misspecified models because of omitted variables, especially the Mundell Effect and some measure of the
distribution of information about future prices (i.e., inflation uncertainty), and (3) dichotomous theory of economic choice which treats private choice as myopic except for the effects of the inflation rate. Demand and supply behavior are treated as if determined by expectations about the future inflation rate, but not by expectations of any other economic, sociopolitical or natural events. This last deficiency is conceptually the most serious, for it is indicative of a failure to develop an integrated and congruent microeconomic theory of information and uncertainty.

It is a widely held belief that financial claims to nonmonetary resources are inflation hedges. Common stocks are an example of a nonmonetary instrument. This inflation hedge concept, when properly defined, is merely alternative expression for the Fisherian Hypothesis. The empirical studies of common stocks as inflation hedges were examined and the lack of any consistent conclusion was once again observed. If the laws and theories of science were determined by democratic vote, we would be compelled to conclude, on the basis of the tests to date, that U.S. common stocks are not complete hedges against unexpected inflation and probably not complete hedges against expected inflation. But science is not democratic. Moreover, these tests should not be taken at face value because they have failed to control for the nonmonetary determinants of the real rate of return.
We do not expect nonmonetary assets, such as common stocks, to be a complete inflation hedge, even if markets are efficient in processing all available information about future price levels. This conclusion, perhaps the most significant of this study, is based on the fact the Fisherian Hypothesis is an incomplete model of market equilibrium. Apart from the fact the model does not account for institutional rigidities and other market imperfections, such as U.S. tax laws, it does not fully incorporate the information parameters about future inflation rates. The Fisherian Hypothesis is a valid model of market equilibrium only if there is no uncertainty about future price-level changes, or if there is uncertainty but investors are risk-neutral. Both assumptions are descriptively false. We have concluded the elasticity of the expected rate of return to the expected inflation rate is indeterminant when risk is considered.

Having examined the Fisherian Hypothesis from alternative specifications, the issue was raised again of the effects of the money supply on interest rates and the role of efficient markets in this relationship. Under the assumption that inflation is always and everywhere a monetary phenomenon, a test of the significance of changes in money supply to changes in interest rates is a test of the Fisherian Hypothesis. Critical evaluation of reported tests leads to the conclusion that changes in the money supply are a determinant of rates of return, as the Monetarist Theory contends; but that the process of adjustment is not one of interest rates lagging in response
to changes in the money supply. The evidence is more consistent with the proposition that rates of return fully reflect all available information regarding the money supply. These results are the most supportive of the Fisherian Hypothesis as a model of capital market equilibrium.

In spirit, the debtor-creditor hypothesis is the counter-positive of the Fisherian Hypothesis, as applied to the capital markets. Technically, the Fisherian Hypothesis may be observed even if a transfer of wealth, in the form of lost purchasing power, from creditors to debtors is caused by the inflation tax. This wealth transfer is generally considered to be among the more serious economic distortions caused by the inflation tax. The debtor-creditor phenomenon is the product of differential inflation rates, which make it inconsistent with an efficient market. The empirical tests of the hypothesis have not been conclusive. Although early studies reported results consistent with the hypothesis, more recent works have incorporated contemporary capital market theory into the analysis of the differential returns earned by debtors, and have found no strong support for the inflation induced transfer of wealth. Without question the debtor-creditor hypothesis is far more widely accepted as fact than it should be based on the empirical validation.
The Fisherian Hypothesis has traditionally been associated solely with the capital markets. However, as stated above, it is a surrogate for the Quantity Theory, and hence it applies to all markets, including the labor market. The efficiency of the labor market, as tested by the neutrality of wage rates to changes in the expected inflation rate, has been measured by three distinct but related hypotheses: (1) the wages lag hypothesis, (2) the standard of living hypothesis, and (3) the Phillips curve. Empirical validation of each has produced mixed results. Most wages lag models and standard of living tests reported a less than unitary elastic response in market wage rates to a change in the general level of prices. However, the interpretation of these findings is handicapped because of their failure to control for structural changes in the labor markets during the periods of inflation — changes which, if ignored and if not themselves caused by inflation, give the appearance of less than rational (efficient) price behavior.

Apart from the original work on the Phillips curve, the literature has been more attentive to the specification of the labor market. As is well known, the empirical results have generally found the structural coefficient for the expected inflation rate is positive but less than unity. However, in most instances this result is probably an artifact of the expectations mechanism used to generate E(ρ). More importantly, the Phillips curve, even its most contemporary versions, is only a partial theory of labor market behavior. It is
theoretically incomplete because it does not encompass a theory of inflation. The Phillips curve is neither a structural model nor a reduced form of the process of labor market behavior to inflation expectations, the direct burden of the inflation tax, inflation uncertainty, or institutional factors which restrict real behavior to nominal measures.

The purpose of this essay was to integrate the concepts of various fields of economics as they pertain to the theory of inflation. We did not contrive and test another model of the effects of inflation. The profession has ample empirical as well as analytic studies of inflation to clearly indicate no unified theory of inflation has been forthcoming. We have instead gone "back to basics." Our objective was to critically evaluate the history of inflation analysis in order to extract those analytical concepts which may prevail over time. Likewise, we tested the proposition that inflation is a tax by examining the methodology of earlier works to determine if their results were likely to have been influenced by a failure to control for all the dimensions of the inflation tax. Our conclusion is that the results were most definitely affected. The Fisherian Hypothesis has not, therefore, been tested because the model has not been fully specified. Conceptually, the Fisherian Hypothesis must be false if it is interpreted as a statement of neutrality. However, it may be a good approximation of the endogeneity of the inflation tax. The difference between neutrality and endogeneity being the process and speed of adjustment to a change in the expected inflation tax rate.
FOOTNOTES

B. THE FISHERIAN HYPOTHESIS

1 Technically, as is well known, the exact Fisherian Hypothesis is \((1+i_t) = (1+r_t)(1+E_t(\rho_{t+1}))\) which produces the joint term \(E_t(\rho_{t+1})r_t\) in the linear expression. However, it is customary to ignore this term. As it approaches zero, \(E_t(\rho_{t+1})r_t\) converges to zero before either \(r_t\) or \(E_t(\rho_{t+1})\).

2 Fisher [1911] clearly distinguished between the lag in interest rate changes to inflation and the lead in equity capital price changes. In the terminology of this study, Fisher believed that common stocks are a hedge against inflation.

   The fact that wages, salaries, the price of gold in nonmonetary forms, etc., and especially the prices of bonded securities, cannot change in proportion to monetary fluctuations, means, then, that the prices of other things, such as commodities in general and stocks, must change much more than in proportion. (p. 190)

Fisher, et. al., [1912] elaborated on the advantages to holding equity capital, vis-a-vis bonds, during inflation in How to Invest When Prices Are Rising. Repeated references are made to what we call the Debtor-Creditor Hypothesis, the Wages-Lag Hypothesis (cf. p. 23) and Common Stock Hedge Hypothesis.

3 Fisher [1930], p. 405.

4 See, Fisher [1930], p. 410.

5 I was unable to replicate Fisher's results using the data reported in [1930].

6 Ibid., p. 417.

7 Wicksell [1935], as Keynes after him, attributed the major source of the change in market interest rates to the savings-investment sector rather than to the monetary sector. In Wicksell's model stochastic fluctuations in the real (natural) rate, relative to the money rate, created an inequality between desired savings and investment, resulting
in inflation or deflation. A key factor in the model was the assumption that bankers are slow to perceive changes in the natural rate, a form of irrational expectations and the basis for a debtor-creditor wealth transfer hypothesis.

Dr. Marian Krzyzaniak has pointed out to me an alternative interpretation of Wickseell, namely that disequilibriums occur because of the impossibility of ever knowing the natural interest rate. In such a case, errors in expectations imply nothing about the rationality of the process generating them.

Fisher [1930], pp. 411-416. The sequence of real rates were constructed by subtracting the actual inflation rate from the observed market interest rate.

Ibid., p. 415.

Ibid., p. 425.

In [1925], Fisher claimed, "So far as I know this is the first attempt to distribute a statistical lag." (p. 183). So far as I know, he was right.

Equation EQ(2) is not found in Fisher, but is equivalent to his function described in [1930], p. 420.

Cf. Fisher [1930], "The fourth relationship stated above must be, I think, regarded as an accidental consequence of the other three." (p. 440).

Generally I was unable to replicate Fisher's empirical results, at least exactly. However, verification of the fourth principle was reasonably successful. For Fisher's yearly British (1898-1924) and U. S. (1900-1927) data, the price level explained approximately 83 percent of the movement in market interest rates. The results for British quarterly data were not as robust, but a significant relationship between the price level and market interest rates was detected.

C. INFLATION AND THE REAL RATE OF INTEREST

1 As noted in Section B, Fisher defined the real rate of interest as the market rate of interest less the actual rate of inflation. Assuming inflation rates are generated by a first order Markov process, non-neutrality follows as:

1. \( E_t(\rho_{t+1}) = \rho_t \)

2. \( \sigma^2_r = \sigma^2_i + \sigma^2_\rho - 2\sigma^2_{i\rho} \)

\[ \Rightarrow \sigma^2_r > \sigma^2_i \]

3. \( \beta_{i\rho} = \sigma^2_i / \sigma^2_\rho \)

\[ \Rightarrow \sigma^2_\rho > 2\sigma^2_{i\rho} \]

\[ \Rightarrow \beta_{i\rho} < 1/2 \]

2 Any model for which the elasticity of the market interest rate to anticipated inflation is less than unity implies real rate changes, which may or may not be caused by inflation rate changes.

3 This brief review of Keynes' transmission theory reveals how similar his theory was to Fisher's and the Monetarists. The only difference is that the Monetarists would preface the argument with the assumption of an increase in the growth of money, which causes a revision in inflation expectations.

4 Mundell [1963], p. 282.

5 Gibson [May-June, 1970], p. 436.

6 Kessel and Alchian [1962], especially pp. 528-530. The mechanics of the process are not described, but are presumably something like the following. An increase in the expected rate of inflation raises market interest rates, holding real rates constant. Individuals attempt to lower their holdings of real balances by substituting for claims to real assets (e.g., common stocks). The increase in the demand for real capital bids up prices because the economy is assumed to be at full employment. The decline in the value of real balances and the rise in prices acts to curtail consumption and stimulate savings, which in turn stimulates investment and drives the real yield on (real) assets down.
The classical position regarding the neutrality of the equilibrium rate of interest to the price level is developed in more detail in Lintner [1973].

Symbolically, Lintner's model is written as:

\[ \Delta F_t = a \Delta S_t + b S_t, \]

where \( F_t \) = total demand for external funds

\( S_t \) = current dollar sales.

The difference between retentions and the current dollar outlays for fixed investment, is the demand for external funds, which is assumed to vary proportionately with the level of sales \( (b S_t) \). Cash balances are assumed to be a fixed percentage of current period sales, as are accounts receivable. An increase in sales thus raises the demand for funds.

Lintner [1975], p. 273.
D. INTEREST RATE TESTS

1 Rutledge [1977] maintains that Fisher's distributed lag estimates did not constitute an attempt to measure the parameters of the inflation forecasting process, which would otherwise imply that long lags are associated with slow revisions of price forecasts. According to Rutledge, Fisher attributed the lag to the transition period disequilibrium adjustments. Fisher, Rutledge argues, held the opinion that the overwhelming impact of inflation during the transition period is on real variables.

2 Roll [1972] remarked on this issue. He stated,

In other words, the conclusion of no Fisher effect between nominal and real interest rates is valid only if commodity markets are efficient in the sense that has been applied to asset markets so that current prices adjust immediately to any relevant new information and thus continually stand at levels equal to expected prices. (p. 253)

We admit to being unclear as to exactly what Roll means by the "Fisher effect." It would appear from this and other statements that Roll interpreted Fisher as having proposed the relationship described by EQ(7). We, on the other hand, interpret Fisher as EQ(8).

3 Cf., Sargent [1971], p. 722.

4 The weights decline geometrically.

5 Cf., Holmes and Kwast [1979] for a rigorous statistical test for structural stability, or what we refer to as the "threshold effect." The authors substantiate a structural shift occurred in the late 1960's.


7 Roll [1972], p. 260, note 18, stated, and we concur, that evidently changes were used in an attempt to reduce autocorrelation.

8 Hendershott and Van Horne [1973] commented that, given the obvious limitation of the Anderson-Carlson model, the results seem impressive for the period 1955-1969. However, referring to a paper by Hendershott, they noted the results depend
crucially on the inclusion of a dummy variable. When the
model was tested without the dummy variable its explanatory
power was greatly reduced (for 1955-1965 $R^2 = .01$).

Roll [1972] has commented that, "one must hesitate to accept
this [Gupta's results] as conclusive evidence because the
variables were constructed so as to maximize the chance of
obtaining the desired results." (p. 275) This criticism
applies to several studies of the Fisherian Hypothesis.

See Johnston [1972], pp. 259-262. The adjustment procedure
assumes only first-order autocorrelation.

Scott [1973] maintains Gupta's empirical tests serve only to
confirm his demand for money equation and are completely
unrelated to the Fisherian Hypothesis. Scott's criticism is
based on Gupta's definition of the expected future inflation
rate as the difference between the nominal and real interest
rates. As a result, the validity of Gupta's demand for money
equation is both necessary and sufficient for his tests - the
results of which hold for any arbitrary real rate.

As discussed in Section C, Fama's studies provide evidence
contradicting the Mundell Effect; or more accurately
consistent with the hypothesis that all the inflationary
affects on the real interest rate are offsetting.

Fama [1975], p. 282.

Ibid., p. 272.

Roll [1972], p. 270. When Roll corrected for the serious
autocorrelation in his estimates not a single explanatory
variable remained significant. The estimated autoregressive
parameter was close to one, characteristic of a random walk.

Sargent reported large confidence regions for the ARIMA
parameters, regions that include implied lag distributions
with very long mean lags. See [February, 1973], pp. 398-400.


Ibid., p. 447.

See comments by, Gibson (pp. 450-453), Hendershott (pp.
454-459), and Gordon (pp. 460-463).


22 Ibid., p. 304.
23 Cf., Sargent [July, 1973].
E. COMMON STOCKS AS INFLATION HEDGES


4. See, Van Horne and Glassmire [1972], p. 1086, Eq(9).

5. In a related study, Fischer [1978] explored the valuation of "index bonds" under conditions of uncertainty and inflation. He employed the Black and Scholes [1973] call pricing model, and concluded that the value of a nominal bond may differ from that of the real bond as a result of differences between the expected real rate of return on default-risk-free index bonds \( r \) and the expected real rate of return on default-risk-free nominal bonds \( r_n \). The values will also differ because of default risk differences. From the option pricing model it followed that the indexed bond will tend to command a higher price if \( r < r_n \) and if \( \sigma^2 < \sigma_n^2 \). When the market rate of return is inversely correlated with the rate of inflation, \( r \) will be less than \( r_n \). Hence Fischer established that an increase in the correlation between the rate of change of the value of the firm and changes in the price level is likely to increase the value of the firm's indexed bonds relative to its nominal bonds.


8. Ibid., p. 255.


10. What if anything the Jaffee and Mandelker [1976] models establish is debatable. The highest coefficient of determination reported was .197. The typical model explained less than 5 percent of the variance of returns to stocks.

11. Keran [1971] sought to extend the St. Louis macroeconomic model by incorporating a stock price determination equation(s). This involved three structural equations: a stock price equation, an interest rate equation, and a corporate earnings equation. The stock price structural
equation did not include any inflation rate variables. Keran estimated the effects of inflation on stock prices by a "semi-reduced form" stock price equation. For unknown reasons Keran chose to exclude four of the eight predetermined variables from the reduced-form equation.

12 Fama and Schwert [1977] appeal to Fama [1975] for this basic model. See Section D.

13 Fama and Schwert [1977], p. 139.
F. CHANGES IN THE MONEY SUPPLY, RATES OF RETURN, AND THE EFFICIENT MARKET HYPOTHESIS

1 Sprinkel [1971], p. 217.

2 Sprinkel is consistent with the Friedman and Schwartz [1963] explanation of the history of money and business cycles.

3 Palmer [1970] attempted to remove the secular growth in the money stock by first differencing the data, as in Friedman and Schwartz [1963]. He also first differenced the stock price series. The lead-lag relationship was shown to be less identifiable for the rates of changes. The turning points for both series were generally quite close, but where a lead relationship existed, Palmer maintained the money supply changes preceded stock price changes by a few months.

4 Reilly and Lewis [1971], p. 20.

5 Hamburger and Kochin [1972], p. 246.

6 Significant correlations at lag 0 and lag 1 were reported. Rogalski and Vinso attributed this to data announcement and revision. The significant correlation at lag zero was thought to indicate the informational content of the estimated or preliminary value of the money supply. Significant correlation at lag 1 indicated the adjustment in stock prices to the actual value of the previous month's money supply, as soon as it was published.

G. THE DEBTOR-CREDITOR HYPOTHESIS

1 Cf. B. Friedman [1978] who makes the same observation and empirically evaluated the role of portfolio behavior of both debtors and creditors in creating the link between interest rates and inflation expectations. Based on partial equilibrium tests Friedman, concluded a one percent increase in expected inflation results in an increase in the equilibrium bond yield of .64 percent, and a slight decrease in the number of bonds outstanding.

2 Who benefits by differential expectations is determined by the ratio of market rate of interest to the "true" rate of interest (i* in Figure III). If the ratio exceeds one, creditors benefit. If the ratio is less than one, debtors gain. This holds even if both lenders and borrowers have erroneous inflation expectations. Such a case is described by B. Friedman [1978], see note 1 above. His results reinterpreted in a Figure III type graph appear as:

Although as Friedman observes the "result follows as the consequence of a slightly greater response by lenders than by borrowers" (p. 844), there will nevertheless be a wealth transfer from creditors to debtors because creditors overpaid for the bonds.

3 Table III uses the complete relationship, i.e.,

\[ r_t = 1 - (1+i_t)[1+E_t(\rho_t+\epsilon)]^{-1} \]
The average rate of inflation is determined by:

\[ \bar{\rho}_t = \prod_{j=0}^{t} (1+\rho_j) t^{-1} \]

This statement is technically incorrect, or more precisely it is true only for the special case where the anticipated one period ahead inflation rate is equal to the anticipated inflation rate for any future period. Otherwise, a change in the expected future inflation rate for one period could be offset by change in the expected rate for another period with the net effect depending on the magnitude of the changes and their timing. These complications do not alter the principle set forth in conclusion number two.

One must clearly distinguish between a "change in expectations of future inflation" and a "change in the expected inflation rate." The two are defined respectively as follows:

\[ E_t(\rho_{t+k}) - E_{t+j}(\rho_{t+k}) \neq 0 \quad \forall j, 0 < j < k \]
\[ E_t(\rho_{t+k}) - E_t(\rho_{t+k+j}) \neq 0 \quad \forall j \neq 0 \]

The former induces wealth adjustments, the latter does not.


Meyer's [1954] theory is identical to Bresciani-Turroni's [1937], although no acknowledgment of the latter is cited in Meyer. See Section H for a brief discussion of Bresciani-Turroni's theory.

For example see Willis and Chapman [1935], pp. 59, 77, 141, and 206.

Ibid., p. 139.

Also see Kessel and Alchian [1960].

Kessel's [1956] tests are subject to criticism for the small sample size (16 banks) and the period of study. The war years are a poor period for study because of the wage-price-interest rate controls. Friedman and Schwartz [1963] report general price control was instituted in early
1942 and suspended in mid-1946. During that period there was a strong tendency for price increases to take a concealed form. Treasury bill rates were also fixed.

Bach and Ando [1957] is the best example we know of the methodological standards in the study of public finance prior to the application of general equilibrium techniques. The authors clearly understood that the incidence of the inflation tax can only be determined by accounting for all income and expenditure adjustments in all sectors. But for the lack of a mechanism to measure these interdependencies, they were forced to limit their analysis to a series of Marshallian partial equilibrium adjustments.

Bach and Stephenson [1974], p. 12.


Rozell [1977] was bolder when he stated the evidence does not suggest net monetary debtors, after abstracting from risk, accumulate windfall gains during inflationary periods. He noted three reasons why this might be so:

1. The effects of unanticipated inflation on stock returns are negligible.

2. The effects of unanticipated inflation on stock returns are undetectible or difficult to isolate given the many other sources of variation in stock returns.

3. Inflation is correctly anticipated, i.e., the market makes unbiased assessments of the moments of the probability distribution of price level change, which are impounded in the beta coefficients of debtor and creditor firms.
H. THE LABOR MARKET AND THE WAGES LAG HYPOTHESIS

1 Hamilton [May 1929], p. 436.

2 Felix [1956], p. 444.

3 Hamilton [1947], p. 215.

4 Also see Alchian and Kessel [1959].

5 Let the computed real wage rate be given as EQ(a). The Fisherian Hypothesis for wages is EQ(b), from which the difference condition EQ(c) is derived.

   EQ(a) \[ \hat{w}_t - \rho_t = \hat{n}_t \]

   EQ(b) \[ \hat{w}_t - E_t(\rho_{t+1}) = n_t \]

   EQ(c) \[ (d \hat{n}_t > d \hat{n}_t \mid d \hat{n}_t < 0) \rightarrow d\rho_t > dE_t(\rho_{t+1}). \]

   EQ(d) \[ d\rho_t > dE_t(\rho_{t+1}) \rightarrow d\rho_t > d\rho_{t+1} + d\epsilon_{t+1} \]

   Equation EQ(d) is obtained by substituting \( \rho_{t+1} + \epsilon_{t+1} \) for \( E_t(\rho_{t+1}) \), where \( \epsilon_{t+1} \) is the stochastic error. Over a large sample, if EQ(d) holds (in expectation form) then the average value of \( d\epsilon \) would be negative, contradictory to the rational expectations interpretation of EQ(b).

