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THE QUESTION OF MONEY SUBSTITUTION IN BRAZIL.
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THE QUESTION OF MONEY SUBSTITUTION IN BRAZIL

by

ROBERT WAYNE SAMOHL

A THESIS SUBMITTED
IN PARTIAL FULFILLMENT OF THE
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APPROVED, THESIS COMMITTEE:

[Signatures and names]

HOUSTON, TEXAS
NOVEMBER, 1978
ABSTRACT

The Question of Money Substitution in Brazil
Robert Wayne Samohyl

Financial markets are the focal point for the efficient operation of any free market economy. When the short-term assets supplied by financial institutions are close substitutes for money narrowly defined as currency and demand deposits, the effect of these short-term assets on monetary variables such as the rate of inflation and the level of interest rates would be similar to the effect of the money supply on these same variables. Therefore, as financial markets develop, the efficiency of given monetary policies may diminish.

The objective of this thesis is to measure the degree of substitution between money and several short-term assets in the Brazilian economy in the period 1973 to 1976.

The methodology utilized is from the path-breaking work of V. K. Chetty who was the first to attempt the estimation of an asset elasticity of substitution. While Chetty's approach is innovative in this area, it is shown that his elasticity of substitution is dimensioned by the length of the period which defines the interest rate, and is, therefore, analytically an inadequate substitution measure.

Proposed here is a model of asset substitution where assets are demanded because of the pecuniary return and services they provide, and paid for by what must be foregone so that they may be held. The model yields an elasticity of substitution that is dimensionless.
Empirical results are that the hypothesis of the presence of money substitution in all short-term assets cannot be accepted. Money is the only medium of exchange in Brazil, no asset easily substitutes for it, and therefore the proper definition of money in Brazil for the period 1973 to 1976 is the simple definition $M_1$, currency and demand deposits.
ACKNOWLEDGEMENTS

It is a difficult task to give credit to everyone who has contributed in one way or another to the completion of this dissertation. Nevertheless, there are a number of individuals who stand out as major influences on this particular work and to whom I am especially indebted.

The Chairman of the Dissertation Committee was Professor Gordon Smith. It was under his guidance that I came to realize what sound, academic performance is all about, and, most importantly, what it demands. The pleasure I experienced working with Professor Smith will remain one of the fondest memories of my last year of graduate work at Rice University.

I am also grateful to Professors Donald Huddle, who served as the second member of the committee, and Ronald Soligo. They kindled my interest in the study of Economic Development and both have always been a constant source of encouragement to me.

The third member of the committee, Professor Robert Dix, made a number of useful suggestions which were incorporated into this last draft.

I am indebted in a special way to my friends and colleagues at the Universidade Federal de Pernambuco in Recife, Brazil, where this research project took on its initial form, for their constant stream of suggestions and support which they so unselfishly gave.

Finally, this list of acknowledgements would be incomplete without my parents, William and Evelyn Samohyl, always steadfast in their
spiritual and emotional support of a son's graduate career that seemed at times to be without end.

I must share with these individuals any merit found in this study, any shortcomings are, of course, my own.
This dissertation is dedicated to the memory of

Tuncay Sunman,

with a special dedication to

Veronica Romao.
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CHAPTER I

THE RECENT DEVELOPMENT OF BRAZILIAN CAPITAL MARKETS
AND THE QUESTION OF MONEY SUBSTITUTION

Crescit amor nummi quantum
ipsa pecunia crescit.
CHAPTER I

THE RECENT DEVELOPMENT OF BRAZILIAN CAPITAL MARKETS
AND THE QUESTION OF MONEY SUBSTITUTION

Since the mid-1960's, Brazil has undergone a fundamental alteration in the structure of its domestic capital markets, and more recently these markets have grown to such proportions that there is no facet of the economy which does not feel their presence. In the period prior to this rapid growth, two predominant factors had been instrumental in suppressing the free flow of resources through financial intermediation. First, the Usury Law of 1938 made illegal any financial transaction between lender and borrower which entailed a nominal rate of interest greater than 12 per cent. Effective rates of interest charged to the borrower by commercial banks were at times greater than this legislated ceiling because commercial banks were in a favored position as a financial intermediary and could require that borrowers hold relatively large compensating balances in the form of non-interest-bearing demand deposits. This requirement on borrowers raised the effective rate of interest of bank loans and was a convenient way of avoiding the repressive effects of the Usury Law. Nevertheless, the appearance of other kinds of intermediaries, unable by law to create demand deposits, was never realized to any great extent. No avenue was open which would have allowed them to compete on an equal level with the privileged position which commercial banks alone maintained.

Secondly, with a persistent rate of inflation of more than 15 percent per year throughout the 1950's, which culminated in 100 per cent in mid-1964, the evolution of financial intermediation was
further constrained. Where the nominal rate of interest is forced to assume a value less than the rate of inflation, a situation which in effect punishes creditors no matter how lucrative the venture their loan supports, one cannot expect the business of making loans to flourish. Consequently, throughout the period in which the Usury Law and galloping inflation were conspiring against the intercourse of lender and borrower, the primitive level at which capital markets functioned was preserved with only the existence of a small number of commercial banks to service the private sector in its need for loanable funds. 1

The recent amplification of financial intermediation in Brazil is a direct result of several modifications in the legal framework of the capital market which had as their primary objective the immediate installation of an efficient financial sector. In the year 1965, a long-term federal government security was offered to the Brazilian investor as a non-inflationary means of financing the ever present budget deficit. Before the introduction of this security, balance in the budget had been achieved, for the most part, through money creation, a practice which contributed to the hyperinflation of 100 percent of the year before. Just as important, however was that for this security to be viable in the marketplace, it had to yield a return which would attract investors even when the inflation rate was anything but an incentive to invest in financial assets. In order to attract investors to this new government bond, its nominal return was tied to the rate of inflation; in other words, it was indexed. In the
same year, indexation was extended to a host of other financial assets: certificates of deposit of commercial banks, the long-term security of the National Housing Bank, and the short-term deposits of the newly created financial institutions of the Brazilian System of Savings and Loans. Finally, the injurious effects of the Usury Law of 1938 would no longer be felt by the Brazilian saver. One year later, commercial banks were allowed to index the rate of return on passbook savings.

Moreover, with the initiation in 1965 of the federal government's National Housing Bank and the Brazilian System of Savings and Loans—created as the major intermediary between the Housing Bank and the Brazilian populace—a network of financial institutions came into being that had the capacity to capture financial resources in the marketplace, and allocate those resources to the construction of personal dwellings. The segmentation of the loan market was further reinforced by the limitation placed on commercial banks that they deal only in credit to agriculture and working capital to the industrial and commercial sectors. The new financial institutions were partially patterned after mutual savings banks and savings and loan associations in the United States. One significant difference, however, was that savings accounts could be opened without the requirement of a minimum balance. For the first time, the small potential investor had the opportunity to obtain an attractive return on his savings.

In 1970, the last of the most important financial reforms occurred. The federal government launched a treasury bill for the explicit purpose of more precise monetary control through open market operations, indi-
cating both that Brazilian capital markets were thought to have reached a level of efficiency which had never been seen in the past, and that the function of the markets was smooth enough to facilitate the control of the money supply by the Central Bank.

The growing importance of financial intermediation in Brazil since 1964 is illustrated in Table I. Short-term non-monetary assets increased from 9 per cent of all financial assets in 1964 to 36 per cent in 1975. On the other hand, the share of money narrowly defined in the total of all financial assets has been declining steadily since 1964 from 68 per cent to 17 per cent in 1975. In the year 1970, short-term non-monetary assets became relatively more important than money as measured by the share in the total of all financial assets, and, up to the present time, these short-term assets have consistently maintained this position of superiority.

Economists argue that the emergence of financial institutions in developing economies may be both a blessing and a burden in attaining economic growth and stability. It is usually emphasized that financial institutions can benefit the growing economy in two ways. First, financial institutions offer an alternative financial asset to money which is attractive to the asset holder because of the explicit rate of return which it yields. Since money does not offer a return beyond the liquidity services it provides, it is a poor savings instrument. The assets of financial institutions, however, may be relatively good savings instruments. If the asset holder feels that the liquid-
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Source: Boletim do Banco Central do Brasil, Volume 12, No. 11, November, 1976, p.124.

Column (1): The total of all financial assets in Brazil excluding those held by the government and financial institutions.

Column (4) A variety of short-term financial assets with fixed nominal return.

Column (6): Includes compulsory savings programs, variable yield securities, and foreign financial assets held in Brazil.

Column (3) + Column (5) + Column (6) = 100.
worth less to him than the pecuniary return that he will receive, it is clear that he will purchase the asset and forego money balances. If the return offered by the financial institution is high enough to reduce consumption in the present period, then not only will the holding of idle money balances be reduced, but savings will be increased.\(^3\) Furthermore, as the number and variety of financial institutions increase throughout the economy, in effect taking their liabilities to the potential saver, transaction costs diminish. And with a broad spectrum of assets to choose from, the potential saver is confronted with less risk through diversification. Once again, the incentive is to save more. One of the severest barriers to the growth of developing areas has been, historically, an inadequate savings propensity; however, the appearance of an efficient network of financial institutions may contribute to the lowering of this barrier.

Secondly, by channeling resources from surplus units (entities that spend less than their income) to deficit units, these institutions may improve the efficiency of the allocation of investment. The individual asset holder usually does not have knowledge of a wide range of investment opportunities, whereas the financial intermediary, concentrating its efforts in a particular area of the economy, can better know where the highest and safest returns can be found.\(^4\)

The presence of financial intermediaries, on the other hand, may lead to a degree of intermarket complexity that, if not properly understood by both private and governmental participants in the economy, could support a series of problems without patent solution.
Financial markets are unique in a free market economy in that they are the pivot around which all sectors of the economy revolve. From saver to investor, from domestic resources to international trade, from public sector to private, the complex of financial markets acts to change the environment around it, and in turn is itself modified by the same facotes which at one time it may have acted to change. It is exactly because of the pivotal position of financial markets in the economy that the mechanism of financial intermediation needs to be analyzed and understood better. Several specific questions of economic importance concerning the impact of financial intermediation are the following:

1. In controlling the money supply, does the monetary authority also control liquidity? Some financial assets may have qualities so close to that of money—narrowly defined as currency and demand deposits—that they easily substitute for money in asset holders' portfolios. If the monetary authority were not aware of the extent of this substitution phenomenon, then a policy which it followed to control liquidity in the economy may not be effective.\(^5\)

2. Do financial markets react to attenuate specific monetary actions of the monetary authority? It has been argued by Gurley and Shaw (1960) that the presence of financial intermediaries that supply a close substitute for money cause the the demand for money to be more elastic with respect to the interest rate than it otherwise would be, implying smaller changes in interest rates for given monetary actions.\(^6\)
3. Do unpredictable changes in taste, technology and competitive environment which originate in the financial sector cause the demand for money to be unstable? If instability were a problem, then the monetary authority could never be very certain about any policy outcome, and in fact trying to counteract the instability it saw could lead to a series of policies so discontinuous that the overall result would be even more unstable.

4. Are monetary policies which are specific to a particular type of financial institution expansionary or contractionary in the aggregate? Brainard and Tobin (1968) have argued that a change in the ceiling rate on time deposits may produce indeterminate results if the degree of substitution between money and the assets and liabilities of the financial institution is unknown.  

5. Do certain sectors of the economy suffer differential effects from a given monetary action by the monetary authority? A given financial institution usually services only a small segment of the economy. One institution may deal exclusively in short-term consumer credit, another in mortgages, and yet another in large business ventures. A given monetary action may affect each institution in different degrees, and likewise each sector of the economy will experience varying degrees of incentive and disincentive.

   It is important to note that the answer to all of the questions above depends, in one way or another, upon the degree of substitution between money and various financial assets—the problem of money substitution. This thesis will have as its focal point the empirical determination of which assets in the Brazilian economy are close
substitutes for money. While there are several ways of measuring this degree of substitution, the method which we shall utilize is the estimation of elasticities of substitution between money and various short-term assets. 8

In the following chapter, various studies which have attempted the estimation of an elasticity of substitution between money and financial assets will be reviewed. The purpose of the chapter is not to express any original criticism of these works but to simply demonstrate the common ground they share and the points which distinguish them from one another. Subsequent chapters will attempt to put forth some original thoughts on the subject.
FOOTNOTES

1. For an interesting discussion of the shrewd maneuvers which commercial banks undertook in this period to sidestep the 12 per cent interest rate ceiling, see Simonsen (1969).

2. A more complete description of Brazilian financial institutions and the liabilities they offer to investors is in Appendix I.

3. Technically, this would occur if the return offered by the financial institution—assuming no risk and no transaction cost—was greater than the rate of time preference of the consumer. See Henderson and Quandt (1958, p. 234-240) for a discussion of the dependence of saving on market interest rates and time preference.

4. An extensive discussion of the benefits which accrue to an economy where financial markets are free to administer to the needs of the productive sector without the threat and occurrence of government intervention may be found in Shaw (1973, chapter 1).

5. See Friedman and Schwartz (1970, part I), for a survey of the research done to estimate liquidity as a weighted average of money narrowly defined and various short-term assets.

6. This does not necessarily imply that the monetary authority experiences weakened control over rates of interest, but only that larger monetary actions are necessary to arrive at specific policy objectives. Marty (1961) has argued theoretically and Cagan and Schwartz (1975) empirically that the presence of substitution between money and short-term assets reduces the elasticity of demand for money and therefore enhances the power of a given monetary action to change the level of interest rates.

7. If money and time deposits are strong substitutes, a reduction in the return on time deposits will increase the demand for money balances and reduce the amount of banks' loanable funds. On the other hand, if time deposits and bonds and equities were the stronger substitutes in asset holders' portfolios, then a reduction in the time deposit rate would be expansionary. See Brainard and Tobin (1968, p. 104).
8. It is beyond the scope of this thesis to survey in detail all of the techniques which have been used in the past to measure money substitution. However, there are two recent surveys of this material which take different but equally interesting approaches to reviewing this area of the literature. Goldfeld (1973) describes the regression techniques and equations from numerous specifications of the demand for money, and then using a common data base for the United States (1952:2-1972:4) reestimates the demand for money following the techniques in the literature. Goldfeld concludes (p. 632):

"Perhaps most interesting is the apparent sturdiness of a quite conventional formulation of the money demand function, however scrutinized. More particularly, such a function yields sensible interest and income elasticities."

This conventional formulation is Goldfeld's equation (4):

\[
\ln m = .271 + .193 \ln y + .717 \ln m_{-1} - .019 \ln RCP - .045 \ln RTD
\]

where \( m \) is real money narrowly defined, \( y \) is real income, \( m_{-1} \) is the dependent variable lagged one period, \( RCP \) is the commercial paper rate, and \( RTD \) is the rate on time deposits at commercial banks. All coefficients are significant at the 99% level. Note that the interest rate coefficients are relatively small as a result one may argue that money substitution is not a particularly pressing problem, at least in the United States during Goldfeld's period of analysis.

Feige and Pearce (1977) review much the same literature that Goldfeld does, but their contribution is in deriving comparable interest rate and income elasticities of the demand for money from the original regression estimates of each study analyzed. Their conclusion (p. 464):

"We, however, interpret the evidence as suggesting that the cross-elasticity estimates ..... are evidence of a relatively low degree of substitution between narrowly-defined money and near-money."

Feige and Pearce do mention several attempts at estimating an elasticity of substitution between money and short-term assets but point out that these estimates are ambiguous in interpretation.
CHAPTER II

THE ELASTICITY OF SUBSTITUTION IN MONETARY ECONOMICS

Even the law of gravitation would be brought into dispute were there a pecuniary interest involved.

Thomas Babington Macauley
CHAPTER II

THE ELASTICITY OF SUBSTITUTION IN MONETARY ECONOMICS

I. INTRODUCTION

The purpose of this chapter is to review the literature concerning the estimation of an elasticity of substitution between money and other short-term assets. It will be apparent from this review that there is a common underlying structure which all studies of this type share. Our attention is focused on four studies--three concern the United States: Chetty (1969), Edwards (1972), and Moroney and Wilbratte (1976), and the other is in reference to Brazil: Contador (1974). While several additional studies are mentioned, these four stand out because of important econometric and theoretical innovations. The motivation for this area of Monetary Economics is to develop a stronger conceptual framework for the definition and measurement of money substitution than is offered by analyses of the determinants of the demand for money. While it may be shown both theoretically and empirically that the level of money demand depends upon certain interest rates, wealth and income, and, perhaps, other variables, it does not necessarily follow from this kind of analysis that information as to degree of substitution may be contained therein, even though degree of substitution may be an important underlying force in the analysis. In other words, any study of money demand is not necessarily a study of money substitution. Indeed, a major contention of this literature is that any attempt at determining the degree of substitution between assets must take into account in an explicit way that substitution is the response of two or more assets to a change in relative price.
where this response is constrained to movements along an indifference curve. Consequently, studies of the elasticity of substitution depend upon strict and explicit assumptions concerning the preferences and budget constraint of the typical asset holder. In fact, once explicit structure has been given to preferences and the budget, substitution studies and demand studies yield the same information. There is no difference between them because the structure of preferences and the budget, if known, is enough to yield an elasticity of substitution and a demand function. That is, any study of money substitution is necessarily a study of money demand.

In the next four sections of this chapter, we shall describe attempts that have been made to estimate an elasticity of substitution between money and other short-term assets. The last section of the chapter is a brief note on the possible effects of the supply side of financial markets on the analysis.

II. THE INTRODUCTION OF THE ELASTICITY OF SUBSTITUTION TO THE MONETARY ECONOMICS LITERATURE

1. Prior to the year in which Chetty's innovative work in Monetary Economics appeared, the literature on money substitution was confusing due to the variety of theoretical models that covered the pages of Economics journals, and because of the diversity of empirical results which these models produced. Two studies from this period which are often cited as evidence of this confusion are by Feige (1964) and Lee (1966). Feige, using cross-section data, found that the rate of interest on nonbank financial assets had little to no effect upon the demand
for money narrowly defined as currency and demand deposits, and he concluded that money and the assets of financial institutions are not close substitutes. Lee's results, however, using Feige's data, were that both savings and loan shares and time deposits at commercial banks are close substitutes for money.² With these two studies being representative of much of the money substitution literature, one can see that work in this area had lead to inconclusive results. A new approach was called for, and it came in the form of Chetty's suggestion to estimate an elasticity of substitution between money and near-mones as a test of the presence and degree of money substitution.

"In order to determine whether the public regards various liquid assets as substitutes for money, one has to determine empirically the shape of the consumer's indifference curves for money and other liquid assets." (Chetty, 1969, p. 270).

One method of empirically demonstrating the shape of the consumer's indifference curve, and the method which Chetty chose to develop, is to estimate an elasticity of substitution. Theoretically, the elasticity of substitution is directly measured from the contour of the consumer's indifference curve. Before Chetty, the traditional substitution measure was the cross-elasticity of demand for money. The strongest argument against the cross-elasticity of demand and in favor of the elasticity of substitution as a substitution measure is that the former is not symmetric. Since it is a combination of substitution and wealth effects, it may happen that the cross-elasticity of demand going in one direction is not the same as its obverse. Following the formulation of Henderson and Quandt (1971, p. 36), define the cross-elasticity of demand for good i given a change in the price of j as
\[ N_{ij} = S_{ij} - R_j W_i \]

where \( S_{ij} \) and \( W_j \) are substitution and wealth effects, respectively, in elasticity terms, and \( R_j \) is the proportion of wealth spent on \( j \). The obverse of \( N_{ij} \) may be written,

\[ N_{ji} = S_{ji} - R_i W_j \]

Clearly, the sign of \( N_{ij} \) need not be the sign of \( N_{ji} \). One cross-elasticity may denote substitution while the other may denote complementarity.\(^3\) The elasticity of substitution does not suffer this criticism because, unlike the cross-elasticity of demand, it is symmetric. A given degree of substitution in one direction will always be identical to substitution in the other direction. In addition to the theoretical advantages of the elasticity of substitution as a measure of substitution, the elasticity of substitution can be a useful tool of the monetary authority.

Using these estimates, a monetary authority is in a position to react to changes in relative price and avoid periods of excess demand or supply of money. For instance, if the price of a money substitute fell, by employing the elasticity of substitution the monetary authority would know what the new relative demands for money and the short-term asset are and could adjust the money supply accordingly to avoid any inflationary pressure.

Lastly, the curvature of the indifference curve between money and other assets is an important ingredient in estimating an adjusted money
supply. From price theory, when an indifference curve is a straight line (elasticity of substitution infinite), substitution is perfect and if one of the assets is money narrowly defined then both may be considered as money. An adjusted money supply, therefore, would have to include both assets. If the indifference curve is right-angled, there is no substitution and only money is money as seen through the preferences of asset holders. Consequently, when less-than-perfect substitution exists between money and another short-term asset, an adjusted money supply would include money and some portion of the short-term asset. We now turn to a review of Chetty (1969).

2. Since the elasticity of substitution relies upon the exact nature of a consumer's preferences as described by his indifference curve mapping, it is necessary in an empirical attempt to estimate an elasticity of substitution to state explicitly the consumer's preference function and to show how this function interrelates with the constraints against which the consumer acts. Chetty proposed the following structure for an asset holder utility function,

"...let us assume that the consumer combines money and time deposits to produce various characteristics like liquidity, store of value, etc. We assume that the consumer combines M and T (money narrowly defined and time deposits) such that for any given budget, he maximizes his satisfaction." (p. 272).

"The utility function of the consumer can be written as

\[ U = (\beta_1 M^{-\rho} + \beta_2 T^{-\rho})^{-1/\rho} \]

where M and T represent money holdings and money value of time deposits in the next period, respectively." (p. 273).
This function is the familiar CES utility function whose formulation is from Arrow et al (1961). From the parameter \( \rho \) the elasticity of substitution may be calculated, \(^5\)

\[
\sigma_{MT} = \frac{\frac{d \log \frac{M}{T}}{\frac{\partial u}{\partial M}}}{d \log \frac{\partial u}{\partial T}} = \frac{1}{1+\rho}
\]

Given a value for \( \rho \), the value of the elasticity of substitution is determined and reflects the curvature of the utility surface. For values of \( \sigma_{MT} \) between 0 and \( +\infty \), the appropriate values for \( \rho \) are between \( +\infty \) and \(-1\), respectively. At \( \rho = -1 \), the function is

\[
U = \beta_1 M + \beta_2 T
\]

and indifference curves between \( M \) and \( T \) are straight lines. If \( \beta_1 + \beta_2 = 1 \), the as \( \rho \to 0 \) the utility function becomes

\[
U = \beta_1 \beta_2 M^T
\]

which is Cobb-Douglas with unitary elasticity of substitution. With \( \rho \) approaching infinity, \( \sigma_{MT} \) goes to zero and the function is

\[
U = U(\min M, \min T)
\]

which is characterized by right-angled indifference curves.
To estimate the parameters of the utility function, one must devise a budget constraint which specifies the amount of resources available to the consumer to allocate to the two financial assets, and the trade-off between the two assets in terms of their relative price. In suggesting the structure of the budget constraint, Chetty argues,

"Suppose the consumer has cash holdings of \( M_0 \) dollars and wants to allocate them between \( M \) and \( T \). If \( T \) represents the cash value of time deposits in the next period and if \( i \) is the rate of interest on time deposits of the current period, then the budget constraint of the consumer can be written as

\[
M_0 = M + \frac{T}{1+i}
\]

The slope of the budget line is \(-(1+i)\). Hence \((1+i)\) can be considered as the ratio of the prices of money to time deposits." (p. 273).

Chetty then shows that the equilibrium condition for maximizing utility is,

\[
\frac{\beta_1}{\beta_2} \left( \frac{M}{T} \right)^{-\sigma - 1} = 1 + i
\]

"Taking logarithms on both sides, rearranging terms, and adding a disturbance term, we have the regression model

\[
\log \frac{M}{T} = -\frac{1}{1+\sigma} \log \frac{\beta_2}{\beta_1} + \frac{1}{1+\sigma} \log \frac{1}{1+i} + \varepsilon.
\]

(p. 273).

The coefficient on the variable \( \log \frac{1}{1+i} \) is the elasticity of substitution. Chetty derives the regression equation from the equilibrium condition of his utility maximizing model and not from its implied demand functions for money (\( M \)) and time deposits (\( T \)). Nevertheless, it can be shown that the same regression model may be
derived through the implied demand functions. The demand equation for
money is

\[ M = \frac{M_0 \left( \frac{\sigma_1}{\sigma_2} \cdot \frac{1}{1+i} \right)^\sigma}{\frac{1}{1+i} + \left( \frac{\sigma_1}{\sigma_2} \cdot \frac{1}{1+i} \right)^\sigma} \]

and the demand equation for time deposits is

\[ T = \frac{M_0}{\frac{1}{1+i} + \left( \frac{\sigma_1}{\sigma_2} \cdot \frac{1}{1+i} \right)^\sigma} . \]

Dividing the first demand equation by the second, we have,

\[ \frac{M}{T} = \left( \frac{\sigma_1}{\sigma_2} \cdot \frac{1}{1+i} \right)^\sigma \]

Taking logarithms and rearranging terms, Chetty's regression
model is derived. Chetty's study of money substitution, therefore, is identical to a study of the relative demand for money under certain strict assumptions concerning the structure of the asset-
holder utility function and budget constraint. The restrictions placed on the typical asset holder are necessary to estimate the elasticity of substitution. If these restrictions were lifted, demand equations would take on a more general form, but it is not clear that an elasticity of substitution, and, hence, the shape of the indifference curve, could be estimated from them. This model, which we label C-I, and its estimates are summarized in Table I.

This model represents the simplest approach offered by Chetty, and its strictest assumption is that the utility function is
TABLE I
MODEL C-I

Utility function:
\[ u = (\beta_1 M^{-\rho} + \beta_2 T^{-\rho})^{-1/\rho} \]

Budget constraint:
\[ M_0 = M + \frac{T}{1+i} \]

Regression equation:
\[ \log \frac{M}{T} = -\frac{1}{1+\rho} \log \frac{\beta_2}{\beta_1} + \frac{1}{1+\rho} \log \frac{1}{1+i} + e \]

Estimated by:
Ordinary least squares

Elasticity of substitution:
\[ \sigma_{MT} = \frac{1}{1+\rho} \]

Empirical results:
\[ \sigma_{MT} = 34.690 \]
\[ \sigma_{M,SL} = 101.851 \]
\[ \sigma_{M,MS} = 27.637 \]

Variables:
- \( U \) = aggregate portfolio utility
- \( T \) = nominal value of time deposits at commercial banks
- \( M \) = nominal value of \( M_1 \)
- \( M_0 \) = initial allocation of wealth
- \( i \) = nominal rate of interest on time deposits
- \( SL \) = nominal value of savings and loan shares
- \( MS \) = nominal value of mutual savings bank shares
homogeneous. Clearly, this assumption on the structure of demand for financial assets is unduly restrictive because the wealth elasticities of demand for the two assets are assumed identical. This implies that the relative holdings of money and time deposits are determined entirely by the relative price of the assets, \(-1 + i\).\(^8\) While the occurrence of equal wealth elasticities is not impossible, it seems unlikely. Consequently, a functional form for \(U\) which would permit a statistical test for the assumption of homogeneity would be more interesting. Toward this end, Chetty proposed the following general structure for the utility function,\(^9\)

\[
u = \left( \beta M^{-\rho} + \sum_{j=1}^{n} \beta_j X_j^{-\rho_j} \right)^{-1/\rho}
\]

where the \(\rho_j\)'s may take on different values, \(M\) is nominal value of money balances and the \(X_j\) are the nominal values of various short-term assets. This functional form allows for non-homogeneity and different elasticities of substitution for different asset pairs due to the possibility of unequal \(\rho_j\)'s. To complete this more general model, the budget constraint must be respecified to include at least all of the assets in the utility function. Nevertheless, the budget constraint remains basically the same as before, where \(r_j\) is the nominal rate of interest on \(X_j\),

\[
M_0 = M + \sum_{j=1}^{n} \frac{X_j}{1+r_j}.
\]
Maximizing the non-homogeneous utility function given the new budget constraint, and after some manipulation, the following regression equation, which Chetty estimates using ordinary least squares, is derived:

\[ \log X_j = -\frac{1}{\rho_j+1} \log \beta_j \rho_j - \frac{1}{\rho_j+1} \log \frac{1}{1+r_j} + \frac{c+1}{\rho_j+1} \log M + e. \]

Since the function is not homogeneous, the elasticity of substitution is a more complicated expression than in the model C-I:

\[ \frac{\partial X_j}{\partial M} = \frac{1}{(1+\rho) + (\rho_j-\rho)} \frac{\beta_j X_j^{-\rho_j}}{1 + \frac{\beta_j X_j^{-\rho_j}}{\beta_j M^{-\rho}}} \]

This expression has been called either the Meade (1959) or Hicks-Allen (1934) partial elasticity of substitution. While they are defined differently, when the utility function is characterized by separability they are equivalent. (See footnote 9.) We shall always refer to this model as C-II, and it is summarized with empirical results in Table II.

As we pointed out above, Chetty estimated both of these models by ordinary least squares. In doing so, the statistical assumption surrounding the variable M is not the same for the two models. In model C-I, M appears on the left hand side of the regression equation. It is assumed to be a random variable with stochastic elements. In model C-II, M appears on the right hand side of the regression equation, and according to the assumptions of ordinary least squares
TABLE II
MODEL C-II

Utility function:

\[ u = \left( \tilde{\Omega}^\rho + \sum_{j=1}^{n} \beta_j x_j^{-\rho} \right)^{-1/\rho} \]

Budget Constraint:

\[ M_o = M + \sum_{j=1}^{n} \frac{x_j}{1+r_j} \]

Regression equation:

\[ \log X_j = -\frac{1}{\rho+1} \log \frac{\beta}{\beta_j} - \frac{1}{\rho+1} \log \frac{1}{1+r_j} + \frac{\rho+1}{\rho+1} \log M + \epsilon \]

Estimated by:

Ordinary least squares

Elasticity of substitution:

\[ \sigma_{MX_j} = \frac{1}{(1+\rho) + (\beta_j x_j^{-\rho})} \]

Empirical results:

\[ \sigma_M = 30.864 \]
\[ \sigma_M, SL = 35.461 \]
\[ \sigma_M, MS = 23.310 \]

Variables:

Same as Table I, and

\[ X_j = \text{nominal value of short-term asset } j. \]
\[ r_j = \text{nominal rate of interest of short-term asset } j. \]
if the estimated coefficients of the regression equation are to be consistent then M is fixed and without stochastic elements. To maintain the assumption of randomness of M from the first model to the second, Chetty suggests a two-stage regression technique.\textsuperscript{10} First, M is purged of its stochastic element, and, second, the purged value of M is used as the variable on the right hand side of the regression equation of the second model. This two-step procedure gives us model C-III. One equation of this two-equation model is the same as the regression equation of model C-II, and the other equation which purges M of its stochastic element is the following:

\[
\log M = a_0 + \sum_{j=1}^{n} a_j \log(1+r_j) + a_{n+1} \log Y + e
\]

where Y is income. The two-step procedure is followed in which the above equation is first estimated, and then the expected value of log M from the estimation is used in the regression equation of model C-II. However, as Chetty himself points out:

"...income and interest rates may not be strictly exogenous variables and hence (estimates from) the two-stage least squares may not be consistent..... Thus the ordinary least squares may be no worse than the two-stage least squares as far as the inconsistency is concerned." (p. 277).

Chetty does on to explain:

"Both the methods were tried in estimating the regression. Since the estimates of the parameters were almost identical, only the ordinary least squares estimates are (reported)." (p. 277).
TABLE III
MODEL C-III

Utility function:

Same as C-II

Budget constraint:

Same as C-II

Regression equations:

Same as C-II, and

\[ \log M = a_0 + \sum_{j=1}^{n} a_j \log(1+r_j) + a_{n+1} \log Y + e \]

Estimated by:

Two-stage least squares

Elasticity of Substitution:

Same as C-II

Empirical results:

None reported.

Variables:

Same as Table I and Table II, and

\[ Y = \text{nominal income.} \]
Furthermore, Chetty goes on to argue that the function \( U \) in either its strict homogeneous form (Model C-I) or its more general non-homogeneous form (Model C-II, Model C-III) is, in fact, an estimator for an adjusted money supply. From each regression equation of the three individual models, all of the parameters of the given utility function may be calculated, with the exception of the coefficient on \( M \) which Chetty (1969, p. 273) argues ought to be set equal to one; and, with these parameter estimates and actual values of the financial assets including money balances, the value of \( U \) itself is derived. It is this value of \( U \) which Chetty calls the adjusted money supply. However, Steinhauer and Chang (1972, p. 223) comment that there is no particular reason for \( U \) to be called an adjusted money supply. Since a given degree of substitution between (say) money balances (\( M \)) and time deposits (\( T \)) may occur because \( M \) is like \( T \), \( U \) could be an estimator for an adjusted time deposit supply. That is, in order to estimate an adjusted money supply from a substitution study, one must not only have estimates for the degree of substitution between \( M \) and \( T \), but, moreover, one must know why a particular degree of substitution obtains. In all likelihood, \( M \) and \( T \) would substitute for one another because they share certain characteristics. For instance, \( M \) is a better means of exchange than \( T \) because of lower transactions costs but \( T \) is a better store of value because it yields a return. Substitution between \( M \) and \( T \) results from substitution between these two characteristics which both possess. It is therefore, unclear what \( U \) is, and as long as the antecedents of a given degree of substitution remain unknown, the identity of \( U \)
remains a mystery.

3. The conclusion which Chetty draws from his estimates is that all of the assets which entered his analysis are close substitutes for money. In light of the fact that the elasticities estimated by Chetty are all substantial, Chetty's analysis leads one to believe that asset holders treat money and short-term assets as almost perfect substitutes.

Feige and Pearce (1977), in their extensive review of the empirical literature on money substitution, approach Chetty's results with caution. While they argue that there is no proposition in the literature which relates any particular value of the cross-elasticity of demand for money or the elasticity of substitution to the presence of money substitution, they do choose to associate substitution with cross-elasticities greater than one for no other reason than this is the traditional benchmark between weak (inelastic) and strong (elastic) response in demand (p. 443). Their caution towards Chetty's results derives from the fact that no cross-elasticity of demand for money in any study they review is greater than one, denoting weak response and little substitution, whereas Chetty's results imply strong substitution possibilities between money and other short-term assets. However, for Chetty's model C-I (two assets and homogeneity in the utility function), Feige and Pearce suggest the following relationship for the elasticity of substitution $\sigma_{ms}$ between money and a possible substitute asset and conventional own- and cross-elasticities of demand for money, $\sigma_{ss}$ and $\sigma_{ms}$, respectively,
\[ ζ_{ms} = \frac{1+r_s}{r_s} \left( N_{ms} - N_{ss} \right) \]

where \( r_s \) is the rate of interest on the substitute asset. The interest rate enters the relationship between the elasticity of substitution and the demand elasticities because the price in the elasticity of substitution is \( \frac{1}{1+r_s} \) and the price in the demand elasticities is simply \( r_s \) alone.\(^{12}\) If \( r_s \) were to double, then \( \frac{1}{1+r_s} \) would fall by only \( \frac{r_s}{1+r_s} \) which, for typical values of the rate of interest, is a relatively small number. Since the percentage change in price is the denominator of the measures, the differential of the demand elasticities is always smaller in absolute value than the elasticity of substitution because a percentage change in the price \( r_s \) of the demand elasticities translates into a much smaller change in the price \( \frac{1}{1+r_s} \) for the elasticity of substitution.

Substituting into the above expression

\[ N_{ms} = -.4, \quad N_{ss} = 1.0, \quad \text{and} \quad r_s = .04, \]

typical values of these variables found in Feige and Pearce, the resulting elasticity of substitution is 36.4, which is in the neighborhood of the values for the elasticity of substitution Chetty found. Feige and Pearce conclude, therefore, that "Chetty's claim (of strong money substitutability) must be regarded with some caution." (p. 460).

It is also important to point out that the ratio \( \frac{1+r_s}{r_s} \) acts as
an arbitrary wedge between the values of the elasticity of substitution and the demand elasticities, and actually determines the relative size of the substitution measures. Due to the unique specification of relative price in Chetty's formulation, the smaller (larger) is the rate of interest, the larger (smaller) is the discrepancy between the elasticity of substitution and the demand elasticities. If Feige and Pearce had used a quarterly interest rate in their example instead of the annual rate of 4 per cent, and the same values of the demand elasticities, the implied elasticity of substitution is approximately 150. In other words, both $\sigma_{ms}$ and $(N_{ms} - N_{ss})$ cannot be constant at the same time for a given change in the rate of interest, even if the change is arbitrary. This result is, of course, contradictory. When it is assumed that consumer preferences are constant, then neither the elasticity of substitution nor the demand elasticities should change in light of an arbitrary change in price. Since both measures are assumed to reflect consumer preferences, it is unacceptable to say that, on the one hand, preferences change, whereas, on the other, they do not. This apparent contradiction in measures will be the major topic of the first part of the next chapter. There we shall show that Chetty's asset-substitution model is flawed because its results depend upon the arbitrary choice of time unit as it contributes to the value of the relative price between assets. We shall show that Chetty's model actively determines the structure of preferences instead of passively reflecting them. In the second part of that chapter, we offer a new approach by
redefining price so that dimensionality is no longer a problem and the substitution measures within the model are not inconsistent.

III. ELASTICITY OF SUBSTITUTION AND CROSS-SECTION DATA.

Edwards (1972) approach to measuring the elasticity of substitution is, in all respects but one, equal to the approach of Chetty— in particular, models C-II and C-III. Edwards differs in arguing that the use of time-series data may bias the measured degree of substitution between money and short-term assets. Changes in the quality of assets undoubtedly occur through time, and if these quality changes are not accounted for in the regression analysis, then the contribution of the rate of interest to asset substitution may be overstated. It is possible, therefore, that the high degree of substitution that Chetty estimated may be due to quality changes of the short-term assets and not necessarily from changes in the rate of interest.

In order to avoid the problem of non-quantifiable quality changes, Edwards proposes that cross-section data would be more appropriate for the estimation of the elasticity of substitution. This type of data base, since it is stationary in time, would not necessarily contain systematic changes in quality (even though quality may be different for different locations), and so the elasticity of substitution measured by using this kind of data base would be less biased than the elasticity of substitution from time-series data.

Nevertheless, Edwards' results are not conclusive. Using Chetty's model C-II (the ordinary least squares model with non-
homothetic preferences), Edwards finds a relatively high degree of substitution between money and bank time deposits, savings and loan shares and mutual savings bank shares. On the other hand, using model C-III (two-stage least squares) the elasticities of substitution were lower and according to Edwards indicate very little substitution between money and the short-term assets. Edwards' results are reported below in Table IV.

IV. EXPECTATIONS, HOMOGENEITY, AND PARTIAL ADJUSTMENT.

1. Contador's estimates of the elasticity of substitution for Brazil are derived from a model which is not fundamentally different from the models of Chetty, but it is modified in several ways to account for differences in data base and inflationary experience. The contributions to this area of Monetary Economics by Contador (1974) may be classified in three distinct areas.

First, being aware of the tendency of the Brazilian economy to suffer from relatively high and unstable rates of inflation, and of the resulting inflationary mentality of the Brazilian populace, Contador proposed that in Brazil actual rates of interest (as used by Chetty) were secondary to expected rates as a decision variable in the allocation of resources to the purchase of financial assets. With interest rates in Brazil strongly dependent upon the rate of inflation, either through the official policy of indexation or through the operation of the free market for short-term securities, it seemed to Contador that expected rates of interest and not actual rates
### TABLE IV

EDWARDS' EMPIRICAL RESULTS

<table>
<thead>
<tr>
<th>Elasticity of Substitution between</th>
<th>Ordinary Least Squares</th>
<th>Two Stage Least Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Money and Bank Savings Accounts</td>
<td>46.942</td>
<td>2.417</td>
</tr>
<tr>
<td>Money and Non-Bank Savings Accounts</td>
<td>6.847</td>
<td>-1.231</td>
</tr>
</tbody>
</table>
determined asset-holding behavior. Contador used Box-Jenkins time series analysis to generate the expected rate of indexation and used it as a proxy for the value of expected interest rates. At the time that his research was being undertaken, an interest rate series was not available for all the potential money substitutes in Brazil, and this lack of interest rate data necessitated the use of the proxy variable. Substituting expected rates for actual interest rates, Contador estimated Chetty's model C-III (2SLS) using monthly Brazilian data for the years 1970 to 1973. His results are in Table V. Why Box-Jenkins time series analysis is important to Contador in the calculation of expectations is best explained by this Brazilian economist himself:

"The models of expectations (in this article) will be, as in conventional in empirical works of this type, based upon only the information found in the values of the time series. Nevertheless, rather than impose a particular process on the expectation mechanism (a very debatable procedure), we will utilize a generic model with autoregressive and moving average components following the ARIMA model recently developed by Box and Jenkins. Using this approach, the generating process of each series will be identified and, from there, expected values computed." (p. 256).