6 Tucker [1933] [1936] argued that real wages started rising after 1776 when laissez-faire became an accepted economic theory. His 1934 paper explored the impact of the gold standard on the price level and wages. In it he advanced the following notion of dynamic responses:

   The prices that are most apt to rise in response to easy money are the prices of speculative commodities and securities, and even those prices do not always respond. If the easy money policy involves depreciating the currency in the foreign exchange markets, the prices that rise the most are those of imported goods. And invariably the last prices to rise are the most important in our social organization - wages of laborers and salaries of the white-collared class. On this point the evidence is overwhelming [1934], p.14.

Bodkin [1966], p. 111.

The unidirectional causality from wages to the price level was strongest for the subperiod 1954 to 1962. Although Mehra elected not to elaborate on this result, it is nevertheless very interesting, for it is consistent with the cost-push theory of inflation that many economists maintain prevailed in the late 1950s, in particular the wage-push version of the cost-push theory.

The negative covariance is also consistent with wages leading profits.

Laidler and Parkin [1975], p. 788.

That the position of the Phillips curve depends on the expected future inflation rate was popularized by Phelps [1967] and M. Friedman [1968] in his presidential address before the American Economic Association. A microeconomic rationale for the Phillips curve based on the expected future inflation rate was presented in Phelps [1970].

Assume no new information regarding future prices is obtained and that $E_t(\rho_{t+1}) = \rho_{t+1}$.  


Cf., Lucas [June, 1972].
# APPENDIX A

## TESTS OF THE FISHERIAN HYPOTHESIS

<table>
<thead>
<tr>
<th>AUTHOR(s)</th>
<th>Reference</th>
<th>Time Period &amp; Frequency of Data</th>
<th>MODEL</th>
<th>$R^2$</th>
<th>$P&lt;.001$.</th>
</tr>
</thead>
<tbody>
<tr>
<td>FISHER</td>
<td>[1930]</td>
<td>A: British, 1899–1924 U.S., 1900–1927 Q: British, 1915–1927</td>
<td>$t_t = b_0 + \sum_{t=0}^{n} a_t p_{t-t}$</td>
<td>.61</td>
<td>.59</td>
</tr>
<tr>
<td>BALL</td>
<td>[1965]</td>
<td>A: British, 1921–1963</td>
<td>$t_t = 8.69 - .005 E_t (c_{t+1}) - .004 { E_t (c_{t+1}) - E_{t-1} (c_t) }$</td>
<td>(.011)</td>
<td>(.008)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>- .081 (M/Y)$_t$</td>
<td>(.013)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$E_t (c_{t+1}) = \rho_t + \lambda (c_t - \rho_{t-1})$</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\lambda = 1$</td>
<td></td>
</tr>
<tr>
<td>SARGENT</td>
<td>[1969]</td>
<td>A: 1902–1940</td>
<td>$t_t = 7.134 - 1.985 (.97)^t + 3.876 \sum_{t=1}^{t-1} .97^{t-1} \rho_{t-t}$</td>
<td>(.630)</td>
<td>(.509)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>- 2.015 $n_t + .010 (y_t - y_{t-1}) - .046 y_t$</td>
<td>(.786)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$t_t = -11.371 + 17.882 (.99)^t + 6.277 \sum_{t=1}^{t-1} .99^{t-1} \rho_{t-t}$</td>
<td>(1.09)</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>- 11.371 - 1.831 $1.73^{t-1} \rho_{t-t} + .010 (y_t - y_{t-1})$</td>
<td>(1.069)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$\lambda = .99$</td>
<td>.6 = .73</td>
</tr>
<tr>
<td>YOH-E-KARNOSKY</td>
<td>[1969]</td>
<td>M: Jan. 1952–Sep. 1969</td>
<td>$t_t = b_0 + \sum_{t=1}^{n} \theta_t p_{t+1-t}$</td>
<td>{ [ \sum_{t=1}^{n} \theta_t = .82; n = 48; \sum_{t=1}^{n} \; \text{mean lag = 16.4 months}}</td>
<td>.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$t_t = b_0 + \sum_{t=1}^{n} \theta_t p_{t+1-t}$</td>
<td>{ 2nd degree Almon polynomial; [ \sum_{t=1}^{n} \theta_t = .86; \sum_{t=1}^{n} \; \text{mean lag = 5.5 quarters}}</td>
<td>.96</td>
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<tr>
<td>Author(s)</td>
<td>Reference</td>
<td>Time Period &amp; Frequency of Data</td>
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<tr>
<td>CPI</td>
<td>-.0009</td>
<td>1.12</td>
<td>.35</td>
<td>.20 ($\theta_2$)</td>
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<td></td>
<td>(.0003)</td>
<td>(.09)</td>
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<td>FOOD</td>
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<td>1.04</td>
<td>.05</td>
<td>.28 ($\theta_2$)</td>
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<td>(.0009)</td>
<td>(.26)</td>
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<tr>
<td>APPAREL</td>
<td>-.0008</td>
<td>.90</td>
<td>.05</td>
<td>.18 ($\theta_2$)</td>
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<td>(.0007)</td>
<td>(.23)</td>
<td></td>
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<td>HOME OWNERSHIP</td>
<td>-.0016</td>
<td>1.64</td>
<td>.35</td>
<td>.37 ($\theta_3$)</td>
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<td></td>
<td>(.0004)</td>
<td>(.14)</td>
<td></td>
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</tr>
<tr>
<td>RENT</td>
<td>.0006</td>
<td>.55</td>
<td>.17</td>
<td>.49 ($\theta_4$)</td>
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<td></td>
<td>(.0002)</td>
<td>(.08)</td>
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</tr>
<tr>
<td>Food: inflation rate</td>
</tr>
<tr>
<td>relative change</td>
</tr>
<tr>
<td>Apparel: inflation rate</td>
</tr>
<tr>
<td>relative change</td>
</tr>
<tr>
<td>Home Ownership: inflation rate</td>
</tr>
<tr>
<td>relative change</td>
</tr>
<tr>
<td>Rent: inflation rate</td>
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<td>relative change</td>
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AUTHOR(s)

Reference

Time Period & Frequency of Data

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<th>$\hat{r}^2$</th>
<th>$D-k$</th>
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<tr>
<td>CARLSON [1977] M: 1953:1 - 1975:12</td>
<td>$\Delta_t = \beta_0 + \beta_1 i_t + \beta_2 (E/N)_t$</td>
<td>.36</td>
</tr>
<tr>
<td></td>
<td>$\Delta_t = -.020 - .639 i_t - .088 (E/N)_t$</td>
<td>(.034)</td>
</tr>
<tr>
<td></td>
<td>$\Delta_t = .069 - .976 i_t$</td>
<td>(.102)</td>
</tr>
<tr>
<td></td>
<td>$\rho_t = -.001 + .77 i_t + .132 \hat{P}<em>{t-1} + .101 \hat{P}</em>{t-2} + .077 \hat{P}_{t-3}$</td>
<td>(.0003)</td>
</tr>
<tr>
<td></td>
<td>$\rho_{12}(e) = .19$</td>
<td>(.036)</td>
</tr>
<tr>
<td></td>
<td>$\rho_{12}(e) = .24$</td>
<td>(.035)</td>
</tr>
<tr>
<td>ELLIOTT [1977] Q: 1964 - 1974</td>
<td>Neo-Keynesian Model: $r_t = \beta_0 + \beta_1 \log \hat{y} - \log y^*_t + \beta_2 z_t$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\hat{y} = \beta_0 \delta_x \log y_t^* + \beta_2 \log M_t + \beta_3 z_t + \beta_4 w_t$</td>
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</tr>
<tr>
<td></td>
<td>$z_t = (\delta_x \Delta_t + (\beta_x)_t + \gamma_t)^2$</td>
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<tr>
<td></td>
<td>$\hat{y} = \Delta_t (y_{t-t+1} - y_{t-t})$</td>
<td></td>
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</tbody>
</table>

344
Neoclassical-Loanable Funds Model:

\[ r_t = \beta_0 + \beta_1 (p^e)_t + (m_t - m_{t-1}) + \beta_2 y_t-1 + \epsilon_t \]

\[ + \beta_3 (r_t - r_{t-1}) + \sum_{t=1}^{12} \alpha_t (y_{t-1} - y_{t-1-1}) + \beta_4 (m_t - m_{t-1}) + \beta_5 (y_t-1 - y_{t-1}) + \beta_6 (y_t-1 - y_{t-2}) + \epsilon_t \]

Fishierian Autoregressive Model:

\[ r_t = I_t - \sum_{t=0}^{12} \alpha_t I_{t-1} \]

\[ r_t = I_t - \rho I \]

Monetary Model of Inflationary Expectations:

\[ J_{p,t} = \beta_0 + \sum_{t=0}^{12} \alpha_t \log (M)_t - I_t \]

Labor Market Model of Inflationary Expectations:

\[ J_{p,t} = \beta_0 + \sum_{t=0}^{12} \alpha_t \left( \log (L)_{t-1} - \log (P)_{t-1} \right) + \beta \log \left( \frac{W}{p} \right)_t \]

\[ + \sum_{t=1}^{12} \alpha_t \left( \log (L)_{t-1} - \log (P)_{t-1} \right) \]

\[ + \beta_3 (m_t - m_{t-1}) + \beta_4 (y_t-1 - y_{t-1}) + \epsilon_t \]
HESS-BICKSLER  
[1979] continued

\[
\rho_t = (1.0 + (r_{t-12} - \hat{\theta}_1 u_{t-1}) - \hat{\theta}_2 u_{t-2} - \hat{\theta}_3 u_{t-3} - \hat{\lambda}_1 u_{t-12} - \hat{\lambda}_2 u_{t-13} + \hat{\lambda}_3 u_{t-14} + \hat{\lambda}_4 u_{t-15} - \hat{\lambda}_5 u_{t-16} - 1 t^{-1} - 1.0) 
\]

\[
\times \beta (t_{t-1} + \rho_{t-12} - \rho_{t-13} - \hat{\theta}_1 u_{t-1} - \hat{\theta}_2 u_{t-2} - \hat{\theta}_3 u_{t-3} - \hat{\lambda}_1 u_{t-12} - \hat{\lambda}_2 u_{t-13}) 
\]

\[
- [1.0 - (r_{t-12} - \hat{\theta}_1 u_{t-1} - \hat{\theta}_2 u_{t-2} - \hat{\theta}_3 u_{t-3} - \hat{\lambda}_1 u_{t-12} - \hat{\lambda}_2 u_{t-13} - \hat{\lambda}_3 u_{t-14} - \hat{\lambda}_4 u_{t-15} - \hat{\lambda}_5 u_{t-16}) - (t_{t-1}) - \hat{\epsilon}_t] 
\]

\[
| \hat{\theta}_1 | \hat{\theta}_2 | \hat{\theta}_3 | \hat{\lambda}_1 | \hat{\lambda}_2 | \beta | t \text{ - values} | \rho_{t-12} | \rho_{t-13} | \hat{\epsilon}_t | \hat{\epsilon}_t |
\]

\[-.106 \quad -.247 \quad .127 \quad .920 \quad .867 \quad .911 \quad 1.86 \quad 55.69 \quad 25.40 \quad 49.03 \quad 2.21 \quad .31 \quad \.82 \quad .02 \]

NELSON-SCHWARTZ  
[1977] 
H: 1953,7 - 1971,7

\[
\hat{\rho}_t = \sum_{\tau=0}^{\infty} \theta^\tau (1 - \theta) \rho_\tau - 1_t \rho_t - \lambda_{t-1} \rho_t 
\]

\[
\theta = .89
\]

\[
\rho_t = - .641 + .651 1_t + .363 \hat{\rho}_t 
\]

(\(.359\) \(.359\) \(.165\) \(.158\))

\[
(\rho_t - \rho_{t-1}) = - .546 + .633 (1_t - \hat{\rho}_{t-1}) + .317 (\hat{\rho}_t - \rho_{t-1}) 
\]

(\(.206\) \(.152\) \(.170\) \(.197\))
HENDERSHOTT-VAN HORNE
[1973]
O: 1960.1 - 1970.1V

\[ t - \left( P_{\alpha} \right)_t = -2.36 + 1.04 \sum_{\tau=0}^{13} \alpha_{t-\tau} + 1.01 \sum_{\tau=0}^{7} \alpha'_{t-\tau} \]
\[ (\hat{\alpha}_{0}) \]
\[ + .121 \sum_{\tau=0}^{13} \alpha''_{t-\tau} + .597 (P_{\alpha})_t + .008 \hat{P}_{\alpha} \]
\[ (\hat{\alpha}_{0}) \]
\[ \alpha_{t-\tau} \in [1960.1 - 1967.1] \]
\[ \alpha'_{t-\tau} \in [1967.11 - 1970.1V] \]
\[ \Sigma_{\hat{\alpha}} = \Sigma_{\hat{\alpha}'} = \Sigma_{\hat{\alpha}''} = 1.0 \]

FAMA
[1975]
M: 1953.1 - 1971.7

\[ \Delta_t = .0007 - .98 \hat{t}_t \]
\[ (.0003) \]
\[ (.10) \]

\[ \Delta_t = \hat{\beta}_0 + \hat{\beta}_1 \Delta_{t-1} \]
\[ (.13) \]

\[ \Delta_t = .0006 - .87 \hat{t}_t + .11 \Delta_{t-1} \]
\[ (.0003) \]
\[ (.12) \]
\[ (.07) \]

\[ r_t = \hat{\beta}_0 + \hat{\beta}_1 r_{t-1} \]
\[ (.01) \]

HESS-BICKSLER
[1975]
M: 1958.1 - 1971.6

\[ \rho_t - (1.0 + \alpha_0 - \hat{t}_t)^{-1} = \hat{\beta}_1 \rho_{t-1} + \rho_{t-12} - \rho_{t-13} - \hat{\beta}_1 \hat{t}_{t-1} \]
\[ - \lambda_{11} \rho_{t-12} + \lambda_{12} \hat{t}_{t-13} \]
\[ - (1.0 + \alpha_0 - \hat{t}_t)^{-1} - 1.0) + \hat{e}_t + 1.0 \]

\[ \hat{\theta}_1 \]
\[ \lambda_1 \]
\[ \beta \]
\[ t-values \]
\[ .867 \]
\[ 25.40 \]
\[ .911 \]
\[ 49.03 \]
\[ .48 \]
\[ 5.6 \]
\[ \rho_{t-1} \in (\Phi) \]
\[ .16 \]
\[ \text{na} \]
\[ .44 \]
\[ 5.4 \]
\[ \rho_{t-1} \notin (\Phi) \]
\[ .15 \]
\[ \text{na} \]
\[ i_t = -8.986 + 1.094 p_t^{(.81)} - 4.521 \log \left( \frac{MB}{PN} \right)_t \]  
\[ (.886) \quad (.056) \quad (.730) \]  
\[ + 5.885 \log \left( \frac{Y}{PN} \right)_t + .062 (i_{t-1} - i_{t-2}) \]  
\[ (.350) \quad (.011) \]  
\[ \rho^{(.81)} = \frac{(1 - .81)^2}{(1 - .81)^2} \quad [400 (\frac{PN}{P_t} - 1)], \text{ second order} \]  
Pascal lag  
\[ L = \text{lag operator} \]  
\[ i_t = -15.00 + 1.324 p_t^{(.83)} - 7.385 \log \left( \frac{MB}{PN} \right)_t \]  
\[ (.93) \quad (.066) \quad (.647) \]  
\[ + 7.359 \log \left( \frac{Y}{PN} \right)_t + .0454 (i_{t-1} - i_{t-2}) \]  
\[ (.322) \quad (.0095) \]  
\[ + .0502 g^{(.80)} \]  
\[ (.0094) \]  
\[ g^{(.80)} = \frac{(1 - .80)^2}{(1 - .80)^2} \quad [400 (\frac{SP_{t-1}}{SP_{t-2}} - 1)] \]  
\[ i_t = -14.13 + 1.072 p_t^{(.80)} - 6.390 \log \left( \frac{MB}{PN} \right)_t \]  
\[ (1.07) \quad (.063) \quad (.646) \]  
\[ + 9.443 \log \left( \frac{Y}{PN} \right)_t + .0386 (i_{t-1} - i_{t-2}) \]  
\[ (.922) \quad (.0091) \]  
\[ + .0181 g^{(.70)} - .182 \lambda^{(.57)} \]  
\[ (.0065) \quad (.052) \]  
\[ \lambda^{(.57)} = \frac{(1 - .57)^2}{(1 - .57)^2} \quad [400 (\frac{MB/PH}{PN}_t - 1)] \]
<table>
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<th>AUTHOR(s)</th>
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<tbody>
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<th>D-W</th>
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<td></td>
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<td>$\alpha$</td>
<td>$\alpha'$</td>
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<tr>
<td>1</td>
<td>.33 *</td>
<td>.56 *</td>
<td>.14 *</td>
</tr>
<tr>
<td>2</td>
<td>.245*</td>
<td>.26</td>
<td>.10 *</td>
</tr>
<tr>
<td>3</td>
<td>.17 *</td>
<td>-.31</td>
<td>.075*</td>
</tr>
<tr>
<td>4</td>
<td>.11 *</td>
<td>.13</td>
<td>.05</td>
</tr>
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<td>5</td>
<td>.065</td>
<td>.32</td>
<td>.032</td>
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<td>6</td>
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<td>.075</td>
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<td>7</td>
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<td>.007</td>
<td>.001</td>
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<td>8</td>
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<tr>
<td>$\Sigma$</td>
<td>.96 (.20)</td>
<td>1.04 (.18)</td>
<td>.43 (.165)</td>
</tr>
</tbody>
</table>

*twice standard error


\[
i_t = 55.4 + .112 P_t + 4.08 \sum_{t=1}^{23} \alpha_{t-1} - 6.47 \log \left( \frac{MB}{PPN_t} \right) + 5.66 \log \left( \frac{V}{PPN_t} \right) + 4.70 \left( \frac{B_C}{PPN_t} \right) + .147 \left( \frac{MB}{PPN_t} - \frac{MB}{PPN_{t-1}} \right)
\]

\[
+ .366 (i_{t-1} - i_{t-2})
\]

\[
i_t = 4.02 + 24.8 P_t + .216 \sum_{t=1}^{23} \alpha_{t-1} + 1.69 \log \left( \frac{MB}{PPN_t} \right) + 1.26 \log \left( \frac{V}{PPN_t} \right) - 5.26 \left( \frac{B_C}{PPN_t} \right) + .0334 \left( \frac{MB}{PPN_t} - \frac{MB}{PPN_{t-1}} \right)
\]

\[
+ .687 (i_{t-1} - i_{t-2})
\]

\[
i_t = -.859 + 19.0 P_t + .0965 P_t + 4.08 \left( \frac{MB}{PPN_t} \right) - 1.52 \log \left( \frac{V}{PPN_t} \right) + 9.13 \left( \frac{B_C}{PPN_t} \right) - .0368 \left( \frac{MB}{PPN_t} - \frac{MB}{PPN_{t-1}} \right)
\]

\[
+ .556 (i_{t-1} - i_{t-2})
\]
<table>
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<th>AUTHOR(s)</th>
<th>Reference</th>
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<th>MODELS</th>
<th>$r^2$</th>
<th>D-W</th>
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</thead>
</table>
\begin{align*}
\log M_t - \lambda_1 \log M_{t-1} &= -0.211 - 0.127 (i_t - \lambda_1 i_{t-1}) \\
&\quad + 1.243 (\log Y_t - \lambda_1 \log Y_{t-1}) \\
\log M_t - \lambda_2 \log M_{t-1} &= -0.250 - 0.147 (r_t - \lambda_2 r_{t-1}) \\
&\quad - 0.154 [E_t (\sigma_{t+1}^2) - \lambda_2 E_t (\sigma_{t+2}^2)] \\
&\quad + 1.146 (\log Y_t - \lambda_2 \log Y_{t-1}) \\
\end{align*}
\] | 0.91 | 1.431 |

\begin{align*}
i_t &= 2.207 + 0.930 E_t (\sigma_{t+1}^2) \\
&\quad (0.170) \quad (0.085) \\
i_t &= 1.889 + 1.087 E_t (\sigma_{t+1}^2) \\
&\quad (0.312) \quad (0.127) \\
\end{align*}
\] | 0.76 | na |

\begin{align*}
i_t &= 0.019 + 1.11 E_t (\sigma_{t+1}^2) \\
&\quad (0.002) \quad (0.080) \\
i_t &= 0.019 + \sum_{\tau=1}^{8} \sigma_{t+\tau}^2 \\
&\quad (.004) \\
i_t &= 0.017 + \sum_{\tau=1}^{6} \sigma_{t+\tau}^2 \\
&\quad (.004) \\
i_t &= 0.26 + 0.91 E_t (\sigma_{t+1}^2) - 0.032 \log (\text{MB\textsubscript{t+1}}) + 0.021 \log (\text{FP\textsubscript{t+1}}) \\
&\quad (0.30) \quad (0.18) \quad (0.021) \quad (0.020) \\
i_t &= -0.16 + \sum_{\tau=1}^{8} \sigma_{t+\tau}^2 \\
&\quad (0.34) \quad (0.025) \\
\end{align*}
\] | 0.88 | 1.34 |

(2nd degree polynomial lag)
GIBSON
[1970, March]
A: 1869-1963
Q: 1946-1963

\[ i_t = \beta_0 + \sum_{\tau=1}^{\tau} \beta_{\tau} q_{t-\tau} + \beta_{t-1} q_{t-1} + \beta_{t-2} q_{t-2} + \beta_{t-3} t_{t-3} + \beta_{t-4} t_{t-4} \]

\[ p^2 = .21 \quad D-K = .144 \]