Contador goes on to briefly comment upon the expectations model of Cagan:

"The expectations model developed by Cagan is a simplification of the ARIMA model where the process is restricted to a moving average process of the first order. Naturally, in the real world, expectations are formed in a much more complex way than Cagan suggests, and it is difficult to justify Cagan's method, unless it is simply for statistical convenience." (p. 257).
Secondly, Contador suggested a straightforward way of testing for homogeneity of the utility surface. Consider the simplest of the regression models of Chetty, model C-I, and note that wealth or a proxy for wealth does not enter the equation in explaining the ratio of $M$ to $T$. This is because of the assumption of homogeneity. Under this assumption, the level of wealth has no affect upon the relative allocation of resources—if wealth increases by a given percentage then all asset holdings increase by the same percentage, and relative holdings of assets remain constant. Consequently, to test for homogeneity, Contador locates in the regression equation a proxy for wealth and tests for the significance of its coefficient. That is, the regression equation modified from Chetty's model C-I may be written in the following way:

\[
\log \frac{M}{X_j} = -\frac{1}{1+\phi} \log \frac{\beta_2}{\beta_1} + \frac{1}{1+\phi} \log \frac{1}{1+i_j} + \alpha \log Y + \epsilon
\]

where $M$ is money balances, $X_j$ is the nominal value of a short-term asset, $i_j$ is the rate of interest on the short-term asset (the nominal rate), and $Y$ is income.

What interpretation can be given to the coefficient $\alpha$? Clearly, if $\alpha = 0$, then the utility function may be characterized as homogeneous since the level of wealth has no effect upon the relative holdings of $M$ and $X_j$. When $\alpha \neq 0$, the utility function cannot be characterized as homogeneous. In the case where $\alpha > 0$, $M$ responds to a change in wealth more than $X_j$, and one would conclude that the wealth...
elasticity of demand for \( M \) is greater than that for \( X_j \); and vice versa for \( \alpha < 0 \). Regression equation (1) was not estimated by Contador because it deserved yet one more modification before it could be applied to the monthly data base which he used.

The third way in which Contador modified the approach of Chetty was to assume explicitly that total adjustment of demand may require more time than would be allowed by the length of the time unit of the data base. Contador assumed that adjustment from desired to actual values of the assets takes the following form:

\[
(2) \quad \log \left( \frac{M}{X_j} \right)_t - \log \left( \frac{M}{X_j} \right)_{t-1} = \psi_j \{ \log \left( \frac{M}{X_j} \right)_t - \log \left( \frac{M}{X_j} \right)_{t-1} \}
\]

where \( \left( \frac{M}{X_j} \right)_t^D \) is the demanded level of the ratio of the assets \( M \) and \( X_j \). The value of \( \psi_j \) is theoretically in the range \((0,1)\), and measures the proportion of adjustment from desired to actual values that takes place from period \( t-1 \) to period \( t \). Since equation (1) explains the demanded ratio of \( M \) to \( X_j \), we may insert equation (1) into (2) and the result is the following regression equation:

\[
(3) \quad \log \left( \frac{M}{X_j} \right)_t = -\frac{\psi_i}{1+\rho} \log \frac{\beta_2}{\beta_1} + \frac{\psi_i}{1+\rho} \log \frac{1}{1+1} + \psi_j \alpha \log Y + \\
+ (1-\psi_j) \log \left( \frac{M}{X_j} \right)_{t-1} + \epsilon
\]

where \( \psi_j \) is the adjustment coefficient. This regression equation was estimated by ordinary least squares. The empirical results are summarized in Table V.
### TABLE V

**CONTADOR'S EMPIRICAL RESULTS**: ELASTICITIES OF SUBSTITUTION AND ADJUSTMENT COEFFICIENTS

<table>
<thead>
<tr>
<th>ASSETS*</th>
<th>Chetty's model C-III</th>
<th>Contador's (3)</th>
<th>σ</th>
<th>Short run</th>
<th>Long run</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank time deposits with and without certificate.</td>
<td>8.20</td>
<td>37.40</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-bank savings deposits.</td>
<td>8.14</td>
<td>38.60</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Bills of exchange.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Government bonds and bills.</td>
<td></td>
<td>0.19</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Housing bonds</td>
<td>5.56</td>
<td>3.00</td>
<td>82.30</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>Mutual funds.</td>
<td>26.40</td>
<td></td>
<td></td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

* See Appendix I for more detailed information concerning the assets.

**Estimates are not reported either because the sign was inconsistent with the theory, or because the adjustment coefficient was outside the stable range (0,1).**
2. We can briefly summarize Contador's contribution to the asset
elasticity of substitution literature by emphasizing that his modi-
fication of the Chetty models are basically in three areas:
(1) the recognition of the importance of expectations in the model
(2) the necessity of changing the regression equation so that it
would allow for less than instantaneous adjustment between actual
and demanded asset values, and (3) the proposal of a simple test
for homogeneity of the utility surface.

Contador's work was published in Portuguese and has never been
translated into English. Unknown to most North American economists,
it is one of the pioneering ventures into the use of Box-Jenkins time
series analysis to generate expected prices. Moreover, as we shall
see in the next section, Contador's use of a partial adjustment
mechanism pre dates the appearance of the same procedure in the North
American literature on the asset elasticity of substitution.

V. RELATIVE MONEY-SUBSTITUTION.

1. As in the other studies reviewed in this chapter, the purpose of
the work of Moroney and Wilbratte (1976) is to measure the degree of
substitution between money and various financial assets. Their
approach differs in essentially two ways.

"First, we incorporate distinctly different theoretical assumptions..... Second, the key economic
variables of our wealth maximizing model are more comprehensive than those considered by Chetty." (p. 183).
In this study, four assets are tested for substitutability with money balances: (1) commercial bank time deposits, (2) short-term government bonds, (3) long-term corporate bonds, (4) savings-and-loan shares plus mutual savings bank deposits. The data is quarterly and for the household sector of the United States during the period 1956.IV-1970.IV.

2. Moroney and Wilbratte assume that the typical household maximizes wealth, and especially that wealth which comes from the possession of money balances and interest-bearing assets. They express this component of household sector wealth as the following,

\[ W_t = M_t + \sum X_{it} (1 + r_{it}) \]

where \( W_t \) is wealth in period \( t \), \( M_t \) is nominal money balances, and \( X_{it} \) denotes the nominal dollar holdings of the \( i \)-th interest-bearing asset with nominal interest rate \( r_{it} \).\(^{17}\) The maximization of wealth is constrained by "a monetary transactions constraint,

\[ T_t = f(M_t, X_{it}) \quad i = 1, \ldots, 4, \]

where \( f \) summarizes the prevailing transactions technology." (p. 185). The constraint shows the ability of an asset to be traded easily and without capital loss. The functional form which Moroney and Wilbratte choose for the transactions constraint is

\[ T_t = \left( \beta_t M_t^{-\rho} + \sum \beta_{it} X_{it}^{-\rho_i} \right)^{-1/\rho} \]

This is the same functional form that Chetty used for utility in
his models C-II and C-III. Furthermore, they allow for $\beta$ and $\beta_i$ to change over time. When $\rho < 0$, both $\beta$ and $\beta_i$ are positively related to $T$. "In this sense increases in $\beta$ and $\beta_i$ may be conceived as asset-augmenting technological changes." (p. 186). The parameters $\beta$ and $\beta_i$ are assumed to be dependent upon permanent income

$$
\beta_t = \beta Y_t^\theta \\
\beta_{it} = \beta_i Y_t^\theta
$$

for two reasons. First, permanent income is a proxy for wealth, and as wealth grows the marginal rate of substitution between money and interest-bearing assets changes. Secondly, since permanent income is dominated by trend, it ought to reflect the changing transactions demand for money. Inserting (6) into (5) and maximizing (4) against this constraint, the following demand equations are obtained:

$$
\log X_{it} = \frac{-1}{1+\rho} \log \frac{\rho \beta}{\beta_i^{\rho_i}} + \frac{\theta - \theta_i}{-(\rho_i + 1)} \log Y_t + \frac{\rho + 1}{\rho_i + 1} \log M_t \\
- \frac{1}{\rho_i + 1} \log \frac{1}{1+r_{it}}; \quad i = 1, \ldots, 4.
$$

When an error term is added to the above expression, we have Moroney and Wilbratte's first regression model, and the elasticity of substitution may be calculated in the same way as Chetty proposed. 3. Brox (1978) as we stated in footnote 1 has demonstrated that the theoretical model of Moroney and Wilbratte is not consistent with their regression model. The inconsistency lies in the fact that all of the signs in the demand equations (7) are not correct, in particular the sign on the coefficient of the interest rate variable.
Moroney and Wilbratte report the sign as negative, when in fact the theoretical model implies that the sign is positive. In order to undo this contradiction, Brox shows that the wealth expression must take the present value form of Chetty,

\[ W_t = M_t + \sum_{i=1}^{n} \frac{X_{i,t+1}}{1+r_{it}}. \]

Undeniably, this slip by Moroney and Wilbratte in the mathematics of their model costs them to lose one of the two reasons for the study. They are no longer able to claim "distinctively different theoretical assumptions" because their model properly revised is identical to Chetty.

4. Regression equation (7) is almost identical to the regression equation from Chetty's model C-II with the one exception that (7) contains permanent income as a wealth proxy. Moroney and Wilbratte do not treat the problem which arises from \( M_t \) being stochastic and on the right-hand-side of the regression (Chetty treats this problem in C-III). Elasticity of substitution estimates are in Table VI.

Moroney and Wilbratte go on to argue that all of the \( \theta_i \)'s in the transactions constraint, may have the same value. When the \( \theta_i \)'s are equal, the regression equation takes the form,

\[
\log \frac{M_i}{X_i} = \sigma_i \log \frac{\beta_i}{\beta_i} + \sigma_i (\theta_i - \theta_i) \log Y_t \\
+ \sigma_i \log \frac{1}{1+r_{it}} + e; \quad i = 1, \ldots, 4.
\]
where $\sigma_i$ is the elasticity of substitution between $M$ and $X_i$. In estimating this equation for the four financial assets, Moroney and Wilbratte find no statistical difference among the four elasticities of substitution. Money substitutes equally well with long-term and short-term financial assets.

Moroney and Wilbratte then modify this last equation to test for partial adjustment from actual to demanded values of the asset ratio. The partial adjustment mechanism they use is identical to that of Contador. Further, they allow for the possibility of autocorrelation in the residuals by employing a maximum likelihood technique in estimation. The regression equation is the following:

\[
(9) \quad \log\left(\frac{M}{X_i/t}\right) - \gamma_i \log\left(\frac{M}{X_i/t-1}\right) = C_{1i} + C_{2i}(\log Y_t - \gamma_i \log Y_{t-1}) + C_{3i} \frac{1}{1+r_{it}} - \gamma_i \frac{1}{1+r_{it-1}} + C_{4i} \log \frac{M}{X_i/t-1} - \gamma_i \log \frac{M}{X_i/t-2} + e_{it}
\]

where $\gamma_i$ is the first-order autocorrelation coefficient in the residuals. The coefficient of partial adjustment is $(1 - C_{ri})$ and measures the per cent of adjustment that occurs in a quarter. The long-run elasticity of substitution is $\frac{C_{3i}}{1 - C_{4i}}$, while the short-run elasticity is simply $C_{3i}$. In order to estimate the above equation, the procedure is followed whereby $\gamma_i$ is allowed to take on a value between -1 and +1, and then the equation is estimated using ordinary least squares. Moroney and Wilbratte use the range (-0.9, -0.8, ..., 0.0, 0.1, ...)
..., 0.9), and so there are 19 regression equations for each asset. The regression with the smallest residual sum of squares is chosen as the best estimate. Estimates for all three regression models may be found in Table VI.

The major conclusion which Moroney and Wilbratte draw from their statistical estimates is impressive: long-term assets seem to be as substitutable for money as short-term assets.

"...Those who stress liquidity as the essential property of money typically reason that short-dated assets are more highly substitutable than long-dated assets are for money. Our findings do not sustain this viewpoint. They are instead consistent with a classical monetary view: a proportionate increase in the spectrum of interest rates would indeed cause households to economize on money balances, but the adjustment would entail a roughly proportionate increase in the holdings of several types of assets. The responsiveness of interest-bearing deposits does not appear to be unique. And if portfolio substitutability is the salient criterion for choosing 'near-monies', short- and long-term bonds qualify on an equal footing with time deposits." (p. 194).

In the next chapter, we shall look more closely into the nature of this substitution measure. Certain inadequacies of the measure will be highlighted, and a new way of defining it proposed.

VI. A NOTE ON SUPPLY.

None of these studies explicitly take into account the supply side of the market for short-term financial assets. All assume that the interest rate is in some way predetermined and independent of the vagaries of demand. Of course, with interest rates assumed fixed and independent of the demand side of the marketplace, the
<table>
<thead>
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<th>Elasticity of substitution regression equations</th>
<th>adjustment coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(7)</td>
<td>(8)</td>
</tr>
<tr>
<td>Commercial bank time deposits</td>
<td>8.59</td>
<td>22.48</td>
</tr>
<tr>
<td>Short-term government bonds</td>
<td>28.57</td>
<td>23.96</td>
</tr>
<tr>
<td>Long-term corporate bonds</td>
<td>34.26</td>
<td>27.53</td>
</tr>
<tr>
<td>Savings-and-loan shares and mutual savings deposits</td>
<td>2.79</td>
<td>19.12</td>
</tr>
</tbody>
</table>
placement of the interest rate on the right-hand side of the regression equation is consistent with the assumptions of ordinary least squares. This independence assumption on the interest rate is not very damaging to the underlying structure of the model, if the model is applied to data of the very short run. Given a change in the demand for a financial asset, financial institutions will change the return on the asset, accordingly. An increase in demand would result in financial institutions lowering return and vice versa. However, we could only expect this reaction of the financial institution in changing the interest rate to take place some time after the change in demand. First, a modification in asset holder behavior may be only temporary, dissipating rapidly, and a financial institution would not amend its own interest rate policy only to modify it once again soon after the initial change. And secondly, even if after some time the permanence of the modification in behavior is recognized, the financial institution must analyze the extent and intensity of the new level of demand, decide upon the new level of return which it will offer on its liabilities, and then eventually put into effect the new policy it has decided upon. All of which takes time. Consequently, only in the long run would changes in price be demand induced. In the short run, therefore, changes in price are not demand induced but are independent of demand. In other words, asset holders can always obtain any amount of an asset they desire without immediately affecting the level of the return of the asset.
While the distinction between short run and long run may be relatively clear theoretically, once the theory is applied to the actual situation at hand the distinction becomes ambiguous. Without a detailed analysis of the behavior of financial institutions, and information concerning reaction time to demand changes, one can make only an educated guess as to what length of the time period could be characterized as short or long. Certainly, in the case of a monthly analysis, it can be assumed that the time period is short enough to be free of demand-induced price changes. Moroney and Wilbratte (1976) argue that models based on quarterly data are equally free on demand-induced price changes. However, once the assumption is extended out to a time period equal to one year—the data base for Chetty (1969)—we must question the validity of statistical results which do not take into account explicitly the supply side of the marketplace.
FOOTNOTES

1. The only study which deviates from the common framework is that of Moroney and Wilbratte (1976). However, as we shall discuss later on in this chapter, it has been pointed out by Brox (1978) that the theoretical model of Moroney and Wilbratte is not consistent with the empirical model, whereas the empirical model does follow the common framework of the literature. Hence, we can say that no study deviates from the common underlying structure.

2. Lee's specification of the demand for money function has real money balances per capita as the dependent variable regressed on real per capita permanent income and two interest rate differentials: the rate spread between government securities and demand deposits and the spread between non-bank savings accounts and demand deposits. Feige's specification is more straightforward and regresses money balances per capita income and various short-term interest rates.

3. Feige and Pearce (1977) were able to find only one study, by Gramlich and Kalchbrenner (1970), which imposed both balance sheet restrictions suggested by Brainard and Tobin (1968) and symmetry conditions suggested by Feige (1964). Their conclusion for the U.S. was that no short-term asset, including time deposits at commercial banks, was a close substitute for money and, therefore, only money narrowly defined was the proper analytical monetary aggregate.

4. Another way of estimating an adjusted money supply is to define it as that monetary aggregate which shows the highest correlation to nominal income and whose components are not better correlated with nominal income than it is. These criterion are put forth by Friedman and Meiselman (1963).

5. In this context, the elasticity of substitution measures the percentage change in the ratio of the two assets that results from a percentage change in the slope of the indifference curve. The slope of the indifference curve is the marginal rate of substitution between the assets and is the term in the denominator of the formula in the text.

6. The trade-off embodied in the budget constraint is between present money balances and future time deposits, and is the basis for the slope of the budget constraint, -(1+i). In other words, on line with the budget constraint, an asset holder may have either one dollar's worth of money balances today or 1+i dollar's worth of time deposits tomorrow. Consequently, the budget constraint is conceptually unclear as to its empirical application because the assets do not exist at the same point in time but are separated by the distance in time which defines the interest rate. This endpoint ambiguity has generated several comments in the literature, and will be more fully discussed in the next chapter. See Steinhauer and Chang (1972, p. 222), and Chetty (1969, p. 273; 1972, p. 227).
7. The equilibrium condition for utility maximization is

\[
\frac{\text{MRS}}{\text{MT}} = \frac{\partial U}{\partial M} \frac{\partial \hspace{1pt} M}{\partial \hspace{1pt} T} = \frac{\beta_1}{\beta_2} \left( \frac{M}{T} \right)^{-\bar{\sigma} - 1} = 1 + i
\]

which states that the marginal rate of substitution between M and T must equal the relative price \((1 + i)\). In other words, the indifference curve must be tangent to the budget constraint. From the utility function, calculate the derivatives

\[
\frac{\partial U}{\partial M} = \frac{\beta_1 M^{\bar{\sigma} - 1}}{\bar{\sigma} - 1}
\]

\[
\frac{\partial U}{\partial T} = \frac{\beta_2 T^{\bar{\sigma} - 1}}{\bar{\sigma} - 1}
\]

Dividing the first derivative by the second, the expression above is derived. The parameter \(\bar{\sigma}\) is a reflection of the degree of curvature of the indifference curve. The straighter the indifference curve, the more "constant" is the \(\text{MRS}_{\text{MT}}\) for changes in the asset ratio \(M/T\), and the closer to -1 is the value of \(\bar{\sigma}\) (\(\sigma\) approaches \(\infty\)). When the \(\text{MRS}_{\text{MT}}\) varies abruptly with changes in the asset ratio, the indifference curves approach a right-angled shape, and \(\bar{\sigma}\) is very large (\(\sigma\) approaches zero).

8. In more general terms, the homogeneity assumption is that \(U\) may be characterized as

\[
\lambda U = U(\lambda M, \lambda T)
\]

where \(\lambda\) is any scalar.

9. This functional form was originally proposed by Dhrymes and Kurz (1964). It is characterized by separability in its arguments as is the simpler CES form. In the context of the asset model, separability means that the marginal rate of substitution between two assets is not affected by any third asset:

\[
\frac{\partial \hspace{1pt} \left( \frac{U_i}{U_j} \right)}{\partial X_k} = 0
\]

An important implication of this characteristic is that the only prices allowed to affect the ratio of two assets in demand are the two asset's prices and no third price. In other words, expansion paths between any two assets will not shift when the prices of other assets change.
10. It is arbitrary whether \( M \) or \( T \) is on the right-hand-side of the regression equation. The point is that both \( M \) and \( T \) are random and both cannot be on the left.

11. Lee (1972) uses a maximum likelihood technique attributed to Dhrymes (1966) to estimate the parameters of model C-II with 1951-1966 data. Lee criticizes Chetty's use of time series data prior to 1951. Chetty's data base for the various statistical applications of the models he proposed is yearly time series for the period 1945-1966. In the fall of 1950, the insurance provision on savings and loans in case of default of the financial institution became identical to that governing time deposits at commercial banks, and, as a result, one would expect that the degree of substitutability of savings and loan shares for money would have increased with this institutional change. In fact, Lee's estimates of the elasticity of substitution are greater than Chetty's for savings and loan shares.

12. Derive the expression as follows. State the elasticity of substitution between money \( M \) and a substitute asset \( S \) as,

\[
\frac{\partial M}{S} \cdot \frac{1}{1+r_s} = \sigma_{MS}
\]

where \( r_s \) is the rate of interest on the substitute asset. Taking the derivative,

\[
S \frac{\partial M}{1+1+r_s} - \frac{\partial S}{1+1+r_s} \cdot \frac{M}{S} \cdot \frac{1}{1+r_s} = \sigma_{MS}
\]

and

\[
\frac{\partial M}{1+1+r_s} \cdot \frac{1}{M} - \frac{\partial S}{1+1+r_s} \cdot \frac{1}{S} = \sigma_{MS}
\]

In order to arrive at conventional cross-elasticities with respect to \( r_s \)

\[
N_{MS} - N_{SS} = \sigma_{MS} \frac{r_s}{1+r_s} \cdot \frac{1}{1+r_s}
\]

and

\[
N_{MS} - N_{SS} = \sigma_{MS} \frac{r_s}{1+r_s}
\]
and, therefore,

\[(N_{MS} - N_{SS}) \frac{1+r_S}{r_S} = \sigma_{MS} \]

If \(\sigma_{MS}\) and the demand elasticities were defined with the same prices, then the relationship reduces to,

\[N_{MS} - N_{SS} = \sigma_{MS}\]

and the value of \(\sigma_{MS}\) no longer depends upon the level of the interest rate. In Chapter III, we argue that price is misspecified in the Chetty-type studies.

13. The Chetty-type study without major modifications has also been done by Short and Villanueva (1977) for Canada and by Subrahmanyam (1977) for India. Short and Villanueva using Chetty's models C-II and C-III found large values for the elasticity of substitution between money and almost all other short-term assets in Canada. Values range from 6 to 47. Subrahmanyam estimated Chetty's model C-I between money and time deposits in India and found an elasticity of substitution equal to 24. Once again, strong substitution is implied.

14. In Chapter V, Section II, is an explanation of how the nominal rate of interest is calculated for 12 brazilian short-term assets.

15. My translation. Contador could have tested the ARIMA model against actual realized rates of interest as predictors of future values. Tests of the efficient market hypothesis as developed by Fama (1970) generally produce the result that in auction-type markets, of which some financial markets are a subset, today's prices are the best predictors of tomorrow's prices.

16. Actually, the proxy for wealth is an estimate of monthly income which Contador derives by using the technique of principle components on four Brazilian time series that reflect the general movements in economic activity. The time series are the following:

1. industrial consumption of electricity,
2. loans in real terms to the industrial sector,
3. loans in real terms to the agricultural sector,
4. value added tax receipts in the state of Sao Paulo.

17. Note that this formulation of household wealth is incorrect because the time subscripts are not consistent. The inconsistency arises from the summation of stocks (the \(X_{it}\)) and flows (the \(r_{it}\)). See the comment by Brox (1978, p.112).
CHAPTER III

A CRITICISM OF THE ELASTICITY OF SUBSTITUTION IN MONETARY ECONOMICS
AND A NEW APPROACH

Give me a lever long enough,
and a prop strong enough,
and I can single-handed move the world.

Archimedes
CHAPTER III

A CRITICISM OF THE ELASTICITY OF SUBSTITUTION IN MONETARY ECONOMICS AND A NEW APPROACH

I. INTRODUCTION

In the previous chapter, we saw that the asset elasticity of substitution was introduced as a device for measuring more precisely the intensity of money substitution among various financial assets. Since it supposedly relies upon the curvature of the indifference curve in a direct way, it is theoretically preferable to the cross-elasticity of demand as a measure of substitution. The object of this chapter is twofold: (1) to show that the generic asset elasticity of substitution model is ambiguously defined and that this ambiguity has lead to a misspecification in price which results in the measure itself being poorly defined; and, furthermore, (2) to present a model of asset-holder behavior which does not suffer the criticisms of the earlier approach. The purpose of the first part of this paper will be served by continuously referring to the simplest Chetty model C-I. Doing this will simplify the exposition and will not detract from the generality of the results. In the section to follow, we demonstrate the ambiguities of the model and the misspecification in the elasticity of substitution which it yields. In the third section, our reformulation of the problem will be put forth.

II. AMBIGUITIES AND MISSPECIFICATION

1. Chetty's simplest model (C-I) of asset-holding behavior, presented in Section II of Chapter II, which assumes a homogeneous utility function and only two assets, money balances M and time deposits at commercial banks T, is the following:
maximize

(1) \[ U = (\beta_1 M^{-\rho} + \beta_2 T^{-\rho})^{-1/\rho} \]

subject to

\[ W_0 = M + \frac{T}{1+i} \]

where \( U \) is some notion of utility, \( W_0 \) is the nominal amount of resources at the disposal of the asset holder at the beginning of the period for acquisition of \( M \) and \( T \), and \( i \) is the rate of interest on \( T \). From the equilibrium conditions of (1), Chetty derives a regression model of the following type,

(2) \[ \log \frac{M_t}{T_t} = -\frac{1}{1+\rho} \log \frac{\beta_2}{\beta_1} + \frac{1}{1+\rho} \log \frac{1}{1+i_t} + \epsilon \]

where \( \frac{1}{1+\rho} \) is an estimate of the elasticity of substitution between \( M \) and \( T \). The ambiguity lies in the aspect of time as it applies to the theoretical model summarized in (1) and its empirical counterpart (2). Restating (1) with the time subscripts from (2), we have

maximize

(3) \[ U_t = (\beta_1 M_t^{-\rho} + \beta_2 T_t^{-\rho})^{-1/\rho} \]

subject to

\[ W_t = M_t + \frac{T_t}{1+i_t} \]

The theoretical model in (3) is clearly misspecified because it is impossible for all of the variables in the budget constraint to have the same time subscript. In other words, Chetty's theoretical and empirical models are inconsistent; the empirical model locates
the assets at the same point in time \((M_t, T_t)\) but the theoretical model locates the assets at different points in time \((M_t, T_{t+1})\).

Chetty (1969, p. 273, footnote 1) suggests that this inconsistency may be remedied by redefining the problem as

maximize

\[ U = U(M_t, T_t(1+i_t)) \]

subject to

\[ W_t = M_t + T_t \]

where all asset values are at the same point in time. However, the inconsistency persists. The equilibrium condition for this problem is:

\[
\frac{U_{T_t(1+i_t)}}{U_{M_t}} = \frac{1}{1+i_t}
\]

Since \(U_{T_t(1+i_t)} = U_{T_{t+1}}\), the equilibrium condition may be restated as

\[
\frac{U_{T_{t+1}}}{U_{M_t}} = \frac{1}{1+i_t}
\]

where it is seen that time deposits \((T)\) and money balances \((M)\) are still separated by the length of one time period, and, therefore, the theoretical model and the empirical model are still not compatible in terms of the time subscripts on the assets.

The most important implication of this inconsistency is that, while relative price may be properly specified for the theoretical model, it is misspecified for the empirical model. The relative price
between \( M_t \) and \( T_t \), the asset values of the empirical model, is not
\[
\frac{1}{1 + i_t}
\]
but rather simply 1 as Steinhauer and Chang (1972, p. 222) have pointed out. However, what is the result of this misspecification in price on the estimate of the elasticity of substitution? In the following paragraphs, we attempt to answer this question.

2. An important characteristic of the elasticity of substitution is that it not depend upon the dimensions of the "goods" and prices used in its calculation. Whether the measure is stated in terms of quarters or dollars, bushels or dozens, it should always be the same. The slope of the indifference curve between two goods or two assets, and, concurrently, the value of the elasticity of substitution, should not change with arbitrary changes in dimension. In order to look more closely into this question of dimensionality, define the elasticity in general terms as,

\[
\log \frac{q_1}{q_2} = \left( \log \frac{q_1}{q_2} \right) + \sigma_{12} \log \frac{p_1}{p_2}.
\]

The intercept term, \( \left( \log \frac{q_1}{q_2} \right) \), has a zero subscript to denote that \( q_1 \) and \( q_2 \) are evaluated at the intercept of the equation. It can be seen that the elasticity \( \sigma_{12} \) is in fact independent of the definitional units of the variables, and, therefore, is a pure number and dimensionless. Prices may be expressed in any monetary unit, and goods or assets in any quantity unit, and the elasticity will not change value.
We now propose a similar exercise to the one above, but here our concern is with the elasticity of substitution as it has been defined and used in Monetary Economics. In the previous exercise, the variables which determined the measure were stocks and the prices of stocks, and for this reason dimension did not explicitly enter the calculation. The model from which the elasticity of substitution is estimated in the Monetary Economics literature, however, is not strictly speaking a model of stock adjustment. The interest rate on the short-term asset is a flow and is dimensioned by the length of time that defines it. The interest rate that persists over a month is approximately one-twelfth the rate that persists over the period of a year. We must suspect, therefore, that the asset elasticity of substitution is not dimensionless but is partially determined by the length of the period in which the interest rate is defined.

The conventional asset elasticity of substitution $\sigma_{MT}$ may be defined in the following way, analogous to equation (5):

\begin{equation}
\log \frac{M}{T} = \left( \log \frac{M}{T} \right)_0 + \sigma_{MT} \log(1+r)
\end{equation}

where $M$ and $T$ are the nominal money values of the assets money and time deposits, respectively, and $r$ is the rate of interest on $T$. Chetty actually uses $\log \frac{1}{1+i_t}$ as the independent variable but in this chapter we use the more straightforward $\log(1+r)$ to only simplify the analysis. No substantive change has occurred. Express the rate of interest as
\begin{equation}
\log \frac{M}{T} = \left( \log \frac{M}{T} \right)_0 + \sigma_{MT} \log \left( 1 + \frac{R/t}{P} \right).
\end{equation}

In the above equation, it is obvious that the elasticity of substitution is not dimensionless because of the persistence of the time dimension in the rate of interest. While it is true that all monetary significators cancel out, the time dimension of the interest rate does contribute to the determination of the value of \( \sigma_{MT} \). From the day of its introduction, the question of the dimensionality in the asset elasticity of substitution has remained unanswered in the literature.  

One particularly enlightening way of specifying the dependence of the asset elasticity of substitution on the time unit of the interest rate is to emphasize that \( \sigma_{MT} \) is a coefficient in a regression equation, and to show that the structure of the equation changes as the time unit changes.

The regression model for Chetty's model C-I is the following,

\begin{equation}
\log \frac{M}{T} = \alpha + \sigma \log(1+r_c) + e
\end{equation}

where \( M_t \) and \( T_t \) are the nominal value of money holdings and time.
deposits at time $t$, and $r_t$ is the rate of interest on $T$ for time period $t$. The coefficient $\sigma$ is the elasticity of substitution between $M$ and $T$. We propose to show that the value of the coefficient $\sigma$ on the independent variable $\log(1+i)$ is sensitive to the way in which the interest rate $r_t$ is defined in time. More exactly, we will show that a quarterly interest rate will determine a value for $\sigma$ which is different from the value determined from a yearly interest rate.

Let us present the problem in the following way. We have a series of ratios through time between the assets money and time deposits, which is being explained by a series of interest rate variables which take the form $(1+r)$. In Table I, a quarterly time series on the data is listed. In the left-hand column is quarterly data on the ratio of $M$ to $T$, in the second column is the variable $(1+r_t^V)$ where the interest rate is that rate which would persist for a year but is recorded quarterly, and in third column is the same interest rate on $T$ with one significant difference: it is that rate which would persist for a quarter. The question posed is whether the regression equation of column (1) on column (2) is fundamentally different in coefficients than the regression equation of column (1) on column (3). The former regression model may be stated in log-log form as

$$
\log\left(\frac{M}{T}\right)_t = \alpha + \sigma_y \log(1+r_t^V) + \epsilon
$$
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<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$(1+r^y_t)$</td>
<td>$(1+r^q_t)$</td>
</tr>
<tr>
<td>$\left(\frac{M}{T}\right)_{1.1}$</td>
<td>$(1+r^y_{1.1})$</td>
<td>$(1+r^q_{1.1})$</td>
</tr>
<tr>
<td>$\left(\frac{M}{T}\right)_{1.2}$</td>
<td>$(1+r^y_{1.2})$</td>
<td>$(1+r^q_{1.2})$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$\left(\frac{M}{T}\right)_{1.4}$</td>
<td>$(1+r^y_{1.4})$</td>
<td>$(1+r^q_{1.4})$</td>
</tr>
<tr>
<td>$\left(\frac{M}{T}\right)_{2.1}$</td>
<td>$(1+r^y_{2.1})$</td>
<td>$(1+r^q_{2.1})$</td>
</tr>
<tr>
<td>$\vdots$</td>
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<td>$\vdots$</td>
</tr>
<tr>
<td>$\left(\frac{M}{T}\right)_{2.4}$</td>
<td>$(1+r^y_{2.4})$</td>
<td>$(1+r^q_{2.4})$</td>
</tr>
</tbody>
</table>

*(M/T)_{i,j} indicates the ratio of M to T for year i and quarter j.*

$r^y_{i,j}$ is the rate of interest on T that prevails for a year (superscript) that is recorded for year i and quarter j.

$r^q_{i,j}$ is the rate of interest on T that prevails for a quarter that is recorded for year i and quarter j.
and the second model as,

\[(10) \quad \log\left(\frac{M}{T}\right)_t = \alpha + \sigma_q \log(1+r_t^q) + e\]

where \(\sigma_y\) is the coefficient estimated from the yearly interest rate and \(\sigma_q\) is the coefficient estimated from the quarterly interest rate. Since the choice of the length of the period which defines the interest rate is arbitrary—the choice between monthly, quarterly, or yearly interest rates usually hinges on how the data is recorded—it would be hoped that the two regression models would give the same results. However, the length of the time period on the interest rate does determine the results of the regression models, and the exact effect can be circumscribed by comparing regression models. First, state the mathematical relationship between the interest rate that would persist for a quarter and the interest rate that would persist for a year as

\[(11) \quad (1+r_t^q)^4 = (1+r_t^y).\]

Substituting this expression into (9), the result is

\[(12) \quad \log\left(\frac{M}{T}\right)_t = \alpha + \sigma_y 4 \log(1+r_t^q) + e\]

Upon inspection of equation (12) and equation (10), we see that the two regression models are equivalent except that the coefficients on the interest rate variable differ. When the quarterly rate model is compared to the yearly rate model, we see,

\[4\sigma_y = \sigma_q.\]
The coefficient from the model with quarterly interest rates is four time larger than the coefficient from the yearly model.\textsuperscript{3,4} In other words, the shorter the period in which the interest rate is defined, the larger is the value of the elasticity of substitution, \textit{ceteris paribus}. With the time dimension dominating the measure, the elasticity of substitution loses its conventional attraction as a meaningful indicator of substitution. As the time unit on the interest rate declines, indifference curves flatten out. The implication is that it is the model which determines the degree of substitution between assets instead of the substitution relationship itself determining the outcome of the model.

This result impairs Chetty's utilization of the function $U$ as an estimate of an adjusted money supply. In addition to the criticism of Steinhauer and Chang which was summarized in the previous chapter (see Chapter II, Section II), we now see that the interest rate is inversely related to $U$ through its affect upon $\varphi$, thus giving Chetty's adjusted money supply the unusual trait that its size depends upon the time unit that defines the interest rate. The function $U$ (for the simple model C-I, but equally applicable to C-II and C-III) should be more properly stated as

$$U = (\beta_1 M^{-\rho(r)} + \beta_2 T^{-\rho(r)})^{-1/\rho(r)}$$

where $\varphi$ depends upon the interest rate ($r$) on the asset $T$.

Nevertheless, the question of substitution remains an important one for the various reasons presented in the introductory chapter.
In the section to follow, we shall present an economic model of asset-holder behavior from which one may derive an asset elasticity of substitution that is conceptually identical to the elasticity of substitution of basic price theory. In Chapter IV, the model will be empirically tested using data from the recent Brazilian monetary experience.

III. A MODEL OF ASSET-HOLDER BEHAVIOR.

1. Two interrelated aspects of the question of money substitution have been the major themes of the first three chapters. First, at the same time that financial markets may enhance the efficient allocation of capital, their presence may lead to a degree of intermarket complexity that may render monetary policy relatively ineffective in reaching economy-wide objectives or, on the other hand, objectives limited to specific sectors of the economy or regions of the country. In studying the connection between monetary policy and financial markets, the fact must be emphasized that some market instruments may be so similar to money narrowly defined that they easily substitute for money in asset-holder's portfolios; and, consequently, any given monetary action which concentrates on $M_1$ may result in asset-holder reactions contrary to the goals of the authorities. Secondly, while it is true that most economists recognize the theoretical importance of the money substitution question, it is not at all clear how the question can be lifted out of the axioms of theory and located in the realm of empirical testing.

The elasticity of substitution between financial assets, at least
in the form in which it has appeared in Monetary Economics up to this time, is inadequate as a substitution measure. It is ill-defined and fails to differentiate among differing degrees of substitution between money and other financial assets. The purpose of this section is to present a model of asset-holder behavior from which a well-defined elasticity of substitution may be derived.

2. On the demand side, the typical asset holder maximizes the expected return from his financial asset portfolio within the limits of the amount of wealth available to him for the purchase of financial assets. The return maximized by the asset holder may be of two kinds. First, there is pecuniary return: the explicit monetary yield of the assets. Included in this explicit yield is interest payments and any capital gains or losses that may accrue during the period in which the asset is held. Secondly, there is non-pecuniary monetary service return. Money is held in the portfolio because of the non-pecuniary return that it yields since its pecuniary return is zero. It provides valuable services to the investor because it is the only financial instrument which readily exchanges for goods, services, and other financial assets. It reduces transaction time and cost. Nevertheless, money narrowly defined is not the only financial asset which provides a non-pecuniary monetary service return. Various financial assets may be similar to money in the characteristic that they have zero or very low transactions costs, and with little effort and practically no loss in value may be readily exchanged for an equal value of money balances. Accordingly, one may suspect that short-term financial
assets are held in the portfolio for the two kinds of return which they yield, the pecuniary and non-pecuniary return. Any theory which attempts to explain asset holder behavior ought to take into account in an explicit way that these two distinct kinds of return exist. It follows that the demand for financial assets including money depends upon at least three separate factors: pecuniary return, non-pecuniary return, and the amount of resources available for asset purchase.

3. On the supply side, we assume that interest rates are predetermined and independent of changes in demand. This assumption implicitly depends upon reaction time between changes in asset demand and resulting changes in interest rates. If the analysis is long-run, changes in demand may indeed lead to changes in the rate of interest. For instance, an upward shift in the demand for money, with time, tends to lift the level of interest rates on other financial assets as financial institutions adjust to restore efficient portfolio positions. However, the reaction on the part of the financial institution may not be immediate, and, in the short run, asset holders are faced with a supply of financial assets which is perfectly elastic. Therefore, interest rates may be treated as predetermined and independent of the market place. This assumption on the independence of interest rates greatly simplifies the analysis because it allows us to abstract from the supply side of the market, and to treat the demand side as self-contained. This assumption on perfectly elastic supply is not necessarily true for government securities. In this case, the supply of securities is fixed and therefore determines price in the marketplace.
Due to the fact that the supply of government securities does not respond to changes in price (assuming of course that the government is not in the business of maximizing the return on its portfolio but rather has economy-wide growth and stabilization goals--two aspects of portfolio management not necessarily consistent with one another), the demand side of the market, as in the case of other assets, still may be treated as self-contained. As can be seen in Chapter II, all asset elasticity of substitution studies have assumed the same.

4.1 The typical asset holder maximizes a combination of expected pecuniary and non-pecuniary monetary service return in his financial asset portfolio. The objective function to be maximized is,

\[(13a) \quad V = V(Z_t, R_t)\]

where \(Z_t\) is the total of all non-pecuniary monetary service returns yielded by the assets of the portfolio in period \(t\), and \(R_t\) is expected pecuniary return which may be expressed as,

\[(13b) \quad R_t = r_{1t}X_{1t} + r_{2t}X_{2t} + \ldots + r_{nt}X_{nt} + r_{at}A_t\]

where \(r_{it}\) is the expected rate of interest on the nominal value of the \(i\)-th short-term asset \(X_{it}\), and the interest rate \(r_{at}\) is the yield on the nominal value of the long-term non-liquid asset \(A_t\). In determining the level of non-pecuniary return \(Z_t\), we shall rely on the relatively general assumption that this kind of return is positively related to the nominal amount of the short-term assets in the portfolio, of course, including money.
(13c) \[ Z_t = z(M_t, x_{1t}, x_{2t}, \ldots, x_{nt}); \quad z' > 0, \quad z'' < 0. \]

Recent work in monetary theory has attempted with some success to develop a more precise definition of non-pecuniary monetary service and their determinants, and it is the contributions of this work that allows us to make certain assumptions about the monetary service function. Prior to this recent work, general (to the point of being vacuous) statements as to what constitutes the services of money lacked insight and contributed only confusion to the issue. For instance, Friedman's statement that "Intuitively, money seems to be a more efficient carrier of non-pecuniary services...than bonds..." (1969, p. 25), seems at best tautological.