FELDSTEIN-ECKSTEIN
[1970]
Q: 1954.1 - 1969.11

\[ i_t = 3.45 + .041 p_t \]

\[ (\hat{.028}) \quad (\hat{.029}) \]

\[ p^2 = .03 \quad D-K = \text{na} \]

WILK [1970]
A: 1900-1968

\[ i_t = 1.99 + 1.146 p_t \]

\[ (\hat{.271}) \quad (\hat{.123}) \]

\[ p^2 = .94 \quad D-K = \text{na} \]

ANDERSON-CARLSON
[1970]
Q: 1955.1 - 1969.1V

\[ i_t = 1.28 + \sum_{\tau=0}^{16} \alpha_{\tau} q_{t-\tau} + \sum_{\tau=0}^{16} \lambda_{\tau} q_{t-\tau} \]

\[ (\hat{.246}) \quad (\hat{.129}) \]

\[ \frac{1}{16} \hat{\alpha} = .96 \quad \frac{1}{16} \hat{\lambda} = .20 \]

\[ (\hat{.050}) \quad (\hat{.069}) \]

\[ p^2 = .92 \quad D-K = .69 \]
APPENDIX B
COMMON STOCKS AS INFLATION HEDGES

AUTHOR(s)
Reference
Time Period & Frequency of Data

KERAN
[1971]
Q: 1956.1 - 1970.11

\[ SP_t = -30.68 + .57 \hat{n}_t + .51 \hat{n}_{t-1} + .21 \hat{n}_{t-2} \]
\[ (3.12) \quad (.157) \quad (.126) \quad (.162) \]
\[ + \sum_{\tau=0}^{16} \lambda_{t-t} \hat{\Delta}_{t-t} + \sum_{\tau=0}^{16} \hat{\beta}_{t-t} \hat{\Delta}_{t-t} + \sum_{\tau=0}^{16} \hat{\gamma}_{t-t} \hat{\Delta}_{t-t} \]
\[ \hat{\Sigma} \lambda_t = -5.37 \]
\[ (.947) \]
\[ \hat{\Sigma} \beta_t = -11.96 \quad E_t(\sigma_{t+1}) = \sum_{t=0}^{16} \hat{\epsilon}_{t-t} \]
\[ (.1508) \]
\[ \hat{\Sigma} \gamma_t = 4.80 \]
\[ (.240) \]

structural equations:
\[ 1_{B_t} = \delta_0 + \delta_1 \hat{b}_t + \delta_2 \hat{b}_{t-1} + \delta_3 \hat{Y}_t + \delta_4 Y_t + \delta_5 Y_t + \delta_6 E_t(c_{t+1}) \]
\[ 1_{S_t} = \alpha_0 + \alpha_1 \hat{s}_t + \alpha_2 \hat{s}_{t-1} + \alpha_3 \hat{Y}_t + \alpha_4 Y_t + \alpha_5 Y_t + \alpha_6 E_t(s_{t+1}) \]

structural estimation:
\[ 1_{S_t} = 6.47 - 1.32 \hat{b}_t + 1.34 \hat{b}_{t-1} + 1.26 \hat{Y}_t + \frac{MB}{\hat{Y}_t} \]
\[ (3.126) \quad (.460) \quad (.487) \quad (.534) \]
\[ - .01 \hat{V}_{\hat{Y}_t} - .02 \hat{NIS}_{\hat{Y}_t} + .06 \hat{SPR}_t + 3.30 \frac{S/Y_t}{(5/Y)_t} \]
\[ (.020) \quad (.068) \quad (.666) \quad (.611) \]
\[ - \sum_{\tau=1}^{16} \lambda_{t-t} \hat{SPR}_{t-t} : \hat{\Sigma} \lambda_t = -8.47 \]
\[ (2.852) \]
reduced form estimation:

\[ i_t = 4.63 + 0.14 t + 2.50 (\text{MP})_t - 0.02 (\text{SP})_t \]
\[ \times 1.96 (0.63) \quad (0.033) \]
\[ - 0.01 (\text{NIS})_t - 1.29 \text{SPR}_t + \sum_{r=1}^{10} \lambda_r \text{SPR}_{t-r} \]
\[ \times 0.004 \quad (0.449) \]
\[ + \sum_{r=1}^{10} \theta_r p_{t-r} - 6.99 (\text{t})_t \]
\[ \times 5.377 \]

\[ \sum \lambda = 9.34 \quad \sum \theta = -0.92 \]
\[ (2.966) \quad (0.275) \]

LINTNER
[1973]
A: 1950 - 1970

\[ i_t = 16.93 + 0.115 (\text{t} - 1) + 3.301 (\frac{\hat{p}_t - \hat{p}_{t-1}}{\hat{p}_{t-1}}) \]
\[ \times 0.053 \quad (0.743) \]
\[ - 0.669 (\frac{\hat{p}_t - \hat{p}_{t-1}}{\hat{p}_{t-1}}) + 0.542 t \]
\[ \times 0.235 \quad (0.259) \]

\[ (\frac{\hat{p}_t - \hat{p}_{t-1}}{\hat{p}_{t-1}}) > \frac{\hat{p}_t - \hat{p}_{t-1}}{\hat{p}_{t-1}} \quad < 0.02 \]

BRANCH
[1974]
A: 1953 - 1969

\[ i_{m,t} = 0.0879 + 0.3920 p_t - 0.6756 y_t \]
\[ \times 0.199 \quad (0.463) \]

\[ i_{m,t} = 0.0453 + 0.4482 p_t \]
\[ \times 0.209 \]
<table>
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<th>Author(s)</th>
<th>Reference</th>
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<td>$(1.206)$</td>
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<td>BODIE</td>
<td>[1976]</td>
<td>A,M: 1953 - 1972</td>
<td>$i_{m,t} = \beta_0 + \beta_1 \rho_{t+1} + \beta_2 [\rho_{t+1} - E(\rho_{t+1})]$</td>
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<td>Annual: -3.631 - 4.885 - 13.193</td>
<td>.40</td>
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<td>$(2.802) (2.865) (4.007)$</td>
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<td>Monthly: -5.843 - 6.866 - 5.253</td>
<td>.10</td>
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<td>$(1.437) (1.439) (1.278)$</td>
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<td>NELSON</td>
<td>[1976]</td>
<td>M: 1953 - 1972</td>
<td>$i_{m,t} = \beta_0 + \beta_1 \rho_{t+1}$</td>
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<td>Nelson</td>
<td>[1976]</td>
<td>continued</td>
<td>$i_{m,t} = 0.02 + \sum_{t=4}^{4} \lambda_t P_{t-t} + \sum \lambda_t - 4.93$</td>
<td>0.08</td>
<td>1.76</td>
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<tr>
<td>Fama-Schwert</td>
<td>[1977]</td>
<td>M: 1953,1 - 1971,7</td>
<td>$i_{m,t} = 0.0235 - 5.70 i_{TB,t} + 2.35 (\Delta_t - i_{TB,t})$</td>
<td>0.03</td>
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<td>Q:</td>
<td>$i_{m,t} = 0.0549 - 4.95 i_{TB,t} + 6.50 (\Delta_t - i_{TB,t})$</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>M:</td>
<td>$i_{m,t} = 0.0234 - 5.50 i_{TB,t}$</td>
<td>0.03</td>
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THE INFLATION TAX AND THE THEORY OF THE FIRM

A. INTRODUCTION

The authors of one of the most frequently referenced studies of the redistributional effects of inflation, Bach and Ando [1957], made the following observation.

Unfortunately, the existing theory of the firm tells us so little about the internal mechanics of intrafirm prices and costs in the real world that we have no satisfactory theoretical model to serve as a framework for investigating in detail the leads and lags within the firm during inflation periods. (p. 9)

This paper presents just such a model derived from contemporary capital market theory.

The purpose of this essay is to develop an equation of the rate of return to the firm as an entity (i.e., return on total investment). No such profitability measure is currently reported in any publicly available source. The equation is derived for the microeconomic unit
called the (homogeneous) "firm". It incorporates specific variables to account for the inadequacies of the income measure used as the base for the corporation income tax, which, during inflationary periods, distort the investment and financing decisions of firms.

An equilibrium expected rate of return to the firm equation is the cornerstone, both conceptually and econometrically, to any model of the business sector. The theory behind the valuation of the firm model presented here presumes a highly rational and anticipatory decision making process. As was demonstrated in the previous essays in this study, the endogeneity of the inflation tax also depends on the degree to which all markets form rational expectations. Thus the model presented here is conceptually consistent with the null hypothesis that changes in the inflation tax are endogenous at the time of imposition. That is, the model presumes efficient markets where information which is pertinent to economic decisions is fully reflected in prices and choices. This includes all information regarding the future general level of prices.

Tests of the effects of the inflation tax on the investment-financing decisions of firms are simultaneously tests of both the theory of the firm and the endogeneity of the inflation tax. For this reason, the results may fail to confirm the endogeneity hypothesis, either because the inflation tax is indeed not endogenous, or because the model is misspecified.1
The equilibrium expected rate of return to the firm equation is relatively complex. Therefore, a meticulous derivation of the equation is desirable so as to insure the reader of its validity and meaning. Several issues involving the proper measurement of the inflationary effects on investment return must be spelled out in detail to reveal the theoretical implications of these measurements.

Very little attention has been given to the question of whether the rate of return earned by the firm conforms to the Fisherian Hypothesis. Virtually all studies of the behavior of rates of return with respect to inflation have concentrated on the return to individual investments, e.g., Treasury bills, common stocks, etc. As the previous essay revealed, the real rate of interest is often either explicitly or implicitly assumed to be constant. In those instances where determinants of the real rate are specified, the authors rarely define the microeconomic foundations for their structural equations. In this study the microeconomic relationships and behavioral assumptions are made an integral part of any assessment of the costs and effects of inflation.

The rate of return to the firm as defined in this study is an economic return to capital. It is, therefore, a measure of the user cost of capital, analogous to the concept in the neoclassical model of the demand for capital assets (cf., Jorgenson [1963], Hall and Jorgenson [1967]). There has been a resurgence of interest in the
study of fixed business investment, primarily because of the desire to stimulate supply capacity needed to reduce inflationary pressures. The more contemporary literature has extended the traditional accelerator and neoclassical models to explicitly control for distortionary taxes (especially taxes on capital income), inflation caused redistribution of capital (what we call the "Debtor-Creditor Hypothesis"), and non-risk neutral investment behavior (cf., Kopcke [1977] [1981], Hendershott and Hu [1979], and Feldstein [1980]). The literature treats inflation variables as either direct determinants of capital investment demand, or as corrections of the user cost of capital. The position adopted here is that the user cost of capital, or what we call the "rate of return to the firm," incorporates all expected future information about the earnings of the firm for the existing fixed capital stock. Included in this information are all expected future inflation tax affects on the firm's earnings. Since inflationary effects are incorporated in the user cost of capital, a model of fixed business investment demand developed as a continuation of this essay would include only three conceptual terms: the user cost of capital, the cost of financial capital, and some expression for the marginal product of fixed business investment.

The derivation of the equilibrium expected rate of return to the firm equation proceeds as follows. The theory of the firm is examined in the next section. The rudimentary principles of investment - production and financing decisions are spelled out. In Section C
certain basic income and valuation concepts are defined. These concepts are then exploited in Section D to derive the fundamental expression for the equilibrium expected rate of return to the firm. The economic interpretation of several of the explanatory variables is analyzed in the last Section, E. This analysis, along with the resolution of certain measurement problems identified in that section, leads to the final form of the equilibrium expected rate of return to the firm model. This model is presented in Section E.5. Summary remarks are given in Section F. Two appendixes are included. The first is an index of the variables used in this essay. Appendix B contains the proofs of the four propositions used to derive the equilibrium equation.

A caveat is in order before proceeding. The expression "economic income" or "economic net income" is used throughout this essay in reference to the economic rent on capital. It should not be interpreted as pure economic profit which is driven to zero with competition.
B. THEORY OF THE FIRM

The managements of firms, in the aggregate, are assumed to always act in the interest of the firm as a whole, and accordingly adopt those policies and undertake those investments which maximize the expected value of the firm. This dictum is called the Value Maximization Rule. It presumes that the only market value rule consistent with a stable equilibrium is the one in which management strives to maximize not just stockholder wealth, but the combined bondholder and stockholder wealth.\(^1\)

It has long been recognized that in a world of perfect and complete markets and perfect information, the firm achieves value maximization by undertaking investments to the point where the marginal cost of capital is equal to the marginal efficiency of capital. Furthermore, this investment rule is unanimously preferred by equity holders, with the resulting competitive equilibrium being Pareto optimal. Both of these results can be traced to Fisher [1930]\(^2\). Modigliani and Miller [1958] [1963] have further demonstrated that in such a world the equilibrium value of the firm is independent of its financial structure.\(^3\) That is, the investment decision is separate from the financing decision - the so called Separation Theorem.\(^4\) The dividend policy is likewise independent.\(^5\)
The value maximization rule has been challenged on two counts. Simon [1959] [1976], Cyert and March [1963], O. Williamson [1964], J. Williamson [1966], and others identified as "managerialists," argue that value or profit maximization is nondescriptive of firm behavior. These attempts at reformulating the theory of the firm have, in general, been motivated by the conviction that the separation of owners and management in large corporations necessarily implies managements will not always act in the interest of stockholders or the firm. Some have even rejected the fundamental principle of maximizing behavior. Jensen and Meckling [1976] argue that owners and management impose contractual constraints on each other to insure their property rights are protected. Separation of ownership and control thus implies constrained optimizing behavior by managers.

The competing theories of firm behavior lack the generality of the Neoclassical profit maximization theory or its equivalent the value maximization rule. In any event, the issue is not whether the assumptions of the value maximization model of the firm are "true," but whether the model explains firm behavior. In this regard, the profit maximization model of the firm has been proven to be very robust, especially as applied to the financial capital markets.

The second assault has questioned the optimality of aggregate investment-production decisions achieved under a value maximization objective function. The contention is that, except for the case of
perfect markets with certainty, value maximizing production decisions need not be unanimously preferred by the owners for Pareto optimality. Jensen and Long [1972], Fama [1972], Stiglitz [1972], Ekern and Wilson [1974], Leland [1974], Merton and Subrahmanyam [1974], and Grossman and Stiglitz [1977] among others, have examined the problem of whether value maximization is optimal if the certainty assumption is relaxed or if the markets are either incomplete or noncompetitive.

Jensen and Long argued the level of investment which would be optimal in terms of social welfare is less than the level which will maximize utility for all investors. They demonstrated that only if all new investment project returns are uncorrelated with the pre-investment cash flows of all other firms will the private and social welfare criteria give identical investment levels. The reason is the failure of the firm to "internalize" all the risk generated by new investment. (This is just a special case of market failure. The standard textbook case is the failure of the firm to capture all gains to investment, hence the argument for patents, copyrights, etc.)

Fama [1972] examined the source of the Jensen and Long inconsistency between value maximization and Pareto optimality and concluded that Jensen and Long, along with Stiglitz [1972], had violated what he called the "Reaction Principle of a Perfectly Competitive Market." In short, this principle maintains that for a
given initial general equilibrium, the response of firms to production changes by an individual firm are precisely offsetting (a type of Walras condition). In the absence of this principle, markets are monopolistic competitive, and as Fama observed, it is hardly surprising that resource allocation in production is found to be Pareto nonoptimal. There are natural externalities in the production decisions of firms which can prevent the capital market from being perfectly competitive. However, the empirical evidence on the Capital Asset Pricing Model and information efficiency of the capital markets is consistent with the assumptions of a perfectly competitive market. Nevertheless, even with perfectly competitive capital markets there can still be externalities in production decisions which prevent the allocations achieved by value maximizing firms from being Pareto optimal. Failure of resource allocation under the value maximization rule to achieve either a competitive equilibrium or Pareto optimality is not an inherent consequence of uncertainty. Generally, as in the case of certainty, the problem is the failure to satisfy the conditions for perfect competition, such as for example if certain firms have monopolistic access to production techniques.

In the opinion of the Bell Journal Symposium papers, by Ekern and Wilson [1974], Leland [1974], and Merton and Subrahmanyam [1974], value maximization yields Pareto efficient resource allocations provided: (i) markets are purely competitive, and (ii) any investment
proposal does not alter the state-distribution of returns available in
the economy, or shareholders' utility behaviors are ordinarily
described by mean-variance portfolio choice.

It has frequently been suggested that the existence of risky debt
(i.e., not default risk free) makes investment decision rule
ambiguous. Maximizing stockholders' wealth, maximizing bondholders' wealth, or maximizing the value of the firm (combined stockholder - bondholder wealth) could imply three different investment decisions. Fama [1978] established that indeed different investment decisions can be implied by these three rules, but that if the firm does not adopt what this study refers to as the "value maximization rule," then it pays outsiders to acquire the firm and switch to this strategy. A well functioning market, in pricing a firm's securities, will generally charge the firm in advance for suspected future departures from currently declared decision rules, including investment - production plans which may deviate from value maximization. Therefore, firms have incentives to self-impose constraints and covenants to assure the market that its policy is to pursue investment - production decisions which will maximize the ex-ante value of the firm. In short, maximizing combined stockholder and bondholder wealth is the only market value rule consistent with a stable equilibrium, and the market naturally provides incentives for firms to elect this rule.
This brief review of the controversy surrounding the theory of the firm, or more precisely regarding the value maximization rule, provides a background against which to assess the model presented in this study. Certainly, as has been implied, sufficient assumptions can always be introduced to derive the result that value maximizing investment - production decisions are both unanimously preferred by all equity holders and socially optimal. Ultimately the question is whether or not the empirical evidence is consistent with the model of the firm which assumes that in the aggregate managements act as if they seek to maximize the value of the firm. Still, the assumptions of the model cannot be ignored. The form of a model is defined by its assumptions. Moreover, one standard by which a model or theory is judged is the logical coherence of the assumptions. This necessitates that the assumptions be made explicit. Therefore, the following assumptions are introduced:

A(1) **Perfect Capital Markets**

There are no transactions costs to investors and firms when they trade in securities. Bankruptcy is costless. There are no taxes. Management can be costlessly compelled to carry out the decision rules set by the firm's security holders.

A(2) **Equal Access**

Information is costlessly available to all individuals and firms, and there are no other barriers preventing equal access to the capital markets.
A(3) Homogeneous Expectations

All investors and firms agree in their estimates of the return prospects for the various securities. This implies that all agents correctly assess the implications of the information for the future prospects of firms.

A(4) Nonsatiation

All consumers and hence all investors, are characterized by a nonsatiable demand for goods and services. Given the consumption \( c_i \) for individual \( i \) which belongs to the consumption set \( C_i \), then there exists \( \hat{c}_i \in C_i \) such that \( \hat{c}_i \) is strictly preferred to \( c_i \).

A(5) Economically Independent Projects

The probability of any outcome from investments of the firm which are not included in the project under evaluation are not altered by the adoption of the project.

The first four assumptions are frequently used in the contemporary financial literature. The nonsatiation assumption A(4) implies the maximization of quasiconcave utility functions is equivalent to wealth maximization. Equity holders are concerned with the investment – production decisions of the firm only as to the effects on their wealth position, because maximizing wealth maximizes the ability to consume - from which all utility is (presumably) derived.

A clear distinction between economic independence and statistical independence is critical to an understanding of assumption A(5). The distinction pertains to the definition of a project. A project is
defined as an investment - production opportunity whose acceptance will not affect the likelihood of accepting any other opportunity available to the firm. Under uncertainty this definition requires the probability distributions of the cash flows of all opportunities disjoint to the project be unaffected by the acceptance of the project. The potential net cash flows of all other projects must be independent of the decision to undertake any particular project. This is what is meant by "economically independent" projects. Economic independence does not imply, however, that the cross correlations of project cash flows are zero. This condition describes statistically independent economic series. The key result is that regardless of how the cash flows from projects are correlated, if they are economically independent they can still be evaluated separately. Any covariance in the cash flows with those of the market is relevant only as information to the determination of project risk.

Given assumptions A(1)-A(5) it has been shown that, with complete markets, competitive profit-maximizing production is Pareto efficient in the production side. It is also Pareto optimal in incomplete markets if either the returns of productive investments are spanned by portfolios of existing securities, or there is universal portfolio separation, in which all consumers divide their wealth between the same two assets, namely the market portfolio and the risk-free security. Moreover, the aggregate equilibrium value of any stream received by investors in the market is independent of how it is
divided into the debt or equity streams of one or more firms. This is the Value Additivity Principle introduced by Schall [1972] and Haley and Schall [1973]. The principle states that the value of an individual cash flow stream to bondholders or stockholders equals the sum of the values of any set of streams into which the stream might be divided. In other words, the market is synergy-free. Perfect market arbitragers always pay for any stream the amount which they can receive by reselling that stream to other investors in the form most desired by those investors. Hence, regardless of how the firm initially offers its earnings to the market (i.e., interest or dividend payments) the firm will have the same value.

The Value Additivity Principle implies that a valuation equation can be derived at the microeconomic level of the firm and then empirically evaluated at the macroeconomic level of the business sector without introducing equation misspecification. Economic models are frequently derived for a microeconomic unit and then tested with aggregated data by explicitly assuming homogeneous microeconomic units but implicitly assuming economically independent behavior. We have merely identified sufficient conditions to guarantee additivity.

For a world defined by assumptions A(1) - A(5) the Value Additivity Principle implies the firm's financing decisions are irrelevant to its total market value. This conclusion was originally derived by Modigliani and Miller [1958].
Modigliani and Miller proved that, in the absence of corporate income taxes and with costless bankruptcy, the cost of capital is independent of the financial capital structure of the firm. The most serious challenge to this theorem arises when the perfect markets assumption A(1) is modified to allow for taxes. If only the corporation is taxed, the choice of capital structure is no longer irrelevant. Modigliani and Miller [1963] demonstrated the equilibrium value of a levered firm is equal to the value of the firm were it unlevered plus the equilibrium value of the firm's debt multiplied by the tax rate.\footnote{12} By increasing the amount of debt outstanding the firm can increase its aggregate value over what the value would be if it were unlevered. That is, the cost of capital can be continuously lowered with increased leverage (corner solution).

The Modigliani-Miller theorems play a crucial role in the derivation of the business sector model presented in this study. Consequently, it is important that the limitations of the theorems be fully explored. The conclusion that the cost of capital can be continually lowered with leverage fails to take into account Section 385 of the U. S. Corporate Tax law. According to this section, debt will be classified as de facto equity for tax purposes if the debt to equity rate is extreme.\footnote{13} This suggests an optimal debt to equity ratio, and hence an optimal financial structure, may exist. Likewise, if assumption A(1) is further relaxed to permit bankruptcy costs, and if the probability of bankruptcy increases at an increasing rate with
the degree of leverage, then an optimal interior financial capital structure for the firm will obtain. Both Section 385 and bankruptcy costs imply investment - production decisions cannot be separated from the source of funds decision. Recently, however, Miller [1977] suggested that in a general equilibrium context which includes a progressive personal tax system, the individual corporation derives no net tax benefit from debt financing. This result occurs in spite of the fact an optimal interior debt to equity ratio exists for the corporate sector as a whole. The basic rationale for this result was explained by Miller,\(^\text{14}\)

Let me assure you that this result is no mere sleight-of-hand due to hidden trick assumptions. The gain evaporates or turns into a loss because investors hold securities for the "consumption possibilities" they generate and hence will evaluate them in terms of their yields net of all tax drains. If, therefore, the personal tax on income from common stocks is less than that on income from bonds, then the before-tax return on taxable bonds has to be high enough, other things equal, to offset this tax handicap. Otherwise, no taxable investor would want to hold bonds. Thus, while it is still true that the owners of a levered corporation have the advantage of deducting their interest payments to bondholders in computing their corporate income tax, these interest payments have already been "grossed up," so to speak, by any differential in the taxes that the bondholders will have to pay on their interest income. The advantage of deductibility at the one level thus merely serves to offset the disadvantages of includability at the other.

But we can say more than this. Any situation in which the owners of corporations could increase their wealth by substituting debt for equity (or vice versa) would be incompatible with market equilibrium. Their attempts to exploit these opportunities would lead, in a world with progressive income taxes, to changes in the yields on stocks and bonds and in their ownership patterns. These
changes, in turn, restore the equilibrium and remove the incentives to issue more debt, even without invoking bankruptcy costs or lending costs as a deus ex machina. (pp. 267-268)

Several other authors have examined the effects of taxes on the optimal financial strategies of firms. Many of these works are interesting and enlightening, but they do not deserve discussion here because the basic point has been established: In the context of corporate and personal taxes, the general equilibrium effect of the firm's financing decisions on the optimal investment-production decisions is unknown. Because of this, the rate of return to the firm model developed below is a general specification. Conceptually, it permits either the strong form of the Modigliani-Miller theorem (irrelevance of financial mix), the corner solution case (100 percent debt financing), or an interior solution (optimal debt to equity mix). Any subsequent empirical estimation of the equation would signify which specification best explains actual market conditions.

It should be clear that the same assumptions which support the irrelevance of financial structure theorem are sufficient for inflation tax neutrality, save the Mundell Effect (i.e., the excess burden caused by portfolio adjustments to the own-price effect of money demand corrections for changes in the expected future inflation tax rate). Certainly it is true with equal access, perfect capital markets (except for the inflation tax), and homogeneous expectations, there can be no ex-ante debtor-creditor wealth transfer. The weak
form Fisherian Hypothesis for interest rates must prevail. If, on the other hand, all investors do not have equal access to information, then as was demonstrated in the previous Essay, an inflation caused wealth transfer can occur as between debtors and creditors. In effect, the rate of return earned by one group, say debtors, would exceed that earned by the other group, creditors. But this is just a special case of divergent borrowing and lending rates, which Hirshleifer [1958] has shown destroys the separation theorem (cf., note 1, Section A).

The purpose of theory is to explain and predict. A theory which explains diverse phenomenon by appeal to diverse sets of assumptions is less robust for its lack of coherence than a theory which draws from a single set of premises. Theory is systematically organized knowledge applicable to a wide range of circumstances. It is important, therefore, to recognize the communality between the sufficient assumptions for the optimality of value maximization investment - production decisions and the irrelevance of financial structure on the one hand, and the assumptions for inflation tax endogeneity on the other. The null hypothesis is that the inflation tax is endogenous at the time of imposition. A model of the firm which implies or assumes unequal access to information and markets, heterogeneous expectations, or satiation would be inherently self-contradictory to the null hypothesis. This is undesirable, not just because of Ockham's razor and the desire for congruent
theoretical structure, but because of the limitations to explanatory power, especially in empirical testing. If the assumption set for the theory of the firm is a subset of the assumption set for inflation tax endogeneity, then a failure to observe endogeneity in the estimated model will permit the rejection of the null hypothesis. The fact that the model of the firm is also rejected would be inconsequential to the null hypothesis. Hence, the microeconomic foundations for the model of the firm are of paramount importance in the evaluation of the effects of inflationary finance.
C. CONCEPTS AND PRELIMINARY RELATIONSHIPS

A world where the managements of firms adhere to the value maximization rule is characterized as one where the objective function of the firm is to maximize the rate of return to the entity. In this section certain concepts, definitions, and relationships are introduced which are later exploited to derive an equation of the equilibrium expected rate of return to the firm. The equation is derived by initially assuming the general price level is constant over time. After the basic rate of return equation is developed, the no inflation assumption is relaxed and inflationary expectations are introduced.

The equilibrium expected rate of return to the firm equation begins with the income statement. Symbolically the income statement may be written as:

\[ EQ(1) \pi_t = (S_t - O_t - W_t - \delta KC_{t-1} - R_t)(1 - \tau) \]

where, 
- \( S_t \) = Sales
- \( W_t \) = Wage and salary expense
- \( \delta KC_{t-1} \) = Depreciation expense based on the historical cost (c) of depreciable capital
- \( \delta \) = Depreciation rate
\[ \text{\(KC_{t-1}\) = Beginning of the period} \]
\[ \text{\(O_t\) = Other operation expenses} \]
\[ \text{\(R_t\) = Interest expense} \]
\[ \text{\(\tau\) = Corporate tax rate, assumed constant} \]
\[ \text{\(\pi_t\) = Net accounting income} \]

Certain additional assumptions will simplify the derivation of the equilibrium rate of return to the firm model. All of these, with the exception of A(7), are subsequently relaxed.

A (6) The general price level is constant over time.
A (7) All economic activity (e.g., cash flows) occurs at the end of the period.
A (8) Firms acquire only one asset, namely real business capital (K).
A (9) All debt and equity instruments are perpetuities.
A(10) The corporate tax rate \(\tau\) is constant and certain.

It is convenient to modify EQ(1) by netting sales (S) and other operating expenses (O) to obtain "net operating cash flow before wages." Netting all other operating expenses against sales, thus leaving only wages, interest, depreciation and taxes as behavior variables, presumes that during inflationary periods the equilibrium expected rate of return to the firm can be accurately modelled without accounting for any inflation tax induced distortions in the cost of
other operating items. There is no evidence to suggest differential inflationary effects between wages and interest on the one hand, and other operating expenses on the other. If wages and interest are distorted by the inflation tax, as the literature maintains, then almost certainly the costs of materials, office expenses, heat-light-and power, advertising, rent and other operating expenses will be distorted in some manner. Nevertheless, these items are netted against revenues in order to keep the rate of return equation to manageable dimensions.

Equation EQ(1) can be rewritten as EQ(2), where \( \pi^* \) stands for "net operating cash flow before wages."

\[
\pi_t = (\pi^*_t - W_t - \delta K_{t-1} - R_t)(1 - \tau)
\]

Because net income is an accrual measure of the income available to stockholders, EQ(2) must be modified to obtain an equation of the net cash flows to the firm irrespective of how the flows are divided between payments to debtholders and stockholders. ¹ Accordingly, let \( \pi_t \) represent the end of the period net cash flow to the firm:

\[
\pi_t = (\pi^*_t - W_t)(1 - \tau) + \tau(\delta K_{t-1} + R_t)
\]
It will prove helpful to define a cash flow variable which is not directly determined by either the financial structure of the firm or the capital-labor ratio. This variable, $\hat{\pi}$, is the tax adjusted net operating cash flow before wages.

$$\text{EQ}(4) \quad \hat{\pi}_t = \pi^*_t (1 - \tau)$$

According to A(8) the firm invests only in real capital ($K$), i.e., it maintains neither money balances nor inventories. If the firm is also unlevered its balance sheet equation is simply,

$$\text{EQ}(5) \quad V^f_t = K_t = V^e_t,$$

where $V^f_t$ = value of the total assets of firm $f$,

$V^e_t$ = the value of stockholders' equity for firm $f$,

$K_t$ = the value of real capital.

Provided $V^f$ is continuously stated at its equilibrium market value, the equilibrium expected rate of return to the firm is defined (without proof for now) by EQ(6). Equation EQ(7) is the equilibrium expected value of the firm equivalent expression to EQ(6). The tilde (\) denotes random variable. The term $\phi_t$ refers to the information set used by the market at time $t$ to derive the expected value of the
EQ(6) \[ E_t(\tilde{r}_{t+1}^f | \phi_t) = \frac{1}{\lambda_t^f} \left[ E_t(\tilde{v}_{t+1}^f | \phi_t) + E_t(\tilde{\pi}_{t+1} | \phi_t) - E_t(\tilde{I}_{t+1} | \phi_t) - \lambda_t^f \right] \]

EQ(7) \[ E_t(\tilde{v}_{t+1}^f | \phi_t) = E_t(\tilde{I}_{t+1} | \phi_t) + \left[ 1 + E_t(\tilde{r}_{t+1}^f | \phi_t) \right] \lambda_t^f - E_t(\tilde{\pi}_{t+1} | \phi_t) \]
variable under the conditional expectations operator $E_t$. It is not necessary for $\varnothing$ to be the same for each variable in the equations above. One symbol is used to conserve notation.

The equilibrium expressions EQ(6) and EQ(8) are corollaries to the present value discounted net cash flow valuation mechanism. That this particular valuation scheme characterizes the determination of equilibrium capital values is assured by the assumption that the value maximization rule is the correct principle for private investment - production decisions. For a given sequence of expected rates of return to the firm over the life of the current real capital, the equilibrium expected value of the firm is given as EQ(8). A formal proof that EQ(6) and EQ(7) are market equilibrium pricing conditions, given the assumptions set-out above, is presented shortly.

\[
\text{EQ}(8) \quad E_t(\tilde{v}_t^f|\varnothing_t, K_t) = \sum_{k=1}^{\infty} E_t(\tilde{\pi}_{t+k}|\varnothing_t, K_t) \prod_{j=1}^{k} E_t(1 + \tilde{r}_{t+j}^f|\varnothing_t)^{-1}
\]

Although by A(6) inflation is assumed to always be zero, it is an appropriate time to point out that the endogeneity of the inflation tax ultimately depends on whether $\varnothing_t$, and other such information sets, encompass all available data, including all data pertinent to estimating future inflation rates. More accurately, the critical issue is whether the set of information that the market uses to
determine the value of the firm at time t is equal to the set of information available at time t which is relevant for determining firm values. This presumes that the joint probability density function for firm values at time \( t+k \) \((k>0)\) assessed by the market at time t on the basis of the information used by the market equals the "true" joint probability density function for values of the firm at time \( t+k \) which is implied by the full information set at time t. In short, the issue is whether or not the capital markets are efficient. This theoretical link between efficient markets (not just the capital market) and inflation tax endogeneity was established in the previous essays. The point which warrants repeating here is that the efficient market assumption is sufficient for the weak form of the Fisherian Hypothesis, while the latter is necessary for the former.

Notwithstanding the conceptual significance of the information set, both its symbol \( \emptyset \) and the random variable notation will be omitted in subsequent equations, except in those instances where clarity is best served by their inclusion. This is to keep the notation to a minimum.

Equilibrium equations such as EQ(6) and EQ(8) are unsuited for assessing the effects of inflation tax in the business sector. The problem is the lack of a behavioral specification. All of the terms in the equilibrium pricing equations are endogenous. Furthermore, there is no way to distinguish between changes in the value of the
firm caused by real variables and changes in \( V^f_t \) caused by the inflation tax. What is desired, therefore, is a model of a form similar to EQ(8) which specifies the real determinants of the value of the firm. The remainder of the study is devoted to reconstructing EQ(6) to suit the needs of this inquiry.

Equation EQ(10) is the capitalized form of EQ(9) which is derived by substituting EQ(4) into EQ(3). The capital value \( V^f_t \) is the

\[
\text{EQ}(9) \quad \pi^*_t = \hat{\pi} + \tau(W_t + \delta K_{t-1} + R_t) - W_t
\]

\[
\text{EQ}(10) \quad V^f_t = V^f_t + \tau PV_t(E_t(R)) + \tau PV_t(E_t(\delta K_{t-1})) + (\delta - 1) PV_t(E_t(W))
\]

where \( V^f_t = \sum_{k=1}^{\infty} E_t(\hat{\pi} + k) \prod_{j=1}^{k} E_t(1 + r_{t+j}^f)^{-1} \)

\( PV_t(E_t(z)) \) = the discounted present value at time \( t \) of the sequence of expected future values of \( z \).

\[
= \sum_{k=1}^{\infty} E_t(z_{t+k}) \prod_{j=1}^{k} E_t(1 + r_{t+j}^f)^{-1}
\]
value the firm would have if it were unlevered and if human capital were included in the balance sheet as an asset. That is, \( \hat{V}_t^f \) is the value of the firm at time \( t \) provided it is wholly financed by stockholders and all expected future wage payments are capitalized at the same rates as applied against \( \pi^* \). What \( \text{EQ}(10) \) says is that the value of the firm at time \( t \) is equal to the value of \( \hat{V}_t^f \) plus the tax adjusted discounted present values of: (1) the expected future interest payments on outstanding debt, (2) the expected future depreciation on the capital investment at time \( t-1 \), and (3) the expected future wage payments. By assumption \( \text{A}(8) \) the firm maintains no open positions in any other asset except real depreciable capital, \( K_t \). Thus \( K_t \) and \( \hat{K}_t \) could be substituted for \( V_t^f \) and \( \hat{V}_t^f \) respectively.

Equation \( \text{EQ}(10) \) is in fact simply an extension of the Modigliani-Miller theorem for the value of a levered firm. Recall that Modigliani and Miller [1963] demonstrated that the equilibrium value of the levered firm \( V_L^f \) may be written as \( \text{EQ}(10-a) \). Because Modigliani and Miller were concerned only with whether the capital structure of a firm, as determined by its financing decisions, affects its cost of capital, they had no reason to explicitly consider the firm's wage bill. They did not separate out \( (1-\tau)PV_t[\varepsilon_t(W)] \) as in \( \text{EQ}(10) \). Likewise, Modigliani and Miller made no provision for the specifics of the U. S. Corporate Tax law, in particular as it
EQ(10-a) \[ V_{L,t}^f = V_{U,t}^f + \tau B_{L,t} \]

where, \( V_{U,t}^f \) = equilibrium value of the firm without leverage,

\( B_{L,t} \) = equilibrium value of the levered firm's debt.

relates to depreciation. By not distinguishing between net cash flows (\( \pi \)) and net income (\( \tilde{\pi} \)), they implicitly assumed that all investment costs are deductible in the period of acquisition. This is the same as assuming that \( E_t(\delta^C_{t+j})_{t+j} \) (\( j \geq 0 \)) in EQ(10) is zero for all future periods. If wage payments are not singled out, and if depreciation is ignored, then EQ(10) would be rewritten as EQ(10-b).

EQ(10-b) \[ V_t^f = V_{U,t}^f + \tau PV_t(E_t[R]) \]

Finally, Modigliani and Miller defined the tax deductible cash flow to bondholders as \( rB_L \), which excludes principal repayment. Because future repayment of principal is a cash flow valued by bondholders, the exclusion in EQ(10-a) of the present value of the future repayment implies the bonds are perpetuities. If \( PV_t(E_t[R]) \) refers to perpetuities then there is no difference between our term and \( B_L \) in EQ(10-a). The perpetuity assumption A(9), however, is not necessary for the validity of EQ(10). What has been demonstrated, therefore, is that the Modigliani and Miller value of the firm equation [EQ(10-a)] is a special case of EQ(10).
Ideally we desire a measure of the expected rate of return to the firm based on \( V^f \) which in equilibrium behaves in a one to one correspondence to the expected rate of return measured with respect to \( \hat{V}^f \) in EQ(10). The significance of \( \hat{V}^f \) is attributable to the fact that by definition it is invariant to the capital structure of the firm. Furthermore, it does not discriminate between capital and labor as assets of the firm. Human capital does not appear as an asset in the traditional balance sheet because of accountants' objectivity principle. According to this principle, financial accounting statements should be based on actual, verifiable events and should be reported in an unbiased manner. There is no means of satisfying this condition with respect to human capital since the firm does not acquire property rights to humans. This of course does not imply an economic stock equivalent to future labor payments does not exist, just as there is a stock equivalent \((V^f)\) to the flows \((\pi)\) earned by capital suppliers.

There is no reason why \( V^f_t \) should be a unique mapping of \( \hat{V}^f_t \). It is noteworthy, therefore, that a one to one correspondence does hold as between two elements of the total return, \( \pi_t/V^f_t \) and \( \hat{\pi}_t/\hat{V}^f_t \). This fundamental relationship, which we title Proposition I, implies that the ratio of net cash flow to the equilibrium value of the firm is independent of the capitalization of
PROPOSITION I

Given: \( V_t^f = K_t \)

Given: \( E_t(r_{t+j}^f) = E_t(r_{t+1}^f) \quad \forall j \geq 1 \)

Given: EQ(7): \( E_t(V_{t+1}^f) = E_t(I_{t+1}) + [1 + E_t(r_{t+1}^f)] V_t^f - E_t(\pi_{t+1}) \)

\( E_t(\hat{V}_{t+1}^f) = E_t(I_{t+1}) + [1 + E_t(r_{t+1}^f)] \hat{V}_t^f - E_t(\hat{\pi}_{t+1}) \)

Then: \( \frac{E_t(\pi_{t+1})}{V_t^f} = \frac{E_t(\hat{\pi}_{t+1})}{\hat{V}_t^f} \)
human capital and the financial capital structure of the firm. The assumptions are sufficient for its proof. The proof appears in Appendix A to this essay.

Assumption A(8), that the firm invests only in real depreciable capital is now relaxed. This introduces several complications, but none of the theoretical relationships established thus far are affected in principle.

With imperfect information the firm will find it necessary to maintain money balances for transactions needs and inventories for precautionary reasons (i.e., avoid stockouts). If either the equal access assumption A(2), or the homogeneous expectations assumption A(3) fails then firms may demand money and inventories for speculative motives. Accounts receivable and accounts payable are also observed, so that the balance sheet equation becomes:

\[
EQ(11) \quad M_t + A^+_t + I_t + K_t + T_t = A^-_t + B_t + V^e_t
\]

where, \( M_t \) = money balances,

\( A^+_t \) = accounts receivable,

\( I_t \) = inventory,

\( T_t \) = land (i.e., "terra"),

\( A^-_t \) = accounts payable (trade debt),

\( B_t \) = bonds,

\( V^e_t \) = stockholders' equity.
The distinction must be made between "investment" in the narrow sense of the word and "investment" in the broad sense, which shall be denoted $I^*$. Used in the narrow sense, investment ($I$) refers to acquisitions of real business capital (equipment, buildings, leasehold improvements). In the broad sense, however, investment ($I^*$) refers to the acquisition of any asset. In the latter sense, money balances are an asset or investment of the firm, just as are inventory and equipment. The firm holds cash because it provides investment benefits, otherwise cash would either be provided to the owners (dividends) or invested in other asset forms.

The equilibrium expected (present) value of the firm was defined above by EQ(7). Behaviorally this value is derived by the discounted present value mechanism as described by EQ(8). These crucial valuation equations must now be rewritten to be consistent with the relaxation of assumption A(8). This is readily accomplished by substituting $I^*$ for $I$ in EQ(7). Equation EQ(8) can be generalized by removing the condition that the future flows $\pi$ pertain only to capital at time $t$. This introduces another term, the expected future investment flows $E_t(I^*_{t+k})$. The valuation equations are therefore given as EQ(12) and EQ(13).