Niehans (1975) puts forth the view that the service which money provides is derived from money's capacity to allow the multi-period budget constraint of the typical consumer to lie farther from the origin, and, consequently, to indirectly augment the level of utility. He argues that, without money, transactions costs are high. That is, when a given set of transactions must be undertaken with barter, a relatively large portion of the initial endowment of the consumer (his time, his shoeleather, etc.) must be expanded in the process of finding and exchanging in order to arrive at the optimal consumable bundle. On the other hand, with money, transactions costs are lower and a smaller portion of the initial endowment is used up in the process of exchange. Brunner and Meltzer's (1971) view is not dissimilar. By extending the choices open to the typical consumer in a stochastic
world to include not only consumption and exchange but also investment in information and transactions costs, "Individuals search for those sequences of transactions, called transaction chains, that minimize the cost of acquiring information and transacting. The use of assets with peculiar technical properties and low marginal cost of acquiring information reduces these costs." (p. 809). Brunner and Meltzer, therefore, have a more delineated view of transactions cost than Niehans because they allow these costs to be divided into cost of exchange, cost of acquiring information, and cost due to the risk of price changes, but the major conclusions of these two studies are identical. Money, and its services, do not generate utility, only goods can do that, but money does allow the optimal goods bundle to be augmented, and, in this way, the budget constraint of the consumer is shifted out and utility permitted to reach a higher level.

Monetary services, therefore, depend upon any asset's ability to increase consumption. The assets chosen to fulfill this role are characterized by small transactions costs and little risk in return. Currency and demand deposits have these characteristics as may certain other assets which are usually short-term. Time deposits and savings and loan shares may produce monetary services. Long-term assets may not qualify as producers of monetary services because their price is relatively unstable and the cost of acquiring information about them is relatively high. If money did not service consumption in this indirect way, then it would produce no service at all, and, lacking a pecuniary return, would not be held under any circumstance. This
argument is in direct opposition to the theoretical work of Patinkin (1965), and the theoretical underpinnings of the empirical work of Klein (1974). These authors assume that money and other liquid assets directly enter the utility function of the consumer, on an equal basis with goods and services, as if these assets had some quality which directly increased consumer satisfaction. Contrary to this view, the monetary services of our theoretical model are based upon a short-term asset's ability to indirectly affect consumer utility, and, therefore, our model must be characterized as a model of partial equilibrium.

One further assumption embodied in the service function Z is that non-pecuniary services of a financial asset increase at a decreasing rate as the amount of the asset in the portfolio increases (i.e., diminishing marginal monetary services). This characteristic of monetary services is implied by both Brunner and Meltzer and Niehans. Monetary services are directly related to their affect upon the optimal consumable bundle, and the first marginal monetary units to enter the portfolio allow the optimal bundle to increase by a relatively large amount. However, as the monetary units increase in the portfolio, the optimal bundle increases at a slower rate since holding money is counteracted and supported by not holding consumable goods and services. Eventually the point is reached where the optimal bundle would actually decline if more monetary units were added to the portfolio. The marginal services of money, therefore, decrease as money itself increases.
There are two constraints against which equation (1) is maximized. A stock constraint,

\[ W_t = M_t + X_{1t} + X_{2t} + \ldots + X_{nt} + A_t \]

and a flow constraint,

\[ R_t = r_{1t}X_{1t} + r_{2t}X_{2t} + \ldots + r_{nt}X_{nt} + r_A A_t. \]

We can combine (13a), (14) and (15) into a LaGrangian-type constrained maximization problem:

\[ L = V + \lambda_1 (R_t - r_{1t}X_{1t} - \ldots - r_{nt}X_{nt}) + \lambda_2 (W_t - A_t - M_t - X_{1t} - \ldots - X_{nt}) \]

where the function \( V \) is maximized against the flow constraint (15) and the stock constraint (14). First-order conditions for the constrained maximization of the objective function \( V \) are the following \( n+4 \) relationships:

\[ \begin{align*}
L_M &= V_Z M - \lambda_2 = 0 \\
L_{X_i} &= V_R r_{i_t} + V_Z X_{i_t} - \lambda_1 r_{i_t} - \lambda_2 = 0 \quad i = 1, \ldots, n. \\
L_A &= V_R r_{a_t} - \lambda_1 r_{a_t} - \lambda_2 = 0 \\
L_{\lambda_1} &= R_t - r_{1t}X_{1t} - \ldots - r_{nt}X_{nt} - r_A A = 0 \\
L_{\lambda_Z} &= W_t - A_t - M_t - X_{1t} - \ldots - X_{nt} = 0
\end{align*} \]

where the \( \lambda \)'s are LaGrangian multipliers and \( V_Z, V_R, Z_m, Z_{X_i} \) and \( L_M \), etc., are first-order partial derivatives. Substituting and rearranging terms, one may derive the following \( n+2 \) equilibrium conditions,
\begin{align*}
Z_M^{X_i} &= \frac{r_{at}}{r_{at} - r_{it}} ; \\
W_t^{r_{at}X_{it} - X_{nt}} - A &= 0 \\
R_t^{r_{it}X_{it} - r_{at}X_{lt}} - r_{at} &= 0
\end{align*}

4.2 What interpretation can be given to the interest rate expression on the right-hand-side of the equilibrium condition above? The denominator of the expression is the differential between the rate of interest on the illiquid asset and the rate of interest on the short-term asset $X_i$. When the short-term asset is held, the asset holder foregoes the interest he could have earned on the long-term asset ($r_{at}$), but gains the yield from the short-term asset ($r_{it}$). In other words, the interest rate differential is the opportunity cost of holding the short-term asset. This differential is the total amount of resources that the asset holder must give up per time period in order to avail himself of the non-pecuniary monetary services yielded by the short-term asset. Since money in this model yields no explicit pecuniary return, the interest rate on the long-term asset is equivalent to the opportunity cost of holding money balances.

These interest rate differentials may also be derived by combining the two budget constraints. Solving the stock constraint for $A_t$ and substituting into the flow constraint, we have,

\begin{align*}
TC_t = W_t r_{at} - R_t &= C_M r_{at} + C_1 X_{lt} + C_2 X_{zt} + \ldots + C_N X_{nt} \\
\end{align*}

where $C_M = r_{at}$, the rate of opportunity cost of holding money balances,
and \( C_i = r_{at} - r_{it} \) the rate of opportunity cost of holding the \( i \)-th asset. The expression \( W_t r_{at} - R_t \) is the total opportunity cost (TC) of holding the given portfolio of \( n \)-assets, and could be interpreted as the constraint against which the asset holder acts. The asset holder lays aside a given amount of wealth \( W_t \) and then decides how much return he is willing to forego to hold a given amount of short-term assets in his portfolio. If a relatively liquid portfolio is his objective, then he will forego much return and suffer a relatively high opportunity cost of the portfolio. Given \( W_t \), the maximum opportunity cost is \( W_t r_{at} \), where \( R=0 \), and is the return foregone in holding a perfectly liquid portfolio of money balances only. In the other extreme, an asset holder may decide not to suffer the loss of any pecuniary return, and, given \( W_t \), would set TC equal to zero. In this situation, all resources would be applied to asset \( A \), and since no short-term assets are held, no opportunity cost is suffered.

The model may also be interpreted as one of cost minimization. The asset holder takes a given pecuniary return (R) and a non-pecuniary return (Z), and then minimizes the opportunity cost (TC) of the portfolio. This interpretation leads to the same equilibrium conditions as before.

4.3 Assuming TC to be the proper constraint of the model, the following \( n \) relative demand equations may be derived:

\[
\frac{X_i}{M} = f\left( \frac{C_i}{C_M}, TC, U \right); \quad i = 1, \ldots, n. \tag{18}
\]
The variable $U$ represents all factors other than the ratio of opportunity costs and total opportunity cost that may affect the asset ratio. Theoretically, one would expect a negative relationship between the ratio of opportunity costs and the ratio of assets. If the opportunity cost of the short-term asset $X_i$ were to fall relative to the opportunity cost of holding money, then the asset holder would decrease his holdings of money balances favoring more of the short-term asset. The relationship of $TC$ to the asset ratio may be positive, negative, or zero.

No other price enters the demand function except the prices of the two assets in question because, in keeping with the traditional structure of elasticity of substitution studies in Monetary Economics, we have further assumed that the monetary service function $Z$ is separable in its arguments. That is, no third price is allowed to directly affect the relative demands of any two assets (see footnote 9, Chapter II). This assumption may tend to overstate the importance of a given price ratio in determining relative asset demand, because possible effects from other prices are simply ignored.

When equation (18) is stated in log-log form, we have,

\[
\log \frac{X_i}{M} = \alpha_{i0} + \sigma_{iM} \log \frac{C_i}{C_M} + \alpha_{i1} \log TC; \quad i = 1, \ldots, n .
\]

Stating the demand equations in log-log form is advantageous because it allows a direct comparison with other asset elasticity of substitution studies as described in Chapter II. In particular, since
both our study and Contador's deals with the recent Brazilian monetary experience, using the log-log form in the regression analysis of the next chapter leads to clearcut comparisons between Contador's results and our own.

It will now be demonstrated that the coefficient on \( \log \frac{C_i}{C_m} \) is the elasticity of substitution between \( X_i \) and \( M \). Recall that the elasticity of substitution measures the response of the ratio \( \frac{X_i}{M} \) along an indifference curve, in this case where monetary services are constant, to a change in relative price (here the ratio of opportunity costs \( \frac{C_i}{C_m} \)). If we can show that constant TC implies constant monetary services \( Z \), then the coefficient on \( \log \frac{C_i}{C_m} \) is the elasticity of substitution. Totally differentiating the monetary service function, we have,

\[
dZ = Z_m \frac{dM}{Z_m} + Z_1 \frac{dX_1}{n_1} + \ldots + Z_n \frac{dX_n}{n_n}.
\]

From the equilibrium condition (16), \( Z_{X_i} = (\frac{C_i}{C_m})Z_m \), \( i = 1, \ldots, n \), and substituting into (20) and rearranging terms, the result is,

\[
\frac{C_m}{Z_m} \frac{dZ}{Z_m} = C_m \frac{dM}{Z_m} + C_1 \frac{dX_1}{n_1} + \ldots + C_n \frac{dX_n}{n_n}.
\]

The right-hand side of this expression is the total change in opportunity cost \( dTC \) when prices are constant, and since \( C_m \) and \( Z_m \) are always positive, a constant amount of TC \( dTC=0 \) is equivalent to a constant amount of monetary services \( Z(dZ=0) \). Fixing TC fixes
the value of Z, and this implies that $\alpha_i M$ in demand equations (19) is the elasticity of substitution between M and $X_i$. This formulation of demand equations (19) is quite general because it assumes only two restrictions on the monetary service function Z: (1) diminishing marginal monetary services, and (2) separability in its arguments.

4.5 If we were to further assume that the monetary service function Z were homogeneous and took the CES form

$$Z = \left( \beta_m M^{-\rho} + \beta_1 X_1^{-\rho} + \beta_2 X_2^{-\rho} + \ldots + \beta_n X_n^{-\rho} \right)^{-1/\rho},$$

then the relative demand equations could be simplified to the following:

$$\log \frac{X_i}{M} = \alpha_i + \sigma \log \frac{C_i}{C_m}; \quad i = 1, \ldots, n.$$  \hspace{1cm} (22)

As we discussed earlier in Chapter II, Section II, with respect to Chetty's function U, when the functional form is of the CES type, two rather strict implications are that (1) only relative price is allowed to determine the asset ratio, and (2) since all $\rho$ are identical, the elasticity of substitution is the same between all asset pairs; that is

$$\sigma_{XiM} = \sigma_{XjM} = \frac{1}{1+\rho}, \quad i \neq j.$$

As a result, demand equations (22) are similar to Chetty's demand equations for his simplest model C-I with the difference that the independent variable in our formulation is the properly specified ratio of opportunity costs while in Chetty's formulation the independ-
ent variable is Chetty's misspecified relative price \( \frac{1}{1+r} \).

4.6 In addition, with this simple homothetic model, we are able to derive an expression similar to Feige and Pearce (1977)--see Chapter II, Section II--for the relationship between our elasticity of substitution \( \sigma_{X_i M} \) defined in terms of opportunity cost and traditional own- and cross-elasticities of demand for money, \( N_{ii} \) and \( N_{mi} \), respectively, defined in terms of the interest rate on \( X_i \). Define the elasticity of substitution as,

\[
\frac{\partial}{\partial M} \left( \frac{C_i}{C_m} \right) \cdot \frac{C_i}{C_m} = \sigma_{X_i M}
\]

Deriving the above expression, we have,

\[
\frac{\partial X_i}{\partial C_i} - \frac{X_i \partial M}{C_i} \cdot \frac{C_i}{C_m} \cdot \frac{C_i}{C_m} \cdot \frac{X_i}{M} = \sigma_{X_i M}
\]

and, rearranging terms,

\[
\frac{\partial X_i}{\partial C_i} \cdot \frac{C_i}{C_m} \cdot \frac{X_i}{C_i} - \frac{\partial M}{\partial C_m} \cdot \frac{C_i}{C_m} = \sigma_{X_i M}.
\]

In traditional demand elasticity form in terms of \( r_i \), we have the following relationship:
\[
N_{ii} - N_{mi} = \sigma_{X_i M} \frac{C_i}{C_m} \cdot \frac{r_i}{r_a - r_i} = \sigma_{X_i M} \frac{r_i}{r_a - r_i}.
\]

Therefore, the relationship between our elasticity of substitution \(\sigma_{X_i M}\) and traditional own- and cross-elasticities of demand does not depend upon the dimension of the interest rates as it does for Chetty's elasticity of substitution. In Chetty's formulation, the absolute value of the interest rate on \(X_i\) determined the relationship between the relative values of his elasticity of substitution and the values of the demand elasticities. In our formulation, however, the absolute value of the interest rates does not matter in the expression since the dimension aspect of the interest rates cancels in the fraction.  

4.7 One of the major drawbacks of money-substitution studies which rely upon estimates of the cross-elasticity of demand is that they rarely assume the presence of a budget constraint nor the symmetry condition that a particular level of substitution between two assets be the same for the two directions in which the cross-elasticity may be defined. As we stated earlier (see Chapter II, footnote 3), Feige and Pearce (1977) review only one study which assumes both of these conditions and the conclusion of this study by Kalchbrenner and Gramlich (1970) is that no asset in the United States is a close substitute for money. The model put forth in this chapter does assume both of these conditions. A portfolio constraint is one of the two constraints in the model—the stock constraint—and the symmetry condition is implied in the explicit specification of the non-pecuniary monetary
service function $Z$.

5. In this chapter, the criticism was made that the asset elasticity of substitution up to this time in Monetary Economics suffers the problem of dimensionality. That is, it depends upon the time unit which defines the interest rate. Our elasticity does not suffer this drawback. It is a pure number since monetary indicators and time units cancel out. It reflects asset-holder preferences as they are described by his indifference contours instead of determining the shape of the contours as the conventional elasticity of substitution does. In the next chapter, we put the model to work on data from the recent Brazilian monetary experience.
FOOTNOTES

1. Chetty (1969) states in the original article on the subject that the choice of time unit may indeed affect the estimate of the elasticity of substitution, but that this result will change the estimate by so little that it may be ignored. Several years later, in reply to his critics, Chetty (1972) mentions that his initial thoughts have changed, and that he now believes the estimates are not affected at all by the choice of time units. Refering to this dimensionality problem, Friedman and Schwartz (1970, p. 188) state the following:

".....while we believe that this approach is extremely promising, we have serious reservations about how much confidence can be placed in Chetty's specific results.....we believe his formulation has the defect that it makes the results depend on a strictly arbitrary choice of the time unit used in stating interest rates." See footnote 3.

2. In order to illustrate more precisely the difference between the interest rate in column 2 and the interest rate in column 3, a numerical example would be helpful.

If \( r_t^q = \begin{bmatrix} .015 \\ .030 \\ .061 \end{bmatrix} \), then \( r_t^y = \begin{bmatrix} .062 \\ .127 \\ .268 \end{bmatrix} \), respectively. The yearly rate is the compounded quarterly rate; and for that reason the yearly rate is more than four times the quarterly rate.

3. Chetty's proof of the invariance of the elasticity of substitution to the length of the time unit is the following (Chetty, 1972, p. 227): The first-order maximizing condition for the simplest Chetty model is, in Chetty's own notation,

\[ \log \frac{M}{T} = \alpha - \sigma \log (1+i) \]

where \( i \) is a yearly interest rate and "\( T \) is the cash value of time deposits in the next period." This expression may be restated in the following way, taking into account that \( T = T_0(1+i) \):

\[ \log M - \log T_0 - \log (1+i) = \alpha - \sigma \log (1+i) \]

or

\[ \log M - \log T_0 = \alpha - (\sigma - 1) \log (1+i) \].
The monthly interest rate $i_m$ may be expressed as

$$\frac{1}{12} \log(1+i) = \log(1+i_m)$$

Inserting the monthly rate into (3) yields

$$\log \frac{M}{T} = \alpha - (\sigma-1)\frac{1}{12} \log(1+i)$$

and comparing (3) and (5) there is no apparent change in $(\sigma-1)$. The problem with this proof is that (5) should not be compared with (3) but rather with

$$\log \frac{M}{T} = \alpha - (\sigma-1) \log(1+i_m)$$

as we argued in the text. Comparing (5) and (6), the coefficients differ by a factor of 12.

4. If the interest rates were derived in more general terms, not stating numerically the length of the two time periods in question, we may restate equation (10) as the following

$$(1+r_t^S)^{1/s} = (1+r_t^1)$$

where $r_t^S$ is the rate of interest in the shorter period and $r_t^1$ is the rate of interest in the longer period. Using this more general form for the relationship between the two interest rates, a more general form of equation (12) may be derived.

$$\frac{1}{s} \sigma_1 = \sigma_s$$

where $\sigma_1$ is the coefficient when the longer period interest rate is used in the regression model, and $\sigma_s$ is the coefficient from the short period model. For example, if 1 represents 6 months and s 1 month then the following is true

$$6 \sigma_1 = \sigma_s$$

5. Financial assets even of the longest term are not in the real world completely illiquid. They tend to be more illiquid than short-term assets but, nevertheless, may yield significant non-pecuniary monetary services. In this respect, Klein (1974, p. 935) argues:

"The return on the much more illiquid asset human capital may be a superior measure of $r_{at}$, but it is extremely difficult to obtain reliable annual estimates of this return."

And so in Klein's formulation, which is somewhat similar to ours, the rate of return on corporate bonds is used as a proxy for the rate of return on a perfectly illiquid asset.

"The flow of monetary services yielded an individual can then be represented by:

\[ N = N(M/P, S/P, \beta) \]

where \( N \) is the flow of real monetary services yielded per time period, \( M/P \) is the stock of real cash balances held, \( S/P \) is the real value of the money substitute asset held. Equation (1) could be thought of as the individual's production function for monetary services... assumed to have declining marginal productivities."

7. Patinkin is never able to say exactly what it is about money that allows it to enter the utility function with goods and services. The stochastic nature of expenditures and receipts is not only an insufficient argument for allowing money the quality of direct utility generation, it is insufficient for the existence of a medium of exchange. With transactions costs and information costs zero, a debtor could always make a verbal promise to pay.

Klein places monetary services in a consumer's utility function as if they are consumed just like goods and services. "The individual's utility function can then be written:

\[ U = U(X, N) \]

where \( X \) equals the rate of consumption of commodity services and \( N \) equals the rate of consumption of monetary services." (Klein, 1974, p. 932).

8. The discussion in the text suggests that the long-term rate of interest is greater than the short-term rate. The long-term rate must be greater in equilibrium to make up for its lack of non-pecuniary yield.

For Niehans (1974), the opportunity cost of holding cash balances is \((\theta - \rho)\), where \( \theta \) is the rate of time preference and \( \rho \) is the pecuniary rate of interest on money. If we assume that \( \theta = \rho \), that the opportunity cost of our model is identical to Niehans's.

9. A comparison can be made between Chetty's elasticity substitution \( \sigma_c \) and the elasticity of this chapter \( \sigma_s \). Putting both elasticities in terms of a percentage change in the short term interest rate, we have the following identity,

\[
\sigma_c \frac{1}{1+r_i} \frac{r_i}{1+\beta} = \sigma_s \frac{r_a}{r_a-r_i} \frac{r_i}{r_a-r_i}
\]
which may be simplified to

$$\frac{-\sigma_c}{1+r_i} = \sigma_s \frac{r_i}{r_a - r_i}$$

or, more simply, as,

$$\frac{\sigma_c}{\sigma_s} = -\frac{1+r_i}{r_a - r_i}.$$

Chetty's elasticity $\sigma_c$ is normally much larger than our own, and the discrepancy increases as the size of the interest rate decreases.
CHAPTER IV

ESTIMATION OF AN ASSET ELASTICITY OF SUBSTITUTION
FOR THE RECENT BRAZILIAN MONETARY EXPERIENCE
CHAPTER IV

ESTIMATION OF AN ASSET ELASTICITY OF SUBSTITUTION FOR THE RECENT BRAZILIAN MONETARY EXPERIENCE

I. INTRODUCTION.

Based upon the theoretical model of the last chapter, this chapter presents estimates of an elasticity of substitution between money and various short-term assets for Brazil for the period 1973 to 1976. In the next section, we shall discuss the data base from which our estimates come and the procedure we followed for calculating the opportunity cost of holding short-term assets. The third section will compare the work of Contador (1974a) to our own approach. This section is necessary because both studies have as their goal the estimation of an elasticity of substitution for the recent Brazilian monetary experience, and it would be enlightening to show in some detail the principal differences between the two approaches. Section IV is the centerpiece of the chapter. It is here that a formal statement of the hypothesis is made and empirically tested. In the fifth section is presented a summary of results and conclusions that may be drawn from the study.

II. DATA.

1. All data used in this study were secondary data drawn from three sources.

   i. O Mercado Financeiro by Dirceu Chiesa. The rate of interest on the National Treasury Bill (Letras de Tesouro Nacional).


   iii. Boletim do Banco Central do Brasil, various issues. All other asset, interest rate, and price data.
A description of the assets and the financial institutions which supply them may be found in Appendix I.

2. It was necessary to use two methods of calculation for the monthly expected opportunity costs for the short-term assets because of the two distinct ways that the inflation rate is permitted to enter the nominal rate of return.¹ In one situation, the correction for inflation is added to a base rate of return at the time of redemption of the asset. The inflation correction is calculated by the Fundação Getulio Vargas. The formula has undergone several changes since its inception in 1964, but, generally, it is a weighted average of past inflation rates. For further exploration of this calculation, see the Boletim do Banco Central do Brazil - 1976, p. 137.

\[ r_i = r_{bi} + CM; \quad i = 2-8, 10, 12. \]

where \( r_{bi} \) is the base rate of interest on the \( i \)-th asset and CM \( \textit{correcao monetaria a posteriori} \) - ex post monetary correction) is the expected inflation correction. When the rate of interest is calculated in this way, with the inflation correction added to the base rate at the time of redemption, the asset holder does not know at the time of purchase what the total nominal rate of return will be. When the asset is purchased, he does know the base rate,² but can only form an expectation as to the value of the inflation correction (CM) which will be added to the base rate at the time of redemption.

The expected rate of return of the alternative asset is calculated in exactly the same way as above (where inflation correction is added
to a base rate at the time of redemption) with one added complication. In Brazil, the purchase of long-term financial assets is subsidized by the federal government through the income tax. On a yearly basis, three per cent of the investment from long-term assets may be subtracted from the income tax liability of the asset holder. In other words, the total return on the alternative asset is not only its base rate plus the inflation correction, but also the fiscal incentive of three per cent per year (0.14% per month). The expected monthly rate of return for the alternative asset is therefore expressed as,

\[ r_a = r_{ba} + CM + 0.14 \]

where \( r_a \) is the total expected rate of return on the alternative asset and \( r_{ba} \) is its base rate. Inserting the expressions for \( r_a \) and \( r_i \) into the equation for expected opportunity cost (see footnote 1), the result is,

\[ C_i = r_{ba} + CM + 0.14 - r_{bi} - CM = r_{ba} + 0.14 - r_{ri}; \]

\[ i = 2-8, 10, 12. \]

where it is seen that the expected opportunity cost of holding asset \( i \) is equal to the sum of the base rate on the alternative asset \( (r_{ba}) \) and the fiscal incentive minus the base rate of interest on the asset in question \( (r_{bi}) \).

Due to the fact that the asset money bears no nominal rate of return--the base rate for money is zero and there is no inflation correction applied to the holding of money balances--the expected
opportunity cost of holding money is simply the expected rate of interest on the alternative asset,

$$C_m = r_{ba} + 0.14 + CM. \text{\footnote{3}}$$

For short-term assets 9 and 13—bills of exchange and treasury bills, respectively—the nominal rate of return of these assets includes an inflation correction at the time of purchase of the assets. This is the case because the assets are purchased at a discount with the redemption value, and nominal rate of interest, known at the time of purchase (correcao monetaria prefixada). \footnote{4} Both of these assets find active trading in secondary markets. While their nominal term to maturity rates of return are known with certainty, the nominal rate of return for any period less than term depends upon the day to day fluctuations of the secondary market and are not known with certainty. Therefore, unlike the term to maturity rates of return, the rates of return for shorter periods of time are, at best, expected rates of return. The expected opportunity cost of holding these assets is the difference between the expected nominal rate of return of the alternative asset and the expected nominal rate of return of the asset in question. That is,

$$C_i = r_a - r_i = r_{ba} + 0.14 + CM - r_i; \quad i = 9, 13,$$

where \( r_i \) is the expected monthly rate of interest on the asset \( i \) which already includes the inflation correction. \footnote{5}

The basic distinction in the two methods of administering inflation
correction may be drawn in terms of the marketplace. In the first situation, where the inflation correction is administered at the time of redemption of the asset, the marketplace for assets is not allowed to search for a nominal rate of return which would include inflation correction, but is instead given in inflation correction from outside the marketplace. In the second situation, it is the market itself which generates and incorporates inflation correction into the rate of return of the asset.

The rate of return on asset 11 (mutual funds) has yet to be considered because it does not clearly fall into either of the above categories, even though its algebraic formulation is identical to that of assets 9 and 13. This rate of return has no inflation correction which is added to a base rate of interest at the time of redemption, nor is it an asset that is sold at a discount. When shares in mutual funds are redeemed, the nominal rate of return which is gained is calculated by the mutual fund itself to reflect the rate of return of the mutual fund's portfolio. At the time of purchase of the mutual fund share, the nominal rate of return of the share is not known with certainty but is expected. The expression for the expected opportunity cost is,

\[ C_{11} = r_a - r_{11} = r_{ba} + 0.14 + CM - r_{11}^* \]

where \( r_{11} \) is the expected monthly rate of interest on the mutual fund share.
From the discussion above, the generation of expectations was necessary on four variables: the rates of interest on assets 9, 11, and 13 (bills of exchange, mutual funds, and treasury bills, respectively) and on the inflation correction (CM). The expected values of these variables were generated by using Box-Jenkins time series analysis. For a description of this procedure, see Appendix II.

3. Opportunity cost was negative in approximately five per cent of the observations.⁶ Recall from the theoretical chapter (Chapter III, Section III) that one of the implications of our theoretical model is that when portfolio equilibrium persists the rate of interest on the illiquid asset is greater than the rate on a short-term liquid asset.

There are two possibilities why the interest rate data does not conform to the theoretical model. First, the interest rate on the long-term asset may understate the opportunity cost of holding money balances. The long-term asset is not perfectly illiquid and its total return may include a non-pecuniary monetary service component. The proper alternative asset should have only pecuniary return. In this respect, the rate of return on human capital would be preferred to the rate of interest on a long-term asset. However, since a time series on the former is not available, the latter is used as a proxy. Secondly, several of the short-term assets are characterized by relatively high risk. This is especially true of mutual funds whose returns fluctuate a great deal and of the short-term assets with interest rate indexation applied at the time of purchase of the asset. The interest rate on these short-term assets may be relatively high to
compensate for this risk factor.

The problem is that negative opportunity costs are not compatible with the log-log regression equations presented later in this chapter. In those cases where negative values appeared they were replaced by the smallest positive value throughout all the opportunity costs, 0.01. This is an arbitrary procedure. Nevertheless, it can be shown that this biases the estimates of substitution upward, and, therefore, if low substitution estimates result, they can only be indicative of an even lower actual degree of substitution. In Figure I, the true relationship between \( X_1/M \) and \( C_1/C_m \) has been plotted along with the estimated relationship which does not allow for negative values of \( C_1/C_m \). Because the average slope of the estimated relationship has a higher value than the slope of the true, more substitution is measured along the estimated. Since our findings are that no strong substitutes for money exist, the arbitrary procedure of forcing negative opportunity costs to be positive can only further substantiate the results.

III. COMPARISON WITH CONTADOR.

While the approach proposed in this dissertation to measure the extent of money substitution in Brazil is similar to the work of Contador, (see Chapter II, Section IV) ours is an improvement in several respects. First of all, the data for our study is more comprehensive and more complete than Contador's. We have disaggregated the asset data into twelve short-term assets that may substitute for money, whereas Contador's breakdown is to a total of six asset categories. What he characterizes as bank time deposits with and without
FIGURE I

NEGATIVE VS. POSITIVE OPPORTUNITY COST AND THE ESTIMATED ELASTICITY OF SUBSTITUTION
certificate (see Chapter II, Table V), we have divided into four assets—banks are of two types, commercial and investment, and each one accepts deposits with and without the emission of certificates. Contador's one category of non-bank savings deposits, we have divided into three separate assets, one from each of three types of financial institutions that are considered non-bank. Contador also groups together under the heading of one composite asset both short-term government bills and long-term government bonds. We use separate series for each asset. It seems reasonable to suspect closer money substitution from short-term government bills than from long-term government bonds. In effect, Contador's aggregation allows for six possible money substitutes, whereas our disaggregation allows for twelve.

The interest rate data for our study is more complete than Contador's. We use the total nominal rate of interest on short-term assets whereas Contador used a proxy variable, the rate of monetary correction (CM). In other words, Contador's data in this respect neglects some variation in the interest rate series because instead of using the total interest rate which is \( r_{bi} + CM \), the base rate plus correction, he used only CM.

Chapter III was devoted to the argument that the specification of opportunity cost in the conventional model of asset substitution \( \frac{1}{1 + \frac{r}{1}} \), where \( r \) is the rate of interest on a short-term asset) was incorrect for estimating an elasticity of substitution. Contador followed the conventional procedure, and, therefore, his substitution
estimates are not reliable. However, as we argued in Chapter III, a more proper specification of the cost of holding a short-term asset would be the difference between the rate of interest on an illiquid asset and the rate of interest on the short-term asset. Using this form for relative cost gives a substitution estimate where the time dimension makes no difference.

Lastly, the econometric techniques used to estimate the various regression models are more general than the ordinary least squares techniques of Contador, principally because they are maximum likelihood techniques which allow for autoregressive behavior in the residuals.

In these ways, we have expanded and improved the data sources and specification of the work of Contador, and, as a result, our estimates of the asset elasticity of substitution should be more accurate and better reflect the real value of these variables.

IV. ESTIMATION.

1. At the end of Chapter III, we concluded that if the non-pecuniary monetary service function were assumed homogeneous, then asset-holder behavior could be explained by the demand equation,

\[(22) \text{ from Chapter III} \quad \log \frac{X_i}{M} = \alpha_{i0} + \sigma X_{i} \log \frac{C_i}{C_m}.\]

Adding seasonal dummy variables, a variable for trend, and an error term, we write the simplest of the regression models,
\[
\log \frac{X_{it}}{M_{it}} = \alpha_{10} + \sigma_{X_{1M}} \log \frac{C_{it}}{C_{mt}} \\
+ d_{1} DS_{1t} + d_{2} DS_{2t} + d_{3} DS_{3t} + \beta_{i} TIME_{t} \\
+ e_{i} \ ; \quad i = 2, \ldots, 13.
\]

where \( \sigma_{X_{1M}} \) is an estimate of the elasticity of substitution, and DS1, DS2, and DS3 are seasonal dummy variables. The trend variable TIME has been added to the equation (the first month is one, the second month is two, the third month is three, etc.) to indicate the transfer of resources from the older and more traditional financial instruments and institutions to the newer and more innovative ones. Some institutions and instruments have captured a growing share of the short-term asset market because (1) they have spread throughout geographical Brazil at a faster pace—the three institutions that supply the first three assets in the sample increased in number of agencies by 20% from 1972 to 1975 while the number of commercial banks increased by only 8%—or because (2) they have relatively attractive characteristics—an active secondary market for CD's (assets 4 and 6) and government securities (assets 12 and 13) have been utilized more frequently in recent years. Superscripts refer to the number of the regression model. This regression model is similar to model C-I of Chetty (see Chapter II, Section II) except that the price variables have been redefined according to the arguments of Chapter III. The twelver regressions, one for each possible money substitute, are in Table I.

Traditionally, in the money substitution literature, the statis-
TABLE I  
EQUATION (1) 
ORDINARY LEAST SQUARES 
COEFFICIENTS AND (STANDARD ERRORS) 
5 per cent significance*

<table>
<thead>
<tr>
<th>LOG(ASSET RATIO)</th>
<th>INTERCEPT</th>
<th>$\log(\frac{C}{C_m})$</th>
<th>DS1</th>
<th>DS2</th>
<th>DS3</th>
<th>TIME</th>
<th>$r^2$</th>
<th>DURBIN WATSON</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{2\over M}$</td>
<td>-2.487*</td>
<td>0.02*</td>
<td>0.071</td>
<td>0.028</td>
<td>0.033</td>
<td>0.027*</td>
<td>0.969</td>
<td>0.682*</td>
</tr>
<tr>
<td></td>
<td>(0.078)</td>
<td>(0.045)</td>
<td>(0.031)</td>
<td>(0.030)</td>
<td>(0.030)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{3\over M}$</td>
<td>-4.430*</td>
<td>-0.121*</td>
<td>0.019</td>
<td>-0.010</td>
<td>-0.011</td>
<td>0.042*</td>
<td>0.982</td>
<td>0.583*</td>
</tr>
<tr>
<td></td>
<td>(0.102)</td>
<td>(0.041)</td>
<td>(0.036)</td>
<td>(0.035)</td>
<td>(0.035)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{4\over M}$</td>
<td>-4.883*</td>
<td>0.013*</td>
<td>0.038</td>
<td>-0.003</td>
<td>0.022</td>
<td>0.026*</td>
<td>0.978</td>
<td>0.690*</td>
</tr>
<tr>
<td></td>
<td>(0.080)</td>
<td>(0.035)</td>
<td>(0.025)</td>
<td>(0.024)</td>
<td>(0.023)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{5\over M}$</td>
<td>-5.985*</td>
<td>0.254*</td>
<td>0.641</td>
<td>0.398</td>
<td>0.220</td>
<td>0.061*</td>
<td>0.573</td>
<td>0.317*</td>
</tr>
<tr>
<td></td>
<td>(0.446)</td>
<td>(0.120)</td>
<td>(0.343)</td>
<td>(0.340)</td>
<td>(0.335)</td>
<td>(0.008)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{6\over M}$</td>
<td>-1.801*</td>
<td>-0.125*</td>
<td>-0.467*</td>
<td>-0.273</td>
<td>-0.148</td>
<td>-0.041*</td>
<td>0.662</td>
<td>0.466*</td>
</tr>
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<td></td>
<td>(0.240)</td>
<td>(0.065)</td>
<td>(0.184)</td>
<td>(0.183)</td>
<td>(0.180)</td>
<td>(0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{7\over M}$</td>
<td>-2.482*</td>
<td>0.206*</td>
<td>0.031</td>
<td>-0.007</td>
<td>0.078</td>
<td>0.029*</td>
<td>0.945</td>
<td>0.760*</td>
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<td>(0.131)</td>
<td>(0.045)</td>
<td>(0.044)</td>
<td>(0.043)</td>
<td>(0.042)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{8\over M}$</td>
<td>-2.183*</td>
<td>-0.120*</td>
<td>0.024</td>
<td>0.039</td>
<td>0.008</td>
<td>-0.021*</td>
<td>0.919</td>
<td>0.622*</td>
</tr>
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<td>(0.118)</td>
<td>(0.040)</td>
<td>(0.039)</td>
<td>(0.038)</td>
<td>(0.038)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{9\over M}$</td>
<td>-0.898*</td>
<td>0.004*</td>
<td>0.052*</td>
<td>0.023</td>
<td>0.025</td>
<td>-0.004*</td>
<td>0.634</td>
<td>0.454*</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.007)</td>
<td>(0.019)</td>
<td>(0.019)</td>
<td>(0.019)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{10\over M}$</td>
<td>-2.609*</td>
<td>-0.067*</td>
<td>0.059</td>
<td>0.014</td>
<td>0.014</td>
<td>-0.013</td>
<td>0.874</td>
<td>0.342*</td>
</tr>
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<td>(0.077)</td>
<td>(0.044)</td>
<td>(0.031)</td>
<td>(0.030)</td>
<td>(0.029)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{11\over M}$</td>
<td>-3.345*</td>
<td>-0.014*</td>
<td>0.030</td>
<td>0.054</td>
<td>0.112*</td>
<td>-0.035*</td>
<td>0.946</td>
<td>0.895*</td>
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<td>(0.055)</td>
<td>(0.007)</td>
<td>(0.052)</td>
<td>(0.051)</td>
<td>(0.051)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{12\over M}$</td>
<td>-1.635*</td>
<td>-0.029*</td>
<td>-0.070*</td>
<td>0.025</td>
<td>0.034*</td>
<td>0.011*</td>
<td>0.864</td>
<td>0.462*</td>
</tr>
<tr>
<td></td>
<td>(0.059)</td>
<td>(0.042)</td>
<td>(0.028)</td>
<td>(0.028)</td>
<td>(0.027)</td>
<td>(0.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{13\over M}$</td>
<td>-2.460*</td>
<td>-0.416*</td>
<td>-0.062</td>
<td>0.133</td>
<td>0.184*</td>
<td>0.009*</td>
<td>0.523</td>
<td>0.674*</td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td>(0.103)</td>
<td>(0.085)</td>
<td>(0.080)</td>
<td>(0.079)</td>
<td>(0.002)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
tical test for the presence of strong substitution between money and other short-term assets has been whether the elasticity of substitution or the cross-elasticity of demand is greater than or less than one. As the argument goes, when these measures of substitution are greater than one, asset response to a change in relative price is elastic and, therefore, denotes strong substitution; less than one denotes inelastic response, and weak substitution. The fact that there is no particularly convincing argument in the literature for using the value of one, or, for that matter, any other value to indicate substitution remains a deficiency in all studies of this type.\(^7\)

The resolution of the problem "how much substitution is strong substitution?" is beyond the scope of this work. Nevertheless, we shall test the null hypothesis

\[ H_0: \sigma_{X_1M} < -1. \]

It is true that when the elasticity of substitution is equal to or less than one for all asset pairs, then all cross-elasticities are zero or negative, respectively.\(^8\) The rejection of the null hypothesis above is a stronger denial of substitution than that proposed by Feige and Pearce (1977), namely,

\[ H_0: N_{MX_1} > 1 \]

where \(N_{MX_1}\) is the cross-elasticity of demand for money with respect to
a change in the cost of holding $X_i$. For this reason, it seems appropriate to use the value of -1 in the elasticity of substitution as a reasonable benchmark for a test of substitution.

According to the statistical results of Table I, no asset is a strong substitute for money. The Durbin-Watson statistic states that the hypothesis of serial independence in the residuals must be rejected at the 5% significance level for all assets. This problem of serial correlation will be dealt with later. It is also important to point out that the assets which show positive trend are the newer, more innovative assets of the Brazilian financial system.

In assuming that the non-pecuniary monetary service function takes a homogeneous form, it is possible that the true elasticity of substitution and the estimated elasticity of substitution differ from one another. This divergence of true and estimated values depends upon the relative sensitivity of money demand and short-term asset demand to changes in total opportunity cost (TC). If this TC effect is not accounted for explicitly in the statistical analysis, the estimated elasticity of substitution may diverge from the true.

In light of this argument, we place TC in the more general demand equation in Chapter III and wrote,

\[
(19) \text{ from Chapter III} \quad \frac{X_i}{M} = \frac{\alpha_i}{\sigma_{X_i M}} + \frac{C_i}{\sigma_{C M}} + \alpha_{i1} \log TC
\]

A problem arises in any attempt to estimate this demand equation because no time series of TC exists. For this reason, we must rely upon a proxy variable. In Brazil, there are no monthly wealth or
income series to approximate in this study movements in TC, but there is available a monthly series, published by *Conjuntura Economica*, called the barometer of retail sales. This series reflects all retail sales throughout the country, and, therefore, should exhibit movements similar to income.

Using retail sales as a proxy for TC and, as before, adding seasonal and trend dummy variables and an error term, we have the regression model,

\[
\begin{align*}
\log_{M_t} X_{i_t} &= \alpha_i + \sigma_{X_iM} \log_{C_t M_t} + \sigma_{X_i} \log_{Y_t} \\
+ d_{i1}^2 DS1_t + d_{i2}^2 DS2_t + d_{i3}^2 DS3_t + \beta_i^2 TIME_t \\
+ e_i^2; & \quad i = 2, \ldots, 13.
\end{align*}
\]

Results for this model are in Table II. The retail sales variable is significant for all but three assets, but this has not changed our elasticity of substitution estimates from Table I. It is still the case that no asset strongly substitutes for money. Autocorrelated residuals persist.