The possibility of non-zero balances for cash, receivables, inventory, and payables does not change the basic principle that the equilibrium expected value of the firm is equal to the discounted
present value of its expected future net earnings. The problem is with the definition of "future net earnings." In addition to simplifying the firm's balance sheet equation, assumption A(8) simplifies the definitions of income. Income earned by the firm is equal to the earnings which could be distributed to the holders of claims to the firm's assets, that is to bondholders and stockholders. With assumption A(8) this income measure is equal to the accountants' net income (\( \bar{\pi} \)), corrected for the accrual adjustment of depreciation expense (\( \delta K^C \)), plus interest payments to bondholders (R), i.e., \( \pi = \bar{\pi} + \delta K^C + R \). The income measure \( \pi \) is just cash earnings from operations. When A(8) is relaxed, however, the real equivalence of \( \pi \) and \( \bar{\pi} \) is seriously distorted by the numerous accrual adjustments made to \( \bar{\pi} \). For example, "revenues", an element of \( \bar{\pi} \), includes cash sales and credit (accrual) sales. Cash receipts from sales, however, is equal to total sales for the period less the change in accounts receivable. Now it is a relatively trivial exercise to convert accountants' accrual net income into cash earnings from operations. The problem is that such an adjustment introduces an arbitrary distinction between asset types. Investment in the broad sense \( I^* \) must be redefined as changes in money balances, plus investment in depreciable assets, plus investment in land. Changes in accounts receivable and inventory, on the other hand, would be treated as adjustments to \( \pi \). Similarly for financing. Changes in bondholdings and shares outstanding would constitute sources of funds from financing. But changes in accounts payable would be offset against
\[ \text{EQ(12)} \quad E_t(v^f_{t+1} | \phi_t) = \sum_{k=1}^{\infty} \left \{ E_t(\pi_{t+k} | \phi_t) - E_t(I^*_t | \phi_t) \right \} \cdot \prod_{j=1}^{k} E_t(1 + r^c_{t+j} | \phi_t)^{-1} \] 

\[ \text{EQ(13)} \quad E_t(v^f_{t+1} | \phi_t) = E_t(I^*_t | \phi_t) + [1 + E_t(r^f_{t+1} | \phi_t)] V^f_t - E_t(\pi_{t+1} | \phi_t) \] 

where \( I^*_t = \Delta M_t + \Delta A^*_t + \Delta I_t + \Delta I_t + \Delta T_t \) 

\[ \text{EQ(14)} \quad E_t(\beta^f_{t+1}) = \frac{1}{V^f_t} \left [ E_t(\beta_{t+1}) + E_t(R_{t+1}) - E_t(v_{t+1}) \right ] \] 

where \( V_t = \text{economic depreciation} - \text{accounting depreciation} \) 

\[ E_t(v_{t+1}) = [E_t(I_{t+1}) - E_t(K_{t+1}) + K_t] - \delta K^c_t \]
π. Such divisions would imply, for instance, that the firm's motive for holding money is somehow inherently different than the motive for holding inventories. This is not the case. Consequently, aggregate investment (I*) is defined as the sum of the change in all assets. This, however, requires that the earnings measure π be adjusted by subtracting investment, as in EQ(12) and EQ(13). Yet, as will be shown shortly, the equilibrium expected rate of return to the firm is independent of the definition of I*.

The firm, as an entity, derives its value from the expected future services it will provide. The economic value of these services can be written as a sequence of payments to the firm's equity holders (bondholders and stockholders). The current value of the firm is merely one of an infinite number of point representations of this payments sequence. If the firm should issue more debt, the present value of the payments stream will increase if and only if the funds provided from the debt issuance are used in such a manner that the incremental net operating flows (\hat{π}) are positive. In other words, changing the composition of the financial structure of the firm does not necessarily raise the value of the firm. The value of the firm is affected only by the operating efficiency of the firm vis-a-vis that for similar risk investments.
The equilibrium expected rate of return to the firm is defined by the valuation equation EQ(13), which was submitted without proof, as were the more restrictive valuation equations EQ(6) and EQ(7). Proving that the capital asset pricing equation EQ(13) is a market equilibrium condition requires two separate proofs. The first step is to establish closure. This is equivalent to demonstrating that the numerator in the equilibrium expected rate of return equation equals the total combined earnings of bondholders and stockholders. The second step is to demonstrate the existence of a competitive equilibrium characterized by EQ(13). Traditionally this is done with an arbitrage proof such as the one found in Modigliani and Miller [1958]. This step is omitted here since such proofs are readily found in investment and corporate finance textbooks.

To establish closure for EQ(13) first requires that the periodic income of the firm's equity holders be defined. The general rule applies: economic income is equal to the earnings due to capital plus (minus) any capital appreciation (depreciation). Bondholders' income, therefore, is equal to interest received plus changes in the value of the beginning of the period bond values. Stockholders' income, on the other hand, is equal to dividends plus any capital gain or loss in the market value of their beginning of the period equity. What closure must establish is that the sum of the earnings of bondholders and stockholders so defined is equal in equilibrium to the capital appreciation (depreciation) of the firm, \( V^f_t - I^*_t - V^f_{t-1} \).
plus the earnings $\pi_t$. Moreover, it must also be demonstrated that $\pi$, when corrected for economic depreciation, exhausts the earnings of the firm. In other words, if EQ(12) and EQ(13) correctly model market equilibrium, then it must be the case that EQ(14) is also a statement of equilibrium capital asset pricing. If this were not the case, which is to say that in a competitive market, as characterized by assumptions A(1) through A(5), the sum of interest to bondholders plus the net income of the firm, corrected by charging economic depreciation (instead of accounting depreciation) against operations, was too high in comparison to the value of the firm, then competition for these earnings streams would raise the price of the firm to the point where the equilibrium expected rate of return was achieved. Similarly if economic net income plus interest payments were too low for the risk level of the firm's investment portfolio, equity holders would have the incentive to divest of their claims to their earnings streams and reinvest in some equally risky investment, but one earning the equilibrium expected market rate of return. One key factor in this is that collectively economic net income plus interest payments exhaust the firms' earnings.

The closure proof is simplified by introducing EQ(15) which is the sources and uses of funds conservation condition. This condition states that funds from operations ($\pi$) plus funds provided by financing ($F$) must be equal to funds expended for investment ($I^*$) and distributions of profit, as in interest payments ($R$) and dividends ($D$).
EQ(15) \( \pi_t + F_t = I^*_t + D_t + R_t \)

where \( F_t = \Delta A^-_t + \Delta B^-_t + \Delta V^E_t \),

given that \( dP_B \) and \( dP_e \) are constant.

Proposition II is the closure proof for EQ(13). The terms \((aB/aP_B)dP_B\) and \((aV^E/aP_e)dP_e\) are the capital gain (loss) for bonds and for shares respectively. In addition to being an integral part of the proof that EQ(13) describes market equilibrium, Proposition II is important in its own right. Bond values and share values are ultimately determined by interest payments and dividends respectively, and the market rate of interest \( r_f \). Yet the value of the firm and the equilibrium expected rate of return to the firm can be written as functions which do not include either interest (R) or dividends. Consequently, as we commented above, the dividend-financing decision does not affect the equilibrium expected value of the firm. The proof of Proposition II is provided in Appendix A.
PROPOSITION II

Given: \[ EQ(11) \quad M_t + A^+ + \hat{I}_t + K_t + T_t = A^- + B_t + V^e_t \]

Given: \[ EQ(15) \quad \pi_t + F_t = I^*_t + D_t + R_t \]

Given: \[ E_t(V_{t+1}) = [E_t(I_{t+1}) - E_t(K_{t+1}) + K_t] - \delta K^C_t \]

Then: \[ E_t(\pi_{t+1}) + E_t(R_{t+1}) - E_t(V_{t+1}) = \]

\[ = \frac{\partial B}{\partial P_B} E_t(dP_B)_{t+1} + \frac{\partial V^e}{\partial P_e} E_t(dP_e)_{t+1} + E_t(D_{t+1}) + E_t(R_{t+1}) \]

\[ = E_t(V^f_{t+1}) + E_t(\pi_{t+1}) - E_t(I^*_t + 1) - V^f_t \]
D. THE EQUILIBRIUM EXPECTED RATE OF RETURN TO THE FIRM:

THE BASIC MODEL

The previous section presented the basic concepts and relationships relevant to a derivation of the equation of the equilibrium expected rate of return to the firm equation. We now derive the general theoretical expression for this variable. Several of the variables are only introduced as conceptual terms. Their derivation and properties are examined in the following section. All the assumptions A(1) - A(10) are in force, with the exception of A(8), that firms acquire only one asset (real business capital).

A notation convention will be adopted. Rather than distinguish between the conditional expectations of a variable and its particular value, such as in $E_t(\pi_{t+1}|\varrho_t)$ and $\pi_t$, we shall omit the expectations operator and adopt the convention of allowing the time subscript to indicate expectation. Thus, any variable with the time subscript t, such as $V^f_t$, will be understood to refer to the time t-1 conditional expectations of that variable, $E_{t-1}(V^f_t|\varrho_{t-1})$. On the other hand, any variable with the time subscript t-1 is known for certain at time t and requires no expectational operator.
From EQ(13) the equilibrium expected rate of return to the firm is given by a modified EQ(6) to reflect the relaxation of assumption A(8). According to Proposition I, \( \hat{V}^f \) and \( \hat{\pi} \) can be substituted for

\[
EQ(6) \quad r^f_t = \left( V^f_t + \pi^* - I^*_t - V^f_{t-1} \right) / V^f_{t-1}
\]

their counterparts in EQ(6). Then solving EQ(6) for \( \hat{\pi} \) and making the appropriate substitutions with EQ(2) and EQ(4) we obtain EQ(16). This equation describes the determinants of the firm's expected net income. Equation EQ(17) is obtained by substituting the present discounted valuation equation EQ(10) for \( \hat{V}^f_t \) in EQ(16) and dividing through by \( V^f_{t-1} \). The result is the net income rate of return equation.

As Proposition II revealed, the equilibrium expected rate of return to the firm may be written as a distributional return to equity holders. Applying EQ(14), the net income rate of return equation is transformed into an equation of the rate of return earned by the firm as an entity, which is denoted \( \zeta^*_t \).
Equation EQ(18) is a behavioral model of the equilibrium expected rate of return to the firm, given the assumptions set out in the previous section. However, EQ(18) can be simplified in an important way by applying Proposition III. This proposition has no meaning in its own right. Its sole purpose is to establish an equality which can then be substituted into EQ(18) to eliminate all end of the period present value terms, such as $\psi_t$, thereby reducing the number of variables for which one period ahead expectations are required.

Because Proposition III is merely a means to an end, more stringent premises are used than are required for its proof. Premises (i) and (ii) are annuity assumptions, whereas the proof, although considerably more involved, achieves the same result even if the sequences of expected future values for $\delta K_{t-1}^C$, $R_t$, $W_t$, and $r_t^f$ are each comprised of different elements. Of course it may well be that the annuity assumption is a good approximation of how the market's expectations are generated. The third premise imposes no restrictions. It is merely a symbolic expression that the correspondence between stock and flow values pertains only to the stream of expected future flows generated by the asset (liability) on hand at time $t$. The proof of Proposition III is presented in Appendix B.
EQ(17) \[ \frac{\bar{\pi}_t}{V^f_{t-1}} = \frac{(1+r^f_t)}{V^f_{t-1}} \left[ V^f_{t-1} \cdot \tau^\psi_{t-1} - \tau^\Lambda_{t-1} + (1-\tau)\Gamma_{t-1} \right] \]

\[ - \frac{1}{V^f_{t-1}} \left[ V^f_{t} - \tau^\psi_{t} - \tau^\Lambda_{t} + (1-\tau)\Gamma_{t} \right] \]

\[ - \frac{(1-\tau)}{V^f_{t-1}} \left[ W_t + R_t + \delta K^C_{t-1} \right] + \frac{I^*_t}{V^f_{t-1}} \]

where, \( \psi_t \) = the time t-1 expectation of the present value at time t of the future depreciation sequence for \( K^C_{t-1} \)

\( \Lambda_t \) = the time t-1 expectation of the present value of time t of the sequence of future interest rates for \( B_{t-1} \)

\( \Gamma_t \) = the time t-1 expectations of the present value at time t of the sequence of wage payments for labor employed at t-1
EQ(18) \( \zeta_t^* = (1 + \tau_t^f) [ 1 - \tau \psi_{t-1} - \tau \lambda_{t-1} + (1-\tau) \gamma_{t-1} ] \)

\[- \frac{1}{V_t^f} \left[ V_t^f - \tau \psi_t - \tau \Lambda_t + (1-\tau) \Gamma_t \right] \]

\[- (1-\tau) \frac{W_t}{V_t^f} - \frac{V_t}{V_t^f} - (1-\tau) \frac{\delta K_t^C}{V_t^f} \]

\[+ \frac{I_t^*}{V_t^f} + \frac{\tau R_t}{V_t^f} \]

where, \( \psi_{t-1} = \frac{\psi_t}{V_t} \)

\( \lambda_{t-1} = \frac{\Lambda_t}{V_t^f} \)

\( \gamma_{t-1} = \frac{\Gamma_t}{V_t^f} \)

\( \nabla_t = \text{economic depreciation - accounting depreciation} \)
PROPOSITION III

Given

(i) \( E_t(\delta K_{t-1,t+j}) = \delta K_{t-1} \quad \forall \quad j \geq 0 \)
\( E_t(R_{t+j}) = R_t \quad \forall \quad j \geq 0 \)
\( E_t(W_{t+j}) = W_t \quad \forall \quad j \geq 0 \)
\( E_t(\pi_{t+j}) = \pi_t \quad \forall \quad j \geq 0 \)

(ii) \( E_t(r^f_{t+j}) = r^f_t \quad \forall \quad j \geq 0 \)

(iii) \( \psi_t = \psi_{t-1}(1 + r^f_t) - \delta K_{t-1} \)
\( \Lambda_t = \Lambda_{t-1}(1 + r^f_t) - R_t \)
\( \Gamma_t = \Gamma_{t-1}(1 + r^f_t) - W_t \)

Then
\[
\frac{I^*}{V^f_{t-1}} = \frac{1}{V^f_{t-1}} \left[ V^f_t - \tau \psi_t - \tau \Lambda_t + (1-\tau)\Gamma_t \right]
+ \frac{1 - \tau \psi_{t-1} - \tau \lambda_{t-1} + (1-\tau)\gamma_{t-1}}{V^f_{t-1}}
= \left[ 1 - \tau \psi_{t-1} - \tau \lambda_{t-1} + (1-\tau)\gamma_{t-1} \right] \left[ \frac{\pi_t}{V^f_{t-1}} - r^f_t \right]
\]
The basic theoretical equation for the equilibrium expected rate of return to the firm is obtained by substituting Proposition III for the appropriate terms in EQ(18). The result is EQ(19), where \( \zeta_t^* \) is the equilibrium expected real rate of return. Notice that as a byproduct of Proposition III the investment variable \( I^* \) drops out of the equation. This implies that the value of the firm does not depend on any special assumptions about what future benefit stream the market "really" capitalizes. This is an important corollary to Proposition III, which we would like to claim is a novel discovery. In point of fact, however, the perceptive reader will recognize this result as just a special case of the Miller and Modigliani [1961] "irrelevance of the dividend policy" theorem.

The equilibrium rate of return to the firm equation is also the objective function for the firm. Abstracting from financing decisions, the firm chooses \( I^* \) which will maximize \( \zeta^* \), subject to the budget constraint represented by EQ(15), the sources and uses of funds condition. The well known solution to this value maximization problem is to invest until the marginal product of capital is just equal to the cost of funds. Clearly, the theory of capital formation is fully contained in the constrained maximization problem characterized by EQ(19) and EQ(15).
\[ \tau_t^* = \frac{\pi_t}{v_{t-1}^f} [1 - \tau \psi_{t-1} - \tau \lambda_{t-1} + (1-\tau)\gamma_{t-1}] \]

\[ - (1-\tau) \frac{\delta K_{t-1}^C}{v_{t-1}^f} - (1-\tau) \frac{w_t}{v_{t-1}^f} + \frac{\tau R_t}{v_{t-1}^f} - \frac{v_t}{v_{t-1}^f} \]
Certain salient features of EQ(19) warrant comment. The first, and perhaps most important, is to note the anticipatory nature of the equation. As observed in the previous essays in this volume, the costs and effects of inflation depend primarily on the errors in expectations of future inflation rates. Consequently, individuals have incentives to collect and process information relevant to the future course of prices in general. For exactly the same reason, they have incentives to anticipate and plan for changes in the future real flows which determine utility by altering the real value of current wealth. Households, managers, and others will exhaust all available information about the joint distribution of future prices so as to maximize their consumption possibilities over time. This implies the firm has incentives to provide assurances, at the lowest possible cost, that statements of investment—production policy can be taken at face value.\(^3\)

In a steady state equilibrium, the expected rate of return to the firm \(\zeta^*\) is equal to the expected cost of capital \(r^C\), and both of these rates are equal to the discount rate \(r^f\). The equality expressed by EQ(19) reflects this. The observed rate of return, however, is described by the behavioral specification of EQ(19) and an unexplained parameter. Stochastic disturbances unaccounted for by the equation will separate \(\zeta^*\) and \(r^C\).\(^4\) Moreover, the markets will not always be in a long-run equilibrium. Consequently, EQ(19) is not an identity.
E. INFLATION AND THE EXPECTED RATE OF RETURN TO THE FIRM

The previous section derived an expression for the equilibrium expected real rate of return to the firm, EQ(19). This equation singles out those factors which will be of immediate interest to a study of the effects of the inflation tax on the business sector, namely, wages, interest, and depreciation. The wages variable is included to capture any wages lag (or lead) effects. The debtor-creditor hypothesis can be investigated by the interest rate variables. The depreciation variables may be used to study the effects, during inflationary periods, of the depreciation provisions of the U. S. income tax code. We now address the question of how inflation affects the firm's rate of return as measured by EQ(19).

This section is subdivided as follows. The first three subsections analyze the economic significance of the capitalized depreciation, wages, and interest variables and address the question of how these variables should behave in an inflationary state. Section E.1 defines depreciable real capital \( K \), and derives a substitute expression for the present value of future depreciation \( \Psi_{t-1} \). The variable \( \Psi \) is decomposed into a term for the present value of inflation adjusted (indexed) depreciation, and another term for the present value of future depreciation not allowed for tax purposes. Section E.2 decomposes the present value of future wage payments \( \Gamma \) into its wage rate and labor quantity components. From
this it is shown that the wages lag hypothesis is a more complex issue than the literature traditionally suggests. Section E.3, considers the affects inflation has on the return earned by the firm as influenced by the firm's monetary liability position. In addition, the assumption that all debt and equity instruments are perpetuities is relaxed. This raises the question of how expected future refinancing should be treated in EQ(19). A well specified model of the rate of return earned by the firm for a noninflationary economy need not behave well if inflation is frequent or material enough to warrant inflation expectations. Section E.4, therefore, evaluates the identifiable effects of inflation on $\xi^*$ which are not captured by the explanatory variables in EQ(19). Finally, in Section E.5 the equilibrium expected rate of return to the firm equation is repeated with all modifications for inflation effects.

E.1 The Measurement of Real Depreciable Capital and The Decomposition of the Present Value of Future Depreciation

Early literature which advocated charging periodic income with an allowance for capital consumption clearly intended for the allowance to protect the integrity of invested capital. By allocating the cost of a wasting asset over the life of the asset, management is prohibited from returning investors' principal as apparent distributed earnings. In a noninflationary world, this constraint on management is nearly satisfied by allocating the historical cost of the wasting
assets. With zero inflation, relative price changes tend to be offsetting the more diverse the portfolio of wasting assets. This is not the case in an inflationary environment, however. With inflation the allocation of historical cost never compensates for the consumption of real investment. This is just a special case of the failure of tax neutrality which Samuelson [1964] succinctly stated in his fundamental theorem of tax-rate invariance.

If, and only if, true loss of economic value is permitted as a tax-deductible depreciation expense will the present discounted value of a cash-receipt stream be independent of the rate of tax. (p. 604)

The firm can try to maintain its pre-inflation rate of return and protect invested capital during inflationary periods by charging income with general price level indexed depreciation. If real capital held at the beginning of the period \(K_{t-1}\) was acquired at time \(t_0\) when the general price level was \(P_0\), then the period \(t\) indexed depreciation would be \(\frac{P_t}{P_0} K_{t-1,0}^C\). In one fashion or another this type of indexing is actually permitted for tax purposes in several countries\(^1\). Without an inflation index correction, however, the real rate of return to the firm declines with inflation. For any given value of the decline in real return caused by the inflation tax on capital, there is a corporate income tax rate which yields the same net-of-tax rate of return. Consequently, the inflation tax effect is equivalent to a rise in the corporate income tax rate, and can be analyzed as such. The effects and incidence of
such an (implicit) increase in the corporate income tax rate are adequately described elsewhere and need not be repeated here. The crucial point is that inflationary finance induces real indirect costs through the corporate income tax law which occur regardless of the speed and accuracy with which the economy anticipates future inflation.

Consider the sequence of expected future depreciation for given investment in depreciable assets at time \( t \). Let \( \delta(k,j);t+k \) denote the \( t+k \) period depreciation rate for a class \( k \) asset acquired at time \( j \) (asset classes are defined shortly). This depreciation rate is applied to investment \( I \) of type \( k \) made at time \( j \), which remains an element of the time \( t \) capital assets \( K_t \). Let \( n_k \) be a depreciation life. The time \( t \) discounted value of expected future allowable depreciation then is defined by EQ(20-a). On the other hand, the present value of future inflation indexed depreciation is denoted \( \psi^* \) and defined by EQ(20-b). Both \( \psi \) and \( \psi^* \) are market values.

The present discounted value of future inflation adjusted depreciation for investment \( I_j \) is measured by the expression enclosed by \{ \}, or symbolically as \( (\psi^*_t \mid I_j) \). Summing this expression over all depreciable assets gives \( (\psi^*_t \mid K_t) \), or simply \( \psi^*_t \).
EQ(20-a)

\[ \psi_t = \sum_{k=1}^{m} \sum_{j=t+1-n_k}^{t} \left\{ \sum_{k=1}^{\hat{k}} E_t [\delta_{(k,j)}; t+\hat{k}] I_{k,j} \right\} \left( \prod_{j=1}^{\hat{k}} f_j^{-1} \right) \]

where \( f_j^\hat{c} = E_t (1 + r_{t+j}^f) \)

EQ(20-b)

\[ \psi^*_t = \sum_{k=1}^{m} \sum_{j=t+1-n_k}^{t} \left\{ \sum_{k=1}^{\hat{k}} E_t [\delta^*_{(k,j)}; t+\hat{k}] I_{k,j} \right\} E_t (P_{t+\hat{k}}) \left[ \prod_{j=1}^{\hat{k}} f_j^{-1} \right] \]

NOTE: The \( \delta \) sequence is indexed to the date of investment, but begins in period \( j+1 \).