2. With the presence of autocorrelated residuals established in the majority of the regressions reported in Tables I and II, it becomes necessary to utilize a regression technique with the power to estimate an autocorrelation coefficient in the error terms. A well-known proposition in econometrics is that the application of ordinary least squares to regression models exhibiting autocorrelated residuals
TABLE II  
EQUATION (2)  
ORDINARY LEAST SQUARES  
COEFFICIENTS AND (STANDARD ERRORS)  
5 per cent significance*  

<table>
<thead>
<tr>
<th>LOG ASSET RATIO</th>
<th>INTERCEPT</th>
<th>LOG(C/(\bar{C})_m)</th>
<th>LOG(RETAIL SALES)</th>
<th>DS1</th>
<th>DS2</th>
<th>DS3</th>
<th>TIME</th>
<th>R^2</th>
<th>DURBIN WATSON</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_2/M</td>
<td>4.250*</td>
<td>.022*</td>
<td>-1.444</td>
<td>.063*</td>
<td>.023</td>
<td>.033</td>
<td>.032*</td>
<td>.984</td>
<td>1.283</td>
</tr>
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<td>(1.080)</td>
<td>(.033)</td>
<td>(.231)</td>
<td>(.023)</td>
<td>(.022)</td>
<td>(.021)</td>
<td>(.001)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>X_3/M</td>
<td>-.127</td>
<td>-.083*</td>
<td>-.908*</td>
<td>.011</td>
<td>-.013</td>
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<td>.044*</td>
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<td>(.026)</td>
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<td>(.088)</td>
<td>(.081)</td>
<td>(.080)</td>
<td>(.003)</td>
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biases the variances of the coefficients but not the coefficients themselves. To estimate a model with autocorrelated residuals, we use a maximum likelihood technique due originally to Cochrane and Orcutt (1949) and recently reformulated by Beach and MacKinnon (1978). Reestimating equation (2) produces the results that are displayed in Table III. The estimation of a first-order autocorrelation coefficient has not significantly changed our previous results as we should expect since autocorrelated residuals do not bias regression coefficients. In Table III, there is no strong substitution between money and other short-term assets. Equations with second-order autocorrelation coefficients were also estimated, but since there was no change from the results in Table III, they are not reported.

Contador (1974) and Moroney and Wilbratte (1976) independently reasoned that if asset data is very short-run, then total adjustment to a change in demand may not occur in the space of one period in the time series. Both used an adjustment mechanism of the kind

\[ \log \frac{X_i}{M_t} - \log \frac{X_i}{M_{t-1}} = \psi_i \log \frac{X_i^D}{M_t} - \log \frac{X_i}{M_{t-1}} \]

Consequently, we have reestimated regression model (2) under the assumption of partial adjustment. This yields the following regression equation:
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<th>( \text{LOG(C_i/C_m)} )</th>
<th>( \text{LOG(RETAIL SALES)} )</th>
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<th>D&lt;sup&gt;2&lt;/sup&gt;</th>
<th>DS3</th>
<th>TIME</th>
<th>rho</th>
<th>( R^2 )</th>
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<td>-.836</td>
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<td>.012</td>
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<td>.987</td>
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<td>(.019)</td>
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<td>-.279*</td>
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<td>.829*</td>
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<td>(.081)</td>
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TABLE III... (Continued)

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<th>( \text{LOG} (\frac{\text{ASSET \text{RATIO}}}{\text{M}}) )</th>
<th>( \text{INTERCEPT} )</th>
<th>( \text{LOG}(\frac{C}{C_{m}}) )</th>
<th>( \text{LOG}(\text{RETAIL SALES}) )</th>
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<th>DS3</th>
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<td>(.040)</td>
<td>(.007)</td>
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\[ \log \frac{X_i}{M} t = \alpha_i^3 - \sigma_{X,iM} \log \frac{C_i}{C_m} t \]

\[ + \alpha_i^3 \log \frac{X_i}{M} t-1 + \alpha_i^3 \log Y_t \]

\[ + d_{i1}^3 DS1_t + d_{i2}^3 DS2_t + d_{i3}^3 DS3_t \]

\[ + e_{it}^3; \; i = 2, \ldots, 13. \]

Trend is not permitted to enter this equation because it is highly correlated with the lagged dependent variable, therefore causing multi-collinearity problems. Estimation results are in Table IV. Our regression model (3) is similar to Contador's equation (3) (see Chapter II, Section IV) except that the price variables differ. The Durbin-Watson statistic is not appropriate for models with stochastic explanatory variables like \( \log \frac{X_i}{M} t-1 \). However, Durbin's h-statistic is appropriate and demonstrates the persistence of residual auto-correlation in some asset demand equations. As before, in Table IV no asset appears as a money substitute in the short-run.

The results of Table IV and later Table V should not be accepted without caution. Recent work\(^{10}\) in the area of rational expectations suggests that all auction-type markets, including financial markets, adjust rapidly to changes in price expectations. However, while no conclusive evidence has yet come from Brazil, it seems unreasonable to assume that market expectations are any less informed there than anywhere else. Why then should the adjustment mechanism be so statistically important as the estimates show? One reason probably is
<table>
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<th>ASSET RATIO</th>
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<th>( C_i / C_m )</th>
<th>LAGGED ASSET RATIO</th>
<th>RETAIL SALES</th>
<th>DS1</th>
<th>DS2</th>
<th>DS3</th>
<th>( R^2 )</th>
<th>DURBIN'S ( n )</th>
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<td>( .691* )</td>
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<td>( 1.013* )</td>
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<td>LAGGED ASSET RATIO</td>
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<td>-0.211* (.103)</td>
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<td>0.005 (.013)</td>
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<td>2.470*</td>
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<td>1.019* (.067)</td>
<td>-0.003 (.217)</td>
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<td>-0.003 (.016)</td>
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<td>-0.013* (.006)</td>
<td>0.919* (.042)</td>
<td>-0.848 (.425)</td>
<td>0.089 (.044)</td>
<td>0.060 (.043)</td>
<td>0.066 (.043)</td>
<td>0.963</td>
<td>2.059*</td>
</tr>
<tr>
<td>( X_{12} ) M</td>
<td>-0.415 (.754)</td>
<td>-0.021* (.025)</td>
<td>0.941* (.057)</td>
<td>0.061 (.151)</td>
<td>0.041 (.018)</td>
<td>0.010 (.017)</td>
<td>0.047* (.017)</td>
<td>0.949</td>
<td>1.276</td>
</tr>
<tr>
<td>( X_{13} ) M</td>
<td>-0.737 (1.621)</td>
<td>-0.105* (.057)</td>
<td>0.923* (.074)</td>
<td>0.100 (.329)</td>
<td>0.087 (.044)</td>
<td>0.062 (.040)</td>
<td>0.057 (.040)</td>
<td>0.886</td>
<td>0.378</td>
</tr>
</tbody>
</table>
that while demand may adjust immediately to changes in relative cost, this may not be true of supply, and therefore what we see in the regressions may be mostly slow supply response on the part of financial institutions or the monetary authority. As Griliches (1967) has pointed out, the coefficient on the lagged dependent variable may be statistically significant for no other reason than trend in the series. In other words, this procedure does not actually test for partial adjustment but rather assumes its presence. Nevertheless, the regressions with lagged dependent variables tend to support the findings of the other simpler regressions: that there is little money substitution in the Brazilian economy.

Table V presents the results for equation (3) when residuals are allowed to follow a first-order autocorrelated process. The estimation procedure is from Dhrymes (1971, Theorem 7.1).

3. The use of the adjustment mechanism allows us to estimate a short-run partial elasticity of substitution and a long-run total elasticity (see footnote 9). The relationship between the two elasticities is the following,

\[ \psi_i \sigma_{X_iM}^{LR} = \sigma_{X_iM}^{SR} \]

where \( \psi_i \) is the adjustment coefficient (per cent adjustment per month), \( \sigma_{X_iM}^{LR} \) is the long-run elasticity, and \( \sigma_{X_iM}^{SR} \) is the short-run elasticity. The adjustment coefficient is simply

\[ \psi_i = 1 - \alpha_{ii}^3 \]
<table>
<thead>
<tr>
<th>ASSET RATIO</th>
<th>INTERCEPT</th>
<th>C_i/C_m</th>
<th>LAGGED ASSET RATIO</th>
<th>RETAIL SALES</th>
<th>DS1</th>
<th>DS2</th>
<th>DS3</th>
<th>rho</th>
<th>R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_{kt}/M</td>
<td>-0.195</td>
<td>-0.033*</td>
<td>0.981*</td>
<td>0.021</td>
<td>0.043*</td>
<td>0.007</td>
<td>0.032</td>
<td>-0.380*</td>
<td>0.990</td>
</tr>
<tr>
<td></td>
<td>(0.697)</td>
<td>(0.024)</td>
<td>(0.022)</td>
<td>(0.143)</td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.017)</td>
<td>(0.134)</td>
<td></td>
</tr>
<tr>
<td>X_{3t}/M</td>
<td>-0.320</td>
<td>-0.012*</td>
<td>0.971*</td>
<td>0.050</td>
<td>0.026</td>
<td>0.0003</td>
<td>0.022</td>
<td>-0.340*</td>
<td>0.995</td>
</tr>
<tr>
<td></td>
<td>(0.877)</td>
<td>(0.022)</td>
<td>(0.014)</td>
<td>(0.174)</td>
<td>(0.018)</td>
<td>(0.017)</td>
<td>(0.018)</td>
<td>(0.136)</td>
<td></td>
</tr>
<tr>
<td>X_{4t}/M</td>
<td>-0.323</td>
<td>-0.025*</td>
<td>0.934*</td>
<td>0.042</td>
<td>0.048*</td>
<td>0.003</td>
<td>0.038*</td>
<td>-0.360*</td>
<td>0.994</td>
</tr>
<tr>
<td></td>
<td>(0.647)</td>
<td>(0.020)</td>
<td>(0.021)</td>
<td>(0.124)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.135)</td>
<td></td>
</tr>
<tr>
<td>X_{5t}/M</td>
<td>-37.563*</td>
<td>0.008**</td>
<td>0.038</td>
<td>7.090*</td>
<td>0.494*</td>
<td>0.355*</td>
<td>0.091</td>
<td>0.960*</td>
<td>0.478</td>
</tr>
<tr>
<td></td>
<td>(16.713)</td>
<td>(0.023)</td>
<td>(0.128)</td>
<td>(3.513)</td>
<td>(0.116)</td>
<td>(0.125)</td>
<td>(0.107)</td>
<td>(0.042)</td>
<td></td>
</tr>
<tr>
<td>X_{6t}/M</td>
<td>29.779</td>
<td>-0.004*</td>
<td>0.481*</td>
<td>-6.610*</td>
<td>-0.304*</td>
<td>-0.176</td>
<td>-0.055</td>
<td>0.400*</td>
<td>0.785</td>
</tr>
<tr>
<td></td>
<td>(12.327)</td>
<td>(0.027)</td>
<td>(0.225)</td>
<td>(2.734)</td>
<td>(0.118)</td>
<td>(0.120)</td>
<td>(0.106)</td>
<td>(0.269)</td>
<td></td>
</tr>
<tr>
<td>X_{7t}/M</td>
<td>2.036</td>
<td>0.003**</td>
<td>1.0175*</td>
<td>-4.25</td>
<td>0.057*</td>
<td>0.041</td>
<td>0.062*</td>
<td>-0.100</td>
<td>0.981</td>
</tr>
<tr>
<td></td>
<td>(1.092)</td>
<td>(0.026)</td>
<td>(0.028)</td>
<td>(0.224)</td>
<td>(0.025)</td>
<td>(0.024)</td>
<td>(0.024)</td>
<td>(0.146)</td>
<td></td>
</tr>
<tr>
<td>X_{8t}/M</td>
<td>-0.160</td>
<td>0.041**</td>
<td>1.022*</td>
<td>0.064</td>
<td>0.024</td>
<td>-0.009</td>
<td>0.008</td>
<td>-0.350*</td>
<td>0.983</td>
</tr>
<tr>
<td></td>
<td>(0.714)</td>
<td>(0.020)</td>
<td>(0.028)</td>
<td>(0.161)</td>
<td>(0.020)</td>
<td>(0.019)</td>
<td>(0.019)</td>
<td>(0.136)</td>
<td></td>
</tr>
<tr>
<td>ASSET RATIO</td>
<td>INTERCEPT</td>
<td>( \frac{C_i}{C_m} )</td>
<td>LAGGED ASSET RATIO</td>
<td>RETAIL SALES</td>
<td>DS1</td>
<td>DS2</td>
<td>DS3</td>
<td>rho</td>
<td>R²</td>
</tr>
<tr>
<td>-------------</td>
<td>-----------</td>
<td>----------------</td>
<td>-----------------</td>
<td>------------</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
<td>-----</td>
</tr>
<tr>
<td>( X_{9} ) ( \frac{1}{M} )</td>
<td>.59* (2.96)</td>
<td>-.009* (.003)</td>
<td>.883* (.060)</td>
<td>-.139* (.068)</td>
<td>.039* (.009)</td>
<td>.009</td>
<td>.026* (.009)</td>
<td>-.390* (.138)</td>
<td>.948</td>
</tr>
<tr>
<td>( X_{10} ) ( \frac{1}{M} )</td>
<td>-.434 (.641)</td>
<td>-.003* (.016)</td>
<td>1.065* (.051)</td>
<td>.122 (.160)</td>
<td>.037* (.012)</td>
<td>-.006 (.011)</td>
<td>.039* (.012)</td>
<td>-.290* (.143)</td>
<td>.987</td>
</tr>
<tr>
<td>( X_{11} ) ( \frac{1}{M} )</td>
<td>3.374* (1.353)</td>
<td>-.015* (.006)</td>
<td>.932* (.029)</td>
<td>-.799* (.304)</td>
<td>.086* (.032)</td>
<td>.056 (.032)</td>
<td>.074* (.032)</td>
<td>-.310* (.139)</td>
<td>.983</td>
</tr>
<tr>
<td>( X_{12} ) ( \frac{1}{M} )</td>
<td>-.331 (.587)</td>
<td>-.023* (.021)</td>
<td>.951* (.046)</td>
<td>.045 (.117)</td>
<td>.039* (.014)</td>
<td>.010 (.014)</td>
<td>.043* (.014)</td>
<td>-.200 (.146)</td>
<td>.966</td>
</tr>
<tr>
<td>( X_{13} ) ( \frac{1}{M} )</td>
<td>-.630 (1.448)</td>
<td>-.097* (.054)</td>
<td>.937* (.072)</td>
<td>.084 (.292)</td>
<td>.090* (.040)</td>
<td>.065 (.035)</td>
<td>.055 (.036)</td>
<td>-.070 (.159)</td>
<td>.899</td>
</tr>
</tbody>
</table>
where $\alpha_{il}^3$ is the regression coefficient on the lagged dependent variable $-\log(X_i/M)_{t-1}$ in regression model (3), and the expression $\psi_i \sigma_{X_i M}^3 (LR)$ is the regression coefficients on the cost ratio. These short-run and long-run elasticities are presented in Tables VI and VII.

Note that in several cases the long-run elasticity denotes a high degree of substitution between money and several short-term assets. But is this value for the long-run statistically significant? We may test this hypothesis in the following way. Clearly, if the regression coefficient on the opportunity cost ratio $-\psi_i \sigma_{X_i M}^3 (LR)$ in equation (3) is more negative than the negative value of the adjustment coefficient $\psi_i$, then the long-run coefficient is more negative than a minus one. This would imply substitution in the long-run.

That is,

$$\psi_i \sigma_{X_i M}^3 (LR) < -\psi_i = \sigma_{X_i M}^3 (LR) < -1$$

We can rewrite this in terms of regression coefficients as,

$$\psi_i \sigma_{X_i M}^3 (LR) - \alpha_{il}^3 < -1 \Rightarrow \sigma_{X_i M}^3 (LR) < -1.$$ 

This last expression suggests that the hypothesis to test for substitution in the long run should be,

$$H_0 : \psi_i \sigma_{X_i M}^3 (LR) - \alpha_{il}^3 = -1$$

$$H_A : \psi_i \sigma_{X_i M}^3 (LR) - \alpha_{il}^3 < -1$$
## TABLE VI
MEASURES OF SUBSTITUTION AND ADJUSTMENT FROM TABLE IV.

<table>
<thead>
<tr>
<th>ASSET 2.</th>
<th>SHORT-RUN $\sigma_{X_iM}$</th>
<th>ADJUSTMENT COEFFICIENT $\psi_i$</th>
<th>LONG-RUN $\sigma_{X_iM}$</th>
<th>90% ADJ. AFTER</th>
<th>MONTHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.</td>
<td>.007</td>
<td>.033</td>
<td>.212</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>4.</td>
<td>.017</td>
<td>.021</td>
<td>.809</td>
<td></td>
<td>31</td>
</tr>
<tr>
<td>5.</td>
<td>.014</td>
<td>.062</td>
<td>.226</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>6.</td>
<td></td>
<td>.309</td>
<td></td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>7.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>.007</td>
<td>.194</td>
<td>.036</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>10.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>.013</td>
<td>.081</td>
<td>.160</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>12.</td>
<td>.021</td>
<td>.059</td>
<td>.356</td>
<td></td>
<td>11</td>
</tr>
<tr>
<td>13.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$$\text{LONG-RUN } \sigma_{X_iM} = \frac{\text{SHORT-RUN } \sigma_{X_iM}}{\psi_i}$$

Values not reported are theoretically with the wrong sign, or the adjustment coefficient $\psi_i$ is outside of the stable range $(0,1)$.

MONTHS FOR 90% ADJ. = $\frac{\log(1 + .90)}{\log(1 + \psi_i)}$. 

### TABLE VII

**Measures of Substitution and Adjustment from Table V**

<table>
<thead>
<tr>
<th>Asset</th>
<th>Short-Run $\sigma_{X_1M}$</th>
<th>Adjustment Coefficient $\nu_1$</th>
<th>Long-Run $\sigma_{X_1M}$</th>
<th>90% Adj. After ______ Months</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.</td>
<td>.033</td>
<td>.019</td>
<td>1.737</td>
<td>34</td>
</tr>
<tr>
<td>3.</td>
<td>.012</td>
<td>.029</td>
<td>.414</td>
<td>22</td>
</tr>
<tr>
<td>4.</td>
<td>.025</td>
<td>.016</td>
<td>1.562</td>
<td>40</td>
</tr>
<tr>
<td>5.</td>
<td></td>
<td>.962</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>6.</td>
<td>.009</td>
<td>.519</td>
<td>.017</td>
<td>2</td>
</tr>
<tr>
<td>7.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>.009</td>
<td>.117</td>
<td>.077</td>
<td>6</td>
</tr>
<tr>
<td>10.</td>
<td>.003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11.</td>
<td>.015</td>
<td>.068</td>
<td>.221</td>
<td>10</td>
</tr>
<tr>
<td>12.</td>
<td>.023</td>
<td>.049</td>
<td>.469</td>
<td>13</td>
</tr>
<tr>
<td>13.</td>
<td>.097</td>
<td>.063</td>
<td>1.540</td>
<td>10</td>
</tr>
</tbody>
</table>

See comments to Table VI.
The test was performed at the five per cent level of significance and it was found that we could not reject the null hypothesis for any asset. The value of the long-run elasticity of substitution for all appropriate assets should be accepted therefore as −1. We are able to conclude that in Brazil most short-term assets tend to be weak substitutes in the long run. However, the elasticity of substitution for money and any other short-term asset is never statistically greater than one. Furthermore, in light of the rather long adjustment periods estimated from equation (3), the regression results imply little substitution either in the short run or the long run in the Brazilian economy during our period of analysis.

V. CONCLUSIONS TO THE STUDY.

The improvements made by this study are both theoretical and empirical. In Chapter III, a model of asset-holder behavior was developed which provided an elasticity of substitution between money and other short-term assets that does not depend upon the dimension of individual interest rates. The empirical results of Chapter IV demonstrate that money substitution by short-term assets is not a probable occurrence in Brazil.

With Brazilian inflation erratic and usually substantial, only those assets which maintain their real values through time as the price level increases would be held in portfolios as adequate stores of value. The average annual rate of inflation in Brazil in the period 1966 to 1976 as reported by the Central Bank was 26 per cent with a
standard deviation of almost 10 per cent. All of the non-monetary short-term assets of this study are indexed to the rate of inflation, and, therefore, may serve as adequate stores of value. With no nominal rate of return, money balances are a prohibitively costly store of value in inflationary environments, and if money balances are held in short-term portfolios it must be because they serve a special function which no other asset can. This special function is, of course, that only money balances are means of exchange. No other asset has this characteristic, and, consequently, the empirical results of the previous sections of this chapter are not so surprising. No asset in the Brazilian economy is a substitute for money because no asset other than money can be used as medium of exchange. The relative holdings of money and other short-term assets are insensitive to small changes in the rate of interest because money is held for transactions purposes only. Money balances are pushed to a minimum by asset-holders, and as long as money is not protected from a continuous devaluation from inflation, it will remain at this minimum.

Given this low degree of money substitution, the increase in interest rates throughout the year 1976, especially that of the national treasury bill which went from 25 per cent per year in December of 1975 to 40 per cent in December of 1976, can be attributed not only to the accelerating rate of inflation but also to the policy of the Central Bank to reduce liquidity. Between December of 1975 and October of 1976, $M_1$ grew 19 per cent and prices 40.3 per cent. This is in contrast to 1975 when $M_1$ grew 40.7 per cent and prices 40.4
per cent. Part of this slowdown in monetary growth was accomplished through open market operations in national treasury bills. This debt instrument of the government increased in private portfolios from approximately 22 billion cruzeiros at the end of 1975 to almost 65 billion cruzeiros at the end of 1976. Clearly, to persuade asset holders to forego transactions balances of money, the interest rate on treasury bills had to increase by a large and marginally attractive amount. This implication of our statistical results points out a dilemma in monetary policy of which the Brazilian monetary authority should be aware. Namely, in periods of accelerating inflation and high interest rates, restriction in money supply growth will necessarily lead to even higher interest rates in the short-run. Due to the fact that money and the treasury bill are not very substitutable, the monetary authority may have to suffer a level of interest rates higher than it expected in order to combat effectively inflationary tendencies in the economy.

A final policy conclusion of this study is that there is no reason to extend the definition of money in Brazil past the simple definition $M_1$, currency and demand deposits. There are two other money definitions reported by the Central Bank. The concept of money $M_2$ is defined as $M_1$ plus demand deposits at some savings and loans and treasury bills held by the non-bank private sector of the economy. The concept $M_3$ is $M_2$ plus 50 per cent of the total of bills of exchange, housing bonds and savings deposits. However, since $M_1$ is not substitutable for the other assets in $M_2$ and $M_3$, these last two con-
cepts do not produce extended money supply definitions. In the Brazilian economy, money is held as means of exchange, other assets as stores of value, and, as a result, the only proper definition of money is $M_1$ itself. Strictly speaking, narrowly defined money tends to be the only widely used liquid asset in Brazil and this study calls for its tight control.

Feige and Pearce conclude their extensive review of the literature for U. S. time series studies on the demand for money with the following statement:

"The major conclusion to be drawn from (this review) is that point estimates of cross-elasticities between money and near-monies are surprisingly consistent and display relatively weak substitution relationships....Once the empirical findings generated by the diverse econometric procedures are put on a comparable basis, it is difficult to escape the conclusion that there does indeed exist an unacknowledged empirical consensus on the inelasticity of responses of the demand for money to changes in the rates of return on 'money substitutes'." (p. 463).

It is always reassuring to find similar conclusions from independent studies of the same problem even if the studies differ by continents. This study could be improved in several ways. The monetary service function could be generalized to allow for non-separability of the arguments. This would permit more variables to enter the regression equations and a wider range of values for the elasticity of substitution. More general econometric techniques could be used. Since many of the assets of this study are similar to one another,
they may not be independent in demand as we have assumed. A system approach to estimation may produce interesting results. Lastly, if the data on asset holding were disaggregated by household, business sector, and financial sector, one may find varying degrees of money substitution by asset-holder classification.
1. Recall from Chapter III that the expected opportunity cost of holding an asset \( (C_i) \) is the difference between the expected nominal rate of return on an alternative illiquid asset \( (r_a) \) and the expected nominal rate of return on the asset in question \( (r_i) \). That is,

\[
C_i = r_a - r_i.
\]

In that chapter it was argued that the proper decision variable for allocating a given portfolio among various assets was the expected opportunity cost of holding each asset. While it may be true that the actual rate contributes to the formation of the expected rate of interest, it is the expected rate upon which the asset holder acts in the decision to allocate. The actual rate of interest is not necessarily an indication of the future return on the asset (the rate may change), and to assume that an asset holder is lacking in any foresight at all leads us to question why assets would be held in the first place.

2. The value of the base rate is information provided by the financial institution, and while it may change, it has been almost constant for all assets throughout the period we are considering.

3. The expression for the ratio of expected opportunity cost used in the regression is,

\[
\frac{C_i}{C_{mt}} = \frac{r_{bat}^{+0.14} - r_{bit}}{r_{bat}^{+0.14 + CM_t}}; \quad i = 2, 8, 10, 12.
\]

4. The term of maturity of bills of exchange is 180, 360, 540, or 720 days. A monthly rate on the 360 day bill is reported by the Central Bank and it is this rate that is used in the statistical analysis. The term to maturity of treasury bills is 91, 182, or 365 days. The rate on the 91 day bill was the rate used.

5. The expression for the ratio of opportunity costs in the regression analysis is,

\[
\frac{C_i}{C_{mt}} = \frac{r_{bat}^{+0.14} + CM_t - r_{it}}{r_{bat}^{+0.14} + CM_t}; \quad i = 9, 13.
\]

6. Negative opportunity costs may be observed in four assets: deposits at commercial banks with and without certificate, assets 5 and 6, respectively; bills of exchange, asset 9; and mutual funds, asset 11.

8. Stating the monetary service function for the case of 3 assets as

\[ Z = (\frac{\beta_{m}^{-\sigma} + \beta_{1}^{-\sigma} + \beta_{2}^{-\sigma}}{\beta_{m}^{1+\sigma} + \beta_{1}^{1+\sigma} + \beta_{2}^{1+\sigma}})^{-1/\rho} \]

the demand equation for money may be written

\[ M = \frac{\text{TC} \cdot \text{CM} \beta_{m}^{\sigma}}{\beta_{m}^{1+\sigma} \beta_{1}^{1+\sigma} + \beta_{m}^{1+\sigma} \beta_{2}^{1+\sigma}} \]

\[ \sigma = -\frac{1}{1+\rho} \]

When \( \sigma = -1 \), the only price in the demand equation is \( m \). Any cross-elasticity of demand for money is, therefore, zero. When \( \sigma > -1 \), then any cross-elasticity of demand is negative.

9. Given the regression equation for the demand of \( y \),

(i) \[ y_{t}^{D} = \alpha_{o} + \alpha_{1}x_{t} + e \]

and the equation for partial adjustment

(ii) \[ y_{t} - y_{t-1} = \psi(y_{t}^{D} - y_{t-1}) \]

and substituting (i) into (ii), we have,

(iii) \[ y_{t} = \psi \alpha_{o} + \psi \alpha_{1}x_{t} + (1-\psi)y_{t-1} + e \].

The adjustment coefficient \( \psi \) is the coefficient on \( y_{t-1} \) minus one. The long-run coefficient on \( x_{t} \) is \( \alpha_{1} \) and the short-run coefficient \( \psi \alpha_{1} \). The rationalization for this adjustment mechanism was first given by Nerlove (1958). Assuming a quadratic cost function of the following kind

\[ \text{COST} = \alpha(y_{t} - y_{t}^{D})^2 + b(y_{t} - y_{t-1})^2 \]

where the first component of cost is the cost of being out of equilibrium and the second component is the cost of adjusting to equilibrium, we can derive,
\[ y_t - y_{t-1} = \frac{a^D}{a+b}(y_t - y_{t-1}) \]

which is equivalent to equation (ii) above. For an excellent review of lag structures in stochastic models see Griliches (1967).

10. Gordon Smith (1978) reviews the literature for international commodity markets. See also Poole (1976) and Fama (1976).
APPENDIX I

FINANCIAL INSTITUTIONS AND ASSETS
APPENDIX I
FINANCIAL INSTITUTIONS AND ASSETS

A. BRAZILIAN SHORT-TERM ASSETS.

1. The money supply. This is the definition $M_1$, currency and
demand deposits.

There are twelve short-term assets that potentially are money
substitutes.

2. Savings accounts (Caderetnas de Poupanca) at State and Federal
Savings Banks (Caixas Economicas).

3. Savings accounts at Housing Credit Companies (Sociedades de
Credito Imobiliario).

4. Savings accounts at Savings and Loan Associations (Associacaoes
de Poupanca e Emprestimos).

These savings accounts yield a rate of interest corrected ex
post for the loss in real value which occurs because of inflation
(correcao monetaria a posteriori). These funds are used primarily
for housing credit.

5. Time deposits at commercial banks with certificate of deposit.

6. Time deposits at commercial banks without certificate of deposit.

7. Time deposits at investment banks with certificate of deposit.

8. Time deposits at investment banks without certificate of deposit.
These deposits may yield either monetary correction ex post or ex ante. In this study, we use only the deposits with expost correction. The deposits with ex ante correction are relatively small. The deposits with certificate are traded in a secondary market.


This a short-term savings instrument with ex ante monetary correction (because it is purchased at a discount). It is designed to capture short-term idle balances for the financing of consumer credit through Finance Companies.

10. Housing Bonds (Letras Imobiliarios).

These are bonds of small denomination (100 dollars to 5,000 dollars) with terms of 3 to 10 years and are traded actively in a secondary market. The funds they capture are used for the financing of mortgages. These bonds are supplied primarily by the Housing Credit Companies.

11. Mutual Funds (Fundos Mutuos).

Deposits in financial institutions which deal primarily in the acquisition of securities from the Brazilian stock exchanges. The debt instruments of the federal government.

12. Treasury Bonds (Obrigaçoens Readjustáveis do Tesouro Nacional).

Long-term government bonds whose offer is controlled by the Central Bank and is the primary budget-balancing instrument of the federal government. They yield a rate of return that includes ex post
monetary correction.


Government Bills offered by the Central Bank in weekly and monthly auctions. They are the chief open-market instrument.

The illiquid long-term asset.

Long-term bonds of the State of Minas Gerais (Obrigaçoess Readjustaveis do Tesouro do Estado de Minas Gerais).

These state bonds yield a rate of interest that is corrected ex post for the real loss incurred due to inflation. They are sold with the anticipation that they will be redeemed with future tax receipts and are regulated by the Federal Senate and the Central Bank.
### TABLE I

**NOMINAL VALUE OF SHORT-TERM ASSETS**

**MILLIONS OF CURRENT CRUZEIROS AT END OF YEAR**

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<td>10,358</td>
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<td>$X_8$</td>
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<td>$X_9$</td>
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*All data is from the Bulletin of the Central Bank of Brazil, various issues.*
B. **PRINCIPAL FINANCIAL INSTITUTIONS IN BRAZIL**

Three financial institutions are charge with the responsibility of supply funds to the housing market. These financial institutions are:

2. Housing Credit Companies.
3. Savings and Loan Associations.

The asset side of the portfolio of these three institutions is made up of government bills and bonds and loans for the construction and purchase of housing.

Commercial Banks function in much the same way as commercial banks in the United States. Loans are both short and long term to industry and agriculture, but in Brazil few loans from the commercial banking sector are for the purchase of consumer durables. Moreover, a certain per cent of the compulsory reserves of commercial banks may be held in the form of government bills and bonds.

Investment Banks are the principal suppliers of working capital to industry. They hold government bills and bonds in reserve, also negotiable securities from the stock exchanges. Investment Banks are the chief conduit for loans from official government banks to industry.

Finance Companies (Financeiras)—The most important supplier of bills of exchange—supply loans for the purchase of consumer durable goods.
Mutual Funds contain basically three items on the asset side of their portfolio: securities from the stock exchanges, and government bills and bonds. Funds which they capture in the marketplace are applied in the stock exchanges in an attempt to maximize both security and rate of return for the holders of their liabilities.
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<td>Housing Credit Companies</td>
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<td>236</td>
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<td>254</td>
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APPENDIX II

THE BOX-JENKINS APPROACH TO TIME SERIES ANALYSIS AND EXPECTATIONS
APPENDIX II
THE BOX-JENKINS APPROACH TO TIME SERIES ANALYSIS
AND EXPECTATIONS

I. INTRODUCTION

The expected rates of return used in the econometric chapter were
generated by using the Box-Jenkins approach to time series analysis.
The purpose of this appendix is to develop briefly the economic rationale for this approach in generating expectations, and then to summarize the approach itself. In the final section of this appendix, expectations will be developed for four Brazilian time series: (1) the index called monetary correction which is used for the indexation of interest rates, (2) the rate of return on mutual funds, (3) the rate of return on treasury bills, and (4) the rate of return on a short-term private asset known as a bill of exchange.

II. RATIONALE

John Hicks in Value and Capital argues that expectations on economic variables may be the result of both economic and non-economic events. Natural disasters, death, peace or any event of this kind may affect the way an economic agent looks to the economic future. And more specifically, Hicks continues to argue, any economic event may affect the expectations on other economic variables. For example, a change in Federal Reserve policy toward growth in the monetary base may generate new expectations about the rate of unemployment, inflation, and so on. Lastly, it may be that expectations on a particular economic variable result from nothing more than the past history of that
variable. In most economic analysis, this is the assumption which is usually followed.

However, the most common technique for generating expectations which is the estimation of distributed lag models in regression analysis may be faulted on the grounds that using this technique assumes that a particular structure of expectations is dependent upon the hypothesis which is being tested. For example, the structure of price expectations derived from a demand for money model with permanent income as an independent variable would differ from that which would result from a model using actual income. Or, likewise, price expectations calculated from a consumption function model would differ from those of an asset demand model. In fact, in the aggregate, expectations ought to be the same regardless of the model or market being explained.

It is because of this need to separate expectations from the particular model being tested that the Box-Jenkins approach to time series analysis become attractive. Using this technique, the structure of expectations may be calculated first, and then inserted as an explanatory variable in any relevant hypothesis.

Essentially, what must be assumed is that, historically, all pertinent information which is used in the generation of expectations on a given variable is contained in the time series of that variable. If patterns can be found in the series, then under certain standard statistical assumptions it may be possible to make the best possible prediction (and, therefore, expectation) from only the series itself.
Deriving an explicit form for these time series patterns is the objective of the Box-Jenkins technique.

III. THE TECHNIQUE

A. STATIONARITY

The most important assumption which Box-Jenkins analysis makes is that, while the random values of a time series may differ, the marginal distribution functions for any two observations in a series must be identical. That is,

$$p(z_t) = p(z_{t+m})$$

where $z_t$ is the observation of $z$ at time $t$, and $m = 1, 2, \ldots, n$.

From this assumption it follows that,

$$E(z_t) = E(z_{t+m})$$

Therefore, the kind of time series which may be analyzed is characterized by temporary movements away from the mean. In Figure I, three different time series have been plotted against time, and only the series $z$ is stationary. The series $y$ increases through time at a constant rate, whereas series $x$ increases at an increasing rate. In order to achieve stationarity for $y$, the first difference of the series must be calculated. For $x$, the second difference. In the last section of this appendix it will be seen that second differences were necessary to achieve stationarity in three of the series analyzed.
FIGURE I
B. PROCESSES

According to Box and Jenkins, any stationary time series may be described by (1) an autoregressive process (AR), or (2) a moving average process (MA), or a combination of the two.

Autoregressive processes may be written as,

\[ w_t = \delta + w_{t-1} + \phi_1 w_{t-2} + \ldots + u_t \]

or, more conventionally, using the backshift operator \( B \), as,

\[ w_t (1 - \phi_1 B - \phi_2 B^2 - \ldots) = \delta + u_t \]

where \( w_t \) is the stationary time series,
\( \delta \) is the mean of the series,
\( \phi_i \) is the autoregressive coefficient for lag \( i \), and
\( u_t \) is the error term.

The largest exponent of the process is usually denoted by \( p \), and the process is said to be autoregressive to the order of \( p \); that is, AR(\( p \)).

Moving Average processes may be written in the following way:

\[ w_t = (1 - \Theta_1 B - \Theta_2 B^2 - \ldots) u_t + \delta \]

The highest order of a moving average process is denoted by \( q \), and in conventional notation we have MA(\( q \)).
C. AUTOCORRELATION

Box and Jenkins show that these processes are distinctly related to the autocorrelations and partial autocorrelations of the time series. Moving average processes are characterized by relatively large autocorrelations (spikes) at the beginning of the series which die out immediately and partial correlations which tail off very slowly. Autoregressive processes are characterized by autocorrelations which tail off slowly and partial autocorrelations that exhibit spikes and then die out immediately.

In turn, autocorrelations and the coefficients of an MA process (the $\theta_i$'s) are related functionally as

$$p_j = \frac{-\theta_j + \theta_{j+1} + \ldots + \theta_{q-j} \theta_q}{1 + \theta_1^2 + \ldots + \theta_q^2} \quad j = 1, \ldots, q$$

$$0 \quad j > q$$

where $p_j$ is the autocorrelation at lag $j$ and $q$ is the order of the process.

Autocorrelations and the coefficients of an AR process are related by

\[
p_1 = \phi_1 + \phi_2 p_1 + \ldots + \phi_p p_{p-1} \\
\ldots \ldots \ldots \ldots \ldots \ldots \\
p_p = \phi_1 p_{p-1} + \phi_2 p_{p-2} + \ldots + \phi_p
\]

This system of equations is known as the Yule-Walker equations.
The method proposed by Box and Jenkins for identifying and estimating patterns in the series is as follows:

1. Autocorrelations are calculated and inspected for spikes and for tailing off.

2. The process is then characterized as AR, MA, or a mixed process. If the process is mixed and the original series is differenced, it is referred to as ARIMA—an integrated autoregressive moving average process.

3. Once the series is identified the relevant coefficients are estimated using maximum likelihood estimation.

D. TESTS

There are four ways in which the time series models may be tested for adequacy.

1. The estimated coefficients may be tested for significance by using the standard t-test.

2. To test the hypothesis that the residuals are distributed randomly, the following statistic is calculated:

\[ Q = (n - d) \sum_{k=1}^{T} r_k^2 \]

where \( n \) is the number of observations in the series,

\( d \) is the degree of differencing, and

\( r_k \) is the autocorrelation of the residuals at lag \( k \).

The statistic \( Q \) is distributed chi-square with \( T-p-q \) degrees of
freedom. The null-hypothesis is that the residuals are distributed randomly, and would be rejected for a given level of significance when Q is greater than the tabled chi-square value for T-p-q degrees of freedom.

3. While the Q statistic is a test for the randomness of all disturbances at all lags, it is also useful to test for the significance of any particular autocorrelation. If an autocorrelation at lag k were significant (this is usually for significance at the 5% level), then this would indicate that a moving average term at lag k should be added to the regression model.

4. As a fourth test on the adequacy of the model, if the functional form for the time series is adequate, then the residuals may be correlated with future values of the stationary series, but observations in the series should not be correlated with future values of the residuals. Therefore, inspection of these correlations may turn up model inadequacy.

IV. FOUR BRAZILIAN TIME SERIES

In Table I are shown the results of the analysis for four Brazilian time series. All coefficients and Q statistics are significant at the 5% level. It was from these equations that the expectations (the one step ahead forecasts) were calculated.

In Table II it is shown that there are no significant autocorrelations in the residuals further verifying the hypothesis of randomness in the residuals.
In Table III, all cross correlations between values of the stationary series and future values of the residuals are relatively small which indicates model adequacy and unbiasedness in the coefficients.
**TABLE I**

ESTIMATES FROM BOX-JENKINS TIME SERIES ANALYSIS

JANUARY, 1972 to DECEMBER, 1976

MONTHLY RATES OF RETURN

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<th>TIME SERIES</th>
<th>COEFFICIENTS ESTIMATED BY MAXIMUM LIKELIHOOD (student-t)</th>
<th>Q(12)</th>
<th>Q(24)</th>
<th>Q(36)</th>
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<tr>
<td>MONETARY CORRECTION</td>
<td>$z_t = (1 - 0.270B + 0.316B^3)(1 - B)^2 u_t$</td>
<td>ARIMA(3, 2, 1)</td>
<td>5.4</td>
<td>12.6</td>
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<tr>
<td>MUTUAL FUNDS</td>
<td>$z_t = (1 + 0.735B^{11})u_t$</td>
<td>MA(11)</td>
<td>8.6</td>
<td>20.6</td>
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<tr>
<td>LTN$^2$</td>
<td>$z_t(1 + 0.683B + 0.392B^2)(1 - B)^2 = u_t$</td>
<td>ARIMA(2, 2)</td>
<td>13.9</td>
<td>33.0</td>
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<td>BILLS OF EXCHANGE</td>
<td>$z_t(1 + 0.763B + 0.265B^2)(1 - B)^2 = u_t$</td>
<td>ARIMA(2, 2)</td>
<td>12.8</td>
<td>27.2</td>
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1. Q statistics for 12, 24, and 36 month lags.
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BIBLIOGRAPHY


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