EQ(21-a)

\[ T_t = \psi^*_t - \psi_t \]

EQ(21-b)

\[ \psi_t = \psi^*_t - v_t \quad \psi^*_t = \frac{\psi^*_t}{v_t^f} \quad v_t = \frac{T_t}{v_t^f} \]
With the aid of EQ(21) it is a trivial matter to decompose $\psi$ into its two principal components. One of these components is of course just $\psi^*$ itself. This is the present value of the future depreciation of real investment, a function solely of the time sequence of scheduled depreciation. On the other hand, $\psi$ is the present value of the sequence of allowable depreciation deductions. Consequently, the difference between these two measures is the present value of the expected future "lost" depreciation, i.e., the tax penalty imposed on invested capital during inflationary periods, which is denoted $T_t$. The decomposition of $\psi_t$ is, therefore, simply defined as previously described in EQ(21-a) and EQ(21-b).

As noted in the Introduction to this essay, there is presently a resurgence of interest in the study of fixed business investment. Several studies have attempted to estimate the effects of inflation on the demand for fixed investment, both as to aggregate investment demand ($I^*$), and to the composition of the demand between structures and equipment. A key determinant in this demand is the discrepancy cost imposed on the firm between replacement cost depreciation and income tax law allowable depreciation, i.e., $T$. Because this cost has been shown to be important in these models it is desirable to briefly address how $T$, or EQ(20), would be estimated. However, as the topic of this study is the theoretical specification and not the estimation of the rate of return equation, the reader may prefer to skip the remainder of this section and continue with Section E.2.
The estimation of Eq(20) requires, among other things, a specification of the depreciation life $n$ and the process by which the planned depreciation sequence is determined. The life of the assets also determines the sequence of past investments which constitute the depreciable capital stock $K^c_t$. The problem then is to simultaneously determine the lives of assets $(n)$, the depreciation rates, and the capital stock.

The first step is to distinguish between machinery and equipment (or "equipment" for short) on the one hand, and structures on the other. The average lives of these investment forms are materially different, in addition to the fact that they are treated under separate tax provisions. Double declining balance can be used for equipment, whereas only the limited declining balance method is allowable for accelerated depreciation of structures. Firms have the incentive to use whichever depreciation method minimizes the present value of future tax payments. This is assured by the value maximization objective function. Generally, this implies the firm will use accelerated depreciation for some subperiod of the life of an asset, and then switch to straight-line depreciation for the remainder of the asset's useful life. The optimal switch-over point is a function of the elected method of depreciation, the life of the asset, and the discount rate. Simulations reveal that, in the absence of
reinvestment, the asset life and the accelerated depreciation rate are the important factors. The optimal switch-over point is relatively insensitive to the discount rate.\textsuperscript{4}

It is assumed then the double declining balance method is used for equipment and a declining balance of 1.50 times the straight line rate is elected for real property. The additional first-year depreciation is ignored.\textsuperscript{5} All equipment have lives of \( n_1 \) periods, and all structures have lives \( n_2 \) periods long. The values for \( n_1 \) and \( n_2 \) must be derived. The algorithm for this is as follows.

Given values of \( n_1 \) and \( n_2 \), the optimal switch-over points \( n^*_1 \) and \( n^*_2 \) are determined. This is defined as the point where the present value of the depreciation sequence is maximized. The depreciation rates are then computed as in EQ(22-a) for equipment and EQ(22-b) for structures. The calculated sequence of depreciation charges for equipment are then computed for observed equipment investment over the chosen sample period. A similar calculation is conducted for structures. The two constructed depreciation sequences are then summed by period and compared to the reported depreciation statistic, such as reported in the Survey of Current Business.\textsuperscript{6} The process is repeated over a range of "realistic" values of asset lives. The optimal pair \((n_1, n_2)\) is defined as the lives for
EQ (22-a) \[ \delta_{e; t+j} = \left( \frac{2}{n_1} \right) (1 - \frac{2}{n_1})^{j-1} \quad 0 \leq j \leq n_1^* \]

\[ = \left( \sum_{j=0}^{n_1^*} \delta_{e; t+j} \right) \frac{(n_1 - n_1^*)^{-1}}{} \quad n_1^* < j \leq n_1 \]

EQ (22-b) \[ \delta_{s; t+j} = \left( \frac{1.50}{n_2} \right) (1 - \frac{1.50}{n_2})^{j-1} \quad 0 \leq j \leq n_1 \]

\[ = \left( \sum_{j=0}^{n_2^*} \delta_{s; t+j} \right) \frac{(n_2 - n_2^*)^{-1}}{} \quad n_2^* < j \leq n_2 \]
EQ(23)

\[ \kappa_t^c = \left( \sum_{k=1}^{n_k} \sum_{j=1}^{n_k-1} \left( \frac{p_{k,t} \cdot I_{k,t-j}}{p_{k,t-j}} \right) \right) - \left( \sum_{k=1}^{j} \delta_{(k,t-j); t-j+k} \cdot I_{k,t-j} \right) \]

where

\[ p_k = \text{price of capital type } k. \]

\[ \delta_{(k,j)} = \text{depreciation rate applied to investment type } k \text{ acquired at time } t-j. \]
which the average absolute value of the difference between the constructed depreciation series and the actual depreciation series is minimized.

Once determined, \( n_1 \) and \( n_2 \) are used to construct the real capital stock \( K^c \) sequence (at historical cost). At any point in time \( t \), real depreciable capital is defined as \( E_0(23) \). Capital \( K \) is the sum of equipment and structures.

E.2 Decomposition of the Present Value of Future Wage Payments

The present discounted value of future wage payments \( \Gamma_t \) is a function of the firm's current labor stock \( L_t \) and the expected future sequence of wage payments. It is the labor currently employed by the firm and not the expected future labor usage which enters into the rate of return to the firm equation. The firm is, at any point in time, its stock of real capital and labor. The value of the firm is the present economic equivalent to the expected future benefit streams to be created by existing capital and labor. Plans to acquire more capital at some future date do not make the current capital more (or less) valuable, save for the informational value such plans may have on the likelihood estimation of the future earnings from currently held capital. The same applies for plans to employ more (or less) labor.
The decomposition of the present value of future wage payments is straight forward. The expected value at time \( t \) of the wage payment at time \( t+1 \) is shown in EQ(24). The discounted present value of the expected future wage rates, denoted \( \hat{\Gamma} \), can be substituted for \( w \), as in EQ(25). \( \hat{\Gamma} \) is in fact a labor price. If the labor market were perfectly competitive and firms were able to buy labor in the same way they buy machinery, then \( \hat{\Gamma} \) would be the equilibrium price for a "unit" of labor.\(^7\) Abolition notwithstanding, labor has a price. This price is customarily represented by the wage rate \( w_t \). The only distinction between \( w_t \) and \( \hat{\Gamma}_t \) is the time horizon for the contract period during which labor services are to be supplied.

It is possible for the wage bill (\( \Gamma \)) to be infinite, in which case \( \tau^* \) is undefined. For \( \Gamma \) to be finite, either the expectations time horizon must be finite, or the average growth rate of real wages must be less than the average discount rate. The latter, in the Neoclassical model, implies real wages grow by the rate of average labor productivity (Harrod natural rate), which is less than the interest rate. The labor force for the firm is held constant, as indicated above.

Treating \( \hat{\Gamma} \) as a labor price allows an important simplification to be made to the equilibrium expected rate of return to the firm equation. As discussed in Section C, the value of the firm \( V_f \) is equal to the value of total investment. Money balances, receivables,
EQ(24) \[ E_t(w_{t+1}) = L_t E_t(w_{t+1}) \]

where \( w_t \) = period \( t \) composite wage rate.

\( L_t \) = period \( t \) labor usage (e.g. employee hours.)

EQ(25) \[ \gamma_t = \hat{L}_t \hat{r}_t \]

where, \( \hat{r}_t = \sum_{j=1}^{\infty} E_t(w_{t+j}) \prod_{k=1}^{j} E_t(1 + r_{t+k} f) \)

EQ(26) \[ \gamma_t = \frac{r_t}{\nu_t} = \frac{\hat{r}_t L_t}{\nu_t} \]

[cf., EQ(18)]
inventory, and depreciable capital are all investment forms in the broader sense of the word "investment." Therefore, the ratio of $\Gamma_t$ to $V_t^f$ is a ratio of labor value to capital value, and the equilibrium expected rate of return to the firm is a function of the relative factor payment $\hat{F}$ and the labor-capital intensity $(L/V^f)$.

The relative price $\hat{\Gamma}_t$ is the key factor by which differential inflation expectations in the labor market influence the rate of return earned by the firm. The specification of this price reveals that the wages lag hypothesis is a more complex phenomenon than customarily represented in the literature. The labor price $\hat{\Gamma}$ is a function both of expected future wage rates and expected future interest rates. For ease of analysis assume a one-period world in which the real discount rate is known with certainty. The relative labor-capital price is defined as $\text{EQ}(27-a)$. Now suppose the same economy is duplicated except that the general level of prices increases over the period. Equation $\text{EQ}(27-b)$ represents the relative price in terms of actual inflation rates and expectation coefficients ($\alpha$ and $\beta$). The expression $\alpha(1 + \rho_{t+1})$ denotes the labor market equilibrium (beginning of the period) inflation expectations adjustment. If $\alpha = 1$ the expected inflation rate is just equal to the actual inflation rate, and the market wage rate is fully corrected for inflation. If $\alpha < 1$ then wages lag prices, and conversely for $\alpha > 1$. The coefficient $\alpha$ is the image of differential inflation expectations under a mapping of labor supply and demand. It is the same as the
\[ \text{EQ}(27-a) \quad \hat{r}_t = E_t(w_{t+1}) \left(1 + r_f^t\right)^{-1} \]

\[ \text{EQ}(27-b) \quad \hat{r}_t = \frac{E_t(w_{t+1}) \{\alpha \left(1 + \rho_{t+1}\right)\}}{(1 + r_f^t) \{\beta \left(1 + \rho_{t+1}\right)\}} \]

\[ \text{EQ}(27-c) \quad \hat{r}_t = \frac{E_t(w_{t+1})}{\prod_{j=1}^{\infty} \left[1 + E_t(\hat{\rho}_{t+1})\right]^j} \]

where \( E_t(\hat{\rho}_{t+1}) \) applies to the labor market

\( E_t(\rho^*_t) \) applies to the capital market
symbol used in the previous essay to explain the wages lag hypothesis. The expression $\beta(1 + \rho_{t+1})$ in the denominator of EQ(27-b) is the analogous capital market inflation expectation. Perfect expectations occur when $E_t(\rho_{t+1})$ equals $\rho_{t+1}$, which obtains if and only if $\beta$ equals unity.

As EQ(27-b) clearly indicates, inflation is endogenous, at least as regards to wages and interest rates, when $\alpha = \beta = 1$; for only in this case does EQ(27-b) collapse to EQ(27-a). For the inflation tax to be endogenous in the labor market requires more than the endogeneity of inflation to wage rates. If $\alpha = 1$ but $\beta < 1$ then the expected real wage rate is higher with inflation just as it is in EQ(27-a) for some unique value $\hat{r} < r$. For example, suppose $E_t(w_{t+1})$ is $\$5.50$, and $r_t$ is 10 percent. Then in the absence of inflation $\hat{\Gamma}_t$ is just 5.00. On the other hand, if the equilibrium expected inflation rate in the labor market is 6 percent, then the expected market wage rate will be $\$5.83$. If for whatever reason the market interest rate is 14.4 percent, implying an expected inflation rate of only 4 percent, the labor price will climb to 5.096. This has the same impact on profits as in the noninflationary economy should the discount rate fall to 7.92 percent or the wage rate be bid up to $\$5.606$. Now presume the actual inflation rate for the period turns out to be 4 percent. The market wage rate will be too high causing firms to lower their demand for labor relative to their demand for real capital. However, if the actual inflation rate is 6 percent
there is no wage rate distortion from inflation (i.e., no wages lag). Nevertheless, firms are induced to demand more labor because the labor contracted at the beginning of the period was based on an erroneous and overstated expectation about the real (end-of-period) wage rate. Exactly what investment-production decisions are affected by the expectation errors depends on the particular structural specification for these decisions. But as this illustration shows, it is conceivable that the burden of the debtor-creditor wealth transfer caused by imperfect inflation expectations in the capital market will be borne in part by labor.

Equation EQ(26) is a statement about labor and capital market relations in yet another important sense. If as the expectations horizon \( n \) in EQ(25) goes to infinity, \( \hat{r}_t \) converges to a finite real number, then any otherwise stable competitive equilibrium in the labor market is also stable with inflation (i.e., when the no inflation assumption A(6) is dropped). This is true for the capital market as well. Hence EQ(26) is also a stability condition. The stability proof is simplest for the case of annuity inflation expectations. Generalizing EQ(27-b) for the multiperiod case gives EQ(27-c), where \( E(\hat{\rho}) \) and \( E(\rho^*) \) are respectively the expected inflation rates in the labor market and the capital market. Clearly for whatever nonzero value is assumed by the ratio \( E_t(w_{t+1})/E_t(r_{t+1}) \) the value taken by \( \hat{r}_t \) is determined solely by the constant \( [1 + E_t(\hat{\rho}_{t+1})]/[1 + E_t(\rho^*_{t+1})] \) and its power \( n \). For \( \hat{r}_t \) to be a
finite real number for any \( n \), this ratio must be a real number \( z \), such that \( 0 \leq z \leq 1 \) (deflation ignored). The proof is more complicated if the series of expected inflation rates are multivalued, requiring as it does the use of fixed point theorems, but the basic idea is the same. This implies there is a limit to which markets or segments of markets can sustain differential inflation expectations.

E.3 The Present Value of Future Interest Payments

It has been shown that the equilibrium expected rate of return to the firm \( \zeta^* \) is a function of the firm's issued debt. This is not necessarily inconsistent with the separation of the investment and financing decisions, a point discussed shortly. The debt-effect on \( \zeta^* \) is captured by the ratio of the present value of the expected future interest payments, \( \Lambda \), to the value of the firm \( V^F \). Clearly this ratio is an indicator of the firm's financial leverage. Indeed, the difference between it and the more traditional indicator, \( B_{t-1}/V_{t-1}^F \), the debt-to-assets ratio, goes to zero as the life of the debt goes to infinity (since the present value of the future principal repayment approaches zero). Parenthetically, it should be noted that this convergence condition holds only if the entire periodic payment to holders of perpetuities is tax deductible to the firm.
Relaxing the perpetuity assumption A(9) raises a complication, namely the necessity to anticipate future capital market conditions for scheduled refinancing dates. Outstanding debt can be refinanced by being redeemed or replaced. In the absence of A(9), outstanding debt at time t will in general have a maturity date prior to the expected future terminal (liquidation) date for the firm, in which case some form of refinancing will usually be required. As a rule firms tend to replace maturing debt with similar obligations. This causal observation suggests that managements link the planned financing stream so as to effectively create permanent funding over the life of the firm. Consequently, when the life of outstanding debt is shorter than the anticipated life of the firm, the market must seemingly discount not only the interest and principal payments for the existing debt, but it must also forecast and discount the interest and principal payments for any replacement instruments.

With perfect capital markets, A(1), all financial transactions, such as refunding, are costless. Given assumption A(1), equal access A(2), and homogeneous expectations A(3), it can be demonstrated that refunding per se has no effect on the value of the firm. This irrelevance proposition is formally expressed by Proposition IV, the proof of which is provided in Appendix B. It is imperative that this Proposition not be misinterpreted. It merely states that the discounted present value of the stream of expected future net cash flows (principal and interest) with maturity n periods ahead is equal
PROPOSITION IV

Let \( r_{b_1} \) = coupon rate for bond 1, and \( r_{b_2} \) = coupon rate for bond 2.

\[ p_1 = \text{principal for bond 1,} \quad \text{and} \quad p_2 = \text{principal for bond 2.} \]

\( i_{m_t} \) = market rate at time \( t \) for risk-class bonds \( b_1 \) and \( b_2 \).

\( n_1, n_2 \) = lives of the bonds.

Given:

\[
B_{10} = \sum_{t=1}^{n_1} r_{b_1} p_1 \prod_{j=1}^{t} \left[ 1 + E_0(r_{m_j}) \right]^{-1} + \prod_{j=1}^{n_1} \left[ 1 + E_0(r_{m_j}) \right]^{-1}
\]

\[
B_{20} = \sum_{t=n_1+1}^{n_1+n_2} r_{b_2} p_2 \prod_{j=n_1+1}^{t} \left[ 1 + E_0(r_{m_j}) \right]^{-1} + \prod_{j=n_1+1}^{n_1+n_2} \left[ 1 + E_0(r_{m_j}) \right]^{-1}
\]

where \( B_{10} \) = value of outstanding bonds at \( t=0 \).

\( B_{20} \) = expected value at time \( t=0 \) of the refinancing bond issued at time \( n_1 \).

continued
PROPOSITION IV  continued

Let \[ \alpha_t = \begin{cases} \frac{r_{b_1}}{\bar{r}_{P_1}} & 1 \leq t \leq n_1 \\ \frac{r_{b_2}}{\bar{r}_{P_2}} & (n_1 + 1) \leq t \leq (n_1 + n_2) \end{cases} \]

Define \( \theta_0 = \) present discounted value of the stream of all bond related net cash flows (from \( t = 1 \) to \( t = n_1 + n_2 \)).

\[
\theta_0 = \sum_{t=1}^{n_1+n_2} \alpha_t \prod_{j=1}^{t} \left[ 1 + E_0(r_{m_j}) \right]^{-1} + \left( \frac{\bar{r}_{P_1}}{\bar{r}_{P_2}} \right) \prod_{j=1}^{n_1} \left[ 1 + E_0(r_{m_j}) \right]^{-1} + \prod_{j=1}^{n_1+n_2} \left[ 1 + E_0(r_{m_j}) \right]^{-1}
\]

Then: \( B_{10} = \theta_0 \)
to the discounted present value of the combined stream of the future net cash flows for outstanding debt and any new (refinancing) debt which together span \( n \) periods. In short, the market value of a currently outstanding bond \( B_t \) is equal to the market value of the claim to the net cash flows associated with \( B_t \) and any sequence of bonds with the same face value which may be issued to refinance \( B_t \).

The Proposition does not imply that any two bonds with identical properties, save their maturity dates, will have the same market value. The significance of Proposition IV is that it implies the market may properly ignore any differences between the lives of the firm's bonds and the firm's income streams. Only the expected future interest payments on outstanding debt matter in the determination of the equilibrium expected rate of return and value of the firm.

The key question for this study is: How does \( \Lambda \) behave in an inflationary environment? To answer this question it is necessary to decompose \( \Lambda \) in much the same way that the labor value \( \Gamma \) was decomposed above. In this case the decomposed equation has even fewer variables because the expected future interest coupon payments are constant over time (assuming the impossibility of default). With the possibility of inflation, \( \Lambda \) is determined as in EQ(28-a). The nominal coupon rate \( \bar{i}_t \) is approximately equal to the real coupon rate \( \bar{r} \) and the capital market's expected inflation rate at the original date of bond issue, denoted simply as \( E(\bar{p}) \). The expected future (market) discount rate is roughly the sum of \( E_t(r^C_{t+k}) \) and \( E_t(p^*_{t+k}) \). Abstracting
from the complications of variable expected future discount rates, EQ(28-a) says that \( \Lambda \) is essentially equal to the ratio of the coupon rate to the market interest rate, weighted by the principal balance \( B \), in other words (EQ(28-b). Conceptually, if not mathematically, this is equivalent to weighting the ratios of the real rates plus the ratio of the expected inflation rates. Accordingly, \( \Lambda \) will change over time if the real discount rate \( r^C \) varies from the coupon rate \( \bar{r} \). This by definition has nothing to do with inflation. It is simply the portfolio adjustment effect. On the other hand, \( \Lambda \) is also a function of differential inflation expectations in a perfectly analogous fashion to the labor market case presented above. At any point in time the capital market may be segmented for whatever reason such that one subgroup of participants, say debtors, has a superior bargaining position vis-a-vis the market's other participants, for example creditors. If this advantage is with respect to inflation forecasting then as time passes \( \Lambda \) will (literally) capitalize this advantage as the spread between \( E(\bar{p}) \) and \( E(\rho^*) \) materializes. An increase in the expected inflation rate will lower the value of \( \Lambda \), cet. par. 9

What has been said here and in the previous essay reasserts the principle that the market value of any financial claim is generated by its relative real return and by differential inflation expectations across markets or across time (i.e., the debtor-creditor hypothesis).
EQ(28-a) \( \lambda_t = \overline{I}_t \overline{B}_t \sum_{j=1}^{n} \prod_{k=1}^{j} \left[ E_t(1 + r_{t+k}^f) E_t(1 + \rho_{t+k}^* -1) \right] \)

where \( \overline{I}_t \) = weighted average nominal coupon rate of return on aggregate principal obligation \( \overline{B} \).

EQ(28-b) \( \lambda_t = \frac{\overline{B}_t \left[ 1 + \overline{I}_t + E(\overline{p}) \right]}{\left[ 1 + E_t(r_{t+1}^c) + E_t(\rho_{t+1}^*) \right]} \)

EQ(29) \( v_t = [v_t^f - v_{t-1}^f] - [\alpha(v_t^f) - \alpha(v_{t-1}^f)] \)

where, \( r_t^f v_t^f = v_t^f + \pi_t - \lambda_t - v_{t-1}^f \)

\( \alpha(r_t^f) \alpha(v_t^f) = \alpha(v_t^f) + \alpha(\pi_t) - \lambda_t - \alpha(v_{t-1}^f) \)

\( \alpha(z) = \) generally accepted accounting principles valuation of \( z \).
E.4 The Expected Market Rate of Return

As stated at the opening of this Section, a model of the rate of return earned by the entity "firm", derived under the assumption of no inflation, \( A(6) \), is not likely to be well specified when the assumption is relaxed. Respecification of the rate of return model for inflationary distortions must imply specific assumptions about how the inflation tax affects business investment-production decisions and financing decisions. This specification problem is not a theoretical one; it is an empirical (or statistical) one. Therefore, the nominal or market rate of return model is expressed in general functional form.

The rate of return \( \xi^* \) is an economic return. Consequently, it must be measured as the ratio of the firm's economic earnings to its economic value. This suggests the immediate modification of \( \nabla \), the difference between economic depreciation and accounting depreciation. In general, \( \nabla \) should be redefined as the broader concept: the difference between the actual (or economic) consumption of assets and the reported (accrual) consumption. Land, for example, is reported at historical cost, unadjusted for any change in value. Accordingly, \( \nabla \) should capture any economic depreciation (or appreciation) of the firm's investment in land. A similar situation arises with respect to inventories. As is well recognized, the firm which uses LIFO cost for inventory valuation will approximately match the current cost of inventory consumption (i.e., cost of goods sold) with the current
dollar reward for that conversion (i.e., revenue). But with inflation the inventory value reported in the balance sheet will understate the opportunity cost of the inventory investment.

The redefinition of \( \nabla \) is formalized by EQ(29), where the symbol \( \alpha \) stands for the accountant's valuation function. The difference between economic profit \( r_t^f V_{t-1}^f \) and accountant's profit \( \alpha (r_t^f) \alpha (V_{t-1}^f) \) is just \( \nabla_t \), provided the accountant's measure of earnings \( \alpha (\pi_t) \) is corrected to equal \( \pi_t \).

In words, \( \nabla_t \) is the difference between the change in the economic value of the firm and the change in the value of the firm as reported in traditional financial statements. Parenthetically, it should be noted that profits adjusted for \( \nabla \) are equivalent, at least conceptually, to a replacement cost (entry value) definition of income. Moreover, in the absence of market imperfections, and with perfect information, the obtained profit measure also equals the current cash equivalent (exit value) definition of income.

The equilibrium capital asset pricing equation EQ(19) describes the determinants of the expected real rate of return to the firm. With the no-inflation assumption A(6) removed it follows that the expected market rate of return to the firm \( \xi \) (distinguished from \( \xi^* \) by *) is just the real rate plus the expected inflation rate.
EQ(30) \[ \zeta_t = f(X_t, E_t(\rho_{t+1}), \sigma_t) \]

where \( X_t \) is the vector of real variables defined by EQ(19).

The expected inflation rate uncertainty, \( \sigma_t \), is also introduced for the reasons cited in the previous essays in this volume.

The null hypothesis is that the partial derivative of \( \zeta \) with respect to \( E_t(\rho_{t+1}) \) equals unity. This is the weak form of the Fishierian Hypothesis. The partial derivative of \( \zeta_t \) with respect to \( \sigma_t \) could be positive, zero, or negative. The sign of the derivative depends on who bears the burden of the inflation uncertainty. The rate of return to the firm will increase if the firm is able to shift the inflation risk forward to consumers in higher product prices, or backwards to labor in lower real wages. To the extent the inflation risk cannot be shifted the rate of return to the firm will decline because the demand for claims to the firm's assets will fall, thus raising the cost of financial capital \( (r^C) \). In the presence of expected future inflation rate uncertainty, the firm's real rate of return may also be higher due to a failure of equal access to information (assumption A(2)), or if investors have heterogeneous expectations (failure of A(3)). It is commonly thought that firms have a comparative advantage in access to information, at least compared to the mean household. Ignoring the cost of information, this is undoubtedly true. The general belief is stronger than that, however. Apparently it is thought that businesses can exploit
economies of scale in information gathering, analysis, and management. Consequently, greater variation in the inflation rate may prove beneficial to businesses if they form more efficient forecasts than their factor suppliers. The effects of such differential expectations were analyzed in Section E of the previous essay in the context of the debtor-creditor hypothesis and in Section G regarding the wages-lag hypothesis.

E.5 Recapitulation

The equilibrium expected real rate of return to the firm was previously defined by EQ(19). Having expanded some of the explanatory variables in that equation it is desirable to repeat the equation in its final form. The expectations operator, $E$, is resurrected and it is used in two different ways. The expression $E_t(z_{t+1})$ refers to the expectation at time $t$ of $z$ at time $t+1$. The expression $E_t(z_t)$ denotes the expectation at time $t$ of $z$ measured as the point estimate of the expected future benefit stream associated with $z$. In other words, $E_t(z_t)$ is a constructed discounted present value at time $t$. This new use of the expectations operator is introduced to remind the reader that, although the time $t$ value of $z$ enters into the determination of the time $t$ value of $z^*$, nonetheless $z_t$ is the product of a mapping of an expected future sequence.
The equilibrium expected rate of return to the firm for the period t to t+j is defined here as follows. It should be stressed that for longer than one-period ahead yields the variables with t+j subscripts refer to the cumulative flow over the period t to t+j or effective rate in the case of T.

The reader is reminded of Appendix A which contains the definitions of all the key variables (symbols) used in this essay.
\[ E_t(\xi_{t+j}) = \frac{E_t(\pi_{t+j})}{V_t^f} \left\{ \left[ 1 - E_t(\tau_{t+j}) \left( E_t(\psi^*_t) - E_t(\nu_t) \right) \right] \right\} \]

\[ - E_t(\tau_{t+j}) \frac{t(\Lambda_t)}{V_t^f} + \left[ 1 + E_t(\tau_{t+j}) \right] \frac{E_t(\hat{\Gamma}_t)L_t}{V_t^f} \]

\[ - \left[ 1 - E_t(\tau_{t+j}) \right] \frac{E_t(\delta K^c_{t-1})t_{t+j}}{V_t^f} - \left[ 1 - E_t(\tau_{t+j}) \right] \frac{E_t(\omega_{t+j})L_t}{V_t^f} \]

\[ + E_t(\tau_{t+j}) \frac{E_t(R_{t+1})}{V_t^f} - \frac{E_t(\nu_{t+j})}{V_t^f} \]

\[ E_t(\sigma_{t+j}) = f \left[ E_t(\xi^*_{t+j}), E_t(\rho_{t+j}), \sigma_t \right] \]
F. SUMMARY

At the outset of this paper we stated the purpose was to develop an equation for the expected rate of return to the firm. This equation was to integrate contemporary corporate finance and investment theory with the theory that inflation is a tax on the purchasing power of money balances. More specifically, a conceptual expression for the expected return to the firm was sought which conformed to contemporary theories of corporate finance, and at the same time identified those variables which may be distorted (in real value) by a non-neutral inflation tax. This equation was given in the previous section.

Considerable attention was given to the sufficient conditions and theorems used to achieve the equation of return. It is fitting, therefore, that this essay end with a summary interpretation of the equation.

Essentially the terms in the rate of return to the firm equation can be collected into five factors. The first is the expected net cash flow rate of return $E_t(\pi_{t+1})/V_t^f$. This factor is the rate of return to the firm in a world free of inflation and corporate income taxes. Every other variable in the model is merely a modification to this basic economic rate of return.¹
The second factor is the real capital consumption allowance. It is the sum of the tax deduction for the current period's depreciation, plus the present value of future depreciation adjusted for taxes and weighted by the net cash flow rate of return. The present value of future capital consumption is equal to the present value of the expected general price-level adjusted capital consumption, less the present value of the expected future lost real capital consumption arising during inflationary periods from corporate income taxes levied on the nominal value of the historical cost of assets. It is of course this difference which distorts the equilibrium expected rate of return to the firm. This distortion, however, is purely the consequence of institutional conditions. Even if the future inflation rate is correctly anticipated, and hence endogenous, the rate of return earned by the firm will decline, cet. par. The question not addressed here or elsewhere in the literature is how this burden is shifted forward to consumers (in the way of higher product prices) or backwards to the factors of supply. Again, ζ* is the return to the firm which is shared by all suppliers of capital. The popular belief is that any lost real capital consumption (υ) is borne solely by equity holders (v^e). This is a simplistic assumption, which at best describes the short-run. According to our model of the firm the cost of lost real capital consumption from inflation will not be shifted forward because all markets are assumed to be competitive and driven by the value maximization rule. However, product prices may be indirectly affected from substitution of labor for capital.
The same depreciation provisions of the U.S. tax laws apply to all forms of business. To this extent, a change in the loss due to depreciation \( (v) \) is equivalent to some well defined increase in the tax on capital. As a general tax on a single factor in both the corporate and noncorporate sectors, the tax is borne entirely by the taxed factor (capital) regardless of the mobility of labor or capital. It is presumed that the tax on capital is shared proportionally by all forms of capital (debt and equity), such that the net-of-tax returns are equal.

While the same depreciation provisions apply to all forms of business, not all forms of business are subject to the same income tax rates. Thus, there is also a differential in the tax on capital which is equivalent to a change in \( v \). This differential tax is a tax on one factor (capital) in one use. In the short-run, capital is fixed and immobile, hence, the tax is borne entirely by capital in the taxed sector. With fixed but mobile capital, the Harberger [1962] intermediate-run, the tax burden is spread to all capital invested in all sectors. Hence, the burden is shifted in part to capital in sectors with differentially lower income tax rates. In the long-run, however, the differential tax will exert opposing and indeterminate effects on factor prices. Product prices in the taxed sector, on the other hand, will rise relative to other product prices. In short, both labor and capital will bear the burden of lost real consumption allowance, although capital may be expected to bear at greater share.
The third component of the expected rate of return to the firm may be termed the "financial structure factor." Mathematically, this factor is the period's net-of-tax interest expense, as a share of the total value of the firm, less a measure of the debt-to-total assets rates, weighted by the corporate tax rate and the net cash flow rate of return. The debt-to-total assets variable, $\Lambda/V^f$, captures the debtor-creditor hypothesis. It was demonstrated that $\Lambda$, the present value of future interest payments on outstanding debt, is a function of differential inflation expectations. The expected rate of return to the firm will increase if the firm is able to understate the coupon rate on its debt compared to the market-wide rate of return for equivalent risk. This is achieved by, in effect, "selling" a lower expected inflation rate forecast. In this instance, equity holders unquestionably gain at the expense of the debt holders. However, with competitive capital markets, this gain will occur only once before the bond price adjusts to yield the market return, which incorporates the market inflation rate forecast.

Parenthetically, we note that writing the expected rate of return to the firm as a function of the debt-to-asset ratio would be inconsistent with the separation of the investment-production decision and the financing decision were it not for the fact that the cost of capital $r^C$ is endogenous in the model. Nothing has been assumed about the behavior of $r^C$ over time or with respect to the
financing mix. Whereas the rate of return equation does permit the possibility of an optimal financial structure, the equation specification does not require this result.

The fourth component of the expected rate of return model is the labor-capital ratio. As was shown above, this factor captures the wages lag hypothesis. The ratio is computed as the present value of future wage payments as a ratio to the value of the firm, multiplied by the corporate income tax and net cash flow rate of return, all less the current period labor-capital ratio (measured as the ratio of net-of-tax wage payments to the total value of the firm). Differential inflation rate expectations as between the firm and its labor are reflected explicitly in \( \hat{\Gamma} \), the present value of future wage rates. If wages lag prices in general, then \( \hat{\Gamma} \) will be less than if wages are inflation neutral. A decline in \( \hat{\Gamma} \) causes a decline in \( \xi^* \). This effect is offset, however, by a positive change in \( \xi^* \) caused by an increase in \( \pi \). For any positive rate of interest, the partial derivative of \( \xi^* \) with respect to \( \hat{\Gamma} \) is positive—just as the wages lag hypothesis predicts.

For \( \xi^* \) to be endogenous to the effects of inflation on labor intensity, it is necessary but not sufficient that the partial derivative of wage rates with respect to expected future inflation tax rates be unitary. A wages lag condition occurs if the discount rate
(i.e., the expected future cost of capital) is overstated as a result of a positive bias in the expected rate of inflation. This condition had not been demonstrated prior to this paper (cf., Section E.2).

The final component of $\zeta^*$ is the difference between actual (or economic) consumption of assets and reported (or accrual) consumption. This adjustment is required because of the divergence between the asset valuation theory which constitutes "generally accepted accounting principles," namely historical cost, and the economic theory of profit which values assets at current "realizable" prices. The magnitude of the adjustment can be expected to increase from period to period with higher inflation rates. In one sense this correction reflects an element of money illusion on the part of the accounting profession, and in this manner violates the unbiased expectations condition for a neutral inflation tax. At the same time, the principle of historical cost is grounded in years of accounting tradition which has no relevance to inflation. In this sense the adjustment to $\zeta^*$ reflects an institutional distortion.

When inflation is explicitly considered, $\zeta^*$ is converted to $\zeta$, the expected market (nominal) rate of return to the firm. Our analysis clearly indicates that even the weak form of the Fisherian Hypothesis is a complex set of inter-relationships. The expected return earned by the competitive, value maximizing firm is determined by:
exogenous disturbances to labor-capital productivity arising from changes in social-political conditions;

institutional conditions which designate resource usage on a basis of absolute prices instead of relative prices;

differential inflation rate expectations among the suppliers of capital;

by differential inflation rate expectations between suppliers of labor and suppliers of capital, and by the uncertainty about any inflation rate expectation.

This paper has sought only the modest goal of presenting a logically derived basic model of the expected rate of return to the firm. The next step would be to isolate and empirically measure the effects of inflationary finance on these returns. From what has been learned in this work, it is clear this undertaking will require a complex model of the business sector, in which the rate of return equation would be the cornerstone. But even more important than the equation is the theory surrounding it. The theory assumes highly anticipatory markets. Resource allocations, product selection and pricing, as well as all other microeconomic behavior are determined, not by what has happened, but by what is expected to occur. To be conceptually congruent with the model of the firm presented here any future empirical estimation of the incidence of the inflation tax will necessary have to assume investment-production and financing decisions are based solely on the expected future values of the behavioral variables. This focus of attention on the anticipatory nature of decision-making is perhaps the single noteworthy contribution of this work.
FOOTNOTES

A. INTRODUCTION

1 The theory of the firm presented in this study assumes a separation theorem between investment and financing decisions. This separation theorem is derived by assuming perfect capital markets, i.e., only one rate of interest for any given risk level. If the inflation tax is exogenous to interest rates, then the real rate of interest varies not just by real risk but according to whether the firm is a debtor or creditor. That is to say, the separation theorem does not hold, and the valuation equation may be incorrectly specified.

Bondholders are assumed to be just a class of equity. Bondholders vote by letting the firm use more or less capital.
B. THE THEORY OF THE FIRM


2. Fisher [1930] explored the theory of choice as it applies to resource allocation over time, i.e., the nature of capital and interest. The basic principles of the capital valuation and allocation are described in Hirshleifer [1958]. A general equilibrium model under the condition of uncertainty is found in Hirshleifer [1965] [1966].

3. Technically, Modigliani and Miller assumed imperfect knowledge. However, while rates of return were assumed to be affected by investment risk, the structural specification was exogenous to their model. In addition, firms were assumed to always invest in the same risk class. Their model is, therefore, indistinguishable from one based on perfect information, except that a corner solution of all debt (or all equity) is ruled out.

4. The separation theorem asserts that there exists a hyperplane which separates two disjoint convex sets. One version of the separation theorem states that if $X$ and $Y$ are nonempty convex sets in $\mathbb{R}^n$ (reals), such that $X \cap Y = \emptyset$, then there exists $\lambda \in \mathbb{R}^n$, $\lambda \neq 0$, and $\alpha \in \mathbb{R}$ such that $\lambda x \geq \alpha$ for all $x \in X$ and $\lambda y \leq \alpha$ for all $y \in Y$ (Minkowski Theorem). The set $X$ can be thought of as the production possibility set, and the set $Y$ as the production impossibility set. The supporting hyperplane to $X$ is the capital transformation function over time (lending and borrowing).


6. Jensen and Meckling [1976] attempt a reconciliation of the managerial theories of the firm and neoclassical profit maximization. The authors apply the Coase Theorem to a model of the firm which explicitly accounts for the fact that stockholders and bondholders have property rights which they seek to protect by contracts in a principal-agent relationship with management.
"Satisficing" can also be consistent with constrained optimization. Indeed, Simon argued that the complexity of choice forces individuals to make recursive decisions wherein alternatives are accepted provided they exceed certain aspiration (reservation) levels. This type of behavior is called "satisficing." Kapteyn, Wansbeck, and Buyze [1979] report empirical results which indicate individuals make purchase decisions (for durables) according to satisficing rules rather than utility maximization.

A complete market is one in which every time-state claim (object of choice) is represented by a marketable instrument. That is, every contingency corresponds to a specific and distinct marketable commodity.

Homogeneous expectations, a condition for market efficiency, defines the state where information \( \varnothing \) is "fully reflected" in prices. However, \( \varnothing \) need not be explicitly revealed through prices. Cf., Verrecchia [1979].

Assumption A(4) has the same purpose as Fama's [1978] "Only Wealth Counts" assumption. As Fama stated, "For our purposes it is sufficient to assume that the capital market satisfies whatever conditions are necessary to ensure the desired correspondence between wealth and welfare." (p. 273)

The term "universal portfolio separation" is borrowed from Rubenstein [1977], although the concept precedes him by several years and countless authors.

Universal portfolio separation does not require households to literally own only two securities. The assumption is that households invest in the risk-free asset and in all other assets in proportion to the total market value of each security.

Also see, Miller and Modigliani [1966].

The Modigliani and Miller papers were published before Section 385 of the Internal Revenue Code was enacted (1969). The Section lists several factors used to determine whether a shareholder-corporation relationship or a debtor-creditor relationship exists. One of the more important factors is the debt to equity ratio. However, even prior to Section 385, the tax courts looked to the debt to equity ratio for guidance in interpreting the relationship. In the case of Brititz Construction Co., v. U. S., 68-1 USTC 9118, 20 AFTR2d 5891, 387 F.2d 451 (CA-8,1967), the Court concluded that a 2:1 ratio was not inordinately high.
In Miller and Scholes [1978], the Miller [1977] irrelevance of financial structure proposition is considered in conjunction with a dividend irrelevance proposition. Sufficient conditions for both are shown to be rather stringent.

Cf., Brennan and Schwartz [1979] who maintain that the tax "shelter" from interest payments is an uncertain stream because the tax savings cease once the firm goes bankrupt. Based on this observation, they show that the issue of additional debt has two effects. It increases the tax shelter enjoyed so long as the firm is a going concern. On the other hand, it also reduces the probability of survival. Whichever of these conflicting influences dominates determines whether the value of the firm increases or decreases with additional debt.
C. CONCEPTS AND PRELIMINARY RELATIONSHIPS

1. The accrual accounting net income $\pi$ is essentially a cash flow measure, given assumption A(8). This assumption is subsequently relaxed, in which case $\pi$ is different from $\pi$ in ways not captured by EQ(4). See Proposition II.


3. Any tax shelter missed through deficient debt coupon payments is lost. Therefore, the tax shelter is gained only for actual debt payments rather than promised coupons. It is assumed, however, that creditors impose sufficient constraints on management to make the actual interest payments equal the promised payments. In the absence of this assumption EQ(11-a) and EQ(11-b) are misspecified. See, Bierman and Oldfield [1979] for further discussion of this point.

4. The term "money" is used throughout this essay as a synonym for any type of financial asset, other than accounts receivable.

5. Money is treated as an investment in this essay in an analogous manner to its treatment as a commodity in Section C of "The General Theory of Inflationary Finance," the first essay of this volume.
D. THE EQUILIBRIUM EXPECTED RATE OF RETURN TO THE FIRM:
THE BASIC MODEL

1 Recall that $\delta K^C_{t-1}$ refers to the depreciation of the beginning of the period capital $K^C_{t-1}$, at which time the depreciation rate may be specified but it is still subject to change. Consequently $\delta K^C_{t-1}$ should be interpreted as $E_{t-1}(\delta K^C_{t-1})_t$.

2 We emphasize that terms such as $\Psi_t$ refer to $E_{t-1}(\Psi_t)$. Without the expectations operator the equations of premise (iii) are valid only if no new information is available at time $t$ which was not available at time $t-1$ concerning the respective sequences of expected future values - certainly a stringent assumption.


4 Such stochastic disturbances include what accountants refer to as "extraordinary events."
E. INFLATION AND THE EXPECTED RATE OF RETURN TO THE FIRM

E.1 The Measurement of Real Depreciable Capital and Decomposition of the Present Value of Future Depreciation

1 Cf. Lent [1976] and Tanzi [1976]. Several studies have examined the problem, which arises during inflationary periods, of the tax regulations that restrict depreciation to the historical cost of depreciable assets. For example, see Samuelson [1964], Aaron [1976], and Tideman and Tucker [1976]. Gramlich [1976] concluded that, "the procedures for computing taxable income should be revised. Depreciation deductions should be based on replacement cost. The use of historical cost deters investment by significant and widely varying amounts, depending on the anticipated rate of inflation and the durability of assets." (p. 290). Also see Tatom and Turley [1978] for several examples of the reduced yield on investment caused by increased taxation from fixed nominal depreciation. Hong [1977] examined the issue empirically and concluded that understating both depreciation expense and the cost of goods sold affects stock prices through the additional tax burden borns by firms. Nelson [June, 1976] provided an analytic study of the effects of inflation and the U. S. tax code on capital budgeting decisions. He derived the following five propositions:

Proposition A. The optimal level of capital investment will depend in general on the rate of inflation. The amount invested will typically be smaller the higher the rate of inflation.

Proposition B. The rate of inflation will influence the firm's choice of technologies of production through its choice of a capital/labor ratio. Higher rates of inflation will typically be associated with lower capital/labor ratios.

Proposition C. The net present value ranking of mutually exclusive investment projects will depend in general on the rate of inflation.

Proposition D. Net present value rankings of mutually exclusive projects which differ with respect to durability will depend on the rate of inflation. Typically, rankings will change in favor of projects with lower durability at higher rates of inflation.
Proposition E. Replacement policy will depend in general on the rate of inflation. The higher the rate of inflation the more likely will replacement be deferred to a future period.


3 For depreciable real property see Internal Revenue Code Sec. 167(j).

4 The present discounted value of future depreciation is not insensitive to the discount rate, only the optimal switch-over point is. For example, if an asset has a ten year life the optimal switch-over point occurs between the third and fourth year for 125 percent declining balance if the interest rate is 5 percent or 15 percent. For double declining balance the optimal switch-over is between 6 and 7 years for either a 5 or 15 percent discount rate.

5 Cf., Internal Revenue Code Sec. 179. The additional first-year depreciation allowance is limited to $2,000 per taxpayer.

6 Data are not available on the breakdown of depreciation (i.e., capital consumption allowance) between equipment depreciation and structures depreciation.

E.2 Decomposition of the Present Value of Future Wage Payments

7 The distinction between $\hat{\pi}$ and other labor prices is solely a function of the chosen "unit" of labor. Traditionally the labor price is expressed as a rate per one man-hour. $\hat{\pi}$, on the other hand, is the price for one unit of labor to be delivered forever (or for whatever the term horizon $n$ is).

8 In this simple one-period model, equilibrium achieves at the end of the period only if economic agents respond to forecast errors as well as to expectations. With a multiperiod horizon the forecast errors may be ignored, especially if they have no informational content, i.e., white noise.
E.3 The Present Value of Future Interest Payments

Cf., Section G in "The Question of Inflation Tax Neutrality: A Review of Selected Literature," this volume, for a discussion of the behavior of $\Lambda$ (actually of $B$) with respect to nonconstant inflation rate expectations.

F. SUMMARY

$\zeta^*_t$ is equal to $\pi_t / Y^f_t$ if, in addition to the absence of the corporate income tax and inflation, the net cash flow is a perpetual annuity.
APPENDIX A

Index of Variables

The following index is offered for ease of reference. Let \( j \) be any semi-positive number.

**Endogenous Variables**

\[ V_t^f \] = Economic or market value of the firm.

\[ E_t (\pi_{t+j}) \] = Expected future funds from operations.

\[ E_t (\tau_{t+j}) \] = Expected future effective corporate income tax rate.

\[ E_t (\psi_t) \] = Present value of expected future actual (allowable) depreciation, cf. EQ(17), EQ(20-a).

\[ E_t (\psi^*_t) \] = Present value of expected future general price level indexed depreciation, cf. EQ(20-b).

\[ E_t (T_t) \] = Present value of expected future lost capital consumption, cf. EQ(21-b).

\[ E_t (\Lambda_t) \] = Present value of expected future interest payments on outstanding debt, cf. EQ(17), EQ(28-a).

\[ E_t (\Gamma_t) \] = Present value of expected future labor services, cf. EQ(17), EQ(25).

\[ \hat{E}_t (\Gamma_t) \] = Expected price of future labor services from \( L_t \) labor units.

\[ E_t (u_t) = \frac{E_t (T_t)}{V_t^f} \]
\[ E_t(\gamma_t) = E_t(\Gamma_t)/V_t^f \]

\[ E_t(\lambda_t) = E_t(\Lambda_t)/V_t^f \]

\[ E_t(\delta K_{t-1}^C)_{t+j} = \text{Expected future (allowable) depreciation.} \]

\[ E_t(w_{t+j}) = \text{Expected future wage rate.} \]

\[ E_t(R_{t+j}) = \text{Expected future interest payment on outstanding debt obligations.} \]

\[ E_t(v_{t+j}) = \text{Expected future difference between economic consumption of assets and reported accrual consumption of assets, cf. EQ(29).} \]

\[ I_t^* = \text{Aggregate investment, cf. EQ(13).} \]

\[ F_t = \text{External financing, cf. EQ(15).} \]

\[ M_t = \text{Money balances.} \]

\[ A_t = \text{Accounts receivable.} \]

\[ \hat{I}_t = \text{Inventory.} \]

\[ K_t^C = \text{Real depreciable capital investment at adjusted historical cost, cf. EQ(23).} \]

\[ T_t = \text{Land.} \]

\[ K_t = \text{Real depreciable capital investment at market value.} \]

\[ A_t = \text{Accounts payable.} \]

\[ B_t = \text{Debt.} \]

\[ V_t^e = \text{Stockholders' equity at market value.} \]
\[ r_t^C = \text{Discount rate, cost of capital.} \]
\[ L_t = \text{Labor services (units of labor).} \]
APPENDIX B

This Appendix presents the proofs of the four propositions used in this essay. The symbol convention adopted in Section D is continued. The conditional expectations of $X$, otherwise written $E_{t-1}(X_t | \phi_{t-1})$, is denoted as an expected value by the time subscript $t$, as in $X_t$. Predetermined values are denoted by the time subscript $t-1$, as $X_{t-1}$. 
PROPOSITION 1

Given: \( V_t^f = K_t \)

Given: \( E_t(r_{t+j}^f) = E_t(r_{t+j+1}^f) \quad \forall j \geq 1 \)

Given: \( EQ(8) \quad V_t^f = \sum_{k=1}^{\infty} E_t(\pi_{t+k}) \prod_{j=1}^{k} E_t(1 + r_{t+j}^f)^{-1} \)
\( \hat{V}_t^f = \sum_{k=1}^{\infty} E_t(\hat{\pi}_{t+k}) \prod_{j=1}^{k} E_t(1 + r_{t+j}^f)^{-1} \)

Given: \( EQ(7) \quad \dot{V}_t^f = I_t + (1 + r_t^f) \dot{V}_{t-1}^f - \pi_t \)
\( \hat{V}_t^f = I_t + (1 + r_t^f) \hat{V}_{t-1}^f - \hat{\pi}_t \)

Then: \( \frac{\pi_t}{\dot{V}_{t-1}^f} = \frac{\hat{\pi}_t}{\hat{V}_{t-1}^f} \)

PROOF

\( EQ(7) \quad 1. \quad \frac{\dot{V}_t^f - I_t}{\dot{V}_{t-1}^f} + \frac{\pi_t}{\dot{V}_{t-1}^f} = \frac{\hat{V}_t^f - I_t}{\hat{V}_{t-1}^f} + \frac{\hat{\pi}_t}{\hat{V}_{t-1}^f} \)

\( EQ(7) \quad 2. \quad V_t^f - I_t = (1 + r_t^f) V_{t-1}^f - \pi_t \)
\( \dot{V}_t^f - I_t = (1 + r_t^f) \dot{V}_{t-1}^f - \hat{\pi}_t \)

substitute 2 into 1

\( EQ(7) \quad 1.b \quad \frac{(1 + r_t^f) V_{t-1}^f - \pi_t}{\dot{V}_{t-1}^f} + \frac{\pi_t}{\dot{V}_{t-1}^f} = \frac{(1 + r_t^f) \hat{V}_{t-1}^f - \hat{\pi}_t}{\hat{V}_{t-1}^f} + \frac{\hat{\pi}_t}{\hat{V}_{t-1}^f} \)

\( \odot \odot \quad \frac{\pi_t}{\dot{V}_{t-1}^f} = \frac{\hat{\pi}_t}{\hat{V}_{t-1}^f} \)
PROPOSITION 11

Given:  
\[ M_t + A_t^+ + I_t + K_t + T_t = A^e_t + B_t + \psi_t^e \]

Given:  
\[ \pi_t + F_t = I_t^* + D_t + R_t \]

Given:  
\[ V_t = [I_t - K_t + K_{t-1}] - \Delta K_{t-1} \]

Then:  
\[ \bar{V}_t + R_t - V_t = \frac{2\hat{B}}{3\hat{P}} dP_{b,t} + \frac{2\psi}{3\hat{P}} dP_{e,t} + D_t + R_t \]

\[ V_t^f + \eta_t - I_t^* - V_{t-1}^f \]

PROOF

Let \[ \Delta B = \frac{2B}{3\hat{P}} dQ_B + \frac{2B}{3\hat{P}} dP_B \quad \Delta V^e = \frac{2\psi}{3\hat{P}} dQ_e + \frac{2\psi}{3\hat{P}} dP_e \quad F = \frac{2B}{3\hat{P}} dQ_B + \frac{2\psi}{3\hat{P}} dQ_e \]

where \( Q \) = "quantity"

\[ \Theta \Theta \quad \text{EQ}(15) \quad \pi_t + \Delta A_t^e + \frac{2B}{3\hat{P}} dQ_{b,t} + \frac{2\psi}{3\hat{P}} dQ_{e,t} = \Delta M_t + \Delta A_t^e + \Delta I_t + \Delta T_t + I_t + D_t - \pi_t \]

Subtract the difference equation \( \Delta M_t + \Delta A_t^e + \Delta I_t + \Delta K_t + \Delta T_t = \Delta A_t^e + \Delta B_t + \Delta V_t^e \)

(1)  
\[ \Delta K_t + \pi_t - I_t = \frac{2\hat{B}}{3\hat{P}} dP_B + \frac{2\psi}{3\hat{P}} dP_e + D_t + R_t \]

Substitute \( V_t \)

(2)  
\[ \pi_t - \Delta K_{t-1} - V_t = \frac{2\hat{B}}{3\hat{P}} dP_B + \frac{2\psi}{3\hat{P}} dP_e + D_t + R_t \]

Recall that \( \pi_t \cong \bar{\pi}_t + \Delta K_{t-1} + R_t \)

\[ \Theta \Theta \quad \bar{V}_t + R_t - V_t = \frac{2\hat{B}}{3\hat{P}} dP_B + \frac{2\psi}{3\hat{P}} dP_e + D_t + R_t \]

Q. E. D. (Part 1)
PROPOSITION 11

PROOF - continued

\[ EQ(11) \rightarrow \Delta \nu_t^f - (\Delta M_t + \Delta A_t^* + \Delta I_t + \Delta T_t) = \Delta K_t \]

from (1) above: \[ \Delta K_t = \frac{\partial B}{\partial B} dP_B + \frac{\partial \nu^e}{\partial e} dP_e + D_t + R_t - \pi_t + I_t \]

\[ \circ \circ \circ \]

\[ \Delta \nu_t^f - (\Delta M_t + \Delta A_t^* + \Delta I_t + \Delta T_t) - I_t + \pi_t = \frac{\partial B}{\partial B} dP_B + \frac{\partial \nu^e}{\partial e} dP_e + D_t + R_t \]

But \[ I_t^* = \Delta M_t + \Delta A_t^* + \Delta I_t + \Delta T_t + I_t \]

and from Part 1 \[ \frac{\partial B}{\partial B} dP_B + \frac{\partial \nu^e}{\partial e} dP_e + D_t + R_t = \pi_t + R_t - \nu_t \]

\[ \circ \circ \circ \]

\[ \nu_t^f + \pi_t - I_t^* - \nu_{t-1}^f = \pi_t + R_t - \nu_t \]

Q. E. D. (Part 2)
PROPOSITION III

Given (i) \[ E_t(\delta k_{t-1}^C, t+\delta) = \delta K_{t-1} \quad \forall \ j \geq 0 \]
\[ E_t(R_{t+\delta}) = R_t \]
\[ E_t(W_{t+\delta}) = W_t \]
\[ E_t(w_{t+\delta}) = w_t \]

Given (ii) \[ E_t(r_{t+\delta}^f) = r_{t}^f \quad \forall \ j \geq 0 \]

Given (iii) \[ v_t = \psi_{t-1}(1 + r_{t}^f) + \delta K_{t-1}^C \]
\[ \Lambda_t = \Lambda_{t-1}(1 + r_{t}^f) - R_t \]
\[ \Gamma_t = \Gamma_{t-1}(1 + r_{t}^f) - W_t \]

Then:
\[
\frac{1}{V_{t-1}^f} - \frac{1}{V_{t-1}^f} \left[ \frac{V_t^f}{V_{t-1}^f} - \tau \psi_t - \tau \Lambda_t + (1-\tau)\Gamma_t \right] + 1 - \tau v_{t-1} - \tau \lambda_{t-1} + (1-\tau)\gamma_{t-1}
\]
\[
= \left[ 1 - \tau v_{t-1} - \tau \lambda_{t-1} + (1-\tau)\gamma_{t-1} \right] \times \left[ \frac{V_t^f}{V_{t-1}^f} - r_{t}^f \right]
\]

PROOF

Recall EQ(6) \[ r_{t}^f = (v_t^f + \psi_t^f - v_{t-1}^f - v_{t-1}^f) / V_{t-1}^f \]

Substitute EQ(6) for \[ r_{t}^f \] in the consequent.

\[
\frac{1}{V_{t-1}^f} \left[ \frac{V_t^f}{V_{t-1}^f} - \tau \psi_t - \tau \Lambda_t + (1-\tau)\Gamma_t \right] - \tau v_{t-1} - \tau \lambda_{t-1} + (1-\tau)\gamma_{t-1}
\]
\[
= \left( \frac{V_t^f - v_{t-1}^f - v_{t-1}^f}{V_{t-1}^f} \right) \times \left[ 1 - \tau v_{t-1} - \tau \lambda_{t-1} + (1-\tau)\gamma_{t-1} \right]
\]

Multiply through by \[ V_{t-1}^f / \tau \] and substitute (iii).

\[
(1 + r_{t}^f) \left[ \psi_{t-1} + \Lambda_{t-1} - \frac{(1-\lambda - 1)}{R_{t-1}} \right] - \left[ \delta K_{t-1}^C + R_t + (\frac{1}{\tau} - 1)W_t \right] - \left[ \psi_{t-1} + \Lambda_{t-1} - \frac{(1-\lambda - 1)}{R_{t-1}} \right]
\]
\[
= \left( \frac{V_t^f - v_{t-1}^f - v_{t-1}^f}{V_{t-1}^f} \right) \times \left[ - \psi_{t-1} - \Lambda_{t-1} + \frac{(1-\lambda - 1)}{R_{t-1}} \right]
\]
PROPOSITION III - continued

Substitute EO(6) and collect terms,

\[
\frac{\pi_t}{V^f_{t-1}} [ V_{t-1} + \Lambda_{t-1} - \left(\frac{1}{t} - 1\right)\pi_{t-1} ] = \delta K^C_{t-1} + R_t - \left(\frac{1}{t} - 1\right)W_t
\]

Now establish that this equation implies the fundamental valuation equation.

Recall EO(8) \( V^f_t = \sum_{k=1}^{\infty} E_t(\pi_{t+k}) \prod_{j=1}^{k} E_t(1 + r^f_{t+j})^{-1} \).

Along with assumption (i) \( V^f_t = (1 + r^f_t) V^f_{t-1} + \pi_t \).

The sequences \( \{E_t(\pi_{t+1}) \}, \{E_t(\pi_{t+2}) \}, \ldots \) and \( \{E_t(1 + r^f_{t+1}) \}, \{E_t(1 + r^f_{t+2}) \}, \ldots \)

are mapped into \( V^f \) by the function \( f \),

\[ f: \{ \{E_t(\pi_{t+j}) \} \}, \{E_t(1 + r^f_{t+j}) \} \}. \]

Given assumption (ii) above, \( \exists \) an image set \( H \), such that there is a one-to-one mapping of \( f : h \in H \),

where \( h \) is a real number.

\( \Theta \circ \Theta \quad h(\pi_t | r^f_t) = V^f_t \)

Similarly by assumption (ii):

\( h(\delta K^C_t | r^f_t) = V_t \)

\( h(\Lambda_t | r^f_t) = \Gamma_t \)

\( h(W_t | r^f_t) = \Gamma_t \)

Substituting these relations into (iv) above:

\[
\frac{\pi_t}{V^f_{t-1}} (h) [ \delta K^C_{t-1} + R_t - \left(\frac{1}{t} - 1\right)W_t ] = \delta K^C_{t-1} + R_t - \left(\frac{1}{t} - 1\right)W_t ; \text{ given } r^f_t
\]

\( \Theta \circ \Theta \quad h(\pi_t) = V^f_{t-1} ; \text{ given } r^f_t \)

Q. E. D.
PROPOSITION IV

Let

\[ r_{b1} \] coupon rate for bond 1.
\[ r_{b2} \] coupon rate for bond 2.
\[ r_{m1} \] market rate at time t for risk-class bonds \( b_1 \) and \( b_2 \).
\[ q_1 \] principal for bond 1.
\[ q_2 \] principal for bond 2.
\[ n_1, n_2 \] lives of the bonds.

Given:

\[ B_{10} = \sum_{t=1}^{n_1} r_{b1} q_1 \prod_{j=1}^{t} (1 + E_0(r_{m1}))^{-1} + q_1 \prod_{j=1}^{n_1} (1 + E_0(r_{m1}))^{-1} \]
\[ B_{2n_1} = \sum_{t=n_1+1}^{n_1+n_2} r_{b2} q_2 \prod_{j=n_1+1}^{t} (1 + E_0(r_{m1}))^{-1} + q_2 \prod_{j=n_1+1}^{n_1+n_2} (1 + E_0(r_{m1}))^{-1} \]

where \( B_{10} \) = value of outstanding bonds at \( t = 0 \).
\[ B_{2n_1} \] expected value at time \( t = 0 \) of the refinancing bond issued at time \( n_1 \).

Let

\[ a_t = \begin{cases} r_{b1} q_1 & 1 \leq t \leq n_1 \\ r_{b2} q_2 & (n_1+1) \leq t \leq (n_1+n_2) \end{cases} \]

Define \( \theta_0 \) = present discounted value of the stream of all bond related net cash flows (from \( t = 1 \) to \( t = n_1 + n_2 \)).

\[ \theta_0 = \sum_{t=1}^{n_1} a_t \prod_{j=1}^{t} (1 + E_0(r_{m1}))^{-1} \]  

\[ \quad + q_1 q_2 \prod_{j=1}^{n_1} (1 + E_0(r_{m1}))^{-1} + \prod_{j=1}^{n_1+n_2} (1 + E_0(r_{m1}))^{-1} \]

Then: \( B_{10} = \theta_0 \)
PROPOSITION IV - continued

PROOF

Expand a term:

\[
\sum_{t=1}^{n_1+n_2} \alpha_t \prod_{j=1}^{t} [1 + E_0(r_{m_j})]^{-1} = \sum_{t=1}^{n_1} \alpha_t \prod_{j=1}^{t} [1 + E_0(r_{m_j})]^{-1} + \sum_{t=n_1+1}^{n_1+n_2} \alpha_t \prod_{j=1}^{t} [1 + E_0(r_{m_j})]^{-1}
\]

Note that:

\[
\sum_{t=1}^{n_1+n_2} \alpha_t \prod_{j=1}^{t} [1 + E_0(r_{m_j})]^{-1} = \sum_{t=1}^{n_1} \beta_2 \prod_{j=1}^{t} [1 + E_0(r_{m_j})]^{-1} + \sum_{t=n_1+1}^{n_1+n_2} \beta_2 \prod_{j=1}^{t} [1 + E_0(r_{m_j})]^{-1}
\]

Note that:

\[
\sum_{j=1}^{n_1} [1 + E_0(r_{m_j})]^{-1} = \sum_{j=1}^{n_1} [1 + E_0(r_{m_j})]^{-1} + \sum_{j=n_1+1}^{n_2} [1 + E_0(r_{m_j})]^{-1}
\]

Note that:

\[
\sum_{j=1}^{n_1} [1 + E_0(r_{m_j})]^{-1} = \sum_{j=1}^{n_1} [1 + E_0(r_{m_j})]^{-1} + \beta_2 \prod_{j=n_1+1}^{n_2} [1 + E_0(r_{m_j})]^{-1}
\]

Rewrite \( \theta_0 \) as follows:

\[
\theta_0 = \sum_{t=1}^{n_1} \alpha_t \prod_{j=1}^{t} [1 + E_0(r_{m_j})]^{-1} + \sum_{j=1}^{n_2} [1 + E_0(r_{m_j})]^{-1}
\]

\[
- \sum_{j=1}^{n_1} [1 + E_0(r_{m_j})]^{-1} \beta_2 \prod_{j=n_1+1}^{n_2} [1 + E_0(r_{m_j})]^{-1}
\]

\[
+ \sum_{j=1}^{n_1} [1 + E_0(r_{m_j})]^{-1} \sum_{t=n_1+1}^{n_1+n_2} \alpha_t \prod_{j=1}^{t} [1 + E_0(r_{m_j})]^{-1}
\]

\[
+ \sum_{j=n_1+1}^{n_2} [1 + E_0(r_{m_j})]^{-1} \beta_2 \prod_{j=n_1+1}^{n_2} [1 + E_0(r_{m_j})]^{-1}
\]
PROPOSITION IV

PROOF - continued

Rearranging terms:

\[
0_0 = \left\{ \frac{n_1}{t+1} \sum_{j=1}^{t} \left[ 1 + \rho_0(r_{mj})^{-1} \right] + \frac{n_1}{t+1} \left[ 1 + \rho_0(r_{mj})^{-1} \right] \right\} \\
- \left\{ \frac{n_1}{t+1} \left[ 1 + \rho_0(r_{mj})^{-1} \right] \sum_{j=1}^{t} \frac{n_1}{t+1} \sum_{j=1}^{t} \left[ 1 + \rho_0(r_{mj})^{-1} \right] + \sum_{j=1}^{t} \frac{n_1}{t+1} \left[ 1 + \rho_0(r_{mj})^{-1} \right] \right\}
\]

\[
\Theta_0 = 0_0 - (B_{10}) - \left\{ \frac{n_1}{t+1} \left[ 1 + \rho_0(r_{mj})^{-1} \right] \right\}
\]

\[
\Theta_0 = B_{10}
\]

Q. E. D.
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